Systemic Risk and Network Formation in the Interbank Market

Ethan Cohen-Cole*
University of Maryland - College Park

Eleonora Patacchini
University of Rome - La Sapienza, EIEF and CEPR

Yves Zenou
Stockholm University, IFN and GAINS

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Abstract

We propose a novel mechanism to facilitate understanding of systemic risk in financial markets. The literature on systemic risk has focused on two mechanisms, common shocks and domino-like sequential default. We provide a new model that draws on the games-on-networks literature. Transmission in our model is not based on default. Instead, we provide a simple microfoundation of banks' profitability based on competition incentives and the outcome of a strategic game. As competitors' loans change, both for closely connected ones and the whole market, banks adjust their own decisions as a result, generating a 'transmission' of shocks through the system. Our approach permits us to measure both the degree that shocks are amplified by the network structure and

*Ethan Cohen-Cole: Robert H Smith School of Business, 4420 Van Munching Hall, University of Maryland, College Park, MD 20742, USA. Email: ecohencole@rhsmith.umd.edu, Tel: +1 (301) 541-7227. Eleonora Patacchini: University of Roma La Sapienza, Box 83 Roma 62, P.le A. Moro 5, I-00185 Rome, Italy. Email: eleonora.patacchini@uniroma1.it, Tel: +39-06-44284 200. Yves Zenou: Stockholm University, Department of Economics, 106 91 Stockholm, Sweden. E-mail: yves.zenou@ne.su.se, Tel: +46-8-16-28-80. We are very grateful to Carefin – Bocconi Centre for Applied Research in Finance for financial support. Cohen-Cole also thanks the CIBER and the US Department of Education for support. All remaining errors are our own.

the manner in which losses and gains are shared. We provide a unique equilibrium characterization of a static model, and embed this model into a full dynamic model of network formation. Because we have an explicit characterization of equilibrium behavior, we have a tractable way to bring the model to the data. Indeed, our measures of systemic risk capture the propagation of shocks in a wide variety of contexts; that is, it can explain the pattern of behavior both in good times as well as in crisis.

Keywords: Financial networks, interbank lending, interconnections, network centrality, spatial autoregressive models.

JEL Classification: G10, C21

1 Introduction

Since the onset of the financial crisis in August of 2007, the discourse about bank safety has shifted strongly from the riskiness of financial institutions as individual firms to concerns about systemic risk.[1] As the crisis evolved, the debate did as well, with concerns about systemic risk growing from too-big-to-fail (TBTF) considerations to too-interconnected-to-fail (TITF) ones. The spectacular collapse of Lehman Brothers in September of 2008 and the subsequent rescue of AIG brought this to the forefront of academic and policy debates.[2]

This paper has two goals. First, to our knowledge, this paper is unique to the finance literature in providing a description of the propagation of financial risk that explicitly models agent incentives and behavior on a network. It is well known and accepted that banks acts strategically given the market and regulatory incentives they face; however, the existing network models in the literature assume that banks do not take into account the optimization problems of other banks in the system. We apply the new methods of optimization in networks (Goyal, 2007; Jackson, 2008; Jackson and Zenou, 2012) to the interbank market with a point-in-time model of homogeneous banks and no defaults.[3] Using these methods, we are able to precisely identify the equilibrium quantity of lending due to the network

[1] Indeed, even prior to the crisis there is a wide range of research on the importance of interbank markets, including some that address the systemic risk inherent to these markets. Some examples include Freixas et al. (2000), Iori and Jafarey (2001), Boss et al (2004), Furfine (2003), Iori et al. (2006), Soramäki et al. (2007), Pröpper et al. (2008), Cocco et al. (2009), Mistrulli (2011), and Craig and Von Peter (2010). Each of these discuss some network properties or discuss the importance of these markets to systemic risk evaluation.

[2] A general perception and intuition has emerged that the interconnectedness of financial institutions is potentially as crucial as their size. A small subset of recent papers that emphasize such interconnectedness or try to explain it include Allen et al., Babus and Carletti (2010), Amini et al. (2010), Cohen-Cole et al. (2011), Boyson et al. (2010), Adrian and Brunnermeier (2009), and Danielsson et al. (2009).

[3] We relax both assumptions towards the end of the paper. The homogeneous agents model does a good job of describing patterns while conveying our methodology. It also greatly reduces mathematical complexity and thus exposition.

structure.

Second, we use this new model to describe a form of systemic risk. Our measure will highlight how the structure of a network can propagate incentives. What we mean by this is that small changes in *uncertainty*, *risk*, or *behavior* can propagate through a network even without defaults. This propagation is well understood from an institutional perspective; what remains is to link this type of phenomena explicitly to network theoretical tools so that these phenomena can be understood structurally. This structural view is important because the exact topology of the network can fundamentally alter incentives, prices, volatility and more. We provide both a measure of total systemic risk, λ, as well as illustrate a method to calculate the contribution of individual banks to this total. Importantly, both of these measures emerge directly from the optimization problem of banks.

We obtain the following results: (i) The proposed method succeeds at characterizing the variation in lending in the European interbank market, both before and after the crisis. The dynamic model is able to explain more than 60% of the change in network structure over time across our period of study. (ii) We find that aggregate systemic risk was relatively constant over time. (iii) the contribution to systemic risk change significantly from 2002 to 2007, reflecting a relatively even distribution at the beginning of the period, and a highly skewed one just before the crisis. (iv) We show that the structure of the networks can have a large influence on the price level and volatility. In a simple example, we illustrate that a hypothetical star-shaped network of 4 banks has double the average transaction price and 30 times the volatility of a similarly sized, circle-shaped network.

With these results in hand, we highlight a number of features of the models and their extensions that have implications for financial policy. Because of the tight link between network structure and market prices / volatility, policymaker knowledge of financial network structure provides a potential tool in ensuring financial stability.

The current financial networks literature is largely based on *random network* and *preferential attachment* models both using models from the applied mathematics and physics literature (Albert and Barabási, 2002; Easley and Kleinberg, 2010; Newman, 2010). The preferential attachment model (Barabási and Albert, 1999) is effectively based on a random network approach since agents form links in a probabilistic way leading to more popular nodes being more likely to be chosen (the so-called rich-get-richer model). To this class of models belongs simplified networks with simple shocks, including models of cascading default. See, for example, Allen and Gale (2000), Herring and Wachter (2001), and Amini et al. (2010). One mechanism to generate systemic risk comes from Herring and Wachter (2001), in which agents are simultaneously impacted by a shock to underlying asset prices. While not a network approach per se, this paper typifies a large body of research which looks at common bank incentives in the face of a shock. The second mechanism is the Allen and Gale (2000) one in which the default of a given entity can lead to a domino-like series of subsequent defaults based on exposures to the defaulting entity. A newer class of model updates the networks approach to specify that links between banks are based on preferential attachment; that is, while links are still random, banks may be more likely to link with banks that have already many links. For example, Allen et al. (2012) illustrate using this approach how the accumulation of exposure to shocks depends on the incentives for individual banks to diversify holdings.

A key emphasis in our paper is that we will extend the current literature on systemic risk to include the *strategic interactions* of banks in a network. We highlight in our model how the integration of strategic action in finance networks produces distinct results from the other methods. Most importantly, building up from first principles, it shows how incentives and uncertainty can propagate through financial networks, thus generating systemic risk. This occurs even in the absence of defaults or risk. A key feature of our model is the existence of a unique equilibrium outcome of bank lending behavior for any network pattern. This uniqueness allows us to directly estimate from an analog to the first order condition. Indeed, the model can capture the propagation of shocks in a wide variety of contexts; that is, it can explain the pattern of behavior both in good times as well as in crisis.

We take the modeling exercise another step forward. As shocks hit a system, the existing pattern of network links will evolve. As such, reduced form and/or static models of systemic risk may be insufficient for understanding the importance of interconnectedness on financial markets. With this in mind, we explicitly embed our static model into a complete dynamic model of network formation. Thus, we are able to characterize not only the equilibrium pattern of behavior at each point in time, but also how this behavior evolves over time. As banks form and break links, the structure of the network will change, and the nature of systemic risk with it. Our model is useful in that we can discuss how systemic events emerge even in the absence of defaults (e.g. runs on the bank, flights to quality, etc.).

Once we have developed the static and dynamic models and shown their ability to match the empirical patterns in European interbank data, we provide a set of extensions. In one case, we show that because the structure of the network impacts incentives, whether capital requirements bind for a given bank will depend on market structure. Once we separate the portion of lending due to network structure, we can show that an identical bank can be capital constrained in one network and not constrained in another. This finding suggests an alternate route to bank regulatory policy.

While our core model is developed with homogeneous agents and no default in order to illustrate the ability of the techniques to match the data, we extend the model to include ex-ante heterogeneity and a bank-specific risk premium. This allows us to derive bank-specific loan prices. We illustrate that the model has the Nash equilibrium form as in the homogeneous agent, zero default case, such that the rest of our results follow accordingly.

2 Stylized facts and Interbank Lending

A now widely discussed feature of the banking system is the presence of an interbank lending market. Bank balance sheets are typically composed of loans on the asset side and deposits (plus equity) on the liability side.

[*Insert Figure* 1 *here*]

Regularly, the natural businesses of banks leads to higher loans or deposits on a given day. These imbalances can be rectified in the short term through the interbank market. For example, a bank with $1100 in loans, $900 in deposits and $100 in equity can use the interbank market to borrow an additional $100 to fully fund its balance sheet. Towards the end of day, the treasury department of a bank seeks to find available funds, or lend excess deposits. When the interbank market was very liquid, some banks used the market to fund a large portion of their balance sheet, effectively relying on the presence of the market in each subsequent day. Instead of collecting deposits, a bank could simply issue loans and fill the liability side of the balance sheet with interbank deposits.

When the crisis arrived, a combination of credit quality fears and liquidity shortages led to difficulties in the interbank market (Afonso et al., 2011). US and European central banks intervened at various points in time to ensure that banks would have continued access to funding. The Federal Reserve began this effort with the TAF in December of 2007. Eventually, the Federal Reserve created a wide variety of related programs and the European Central Bank moved on October 15 of 2008 to a 'full-allotment' policy in which it provided unlimited credit to banks in the euro area.

For our purposes, the key question is the degree to which the network features of the interbank market are important in determining access, profitability and liquidity. It has been widely acknowledged that the markets are not complete networks; many banks would establish relationships with other banks either through repeated transactions or through commitments to future lending. While banks in crisis will call around to look for additional liquidity, the established lending relationships are a primary source of funding. As an example, many banks during normal times would simply roll-over existing loans at expiry.

We use transaction level data on interbank lending from an electronic interbank market. The e-MID SPA (or e-MID) was the reference marketplace for liquidity trading in the Euro area during the time period studied. It was the first electronic marketplace for interbank deposits (loans), a market that has traditionally been conducted bilaterally. Our data includes every interbank loan transaction conducted on the e-MID during the time period from January 2002 to December 2009. During this time period, transactions on this exchange represented about 17 percent of the Euro area market. As such, during this time period, it served as a good representation of general market activity. Indeed, the 2008 Euro Money Market Study published by the ECB in February of 2009 confirmed that e-MID prices tracked the Euro overnight index average (EONIA) closely until the crisis started in August of 2007 (Euro Study 2009).

The e-MID market is an open-access one. All banks in the European interbank market can participate. The market opens at 8am and closes at 6pm Central European Time. Both bids and asks can be posted on the exchange along with a price and quantity. Each trader may decide to initiate a transaction with any of the counterparties present on the book. Once a trader chooses the transaction, the two parties bilaterally negotiate the trade. The benefit of the bilateral negotiation is that it allows each party the ability to refuse the transaction, change the quantity and/or the price. Such bilateral negotiation allows banks to maintain lending limits for each specific counterpart. Outside of e-MID, banks privately negotiate lines of credit (liquidity guarantees) with each other and conduct regular transactions with each other based on these lines. As a result, e-MID can support the continuation of the bilateral lending arrangements without forcing banks to accept / give loans outside their prior guidelines.

Table 1 reports descriptive statistics for the e-MID market. We report the average daily volume for overnight and total lending. As well, we include the proportion of lending made by the 25 largest market participants, which averages about 20% prior to the crisis and 5-10% after. In addition to total volumes dropping, the market shifts from being highly centralized to considerably less so after the crisis. This finding is consistent with our estimation of the role of centrality over time; we find below that the importance of being central declines after the crisis.

[*Insert Table* 1 *here*]

We also show in Figure 2, the daily volume of lending for overnight and long-term loans. This shows the stylized patterns of lending in this market. Notice that lending volume of long-term loans drops precipitously beginning with the onset of the crisis in August of 2007. Both overnight and long-term loans decline following the beginning of the ECB full allotment policy in October of 2008. Our model and empirical analysis will seek to explain some of the variation in lending quantities during this volatile time period.

[*Insert Figure* 2 *here*]

In Figure 3 we report the daily price volatility, as the standard deviation of prices. The reverse pattern is observed here. After a long period with relatively low volatility dating back to the beginning of 2002, the onset of the crisis saw price volatility increase dramatically. Two significant increases are apparent, in August of 2007 and in October of 2008. Towards the end of the paper, we will illustrate that the network models we use provide a mapping from network structure to price level and price volatility.

Each of these two figures shows the daily value as well as a two-month moving average.

[*Insert Figure* 3 *here*]

Our data includes approximately 250 institutions that participated in at least one transaction during the time period. Loans in the database range from overnight to two years in length, though about 70% of the loans are for overnight alone. Our information spans 945,566 loans of all types, of which 752,901 are overnight loans. We will focus on overnight loans for simplicity.[4]

The principal purpose of the interbank market is provide a mechanism for banks to reallocate deposit imbalances. For larger shocks or gross liquidity needs, most institutions borrow directly from the ECB.

[4] Results of similar models estimated with longer term loans as well is available on request from the authors.

3 Static Model

3.1 Notation and model

We begin with a simple static model of a bank whose balance sheets is given in Figure 1. On the asset side of the balance sheet, we include cash, loans and interbank loans. On the liability side: deposits, interbank borrowing and equity. Our primary object of interest will be either interbank loans or interbank deposits. In addition, we specify a basic leverage constraint for each bank as:

$$\xi \, e_i \geq assets$$

where e_i is the equity of bank i and ξ is the leverage constraint. For simplicity, we group cash and loans into a single variable X_i (i.e. loans + cash = X_i). Then we can write that interbank loans at each point in time must satisfy two criterion. One, given a value for liabilities and for X_i,

$$q_i = liabilities_i - X_i$$

The equality conditions simply means that banks must match assets and liabilities. The assumption that interbank loans are the remaining choice on the balance sheet reflects the nature of this market. Precise deposits balances are determined by customer preferences, loans are typically much longer maturity and cannot be underwritten or sold on a moment's notice with any reliability, and equity takes weeks or months to issue. Thus, in the perspective of a day or two, one of the only free variables for a bank to clear its balance sheet is the interbank market.[5]

[5] We abstract for now from the ability to borrow from the central bank. This is an alternate mechanism to match the balance sheet. However, this type of borrowing typically comes at a penalty rate. We return to penalty rate borrowing in the policy section at end.

Two, the leverage constraint requires that

$$\xi\, e_i \geq X_i + q_i \tag{1}$$

This reflects the fact that banks cannot lend more funds than some multiple of their equity. In the time period we address, European banks were not bound explicitly by a leverage constraint. However, Basel capital constraints formed a type of upper bound on the quantity of lending possible. We include this feature particularly because new Basel III regulations explicitly discuss additional capital requirements for Systemically Important Financial Institutions (SIFIs). The borrowing side obviously has no such constraint.

