"Firm-Level Comparative Advantage" *

Matthieu Crozet† and Federico Trionfetti‡

First version, December 2008.
This version, September 2013

Abstract

We study the consequences of heterogeneity in factor intensity on firm performance. We present a standard Heckscher-Ohlin model augmented with factor intensity differences across firms within a country-industry pair. We show that for any two firms, each of whose capital intensity is, for instance, one percent above (below) its respective country-industry average, the relative marginal cost of the firm in the capital-intensive industry of the capital-abundant country is lower (higher) than that of the other firm. Our empirical analysis, conducted using data for a large panel of European firms, supports this prediction. These results provide a novel approach to the verification of the Heckscher-Ohlin theory and new evidence on its validity.

JEL codes: F1

Keywords: Factor intensity, Firm heterogeneity, Test of trade theories.

*This paper has previously circulated under the title "Comparative Advantage and Within-Industry Firms Performance". We thank CEPREMAP and GREQAM and PSE for financial support. We also thank conference and workshop participants at the ETSG 2010 (University of Lausanne), the DEGIT XVI (Saint Petersburg), the CEPII, the ITSG & AIEL 2013 (University of Padova), as well as seminar participants at Bocconi University, Kiel University, and La Sapienza university for helpful suggestions.

†University Paris Sud and CEPII. matthieu.crozet@cepii.fr. CEPII, 113 rue de grenelle, 75007 Paris, France.

‡Aix-Marseille University (Aix-Marseille School of Economics), CNRS & EHESS. Federico.Trionfetti@univ-amu.fr

1 Introduction

There is a wealth of empirical evidence that factor intensity differs across firms even within the same country-industry groups. For instance, in our data, comprising more than 300,000 European firms, only 30 percent of the total variance in firm-level capital/labor ratios is between country-industry groups, 70 percent is within the same country-industry groups. Empirical literature in international trade has documented that differences in factor intensities matter for firms' performances (e.g. Bernard, Jensen, Redding and Schott 2007). These observations contrasts with the assumption usually adopted in trade models whereby firms are either assumed to be identical or are assumed to differ by Hicks-neutral productivity. In either case, the resulting factor intensities are identical across firms within any industry. Following this empirical observation, we consider a Heckscher-Ohlin model in which firms differ in the relative marginal productivity of factors. As a result, firms have different factor intensities even within the same country-industry group.

The main result emerging from this model is that countries' comparative advantage begets a comparative advantage at firm level. The three key firm-level variables in the model are relative capital/labor ratio (κ), relative marginal costs, and relative sales; all three relative to the country-industry average. We say that a firm is capital- (labor)-intensive with respect to its country-industry average if $\kappa > 1$ ($\kappa < 1$). We also say that a firm has a comparative advantage over another if the first firm's relative marginal cost is lower than the second firm's. Our main theoretical result may be stated as follows:

Consider any two firms with same κ but belonging to different countries and industries. The firm that is intensive in the factor intensively used in its industry and of which its country is relatively well-endowed has a comparative advantage over the other firm. Because of the comparative advantage, the firm will also have higher relative sales.

The statement above is the natural generalization of the Heckscher-Ohlin theorem to an environment with heterogeneous firms. In the Heckscher-Ohlin theorem, the comparative advantage of a country is determined by the matching between the characteristics of countries and those of industries. In the statement above, the comparative advantage of a firm is determined by the matching between its characteristics (κ) and those of the industry and country to which it belongs. In the Heckscher-Ohlin theorem, the comparative advantage of countries gives rise to differences in the relative size of industry output (international specialization). In the statement above, the comparative advantage of firms gives rise to differences in the relative size of firm output (relative sales).

As an example, consider two firms in different industries and different countries but with an identical κ. Assume, for instance, $\kappa > 1$. Then, the firm in the capital intensive industry and capital abundant country will have a comparative advantage (lower relative marginal cost) over the firm in another industry and country. The firm's comparative advantage will show up in larger relative sales. If, instead, we consider two firms whose capital intensity is lower than their respective country-industry average ($\kappa < 1$) then the firm in the capital intensive industry and capital abundant country will have a comparative disadvantage and lower relative sales. This prediction does not obtain in models where heterogeneity is Hicks-neutral. In these models, two firms with same relative productivity but in different countries and industries have identical relative marginal cost and identical relative sales.

Our empirical investigation, conducted on a dataset which comprises about 300,000 firms in 21 European countries and 95 industries, strongly supports the theoretical result. Both structural and non-structural estimates show that the relationship between relative sales and relative factor intensity is affected by comparative advantage in the way predicted by the model. For instance, the non-structural estimates show that firms with capital intensity ten percent above their respective country-industry average ($\kappa = 1.1$) have different relative sales: the sales of firms in a capital intensive industry and a capital abundant country are 2.68 percent larger than the average firm in the same country-industry whereas the sales of firms in a labor intensive industry and a labor abundant country are only 1.66 percent larger than the average firm in the same country-industry.

We are not alone to assume heterogeneity in factor intensity. Costinot and Vogel (2010) and Burstein and Vogel (2012) are notable examples. Their models differ from ours in terms of the market structure, technology, and preferences. The focuses are also very different; they (as well as Vannoorenberghe, 2012, who uses a model structure similar to ours) study the effect of trade liberalization on wage inequality, we study instead how countries comparative advantage begets comparative advantage at firm level. There are also differences in the mechanisms driving the results; in these three papers results rest on skilled biased heterogeneity, ours do not. Yet, in their works like in ours, heterogeneity in factor intensity harnesses to a better extent the potentials of heterogenous firms models in the understanding of international trade issues.

Our paper contributes to the literature in two ways. First, it provides a novel approach to the verification of the Heckscher-Ohlin (HO) model. Seminal contributions, e.g., Leamer (1980), Trefler (1993, 1995), Davis and Weinstein (2001), Romalis (2004) have provided solid evidence on the empirical merits of the factors proportion theory. In their works, comparative advantage is revealed by the effect it has on aggregate variables (the factor content of trade or industry specialization). We propose a different approach. In our model, comparative advantage is revealed by the effect it has on firm-level variables. Comparative advantage is the mechanism driving the HO theorem but it remains behind the scenes in homogenous-firms models, as well as in Hicks-neutral heterogeneity models, because it does not give rise to a comparative advantage at firm level. Being able to observe the firm-level comparative advantage generated by the comparative advantage of countries brings to light the fundamental mechanism driving the HO theorem. Approaches based on aggregate variables are, of course, unsuited to bring this mechanism to light.

Our second contribution is to show that countries' comparative advantage matters for explaining relative performances within industries. Seminal contributions by Bernard, Eaton, Jensen and Kortum (2003) and Melitz (2003) as well as many subsequent important developments use a one factor model and take out countries' comparative advantage.[1] They are therefore unsuited to study the effect of country' comparative advantage on firms performances. Bernard, Redding and Schott (2007) are the first to introduce firm heterogeneity in a HO model but they consider only Hicks-neutral productivity differences. When productivity differences are Hicks-neutral the comparative advantage of countries does not influence firms relative sales within an industry. Thus, the one-factor assumption or Hicks-neutral heterogeneity make that within industry relative performances are independent from comparative advantage and depend only on exogenously given differences in

[1] See, e.g., Yeaple (2005), Chaney (2008), Melitz and Ottaviano (2008), Arkolakis, Costinot and Andrés Rodríguez (2012), Bustos (2011), Eaton, Kortum and Kramarz (2011), Crozet, Head and Mayer (2012). See also Manasse and Turrini (2001), Egger and Kreickemeier (2009), Helpman, Itskhoki and Redding (2010), Davis and Harrigan (2011), Amiti and Davis (2011), Harrigan and Reshef (2012), for particular focus on the distribution effects of trade integration.

productivity. As a result, one may be left with the impression that there is a dichotomy between within-industry effects and across-industry effects, the former being driven by firm-level differences and the latter by countries' comparative advantage. Our work, instead, highlights precisely how within-industry effects are determined jointly by firm-level characteristics (κ) and countries' comparative advantage.

We conclude this section by mentioning that the four core theorems of international trade remain valid in our model. But the degree of international specialization, the intensity of the Stolper-Samuelson and Rybczynski magnification effects, and the size of the factor price equalization set are all affected. We briefly present these results in Sect. 9.2.9 of the online supplement.

The remainder of the paper is as follows. Section 2 outlines the model, Section 3 describes the theoretical results, Section 4 derives the estimable equation, Section 5 presents the data, Sections 6 and 7 present the empirical results for the structural and non-structural estimates respectively, and Section 8 concludes. The online supplement, Sect. 9, contains the detailed description of the model, proofs of the results, and further technical matters.

2 Heterogeneity

We extend the model in Bernard, Redding and Schott (2007) by assuming firm heterogeneity in factor intensity. In this section we focus on the essential features of the model which we detail in Sect. 9.1 of the online supplement.

The world economy is composed of two countries indexed by $c = H, F$; it produces two differentiated goods indexed by $i = Y, Z$, by using two primary factors indexed by $j = K, L$. Each country is endowed with a share $\nu_j^c > 0$ of world's endowments, $\overline{K}$ and $\overline{L}$. Production requires fixed and variable inputs in each period. The variable input technology takes the CES form here represented by the marginal cost which, for a firm in industry i of country c, is

$$mc_i^c(\beta, \phi) = \frac{1}{\phi}\left[(\lambda_i)^\sigma (w^c)^{1-\sigma} + (1-\lambda_i)^\sigma (r^c)^{1-\sigma} \beta^{\sigma-1}\right]^{\frac{1}{1-\sigma}}. \tag{1}$$

where $\lambda_i \in (0,1)$ is a constant technology parameter of industry i, the variables r^c and w^c denote, respectively, the price of K and L in country c, and $\sigma > 1$ measures gross substitutability between factors.[2] The variable ϕ captures Hicks-neutral productivity differences across firms. The variable β captures differences across firms in the relative marginal productivity of factors. Models that focus on Hicks-neutral heterogeneity let ϕ vary across firms but assume β identical across firms. We instead let both ϕ and β vary across firms. We let ϕ and β be independent random variables with cumulative distribution $V(\phi)$ and $G(\beta)$ both with support $(0, \infty)$.

The optimal *K-intensity* in production, $\theta_i^c(\beta)$, is

$$\theta_i^c(\beta) = (\omega^c)^{\sigma} \Lambda_i^{\sigma} (\beta)^{\sigma-1}, \tag{2}$$

where $\omega^c \equiv w^c/r^c$ and $\Lambda_i \equiv (1-\lambda_i)/\lambda_i$. Obviously, $\theta_i^c(\beta)$ is independent from the Hicks-neutral shifter ϕ.

