Payout Policy under Heterogeneous Beliefs: A Theory of Dividends versus Stock Repurchases, Price Impact, and Long-Run Stock Returns

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Abstract

We analyze a firm's choice between dividends and stock repurchases in an environment of heterogeneous beliefs and short sale constraints. We study a setting in which the insiders of a firm, owning a certain fraction of its equity and having a certain amount of cash to distribute to shareholders, choose between paying out cash dividends and buying back equity, as well as the scale of investment in their firm's new project. Outside equity holders in the firm have heterogeneous beliefs about the probability of success of the firm's project and therefore its long-run prospects; they may also disagree with firm insiders about this probability. We show that, depending on the beliefs of firm insiders versus outsiders, the firm may distribute value through cash dividends alone; through a repurchase alone; or through a combination of a cash dividend and a stock repurchase. We also show that, in many situations, it is optimal for firm insiders to underinvest in the firm's positive net present value project and undertake a stock repurchase with the amount of cash saved by underinvesting. We then analyze the price impact of a cash dividend versus a share repurchase, where the price impact is defined as the abnormal return to the firm's equity upon the actual payment of a cash dividend or the implementation of a share repurchase respectively (rather than upon the announcement of these events). Finally, we analyze the long-run returns to a firm's equity following dividend payments and stock repurchases. Our model generates a number of testable predictions different from asymmetric information models of a firm's choice between dividends versus stock repurchases.

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1 Introduction

In recent years, the number of firms undertaking stock repurchases has increased dramatically, while the proportion of firms distributing value through cash dividends has declined (see, e.g., Fama and French (2001)). The popularity of share repurchases has not been mitigated even after the passage of the Jobs and Growth Tax Relief Act of 2003 in the U.S., which cut the dividend tax rate to 15%, thus substantially reducing the tax disadvantage of dividend payments to investors (see, e.g., Chetty and Saez (2006)). There have also been several articles in the popular and practitioner oriented press indicating that, in some situations, firms cut back on some positive net present value projects and use the cash saved for stock repurchases.[1] Other such articles indicate that firms that do not have cash on hand go to the extent of borrowing money to undertake stock repurchase programs.[2] Finally, a growing number of firms seem to be using a combination of dividends and stock repurchases to distribute value to shareholders.

The above evidence and anecdotes raise several interesting questions: What are the advantages and disadvantages of dividends and share repurchases from the point of view of maximizing shareholder wealth? Are there any situations where shareholders are better off if the firm underinvests in positive net present value projects and uses the cash to buy back equity? Is there an optimal combination of share repurchases and dividend payments that a firm should undertake? Is it ever optimal for a firm to fund a repurchase or a dividend payment by raising external financing? What are the longer term implications (price impact and long-run stock returns) of dividend payments and stock repurchase programs?

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[1] See, e.g., the *New York Times* article, "As Layoffs Rise, Stock Buybacks Consume Cash," November 21, 2011, describing how, while Pfizer cut back on its research budget and laid off 1,100 employees, it added $5 billion to the $4 billion it already earmarked for stock repurchases in 2011 and beyond. There is also some empirical evidence of firms implementing repurchase programs under certain conditions cutting back on investment: see Almeida, Fos, and Kronlund (2013) for details.

[2] See, e.g., the *Wall Street Journal* article, "Intel Borrows $6 Billion to Help Fund Stock Buyback," December 4, 2012, mentioning that Intel borrowed $6 billion in 2012 partly to fund a stock repurchase.

Given that, under perfect capital markets, stock repurchases and dividends are equivalent from the point of view of market value maximization, the usual theoretical justification given for stock repurchases rests on asymmetric information. In particular, a number of authors have advanced signaling models of stock repurchases: see, e.g., Ofer and Thakor (1987), and Constantinides and Grundy (1989).[3,4] While the details vary across these models, the basic idea underlying signaling models is that firm insiders have private information about its future prospects, and buy back equity when they believe that its equity is undervalued, thus signaling their private information to outside shareholders. Thus, signaling models can explain the announcement effects of share repurchase programs, especially in the context of tender offers: see, e.g., Dann (1981), Vermaelen (1981), Asquith and Mullins (1986), Comment and Jarrell (1991), D'Mello and Shroff (2000), or Louis and White (2007) for evidence. However, while share repurchases can serve as a credible signal in the context of tender offer repurchases, it is more difficult to believe that open market repurchases can serve as a credible signal, given that the repurchasing firm does not need to commit to repurchasing the entire amount of the share repurchase authorized by its board.[5] It is worth noting here that open market repurchases constitute around 90% of the stock repurchases consummated in recent years: see, e.g., Comment and Jarrell (1991) or Grullon and Michaely (2004). Further, asymmetric information models with fully rational investors cannot explain the positive abnormal long-run stock returns following stock repurchases that have been documented by the empirical literature (see, e.g., Ikenberry, Lakonishok, and Vermaelen (1995), and Peyer and

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[3]Asymmetric information models of firms' choice between dividends and repurchases that do not involve signaling are Brennan and Thakor (1990) and Chowdhry and Nanda (1994). Brennan and Thakor (1990) develop a model with heterogeneously informed outside investors: they show that, since uninformed investors are put at a disadvantage when a firm repurchases shares, under some circumstances these investors require a firm to make a taxable dividend payment rather than undertake a stock repurchase. Chowdhry and Nanda (1994) develop a dynamic model of a firm's choice between taxable dividends and tender offer repurchases where managers with private information choose not only the method of distributing value, but also the timing of a stock repurchase.

[4]Signaling models of dividends include Allen, Bernardo, and Welch (2000), John and Williams (1984), Miller and Rock (1984), and Bhattacharya (1979).

[5]See, however, Oded (2005), who develops a theoretical model demonstrating that, under suitable assumptions, open market share repurchase programs can signal firm insiders' private information even in the absence of a commitment by the firm to repurchase the entire amount of equity authorized to be repurchased by its board.

Vermaelen (2009)).[6] The above findings suggest the need for alternative theories that can better explain many aspects of a firm's choice between dividends and share repurchases, and the long-run impact of dividend payments and share repurchase programs on shareholder wealth.[7]

The objective of this paper is to fill the above gap in the literature by developing a new theory of a firm's choice between dividends and stock repurchases in a setting of heterogeneous beliefs and short sale constraints. We study a setting in which the insiders of a firm, owning a certain fraction of its equity and having a certain amount of cash to distribute to shareholders, choose between paying out cash dividends and buying back equity from shareholders, as well as the optimal scale of investment in their firm's new project. Outside equity holders in the firm have heterogeneous beliefs about the probability of success of the firm's project and therefore its long run prospects; they may also disagree with firm insiders about this probability.

Our theoretical analysis is in four parts. In the next section (section 2) we develop our basic model where we do not allow the firm to raise external financing to fund a cash dividend payment or a

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[6]Asymmetric information models will not be able to generate a significant price impact for a stock repurchase as well, since there is no new information flow from firm insiders to outsiders when a stock repurchase (that has already been announced) is actually implemented.

[7]A recent episode that is hard to explain in the context of asymmetric information models of share buybacks is the attempt by the activist investor Carl Icahn to persuade the CEO of Apple, Tim Cook, to increase the amount of Apple's buyback to around $150 billion (more than double the amount proposed by the company itself): see, e.g., the *Wall Street Journal* article, "Icahn Presses Apple for $150 billion Buyback," October 1, 2013. As Icahn explained in his letter to Apple CEO Tim Cook, he feels that Apple shares are heavily undervalued. To quote Icahn's letter: "Per my investment thesis, commencing this buyback immediately would ultimately result in further stock appreciation of 140% for the shareholders who choose not to sell into the proposed tender offer. Furthermore, to invalidate any possible criticism that I would not stand by this thesis in terms of its long term benefit to shareholders, I hereby agree to withhold my shares from the proposed $150 billion tender offer. There is nothing short term about my intentions here." Given that Icahn is an outsider to the firm, and considering that he is writing to an insider of the firm (Tim Cook), it is difficult to characterize Mr. Icahn's strong urging of a much larger stock repurchase of Apple shares (and other actions such as accumulating a much larger additional equity stake in the firm on his own account) as driven by private information about its future value. Rather, it would be more appropriate to view it as driven by disagreement (heterogeneity in beliefs) between Mr. Icahn and other shareholders who are much more pessimistic about Apple's future prospects. In this case, it seems to be the case that Icahn is more optimistic about the firm's future prospects than firm insiders (Tim Cook) on the one hand, and other outside shareholders on the other. This is consistent with the economic setting studied in our model, where there is heterogeneity in beliefs between firm insiders and outside shareholders, and also among outside shareholders. Incidentally, Icahn's undervaluation thesis and his advice to the company to undertake the repurchase even at the cost of borrowing money to undertake the larger repurchase also rules out the possibility that his desire for a larger buyback is driven purely by agency considerations (i.e., the possibility that the push for a larger buyback is driven by his desire to get the company to disgorge excess cash that could be more productively utilized outside the firm).

stock repurchase. In this section the decision facing firm insiders is regarding how to allocate the cash available in the firm between investing in the firm's project (they may choose to undertake it up to the full investment level or to underinvest in it) and distributing it to shareholders; they also decide on the optimal combination of a cash dividend and a stock repurchase to distribute this value to shareholders. We show that, depending on the beliefs of firm insiders versus different groups of outsiders, the firm may distribute value by cash dividends alone; through a repurchase alone; or through a combination of a cash dividend and a stock repurchase. We also show that, in many situations, it is optimal for firm insiders to underinvest in the firm's positive net present value project and undertake a stock repurchase with the amount of cash saved by underinvesting.

In the following section (section 3), we analyze the price impact of cash dividends and stock repurchases. Here the price impact is defined as the abnormal return to the firm's equity upon the actual payment of a cash dividend or the implementation of a share repurchase respectively (not the abnormal stock return upon the announcement of these events). Here we show that, while the price impact of a dividend payment will be zero, the price impact of a stock repurchase will be positive on average; further, the larger the number of shares the firm repurchases, the greater the price impact on average.

In the subsequent section (section 4), we study an extension of our basic model where we allow the firm to raise external financing (in the form of equity or debt) that may be used to fund either its dividend payment or stock repurchase. We show that, in some situations, it is optimal for a firm to issue equity (but never debt) to fund a dividend payment; in other situations, it is optimal for it to issue debt (but never equity) to fund a stock repurchase. As in our basic model, the choice between dividends and repurchases is driven by the relative optimism of firm insiders and different groups of outside shareholders about the firm's future prospects.

Finally (in section 5), we study an extension to our basic model where we analyze the long-run

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returns to a firm's equity following dividend payments and stock repurchases. Here we characterize the conditions under which the long run stock returns following dividend payments and repurchases will be positive and those under which it will be negative. We show that, if a firm does not underinvest in its project, the long-run stock returns following a stock repurchase will always be positive. Further, the long-run stock returns of a sample of firms that do not underinvest in their project and distribute value to shareholders through a stock repurchase will, on average, exceed that of a similar sample of firms that distributes value through cash dividends.

Our model generates a number of new testable predictions (detailed in section 6) regarding a firm's choice between dividends and stock repurchases. To the best of our knowledge, all the predictions of our model regarding the price impact of dividends and stock repurchases and the long-run stock returns following dividend payments and stock repurchases are new to the literature. While some of these predictions provide a theoretical rationale for observed empirical regularities, there is no evidence so far in the literature regarding some of our other model predictions: they can therefore serve to generate testable hypotheses for new empirical tests.

The paper in the existing literature closest in spirit to our paper is that of Huang and Thakor (2013). While developing testable hypotheses for their empirical analysis, they build on the existing theoretical literature on the consequences of disagreement between managers and outsiders (e.g., Boot, Gopalan, and Thakor (2006)) to argue that open market share repurchases may be a way for managers to buy back equity from outsiders who disagree with them, thus increasing firm value. The predictions here are that firms are more likely to repurchase equity when there is a greater disagreement regarding financial policies between firm managers and outsiders, and that this disagreement is reduced following a share repurchase. They find support for these predictions in their empirical analysis. Huang and Thakor (2013) assume that disagreement between managers and outsiders reduces firm value; in contrast,

in our model any valuation effects of differences in beliefs between firm insiders and outsiders arise endogenously. However, the empirical finding of Huang and Thakor (2013) that stock repurchases are more likely when there is more disagreement between firm insiders and outside equity holders provides support for the theoretical predictions of our model as well. It is worth noting that, while Huang and Thakor (2013) provide a disagreement-based explanation for open market stock repurchases, they do not analyze a firm's choice between cash dividends and stock repurchases. Neither do they analyze the price impact or the long-run stock returns following cash dividends or stock repurchases.

Apart from the literature on payout policy discussed above, our paper is also related to the emerging literature in economics and finance on the effect of heterogeneity in investor beliefs on long-run stock returns and valuations and on trading among investors. Starting with Miller (1977), a number of authors have theoretically examined the stock price implications of heterogeneous beliefs and short sale constraints on stock valuations. Miller (1977) argues that when investors have heterogeneous beliefs about the future prospects of a firm, its stock price will reflect the valuation that optimists attach to it, because the pessimists will simply sit out the market (if they are constrained from short-selling). A number of subsequent authors have developed theoretical models that derive some of the most interesting cross-sectional implications of Miller's logic. In an important paper, Morris (1996) shows that the greater the divergence in the valuations of the optimists and the pessimists, the higher the current price of a stock in equilibrium, and hence lower the subsequent returns (see also Allen and Morris (1998)). In another important paper, Duffie, Gârleanu, and Pedersen (2002) show that, even when short-selling is allowed (but requires searching for security lenders and bargaining over the lending fee), the price of a security will be elevated and can be expected to decline subsequently in an environment of heterogeneous beliefs among investors if lendable securities are difficult to locate. Another important implication of heterogeneous beliefs among investors is that it can lead to a significant amount of speculative trading:

see, e.g., Harris and Raviv (1993), who use differences in opinion among investors to explain empirical regularities about the relationship between stock price and volume.[8]

The corporate finance implications of heterogeneous beliefs have been relatively less widely studied. Allen and Gale (1999) examine how heterogeneous priors among investors affect the source of financing (banks versus equity) of new projects. Garmaise (2001) analyzes the optimal design of securities by a cash-constrained firm facing investors with diverse beliefs: however, his focus is on comparing optimal designs when investors have rational beliefs (in the sense of Kurz (1994)) versus rational expectations. Dittmar and Thakor (2007) study a firm's choice between issuing debt and equity when insiders and outsiders disagree about the firm's choice of project to invest in. Finally Bayar, Chemmanur, and Liu (2011), develop a model of equity carve-outs and negative stub values in a setting of heterogeneous beliefs.

The rest of the paper is organized as follows. In Section 2, we describe the structure of our basic model and analyze a firm's choice between cash dividends and stock repurchases. Section 3 analyzes the price impact of cash dividends and stock repurchases on a firm's equity. Section 4 extends the analysis in our basic model to allow the firm to raise external financing in the form of equity or debt to fund its new project or its dividend payments or stock repurchases. Section 5 extends our basic model to analyze the long-run stock returns to the firm following a dividend payment or a stock repurchase. Section 6 describes the testable implications of our model, and Section 7 concludes. The proofs of all propositions are confined to the Appendix.

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[8]Several other authors have also examined the asset pricing and trading implications of heterogeneous beliefs (see, e.g., Harrison and Kreps (1978), Varian (1985, 1989), Kandel and Pearson (1995), and Chen, Hong, and Stein (2002) for contributions to this literature, and Scheinkman and Xiong (2004) for a review.)

At time 0, insiders of a firm own a fraction α of the firm's equity. The remaining 1- α is held by a group of outside shareholders.

The total number of shares outstanding in the firm is normalized to 1.

Time 0	Time 1	Time 2

The firm comes to know its time-1 earnings E; it chooses the scale of investment in its project; amount of dividends or share repurchase (or both) is announced.

Dividends are paid or share repurchase is initiated (or both); new project is implemented.
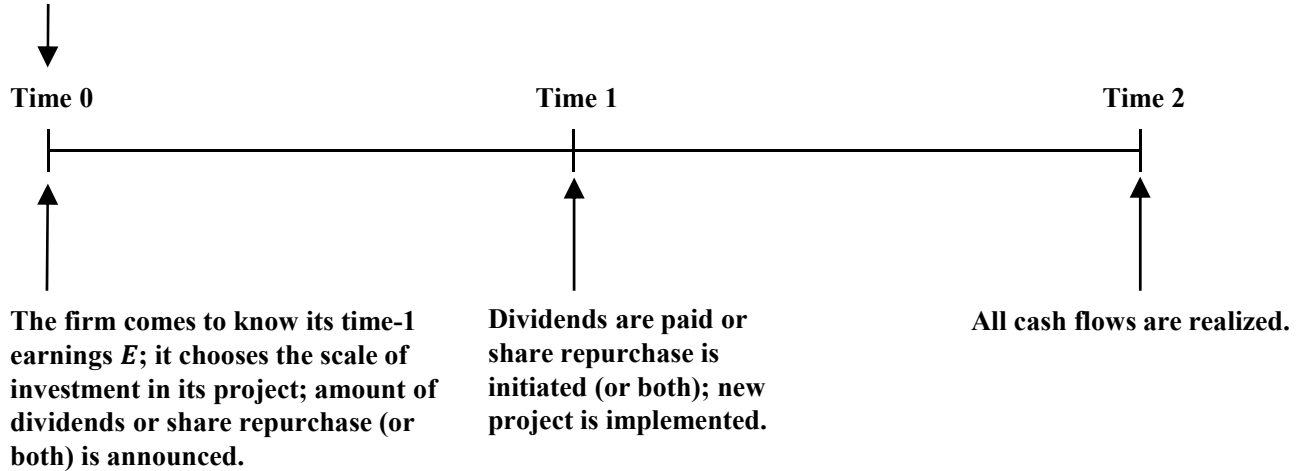
All cash flows are realized.

Figure 1: Sequence of Events in the Basic Model

2 The Basic Model

There are three dates in the model: times 0, 1, and 2. At time 0, insiders of a firm own a fraction α of the firm's equity. The remaining fraction $(1 - \alpha)$ is held by a group of outside shareholders. The total number of shares in the firm is normalized to 1, so that α can be thought of as either the fraction of shares or the number of shares held by insiders. At time 0, the firm learns about its time-1 earnings E and chooses the scale of investment in its project.[9] It then announces its time-1 cash dividend payment amount D_c and/or stock repurchase amount D_b. At time 1, the firm distributes the cash dividend and repurchases stock as announced at time 0, and simultaneously invests in its project. At time 2, the cash flows from the firm's project are realized and become common knowledge to all market participants. The sequence of events is shown in Figure 1.

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[9]Even though we refer to E as the firm's time-1 earnings, all our results will go through if the firm has some cash retained from prior period earnings as well. In that case, we can view E as the sum of cash retained from prior periods and time-1 earnings.
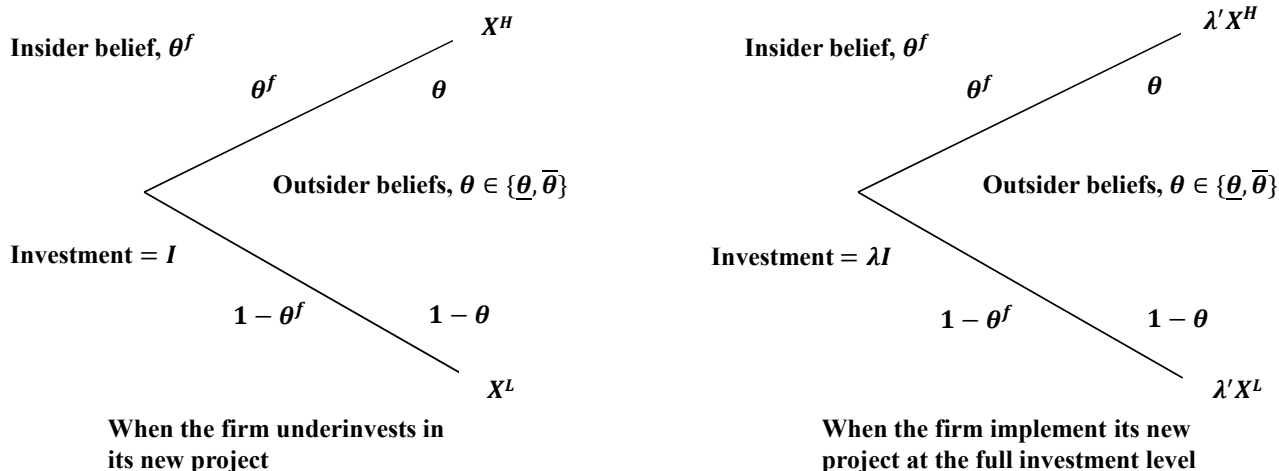
Figure 2: The Beliefs of Firm Insiders and Outsiders and Project Payoffs

The firm can invest in two different scales. If the firm invests the smaller amount I in the project, the cash flow from the firm's project will be either X^H or X^L, where $X^H > X^L \geq 0$. The firm can also choose to invest a larger amount λI in the project, and in this case, the cash flow from the firm's project will be either $\lambda' X^H$ or $\lambda' X^L$, where λ and λ' are known constants with $\lambda \geq \lambda' > 1$. The condition $\lambda \geq \lambda'$ implies that the project has decreasing return to scale. From now onwards, we will refer to the case where the firm invests the larger amount as "investing up to the full investment level" and to the case where the firm invests only up to the smaller investment level as "underinvesting."

The payout policy of the firm involves the following decisions. First, what amount of the time-1 earnings E should the firm distribute to its shareholders, and what amount should the firm reinvest? Second, how should the firm distribute value to its shareholders: cash dividend, stock repurchase, or a combination of the two?[10]

The capital market is characterized by heterogeneous beliefs and short-sale constraints. Specifically, the beliefs about the future (time 2) cash flows of the firm's project are different between firm insiders

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[10]Throughout the paper, we assume that the firm carries out its stock repurchase through an open market repurchase program, thus allowing it to pay different prices to the two groups of shareholders (optimists and pessimists). Note that, in practice, more than 90% of stock repurchases currently occur through an open market repurchase program: see, e.g., Grullon and Michaely (2004).

and outsiders, and these beliefs are also different among outsiders. Firm insiders believe that with probability θ^f, the cash flow will be X^H, and with probability $(1 - \theta^f)$, the cash flow will be X^L if the smaller amount I is invested. If the larger amount λI is invested, insiders believe that with probability θ^f, the firm's time-2 cash flow will be $\lambda' X^H$, and with probability $(1 - \theta^f)$, the cash flow will be X^L. We assume that regardless of whether the firm invests the larger amount λI or the smaller amount I in its project, the net present value of its project is positive conditional on insider beliefs about the probability of project success: i.e.,

$$\lambda' \left(\theta^f X^H + (1 - \theta^f) X^L \right) - \lambda I > 0, \quad \theta^f X^H + (1 - \theta^f) X^L - I > 0. \tag{1}$$

Further, given our assumption of decreasing returns to scale, it follows that the net present value per dollar of investment will be smaller if the firm undertakes its project at the larger scale (i.e., full investment level) compared to the case where it makes only a smaller investment in its project (i.e., it underinvests). However, the incremental investment from the underinvestment level up to the full investment level still has a positive net present value. In other words, we assume that

$$(\lambda' - 1) \left(\theta^f X^H + (1 - \theta^f) X^L \right) > (\lambda - 1) I, \tag{2}$$

which can be equivalently stated as $\theta^f > \theta_z$, where the threshold belief θ_z is defined as follows:

$$\theta_z \equiv \frac{(\lambda - 1) I - (\lambda' - 1) X^L}{(\lambda' - 1) (X^H - X^L)}. \tag{3}$$

The firm's current shareholders have heterogeneous beliefs about its future prospects. One group of outsiders, who hold a fraction δ of the firm's shares $(\delta < 1 - \alpha)$, are relatively less optimistic about the success probability of the firm's project: they believe that this probability is $\underline{\theta}$. The second group of

outsiders, who hold the fraction $(1 - \alpha - \delta)$ of the firm's equity, are relatively more optimistic about the success probability of the firm's project: they believe that this probability is $\overline{\theta}$, where $0 < \underline{\theta} < \overline{\theta} < 1$ (recall that firm insiders hold the remaining fraction α of the firm's equity). Finally, we assume that outside investors in the equity market who currently do not own shares in the firm also have a probability assessment $\underline{\theta}$ about the success probability of the firm's project.[11] The structure of insider and outsider beliefs and project payoffs are depicted in Figure 2.

