The Design of Mortgage-Backed Securities and Servicer Contracts*

Robert Mooradian† Pegaret Pichler‡

December 12, 2013

Abstract

We develop a unified model of mortgage and servicer contracts. We show that renegotiating mortgage contracts following default is strictly Pareto improving, if the lender gathers updated borrower information. To align servicer incentives with investor interests, we demonstrate that servicers must hold risk positions in MBSs that include "vertical" components. However, offering incentive compatible contracts is not possible if foreclosure is highly inefficient and servicers do not sufficiently value investment in MBSs. In this case, forming a nondiversified pool to preserve pool-wide information may increase MBS value.

Keywords: security design, mortgage contracts, renegotiation
JEL Classifications: G21, D86, C72

*We thank Jonathan Berk, Charles Kahn, Gyöngyi Lóránth, Alexander Mürmann, Carol Osler, Tomasz Piskorski, Matthew Spiegel and participants of the Utah Winter Finance Conference, the Swiss Winter Conference on Financial Intermediation, and seminars at the Vienna Graduate School of Finance and Northeastern University for helpful discussions and suggestions.

†Northeastern University, 413 Hayden Hall, 360 Hungtington Ave, Boston, MA 02115. 617-373-5955, fax: 617-373-8798 r.mooradian@neu.edu

‡Northeastern University, 413 Hayden Hall, 360 Hungtington Ave, Boston, MA 02115. 617-373-4572, fax: 617-373-8798 p.pichler@neu.edu.

1 Introduction

At least five million homeowners have lost their homes to costly foreclosure in the United States from 2006-2012.[1] Mortgage modification programs have been promoted, yet mortgage backed security (MBS) servicers are generally reluctant to engage in mortgage modification. Any renegotiation of mortgages in default suffers from two problems: an asymmetric information problem that occurs regardless of whether the mortgage is securitized and a moral hazard problem due to the unbundling of funding and servicing that occurs in securitization. To address the second problem, servicers must hold risk positions in the MBSs they service.

We show that renegotiating mortgage contracts following default is strictly Pareto improving, if the lender gathers updated borrower information. To align servicer incentives with investor interests with regard to information collection and offering loan concessions, servicers must hold risk positions in MBSs that include "vertical" components. If foreclosure is highly inefficient and information gathering is costly, it is in investors' interest to offer these risk positions only in exchange for servicer investment in the MBS. If servicers are unwilling to make sufficient investment, the servicer contract is not incentive-compatible; organizing mortgages into diversified MBS pools precludes renegotiation with borrowers in default. The "second-best" MBS design, a non-diversified mortgage pool, preserves pool-wide information useful for renegotiation with borrowers.

To obtain our results we analyze the joint problems of the design of mortgage-backed securities, servicer contracts and mortgage contracts. Our work is particularly relevant for the securitization of mortgages that exhibit significant default risk. Many of the early MBSs were formed by government sponsored entities (GSE's), such as Fannie Mae, that provide guarantees against default risk. However, by 2007 nearly 20% of outstanding mortgage credit was through non-agency

[1] www.realtytrack.com, posted on March 22, 2013.

securitization, without guarantees against default risk.[2] We have thus seen a large growth in MBSs that exhibit credit risk.

There are two primary contributions of this paper. First, we build on work by Kahn and Huberman (1989), Aghion, Dewatripont and Rey (1994) and Hart and Moore (1998) to determine optimal renegotiation policies with one borrower and one lender. The renegotiation design explicitly takes into account the trade-off for the lender between the benefit of avoiding costly foreclosure and the cost of potential wealth transfers to borrowers. Wealth transfers result from making concessions larger than the minimum necessary to avoid foreclosure and from borrowers strategically defaulting to obtain concessions. To simplify the analysis we assume a two-period debt model in which the underlying collateral (house) is the only asset that a lender can seize in the case of nonpayment. In this setting a borrower defaults if the value of the collateral has fallen below the required debt payment.[3] However, it is not so obvious what the borrower does if the collateral value is higher than the required payment. Strategic default in our model is defined as default that occurs not because the collateral value has dropped too low, but because a borrower is exploiting an information asymmetry in an attempt to obtain a concession from the lender.[4] We explicitly consider the connection between renegotiation policy and strategic default.

Our results link renegotiation, the likelihood of foreclosure and the cost of credit to the quality of information gathering. We demonstrate that if the lender gathers

[2] See Krainer (2009), page 2. Krainer also points out that non-agency mortgage securities differ further in that they are more likely to include subprime mortgages.

[3] Foote, Gerardi and Willen (2008) and Krainer and LeRoy (2010) provide evidence that many borrowers with mortgages in which the principal balance exceeds the house value do not default. Borrowers who have the cash to continue servicing their mortgages hold an option to default later. We assume away these dynamic aspects. We also ignore any costs to default that would lead a borrower to default only if the principal is some fixed amount above the house value. Adding these complexities will not change our main qualitative results.

[4] The term "strategic default" is often used in practice to refer to default by a borrower who has sufficient cash flow to make required payments, but chooses not to. The term is also used when discussing concerns about loan renegotiations encouraging default. Our use of the term is most consistent with this latter usage.

no information following borrower default, the lender would like to commit to no loan renegotiation. If the pre-commitment can be made, then only nonstrategic default occurs, and all defaults result in foreclosure. If the lender gathers some information following default, the lender renegotiates with defaulting borrowers. The extent of renegotiation is increasing, the incidence of foreclosure and strategic default are decreasing, the availability of credit is increasing, and the cost of credit is decreasing in the quality of information gathering.

The second and principal contribution of the paper is to introduce a servicer, the investors' agent, into the model; MBS investors cannot observe servicer actions. By compensating the servicer with a share of MBS proceeds, servicer incentives can be aligned with investor interests. The incentive-compatible contract may be nonlinear; it may include a cut-off value for the MBS proceeds below which the servicer receives no compensation, but does not approximate a horizontal first-loss position. For a single mortgage the cut-off must be set low enough that servicer expected compensation exceeds the expected cost of exerting effort. The servicer moral hazard problem is thus costly for the investor. This cost results from the interaction of the servicer's limited liability (servicer compensation cannot be made negative if MBS proceeds are small) and the need to provide incentives for the servicer to both exert information gathering effort and set the correct loan concession after exerting effort. Pooling mortgages may ease the limited liability constraint and decrease this cost, but may also increase the cost as the timing of borrower defaults, followed by servicer action and information revelation, approaches sequential as compared to simultaneous.

For a wide range of parameter values it is in investors' interest to offer the servicer an incentive-compatible contract only if the servicer is willing to make a sufficient investment in the MBS in exchange for the contract. If the servicer is unwilling to do so, and mortgages are organized in diversified pools, investors optimally offer a servicing contract that precludes all loan renegotiation. Securitization becomes a no-renegotiation commitment device, and as a result, all defaults result in foreclosure. Alternatively, investors may form non-diversified pools of mortgages. In contrast to a diversified pool where the servicer controls information gathering, investors are able to obtain (or verify) pool-wide information relevant for all mortgages of a non-diversified pool. Making renegotiation decisions based on pool-wide information is not as efficient as making such decisions based on mortgage-specific information, but it is more efficient than not having any renegotiation-relevant information. If renegotiation decisions cannot be made based on mortgage-specific information, the availability of credit is greater (and the cost of credit lower) if mortgages are securitized into non-diversified, rather than diversified, pools.

We next expand the model to take into account the potential contagion effect in home mortgages: foreclosures adversely affect the value of other houses leading to more foreclosures. In the presence of contagion effects investors are better off if they can coordinate to limit foreclosures. Organizing mortgages into non-diversified MBS pools enhances the ability to achieve Pareto improving coordination.

Ours is the only work we know of that jointly considers the design of mortgage-backed securities, servicer contracts and the renegotiation of mortgage contracts. A key aspect in which our work differs from earlier studies of security design is that we analyze an agency problem that occurs *after* securitization takes place, instead of before. Wang, Young and Zhou (2002) model the loan renegotiation problem for a single borrower and lender, but with a somewhat different model and results. Their borrower has private information about her personal cost of default, rather than about collateral value. They find that an uninformed lender randomizes between foreclosure and modification of a loan in default, whereas our lender strictly prefers to foreclose when certain to be uninformed.[5] The method in

[5]Our paper also differs in that most of our solutions are perfect Bayesian equilibria without pre-commitment on the part of the lender. We require pre-commitment only for the pure no-renegotiation strategy, and we point out how securitization can enable this commitment.

which loans are renegotiated in our non-diversified MBS design is consistent with the Posner and Zingales (2009) automated loan modification plan based on average house prices within a zip code. Our incentive-compatible servicer contract shares some similarities with a servicer fee structure recommended by Mayer, Morrison and Piskorski (2009).

Hartman-Glaser, Piskorski and Tchistyi (2012) show that the optimal incentive-compatible contract that induces originators to screen borrowers prior to loan securitization pays a lump sum if no defaults occur prior to a pre-determined date; requiring the originator to retain a first loss piece approximates this contract.[6] Our servicer contract requires the servicer to retain part of the security, but holding the first-loss piece is generally not sufficient, because the servicer moral hazard problem differs from the originator problem: The originator takes actions prior to securitization and the consequences of those actions are revealed over time, whereas the servicer takes actions after securitization and after observing a number of defaults. After exerting effort the servicer must choose a loan concession that is in the investors' interest. Servicer actions should maximize expected cash flow following default without encouraging excess default.[7]

A number of papers, including Cordell, Dynan, Lehnert, Liang and Mauskopf (2008), Eggert (2004), and Pennington-Cross and Ho (2006), describe problems with current servicing arrangements and influence our choice of modeling assumptions. Prior research suggests gains from mortgage modification, impediments to modification, and strategic default with modification programs. For example, Cordell, et al (2008) state, "Given loss rates to investors from foreclosed subprime mortgages of 50 percent or more, both investors and borrowers could be better off with more effective loss mitigation" (p.3). Mayer, Morrison, Piskorski and Gupta

[6] Malamud, Rui and Whinston (2013) provide an extension and generalization of these results.

[7] DeMarzo (2005) and Riddiough (1997) also address a moral hazard in origination problem. DeMarzo considers different types of risk. He recommends nondiversification with respect to risks about which the intermediary is better informed. Riddiough (1997) finds that diversification is optimal.

(2011) present evidence that homeowners increase default rates in response to mortgage modification programs. Elul, Souleles, Chomsisengphet, Glennon, and Hunt (2010) find both borrower/collateral specific factors and factors common to subsets of borrowers with explanatory power for default rates.

