Incidence of environmental taxes under quadratic almost ideal demand system

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Abstract

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This paper uses two different estimation procedures to calculate the incidence of environmental taxes and compares the results. Both estimation procedures assume non-separability of leisure and so the labor response is included in estimates of household behavior. The first method is the Almost Ideal Demand System (AIDS) model of Deaton and Muellbauer. The AIDS model assumes linear Engel curves and if this assumption is violated then welfare estimates are biased. The Quadratic Almost Ideal Demand System (QUAIDS) model of Banks, Blundell and Lewbel extends the AIDS model by allowing for non-linear Engel curves. Households consume three goods - a composite clean good, a composite energy good and leisure. Data on household consumption is from the Consumer Expenditure Survey.

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1 Introduction

Growing concern about the environmental costs of household energy consumption has led to increasing support for higher environmental taxes. Environmental taxes, such as the gasoline tax, are regressive and if one is also concerned about equity then the optimal environmental tax rate should be calculated within an optimal income tax framework. A complete optimal tax model includes the cross-price elasticity for leisure, yet many previous empirical articles assume either the separability of leisure or that labor is constant. The Almost Ideal Demand System (AIDS) model of Deaton and Muellbauer (1980) allows for non-separability of leisure so that the cross-price elasticities for leisure are calculated. The AIDS model assumes Engel curves are linear which could bias welfare estimates if the assumption is violated. Therefore Banks, Blundell and Lewbel (1997) extend the AIDS model by assuming non-linear Engel curves. They name their extension the Quadratic Almost Ideal Demand System (QUAIDS). The objective of this paper is to estimate household demand using both the AIDS model and the QUAIDS model, and compare the corresponding environmental tax incidence calculations.

Household consumption is separated into three goods - a composite clean good, a composite energy good and leisure. The composite energy good consists of gasoline consumption and household energy consumption - electricity, natural gas or home heating fuels and oils. The composite clean good consists of the difference between total consumption and energy consumption. Leisure consumption is based on a time endowment of 14 hours per day per working spouse. Household consumption data is from the 1996–1999 Consumer Expenditure Survey while the price for the clean and energy goods comes from the Bureau of Labor Statistics.

This is the procedure used by West and Williams (2007), who estimate a similar model on a similar data set. Their model differs in the definition of the energy good, West and Williams consider gasoline consumption only. West and Williams use the 1996–1998 CEX data set and the same AIDS estimation procedure to estimate the cross-price elasticity of leisure with respect to gasoline. In addition, their data set is of a smaller size, 20,759 households.

The sample is separated into three subsamples based on marital status and the number of working adults – single households, married one worker households and married two worker households. The QUAIDS specification is estimated for all three subsamples and is found to be appropriate for the single household and married two worker sample. In general the cross-price elasticities under the QUAIDS model are relatively more inelastic. Moreover assuming linear Engel curves biases the welfare estimates. We calculate the equivalent variation under both the AIDS specification and the QUAIDS specification. Relative to QUAIDS, the AIDS specification underestimates the welfare loss for low income households and overestimates for high income households. Lastly, the importance of assuming a non-separable demand model is seen in the compensated and uncompensated cross-price elasticities of both the clean good and energy good w.r.t. labor supply, which are found to be significant.

2 The model

The economy is populated with households who have identical tastes but different income levels. Households have either one or two working adults. In the case of households with one working adult, these are single households or married households where only one spouse works. The two working adults case is for married households where both spouses work.[1] We distinguish between the two working adults by referring to them as the "primary" and the "secondary" workers. We also apply the "primary" label to the worker in the single households and the married one worker households. Each working adult has one unit of time which he divides between working in the market or "leisure".[2]

[1] West and Williams separate their sample into two-subsamples, households with one working adult and households with two working adults. However given that there is most likely a joint labor-leisure decision in married households it seems prudent to further sub-divide the one worker household sample.

[2] Time not working outside the house does not constitute leisure. To arrive at leisure, we subtract time customarily spent sleeping and doing household chores from time not working outside the house.

Denote the labor supply by L, leisure consumption by l, and the net-of-tax wage by w. Distinguish between variables pertaining to the primary and secondary workers through subscripts p and s. To economize on notation, let $\mathbf{L} = L_p, \mathbf{l} = l_p, \mathbf{w} = w_p$ when there is one worker in the household and $\mathbf{L} = (L_p, L_s), \mathbf{l} = (l_p, l_s), \mathbf{w} = (w_p, w_s)$ when there are two workers in the household. In a similar fashion, denote households' non-labor income by $\mathbf{m} = m_p$ and $\mathbf{m} = m_p + m_s$ depending on whether there is one or there are two workers in the household. Observe that $\mathbf{m}$ includes the "virtual income" required for linearizing the household's budget constraint

Households consume two categories of goods: One is "energy" comprised of household expenditures on electricity, natural gas, home heating fuels and oils, and gasoline. The other, called "non-energy" or "clean good", comprises all other household expenditures. All consumer goods are produced by a linear technology subject to constant returns to scale in a competitive environment. Denote consumption goods by $\mathbf{x} = (x_1, x_2)$ and their corresponding *consumer* prices by $\mathbf{p} = (p_1, p_2)$. The household's linearized budget constraint is given by

$$\mathbf{px} = \mathbf{wL} + \mathbf{m},$$

which one can rewrite as[3]

$$\mathbf{px} + \mathbf{wl} = \mathbf{w1} + \mathbf{m} \equiv \mathbf{I}. \tag{1}$$

Observe that $\mathbf{I} = w_p + m_p$ when there is one worker in the household and $\mathbf{I} = w_p + m_p + w_s + m_s$ when there are two workers. We shall refer to $\mathbf{I}$ as "potential income".

Households have preferences over $(\mathbf{x}, \mathbf{l})$ and E, the total level of emissions generated by consuming energy goods. We assume that preferences are separable in $(\mathbf{x}, \mathbf{l})$ and E so that the non-emission component of preferences can be represented by the indirect utility function $v = v(\mathbf{w}, \mathbf{p}, \mathbf{I})$.[4] We further assume that this component subscribe to the Quadratic Almost Ideal Demand System (QUAIDS) introduced by Banks et al.

[3] $\mathbf{1} = (1, 1)$ with two workers and $\mathbf{1} = 1$ with one worker.

[4] To avoid cluttered notation, references to households are suppressed. However, it is clear that $v, \mathbf{w}, \mathbf{I}$, etc. differ across households, h.

(1997). The advantage of this formulation is that it allows Engel curves to vary with $\ln \mathbf{I}$ linearly for some goods and nonlinearly for others—a property often displayed by empirical Engel curves.[5]

Thus

$$\ln v = \left\{ \left[\frac{\ln \mathbf{I} - \ln a(\mathbf{w}, \mathbf{p})}{b(\mathbf{w}, \mathbf{p})} \right]^{-1} + \lambda(\mathbf{w}, \mathbf{p}) \right\}^{-1}, \tag{2}$$

where, with a two-worker household,

$$\begin{aligned}
\ln a(\mathbf{w}, \mathbf{p}) \quad &\equiv \quad \alpha_0 + \alpha_p \ln w_p + \alpha_s \ln w_s + \sum_{i=1}^{2} \alpha_i \ln p_i \qquad (3) \\
&+ \ln w_p \sum_{i=1}^{2} \gamma_{ip} \ln p_i + \ln w_s \sum_{i=1}^{2} \gamma_{is} \ln p_i \\
&+ \left[\frac{1}{2} \gamma_{pp} (\ln w_p)^2 + 2\gamma_{ps} (\ln w_p)(\ln w_s) + \gamma_{ss} (\ln w_s)^2 \right. \\
&\left. + \sum_{i=1}^{2} \sum_{j=1}^{2} \gamma_{ij} \ln p_i \ln p_j \right], \\
b(\mathbf{w}, \mathbf{p}) \quad &\equiv \quad (w_p)^{\beta_p} (w_s)^{\beta_s} \prod_{i=1}^{n} p_i^{\beta_i}, \qquad (4) \\
\lambda(\mathbf{w}, \mathbf{p}) \quad &\equiv \quad \lambda_p \ln w_p + \lambda_s \ln w_s + \sum_{i=1}^{n} \lambda_i \ln p_i, \qquad (5)
\end{aligned}$$

with $\alpha_0, \alpha_p, \alpha_s, \alpha_i, \beta_p, \beta_s, \beta_i, \lambda_p, \lambda_s, \lambda_i, \gamma_{ip}, \gamma_{is}, \gamma_{pp}, \gamma_{ps}, \gamma_{ss}$, and γ_{ij} $(i, j = 1, 2)$ being constants. In the case of one-worker households, equations (3)–(5) continue to apply as well with the further stipulation that $\alpha_s = \beta_s = \lambda_s = \gamma_{is} = \gamma_{ps} = \gamma_{ss} = 0$. Let $n+1$ stand for subscript p and $n+2$ for s. Imposing restrictions $\sum_{i=1}^{n+2} \gamma_{ij} = \sum_{j=1}^{n+2} \gamma_{ij} = 0$, $\sum_{i=1}^{n+2} \beta_i = \sum_{i=1}^{n+2} \lambda_i = 0$ on the parameters of (3)–(5) ensures the demand system's homogeneity of degree zero in income and prices of the demand system, and imposing $\sum_{i=1}^{n+2} \alpha_i = 1$ its adding up property. The symmetry restriction, of the Slutsky matrix,

[5] If $\lambda_i = 0$, for all $i = 1, 2, \ldots, n+2$, the indirect utility function (2) will be reduced to Deaton and Muellbauer's (1980) Almost Ideal Demand System. In this case, Engel curves will be linear in $\ln \mathbf{I}$.

requires $\gamma_{ij} = \gamma_{ji}$, for all $i \neq j = 1, 2, ..., n+2$, and is also imposed on the estimated parameters.

It will be simpler, however, to estimate the goods' expenditure shares rather than their quantity demanded. We have, from Roy's identity, for consumption good i and leisure of primary and secondary workers $k = p, s$,

$$\omega_i \equiv \frac{p_i x_i}{\mathbf{I}} = \frac{p_i}{\mathbf{I}} \left(\frac{-\partial v / \partial p_i}{\partial v / \partial \mathbf{I}} \right) = -\frac{p_i}{\mathbf{I}} \frac{\partial \ln v / \partial p_i}{\partial \ln v / \partial \mathbf{I}}, \tag{6}$$

$$\omega_k \equiv \frac{w_k l_k}{\mathbf{I}} = \frac{w_k}{\mathbf{I}} \left(1 - \frac{\partial v / \partial w_k}{\partial v / \partial \mathbf{I}} \right) = \frac{w_k}{\mathbf{I}} \left(1 - \frac{\partial \ln v / \partial w_k}{\partial \ln v / \partial \mathbf{I}} \right). \tag{7}$$

where ω_i denotes the expenditure share for good $i = 1, 2$ and ω_k, $k = p, s$, denotes the share of the primary and secondary workers' leisure in a households budget. Partially differentiate $\ln v$ with respect to $p_i, \mathbf{I}, w_k$, and simplify through equations (3)–(5); then substitute the resulting partial derivatives in equations (6)–(7) and simplify. One arrives at, for $i = 1, 2$, and $e \neq k = p, s$,

$$\begin{aligned} \omega_i &= \alpha_i + \gamma_{ip} \ln w_p + \gamma_{is} \ln w_s + \sum_{j=1}^{2} \gamma_{ij} \ln p_j \\ &\quad + \beta_i \ln \frac{\mathbf{I}}{a(\mathbf{w}, \mathbf{p})} + \frac{\lambda_i}{b(\mathbf{w}, \mathbf{p})} \left[\ln \frac{\mathbf{I}}{a(\mathbf{w}, \mathbf{p})} \right]^2, \end{aligned} \tag{8}$$

$$\begin{aligned} \omega_k &= \alpha_k + \gamma_{kk} \ln w_k + \gamma_{ke} \ln w_e + \sum_{j=1}^{2} \gamma_{kj} \ln p_j \\ &\quad + \beta_k \ln \frac{\mathbf{I}}{a(\mathbf{w}, \mathbf{p})} + \frac{\lambda_k}{b(\mathbf{w}, \mathbf{p})} \left[\ln \frac{\mathbf{I}}{a(\mathbf{w}, \mathbf{p})} \right]^2. \end{aligned} \tag{9}$$

3 Data and the Engel curves

The data come from the Interview Survey component of the Consumer Expenditure Survey (CEX) covering the period 1996–1999. This is a quarterly data set that tracks the same households in any given year. However, since four quarters is not a long enough time frame to create a panel, we treat each quarter as an independent sample and the sixteen quarters form a pooled cross-sectional data set.[6] The unit of analysis in the survey is the household. Given our interest in the labor-leisure choice, we restrict the data to households with at least one employed worker. To work with a homogeneous population in terms of labor-leisure decision, we also restrict the data to households between the ages 18–65. Nor do we include, for the same reason, households whose occupation codes appear as armed forces, self-employed, farming, forestry, or fishing. This yields a sample size of 27,906 households.