By paying a fixed entry cost, a firm draws randomly one of the random variables and remains married to it until death of the firm do them apart (like in all the heterogeneous firms literature). The other variable instead is a random shock hitting continuously the production process of the firm. This assumption captures the idea that although an initial investment in fixed entry cost gives the firm an information on its marginal cost (this is represented in the model by the costly draw of one of the stochastic variables), the actual marginal cost of every production experience is not known until production is actually taking place (random shock to the production process). It is irrelevant to the results of the model which variable has a life-time association with the firm. For the sake of expositional clarity, we assume that by paying a fixed entry cost a firm draws β and remains associated with it until death. Furthermore, at any point in time the firm is hit by a particular realization of ϕ and as a probability of death equal to δ.[3]

Turning to the fixed input technology, whether it is homogenous or heterogenous across firms gives qualitatively the same results. We assume ho-

[2]When factors are gross complement ($\sigma < 1$) our main result, namely, that countries comparative advantage begets comparative advantage at firm level, remains valid (see Sect. 9.2.5 of the online supplement).

[3]Other modelling choices are possible and would leave the results unchanged. For instance, we could assume that both ϕ and β remain attached to the firm until death (as in Harrigan and Reshef, 2012). This assumption is particularly suitable for models where there is no selection into entry. If selection into entry is instead a desired feature of the model, then it is sufficient that the firm has a random life-time association with only one of the random variables. The other random variable may then be used to add a different element of stochasticity. In our model such element is a continuous random shock to Hicks-neutral productivity.

mogeneous fixed costs since this assumption allows focusing on heterogeneity in the production process (which is the heart of the matter). This is the assumption most commonly retained in the literature (Melitz, 2003; Bernard, Redding and Schott, 2007; and many others). Specifically we assume that the fixed input technology is represented by the cost function $\widetilde{mc}_i^c$ described below in equation (3). Thus, the fixed production cost is $F_i\widetilde{mc}_i^c$ where F_i is a positive constants. This assumption represents the fixed input as a homogenous, non-traded, composite good produced in a perfectly competitive market by assembling in a CES all varieties of the domestic industry output (similarly to Ethier, 1980). But it may also be interpreted as in Yeaple (2005) who assumes that the fixed cost is represented by output that must be produced by the firm and that ultimately cannot be sold; with the difference that in our model this output requires a unit cost function $\widetilde{mc}_i^c$. Analogously to fixed production cost, the fixed exporting cost is $F_{ix}\widetilde{mc}_i^c$ and the fixed entry cost is $F_{ie}\widetilde{mc}_i^c$ where F_{ix} and F_{ie} are positive constant. Fixed entry cost is paid in order to draw β, fixed exporting costs are paid in order to access the foreign market.

Two remarks are useful at this point. The first remark concerns all the fixed cost. Fixed exporting costs are not necessary in our model. Thus, we present our results in the main text under the assumption that $F_{ix} = 0$. This makes the reading smoother by simplifying the notation but, more importantly, highlights that our results do not depend on the partitioning of firms by export status. We think this is quite interesting because many results in the literature (especially the literature on the skill premium) hinge on the existence of the partitioning of firms by export status.[4] For completeness of investigation we shall reintroduce fixed exporting costs in Sect. 9.3 of the online supplement where we show that our results remain valid. Fixed entry and fixed production costs result in firm selection. This selection is not necessary in our model but we keep it since we find interesting the fact that the entry process makes firm-level comparative advantage hold *a fortiori*. In any case, if we assume that all firms survive in the market (as in Burstein and Vogel, 2012) the results would remain unchanged (see Sect. 9.2.6 of the online supplement). The second remark concerns the relationship between sales and *K-intensity*. We see from expression (1) and (2) that higher K-

[4]See, e.g., Manasse and Turrini (2001), Yeaple (2005), Costinot and Vogel (2010), Helpman, Itskhoki and Redding (2010), Amiti and Davis (2011), and Burstein and Vogel (2012).

intensity corresponds to a lower marginal cost and, therefore, to higher sales. This positive correlation between sales and *K-intensity* is what we find in our data.[5] However, nothing in our model hinges on this positive relationship. In the appendix we show that assuming a negative relationship between *K-intensity* and sales leaves our results unchanged (see Sect. 9.2.8 of the online supplement).

After drawing β the firm stays in the market if the expected realization of profits is non negative and exits otherwise.[6] Let β_i^{*c} be the least value of β in industry i of country c such that the expected realization of profit after having drawn β is zero. Let $\widetilde{\phi} \equiv \left(\int_0^\infty \phi^{\sigma-1} dV\right)^{\frac{1}{\sigma-1}}$ and $\widetilde{\beta}_i^c \equiv \left(\int_0^\infty \beta^{\sigma-1} dG\right)^{\frac{1}{\sigma-1}}$. Furthermore, let the average *K-intensity* and the average marginal cost in industry i of country c be defined respectively as $\widetilde{mc}_i^c \equiv \left\{\int_0^\infty \int_0^\infty \left[mc_i^c(\phi,\beta)\right]^{1-\sigma} dGdV\right\}^{\frac{1}{1-\sigma}}$ and $\overline{\theta}_i^c \equiv \int_0^\infty \theta_i^c(\beta)\, dG$. Then, we have

$$\widetilde{mc}_i^c = \frac{1}{\widetilde{\phi}}\left[(\lambda_i)^\sigma (w^c)^{1-\sigma} + (1-\lambda_i)^\sigma (r^c)^{1-\sigma}\left(\widetilde{\beta}_i^c\right)^{\sigma-1}\right]^{\frac{1}{1-\sigma}}, \tag{3}$$

$$\overline{\theta}_i^c = (\omega^c)^\sigma \Lambda_i^\sigma \left(\widetilde{\beta}_i^c\right)^{\sigma-1}, \tag{4}$$

where, given that some firms decide not to stay in the market, $\widetilde{\beta}_i^c$ becomes equal to $\left[\frac{1}{1-G(\beta_i^{*c})}\int_{\beta_i^{*c}}^\infty \beta^{\sigma-1} dG\right]^{\frac{1}{\sigma-1}}$. Incidentally, for further reference note that $\widetilde{mc}_i^c = mc_i^c\left(\widetilde{\phi}, \widetilde{\beta}_i^c\right)$. A further remark is useful at this point. In models where firms are homogeneous or heterogenous with identical factor intensity the average factor intensity is $(\omega^c)^\sigma \Lambda_i^\sigma$. We can see from expression (4) that, with respect to those models, there is a factor bias in our model whenever $\widetilde{\beta}_i^c \neq 1$. The bias is endogenous since it depends on the cut-off values β_i^{*c}. It may go in either direction - to a *K-bias* or to a *L-bias* - and the direction may differ in different industries or countries. None of our results depend on the direction or on the existence of such bias (see through Sect. 9.2 of the online supplement).

[5] In our data the correlation between $\ln RS$ and $\ln \kappa$ is 0.2186.

[6] Given that the probability distribution of ϕ and β and the probability of death δ are all constant over time, it is irrelevant for the equilibrium value of the endogenous variables whether the firm decides to stay on the basis of expected profit or on actual (instant) profit. See also the discussion of Eq. 16 in Sect. 9.1 of the online supplement.

Turning to demand, the representative consumer has Dixit-Stiglitz preferences represented by a Cobb-Douglas index, with shares $\gamma_i \in (0,1)$, $\gamma_Y + \gamma_Z = 1$, and defined over CES aggregates whose elasticity of substitution between varieties is $\varsigma > 1$.

3 Comparative Advantage

To fix ideas, throughout the paper it is assumed that country H is *K-abundant,* i.e., $\nu_K^H > \nu_L^H$, and that industry Y is *K-intensive*, i.e., $\Lambda_Y > \Lambda_Z$.[7] Our objective is to show that countries' comparative advantage generates a comparative advantage at firm level. We begin by four definitions.

Let $a_i^c = (\Lambda_i)^\sigma (\omega^c)^{\sigma-1} \left(\widetilde{\beta}_i^c\right)^{\sigma-1}$. Note $a_Y^H > a_Z^F$ since H is *K-abundant,* Y is *K-intensive* and (as shown in Sect. 9.2.1 of the online supplement) $\widetilde{\beta}_Y^H > \widetilde{\beta}_Z^F$.[8]

Definition 1 *Let $a \in A = \left\{a_Y^H, a_Z^F\right\}$. Then a is an index of comparative advantage of countries.*

Definition 2 *Let κ denote the relative K-intensity and let φ denote the relative Hicks-neutral productivity, i.e.,*

$$\kappa \equiv \frac{\theta_i^c}{\overline{\theta}_i^c} = \left(\frac{\beta}{\widetilde{\beta}_i^c}\right)^{\sigma-1}, \qquad \varphi \equiv \frac{\phi}{\widetilde{\phi}}. \tag{5}$$

Two firms are called ***twin firms*** *if they have the same κ, the same φ, and belong to different countries and industries.*

Definition 3 *A firm is K-intensive if $\kappa(\beta) > 1$. A firm is L-intensive if $\kappa(\beta) < 1$.*

[7] Firms with a very high β in industry Z may have a higher *K-intensity* than firms with a low β in industry Y. Yet, $\Lambda_Y > (<)\, \Lambda_Z$ is sufficient condition for the average *K-intensity* to be larger (smaller) in industry Y than in Z. Thus, there is no average factor intensity reversal (see Sect. 9.2.7 of the online supplement).

[8] If we had assumed that all firms survived in the market then β_i^{*c} and $\widetilde{\beta}_i^{*c}$ would be the same for all c and i; still $a_Y^H > a_Z^F$.

Definition 4 *Let, $\mu(a,\varphi,\kappa)$, be an index of comparative advantage of firms defined as the marginal cost of a firm relative to the marginal cost of the average firm,*

$$\mu(a,\varphi,\kappa) \equiv \frac{mc_i^c}{\widetilde{mc}_i^c} = \varphi^{-1}\left(\frac{1+a\kappa}{1+a}\right)^{\frac{1}{1-\sigma}}. \tag{6}$$

We say that a firm has a comparative advantage over another firm iff it has lower relative marginal cost. We can now state the following theorem.