In our basic model, we assume that the firm cannot raise any external financing to fund its investment in its new project. We will relax this assumption in our extended model. Further, we assume that the earnings E at time 1 are large enough for the firm to fund the project at the full investment level: i.e., $E \geq \lambda I$. Consistent with much of the literature on heterogeneous beliefs, we assume that all investors are subject to a short-sale constraint; i.e., short selling in the firm's equity is not allowed in this economy.[12]

The objective of firm insiders is to choose the optimal payout policy in order to maximize the expected sum of the time-1 and time-2 payoffs to firm insiders, based on insiders' belief, θ^f, about the firm's future prospects. There is a risk-free asset in the economy, the net return on which is normalized to 0. All agents are assumed to be risk-neutral.

To avoid notational clutter, we will make use of the following parameter definitions in the remainder

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[11]This assumption is appropriate, since one would expect that outsiders who are more optimistic about the firm's future prospects to be the first to buy shares in the firm (at any given price), so that investors who are not current shareholders would be those who are relatively less optimistic about the firm's future prospects.

[12]As in the existing literature on heterogeneous beliefs (see, e.g., Miller (1977) or Morris (1996)) we assume short-sale constraints throughout, so that the effects of differences in beliefs among investors are not arbitraged away. The above standard assumption is made only for analytical tractability: our results go through qualitatively unchanged as long as short selling is costly (see, e.g., Duffie, Gârleanu, and Pedersen (2002)).

of the text:

$$X^f \equiv \theta^f X^H + (1 - \theta^f)X^L, \tag{4}$$

$$\underline{X} \equiv \underline{\theta} X^H + (1 - \underline{\theta})X^L, \tag{5}$$

$$\overline{X} \equiv \overline{\theta} X^H + (1 - \overline{\theta})X^L. \tag{6}$$

Further, for ease of exposition, we will specify two ranges of the amount of earnings E available at time 1 to the firm: small or large. We assume that if the amount of earnings available to the firm is small, then if the firm implements its project at the full investment level (i.e., invests an amount λI in its project), the amount of cash it will have left over will be adequate to buy back only a fraction of equity less than the fraction δ held by pessimistic shareholders. On the other hand, if the firm underinvests (i.e., it invests only an amount I in its project), then the amount left over to distribute to shareholders is large enough to buy back all the shares held by pessimistic outside shareholders, but also (some shares) from optimistic outside shareholders. Thus, when the amount of earnings available to the firm is small, the following parametric restriction holds:[13]

$$\frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I} < \delta < \frac{E - I}{E + \underline{X} - I}. \tag{7}$$

We assume that if the amount of earnings available to the firm at time 1 is large, even if the firm implements its project at its full investment level, the amount of cash left over will be adequate to buy back the entire equity δ held by pessimistic outside shareholders, and also some shares from optimistic

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[13]If the firm invests to the full extent in its project, the expected cash flow from the project based on the belief $\underline{\theta}$ of pessimistic current shareholders is $\lambda' \underline{X}$, and the earnings after investment is $(E - \lambda I)$. Further, the NPV of the new project based on the belief of pessimistic shareholders is then $(\lambda' \underline{X} - \lambda I)$. Thus, in the case where E is small and the firm implements its project at the full investment level, the repurchase price will be $(E + \lambda' \underline{X} - \lambda I)$. If the firm underinvests in its project, then the firm buys back more than δ shares regardless of the level of time-1 earnings E. In this case, the repurchase price will be $\frac{\underline{X}}{\delta \underline{X} + (1 - \delta)\overline{X}}(E + \overline{X} - I)$ for the first δ shares and $\frac{\overline{X}}{\delta \underline{X} + (1 - \delta)\overline{X}}(E + \overline{X} - I)$ for the remaining shares.

outside shareholders. In other words, when the amount of earnings available to the firm is large, the relevant parametric restriction is:

$$\delta < \frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I} < \frac{E - I}{E + \underline{X} - I}.^{14}$$ (8)

Finally, we also assume that, regardless of whether the amount of earnings E available inside the firm at time 1 is small or large, it is not large enough to buy back all the equity held by outsiders (i.e., optimistic and pessimistic shareholders combined).[15]

2.1   The Choice Between Stock Repurchase and Cash Dividend

In this section, we analyze the firm's choice between a stock repurchase and a cash dividend. The firm has to choose between three possible ways of paying out excess cash to shareholders: (i) Stock repurchase alone; (ii) Paying out dividends alone; (iii) A combination of a stock repurchase and dividend payout.

While determining its payout policy, the firm also simultaneously chooses its investment policy: i.e., it decides whether to invest in its new project up to the full investment level, or to underinvest in it. Clearly, if the firm undertakes its project at the full investment level, it will pay out only a smaller amount compared to the case where its underinvests in its project.

Five possible payout and investment choices made by the firm in equilibrium can be summarized as

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[14]As one can see in equations (7) and (8), the number of shares the firm can potentially repurchase from pessimistic outside shareholders when the firm implements its project at the full investment level is always less the number of shares the firm can repurchase from pessimistic outside shareholders when the firm underinvests: i.e., $\frac{E-\lambda I}{E+\lambda'\underline{X}-\lambda I} < \frac{E-I}{E+\underline{X}-I}$. This is an implication of our assumption that the firm's new project has decreasing returns to scale.

[15]In practice, outsiders' beliefs are likely to be continuously distributed (instead of having only two discrete beliefs, which we assume for tractability). In such a setting, as firms repurchase more and more shares, they have to buy from investors with higher and higher beliefs about the firm's prospects and this will drive up the repurchase price. The assumptions we make here on the small versus large earnings amount E available to the firm are designed to capture the essence of the above realistic scenario where a firm that devotes a larger amount of resources to repurchasing shares has to buy back some stock from more optimistic shareholders as well, thus paying a higher repurchase price on average (while maintaining our two-level belief structure). Thus, our parametric restriction (7) captures the notion that, when E is small, the firm is able to devote more resources to a share repurchase (and therefore pays a higher average repurchase price) only when it underinvests in its project. On the other hand, our parametric restriction (8) captures the notion that, when E is large, the firm is able to devote a similar amount of resources to a share repurchase even when it fully invests in its project.

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follows: (i) the firm chooses to invest up to the full investment level in its project, and distributes the remaining cash available to it at time 1 as dividend alone to outside shareholders; (ii) the firm chooses to invest up to the full investment level in its project, and distributes the remaining cash available to it at time 1 in the form of a stock repurchase to outside shareholders; (iii) the firm chooses to underinvest in its project, and distributes the remaining cash available to it at time 1 in the form of a stock repurchase; (iv) the firm chooses to underinvest in its project, and distributes the remaining cash available to it at time 1 through a combination of a stock repurchase and a dividend payment; (v) the firm chooses to invest up to the full investment level in its project, and distributes the remaining cash available to it at time 1 through a combination of a stock repurchase and a dividend payment.[16]

The three factors that drive firm insiders' equilibrium choices are the following. First, the incremental NPV of the firm's project implemented at its full investment level relative to the same project implemented at its underinvestment level. Second, whether the NPV of a stock repurchase is positive or negative, and if positive, the magnitude of this NPV. This, in turn, will depend upon the relative levels of the beliefs of firm insiders, θ^f, and that of the two groups of outside shareholders (optimists and pessimists), $\bar{\theta}$ and $\underline{\theta}$, respectively. Note that the latter two beliefs $\bar{\theta}$ and $\underline{\theta}$ will determine the price firm insiders need to pay to repurchase equity from optimistic and pessimistic outside shareholders, respectively. Third, the NPV of a dividend payment by the firm, which is zero regardless of the beliefs of either of the two groups of outside shareholders.

It is useful to first consider the firm's optimal investment policy conditional on its choice of payout policy. One should note that the incremental NPV of implementing the firm's project at the full investment level is positive based on insiders' belief (see equation (2)), while the NPV of paying out dividends is zero. Thus, when the firm distributes value to shareholders through a dividend payment

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[16]We will show that it is not optimal for the firm to underinvest in its new project and distribute the remaining cash available to it by making a dividend payment only.

only, it invests an amount λI in its project (i.e., up to the full investment level) and distributes its excess cash $(E - \lambda I)$ to shareholders as dividends. In this case, the equity valuation of outside shareholders (and therefore their beliefs about the firm's future prospects) does not affect its investment policy. In other words, in this setting, the investment and distribution policy of the firm is very similar to the benchmark case where there is no heterogeneity in beliefs either among outsiders or between firm insiders and outsiders: i.e., the firm invests to the fullest extent in any positive NPV project available to it, and distributes the remaining cash to outsiders in the form of dividends.

If the earnings E available to the firm is small and the firm distributes value to its shareholders through a stock repurchase only, it has to repurchase shares from pessimists alone when it implements its new project at the full investment level. On the other hand, the firm has to repurchase shares from both optimists and pessimists when it underinvests in its project. Therefore, if the optimists' belief $\overline{\theta}$ is significantly higher than the belief $\underline{\theta}$ of pessimists, the NPV of repurchasing shares from both optimists and pessimists will be significantly smaller than that of repurchasing shares from pessimists alone (since the price paid to repurchase equity from pessimists is much lower). Firm insiders will prefer to implement the firm's project at the full investment level, and repurchase shares only from pessimistic outside shareholders if the following condition holds:

$$\alpha \left(\frac{E + \lambda' \underline{X} - \lambda I}{\lambda' \underline{X}} \right) \lambda' X^f \geq \alpha \left(\frac{E + \overline{X} - I}{\delta \underline{X} + (1 - \delta)\overline{X}} \right) X^f. \tag{9}$$

In this case, the incremental NPV obtained from increasing the scale of the firm's project from I to λI will be greater than the NPV of repurchasing shares from optimistic as well as pessimistic outside shareholders. After rearranging (9), we equivalently obtain a threshold on optimists' belief $\overline{\theta}$, which is equal to

$$\overline{\theta}_u = \frac{\underline{\theta}(E - I - \delta(E + \lambda'\underline{X} - \lambda I)) + \frac{X^L}{(X^H - X^L)} ((\lambda - 1)I - (\lambda' - 1)\underline{X})}{(1 - \delta)(E + \lambda'\underline{X} - \lambda I) - \underline{X}}. \tag{10}$$

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If optimistic shareholders' belief $\overline{\theta}$ is above this threshold, the firm will implement its new project at the full investment level and repurchase $\frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I}$ shares from pessimistic shareholders only.[17]

If, on the other hand, the optimists' belief $\overline{\theta}$ is lower than the threshold given in (10), so that it is closer to that of pessimists' belief $\underline{\theta}$, then the price paid to repurchase shares from optimistic outside shareholders will also be lower (and closer to the price paid to repurchase shares from the pessimists). In this case, the NPV of repurchasing shares from both pessimists and optimists will be significantly larger than that in the scenario where the optimists' belief $\overline{\theta}$ is higher than $\overline{\theta}_u$, and it will exceed the incremental NPV obtained from increasing the scale of the firm's project from the underinvestment to the full investment level. Hence, in this case, the firm will choose to underinvest in its project, buy back all the shares (δ) held by pessimistic outside shareholders and also some of the shares held by optimistic shareholders.[18]

We now characterize the conditions under which the firm uses a stock repurchase alone, a dividend payment alone, or a combination of a stock repurchase and a dividend payment to distribute value to shareholders. We first analyze the case where the earnings E available to the firm at time 1 is small.

Proposition 1. *(The Choice Between Stock Repurchase and Cash Dividend when E is* ***small****) Let the earnings E available to the firm at time 1 be small, so that the parametric restriction (7) holds. Then, the optimal investment and payout policy of the firm will depend on the belief levels $\underline{\theta}$ and $\overline{\theta}$ of optimists and pessimists respectively relative to the threshold beliefs θ_z and $\overline{\theta}_u$ and insider belief θ^f, as follows:*

(i) *If $\theta_z \leq \underline{\theta} < \overline{\theta}$, where θ_z is defined in (3), then:*

    (a) *If $\theta^f \leq \underline{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a dividend payment $(E - \lambda I)$ alone.*

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[17]Note that the threshold $\overline{\theta}_u$ defined in (10) is increasing in δ, λ, and I, whereas it is decreasing in λ' and E. In other words, the higher the NPV of increasing the scale of the new project, the lower is the threshold $\overline{\theta}_u$. Further, the greater the number of shares δ held by pessimistic shareholders relative to the earnings E available to the firm at time 1, the higher is this threshold.

[18]Note that for the parameter condition $\underline{\theta} < \overline{\theta} < \overline{\theta}_u$ to be satisfied, it must be the case that $\overline{\theta}_u > \underline{\theta}$. This, in turn, implies that the pessimistic shareholders' belief $\underline{\theta}$ must be less than the threshold belief θ_z defined in (3). In other words, a necessary condition for the firm to underinvest and repurchase shares from both pessimistic and optimistic groups of shareholders is that the NPV of the new project based on pessimistic shareholders' belief $\underline{\theta}$ is negative.

(b) If $\underline{\theta} < \theta^f$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone, repurchasing $\frac{E-\lambda I}{E+\lambda'\underline{X}-\lambda I}$ shares from pessimistic shareholders with belief $\underline{\theta}$.

(ii) If $\underline{\theta} < \theta_z < \overline{\theta}_u < \overline{\theta}$, where $\overline{\theta}_u$ is defined in (10), then:

(a) If $\theta^f \leq \theta_u^f$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a combination of a dividend payment and a stock repurchase, repurchasing δ shares from pessimistic outside shareholders with belief $\underline{\theta}$ and paying a cash dividend of $(E - I - \delta(E + \underline{X} - I))$.

(b) If $\theta^f > \theta_u^f$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone, repurchasing $\frac{E-\lambda I}{E+\lambda'\underline{X}-\lambda I}$ shares from pessimistic shareholders with belief $\underline{\theta}$.

(iii) If $\underline{\theta} < \theta_z < \delta\underline{\theta} + (1-\delta)\overline{\theta} < \overline{\theta} < \overline{\theta}_u$, then:

(a) If $\theta^f \leq \delta\underline{\theta} + (1-\delta)\overline{\theta}$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a combination of a dividend payment and a stock repurchase, repurchasing δ shares from pessimistic outside shareholders with belief $\underline{\theta}$ and paying a cash dividend of $(E - I - \delta(E + \underline{X} - I))$.

(b) If $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E-I-\delta(E+\underline{X}-I)}{E+\overline{X}-I}\right)$ from optimistic shareholders with belief $\overline{\theta}$.

(iv) If $\underline{\theta} < \delta\underline{\theta} + (1-\delta)\overline{\theta} \leq \theta_z < \overline{\theta} < \overline{\theta}_u$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E-I-\delta(E+\underline{X}-I)}{E+\overline{X}-I}\right)$ from optimistic shareholders with belief $\overline{\theta}$.

(v) If $\underline{\theta} < \overline{\theta} \leq \theta_z$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E-I-\delta(E+\underline{X}-I)}{E+\overline{X}-I}\right)$ from optimistic shareholders with belief $\overline{\theta}$.

Part (i) of Proposition 1 characterizes a situation where both groups of outside shareholders are very optimistic about the prospects of the firm's new project so that even the belief of pessimistic shareholders, $\underline{\theta}$, is higher than the critical threshold θ_z given in (3). If this condition holds, the firm will choose to invest up to the full investment level λI in its project regardless of whether it distributes value to shareholders through a stock repurchase or through a dividend payment. In this case, firm

insiders prefer a dividend payment to a stock repurchase if the following condition holds:

$$\alpha \left(E - \lambda I + \lambda' X^f \right) > \alpha \left(\frac{E + \lambda' \underline{X} - \lambda I}{\lambda' \underline{X}} \right) \lambda' X^f, \tag{11}$$

which is equivalent to $\theta^f \leq \underline{\theta}$. Intuitively, if firm insiders are more pessimistic about the new project than both groups of outside shareholders (i.e., if $\theta^f \leq \underline{\theta} < \overline{\theta}$), they assess that the firm is overvalued by outside shareholders. In other words, the NPV of a stock repurchase will be negative based on insiders' belief. Therefore, the firm will choose to distribute the remaining cash available to it at time 1 as a dividend payment alone to outside shareholders. If, however, firm insiders are more optimistic than the pessimistic group of shareholders, repurchasing shares from these shareholders is a positive-NPV transaction for firm insiders, and the firm chooses to distribute value through a stock repurchase alone, repurchasing $\frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I}$ shares from pessimistic shareholders only.

In the situation characterized in part (ii) of Proposition 1, the belief of pessimistic outside shareholders, $\underline{\theta}$, is less than the threshold θ_z, and the belief of optimistic outside shareholders, $\overline{\theta}$, is greater than the threshold $\overline{\theta}_u$.[19] In this case, since insiders are more optimistic than pessimistic shareholders, i.e., $\theta^f > \theta_z > \underline{\theta}$, the NPV of repurchasing shares from pessimists is positive, and therefore, paying dividends alone, which is a zero-NPV transaction, is not an optimal choice for the firm. Further, we know that since $\overline{\theta} > \overline{\theta}_u$, the firm will prefer to implement its project at the full investment level in case it distributes value through a stock repurchase alone. However, this proposition shows that if the incremental NPV obtained by increasing the scale of the firm's project is relatively small (i.e., λ' and θ^f are relatively small), the firm will prefer to underinvest in its project, and distribute value to shareholders through a

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[19]Since we assume that $\theta^f > \theta_z$ throughout the paper, it follows that $\theta^f > \underline{\theta}$ in parts (ii) to (v) of Proposition 1.

combination of a stock repurchase and a dividend payment if the following condition holds:

$$\alpha \left(\frac{E + \lambda'\underline{X} - \lambda I}{\lambda'\underline{X}} \right) \lambda'X^f \leq \frac{\alpha}{1-\delta} \left[X^f + E - I - \delta(E + \underline{X} - I) \right], \tag{12}$$

which, in turn, is equivalent to the following condition:

$$\theta^f \leq \theta_u^f \equiv \frac{\theta(E - I - \delta(E + \underline{X} - I)) + \frac{(1-\delta)X^L}{(X^H - X^L)}((\lambda-1)I - (\lambda'-1)\underline{X})}{(1-\delta)(E + \lambda'\underline{X} - \lambda I) - \underline{X}}. \tag{13}$$

If $\theta^f \leq \theta_u^f$, the incremental NPV from repurchasing all the shares (δ) held by pessimistic investors will exceed the incremental NPV of increasing the scale of the project.[20] By conducting a stock repurchase program and making a dividend payment simultaneously, the firm can first repurchase all the δ shares held by pessimistic shareholders ($\underline{\theta}$) and then avoid repurchasing shares from optimistic shareholders at a higher price by paying out the remaining earnings as cash dividends. Therefore, if $\theta^f \leq \theta_u^f$, the firm will underinvest in its project, and use the remaining cash available to it ($E - I$) at time 1 to distribute value through a combination of a stock repurchase and a dividend payment. If, however, $\theta^f > \theta_u^f$, the incremental NPV obtained by increasing the scale of the project will be sufficiently large so that the firm will implement its project at the full investment level and distribute value through a stock repurchase alone.

Parts (iii) and (iv) of Proposition 1 characterize a situation, where the belief of pessimistic outside shareholders, $\underline{\theta}$, is below the threshold belief θ_z as in part (ii), but the belief of optimistic outside shareholders, $\overline{\theta}$, is less than the threshold belief $\overline{\theta}_u$. Since $\theta^f > \underline{\theta}$, a dividend payment alone is not optimal for firm insiders, who have an incentive to buy back undervalued shares from pessimistic current shareholders. Further, we also know that since $\overline{\theta} < \overline{\theta}_u$, the firm will prefer to underinvest in its new

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[20]Note that when E is small, if the firm implements its project at the full investment level, the remaining excess earnings $E - \lambda I$ is not enough to repurchase all the shares held by pessimistic outside shareholders.

project in case it distributes value through a stock repurchase alone. However, this proposition shows that, while the firm still chooses to underinvest in its project in the case outlined in parts (iii) and (iv) of Proposition 1, it will prefer to distribute value to its shareholders through a combination of a stock repurchase and a dividend payment if the following condition holds:

$$\alpha \left(\frac{E + \overline{X} - I}{\delta \underline{X} + (1 - \delta)\overline{X}} \right) X^f \leq \frac{\alpha}{1 - \delta} \left[X^f + E - I - \delta(E + \underline{X} - I) \right], \tag{14}$$

which is equivalent to the condition that $\theta^f \leq \delta\underline{\theta} + (1 - \delta)\overline{\theta}$. Thus, if firm insiders' belief θ^f is between the pessimistic shareholders' belief $\underline{\theta}$ and the optimistic shareholders' belief $\overline{\theta}$, and it is not very high, the firm will find that the NPV of repurchasing all the shares held by pessimistic shareholders alone will be greater than the NPV of repurchasing shares from both pessimistic and optimistic shareholders. In this case, the firm will be better off by first repurchasing all the shares (δ) held by pessimistic shareholders and then distributing the remaining cash through a dividend payment. On the other hand, if $\theta^f > \delta\underline{\theta} + (1 - \delta)\overline{\theta}$, repurchasing shares from optimistic shareholders is also a positive-NPV transaction. Therefore, the firm will find it optimal to distribute value through a stock repurchase alone while it underinvests in its new project in the cases described in parts (iii)(b) and (iv) of Proposition 1.

Part (v) of Proposition 1 characterizes a situation where both groups of outside shareholders are very pessimistic about the prospects of the firm's new project so that $\underline{\theta} < \overline{\theta} \leq \theta_z < \theta^f$. Since both groups of current shareholders are more pessimistic about the future prospects of the firm than firm insiders, the NPV of repurchasing shares from both groups of shareholders will clearly be larger than the NPV of repurchasing shares from pessimistic shareholders alone, and it will exceed the NPV of increasing the scale of the firm's new project from I to λI. Hence, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone in this situation.

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We next analyze the case where the earnings E available to the firm at time 1 is large, so that the parameter restriction given in (8) holds. In this case, if the firm implements its new project at the full investment level λI, the remaining cash available to it at time 1, $E - \lambda I$, is more than enough to repurchase all the shares (δ) held by pessimistic shareholders. Thus, if the firm uses a stock repurchase alone to distribute value to its shareholders, it will repurchase shares from both optimists and pessimists even when it implements its new project at the full investment level λI. However, the total number of shares the firm repurchases from optimists in the case it implements its project at the full investment level will still be substantially less than the number of shares it can repurchase from optimists in the case when it underinvests in its project:

$$\frac{E - \lambda I - \delta(E + \lambda'\underline{X} - \lambda I)}{E + \lambda'\overline{X} - \lambda I} < \frac{E - I - \delta(E + \underline{X} - \lambda I)}{E + \overline{X} - I}. \tag{15}$$

Thus, if the amount of earnings E is large, firm insiders will prefer to implement the firm's project at the full investment level, and repurchase a smaller number of shares from optimistic outside shareholders if the following condition holds:

$$\alpha \left(\frac{E + \lambda'\overline{X} - \lambda I}{\delta\lambda'\underline{X} + (1 - \delta)\lambda'\overline{X}} \right) \lambda'X^f > \alpha \left(\frac{E + \overline{X} - I}{\delta\underline{X} + (1 - \delta)\overline{X}} \right) X^f. \tag{16}$$

After rearranging (16), we find that the firm will prefer to implement its project at the full investment level λI and repurchase a smaller number of shares if the belief of optimistic shareholders is sufficiently high, i.e., if

$$\overline{\theta} \geq \theta_z. \tag{17}$$

Otherwise, if $\overline{\theta} < \theta_z$, the beliefs of both pessimistic and optimistic shareholders ($\underline{\theta}$ and $\overline{\theta}$) will be substantially less than the belief θ^f of firm insiders. In this case, firm insiders find that the NPV from

a stock repurchase (when the firm underinvests) is large enough that it is greater than the incremental NPV obtained from increasing the scale of the firm's project from the underinvestment to the full investment level. Therefore, the firm chooses to underinvest in its project, and repurchases a larger amount of equity.