Household composition falls into three categories: single, married two worker and married one worker. Of the 27,906 households in our data, 12,611 are single households, 11,013 are married two worker households and 4,282 are married one worker households. This last group, though not negligible in size, is far smaller than the other two groups. Estimating the demand equations for this group also poses difficulties not encountered when estimating the demand equations for the other two groups. The question is what is the appropriate amount of leisure hours for the non-working spouse. There is of course an extensive literature on labor participation decision which one can use to model the behavior of these households. In the context of our study, it is crucial to know the leisure consumption of both working and non-working adults. Yet it is difficult to identify these based on the time endowments of both adults but the working hours of the sole adult who works. One may be tempted to assume that the non-working adult's leisure is equal to that adult's entire time endowment. However it is more likely that the non-working spouse will take over some of the household chores that, had he/she been employed, the primary earner would have done. It is most likely inappropriate to assign the non-working spouse the full time endowment. We therefore assign no time endowment to the non-working spouse. We assume there is no "secondary" worker and the QUAIDS model for married one worker households follows the single household specification. [7]

[6] http://www.bls.gov/opub/hom/pdf/homch16.pdf

[7] A second approach is to assign a time endowment to the non-working spouse and use the Heckman selection model for the purpose of imputing a net wage rate. This procedure, used also by West

The CEX reports both total household expenditures and detailed expenditures on certain specific categories such as food, apparel, entertainment, housing and transportation. It also reports detailed information on the actual items that comprise each category. We used these to create two broad categories of energy and clean goods. The first comprises all household expenditures on electricity, natural gas, home heating fuels and oils, and gasoline. The other is found as the difference between total household expenditures and energy good expenditures. We used week as our unit of time, but the CEX reports are quarterly expenditures. We thus converted the CEX figures to a weekly basis by assuming thirteen weeks in a quarter.[8]

The expenditure on leisure is the product of leisure consumption and the net wage. To compute leisure hours, we assign a time endowment of 14 hours per day and 5 days per week for a total of 70 hours to every working adult in a household.[9] Subtracting the working hours, which the CEX also reports, from the time endowment yields leisure hours per week. [10]

and Williams (2004, 2007), estimates both a selection and a wage equation based on demographic characteristics. The male and female net wages are estimated separately. The selection equation includes exclusion restrictions that consist of the demographic characteristics – number of children, clean and energy good prices, state unemployment rates and spouse's salary. In this scenario, the QUAIDS model follows the married two wage earner specification. However, the results were "unsatisfactory" in that the signs of the compensated own-price elasticities were incorrect. Given that the non-working spouse will spend part of their time on non-market work, i.e. household chores, the non-working spouse was assigned various time endowments ranging from 12 hours to 6 hours. Only the 8 hour time endowment provided "satisfactory" results with the correct compensated own-price elasticity signs.

[8]A small number of households, 552, reported zero energy expenditures. This cannot be correct. With only 2.54% of these households owning a home, the most likely explanation for this reporting is that their rent included utilities. Not knowing their actual energy expenditures, we drop these households from our data.

[9]An individual's time can be separated into four different components: taxable work or labor, non-taxable work, leisure and sleep. Non-taxable work consists of commuting to work, household chores and other tasks such as grocery shopping etc. (Household chores are considered labor only if one is paid for them, such as working as a maid.) It is likely that one does not have much flexibility with adjusting the time one spends on sleeping, commuting to work, and household chores. The 14 hours a day allotment assumes that one spends 10 hours a day on these activities. This being somewhat uncertain, we used several different time endowments (12 hours to 18). There was no significant change in the results due to time endowment variations.

[10]A total of 97 single households, 75 married one wage earner households and 198 married two wage earner households were found to have zero or negative leisure hours because their weekly hours of work

Turning to the calculation of the net wage, we first calculate a gross hourly wage for each spouse based on the annual salary information, the hours worked per week, and weeks worked per year; all reported by CEX. To translate this into a net wage, one requires the household's marginal income tax rate. We use the NBER TaxSim program to calculate the household's effective federal and state marginal income tax rates.[11] This effective marginal tax rate is used to calculate an hourly net wage for each working spouse. For single households and married one worker households this is straightforward since the household's effective marginal tax rate is the worker's tax rate. For married two worker households, we assume they file a joint tax return and therefore the household effective marginal tax rate is applied to each of the spouses. [12]

Data on prices comes from the Bureau of Labor Statistics (BLS). The BLS has a price index for "all items less energy," which we use for the clean good price, and the "energy" price index which we use for the energy good price.[13] The indices are divided by 100 so they can be used as a dollar price. Both price indices are national indices reported on a monthly basis. We calculate their three month averages to correspond to the household's three-month reporting period in the CEX.

Table 1 reports sample statistics for the single household and married one worker sample, while Table 2 reports sample statistics for the married two worker sample. For single households and married one worker households, over half of potential income is spent on consumption of clean goods, 58.56% for single households and 53.03% for

exceeded their time endowment. These households violate the time endowment constraint. Given their tiny size, we also drop them from the sample. Dropping these 370 households, and the 552 households with zero energy expenditures, results in our final sample size of 27,906.

[11] www.nber.og/taxsim

[12] Denote the marginal income tax rates household h faces by θ_p^h for the primary worker and by θ_s^h for the secondary worker. Denoting their corresponding gross-of-tax wage rates by wg_p^h and wg_s^h, we have $w_p^h \equiv wg_p^h(1-\theta_p^h)$ and $w_s^h = wg_s^h(1-\theta_s^h)$. In conformity with our notation, $\theta^h = \theta_p^h, \mathbf{wg}^h = wg_p^h$ for single households and married one worker households, and $\theta^h = \theta_p^h = \theta_s^h$, $\mathbf{wg}^h = \left(wg_p^h, wg_s^h\right)$ for two-worker married households.

[13] Appendix 3, Chapter 17 of the BLS Handbook of Methods lists the components of various aggregate price indices. The "energy" price index is comprised of gasoline, electricity, natural gas, and home heating fuels and oils.

married one worker households as reported in Table 1. Leisure consumption is also a sizable expenditure, 36.81% for single households and 42.03% for married one worker households. For married two worker households, expenditure on leisure consumption represents half of potential income, 25.64% for the male spouse and 24.61% for the female spouse (Table 2). [14] For all three household types, expenditure on the household energy good never exceeds 5%. Married one worker households spend 4.94% of potential income (Table 1), while married two worker households spend 3.50% of potential income (Table 2).

The effective marginal tax rate reported in the tables combines the federal, state and FICA tax rates. For single households the average marginal tax rate is 34.93% (Table 1). This consists of an average federal income tax rate of 16.25%l, an average state income tax rate of 3.87% and FICA tax rate of 14.80%. The hourly after-tax wage rate for single households is $8.12, for married one worker households it is $10.91, for married two worker households it is $10.22 for the male earner and $7.88 for the female earner.

3.1 Engel curves

As a first step to examining whether or not a linear specification for Engel curves is appropriate, we estimate simple quadratic polynomial regressions. Each of the three goods is first regressed on ln of potential income (linear specification), and then regressed on ln of potential income and its square (quadratic specification). These results are reported in Table 3. The top frame gives results for the single households, the middle frame gives results for the married one worker households, and the bottom frame gives results for the married two worker households. For single households the potential

[14] Our theoretical model for married two worker households distinguishes between a "primary" worker and a "secondary" worker, where the primary worker is the higher earner. The empirical labor supply literature has traditionally distinguished between male labor supply and female labor supply. Therefore we calculate labor supply elasticities for the male spouse and the female spouse rather than identifying the higher wage earner and labeling he/she the "primary" worker.

Table 1: Summary Statistics

	Single Households		Married One Worker	
Variable	Mean	Std. Dev.	Mean	Std. Dev
Clean Good Exp ($)	409.03	309.44	373.64	286.83
Energy Good Exp ($)	27.20	17.96	29.01	15.58
Leisure Exp ($)	235.24	154.98	285.20	195.02
Total Expenditures ($)	671.47	376.76	687.86	393.58
Clean Good Share (%)	58.56	15.77	53.03	15.26
Energy Good Share (%)	4.63	3.11	4.94	2.99
Leisure Share (%)	36.81	15.94	42.03	15.66
Clean Good Price ($)	1.69	0.04	1.69	0.04
Energy Good Price ($)	1.07	0.04	1.07	0.04
Hourly Gross Wage ($)	13.44	8.01	17.10	9.97
Marginal Tax Rate (%)	34.93	17.20	33.19	15.59
Hourly Net Wage ($)	8.12	4.45	10.91	6.12
Hours Worked (Wkly)	40.50	10.11	43.31	9.00
Hours Leisure (Wkly)	29.50	10.11	26.69	9.00
Ln(Clean Price) ($)	0.52	0.03	0.52	0.03
Ln(Energy Price) ($)	0.07	0.04	0.07	0.04
Ln(Net Wage) ($)	1.93	0.65	2.20	0.70
Age	37.52	11.58	42.41	11.56
No HS Diploma (%)	8.05	–	17.91	–
HS Diploma (%)	24.74	–	29.71	–
Some College (%)	35.52	–	24.45	–
Bachelor's Degree (%)	22.45	–	17.05	–
Graduate Degree (%)	9.25	–	10.88	–
Male (%)	42.11	–	82.46	–
White (%)	79.41	–	87.48	–
Black (%)	16.18	–	6.33	–
Asian (%)	3.74	–	5.42	–
Other (%)	0.67	–	0.77	–
No Children (%)	76.27	–	35.22	–
One Child (%)	12.12	–	19.52	–
Two Children (%)	8.02	–	26.20	–
Three or More (%)	3.59	–	19.06	–
Own Home (%)	37.27	–	69.34	–
No Cars (%)	32.27	–	19.99	–
One Car (%)	56.11	–	47.41	–
Two Cars (%)	9.46	–	25.34	–
Three or More (%)	2.16	–	7.26	–
# of Observations	12,611	–	4,282	–

*Data is from the 1996 – 1999 CEX. The energy good is gasoline, electricity, natural gas and home heating fuels and oils. The clean good is remaining household expenditures.