As we develop the model, two key features will emerge: *global strategic substitutability* and *local strategic complementarities*. These will show that, as total quantities in the market increase, prices will fall. However, at the local level, between two agents, there will be an incentive to increase prices when quantities increase due to the complementarity effect. The model will find an equilibrium where these effects are balanced.

To our knowledge, the fact that we incorporate both local and global components is unique to the financial networks literature; by incorporating both the direct network influences as well as the system-wide effects, our model is particularly suited to the description of financial markets. These markets are influenced both by prices (global) and as well by network impacts (local).

We look at a population of banks. We define for this population a *network* $g \in \mathcal{G}$ as a set of ex-ante identical banks $N = \{1, \ldots, n\}$ and a set of *links* between them. We assume at all times that there are least two banks, $n \geq 2$. Links in this context can be defined in a variety of ways. In other work, they have represented the exchange of a futures contract (Cohen-Cole et al., 2011). In the banking networks that we study, the links will represent the presence of a interbank loan.

The set of bank i's direct links is: $N_i(g) = \{j \neq i \mid g_{ij} = 1\}$. The cardinality of this set is denoted by $n_i(g) = |N_i(g)|$.[6] The $n-$square adjacency matrix $\mathbf{G}$ of a network g keeps track of the direct connections in this network. By definition, banks i and j are directly connected in g if and only if $g_{ij} = 1$, (denoted by ij), and $g_{ij} = 0$ otherwise. Links are taken to be reciprocal, so that $g_{ij} = g_{ji}$ (*undirected* graphs/networks). By convention, $g_{ii} = 0$. Thus $\mathbf{G}$ is a *symmetric* $(0,1)-$matrix. All our theoretical results hold with directed (i.e. $g_{ij} \neq g_{ji}$) and weighted networks, which would imply that $\mathbf{G}$ is an *asymmetric* matrix with weights between 0 and 1.

We hypothesize that these direct links produce some type of reduction in costs of the collaborating banks. For example, as the size of the loan increases, the cost per dollar of loan is reduced for both parties to the loan. It is a straightforward assumption that the operational costs of a trading floor or treasury operation decline per dollar of loan as loan size increases.

We will model the quantity choice based on competition in quantities of lending a la Cournot between n banks with a single homogenous product (a loan). We will then look at quantities of borrowing on the same market. This distinction is useful for three reasons. First, it allows us to look separately at what happens to each side of the market. As will be apparent below, the two markets evolve differently during the 8 years we study. Second, it allows us to use well-established competition frameworks, such as Cournot. These are based on the idea of a group of firms competiting for customer business. Looking only at one side of the market allows this view. Third, looking at each side of the market captures the fact that we need gross lending amounts to understand competitive forces. If a bank borrows

[6] Vectors and matrices will be denoted in bold and scalars in normal text.

\$100 and lends \$99, to understand the network, we need to know both quantities; a netted \$1 borrowing does not capture the complexity and scale of the interactions. Recall that we will precisely identify who lends and borrows from which other banks.

We assume the following standard linear inverse market demand where the market price is given by:

$$p = \theta - \sum_{j \in N} q_j \tag{2}$$

where $\theta > 0$. The marginal cost of each bank $i \in N$ is $c_i(g)$. The profit function of each bank i in a network g is therefore given by:

$$\begin{aligned} \pi_i(g) &= pq_i - c_i(g)q_i \\ &= \theta q_i - \sum_{j \in N} q_i q_j - c_i(g)q_i \end{aligned}$$

where q_i is the loan quantity produced by bank i. We assume throughout that θ is large enough so that price and quantities are always strictly positive.

Our specification of inter-related cost functions is as follows. The cost function is assumed to be equal to:

$$c_i(g) = c_0 - \phi \left[\sum_{j=1}^{n} g_{ij} q_j \right] \tag{3}$$

where $c_0 > 0$ represents a bank's marginal cost when it has no links while $\phi > 0$ is the cost reduction induced by each link formed by a bank. The parameter ϕ could be bank specific as well so that ϕ_i, but for simplicity of notation, we do not report this case.

Equation (3) means that the marginal cost of each bank i is a decreasing function of the quantities produced by all banks $j \neq i$ that have a direct link with bank i. As stated above, this is because the operational costs of a trading floor or treasury operation decline per dollar of loan as loan size increases. This is the specification that drives the functional relationships between banks.

To ensure that we obtain a reasonable solution, we assume that c_0 is large enough so that $c_i(g) \geq 0$, $\forall i \in N$, $\forall g \in \mathcal{G}$. The profit function of each bank i can thus be written as:

$$
\begin{aligned}
\pi_i(g) &= pq_i - c_i(g)q_i \\
&= \theta q_i - \sum_{j \in N} q_i q_j - c_0 q_i + \phi \sum_{j=1}^{n} g_{ij} q_i q_j \\
&= aq_i - q_i^2 - \sum_{j \neq i} q_i q_j + \phi \sum_{j=1}^{n} g_{ij} q_i q_j \qquad (4)
\end{aligned}
$$

where $a \equiv \theta - c_0 > 0$.

We highlight a few features of equation (4). First, we can see that profits are a negative function of total loans. This we call *global strategic substitutability,* as the effect operates only through the market and not through the direct links that form the network. So as q_j increases, $\frac{\partial \pi_i(g)}{\partial q_i}$ is reduced as demand falls.

Second, we can see that profit is increasing in the quantity of direct links, via the cost function impact. This we refer to as *local strategic complementarities* since if j is linked with i, then if q_j increases $\frac{\partial \pi_i(g)}{\partial q_i}$ is increased because of the reduction in the cost. Total profits are of course, dependent on the two jointly.

Third, we can define σ_{ij} as the cross partial of profitability with respect to a bank's quantity change and another bank's quantity change. We have:

$$
\sigma_{ij} = \frac{\partial^2 \pi_i(g)}{\partial q_i \partial q_j} = \begin{cases} \overline{\sigma} = -1 + \phi \sum_{j=1}^{n} g_{ij} q_j & \text{if} \quad g_{ij} = 1 \\ \underline{\sigma} = -1 & \text{if} \quad g_{ij} = 0 \end{cases} \qquad (5)
$$

so that $\sigma_{ij} \in \{\underline{\sigma}, \overline{\sigma}\}$, for all $i \neq j$ with $\underline{\sigma} \leq 0$.

This last feature highlights the mechanism of the model. A shock to a connected bank changes the incentives of a bank to lend, precisely through the function (5). Notice that the model generates systemic risk insofar as shocks that impact a given bank, such as an exogenous decrease in capital and ability to lend, pass through to the rest of the market through a competition mechanism. The global effect of the reduction in lending by a single bank is an increase by others. The local effect, however, that passes through the network linkages, is that costs increase. As a result, loans volumes of direct network links decline as well. Once network links change their choices, their links do so as well, and so on.

3.2 Equilibrium loans

Consider a Cournot game in which banks chose a volume of interbank lending conditional on the actions of other banks. This game requires common knowledge of the actions of other banks. We describe below that our data will allow this assumption; all bid and asks are posted on the system. We expand the standard game to fit the model above. Agents have the defined profit function in (4), which implies that cost is intermediated by the network structure.

It is easily checked that the first-order condition for each bank i is given by:

$$q_i^* = \frac{1}{2}a - \frac{1}{2}\sum_{j \neq i} q_j + \frac{1}{2}\phi \sum_{j=1}^{n} g_{ij} q_j \tag{6}$$

Formally, we show below that this game has a unique Nash equilibrium.

We use a network centrality measure due to Katz (1953), and latter extended by Bonacich (1987), that proves useful to describe the equilibrium of our network model.

The Katz-Bonacich network centrality The Bonacich centrality will provide a measure of direct and indirect links in the network. Effectively, a relationship between two banks is not made in isolation. If bank A lends money to bank B, and bank B already lends to bank C, the strategic decisions of bank A will depend, in part on the strategic decisions of B. Of course, B's decisions will also be a function of C's. The Katz-Bonacich measure will help keep track of these connections and, as we will see in the subsequent section, has a natural interpretation in the Nash solution. Denote by $\omega(\mathbf{G})$ the largest eigenvalue of $\mathbf{G}$.

Definition 1 *Consider a network g with adjacency $n-$square matrix $\mathbf{G}$ and a scalar ϕ such that $\mathbf{M}(g,\phi) = [\mathbf{I}-\phi\mathbf{G}]^{-1}$ is well-defined and non-negative. Let $\mathbf{1}$ be the $n-$dimensional vector of ones. Then, if $\phi\omega(\mathbf{G}) < 1$, the Katz-Bonacich centrality of parameter ϕ in g is defined as:*

$$\mathbf{b}(g,\phi) = \sum_{k=0}^{+\infty} \phi^k \mathbf{G}^k \mathbf{1} = [\mathbf{I}-\phi\mathbf{G}]^{-1}\mathbf{1} \tag{7}$$

where $\phi \geq 0$ is a scalar and $\mathbf{1}$ is a vector of one.

An element i of the vector $\mathbf{b}(g,\phi)$ is denoted by $b_i(g,\phi)$. For all $\mathbf{b}(g,\phi) \in \mathbb{R}^n$, $b(g,\phi) = b_1(g,\phi) + ... + b_n(g,\phi)$ is the sum of its coordinates. We provide additional description in Appendix 1. Observe that, by definition, the Katz-Bonacich centrality of a given node is zero when the network is empty and is greater than 1 if the network is not empty. It is also null when $\phi = 0$, and is increasing and convex with ϕ.

We now characterize the Nash equilibrium of the game.

Proposition 1 *Consider a game where the profit function of each bank i is given by (4). Then this game has a unique Nash equilibrium in pure strategies if and only if $\phi\omega(\mathbf{G}) < 1$. This equilibrium $\mathbf{q}^*$ is interior and given by:*

$$\mathbf{q}^* = \frac{a}{1 + b(g,\phi)} \mathbf{b}(g,\phi) \tag{8}$$

This result is a direct application of Theorem 1 in Ballester et al. (2006). Appendix 1 shows in more detail how the first order condition can be written as a function of Katz-Bonacich centrality. It also provides an example.

This solution is useful for a couple of reasons: One, notice that this equation provides a closed form solution to the game with any number of banks and to calculate output, only the matrix of interconnections $\mathbf{G}$ and the bank specific cost functions are needed. Two, this equation provides the basis for estimation of any network linked bank decision. We explore this implication below in more detail.

We can now calculate the equilibrium profit of each bank by replacing the equilibrium value of q_i^* into the profit function (4). It is easily verified that we obtain:

$$\pi_i^* = (q_i^*)^2 = \frac{a^2 b_i^2 (g, \phi)}{[1 + b(g, \phi)]^2} \tag{9}$$

so that the profit function of each bank is an *increasing* function of its Bonacich centrality.

A key parameter in this equilibrium result is ϕ, the coefficient in the equation (7) that measures how much of a shock to agents is passed on to connected agents. We estimate the coefficient below ϕ. Here we illustrate that ϕ is a multiplier and, in our context, is a measure of *systemic risk* that propagates risk through incentives. Consider the same n banks but without a network (i.e. $\phi = 0$) so that there are no links (or loans) between them and $c_i = c_0$. In that case, the profit of each firm is given by:

$$\pi_i = \theta q_i - \sum_{j=1}^{n} q_i q_j - c_0 q_i$$

The Nash equilibrium is such that:

$$q_i^* = a - \sum_{j=1}^{n} q_j^*$$

where $a \equiv \theta - c_0$. Summing the n first-order conditions, we obtain:

$$q^{NO*} = \left(\frac{n}{1+n} \right) a \tag{10}$$

where $q^* = \sum_{j=1}^{n} q_j^*$, so that

$$q_i^{NO*} = \frac{a}{1+n} \tag{11}$$

Comparing this result with the network case, i.e. (6) and some additional math, shown in Appendix 2, finds an additional term, which is a positive function of ϕ. Indeed,

$$q_i^{NET*} = \underbrace{\frac{a}{(1+n)}}_{\text{quantity produced with no network}} + \underbrace{\frac{\phi}{(1+n)}\left[n\sum_{j=1}^{n} g_{ij}q_j^* - \sum_{k\neq i}^{n}\sum_{j=1}^{n} g_{kj}q_j^*\right]}_{\text{extra quantity due to multiplier effects}}$$

The intuition is clear. Total output is higher with networks than without networks and the difference is measured as $q^{NET*} - q^{NO*} = \left(\frac{\phi}{1+n}\right)\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}q_j^* > 0$. In other words, total output increases by this value when network effects are present. This implies that prices of loans are much lower with networks since $p^{NO*} = p^{NET*} + \left(\frac{\phi}{1+n}\right)\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}q_j^*$, which creates even more interactions (i.e. loans). As a result, profits are much higher with networks.

To better understand the multiplier effect due to networks, consider the case of two banks A and B ($n = 2$). Assume first that there is no network (i.e. $\phi = 0$) so that no bank gives a loan to the other. In that case, using (11), each bank will produce

$$q^{NO*} = q_A^{NO*} = q_B^{NO*} = \frac{a}{3}$$

Consider now the simplest possible network, that is each bank gives loans to the other bank, i.e., $g_{12} = g_{21} = 1$. The adjacency matrix is:[7]

$$\mathbf{G} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

[7] There are two eigenvalues: $1, -1$ and thus $\omega(\mathbf{G}) = 1$. Thus the condition from Proposition 1, $\phi\omega(\mathbf{G}) < 1$, is now given by: $\phi < 1$.

We easily obtain:

$$\mathbf{b}(g,\phi) = \frac{1}{(1-\phi)}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

and thus the unique Nash equilibrium is given by:

$$\mathbf{q}^* = \frac{a}{1+\frac{2}{1-\phi}}\mathbf{b}(g,\phi) = \frac{a}{3-\phi}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

that is

$$q^{NET*} = q_A^{NET*} = q_B^{NET*} = \frac{a}{3-\phi}$$

Since $\phi < 1$, then this solution is always positive and unique and

$$q^{NET*} = \frac{a}{3-\phi} > \frac{a}{3} = q^{NO*}$$

In fact, we have:

$$q^{NET*} = q^{NO*} + \frac{a\,\phi}{3(3-\phi)}$$

or equivalently

$$q^{NET*} = \frac{3}{(3-\phi)}q^{NO*}$$

In this example, the multiplier is equal to $3/(3-\phi) > 1$. One can see that this multiplier increases in ϕ so that the higher is ϕ, the higher is the quantity of loans that will be given to each bank. This means, in particular, that if there is a shock to this economy, ϕ, the *systemic risk*, will propagate the risk at a factor $3/(3-\phi)$.

3.3 Equilibrium prices

One of the powerful features of the model is that it provides a structural link between the network pattern and the equilibrium market price for interbank loans. Changing the network structure changes equilibrium prices. This can be observed through two features of the model. First, the equilibrium quantity for each bank is expressed precisely in (8). As the sum of these quantities change, the global effects will be to influence prices as in any market. This is what we labeled *global strategic substitutability*, above. Two, the individual patterns of links in the network will influence the local loan decisions. This *local strategic complementarities,* also influences aggregate prices.

To be more precise, using the linear inverse market demand (2) and (8), we obtain the following equilibrium price of loan transactions:

$$p^* = \theta - \sum_{j\in N} q_j^* = \theta - \frac{b(g,\phi)}{1+b(g,\phi)} \tag{12}$$

Example 1 Consider the two following directed networks:

[*Insert Figure* 4 *here*]

The network on the left panel is a circle (and its adjacency matrix is denoted by $\mathbf{G}_C$) while the network on the right panel is a star (and its adjacency matrix is denoted by $\mathbf{G}_S$). We have:[8]

$$\mathbf{G}^C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{G}^S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

where the first row corresponds to bank A, the second row to bank B, etc. Observe that we have a directed network (since loans are by definition directed) and thus the adjacencies matrices are asymmetric. We focus on *outdegrees* only, i.e. links (i.e. loans) that go from one bank to the other one. In other words, we analyze the *lending* market. The two networks have the same total number of banks (4) and have the same total numbers of loans (5) but have a different structure.