Proposition 2. *(The Choice Between Stock Repurchase and Cash Dividend when E is large)* *Let the earnings E available to the firm at time 1 be large, so that the parametric restriction* (8) *holds. Then, the optimal investment and payout policy of the firm will depend on the belief levels $\underline{\theta}$ and $\overline{\theta}$ of optimists and pessimists respectively relative to the threshold belief θ_z and insider belief θ^f, as follows:*

(i) *If $\theta_z \leq \underline{\theta} < \overline{\theta}$, where θ_z is defined in* (3), *then:*

    (a) *If $\theta^f \leq \underline{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a dividend payment $(E - \lambda I)$ alone.*

    (b) *If $\underline{\theta} < \theta^f < \delta\underline{\theta}+(1-\delta)\overline{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a combination of a dividend payment and a stock repurchase, repurchasing δ shares from pessimistic outside shareholders with belief $\underline{\theta}$ and paying a cash dividend of $(E - \lambda I - \delta(E + \lambda'\underline{X} - \lambda I))$.*

    (c) *If $\delta\underline{\theta} + (1-\delta)\overline{\theta} \leq \theta^f$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$, and the remaining $\left(\frac{E - \lambda I - \delta(E+\lambda'\underline{X}-\lambda I)}{E+\lambda'\overline{X}-\lambda I}\right)$ from optimistic shareholders with belief $\overline{\theta}$.*

(ii) *If $\underline{\theta} < \theta_z < \delta\underline{\theta} + (1-\delta)\overline{\theta} < \overline{\theta}$, then:*

    (a) *If $\theta^f < \delta\underline{\theta} + (1-\delta)\overline{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a combination of a dividend payment and a stock repurchase, repurchasing δ shares from pessimistic outside shareholders with belief $\underline{\theta}$ and paying a cash dividend of $(E - \lambda I - \delta(E + \lambda'\underline{X} - \lambda I))$.*

    (b) *If $\delta\underline{\theta} + (1-\delta)\overline{\theta} \leq \theta^f$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E - \lambda I - \delta(E+\lambda'\underline{X}-\lambda I)}{E+\lambda'\overline{X}-\lambda I}\right)$ from optimistic shareholders with belief $\overline{\theta}$.*

(iii) *If $\underline{\theta} < \delta\underline{\theta} + (1-\delta)\overline{\theta} \leq \theta_z < \overline{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E - \lambda I - \delta(E+\lambda'\underline{X}-\lambda I)}{E+\lambda'\overline{X}-\lambda I}\right)$ from optimistic shareholders with belief $\overline{\theta}$.*

(iv) *If $\underline{\theta} < \overline{\theta} \leq \theta_z$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$, and the remaining $\left(\frac{E - I - \delta(E+\underline{X}-I)}{E+\overline{X}-I}\right)$ from optimistic shareholders with belief $\overline{\theta}$.*

Part (i) of Proposition 2 characterizes a situation where both groups of outside shareholders are relatively optimistic about the future prospects of the firm so that $\theta_z \leq \underline{\theta} < \overline{\theta}$. If insiders are more pessimistic about the firm's future prospects than pessimistic outside shareholders, i.e., if $\theta^f \leq \underline{\theta}$, the firm will choose to distribute its excess earnings through a dividend payment alone, since the NPV of a stock repurchase will be negative if both groups of outside shareholders overvalue the firm's shares relative to firm insiders. However, if insiders are more optimistic about the firm's future prospects than pessimistic outside shareholders (as in parts (i)(b) and (i)(c) of Proposition 2), repurchasing shares from pessimistic shareholders will be optimal for the firm, since the NPV of doing so will be positive based on insiders' belief.

If $\underline{\theta} < \theta^f < \delta\underline{\theta} + (1-\delta)\overline{\theta}$, the firm will prefer to distribute value to its shareholders through a combination of a stock repurchase and a dividend payment. It will repurchase all the shares (δ) held by pessimistic shareholders and distribute the remaining cash through a dividend payment, since repurchasing shares from optimistic shareholders will be too costly for the firm, when insiders' belief θ^f is closer to the belief $\underline{\theta}$ of pessimistic shareholders than it is to that of optimistic shareholders ($\overline{\theta}$). If insiders' belief θ^f is sufficiently high, however, so that it is greater than $\delta\underline{\theta} + (1-\delta)\overline{\theta}$, the firm will prefer to distribute its excess earnings to its shareholders through a stock repurchase alone, since, in this case, the NPV of repurchasing shares from both pessimistic and optimistic shareholders will be greater than the NPV of repurchasing shares from pessimistic shareholders alone. The same intuition also applies in parts (ii) and (iii) of Proposition 2.[21]

Part (iv) of Proposition 2 characterizes a situation where both groups of outside shareholders are very pessimistic about the prospects of the firm's new project so that $\underline{\theta} < \overline{\theta} \leq \theta_z$. In this case, the NPV of repurchasing shares from both groups of shareholders will be substantially large, since $\underline{\theta} < \overline{\theta} < \theta^f$. Thus, the firm will have an incentive to maximize the number of shares it repurchases by underinvesting

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[21]Since we assume that $\theta^f > \theta_z$ throughout the paper, it follows that $\theta^f > \underline{\theta}$ in parts (ii) to (iv) of Proposition 2.

in its project, since the NPV of repurchasing a larger number of shares from both pessimistic and optimistic shareholders will exceed the NPV of increasing the scale of the firm's new project from I to λI. Hence, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone in this situation.

In summary, it is worth noting the two broad ingredients driving our results discussed above. First, from the insiders' point of view, buying back shares from outside shareholders who are more pessimistic than themselves is a positive NPV transaction, while buying back equity from outsiders more optimistic than themselves is a negative NPV transaction. Further, paying out a cash dividend is a zero NPV transaction. This means that, if the firm has cash left over after buying back equity from shareholders more pessimistic than insiders (and after investment requirements are satisfied), then it will choose to pay it out as a cash dividend.[22] Second, because buying back equity from shareholders more pessimistic than insiders is a positive NPV transaction, it makes sense for the firm to underinvest in its project if the NPV of devoting additional resources to a stock repurchase is greater than the NPV of using these resources to scale up investment in the firm's project.

3 The Price Impact of Cash Dividends and Stock Repurchases

In this section, we investigate the price impact of cash dividend payments or stock repurchases on the current stock price of the firm on the day the payout becomes effective ("the execution day" from now on). The price impact is measured as the abnormal return to the firm's equity from the price prevailing before a cash dividend payment or a stock repurchase (not the announcement date) to the

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[22]Note that this trade-off driving our results does not depend on the two-level (discrete) belief structure that we have assumed in our model for tractability. Thus, if outside shareholders' beliefs are distributed continuously, with some outsiders more pessimistic than firm insiders and others more optimistic than them, it will be optimal for the firm to use part of their earnings left over after investment to repurchase shares only from the pessimists while using the remaining amount to pay a cash dividend.

price prevailing after the corporate payout.[23, 24] Thus, the price impact by the total return on the execution day, which is the sum of dividend yield and capital gains yield.[25]

Proposition 3. *(The Price Impact of Cash Dividends and Stock Repurchases)*

  (i) *If the firm pays a cash dividend, then the price impact will be zero.*

 (ii) *If the earnings E available to the firm at time 1 is small, so that the parametric restriction* (7) *holds, then:*

   (a) *The price impact of a stock repurchase will be weakly positive.*

   (b) *The price impact of a stock repurchase will be weakly greater when the firm repurchases a larger number of shares compared to the situation where it repurchases a smaller number of shares.*

(iii) *If the earnings E available to the firm at time 1 is large, so that the parametric restriction* (8) *holds, the price impact of a stock repurchase will be strictly positive.*

Before the cash dividend or stock repurchase comes into effect, the firm's stock price at time 0 is either $(E + \underline{X} - I)$ or $(E + \lambda' \underline{X} - \lambda I)$, depending on whether the firm is known to implement its project at the underinvestment or the full investment level. Note that the pricing of equity at time 0 is driven by the belief (and therefore, the valuation) of the marginal investor in the firm's equity, which is $\underline{\theta}$.[26]

If the firm makes a cash dividend payment, the share price at time 1 will decrease by the amount of the cash dividend. Without loss of generality, if the firm chooses to implement its project at the full

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[23]Note that, empirically, the price impact is quite different from the announcement effect measured on the day of the announcement of the corporate payout (time 0 in our model), while the price impact is the abnormal return measured on the day the cash dividend or stock repurchase actually comes into effect (time 1 in our model).

[24]We do not allow strategic trading among outsiders in this model. One may wonder, however, whether the entire price impact of a stock repurchase will move earlier, to the announcement date, in practice if outsiders are allowed to strategically buy shares on the announcement date in order to take advantage of the anticipated increase in share price upon repurchase. It is possible that a fraction of the price impact will come into effect on the announcement date if such strategic trading is allowed. However, given that, in practice, firms do not have to commit to implementing an open market repurchase program authorized by its board fully, and even when fully implemented, such a share repurchase occurs over a long (indeterminate) period of time at various different prices, there is significant uncertainty associated with the share price increase associated with a stock repurchase. Consequently, it is unlikely that the entire price impact of a share repurchase will move earlier to the announcement date of the repurchase even when strategic trading among outsiders is allowed.

[25]In the case of a stock repurchase, the dividend yield is zero and the capital gains yield is potentially nonzero. In contrast, in the case of a cash dividend payment, both the dividend yield and capital gains yield are potentially nonzero. Therefore, we must take both the dividend yield and capital gains yield into account if we are to compare the total returns on the execution day for a dividend payment versus a stock repurchase.

[26]In any equity market, the price at which a trade takes place will be the lowest price at which sellers are willing to sell and the highest price at which buyers are willing to buy. In our setting, this will clearly be determined by the belief $\underline{\theta}$, since this is the belief of current pessimistic equity holders about the firm's future prospects, which also equals the belief of non-shareholders.

investment level and makes a dividend payment of $(E - \lambda I)$, the firm's stock price will be $(E + \lambda'\underline{X} - \lambda I)$ at time 0 before the dividend payment and $\lambda'\underline{X}$ at time 1 after the payment. However, the total stock return at time 1 will be zero, because the capital gains yield will be $-\left(\frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)$, and the dividend yield will be $\left(\frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)$, so that the total return, which is the sum of the two, will be zero.

If the earnings E available to the firm at time 1 is small and the firm chooses to undertake a stock repurchase, the price impact of the stock repurchase will depend on the number of shares the firm repurchases. If the firm implements its project at the full investment level, the firm's share price will be $(E + \lambda'\underline{X} - \lambda I)$ before the stock repurchase. After the stock repurchase, the total firm value will be $\lambda'\underline{X}$, but the total number of shares outstanding will be reduced from 1 to $\left(1 - \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)$, so that the share price will be

$$\frac{\lambda'\underline{X}}{1 - \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}} = E + \lambda'\underline{X} - \lambda I, \tag{18}$$

yielding a total return of zero on the firm's equity (i.e., the price impact is zero). Notice that, in this case, the number of shares repurchased by the firm is less than δ, and the firm repurchases shares from pessimistic outside shareholders only. Thus, the price of the firm's equity does not change after the stock repurchase, since the marginal investor in the firm's equity after the repurchase is the same as that before the repurchase, namely, a pessimistic investor with belief $\underline{\theta}$.

On the other hand, if the firm undertakes a stock repurchase while it underinvests in its new project, it will have to repurchase a larger number of shares (from both pessimistic and optimistic outside shareholders) compared to the above case where it undertakes a stock repurchase while implementing its project at the full investment level. Before the stock repurchase, the firm's share price will be $(E + \underline{X} - I)$. After the repurchase, the total firm value will be $\overline{X}$, but the total number of shares

outstanding will be reduced from 1 to $\left(1 - \delta - \frac{E-I-\delta(E+\underline{X}-I)}{E+\overline{X}-I}\right)$, so that the share price will be

$$\frac{\overline{X}}{1 - \delta - \frac{E-I-\delta(E+\underline{X}-I)}{E+\overline{X}-I}} = \frac{\overline{X}}{\delta\underline{X} + (1-\delta)\overline{X}}(E + \overline{X} - I). \tag{19}$$

Thus, the total return on the firm's equity will be

$$\frac{\frac{\overline{X}}{\delta\underline{X}+(1-\delta)\overline{X}}(E + \underline{X} - I) - (E + \underline{X} - I)}{E + \underline{X} - I} > 0, \tag{20}$$

yielding a positive price impact. Here, the share price after the stock repurchase is higher than that prevailing before the stock repurchase, since the marginal investor in the firm's equity after the stock repurchase is an optimistic investor with belief $\overline{\theta}$, while the marginal investor prior to the repurchase was a pessimistic investor with belief $\underline{\theta}$.[27, 28]

The intuition behind part (ii)(b) of Proposition 3 is that, when a firm repurchases a larger number of shares, it is more likely that the marginal investor in the firm's equity changes to a more optimistic investor after the repurchase compared to that before the repurchase.[29] Thus, if we compare the two situations of stock repurchase characterized above, the price impact when the firm invests up to the full investment level and repurchases a smaller number of shares is zero, while it is positive when the firm

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[27]Note that when a firm undertakes a stock repurchase only and underinvests in its new project, the price impact is given by (20) regardless of whether the earnings E is small or large.

[28]When E is small, if the firm chooses to distribute value through a combination of a stock repurchase and a dividend payment, and underinvest in its new project, it first repurchases δ shares from pessimistic shareholders, and then pays a cash dividend of $(E - I - \delta(E + \underline{X} - I))$ to the remaining shareholders. The share price at time 0 will be $(E + \underline{X} - I)$. After the repurchase, the total number of shares outstanding will be $(1 - \delta)$. If the number of shares repurchased by the firm is slightly (ϵ) less than δ, the marginal investor in the firm's equity will still be a pessimistic investor with belief $\underline{\theta}$ after the payout at time 1. Therefore, the share price at time 1 after the stock repurchase and the dividend payment is $\frac{\underline{X}}{1-\delta}$, which implies a capital gain of $\left(\frac{\underline{X}}{1-\delta} - (E + \underline{X} - I)\right)$, while the dividend per share is $\left(\frac{E-I-\delta(E+\underline{X}-I)}{1-\delta}\right)$. The total return on the firm's equity, and therefore, the price impact is zero, since the dividend payment and the capital gain cancel each other out.

[29]In other words, when the firm repurchases a larger number of shares, the firm will be more likely to go up the ladder of shareholder beliefs, and therefore, the price impact will be greater. The price impact of a stock repurchase is not continuously increasing in our model due to our assumption of two discrete groups of investors, one with more pessimistic beliefs than the other. If instead we assume that there are atomistic investors and their beliefs are continuously distributed, then any number of shares repurchased will change marginal investor beliefs and therefore, the price of the firm's stock. In such a setting, the price impact will be strictly increasing in the number of shares repurchased.

underinvests and repurchases a larger number of shares.

Part (iii) of Proposition 3 shows that if the earnings E is large and the firm chooses to undertake a stock repurchase alone to distribute value to shareholders, the price impact of the stock repurchase will be strictly positive, since the marginal investor in the firm's equity changes from pessimistic (before the repurchase) to optimistic (after the repurchase) regardless of the amount the firm chooses to invest in its new project. Unlike in the above case where E is small, if the firm chooses to implement its project at the full investment level when E is large, the firm will repurchase shares from both pessimistic and optimistic outside shareholders. In this case, we show that the total return on the firm's equity will be equal to

$$\frac{\frac{\overline{X}}{\delta \underline{X} + (1-\delta)\overline{X}}(E + \lambda'\underline{X} - \lambda I) - (E + \lambda'\underline{X} - \lambda I)}{E + \lambda'\underline{X} - \lambda I} > 0, \tag{21}$$

yielding a positive price impact in the case where the firm chooses a stock repurchase alone to distribute value while implementing its project at the full investment level. Thus, when E is large, a stock repurchase has a positive price impact regardless of whether the firm underinvests in its project or implements it at the full investment level, since after the repurchase, the belief of the marginal investor in the firm's equity increases from $\underline{\theta}$ to $\overline{\theta}$.

4   Analysis of the Case Where the Firm May Raise External Financing

In this section, we allow the firm to issue debt or equity to raise external financing from outside investors. Our objective here is to analyze whether the firm has an incentive to raise external financing by issuing debt or equity while distributing value to shareholders, and if so, to study the method the firm would adopt to implement this payout (cash dividend or stock repurchase).

4.1 Analysis of the Case Where the Firm Chooses to Issue Debt

In this subsection, we show that when outside investors and some of the firm's current outside shareholders are more pessimistic about the firm's future prospects than firm insiders, i.e., $\underline{\theta} < \theta^f$, issuing debt (the less belief-sensitive security) and using the proceeds from the debt issue to buy back more of the firm's (undervalued) equity (the more belief-sensitive security) can benefit firm insiders. The debt issued by the firm can be either risk-free or risky.[30]

There are mainly two reasons why the firm may choose to issue some debt to increase the size of its stock repurchase program. First, if the internally generated earnings E available to the firm at time 1 is small and the firm implements its new project at the full investment level, the firm's excess earnings after investment, $E - \lambda I$, will not be large enough to buy back all the undervalued equity (δ shares) held by pessimistic outside shareholders, even though the NPV of repurchasing these shares is positive when $\underline{\theta} < \theta^f$. In this case, the firm has an incentive to issue some debt to repurchase more undervalued shares from pessimists. The firm can issue risk-free debt against the cash flows of the new project to repurchase all the δ shares held by pessimistic shareholders, if δ is at or below a threshold δ^*, which is defined by the following equality:

$$\delta^* \left(E - \lambda I + \lambda' \underline{X} \right) - (E - \lambda I) = \lambda' X^L. \tag{22}$$

If $\delta > \delta^* = \frac{\lambda' X^L + E - \lambda I}{E - \lambda I + \lambda' \underline{X}}$, the firm has to issue risky debt in order to repurchase more than δ^* shares.

Second, if firm insiders are much more optimistic about the firm's future prospects relative to both groups of current shareholders so that $\theta^f > \delta \underline{\theta} + (1 - \delta)\overline{\theta}$, the NPV of repurchasing shares from optimistic current shareholders is also positive. In this case, regardless of its investment policy, the firm will also have an incentive to increase the size of its stock repurchase program if it is able to issue some risk-free

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[30]Note that risk-free debt is not belief-sensitive at all; i.e., it is belief-neutral.

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debt.

We will now fully characterize the conditions under which the firm issues new debt to raise external financing while distributing value to shareholders.

Proposition 4. *(The Case Where the Firm Chooses to Simultaneously Issue Debt and Distribute Value)* *Let the earnings E available to the firm at time 1 be small, so that the parametric restriction (7) holds. If $\theta^f > \underline{\theta}$, then:*

(i) *If $\theta^f \leq \delta\underline{\theta} + (1-\delta)\overline{\theta}$ and $\delta \leq \delta^*$, the firm will issue new risk-free debt worth $P_D = \delta\left(E - \lambda I + \lambda'\underline{X}\right) - (E - \lambda I)$, repurchase all the shares (δ) held by its current outside shareholders with belief $\underline{\theta}$, and implement its project at the full investment level (λI).*

(ii) *If $\theta^f \leq \delta\underline{\theta} + (1-\delta)\overline{\theta}$ and $\delta > \delta^*$, the firm's optimal policies are as follows:*

   (a) *The firm will issue new risk-free debt worth $P_D = \lambda'X^L$, repurchase δ^* shares held by its current shareholders with belief $\underline{\theta}$, and implement its project at the full investment level (λI), if the following condition holds:*

$$\theta^f > \frac{\underline{\theta}\left(E - I + X^L - \delta\left(E - I + \underline{X}\right)\right)}{(1-\delta)(E - \lambda I + \lambda'\underline{X}) - \underline{\theta}(X^H - X^L)}. \tag{23}$$

   (b) *If the inequality given in (23) is reversed, the firm will choose not to issue any debt, and will implement its project at the underinvestment level (I), and distribute value through a combination of a dividend payment and a stock repurchase, repurchasing δ shares from pessimistic outside shareholders with belief $\underline{\theta}$ and paying a cash dividend of $(E - I - \delta(E + \underline{X} - I))$.*

(iii) *If $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$ and $\delta \leq \delta^*$, the firm's optimal policies are as follows:*

   (a) *If $\overline{\theta} \geq \theta_z$, where θ_z is given in (3), the firm will issue new risk-free debt worth $P_D = \lambda'X_L$, repurchase all the shares (δ) held by its current shareholders with belief $\underline{\theta}$ and some shares from its current shareholders with belief $\overline{\theta}$, and implement its project at the full investment level (λI).*

   (b) *If $\overline{\theta} < \theta_z$, the firm will issue new risk-free debt worth $P_D = X_L$, repurchase all the shares (δ) held by its current shareholders with belief $\underline{\theta}$ and some shares from its current shareholders with belief $\overline{\theta}$, and implement its project at the underinvestment level (I).*

(iv) *If $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$ and $\delta > \delta^*$, the firm's optimal policies are as follows:*

   (a) *The firm will issue new risk-free debt worth $P_D = \lambda'X^L$, repurchase δ^* shares held by its current shareholders with belief $\underline{\theta}$, and implement its project at the full investment level (λI), if the following condition holds:*

$$\overline{\theta} > \frac{\underline{\theta}\left(E - I + X^L - \delta\left(E - \lambda I + \lambda'\underline{X}\right)\right)}{(1-\delta)(E - \lambda I + \lambda'\underline{X}) - \underline{\theta}(X^H - X^L)}. \tag{24}$$

   (b) *If the condition given in (24) does not hold, the firm will issue new risk-free debt worth $P_D = X_L$, repurchase all the shares (δ) held by its current shareholders with belief $\underline{\theta}$ and some*

> shares from its current outside shareholders with belief $\overline{\overline{\theta}}$, and implement its project at the underinvestment level (I).

(v) *The firm will never choose to issue equity and simultaneously repurchase shares from its current shareholders. Further, the firm will never choose to raise external financing and simultaneously pay dividends to its current shareholders.*

Part (i) of Proposition 4 shows that the firm will optimally raise external financing by issuing some risk-free debt in order to repurchase all the δ shares held by pessimistic outside shareholders while implementing its new project at the full investment level. If $\theta^f \leq \delta\underline{\theta} + (1 - \delta)\overline{\theta}$, firm insiders have an incentive to repurchase shares held by pessimistic outside shareholders with belief $\underline{\theta}$ alone, since the NPV of repurchasing shares from optimistic outside shareholders with belief $\overline{\theta}$ is negative. Therefore, the firm does not buy back more than δ shares by issuing more risk-free debt even though it is able to do so (since $\delta \leq \delta^*$).

In part (ii) of Proposition 4, the firm has to issue risky debt in order to finance the repurchase of all δ shares held by pessimistic shareholders since $\delta > \delta^*$. However, the valuation of risky debt is sensitive to outside investors' beliefs, and therefore, issuing risky debt to buy back more than δ^* shares has a cost to firm insiders arising from the undervaluation (with respect to firm insiders' beliefs) of any risky debt issued. In other words, the positive NPV of repurchasing undervalued shares from pessimistic shareholders with belief $\underline{\theta}$ is exactly offset by the cost of issuing undervalued risky debt to outside investors with belief $\underline{\theta}$. If insiders' belief θ^f is greater than the threshold value given in (23), the incremental NPV of increasing the scale of the firm's project is sufficiently large so that the firm uses risk-free debt financing to repurchase δ^* shares from pessimistic current shareholders while implementing its project at the full investment level. If θ^f is below this threshold, the firm does not issue any debt, and chooses to underinvest in its project while distributing value through a combination of a dividend payment and a stock repurchase (since the incremental NPV of repurchasing all δ shares held by pessimistic shareholders exceeds the incremental NPV of increasing the scale of the firm's project).

Parts (iii) and (iv) of Proposition 4 show that the firm might find it optimal to issue debt to buy back additional shares from optimistic shareholders with belief $\overline{\theta}$ as well, since the NPV of repurchasing shares from optimistic shareholders is also positive when $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$. When firm insiders are more optimistic relative to both groups of outside shareholders, their incentive to scale up their repurchase of undervalued equity from current shareholders through debt financing from outside investors becomes stronger. For instance, in the case the firm underinvests in its project, it also issues some risk-free debt to repurchase more shares from optimistic current shareholders. Similarly, if the firm implements its project at the full investment level and has the capacity to issue risk-free debt to repurchase all δ shares held by pessimists (as in part (iii) of the proposition where $\delta \leq \delta^*$), it increases its borrowing of risk-free debt to repurchase some shares from optimistic shareholders as well.