Table 2: Summary Statistics

	Married Two Worker Households			
Variable	Male	Std. Dev.	Female	Std. Dev
Clean Good Exp ($)	500.61	328.18	–	–
Energy Good Exp ($)	32.88	15.69	–	–
Leisure Exp ($)	263.99	161.62	256.32	181.60
Total Expenditures ($)	1,053.81	476.35	–	–
Clean Good Share (%)	46.24	13.72	–	–
Energy Good Share (%)	3.50	1.87	–	–
Leisure Share (%)	25.64	11.55	24.61	11.54
Clean Good Price ($)	1.69	0.04	–	–
Energy Good Price ($)	1.07	0.04	–	–
Hourly Gross Wage ($)	17.53	9.15	13.60	7.96
Marginal Tax Rate (%)	40.17	9.33	–	–
Hourly Net Wage ($)	10.22	5.17	7.88	4.34
Hours Worked (Wkly)	44.14	8.35	37.36	10.37
Hours Leisure (Wkly)	25.86	8.35	32.64	10.37
Ln(Clean Price) ($)	0.52	0.03	–	–
Ln(Energy Price) ($)	0.07	0.04	–	–
Ln(Net Wage) ($)	2.18	0.63	1.90	0.66
Age	40.01	9.86	38.17	9.50
No HS Diploma (%)	8.39	–	6.96	–
HS Diploma (%)	27.59	–	27.45	–
Some College (%)	29.17	–	32.75	–
Bachelor's Degree (%)	22.95	–	22.78	–
Graduate Degree (%)	11.90	–	10.06	–
White (%)	86.82	–	86.87	–
Black (%)	8.18	–	7.60	–
Asian (%)	4.27	–	4.98	–
Other (%)	0.73	–	0.55	–
No Children (%)	40.45	–	–	–
One Child (%)	23.48	–	–	–
Two Children (%)	24.95	–	–	–
Three or More (%)	11.11	–	–	–
Own Home (%)	77.25	–	–	–
No Cars (%)	15.28	–	–	–
One Car (%)	45.11	–	–	–
Two Cars (%)	30.50	–	–	–
Three or More (%)	9.11	–	–	–
# of Observations	11,013	–	–	–

*Data is from the 1996 – 1999 CEX. The energy good is gasoline, electricity, natural gas and home heating fuels and oils. The clean good is remaining household expenditures.

Table 3: Quadratic Polynomial Regression Results

	Single Households					
	Clean Good		Energy Good		Leisure	
	Linear	Quadratic	Linear	Quadratic	Linear	Quadratic
ln M	0.0760‡	–0.4910‡	–0.0221‡	–0.0327‡	–0.0539‡	0.5237‡
	(0.0027)	(0.0453)	(0.0005)	(0.0086)	(0.0028)	(0.0465)
ln M^2	–	0.0443‡	–	0.0008	–	–0.0451‡
	–	(0.0035)	–	(0.0007)	–	(0.0036)

	Married One Worker Households					
	Clean Good		Energy Good		Leisure	
	Linear	Quadratic	Linear	Quadratic	Linear	Quadratic
ln M	0.0284‡	–0.7726‡	–0.0259‡	–0.0503‡	–0.0025	0.8230‡
	(0.0044)	(0.0754)	(0.0008)	(0.0133)	(0.0045)	(0.0777)
ln M^2	–	0.0626‡	–	0.0019	–	–0.0645‡
	–	(0.0059)	–	(0.0010)	–	(0.0061)

	Married Two Worker Households							
	Clean Good		Energy Good		Leisure (M)		Leisure (F)	
	Linear	Quadratic	Linear	Quadratic	Linear	Quadratic	Linear	Quadratic
ln M	0.0525‡	–0.8664‡	–0.0209‡	–0.0681‡	–0.0208‡	0.6724‡	–0.0108‡	0.2621‡
	(0.0029)	(0.0614)	(0.0003)	(0.0074)	(0.0025)	(0.0523)	(0.0025)	(0.0528)
ln M^2	–	0.0672‡	–	0.0034‡	–	–0.0507‡	–	–0.0200‡
	–	(0.0045)	–	(0.0005)	–	(0.0038)	–	(0.0038)

*Regression results test the Working-Leser Engel curve specification versus a quadratic log income specification. † significance at 5% level. ‡ significance at 1% level.

income term is significant for all three goods in the linear specification. However the squared term is insignificant for the energy good under the quadratic specification. For married one worker households the potential income term is significant for the clean and energy goods in the linear specification, but not for the leisure good. Again the squared term is insignificant for the energy good under the quadratic specification. For married two worker households both potential income and its squared term are significatn for all four goods under both specifications. These estimates provide prima facie evidence for using the QUAIDS specification rather than the original AIDS specification of Deaton and Muellbauer.

4 Estimation

Expenditure shares $\omega_i, i = 1, 2$, and $\omega_k, k = p, s$, given by (8)–(9) constitute our estimating equations. A particular difficulty with estimating these equations is their being nonlinear in parameters. However, they are conditionally linear if the value of the price indices $a(\mathbf{w}, \mathbf{p})$ and $b(\mathbf{w}, \mathbf{p})$ are known the equations become linear in parameters. We thus follow the Iterated Linear Least Squares Estimator (ILLE) procedure of Blundell and Robin (1999).[15] A three-stage least squares procedure is then used to estimate the model. The process is repeated until the parameter estimates converge.

The three-stage least squares procedure is needed because the net wage rate is endogenous. The marginal tax rate, which is used to calculate the net wage rate, is based on household income. The net wage rate is instrumented using a sample average net wage rate based on occupation–, state– and gender- specific sample cells. In addition error terms are potentially correlated across equations since the right-hand side variables are identical.

The three-stage least squares procedure combines a two-stage least squares model with a seemingly unrelated regression model. This latter model controls for the endogenous error term by taking into account the correlated error structure and also allows the imposition of the cross-equation restrictions. The two-stage least squares component allows for the use of instruments in controlling for endogeneity. The demographic variables included are age, age squared, education dummy variables, ethnicity dummy variables, dummy variables for the number of children, a dummy variable for home ownership, dummy variables for the number of cars owned and state and month fixed effects. A dummy variable for gender is included in the single household estimation, while demographic characteristics for both the male and female spouses are included in the married household estimation.

[15] The price indices are calculated using an initial parameter guess. The initial guess is provided by the AIDS specification while approximating $\ln a(\mathbf{w}, \mathbf{p})$ with Stone's Index.

The energy good is a combination of home energy consumption and gasoline consumption. Dummy variables for home ownership and the number of cars are included to control for energy consumption differences based on owning versus renting and whether one drives. State fixed effects are included to control for differences in energy consumption across states.[16] Weather also differs by region. Month fixed effects are included to control for seasonal variation in energy consumption.[17] Cross-sectional wage variation, within each state, is used to estimate the cross-price elasticity of labor supply; and variation in prices over time is used to estimate the cross-price elasticity of the energy good.

We drop the clean goods equation from the set of equations that are directly estimated, computing its parameter estimates for α, β, and λ from the adding up restrictions. (It nevertheless leaves undetermined the γ_{1j} $j = 1, 2$, and γ_{1k} $k = p, s$, estimates, referring to clean good as the first good). This procedure ensures that adding up restriction is satisfied. However, the homogeneity conditions $\sum_{i=1}^{n+2} \gamma_{ij} = \sum_{j=1}^{n+2} \gamma_{ij} = 0$ and symmetry restriction $\gamma_{ij} = \gamma_{ji}$ will have to be imposed. The three-stage least squares procedure that we use allows us to incorporate these restrictions. Standard errors are calculated using a bootstrap procedure consisting of 1,500 replications.

Table 4 reports the estimated parameters for single households, Table 5 for married one worker households and Table 6 for married two worker households. Initially we ran all the estimations with no restriction on the ln income squared coefficient, λ, in any of the equations. In the case of single households, the data supported a statistically non-zero value for λ for the clean good and leisure only. Table 4 reports the results when the estimation is re-run according to this specification. For married one worker households, the data does not suport a non-zero λ value for any of the three goods.

[16] Public transportation is used regularly in New York City and Washington D.C., while driving is essential in the West. Cities such as Denver and Seattle have strong bicycling cultures.

[17] Heating and air-conditioning increase energy consumption in the winter and summer months, while families may drive more during the summer for family vacations.

Table 4: Parameter Estimates Single Households

	Clean Good		Energy Good		Leisure	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
Constant	0.5435‡	(0.0952)	0.0834‡	(0.0075)	0.3731‡	(0.1007)
Ln(Clean Price)	–0.0131	(0.0322)	–0.0205‡	(0.0046)	0.0336	(0.0321)
Ln(Energy Price)	–0.0205‡	(0.0046)	0.0203‡	(0.0043)	0.0001	(0.0028)
Ln(Net Wage)	0.0336	(0.0321)	0.0001	(0.0028)	–0.0337	(0.0321)
Ln(Real Income)	–0.2481‡	(0.0835)	–0.0280‡	(0.0021)	0.2761‡	(0.0834)
Ln(Real Income^2)	0.1011‡	(0.0279)	–	–	–0.1011‡	(0.0279)

*System of 3 demand equations, the clean good equation is dropped. Parameters for the clean good are calculated based on cross-equation restrictions. $\lambda = 0$ for the energy good only. Standard errors are calculated using a bootstrap procedure (1,500 replications).
‡ significance at 5% level.

Table 5: Parameter Estimates Married One Worker Households

	Clean Good		Energy Good		Leisure	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
Constant	0.5412‡	(0.0366)	0.0894‡	(0.0100)	0.3694‡	(0.0395)
Ln(Clean Price)	0.0957‡	(0.0277)	–0.0257‡	(0.0072)	–0.0700‡	(0.0281)
Ln(Energy Price)	–0.0257‡	(0.0072)	0.0262‡	(0.0075)	–0.0005	(0.0055)
Ln(Net Wage)	–0.0700‡	(0.0281)	–0.0005	(0.0055)	0.0705‡	(0.0292)
Ln(Real Income)	0.0732‡	(0.0271)	–0.0327‡	(0.0052)	–0.0405	(0.0282)

*System of 3 demand equations, the clean good equation is dropped. Parameters for the clean good are calculated based on cross-equation restrictions. $\lambda = 0$ for all three goods. Standard errors are calculated using a bootstrap procedure (1,500 replications).
‡ significance at 5% level.

Therefore the estimation procedure is re-run for the married one worker sample where we restrict λ to be zero for all three goods, the AIDS specification. Table 5 reports these results. The data supports non-zero λ values for all four goods for the married two earner sample, Table 6.[18]

5 Elasticities

Denote the income elasticity of demand for good $i = 1, 2$ and for leisure $k = p, s$ by η_i and η_k; the own- and cross-price elasticities of demand for good $i = 1, 2$ with respect to good $j = 1, 2$ by ε_{ij}; the cross-price elasticity of demand for good $i = 1, 2$ with respect to leisure $k = p, s$ by ε_{ik}; its cross-price elasticity of demand for leisure $k = p, s$ with

[18]The AIDS specification, where $\lambda = 0$ for all goods, is estimated for all three subsamples. Results are not reported here but are available from the authors upon request, as are the results for the initial single household specification, where λ is non-zero for all goods.