Theorem 1 *A firm has a comparative advantage on its twin if and only if it is intensive in the factor intensively used in its industry and of which its country is relatively well endowed. In our notation:*

$$\mu\left(a_Y^H,\varphi,\kappa\right) \lesseqqgtr \mu\left(a_Z^F,\varphi,\kappa\right) \text{ as } \kappa \gtreqqless 1, \text{ for any } \varphi. \tag{7}$$

Proof. *See Sect. 9.2.2 of the online supplement.* ■

Thus, for instance, a *K-intensive* firm ($\kappa > 1$) has a comparative advantage over its twin if it is in the *K-intensive* industry of the *K-abundant* country. Likewise, *mutatis mutandi*, for twins with $\kappa < 1$.

Theorem 1 relates firms comparative advantage to countries comparative advantage. Thanks to heterogeneity in factor intensity we can *observe* HO comparative advantage *in action*, as it gives rise to the comparative advantage of firms. Indeed the ranking of a_i^c (the index of comparative advantage of countries) determines the ranking of $\mu(a,\varphi,\kappa)$ (the index of comparative advantage of firms). Verifying the existence of this relationship in the data provides a new empirical verification of the HO theory which goes to the very heart of the HO functioning mechanism.

Incidentally, note that the inequality (7) depends on κ but does not depend on φ. Indeed, using (1), (3) and (6) shows that φ cancels out of (7). As it is obvious, Hicks-neutral productivity differences are irrelevant to the determination of the comparative advantage of firms.

For our empirical purposes will shall use the following proposition. Let $RS(a,\varphi,\kappa)$ denote relative sales, i.e., the sales of a firm relative to the average firm in the same industry and country. It is well known that with Dixit-Stiglitz preferences, relative sales are equal to relative marginal cost powered to 1 minus the elasticity of substitution between varieties (see Eq. 14 in Sect. 9.1 of the online supplement). Thus, using (6) we obtain

$$RS\left(a,\varphi,\kappa\right)=\varphi^{\varsigma-1}\left(\frac{1+a\kappa}{1+a}\right)^{\frac{1-\varsigma}{1-\sigma}}. \tag{8}$$

Lastly, let $B_i^c \equiv \left\{\kappa : \kappa = \left(\beta/\widetilde{\beta}_i^c\right)^{\sigma-1} \wedge \beta_i^{*c} \leq \beta < \infty\right\}$. and let the set B be the intersection of the four sets B_i^c. Since we compare twin firms we consider values of $\kappa \in B$. Then:

Proposition 1 *The function RS: $A \times B \rightarrow \mathbb{R}^+$ is strictly log-supermodular in (a,κ). Further, for twin firms*

$$RS\left(a_Y^H,\varphi,\kappa\right) \gtreqless RS\left(a_Z^F,\varphi,\kappa\right) \text{ as } \kappa \gtreqless 1, \text{ for any } \varphi. \tag{9}$$

Proof. *See Sect. 9.2.3 of the online supplement.*[9] ■

Proposition 1 says that for any pair of twin firms, the firm which is intensive in the factor intensively used in its industry and of which its country is relatively well endowed has larger relative sales. Note, again, the irrelevance of Hicks-neutral productivity difference, φ, which cancels out of (9).

Fig. 1 offers a graphical representation of the relationship between relative sales (RS) and relative *K-intensity* (κ) stated in Proposition 1.

Intuition for Proposition 1 is provided by analyzing the two underlying mechanisms giving rise to it, which we do next.

1. Factor-intensity and industry technology. The first mechanism relates firm factor intensity (κ) to the technology of the industry (Λ_i). Consider two equally *K-intensive* firms in the same country but in different industries. Although these firms have same κ, the relative marginal cost is lower and relative sales are higher for the firm in the *K-intensive* industry, because the factor whose relative marginal productivity is higher for both firms with respect to the industry average (K) is used more intensively in the *K-intensive* industry. Consider now two equally *L-intensive* firms in the same country but in different industries. The relative marginal cost is higher

[9] If a were a continuos variable, supermodularity would boil down to a condition on cross partial derivatives. But a is discrete, hence the need to state the results using supermodularity.

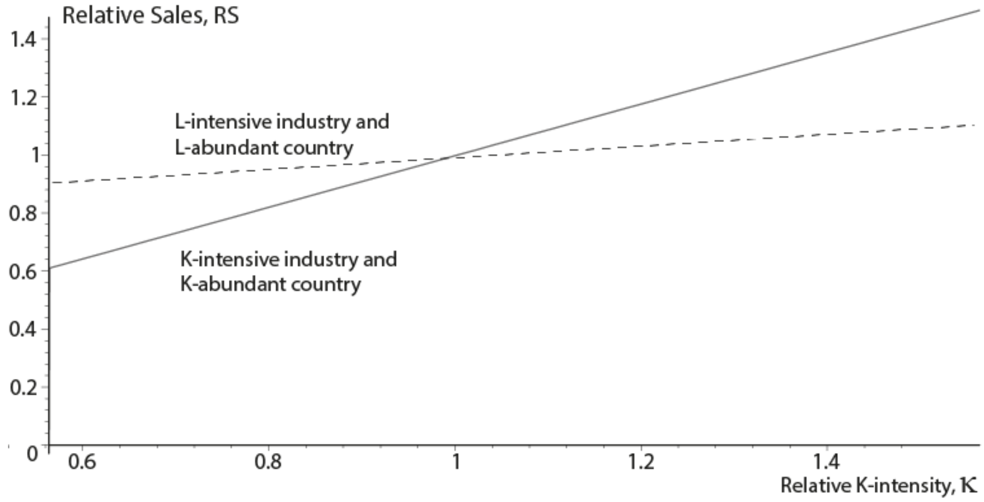

Figure 1: Graphical representation of Proposition 1

and relative sales lower for the firm in the *K-intensive* industry, because the factor whose relative marginal productivity is lower for both firms with respect to the industry average (K) is intensively used in this industry.

Formally: $RS\left(a_Y^c, \varphi, \kappa\right) \gtreqless RS\left(a_Z^c, \varphi, \kappa\right)$ *as* $\kappa \gtreqless 1$, for any φ.

2. Factor intensity and factors abundance. The second mechanism relates firm factor intensity (κ) to countries' relative factors endowments via relative factors price (ω). Consider two equally *K-intensive* firms in the same industry but in different countries. Relative marginal costs are lower and relative sales are higher for the firm in the *K-abundant* country, because the factor that both firms use intensively with respect to the industry average (K) is relatively cheaper in the *K-abundant* country. Consider now two equally *L-intensive* firms in the same industry but in different countries. The relative marginal cost is higher and sales lower for the firm in the *K-abundant* country, because the factor that both firms save with respect to the industry average (K) is relatively cheaper in the *K-abundant* country.

Formally: $RS\left(a_i^H, \varphi, \kappa\right) \gtreqless RS\left(a_i^F, \varphi, \kappa\right)$ *as* $\kappa \gtreqless 1$, for any φ.

We conclude this section with the following proposition.

Proposition 2 *When heterogeneity is only Hicks-neutral, relative sales do not depend on country-industry characteristics.*

Proof. *Assume* β *to be constant and identical for all firms, then* $\kappa = 1$*. Thus, from (8) we have*

$$RS\left(a, \varphi, 1\right) = \varphi^{\varsigma - 1} \tag{10}$$

which proves the Proposition. ■

4 Empirical Implementation

In the empirical analysis a firm is an element of our dataset and is identified by the index ξ. To every firm ξ in country c and industry i there correspond a relative Hicks-neutral component of TFP $\varphi(\xi) = \phi\left(\xi\right)/\widetilde{\phi}_i^c$ and a relative *K-intensity* $\kappa(\xi) = \theta_i^c\left[\beta(\xi)\right]/\overline{\theta}_i^c$. From (8) the log of relative sales of firm ξ is given by:

$$\ln RS(\xi) = (\varsigma - 1)\ln\varphi(\xi) + \frac{1-\varsigma}{1-\sigma}\ln\left[\frac{1+\kappa(\xi)a_i^c}{1+a_i^c}\right], \tag{11}$$

We use the data described in Sect. 5 to estimate equation (11) over different groups of countries and industries.

In Sect. 6 we instead estimate a non-structural log-linear formulation of equation (11):

$$\ln RS(\xi) = \eta \ln\varphi(\xi) + \psi_i^c \ln\kappa(\xi) + F + \epsilon(\xi), \tag{12}$$

where F is an intercept and $\epsilon(\xi)$ is an error term. The dependant variable is the sales of a firm ξ relative to its country-industry average. The first term on the right hand side is the relative Hicks-neutral productivity of ξ, and the second term is its relative *K-intensity.* According to the model the estimated value of the coefficients ψ_i^c should be larger for *K-intensive* industries and *K-abundant* countries. This non-structural approach is particularly useful to verify the validity of each constitutive mechanisms of Proposition 1. It also provides an empirical assessment not only of our model but, potentially, of an entire class of models exhibiting heterogeneity in factor intensity.

In Sect. 7 we proceed to a structural non-linear estimation of equation (11) which gives estimated values of ς and a_i^c. According to Proposition 1 we should find that the estimates a_i^c are larger for *K-intensive* industries and *K-abundant* countries.

5 Data

Our empirical examination combines two main sources of data: firm-level balance sheets and country-level capital and labor endowments. Firm-level data are provided by Bureau Van Dijk's Amadeus database.[10] Amadeus compiles balance sheet information for a very large number of companies located in 41 European countries. Its coverage is increasing progressively. To get the most comprehensive database, we use the most recent year available at the time of writing, 2007. We proxy the capital intensity by the ratio of total assets to total employment. Sales is measured by the turnover of the firm, without distinction between exports and domestic sales. Firms in Amadeus are classified according to their primary activity. Each company is assigned to a single 3-digit NACE-Rev2 code. We restrict our empirical analysis to manufacturing sectors (including agrifood), i.e., to firms with a primary activity code between 101 and 329.[11] We keep firms with more than one employee and non-extreme values of capital intensity (we drop firms whose K-intensity is 200 times larger or 200 times smaller than the country-industry median). Moreover, we drop all country-industry pairs which contain too few observation to perform robust regressions. We fix arbitrary limits and retain countries with more than 300 firms, industries with more than 50 active firms in 5 countries at least, and country-industry pairs with more than 5 firms.

Capital abundance for each country (K^c/L^c) is derived from several sources. We use ILO and United Nations data for workforce figures. Capital stocks are estimated by the perpetual inventory method, using investment data from the World Bank and national sources.[12] Industry-level capital intensity is computed directly with our data. For each country and industry, we compute the average firm-level capital-labor ratio, weighted by firms' sales. Then, *K-intensity* for industry i, (K_i/L_i), is the industry-level average of these values across all countries, weighted by countries' output in industry i.