If the incremental NPV from increasing the scale of the project is small (large) and the belief of optimistic outside shareholders is sufficiently low (high), the firm will choose to underinvest (fully invest) in its new project while issuing some debt to outside investors and distributing value to its current shareholders through a stock repurchase simultaneously. Note that in the cases described in parts (iii) and (iv), the firm does not to choose issue risky debt to buy back more shares from optimistic current shareholders, since the cost of issuing undervalued risky debt to outside investors with belief $\underline{\theta}$ exceeds the present value of repurchasing undervalued shares from optimistic current shareholders with belief $\overline{\theta}$. However, in both cases, the firm uses its capacity to issue risk-free debt to the greatest extent possible in order to increase the number of shares it repurchases from current outside shareholders, since issuing belief-insensitive risk-free debt involves no undervaluation cost.

Finally, the above proposition (part (v)) shows that it is not optimal for firm insiders to issue new equity to outside investors with belief $\underline{\theta}$, and then buy back stock from current outside shareholders with belief $\underline{\theta}$, since the valuation effects of these two transactions will cancel each other out. Further,

if $\theta^f > \underline{\theta}$, the firm will never choose to raise any external financing (by issuing either debt or equity), and simultaneously pay dividends to its current outside shareholders, since in this case, the NPV of repurchasing shares is positive while the NPV of paying out dividends is zero.

4.2 Analysis of the Case Where the Firm Chooses to Issue Equity

We now assume that the total wealth of outsiders who are interested in investing in the firm's equity is W. Recall also our assumption that outside investors who currently do not hold equity in the firm believe that the success probability of the firm's project is $\underline{\theta}$. We now characterize the conditions under which the firm simultaneously issues equity while distributing value to shareholders.

Proposition 5. *(The Case Where the Firm Chooses to Simultaneously Issue Equity and Distribute Value) Let the earnings E available to the firm at time 1 be small, so that the parametric restriction (7) holds. If $\theta^f < \underline{\theta}$, then:*

(i) The firm will issue new equity and make a dividend payment to its current shareholders simultaneously. In particular, the firm will implement its project at the full investment level (λI), issue $\frac{W}{E + \lambda' \underline{X} - \lambda I}$ new shares, and pay a cash dividend of $E - \lambda I + W$ to its current shareholders.

(ii) The firm will never choose to issue risk-free debt and simultaneously pay dividends to its current shareholders. Further, the firm will never raise external financing and simultaneously repurchase shares from current shareholders.

We showed earlier (in propositions 1 and 2), we showed that if $\theta^f < \underline{\theta}$, so that firm insiders are more pessimistic than both groups of outside shareholders about the future prospects of the firm, the firm will choose to distribute value to its shareholders through a dividend payment of magnitude $(E - \lambda I)$ alone while implementing its project at the full investment level (λI) at time 1.[31] The NPV of repurchasing shares is negative in this case, so that the firm prefers to make a dividend payment alone (which is a zero-NPV transaction) to distribute its excess earnings to shareholders. The intuition underlying Proposition 5 is that if $\theta^f < \underline{\theta}$, firm insiders will also have an incentive to issue new equity in order

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[31]Recall also that the firm chooses to implement its new project at the full investment level, since the incremental NPV of increasing the project's scale is positive.

to take advantage of the optimism of outside investors by selling new shares to them at a price higher than the insiders' own valuation of the firm, thereby increasing the firm's dividend payout to its current shareholders.[32]

Consider the case when the firm implements its project at the full investment level, issues β_b new shares to outsiders, and pays an amount D_c as a cash dividend. Since all outsiders who currently do not hold equity in the firm have belief $\underline{\theta}$, the price per share in the equity issue will be $(E + \lambda'\underline{X} - \lambda I)$. Before the stock issue, the firm has one share of stock outstanding. After the stock issue, the firm will have $(1 + \beta_b)$ shares outstanding. After investing an amount λI in its new project, the firm pays out a cash dividend of

$$D_c = E - \lambda I + \beta_b \left(E + \lambda'\underline{X} - \lambda I \right) \tag{25}$$

to its shareholders. The objective function of firm insiders is now given by:

$$\max_{\beta_b} \frac{\alpha}{1 + \beta_b} \left[E - \lambda I + \beta_b \left(E + \lambda'\underline{X} - \lambda I \right) + \lambda'X^f \right], \tag{26}$$

where $\beta_b \in [0, \frac{W}{E + \lambda'\underline{X} - \lambda I}]$.[33] The partial derivative of this objective function with respect to the choice variable β_b is equal to

$$\frac{\alpha \left(\underline{\theta} - \theta^f \right) \lambda' \left(X^H - X^L \right)}{(1 + \beta_b)^2}. \tag{27}$$

Note that if $\theta^f < \underline{\theta}$, this partial derivative is positive, and the optimal choice for the firm is to issue as many new shares as possible by setting $\beta_b = \frac{W}{E + \lambda'\underline{X} - \lambda I}$. On the other hand, if $\theta^f \geq \underline{\theta}$, the optimal choice for the firm is to issue no new equity by setting $\beta_b = 0$.[34]

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[32]Since outside investors' belief is $\underline{\theta}$, the NPV of issuing new equity at time 1 will be positive based on insiders' belief if and only if $\theta^f < \underline{\theta}$.

[33]If it implements the project at the full investment level, the maximum number of shares the firm can issue to outside investors is $\frac{W}{E + \lambda'\underline{X} - \lambda I}$.

[34]Note that in this case, i.e., when $\theta^f > \underline{\theta}$, if we don't restrict β_b to be nonnegative and E is sufficiently large so that (8) holds, the optimal value of β_b will be equal to $-\delta$, and the firm will optimally repurchase the δ shares held by pessimistic shareholders, and distribute the remaining earnings as a dividend payment. The optimality of a combination of a share

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Finally, the above proposition (part (ii)) shows that the firm chooses to increase its dividend payout by simultaneously issuing (overvalued) equity rather than risk-free debt when $\theta^f < \underline{\theta}$, since issuing risk-free debt is a zero-NPV transaction.[35] Further, in this setting where firm insiders are more pessimistic than either group of outside shareholders, the firm will not use external financing to simultaneously repurchase (overvalued) equity as a means of distributing value to its shareholders. Earlier, we showed in Propositions 1, 2, and 4 that the firm will have an incentive to repurchase equity from current outside shareholders only if insiders are more optimistic than (at least) pessimistic shareholders, i.e., if $\theta^f > \underline{\theta}$. On the other hand, firm insiders have an incentive to issue new equity only if they are more pessimistic than the marginal outside investor in the firm's equity with belief $\underline{\theta}$, i.e. if $\theta^f < \underline{\theta}$. Thus, it follows that insiders never find it optimal to issue new equity and repurchase shares from current shareholders at the same time. In other words, when the NPV of issuing new equity based on insiders' beliefs is positive, the NPV of repurchasing shares from current shareholders is negative, and vice versa. On the other hand, the NPV of a dividend payment is always zero, and if firm insiders are more pessimistic than outside investors about the firm's future prospects (so that the NPV of issuing new equity is positive), it will be optimal for the firm to issue equity and distribute value through a dividend payment simultaneously.

5   Long-Run Stock Returns following Dividend Payments and Stock Repurchases

In this section, we will extend our basic model to analyze the long-run stock returns of firms following cash dividend payments and stock repurchases. In extending our basic model, we will make the ad-

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repurchase and a dividend payment (when E is large) in this case was already shown in Proposition 2.

[35]In our model with two states of the world, the firm can also choose to issue overvalued risky debt to increase its dividend payout by choosing a face value F that is as close as possible to the project cash flow $\lambda' X^H$ in the good state of the world, i.e., by designing the risky debt issue to be as belief-sensitive or equity-like as possible. By doing so, insiders will receive an expected payoff equal to that received by issuing equity. However, we can show that for any $F < \lambda' X^H$, there exists a threshold W^* so that if $W > W^*$, firm insiders will prefer to issue equity rather than risky debt in order to increase its dividend payout when $\theta^f < \underline{\theta}$.

At time 0, insiders of a firm own a fraction α of the firm's equity. The remaining 1- α is held by a group of outside shareholders.

The total number of shares outstanding in the firm is normalized to 1.

Additional (noisy) information about the firm's prospects arrives.

Time 0   **Time 1**   **Time 2**   **Time 3**

The period over which long-term stock returns are measured.

The firm comes to know its time-1 earnings *E*; it chooses the scale of investment in its project; amount of dividends or share repurchase (or both) is announced.

Dividends are paid or share repurchase is initiated (or both); new project is implemented.
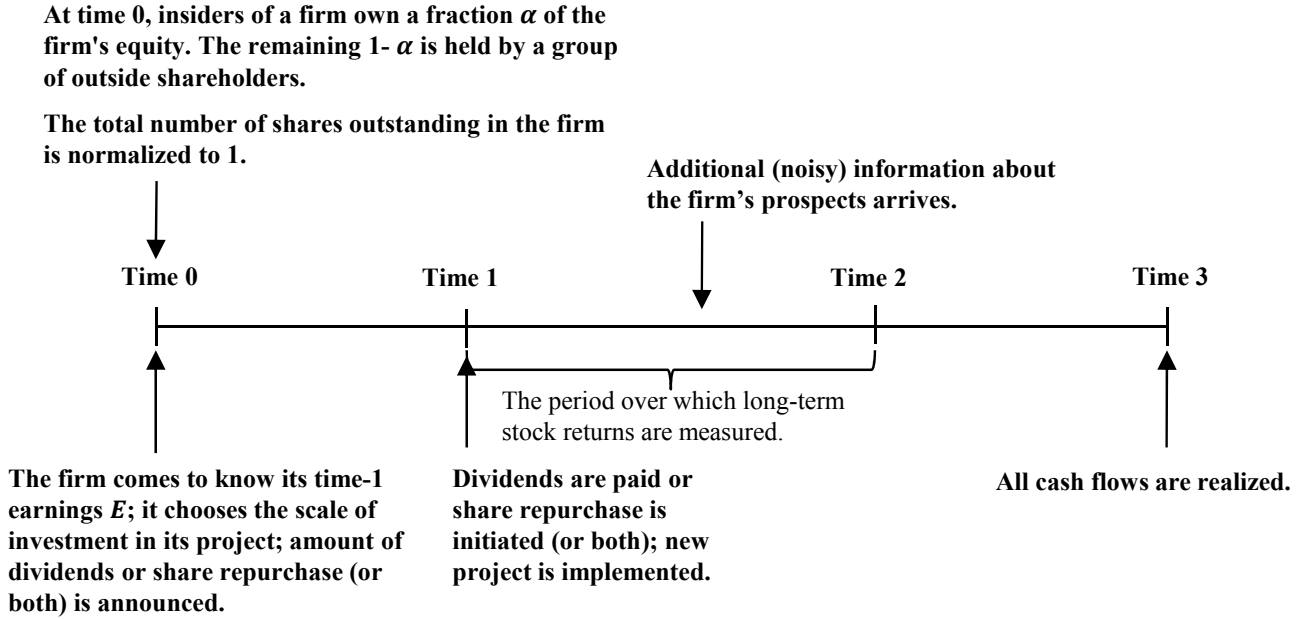
All cash flows are realized.

Figure 3: Sequence of Events in the Extended Model

ditional assumption that, in the long run, some additional noisy public information arrives about the firm's future performance. We denote the final date of our extended model as time 3, and introduce another date (time 2) between the corporate payout date (time 1) and the final cash flow realization date (time 3); noisy new information about the firm's future operating performance becomes available to outsiders at time 2. The sequence of events in this extended model is given in Figure 3.

The noisy new information arriving at time 2 is hard information, and may be collected from the firm's annual reports, earnings announcements, and other public sources. Thus, due to the arrival of this new information, we assume that investor beliefs about the firm's cash flows become less heterogeneous in the long run. The motivation for this assumption is that the beliefs of each agent is a function of their prior beliefs and the additional information they receive from various sources. Thus, at time 0, the beliefs of each of the three groups of agents in our model (firm insiders, optimistic outside shareholders, and pessimistic outside shareholders) is driven by their (heterogeneous) prior beliefs. On the other hand, at time 2, the beliefs of these three groups of agents is determined not only by their prior beliefs,

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but also by the common (hard) information that arrives between the payout date and time 2. Since the proportion of information common to the above three groups of agents increases at time 2, their beliefs become less heterogeneous at this date. Therefore, we assume that in the long run (i.e., at time 2), the three different beliefs θ^f, $\overline{\theta}$, and $\underline{\theta}$ will converge toward each other. Let us define the beliefs of the insiders, optimistic outsiders, and pessimistic outsiders at time 2 as ν^f, $\overline{\nu}$, and $\underline{\nu}$, respectively. These beliefs converge toward each other in the long run (i.e., at time 2) as follows:

$$\nu^f = \theta^f + \frac{d}{3}\left(\overline{\theta} - \theta^f\right) + \frac{d}{3}\left(\underline{\theta} - \theta^f\right), \tag{28}$$

$$\overline{\nu} = \overline{\theta} + \frac{d}{3}\left(\theta^f - \overline{\theta}\right) + \frac{d}{3}\left(\underline{\theta} - \overline{\theta}\right), \tag{29}$$

$$\underline{\nu} = \underline{\theta} + \frac{d}{3}\left(\theta^f - \underline{\theta}\right) + \frac{d}{3}\left(\overline{\theta} - \underline{\theta}\right), \tag{30}$$

where $d \in [0,1]$ measures the degree of convergence. If $d = 0$, there is no convergence, and we have $\nu^f = \theta^f$, $\overline{\nu} = \overline{\theta}$, and $\underline{\nu} = \underline{\theta}$. On the other extreme, if $d = 1$, there is full convergence, and we have $\nu^f = \overline{\nu} = \underline{\nu} = \frac{1}{3}\left(\theta^f + \overline{\theta} + \underline{\theta}\right)$.

We first characterize the long-run stock return in a situation where the firm uses only a dividend payment to distribute value to shareholders.

Proposition 6. *(Long-run stock returns following cash dividends) If the firm makes a dividend payment and undertakes its new project at the full investment level, the long-run stock return will be $\frac{d\left(X^H - X^L\right)}{3\underline{X}}\left[\left(\theta^f - \underline{\theta}\right) + \left(\overline{\theta} - \underline{\theta}\right)\right]$. The long-run stock return will be positive if $\underline{\theta} - \left(\overline{\theta} - \underline{\theta}\right) < \theta^f \le \underline{\theta}$, and it will be negative if $\theta^f < \underline{\theta} - \left(\overline{\theta} - \underline{\theta}\right)$.*

If the firm distributes value through a dividend payment alone while implementing its new project at the full investment level, the firm's stock price will be $P_1 = \lambda'\underline{X}$ at time 1 after the dividend payment, and the number of shares outstanding will still be 1. At time 2, after the marginal equity investor

updates her belief, the stock price will be

$$P_2 = \lambda' X^{\underline{\nu}} \equiv \lambda' \left(\underline{\nu} X^H + (1 - \underline{\nu}) X^L \right). \tag{31}$$

The long-run stock return following the dividend payment will then be given by:

$$\frac{P_2 - P_1}{P_1} = \frac{d \left(X^H - X^L \right)}{3\underline{X}} \left[\left(\theta^f - \underline{\theta} \right) + \left(\overline{\theta} - \underline{\theta} \right) \right]. \tag{32}$$

We know that a firm will choose to distribute value through a dividend payment alone (while imple-menting its new project at the full investment level) only if $\theta^f < \underline{\theta}$ (at time 1). Thus, the long-run stock return will be positive if $\underline{\theta} - \left(\overline{\theta} - \underline{\theta} \right) < \theta^f \leq \underline{\theta}$. However, if firm insiders are much more pessimistic than the firm's current shareholders at the time of dividend payment so that $\theta^f < \underline{\theta} - \left(\overline{\theta} - \underline{\theta} \right)$, then the long-run stock return will be negative.

The intuition behind the above long-run stock return result is the following. Recall that the firm chooses to distribute value through a cash dividend alone only when both the optimists' and pessimists' beliefs are above that of firm insiders at time 1. Once additional information arrives about the firm's future performance, all three of the above sets of beliefs converge toward each other. If the pessimists' beliefs are much higher than the insiders' beliefs at time 1 (and closer to optimists' beliefs) then the pessimists' beliefs move downward between time 1 and time 2 as a result of the arrival of new information, resulting in a negative long-run stock return (since the pessimists are the marginal investors in the firm's equity upon a dividend payment). If, however, the pessimistic investors' beliefs are closer to insiders' beliefs (and farther away from optimists' beliefs) at time 1, then pessimists' beliefs move upward between time 1 and time 2 upon convergence toward the other two sets of beliefs as a result the arrival of information. The long-run stock return subsequent to a dividend payment will be positive in this case.

We now characterize the long-run stock return in a situation where the firm uses only a stock repurchase to distribute value to shareholders.

Proposition 7. *(Long-run stock returns following stock repurchases)*

(i) *If E is small, and the firm implements a stock repurchase and undertakes its project at the full investment level, then its long-run stock return will be $\frac{d(X^H - X^L)}{3\underline{X}}\left[\left(\theta^f - \underline{\theta}\right) + \left(\overline{\theta} - \underline{\theta}\right)\right]$, which is positive.*

(ii) *If the firm implements a stock repurchase and underinvests in its project, then its long-run stock return will be $\frac{d(X^H - X^L)}{3\overline{X}}\left[\left(\theta^f - \overline{\theta}\right) + \left(\underline{\theta} - \overline{\theta}\right)\right]$. This long-run stock return will be positive if $\theta^f > \overline{\theta} + \left(\overline{\theta} - \underline{\theta}\right)$, and it will be negative if $\underline{\theta} < \theta^f < \overline{\theta} + \left(\overline{\theta} - \underline{\theta}\right)$.*

(iii) *If E is large, and the firm implements a stock repurchase and undertakes its project at the full investment level, then its long-run stock return will be $\frac{d(X^H - X^L)}{3\overline{X}}\left[\left(\theta^f - \overline{\theta}\right) + \left(\underline{\theta} - \overline{\theta}\right)\right]$. This long-run stock return will be positive if $\theta^f > \overline{\theta} + \left(\overline{\theta} - \underline{\theta}\right)$, and it will be negative if $\underline{\theta} < \theta^f < \overline{\theta} + \left(\overline{\theta} - \underline{\theta}\right)$.*

When E is small and the firm chooses to implement its project at the full investment level and distribute value through a stock repurchase alone (as in part (i) of Proposition 7), the total firm value is $\lambda'\underline{X}$, but the number of shares outstanding after the repurchase is reduced to $\left(1 - \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)$, so that the price per share at time 1 is

$$P_1 = \frac{\lambda'\underline{X}}{\left(1 - \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)}. \tag{33}$$

At time 2, after the marginal equity investor updates her belief to $\underline{\nu}$, the price per share will change to

$$P_2 = \frac{\lambda'X^{\underline{\nu}}}{\left(1 - \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)}. \tag{34}$$

Therefore, the long-run stock return is equal to

$$\frac{P_2 - P_1}{P_1} = \frac{d\left(X^H - X^L\right)}{3\underline{X}}\left[\left(\theta^f - \underline{\theta}\right) + \left(\overline{\theta} - \underline{\theta}\right)\right]. \tag{35}$$

Since the firm chooses to implement its project at the full investment level and distribute value through a stock repurchase alone only if $\theta^f > \underline{\theta}$, it follows that the long-run stock return will be positive in

this case. Note that, in this case, the marginal investor in the firm's equity is the pessimistic group of shareholders, both before and after a share repurchase.

If the firm chooses to distribute value through a stock repurchase alone while underinvesting in its new project (as in part (ii) of Proposition 7), the firm's stock price per share will be $E + \underline{X} - I$ before the stock repurchase (because the firm value is $E + \underline{X} - I$ and the firm has one share outstanding). In this case, since the firm buys back all the δ shares held by pessimistic shareholders and some shares held by optimistic shareholders, the marginal shareholder in the firm's equity after the repurchase is the optimistic group of investors. Therefore, after the repurchase, the total firm value (based on the belief of the new marginal equity investor with belief $\overline{\theta}$) will be $\overline{X}$, but the number of shares outstanding will be reduced to $\left(1 - \delta - \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}\right)$, so the price per share at time 1 will be

$$P_1 = \frac{\overline{X}}{\left(1 - \delta - \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}\right)}. \tag{36}$$

At time 2, after the marginal equity investor updates her belief to $\overline{\nu}$, the stock price will change to

$$P_2 = \frac{X^{\overline{\nu}}}{\left(1 - \delta - \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}\right)}. \tag{37}$$

Therefore, in this case, the long-run stock return will be given by:

$$\frac{P_2 - P_1}{P_1} = \frac{d\left(X^H - X^L\right)}{3\overline{X}}\left[\left(\theta^f - \overline{\theta}\right) + \left(\underline{\theta} - \overline{\theta}\right)\right]. \tag{38}$$

Note that if both groups of outsiders (with beliefs $\underline{\theta}$ and $\overline{\theta}$, respectively) are much more pessimistic about the firm's future prospects than firm insiders with belief θ^f at time 1, so that $\theta^f > \overline{\theta} + \left(\overline{\theta} - \underline{\theta}\right)$, then the long-run stock return following the stock repurchase will be positive in this case. This is because, in this case, the beliefs of optimistic shareholders (who are the marginal investors in the firm's equity after the

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repurchase) move upward between time 1 and time 2 as a result of the convergence in beliefs across all three groups of agents due to the arrival of new information. If, however, the optimistic shareholders' beliefs are much closer to that of firm insiders (relative to their distance from pessimistic investors' beliefs), so that $\underline{\theta} < \theta^f < \overline{\theta} + (\overline{\theta} - \underline{\theta})$, then the long-run stock return following a stock repurchase will be negative. This is because, in this case, the beliefs of optimistic shareholders will move downward between time 1 and time 2 as a result of the convergence in beliefs across all three groups of agents due to the arrival of new information.

Finally, when E is large, if the firm chooses to distribute value through a stock repurchase alone while implementing its project at the full investment level (as in part (iii) of Proposition 7), the marginal investor in the firm's equity after the repurchase at time 1 will also have the belief $\overline{\theta}$ as in part (ii) of Proposition 7. Therefore, in this case, the long-run stock return following the stock repurchase will be positive if both groups of outsiders are much more pessimistic about the firm's future prospects than firm insiders, so that $\theta^f > \overline{\theta} + (\overline{\theta} - \underline{\theta})$. If, however, the optimistic shareholders' beliefs are much closer to that of firm insiders (relative to their distance from pessimistic investors' beliefs), so that $\underline{\theta} < \theta^f < \overline{\theta} + (\overline{\theta} - \underline{\theta})$, then the long-run stock return following a stock repurchase will be negative. The intuition here is similar to that behind part (ii) of Proposition 7, as discussed above.

We now characterize the long-run stock return in a situation where the firm uses a combination of a dividend payment and a stock repurchase to distribute value to shareholders.

Proposition 8. (Long-run stock returns following a combination of a stock repurchase and a cash dividend) *If the firm chooses a combination of a stock repurchase and a dividend payment to distribute value to shareholders, the long-run stock return will be $\frac{d(X^H - X^L)}{3\underline{X}} \left[(\theta^f - \underline{\theta}) + (\overline{\theta} - \underline{\theta}) \right]$, which is positive.*

For a firm to optimally choose a combination of a stock repurchase and a dividend payment to distribute value to its shareholders, it has to be the case that $\theta^f > \underline{\theta}$. In this case, the firm first repurchases δ shares from pessimistic shareholders and then makes a dividend payment of $(E - I - \delta(E + \underline{X} - I))$

to the remaining shareholders (when it underinvests in its project).[36] The price per share at time 1 after the stock repurchase and the dividend payment will be $P_1 = \frac{X}{1-\delta}$.[37] At time 2, after the arrival of additional information, the price will be $P_2 = \frac{X\nu}{1-\delta}$. The marginal investor in the firm's equity immediately after repurchase (at time 1) belongs to the pessimistic group of shareholders with belief $\underline{\theta}$, which will be below the belief θ^f of firm insiders and the belief $\overline{\theta}$ of optimistic current shareholders. This marginal investor belief will move upward between time 1 and time 2 as a result of the convergence in beliefs across all three groups of agents due to the arrival of new information. The long-run stock return following a combination of a stock repurchase and a cash dividend will therefore be unambiguously positive.

5.1 Comparison of Long-Run Stock Returns Following Dividends versus Stock Repurchases

We now compare the long-run stock returns of firms following dividend payments versus following stock repurchases.