Table 6: Parameter Estimates Married Two Worker Households

	Clean Good		Energy Good	
	Estimate	Std. Error	Estimate	Std. Error
Constant	0.4827‡	(0.0634)	0.0715‡	(0.0105)
Ln(Clean Price)	–0.0009	(0.0311)	–0.0292‡	(0.0046)
Ln(Energy Price)	–0.0292‡	(0.0046)	0.0194‡	(0.0032)
Ln(Net Wage) (M)	0.0146	(0.0193)	0.0032	(0.0029)
Ln(Net Wage) (F)	0.0155	(0.0160)	0.0065‡	(0.0023)
Ln(Real Income)	–0.2469‡	(0.0703)	–0.0555‡	(0.0095)
Ln(Real Income^2)	0.1471‡	(0.0335)	0.0137‡	(0.0039)
	Male Leisure		Female Leisure	
	Estimate	Std. Error	Estimate	Std. Error
Constant	0.1555‡	(0.0429)	0.2903‡	(0.0347)
Ln(Clean Price)	0.0146	(0.0193)	0.0155	(0.0160)
Ln(Energy Price)	0.0032	(0.0029)	0.0065‡	(0.0023)
Ln(Net Wage) (M)	0.0688‡	(0.0144)	–0.0865‡	(0.0091)
Ln(Net Wage) (F)	–0.0865‡	(0.0091)	0.0645‡	(0.0103)
Ln(Real Income)	0.1658‡	(0.0450)	0.1367‡	(0.0349)
Ln(Real Income^2)	–0.0951‡	(0.0215)	–0.0656‡	(0.0161)

*System of 4 demand equations, the clean good equation is dropped. Parameters for the clean good are calculated based on cross-equation restrictions. $\lambda \neq 0$ for all goods. Standard errors are calculated using a bootstrap procedure (1,500 replications). ‡ significance at 5% level.

respect to good $j = 1, 2$ by ε_{kj}; and the own- and cross-price elasticities of demand for leisure $k = p, s$ with respect to leisure $e = p, s$ by ε_{ke}. To relate these elasticity terms to the estimating equations, one can rewrite them in terms of the budget shares. We have, for i and $j = 1, 2$, and for k and $e = p, s$,

$$\eta_i \equiv \frac{\partial x_i}{\partial \mathbf{I}} \frac{\mathbf{I}}{x_i} = \frac{1}{\omega_i} \frac{\partial \omega_i}{\partial \ln \mathbf{I}} + 1, \tag{10}$$

$$\eta_k \equiv \frac{\partial l_k}{\partial \mathbf{I}} \frac{\mathbf{I}}{l_k} = \frac{1}{\omega_k} \frac{\partial \omega_k}{\partial \ln \mathbf{I}} + 1, \tag{11}$$

$$\varepsilon_{ij} \equiv \frac{\partial x_i}{\partial p_j} \frac{p_j}{x_i} = \frac{1}{\omega_i} \frac{\partial \omega_i}{\partial \ln p_j} - \delta_{ij}, \tag{12}$$

$$\varepsilon_{kj} \equiv \frac{\partial l_k}{\partial p_j} \frac{p_j}{l_i} = \frac{1}{\omega_k} \frac{\partial \omega_k}{\partial \ln p_j}, \tag{13}$$

$$\varepsilon_{ie} \equiv \frac{\partial x_i}{\partial w_e} \frac{w_e}{x_i} = \frac{1}{\omega_i} \frac{\partial \omega_i}{\partial \ln w_e} + \frac{w_e}{\mathbf{I}}, \tag{14}$$

$$\varepsilon_{ke} \equiv \frac{\partial l_k}{\partial w_e} \frac{w_e}{l_k} = \frac{1}{\omega_k} \frac{\partial \omega_k}{\partial \ln w_e^h} + \frac{w_e}{\mathbf{I}} - \delta_{ke}, \tag{15}$$

where δ_{ij} and δ_{ke} denote the Kronecker delta. Then partially differentiate ω_i and ω_k, as given by equations (8)–(9), with respect to $\ln \mathbf{I}, \ln p_j, \ln w_e$ and substitute in (10)–(15).

This yields, for all i and $j = 1, 2$, and k and $e = p, s$,

$$\begin{aligned}
\eta_i &= 1 + \frac{1}{\omega_i}\left[\beta_i + \frac{2\lambda_i}{b(\mathbf{w},\mathbf{p})}\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right], && (16)\\
\eta_k &= 1 + \frac{1}{\omega_k}\left[\beta_k + \frac{2\lambda_k}{b(\mathbf{w},\mathbf{p})}\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right], && (17)\\
\varepsilon_{ij} &= -\delta_{ij} + \frac{\gamma_{ij}}{\omega_i} - \frac{1}{\omega_i}\left[\alpha_j + \gamma_{jp}\ln w_p + \gamma_{js}\ln w_s + \sum_{i=1}^{2}\gamma_{ij}\ln p_i\right]\\
&\quad \times\left[\beta_i + \frac{2\lambda_i}{b(\mathbf{w},\mathbf{p})}\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right] - \frac{\lambda_i\beta_j}{\omega_i b(\mathbf{w},\mathbf{p})}\left[\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right]^2, && (18)\\
\varepsilon_{kj} &= \frac{\gamma_{kj}}{\omega_k} - \frac{1}{\omega_k}\left[\alpha_j + \gamma_{jp}\ln w_p + \gamma_{js}\ln w_s + \sum_{i=1}^{2}\gamma_{ij}\ln p_i\right]\\
&\quad \times\left[\beta_k + \frac{2\lambda_k}{b(\mathbf{w},\mathbf{p})}\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right] - \frac{\lambda_k\beta_j}{\omega_k b(\mathbf{w},\mathbf{p})}\left[\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right]^2, && (19)\\
\varepsilon_{ie} &= \frac{w_e}{\mathbf{I}} + \frac{\gamma_{ie}}{\omega_i} - \frac{1}{\omega_i}\left[\alpha_e + \gamma_{ee}\ln w_e + \gamma_{ek}\ln w_k + \sum_{i=1}^{2}\gamma_{ie}\ln p_i\right]\\
&\quad \times\left[\beta_i + \frac{2\lambda_i}{b(\mathbf{w},\mathbf{p})}\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right] - \frac{\lambda_i\beta_e}{\omega_i^h b(\mathbf{w},\mathbf{p})}\left[\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right]^2, && (20)\\
\varepsilon_{ke} &= \frac{w_e}{\mathbf{I}} - \delta_{ke} + \frac{\gamma_{ke}}{\omega_k^h} - \frac{1}{\omega_k}\left[\alpha_e + \gamma_{ee}\ln w_e + \gamma_{ek}\ln w_k + \sum_{i=1}^{2}\gamma_{ie}\ln p_i\right]\\
&\quad \times\left[\beta_k + \frac{2\lambda_i}{b(\mathbf{w},\mathbf{p})}\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right] - \frac{\lambda_k\beta_e}{\omega_k b(\mathbf{w},\mathbf{p})}\left[\ln\frac{\mathbf{I}}{a(\mathbf{w},\mathbf{p})}\right]^2. && (21)
\end{aligned}$$

Observe that if $\lambda_i = 0$, both the income and the cross price elasticity of demand for good i is independent of potential income.

Using the parameter estimates given in Tables 4–6, we calculate and report the income and price elasticities of demand for all three subsamples. The AIDS specification elasticities for single households are reported in Table 7 while the corresponding QUAIDS elasticities are reported in Table 8. The married two worker household AIDS elasticities are reported in Table 9 and the corresponding QUAIDS elasticities are reported in Table 10. Table 11 reports the AIDS specification elasticities for married one worker households.

The importance of relaxing the linear Engel curve assumption can be seen by comparing Tables 7 and 8. For single households the uncompensated cross-price elasticity of the energy good w.r.t. the clean good is –0.3056 under the AIDS specification but –0.0800 under the QUAIDS specification, although this latter value is insignificant. The pattern of the QUAIDS values generally being more inelastic is again evident when looking at either the compensated or uncompensated cross-price elasticity of the energy good w.r.t. labor supply. The compensated value is 0.6800 under the AIDS specification but 0.3296 under the QUAIDS specification.

Comparing the elasticity results under the AIDS specification with the QUAIDS spefication results again show the same general pattern. For instance consider either the cross-price elasticity of the energy good w.r.t. to the clean good or w.r.t. male labor supply. The QUAIDS result is more inelastic in both the compensated and uncompensated case. The compensated cross-price elasticity of the energy good w.r.t. the clean good is –0.4418 under AIDS and –0.1793 under QUAIDS. With respect to male labor supply, the compensated elasticity is 0.3509 under AIDS and 0.1675 under QUAIDS. Exceptions can occur if one of the estimates is insignificant. The other exception is the cross-price elasticity between male and female labor supply. In this case the AIDS estimate is relatively more inelastic but only in the uncompensated case.

Whether the QUAIDS or AIDS specification is appropriate, both results show the importance of specifying a demand model that does not impose separability and thus allows for the estimation of the full complement of cross-price elasticities. For single households the compensated and uncompensated cross-price elasticity w.r.t. labor supply is significant for both the clean good and the energy good. This is true whether the AIDS model is specified or the QUAIDS model is specified. Similarly for married two worker households, only the compensated cross-price elasticity of the energy good w.r.t. male labor supply is insignificant and only under QUAIDS. In all other cases

Table 7: AIDS Elasticity Estimates Single Households

	Clean Price	Energy Price	Net Wage
Income Elasticities			
Clean Good	1.0404		
	(0.9418, 1.1390)		
Energy Good	0.2512		
	(0.0197, 0.4827)		
Labor Supply	–0.7502		
	(–0.8700, –0.6305)		
Compensated Elasticities			
Clean Good	–0.3926	–0.0001	0.3510
	(–0.5269, –0.2583)	(–0.0216, 0.0215)	(0.2132, 0.4888)
Energy Good	–0.1585	–0.5316	0.6800
	(–0.4654, 0.1485)	(–0.7360, –0.3272)	(0.3719, 0.9881)
Labor Supply	–0.4695	–0.0488	0.5483
	(–0.6206, –0.3184)	(–0.0717, –0.0258)	(0.3909, 0.7057)
Uncompensated Elasticities			
Clean Good	–1.0019	–0.0482	0.8903
	(–1.0805, –0.9233)	(–0.0702, –0.0263)	(0.8029, 0.9777)
Energy Good	–0.3056	–0.5432	0.8102
	(–0.5316, –0.0796)	(–0.7449, –0.3415)	(0.6126, 1.0078)
Labor Supply	–0.0302	–0.0140	0.1594
	(–0.1141, 0.0537)	(–0.0368, 0.0087)	(0.0626, 0.2563)

*95% confidence intervals are calculated using a bootstrapping procedure (1,500 replications). AIDS specification, $\lambda = 0$, for all three goods.

the compensated and uncompensated cross-price elasticity w.r.t. either male or female labor supply is significant for both goods.