[8] The largest (noncomplex) eigenvalue for $\mathbf{G}^C$ is 1 and for $\mathbf{G}^S$, it is also 1. As a result, the eigenvalue condition $\phi\omega(\mathbf{G}) < 1$ is $\phi < 1$ for both networks.

In this framework, the Bonacich centrality is a measure of "popularity" since the most central bank (i.e. node) is the one who gives the higher number of loans (i.e. links). Fortunately, the symmetry of the adjacency matrix does not play any role in the proof of Proposition 1 and thus the results are true for both directed and undirected networks. We can see here how prices vary as a function of even relatively small changes in network structure. Using Proposition 1, the unique Nash equilibrium is given by:

$$\begin{bmatrix} q_A^{C*} \\ q_B^{C*} \\ q_C^{C*} \\ q_D^{C*} \end{bmatrix} = \frac{(\theta - c_0)}{5 + 5\phi + 6\phi^2 + 2\phi^3 - \phi^4} \begin{bmatrix} 1 + \phi + 2\phi^2 + \phi^3 \\ 1 + 2\phi + 2\phi^2 + \phi^3 \\ 1 + \phi + \phi^2 \\ 1 + \phi + \phi^2 + \phi^3 \end{bmatrix}$$

for the circle network and

$$\begin{bmatrix} q_A^{S*} \\ q_B^{S*} \\ q_C^{S*} \\ q_D^{S*} \end{bmatrix} = \frac{(\theta - c_0)}{5} \begin{bmatrix} 1 \\ 1 + \phi \\ 1 \\ 1 \end{bmatrix}$$

for the star-shaped network. The circle network where bank B has the highest Bonacich centrality (and thus gives the highest loans quantities) is less symmetric than the star-shaped one. Bank A is the second most active bank because it gives a loan to B. Then come bank D and then bank C. On the contrary, for the star-shaped network, banks A, C, and D give the same loan quantities while bank B has the highest Bonacich centrality. This is because they all lend to the same bank D.

What is interesting here is the impact of network structure on the aggregate equilibrium price of loans. In the circle network, each loan is priced at

$$p^{C*} = \frac{\left(1 - \phi^3 - \phi^4\right)\theta + (4 + 5\phi + 6\phi^2 + 3\phi^3)c_0}{5 + 5\phi + 6\phi^2 + 2\phi^3 - \phi^4}$$

where θ is the market demand from equation (2) and ϕ is the coefficient in the equation (7).

For the star-shaped network, we obtain:

$$p^{S*} = \frac{(1-\phi)\,\theta + (4+\phi)\,c_0}{5}$$

As a result, with four banks A, B, C and D, depending on the network structure, the price for loans can differ. Indeed, it is easily verified that $p^{S*} > p^{C*}$. This reflects the fact that the star-shaped network induces less competition and thus less loan output than the star-shaped network.

3.4 Equilibrium behavior with leverage constraints

Remember that we have a leverage constraint given by (1). We need to check that the Nash equilibrium satisfies this condition. Define

$$\overline{q}_i \equiv \xi\, e_i - X_i$$

Since ξ, e_i and X_i are purely exogenous variables, we consider three possibilities. The first is that no banks are constrained by the leverage constraint, i.e., the equilibrium quantity of loans q_i^*, defined by (8), is such that $q_i^* \leq \overline{q}_i$, for all $i = 1, ..., n$. In that case, all banks play the Nash equilibrium described above and give a quantity of loans q_i^* defined by (8). The second is that all banks are constrained so that $q_i^* > \overline{q}_i$, for all $i = 1, ..., n$. In that case, the equilibrium is such that all banks provide loans equal to $q_i^* = \overline{q}_i \equiv \xi\, e_i - X_i$. Finally, there is an intermediary case for which some banks are constrained by the leverage constraint and some are not. To characterize this equilibrium, we rank banks by their position in the network, i.e. their Bonacich centrality. Then, those for which $q_i^* \leq \overline{q}_i$ will give loans equal to q_i^* defined by (8) while those for which $q_i^* > \overline{q}_i$, which have $q_i^* = \overline{q}_i \equiv \xi\, e_i - X_i$.

This difference between constrained and unconstrained suggests that increasing the fraction of constrained banks leads to a smaller fraction of banks propagating incentives through the network. This can have a range of potential impacts on total systemic risk and the allocation of risk in the system. Indeed, when ξ increases, more and more banks are constrained in their loan possibilities and are more likely to hit the leverage constraint so that $q_i^* = \overline{q}_i \equiv \xi\, e_i - X_i$. Interestingly, this depends on the network structure so that the same bank with the same leverage constraint can behave differently depending on the network it belongs to. Consider again Example 1 (Section 3.3) and assume for bank C that[9]

$$\frac{(\theta - c_0)\left(1+\phi+\phi^2\right)}{5+5\phi+6\phi^2+2\phi^3-\phi^4} < \overline{q}_C < \frac{(\theta - c_0)}{5}$$

This implies that $q_C^{S*} > \overline{q}_C$ and $q_C^{C*} < \overline{q}_C$ and thus, in equilibrium,

$$q_C^{C*} = \frac{(\theta - c_0)\left(1+\phi+\phi^2\right)}{5+5\phi+6\phi^2+2\phi^3-\phi^4} \text{ and } q_C^{S*} = \overline{q}_C \equiv \xi\, e_C - X_C$$

In other words, the same bank C in the circle network will produce its Nash equilibrium quantity of loans while, in the star-shaped network, will hit the leverage constraint and will lend loans so that $q_C^{S*} = \overline{q}_C$. This is true for a given ξ. When ξ increases then banks are more likely to hit the leverage constraint and will not give their "optimal" (i.e. Nash equilibrium) quantity of loans.

4 Dynamic Model

In this section, we extend the model of Section 3 to include strategic link formation amongst banks. This step is crucial in that it permits us to include in our analysis not only the quantity and price choices amongst banks conditional on their existing network, but also

[9] Observe that, since $\phi < 1$, we have:

$$\frac{(\theta - c_0)\left(1+\phi+\phi^2\right)}{5+5\phi+6\phi^2+2\phi^3-\phi^4} < \frac{(\theta - c_0)}{5}$$

their decisions on how to change the network structure itself. The model here will show the equilibrium outcome network structure conditional on these strategic choices. Such a model gives us the ability to validate that our static model results are reasonable insofar as they are not contradicted by strategic network formation incentives. It also allows us to investigate how strategic behavior can impact network structure and liquidity availability.

Our central modeling assumptions will be that links are formed based on the profitability tradeoff that emerges from the game in the static model. Effectively, banks know that the game will be played in the subsequent period and that all other banks are also making network formation decisions. Based on these, banks can choose whether or not to form a link; that is, to make a loan to a new customer. We will also specify an exogenous probability of link formation, α.[10]

To describe the network formation process we follow König et al. (2010a). Let time be measured at countable dates $t = 1, 2, ...$ and consider the network formation process $(G(t))_{t=0}^{\infty}$ with $G(t) = (N, L(t))$ comprising the set of banks $N = \{1, ..., n\}$ together with the set of links (i.e. loans) $L(t)$ at time t. We assume that initially, at time $t = 1$, the network is empty. Then every bank $i \in N$ optimally chooses its quantity $q_i \in \mathbb{R}_+$ as in the standard Cournot game with no network. Then, a bank $i \in N$ is chosen at random and with probability $\alpha \in [0, 1]$ forms a link (i.e. loan) with bank j that gives her the highest payoff. We obtain the network $G(1)$. Then every bank $i \in N$ optimally chooses its quantity $q_i \in \mathbb{R}_+$, and the solution is given by (8). The profit of each bank is then given by (9) and only depends on its Bonacich centrality, that is its position in the network. At time $t = 2$, again, a bank is chosen at random and with probability α decides with whom she wants to

[10] While we don't discuss in detail, this asssumption can be relaxed in a number of ways. For example, König et al. (2010b) show that a capacity constraint, what this model would interpret as a capital constraint, generates similar network patterns.

form a link while with probability $1-\alpha$ this bank has to delete a link if she has already one. Because of (9), the chosen bank will form a link with the bank that has the highest Bonacich centrality in the network. And so forth.

As stated above, the randomly chosen bank does not create or delete a link randomly. On the contrary, it calculates all the possible network configurations and chooses to form (delete) a link with the bank that gives her the highest profit (reduces the least her profit). It turns out that connecting to the bank with the highest Bonacich centrality (deleting the link with the agent that has the lowest Bonacich centrality) is a best-response function for this bank. Indeed, at each period of time the Cournot game described in Section 3 is played and it rationalizes this behavior since the equilibrium profit is increasing in her Bonacich centrality (see 9).

To summarize, the dynamics of network formation is as follows: At time t, a bank i is chosen at random. With probability α bank i creates a link to the most central bank while with complementary probability $1-\alpha$ bank i removes a link to the least central bank in its neighborhood.

Characterization of equilibrium We would like to analyze this game and, in particular, to determine, in equilibrium, how many links banks will have. More importantly, we would like to describe the entire distribution of links for banks in the network. This degree distribution gives the percentage of banks with number of links (degree) $d = 1, ..., n$. Recall that the decision to add or delete a node is made based on bank optimization decisions that emerge from our static model.

Our results follow the work in König et al. (2010a) who show that, at every period, the emerging network is a *nested split graph* or a *threshold network*, whose matrix representation is stepwise. This means that agents can be rearranged by their degree *rank* and, conditional on degree $d \neq 0$, agents with degree d are connected to all agents with degrees larger than d. Moreover, if two agents i, j have degrees such that $d_i < d_j$, this implies that their neighborhoods satisfy $\mathcal{N}_i \subset \mathcal{N}_j$. Below, we will show how closely the theoretical patterns implied by this model are replicated in the data.

Denote by $N(d,t)$ the number of agents with degree $d \leq K/2$ at time t. It can be shown that the dynamic evolution is given by:

$$N(d,t'+1) - N(d,t') = \left(\frac{1-\alpha}{n}\right) N(d+1,t') + \frac{\alpha}{n} N(d-1,t') - \frac{1}{n} N(d,t') \tag{13}$$

$$N(0,t'+1) - N(0,t) = \left(\frac{1-2\alpha}{n}\right) - \frac{\alpha}{n} N(0,t) + \left(\frac{1-\alpha}{n}\right) N(1,t) \tag{14}$$

These equations mean that the probability to add nodes to banks with degree d is proportional to the number of nodes with degree $d-1$ (resp. $d+1$) when selected for node addition (deletion). The dynamics of the adjacency matrix (and from this the complete structure of the network) can be directly recovered from the solution of these equations.

Since the complement $\overline{G}$ of a nested split graph G is a nested split graph, we can derive the stationary distribution of networks for any value of $1/2 < \alpha < 1$ if we know the corresponding distribution for $1-\alpha$. With this symmetry in mind we restrict our analysis in the following to the case of $0 < \alpha \leq 1/2$. Let $\{N(t)\}_{t=0}^{\infty}$ be the degree distribution with the d-th element $N_d(t)$, giving the number of nodes with degree d in $G(t)$, in the t-th sequence $N(t) = \{N_d(t)\}_{d=0}^{n-1}$. Further, let $n_d(t) = N_d(t)/n$ denote the proportion of nodes with degree d and let $n_d = \lim_{t\to\infty} \mathbb{E}(n_d(t))$ be its asymptotic expected value (as given by μ). In the following proposition (König et al., 2010a), we determine the asymptotic degree distribution of the nodes in the independent sets for n sufficiently large.

Proposition 2 *Let $0 < \alpha \leq 1/2$. Then the asymptotic expected proportion n_d of nodes in the independent sets with degrees, $d = 0, 1, ..., d^*$, for large n is given by*

$$n_d = \frac{1-2\alpha}{1-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^d, \tag{15}$$

where

$$d^*(n, \alpha) = \frac{\ln\left(\frac{(1-2\alpha)n}{2(1-\alpha)}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}.$$

These equations precisely define the equilibrium degree distribution in the interbank market. In the empirical section below, we will test the correspondence to the observed empirical distribution in the European interbank market.

5 Empirical analysis

5.1 Empirical model and identification

We begin by defining a network of banks. Banks conduct transactions with other banks nearly continuously; as such, we make an assumption about what defines a network. Since the vast majority of transactions are overnight transactions and banks use the interbank loan market for rectifying deposit imbalances, one can surmise that a reasonable network is characterized by the transactions that occur in a short time frame. A one-day time period is a natural time period to start with. That said, many overnight interbank loans are rolled over the following day. While the lending bank typically has the option to withdraw funding, the persistence in relationships implies that the networks that determine lending choices may be slightly longer than a day. We will use one day as a benchmark measure of networks.[11]

[11] We have conducted extensive sensitivity analysis on this assumption that is available on request from the authors. Slightly moving the time window used to define a network does not change the substance of our results.

Further supporting the use of a short-term network measure, the model above takes the interbank lending/borrowing decision as the tool to balance the bank's assets and liabilities, taking the remainder of the balance sheet as given. Once we consider other assets and liabilities with longer maturies, alternate network measures may be important.

Assume that there are K networks in the economy, defined by the number of days. Each network contains n_k banks. We can then estimate the direct empirical counterpart of the first-order condition in the static model above, equation (6):

$$q_{i,\kappa} = c + \phi \frac{1}{g_{i,k}} \sum_{j=1}^{n_\kappa} g_{ij,\kappa} q_{j,\kappa} + \upsilon_{i,\kappa}, \qquad \text{for } i = 1, ..., n_\kappa \ ; \ \kappa = 1, ..., K. \tag{16}$$

where $c = \frac{1}{2}a - \frac{1}{2}\sum_{j=1, j\neq i}^{n} q_j$. This equation indicates that the equilibrium quantity choice of a bank is a function of quantity choices of others in the same market. We denote as $q_{i,\kappa}$ the lending or borrowing of bank i in the network k, $g_{i,k} = \sum_{j=1}^{n_\kappa} g_{ij,\kappa}$ is the number of direct links of i, $\frac{1}{g_{i,k}} \sum_{j=1}^{n_\kappa} g_{ij,\kappa} q_{j,\kappa}$ is a *spatial lag* term and $\upsilon_{i,k}$ is a random error term. The spatial lag term is equivalent to an autoregressive term in a time regression: a length-three connection in this model through lending connections is akin to a three period lag in a time series autoregressive model. This model is the so-called *spatial lag model* in the spatial econometric literature and can be estimated using Maximum Likelihood (see, e.g. Anselin 1988).[12]

Our goal is to make the claim that we are estimating the ϕ that corresponds to the equilibrium outcome of the game described in Section 3.

[12] In the empirical model, we work with a row-standardized adjacency matrix, i.e. if we normalize the spatial lag term by $g_{i,\kappa} = \sum_{j=1}^{n_\kappa} g_{ij,\kappa}$, the number of direct links of i. Because a row-standardized matrix implies that the largest eigenvalue is 1, we present the analysis using this approach to ease the interpretation of the results. Indeed, by providing a common upper bond for ϕ, it allows a comparison of the importance of systemic risk in different network structures, i.e. for different $\mathbf{G}$ matrices.