There is conflicting evidence on the relation between securitization, foreclosure and loan modification. Piskorski, Seru and Vig (2010) find a higher rate of foreclosure on securitized delinquent loans compared to nonsecuritized loans. This result is stronger in periods of house price declines, consistent with our model of contagion. Adelino, Gerardi and Willen (2009) find that securitized mortgages are not modified less than nonsecuritized mortgages. Agarwal, Amromin, Ben-David, Chomsisengphet and Evanoff (2011), however, find that bank-held loans are more likely to be renegotiated than securitized loans. Ghent (2011) finds that mortgages were rarely modified during the great depression (1929 to 1935). Most mortgages in this period were not securitized, but there was a federal refinancing program that may have discouraged lenders from offering concessions.[8] Finally, there is empirical evidence relating securitization design to loan performance. Ambrose, Sanders and Yavas (2010) find that foreclosure is less likely if commercial mortgage-backed securities (CMBS) are less diversified across property types. Loutskina and Strahan (2011) find empirical evidence that, compared to lenders with diversified pools, mortgage originators who concentrate tend to have higher profits.

2 Optimal renegotiation of mortgage contracts with one borrower and one lender

In this section we develop basic results on mortgage contract renegotiation. Renegotiation involves a cost-benefit tradeoff for the lender: the benefit is avoidance of costly foreclosure; the cost is a wealth transfer to the borrower. The cost is

[8] Also, more than half of the loans in the sample, and a preponderance of loans that went into foreclosure, were held by insurance companies that neither originated nor serviced the loans.

larger if renegotiation encourages strategic default. The main result of this section is that loan renegotiation is strictly Pareto improving only if the lender gathers information about the borrower, subsequent to the original contracting. If there is no information gathering, the lender optimally refuses to renegotiate. We also show that the availability of credit is increasing in the quality of post-contracting information gathering.

The model is one of incomplete collateralized debt contracts with renegotiation. Incompleteness in mortgage contracts is due to the inability of contracting players to write enforceable contracts on relevant information such as the borrower's available cash flow and collateral value. In some cases this inability is due to asymmetries of information between the borrower and lender; but even with no information asymmetries, the information cannot be verified by a third party to make such contracts enforceable.[9] Because contracts are incomplete, both parties may wish to renegotiate in the future. Aghion, Dewatripont and Rey (1994) pointed out that if the original contract specifies how renegotiation will work, renegotiation may be beneficial. Their solution assigns all bargaining power in renegotiation to one party and specifies a default outcome if renegotiation fails. We follow this strategy in our model of mortgage contracts. Transfer of collateral, i.e., foreclosure, is the default outcome in case renegotiation fails.

There are two time periods, 0 and 1. At time 0 the initial contract is signed. The contract specifies the time 0 amount loaned and the borrower's time 1 promised payment to the lender. We use the notation r_0 for this promised payment to indicate that it is based on the time 0 contract. The contract specifies consequences if payment is not made. The contract may also specify how renegotiation may be done. $\tilde{v}$ is the time 1 value of the collateral (house). If foreclosure occurs at time 1, the lender realizes a payoff of δv, where $\delta \in (0, 1)$ is the foreclosure discount factor. The deadweight loss in foreclosure is thus $(1 - \delta)v$.

The contract cannot be written on the realization of $\tilde{v}$. Even if both the bor-

[9] Our notion of incompleteness is the same as that used by Hart and Moore (1988).

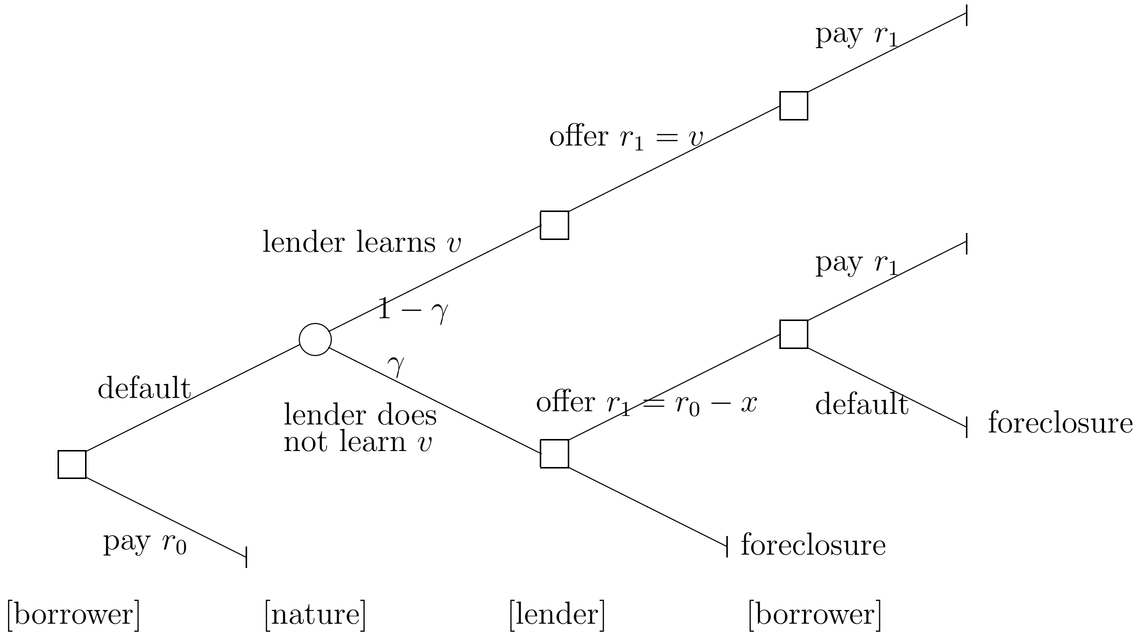

Figure 1: The time 1 game tree

rower and lender observe this realization, a third party cannot verify it. The model looks somewhat like Townsend (1979), but with the following differences: i) we assume inefficient collateral, i.e., the possibility of foreclosure where some value is lost; ii) even if the lender can determine the borrower's cash flow, a contract cannot be written on this amount, and the lender cannot take possession of an individual borrower in the way that a lender can take possession of a firm. We thus ignore cash flow altogether because in a two period model it is irrelevant. The model is also similar to that of Hart and Moore (1998).[10]

At time 1 the borrower observes the realization v. Figure 1 illustrates the game played following this realization. The borrower moves first. If she makes the promised payment, r_0, she keeps the collateral, realizes a payoff of $v - r_0$ and the game ends. If the borrower defaults, the lender gathers information and with probability $1 - \gamma$ observes the realization v.[11] After realizing the information

[10]Hart and Moore have three periods, but in their model the collateral has no remaining value in the final period so payment is made only in one period as in our model.

[11]γ is an exogenously given parameter. In this section information gathering, of quality $1-\gamma$, is cost-free. Our objective here is to determine the value of information gathering.

gathering outcome, the lender decides whether to renegotiate with the borrower. In renegotiation the lender makes a take-it-or-leave-it offer, r_1, to the borrower. If the borrower accepts the offer, she pays r_1 to the lender. If the offer is refused, foreclosure occurs.[12] The lender has all of the power in the renegotiation bargaining game: If the lender has observed v, the lender makes a renegotiated offer of $r_1 = v$ and the borrower accepts the offer.[13] If the lender has not observed v, the lender either forecloses or makes an offer $r_1 = r_0 - x$, where x may be positive or negative. The borrower accepts the offer and pays r_1, or refuses, resulting in foreclosure.

The borrower's last decision in Figure 1 is automatic: if $v \geq r_1$, the borrower pays r_1. Otherwise, she rejects the offer and foreclosure occurs. The borrower's first decision is not automatic. A borrower who realizes $v < r_0$ defaults, because the worst default outcome is a zero payoff. A borrower with a realization $v > r_0$ may pay r_0 or default in the hope of a renegotiated offer less than r_0. We refer to such a default as a "strategic default".[14]

We begin with two polar cases: first, the lender always learns a defaulting borrower's realized collateral value v ($\gamma = 0$), and second, the lender has zero probability of obtaining any information beyond the prior distribution on $\tilde{v}$ ($\gamma = 1$). If the lender knows the realization v, the lender sets $r_1 \leq v$ and the offer is accepted. If $v < r_0$, following default the lender makes a concession: $r_1 < r_0$. If $v > r_0$, the lender imposes fees on the borrower following default: $r_1 > r_0$. With an informed lender, a borrower cannot gain from strategic default. If $\gamma = 0$, all defaults are nonstrategic and there is no foreclosure.

[12] In practice borrowers have some bargaining power in that legal systems in many US states prevent foreclosure from taking place immediately. The model can be expanded along the lines of Hart and Moore (1998) so that with probability $1 - b_L$ the borrower has the bargaining power in renegotiation. In the interest of succinctness, we assume b_L =1.

[13] In practice, lenders charge fees to extract surplus from defaulting borrowers. Regulation may cap such fees. We later explain why a cap on fees does not qualitatively change our results.

[14] In practice the term strategic default is applied to cases in which borrowers have sufficient cash to pay the contracted amount, but choose not to pay. We use the term for the case in which a borrower behaves strategically, using an information asymmetry to increase her expected surplus.

2.1 The lender never gathers information following default

Suppose now that $\gamma = 1$: the lender is certain to be uninformed. Consider a borrower with a realization $v > r_0$. If this borrower makes the promised payment, her payoff is the surplus $v - r_0 > 0$. If the borrower defaults, a strategic default, then either the borrower receives and accepts a renegotiation offer, or foreclosure occurs. If there is a positive probability of a renegotiation offer with a lower payment and a positive probability of foreclosure, the borrower defaults only if the expected gain from a lower payment in renegotiation is greater than the expected loss from foreclosure. The expected loss from foreclosure is increasing in the collateral value, v, of the borrower. The following lemma demonstrates that there exists a "default cut-off value", v^D, that determines the borrower's default strategy.

Lemma 1. i) There exists a "default cut-off value", $v^D \geq r_0$, such that any borrower with realization $v < v^D$ defaults and any borrower with realization $v > v^D$ does not default. ii) If $\gamma = 1$ (the lender has zero probability of learning v following a default) and there is any possibility of successful renegotiation with a lowered payment, then $v^D > r_0$, and some strategic default occurs.

Proof: *See the Appendix.*

The default cut-off value, v^D, is a function of the lender's information gathering and renegotiation policy. If the lender is certain to not learn a defaulting borrower's collateral value ($\gamma = 1$), then any possibility of a renegotiated offer with a lower payment ($r_1 < r_0$) leads to a positive probability of strategic default (i.e., v^D is strictly greater than r_0). The proof of Lemma 1 shows that the lender can decrease the likelihood of strategic default by randomizing in renegotiation. Randomizing in renegotiation introduces the possibility of foreclosure, which reduces the expected payoff to borrowers who default. Borrowers with collateral values only slightly above the promised payoff have little to lose and so engage in default as long as loan renegotiation is at all possible.

We now determine the equilibrium outcome for the case in which the lender has zero probability of learning the collateral value of a defaulting borrower ($\gamma = 1$). When deciding whether to make a renegotiation offer and what offer to make, the lender faces a trade-off. If the lender offers r_1 higher than v, the borrower refuses the offer, foreclosure occurs and the lender receives δv. If the lender offers r_1 lower than v, foreclosure is avoided, but the lender does not extract all of the surplus from the borrower. In addition, any offer $r_1 < r_0$ encourages strategic default.