6 Welfare

To gauge how a change in the energy tax affects the welfare of different income groups, one should first specify what the government would do with the additional tax proceeds. To separate the two aspects, we first look at the welfare effect of the change in energy taxes while assuming that there is no feedback from the expenditures financed by the extra revenues (on consumers demands and welfare). Suppose the energy tax increases from t_1 to t_1'. Assuming no concomitant changes in other tax instruments, the equivalent

Table 8: QUAIDS Elasticity Estimates Single Households

	Clean Price	Energy Price	Net Wage
Income Elasticities			
Clean Good	1.0121		
	(0.9155, 1.1088)		
Energy Good	0.3956		
	(0.3047, 0.4866)		
Labor Supply	–0.7697		
	(–0.8822, –0.6572)		
Compensated Elasticities			
Clean Good	–0.3303	0.0230	0.2668
	(–0.4177, –0.2429)	(0.0057, 0.0402)	(0.1766, 0.3569)
Energy Good	0.1517	–0.4972	0.3296
	(–0.0945, 0.3979)	(–0.6796, –0.3147)	(0.1252, 0.5341)
Labor Supply	–0.3689	–0.0189	0.4186
	(–0.4749, –0.2628)	(–0.0334, –0.0044)	(0.3055, 0.5317)
Uncompensated Elasticities			
Clean Good	–0.9230	–0.0239	0.7914
	(–1.0333, –0.8127)	(–0.0437, –0.0041)	(0.6810, 0.9017)
Energy Good	–0.0800	–0.5155	0.5347
	(–0.3130, 0.1530)	(–0.6974, –0.3336)	(0.3532, 0.7162)
Labor Supply	0.0819	0.0167	0.0196
	(–0.0559, 0.2197)	(–0.0019, 0.0353)	(–0.1213, 0.1606)

*95% confidence intervals are calculated using a bootstrapping procedure (1,500 replications). QUAIDS specification, $\lambda \neq 0$, for clean good and leisure.

Table 9: AIDS Elasticity Estimates Married Two Worker Households

	Clean Price	Energy Price	Net Wage Male	Net Wage Female
Income Elasticities				
Clean Good	0.9919			
	(0.8555, 1.1283)			
Energy Good	0.0400			
	(–0.2007, 0.2808)			
Labor Supply (M)	–0.6049			
	(–0.6933, –0.5164)			
Labor Supply (F)	–0.9769			
	(–1.0917, –0.8621)			
Compensated Elasticities				
Clean Good	–0.5484	–0.0294	0.2668	0.2679
	(–0.7103, –0.3865)	(–0.0486, –0.0101)	(0.1711, 0.3625)	(0.1819, 0.3539)
Energy Good	–0.4418	–0.4069	0.3509	0.4960
	(–0.7089, –0.1746)	(–0.5876, –0.2262)	(0.1824, 0.5194)	(0.3537, 0.6383)
Labor Supply (M)	–0.3099	–0.0293	0.2965	0.0689
	(–0.4133, –0.2065)	(–0.0423, –0.0162)	(0.2255, 0.3674)	(0.0180, 0.1199)
Labor Supply (F)	–0.4738	–0.0534	0.1119	0.4577
	(–0.5973, –0.3503)	(–0.0681, –0.0387)	(0.0441, 0.1796)	(0.3784, 0.5371)
Uncompensated Elasticities				
Clean Good	–1.0071	–0.0641	0.7116	0.5607
	(–1.1080, –0.9062)	(–0.0833, –0.0450)	(0.6654, 0.7578)	(0.5090, 0.6124)
Energy Good	–0.4603	–0.4083	0.3689	0.5078
	(-0.6675, –0.2530)	(–0.5862, –0.2304)	(0.2698, 0.4679)	(0.4149, 0.6007)
Labor Supply (M)	–0.0302	–0.0080	0.0252	–0.1096
	(–0.0944, 0.0340)	(–0.0209, 0.0048)	(–0.0138, 0.0642)	(–0.1420, –0.0772)
Labor Supply (F)	–0.0221	–0.0191	–0.3262	0.1693
	(–0.0956, 0.0515)	(–0.0335, –0.0048)	(–0.3628, –0.2896)	(0.1171, 0.2216)

*95% confidence intervals are calculated using a bootstrapping procedure (1,500 replications).
AIDS specification, $\lambda = 0$, for all four goods.

Table 10: QUAIDS Elasticity Estimates Married Two Worker Households

	Clean Price	Energy Price	Net Wage Male	Net Wage Female
Income Elasticities				
Clean Good	1.0044			
	(0.8925, 1.1163)			
Energy Good	0.0766			
	(–0.1460, 0.2993)			
Labor Supply (M)	–0.5966			
	(–0.6697, –0.5234)			
Labor Supply (F)	–0.9647			
	(–1.0643, –0.8651)			
Compensated Elasticities				
Clean Good	–0.4438	–0.0066	0.1982	0.2086
	(–0.5232, –0.3643)	(–0.0284, 0.0152)	(0.1475, 0.2489)	(0.1616, 0.2555)
Energy Good	–0.1793	–0.3453	0.1675	0.3537
	(–0.5099, 0.1514)	(–0.5126, –0.1781)	(–0.0345, 0.3696)	(0.1901, 0.5173)
Labor Supply (M)	–0.2377	–0.0126	0.2485	0.0277
	(–0.2924, –0.1830)	(–0.0265, 0.0012)	(0.2029, 0.2942)	(–0.0022, 0.0576)
Labor Supply (F)	–0.3815	–0.0342	0.0509	0.4067
	(–0.4588, -0.3043)	(–0.0514, –0.0170)	(0.0023, 0.0995)	(0.3487, 0.4648)
Uncompensated Elasticities				
Clean Good	–0.9082	–0.0418	0.6486	0.5051
	(–0.9842, –0.8323)	(–0.0659, –0.0178)	(0.5841, 0.7131)	(0.4575, 0.5526)
Energy Good	–0.2147	–0.3480	0.2019	0.3764
	(–0.5374, 0.1079)	(–0.5145, –0.1816)	(0.0043, 0.3995)	(0.2194, 0.5333)
Labor Supply (M)	0.0382	0.0083	–0.0190	–0.1484
	(–0.0161, 0.0924)	(–0.0071, 0.0236)	(–0.0689, 0.0309)	(–0.1838, –0.1130)
Labor Supply (F)	0.0646	–0.0004	–0.3817	0.1219
	(–0.0086, 0.1378)	(–0.0195, 0.0187)	(–0.4453, –0.3181)	(0.0674, 0.1765)

*95% confidence intervals are calculated using a bootstrapping procedure (1,500 replications).
QUAIDS specification, $\lambda \neq 0$, for all four goods.

Table 11: AIDS Elasticity Estimates Married One Worker Households

	Clean Price	Energy Price	Net Wage
Income Elasticities			
Clean Good	1.1381		
	(1.0379, 1.2382)		
Energy Good	0.3370		
	(0.1316, 0.5424)		
Labor Supply	–0.5569		
	(–0.6386, –0.4751)		
Compensated Elasticities			
Clean Good	–0.2761	–0.0028	0.2314
	(–0.4003, –0.1518)	(–0.0301, 0.0244)	(0.1015, 0.3613)
Energy Good	–0.0530	–0.4026	0.4415
	(–0.3415, 0.2355)	(–0.6885, –0.1166)	(0.2102, 0.6728)
Labor Supply	–0.2185	–0.0314	0.2730
	(–0.3114, –0.1256)	(–0.0480, –0.0147)	(0.1754, 0.3707)
Uncompensated Elasticities			
Clean Good	–0.8796	–0.0590	1.0642
	(–0.9552, –0.8039)	(–0.0863, –0.0318)	(1.0019, 1.1265)
Energy Good	–0.2317	–0.4192	0.6881
	(–0.4996, 0.0362)	(–0.7020, –0.1364)	(0.5748, 0.8014)
Labor Supply	0.0768	–0.0038	–0.1344
	(0.0235, 0.1302)	(–0.0203, 0.0126)	(–0.1778, –0.0911)

*95% confidence intervals are calculated using a bootstrapping procedure (1,500 replications). AIDS specification, $\lambda = 0$, for all three goods.

variation of this tax regime change for household h, EV^h, is implicitly defined by[19]

$$v\left(\mathbf{w}^h, \mathbf{p}, \mathbf{I}^h + EV^h\right) = v\left(\mathbf{w}^h, \mathbf{p}', \mathbf{I}^h\right),$$

where $\mathbf{p}'$ is the new vector of consumer prices (with the new energy tax t_1').

Table 12 reports welfare calculations for the single household sample assuming that there is an additional 25c per unit tax levied on the energy good. First we report the EV under AIDS, next we report the EV under QUAIDS and finally the bias that results from incorrectly specifying the AIDS model. The population is separated into deciles and values are calculated for a representative household in each decile. The AIDS model underestimates the change in welfare for low income households but overestimates for high income households. The transition occurs at the 3rd decile, for wealthier households the welfare loss under AIDS is higher.

These calculations are repeated for tax increases of 50c, \$1.00 and \$1.50. The bias from using AIDS is reported in Table 13. The pattern remains the same. The AIDS model underestimates for low income households. However after the 3rd or 4th decile, the AIDS model overestimates the welfare loss. Tables 14 and 15 repeat the same calculations for the married two worker households. The end result is the same as for the single household sample.

[19]Recall that $\mathbf{w}^h = w_p^h, \mathbf{I}^h = w_p^h + m_p^h, \theta^h = \theta_p^h, \mathbf{wg}^h = wg_p^h, \mathbf{m}^h = m_p^h$ when there is one worker in the household and $\mathbf{w}^h = w_p^h + w_s^h, \mathbf{I}^h = w_p^h + m_p^h + w_s^h + m_s^h, \theta^h = \left(\theta_p^h, \theta_s^h\right), \mathbf{wg}^h = \left(wg_p^h, wg_s^h\right), \mathbf{m}^h = m_p^h + m_s^h$ when there are two workers.

Table 12: Equivalent Variation Results Single Households

	25c Increase In Energy Price		
Income Percentiles	AIDS EV ($)	QUAIDS EV ($)	Bias (%)
Tenth	–0.0071	–0.0081	–12.21
Twentieth	–0.0065	–0.0068	–4.86
Thirtieth	–0.0061	–0.0061	0.64
Fortieth	–0.0060	–0.0055	8.12
Fiftieth	–0.0057	–0.0050	14.04
Sixtieth	–0.0055	–0.0045	22.67
Seventieth	–0.0052	–0.0040	31.35
Eightieth	–0.0048	–0.0033	44.51
Ninetieth	–0.0043	–0.0025	72.04
One Hundredth	–0.0025	–0.0004	540.11

Table 13: % Bias When Using AIDS (Single Households)

% Increase in energy prices →	$0.25	$0.50	$1.00	$1.50
Income percentiles ↓				
Tenth	–12.21	–11.87	–11.38	–11.02
Twentieth	–4.86	–4.86	–4.85	–4.83
Thirtieth	0.64	0.29	–0.15	–0.44
Fortieth	8.12	7.23	6.06	5.32
Fiftieth	14.04	12.61	10.77	9.60
Sixtieth	22.67	20.39	17.47	15.64
Seventieth	31.35	27.98	23.77	21.18
Eightieth	44.51	39.14	32.66	28.81
Ninetieth	72.04	61.37	49.37	42.67
One Hundredth	540.11	261.73	141.90	103.76

Table 14: Equivalent Variation Results Married Two Worker Households

	Single Households (25c Increase In Energy Price		
Income Percentiles	AIDS EV ($)	QUAIDS EV ($)	Bias (%)
Tenth	–0.0058	–0.0066	–11.43
Twentieth	–0.0057	–0.0056	1.18
Thirtieth	–0.0056	–0.0051	9.13
Fortieth	–0.0052	–0.0045	15.41
Fiftieth	–0.0051	–0.0042	21.74
Sixtieth	–0.0047	–0.0037	28.05
Seventieth	–0.0045	–0.0033	37.62
Eightieth	–0.0042	–0.0028	53.09
Ninetieth	–0.0037	–0.0020	83.64
One Hundredth	–0.0027	–0.0007	301.84