Proposition 9. *(Comparison of the long-run stock return following a cash dividend versus that following a stock repurchase) Let the earnings E available at time 1 be small, and let firm insiders' belief θ^f and $k = \overline{\theta} - \underline{\theta}$ be constant across all firms. Consider two samples of firms: (i) a sample of firms, which distribute value through dividend payments alone and implement their projects at the full investment level; (ii) a sample of firms, which distribute value through stock repurchases alone and implement their projects at the full investment level. Then, if the beliefs of pessimistic as well as optimistic shareholders of the firms in the two samples are uniformly distributed, the average long-run stock return following payout of the latter sample of firms will be greater than the average long-run stock return following payout of the former sample of firms.*

Earlier we showed that if E is small, conditional on implementing its new project at the full investment level, a firm's long-run stock return following a pure cash dividend payment or a pure stock

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[36]It can be shown that the long-run stock return following a combination of a stock repurchase and a dividend payment will be positive regardless of the investment level at which the firm chooses to implement its project.

[37]We assume that the number of shares repurchased by the firm is slightly (ϵ) less than δ so that the marginal investor in the firm's equity has still the belief $\underline{\theta}$ after the payout at time 1.

repurchase is equal to

$$\frac{d\left(X^H - X^L\right)}{3\underline{X}}\left[\left(\theta^f - \underline{\theta}\right) + \left(\overline{\theta} - \underline{\theta}\right)\right].\tag{39}$$

Assuming that $k = \overline{\theta} - \underline{\theta}$, this long-run stock return is equal to $\frac{d(X^H - X^L)}{3\underline{X}}\left[\left(\theta^f - \underline{\theta}\right) + k\right].$[38]

Proposition 6 showed that, for a firm which distributes value through a dividend payment alone (while implementing its new project at the full investment level), this long-run stock return is positive if $\underline{\theta} - \left(\overline{\theta} - \underline{\theta}\right) < \theta^f \leq \underline{\theta}$, and it is negative if $\theta^f < \underline{\theta} - \left(\overline{\theta} - \underline{\theta}\right)$. We also know that a firm will choose to distribute value through a dividend payment only if $\theta^f \leq \underline{\theta}$ (see Proposition 1(i)(a)). If we assume that for the cross section of firms paying dividends only and implementing their projects at the full investment level, optimistic shareholders' belief $\overline{\theta}$ is uniformly distributed over the interval $[\theta^f + k, 1)$, it follows that for the same cross section of firms, pessimistic shareholders' belief $\underline{\theta}$ is uniformly distributed over the interval $[\theta^f, 1 - k)$. While the long-run stock return will be positive for some fraction of these firms ($\theta^f < \underline{\theta} \leq \theta^f + k$), it will be negative for the remaining fraction of them ($\theta^f + k < \underline{\theta} < 1 - k$). Thus, the *average* long-run stock return $\overline{r}_{div}$ of firms paying dividends only and implementing their projects at the full investment level may be positive or negative, and it is given by:[39]

$$\overline{r}_{div} = \frac{d\left(X^H - X^L\right)}{3}\int_{\theta^f}^{1-k}\frac{\left(\theta^f - \theta + k\right)}{X^L + \theta\left(X^H - X^L\right)}\frac{1}{\left(1 - k - \theta^f\right)}d\theta.\tag{40}$$

In contrast, a firm will optimally choose to distribute value through a stock repurchase alone if firm insiders are more optimistic than the pessimistic group of outside shareholders, i.e., if $\theta^f > \underline{\theta}$. Further, we know from Proposition 7(i) that, in this case (when E is small), the marginal equity investor's belief will be $\underline{\theta}$ after the stock repurchase. Thus, the long-run valuation of the firm at time 2 will be determined by the updated time-2 belief $\underline{\nu}$ of the marginal investor in the firm's equity at time 1, who

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[38]Note that the long-run stock return given in (39) is decreasing in $\underline{\theta}$ (regardless of whether $k = \overline{\theta} - \underline{\theta}$ is fixed or not).

[39]Note that $\theta^f - \theta < 0$ in the expression for $\overline{r}_{div}$ given in (40).

has the belief $\underline{\theta}$ at time 1. Note that for a firm distributing value through a stock repurchase alone, the pessimistic shareholders' belief $\underline{\theta}$ is always lower than both the optimistic shareholders' belief $\overline{\theta}$ and the firm insiders' belief θ^f. When the three beliefs converge in the long run, the ex-post value of $\underline{\theta}$ will be above the ex-ante value of $\underline{\theta}$. Therefore, the long-run stock return of a firm that distributes value through a stock repurchase alone and implements its new project at the full investment level will be positive as described in part (i) of Proposition 7. When E is small, for a firm to repurchase shares only and fully invest in its new project, we know from Proposition 1(i)(b) that $\theta_z \le \underline{\theta} < \theta^f$.[40] Thus, the *average* long-run stock return $\bar{r}_{rep}$ of firms repurchasing shares only and implementing their projects at the full investment level will be positive, and it is given by:

$$\bar{r}_{rep} = \frac{d\left(X^H - X^L\right)}{3} \int_{\theta_z}^{\theta^f} \frac{\left(\theta^f - \theta + k\right)}{X^L + \theta\left(X^H - X^L\right)} \frac{1}{\left(\theta^f - \theta_z\right)} d\theta. \tag{41}$$

It is straightforward to verify that $\bar{r}_{rep} > \bar{r}_{div}$. Thus, the average long-run stock return of firms that distribute value through a stock repurchase alone and implement their new projects at the full investment level will be greater than the average long-run stock return of firms that distribute value through a dividend payment only and implement their new projects at the full investment level.

In summary, the broad intuition driving the above result is the following. Under the conditions of the above proposition, the marginal investor in the firm's equity after payout following either a cash dividend or a stock repurchase is the pessimistic investor with belief $\underline{\theta}$. Further, the lower the value of $\underline{\theta}$, the higher the long-run stock return. Therefore, given that the value of $\underline{\theta}$ will always be lower for firms choosing to distribute value through a stock repurchase compared to that for firms choosing to

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[40]Note that strictly speaking, for some parameter specifications, a small fraction of the share-repurchasing (and fully-investing) firms may actually satisfy the condition given in Proposition 1(ii)(b) rather than the condition given in Proposition 1(i)(b). For these firms, it holds that $\theta_z - k < \underline{\theta} < \theta_z$. Since the long-run stock return given in (39) is decreasing in $\underline{\theta}$, the average long-run stock return of these firms is greater than the average long-run stock return of firms satisfying the condition that $\theta_z \le \underline{\theta} < \theta^f$. Since we can show that the average long-run stock return of share-repurchasing firms, for which $\theta_z \le \underline{\theta} < \theta^f$, is greater than the average long-run stock return of dividend-paying firms satisfying the condition given in Proposition 1(i)(a), the result in Proposition 9 holds for these parameter values as well.

do so through a cash dividend, the average long-run stock return for the former group of firms will be greater than that for the latter group of firms.[41]

6   Testable Implications

In this section, we describe some of the testable implications of our model. In developing testable implications based on our model, one has to interpret dividend payment in our model carefully. This is because our model of a firm's choice of payout policy is a static one, and it is well known that, in practice, many dynamic considerations enter into a firm's choice of dividend policy. For example, as documented by Lintner (1956) based on a survey of how managers choose dividend policy in practice, firm managers are usually reluctant to cut dividends (see, e.g., Lambrecht and Myers (2012) for a formal model of the dynamics of a firm's dividend policy). This means that, in many cases, dynamic considerations may prevent firms from reducing dividends and committing all available cash to make a stock repurchase, even though the purely static considerations based on heterogeneous beliefs that we model here may motivate them to do so. However, these problems arising from the interaction between dynamic and static considerations that drive a firm's choice of dividend policy can be almost fully addressed if we think of a "dividend payment" in our model as being, in practice, a dividend increase above its existing dividend level (or a dividend initiation, if the firm is not currently paying any dividends). Under this interpretation, if, for example, our model predicts that a firm will choose not to pay any dividends in equilibrium (i.e., it will choose to pay out all its cash through a repurchase), this translates, in practice, to the firm choosing not to raise its dividend above its existing level (so that the firm will maintain

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[41]While we make very specific assumptions to mathematically prove the result in Proposition 9, the broad intuition underlying this result is much more general, and is as follows. Regardless of whether the marginal investor in the firm's equity is an optimist or a pessimist, the belief of the marginal investor for a firm distributing value through a stock repurchase is in general lower than that for a firm distributing value through a cash dividend. Given that the long-run stock return is decreasing in the marginal investor belief immediately after payout, the average long-run stock return for firms distributing value through stock repurchases will be greater than that for firms distributing value through cash dividends.

its current dividend level, and pay out all available remaining cash to its shareholders through a stock repurchase). In reading our testable implications below, it is useful to keep in mind this "dynamic" interpretation of the static results in our model.

(i) Relationship between insider and outsider beliefs and payout policy: Our model predicts that, if the level of optimism (average belief) among outside shareholders and the dispersion in beliefs among outsiders is such that a significant proportion of outside shareholders are less optimistic than firm insiders, then the firm is more likely to distribute value through a stock repurchase rather than paying a cash dividend. Conversely, if the bulk of outside shareholders are at the same or a higher optimism level about the firm's future prospects compared to firm insiders, the firm is more likely to distribute value through a cash dividend. This is because the net present value of a stock repurchase to firm insiders will be greater (more positive) when outsiders are more pessimistic while a dividend payment is a zero net present value transaction regardless of outsider beliefs. The above prediction can be tested using proxies for investor optimism developed by Baker and Wurgler (2006), and the two standard proxies for heterogeneity in investor beliefs used in the literature, namely, the dispersion in analyst earnings forecasts and abnormal share turnover.

(ii) Relationship between dividends and stock repurchases and investment policy: Our model predicts that firms implementing stock repurchases are more likely to cut back on positive net present value projects compared to those initiating or increasing dividends. This is because, while a stock repurchase can be a positive net present value transaction for firm insiders (if outside investors in the firm are less optimistic), paying dividends is a zero net present value transaction in our setting regardless of the relative optimism of firm insiders and outside shareholders about the firm's future prospects. This means that, when a firm is cash–constrained, firm insiders may compare the NPV of the real investment opportunities available to it to the NPV of undertaking a stock repurchase, and may prefer to implement

a stock repurchase at the expense of undertaking the real investment opportunity if the latter NPV is greater. Some evidence that certain categories of firms cut back on investment in their projects to finance a stock repurchase is provided by Almeida et al. (2013).

(iii) Effects of macro-events on the propensity for a stock repurchase: Our model predicts that stock repurchases will increase after market crashes. A market crash is likely to make outside investors more pessimistic about the future prospects of firms (while insiders' beliefs are likely to be relatively less affected) resulting in an increase in the difference in beliefs of the two groups of agents (insiders versus outsiders). This, in turn, will lead to an increase in the number of firms repurchasing equity in periods where the difference in optimism between firm insiders and outsiders is greater.[42]

(iv) Choice between stock repurchases and cash dividend payments under external financing: Our model makes two predictions regarding a firm's payout policy under external financing. First, it predicts that, when a firm undertakes a share repurchase program, it will finance it by issuing debt rather than by issuing equity. The intuition here is that, since a firm undertakes a share repurchase program only when outsiders are more pessimistic than firm insiders (so that their firm's securities are undervalued), it is optimal for them to finance the repurchase by issuing a security that is least undervalued under these circumstances, which is debt. Second, our model implies that, when outside shareholders are relatively more optimistic about the firm's future prospects compared to firm insiders, the firm will simultaneously issue equity and pay out dividends since this allows them to take advantage of the optimism of outside investors. Further, our model implies that the firm is less likely to finance the payment of cash dividends by issuing debt. The intuition here is that, since a firm makes a cash dividend payment only when

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[42]Mitchell and Netter (1989) document that, during the two weeks following the stock market crash of October 19, 1987, almost 600 firms announced open market repurchase programs, compared to only 350 firms announcing such programs from January 1, 1987 to the crash date. It is unlikely that the above increase in the number of firms announcing open market repurchase programs was driven purely by an increase in the information asymmetry facing these firms over the two weeks following the crash. More likely, a change in outside investor beliefs about the prospects of the economy as a whole following the crash, resulting in their becoming more pessimistic about these firms' future cash flows and yielding lower stock prices led to this dramatic increase in share repurchase announcements immediately after the crash.

outsiders are more optimistic than firm insiders (so that their firm's securities are overvalued), it is optimal for them to finance the dividend payment by issuing a security that is more overvalued under these circumstances, which is equity rather than debt.

(v) *Relationship between outside investor beliefs, cash holdings, and payout policy:* Our model predicts that when the bulk of outside shareholders in the firm are more pessimistic about the firm's prospects compared to firm insiders, then the firm will distribute value through either a stock repurchase alone, or a combination of a stock repurchase and a dividend increase (depending upon the amount of cash available to distribute to shareholders). If the amount of cash available to distribute is relatively small, the firm will undertake only a repurchase, since, in this case, a repurchase will be a positive NPV transaction. If, however, the amount of cash available to distribute is large, the firm will distribute only a fraction of this using a stock repurchase, using the remaining cash to increase its dividend. The intuition here is that, while repurchasing shares from outsiders who are more pessimistic than them is a positive-NPV transaction from insiders' point of view, this NPV becomes smaller as the firm repurchases a larger amount of equity, since it has to go up the belief ladder and buy up equity from outside shareholders who are more and more optimistic; in particular, a share repurchase becomes a negative-NPV transaction if the firm has to repurchase equity from shareholders who are more optimistic than firm insiders. Consequently, when a firm has a large amount of cash to distribute to shareholders, it devotes only a fraction of this toward a share repurchase, paying the remainder as cash dividend (recall that dividend payments are zero-NPV transactions regardless of outsider beliefs).

(vi) *Price impact of dividend payments and stock repurchases:* Our model predicts that the price impact of a dividend payment (measured as the sum of any dividend yield and capital gain on the stock) will be zero while the price impact of a stock repurchase will be positive. Further, our model predicts that, the larger the number of shares repurchased by the firm, the larger the price impact. The intuition

behind the latter prediction is that, as the firm repurchases more and more shares, it goes up the ladder of outside shareholder beliefs (repurchasing shares from outsiders with higher and higher valuations) so that the marginal investor after the completion of the repurchase program will have a higher valuation for the firm's equity if the number of shares repurchased is greater. Since, in practice, open market repurchases are completed over a period of months rather than days, the above price impact needs to be measured empirically over such a period. This prediction can serve to distinguish our model from the predictions of asymmetric information models for stock repurchases: if the positive effects of stock repurchases on the repurchasing firm's share price is due to the information release associated with the announcement of a repurchase, one would expect the effect of this release to be incorporated into the firm's share price in a matter of days, not months.

(vii) Long-run stock returns following dividend payments and stock repurchases: Our model makes three predictions regarding the above. First, the long-run stock return following dividend increases and stock repurchases may be positive or negative. However, the long-run stock return following a stock repurchase undertaken when a significant proportion of outsiders are pessimistic about the firm's prospects will be positive; the more pessimistic these outsiders, the larger the long-run stock return following the stock repurchase. Second, holding the real investment level constant, the long-run stock return following a stock repurchase will be greater than that following a dividend initiation or increase. Third, the long-run stock return following a dividend initiation (or increase) and stock repurchase combination (i.e., where both methods of payout are undertaken roughly around the same time) will be positive on average.

7   Conclusion

We analyzed a firm's choice between dividends and stock repurchases in an environment of heterogeneous beliefs and short sale constraints. We studied a setting in which the insiders of a firm, owning a certain fraction of its equity and having a certain amount of cash to distribute to shareholders, choose between paying out cash dividends and buying back equity, as well as the scale of investment in their firm's new project. Outside equity holders in the firm have heterogeneous beliefs about the probability of success of the firm's project and therefore its long run prospects; they may also disagree with firm insiders about this probability. We showed that, depending on the beliefs of firm insiders versus outsiders, the firm may distribute value through cash dividends alone; through a repurchase alone; or through a combination of a cash dividend and a stock repurchase. We also showed that, in many situations, it is optimal for firm insiders to underinvest in the firm's positive net present value project and undertake a stock repurchase with the amount of cash saved by underinvesting. We then analyzed the price impact of a cash dividend versus a share repurchase, where the price impact is defined as the abnormal return to the firm's equity upon the actual payment of a cash dividend or the implementation of a share repurchase respectively (rather than the abnormal return upon the announcement of these events). Finally, we analyzed the long-run returns to a firm's equity following dividend payments and stock repurchases. Our model generates a number of testable predictions different from asymmetric information models of a firm's choice between dividends versus stock repurchases.

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Appendix: Proofs of Propositions (Some of these proofs may be confined to an unpublished online appendix)

Proof of Proposition 1. We first derive some results regarding the optimal investment policy of the firm conditional on its payout policy and the level of its earnings E available at time 1.

First, consider the case where the firm distributes value through a dividend payment alone. If the investment level is I (underinvestment), the firm insiders' expected payoff is

$$\alpha D_c + E_1[\alpha CF_2^{equity} \mid \theta^f] = \alpha(E - I) + \alpha X^f = \alpha\left(E - I + X^f\right). \tag{A.1}$$

If the investment level is λI (full investment), the firm insiders' expected payoff is

$$\alpha D_c + E_1[\alpha CF_2^{equity} \mid \theta^f] = \alpha(E - \lambda I) + \alpha\lambda' X^f = \alpha\left(E - \lambda I + \lambda' X^f\right). \tag{A.2}$$

Comparing the expected payoffs in the two cases, we find that the firm will implement its project at the full investment level λI if and only if

$$X^f - I < \lambda' X^f - \lambda I. \tag{A.3}$$

From (2), it follows that the firm will invest an amount λI in its project and distribute its excess cash, $E - \lambda I$, to shareholders as dividends.

Now, consider the case where the firm distributes value through a stock repurchase alone. Let E be small, so that the parametric restriction (7) holds. If the firm implements its project at the full investment level by investing an amount λI in the project and spends a dollar amount of $E - \lambda I$ on stock repurchases from outside shareholders, the number of shares the firm buys back is

$$\alpha_b = \frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I}. \tag{A.4}$$

In this case, the firm insiders' expected payoff is given by:

$$\frac{\alpha}{1 - \frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I}} \lambda' X^f = \alpha \frac{X^f}{\underline{X}} \left(E + \lambda' \underline{X} - \lambda I\right). \tag{A.5}$$

If the firm invests an amount I in its project (underinvestment) and repurchases shares worth $E - I$ from outside shareholders, the number of shares the firm buys back is

$$\alpha_b = \delta + \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}. \tag{A.6}$$

In this case, we can show that the firm buys the first δ shares at a price of $\frac{\underline{X}}{\delta \underline{X} + (1-\delta)\overline{X}}\left(E + \overline{X} - I\right)$ from outside shareholders with belief $\underline{\theta}$ and the remaining shares at a price of $\frac{\overline{X}}{\delta \underline{X} + (1-\delta)\overline{X}}\left(E + \overline{X} - I\right)$ from outside shareholders with belief $\overline{\theta}$. The firm insiders' expected payoff is given by:

$$\frac{\alpha}{1 - \delta - \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}} X^f = \alpha \frac{X^f}{\delta \underline{X} + (1-\delta)\overline{X}} \left(E + \overline{X} - I\right). \tag{A.7}$$

From the comparison of insiders' expected payoffs, it follows that firm insiders will choose to implement the firm's project at the full investment level if the following condition holds:

$$\frac{E + \lambda' \underline{X} - \lambda I}{\underline{X}} > \frac{E + \overline{X} - I}{\delta \underline{X} + (1-\delta)\overline{X}}. \tag{A.8}$$

Otherwise, it is optimal for the firm to underinvest and repurchase a larger number of shares. When is condition

(A.8) more likely to be satisfied? First, the LHS of (A.8) is increasing in λ' (the incremental NPV of the project at the full investment level). Second the RHS of (A.8) is decreasing in $\overline{\theta}$, since

$$\frac{\partial\left[\frac{E+\overline{X}-I}{\delta\underline{X}+(1-\delta)\overline{X}}\right]}{\partial\overline{\theta}} = \left(X^H - X^L\right)\frac{\delta(E+\underline{X}-I)-(E-I)}{\left(\delta\underline{X}+(1-\delta)\overline{X}\right)^2} < 0. \tag{A.9}$$

This inequality follows from the restriction that $\delta < \frac{E-I}{E+\underline{X}-I}$ given in (7) or (8). This means that when $\overline{\theta}$ is high, condition (A.8) is more likely to be satisfied so that it will be optimal for the firm to implement its project at the full investment level. After rearranging (A.8), we obtain the following equivalent restriction on $\overline{\theta}$:

$$\overline{\theta} \geq \overline{\theta}_u \equiv \frac{\underline{\theta}(E-I-\delta(E+\lambda'\underline{X}-\lambda I))+\frac{X^L}{(X^H-X^L)}((\lambda-1)I-(\lambda'-1)\underline{X})}{(1-\delta)(E+\lambda'\underline{X}-\lambda I)-\underline{X}}, \tag{A.10}$$

where $\overline{\theta}_u > \underline{\theta}$ if and only if $\underline{\theta} < \theta_z \equiv \frac{(\lambda-1)I-(\lambda'-1)X^L}{(\lambda'-1)(X^H-X^L)}$.

Now, consider the case where E is large, so that the parametric restriction (8) holds. If the firm implements its project at the full investment level by investing an amount λI in the project and spends a dollar amount of $E - \lambda I$ on stock repurchases from outside shareholders, the number of shares the firm buys back is

$$\alpha_b = \delta + \frac{E-\lambda I-\delta(E+\lambda'\underline{X}-\lambda I)}{E+\lambda'\overline{X}-\lambda I}. \tag{A.11}$$

In this case, the firm buys the first δ shares at a price of $\frac{\underline{X}}{\delta\underline{X}+(1-\delta)\overline{X}}\left(E+\lambda'\overline{X}-\lambda I\right)$ from outside shareholders with belief $\underline{\theta}$ and the remaining shares at a price of $\frac{\overline{X}}{\delta\underline{X}+(1-\delta)\overline{X}}\left(E+\lambda'\overline{X}-\lambda I\right)$ from outside shareholders with belief $\overline{\theta}$. The firm insiders' expected payoff is given by:

$$\frac{\alpha}{1-\delta-\frac{E-\lambda I-\delta(E+\lambda'\underline{X}-\lambda I)}{E+\lambda'\overline{X}-\lambda I}}X^f = \alpha\frac{X^f}{\delta\underline{X}+(1-\delta)\overline{X}}\left(E+\lambda'\overline{X}-\lambda I\right). \tag{A.12}$$

Comparing this expected payoff (A.12) to that of underinvestment in (A.7), the firm insiders will prefer to implement the project at the full investment level if the following condition holds:

$$\lambda'\overline{X} - \lambda I > \overline{X} - I, \tag{A.13}$$

which is equivalent to the following parameter restriction: $\overline{\theta} > \theta_z$.

We now analyze the conditions under which the firm distributes value to its shareholders through a combination of a dividend payment and a stock repurchase conditional on its investment policy and its level of earnings E. If E is small and the firm implements its new project at the full investment level, the firm's share price at time 1 prior to the value distribution is $(E+\lambda'\underline{X}-\lambda I)$ and the total number of shares outstanding is 1. Therefore, the maximum number of shares the firm can repurchase is equal to $\frac{E-\lambda I}{E+\lambda'\underline{X}-\lambda I}$, which is strictly less than δ (when E is small).

If the firm buys back shares worth a dollar amount of $D_b = E - \lambda I - D_c$ from its outside shareholders at the price $(E+\lambda'\underline{X}-\lambda I)$ per share, the number of shares repurchased is given by:

$$\alpha_b = \frac{E-\lambda I-D_c}{E+\lambda'\underline{X}-\lambda I}. \tag{A.14}$$

After the repurchase, the firm will make a dividend payment of D_c to the existing shareholders who will collectively hold $1-\alpha_b = 1 - \frac{E-\lambda I-D_c}{E+\lambda'\underline{X}-\lambda I}$ shares outstanding. So, the firm insiders' objective is given by

$$\max_{D_c\in[0,E-\lambda I]} \frac{\alpha}{1-\frac{E-\lambda I-D_c}{E+\lambda'\underline{X}-\lambda I}}\left[D_c+\lambda'X^f\right], \tag{A.15}$$

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which is equivalent to

$$\max_{D_c \in [0, E - \lambda I]} \alpha \left(E + \lambda' \underline{X} - \lambda I \right) \left[1 + \frac{\lambda' \left(X^f - \underline{X} \right)}{D_c + \lambda' \underline{X}} \right]. \tag{A.16}$$

The optimal solution for the firm is

$$D_c = \begin{cases} E - \lambda I & \text{if } \theta^f \leq \underline{\theta}, \\ 0 & \text{if } \theta^f > \underline{\theta}. \end{cases}$$

Thus, if $\theta^f \leq \underline{\theta}$, the firm finds it optimal to distribute value through a dividend payment only. Otherwise, if $\theta^f > \underline{\theta}$, the firm finds it optimal to distribute value through a stock repurchase only, and buys back as many shares as possible. Therefore, if the firm implements its project at the full investment level and E is small, the firm will never choose to distribute value through a combination of a stock repurchase and a dividend payment.

