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Financial Market Shocks and the Macroeconomy

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Abstract

Financial Market Shocks and the Macroeconomy

Feedback from stock prices to cash flows occurs because information revealed by firms' stock prices influences the actions of competitors. We explore the implications of feedback within a noisy rational expectations setting with incumbent publicly traded firms and privately held new entrants. In this setting the equilibrium relation among stock prices and both future dividends and aggregate output depends on the strategic environment in which these firms operate. In general, under reasonable conditions, the relations between prices, dividends, and economic output in our framework are consistent with empirical evidence in the macroliterature. We also generate new, potentially testable, implications. (*JEL* E3, G12, G14)

Macroeconomic forecasts, such as the NBER macroforecast, typically use stock returns as a leading economic indicator. This is intuitive. Stock prices are forward looking and should thus reflect expectations of future economic activity. Indeed, a relation between stock returns and expectations of future economic activity is a central feature of the recent literature on consumption-based asset pricing (e.g., Bansal and Yaron (2004)). Papers by Fama (1990) and Schwert (1990) are prominent examples that provide evidence on the link between stock returns and industrial output.[1]

Although the "forward-looking" interpretation of the stock returns/economic activity relation is intuitive, it is not really complete. Stock returns are not claims, for example, on industrial production, which they do predict, but are claims on future dividends. The rationale for the forward-looking view is that returns predict aggregate economic activity because dividends tend to grow when the economy grows. As Cochrane (2011) and others have emphasized,[2] however, the return on the market does not predict future changes in dividends on the market portfolio.[3] If we believe that stock returns predict aggregate economic activity because they are forward looking, then the fact that they do not predict changes in future dividends is somewhat puzzling.[4]

[1]See also Choi, Hauser, and Kopecky (1999). Barro (1990) and Farmer (2012) relate stock prices to aggregate investment and unemployment, respectively. The Conference Board (www.conference-board.org/data/bci/index.cfm?id=2160) uses the stock market return as a leading indicator for the overall macroeconomy.

[2]See also Campbell and Shiller (1989) and Cochrane (1996).

[3]See, for example, page 1053 (Table III) in Cochrane (2011). Although dividends have been the focus of the predictability literature, the findings of Larrain and Yogo (2008) suggest that the ability of stock returns to predict shifts in net payouts (defined as dividends plus interest plus net repurchases of equity and debt) is also quite limited (see their Table 4).

[4]In a back-of-the-envelope calculation to confirm the results of Fama (1990), Schwert (1990), and Cochrane (2011), we retrieve data on the industrial production index from http://research.stlouisfed.org/fred2/data/INDPRO.txt and retrieve S&P 500 index levels and dividend levels from www.econ.yale.edu/~shiller/data/ie_data.xls for the period January 1919 to March

The lack of correlation between stock returns and dividends indicates that a substantial portion of observed stock market volatility is due to changes in discount rates.[5] Thus, if an increase in stock prices does not reflect an increase in expected dividends, then it must necessarily reflect a decline in the discount rate. Given that discount rate changes can have economic consequences (e.g., through changes in investment choices), it is possible that stock returns are a leading economic indicator because they cause, rather than just predict, macroeconomic change.

Although this story is intuitive, it is also incomplete. Specifically, it does not explain why discount rates (or risk premia) can change without shocks to economic fundamentals. Moreover, in addition to a rationale for exogenous discount rate changes, a complete story requires that we establish a causal relation between stock prices and economic activity and finally, a reason why the profits and dividends of public corporations are only imperfectly correlated with aggregate economic activity.[6]

To address these issues, we develop a model that includes a publicly traded firm,

2013. We then regress quarterly growth in industrial production and in dividends, in turn, on contemporaneous and lagged values of the quarterly return on the S&P 500 index. (The forecasting horizon accords with the quarterly returns used in Fama (1990) and Schwert (1990).) In the regression for dividend growth, the coefficient of the lagged S&P 500 return is insignificant with a t-statistic of -1.41. In the regression for the growth in industrial production, the lagged S&P 500 return is significant with a t-statistic of 4.78. This simple calculation confirms the findings suggested by Fama (1990) and Cochrane (2011), that is, a significant link between aggregate output growth and stock returns, but a very tenuous link between dividend growth and stock returns.

[5]Using a variance decomposition approach, Campbell (1991) finds that cash flow news and discount rate news contribute equally (one-third each) to stock return volatility, with the balance attributable to the covariance between the two types of news.