The following proposition indicates that, unless the promised payment, r_0, is high relative to possible collateral values, the lender's cost of strategic default overwhelms the renegotiation benefit. For most of the ensuing analysis collateral value is assumed to be uniformly distributed:

$$\tilde{v} \sim U[v_0 - \Delta, v_0 + \Delta] \quad \text{and} \quad v_0 = \Delta\,. \tag{1}$$

The uniform distribution assumption is useful in that it provides intuitive results, given the nature of our problem which involves truncations of probability distributions at the default thresholds.[15] The equilibrium concept employed throughout the paper is the perfect Bayesian equilibrium (PBE). We solve for the PBE under two different assumptions: the lender is able to pre-commit to a renegotiation policy at time zero and the lender is not able to pre-commit. The latter assumption is consistent with the game tree of Figure 1.

Proposition 1. Suppose the lender has zero probability of learning the collateral value of any borrower who defaults ($\gamma = 1$). There exists a "renegotiation cut-off value",

$$r^{noInf} = \frac{v_0 + \Delta}{2 - \delta} = \frac{2\Delta}{2 - \delta}, \tag{2}$$

[15] Setting the lower bound of the distribution to zero greatly simplifies the math and is done for the most part without loss of generality. Many of the results of this section have been obtained for a triangular distribution. We do not include these derivations as they are much more complicated than those employing the uniform distribution, and the main qualitative results are the same.

such that the lender's bond value is maximized if the required payment is equal to r^{noInf}.

i) If $r_0 > r^{noInf}$, the lender's PBE strategy, with and without pre-commitment, is to offer $r_1 = r^{noInf}$ to all borrowers.

ii) If $r_0 \leq r^{noInf}$, the lender, if able to pre-commit to a renegotiation policy, commits to no renegotiation: all defaults are nonstrategic and result in foreclosure. If the lender is unable to pre-commit, there does not exist a pure strategy PBE. The lender is ex ante better off if able to pre-commit to no renegotiation.

Proof: *See the Appendix.*

If the original payment, r_0, is set too high, a lender who is certain to be uninformed offers a lower payment to any defaulting borrower. Given this policy, all borrowers default to obtain a lower payment. The lender thus offers the lower payment to all borrowers. If r_0 is below the renegotiation cut-off value, the lender would like to commit to a no renegotiation policy, because the cost of strategic default overwhelms any renegotiation benefit. If borrowers expect every default to lead to foreclosure, no borrowers default strategically. But, if the lender makes the renegotiation decision only after a default has taken place, then a lender who believes the default is nonstrategic offers a renegotiation to avoid costly foreclosure. *In the absence of information gathering, the only way to achieve the outcome the lender prefers is for the lender to pre-commit to a no-renegotiation policy.*[16]

The following Corollary follows directly from Proposition 1 and the logic that if the lender offers the cut-off value r^{noInf} to all borrowers whenever r_0 is greater than r^{noInf}, then the initial promised payment is not set greater than r^{noInf}.

Corollary 1. If the lender is certain to be uninformed ($\gamma = 1$) and the lender is able to pre-commit to a no-renegotiation policy, the initial mortgage contract calls for a required payment $r_0 \leq r^{noInf}$ and the lender pre-commits to no renegotiation.

[16] There does exist a mixed strategy PBE in the case of no commitment. We show, however, that the lender strictly prefers the outcome that follows from a pre-commitment to not renegotiate. This result is in contrast to Wang, Young and Zhou (2002).

Only nonstrategic defaults occur, and all defaults result in foreclosure.

We next determine the loan proceeds, given a promised payment of r_0 and no information gathering. We assume many potential lenders compete to loan money at time zero; the lender thus loans an amount equal to his time zero expected bond value. If no information is gathered following a default, the bond value is:[17]

$$B^{noInf} \;=\; r_0\left(1 - \frac{(2-\delta)r_0}{4\Delta}\right) < r_0\,. \tag{3}$$

If foreclosure is fully efficient ($\delta = 1$), the maximum possible value of B^{noInf}, obtained by setting $r_0 = r^{noInf}$, is v_0. That is, if $\delta = 1$, the borrower can borrow as much as the expected value of the house, v_0.[18] If $\delta < 1$, the most she can borrow is strictly less than v_0. In addition, the maximum loan amount is decreasing in $1-\delta$, the proportion of the house value that is deadweight loss in foreclosure.

We have so far established that the lender always renegotiates with defaulting borrowers if the lender is informed, and prefers to never renegotiate if certain to be uninformed. In both of these polar cases, only nonstrategic defaults occur.

2.2 The lender becomes informed with probability $1-\gamma$ following default

We now examine the intermediate case: following a default the lender learns v with probability $1-\gamma$, where $0 < \gamma < 1$. Referring to Figure 1, if the lender learns the defaulter's value v, the lender demands this full value. Given a strategic default, the lender demands and receives an amount greater than r_0; in practice this means charging fees to borrowers who default. If the lender does not learn the defaulter's value, the lender can either foreclose without renegotiating, or renegotiate by offering a new payment, $r_1 = r_0 - x$. Suppose the lender renegotiates when uninformed. Any borrower who considers strategically defaulting faces a trade-off. With probability $1-\gamma$ she loses her surplus, $v - r_0$; with probability γ she

[17] B^{noInf} is obtained by inserting r_0 into equation (24) in the proof of Proposition 1.

[18] In the parlance of mortgage loans, the loan-to-value ratio can be as high as one. This result was obtained by combining equations (1), (2) and (3).

increases her surplus by x. If x is nonnegative, the indifference point for this trade-off occurs for a collateral value equal to $r_0 + \gamma x/(1-\gamma)$. This means that any borrower with a realized collateral below the following default cut-off value defaults:[19]

$$v^D = \min \left[r_0 + \frac{\gamma x}{1-\gamma} \, , \, 2\Delta \right]. \tag{4}$$

Any borrower with a realized collateral value above v^D does not default. From equation (4), as the quality of information gathering improves (γ decreases), the default cut-off value decreases and approaches r_0. For any borrower considering a strategic default, the prospect of being discovered and losing one's surplus, deters the act of strategic default.

We now discuss further the lender response to borrower default. In practice, lenders may be limited in the surplus they can extract from defaulting borrowers. There may be a cap on the fees charged following a default. As long as the cap is greater than $v^D - r_0$, the cap does not affect our results. If the cap is binding (less than $v^D - r_0$), the qualitative results still hold, but the lender offers a smaller concession, fewer strategic defaults occur and a greater proportion of defaults end in foreclosure. We proceed assuming that if there is a fee cap, it is greater than $v^D - r_0$, which for our analysis is equivalent to no cap at all.

The following proposition indicates that, if there is a positive probability that the lender becomes informed following a default ($\gamma < 1$), the lender offers a strictly positive renegotiation concession when uninformed.

Proposition 2. If, γ, the probability the lender is uninformed is strictly less than one, the perfect Bayesian equilibrium is as follows: When informed the lender

[19] 2Δ is the upper bound of the distribution on $\tilde{v}$.

offers $r_1 = v$. When uninformed the lender offers $r_1 = r_0 - x^*$, where[20]

$$x^* = \frac{(1-\gamma)(1-\delta)r_0}{1+(1-\gamma)(1-\delta)} \; > 0 \,. \tag{5}$$

The default cut-off value is

$$v^D(x^*) = r_0 + \frac{\gamma(1-\delta)r_0}{1+(1-\gamma)(1-\delta)} \; = \; \frac{(2-\delta)r_0}{1+(1-\gamma)(1-\delta)} > r_0 \,. \tag{6}$$

The likelihood of both strategic default and foreclosure are decreasing in $1-\gamma$.

Proof: *See the Appendix.*

The concession x^* is the lender's optimal choice with and without pre-commitment. It follows from equation (6) that necessary and sufficient conditions for strategic default are inefficient foreclosure ($\delta < 1$) and a positive probability that the lender remains uninformed ($\gamma > 0$). Strategic default occurs if the lender is willing to renegotiate ($x^* > 0$) and the borrower has the possibility to exploit private information. Proposition 2 further indicates that better information gathering (smaller value of γ) leads to less strategic default and less foreclosure.

We can now relate the original mortgage contract terms to the quality of the lender's information gathering. The lender's time zero bond value and thus the amount the lender is willing to loan, if information is obtained with probability $1-\gamma$ is:[21]

$$B^\gamma \;=\; r_0\left(1 - \frac{(2-\delta)r_0}{4\Delta(1+(1-\gamma)(1-\delta))}\right) = r_0\left(1 - \frac{(2-\delta)(r_0 - x^*)}{4\Delta}\right) . \tag{7}$$

B^γ is the expected value of loan proceeds in the second-best solution. If foreclosure is fully efficient ($\delta = 1$) or the lender is certain to be informed ($\gamma = 0$), equation (7) simplifies to the first-best bond value:

$$B^{FB} = r_0\left(1 - \frac{r_0}{4\Delta}\right) . \tag{8}$$

[20]For equations (5), (6) and (7) we assume that $r_0 \leq r^{noInf}$. If $r_0 > r^{noInf}$, then $r_0 - x^* = r^{noInf}$. The assumption that $r_0 \leq r^{noInf}$ also ensures that the upper boundary in (4) is not strictly binding.

[21]See the Appendix. This expression assumes zero cost of information gathering.

The following corollary presents some characteristics of the second-best bond value.

Corollary 2. Assume foreclosure is inefficient ($\delta < 1$) and $\gamma \in (0, 1)$. Given a promised payment r_0, loan proceeds are strictly less than the first-best bond value and strictly greater than loan proceeds with no information gathering: $B^{noInf} < B^{\gamma} < B^{FB}$. B^{γ} is increasing in the quality of information gathering $(1 - \gamma)$.

Proof: *Follows from equations (3), (7) and (8).*

Up to this point we assume information gathering is costless. The following corollary presents the highest information gathering cost such that the lender gathers information.

Corollary 3. If the cost of information gathering is not greater than c_{max} where

$$c_{max} = \frac{(1-\gamma)(1-\delta)r_0}{2(1+(1-\gamma)(1-\delta))} = \frac{x^*}{2}\,, \tag{9}$$

then equations (5) and (6) describe a perfect Bayesian equilibrium (PBE) for the borrower and lender at the time the loan is due.

Proof: *See the Appendix.*

c_{max} can be thought of as the ex post (after default) value of information gathering. The ex ante value of information gathering can be represented as:

$$B^{\gamma} - B^{noInf} = \frac{(2-\delta)r_0 x^*}{4\Delta} \tag{10}$$

where B^{noInf}, given in equation (3), is the value of B^{γ} with $\gamma = 1$. The ex ante value of information gathering is decreasing in both γ and δ: better information (smaller γ) and information gathering are more valuable if foreclosure is more inefficient (δ smaller).[22]

Discussion of one borrower/one lender results: If it is certain that the lender becomes informed following a default, all defaults are nonstrategic and

[22] $prob\{\text{default}\} \times c_{max} \leq B^{\gamma} - B^{noInf} \leq c_{max}$. The second inequality holds if $r_0 \leq r^{noInf}$.

successful renegotiation follows each default. If it is certain that the lender remains uninformed following a default and the lender is able to commit to a no-renegotiation strategy, foreclosure follows each default. Information gathering adds value, even if imperfect. Common knowledge that the lender attempts to obtain information creates a credible threat that limits strategic default, making it profitable for the lender even when he remains uninformed to offer a concession to a defaulting borrower. Information gathering is valuable for both the lender and borrower. For the lender it directly increases the expected payoff following default and indirectly limits the incidence of strategic default. For the borrower it increases the availability and decreases the cost of credit.