Now, consider the case where the firm underinvests in its new project. In this case, E is large enough so that the firm is able to repurchase more than δ shares. However, if the firm repurchases more than δ shares, it also has to buy back shares from optimistic outside shareholders at a higher price. First, consider the case where we restrict the number of shares that the firm can repurchase to be less than or equal to δ, i.e., $0 \leq \alpha_b \leq \delta$. Then, the amount of cash dividend D_c that the firm can choose to distribute will be in the closed interval $[E - I - \delta(E + \underline{X} - I), E - I]$.

Before the stock repurchase and the dividend payment, the firm value is $E + \underline{X} - I$ and the total number of shares outstanding is 1. The firm buys back shares at a dollar amount of $D_b = E - I - D_c$ from outsiders, at the price of $E + \underline{X} - I$ per share, so that the number of shares repurchased is equal to

$$\alpha_b = \frac{E - I - D_c}{E + \underline{X} - I}.$$

After the repurchase, the firm will make a dividend payment of D_c to the existing shareholders who will collectively hold $1 - \alpha_b = 1 - \frac{E - I - D_c}{E + \underline{X} - I}$ shares outstanding. So, the firm insiders' objective is given by

$$\max_{D_c \in [E - I - \delta(E + \underline{X} - I), E - I]} \frac{\alpha}{1 - \frac{E - I - D_c}{E + \underline{X} - \lambda I}} \left[D_c + X^f \right],$$

which is equivalent to

$$\max_{D_c \in [E - I - \delta(E + \underline{X} - I), E - I]} \alpha \left(E + \underline{X} - I \right) \left[1 + \frac{\left(X^f - \underline{X} \right)}{D_c + \underline{X}} \right].$$

The optimal solution for the firm is

$$D_c = \begin{cases} E - I & \text{if } \theta^f \leq \underline{\theta}, \\ E - I - \delta(E + \underline{X} - I) & \text{if } \theta^f > \underline{\theta}. \end{cases}$$

Thus, if $\theta^f \leq \underline{\theta}$, the firm finds it optimal to distribute value through a dividend payment only. Otherwise, if $\theta^f > \underline{\theta}$, the firm finds it optimal to distribute value through a combination of a stock repurchase and a dividend payment, by repurchasing δ shares from pessimistic shareholders and distribute the remaining earnings as a cash dividend payment.

If we allow the firm to repurchase more than δ shares (but not less) when it underinvests in its project, we can describe the firm insiders' optimization problem as follows. The firm buys back δ shares from pessimistic shareholders at price $\frac{X + D_c}{\delta \underline{X} + (1 - \delta) \overline{X} + D_c} \left(E + \underline{X} - I \right)$, and also some shares from optimistic shareholders at price $\frac{\overline{X} + D_c}{\delta \underline{X} + (1 - \delta) \overline{X} + D_c} \left(E + \underline{X} - I \right)$. It is straightforward to show that the total number of shares repurchased by the firm is equal to

$$\alpha_b = \delta + \frac{E - I - D_c - \delta(E + \underline{X} - I)}{E + \overline{X} - I}.$$

After the repurchase, the firm pays a cash dividend of D_c to existing shareholders (who collectively hold $1 - \alpha_b = 1 - \delta - \frac{E - I - D_c - \delta(E + \underline{X} - I)}{E + \overline{X} - I}$ shares. The dividend payment D_c can take a value in the closed interval $[0, E - I - \delta(E + \underline{X} - I)]$.

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Thus, the firm insiders' objective function is

$$\max_{D_c \in [0, E-I-\delta(E+\underline{X}-I)]} \frac{\alpha}{\left(1 - \delta - \frac{E-I-D_c-\delta(E+\underline{X}-I)}{E+\overline{X}-I}\right)} \left[D_c + X^f\right],$$

which is equivalent to

$$\max_{D_c \in [0, E-I-\delta(E+\underline{X}-I)]} \alpha \left(E + \overline{X} - I\right) \left[1 + \frac{\left(X^f - \left(\delta\underline{X} + (1-\delta)\overline{X}\right)\right)}{D_c + \delta\underline{X} + (1-\delta)\overline{X}}\right].$$

The optimal solution for the firm is

$$D_c = \begin{cases} E - I - \delta(E + \underline{X} - I) & \text{if } \theta^f \le \delta\underline{\theta} + (1-\delta)\overline{\theta}, \\ 0 & \text{if } \theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}. \end{cases}$$

Thus, if $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$, the firm finds it optimal to distribute value through a stock repurchase alone. Otherwise, if $\theta^f \le \delta\underline{\theta} + (1-\delta)\overline{\theta}$, the firm will choose to distribute value through a combination of a stock repurchase and a dividend payment, by repurchasing δ shares from pessimistic shareholders and distributing the remaining earnings as a cash dividend payment.

In summary, conditional on the firm underinvesting in its project, its optimal combination of a stock repurchase and a dividend payment is as follows: the firm will repurchase exactly δ shares from pessimistic shareholders, and pay out a cash dividend in the amount of $E - I - \delta(E + \underline{X} - I)$. In this case, the expected payoff of firm insiders is equal to

$$\frac{\alpha}{1-\delta} \left[X^f + E - I - \delta\left(E + \underline{X} - I\right)\right]. \tag{A.17}$$

Finally, if we consider the case where the firm implements its new project at the full investment level and E is large, the proof is very similar to that of the above case where the firm underinvests. Conditional on the firm fully investing in its project and E being large, the optimal combination of a stock repurchase and a dividend payment is as follows: the firm will repurchase exactly δ shares from pessimistic shareholders, and pay out a cash dividend in the amount of $E - \lambda I - \delta\left(E + \lambda'\underline{X} - \lambda I\right)$. In this case, the expected payoff of firm insiders is equal to

$$\frac{\alpha}{1-\delta} \left[\lambda'X^f + E - \lambda I - \delta\left(E + \lambda'\underline{X} - \lambda I\right)\right]. \tag{A.18}$$

We now prove the results regarding the firm's simultaneous choice of optimal payout and investment policies outlined in Proposition 1. When E is small, the firm insiders have the following menu of choices: 1) Implement the new project at the full investment level and make a dividend payment only; 2) Underinvest in the new project and make a dividend payment only; 3) Implement the new project at the full investment level and repurchase shares only; 4) Underinvest in the new project and repurchase shares only; 5) Underinvest in the new project and undertake a combination of a stock repurchase and a dividend payment.

Since $\theta_z < \theta^f$, we know from the above discussion of firm's investment policy in the case of a dividend payment that choice 1 strictly dominates choice 2. Further, we also know from the above discussion of firm's investment policy in the case of a stock repurchase that when E is small, choice 4 is preferred to choice 3 if and only if the following constraint on $\overline{\theta}$ holds:

$$\underline{\theta} < \overline{\theta} < \overline{\theta}_u = \frac{\underline{\theta}(E - I - \delta(E + \lambda'\underline{X} - \lambda I)) + \frac{X^L}{(X^H - X^L)}((\lambda-1)I - (\lambda'-1)\underline{X})}{(1-\delta)(E + \lambda'\underline{X} - \lambda I) - \underline{X}}. \tag{A.19}$$

For this constraint to be feasible, i.e., $\overline{\theta}_u > \underline{\theta}$, the following condition must hold:

$$\underline{\theta} < \theta_z = \frac{(\lambda-1)I - (\lambda'-1)X^L}{(\lambda'-1)\left(X^H - X^L\right)} \tag{A.20}$$

If (A.20) is not satisfied, choice 3 will strictly dominate choice 4. Further, it is straightforward to show that if E

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is small so that (7) holds, and $\underline{\theta} < \theta_z$, then the following relationship will hold:

$$\theta_z < \overline{\theta}_u. \tag{A.21}$$

Let us first consider the firm's optimal choices conditional on the firm implementing its new project at the full investment level. The firm will prefer choice 3 to choice 1 if and only if insiders' expected payoff from choice 3 is greater than insiders' expected payoff from choice 1:

$$\alpha \frac{X^f}{\underline{X}} \left(E + \lambda' \underline{X} - \lambda I \right) > \alpha \left(E + \lambda' X^f - \lambda I \right), \tag{A.22}$$

which is equivalent to the following condition:

$$\theta^f > \underline{\theta}. \tag{A.23}$$

Let us next consider the firm's optimal choices conditional on the firm underinvesting in its new project. The firm will prefer choice 4 to choice 2 if and only if insiders' expected payoff from choice 4 is greater than insiders' expected payoff from choice 2:

$$\alpha \frac{X^f}{\delta \underline{X} + (1 - \delta)\overline{X}} \left(E + \overline{X} - I \right) > \alpha \left(E + X^f - I \right), \tag{A.24}$$

which is equivalent to the following condition:

$$\theta^f > \frac{\left(\delta \underline{\theta} + (1 - \delta)\overline{\theta} \right) (E - I) - \delta \left(\overline{\theta} - \underline{\theta} \right) X^L}{E - I + \delta \left(\overline{\theta} - \underline{\theta} \right) (X^H - X^L)}, \tag{A.25}$$

where $\frac{\left(\delta \underline{\theta} + (1 - \delta)\overline{\theta} \right)(E - I) - \delta \left(\overline{\theta} - \underline{\theta} \right) X^L}{E - I + \delta \left(\overline{\theta} - \underline{\theta} \right)(X^H - X^L)} > \underline{\theta}$, since $\delta < \frac{E - I}{E + \underline{X} - I}$ by assumption. The firm will prefer choice 4 to choice 5 if and only if insiders' expected payoff from choice 4 is greater than their expected payoff from choice 5:

$$\alpha \frac{X^f}{\delta \underline{X} + (1 - \delta)\overline{X}} \left(E + \overline{X} - I \right) > \frac{\alpha}{1 - \delta} \left(X^f + E - I - \delta (E + \underline{X} - I) \right), \tag{A.26}$$

which is equivalent to the following condition:

$$\theta^f > \delta \underline{\theta} + (1 - \delta)\overline{\theta}. \tag{A.27}$$

The firm will prefer choice 5 to choice 2 if and only if insiders' expected payoff from choice 5 is greater than their expected payoff from choice 2:

$$\frac{\alpha}{1 - \delta} \left(X^f + E - I - \delta (E + \underline{X} - I) \right) > \alpha \left(E + X^f - I \right), \tag{A.28}$$

which is equivalent to the following condition:

$$\theta^f > \underline{\theta}. \tag{A.29}$$

Let us also compare choice 5 and choice 3 with investment levels I and λI, respectively. The firm will prefer choice 5 to choice 3 if and only if insiders' expected payoff from choice 5 is greater than their expected payoff from choice 3:

$$\frac{\alpha}{1 - \delta} \left(X^f + E - I - \delta (E + \underline{X} - I) \right) > \alpha \frac{X^f}{\underline{X}} \left(E + \lambda' \underline{X} - \lambda I \right), \tag{A.30}$$

which is equivalent to the following condition:

$$\theta^f < \theta_u^f = \frac{\theta(E - I - \delta(E + \underline{X} - I)) + \frac{(1 - \delta)X^L}{(X^H - X^L)} \left((\lambda - 1)I - (\lambda' - 1)\underline{X} \right)}{(1 - \delta)(E + \lambda'\underline{X} - \lambda I) - \underline{X}}, \tag{A.31}$$

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where $\theta_u^f > \underline{\theta}$ if and only if $\underline{\theta} < \theta_z$. Further $\theta_u^f < \overline{\theta}_u$ if and only if $\underline{\theta} < \theta_z$.

In part (i) of Proposition 4, we consider the case where $\theta_z \leq \underline{\theta} < \overline{\theta}$. Since $\underline{\theta} \geq \theta_z$, it follows from the above discussion (see condition (A.19)) that in this case, choice 3 strictly dominates choice 4.

If $\theta^f \leq \underline{\theta}$, it follows from (A.23) that choice 1 is preferred to choice 3. Choice 1 is preferred to choice 4 by transitivity (choice 1 ≻ choice 3 ≻ choice 4). Similarly, from inequality (A.29), it follows that choice 1 is preferred to choice 5 by transitivity (choice 1 ≻ choice 2 ≻ choice 5). Thus, choice 1 is optimal in this case.

If $\theta^f > \underline{\theta}$, it follows from (A.23) that choice 3 is preferred to choice 1. Since $\theta_z \leq \underline{\theta}$ in this case, it follows from (A.31) that $\theta_u^f \leq \underline{\theta}$, and therefore, choice 3 is preferred to choice 5. Choice 3 is preferred to choice 2 by transitivity (choice 3 ≻ choice 1 ≻ choice 2). Thus, choice 3 is optimal in this case.

In part (ii) of Proposition 1, we consider the case where $\underline{\theta} < \theta_z < \overline{\theta}_u \leq \overline{\theta}$. Note that in this case, $\underline{\theta} < \theta^f$ due to our global assumption that $\theta_z < \theta^f$. Thus, it follows from (A.23) that choice 3 is preferred to choice 1, and choice 3 is preferred to choice 2 by transitivity (choice 3 ≻ choice 1 ≻ choice 2). Further, since $\overline{\theta}_u \leq \overline{\theta}$, it follows from (A.19) that choice 3 is preferred to choice 4.

If $\theta^f > \theta_u^f$, it follows from (A.31) that choice 3 is preferred to choice 5. Thus, choice 3 is optimal in this case.

If $\theta^f \leq \theta_u^f$, it follows from (A.31) that choice 5 is preferred to choice 3. Since $\underline{\theta} < \theta^f$, it follows from (A.29) that choice 5 is preferred to choice 2. Choice 5 is preferred to choice 1 and choice 4 by transitivity (choice 5 ≻ choice 3 ≻ choice 1, and choice 5 ≻ choice 3 ≻ choice 4). Thus, choice 5 is optimal in this case.

In part (iii) of Proposition 1, we consider the case where $\underline{\theta} < \theta_z < \delta\underline{\theta} + (1-\delta)\overline{\theta} < \overline{\theta} < \overline{\theta}_u$. Note that in this case again, $\underline{\theta} < \theta^f$ due to our global assumption that $\theta_z < \theta^f$. Thus, it follows from (A.23) that choice 3 is preferred to choice 1, and choice 3 is preferred to choice 2 by transitivity (choice 3 ≻ choice 1 ≻ choice 2). Further, since $\overline{\theta} < \overline{\theta}_u$, it follows from (A.19) that choice 4 is preferred to choice 3.

If $\theta^f \leq \delta\underline{\theta} + (1-\delta)\overline{\theta}$, it follows from (A.27) that choice 5 is preferred to choice 4. Then, choice 5 is preferred to choice 3 and choice 1 by transitivity (choice 5 ≻ choice 4 ≻ choice 3, and choice 5 ≻ choice 3 ≻ choice 1). From (A.29) and $\theta^f > \underline{\theta}$, it follows that choice 5 is preferred to choice 2 as well. Thus, choice 5 is optimal in this case.

If $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$, it follows from (A.27) that choice 4 is preferred to choice 5. Then, choice 4 is preferred to choice 1 by transitivity (choice 4 ≻ choice 3 ≻ choice 1). From (A.25), it follows that choice 4 is preferred to choice 2 since $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$. Thus, choice 4 is optimal in this case.

In part (iv) of Proposition 1, we consider the case where $\underline{\theta} < \delta\underline{\theta} + (1-\delta)\overline{\theta} \leq \theta_z < \overline{\theta} < \overline{\theta}_u$. Note that in this case, $\theta^f > \underline{\theta} + (1-\delta)\overline{\theta}$ due to our global assumption that $\theta_z < \theta^f$. In this case, the optimality of choice 4 follows from the above proof for part (iii) of Proposition 1.

Finally, in part (v) of Proposition 4, we consider the case where $\underline{\theta} < \overline{\theta} \leq \theta_z$. Note that in this case, $\theta^f > \overline{\theta}$ due to our global assumption that $\theta_z < \theta^f$. Since $\theta_z < \overline{\theta}_u$ in this case as well, it follows that $\overline{\theta} < \overline{\theta}_u$. Therefore, choice 4 is preferred to choice 3. Since $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$, the overall optimality of choice 4 in this case follows from the above proofs for part (iii) and (iv) of Proposition 1. ∎

Proof of Proposition 2. When E is large, the firm insiders have the following menu of choices: 1) Implement the new project at the full investment level and make a dividend payment only; 2) Underinvest in the new project and make a dividend payment only; 3) Implement the new project at the full investment level and repurchase shares only; 4) Underinvest in the new project and repurchase shares only; 5) Underinvest in the new project and undertake a combination of a stock repurchase and a dividend payment; 6) Implement the new project at the full investment level and undertake a combination of a stock repurchase and a dividend payment.

Since $\theta_z < \theta^f$, we know from that choice 1 strictly dominates choice 2. Further, we also know that choice 3 is preferred to choice 4 if $\theta_z \leq \overline{\theta}$, and choice 4 is preferred to choice 3 if $\overline{\theta} < \theta_z$.

Let us first consider the firm's optimal choices conditional on the firm implementing its new project at the full investment level. The firm will prefer choice 3 to choice 1 if and only if insiders' expected payoff from choice 3 is greater than their expected payoff from choice 1:

$$\alpha \frac{X^f}{\delta\underline{X} + (1-\delta)\overline{X}} \left(E + \lambda'\overline{X} - \lambda I \right) > \alpha \left(E + \lambda'\underline{X} - \lambda I \right), \tag{A.32}$$

which is equivalent to the following condition:

$$\theta^f > \frac{\left(\delta\underline{\theta} + (1-\delta)\overline{\theta} \right)(E - \lambda I) - \delta\lambda'\left(\overline{\theta} - \underline{\theta}\right)X^L}{E - \lambda I + \delta\lambda'\left(\overline{\theta} - \underline{\theta}\right)\left(X^H - X^L\right)}, \tag{A.33}$$

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where $\frac{\left(\delta\underline{\theta}+(1-\delta)\overline{\theta}\right)(E-\lambda I)-\delta\lambda'\left(\overline{\theta}-\underline{\theta}\right)X^L}{E-\lambda I+\delta\lambda'\left(\overline{\theta}-\underline{\theta}\right)(X^H-X^L)} > \underline{\theta}$ since $\delta < \frac{E-\lambda I}{E+\lambda'\underline{X}-\lambda I}$ if E is large (see inequality (8)). The firm will prefer choice 3 to choice 6 if and only if insiders' expected payoff from choice 3 is greater than their expected payoff from choice 6:

$$\alpha\frac{X^f}{\delta\underline{X}+(1-\delta)\overline{\overline{X}}}\left(E+\lambda'\overline{X}-\lambda I\right) > \frac{\alpha}{1-\delta}\left(\lambda'X^f+E-\lambda I-\delta\left(E+\lambda'\underline{X}-\lambda I\right)\right), \tag{A.34}$$

which is equivalent to the following condition:

$$\theta^f > \delta\underline{\theta}+(1-\delta)\overline{\theta}. \tag{A.35}$$

The firm will prefer choice 6 to choice 1 if and only if insiders' expected payoff from choice 6 is greater than their expected payoff from choice 1:

$$\frac{\alpha}{1-\delta}\left(\lambda'X^f+E-\lambda I-\delta\left(E+\lambda'\underline{X}-\lambda I\right)\right) > \alpha\left(E+\lambda'\underline{X}-\lambda I\right), \tag{A.36}$$

which is equivalent to the following condition:

$$\theta^f > \underline{\theta}. \tag{A.37}$$

Let us next consider the firm's optimal choices conditional on the firm underinvesting in its new project. The firm will prefer choice 4 to choice 2 if and only if insiders' expected payoff from choice 4 is greater than their expected payoff from choice 2:

$$\alpha\frac{X^f}{\delta\underline{X}+(1-\delta)\overline{\overline{X}}}\left(E+\overline{X}-I\right) > \alpha\left(E+X^f-I\right), \tag{A.38}$$

which is equivalent to the following condition:

$$\theta^f > \frac{\left(\delta\underline{\theta}+(1-\delta)\overline{\theta}\right)(E-I)-\delta\left(\overline{\theta}-\underline{\theta}\right)X^L}{E-I+\delta\left(\overline{\theta}-\underline{\theta}\right)(X^H-X^L)}, \tag{A.39}$$

where $\frac{\left(\delta\underline{\theta}+(1-\delta)\overline{\theta}\right)(E-I)-\delta\left(\overline{\theta}-\underline{\theta}\right)X^L}{E-I+\delta\left(\overline{\theta}-\underline{\theta}\right)(X^H-X^L)} > \underline{\theta}$, since $\delta < \frac{E-I}{E+\underline{X}-I}$ by assumption. The firm will prefer choice 4 to choice 5 if and only if insiders' expected payoff from choice 4 is greater than their expected payoff from choice 5:

$$\alpha\frac{X^f}{\delta\underline{X}+(1-\delta)\overline{\overline{X}}}\left(E+\overline{X}-I\right) > \frac{\alpha}{1-\delta}\left(X^f+E-I-\delta\left(E+\underline{X}-I\right)\right), \tag{A.40}$$

which is equivalent to the following condition:

$$\theta^f > \delta\underline{\theta}+(1-\delta)\overline{\theta}. \tag{A.41}$$

The firm will prefer choice 5 to choice 2 if and only if insiders' expected payoff from choice 5 is greater than their expected payoff from choice 2:

$$\frac{\alpha}{1-\delta}\left(X^f+E-I-\delta\left(E+\underline{X}-I\right)\right) > \alpha\left(E+X^f-I\right), \tag{A.42}$$

which is equivalent to the following condition:

$$\theta^f > \underline{\theta}. \tag{A.43}$$

Finally, we compare choice 5 and choice 6 with investment levels I and λI, respectively. The firm will prefer choice 6 to choice 5 if and only if insiders' expected payoff from choice 6 is greater than their expected payoff

from choice 5:

$$\frac{\alpha}{1-\delta}\left(\lambda'X^f + E - \lambda I - \delta\left(E + \lambda'\underline{X} - \lambda I\right)\right) > \frac{\alpha}{1-\delta}\left(X^f + E - I - \delta\left(E + \underline{X} - I\right)\right), \tag{A.44}$$

which is equivalent to the following condition:

$$\theta^f > \delta\underline{\theta} + (1-\delta)\theta_z. \tag{A.45}$$

In part (i) of Proposition 2, we consider the case where $\theta_z \leq \underline{\theta} < \overline{\theta}$. Since $\theta_z < \overline{\theta}$, it follows that firm insiders prefer choice 3 to choice 4.

If $\theta^f \leq \underline{\theta}$ it follows from inequalities (A.33) and (A.37) that choice 1 is preferred to both choice 3 and choice 6. Choice 1 is preferred to choice 4 by transitivity (choice 1 > choice 3 > choice 4). Similarly, from inequality (A.43), it follows that choice 1 is preferred to choice 5 by transitivity (choice 1 > choice 2 > choice 5). Thus, choice 1 is optimal if $\theta^f \leq \underline{\theta}$.

If $\underline{\theta} < \theta^f < \delta\underline{\theta} + (1-\delta)\overline{\theta}$, it follows from inequalities (A.35) and (A.37) that choice 6 is preferred to both choice 3 and choice 1. Since $\theta_z \leq \underline{\theta} < \theta^f$ in this case, it follows from (A.45) that choice 6 is preferred to choice 5. From (A.43), choice 6 is preferred to choice 2 by transitivity (choice 6 > choice 5 > choice 2). Thus, choice 6 is optimal in this case.

If $\delta\underline{\theta} + (1-\delta)\overline{\theta} \leq \theta^f$, it follows from inequalities (A.33) and (A.35) that choice 3 is preferred to both choice 1 and choice 6. From (A.39) and (A.41), it follows that choice 3 is preferred to both choice 2 and choice 5 by transitivity (choice 3 > choice 4 > choice 2, and choice 3 > choice 4 > choice 5). Thus, choice 3 is optimal in this case.