Finally, we need a measure of $\phi(\xi)$. Here, we are strongly constraint by the data. The Amadeus database provides firm-level information for a quite large number of countries and industries, which is crucial for us because our identification strategy is based on a comparison, across country-industry pairs, of the magnitude of the relationship between firm-level relative K-

[10] http://www.bvdep.com

[11] We exclude producers of tobacco, coke and refined petroleum products, and printing.

[12] We are indebted to Jean Fouré for giving us these country-level data. See Bénassy-Quéré *et al.* (2010) for a description of the source data and the methodology.

intensity and relative sales. But, unfortunately, this database contains many missing observations, so that we can rely on a very small number of variables to estimate the Hicks-neutral component of TFP. Given these restrictions, we propose a simple methodology based on the estimation of a CES production function allowing for the existence of factor-biased productivity. Letting $\widetilde{\beta}_i^c = 1\ \forall(i,c)$, Eq. (5) gives $\beta(\xi) = \left[\theta_i^c[\beta(\xi)]/\overline{\theta}_i^c\right]^{1/(\sigma-1)}$. [13] To compute $\beta(\xi)$ from the observed capital intensity, we have to pin down σ. We obtain this parameter from Eq. (2). Consistently with our framework, we assume that σ is the same for all industries, and that all firms in a given country face the same factor prices. Then, taking log and first difference (between 2006 and 2007) of Eq. (2), σ is obtained by regressing the first difference of the firm-level relative capital intensity on the first-difference of the relative input prices $\omega^c = w^c/r^c$. We compute the national manufacturing wage, w^c, by averaging at the country-level the individual wages reported in the Amadeus dataset. We take the price of investment goods from the Penn World Tables as a proxy for r^c, which is also used to deflate the firms' assets.[14] This estimation gives a σ of 1.2079, which is used to compute $\beta(\xi)$.[15] The last step consists in plugging the $\beta(\xi)$ and the estimated σ to recover the Hicks-neutral productivity from a non-linear estimation of the CES production function with factor-biased productivity. This production function is estimated for

[13] As already discussed above assuming $\widetilde{\beta}_i^c = 1\ \forall(i,c)$ leaves the results unchanged, see also section 9.2.1 of the online supplement.

[14] In order to control for possible endogeneity between the annual change in the aggregate wage and the firm-level capital intensity, and to alleviate measurement errors, we instrument the log change in the factor price ratio w^c/r^c by the initial level of this variable. In this regression, we also restrict the sample of firms to the ones that serve to compute national-level wages, i.e. the ones that are present in Amadeus in both 2006 and 2007 and report non-missing wages for these two years.

[15] The point estimate for σ is larger than one, which implies substitutability between labor and capital. However, with standard errors clustered at the country level, it is not statistically different from 1, while significantly positive. Our estimate is larger than most of the estimates reported in the existing literature, which generally concludes in favor of a complementarity between the two factors (see Chirinko, 2008 for a survey). Nevertheless, our σ is smaller than the highest estimates surveyed in Chirinko (2008). It is also consistent with the vast literature which finds evidence that sales are positively correlated with the K-intensity of the firm; this positive correlation implies a $\sigma > 1$. Lastly, $\sigma > 1$ is consistent with our findings reported below in Table 1 which shows a positive impact of relative *K-intensity* on relative sales. See Sect. 9.2.5 of the online supplement for the discussion on complementarity and substitutability of factors.

each industry separately to allow the sectoral *K-intensity* to vary across industries.

The final database is a panel of 300,864 firms in 95 industries and 21 European countries.[16] The country-industry panel is unbalanced because all of the 95 industries are not active in all countries. We have data for 1,490 country-industry pairs, for a total of 1,995 possible combinations. The average number of firms per country-industry pair is 1571, but the population within each group varies greatly. The median country-industry pair has only 796 firms, and the largest group contains 8,024 observations.[17]

6 Non-Structural Estimates

Table 1 reports estimates from our firm-level regressions corresponding to Eq. (12). All regressions include country-industry fixed effects. Column (1) regresses firms' relative *K-intensity* on firms' relative sales. The positive and very significant coefficient shows that firms with higher relative *K-intensity* have significantly higher relative sales. A firm with a *K-intensity* 10% above the country-industry mean would have relative sales that are 3.3% higher than the average firm. Column (2) introduces firms' relative ϕ. Not surprisingly, Hicks-neutral productivity has a great influence on firms' performances. The coefficient on the relative Hicks-neutral productivity is highly significant and very large in magnitude. The introduction of this variable also improves greatly the global fit of the regression, multiplying the R^2 by a factor of 5. Column (3) verifies more directly the log-supermodularity of the RS function interacting the firm-level relative *K-intensity* with the product of the relative *K-abundance* of the country and the relative *K-intensity* of the industry. The coefficient on the interaction term is significantly positive, which supports our main theoretical prediction summarized in Proposition 1: the marginal impact of firms' relative *K-intensity* on firms' relative sales is larger in *K-intensive* industries of *K-abundant* countries. Column (4) replicates the same exercise but interacting the relative *K-intensity* of the firm with dummies characterizing *K-intensive* industries and *K-abundant* countries instead of a continuous variables. We assign each firm to one of three mutually exclu-

[16] Belgium, Bosnia and Herzegovina, Bulgaria, Croatia, Czech republic, Estonia, Finland, France, Germany, Hungary, Italy, Latvia, Poland, Portugal, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden and Ukraine.

[17] Spain, "Manufacture of structural metal products".

sive groups. The first groups contains firms in countries whose *K-abundance* is above the median and in industries whose *K-intensity* is above the median. We shall refer to this sub-sample as the *KK-group*. Similarly, the *LL-group* contains firms belonging to countries with lower-than-median *K-abundance* and industries with lower-than-median *K-intensity*. The last group contains all remaining firms; these firms belong to either *K-intensive* industries of *L-abundant* countries or to *L-intensive* industries of *K-abundant* countries. The three coefficients associated to with these interaction terms are significantly positive and, more importantly, they rank as predicted by Proposition 1: higher relative capital intensity has a stronger marginal impact in the *KK-group* than on the *LL-group*.[18] Columns (5) and (6) replicate the tests shown in Column (2) on the *KK-group* and the *LL-group* separately. Here, by letting all the coefficients to vary across groups of country-industry pairs, we explicitly control for the possible influence of heterogeneity in ς across country-industry groups on the marginal effect of firms' *K-intensity* on firms' sales. The results confirm the ones shown in Columns (3) and (4). The estimated coefficient on relative *K-intensity* is significantly larger for the *KK-group* than for the *LL-group*, as expected. The coefficient on the relative Hicks neutral productivity term is also significantly larger for the KK-group. This result, which runs against Proposition 2, seems to reveal a composition effect across industries with different ς.

We now move to testing each of the two mechanisms giving rise to proposition 1. We do this in a two-steps procedure. The first step consists in estimating the non-structural equation (12) separately for each country-industry pairs and collecting the corresponding estimated coefficients on firms' relative *K-intensity*. In a second step, we test whether these estimated coefficients, $\widehat{\psi}_i^c$ vary with the industry-level *K-intensity* and the country-level *K-abundance*. According to mechanism 1, we expect $\widehat{\psi}_i^c$ to be significantly larger for *K-intensive* industries. Similarly, mechanism 2 suggests a positive relationship between $\widehat{\psi}_i^c$ and countries' *K-abundance*.

Estimates of Eq. (12) for each of the 1,490 country-industry pairs are quite robust. Only 13 country-industry pairs (0.9 percent) show an unexpected significantly negative coefficient $\widehat{\psi}_i^c$. In 594 cases (39.9 percent), the coefficient is not significantly different from zero at the 10% level (130 are

[18] The coefficient for firms in the *No KK - No LL* group should lie in between the other two coefficients. The point estimate of this coefficient does but the confidence interval overlaps with that of the *LL-group*.

Table 1: Impact of relative Hicks-neutral productivity and K-intensity on relative sales: non-structural log-linear model

	(1)	(2)	(3)	(4)	(5)	(6)
Dependant variable: ln firms' relative sales, $\ln(RS_i^c(\xi))$						
Countries	All	All	All	All	K-abund.	L-abund.
Industries	All	All	All	All	K-intens.	L-intens.
$\ln\varphi(\xi)$		1.0769^a	1.0484^a	1.0766^a	1.1258^a	0.9701^a
		(0.011)	(0.015)	(0.011)	(0.017)	(0.029)
$\ln\kappa(\xi)$	0.3329^a	0.2064^a	0.1562^a		0.2682^a	0.1660^a
	(0.009)	(0.008)	(0.010)		(0.011)	(0.017)
$\ln\kappa(\xi)\times$ K-intensity			0.0462^a			
$\times$ K-abundance			(0.005)			
$\ln\kappa(\xi)\times$				0.2754^a		
KK-group				(0.011)		
$\ln\kappa(\xi)\times$				0.183^a		
No KK - No LL				(0.010)		
$\ln\kappa(\xi)\times$				0.1537^a		
LL-group				(0.017)		
Observations	300864	300864	300864	300864	89635	34048
F-test ($\ln\varphi(\xi)$)					21.98	
F-test ($\ln\kappa(\xi)$)					25.79	
R^2	0.077	0.389	0.391	0.391	0.406	0.365

Notes: $\varphi(\xi)$ is the Hicks-neutral productivity of firm ξ relative to the country-industry average. $\kappa(\xi)$ is the K-intensity of firm ξ relative to the country-industry average. Country-Industry fixed effects for all columns. Robust standard errors adjusted for country-industry clusters in parentheses. Within R^2 are reported. Significance levels: ^{a} $p<0.01$. The F-tests test the equality of the estimated coefficients on the relative Hick-neutral productivity and the relative *K-intensity* reported in Columns (5) and (6).

negative, and 473 are positive). Finally, we obtain significantly positive coefficients for a majority of country-industry pairs (883 cases, representing 59 percent of the country-industry pairs).[19]

The econometric tests of the two mechanisms are shown in Table 2. The top half of the table tests the validity of mechanism 1. Here, the estimated slope of the relationship between firms' relative *K-intensity* and firms' relative sales, $\widehat{\psi}_i^c$, is regressed on industry-level capital intensity and country fixed effects. Columns (1) and (2) show regressions performed using all $\widehat{\psi}_i^c$. Columns (3) and (4) retain only significantly positive $\widehat{\psi}_i^c$. Because our dependant variable is an estimated parameter, our standard errors are likely to be affected by heteroschedasticity. We fix this problem in Columns (2) and (4). These columns report weighted least squares estimates with weights equal to the inverse of the standard error reported for each $\widehat{\psi}_i^c$ (Saxonhouse 1976). The positive coefficients shown in all columns of the top panel explicitly validate Mechanism 1 of Proposition 1. They confirm that, in a given country, the positive impact of relative *K-intensity* on relative sales is stronger the higher the *K-intensity* of the industries. Empirical tests of Mechanism 2 of Proposition 1 are shown in the bottom half of Table 2. Here, we regress $\widehat{\psi}_i^c$ on countries' *K-abundance* and industry fixed effects. Again, the positive coefficient on *K-abundance* in all columns supports Mechanism 2 of Proposition 1. In a given industry, the positive impact of relative *K-intensity* on relative sales is stronger the higher the *K-abundance* of the country.