[6]There is a large class of models in which discount rates change as a function of shocks to consumption and expected consumption. See for example, Campbell and Cochrane (1988) and Bansal and Yaron (2004). Researchers, however, have questioned whether these preferences can explain the magnitude of the observed volatility of risk premia (e.g., Hansen, Heaton, and Li (2008); Hansen and Sargent (2007); Marakani (2009)), and no distinction is drawn between consumption and dividends in these models of changing discount rates.

which represents existing incumbent firms, and also private firms, which represent potential new entrants. Because public stock prices convey information, the investment choices of these entrants are influenced by the public firm's stock prices.[7] For tractability reasons, we assume that all new investment comes from the entrants rather than from the existing public firms. However, because the public firms compete with the new entrants, a higher stock price, which increases investment by the entrants, can reduce the public firm's cash flows. To understand this, consider the situation faced by incumbent computer firms like IBM and Digital Equipment Corporation, that faced competition from emerging entrants like Microsoft, Apple, and Dell in the 1980s. It is plausible that the bull market in the 1980s facilitated investment in personal computer and operating system technology by the entrants, thereby ultimately reducing the profitability of the incumbent mainframe technology.

In addition to including two types of firms, the model includes two types of investors, informed and uninformed, and two types of shocks. The first, a technology shock observed by the informed investors, affects the productivity of both the public and private firms. The second type of shock, which exogenously affects the overall demand for traded shares, may be interpreted as a shock to the participation of unmodeled "noise traders," and will be henceforth described as a "participation shock." These shocks can be generated by technological innovations, such as the internet and web-based trading, and policy changes, such as a shift from defined benefit to defined contribution pension funds, which can affect the flow of funds into and out of the

[7] See Goldstein and Guembel (2008), Goldstein, Ozdenoren, and Yuan (2013), Ozdenoren and Yuan (2008), Subrahmanyam and Titman (2001), and Fishman and Hagerty (1989) for other models in which there is feedback from stock prices to cash flows via corporate investment. Chen, Goldstein, and Jiang (2007) show that real investment is more sensitive to stock prices when proxies for informed trading are higher, supporting an implication of these models.

stock market.

Because of the trades of the informed investors, stock prices reflect information about the future earnings of the public firms. The information content of the prices, however, is muddled by the shock to market participation that adds noise to the prices. Nevertheless, prices are still a useful input into the investment choices of the private firms. But because the uninformed investors and the nontraded sector cannot discern the extent to which a high price is due to informed trade or a high realization of the participation shock, equilibrium levels of capital investment by the private firms are sensitive to both the technology and participation shocks.

Further, because new entrants use the stock prices of the public firms as an indication of future demand, a participation shock that increases the stock prices of public firms increases aggregate investment by the new firms. Moreover, because these new firms compete with the existing public firms, the increased investment due to the positive participation shock can reduce the profits of public firms. This last component of our model, which implies that there can be negative feedback from the prices of public stocks to their future dividends, dampens the relation between public stock returns and future dividends. Overall, our key observation is that the relation between stock prices and dividends is not always positive, as would be suggested by casual intuition, but depends on the strategic environment in which public firms operate.

Under the assumption of constant absolute risk aversion (CARA preferences) the model can be solved in closed form, and it generates, at least qualitatively, the three observations that we mentioned at the outset. In particular, the model can generate a positive relation between public stock prices and both aggregate economic activity and investment, but a zero or even negative correlation between the public stock prices and the earnings and dividends of the public companies. In addition, our model generates an explicit relation between shocks to stock market participation and future economic activity, illustrating that activity in the financial markets can cause, as well as reflect, expectations about future aggregate output.

In summary, a combination of information asymmetries about technology and stochastic participation generates our results. Although changes in participation can lead to changes in discount rates and investment choices in settings with symmetric information, the interaction between asymmetrically observed technology shocks and unobserved participation amplifies the participation shocks. Indeed, as we show, shocks to participation have a larger effect on the real economy when the volatility of the technology shocks is larger. We also show that overconfidence of informed agents increases their trading aggressiveness and magnifies the impact of negative feedback on asset prices. Finally, we demonstrate that beliefs about the magnitude of participation shocks can influence their effect on the macroeconomy. Specifically, stronger priors about market efficiency (i.e., that price changes are generated primarily from cash flow news rather than participation shocks) result in magnified price volatility because of greater feedback from participation shocks to the macroeconomy.

Although our model is motivated by a puzzle from the macrofinance literature, it draws on other literature as well. For example, the idea that innovations from emerging firms can push out existing technologies has been around since Schumpeter

(1911), but we believe that we are the first to consider how this activity is influenced by activities in the public capital markets. In addition, we draw on the noisy rational expectations literature started by Grossman (1976) and Grossman and Stiglitz (1980), who describe the inference problem that arises when there are shocks to supply and shocks to expected cash flows. This literature and extensions to the literature have been applied to issues that relate to market microstructure, i.e., studies of stock market liquidity, insider trading, and selling versus trading on information (viz. Holden and Subrahmanyam (1996); Leland (1992); Admati and Pfleiderer (1986)).[8] In this literature, supply shocks and information tend to be interpreted as being very short in duration and affecting individual securities. In contrast, our model is based on the idea that there can be systematic and longer term changes to the demand and supply of stocks (for example, because of financial innovations and policy shifts) that may take a while before their effects are readily apparent to market participants. Whereas these innovations and policy changes may be observable, their implications for the asset allocation choices of investors may not be apparent until much later.