The basic identification issue that arise when estimating model (16) is the possible presence of unobservable factors affecting both an individual bank's behavior and neighboring banks' behavior. It is indeed difficult to disentangle the endogenous peer effects from the correlated effects, i.e. effects arising from the fact that individuals in the same network tend to behave similarly because they are subject to similar shocks. If banks are not randomly assigned into networks, this problem might also originate from the possible sorting of banks into networks according to unobserved group characteristics. Because the spatial lag term contains the dependent variable for neighboring observations, which in turn contains the spatial lag for their neighbors, and so on, a nonzero correlation between the spatial lag and the error terms is a major source of bias. A number of papers using network data have dealt with the estimation of network effects with correlated effects (e.g., Clark and Loheac 2007; Lee 2007; Calvó-Armengol et al., 2009; Lin, 2010; Lee et al., 2010). This approach is based on the use of network fixed effects to control for unobserved heterogeneity and exploits the spatial allocation of agents into networks. In other words, having network data one can exploit two sources of variation: between networks and within networks (i.e. across individuals in a given network). Similarly to a panel data context where we observe individuals over time, we can thus estimate a fixed-effects model where the effects are fixed across individuals in the same networks (instead of fixed across time). Indeed the architecture of network data allows an estimation procedure similar to a panel within-group estimator to control for correlated effects.[13] This is thus able to provide a reliable estimate of the relevant structural parameter ϕ.

This strategy is adopted in our analysis and it is made more compelling in our case

[13] A more technical exposition of this approach can be found in Liu and Lee (2010). Bramoullé et al. (2009) also deal with this problem and show formally the extent to which by subtracting from the variables the network average, social effects are identified.

study by distinguishing between lending and borrowing networks. Indeed agents in the same network, k, could be lending or borrowing due to different shocks or incentives coming from the demand or supply side of the market. By estimating model (16) separately for borrowers and lenders we reduce the heterogeneity in the error term which is captured by the network fixed effect.

5.2 Estimation results

Table 2 reports the estimation results of model (16) for each year between 2002 and 2009; each year's results is obtained pooling the different networks of transactions that arise in each day of the year and including network fixed effects. Observe that network fixed effects do not coincide with day fixed effects as there can be different separated networks of transactions in each day. Panel A shows the results from lending networks. Panel B shows the results from borrowing networks.

[*Insert Table* 2 *here*]

The positive and statistically significant estimates of ϕ point towards the existence of a cross-sectional dependence in quantities that is not explicitly mediated by the market. Looking at the $R-$squared values, one can see that this network model explain more then 10% of the variation of individual bank lending and borrowing, and, in some cases, as much as 30%. Network structure thus appears to be important in explaining a bank's lending and borrowing activity. The estimates ϕ are roughly constant over time. Indeed, after the influence of (unobserved) factors shaping the heterogeneity between networks have been controlled for, the *average* correlation in outcomes between connected agents is about 0.5, both for lending networks (panel A) and for borrowing networks (panel b). The goal of our analysis is to show the role of network position *within each network* to understand the precise *cascade in behavior* that occurs between interconnected agents following an idiosyncratic shock. Given an average ϕ of 0.5, what is the specific impact of a shock to a given bank for others further away in the network structure and what is the aggregate impact across the network? To understand systemic risk, we need to understand both the total risk to the system, as well as the contributions of each agent to that risk.

We begin by considering the average impact to the system.

Our behavioral framework shows that the equilibrium quantity choice is related to the position of each agent within the network of contacts, as captured by the Bonacich centrality, and thus depends on the evolution of the network through the linking choices of each agent.

For each network of one day of transactions, we obtain a range of estimates for each year between 2002 and 2009 (one for each day of each year). The results shows that not only the difference between the minimum and maximum estimate for each year is small, but also that they are almost constant across the years. Table 2 reports the maximum value in each year and shows that the *average* correlation in outcomes between connected agents is about 0.5 between 2002 and 2009, both for lending networks (panel A) and for borrowing networks (panel b) . The goal of our analysis is to show the role of network position within each network to understand the precise *cascade in behavior* that occurs between interconnected agents following an idiosyncratic shock. Given an average ϕ of 0.5, what is the specific impact of a shock to a given agent for others further away in the network structure and what is the aggregate impact across the network?

Panel A shows the results from lending networks. Panel B shows the results from borrowing networks.

The systemic risk multiplier

To understand systemic risk, we want to understand both the total risk to the system λ, as well as the contributions of each agent to that risk. We begin with the total risk.

Table 2 shows our measure of systemic risk λ, which simply converts the estimates of our target parameter ϕ into $\lambda = 1/(1-\phi)$. As highlighted in the theoretical model, the parameter ϕ captures the strength of network interactions that stems from the network architecture. This strength is easily interpreted as the quantity of loans that are re-lent or re-borrowed into the network. For example, in the simplest case, if bank i lends \$100 to bank j and j relends \$50 to bank k, and bank k to bank s, the ϕ parameter will equal 0.5.

To see how this converts into a measure of systemic risk, notice that in a complete network (one in which every agent is connected to every other), a parameter of 0.5 implies that each loan of \$100 is re-lent in some proportion to every other bank such that the total re-lent is 50%. One can show then that the multiplier effect, the total re-lent for each \$1 lent is calculated as $\lambda = 1/(1-\phi)$. Why? Because the first dollar was relent out at .5, that .5 out at .25, etc. The infinite sum converges to $\lambda = 1/(1-\phi)$. Of course, the total effect is precisely what we wish to understand as a measure of the amplification of a shock to a given agent. If $\phi = 0.5$ each \$1 change will be amplified on aggregate 2 times, $\lambda = 1/(1-0.5) = 2$.

In the incomplete networks that we study, risk is propagated through the network via realized loans. In such a context, the impact of a given bank must pass through a limited number of other agents, as described by the transaction pattern. As the effect dissipates in each successive link, the impact on directly connected agents is necessarily greater (see equation 7). One can see then that for directly connected agents, the impact of systemic risk is much larger; being 'close' to an impacted party leads to a greater risk of impact.[14] Thus, while the aggregate impact is 2 times the initial shock, the maximum shock a bank may face can be many times larger.

To understand the individual contribution to the total risk of the system, i.e. the diffusion process across the network, we use our theoretical model, which considers explicitly the network structure where each bank operates.

Individual contribution to systemic risk

Our behavioral framework shows that the equilibrium quantity choice is related to the position of each bank within the network of loans, as captured by the Bonacich centrality, and thus depends on the evolution of the network through the linking choices of each agent.

Accordingly to equation (7), we calculate the Bonacich centrality for each bank in our networks. This calculation generates a distribution of individual centralities depending on the strength of network interactions and on the heterogeneity of network links (as captured by the estimate of ϕ and the matrix $\mathbf{G}$ in formula (7), respectively). This measure of individual centrality provides regulators with an ability to understand which banks are the largest risk to the system. Note that, according to this measure, the more central banks will not be the most connected or the largest institution, but rather the one that contributes the most to the propagation of shocks. This contribution to propagation depends not only on number of links, but also on the number of links of connected agents, and their links and so on. For example, a bank bridging two otherwise separate networks might be a key actor in the diffusion of shocks even if it is connected with just two banks (each with many links). Not

[14] One can think of this as how banks' liquidity is impacted by the systemic risk in the network. A bank will gain (lose) when the banks linked to her gain (lost). Below, in this section, we look at the extent to which banks' liquidity is affected by changes in banks' centrality, which is a trasformation of the systemic risk parameter (equation 7). That is, does it help to change position in the network?

only the quantity but also the quality of contacts is crucial.

Table 2 shows two statistics, the first is the impact of a one-unit change in the Bonacich centrality. The second is the variance of the Bonacich centrality. Our results shows that the network structure as well as its influence on banks' outcomes vary greatly over time. So, for example, a unit increase in Bonacich centrality, i.e. a better bank positioning in the network, raises lending by about 6 million in 2004 (before the crisis), 32 million in 2006, and 2 million in 2009 (after the crisis). The one-unit impact of a change in individual centrality moves significantly over time, as does the variance of individual centrality for each network. This variance reflects the distributional impacts of a shock. As the variance of Bonacich centrality changes, it reflects changes in how shocks are absorbed by the market. A high variance suggests that a concentrated group of banks will absorb the total effect of the shock. Example here are the contagious default model where successive agents bear the full cost of the default and a zero variance case in which the shocks will be equally distributed across all agents in the network. Table 2 shows that the variance of Bonacich is higher during the crisis, indicating that the difference between the maximum and minimum shock absorbed by the different banks rises.

Lending vs Borrowing

Even as we continue to capture the relevance of network structure, its role is not constant. Why? As we approach crisis in 2006, the market was very, very liquid, but the central lenders became increasingly important. The importance of centrality became very large; moving from the periphery to the center meant very large changes in liquidity provision. These changes were 4/5 times larger than in 2002/2003. With the onset of the crisis in 2007, and particularly in 2008, the role of the central lenders declined.

The borrowing market looked a bit different. Here the importance of central players and the variance of centrality both declined secularly over time, with a slight up-tick during the crisis. We interpret this as the converse of the lending market. As lenders became more centralized, borrowers became more dispersed, with many relying a few key lenders. As the crisis hit and lenders dispersed, we begin to see some additional concentration on the borrowing side.

Thus we view our results as illustrating that the model can explain the role of network structure consistently over time, even as the market changes along many dimensions. The first, the average impact on the network of a shock, is captured in our systemic risk estimates. The second reflects the distributional impacts of a shock. As the variance of Bonacich centrality changes, it reflects changes in how shocks are absorbed by the market. A high variance suggests that a concentrated group of banks will absorb the total effect of the shock. Examples here are the contagious default model where successive agents bear the full cost of the default and a zero variance case in which the shocks will be equally distributed across all agents in the network. The effect of the crisis in making networks more dispersed in the lending market is well-captured by the huge drop in the importance of centrality and its variance in 2009. In the lending market, the impact of network position and its variance dropped by 17 and 7 times respectively.

5.3 Network formation

Our theoretical model of Section 4 provides a set of predictions for network structure that depend on a single parameter. This result emerges because agents can adopt a very simple link formation rule that depends only on the Bonacich centrality of the instantaneous network structure. Because profits are greater when one links with those with higher Bonacich centrality, link formation patterns can be described in a parsimonious way.

That link formation is a function of profitability is crucial to understanding systemic risk in the context of this type of model. The incentive process produces predictable network patterns, as we discussed above. Thus, as a result, the model provides additional ability to understand and explain systemic risk, over and above what would be feasible with reduced form approaches.

The key parameter for determining network structure, is the probability of creating a link, α. This parameter is exogenous to the dynamic model, and provides us with a way to determine the efficacy of the dynamic model in describing the observed pattern. Notice that Proposition 2 describes the precise relations between the α and the degree distribution on the entire network. Recall that the each agent has a degree, which is the count of number of links to other agents. Recall as well that the degree distribution is simply the distribution over agents in a particular network of their respective degrees. So, the model generates a prediction for the degree distribution that is a precise function of α.

Figure 5 shows the theoretical degree distributions that are obtained when calibrating the model for different values of α.

[*Insert Figure* 5 *here*]

Empirically the probability of creating a link (α) can be estimated by considering the ratio between the number of actual links and the possible ones (in a given network).

Figure 6 shows the estimated values of α on a daily basis between 2002 and 2009.[15] The graph thus shows that at the beginning of the period (January 2002) the estimated

[15] If more than one network is detected in a given day, the average value of α is reported.

link creation is about 0.4, whereas at the end of the period (December 2009) it has fallen to about 0.25. Consistently with our results in the previous section, the fall reflects that the network has become more sparse once the crisis took hold. This occurred both because the ECB began to provided unlimited access to funds through its full allotment policy and because market participants may have become less willing to lend.

[*Insert Figure* 6 *here*]

Figures 7a and 7b then plot the degree distribution in our data in different years and the one which is predicted by our model for the corresponding α value. Figure 7a shows $\alpha = 0.4$, corresponding to January 2002 and Figure 7b shows $\alpha = 0.25$, corresponding to December 2009. The important evidence revealed by these two figures is that the dynamic model captures the change in degree distribution that occurs as a result of the crisis. As the networks become more sparse (α declining), the incentives to form new links change. We can see in these two figures that the change network formation behavior observed in these markets is closely matched by the model's predictions.

[*Insert Figures* 7*a and* 7*b here*]

Figure 8 shows that this close alignment between model and data occurs over the entire time period. We plot the percentage of banks having 0 links for 250 networks over the 8 year time period (zero degree component of the corresponding degree distributions). A perfect prediction would yield a 45 degree line; here we find a slight divergence, but a very consistent ability of the model to predict the network structure in the data.

[*Insert Figure* 8 *here*]

This ability is not just confined to banks having one link but pertains to the entire degree distribution. Indeed, accompanying these three figures, we regress the observed degree distribution on the predicted one. We break our data in approximately 250 time periods. Each observation in our regression then corresponds to a time period, degree pair. So, an observation may be time period 25, degree 10. For this observation, we will have the estimated and the theoretically predicted percentage of banks with 10 links. Regression results are reported in Table 3. The model goodness of fit provides a more accurate test of our theoretical model against the data.

[*Insert Table* 3 *here*]

Column 1 of Table 3 shows the regression on the entire sample. This specification has an R-squared of 0.79. Column 2 includes a level shift for each of the 50 possible degrees. By viewing each of the degree separately, the specification explains more than 98% of the variation in the data. To evaluate whether this approach is successful before or after the crisis, or both, we break the sample at August 2007 and run our regression before and after August 2007. Post August 2007, the model explains essentially all of the variation.[16]

That the model produces distributions that are empirically so close to the data supports the ability of the static model to generates estimates of systemic risk that are plausible. In particular, it allows us to claim that our results take into account the incentive of agents to change partners.

[16] Our results remain mainly unchanged if we include time-fixed effects.

6 Extension to heterogeneous banks and prices

To this point, our model has assumed that agents are identical in every dimension except for their network position. In this section, we extend our model to allow for variation along two dimensions. The first is in demand, our parameter θ, which we generalize to θ_i. The second is to allow for bank-specific default risk. We introduce d_i to reflect the bank-specific default risk premium paid by a bank for a loan on the interbank market. We assume here that this risk premium is publicly observable and has no uncertainty. This class of models can also permit uncertainty; however, we save a full elaboration of this for future work.

6.1 Equilibrium loans

Allowing for heterogeneity means we can rewrite equation (2) above as:

$$p_i = \theta_i + d_i - \sum_{j \in N} q_j$$

This means that the interest rate (i.e. price) p_i of each loan is going to be bank specific. This implies that the profit function of each firm i in network g can be written as:

$$\pi_i(g) = a_i\, q_i - \sum_{j \in N} q_i q_j + \phi \sum_{j=1}^{n} g_{ij} q_i q_j$$

where $a_i \equiv \theta_i + d_i - c_0$. The first-order condition for each i is:

$$q_i^* = a_i - \sum_{j \in N} q_j^* + \phi \sum_{j=1}^{n} g_{ij} q_j^* \tag{17}$$

To characterize the Nash equilibrium of this new game, we need to generalize Definition 1 of the Katz-Bonacich centrality (see (7)). Indeed, the *weighted* Katz-Bonacich centrality of parameter ϕ in g is defined as:

$$\mathbf{b}(g, \phi) = \sum_{k=0}^{+\infty} \phi^k \mathbf{G}^k \mathbf{u} = [\mathbf{I} - \phi \mathbf{G}]^{-1}\, \mathbf{u} \tag{18}$$

where $\phi \geq 0$ is a scalar and $\mathbf{u}$ can be any $n \times 1$ vector. When $\mathbf{u} = \mathbf{1}$, then we are back to the unweighted Katz-Bonacich centrality defined in (7).