7 Structural Estimates

In this section, we present the results of the structural estimation of our model, based on Eq. (11).

Again, the dependant variable, $\ln RS(\xi)$, is the log of firms' total sales relative to the corresponding country-industry average. The right-hand side variables are the Hicks-neutral productivity and the capital intensity of this firm, both relative to the country-industry averages. The parameters of this equation are ς, σ and a_i^c. Equation (11) can be estimated with non-linear least squares. Since a_i^c and σ cannot be identified independently, we impose $\sigma = 1.2079$, which is the estimated value we have used to compute firms'

[19] The coefficients $\widehat{\psi_i^c}$ range between -0.525 and 2.599, with a mean of 0.30 and a median of 0.26.

Table 2: Verification of Mechanisms 1 and 2 of Proposition 1

Dependant Variable: $\widehat{\psi}_i^c$				
Test of Mechanism 1				
	(1)	(2)	(3)	(4)
Industry K-intensity	2.118^a	2.759^a	3.050^a	3.347^a
	(0.386)	(0.526)	(0.334)	(0.329)
Observations	1490	1490	771	771
R^2	0.868	0.868	0.941	0.941
Fixed effects	Country			
Test of Mechanism 2				
	(1)	(2)	(3)	(4)
Country K-abundance	2.751^a	2.357^a	2.551^a	2.315^a
	(0.359)	(0.160)	(0.335)	(0.146)
Observations	1490	1490	883	883
R^2	0.920	0.940	0.927	0.954
Fixed effects	Industry			

Notes: Robust standard errors in parentheses. Significance levels: ^{a} $p < 0.01$. Overall R^2 are reported. Regressions in Columns (2) and (4) are performed with weight = 1/s.e.($\widehat{\psi}_i^c$). Regressions in Columns (1) and (3) are unweighted. Columns (3) and (4) retain only significantly positive $\widehat{\psi}_i^c$.

Hicks-Neutral productivity.

The model requires $a_i^c > 0$ and $\varsigma > 1$. More importantly, according to Proposition 1, a_i^c should be bigger for *K-abundant* countries and *K-intensive* industries than for *L-abundant* countries and *L-intensive* industries.

Estimation results are shown in Table 3. Column (1) reports the estimates for all firms in our sample. In Column (2) we impose the same ς for all countries-industry pairs but we interact $\kappa(\xi)$ in Eq. (11) with dummies characterizing whether the countries-industry pairs belong to the *KK-group*, the *LL-group*, or none of them. Columns (3) and (4) retain countries-industry pairs in the *KK-group* and the *LL-group* respectively, letting ς to vary across groups of country-industry pairs. All estimates of ς are statistically strictly larger than one. They vary between 1.965 and 2.301. These values of ς are relatively small compared to other estimates in the literature. For instance, Anderson and Van Wincoop (2004), surveying several empirical trade analyses, consider that a reasonable range for the elasticity of substitution between

Table 3: Impact of relative Hicks-neutral productivity and *K-intensity* on relative sales: structural estimates of Eq. (11) - Sample 1

	Dependant variable: ln firms' relative sales ($\ln(RS_i^c(\xi))$)			
Countries	All	All	K-abundant	L-abundant
Industries	All	All	K-intensive	L-intensive
	(1)	(2)	(3)	(4)
ς	2.231^a	2.220^a	2.301^a	1.965^a
	(0.047)	(0.045)	(0.084)	(0.053)
a_i^c	0.025^a		0.055^a	0.013^a
	(0.002)		(0.006)	(0.002)
a_i^c		0.064^a		
$\times$ *KK-group*		(0.009)		
a_i^c		0.022^a		
$\times$ *No KK- No LL*		(0.002)		
a_i^c		0.011^a		
$\times$ *LL group*		(0.001)		
R^2	0.369	0.373	0.386	0.379
Observations	300864	300864	89635	34048

Notes: Equation (11). Non-linear least squared. Starting values: $a_i^c = 1$ and $\varsigma = 2$. Robust standard errors adjusted for country-industry clusters in parentheses. Significance level: ^{a} $p < 0.01$.

varieties in a CES utility function is between 5 and 10. But our results are in line with Broda and Weinstein (2006) who report a median value for this parameter of 2.2 when they conduct their estimates using a 3-digit product classification. Our result are also reasonably close to Imbs and Méjean (2012) who find a values slightly above 3 when they force the elasticities to be equal across sectors, as we do. As in Table 1, this parameter is significantly larger for country-industry pairs in the *KK-group* than for the ones in the *LL-group*, which contradicts Proposition 2.[20]

Turning to the heart of the matter, the values of a_i^c, estimated for each groups of country-industry pairs are significantly different from each other, and can be ranked strictly. The smallest parameter is obtained with the *LL-group* and the largest with the *KK-group*. Furthermore, comparing Columns (4), (3) and (1) we see that the estimates obtained using all firms consistently lies between the estimates of the *KK-* and *LL-group*. These results give strong support to Proposition 1.

8 Conclusion.

In this paper we have shown that the comparative advantage of countries gives rise to the comparative advantage of firms. Two otherwise identical firms (twin firms) have different relative sales if they belong to different industries or countries. This result is due to two distinct mechanisms: the interaction between relative factor intensity and industry technology and between relative factor intensity and factors endowment. These results do not require any assumption about the direction of the technology bias (if any), or about the relationship between productivity and factor intensity.

We have verified empirically the predictions of the model using firm-level data using two different samples of the same dataset. All predictions are confirmed by the empirical evidence: the comparative advantage of a firm is found to be related to the comparative advantage of the country. Each constitutive mechanism of this relationship also finds support in the data. These results contribute to the literature in two ways: they provide the first firm-level verification of the HO model and show that country comparative advantage begets a comparative advantage at firm level. Undoubtedly,

[20] Note that these coefficients are very close to the estimates reported in Columns (5) and (6) of table 1 where, according to Eq. 12, the coefficient on $\ln \varphi(\xi)$ should be equal to $(\varsigma - 1)$.

Heckscher-Ohlin mechanisms not only shape international specialization but also influence firm-level performance.

Firms, countries, industries, these are words that go together well. Our work has studied the theoretical and empirical interactions between these three protagonists of international trade

References

[1] Amiti, Mary, and Donald R. Davis (2011), "Trade, Firms, and Wages: Theory and Evidence," *Review of Economic Studies*, 79, 1-36.

[2] Anderson, James E. and Eric Van Wincoop (2004), "Trade costs," *Journal of Economic Literature* 42(3), 691-751.

[3] Arkolakis, Costas, Arnaud Costinot and Clare Andrés Rodríguez (2012), "New Trade Models, Same Old Gains?," *American Economic Review* 102(1): 94-130,

[4] Bénassy-Quéré, Agnès, Lionel Fontagné and Jean Fouré (2010), "The world economy in 2050: a tentative picture," *CEPII Working Paper* 2010-27.

[5] Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen and Samuel Kortum (2003), "Plants and Productivity in International Trade," *American Economic Review* 93, 1268-1290.

[6] Bernard, Andrew B., J. Bradford Jensen, Stephen J. Redding and Peter K. Schott, (2007), "Firms in International Trade," *Journal of Economic Perspectives* 21(3), 105-130.

[7] Bernard, Andrew B., Stephen J. Redding and Peter K. Schott (2007), "Comparative Advantage and Heterogenous Firms," *Review of Economic Studies* 74, 31-66.

[8] Broda, Christian and David E. Weinstein (2006), "Globalization and the Gains from Variety," *Quarterly Journal of Economics* 121(2), 541-85.

[9] Burstein, Ariel and Jonathan Vogel (2012), "International Trade, Technology, and the Skill Premium" mimeo.

[10] Bustos, Paula (2011), "Trade Liberalization, Exports and Technology Upgrading: Evidence on the impact of MERCOSUR on Argentinean Firms," *American Economic Review* 101 (1), 304-340.

[11] Chirinko, Robert (2008), "σ, The long and short of it," *Journal of Macroeconomics* 30(2), 671-686.

[12] Chaney, Thomas (2008), "Distorted Gravity: Heterogeneous Firms, Market Structure, and the Geography of International Trade," *American Economic Review* 98, 1707-1721.

[13] Costinot, Arnaud, and Jonathan Vogel (2010), "Matching and Inequality in the World Economy," *Journal of Political Economy* 118(4), 747-786.

[14] Crozet Matthieu, Keith Head and Thierry Mayer (2012), "Quality sorting and trade: Firm-level evidence for French wine," *Review of Economic Studies*, 79(2), 609-644..

[15] Davis, Donald and James Harrigan (2011), "Good Jobs, Bad Jobs, and Trade Liberalization," *Journal of International Economics* 84, 26-36.

[16] Davis, Donald and David E. Weinstein (2001), "An Account of Global Factor Trade," *American Economic Review* 91(5),1423-1453.

[17] Eaton, Jonathan, Samuel Kortum and Francis Kramarz (2011), "An Anatomy of International Trade: Evidence from French Firms," *Econometrica* 75(5), 1453-1498.

[18] Egger, Hartmut and Udo Kreickemeier (2009), "Firm Heterogeneity and the Labor Market Effects of Trade Liberalization," *International Economic Review* 50(1), 187-216.

[19] Ethier, Wilfred J. (1982) "National and International Returns to Scale and the Modern Theory of International Trade", *American Economic Review* 72(3):389-405.

[20] Harrigan James and Ariell Reshef (2012) "Skill biased heterogenous firms, trade liberalization, and the skill premium" First version (2011) in NBER Working Paper 17604.