If the post-default information gathering cost is less than its value, information gathering should occur. However, if a mortgage has been securitized, so that the funding and servicing are unbundled, information may not be gathered. We now examine whether securitization can be designed to facilitate information gathering.

3 Mortgage pooling and servicer contracts

In the previous section we determine two types of cut-off values that play key roles in the mortgage problem. For borrowers the default cut-off value is v^D. Any borrower with a realized collateral value below v^D defaults; any borrower with a value above v^D does not. For lenders the renegotiation cut-off value is r^{noInf}. If the lender is certain not to learn a defaulting borrower's collateral value, and r_0 is greater than r^{noInf}, the lender offers $r_1 = r^{noInf}$ to all borrowers. In the previous section we assume the lender never agrees to an original promised payment greater than r^{noInf}, leading to our result that the lender strictly prefers no renegotiation in the absence of information gathering. In this section we apply the concept of r^{noInf} to the case in which a lender does not gather information about individual borrower collateral values, but does obtain information applicable to an entire pool of mortgages. Before doing so, however, we first extend the basic model to analyze servicer contracts.

3.1 Servicer contracts: single mortgage

Once a loan has been securitized the lender, now called the investor, no longer interacts directly with the borrower. All renegotiations are carried out between a "servicer", the investor's agent, and a borrower. At time zero the borrower enters into a mortgage contract.[23] The borrower receives an amount of money, B, and in exchange promises to either pay r_0 or forfeit the collateral at time one. As in the previous section, at time one the borrower observes the outcome v and pays r_0 or defaults. What happens following a default now depends on the nature of the contract between the servicer and the investor.

We now assume the following: i) The servicer can, at cost c, gather information about a borrower's realized collateral value, v. If c is expended, with probability $1-\gamma$ the servicer observes v. ii) The cost c is less than c_{max}, given in equation (9): if servicer incentives align with investor interests, the servicer gathers information. iii) The investor cannot observe servicer actions.[24] iv) The investor observes the total revenue from all borrower payments and all proceeds from foreclosures.

The investor decides whether to assign renegotiation authority to the servicer, and if so, specifies servicer compensation as a function of observable outcomes. Since renegotiation without information gathering is contrary to investor interests, it is also contrary to investor interests to delegate renegotiation authority to the servicer without providing sufficient incentive for the servicer to gather information following a default. We first determine the nature of the least cost contract that induces the servicer to engage in information gathering following a

[23]There are no information asymmetries at time zero. We assume away any problems due to the unbundling of origination and funding of mortgages, so that we can focus on problems stemming from the unbundling of servicing and funding.

[24]According to Federal Housing Finance Agency (2011), private label security "investors do not receive loss mitigation reports, and do not have the right to review the servicer's loss mitigation decisions" (p.28).

default, and to offer the concession x^* when uninformed. We next compare the investor's expected payoff under this incentive-compatible (IC) contract to his expected payoff if renegotiation authority is not delegated and all defaults result in foreclosure. We then present a condition such that the latter (non-IC) contract dominates the IC contract, from the investor's perspective.

In an IC contract, the agent's (servicer's) wages should be conditioned on something that the principal (investor) observes and is most closely related to the actions of the agent. The investor observes the total revenue of the MBS, and may also observe the numbers of defaults and foreclosures. However, paying the servicer a higher amount for fewer foreclosures results in perverse incentives. For example, to minimize the number of foreclosures the servicer can offer large renegotiation concessions without expending resources to gather information; such actions are not in the investor's interest. In what follows the contract may be conditioned on both default numbers and total revenue, or just on total revenue.

An IC contract must provide the servicer with incentives to take two actions following a default: expend c to gather information and offer the concession x^* if information gathering fails. We restrict our analysis to piece-wise linear contracts: most contracts in practice are of this form. In addition, because we have assumed risk neutrality and a uniform distribution on the stochastic asset value, we do not expect a more complex functional form to dominate a piece-wise linear contract. We first consider each action separately. We say that a contract "pays no excess" if the servicer expected payoff from gathering information about a defaulted loan is exactly the cost c.

Lemma 2. Consider a single securitized mortgage.

i) Suppose the cost of information gathering, c, is zero. There exists a servicer contract that pays no excess and induces the servicer to offer to a borrower in default the concession x^* given in equation (5), if information gathering fails.

ii) Suppose the cost of information gathering is positive ($0 < c \leq c_{max}$), and the investor can compel the servicer to offer $r_1 = v$ if informed and $r_1 = r_0 - x^*$ if uninformed. There exists a servicer contract that induces the servicer to expend c to gather information and pays no excess.

Proof: *i) See below. ii) See the Appendix.*

The contract in part i) of Lemma 2 is quite simple. The servicer receives an amount $\varepsilon > 0$ if the realized cash flow from the defaulted mortgage is at least $r_0 - x^*$, and zero otherwise. ε can be allowed to approach zero and the servicer still has positive incentive to set $x = x^*$ if uninformed. The contract in part ii) of Lemma 2 is a piece-wise linear contract that pays the servicer a fraction z of all cash flow above the cut-off $r_0 - x^*$ following a default, and nothing otherwise. The fraction z is chosen such that the servicer has the incentive to expend c to gather information following a default, and his expected wage is equal to c. Lemma 2 indicates that the investor need not pay excess to the servicer to induce him to take one action, expend c or offer the concession x^*. In contrast, the following proposition indicates that to induce both actions, the IC contract provides the servicer excess expected compensation.

Proposition 3. Consider a single securitized mortgage. To induce the servicer to expend c to gather information *and* offer concession x^* if uninformed, the least-cost contract pays the servicer a fraction z^* of all cash flow above the cut-off $\delta(r_0 - x^*)$. The expected value of this cash flow share is strictly greater than the cost c.

Proof: *See the Appendix.*

Proposition 3 indicates that the IC contract may be nonlinear, but within limits.[25] If the cut-off, below which the servicer receives nothing, is strictly greater than $\delta(r_0 - x^*)$, the servicer offers a concession strictly less than x^*. In addition, because of the servicer's limited liability constraint, the servicer's expected payoff is strictly positive even if no effort is exerted. As a result, the expected wage for

[25]We thank Tomasz Piskorski for suggesting that we consider nonlinear contracts.

the servicer who expends c to gather information is strictly greater than c.

3.2 Servicer contracts: pooled mortgages

In Proposition 3 it is shown that for a single mortgage the servicer's limited liability constraint makes it impossible to design an incentive compatible contract that does not pay excess to the servicer. For pooled mortgages both the fraction of cash flow assigned to the servicer, and the cut-off, below which the servicer receives no compensation, may be defined for the entire mortgage pool. As such, pooling may help to relax the servicer's limited liability constraint. This effect is pointed out by Laux (2001) who shows how combining multiple projects relaxes an agent's limited liability constraint. Hartman-Glaser, Piskorski and Tchistyi (2012) also show how pooling can lower the cost of an incentive compatible contract for a mortgage originator.

The servicer contract problem, however, differs from these other contract problems because of the timing of actions and information revelation. Whether pooling can ease the servicer's limited liability constraint depends on this timing. We examine both "sequential" and "simultaneous" timing of mortgage defaults and default resolution. In the sequential model the servicer sequentially makes a decision for each defaulted loan regarding information gathering and the loan concession. The servicer observes the outcome of decisions with respect to a given loan before making decisions for the next defaulted loan. In the simultaneous model all pooled loans with realized values of $\tilde{v} < v^D$ default simultaneously and the servicer simultaneously makes information gathering and loan concession decisions for all defaulted loans. Although in practice servicers make decisions in environments that are hybrids of these two models, we focus on the two extremes, sequential and simultaneous, to better understand potential benefits, or costs, of pooling for the servicer contract problem.

Under the sequential model pooling mortgages does not relax the servicer's limited liability constraint. In Proposition 3 the least cost IC contract pays nothing if the realized cash flow for a defaulted loan falls below $\delta(r_0 - x^*)$, and a fraction z of all cash flow above the cut-off $\delta(r_0 - x^*)$, where z is set such that the incentive compatibility constraint for information gathering is just satisfied. Suppose that with pooled mortgages the cut-off below which the servicer receives nothing is set at $N_T^D \times \delta(r_0 - x^*)$, where T is the ending date for the sequential model and N_T^D is the realized number of loans that default. If at any time $t < T$ the total cash flow obtained from defaulted loans falls below $N_t^D \times \delta(r_0 - x^*)$, the servicer does not gather information for the next loan that defaults, because the servicer's cost of information gathering exceeds his expected payoff, and the servicer sets a concession for that loan that is less than x^*. To ensure that the servicer collects information for each defaulted loan the cut-off below which the servicer receives no payment must be specified on a per loan basis; that is, the loans must be treated as if they are not pooled. Pooling thus provides no benefit if we believe that the sequential model best describes the servicer contract problem.

In the simultaneous model the servicer makes information gathering and loan concession decisions before observing outcomes for any defaulted loans in the pools. Simultaneity allows for the possibility that the effect of limited liability is eased. To determine whether it is possible to eliminate all excess payment in an IC contract we make further idealized assumptions: we assume the pool is fully diversified (collateral values are independently distributed) and the number of mortgages in the pool, N, approaches infinity. These assumptions allow for the maximum possible benefit of pooling in the servicer contract problem. In the following lemma we show that even with this combination of idealized assumptions, if foreclosure and information gathering are sufficiently inefficient (δ small and γ large), the IC contract must pay excess to the servicer.

Lemma 3. Assume that servicing decisions are made "simultaneously", the number of mortgages in the pool approaches infinity and collateral values are independently distributed. The following condition is necessary such that there exists a piece-wise linear incentive-compatible servicer contract that does not pay excess:

$$(1-\gamma)(2-\delta)/2+\gamma\delta \geq 1/2 \tag{11}$$

Proof: *See the Appendix.*

In order to avoid paying excess to the servicer the cutoff, below which the servicer receives no payment, must be set high enough so that the servicer expects zero payment if he does not expend c to gather information about defaulted mortgages. However, in order for the contract to be incentive compatible with respect to setting the concession x^*, the cut-off must not be too high. If the servicer does not gather information and simply offers concession x^* to all borrowers in default, the average payoff for loans in default is $(r_0 - x^*)/2$.[26] Setting the cut-off equal to $(r_0 - x^*)/2$ ensures no excess payment. If, however, condition (11) is not satisfied, then this cut-off is too high for the contract to be IC.[27]

Under the assumptions of simultaneity and full diversification, pooling mortgages eases the servicer's limited liability constraint. For pooled mortgages the negative wages the servicer receives for underperforming mortgages are balanced by positive wages for mortgages that perform above the cut-off. This averaging enables the investor to set an average (per mortgage) cut-off that is higher than with a single mortgage. However, as Lemma 3 indicates, even under highly idealized assumptions, there exist parameter values such that an IC servicer contract cannot be offered that does not pay excess. Notably this occurs for parameter values such that foreclosure is highly inefficient (δ small) and thus proper servicing is more valuable.