Define $b_{\mathbf{1}}(g, \phi) = b_{1,1}(g, \phi) + ... + b_{n,1}(g, \phi)$ as the sum of the unweighted Bonacich centralities and $b_{\mathbf{a}}(g, \phi) = a_1 b_{1,a} + ... + a_n b_{n,a}$ as the sum of *weighted Bonacich centralities* of all banks. Then, using Calvó-Armengol et al. (2009), we can derive the following result:

Proposition 3 *Suppose that* $\mathbf{a} \neq a\mathbf{1}$. *Let* $\overline{a} = \max\{a_i \mid i \in N\}$ *and* $\underline{a} = \min\{a_i \mid i \in N\}$, *with* $\overline{a} > \underline{a} > 0$. *If* $\phi\omega(\mathbf{G}) + n(\overline{a}/\underline{a} - 1) < 1$, *then this game has a unique Nash equilibrium in pure strategies* $\mathbf{q}^*$, *which is interior and given by:*

$$\mathbf{q}^* = \mathbf{b}_{\mathbf{a}}(g, \phi) - \frac{b_{\mathbf{a}}(g, \phi)}{1 + b_{\mathbf{1}}(g, \phi)} \mathbf{b}_{\mathbf{1}}(g, \phi) \tag{19}$$

Let us show how we obtain this result. We can write the first-order condition (17) in matrix form:

$$\mathbf{q}^* = \mathbf{a} - \mathbf{J}\mathbf{q}^* + \phi \mathbf{G}\mathbf{q}^*$$

where $\mathbf{J}$ is a $n \times n$ matrix of 1. Since $\mathbf{J}\mathbf{q}^* = q^*\mathbf{1}$, this can be written as

$$\begin{aligned} \mathbf{q}^* &= [\mathbf{I} - \phi\mathbf{G}]^{-1} (\mathbf{a} - q^*\mathbf{1}) \\ &= \mathbf{b}_{\mathbf{a}}(g, \phi) - q^* \mathbf{b}_{\mathbf{1}}(g, \phi) \end{aligned}$$

Multiplying to the left by $\mathbf{1}^t$ and solving for q^* gives:

$$q^* = \frac{b_{\mathbf{a}}(g, \phi)}{1 + b_{\mathbf{1}}(g, \phi)}$$

Plugging back q^* into the previous equation gives (19).

6.2 Equilibrium volatility

Ex-ante heterogeneity enables us to provide specific loan pricing at the bank level. The equilibrium price of a loan for each bank is:

$$p_i^* = \theta_i + d_i - \sum_{j \in N} q_j^*$$

This allows us to characterize price volatility in equilibrium. Here our measure of volatility is the standard deviation of prices during the day in which the given network is active. The volatility $vol(g)$ can then be expressed as:

$$vol(g) = Var_{prices}(g) = \frac{1}{n} \sum_{i=1}^{n} (p_i^* - \overline{p}^*)^2$$

where $\overline{p}^* = \frac{1}{n} \sum_{i=1}^{n} p_i^*$ is the average price in the loan market during the day.

Let us illustrate this result by using Example 1 (Section 3.3) described above. After tedious calculations, which we derive in Appendix 3, the prices and volatility of the circle and star networks are noticeably different:

$$\overline{p}^{C*} = 1.270 \text{ and } \overline{p}^{S*} = 2.510$$

$$vol^{C*} = Var_{prices}^{C*} = 0.0192 \text{ and } vol^{S*} = Var_{prices}^{S*} = 3.562$$

This shows that the star-shaped network experienced a much larger volatility than the circle network.

7 Extension to revenue spillovers

Our spillover mechanism through the cost channel is not the only mechanism through which networks can have an impact. Let us now assume that there are revenue spillovers due to network effects. We generalize the model above and maintain our prior assumption that there are network effects in the cost function, i.e. $c_i(g)$, is still given by (3):

$$c_i(g) = c_0 - \phi_1 \left[\sum_{j=1}^{n} g_{ij} q_j \right]$$

We will also assume that there are network effects on the revenue side, i.e.

$$\theta_i = \phi_2 \sum_{j=1}^{n} g_{ij} \theta_j$$

so that

$$p_i = \theta_i - \sum_{j \in N} q_j = \phi_2 \sum_{j=1}^{n} g_{ij} \theta_j - \sum_{j=1} q_j$$

The interpretation of this spillover would be that the more a bank gives loans to other banks the higher is the interest rate (price) of the loans. The profit function of each bank i in a network g is therefore given by:

$$\begin{aligned} \pi_i(g) &= \left(\phi_2 \sum_{j=1}^{n} g_{ij} \theta_j - \sum_{j=1} q_j \right) q_i - \left(c_0 - \phi_1 \left[\sum_{j=1}^{n} g_{ij} q_j \right] \right) q_i \\ &= \left(\phi_2 \sum_{j=1}^{n} g_{ij} \theta_j - c_0 \right) q_i - \sum_{j=1} q_i q_j + \phi \sum_{j=1}^{n} g_{ij} q_i q_j \end{aligned}$$

First-order condition gives:

$$q_i = \phi_2 \sum_{j=1}^{n} g_{ij} \theta_j - c_0 - \sum_{j=1} q_j + \phi_1 \sum_{j=1}^{n} g_{ij} q_j$$

Denoting $q^* = \sum_{j=1} q_j$, we can write this first-order condition in matrix form as

$$\mathbf{q} = \phi_2 \mathbf{G} \boldsymbol{\theta} - (c_0 + q^*) \mathbf{1} + \phi_1 \mathbf{G} \mathbf{q}$$

which is equivalent to

$$\begin{aligned} \mathbf{q} &= [\mathbf{I} - \phi_1 \mathbf{G}]^{-1} \phi_2 \mathbf{G} \boldsymbol{\theta} - (c_0 + q^*) [\mathbf{I} - \phi_1 \mathbf{G}]^{-1} \mathbf{1} \\ &= [\mathbf{I} - \phi_1 \mathbf{G}]^{-1} \mathbf{b}_{\phi_2 \mathbf{G} \boldsymbol{\theta}} (g, \phi_1) - (c_0 + q^*) \mathbf{b}_{\mathbf{1}} (g, \phi_1) \end{aligned}$$

We are back to the previous section case with ex ante heterogeneity and can again apply Proposition 3. Denote $\alpha_i = \phi_2 \sum_{j=1}^{n} g_{ij}\theta_j - c_0$ and let $\overline{\alpha} = \max\{\alpha_i \mid i \in N\}$ and $\underline{\alpha} = \min\{\alpha_i \mid i \in N\}$, with $\overline{\alpha} > \underline{\alpha} > 0$. Then if $\phi_1\omega(\mathbf{G}) + n(\overline{\alpha}/\underline{\alpha} - 1) < 1$, there is a unique Nash equilibrium which is interior and given by:

$$\mathbf{q}^* = \mathbf{b}_{\phi_2 \mathbf{G}\boldsymbol{\theta}}(g, \phi_1) - \frac{b_{\phi_2 \mathbf{G}\boldsymbol{\theta}}(g, \phi_1)}{1 + b_{\mathbf{1}}(g, \phi_1)} \mathbf{b}_{\mathbf{1}}(g, \phi_1).$$

While the solution is more complex than the single spillover above, notice that the general form of the equilibrium quantities remains. As above, one can calculate prices and volatility directly from this equation.

8 Discussion and policy implications

Our approach is designed to understand the role of network structure on interbank lending. As with any model and data, there are some limitations to the exercise. For example, while our data is exceptional in providing comprehensive coverage of the European interbank market during most of the time period studied, e-MID particular role in the market limits the capacity of the model to explain some shocks. Because e-MID is transparent, and European banks have access to the ECB for emergency borrowing purposes, e-MID evolved as a way to balance relatively small liquidity shocks. Larger structural shocks would be dangerous to post on a public platform and access to the ECB provided an alternate outlet. As such, our results should be seen as a way to analyze shocks and network impacts on the margin. That said, this should indicate that increases in systemic risk in this market would understate the level of risk in the market as a whole.

From a policy perspective, we emphasize the utility of using a structural approach to networks. To the extent that the model captures bank behavior, it allows policymakers the ability to test interventions with an eye both to how banks will optimize in the short-run and how networks will form and re-form under each assumption.

An example is how one can interpret in our model the imposition of the ECB's full allotment policy. This policy permitted banks to access credit lines from the ECB in unlimited quantities at a fixed rate. We can model this by removing many higher risk banks from the market. It is a straightforward result of the static model that this will lower average demand, $E\theta_i$, in this market and well as reduce average risk, Ed_i.

A second example is the use of exceptional capital cushions for SIFIs. Our approach allows one both to identify the SIFIs that are the largest contributors to systemic risk, λ_i, and what occurs if these banks have increased, and now binding, capital constraints. To identify the largest contributors, the static model indicates simply that the banks with the highest Bonacich centrality are those with the highest contribution. Their share of contribution is calculated above as well.

An alternate approach, which takes into account how the network changes as a result of a shock to an institution is beyond the scope of this paper. However, related work by Liu et al. (2012), has found that the because of network dynamics, the largest contributor is not always the one with the highest centrality. This can occur depending on the type of intervention as well as depending on the precise nature of the network.

Using this information, an avenue for future research would be to evaluate optimal regulatory policy in the presence of networks. Given a particular objective function for the regulator, such as minimizing volatility or minimizing total systemic risk, the approach here could yield a set of capital constraints that solve the regulator's problem. Notice that these constraints would not necessarily have any of the cyclicality problems that a static, fixed constraint does. For example, the regulator could optimize over contribution to systemic risk over a period of time that includes recessions. Then, a capital cushion that depends on the contribution to risk and position in the network would vary against the cycle.

9 Conclusion

We have constructed two models of the interbank loan market, a static and dynamic one. To complement these, we have provided empirical evidence of the models' accuracy in describing the data. Then, using these models, we have presented a measure of systemic risk in this market, which is a precise measure of the aggregate liquidity cost of a reduction in lending by an individual financial institution. This systemic risk measure is presented as an innovation vis-a-vis existing approaches. It is based on the foundation of a microfounded dynamic model of behavior. As well, it provides a tool to understand the transmission of shocks that extends beyond default events and generalized price shocks. In the combination of these lies our tool; the competitive responses that banks make generate the transmission of shocks in our model and provide a tractable method of measuring and understanding systemic risk.

There are a number of tangible benefits to the models and methods presented in this paper. The calculation of spillovers in interbank markets gives regulators an ability to gauge the market's sensitivity to shocks. We find multiples that grow as large as 2.5 times the initial reaction.

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Appendix: Theoretical results

Appendix 1: Nash equilibrium in loans

Katz-Bonacich centrality

Let $\mathbf{G}^k$ be the kth power of $\mathbf{G}$, with coefficients $g_{ij}^{[k]}$, where k is some integer. The matrix $\mathbf{G}^k$ keeps track of the indirect connections in the network: $g_{ij}^{[k]} \geq 0$ measures the number of paths of length $k \geq 1$ in g from i to j.[17] In particular, $\mathbf{G}^0 = \mathbf{I}$.

Given a scalar $\phi \geq 0$ and a network $\mathbf{g}$, we define the following matrix:

$$\mathbf{M}(g, \phi) = [\mathbf{I} - \phi \mathbf{G}]^{-1} = \sum_{k=0}^{+\infty} \phi^k \mathbf{G}^k$$

where $\mathbf{I}$ is the identity matrix. These expressions are all well-defined for low enough values of ϕ. It turns out that an exact strict upper bound for the scalar ϕ is given by the inverse of the largest eigenvalue of $\mathbf{G}$ (Debreu and Herstein, 1953). The parameter ϕ is a decay factor that scales down the relative weight of longer paths. If $\mathbf{M}(g, \phi)$ is a non-negative matrix, its coefficients $m_{ij}(g, \phi) = \sum_{k=0}^{+\infty \phi k} g_{ij}^{[k]}$ count the number of paths in g starting from i and ending at j, where paths of length k are weighted by ϕ^k. Observe that since $\mathbf{G}$ is symmetric then $\mathbf{M}$ is also symmetric.

Nash Equilibrium and Katz-Bonacich centrality

Let us show how the first order condition can be written as a function of Katz-Bonacich centrality. For each bank $i = 1, ..., n$, maximizing (4) leads to:

$$q_i^* = a - \sum_{j=1}^{n} q_j^* + \phi \sum_{j=1}^{n} g_{ij} q_j^* \tag{20}$$

[17] A *path* lof length k from i to j is a sequence $\langle i_0, ..., i_k \rangle$ of players such that $i_0 = i$, $i_k = j$, $i_p \neq i_{p+1}$, and $g_{i_p i_{p+1}} > 0$, for all $0 \leq k \leq k-1$, that is, players i_p and i_{p+1} are directly linked in $\mathbf{g}$. In fact, $g_{ij}^{[k]}$ accounts for the total weight of all paths of length k from i to j. When the network is un-weighted, that is, $\mathbf{G}$ is a $(0, 1)-$matrix, $g_{ij}^{[k]}$ is simply the number of paths of length k from i to j.

We can write this equation in matrix form to obtain:

$$\mathbf{q}^* = a\mathbf{1} - \mathbf{J}\mathbf{q}^* + \phi\mathbf{G}\mathbf{q}^*$$

where $\mathbf{J}$ is a $n \times n$ matrix of 1. Since $\mathbf{J}\mathbf{q}^* = q^*\mathbf{1}$, this can be written as

$$\begin{aligned}\mathbf{q}^* &= [\mathbf{I} - \phi\mathbf{G}]^{-1}(a - q^*)\,\mathbf{1} \\ &= (a - q^*)\,\mathbf{b}(g, \phi)\end{aligned}$$

Multiplying to the left by $\mathbf{1}^t$ and solving for q^* gives:

$$q^* = \frac{ab(g,\phi)}{1 + b(g,\phi)}$$

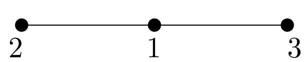
where $b(g,\phi) = \mathbf{1}^t\mathbf{b}(g,\phi)$. Plugging back q^* into the previous equation gives

$$\mathbf{q}^* = \frac{a}{1 + b(g,\phi)}\mathbf{b}(g,\phi)$$

which is (8).

Example

Consider the following network with three banks.



The corresponding adjacency matrix is,

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

The kth powers of $\mathbf{G}$ are then, for $k \geq 1$:

$$\mathbf{G}^{2k} = \begin{bmatrix} 2^k & 0 & 0 \\ 0 & 2^{k-1} & 2^{k-1} \\ 0 & 2^{k-1} & 2^{k-1} \end{bmatrix} \quad \text{and} \quad \mathbf{G}^{2k+1} = \begin{bmatrix} 0 & 2^k & 2^k \\ 2^k & 0 & 0 \\ 2^k & 0 & 0 \end{bmatrix}.$$

For instance, we deduce from $\mathbf{G}^3$ that there are exactly two paths of length three between agents 1 and 2, which are $12 \to 21 \to 12$ and $12 \to 23 \to 32$.