In part (ii) of Proposition 2, we consider the case where $\underline{\theta} < \theta_z < \delta\underline{\theta} + (1-\delta)\overline{\theta} < \overline{\theta}$. Since $\theta_z < \overline{\theta}$, it again follows that firm insiders prefer choice 3 to choice 4. Note that in this case, $\underline{\theta} < theta^f$ due to our global assumption that $\theta_z < \theta^f$. The optimality of choice 6 when $\theta^f < \delta\underline{\theta} + (1-\delta)\overline{\theta}$, and the optimality of choice 3 when $\underline{\theta} + (1-\delta)\overline{\theta} \leq \theta^f$ follow from the discussion above.

In part (iii) of Proposition 2, we consider the case where $\underline{\theta} < \delta\underline{\theta} + (1-\delta)\overline{\theta} \leq \theta_z < \overline{\theta}$. Since $\theta_z < \overline{\theta}$, it again follows that firm insiders prefer choice 3 to choice 4. Further, in this case, it holds that $\delta\underline{\theta} + (1-\delta)\overline{\theta} < \theta^f$. The optimality of choice 3 when $\delta\underline{\theta} + (1-\delta)\overline{\theta} < \theta^f$ follows from the discussion above.

Finally, in part (iv) of Proposition 2, we consider the case where $\underline{\theta} < \overline{\theta} < \theta_z$. Since $\overline{\theta} < \theta_z$, it follows that firm insiders prefer choice 4 to choice 3. Further, in this case, $\theta^f > \overline{\theta}$ due to our global assumption that $\theta_z < \theta^f$. From inequalities (A.39) and (A.41), it follows that choice 4 is preferred to both choice 2 and choice 5. From (A.35) and (A.33), it follows that choice 4 is preferred to both choice 6 and choice 1 by transitivity (choice 4 > choice 3 > choice 6, and choice 4 > choice 3 > choice 1). Thus, choice 4 is optimal in this case. ∎

Proof of Proposition 3. If the firm makes a cash dividend payment, the share price at time 1 will decrease by the amount of the cash dividend. Without loss of generality, if the firm chooses to implement its project at the full investment level and makes a dividend payment of $(E - \lambda I)$, the firm's stock price will be $(E + \lambda'\underline{X} - \lambda I)$ at time 0 before the dividend payment, and it will be $\lambda'\underline{X}$ at time 1 after the payment. However, the total stock return at time 1 will be zero, because the capital gains yield will be

$$\frac{P_1 - P_0}{P_0} = -\left(\frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right), \tag{A.46}$$

and the dividend yield will be

$$\frac{Div_1}{P_0} = \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}, \tag{A.47}$$

so that the total return, which is the sum of the two, will be zero. Hence, the price impact (measured as the sum of the dividend yield and the capital gains yield) of a cash dividend payment is 0.

If E is small and the firm chooses to undertake a stock repurchase, its price impact depends on the number of shares the firm repurchases. If the firm implements its project at the full investment level, the firm's share price will be $P_0 = (E + \lambda'\underline{X} - \lambda I)$ before the stock repurchase (at time 0). After the stock repurchase, the total firm value will be $\lambda'\underline{X}$, but the total number of shares outstanding will be reduced from 1 to $\left(1 \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)$, so that the

share price at time 1 will be

$$P_1 = \frac{\lambda' \underline{X}}{1 - \frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I}} = E + \lambda' \underline{X} - \lambda I, \tag{A.48}$$

yielding a total return of zero on the firm's equity (i.e., the price impact $\frac{P_1 - P_0 + Div_1}{P_0}$ is zero).

On the other hand, if the firm undertakes a stock repurchase while it underinvests in its new project, it will have to repurchase a larger number of shares (from both pessimistic and optimistic outside shareholders) compared to the above case where it undertakes a stock repurchase while implementing its project at the full investment level. Before the stock repurchase, the firm's share price will be $P_0 = (E + \underline{X} - I)$. After the repurchase, the total firm value will be $\overline{X}$, but the total number of shares outstanding will be reduced to $\left(1 - \delta - \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}\right)$, so that the share price at time 1 will be

$$P_1 = \frac{\overline{X}}{1 - \delta - \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}} = \frac{\overline{X}}{\delta \underline{X} + (1 - \delta)\overline{X}} \left(E + \overline{X} - I\right). \tag{A.49}$$

Thus, the total return on the firm's equity will be

$$\frac{P_1 - P_0}{P_0} = \frac{\frac{\overline{X}}{\delta \underline{X} + (1 - \delta)\overline{X}} \left(E + \overline{X} - I\right) - (E + \underline{X} - I)}{E + \underline{X} - I} > 0, \tag{A.50}$$

yielding a positive price impact. Thus, if E is small, the price impact of a stock repurchase is weakly positive, and it increases with the number of shares the firm buys back.

If E is large and the firm chooses to undertake a stock repurchase alone to distribute value to shareholders, the price impact of the stock repurchase will be strictly positive, since the firm will go up the ladder of shareholder beliefs regardless of the optimal level of investment in its new project. Unlike in the above case where E is small, if the firm chooses to implement its project at the full investment level when E is large, the firm will repurchase shares from both pessimistic and optimistic outside shareholders. Before the stock repurchase, the firm's share price will be $P_0 = (E + \lambda' \underline{X} - \lambda I)$. After the repurchase, the total firm value will be $\lambda' \underline{X}$, but the total number of shares outstanding will be reduced to $\left(1 - \delta - \frac{E - \lambda I - \delta(E + \lambda' \underline{X} - \lambda I)}{E + \lambda' \overline{X} - \lambda I}\right)$, so that the share price at time 1 will be

$$P_1 = \frac{\lambda' \overline{X}}{1 - \delta - \frac{E - \lambda I - \delta(E + \lambda' \underline{X} - \lambda I)}{E + \lambda' \overline{X} - \lambda I}} = \frac{\overline{X}}{\delta \underline{X} + (1 - \delta)\overline{X}} \left(E + \lambda' \overline{X} - \lambda I\right). \tag{A.51}$$

In this case, the total return on the firm's equity will be

$$\frac{P_1 - P_0}{P_0} = \frac{\frac{\overline{X}}{\delta \underline{X} + (1 - \delta)\overline{X}} \left(E + \lambda' \overline{X} - \lambda I\right) - (E + \lambda' \underline{X} - \lambda I)}{E + \lambda' \underline{X} - \lambda I} > 0, \tag{A.52}$$

also yielding a positive price impact in the case where the firm chooses a stock repurchase alone to distribute value while implementing its project at the full investment level. Thus, when E is large, a stock repurchase has a positive price impact regardless of whether the firm underinvests in its project or implements it at the full investment level, since after the repurchase, the belief of the marginal investor in the firm's equity increases from $\underline{\theta}$ to $\overline{\theta}$. ∎

Proof of Proposition 4. E is small so that (7) holds. We first analyze the conditions under which the firm raises external financing by issuing debt and simultaneously repurchases shares, when the firm implements its project at the full investment level λI. If the firm implements its project at the full investment level λI, we know that the maximum number of shares that the firm can repurchase without external financing is equal to $\frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I}$, which is strictly less than δ.

We will first prove that in case the firm fully invests in its new project and if $\theta^f > \underline{\theta}$, the firm will raise external financing by issuing debt to increase the number of shares it repurchases from pessimistic shareholders with belief $\underline{\theta}$. If the firm uses debt financing, the amount of funds that needs to be raised, i.e., the amount of

borrowed funds, is equal to

$$P_D = D_b - (E - \lambda I). \tag{A.53}$$

Thus, the firm borrows an amount P_D to make up the shortfall between the dollar amount D_b spent on the stock repurchase and the firm's excess earnings after investment, $E - \lambda I$.

We first consider the case where the firm is able to issue risk-free debt to repurchase all δ shares held by pessimistic current shareholders while the firm invests λI in its project. In this case, the payoff to debtholders will be P_D in either state so that $P_D \leq \lambda' X^L$ (note that the risk-free rate is normalized to 0). We will verify that for the firm to be able to issue risk-free debt to repurchase δ shares, the following constraint must hold:

$$\delta \leq \delta^* \equiv \frac{\lambda' X^L + (E - \lambda I)}{E + \lambda' \underline{X} - \lambda I}, \tag{A.54}$$

where δ^* satisfies the following equation:

$$\delta^* (E + \lambda' \underline{X} - \lambda I) - (E - \lambda I) = \lambda' X^L. \tag{A.55}$$

First, we restrict our analysis to the case where the number of shares repurchased, x, is between $\frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I}$ and δ. When the firms buys back stocks from current shareholders with belief $\underline{\theta}$, they will pay a price of PE_1 for each share so that

$$D_b = x \times PE_1. \tag{A.56}$$

Investors know that the firm value at time 2 will be either $(\lambda' X^H - P_D)$ or $(\lambda' X^L - P_D)$, with $(1 - x)$ shares outstanding. So, at time 1, current shareholders with belief $\underline{\theta}$ will value each share at a price of

$$PE_1 = \frac{\lambda' \underline{X} - P_D}{1 - x}. \tag{A.57}$$

Solving equations (A.53), (A.56), (A.57) for P_D, D_b, and PE_1, we obtain:

$$PE_1 = E + \lambda' \underline{X} - \lambda I, \; D_b = x (E + \lambda' \underline{X} - \lambda I), \; P_D = x (E + \lambda' \underline{X} - \lambda I) - (E - \lambda I). \tag{A.58}$$

Note that when E is small and the firm invests λI in its project, it follows from (A.58) (substitute $x = \delta$ in P_D) that the firm can issue risk-free debt to buy back δ shares, i.e., $P_D \leq \lambda' X^L$, if and only if $\delta \leq \delta^*$ as we asserted in equations (A.54) and (A.55). Firm insiders will maximize their expected payoff by solving the following maximization problem:

$$\max_{x \in \left[\frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I}, \delta \right]} \frac{\alpha}{1 - x} \left(\lambda' X^f + E - \lambda I - x (E + \lambda' \underline{X} - \lambda I) \right).$$

The optimal solution is given by

$$x = \begin{cases} \delta & \text{if } \theta^f > \underline{\theta}, \\ \frac{E - \lambda I}{E + \lambda' \underline{X} - \lambda I} & \text{if } \theta^f \leq \underline{\theta}. \end{cases} \tag{A.59}$$

Thus, if $\theta^f > \underline{\theta}$, the firm will optimally set $x = \delta$ so that $P_D = \delta (E + \lambda' \underline{X} - \lambda I) - (E - \lambda I)$, and the firm insiders' expected payoff from issuing risk-free debt to buy back δ shares while implementing the project at the full investment level is equal to

$$\frac{\alpha}{1 - \delta} \left[E - \lambda I + \lambda' X^f - \delta (E + \lambda' \underline{X} - \lambda I) \right]. \tag{A.60}$$

Second, we analyze and show the circumstances under which the firm will want to buy back more than δ shares by issuing risk-free debt when it implements the project at the full investment level when E is small, $\theta^f > \underline{\theta}$, and $\delta \leq \delta^*$. In this case, the firm buys back δ shares from pessimistic current shareholders with belief $\underline{\theta}$ at a price of PE_1 per share and y shares from optimistic shareholders with belief $\overline{\theta}$ at a price of PE_2. Then, given that the debt is risk-free, the following equations will hold:

$$P_D = \delta \times PE_1 + y \times PE_2 - (E - \lambda I), \; PE_1 = \frac{\lambda' \underline{X} - P_D}{1 - \delta - y}, \; PE_2 = \frac{\lambda' \overline{X} - P_D}{1 - \delta - y}. \tag{A.61}$$

After solving these equations, we obtain

$$P_D = \delta \left(E + \lambda' \underline{X} - \lambda I \right) + y \left(E + \lambda' \overline{X} - \lambda I \right) - (E - \lambda I). \tag{A.62}$$

Note that for the debt to be risk-free, the following condition must hold:

$$\delta \left(E + \lambda' \underline{X} - \lambda I \right) - (E - \lambda I) \le P_D \le \lambda' X^L, \tag{A.63}$$

which can be equivalently expressed in terms of the number of shares repurchased from optimists, y:

$$0 \le y \le y^* \equiv \frac{\lambda' X^L + (E - \lambda I) - \delta \left(E + \lambda' \underline{X} - \lambda I \right)}{E + \lambda' \overline{X} - \lambda I}. \tag{A.64}$$

Then, firm insiders will maximize their expected payoff by solving the following maximization problem:

$$\max_{y \in [0, y^*]} \frac{\alpha}{1 - \delta - y} \left(\lambda' X^f + E - \lambda I - \delta \left(E + \lambda' \underline{X} - \lambda I \right) - y \left(E + \lambda' \overline{X} - \lambda I \right) \right).$$

The optimal solution is given by

$$y = \begin{cases} 0 & \text{if } \theta^f \le \delta\underline{\theta} + (1 - \delta)\overline{\theta}, \\ y^* & \text{if } \theta^f > \delta\underline{\theta} + (1 - \delta)\overline{\theta}. \end{cases} \tag{A.65}$$

Thus, we proved that if $\underline{\theta} < \theta^f \le \delta\underline{\theta} + (1 - \delta)\overline{\theta}$, $\delta \le \delta^*$, and the firm invests λI in its project, the firm has no incentive to repurchase more than δ shares by increasing its borrowing of risk-free debt. Later, we will also prove that if $\underline{\theta} < \theta^f \le \delta\underline{\theta} + (1 - \delta)\overline{\theta}$, the firm has no incentive to raise external financing to repurchase more shares when it underinvests in its project. From the proof of Proposition 1 (see equation (A.27)), we also know that if $\theta^f \le \delta\underline{\theta} + (1 - \delta)\overline{\theta}$ and the firm underinvests in its project, firm insiders prefer a combination of a stock repurchase and a dividend payment to a stock repurchase alone. Comparing the expected payoffs given in (A.60) and (A.17), it is easy to verify that if $\theta^f > \delta\underline{\theta} + (1 - \delta)\theta_z$, the following inequality holds:

$$\frac{\alpha}{1 - \delta} \left[E - \lambda I + \lambda' X^f - \delta \left(E + \lambda' \underline{X} - \lambda I \right) \right] > \frac{\alpha}{1 - \delta} \left[E - I + X^f - \delta \left(E + \underline{X} - I \right) \right], \tag{A.66}$$

so that when $\underline{\theta} < \theta^f \le \delta\underline{\theta} + (1 - \delta)\overline{\theta}$ and $\delta \le \delta^*$, the firm prefers to repurchase δ shares with risk-free debt financing while it invests λI in its project rather than distribute value through a combination of a stock repurchase and a dividend payment while it underinvests I in its project. This proves part (i) of Proposition 4.

From the solution given in (A.65), note that if $\theta^f > \delta\underline{\theta} + (1 - \delta)\overline{\theta}$ and $\delta \le \delta^*$, the firm has an incentive to maximize its borrowing of risk-free debt (i.e., $P_D = \lambda' X^L$) in order to repurchase $(\delta + y^*)$ shares. Then, the firm insiders' expected payoff from issuing risk-free debt (with $P_D = \lambda' X^L$) to buy back $(\delta + y^*)$ shares while implementing the project at the full investment level is equal to

$$\frac{\alpha}{1 - \delta - y^*} \theta^f \left(\lambda' X^H - \lambda' X^L \right) = \frac{\alpha \theta^f}{\delta\underline{\theta} + (1 - \delta)\overline{\theta}} \left(E + \lambda' \overline{X} - \lambda I \right). \tag{A.67}$$

Next, we analyze if the firm has any incentive to issue risky debt to repurchase more than $(\delta + y^*)$ shares when $\delta \le \delta^*$. Note that in this case, the debt will be risky if and only if $P_D > \lambda' X^L$, or equivalently, if and only if $y > y^*$. Then, the following equations must hold:

$$P_D = \underline{\theta} F + (1 - \underline{\theta})\lambda' X^L = \delta \times PE_1 + y \times PE_2 - (E - \lambda I), \tag{A.68}$$

$$PE_1 = \frac{\underline{\theta} \left(\lambda' X^H - F \right)}{1 - \delta - y}, \quad PE_2 = \frac{\overline{\theta} \left(\lambda' X^H - F \right)}{1 - \delta - y}. \tag{A.69}$$

After solving these equations for the face value F of risky debt, we obtain:

$$F = \frac{\delta\left(E + \lambda'\underline{X} - \lambda I\right) + y\left(E + \overline{\theta}\lambda'X^H + (1 - \underline{\theta})\lambda'X^L - \lambda I\right) - \left(E - \lambda I + (1 - \underline{\theta})\lambda'X^L\right)}{\underline{\theta} + y\left(\overline{\theta} - \underline{\theta}\right)}. \tag{A.70}$$

Then, firm insiders will maximize their expected payoff by solving the following maximization problem:

$$\max_{y \geq y^*} \frac{\alpha}{1 - \delta - y}\theta^f\left(\lambda'X^H - F\right).$$

After substituting F from (A.70), we obtain the following equivalent problem:

$$\max_{y \geq y^*} \frac{\alpha\theta^f}{\underline{\theta} + y\left(\overline{\theta} - \underline{\theta}\right)}\left(E + \lambda'\underline{X} - \lambda I\right).$$

Clearly, the optimal value of y is equal to y^*, and the firm has no incentive to issue risky debt to repurchase more than $(\delta + y^*)$ shares while it fully invests in its project when $\delta \leq \delta^*$.

We now consider the case where the firm is not able to issue risk-free debt to repurchase all δ shares held by pessimistic current shareholders while the firm invests λI in its project: i.e., $\delta > \delta^* = \frac{\lambda'X^L + E - \lambda I}{E + \lambda'\underline{X} - \lambda I}$. In this case,

$$\delta\left(E + \lambda'\underline{X} - \lambda I\right) - \left(E - \lambda I\right) > \lambda'X^L, \tag{A.71}$$

so that the firm has to issue risky debt $(P_D > \lambda'X^L)$ to buy back δ shares or more. In fact, it has to issue risky debt to buy back more than δ^* shares.

First, we restrict our analysis to the case where the number of shares repurchased, x, is between $\frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}$ and δ^* so that the firm can raise external financing by issuing risk-free debt. It is easy to show that

$$PE_1 = E + \lambda'\underline{X} - \lambda I, \; P_D = x\left(E + \lambda'\underline{X} - \lambda I\right) - \left(E - \lambda I\right). \tag{A.72}$$

Then, firm insiders will maximize their expected payoff by solving the following problem:

$$\max_{x \in \left[\frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}, \delta^*\right]} \frac{\alpha}{1 - x}\left(\lambda'X^f + E - \lambda I - x\left(E + \lambda'\underline{X} - \lambda I\right)\right).$$

The optimal solution is

$$x = \begin{cases} \delta^* & \text{if } \theta^f > \underline{\theta}, \\ \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I} & \text{if } \theta^f \leq \underline{\theta}. \end{cases} \tag{A.73}$$

Thus, if $\theta^f \leq \underline{\theta}$, the firm has no incentive to issue debt to repurchase additional shares. However, if $\theta^f > \underline{\theta}$, the firm prefers to issue as much risk-free debt as possible to increase the number of shares it repurchases from pessimists. If $x = \delta^*$, the expected payoff of firm insiders is equal to

$$\frac{\alpha\theta^f}{\underline{\theta}}\left(E + \lambda'\underline{X} - \lambda I\right). \tag{A.74}$$

Second, we analyze the case where the firm has to issue risky debt to repurchase shares from current shareholders with belief $\underline{\theta}$ while investing λI in its project, and the number of shares repurchased, x, is between δ^* and δ. In this case, the following equations must hold:

$$P_D = x \times PE_1 - \left(E - \lambda I\right), \; PE_1 = \frac{\underline{\theta}\left(\lambda'X^H - F\right)}{1 - x}, \; P_D = \underline{\theta}F + (1 - \underline{\theta})\lambda'X^L. \tag{A.75}$$

After solving these equations for the face value F of risky debt, we obtain:

$$F = \frac{x\left(E + \lambda'\underline{X} - \lambda I\right) - \left(E - \lambda I\right) - \left(1 - \underline{\theta}\right)\lambda'X^L}{\underline{\theta}}. \tag{A.76}$$

Then, firm insiders will maximize their expected payoff by solving the following maximization problem:

$$\max_{\delta^* \leq x \leq \delta} \frac{\alpha}{1 - x}\left(\theta^f\left(\lambda'X^H - F\right)\right).$$

After substituting F from (A.76) and making simplifications, we obtain the following equivalent problem:

$$\max_{\delta^* \leq x \leq \delta} \frac{\alpha\theta^f}{\underline{\theta}}\left(E + \lambda'\underline{X} - \lambda I\right).$$

Note that if $\delta^* \leq x \leq \delta$, the insiders' expected payoff does not depend on x, and it takes the same value given in (A.74). Thus, if $\theta^f > \underline{\theta}$ and $\delta > \delta^*$, firm insiders do not realize any gains from issuing risky debt to increase the number of shares the firm repurchases from pessimists beyond δ^*, when the firm implements the new project at the full investment level.

Next, we will show that firm has also no incentive to issue risky debt to repurchase more than δ shares when $\delta \leq \delta^*$ while it invests λI in its project. If we denote the number of shares repurchased from optimists by y, the equations given in (A.68) and (A.69) must also hold in this case. Therefore, the face value F of risky debt is also given by (A.70). Then, firm insiders will maximize their expected payoff by solving the following maximization problem:

$$\max_{y \geq 0} \frac{\alpha}{1 - \delta - y}\theta^f\left(\lambda'X^H - F\right).$$

After substituting F from (A.70), we obtain the following equivalent problem:

$$\max_{y \geq 0} \frac{\alpha\theta^f}{\underline{\theta} + y\left(\overline{\theta} - \underline{\theta}\right)}\left(E + \lambda'\underline{X} - \lambda I\right).$$

Clearly, the optimal value of y is equal to 0, and the firm has no incentive to issue risky debt to repurchase more than δ shares while it fully invests in its project when $\delta > \delta^*$.

We now analyze the conditions under which the firm raises external financing by issuing debt and simultaneously repurchases shares, when the firm implements its project at the underinvestment level I. If the firm invests I in its project, we know that the maximum number of shares that the firm can repurchase without external financing is equal to $\delta + \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}$. If the firm uses debt financing, the amount of funds that needs to be raised, i.e., the amount of borrowed funds, is equal to

$$P_D = D_b - \left(E - I\right), \tag{A.77}$$

where $D_b = \delta PE_1 + yPE_2$ is the dollar amount spent on the stock repurchase program, and y is the number of shares the firm repurchases from optimistic current shareholders with belief $\overline{\theta}$.