The plan of our paper is as follows. Section 1 presents the setting with public and private firms.[9] Section 2 presents an analysis of the equilibrium. Section 3 presents our central results by analyzing the relation between cash flows and market prices.

[8] In more recent work, Amador and Weill (2012) discuss the notion that public signal releases can crowd out private signal acquisition to such an extent that agents may be less informed of public releases of information in the long run. Hassan and Mertens (2011) suggest that small, but common, errors made by households in their optimal investment policies may amplify in the aggregate and crowd out the information content of prices. Angeletos, Lorenzoni, and Pavan (2010) argue that if financial markets look to entrepreneurs' investment to set prices and entrepreneurial signal errors are correlated, rationally entrepreneurs will want to overinvest relative to that warranted by their investment of fundamentals in order to obtain a favorable price for their capital.

[9] We present our results in the setting that most simply conveys our intuition. In an internet Appendix, we include various extensions and microfoundations for some of our assumptions.

Section 4 maximizes total output in the economy and contrasts with the noncooperative Nash outcome. Section 5 discusses how the risk aversion of the informed investors, which may differ from the risk aversion of the uninformed investors, affects our results. As we show in this section, changing the risk aversion of informed investors affects our results in a similar way as changing the variance of participation shocks. Section 6 discusses ways in which the effects of participation shocks can be amplified, and Section 7 concludes. All proofs, unless otherwise stated, appear in the Appendix.

1. The Economic Setting

Our model includes a public firm (that represents all listed firms), privately held new entrants, and a riskless and unlimited storage technology. The entrants have no assets at the time of entry, but possess a technology that provides the option to invest. Their actual investment choice is influenced by the information conveyed by the public firm's stock price, and because the investments can complement or compete with the public firm, there is "feedback" from the public firm's stock price to its cash flows. We also assume that the public firm consists of an asset in place but no growth opportunities.

1.1 The public firm

The public firm is born at date 0, investors trade the stock at date 1 at a price P, and its cash flows, which are realized at date 2, are expressed as follows:

$$F = \theta + \epsilon + k\mu. \tag{1}$$

The variables θ and ϵ represent exogenous technology shocks; ϵ is not revealed until date 2, but θ can be observed by informed investors at date 1. These variables have zero mean and are mutually independent and normally distributed. The term $k\mu$, where $\mu \equiv E(\theta|P)$, reflects the impact of the public firm's cash flows from the investment choices of the private firms, conditional on the market price P. We allow the parameter k to take on any value on the real line and model it as a function of the private firms' investment choices in Section 2.

Following Grossman and Stiglitz (1980), we assume there is a mass m of informed agents and $1 - m$ of uninformed agents, each with negative exponential utility with risk aversion R. Informed agents learn the realization of the technology shock θ perfectly after date 0 and prior to trade at date 1. We also assume that there is an exogenous shock that influences participation of unmodeled "noise traders" in the financial market, and, in turn, affects the supply of shares available to the informed and uninformed investors that we model. As we discussed in the introduction, these shocks may be caused, for example, by financial innovations in the brokerage business and policy changes (e.g., online trading, the rise of defined contribution pension plans, and the advent of ETFs). We represent this additional per capita demand by z (or supply by $-z$), which is normally distributed with mean zero and is independent of all other random variables.[10] Throughout the paper, we denote the variance of any generic random variable, η, by v_η.

1.2 The private firms

The model includes two private entrants that complement and compete with the incumbent public firm. They are both endowed with technologies as well as with a sufficient quantity of an asset that can be either transformed into the investment asset or stored to generate a risk-free cash flow at date 2 (the rate of return on the storage technology is normalized to zero without loss of generality). Each risk-neutral private firm i ($i = 1, 2$) invests a level of capital in a nontraded growth opportunity. The amount of capital invested by firm i is denoted by K_i.

Let π_i, $i = 1, 2$ denote the profit of firm i. We postulate that the profit of firm 1 depends on the technology shock θ and the level of investment through the function

$$\pi_1 = C_1 + C_2\theta + C_3K_1\theta - 0.5(K_1^2 - 2C_4K_1K_2). \tag{2}$$

Similarly, the profit of firm 2 is given by

$$\pi_2 = D_1 + D_2\theta + D_3K_2\theta - 0.5(K_2^2 - 2D_4K_1K_2). \tag{3}$$

We assume that all of the constants are positive (i.e., $C_i > 0$, $D_i > 0$ $\forall i = 1, \ldots, 4$), and $C_4 + D_4 < 1$ (the last assumption is to ensure an optimum). The expressions in (2) and (3) thus model the notion that profits depend positively on the technology shock θ and that the marginal productivity of capital is positively related to θ. The

[10]The analysis is unchanged if we model z as an shock to the informed agents' endowment.

term that involves the product of K_1 and K_2 in each profit function captures the strategic complementarity between the two firms, because we have assumed that C_4 and D_4 are positive.