The assumptions of simultaneity and full diversification are, we believe, rather unrealistic assumptions. If we do away with these assumptions and allow the pool to increase in size, a contract that is incentive compatible approaches a linear contract: the average per loan cut-off below which the servicer receives no

[26] See the proof of Lemma 3.

[27] If $\delta = 0$, $\gamma > 1/2$ is sufficient such that (11) is not satisfied; If $\delta = 0.25$, $\gamma > 0.6$ is sufficient.

payment approaches zero. In the following section we examine the linear contract for a pool of mortgages and allow the servicer to make a side payment.

3.2.1 A linear contract with side payments

In a linear contract the servicer receives fraction z of the mortgage pool cash flow. Offering concession x^* when uninformed is an equilibrium action for the servicer. The servicer expends c to gather information about a defaulted loan if z satisfies the following IC constraint:

$$z(1-\gamma)\Big(E[\tilde{v}|\tilde{v} < v^D] \; - \; (r_0 - x^*)prob\{\tilde{v} \geq r_0 - x^*|\tilde{v} < v^D\} \; - \\ \delta E[\tilde{v}|\tilde{v} < r_0 - x^*]prob\{\tilde{v} < r_0 - x^*|\tilde{v} < v^D\}\Big) \; \geq \; c \quad (12)$$

Continuing with the uniform distribution of equation (1) and satisfying the IC constraint with equality, we obtain the IC fraction that the servicer must hold:

$$z^* = \frac{2(1+(1-\gamma)(1-\delta))c}{(1-\gamma)(1-\delta)r_0} \; = \; \frac{2c}{x^*} \; = \; \frac{c}{c_{max}} \quad (13)$$

Given the assumption of efficient information gathering in the absence of a servicer moral hazard problem ($c \leq c_{max}$), $z^* \leq 1$. The individual rationality (IR) constraint requires that the expected servicer compensation be no less than the expected cost of servicing: $Nc \times prob\{\text{default}\}$, where N is the number of mortgages in the MBS. Consistent with Proposition 3, the IR constraint is nonbinding and the IC constraint is strictly binding. We define W^{IR} as the excess expected value of the contract:[28]

$$W^{IR} \equiv z^* N B^{\gamma} - Nc \times prob\{\text{default}\}\,, \quad (14)$$

where B^{γ} is the expected value of the mortgage bond with information gathering, given in equation (7). Because W^{IR} is strictly positive, the servicer is paid

[28]We assume here that because the contract offers an excess payment, the contract must be written on all loans in the pool, not just those that default. Otherwise, the servicer has incentives to encourage excess default.

excess to service the security if given the share z^* of the MBS. A fixed negative component can be added to the contract to apply this incentive scheme without excess expected wages. In practice the servicer invests an amount W in exchange for the servicing contract.

We now determine the minimum value of W acceptable to the investor. The value of the entire MBS absent an IC servicer contract is NB^{noInf}, where B^{noInf}, given in equation (3), is the expected mortgage bond value with no information gathering. The investor offers an IC contract if the servicer is willing to pay at least W^{ask} for share z^*, where:

$$W^{ask} \equiv \max\left[0,\ z^* N(B^\gamma - \psi) - N(B^\gamma - B^{noInf})\right] < W^{IR}\,. \tag{15}$$

The assumption $c \leq c_{max}$ ensures that $W^{ask} < W^{IR}$.[29] If the value to the servicer of the share z^* is at least W^{ask}, the servicer is willing to pay the investor to hold an IC share of the MBS, resulting in information gathering and renegotiation of defaulted mortgages. We assume, however, that the servicer values this share at $\beta \times W^{IR}$ where $\beta \in (0, 1)$. The servicer applies a discount factor to the net value of his investment in the MBS.[30] If this discount factor is too small ($\beta \times W^{IR} < W^{ask}$), the investor does not offer the servicer an incentive-compatible contract, but rather a contract that discourages renegotiation with borrowers who default. Securitization becomes a device to pre-commit to a no renegotiation policy, consistent with the results of Proposition 1.

Discussion of the MBS servicer contract: The servicer incentive-compatible (IC) compensation scheme can be referred to as a "vertical" risk-sharing scheme. If the MBS is divided into multiple tranches with different levels of risk, in a fully vertical (linear) risk-sharing contract the servicer holds a fraction z of each tranch.

[29] This is easily shown by applying equations (6), (9) and (10).

[30] As in DeMarzo (2005) and Hartman-Glaser, Piskorski and Tchistyi (2012) we assume that the intermediary is more impatient than the investor. We also assume that the servicer discounts the net value of the share z^*, not the gross amount. Discounting the gross amount would increase the parameter range in which the servicer is not offered an IC contract. Assuming the servicer is wealth constrained instead of applying a discount factor β leads to the same result.

While it may be possible under some conditions to design an IC contract that is not fully vertical, i.e., excludes the safest tranches, the servicer's risk position cannot consist only of a fully horizontal "first-loss" position. As demonstrated by Proposition 3, such a first-loss position does not induce the servicer to act in investors' interest.

The IC allocation fraction z^* is also a measure of the relative efficiency of information gathering: z^* is the ratio of the cost of information gathering to the value of information gathering. It is plausible, particularly in an economic downturn, that the real estate market becomes less liquid, collateral becomes more costly to evaluate (higher c) and foreclosure becomes more inefficient (smaller δ). It is under these conditions that the IC share of the MBS becomes large, and at the time of securitization, the investor requires a large side payment, W^{ask}. If the servicer does not sufficiently value an investment in the MBS, the investor will not offer an IC contract that the servicer will accept. Notably, this outcome occurs at times when foreclosure is costly (δ small) and thus active servicing is more valuable.

3.3 Design of MBS pools

We now extend our model by assuming "types" of loans, or more relevant to our analysis, types of underlying collateral assets.[31] There are N different types of loans and N loans of each type. Each loan has individual risk and type risk. Loans of identical type experience identical outcomes of type risk. Within each type the individual risks, conditional on the outcome of type risk, are independently distributed. Outcomes of type risk are publicly observed at little or no cost. Only individual borrowers, and possibly the servicer, can observe outcomes of individual risks. Loans are securitized into N MBSs, each containing N loans.[32]

[31] Loans of the same type may, for example, have collateral that is all within similar zip codes.

[32] Our objective is to evaluate different methods of securitization, rather than reasons for doing securitization. Parlour and Plantin (2008) present reasons for securitizing loans.

A diversified MBS contains one loan of each type. A non-diversified MBS contains loans of only one type. The investor learns the composition of the MBS pool; in the non-diversified design the investor knows the loan type.[33]

3.3.1 Diversified MBS

As demonstrated in Lemma 3, the diversified design has a potential benefit in that it may ease the effect of the servicer's limited liability constraint *if* the servicer makes decisions simultaneously. A disadvantage is that the investor depends on the servicer to gather borrower information. In theory an investor can obtain information about component loans. In practice, if the MBS is diversified there are so many types within the MBS that an investor is unable to identify defaulting borrower type.[34] The following result follows directly from Proposition 1 and the analysis of Section 3:

Corollary 4. If a MBS is fully diversified, $W^{ask} > 0$, and the servicer does not sufficiently value an investment in the MBS ($\beta W^{IR} < W^{ask}$), investors offer a servicer contract that discourages all renegotiation. All defaults result in foreclosure.

If the servicer does not sufficiently value investment in the MBS, the a priori value of each securitized mortgage in a diversified MBS is B^{noInf}. Securitizing mortgages in diversified pools when $\beta W^{IR} < W^{ask}$ effectively makes the mortgages renegotiation proof. As demonstrated in Section 2, doing so decreases the availability of credit and increases the cost of credit.

Policy implications: With a diversified MBS, the investor depends on the servicer for collateral value information. If an incentive-compatible (IC) contract cannot be offered to the servicer, it is in the investors' interest to require a no

[33] Alternatively, if it is costly to evaluate type, the investor only needs to expend this cost once in a non-diversified design, because all loans in the pool have the same type.

[34] At the end of this section we discuss the possibility of automating type-based renegotiations and the difficulties of doing so in a diversified MBS.

renegotiation policy. Suppose instead, perhaps as the result of a legal decision or government policy, that securitized loans in default must be renegotiated.[35] Given such a mandate, if servicers do not hold IC positions in MBSs, renegotiation leads to excess strategic default. Renegotiation without information gathering is not in the interest of the investor and results in a wealth transfer from the investor to borrowers. For an investor who anticipates such a mandate, the time 0 mortgage bond value is lower, resulting in a higher cost of credit for borrowers.

3.3.2 Non-diversified MBS

In a non-diversified MBS all mortgages share the same type risk. The advantage of this organizational form is that the investor can easily verify some information that is relevant for the entire pool of mortgages. This idea is related to the notion of "hard" versus "soft" information.[36] In the diversified MBS the relevant information for renegotiation is soft in that the investor is unable to verify the information. Forming a non-diversified MBS effectively hardens some of the information. Because type information is the same for all loans in the pool, the investor can verify this information for any borrower. Type information is only part of the information the investor would like to know before making a renegotiation offer. But, as we show below, investors and borrowers can be made better off with type-based renegotiation.

In the non-diversified design each MBS contains N loans of identical type. We continue to assume the investor cannot communicate directly with borrowers, or determine individual loan characteristics. We extend our example in which uncertainty about borrower collateral value is described by the following distribution:

$$\tilde{v}_i \sim U[v_0 - \Delta, v_0 + \Delta], \quad \Delta = v_0, \quad i \in \{1, 2, ..., N\}. \tag{16}$$

[35] Mian, Sufi, and Trebbi (2010) show that in a crisis politicians respond to constituents in terms of voting for legislation such as the Foreclosure Prevention Act. We are thus likely to see modification programs that are politically motivated rather than in the interest of investors.

[36] See Petersen (2004) for a nice description of hard versus soft information.

At time one each borrower i observes the realization of her collateral value, v_i. If the investor does not obtain type information, (16) continues to represent his time one uncertainty. We assume the type risk outcome is either *bad* or *good* and, conditional on the type outcome, individual realizations are independently, uniformly distributed:

$$\tilde{v}_i|_{bad} \sim U[0, 2(\Delta - \eta)], \quad \tilde{v}_i|_{good} \sim U[2\eta, 2\Delta], \quad \eta < \Delta. \tag{17}$$

If the investor learns the type information, his expected value of $\tilde{v}_i$ for each borrower in the pool shifts to $v_0 + u_T$, where $u_T \in \{-\eta, \eta\}$, and the size of the range of possible values decreases from 2Δ to $2(\Delta - \eta)$.