When ϕ is small enough,[18]

$$\mathbf{M} = [\mathbf{I}-\phi\mathbf{G}]^{-1} = \frac{1}{1-2\phi^2}\begin{pmatrix} 1 & \phi & \phi \\ \phi & 1-\phi^2 & \phi^2 \\ \phi & \phi^2 & 1-\phi^2 \end{pmatrix}$$

and the vector of Katz-Bonacich network centralities is:

$$\mathbf{b}(g,\phi) = \begin{bmatrix} b_1(g,\phi) \\ b_2(g,\phi) \\ b_3(g,\phi) \end{bmatrix} = \frac{1}{1-2\phi^2}\begin{bmatrix} 1+2\phi \\ 1+\phi \\ 1+\phi \end{bmatrix}$$

Not surprisingly, the center (bank 1) is more central than the peripheral banks 2 and 3. The Nash equilibrium is then given by (using (**??**)):

$$\mathbf{q}^* = \begin{bmatrix} q_1^* \\ q_2^* \\ q_3^* \end{bmatrix} = \frac{a}{4(1+\phi)\left(1-2\phi^2\right)}\begin{bmatrix} 1+2\phi \\ 1+\phi \\ 1+\phi \end{bmatrix}$$

Appendix 2: Network externality on loan equilibrium

The first order condition is:

$$q_i^* = a - \sum_{j=1}^{n} q_j^* + \phi\sum_{j=1}^{n} g_{ij}q_j^*$$

or in matrix form

$$\mathbf{q}^* = a\mathbf{1} - \mathbf{J}\mathbf{q}^* + \phi\mathbf{G}\mathbf{q}^*$$

where $\mathbf{J}$ is a $n \times n$ matrix of 1. Since $\mathbf{J}\mathbf{q}^* = q^*\mathbf{1}$, this can be written as

$$\mathbf{q}^* = a\mathbf{1} - q^*\mathbf{1} + \phi\mathbf{G}\mathbf{q}^*$$

[18] Here, the largest eigenvalue of $\mathbf{G}$ is $\sqrt{2}$, and so the exact strict upper bound for ϕ is $1/\sqrt{2}$.

Multiplying to the left by $\mathbf{1}^t$, we get

$$q^* = \left(\frac{n}{1+n}\right) a + \left(\frac{\phi}{1+n}\right) \mathbf{1}^t \mathbf{G}\mathbf{q}^*$$

or equivalently:

$$q^{NET*} = \left(\frac{n}{1+n}\right) a + \left(\frac{\phi}{1+n}\right) \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} q_j \tag{21}$$

By plugging back this equation into the first-order condition, we obtain:

$$q_i^{NET*} = \frac{a}{(1+n)} + \phi \sum_{j=1}^{n} g_{ij} q_j^* - \left(\frac{\phi}{1+n}\right) \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} q_j$$

which is equivalent to:

$$q_i^* = \frac{a}{(1+n)} + \left(\frac{n}{1+n}\right) \phi \sum_{j=1}^{n} g_{ij} q_j^* - \left(\frac{1}{1+n}\right) \phi \sum_{k \neq i}^{n} \sum_{j=1}^{n} g_{kj} q_j^* \tag{22}$$

Appendix 3: Volatility calculation for circle and star networks

Specifying

$$\boldsymbol{\theta} = \begin{bmatrix} 0.5 \\ 0.8 \\ 0.1 \\ 0.9 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} 0.75 \\ 0.4 \\ 1.4 \\ 0.6 \end{bmatrix} \text{ and } \mathbf{c}_0 = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.5 \\ 0.3 \end{bmatrix}$$

we have

$$\mathbf{a} = \begin{bmatrix} 1.05 \\ 1.1 \\ 1 \\ 1.2 \end{bmatrix}$$

The eigenvalue condition $\phi\omega(\mathbf{G}) + n\left(\overline{a}/\underline{a} - 1\right) < 1$ can be written as: $\phi < 0.2$. This is an upper bound so ϕ can still be greater than 2. For the circle network, the Bonacich centralities, unweighted and weighted, are respectively given by:

$$\mathbf{b}_{\mathbf{1}}^{C}(g, \phi) = \frac{1}{\left(1 - \phi^3 - \phi^4\right)} \begin{bmatrix} 1 + \phi + 2\phi^2 + \phi^3 \\ 1 + 2\phi + 2\phi^2 + \phi^3 \\ 1 + \phi + \phi^2 \\ 1 + \phi + \phi^2 + \phi^3 \end{bmatrix}$$

$$\mathbf{b}_{\mathbf{a}}^{C}(g,\phi)=\frac{1}{20\left(1-\phi^{3}-\phi^{4}\right)}\begin{bmatrix}21+22\phi+44\phi^{2}+24\phi^{3}\\22+44\phi+45\phi^{2}+21\phi^{3}\\20+24\phi+21\phi^{2}+2\phi^{3}\\24+21\phi+22\phi^{2}+20\phi^{3}\end{bmatrix}$$

and the equilibrium loan quantities by:

$$\begin{bmatrix}q_{A}^{C*}\\q_{B}^{C*}\\q_{C}^{C*}\\q_{D}^{C*}\end{bmatrix}=\frac{\begin{bmatrix}18+17\phi+39\phi^{2}+6\phi^{3}-35\phi^{4}-56\phi^{5}-63\phi^{6}-24\phi^{7}\\23+45\phi+49\phi^{2}-2\phi^{3}-68\phi^{4}-94\phi^{5}-70\phi^{6}-21\phi^{7}\\13+22\phi+15\phi^{2}-11\phi^{3}-35\phi^{4}-37\phi^{5}-17\phi^{6}-2\phi^{7}\\33+27\phi+29\phi^{2}-13\phi^{3}-60\phi^{4}-56\phi^{5}-49\phi^{6}-20\phi^{7}\end{bmatrix}}{20\left(1-\phi^{3}-\phi^{4}\right)\left(50+50\phi+60\phi^{2}+2\phi^{3}-\phi^{4}\right)}$$

For the star-shaped network, we have:

$$\mathbf{b}_{\mathbf{1}}^{S}(g,\phi)=\frac{1}{1-\phi}\begin{bmatrix}1\\1+\phi\\1\\1\end{bmatrix}$$

$$\mathbf{b}_{\mathbf{a}}^{S}(g,\phi)=\frac{1}{1-\phi}\begin{bmatrix}\frac{21+24\phi-\phi^{2}}{20(1+\phi)}\\1.1\left(1+\phi\right)\\\frac{5+6\phi}{5(1+\phi)}\\\frac{6+5\phi}{5(1+\phi)}\end{bmatrix}$$

and

$$\begin{bmatrix}q_{A}^{S*}\\q_{B}^{S*}\\q_{C}^{S*}\\q_{D}^{S*}\end{bmatrix}=\frac{1}{1-\phi}\begin{bmatrix}0.02\left(9+4\phi-13\phi^{2}\right)/\left(1+\phi\right)\\0.23-0.02\phi-0.21\phi^{2}\\0.01\left(13+8\phi-21\phi^{2}\right)/\left(1+\phi\right)\\0.01\left(33-12\phi-21\phi^{2}\right)/\left(1+\phi\right)\end{bmatrix}$$

Thus the price for each bank for a loan are given by:

$$\begin{bmatrix}p_{A}^{C*}\\p_{B}^{C*}\\p_{C}^{C*}\\p_{D}^{C*}\end{bmatrix}=\frac{1}{20\left(50+50\phi+60\phi^{2}+2\phi^{3}-\phi^{4}\right)}\begin{bmatrix}1163+1139\phi+1368\phi^{2}-17\phi^{3}-25\phi^{4}\\1113+1089\phi+1308\phi^{2}-19\phi^{3}-24\phi^{4}\\1413+1389\phi+1668\phi^{2}-30\phi^{4}-7\phi^{3}\\1413+1389\phi+1668\phi^{2}-30\phi^{4}-7\phi^{3}\end{bmatrix}$$

for the circle network and by

$$\begin{bmatrix} p_A^{S*} \\ p_B^{S*} \\ p_C^{S*} \\ p_D^{S*} \end{bmatrix} = \frac{1}{1+\phi} \begin{bmatrix} 0.38 + \phi + 0.91\phi^2 + 0.21\phi^3 \\ 0.33 + 0.95\phi + 0.91\phi^2 + 0.21\phi^3 \\ 0.63 + 1.25\phi + 0.91\phi^2 + 0.21\phi^3 \\ 0.63 + 1.25\phi + 0.91\phi^2 + 0.21\phi^3 \end{bmatrix}$$

for the star-shaped network.

The mean price in each network is given by:

$$\overline{p}^{C*} = \frac{5102 + 5006\phi + 6012\phi^2 - 50\phi^3 - 109\phi^4}{80\left(50 + 50\phi + 60\phi^2 + 2\phi^3 - \phi^4\right)}$$

and

$$\overline{p}^{S*} = \frac{1.97 + 4.45\phi + 3.64\phi^2 + 0.84\phi^3}{1+\phi}$$

and therefore the variances of prices are:

$$vol^{C*} = Var^{C*}_{prices} = 0.0192$$

$$vol^{S*} = Var^{S*}_{prices} = \frac{2.2 + 9.9\phi + 19.23\phi^2 + 20.09\phi^3 + 11.66\phi^4 + 3.44\phi^5 + 0.40\phi^6}{(1+\phi)^2}$$

For $\phi = 0.2$, we have:

$$\begin{bmatrix} q_A^{C*} \\ q_B^{C*} \\ q_C^{C*} \\ q_D^{C*} \end{bmatrix} = \begin{bmatrix} 0.0185 \\ 0.0273 \\ 0.0144 \\ 0.0318 \end{bmatrix} \text{ and } \begin{bmatrix} q_A^{S*} \\ q_B^{S*} \\ q_C^{S*} \\ q_D^{S*} \end{bmatrix} = \begin{bmatrix} 0.193 \\ 0.272 \\ 0.143 \\ 0.310 \end{bmatrix}$$

and

$$\begin{bmatrix} p_A^{C*} \\ p_B^{C*} \\ p_C^{C*} \\ p_D^{C*} \end{bmatrix} = \begin{bmatrix} 1.158 \\ 1.108 \\ 1.408 \\ 1.408 \end{bmatrix} \text{ and } \begin{bmatrix} p_A^{S*} \\ p_B^{S*} \\ p_C^{S*} \\ p_D^{S*} \end{bmatrix} = \begin{bmatrix} 0.515 \\ 0.465 \\ 0.765 \\ 0.765 \end{bmatrix}$$

Also,

$$\overline{p}^{C*} = 1.270 \text{ and } \overline{p}^{S*} = 2.510$$

$$vol^{C*} = Var^{C*}_{prices} = 0.0192 \text{ and } vol^{S*} = Var^{S*}_{prices} = 3.562$$

Figure 1: Bank balance sheet

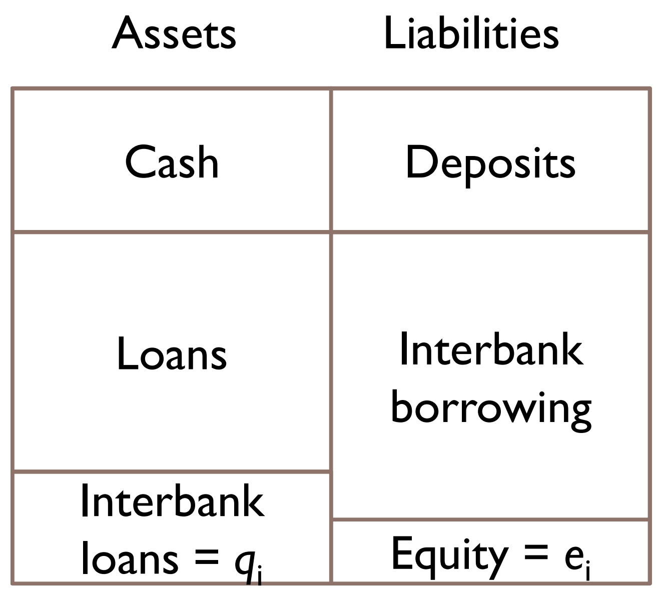


Figure 2: Daily Lending Quantities

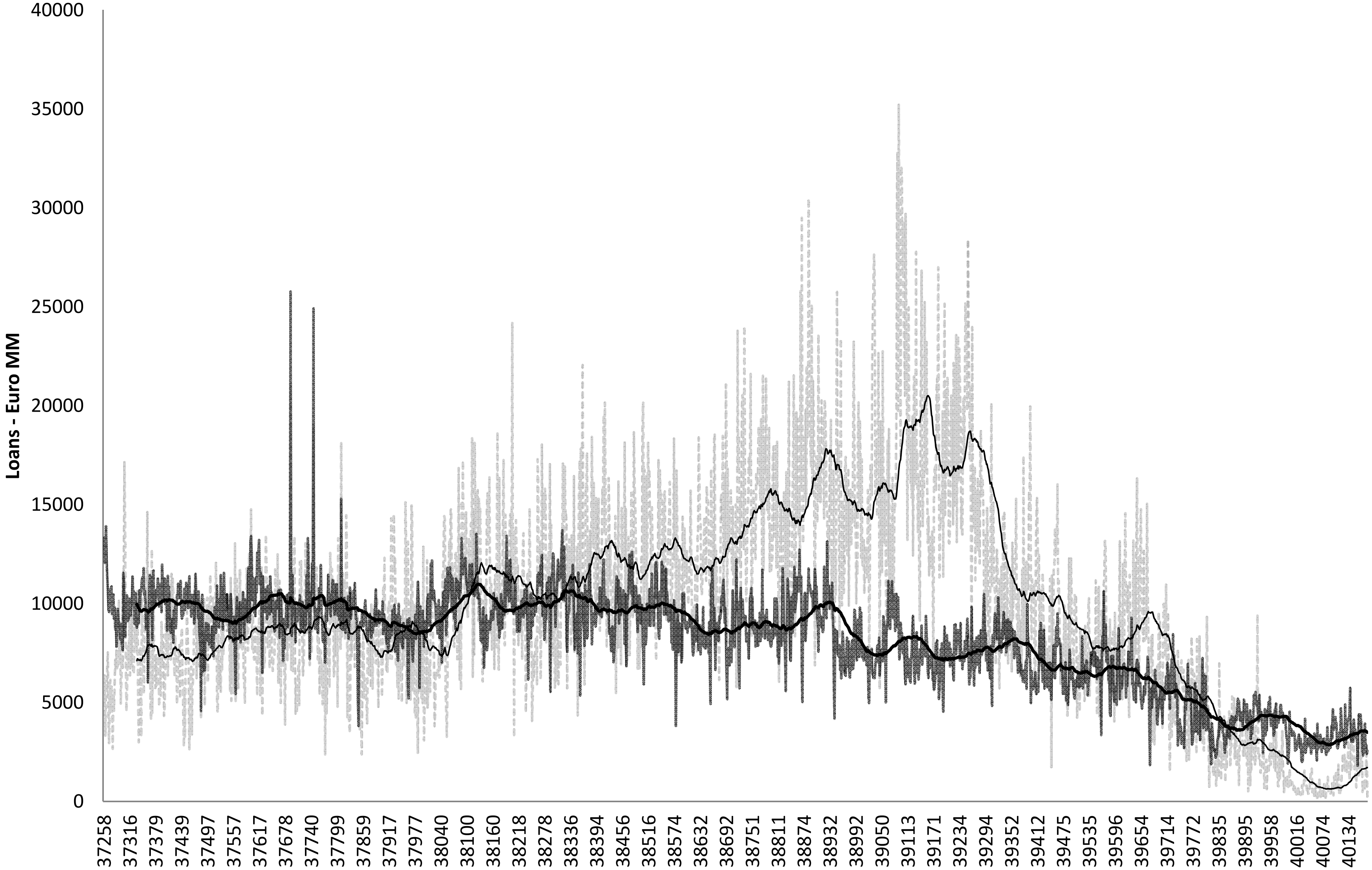


Figure 4 shows daily lending quantities over the sample for overnight and longer term lending. The black solid line reports overnight lending quantities. The grey dashed line reports all other lending. Each has a 2 month moving average trend added.