Table 15: % Bias When Using AIDS (Married Two Worker Households)

% Increase in energy prices →	$0.25	$0.50	$1.00	$1.50
Income percentiles ↓				
Tenth	–11.43	–11.32	–11.18	–11.09
Twentieth	1.18	0.46	–0.52	–1.17
Thirtieth	9.13	7.75	5.92	4.72
Fortieth	15.41	13.34	10.67	8.98
Fiftieth	21.74	18.92	15.35	13.12
Sixtieth	28.05	24.27	19.62	16.78
Seventieth	37.62	32.29	25.92	22.15
Eightieth	53.09	44.84	35.41	30.04
Ninetieth	83.64	67.64	51.03	42.32
One Hundredth	301.84	181.47	108.58	81.44

Appendix A

Derivation of (8)–(9): Step1. Partially differentiate

$$\ln v^h = \left\{ \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right]^{-1} + \lambda(\mathbf{w}^h, \mathbf{p}) \right\}^{-1},$$

with respect to p_i, w_k^h, and I^h:

$$\begin{aligned}
\frac{\partial \ln v^h}{\partial p_i} &= \Lambda \left\{ \frac{\partial}{\partial p_i} \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right]^{-1} + \frac{\partial}{\partial p_i} \lambda(\mathbf{w}^h, \mathbf{p}) \right\}, \\
\frac{\partial \ln v^h}{\partial w_k^h} &= \Lambda \left\{ \frac{\partial}{\partial w_k^h} \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right]^{-1} + \frac{\partial}{\partial w_k^h} \lambda(\mathbf{w}^h, \mathbf{p}) \right\}, \\
\frac{\partial \ln v^h}{\partial I^h} &= \Lambda \left\{ \frac{\partial}{\partial I^h} \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right]^{-1} + \frac{\partial}{\partial I^h} \lambda(\mathbf{w}^h, \mathbf{p}) \right\},
\end{aligned}$$

where

$$\Lambda \equiv - \left\{ \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right]^{-1} + \lambda(\mathbf{w}^h, \mathbf{p}) \right\}^{-2}.$$

Simplifying yields:

$$\begin{aligned}
\frac{\partial \ln v^h}{\partial p_i} &= \Lambda \left\{ - \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right]^{-2} \frac{\partial}{\partial p_i} \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right] + \frac{\partial}{\partial p_i} \lambda(\mathbf{w}^h, \mathbf{p}) \right\}, \\
\frac{\partial \ln v^h}{\partial w_k^h} &= \Lambda \left\{ - \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right]^{-2} \frac{\partial}{\partial w_k^h} \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right] + \frac{\partial}{\partial w_k^h} \lambda(\mathbf{w}^h, \mathbf{p}) \right\}, \\
\frac{\partial \ln v^h}{\partial I^h} &= \Lambda \left\{ - \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right]^{-2} \frac{\partial}{\partial I^h} \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})} \right] + \frac{\partial}{\partial I^h} \lambda(\mathbf{w}^h, \mathbf{p}) \right\},
\end{aligned}$$

Or

$$
\begin{aligned}
\frac{\partial \ln v^h}{\partial p_i} &= \Lambda \frac{\partial}{\partial p_i}\lambda(\mathbf{w}^h,\mathbf{p}) - \Lambda\left[\frac{\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})}{b(\mathbf{w}^h,\mathbf{p})}\right]^{-2} \qquad \text{(A1)}\\
&\times \left[\frac{-b(\mathbf{w}^h,\mathbf{p})\left(\partial \ln a(\mathbf{w}^h,\mathbf{p})/\partial p_i\right) - \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\left(\partial b(\mathbf{w}^h,\mathbf{p})/\partial p_i\right)}{\left(b(\mathbf{w}^h,\mathbf{p})\right)^2}\right],\\
\frac{\partial \ln v^h}{\partial w_k^h} &= \Lambda \frac{\partial}{\partial w_k^h}\lambda(\mathbf{w}^h,\mathbf{p}) - \Lambda\left[\frac{\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})}{b(\mathbf{w}^h,\mathbf{p})}\right]^{-2} \qquad \text{(A2)}\\
&\times \left[\frac{b(\mathbf{w}^h,\mathbf{p})\left(1/I^h - \partial \ln a(\mathbf{w}^h,\mathbf{p})/\partial w_k^h\right) - \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\left(\partial b(\mathbf{w}^h,\mathbf{p})/\partial w_k^h\right)}{\left(b(\mathbf{w}^h,\mathbf{p})\right)^2}\right],\\
\frac{\partial \ln v^h}{\partial I^h} &= \Lambda \frac{\partial}{\partial I^h}\lambda(\mathbf{w}^h,\mathbf{p}) - \Lambda\left[\frac{\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})}{b(\mathbf{w}^h,\mathbf{p})}\right]^{-2} \qquad \text{(A3)}\\
&\times \left[\frac{b(\mathbf{w}^h,\mathbf{p})\left(1/I^h - \partial \ln a(\mathbf{w}^h,\mathbf{p})/\partial I^h\right) - \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\left(\partial b(\mathbf{w}^h,\mathbf{p})/\partial I^h\right)}{\left(b(\mathbf{w}^h,\mathbf{p})\right)^2}\right].
\end{aligned}
$$

Step 2. Partially differentiate $b(\mathbf{w}^h,\mathbf{p})$ and $\lambda(\mathbf{w}^h,\mathbf{p})$ with respect to p_i, w_k^h, and I^h to get

$$
\begin{aligned}
\frac{\partial b(\mathbf{w}^h,\mathbf{p})}{\partial p_i} &= \frac{\beta_i}{p_i} b(\mathbf{w}^h,\mathbf{p}),\\
\frac{\partial b(\mathbf{w}^h,\mathbf{p})}{\partial w_k^h} &= \frac{\beta_k}{w_k^h} b(\mathbf{w}^h,\mathbf{p}),\\
\frac{\partial b(\mathbf{w}^h,\mathbf{p})}{\partial I^h} &= 0,\\
\frac{\partial \lambda(\mathbf{w}^h,\mathbf{p})}{\partial p_i} &= \frac{\lambda_i}{p_i},\\
\frac{\partial \lambda(\mathbf{w}^h,\mathbf{p})}{\partial w_k^h} &= \frac{\lambda_k}{w_k^h},\\
\frac{\partial \lambda(\mathbf{w}^h,\mathbf{p})}{\partial I^h} &= 0.
\end{aligned}
$$

Also note that $\ln a(\mathbf{w}^h,\mathbf{p})$ is independent of I^h so that

$$
\frac{\partial \ln a(\mathbf{w}^h,\mathbf{p})}{\partial I^h} = 0.
$$

Substituting these expressions in (**??**)–(**??**),

$$\frac{\partial \ln v^h}{\partial p_i} = \Lambda \frac{\lambda_i}{p_i} - \Lambda \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})}\right]^{-2} \times \left[\frac{-b(\mathbf{w}^h, \mathbf{p}) \left(\partial \ln a(\mathbf{w}^h, \mathbf{p})/\partial p_i\right) - \left(\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})\right) \beta_i b(\mathbf{w}^h, \mathbf{p})/p_i}{\left(b(\mathbf{w}^h, \mathbf{p})\right)^2}\right], \tag{A4}$$

$$\frac{\partial \ln v^h}{\partial w_k^h} = \Lambda \frac{\lambda_k}{w_k^h} - \Lambda \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})}\right]^{-2} \times \left[\frac{b(\mathbf{w}^h, \mathbf{p}) \left(1/I^h - \partial \ln a(\mathbf{w}^h, \mathbf{p})/\partial w_k^h\right) - \left(\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})\right) \beta_k b(\mathbf{w}^h, \mathbf{p})/w_k^h}{\left(b(\mathbf{w}^h, \mathbf{p})\right)^2}\right], \tag{A5}$$

$$\frac{\partial \ln v^h}{\partial I^h} = -\Lambda \left[\frac{\ln I^h - \ln a(\mathbf{w}^h, \mathbf{p})}{b(\mathbf{w}^h, \mathbf{p})}\right]^{-2} \left[\frac{b(\mathbf{w}^h, \mathbf{p}) \left(1/I^h\right)}{\left(b(\mathbf{w}^h, \mathbf{p})\right)^2}\right]. \tag{A6}$$

Dividing (A4) and (A5) by (A6) then results in

$$\frac{\frac{\partial \ln v^h}{\partial p_i}}{\frac{\partial \ln v^h}{\partial I^h}} = \frac{\frac{\lambda_i}{p_i} - \left[\frac{\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})}{b(\mathbf{w}^h,\mathbf{p})}\right]^{-2} \left[\frac{-b(\mathbf{w}^h,\mathbf{p})\left(\partial \ln a(\mathbf{w}^h,\mathbf{p})/\partial p_i\right) - \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\beta_i b(\mathbf{w}^h,\mathbf{p})/p_i}{\left(b(\mathbf{w}^h,\mathbf{p})\right)^2}\right]}{-\left[\frac{\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})}{b(\mathbf{w}^h,\mathbf{p})}\right]^{-2} \left[\frac{b(\mathbf{w}^h,\mathbf{p})\left(1/I^h\right)}{\left(b(\mathbf{w}^h,\mathbf{p})\right)^2}\right]},$$

$$\frac{\frac{\partial \ln v^h}{\partial w_k^h}}{\frac{\partial \ln v^h}{\partial I^h}} = \frac{\frac{\lambda_k}{w_k^h} - \left[\frac{\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})}{b(\mathbf{w}^h,\mathbf{p})}\right]^{-2} \left[\frac{b(\mathbf{w}^h,\mathbf{p})\left(1/I^h - \partial \ln a(\mathbf{w}^h,\mathbf{p})/\partial w_k^h\right) - \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\beta_k b(\mathbf{w}^h,\mathbf{p})/w_k^h}{\left(b(\mathbf{w}^h,\mathbf{p})\right)^2}\right]}{-\left[\frac{\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})}{b(\mathbf{w}^h,\mathbf{p})}\right]^{-2} \left[\frac{b(\mathbf{w}^h,\mathbf{p})\left(1/I^h\right)}{\left(b(\mathbf{w}^h,\mathbf{p})\right)^2}\right]}.$$

Or

$$\frac{\frac{\partial \ln v^h}{\partial p_i}}{\frac{\partial \ln v^h}{\partial I^h}} = \frac{\left[\frac{-b(\mathbf{w}^h,\mathbf{p})\left(\partial \ln a(\mathbf{w}^h,\mathbf{p})/\partial p_i\right) - \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\beta_i b(\mathbf{w}^h,\mathbf{p})/p_i}{\left(b(\mathbf{w}^h,\mathbf{p})\right)^2}\right] - \frac{\lambda_i}{p_i}\left[\frac{\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})}{b(\mathbf{w}^h,\mathbf{p})}\right]^2}{\left[\frac{1}{I^h b(\mathbf{w}^h,\mathbf{p})}\right]},$$

$$\frac{\frac{\partial \ln v^h}{\partial w_k^h}}{\frac{\partial \ln v^h}{\partial I^h}} = \frac{\left[\frac{b(\mathbf{w}^h,\mathbf{p})\left(1/I^h - \partial \ln a(\mathbf{w}^h,\mathbf{p})/\partial w_k^h\right) - \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\beta_k b(\mathbf{w}^h,\mathbf{p})/w_k^h}{\left(b(\mathbf{w}^h,\mathbf{p})\right)^2}\right] - \frac{\lambda_k}{w_k^h}\left[\frac{\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})}{b(\mathbf{w}^h,\mathbf{p})}\right]^2}{\left[\frac{1}{I^h b(\mathbf{w}^h,\mathbf{p})}\right]}.$$