In the absence of full information about borrower collateral value, type information can be used to make a Pareto improving renegotiation offer. This statement follows from Proposition 1 which presents the concept of a renegotiation cut-off value, r^{noInf}. If the lender knows nothing beyond the original distribution on borrower collateral, the lender sets $r_1 = \min(r_0, r^{noInf})$. We assume no origination problems so $r_0 \leq r^{noInf}$, but after learning type information the renegotiation cut-off value may change. If the revised renegotiation cut-off value is smaller than r_0, the investor and borrowers are made better off with a renegotiated payment. The following Lemma is similar to Proposition 1.

Lemma 4. If a MBS is non-diversified, $W^{ask} > 0$, and the servicer does not sufficiently value an investment in the MBS ($\beta W^{IR} < W^{ask}$), there exists a renegotiation cut-off value that depends on the type outcome:

$$r^{bad} = \frac{2\Delta - 2\eta}{2 - \delta} \quad \text{and} \quad r^{good} = \frac{2\Delta}{2 - \delta} = r^{noInf}. \tag{18}$$

The investor's expected bond value is maximized if all borrowers in the pool have a required payment equal to r^T, $T \in \{bad, good\}$. The servicer is instructed to offer $r_1 = r^T$ if $r_0 > r^T$, and to not renegotiate if $r_0 < r^T$.

Proof: *Identical to the proof of Proposition 1.*

Following from Proposition 1, the renegotiation offer described in Lemma 4 is Pareto improving. For each borrower the required payment is unchanged or reduced; some foreclosures are avoided. If type information is good, default results in foreclosure. But, if type information is bad, the required payment is reduced to r^{bad}. Borrowers with $v \in [r^{bad}, r_0)$ experience foreclosure in the diversified MBS design, but avoid foreclosure in the non-diversified design. For the investor the mortgage bond value is higher if the required payment is reset to r^{bad} following a bad type outcome. At time 0, loan proceeds are greater because the investor is willing to pay more for the mortgage bond. We thus obtain the following proposition.[37]

Proposition 4. If $W^{ask} > 0$ and servicers do not sufficiently value investment in MBSs ($\beta W^{IR} < W^{ask}$), securitizing mortgages into non-diversified MBSs instead of diversified MBSs results in the following changes:

i) Loan proceeds (ex ante bond value), relative to the expected collateral value, are higher, and the cost of borrowing is lower.

ii) The incidence of foreclosure is lower.

Proof: *i): From Lemma 4 and the results of Section 2, the initial bond value, as a function of the promised payment and prior collateral value (v_0), is higher. ii) Some foreclosures are avoided with a lower renegotiated payment.*

Proposition 4 follows directly from the Section 2 results and the assumption that forming non-diversified MBSs facilitates the acquisition of information about underlying mortgage loans. If parameter values are such that the servicer cannot be offered an IC contract and the MBS is diversified, any realized collateral value less than the original contracted payment, $v < r_0$, results in foreclosure. In contrast, the non-diversified design avoids foreclosures that would occur in the diversified design. Furthermore, the renegotiation process in the non-diversified

[37]We do not in this section solve for an IC contract for the servicer, because the investor can verify the type information.

MBS is elegant in that it requires only pool-wide information.

Policy implications: Some non-diversified MBSs may experience large losses while others experience relatively small losses. This is not a problem because securitization enables an investor to diversify on her own by holding a portfolio of MBSs.[38] With diversification, however, MBS investor claims may be diffusely-held, possibly causing coordination problems that interfere with mortgage renegotiation.[39] Including in the original security prospectus rules for mortgage renegotiation may enable the renegotiation proposed in Lemma 4 while avoiding coordination problems.

Rule-based renegotiation may also be included in mortgage contracts at origination. This avoids the need for a non-diversified MBS design, but is inefficient if the loan is not subsequently securitized or is securitized with an incentive compatible (IC) servicer contract. Alternatively, rules could be put in place at the time of securitization. In either case, however, without an IC servicer contract, the servicer must be monitored, and with a diversified design monitoring is costly. A non-diversified MBS makes it easier for the investors' trustee to verify that a renegotiation rule is followed.

4 Policy Implications and Extensions

4.1 Contagion

Contagion occurs because foreclosure adversely affects the value of similar houses, leading to more foreclosures, which then leads to further reduction in house values.[40] Rather than assume that the realized collateral value, v_i, for each borrower is exogenously determined, foreclosure of other properties may now influence col-

[38] Diversified CDOs may also be formed with non-diversified MBSs.

[39] In addition, securities of different priority (tranches) are typically created. A renegotiation may impact each tranche differently resulting in coordination problems. Cordell, Dynan, Lehnert, Liang and Mauskopf (2008) state that "tranche warfare" (p. 22) can increase the time that a servicer needs to get modification approved.

[40] See Frame (2010) and Lee (2008) for reviews of literature on foreclosure spillover effects.

lateral value. We assume that foreclosing on one property has a negative feedback effect on similar properties and may cause additional foreclosures. In the presence of contagion, investors are better off if they coordinate to limit foreclosures. But, coordination may be unachievable if investors do not experience foreclosure feedback effects. Investors face a prisoner's dilemma problem if mortgages are organized into diversified MBSs.

To illustrate this problem we present a numerical example consistent with earlier results and simplify with just two mortgages of each type: $N = 2$. Assume diversified MBS, no information collection following default, and $r_0 \leq r^{noInf}$. According to Proposition 1 an investor insensitive to feedback effects forecloses on any borrower who defaults. Figure 2 illustrates a numerical example consistent with these assumptions. The expected payoff to foreclosure without negative feedback effects is 80. If, however, the other similar property already foreclosed, the expected payoff is only 40. Suppose instead that both mortgages are renegotiated. From Proposition 1, in the absence of feedback effects, renegotiation is suboptimal; the payoff is less than 80. Assume a payoff of 70. The top left cell of Figure 2 presents payoffs if both mortgages are renegotiated. If the investor in mortgage 1 deviates and forecloses, payoffs are represented in the bottom left cell. The deviating investor, if first to foreclose, obtains 80; the other investor obtains 40. If both investors foreclose without renegotiating, each has a 50% chance to be the first to complete the foreclosure, resulting in expected payoffs of 60. If both investors are rational and playing a noncooperative game, (renegotiate, renegotiate) is not an equilibrium. Each investor has a strictly positive incentive to deviate and attempt to be the first to foreclose. The only equilibrium in this game is (foreclose, foreclose).

The prisoners' dilemma game of Figure 2 illustrates the problem with diversified MBS. If each investor makes decisions for multiple unrelated mortgages, the equilibrium outcome for each mortgage type is (foreclose, foreclose). Each mortgage bond is worth 60 and each MBS has a value of 120. Alternatively, with the

		Mortgage 2	
		renegotiate	foreclose
Mortgage 1	renegotiate	70, 70	40, 80
	foreclose	80, 40	60, 60

Figure 2: Prisoners' Dilemma Foreclosure problem

nondiversified design each investor makes decisions for both mortgages of a single type. Investors renegotiate and each MBS has a value of 140. In practice there are many more than two of any given type. Even if all MBSs are nondiversified, a given type may be distributed across multiple MBSs. However, as long as each MBS contains one type, investors recognize some negative feedback effects, and are more willing to renegotiate to limit foreclosure.

4.2 Underinvestment problem

An underinvestment problem occurs if the loan to value ratio is too high. A homeowner with negative equity ($v < r_0$) may refrain from performing maintenance, such as repairing the roof. As a result, the house value drops even lower. This underinvestment problem is exacerbated if lenders base loan renegotiation on individual property revealed value. A homeowner who expects a loan modification only if the revealed house value is low, refrains from even a positive NPV improvement. If renegotiations are instead based only on pool-wide (type) information, an individual borrower's home improvement decision is independent of the lender's loan modification decision. The nondiversified MBS design with type-based renegotiation mitigates the underinvestment problem.

4.3 Policy implications for servicer contracts

In September 2011 the Federal Housing Finance Agency released a discussion paper (FHFA, 2011) regarding current servicer compensation practices and proposed changes. For a loan sold to a third party, servicers retain a minimum servicing fee (MSF) consisting of a claim to part of the interest paid on a performing loan (typically 20 to 50 basis points of the outstanding loan principal). The servicer may also receive ancillary fees, including late fees on delinquent payments and payment for services rendered. The right to service a mortgage and receive fees is the mortgage servicing right (MSR).

The standard servicing fee resembles our linear contract in that it is defined as a percent of the outstanding principal. For a non-performing loan the servicer gets nothing. If the fractional claim z^* is large enough, the servicer has sufficient incentive to act in the investors' interest in renegotiating with a borrower in default. If, however, the fractional claim is too small, the servicer exerts no effort for non-performing loans. Given the cost of retaining the MSR and servicing non-performing loans, a servicer may be unwilling to invest enough to retain an incentive-compatible fraction.

One FHFA (2011) proposal to ameliorate the problem of inadequate servicing of non-performing mortgages is to set up a reserve account for each mortgage backed security (MBS). A part of the current MSF would go into the reserve account. Reserve account funds would offset the cost of servicing non-performing loans. As an extra incentive, "above-average servicer performance that helps negate the need for the reserve account could lead to a partial or full refund of the reserve account to the servicer" (p. 20). A reserve account would decrease the minimum servicing fee to a level sufficient to cover the costs of servicing a performing loan, allow for the possibility of a nonlinear contract for servicing non-performing loans, and is functionally equivalent to our proposal to ease the servicer's limited liability constraint by pooling. As we have demonstrated, with a nonlinear contract it is possible to give the servicer an incentive-compatible fraction of the cash flows at a lower cost than with a linear contract. The FHFA proposal does not describe how funds in the reserve account would be distributed in the case of mortgage defaults. To create the right servicer incentives, our model suggests that this be done based on loan performance, rather than as fees for service.

5 Concluding discussion

Any policy that enhances the efficiency of renegotiation and foreclosure decisions subsequent to securitization increases the value of the securities. Rational lenders and investors take into account the costs related to default and foreclosure: lower costs imply greater mortgage credit availability and less expensive credit terms. If mortgage-backed security (MBS) design decreases the incidence of costly foreclosure, welfare is improved for borrowers and investors.

We demonstrate that renegotiation of mortgages in default is beneficial for both MBS investors and borrowers, but only if servicers gather sufficient information. Renegotiation without information gathering leads to inefficient loan modification and encourages excess strategic default; both are costly for investors. The problem is that MBS investors can neither verify servicer efforts to obtain information nor verify information obtained, as borrower specific information is often "soft" in nature. The simplest solution to this moral hazard problem is to write servicer contracts that discourage all modifications of securitized mortgages. We demonstrate that this is not the most efficient solution and propose two alternative solutions.