The modeling of two entrants (as opposed to a single private firm) and their strategic complementarity is not needed for most of our results. Indeed, our main result, that public stock returns can be positively related to future aggregate output but unrelated to future dividends, holds in a slightly simpler model with just one entrant that competes with the public incumbent firm. However, by modeling two entrants with complementary investment expenditures we can illustrate how such complementarity magnifies feedback, and address how participation shocks can sometimes mitigate the underinvestment problem that arises from the coordination problem that arises in such a setting (Cooper and John 1988) (see Section 4).

In our setting, firms maximize their expected profits conditional on their information set at date 1 in a Nash equilibrium. This information set consists of the price of the publicly listed firm; the price provides noisy information about the technology shock θ. We wish to capture the notion that there are emerging firms in the private sector with products that can either complement or substitute for the products produced by established firms in the public sector. For example, a firm may improve the quality of an input to the traded firm (like a better operating system to a computer manufacturer), thereby raising the cash flow of the latter firm. Or a nontraded firm may increase the quality of a differentiated, but related, good (like a different brand of smartphone) by greater investment, thus reducing the cash flow of the traded firm. We model this by postulating that firm 1's product is complementary to that of the traded firm, whereas firm 2's product is a differentiated substitute. Each unit of capital invested in firm 1 increases the cash flow of the public firm by G_1 units, whereas each unit of capital invested in firm 2 reduces the cash flow of the public firm by G_2 units. Thus, the feedback component of cash flow is given by $G_1K_1 - G_2K_2$, where $G_1 > 0$ and $G_2 > 0$. In the next section, we show that K_1 and K_2 are linear in μ, so that the feedback has the form assumed in (1).

2. Equilibrium

We first solve for an equilibrium in the market for the public firm's shares taking the cash flows in (1) as given. Subsequently we model the feedback as a function of optimal real investment by the private firms.

2.1 Equilibrium in the market for the public firm

We conjecture that prices are normally distributed, and confirm the conjecture in equilibrium. Let the subscripts I and U denote the informed and uninformed, respectively. Further, let W_i and ϕ_i, $i = \{I, U\}$, respectively, denote the wealth and information sets of the two classes of agents. Each agent solves

$$\max E[-\exp(-RW_i)|\phi_i].$$

Let x_i denote the demand of agent i and P the market price. Then $W_i = (F - P)x_i$. It follows from the standard mean-variance objective in our CARA-normal setting that the demand of each informed agent is

$$x_I = \frac{E(F|\theta, P) - P}{R \text{ var}(F|\theta, P)} = \frac{\theta + k\mu - P}{Rv_\epsilon},$$

and that of each uninformed agent is

$$x_U = \frac{E(F|P) - P}{R \text{ var}(F|P)} = \frac{(1+k)\mu - P}{R \text{ var}(F|P)}.$$

Denote $v \equiv \text{var}(F|P) = v_\epsilon + \text{var}(\theta|P)$. The market clearing condition is

$$m\frac{\theta + k\mu - P}{Rv_\epsilon} + (1-m)\frac{(1+k)\mu - P}{Rv} + z = 0. \tag{4}$$

The rational expectations equilibrium of the model, derived in the Appendix, proves the following:

Lemma 1. The closed-form expression for the price P is given by

$$P = H_1\theta + H_2 z, \tag{5}$$

where

$$H_1 = \frac{m[kmv_\theta(m^2v_\theta + mR^2v_\epsilon v_\theta v_z + R^2v_\epsilon^2 v_z) + \{mv_\theta + R^2v_\epsilon v_z(v_\epsilon + v_\theta)\}\{m^2v_\theta + R^2v_\epsilon^2v_z\}]}{(m^2v_\theta + mR^2v_\epsilon v_\theta v_z + R^2v_\epsilon^2v_z)(m^2v_\theta + R^2v_\epsilon^2v_z)} \tag{6}$$

and

$$H_2 = Rv_\epsilon H_1/m. \tag{7}$$

Further,

$$\mu = E(\theta|P) = a_1\theta + a_2 z, \tag{8}$$

where

$$a_1 = \frac{m^2v_\theta}{m^2v_\theta + R^2v_\epsilon^2v_z} \tag{9}$$

and

$$a_2 = Rv_\epsilon a_1/m. \tag{10}$$

Note that H_1, the coefficient of the information variable θ in the equilibrium price, can be positive or negative in equilibrium. Specifically, H_1 is positive if and only if

$$k > -\left[1 + \frac{R^2 v_\epsilon^2 v_z \{m^2 v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta)\}}{m v_\theta (m^2 v_\theta + m R^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z)}\right] \equiv -[1+\nu]. \tag{11}$$