Or

$$\frac{\frac{\partial \ln v^h}{\partial p_i}}{\frac{\partial \ln v^h}{\partial I^h}} = \frac{I^h}{b(\mathbf{w}^h,\mathbf{p})}\left[-b(\mathbf{w}^h,\mathbf{p})\left(\partial \ln a(\mathbf{w}^h,\mathbf{p})/\partial p_i\right) - \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\beta_i b(\mathbf{w}^h,\mathbf{p})/p_i\right]$$
$$-\frac{\lambda_i}{p_i}\left[\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right]^2 \frac{I^h}{b(\mathbf{w}^h,\mathbf{p})},$$
$$\frac{\frac{\partial \ln v^h}{\partial w_k^h}}{\frac{\partial \ln v^h}{\partial I^h}} = \frac{I^h}{b(\mathbf{w}^h,\mathbf{p})}\left[b(\mathbf{w}^h,\mathbf{p})\left(1/I^h - \partial \ln a(\mathbf{w}^h,\mathbf{p})/\partial w_k^h\right) - \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\beta_k b(\mathbf{w}^h,\mathbf{p})/w_k^h\right]$$
$$-\frac{\lambda_k}{w_k^h}\left[\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right]^2 \frac{I^h}{b(\mathbf{w}^h,\mathbf{p})}.$$

Or

$$\frac{\frac{\partial \ln v^h}{\partial p_i}}{\frac{\partial \ln v^h}{\partial I^h}} = -\left[\frac{\partial \ln a(\mathbf{w}^h,\mathbf{p})}{\partial p_i} + \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\frac{\beta_i}{p_i}\right] I^h - \frac{\lambda_i}{p_i}\frac{I^h}{b(\mathbf{w}^h,\mathbf{p})}\left[\ln \frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2,$$
$$\frac{\frac{\partial \ln v^h}{\partial w_k^h}}{\frac{\partial \ln v^h}{\partial I^h}} = 1-\left[\frac{\partial \ln a(\mathbf{w}^h,\mathbf{p})}{\partial w_k^h} + \left(\ln I^h - \ln a(\mathbf{w}^h,\mathbf{p})\right)\frac{\beta_k}{w_k^h}\right] I^h - \frac{\lambda_k}{w_k^h}\frac{I^h}{b(\mathbf{w}^h,\mathbf{p})}\left[\ln \frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2.$$

Step 3. Substitute these expressions in

$$\omega_i^h = -\frac{p_i}{I^h}\frac{\partial \ln v^h/\partial p_i}{\partial \ln v^h/\partial I^h},$$
$$\omega_k^h = \frac{w_k^h}{I^h}\left(1-\frac{\partial \ln v^h/\partial w_k^h}{\partial \ln v^h/\partial I^h}\right),$$

to get:

$$\omega_i^h = p_i\frac{\partial \ln a(\mathbf{w}^h,\mathbf{p})}{\partial p_i} + \beta_i \ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})} + \frac{\lambda_i}{b(\mathbf{w}^h,\mathbf{p})}\left[\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2,$$
$$\omega_k^h = w_k^h\frac{\partial \ln a(\mathbf{w}^h,\mathbf{p})}{\partial w_k^h} + \beta_k \ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})} + \frac{\lambda_k}{b(\mathbf{w}^h,\mathbf{p})}\left[\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2.$$

Step 4. Partially differentiate $\ln a(\mathbf{w}^h,\mathbf{p})$ with respect to p_i and w_k^h:

$$\frac{\partial \ln a(\mathbf{w}^h,\mathbf{p})}{\partial p_i} = \frac{1}{p_i}\left[\alpha_i + \gamma_{ip}\ln w_p^h + \gamma_{is}\ln w_s^h + \sum_{j=1}^{n}\gamma_{ji}\ln p_j\right],$$
$$\frac{\partial \ln a(\mathbf{w}^h,\mathbf{p})}{\partial w_k^h} = \frac{1}{w_k^h}\left[\alpha_k + \sum_{i=1}^{n}\gamma_{ik}\ln p_i + \gamma_{kk}\ln w_k^h + \gamma_{ke}\ln w_e^h\right],$$

Then substitute these expressions in the expressions for ω_i^h and ω_k^h derived in step 3. This yields

$$\begin{aligned}
\omega_i^h &= \alpha_i + \gamma_{ip}\ln w_p^h + \gamma_{is}\ln w_s^h + \sum_{j=1}^{n}\gamma_{ij}\ln p_j + \\
&\quad \beta_i \ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})} + \frac{\lambda_i}{b(\mathbf{w}^h,\mathbf{p})}\left[\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2, \\
\omega_k^h &= \alpha_k + \gamma_{kk}\ln w_k^h + \gamma_{ke}\ln w_e^h + \sum_{j=1}^{n}\gamma_{kj}\ln p_j + \\
&\quad \beta_k \ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})} + \frac{\lambda_k}{b(\mathbf{w}^h,\mathbf{p})}\left[\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2,
\end{aligned}$$

which are equations (8)–(9).

Derivation of (10)–(15): Step 1. From the definition of budget shares, $\omega_i^h \equiv p_i x_i^h / I^h$ and $\omega_k^h \equiv w_k^h l_k^h / I^h$. Rearranging yields

$$\begin{aligned}
x_i^h &= \frac{I^h \omega_i^h}{p_i}, \\
l_k^h &= \frac{I^h \omega_k^h}{w_k^h}.
\end{aligned}$$

Partially differentiating these two relationships with respect to I^h, p_j, and w_e^h yields,

$$\begin{aligned}
\frac{\partial x_i^h}{\partial I^h} &= \frac{\omega_i^h}{p_i} + \frac{I^h}{p_i}\frac{\partial \omega_i^h}{\partial I^h} = \frac{1}{p_i}\left[\omega_i^h + \frac{\partial \omega_i^h}{\partial \ln I^h}\right], \\
\frac{\partial l_k^h}{\partial I^h} &= \frac{\omega_k^h}{w_k^h} + \frac{I^h}{w_k^h}\frac{\partial \omega_k^h}{\partial I^h} = \frac{1}{w_k^h}\left[\omega_k^h + \frac{\partial \omega_k^h}{\partial \ln I^h}\right] \\
\frac{\partial x_i^h}{\partial p_j} &= I^h\left[\frac{\left(\partial \omega_i^h/\partial p_j\right)p_i - \left(\partial p_i/\partial p_j\right)\omega_i^h}{(p_i)^2}\right] = I^h\left[\frac{1}{p_i}\frac{\partial \omega_i^h}{\partial p_j} - \frac{\omega_i^h}{(p_i)^2}\frac{\partial p_i}{\partial p_j}\right] \\
&= \frac{I^h}{p_i}\left[\frac{1}{p_j}\frac{\partial \omega_i^h}{\partial \ln p_j} - \frac{\omega_i^h}{p_i}\frac{\partial p_i}{\partial p_j}\right] = \frac{I^h}{p_i}\left[\frac{1}{p_j}\frac{\partial \omega_i^h}{\partial \ln p_j} - \frac{\omega_i^h}{p_i}\delta_{ij}\right] \\
\frac{\partial l_k^h}{\partial p_j} &= \frac{I^h}{w_k^h}\frac{\partial \omega_k^h}{\partial p_j} = \frac{I^h}{p_j w_k^h}\frac{\partial \omega_k^h}{\partial \ln p_j} \\
\frac{\partial x_i^h}{\partial w_e^h} &= \frac{I^h}{p_i}\frac{\partial \omega_i^h}{\partial w_e^h} + \frac{\omega_i^h}{p_i} = \frac{I^h}{w_e^h p_i}\frac{\partial \omega_i^h}{\partial \ln w_e^h} + \frac{\omega_i^h}{p_i} \\
\frac{\partial l_k^h}{\partial w_e^h} &= I^h\left[\frac{\left(\partial \omega_k^h/\partial w_e^h\right)w_k^h - \left(\partial w_k^h/\partial w_e^h\right)\omega_k^h}{\left(w_k^h\right)^2}\right] + \frac{\omega_k^h}{w_k^h} = I^h\left[\frac{1}{w_k^h}\frac{\partial \omega_k^h}{\partial w_e^h} - \frac{\omega_k^h}{\left(w_k^h\right)^2}\frac{\partial w_k^h}{\partial w_e^h}\right] + \frac{\omega_k^h}{w_k^h} \\
&= \frac{I^h}{w_k^h}\left[\frac{1}{w_e^h}\frac{\partial \omega_k^h}{\partial \ln w_e^h} - \frac{\omega_k^h}{w_k^h}\frac{\partial w_k^h}{\partial w_e^h}\right] + \frac{\omega_k^h}{w_k^h} = \frac{I^h}{w_k^h}\left(\frac{1}{w_e^h}\frac{\partial \omega_k^h}{\partial \ln w_e^h} - \frac{\omega_k^h}{w_k^h}\delta_{ke}\right) + \frac{\omega_k^h}{w_k^h}
\end{aligned}$$

Step 2. Substituting these derivatives in the various definition of elasticity terms results in

$$\begin{aligned}
\eta_i^h &\equiv \frac{\partial x_i^h}{\partial I^h}\frac{I^h}{x_i^h} = \frac{I^h}{x_i^h}\frac{1}{p_i}\left[\omega_i^h + \frac{\partial \omega_i^h}{\partial \ln I^h}\right] = 1 + \frac{1}{\omega_i^h}\frac{\partial \omega_i^h}{\partial \ln I^h}, \\
\eta_k^h &\equiv \frac{\partial l_k^h}{\partial I^h}\frac{I^h}{l_k^h} = \frac{I^h}{l_k^h}\frac{1}{w_k^h}\left[\omega_k^h + \frac{\partial \omega_k^h}{\partial \ln I^h}\right] = 1 + \frac{1}{\omega_k^h}\frac{\partial \omega_k^h}{\partial \ln I^h}, \\
\varepsilon_{ij}^h &\equiv \frac{\partial x_i^h}{\partial p_j}\frac{p_j}{x_i^h} = \frac{p_j}{x_i^h}\frac{I^h}{p_i}\left[\frac{1}{p_j}\frac{\partial \omega_i^h}{\partial \ln p_j} - \frac{\omega_i^h}{p_i}\delta_{ij}\right] = \frac{1}{\omega_i^h}\frac{\partial \omega_i^h}{\partial \ln p_j} - \delta_{ij}, \\
\varepsilon_{kj}^h &\equiv \frac{\partial l_k^h}{\partial p_j}\frac{p_j}{l_k^h} = \frac{p_j}{l_k^h}\frac{I^h}{p_j w_k^h}\frac{\partial \omega_k^h}{\partial \ln p_j} = \frac{1}{\omega_k^h}\frac{\partial \omega_k^h}{\partial \ln p_j}, \\
\varepsilon_{ie}^h &\equiv \frac{\partial x_i^h}{\partial w_e^h}\frac{w_e^h}{x_i^h} = \left[\frac{I^h}{w_e^h p_i}\frac{\partial \omega_i^h}{\partial \ln w_e^h} + \frac{\omega_i^h}{p_i}\right]\frac{w_e^h}{x_i^h} = \frac{1}{\omega_i^h}\frac{\partial \omega_i^h}{\partial \ln w_e^h} + \frac{w_e^h}{I^h}, \\
\varepsilon_{ke}^h &\equiv \frac{\partial l_k^h}{\partial w_e^h}\frac{w_e^h}{l_k^h} = \frac{w_e^h}{l_k^h}\left[\frac{I^h}{w_k^h}\left(\frac{1}{w_e^h}\frac{\partial \omega_k^h}{\partial \ln w_e^h} - \frac{\omega_k^h}{w_k^h}\delta_{ke}\right) + \frac{\omega_k^h}{w_k^h}\right] = \frac{1}{\omega_k^h}\frac{\partial \omega_k^h}{\partial \ln w_e^h} + \frac{w_e^h}{I^h} - \delta_{ke}.
\end{aligned}$$