[21] Helpman, Elhanan, Oleg Itskhoki and Stephen Redding (2010), "Inequality and Unemployment in a Global Economy," *Econometrica* 78(4), 1239–1283.

[22] Imbs, Jean and Isabelle Méjean (2012), "Elasticity Optimism," *mimeo*.

[23] Leamer, Edward E. (1980), "The Leontief Paradox Reconsidered," *Journal of Political Economy* 88, 495-503.

[24] Manasse Paolo and Alessandro Turrini (2001), "Trade, wages, and 'superstars'," *Journal of International Economics* 54, 97-117.

[25] Melitz, Marc J. (2003), "The impact of trade on intra-industry reallocations and aggregate industry productivity," *Econometrica* 71(6), 1695-1725.

[26] Melitz, Marc and Gianmarco Ottaviano (2008), "Market Size, Trade, and Productivity," *Review of Economic Studies* 75, 295-316.

[27] Saxonhouse, Gary R. (1976), "Estimated Parameters as Dependent Variables," *American Economic Review* 66(1), 178-183.

[28] Romalis, John (2004), "Factor Proportions and the Structure of Commodity Trade," *American Economic Review* 91(1), 67-97.

[29] Trefler Daniel (1993), "International Factor Price Differences: Leontief Was Right!," *Journal of Political Economy* 101, 961-987.

[30] Trefler, Daniel (1995), "The Case of the Missing Trade and Other Mysteries," *American Economic Review* 85(5), 1029-1046.

[31] Vannoorenberghe Gonzague (2011) "Trade between symmetric countries, heterogeneous firms and the skill premium", *Canadian Journal of Economics*, vol.44(1): 148-170.

[32] Yeaple, Stephen R. (2005), "A simple model of firm heterogeneity, international trade, and wages," *Journal of International Economics* 65, 1-20.

9 Appendix to "Firm-Level Comparative Advantage" by Matthieu Crozet and Federico Trionfetti

We describe the model in detail and provide analytical and numerical results.

9.1 The Model

Demand. Preferences are represented by a Cobb-Douglas index, with shares $\gamma_i \in (0,1)$, $\gamma_Y + \gamma_Z = 1$, defined over CES aggregates whose elasticity of substitution between varieties is $\varsigma > 1$. The demand function emanating from domestic residents, $s_{id}^H(\beta,\phi)$, and from foreign residents, $s_{ix}^H(\beta,\phi)$, for the output of a firm in industry i of country H is:

$$s_{id}^H(\beta,\phi) = \left(\frac{p_{id}^H}{P_i^H}\right)^{1-\varsigma}\gamma_i I^H; \qquad s_{ix}^H(\beta,\phi) = \left(\frac{p_{ix}^H}{P_i^F}\right)^{1-\varsigma}\gamma_i I^F, \tag{13}$$

where s stands for sales, d for domestic, and x for foreign; p_{id}^H and p_{ix}^H are the price faced by consumers, P_i^c are the price indices and $I^c = w^c\nu_L^c\overline{L} + r^c\nu_K^c\overline{K}$ is national income. Analogous demand functions obtain for the output of a firm in industry i of country F. For any two firms with draws β' and β'' and with realizations ϕ' and ϕ'', the relative sales in the same market at any point in time are

$$\frac{s_{im}^c(\phi',\beta')}{s_{im}^c(\phi'',\beta'')} = \left[\frac{mc_i^c(\phi',\beta')}{mc_i^c(\phi'',\beta'')}\right]^{1-\varsigma}, \qquad m = d, x. \tag{14}$$

Production. With monopolistic competition and under the large-group assumption, the profit-maximizing prices for the domestic and the foreign market are:

$$p_{id}^c(\beta,\phi) = \frac{\varsigma}{\varsigma-1}mc_i^c(\beta,\phi); \qquad p_{ix}^c(\beta,\phi) = \frac{\varsigma}{\varsigma-1}\frac{1}{\tau_i}mc_i^c(\beta,\phi). \tag{15}$$

The second expression in (15) takes into account iceberg transport costs: for one unit of good shipped, only a fraction $\tau_i \in [0,1]$ arrives at its destination. Profits in the domestic and foreign market are, respectively, $\pi_{id}^c(\beta,\phi) = s_{id}^c(\beta,\phi)/\varsigma - F_i\widetilde{mc}_i^c$ and $\pi_{ix}^c(\beta,\phi) = s_{ix}^c(\beta,\phi)/\varsigma - F_{ix}\widetilde{mc}_i^c$. Expected profits after drawing β are equal to $\pi_{id}^c\left(\beta,\widetilde{\phi}\right)$ and $\pi_{ix}^c\left(\beta,\widetilde{\phi}\right)$. After drawing

β a firm decides to stay in the market if $\pi_{id}^{c}\left(\beta,\widetilde{\phi}\right)\geqslant 0$ and decides to export if $\pi_{ix}^{c}\left(\beta,\widetilde{\phi}\right)\geqslant 0$. Thus, the zero expected profit conditions require the expected sales of the cut off firm be equal to the fixed cost,[21]

$$s_{id}^{c}\left(\beta_{i}^{*c},\widetilde{\phi}\right)=\varsigma F_{i}\widetilde{mc}_{i}^{c},\qquad s_{ix}^{c}\left(\beta_{ix}^{*c},\widetilde{\phi}\right)=\varsigma F_{ix}\widetilde{mc}_{i}^{c}.\tag{16}$$

Aggregation. In addition to the average marginal productivity in the industry denoted $\widetilde{mc}_{i}^{c}$ and defined already in the main text as

$\widetilde{mc}_{i}^{c}=\frac{1}{\widetilde{\phi}}\left[\left(\lambda_{i}\right)^{\sigma}\left(w^{c}\right)^{1-\sigma}+\left(1-\lambda_{i}\right)^{\sigma}\left(r^{c}\right)^{1-\sigma}\left(\widetilde{\beta}_{i}^{c}\right)^{\sigma-1}\right]^{\frac{1}{1-\sigma}}$

we make use of the average marginal productivity of exporting firms, $\widetilde{mc}_{ix}^{c}$, computed analogously to $\widetilde{mc}_{i}^{c}$. Given the profit-maximizing prices (15), the average price and the average export price are, respectively:

$$\widetilde{p}_{id}^{c}=\frac{\varsigma}{\varsigma-1}\widetilde{mc}_{i}^{c};\qquad \widetilde{p}_{ix}^{c}=\frac{\varsigma}{\varsigma-1}\frac{1}{\tau_{i}}\widetilde{mc}_{ix}^{c},\tag{17}$$

and the price indices are:

$$P_{i}^{H}=\left[M_{i}^{H}\left(\widetilde{p}_{id}^{H}\right)^{1-\varsigma}+\chi_{i}^{F}M_{i}^{F}\left(\widetilde{p}_{ix}^{H}\right)^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}},\tag{18}$$

$$P_{i}^{F}=\left[M_{i}^{F}\left(\widetilde{p}_{id}^{F}\right)^{1-\varsigma}+\chi_{i}^{H}M_{i}^{H}\left(\widetilde{p}_{ix}^{F}\right)^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}},\tag{19}$$

where M_{i}^{c} is the mass of firms and $\chi_{i}^{c}\equiv\frac{1-G(\beta_{ix}^{*c})}{1-G(\beta_{i}^{*c})}$ is the ex-ante probability of exporting, conditional to successful entry. Let $\underline{\phi}$ be the least value of the support set of ϕ. Then, applying equations (14) and (16) to $s_{id}^{c}\left(\underline{\phi},\beta_{i}^{*c}\right)/(s_{id}^{c}\left(\widetilde{\phi},\beta_{i}^{*c}\right)$ and to $s_{id}^{c}\left(\phi,\beta\right)/s\left(\underline{\phi},\beta_{i}^{*c}\right)$ gives

$$s_{id}^{c}\left(\phi,\beta\right)=\left[\frac{mc_{i}^{c}\left(\phi,\beta\right)}{mc_{i}^{c}\left(\underline{\phi},\beta_{i}^{*c}\right)}\right]^{1-\varsigma}\left(\frac{\widetilde{\phi}}{\underline{\phi}}\right)^{1-\varsigma}\varsigma F_{i}\widetilde{mc}_{i}^{c},\tag{20}$$

[21] We could alternatively assume that the firm stays only if the profit is non negative with certainty. Then the zero cut off profit condition would be $s_{id}^{c}\left(\beta_{i}^{*c},\underline{\phi}\right)=\varsigma F_{i}\widetilde{mc}_{i}^{c}$, where $\underline{\phi}$ is the least value of the of the support set of ϕ. Results would remain unchanged.

and analogously for $s_{ix}^{c}(\phi, \beta)$. The average sales across firms in the same industry and country is

$$\overline{s}_{i}^{c} = \int_{0}^{\infty}\int_{0}^{\infty} s_{id}^{c}(\phi, \beta)\, dGdV + \chi_{i}^{c} \int_{0}^{\infty}\int_{0}^{\infty} s_{ix}^{c}(\phi, \beta)\, dGdV. \tag{21}$$

Note that $\overline{s}_{i}^{c}$ is also the expected sale of a firm prior to entry. Denoting the first and second addendum of (21), respectively, $\overline{s}_{id}^{c}$ and $\overline{s}_{ix}^{c}$, the value of expected future profit prior to drawing β is

$$\overline{\pi}_{i}^{c} = \left[\frac{\overline{s}_{id}^{c}}{\varsigma} - F_{i}\widetilde{mc}_{i}^{c}\right] + \chi_{i}^{c}\left[\frac{\overline{s}_{ix}^{c}}{\varsigma} - F_{ix}\widetilde{mc}_{i}^{c}\right]. \tag{22}$$

which is also the average profit across firms.