Thus, H_1 is positive if $1+k$ exceeds $-\nu$, where ν is a positive constant. The ambiguity in the sign of H_1 leads to an interesting feature of our model: that the price P can be negatively correlated with θ, the variable representing private information. This "perverse" case, which can arise when high θ is bad news for the public firm, happens when $H_1 < 0$, that is, when (11) does not hold, which implies that the negative feedback is so intense that it more than offsets the direct effect of the higher θ on the public firm's output. Because ν is increasing in R and v_z, the bound on the right-hand side of (11) decreases (becomes more negative) as R and v_z increase. Thus, the perverse case (which occurs to the left of the bound) obtains in a narrower parameter range when informed agents are more risk averse or the variance of the participation shock is higher. Higher risk aversion or a higher variance of the participation shock make the price less informative and the "perverse" negative feedback less intense, reducing the tendency for the perverse case to obtain. Although an occurrence of this phenomenon is interesting, within the context of our model, the perverse case, in which good news for the economy is bad news for public companies, does not generate any of our main results.

2.2 Equilibrium real investment

Up to this point, we have just assumed the feedback effect from price to cash flows of the public company (via the term $k\mu$, where $\mu \equiv E(\theta|P)$ in (1)). This subsection formally characterizes this feedback effect by characterizing the capital investments of the two private firms, which are determined within the context of a Nash equilibrium. Specifically, the private firms maximize the expected value of π_i (conditional on the publicly traded firm's price P), taking the other firm's investment choice as given. Taking conditional expectations of (2) and (3), firm 1's objective is

$$\max_{K_1} C_1 + C_2\mu + C_3K_1\mu - 0.5(K_1^2 - 2C_4K_1K_2),$$

whereas firm 2's objective is

$$\max_{K_2} D_1 + D_2\mu + D_3K_2\mu - 0.5(K_2^2 - 2D_4K_1K_2).$$

Performing the maximizations indicated above, and solving for K_1 and K_2 in terms of exogenous parameters yields,

$$K_1 = \frac{\mu(C_3 + C_4D_3)}{1 - C_4D_4}, \tag{12}$$

and

$$K_2 = \frac{\mu(D_3 + D_4C_3)}{1 - C_4D_4}. \tag{13}$$

Thus, real investment is linear in μ, the expectation of the technology shock conditional on the market price.

Substituting for K_1 and K_2 from (12) and (13) into π_1 and π_2, we have

$$\pi_1 = C_1 + \theta C_2 + \frac{(C_3 + C_4D_3)[2\mu\theta C_3(1 - C_4D_4) + \mu^2\{C_4D_3 + C_3(2C_4D_4 - 1)\}]}{2(1 - C_4D_4)^2}, \tag{14}$$

and

$$\pi_2 = D_1 + \theta D_2 + \frac{(D_3 + D_4C_3)[2\mu\theta D_3(1 - C_4D_4) + \mu^2\{D_4C_3 + D_3(2C_4D_4 - 1\}]}{2(1 - C_4D_4)^2}. \tag{15}$$

The realized profits vary both with θ and μ, the latter because the capital invested is a linear function of μ. It easily can be shown that the expected profits conditional on the public firm's stock price P are positive and increasing in the constants C_3, D_3, C_4, and D_4, where the former two and the latter two parameters measure the marginal productivity of capital and the degree of strategic complementarity, respectively.

Recalling from Section 1.2 that the feedback component of cash flow is $G_1K_1 + G_2K_2$, from (12) and (13), we have that the feedback parameter k is given by:

$$k = \frac{G_1(C_3 + C_4D_3) - G_2(D_3 + D_4C_3]}{1 - C_4D_4}. \tag{16}$$

Whether the feedback is negative or positive then depends on the relative sizes of G_1 and G_2. Rather than interpreting (16) in full generality, to gain some intuition, consider the special case in which $C_3 = D_3$ and $C_4 = D_4$, so that the right-hand side of (16) reduces to $[(G_1 - G_2)C_3]/(1 - C_4)]$. The absolute value of this expression is increasing in C_3 and C_4. Thus, in this instance, the sign of the feedback is determined by the sign of $G_1 - G_2$, whereas its absolute magnitude is increasing in the marginal productivity of capital and the strategic complementarity between the private firms; these are intuitive results. Because the right-hand side of (16) is comprised entirely of exogenous variables, we will continue to denote the feedback parameter as k in many places for convenience. Implicitly, however, from this point on, k means the right-hand side of (16).

3. The Relation between Cash Flows and Market Prices

In this section, we turn to the central issues, namely, the correlations among prices and total output as well as the public firm's cash flows ("dividends"). We show that the correlation between dividends and prices can be weak or even negative in certain parameter ranges, owing to the negative feedback effect. However, because the private firms' output is positively correlated with the technology shock, total output in these parameter ranges exhibits a strong positive relation with stock prices.