The first solution is to design contracts that align servicer incentives with investor interests. The servicer must hold a "vertical" risk position in the MBS, a position with positive value following many defaults. In general the value of such a position is greater than the expected cost of servicing the loans, but servicers can be required to make side payments (investments) in exchange for these servicing contracts. If, however, foreclosure is very inefficient and information gathering very costly, then the cost of an incentive-compatible servicer contract may be higher than any amount the servicer is willing to invest. Our second solution bundles mortgages into non-diversified pools. Recent evidence suggests there are common factors related to declines in collateral values that contribute to borrower default. We argue that mortgages, and in particular the collateral, have a "type". If mortgages are securitized into diversified MBSs, valuable type information is lost (or becomes too costly to retrieve). For mortgages pooled into non-diversified MBSs, type information is preserved (or readily verified), enabling renegotiation based on accessible type information. In this case it is not necessary that the servicer have an incentive-compatible contract.

While our main results pertain to MBS design, we also develop a number of results applicable to existing securities. We demonstrate that MBS investors can benefit from the renegotiation of loans in default, but only if the servicer gathers new information about defaulting borrowers or their underlying collateral. For investors the wealth transfer cost of loan modification without proper information gathering can more than outweigh any benefit from avoiding costly foreclosure. Government loan modification mandates can be costly for MBS investors if servicers do not have sufficient incentive to gather the needed information. The cost of renegotiation without information hurts investors in the short run, but ultimately hurts borrowers as investors increase the cost of providing credit. Paying servicers fees that are a function of the number of defaults exacerbates the problem by inducing servicers to encourage more defaults. Fees paid to servicers for information collection must be associated with MBS performance. In particular, the servicer's share of the MBS must have a "vertical" component. A well-designed vertical loss position always retains some value and that value is always sensitive to servicer actions.

References

[1] Adelino, Manuel, Kristopher Gerardi and Paul S. Willen, 2009, Why don't lenders renegotiate more home mortgages? Redefaults, self-cures, and securitization, Public policy discussion paper, Federal Reserve Bank of Boston.

[2] Agarwal, Sumit, Gene Amromin, Itzhak Ben-David, Souphala Chomsisengphet, and Douglas D. Evanoff, 2011, The Role of Securitization in Mortgage Renegotiation, *Journal of Financial Economics* 102, 559–578.

[3] Aghion, Philippe, Mathias Dewatripont and Patrick Rey, 1994, Renegotiation design with unverifiable information, *Econometrica* 62, 257–282.

[4] Ambrose, Brent, Anthony Sanders and Abdullah Yavas, 2010, Servicers and mortgage-backed securities default: theory and evidence, working paper, Penn State University and SSRN.

[5] Cordell, Larry, Karen Dynan, Andreas Lehnert, Nellie Liang, Eileen Mauskopf, 2008, The incentives of mortgage servicers: myths and realities, working paper, Federal Reserve Board.

[6] DeMarzo, Peter M., 2005, The pooling and tranching of securities: a model of informed intermediation, *Review of Financial Studies* 18, 1–35.

[7] Eggert, Kurt, 2004, Limiting abuse and opportunism by mortgage servicers, *Housing Policy Debate* 15, #3, pp753–784.

[8] Elul, Ronel, Nicholas Souleles, Souphala Chomsisengphet, Dennis Glennon, and Rober Hunt, 2010, What "triggers" mortgage default?, *American Economic Review* 100, 490–494.

[9] Federal Housing Finance Agency, 2011, Alternative Mortgage Servicing Compensation Discussion Paper, downloaded from SSRN.

[10] Foote, Christopher L., Kristopher Gerardi and Paul S. Willen, 2008, Negative equity and foreclosure: theory and evidence, *Journal of Urban Economics* 64, 234–245.

[11] Frame, W. Scott, 2010, Estimating the effect of mortgage foreclosures on nearby property values: a critical review of the literature, *Federal Reserve Bank of Atlanta Economic Review*, Number 3.

[12] Ghent, Andra C., 2011, Securitization and mortgage renegotiation: evidence from the great depression, *Review of Financial Studies* 24, 1814–1847.

[13] Hart, Oliver and John Moore, 1988, Incomplete contracts and renegotiation, *Econometrica* 56, 755–785.

[14] Hart, Oliver and John Moore, 1998, Default and renegotiation: a dynamic model of debt, *The Quarterly Journal of Economics* 63, 1–41.

[15] Hartman-Glaser, Barney, Tomasz Piskorski and Alexei Tchistyi, 2012, Optimal Securitization with moral hazard, *Journal of Financial Economics* 104, 186–202.

[16] Kahn, Charles and Gur Huberman, 1989, Default, foreclosure, and strategic renegotiation, *Law and Contemporary Problems* 52, 49–61.

[17] Krainer, John, 2009, Recent developments in mortgage finance, *FRBSF Economic Letter*, 2009-33.

[18] Krainer, John and Stephen LeRoy, 2010, Underwater mortgages, *FRBSF Economic Letter*, 2010-31.

[19] Laux, Christian, 2001, Limited-liability and incentive contracting with multiple projects, *Rand Journal of Economics* 32, 514–526.

[20] Lee, Kai-yan, 2008, Foreclosure's price-depressing spillover effects on local properties: a literature review, *Federal Reserve Bank of Boston Discussion Paper*, 2008-01.

[21] Loutskina, Elena and Philip E. Strahan, 2011, Informed and uninformed investing in housing: the downside of diversification, *Review of Financial Studies* 24, 1447–1480.

[22] Malamud, Semyon, Huaxia Rui and Andrew Whinston, 2013, Optimal incentives and securitization of defaultable assets, *Journal of Financial Economics* 107, 111–135.

[23] Mayer, Christopher, Edward Morrison and Tomasz Piskorski, 2009, A new proposal for loan modifications, *Yale Journal on Regulation*.

[24] Mayer, Christopher, Edward Morrison, Tomasz Piskorski and Arpit Gupta, 2011, Mortgage modification and strategic behavior: evidence from a legal settlement with Countrywide, working paper.

[25] Mian, Atif, Amir Sufi, and Francesco Trebbi, 2010, The political economy of the US mortgage default crisis, *American Economic Review* 100, 1967–1998.

[26] Parlour, Christine A. and Guillaume Plantin, 2008, Loan sales and relationship banking, *Review of Financial Studies* 63, 1291–1314.

[27] Pennington-Cross, Anthony and Giang Ho, 2006, Loan servicer heterogeneity and the termination of subprime mortgages, working paper, Federal Reserve Bank of St. Louis.

[28] Petersen, Mitchell A., 2004, Information: hard and soft, working paper, Northwestern University.

[29] Piskorski, Tomasz, Amit Seru and Vikrant Vig, 2010, Securitizaton and distressed loan renegotiation: evidence from the subprime mortgage cirsis, *Journal of Financial Economics*.

[30] Posner, Eric A. and Luigi Zingales, 2009, A loan modification appproach to the housing crisis, *American Law and Economics Review*, 575-607.

[31] Riddiough, T., 1997, Optimal design and governance of asset-backed securities, *Journal of Financial Intermediation* 6(2), 121–152.

[32] Townsend, R., 1979, Optimal contracts and competitive markets with costly state verification, *Journal of Economic Theory* 21, 265–293.

[33] Wang, Ko, Leslie Young and Yuqing Zhou, 2002, Nondiscriminating foreclosure and voluntary liquidating costs, *Review of Financial Studies* 15, 959–985.

Appendix

Proof of Lemma 1. Suppose the lender plays a pure strategy of always making the renegotiation offer $r_0 - x$. If $x \leq 0$, then only borrowers with $v < r_0$ default, and no renegotiation offers are accepted. If $x > 0$, then all borrowers default. Suppose that following a default, with probability $\alpha \in (0, 1)$, the lender makes an offer $x > 0$, and with probability $1 - \alpha$ the lender forecloses. A borrower will default iff $v - r_0 \leq \alpha(v - r_0 + x)$. We can thus define the default cut-off value $v^D = r_0 + \alpha x/(1 - \alpha)$. Any borrower with a realization $v \leq v^D$ will default. A necessary condition for renegotiation to be possible is that $\alpha > 0$ and $x > 0$. In this case, $v^D > r_0$: there is a strictly positive probability of strategic default. ∎

Proof of Proposition 1. We first determine the lender's optimal policy assuming the lender can pre-commit to a renegotiation policy. We then check if the policy constitutes an equilibrium action without pre-commitment. If the lender commits to a policy of offering $x \leq 0$ to defaulting borrowers, there is no strategic default, all defaults result in foreclosure and the time zero bond value is:

$$B(x = 0) = r_0 \cdot prob\{\tilde{v} \geq r_0\} \; + \; \delta E[\tilde{v}|\tilde{v} < r_0] \cdot prob\{\tilde{v} < r_0\} \tag{19}$$

If the lender commits to renegotiate ($x > 0$) with probability α, the bond value is:

$$\begin{aligned} B(x > 0) \;\; = \;\; & r_0 \cdot prob\{\tilde{v} \geq v^D\} \; + \; \alpha(r_0 - x) prob\{r_0 - x \leq \tilde{v} < v^D\} \\ & + (1 - \alpha)\delta E[\tilde{v}|r_0 - x \leq \tilde{v} < v^D] \cdot prob\{r_0 - x \leq \tilde{v} < v^D\} \\ & + \delta E[\tilde{v}|\tilde{v} < r_0 - x] \cdot prob\{\tilde{v} < r_0 - x\} \end{aligned} \tag{20}$$

$$\text{where} \quad \tilde{v} \sim U[v_0 - \Delta, v_0 + \Delta] = U[0, 2\Delta] \quad \text{and} \quad v^D = \min\left[r_0 + \frac{\alpha x}{1 - \alpha}, 2\Delta\right] \tag{21}$$

Let $\alpha \in (0, 1)$, $x > 0$, and $v^D < 2\Delta$. The benefit to renegotiating is:

$$\begin{aligned} B(x > 0) - B(x = 0) \;\; = \;\; & \frac{\alpha(r_0 - x)x}{2\Delta} - \frac{\alpha\delta(r_0 - x/2)x}{2\Delta} \\ & - \frac{((1 - \alpha)r_0 + \alpha x)\alpha x}{(1 - \alpha)2\Delta} + \frac{\delta(r_0 + \alpha x/2(1 - \alpha))\alpha x}{2\Delta} \end{aligned} \tag{22}$$

$$= \frac{-\alpha x^2}{(1 - \alpha)2\Delta} + \frac{\alpha\delta x^2}{(1 - \alpha)4\Delta} = \frac{(\delta - 2)\alpha x^2}{(1 - \alpha)4\Delta} < 0 \tag{23}$$

The first line of equation (22) represents the benefit of avoiding foreclosure for some borrowers. The second line represents the cost of encouraging strategic default. The inequality in (23) means the lender will not play a mixed strategy with $\alpha > 0$ and $x > 0$.

Now let $v^D = v_0 + \Delta$. In this case, α is set to one because all borrowers default regardless. The expected bond value as a function of the renegotiated offer r is:

$$B(r) = r(2\Delta - r)/2\Delta \ + \ \delta r^2/4\Delta \,. \tag{24}$$

(24) is maximized at $r^* = 2\Delta/(2-\delta)$. A renegotiated offer must be mutually acceptable, so $r_1 = \min[r_0, 2\Delta/(2-\delta)]$. (If $r_0 \leq 2\Delta/(2-\delta)$, no renegotiation offer is made.)