Equilibrium. In addition to profit-maximizing prices and to the zero profit conditions discussed above, there are five additional sets of equilibrium conditions. First, stationarity of the equilibrium requires the mass of potential entrants, M_{ei}^{c}, to be such that at any instant the mass of successful entrants, $[1 - G(\beta_{i}^{*})]\, M_{ei}^{c}$ equals the mass of incumbent firms who die, δM_{i}^{c}:

$$[1 - G(\beta_{i}^{*})]\, M_{ei}^{c} = \delta M_{i}^{c}, \qquad c = H, F \text{ and } i = Y, Z. \tag{23}$$

Second, the presence of an infinity of potential entrants arbitrages away any possible divergence between the expected value of entry and entry cost. Therefore, the free entry condition, is:

$$[1 - G(\beta_{i}^{*c})]\, \overline{\pi}_{i}^{c}/\delta = F_{ei}\widetilde{mc}_{i}^{c}; \quad i = Y, Z; \;\; c = H, F. \tag{24}$$

The left-hand-side is the present value of the expected profit stream until death multiplied by the probability of successful entry, and the right-hand-side is the entry cost. Third, replacing (17) into (13) gives average demands as functions of average prices, $s_{id}^{c}(\widetilde{p}_{id}^{c})$ and $s_{ix}^{c}(\widetilde{p}_{ix}^{c})$, which allows writing the goods markets equilibrium as

$$\overline{s}_{i}^{c} = s_{id}^{c}(\widetilde{p}_{id}^{c}) + \chi_{i}^{c} s_{ix}^{c}(\widetilde{p}_{ix}^{c}); \quad i = Y, Z; \;\; c = H, F. \tag{25}$$

Fourth, the optimal relationship between foreign and domestic sales is

$$\frac{mc_i^H(\beta_{ix}^*)}{mc_i^H(\beta_i^*)} = \left[\tau_i^{\varsigma-1}\left(\frac{P_i^F}{P_i^H}\right)^{\varsigma-1}\frac{I^F}{I^H}\frac{F_i}{F_{ix}}\right]^{\frac{1}{\varsigma-1}}; \quad i = Y, Z. \tag{26}$$

$$\frac{mc_i^F(\beta_{ix}^*)}{mc_i^F(\beta_i^*)} = \left[\tau_i^{\varsigma-1}\left(\frac{P_i^H}{P_i^F}\right)^{\varsigma-1}\frac{I^H}{I^F}\frac{F_i}{F_{ix}}\right]^{\frac{1}{\varsigma-1}}; \quad i = Y, Z. \tag{27}$$

Fifth, equilibrium in factor markets requires that factor demand inclusive of all fixed factors inputs, denoted L_i^c and K_i^c, be equal to factor supply

$$L_Y^c + L_Z^c = \nu_L^c \overline{L}; \qquad c = H, F. \tag{28}$$

$$K_Y^c + K_Z^c = \nu_k^c \overline{K}; \qquad c = H, F. \tag{29}$$

After replacing equations (17), (18)-(22) into (24)-(29) the model counts 15 independent equilibrium conditions and 16 endogenous variables. The equilibrium conditions are the four free-entry conditions (24), any three out of the four goods market equilibrium conditions (25), the four relationships between foreign and domestic sales (26)-(27), and the four factor market equilibrium (28)-(29). The endogenous are $\{\beta_i^{*c}\}$, $\{\beta_{ix}^{*c}\}$, $\{w^c, r^c\}$ and $\{M_i^c\}$. The equilibrium value of all other endogenous variables can be computed from these. The choice of a numéraire makes the model determined.

9.2 Analytical Results

In this section $F_{ix} = 0$, which implies $\chi_i^c = 1$. Furthermore, to isolate the effect of comparative advantage we eliminate any cross-industry differences in fixed cost and trade cost: i.e., $F_i = F$, $F_{ei} = F_e$, and $\tau_i = \tau$ for $i = Y, Z$. We begin by ranking the cut off values. Then, we use the ranking to prove Theorem 1, Proposition 1 and its two underlying mechanisms.

9.2.1 Ranking of cut-off values.

Replacing expressions (21) into (22) and then the latter into (24), for each country and industry we obtain a single condition which combines the free entry and the zero cut-off profit conditions:

$$\underbrace{\int_{\beta_i^{*c}}^{\infty}\left\{\left[\frac{1+(\Lambda_i)^{\sigma}(\omega^c)^{\sigma-1}(\beta)^{\sigma-1}}{1+(\Lambda_i)^{\sigma}(\omega^c)^{\sigma-1}(\beta_i^{*c})^{\sigma-1}}\right]^{\frac{1-\varsigma}{1-\sigma}}-1\right\}dG}_{\Upsilon_i^c(\beta_i^{*c},\Lambda_i,\omega^c)}=\delta\frac{F_e}{F},\quad\forall c,i. \tag{30}$$

Defining the left-hand side of equation (30) as $\Upsilon_i^c(\beta_i^{*c},\Lambda_i,\omega^c)$ will save space below. For later use note that $\Upsilon_i^c(\beta_i^{*c},\Lambda_i,\omega^c)$ is declining in β_i^{*c} for any $\sigma>0$; and $d\left(\widetilde{\beta}_i^c\right)^{\sigma-1}/d\beta_i^{*c}\gtrless 0$ as $\sigma\gtrless 1$. The four equations (30) allow establishing the following lemmas

Lemma 1 *Within a country, the K-intensive industry has the highest average value of β. In our notation:*

$$\left(\widetilde{\beta}_Y^c\right)^{\sigma-1}\gtrless\left(\widetilde{\beta}_Z^c\right)^{\sigma-1}\qquad\forall\sigma\gtrless 1,\ \forall\tau\in[0,1]\ . \tag{31}$$

Proof. Note that $\Upsilon_i^c(\beta_i^{*c},\Lambda_i,\omega^c)$ is increasing in Λ_i for any $\sigma>0$. Therefore the equations $\Upsilon_i^c(\beta_i^{*c},\Lambda_i,\omega^c)=\delta F_e/F$ holds for $i=Y,Z$ if and only if $\beta_Y^{*c}>\beta_Z^{*c}$. Then, recalling that $d\left(\widetilde{\beta}_i^c\right)^{\sigma-1}/d\beta_i^{*c}\gtrless 0$ as $\sigma\gtrless 1$ proofs the lemma. ■

Lemma 2 *Except under free trade, the K-abundant country has a higher zero-profit productivity cut-off in both industries. Furthermore, each cut-off value of the K-abundant country is larger in costly trade than in free trade, whereas each cut-off value of the L-abundant country is smaller in free trade than in autarky. In our notation:*

$$\left(\widetilde{\beta}_i^{*H}\right)^{\sigma-1}_{Costly\ Trade}\geqslant\left(\widetilde{\beta}_i^{*}\right)^{\sigma-1}_{Free\ Trade}\geqslant\left(\widetilde{\beta}_i^{*F}\right)^{\sigma-1}_{Costly\ Trade}\quad\forall i,\ and\ \forall\sigma>0, \tag{32}$$

with equality holding only in free trade.

Proof. Note that in free trade $\omega^H=\omega^F\equiv\omega$. Therefore, in free trade the equations $\Upsilon_i^c(\beta_i^{*c},\Lambda_i,\omega^c)=\delta F_e/F$ imply $\beta_i^{*H}=\beta_i^{*F}\equiv(\beta_i^*)_{Free\ Trade}$. In costly trade instead $\omega^H>\omega^F$since H is *K-abundant*.[22]. $\Upsilon_i^c(\beta_i^{*c},\Lambda_i,\omega^c)$

[22] This is intuitive but we discuss it further in Sect. 9.2.4.

is increasing in ω^c if $\sigma > 1$ and decreasing in ω^c if $\sigma < 1$. Therefore, $\Upsilon_i^c\left(\beta_i^{*c}, \Lambda_i, \omega^c\right) = \delta F_e/F$ holds for $c = H, F$ if and only if $\beta_i^{*H} \gtrless \beta_i^{*F}, \ \sigma \gtrless 1$. Recalling that recalling that $d\left(\widetilde{\beta}_i^c\right)^{\sigma-1}/d\beta_i^{*c} \gtrless 0$ as $\sigma \gtrless 1$ proofs the lemma. ■

9.2.2 Proof of Theorem 1

Using (14) it is clear that proving Theorem 1 is equivalent to proving Proposition 1 since $\varsigma > 1$.

9.2.3 Proof of Proposition 1 and its constitutive mechanisms

In this subsection we continue to assume $\sigma > 1$.

Proposition 1. To set the appropriate condition for log-supermodularity we need first to rank a_Y^H and a_Z^F. This is done by use of Lemmas 1 and 2, which allow establishing that $a_Y^H > a_Z^F$. Then, RS is strictly log-supermodular in (a, κ) iff

$$RS\left(a_Y^H, \kappa''\right)/RS\left({}_Y^H, \kappa'\right) > RS\left(a_Z^F, \kappa''\right)/RS\left(a_Z^F, \kappa'\right) \text{ for any } \kappa'' > \kappa'. \quad (33)$$

which, using (14), becomes

$$\left(\frac{\Lambda_Y}{\Lambda_Z}\right)^{\sigma} \left(\frac{\omega^H}{\omega^F}\right)^{\sigma-1} \left(\frac{\widetilde{\beta}_Y^H}{\widetilde{\beta}_Z^F}\right)^{\sigma-1} > 1. \quad (34)$$

Condition (34) is satisfied since $\widetilde{\beta}_Y^H > \widetilde{\beta}_Z^F$ from Lemma 1, $\left(\omega^H/\omega^F\right) > 1$ (H is *K-abundant*), and $\Lambda_Y > \Lambda_Z$ (Y is $K - intensive$). The inequalities $RS\left(a_Y^H, \varphi, \kappa\right) \gtreqless RS\left(a_Z^F, \varphi, \kappa\right)$ as $\kappa \gtreqless 1$ stated in Proposition 1 follow immediately from log-supermodularity noticing that $RS\left(a, \varphi, 1\right) = \varphi^{\sigma-1}$.

Mechanism 1 The condition $RS\left(a_Y^c, \varphi, \kappa\right) \gtreqless RS\left(a_Z^c, \varphi, \kappa\right)$ *as* $\kappa \gtreqless 1$, for any φ. may be written as

$$\left(\frac{\Lambda_Y}{\Lambda_Z}\right)^{\sigma} \left(\frac{\widetilde{\beta}_Y^c}{\widetilde{\beta}_Z^c}\right)^{\sigma-1} > 1, \quad (35)$$

which is satisfied since $\widetilde{\beta}_Y^C > \widetilde{\beta}_Z^C$ from Lemma 1 and $\Lambda_Y > \Lambda_Z$ since Y is *K-intensive.*

Mechanism 2 The condition $RS\left(a_i^H, \varphi, \kappa\right) \gtreqless RS\left(a_i^F, \varphi, \kappa\right)$ *as* $\kappa \gtreqless 1$, for any φ. may be written as

$$\left(\frac{\omega^H}{\omega^F}\right)^{\sigma-1}\left(\frac{\widetilde{\beta}_i^H}{\widetilde{\beta}_i^F}\right)^{\sigma-1} > 1, \tag{36}$$

which is satisfied since $\widetilde{\beta}_i^H > \widetilde{\beta}_i^F$ from Lemma 2 and, in costly trade, $\omega^H > \omega^F$.