We first turn to the correlation between the realized cash flow of the public firm and the market price. Noting that $\mu \equiv a_1\theta + a_2 z$, and using expressions for a_1 and a_2 in (9) and (10), respectively, the covariance $\text{cov}(F, P)$ can be written as

$$\text{cov}(F, P) = \text{cov}(\theta+\epsilon+k\mu, H_1\theta+H_2 z) = (1+ka_1)H_1 v_\theta + ka_2 H_2 v_z = H_1(1+k)v_\theta. \quad (17)$$

The above covariance can be negative only if H_1 and $1 + k$ are of opposing signs. Because the right-hand side of (11) (the bound on k below which H_1 is negative) is less than -1, $1 + k$ and H_1 cannot be simultaneously positive and negative, respectively. Thus, we immediately have the following proposition, stated without proof.

Proposition 1. The covariance between the public firm's cash flows and the equilibrium price, $\text{cov}(F, P)$, is negative if and only if $k < -1$, and $H_1 > 0$.

The idea that the cash flow and price of the publicly traded firm can be negatively correlated is somewhat counterintuitive and arises from a combination of the participation shocks and the feedback. The feedback parameter has to be in a certain negative range, that is, $-1 > k > -(1+\nu)$, where ν is defined in (11), for the negative correlation to obtain. That is, for $\text{cov}(F, P)$ to be negative, k must be negative but not so negative that the "perverse case" (where P is negatively correlated with θ) obtains.

To understand the intuition, consider the case in which the feedback parameter is less than zero but not too negative, that is, greater than -1. In this case, a positive "shock" (i.e. positive realization of z and/or θ) increases the price, and increases real investment, but the negative feedback from the real investment to the public firm's cash flows is not strong enough to negate the "normal" positive relation between prices and cash flows. On the other hand, suppose the feedback parameter is extremely (or "perversely") negative. Then the increase in real investment due to an initial positive shock is so strong that it decreases both the cash flows and the market price of the public firm. However, if k is between -1 and $-(1 + \nu)$, then a positive shock tends to increase market prices, raise investment, and decrease final cash flows.

Interestingly, so long as $k < -1$, greater risk aversion of agents or a greater variance of the participation shock narrows the parameter space under which the perverse case obtains and increases the parameter range over which $\text{cov}(F, P)$ is negative. This occurs because $-(1+\nu)$ is decreasing in R and v_z (as noted in the discussion following Lemma 1); higher R or v_z lower the information content of the price, thus making the "perversely negative" feedback less intense, and increasing the range of k over which $\text{cov}(F, P)$ is negative.[11]

[11]Provided $k < -1$, as R and v_z become unboundedly large, the covariance between F and P

Note also that the correlation between prices and the public firm's cash flows (i.e., the correlation between future "dividends" and current stock prices) is exactly zero at two levels of feedback, when k equals -1, and when k equals $-(1+\nu)$ (i.e., when $H_1 = 0$). At these points the effect of the negative feedback exactly cancels the standard effect that high prices signal higher future cash flows.

We now analyze the correlation between the stock price of the public firm and total cash flows in the economy. From (1), (14), and (15), the following proposition can be derived.

Proposition 2. Provided that $H_1 > 0$, the correlation between public stock prices and the economy's total cash flows is positive if and only if

$$k > -(1 + C_2 + D_2).$$

Under the above proposition's premise, the more sensitive are the private firms' profits to the technology shock θ (i.e., the higher are C_2 and D_2), the greater is the tendency for these profits to covary positively with the public firm's stock price. In turn, the higher are C_2 and D_2, the greater is the range of k over which the total output in the economy covaries positively with stock prices.

The following proposition on the correlations among prices, total cash flows, the cash flows of the public firm, and investment, which is our central result, can also be derived from the preceding analysis.

remains negative, but the corresponding correlation goes to zero because the price becomes increasingly less informative.

Proposition 3. Suppose that $H_1 > 0$, and $-1 > k > -(1 + C_2 + D_2)$. Then, in equilibrium, (1) the correlation between the traded firm's cash flow and its stock price is negative, that is, $\mathrm{corr}(F, P) < 0$, (2) the correlations between aggregate investment and the public stock price, and between aggregate cash flow and the public stock price are both positive, that is, $\mathrm{corr}(K_1 + K_2, P) > 0$ and $\mathrm{corr}(F + \pi_1 + \pi_2, P) > 0$.

Another way of representing the proposition is that total output and real investment are positively correlated with stock prices, and dividends are negatively correlated with stock prices, as long as $-1 > k > -(1 + C_2 + D_2) > -(1 + \nu)$. These conditions together guarantee that the conditions in Propositions 1 and 2. are simultaneously satisfied. Thus, our model is consistent with both a low or negative correlation between future dividends and current stock prices, and a positive correlation between future economic output and current stock prices. The economic intuition behind Proposition 3 is as follows. Higher prices stimulate greater investment by the nontraded sector, generating competition from the nontraded firms that can lead to a negative relation between the traded sector's cash flows and its stock prices. In the overall economy, the effect of greater investment in the nontraded sector dominates the effect of increased competition in the publicly traded sector, so that the combined (aggregate) cash flow of the traded and the nontraded sector (i.e., the aggregate output) is positively correlated with prices.