Figure 3: Daily Price Volaility

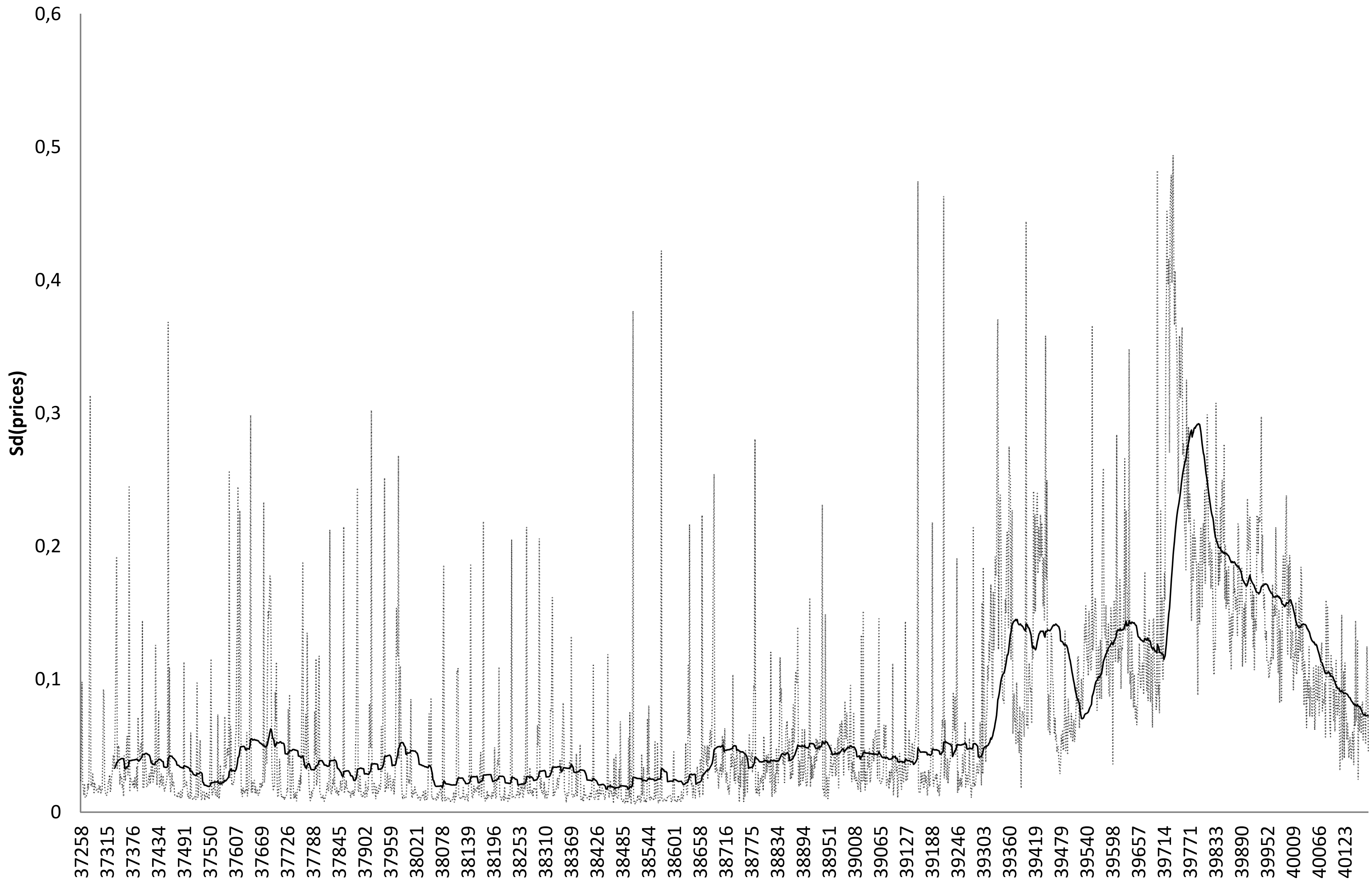


Figure 5 reports the daily standard deviation of prices (taken over prices during the day and normalized). The price volatility itself is reported in a grey dashed line and the 2-month moving average reported in a black solid line.

Figure 4: Circle and star directed networks

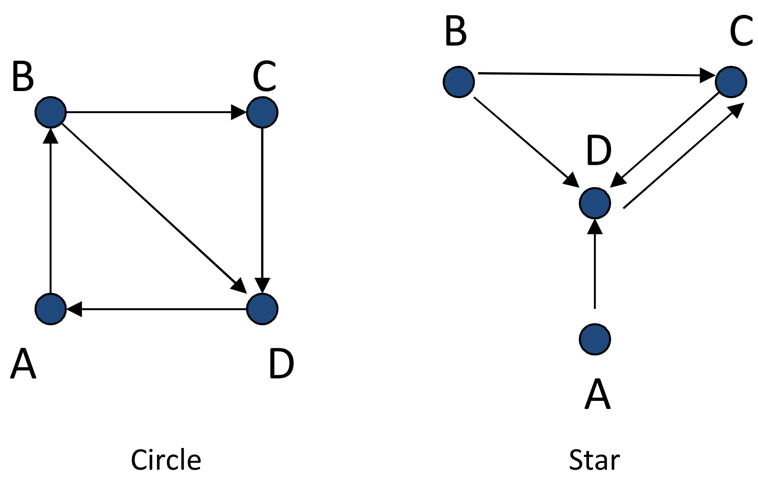


Figure 5: Dynamic Model : Theoretical Degree Distribution

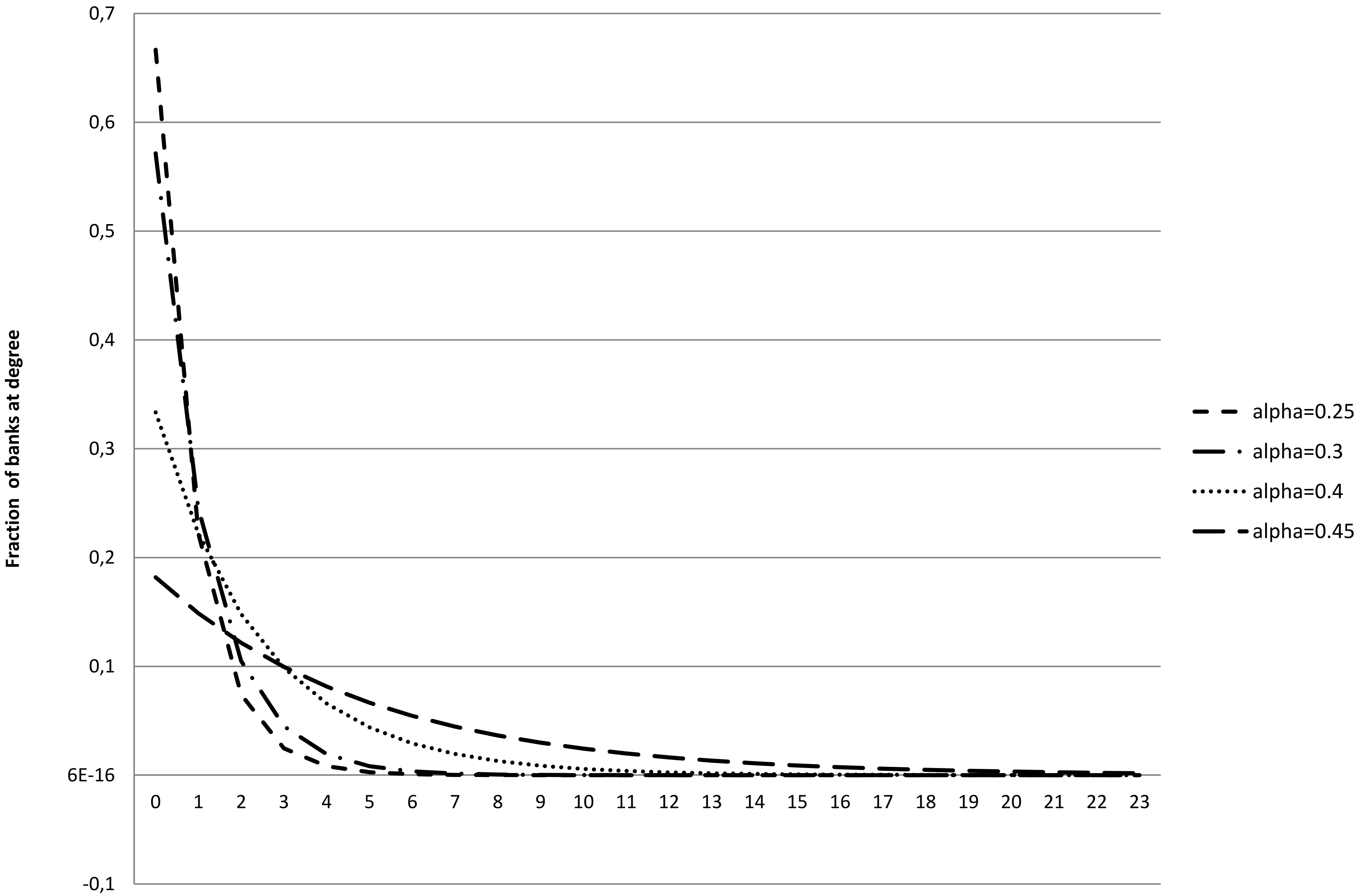


Figure 6 shows the theoretical degree distribution of the dynamic network formation model in the paper For each level of α (link formation probability), the model general an invariant distribution of network links. Precise invariant distribution is described in the text: n(d)=((1-2α)/(1-α))((α/(1-α)))^{d}, where n is the proportion at end degree. We report these distributions for α={.25, .3, .4, .45}.

Figure 6: Estimated Link Formation Probability (Alpha)

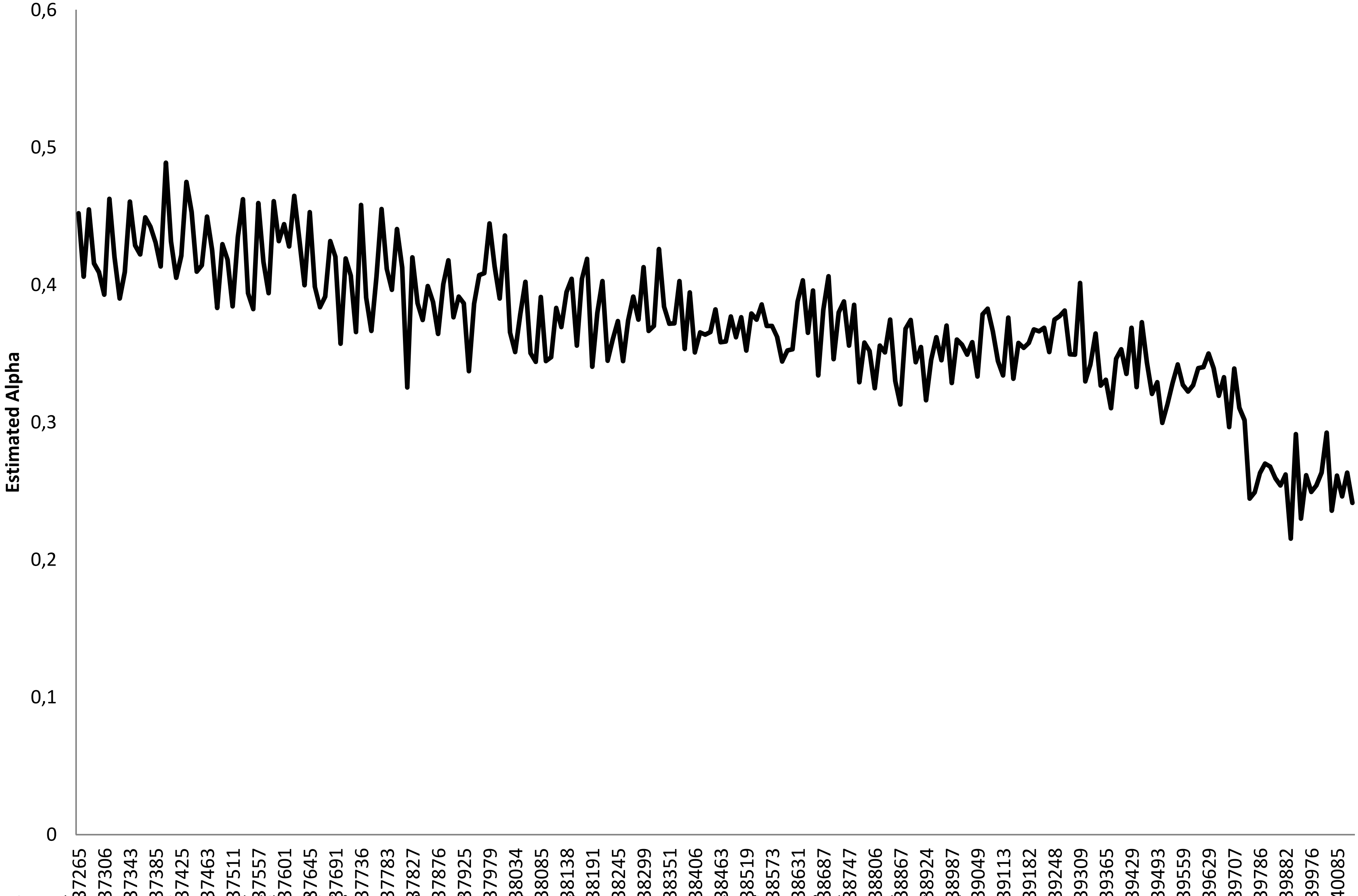


Figure 7b shows the estimated probability of link formation for each network. Recall that we use a network definition of 1 day as a benchmark.

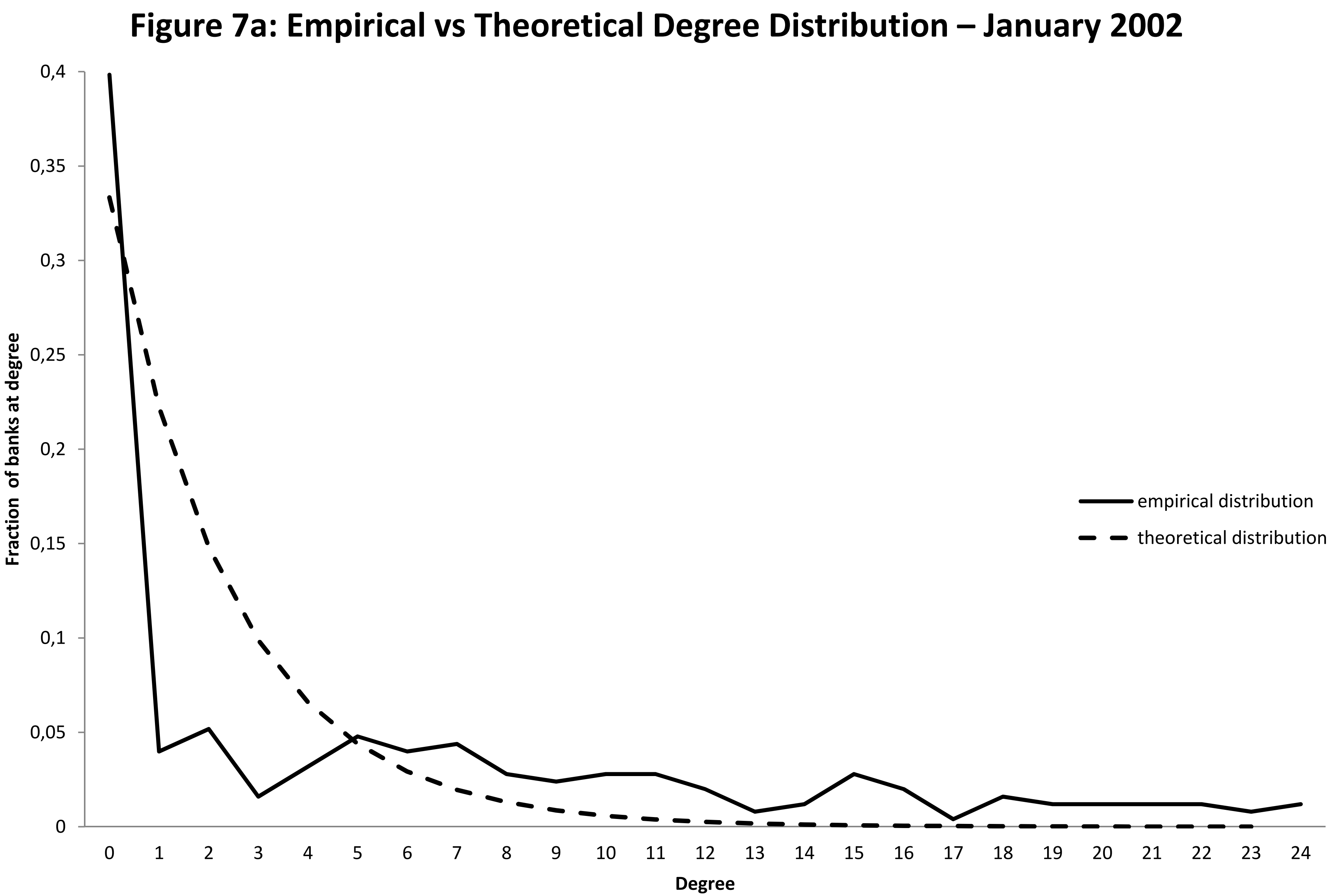


Figure 8a shows the empirical degree distribution of the network that existed on January 9, 2002. On the same figure, we plot the theoretical distribution generated by the empirical link formation probability on that day.