Derivation of (16)–(21): Step 1. Partially differentiate equations (8)–(9) with respect to I^h, p_j, and w_e^h. We have:

$$
\begin{aligned}
\frac{\partial \omega_i^h}{\partial \ln I^h} &= \beta_i + \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}, \\
\frac{\partial \omega_k^h}{\partial \ln I^h} &= \beta_k + \frac{2\lambda_k}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}, \\
\frac{\partial \omega_i^h}{\partial \ln p_j} &= \gamma_{ij} - \beta_i \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j} - \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})} \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j} \\
&\quad - \frac{\lambda_i}{(b(\mathbf{w}^h, \mathbf{p}))^2} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})} \right]^2 \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j} \\
\frac{\partial \omega_k^h}{\partial \ln p_j} &= \gamma_{kj} - \beta_k \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j} - \frac{2\lambda_k}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})} \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j} \\
&\quad - \frac{\lambda_k}{(b(\mathbf{w}^h, \mathbf{p}))^2} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})} \right]^2 \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j}, \\
\frac{\partial \omega_i^h}{\partial \ln w_e^h} &= \gamma_{ie} - \beta_i \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h} - \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})} \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h} \\
&\quad - \frac{\lambda_i}{(b(\mathbf{w}^h, \mathbf{p}))^2} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})} \right]^2 \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h}, \\
\frac{\partial \omega_k^h}{\partial \ln w_e^h} &= \gamma_{ke} - \beta_k \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h} - \frac{2\lambda_k}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})} \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h} \\
&\quad - \frac{\lambda_k}{(b(\mathbf{w}^h, \mathbf{p}))^2} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})} \right]^2 \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h}.
\end{aligned}
$$

Simplifying yields

$$\frac{\partial \omega_i^h}{\partial \ln I^h} = \beta_i + \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}, \tag{A7}$$

$$\frac{\partial \omega_k^h}{\partial \ln I^h} = \beta_k + \frac{2\lambda_k}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}, \tag{A8}$$

$$\begin{aligned}\frac{\partial \omega_i^h}{\partial \ln p_j} &= \gamma_{ij} - \left[\beta_i + \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right] \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j} \\ &\quad - \frac{\lambda_i}{(b(\mathbf{w}^h, \mathbf{p}))^2} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right]^2 \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j}\end{aligned} \tag{A9}$$

$$\begin{aligned}\frac{\partial \omega_k^h}{\partial \ln p_j} &= \gamma_{kj} - \left[\beta_k + \frac{2\lambda_k}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right] \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j} \\ &\quad - \frac{\lambda_k}{(b(\mathbf{w}^h, \mathbf{p}))^2} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right]^2 \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j},\end{aligned} \tag{A10}$$

$$\begin{aligned}\frac{\partial \omega_i^h}{\partial \ln w_e^h} &= \gamma_{ie} - \left[\beta_i + \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right] \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h} \\ &\quad - \frac{\lambda_i}{(b(\mathbf{w}^h, \mathbf{p}))^2} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right]^2 \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h},\end{aligned} \tag{A11}$$

$$\begin{aligned}\frac{\partial \omega_k^h}{\partial \ln w_e^h} &= \gamma_{ke} - \left[\beta_k + \frac{2\lambda_k}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right] \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h} \\ &\quad - \frac{\lambda_k}{(b(\mathbf{w}^h, \mathbf{p}))^2} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right]^2 \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h}.\end{aligned} \tag{A12}$$

Step 2. Partially differentiate equations (3)–(4) with respect to p_j and w_e^h. We have

$$\begin{aligned}\frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j} &= \alpha_j + \gamma_{jp} \ln w_p^h + \gamma_{js} \ln w_s^h + \sum_{i=1}^{n} \gamma_{ij} \ln p_i, \\ \frac{\partial \ln a(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h} &= \alpha_e + \gamma_{ee} \ln w_e^h + \gamma_{ek} \ln w_k^h + \sum_{i=1}^{n} \gamma_{ie} \ln p_i, \\ \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln p_j} &= p_j \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial p_j} = p_j \frac{\beta_j}{p_j} b(\mathbf{w}^h, \mathbf{p}), \\ \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial \ln w_e^h} &= w_e^h \frac{\partial b(\mathbf{w}^h, \mathbf{p})}{\partial w_e^h} = w_e^h \frac{\beta_e}{w_e^h} b(\mathbf{w}^h, \mathbf{p}).\end{aligned}$$

Substituting in equations (A7)–(A12) yields,

$$
\begin{aligned}
\frac{\partial \omega_i^h}{\partial \ln I^h} &= \beta_i + \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}, \\
\frac{\partial \omega_k^h}{\partial \ln I^h} &= \beta_k + \frac{2\lambda_k}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}, \\
\frac{\partial \omega_i^h}{\partial \ln p_j} &= \gamma_{ij} - \left[\beta_i + \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right] \times \\
&\quad \left[\alpha_j + \gamma_{jp} \ln w_p^h + \gamma_{js} \ln w_s^h + \sum_{i=1}^{n} \gamma_{ij} \ln p_i\right] - \frac{\lambda_i \beta_j}{b(\mathbf{w}^h, \mathbf{p})} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right]^2, \\
\frac{\partial \omega_k^h}{\partial \ln p_j} &= \gamma_{kj} - \left[\beta_k + \frac{2\lambda_k}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right] \times \\
&\quad \left[\alpha_j + \gamma_{jp} \ln w_p^h + \gamma_{js} \ln w_s^h + \sum_{i=1}^{n} \gamma_{ij} \ln p_i\right] - \frac{\lambda_k \beta_j}{b(\mathbf{w}^h, \mathbf{p})} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right]^2, \\
\frac{\partial \omega_i^h}{\partial \ln w_e^h} &= \gamma_{ie} - \left[\beta_i + \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right] \times \\
&\quad \left[\alpha_e + \gamma_{ee} \ln w_e^h + \gamma_{ek} \ln w_k^h + \sum_{i=1}^{n} \gamma_{ie} \ln p_i\right] - \frac{\lambda_i \beta_e}{b(\mathbf{w}^h, \mathbf{p})} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right]^2, \\
\frac{\partial \omega_k^h}{\partial \ln w_e^h} &= \gamma_{ke} - \left[\beta_k + \frac{2\lambda_i}{b(\mathbf{w}^h, \mathbf{p})} \ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right] \times \\
&\quad \left[\alpha_e + \gamma_{ee} \ln w_e^h + \gamma_{ek} \ln w_k^h + \sum_{i=1}^{n} \gamma_{ie} \ln p_i\right] - \frac{\lambda_k \beta_e}{b(\mathbf{w}^h, \mathbf{p})} \left[\ln \frac{I^h}{a(\mathbf{w}^h, \mathbf{p})}\right]^2.
\end{aligned}
$$

Step 3. Substituting these relationships in (10)–(15) leads to,

$$
\begin{aligned}
\eta_i^h &= 1+\frac{1}{\omega_i^h}\left[\beta_i+\frac{2\lambda_i}{b(\mathbf{w}^h,\mathbf{p})}\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right],\\
\eta_k^h &= 1+\frac{1}{\omega_k^h}\left[\beta_k+\frac{2\lambda_k}{b(\mathbf{w}^h,\mathbf{p})}\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right],\\
\varepsilon_{ij}^h &= -\delta_{ij}+\frac{\gamma_{ij}}{\omega_i^h}-\frac{1}{\omega_i^h}\left[\beta_i+\frac{2\lambda_i}{b(\mathbf{w}^h,\mathbf{p})}\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]\times\\
&\quad\left[\alpha_j+\gamma_{jp}\ln w_p^h+\gamma_{js}\ln w_s^h+\sum_{i=1}^{n}\gamma_{ij}\ln p_i\right]-\frac{\lambda_i\beta_j}{\omega_i^h b(\mathbf{w}^h,\mathbf{p})}\left[\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2,\\
\varepsilon_{kj}^h &= \frac{\gamma_{kj}}{\omega_k^h}-\frac{1}{\omega_k^h}\left[\beta_k+\frac{2\lambda_k}{b(\mathbf{w}^h,\mathbf{p})}\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]\times\\
&\quad\left[\alpha_j+\gamma_{jp}\ln w_p^h+\gamma_{js}\ln w_s^h+\sum_{i=1}^{n}\gamma_{ij}\ln p_i\right]-\frac{\lambda_k\beta_j}{\omega_k^h b(\mathbf{w}^h,\mathbf{p})}\left[\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2,\\
\varepsilon_{ie}^h &= \frac{w_e^h}{I^h}+\frac{\gamma_{ie}}{\omega_i^h}-\frac{1}{\omega_i^h}\left[\beta_i+\frac{2\lambda_i}{b(\mathbf{w}^h,\mathbf{p})}\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]\times\\
&\quad\left[\alpha_e+\gamma_{ee}\ln w_e^h+\gamma_{ek}\ln w_k^h+\sum_{i=1}^{n}\gamma_{ie}\ln p_i\right]-\frac{\lambda_i\beta_e}{\omega_i^h b(\mathbf{w}^h,\mathbf{p})}\left[\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2,\\
\varepsilon_{ke}^h &= \frac{w_e^h}{I^h}-\delta_{ke}+\frac{\gamma_{ke}}{\omega_k^h}-\frac{1}{\omega_k^h}\left[\beta_k+\frac{2\lambda_i}{b(\mathbf{w}^h,\mathbf{p})}\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]\times\\
&\quad\left[\alpha_e+\gamma_{ee}\ln w_e^h+\gamma_{ek}\ln w_k^h+\sum_{i=1}^{n}\gamma_{ie}\ln p_i\right]-\frac{\lambda_k\beta_e}{\omega_k^h b(\mathbf{w}^h,\mathbf{p})}\left[\ln\frac{I^h}{a(\mathbf{w}^h,\mathbf{p})}\right]^2.
\end{aligned}
$$

Expressions when there is no secondary worker:

$$
\begin{aligned}
\ln a(\mathbf{w}^h,\mathbf{p}) &\equiv \alpha_0+\alpha_p\ln w_p^h+\sum_{i=1}^{n}\alpha_i\ln p_i\\
&\quad+\ln w_p^h\sum_{i=1}^{n}\gamma_{ip}\ln p_i+\frac{1}{2}\left[\gamma_{pp}\left(\ln w_p^h\right)^2+\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}\ln p_i\ln p_j\right],\\
b(\mathbf{w}^h,\mathbf{p}) &\equiv \left(w_p^h\right)^{\beta_p}\prod_{i=1}^{n}p_i^{\beta_i},\\
\lambda(\mathbf{w}^h,\mathbf{p}) &\equiv \lambda_p\ln w_p^h+\sum_{i=1}^{n}\lambda_i\ln p_i,
\end{aligned}
$$

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