We next check if the equilibrium described above is a perfect Bayesian equilibrium (PBE) if the lender is unable to pre-commit to a renegotiation strategy. For this to be the case, the lender must be willing to play the prescribed strategy following a default. For the case such that $r_0 > 2\Delta/(2-\delta)$, the described equilibrium is clearly PBE: if everyone defaults, the lender optimally offers $r_1 = 2\Delta/(2-\delta)$. Consider the case such that $r_0 \leq 2\Delta/(2-\delta)$. In the described equilibrium $v^D = r_0$ and $\alpha \times x = 0$. If the lender plays $\alpha \times x = 0$, the lender's expected bond value, given default (and $v^D = r_0$), is $\delta r_0/2$. If the lender instead plays $\alpha \times x > 0$, the lender increases the expected bond value by:

$$\alpha \left(r_0 - x - \delta(r_0 - x/2)\right) \times prob\{r_0 - x \leq \tilde{v} < r_0\} \tag{25}$$

(25) is clearly optimized for $\alpha = 1$ and some value of $x > 0$. The above solution, with $x = 0$, is thus not a PBE without pre-commitment. But, if the lender plays $\alpha = 1$ and $x > 0$, all borrowers will default so that $x > 0$ is not an equilibrium strategy. In the case that $r_0 \leq 2\Delta/(2-\delta)$, there is no pure strategy PBE without pre-commitment. ∎

Proof of Proposition 2. A defaulting borrower is with probability $1-\gamma$ offered $r_1 = v$, and with probability γ offered $r_1 = r_0 - x$, where $x \geq 0$. A borrower defaults iff $v \leq v^D = \min[r_0 + \gamma x/(1-\gamma), 2\Delta]$. We first assume the lender can pre-commit to a renegotiation strategy, and then check if this solution is a PBE without pre-commitment. Consider the case such that $v^D = r_0 + \gamma x/(1-\gamma)$. The expected bond value is:

$$\begin{aligned}
B &= r_0 \cdot prob\{\tilde{v} \geq v^D\} + (1-\gamma)E[\tilde{v}|\tilde{v} < v^D] \cdot prob\{\tilde{v} < v^D\} \\
&\quad + \gamma(r_0 - x)prob\{r_0 - x \leq \tilde{v} < v^D\} \ + \ \gamma\delta E[\tilde{v}|\tilde{v} < r_0 - x] \cdot prob\{\tilde{v} < r_0 - x\} \\
&= \frac{r_0(2\Delta - v^D)}{2\Delta} + \frac{(1-\gamma)(v^D)^2}{4\Delta} + \frac{\gamma(r_0 - x)(x + \gamma x/(1-\gamma))}{2\Delta} + \frac{\gamma\delta(r_0 - x)^2}{4\Delta} \\
&= r_0 - \frac{r_0^2}{2\Delta} + \frac{(1-\gamma)(r_0 + \gamma x/(1-\gamma))^2}{4\Delta} - \frac{\gamma x^2/(1-\gamma)}{2\Delta} + \frac{\gamma\delta(r_0 - x)^2}{4\Delta} \qquad (26)
\end{aligned}$$

Solving the first-order condition for x, we obtain

$$x^* = \frac{(1-\gamma)(1-\delta)r_0}{1 + (1-\gamma)(1-\delta)} \qquad (27)$$

x^* is strictly positive as long as $\gamma < 1$ and $\delta < 1$. The probability of foreclosure ($prob\{\tilde{v} < r_0 - x\}$) is decreasing in x, and $\partial x^*/\partial(1-\gamma) \geq 0$. Thus, if $\gamma < 1$, the probability of foreclosure is decreasing in $1-\gamma$. Inserting x^* into the equation for v^D, we obtain equation (6). We now check if the solution is a PBE without pre-commitment. The expected bond value, given default and no information is:

$$\frac{(r_0 - x)(v^D - r_0 + x)}{v^D} + \frac{\delta(r_0 - x)^2}{2v^D} \qquad (28)$$

The above is optimized at

$$x = \frac{(1-\gamma)(1-\delta)r_0}{1 + (1-\gamma)(1-\delta)} = x^*$$

Thus, the solution with commitment is also a PBE when there is no pre-commitment. Next consider the boundary case in which $r_0 > r^{noInf}$ and $v^D = 2\Delta$:

$$\begin{aligned}
B &= (1-\gamma)v_0 \ + \ \gamma(r_0 - x)prob\{r_0 - x \leq \tilde{v}\} \ + \ \gamma\delta E[\tilde{v}|\tilde{v} < r_0 - x] \cdot prob\{\tilde{v} < r_0 - x\} \\
&= (1-\gamma)v_0 + \frac{\gamma(r_0 - x)(2\Delta - r_0 + x)}{2\Delta} + \frac{\gamma\delta(r_0 - x)^2}{4\Delta}
\end{aligned}$$

$$x^* = r_0 - 2\Delta/(2-\delta) \qquad \text{and} \qquad r_1 = r_0 - x^* = 2\Delta/(2-\delta) = r^{noInf} \qquad (29)$$

The solution in the boundary case is a PBE even without pre-commitment. ∎

Proof of Corollary 3. Following a default, the most the lender is willing to spend on information gathering is:

$$
\begin{aligned}
c_{max} &= (1-\gamma)E[\tilde{v}|\tilde{v} < v^D] - (1-\gamma)(r_0 - x^*) \times prob\{\tilde{v} \geq r_0 - x^*|\tilde{v} < v^D\} - \\
&\quad (1-\gamma)\delta E[\tilde{v}|\tilde{v} < r_0 - x^*]prob\{\tilde{v} < r_0 - x^*|\tilde{v} < v^D\} \\
&= \frac{(1-\gamma)(2-\delta)r_0}{2(1+(1-\gamma)(1-\delta))} - \frac{(1-\gamma)(2-\delta)r_0}{2(2-\delta)(1+(1-\gamma)(1-\delta))} \\
&= \frac{(1-\gamma)(1-\delta)r_0}{2(1+(1-\gamma)(1-\delta))} = \frac{x^*}{2} \qquad (30)
\end{aligned}
$$

The remainder of the proof follows directly from Proposition 2. ∎

Bond value with information gathering. Combining equations (26), (27) and (6) we obtain the bond value, given that $r_0 \leq r^{noInf}$ and $0 < \gamma < 1$:

$$
\begin{aligned}
B^\gamma &= r_0 - \frac{(2-\delta)r_0^2}{2\Delta(1+(1-\gamma)(1-\delta))} + \frac{r_0^2((1-\gamma)(2-\delta)^2 + 2\gamma(1-\delta) + \gamma\delta)}{4\Delta(1+(1-\gamma)(1-\delta))^2} \\
&= r_0 - \frac{(2-\delta)r_0^2}{4\Delta(1+(1-\gamma)(1-\delta))} = r_0\left(1 - \frac{(2-\delta)(r_0 - x^*)}{4\Delta}\right) \qquad (31)
\end{aligned}
$$

Proof of Lemma 2. ii) Consider a mortgage in default. By assumption, if the servicer does not gather information, the borrower is offered $r_1 = r_0 - x^*$. Consider a contract that pays the servicer a fraction z of all cash flows above $r_0 - x^*$ if a mortgage defaults and nothing otherwise. The servicer's expected compensation is zero if he does not expend c. Thus, setting z so that the expected value of the compensation given information gathering is exactly c provides sufficient incentive to expend c to gather information. ∎

Proof of Proposition 3. Consider a single loan in default. Suppose the servicer gets fraction z of all cash produced by the loan above a cut-off ψ, where $\psi \leq r_0$. If uninformed, the servicer sets concession x' to maximize:

$$
\begin{aligned}
&(r_0 - x' - \psi)prob\{\tilde{v} \geq r_0 - x'|\tilde{v} < v^D\} + \\
&\quad \delta E[\tilde{v} - \psi/\delta|\psi/\delta < \tilde{v} < r_0 - x']prob\{\psi/\delta < \tilde{v} < r_0 - x'|\tilde{v} < v^D\}
\end{aligned}
$$

If $\psi/\delta < r_0 - x'$, the servicer maximizes: $(r_0 - x' - \psi)(v^D - r_0 + x') + \delta(r_0 - x' - \psi/\delta)^2/2$, and the first order condition is:

$2r_0 - \psi - v^D - 2x' - \delta r_0 + \psi + \delta x' = 0 \Longrightarrow x' = r_0 - v^D/(2-\delta) = x^*$.

If $\psi/\delta > r_0 - x'$, the servicer maximizes: $(r_0 - x' - \psi)(v^D - r_0 + x')$, and the first order condition leads to: $x' = r_0 - (\psi + v^D)/2$. If $\psi = \delta(r_0 - x^*)$, then $r_0 - (\psi + v^D)/2 = x^*$. Thus, if $\psi \leq \delta(r_0 - x^*)$, the servicer sets $x' = x^*$. If $\psi > \delta(r_0 - x^*)$, the servicer sets $x' = r_0 - (\psi + v^D)/2 < x^*$. The IC constraint for gathering information is:

$$zE[\max(0, CF - \psi)|\text{info gathering}] \geq zE[\max(0, CF - \psi)|\text{no info gathering}] + c$$

Let $\psi = \delta(r_0 - x^*) = \delta v^D/(2-\delta)$.

$E[\max(0, CF - \psi)|\text{no info gathering}] =$

$$(1-\delta)(r_0 - x^*)prob\{\tilde{v} > r_0 - x^*|\tilde{v} < v^D\} > 0$$

Thus, $zE[\max(0, CF - \psi)|\text{info gathering}] > c$. ∎

Proof of Lemma 3. Define the cut-off below which the servicer receives no payment as $N^D \times \psi$, where N^D is the number of loans in default and ψ is the average effective cut-off for a loan in default.

If the servicer does not gather information and offers concession x^* to all defaulters, the average cash flow per loan in default is

$$\frac{(v^D - r_0 + x^*)(r_0 - x^*)}{v^D} + \frac{\delta(r_0 - x^*)^2}{2v^D} = (r_0 - x^*)\left(\frac{1-\delta}{2-\delta} + \frac{\delta}{2(2-\delta)}\right) = \frac{r_0 - x^*}{2} \quad (32)$$

If $\psi = (r_0 - x^*)/2$, the servicer expects zero payoff for not exerting effort: $W^{IR} = 0$. If $\psi < (r_0 - x^*)/2$, the servicer expects a strictly positive payoff for not exerting effort: $W^{IR} > 0$. Applying Proposition 3, the servicer offers concession x^* when uninformed iff

$$\psi \leq (1-\gamma)E[\tilde{v}|\tilde{v} \leq v^D] + \gamma\delta(r_0 - x^*) = ((1-\gamma)(2-\delta)/2 + \gamma\delta)(r_0 - x^*)\,. \quad (33)$$

Thus, the least cost piece-wise linear IC contract has $W^{IR} = 0$ iff

$$1/2 \leq (1-\gamma)(2-\delta)/2 + \gamma\delta \quad (34)$$

∎