Figure 7b: Empirical vs Theoretical Degree Distribution - Dec 2009

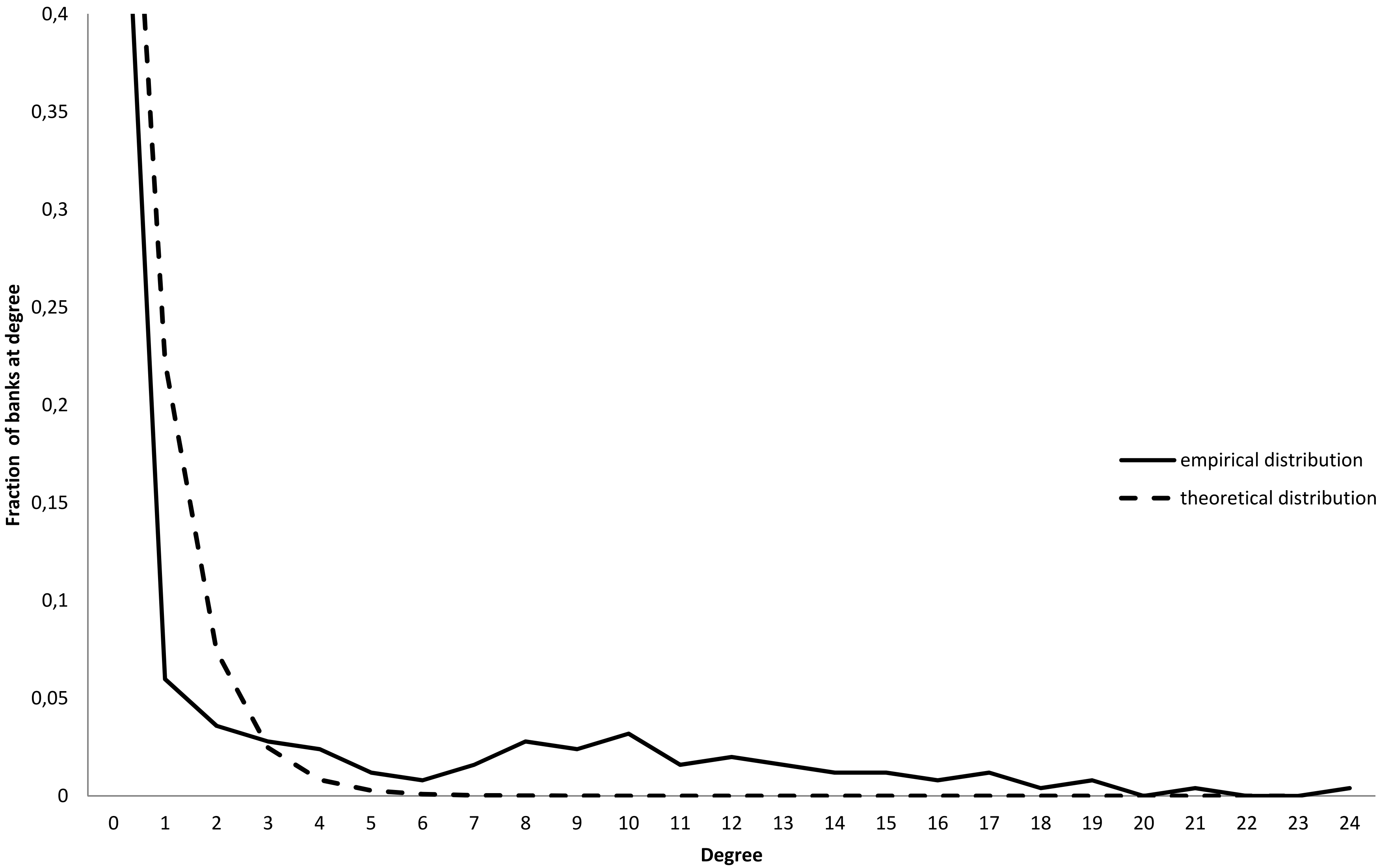


Figure 8b shows the empirical degree distribution of the network that existed on December 31, 2009. On the same figure, we plot the theoretical distribution generated by the empirical link formation probability on that day.

Figure 8: Dynamic Model: Model Fit

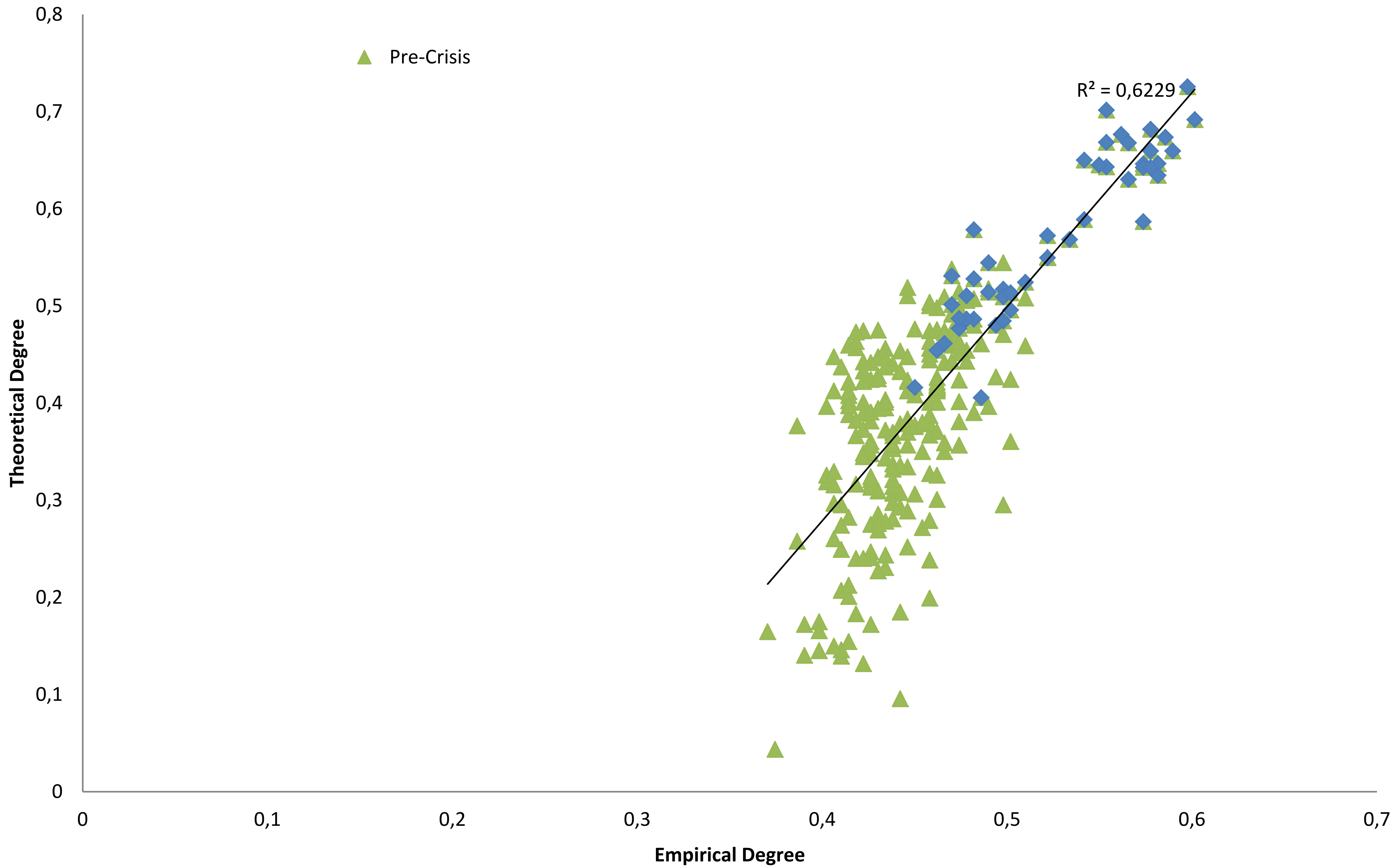


Figure 8c shows a scatterplot of two variables. The first (on the horizontal axis) is the fraction of one-degree participants in network (1 day) in our dataset. The second (on the vertical axis) is the fraction of one-degree participants implied by our dynamic model, conditional on the α for that given network. Recall that α is the empirical probability of link formation. A 45% line implies that the model works perfectly and a high correlation implies that the model is consistent over a wide range of network structures.

Table 1: Summary Statistics

	2002	2003	2004	2005	2006	2007	2008	2009
Overnight Lending Only								
Average Daily Volume (mm euros)	10 046	9 666	10 458	9 577	8 912	7 615	6 028	3 669
Daily standard dev of volume	1 321	1 860	1 360	1 389	1 563	1 146	1 389	816
Daily standard dev of prices	0,12	0,35	0,09	0,09	0,38	0,23	0,53	0,46
Number of Loans	130 614	114 844	104 393	97 551	90 370	86 453	75 933	52 743
largest 25 lenders in 2002								
Total Lending (mm euros)	1 634	1 913	2 162	2 033	2 109	1 326	346	181
Fraction of total	16%	20%	21%	21%	24%	17%	6%	5%
Number of loans	14 755	14 766	15 793	14 645	14 759	9 408	2 635	1 701
Panel B								
All Lending								
Average Daily Volume (mm euros)	17 892	18 369	21 258	22 412	24 745	22 835	13 731	5 516
Daily standard dev of volume	2 606	3 940	4 028	3 580	4 662	6 262	3 628	1 810
Daily standard dev of prices	0,11	0,34	0,08	0,09	0,38	0,23	0,53	0,47
Number of Loans	166 139	143 562	129 082	124 444	118 548	110 596	93 069	60 124
largest 25 lenders in 2002								
Total Lending (mm euros)	2 370	2 975	2 800	2 946	2 587	1 697	1 210	32
Fraction of total	13%	16%	13%	13%	10%	7%	9%	1%
Number of loans	15 108	17 669	14 774	13 085	11 254	6 222	3 626	176

Table 2: Interbank Network Systemic Risk

Note: Panel A shows results from the lending networks. Panel B shows results from the borrowing networks. Each of the two panels shows estimation results from model (14). We report the average estimates for each year between 2002 and 2009. Recall that model (14) estimates the relationship: q_{i,κ}=c+ϕ(1/(g_{i.,k}))∑_{j=1}^{n_{κ}}g_{ij,κ}q_{j,κ}+υ_{i,κ} , i.e. the spatial autoregression of individual loan volume on the network patterns of the loan volume of the rest of the market. Network fixed effects are included. The adjacency matrix of realized trades is a symmetric, non-directed matrix of 1's and 0's with 1's indicating the presence of a loan and 0 the absence. The first row shows the estimates of the parameter ϕ, the systemic risk measure, from the above specification. T-statistics are reported below coefficient estimates. The average systemic risk multiplier λ is the total network impact of a one unit shock to an individual bank loan volume. Summing across the impact for all individuals in the network produces this number, which is equal to 1/(1-φ). Individual centrality is measured using Bonacich centrality (equation 7).

Panel A

	2002	*2003*	*2004*	*2005*	*2006*	*2007*	*2008*	*2009*
Lending Networks								
Network effects (φ)	0,577	0,569	0,544	0,546	0,547	0,550	0,530	0,494
t - statistic	3,373	3,266	2,950	2,976	2,989	3,008	2,792	2,414
R-Squared	0,220	0,310	0,230	0,210	0,280	0,220	0,210	0,230
Average Systemic Risk Multiplier (λ)	2,36	2,32	2,19	2,20	2,21	2,22	2,13	1,98
Impact of Unit Change in Centrality	13,86	7,89	5,69	25,40	31,82	42,10	31,70	1,75
Variance of Centrality	42,50	55,75	29,37	203,54	202,15	118,46	94,29	14,58

Panel B

	2002	*2003*	*2004*	*2005*	*2006*	*2007*	*2008*	*2009*
Borrowing Networks								
Average ϕ Coefficient	0,591	0,567	0,541	0,549	0,528	0,527	0,488	0,478
t - statistic	3,583	3,246	2,936	3,009	2,789	2,819	2,431	2,268
R-Squared	0,260	0,270	0,340	0,210	0,280	0,310	0,120	0,180
Average Systemic Risk Multiplier - λ	2,44	2,31	2,18	2,22	2,12	2,12	1,95	1,92
Impact of Unit Change in Centrality	59,82	21,05	8,64	6,16	21,57	27,99	21,26	5,61
Variance of Centrality	28,32	3,74	0,29	0,71	3,33	4,85	3,67	0,09

Table 3: Dynamic Model Fit

Note: The table shows OLS estimation results obtained from regressing the empirical degree distribution on model-predicted degree distribution. We simply calculate the fraction of 0,1,…,n degree nodes using the dynamic model from the text and the data at each point in time. We break our data in approximately 249 time periods. Each observation in our regression then corresponds to a time period, degree pair. So, an observation may be time period 25, degree 10. For this observation, we will have the estimated and the theoretically predicted percentage of banks with 10 links. Column 1 of table shows the regression on the entire sample. Column 2 includes a level shift (fixed effect) for each of the 50 possible degrees. Columns 3 and 4 break the sample into pre-crisis and post-crisis time periods. Columns 5 and 6 show pre- and post- crisis with a fixed effect by degree. OLS specifications in model 1, 3 and 4 have no constant as implied by the model.

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
sample	***all***	***all***	***post-crisis***	***pre-crisis***	***post-crisis***	***pre-crisis***
Theoretical Distribution	0.838***	0.254***	0.816***	0.852***	0.436***	0.120***
	(0.00369)	(0.00346)	(0.00474)	(0.00504)	(0.00763)	(0.00477)
Constant		0.0146***			0.0111***	0.0173***
		(9.59e-05)			(0.000186)	(0.000121)
Degree Fixed Effect	no	yes	no	no	yes	yes
Observations	12,699	12,699	3,570	9,129	3,570	9,129
R-squared	0.803	0.986	0.893	0.758	0.992	0.986