3.1 Implications

This section describes some potentially testable implications of our analysis. Broadly speaking, an interesting area of future research is on how the informational efficiency of the stock market influences the relation between stock market returns and future economic output and capital investment expenditures. Specifically, we consider the effect of more volatile participation shocks, which make stock prices less informative, and a higher concentration of informed investors, which makes prices more informative. The following proposition, which considers regressions of aggregate output and investment expenditures on stock prices, is proven in the appendix.

Proposition 4. Let $\beta_T \equiv \mathrm{cov}(F+\pi_1+\pi_2, P)/\mathrm{var}(P)$ and $\beta_K \equiv \mathrm{cov}(K_1+K_2, P)/\mathrm{var}(P)$, respectively, represent the sensitivities of aggregate output and real investment expenditures to stock market prices. Then under the conditions of Proposition 3, for $i = \{T, K\}$, we have that $\beta_i > 0$, $d\beta_i/dv_z < 0$, $d\beta_i/dv_\epsilon < 0$, and $d\beta_i/dm > 0$.

As the above proposition demonstrates, the sensitivities of aggregate output and real investment expenditures to stock market prices are positive and declining in the variance of participation shocks v_z and the risk borne by the informed v_ϵ, and increasing in the mass of informed investors m. The intuition is that prices reflect more information when informed trading is more intense (higher m or lower v_ϵ) and the noise in market prices (v_z) is not too high, and this increases the sensitivity of investment as well as output to prices.

Also, in our model the correlation between stock prices and dividends is weakened by negative feedback from the privately held firms, so a large private sector that can potentially compete with the public firms is a necessary condition for a weak correlation between stock prices and dividends. This last observation can be potentially used to generate implications on the relation between stock returns and dividend changes across industries. Specifically, the correlation between stock prices and dividends is likely to be relatively low for those industries in which higher stock prices result in more competition from new entrants and is likely to be relatively high in those industry sectors that do not face competition from new entrants or, alternatively, may attract entrants with complementary investments that positively affect the profits of the public firms.

The above observations can be potentially tested by looking at the relation between stock returns and investment expenditures as well as economic output. Specifically, the preceding discussion suggests the following implications (the preceding material in parentheses indicates the signs of derivatives from Proposition 4 or specific propositions or model features that suggest the implications):

(1) ($d\beta_i/dv_z < 0$, $d\beta_i/dm > 0$) Aggregate output and capital investment are likely to be less sensitive to stock prices in economies with a larger mass of unsophisticated (e.g., retail) investor participation and a smaller mass of informed investors (e.g., institutional).

(2) ($d\beta_i/dm > 0$) In general, economies with institutions that facilitate the dissemination of information should exhibit a stronger relation between stock prices

and aggregate output. For example, the relation between total economic output and stock market prices will be greater for economies with an active community of equity analysts.

(3) ($d\beta_i/dv_\epsilon < 0$) Economies with greater inherent uncertainty (due, for example, to political risk) will have weaker links between total output and stock prices.

(4) (Proposition 1) At the industry or sector level, our analysis implies that industries with entrants that compete with public incumbents will tend to exhibit weaker links between stock prices and dividends than other industries with either greater entry barriers, or with entrants that tend to complement the incumbents.[12]

(5) (Existence of an emerging private sector) The relation between stock prices and future dividends is likely to be weaker in countries with an emerging private sector with investment expenditures that are either directly or indirectly funded by the stock market. For example, the association between stock returns and dividends may be weaker in countries with an active venture capital industry and an active IPO market.[13]

[12] An empirical measure of whether two firms are complements or substitutes is developed by Sundaram, John, and John (1996). They first measure the marginal profit of a firm (the sensitivity of a firm's profit to its sales). Their metric then is the correlation of the marginal profit to the output (sales) of other firms. A positive (negative) correlation indicates complementarity (substitutability).

[13] See Michelacci and Suarez (2004) for a model that links the funding of emerging firms by venture capitalists to activities in the stock market and the market for IPOs.

3.2 Numerical simulations

In this section, we provide simulations to illustrate our central results numerically. Consider the parameter values $C_1 = D_1 = 0$, $C_2 = D_2 = C_3 = 1$, and $D_3 = C_4 = D_4 = 0.4$. Also suppose that the exogenous variances (v_θ, v_ϵ, and v_z) and the risk aversion coefficient R all equal unity and the mass of informed agents $m = 0.5$. Because we use this parameter set in the next two sections as well (and vary k through the exogenous parameters G_1 and G_2), we will conveniently denote the set as Ω.