9.2.4 Factor price and factor abundance.

In the proof of Lemma 2 we have claimed that in costly trade

$$\nu_K^H \gtrless \nu_L^H \Rightarrow \omega^H \gtrless \omega^F. \tag{37}$$

We show that this is indeed the case by the following thought experiment. Assume that factors price equalized in costly trade. Then, from (30) we have $\beta_i^{*H} = \beta_i^{*F}$, $\forall i$. Therefore,

$$mc_i^{*H} = mc_i^{*F} \Rightarrow \widetilde{mc}_i^H = \widetilde{mc}_i^F \Rightarrow \widetilde{p}_{di}^H = \widetilde{p}_{di}^F \Rightarrow \overline{s}_i^H = \overline{s}_i^F. \tag{38}$$

Goods markets equilibrium equations (25) become

$$\overline{s}_i^H = \frac{\gamma_i I^H}{M_i^H + \tau^{\varsigma-1} M_i^F} + \frac{\tau^{\varsigma-1}\gamma_i I^F}{\tau^{\varsigma-1} M_i^H + M_i^F} \quad \forall i, \tag{39}$$

$$\overline{s}_i^F = \frac{\tau^{\varsigma-1}\gamma_i I^H}{M_i^H + \tau^{\varsigma-1} M_i^F} + \frac{\gamma_i I^F}{\tau^{\varsigma-1} M_i^H + M_i^F} \quad \forall i, \tag{40}$$

were the left-hand side is the same for both equations under the assumption of FPE. Let $\widetilde{L}_i^c$ and $\widetilde{K}_i^c$ be average factors demand in each country and industry. Equilibrium in factor markets is

$$\left(\widetilde{L}_Y M_Y^c + \widetilde{L}_Z M_Z^c\right) = v_L^c \bar{L}; \qquad c = H, F. \tag{41}$$

$$\left(\widetilde{K}_Y M_Y^c + \widetilde{K}_Z M_Z^c\right) = v_K^c \bar{K}; \qquad c = H, F. \tag{42}$$

Note that Shepherd's lemma implies that if the marginal costs are the same, so are factors demands; therefore $\widetilde{L}_i^H = \widetilde{L}_i^F = \widetilde{L}_i$ and $\widetilde{K}_i^H = \widetilde{K}_i^F = \widetilde{K}_i$. Using (38) in equations (39)-(40) and solving for the masses gives $M_Y^H / M_Z^H =$

M_Y^F/M_Z^F. This solution is inconsistent with equilibrium in the factor markets. Indeed, $M_Y^H/M_Z^H = M_Y^F/M_Z^F$ implies from (41)-(42) that the relative demand for L is the same in both countries, which cannot satisfy the equilibrium given the differences in relative endowments. Therefore, $\omega^H = \omega^F$ is inconsistent with equilibrium in all markets. In which direction should factors price move to assure equilibrium in all markets? This is easily answered by inspection of equations (41)-(42). If $\omega^H = \omega^F$, then relative demand for L falls short of relative endowment in F and exceeds relative endowment in H. Therefore, factors price must move in a way that $\omega^H/\omega^F > 1$. This change in factor price will make all industries become more *K-intensive* in H and less *K-intensive* in F thus pushing towards the equilibrium in factors markets. Naturally, a change in factors price alone is not sufficient to assure equilibrium. In fact, an increase in ω^H/ω^F pushes marginal costs in different directions in different countries and industries, further the change in factor prices gives rise to the inequality in (32); it can be easily shown by inspection of (39)-(40) that the change in factors price require $M_Y^H/M_Z^H > M_Y^F/M_Z^F$ for the goods market equilibrium to be satisfied. Thus, a costly trade equilibrium is necessarily one in which $\omega^H > \omega^F$ and $M_Y^H/M_Z^H > M_Y^F/M_Z^F$, which is the canonical Heckscher-Ohlin outcome and it occurs in our model for exactly the same reasons as in Heckscher-Ohlin. After all, this is intuitive since our model structure does not violate any of the key assumptions of the Heckscher-Ohlin model.

9.2.5 Substitutability and complementarity of factors

In the theoretical part of the paper we assumed that factors are gross substitutes ($\sigma > 1$). Empirical estimates of σ confirmed that this assumption is tenable. Factor gross substitutability is sufficient but not necessary for the results. Indeed, using (31) and (32) shows that assuming $\sigma < 1$ does not necessarily invalidate inequalities (34)-(36). Therefore, gross complementarity of factors is perfectly compatible with Theorem 1 and Proposition 1. Furthermore, if $\sigma < 1$ resulted in an invalidation of inequality (34) then necessarily RS would be strictly log-submodular. This means that the comparative advantage of countries begets a comparative advantage at firm-level though in the opposite direction with respect to that stated in Proposition 1. Empirical investigation coherently with the model's parameters indicates the direction of the relationship between firms' and countries' comparative advantage.

9.2.6 Entry

The two mechanisms giving rise to Proposition 1 are active even if all firms can survive in the market. If all firms survived the cut off values $\{\beta_i^{*c}\}$ and the averages $\left\{\widetilde{\beta}_i^{*c}\right\}$ would be the same for all c and i. Replacing these identical values in (34)-(36) for all c and i (whatever this value is) shows that Proposition 1 and its two constitutive mechanisms remain valid. Allowing for entry, as we do in the model, makes Proposition 1 hold *a fortiori* as it is apparent by observing the ranking of cut-off values β_i^{*c} obtained in Lemmas 1 and 2 and the resulting ranking of $\widetilde{\beta}_i^c$.

9.2.7 No average factor intensity reversal

From Equation (4) we have:

$$\frac{\overline{\theta}_Y^c}{\overline{\theta}_Z^c} = \left(\frac{\Lambda_Y}{\Lambda_Z}\right)^{\sigma} \left(\frac{\widetilde{\beta}_Y^c}{\widetilde{\beta}_Z^c}\right)^{\sigma-1} > 1, \tag{43}$$

which shows no average factor intensity reversal.

9.2.8 Relationship between *K-intensity* and sales

Let the relationship between K-intensity and sales be negative. This is done by assuming $\alpha \in (0, \infty)$ and $\beta = 1$. Then,

$$mc_i^c(\phi, \alpha) = \frac{1}{\phi}\left[(\lambda_i)^{\sigma} (w^c)^{1-\sigma} \alpha^{\sigma-1} + (1-\lambda_i)^{\sigma} (r^c)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{44}$$

Using the same procedure as in Sect. 9.2.1 shows that:

$$\alpha_Z^{c*} > \alpha_Y^{c*} \quad , \quad \forall \tau \in [0,1], \quad \text{and} \quad \alpha_i^{H*} < \alpha_i^{F*} \ , \quad \forall \tau \in (0,1). \tag{45}$$

Now, going through the same procedure as in Sect. 9.2.3 shows the validity of Theorem 1 and Proposition 1. With reference to Fig. 1, assuming $\alpha \in (0, \infty)$ and $\beta = 1.$ gives a negative slope of RS when plotted against k for all firms, but the slope is everywhere steeper for firms in the *LL-group* than for firms in the *KK-group*.

9.2.9 The Four Core Theorems

The Stolper-Samuelson, Rybczynski, Factor Price Equalization, and Heckscher-Ohlin theorems remain valid but, compared to a model where heterogeneity is only Hicks-neutral, their intensity is affected.

The Stolper-Samuelson and Rybczynski magnification effects are attenuated (amplified) by heterogeneity in β if the average factor intensity is *K-biased* (*L-biased*).[23]

The FPE set is expanded by heterogeneity in RMP. This can be seen in inequality (43), which shows that the diversification cone is expanded by heterogeneity in factor intensities.[24] The expansion of the FPE set does not depend on the direction of the factor bias. It does not depend either on the relationship between factor intensity and sales, indeed in the case of Sect. 9.2.8 we have $a_Z^* > a_Y^*$ and $\frac{\overline{\theta}_Y^c}{\overline{\theta}_Z^c} = \left(\frac{\Lambda_Y}{\Lambda_Z}\right)^\sigma \left(\frac{\widetilde{\alpha}_Z}{\widetilde{\alpha}_Y}\right)^{\sigma-1} > 1$.

The Heckscher-Ohlin specialization occurring when moving from autarky to free trade is attenuated. The attenuation is asymmetric: it is stronger (weaker) for the *L-abundant* (*K-abundant*) country when the average factor intensity is *K-biased*, and vice-versa when the average factor intensity is *L-biased*.

9.3 Positive fixed exporting cost.

When there are fixed exporting costs, establishing analytical results is a taxing exercise. We therefore resort to numerical simulations. With positive fix exporting cost we have to distinguish between domestic and total sales. To understand which of them is relevant for our purposes we should recall the logic of Theorem 1. This theorem states that the comparative advantage of countries influences the relative marginal cost of firms. Now, relative marginal cost are linked one-to-one to relative domestic sales. Therefore, we have to verify that Propositions 1 holds when written in terms of domestic sales.

[23] The average factor intensity is as given in expression (4) and exhibits a bias even when the technology is in itself neutral; i.e., even when $\int_0^\infty (\beta)^{\sigma-1} dG(\beta) = 1$. With this neutral technology, if all firms could survive in the market, the average factor intensity would be exactly $(\omega^c)^\sigma (\lambda_i)^\sigma \ \ \forall i, c$. Yet, because of selection in entry, a factor bias emerges in equilibrium (a *K-bias* in this case) since $\left(\widetilde{\beta}_i^c\right)^{\sigma-1} > 1$ even though $\int_0^\infty (\beta)^{\sigma-1} dG = 1$. Naturally, depending on the form of $G(\beta)$ the average factor intensity could be *L-biased*.

[24] In a two-by-two setting, the size of the FPE set increases with the size of the diversification cone.

Numerical simulations performed for many different set of parameter values have given plots qualitatively identical to those in Fig. 1 thus confirming the numerical validity of Proposition 1. Lastly, Proposition 2 remains valid when $F_x > 0$. This is proven by observing that $s_{im}^{c}(\varphi, 1) / s_{im}^{c}(\varphi, 1) = \varphi^{\sigma-1}$, for $m = d, x; \quad c = H, F; \quad i = Y, Z$.

Lastly, with reference to the empirical section, we report that systematic numerical simulations have shown that the relationship between relative sales depicted in Fig. 1 holds also for total sales.