We first consider the case where the firm is able to issue risk-free debt to repurchase additional shares held by optimistic current shareholders while the firm invests I in its project, i.e., $P_D \leq X^L$. Then, the following equations must hold:

$$P_D = \delta \times PE_1 + y \times PE_2 - \left(E - I\right), \quad PE_1 = \frac{\underline{X} - P_D}{1 - \delta - y}, \quad PE_2 = \frac{\overline{X} - P_D}{1 - \delta - y}. \tag{A.78}$$

Solving these equations, we find that the amount of funds borrowed will be equal to

$$P_D = \delta\left(E + \underline{X} - I\right) + y\left(E + \overline{X} - I\right) - \left(E - I\right). \tag{A.79}$$

Note that $P_D \geq 0$ if and only if $y \geq \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}$. Further, the debt will be risk-free, i.e., $P_D \leq X^L$, if and only if

the following condition holds:

$$y \le \bar{y} \equiv \frac{X^L + (E - I) - \delta(E + \underline{X} - I)}{E + \overline{X} - I}. \tag{A.80}$$

Firm insiders will maximize their expected payoff by solving the following problem:

$$\max_{y \in \left[\frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}, \bar{y}\right]} \frac{\alpha}{1 - \delta - y} \left(X^f + E - I - \delta(E + \underline{X} - I) - y(E + \overline{X} - I)\right).$$

The optimal solution is given by

$$y = \begin{cases} \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I} & \text{if } \theta^f \le \delta\underline{\theta} + (1 - \delta)\overline{\theta}, \\ \bar{y} & \text{if } \theta^f > \delta\underline{\theta} + (1 - \delta)\overline{\theta}. \end{cases} \tag{A.81}$$

Thus, we proved that if $\theta^f \le \delta\underline{\theta} + (1 - \delta)\overline{\theta}$ and the firm underinvests in its project, it has no incentive to issue debt in case it distributes value to its shareholders through a stock repurchase. However, if $\theta^f > \delta\underline{\theta} + (1 - \delta)\overline{\theta}$, the firm uses its full capacity to issue risk-free debt in order to increase its repurchase of shares from optimistic current shareholders. if $y = \bar{y}$, the firm insiders' expected payoff from issuing risk-free debt (with $P_D = X^L$) to buy back $(\delta + \bar{y})$ shares while implementing the project at the underinvestment level is equal to

$$\frac{\alpha}{1 - \delta - \bar{y}} \theta^f \left(X^H - X^L\right) = \frac{\alpha\theta^f}{\delta\underline{\theta} + (1 - \delta)\overline{\theta}} \left(E + \overline{X} - I\right). \tag{A.82}$$

We will now prove that in case the firm underinvests in its project, it will not issue risky debt (with $P_D > X^L$) in order to repurchase more than $(\delta + \bar{y})$ shares. After solving risky debt equations which are very similar to those given in (A.68) and (A.69), we find that the face value F of the risky debt is given by:

$$F = \frac{\delta(E + \underline{X} - I) + y(E + \overline{\theta}X^H + (1 - \underline{\theta})X^L - I) - (E - I + (1 - \underline{\theta})X^L)}{\underline{\theta} + y(\overline{\theta} - \underline{\theta})}. \tag{A.83}$$

Then, firm insiders will maximize their expected payoff by solving the following maximization problem:

$$\max_{y \ge \bar{y}} \frac{\alpha}{1 - \delta - y} \theta^f \left(X^H - F\right).$$

After substituting F from (A.83), we obtain the following equivalent problem:

$$\max_{y \ge \bar{y}} \frac{\alpha\theta^f}{\underline{\theta} + y(\overline{\theta} - \underline{\theta})} \left(E + \underline{X} - I\right).$$

Clearly, the optimal value of y is equal to $\bar{y}$, and the firm has no incentive to issue risky debt to repurchase more than $(\delta + \bar{y})$ shares while it underinvests in its project.

Now, we can summarize our results and complete the proof of parts (ii), (iii), (iv), and (v) of Proposition 4.

If $\theta^f \le \delta\underline{\theta} + (1 - \delta)\overline{\theta}$ and $\delta > \delta^*$, the firm prefers to distribute value through a combination of a stock repurchase and a dividend payment in case it underinvests in its project. On the other hand, in case the firm invests λI in its project, the firm issues risk-free debt with face value $\lambda' X^L$ to repurchase δ^* shares. Comparing the expected payoffs given in (A.74) and (A.17), the firm will prefer to implement its project at the full investment level if and only if the following condition holds:

$$\theta^f > \frac{\underline{\theta}(E - I + X^L - \delta(E - I + \underline{X}))}{(1 - \delta)(E - \lambda I + \lambda'\underline{X}) - \underline{\theta}(X^H - X^L)}. \tag{A.84}$$

This proves part (ii) of Proposition 4.

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If $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$ and $\delta \leq \delta^*$, the firm uses its capacity to issue risk-free debt and simultaneously repurchase shares to the fullest extent regardless of its investment level. Given the expected payoffs in (A.67) and (A.82), the firm will choose to implement its project at the full investment level if and only if $\overline{\theta} \geq \theta_z$, which proves part (iii) of Proposition 4.

If $\theta^f > \delta\underline{\theta} + (1-\delta)\overline{\theta}$ and $\delta > \delta^*$, the firm uses its capacity to issue risk-free debt and simultaneously repurchase shares to the fullest extent if its investment level is I. In case it invests λI in its project, it also uses its capacity to issue risk-free debt and simultaneously repurchase shares to the fullest extent, but can profitably repurchase only δ^* shares from pessimistic shareholders only. Given the expected payoffs in (A.74) and (A.82), the firm will choose to implement its project at the full investment level if and only if the following condition holds:

$$\overline{\theta} > \frac{\underline{\theta}\left(E - I + X^L - \delta\left(E - \lambda I + \lambda'\underline{X}\right)\right)}{(1-\delta)(E - \lambda I + \lambda'\underline{X}) - \underline{\theta}(X^H - X^L)}. \tag{A.85}$$

This proves part (iv) of Proposition 4.

Now, suppose that when the firm fully invests in its project, the firm raises external financing by issuing new equity (y new shares) to outsiders with belief $\underline{\theta}$ to repurchase x shares from pessimistic current shareholders, where $\frac{E-\lambda I}{E+\lambda'\underline{X}-\lambda I} \leq x \leq \delta$. Then, the following equations will hold:

$$P_E = x \times PE_1 - (E - \lambda I), \; PE_1 = \frac{\lambda'\underline{X}}{1 + y - x}, \; P_E = y \times PE_1. \tag{A.86}$$

Solving these equations, we obtain $PE_1 = E + \lambda'\underline{X} - \lambda I$, and $x - y = \frac{E-\lambda I}{E+\lambda'\underline{X}-\lambda I}$. Firm insiders' expected payoff is equal to

$$\alpha\frac{\lambda'X^f}{1-(x-y)} = \alpha\frac{\lambda'X^f}{1 - \frac{E-\lambda I}{E+\lambda'\underline{X}-\lambda I}} = \alpha\frac{X^f}{\underline{X}}\left(E + \lambda'\underline{X} - \lambda I\right), \tag{A.87}$$

which is exactly equal to the firm insiders' expected payoff in the case where the firm invests λI in its project, and undertakes a stock repurchase alone without external financing when E is small. Clearly, if $\theta^f > \underline{\theta}$, the positive NPV of repurchasing additional undervalued shares from current shareholders with belief $\underline{\theta}$ is exactly offset by the negative NPV of issuing undervalued new shares to outside investors with belief $\underline{\theta}$. Similarly, it is straightforward to show that the firm has no incentive to repurchase more than δ shares by issuing new equity regardless of its investment level. If the firm buys back additional shares from optimistic investors with belief $\overline{\theta}$ by raising external financing from outside investors with belief $\underline{\theta}$, this will clearly be a negative-NPV transaction.

Finally, if $\theta^f > \underline{\theta}$, the NPV of repurchasing some shares (from current shareholders with belief $\underline{\theta}$ at least) is positive, while the NPV of paying out dividends is zero, and the NPV of external financing is at most zero. Thus, the firm will not pay out dividends and simultaneously raise external financing (by issuing debt or equity) to further increase its dividend payout if $\theta^f > \underline{\theta}$. ∎

Proof of Proposition 5. First, consider the case where the firm implements its project at the full investment level, issues β_b new shares to outsiders, and pays an amount D_c as cash dividend. Since all outsiders who currently do not hold equity in the firm have belief $\underline{\theta}$, the share price in the equity issue will be $(E + \lambda'\underline{X} - \lambda I)$. Before the stock issue, the firm has one share of stock outstanding. After the stock issue, the firm will have $(1 + \beta_b)$ shares outstanding. After investing an amount λI in its new project, the firm pays out a cash dividend of

$$D_c = E - \lambda I + \beta_b\left(E + \lambda'\underline{X} - \lambda I\right) \tag{A.88}$$

to its shareholders. The objective function of firm insiders is given by:

$$\max_{\beta_b \in \left[0, \frac{W}{E+\lambda'\underline{X}-\lambda I}\right]} \frac{\alpha}{1+\beta_b}\left(E - \lambda I + \beta_b\left(E + \lambda'\underline{X} - \lambda I\right) + \lambda'X^f\right), \tag{A.89}$$

where $\frac{W}{E+\lambda'\underline{X}-\lambda I}$ is the maximum number of shares the firm can issue to outside investors, if it implements the project at the full investment level. The partial derivative of this objective function with respect to the choice

variable β_b is equal to

$$\frac{\alpha\left(\underline{\theta} - \theta^f\right)\lambda'\left(X^H - X^L\right)}{\left(1 + \beta_b\right)^2}. \tag{A.90}$$

Thus, the optimal solution is

$$\beta_b = \begin{cases} \frac{W}{E+\lambda'\underline{X}-\lambda I} & \text{if } \theta^f < \underline{\theta}, \\ 0 & \text{if } \theta^f \geq \underline{\theta}. \end{cases} \tag{A.91}$$

Note that if $\theta^f < \underline{\theta}$, the optimal choice for the firm is to issue as many new shares as possible by choosing $\beta_b = \frac{W}{E+\lambda'\underline{X}-\lambda I}$. On the other hand, if $\theta^f \geq \underline{\theta}$, the firm will not issue new equity and simultaneously pay out dividends.

Second, consider the case where the firm underinvests in its new project, issues β_b new shares to outsiders, and pays an amount D_c as cash dividend. The share price in the equity issue will be $(E + \underline{X} - I)$. After the equity issue, the firm will have $(1 + \beta_b)$ shares outstanding. After investing an amount I in its new project, the firm pays out a cash dividend of $D_c = E - I + \beta_b\left(E + \underline{X} - I\right)$ to its shareholders. The objective function of firm insiders is given by:

$$\max_{\beta_b \in \left[0, \frac{W}{E+\underline{X}-I}\right]} \frac{\alpha}{1+\beta_b}\left(E - I + \beta_b\left(E + \underline{X} - I\right) + X^f\right). \tag{A.92}$$

Then, the optimal solution is

$$\beta_b = \begin{cases} \frac{W}{E+\underline{X}-I} & \text{if } \theta^f < \underline{\theta}, \\ 0 & \text{if } \theta^f \geq \underline{\theta}. \end{cases} \tag{A.93}$$

Now, the firm has to choose the level of investment in its new project when $\theta^f < \underline{\theta}$. If the firm chooses to fully invest in its project, the expected payoff of firm insiders (substitute $\beta_b = \frac{W}{E+\lambda'\underline{X}-\lambda I}$ in (A.89)) is equal to

$$\alpha\left(\frac{E + \lambda'X^f - \lambda I + W}{E + \lambda'\underline{X} - \lambda I + W}\right)\left(E + \lambda'\underline{X} - \lambda I\right). \tag{A.94}$$

If the firm chooses to underinvest in its new project, the expected payoff of firm insiders is equal to

$$\alpha\left(\frac{E + X^f - I + W}{E + \underline{X} - I + W}\right)\left(E + \underline{X} - I\right). \tag{A.95}$$

Since $\theta_z < \theta^f$ by our global assumption and $\theta^f < \underline{\theta}$ in this case, it follows that the expected payoff given in (A.94) is greater than the expected payoff given in (A.95). Therefore, it is optimal for the firm to implement its new project at the full investment level.

Earlier, we showed in Propositions 1, 2, and 4 that the firm will have an incentive to repurchase equity from current outside shareholders only if $\theta^f > \underline{\theta}$, i.e., if some current shareholders undervalue the firm's shares based on insiders' belief. On the other hand, firm insiders have an incentive to issue new equity only if $\theta^f < \underline{\theta}$. Thus, it follows that insiders never find it optimal to issue new equity and repurchase shares from current shareholders simultaneously. From the proof of Proposition 4, we also know that the firm will not issue new debt and simultaneously repurchase shares from current shareholders if $\theta^f \leq \underline{\theta}$.

We will now analyze if the firm has an incentive to issue new debt and simultaneously pay out dividends or not. First, consider the case where the firm invests λI in its project, issues risk-free debt worth P_D, where $P_D \leq \lambda'X^L$, and simultaneously makes a dividend payment $D_c = E - \lambda I + P_D$. Given that the face value of debt F is equal to P_D, firm insiders solve the following problem to maximize their expected payoff:

$$\max_{F \in [0, \lambda'X^L]} \alpha\left(D_c + \lambda'X^f - F\right).$$

Substituting $D_c = E - \lambda I + F$, the equivalent problem is

$$\max_{F \in [0, \lambda'X^L]} \alpha\left(E - \lambda I + F + \lambda'X^f - F\right),$$

and the expected payoff for any $F \in [0, \lambda'X^L]$ is equal to $\alpha(E + \lambda'X^f - \lambda I)$, which is equal to insiders' expected payoff when the firm makes a dividend payment alone without external financing ($F = 0$). Hence, the firm will not issue risk-free debt and pay out dividends simultaneously.

Second, consider the case where the firm issues risky debt worth P_D, where $P_D > \lambda'X^L$, and simultaneously makes a dividend payment $D_c = E - \lambda I + P_D$ while investing λI in the project. Since the debt is risky, outside investors with belief $\underline{\theta}$ will pay $P_D = \underline{\theta}F + (1 - \underline{\theta})\lambda'X^L$ for the firm's debt issue. The, the firm insiders will solve the following problem:

$$\max_{F \in [\lambda'X^L, \lambda'X^H)} \alpha\left(D_c + \theta^f\left(\lambda'X^H - F\right)\right),$$

which can be equivalently stated as

$$\max_{F \in [\lambda'X^L, \lambda'X^H)} \alpha\left(E - \lambda I + \underline{\theta}F + (1 - \underline{\theta})\lambda'X^L + \theta^f\left(\lambda'X^H - F\right)\right).$$

The partial derivative of this objective function with respect to F is equal to $\alpha(\underline{\theta} - \theta^f)$. Thus, if $\theta^f \geq \underline{\theta}$, the optimal value of F is $\lambda'X^L$, and the firm insiders' expected payoff is $\alpha(E + \lambda'X^f - \lambda I)$ so that the firm has no incentive to issue risky debt and distribute value through a dividend payment simultaneously. On the other hand, if $\theta^f < \underline{\theta}$, the firm has an incentive to issue risky debt, which is overvalued based on the insiders' belief θ^f and set the face value F as large as possible while paying out dividends to shareholders at the same time. If $\theta^f < \underline{\theta}$, the insiders' expected payoff from paying out dividends while raising external financing by issuing new equity or risky debt converges to $\alpha(E + \lambda'\underline{X} - \lambda I)$ in both cases. It is straightforward to show that for any $F < \lambda'X^H$, there exists a threshold W^* so that if $W > W^*$, firm insiders will prefer to issue equity rather than risky debt in order to increase the firm's dividend payout when $\theta^f < \underline{\theta}$. ∎

Proof of Proposition 6. In this case, the firm's stock price will be $P_1 = \lambda'\underline{X}$ at time 1 after the dividend payment, and the number of shares outstanding will still be 1. At time 2, after the marginal equity investor updates her belief as $\underline{\nu} = \underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\overline{\theta} - \underline{\theta})$, the stock price will be

$$P_2 = \lambda'X^{\underline{\nu}} = \lambda'\left(\underline{\nu}X^H + (1 - \underline{\nu})X^L\right). \tag{A.96}$$

The long-run stock return, $\frac{P_2 - P_1}{P_1}$ will then be given by $\frac{d(X^H - X^L)}{3\underline{X}}\left[(\theta^f - \underline{\theta}) + (\overline{\theta} - \underline{\theta})\right]$. A firm will choose to distribute value through a dividend payment alone (while implementing its new project at the full investment level) only if $\theta^f < \underline{\theta}$. Thus, the long-run stock return will be positive if $\underline{\theta} - (\overline{\theta} - \underline{\theta}) < \theta^f \leq \underline{\theta}$. Otherwise, if $\theta^f < \underline{\theta} - (\overline{\theta} - \underline{\theta})$, the long-run stock return will be negative. ∎

Proof of Proposition 7. When E is small, if the firm chooses to implement its project at the full investment level and distribute value through a stock repurchase alone, after the repurchase, the total firm value is $\lambda'\underline{X}$, but the number of shares outstanding is reduced to $\left(1 - \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)$ so that the price per share at time 1 is

$$P_1 = \frac{\lambda'\underline{X}}{\left(1 - \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)}. \tag{A.97}$$

At time 2, after the marginal equity investor updates her belief as $\underline{\nu} = \underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\overline{\theta} - \underline{\theta})$, the price per share will change to

$$P_2 = \frac{\lambda'X^{\underline{\nu}}}{\left(1 - \frac{E - \lambda I}{E + \lambda'\underline{X} - \lambda I}\right)} \tag{A.98}$$

Therefore, the long-run stock return, $\frac{P_2 - P_1}{P_1}$, is equal to $\frac{d(X^H - X^L)}{3\underline{X}}\left[(\theta^f - \underline{\theta}) + (\overline{\theta} - \underline{\theta})\right]$. A firm conducts a stock repurchase only if $\theta^f > \underline{\theta}$. Thus, the long-run stock return will be positive in this case.

If the firm chooses to distribute value through a stock repurchase alone while underinvesting in its new project, the firm's stock price per share will be $(E + \underline{X} - I)$ before the stock repurchase. After the repurchase, the belief of the marginal investor in the firm's equity will change from $\underline{\theta}$ to $\overline{\theta}$. Therefore, the total firm value will be $\overline{X}$, but the number of shares outstanding will be reduced to $\left(1 - \delta - \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}\right)$, so the share price at time 1 will

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be

$$P_1 = \frac{\overline{X}}{\left(1 - \delta - \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}\right)}. \tag{A.99}$$

At time 2, after the marginal equity investor updates her belief as $\overline{\nu} = \overline{\theta} + \frac{d}{3}\left(\theta^f - \overline{\theta}\right) + \frac{d}{3}\left(\underline{\theta} - \overline{\theta}\right)$, the stock price will change to

$$P_2 = \frac{X^{\overline{\nu}}}{\left(1 - \delta - \frac{E - I - \delta(E + \underline{X} - I)}{E + \overline{X} - I}\right)}. \tag{A.100}$$

Therefore, in this case, the long-run stock return, $\frac{P_2 - P_1}{P_1}$, is equal to $\frac{d(X^H - X^L)}{3\overline{X}}\left[\left(\theta^f - \overline{\theta}\right) + \left(\underline{\theta} - \overline{\theta}\right)\right]$. If $\theta^f > \overline{\theta} + \left(\overline{\theta} - \underline{\theta}\right)$ in this case, then the long-run stock return following the stock repurchase will be positive. Otherwise, if $\underline{\theta} < \theta^f < \overline{\theta} + \left(\overline{\theta} - \underline{\theta}\right)$, it will be negative.

When E is large, if the firm chooses to distribute value through a stock repurchase alone while implementing its project at the full investment level, the marginal investor in the firm's equity after the repurchase at time 1 will also have the belief $\overline{\theta}$ as in part (ii) of Proposition 7. Thus, the share price at time 1 (after the repurchase) will be

$$P_1 = \frac{\lambda' \overline{X}}{\left(1 - \delta - \frac{E - \lambda I - \delta(E + \lambda' \underline{X} - \lambda I)}{E + \lambda' \overline{X} - \lambda I}\right)}. \tag{A.101}$$

At time 2, after the marginal equity investor updates her belief as $\overline{\nu} = \overline{\theta} + \frac{d}{3}\left(\theta^f - \overline{\theta}\right) + \frac{d}{3}\left(\underline{\theta} - \overline{\theta}\right)$, the stock price will change to

$$P_2 = \frac{\lambda' X^{\overline{\nu}}}{\left(1 - \delta - \frac{E - \lambda I - \delta(E + \lambda' \underline{X} - \lambda I)}{E + \lambda' \overline{X} - \lambda I}\right)}. \tag{A.102}$$

Therefore, in this case, the long-run stock return, $\frac{P_2 - P_1}{P_1}$, is equal to $\frac{d(X^H - X^L)}{3\overline{X}}\left[\left(\theta^f - \overline{\theta}\right) + \left(\underline{\theta} - \overline{\theta}\right)\right]$ as well. It will be positive if $\theta^f > \overline{\theta} + \left(\overline{\theta} - \underline{\theta}\right)$. Otherwise, if $\underline{\theta} < \theta^f < \overline{\theta} + \left(\overline{\theta} - \underline{\theta}\right)$, it will be negative. ∎

Proof of Proposition 8. Without loss of generality, consider the case where the firm underinvests in its new project while it distributes value through a combination of a stock repurchase and a dividend payment when E is small. The firm first repurchases δ shares from pessimistic shareholders and then makes a dividend payment of $(E - I - \delta(E + \underline{X} - I))$ to the remaining shareholders. The share price at time 0 is $(E + \underline{X} - I)$. Recall that after the repurchase, the total number of shares outstanding is $1 - \delta$. If the number of shares repurchased by the firm is slightly (ϵ) less than δ, then the marginal investor in the firm's equity will still have the belief $\underline{\theta}$ after the payout at time 1. Therefore, the share price at time 1 after the stock repurchase and the dividend payment is

$$P_1 = \frac{\underline{X}}{1 - \delta}. \tag{A.103}$$

At time 2, after the marginal investor in the firm's equity updates her belief as $\underline{\nu} = \underline{\theta} + \frac{d}{3}\left(\theta^f - \underline{\theta}\right) + \frac{d}{3}\left(\overline{\theta} - \underline{\theta}\right)$, the share price will be

$$P_2 = \frac{\underline{X}^{\underline{\nu}}}{1 - \delta}. \tag{A.104}$$

Therefore, the long-run stock return, $\frac{P_2 - P_1}{P_1}$, is equal to $\frac{d(X^H - X^L)}{3\underline{X}}\left[\left(\theta^f - \underline{\theta}\right) + \left(\overline{\theta} - \underline{\theta}\right)\right]$. Since $\left(\overline{\theta} - \underline{\theta}\right) > 0$ by assumption, the long-run stock return is positive if $\theta^f > \underline{\theta}$. Note that for a firm to optimally choose a combination of a stock repurchase and a dividend payment to distribute value to its shareholders, it has to be the case that $\theta^f > \underline{\theta}$. Therefore, the long-run stock return will be positive in this case. It is straightforward to show that the same result will be obtained, even when E is large and the firm implements its project at the full investment level. ∎

Proof of Proposition 9. We assume that E is small so that the condition in (7) holds, θ^f is fixed, $k = \overline{\theta} - \underline{\theta}$ is a constant, and $\underline{\theta}$ is uniformly distributed on the interval $(0, 1 - k)$. From Proposition 1(i)(a), it follows that for the sample of pure dividend-paying (and fully-investing) firms, the following parameter condition holds: $\theta^f \leq \underline{\theta} < 1 - k$. Then, the *average* long-run stock return $\bar{r}_{div}$ of firms paying dividends only and implementing their projects at

the full investment level is given by:

$$\bar{r}_{div} = \frac{d\left(X^H - X^L\right)}{3} \int_{\theta^f}^{1-k} \frac{\left(\theta^f - \theta + k\right)}{X^L + \theta\left(X^H - X^L\right)} \frac{1}{\left(1 - k - \theta^f\right)} d\theta. \tag{A.105}$$

When E is small, we know from Proposition 1(i)(b) that firms, for which $\theta_z \leq \underline{\theta} < \theta^f$, will repurchase shares only and fully invest in their new projects. Thus, the *average* long-run stock return $\bar{r}_{rep}$ of these firms is given by:

$$\bar{r}_{rep} = \frac{d\left(X^H - X^L\right)}{3} \int_{\theta_z}^{\theta^f} \frac{\left(\theta^f - \theta + k\right)}{X^L + \theta\left(X^H - X^L\right)} \frac{1}{\left(\theta^f - \theta_z\right)} d\theta. \tag{A.106}$$

Since the long-run stock return $\frac{d\left(X^H - X^L\right)}{3\left(X^L + \underline{\theta}\left(X^H - X^L\right)\right)}\left[\left(\theta^f - \underline{\theta}\right) + k\right]$ is decreasing in $\underline{\theta}$, it follows that $\bar{r}_{rep} > \bar{r}_{div}$.

For some parameter specifications, a small fraction of the share-repurchasing (and fully-investing) firms may actually satisfy the condition given in Proposition 1(ii)(b) rather than the condition given in Proposition 1(i)(b). For these firms, it holds that $\theta_z - k < \underline{\theta} < \theta_z$. Since the long-run stock return given in (39) is decreasing in $\underline{\theta}$, the average long-run stock return of these firms is greater than the average long-run stock return $\bar{r}_{rep}$ (given in (A.106)) of firms satisfying the condition that $\theta_z \leq \underline{\theta} < \theta^f$. Therefore, it suffices to show that the average long-run stock return of share-repurchasing firms (given in (A.106)), for which $\theta_z \leq \underline{\theta} < \theta^f$, is greater than the average long-run stock return $\bar{r}_{div}$ of dividend-paying firms (given in (A.105)) satisfying the condition given in Proposition 1(i)(a). ∎