To illustrate the different signs and magnitudes of $\mathrm{corr}(F, P)$ and $\mathrm{corr}(F + \pi_1 + \pi_2, P)$ as a function of the level of feedback, we use the parameter set Ω, and let $G_1 = 1$, while varying G_2 from 2 to 3, implying a variation in k from -0.52 to -1.48. We use one million Monte Carlo draws of the triplet $[\theta, \epsilon, z]$ to simulate the model. Figure 1 demonstrates how the correlation between traded cash flows and prices, $\mathrm{corr}(F, P)$, switches from positive to negative as k decreases, but the correlation between total cash flows and prices, $\mathrm{corr}(F + \pi_1 + \pi_2, P)$, is positive throughout this range of k.

Although a full-fledged calibration of our model is beyond the scope of this paper, within the context of this numerical exercise, we can roughly gauge the magnitudes of the regression coefficients our model is able to generate. We can then compare the regression coefficients generated from this exercise to regressions of dividends and industrial production on stock prices that are estimated by Cochrane (2011), Fama (1990), and Schwert (1990). We consider the parameter set Ω, and let $G_1 = 1$, and $G_2 = 2.4$. For these parameter values, the regression coefficient of F on P ("dividends" on market prices) is 0.035. The coefficient of 0.035 compares favorably with Cochrane's (2011) coefficient of 0.04 when future dividend growth rates are regressed on current dividend yields (see his Table III, right panel). Keeping all parameters except m fixed and varying m from 0.1 to 0.6 makes the regression coefficient of F on P vary from 0.01 to 0.04, whereas the coefficient of total cash flows on market price (β_T) varies in a much higher range: from 0.11 to 0.79. The range for β_T spans the 0.12–0.46 range obtained by Schwert (1990) for coefficients of lagged returns when annual and quarterly growth rates in industrial production are the dependent variable. Similar magnitudes obtain for a wide range of parameter values other than in the set Ω. Whereas the empirical studies forecast growth rates in dividends and industrial production using returns and/or dividend yields, and are not directly comparable to our model, which uses levels of cash flows and prices, our numerical example suggests that a calibration of parameters (possibly via detailed empirical studies of feedback) can potentially replicate the relations found in the data.

4. The Effect of Participation Shocks on Total Output

This section examines how participation shocks affect the overall output in an economy. Our intuition illustrated in this section, is that positive participation shocks, which lead to increased investment by the private entrants, can increase total output if investment expenditures by private firms are sufficiently complementary.

We start by comparing the level of investment that maximizes the total output in the economy to the Nash equilibrium described in the previous section. We do this by first examining whether the investment expenditures in the Nash equilibrium are above or below the levels that maximize total output in the case with complete information. We then compare the Nash equilibrium with noisy rational expectations to the full revelation level of investment that maximizes total output. We particularly focus on a case in which the Nash equilibrium generates underinvestment when the participation shock is exactly zero. When this is the case, a small positive participation shock results in higher investment and increases the economy's cash flows, thus moving investment expenditures closer to the level that maximizes total output. In this case, a small negative participation shock has the opposite effect, but a sufficiently large positive participation shock can lead to overinvestment relative to the level that maximizes total output.

To illustrate these results, we consider a planner who maximizes the sum of the firms' conditional expected profits.[14] Thus, the planner solves

$$\max_{K_1,K_2} E(F+\pi_1+\pi_2|P) = \max_{K_1,K_2} \mu\,[C_3'K_1 + D_3'K_2] - 0.5[K_1^2 + K_2^2 - 2(C_4+D_4)K_1K_2], \tag{18}$$

where we define $C_3' \equiv C_3 + G_1$ and $D_3' \equiv D_3 - G_2$. Setting the partial derivatives of the above expression with respect to K_1 and K_2 to zero yields

$$K_1 \;=\; \mu C_3' + (C_4+D_4)K_2 \tag{19}$$

[14]It can be easily shown that the profits of the informed and uninformed and the losses of noise traders (with demand z) do not depend on the level of feedback and, in turn, on the level of investment. The feedback-dependent part of the wealth is $k\mu - P = (ka_1 - H_1)\theta + (ka_2 - H_2)z$. The losses of the noise traders are $(F-P)z = (ka_2 - H_2)z^2$. From (6), (7), (9), and (10), $ka_1 - H_1$ and $ka_2 - H_2$ do not involve k. Therefore, the expected utilities of the agents who trade the public claim do not form part of the social objective. So the total optimum is identical to the social optimum.

and

$$K_2 = \mu D_3' + (C_4 + D_4)K_1. \tag{20}$$

Substituting for K_2 from (20) into (19), yields[15]

$$K_{o1} = \frac{\mu\left[C_3' + (C_4 + D_4)D_3'\right]}{1-(C_4+D_4)^2}, \tag{21}$$

and similarly

$$K_{o2} = \frac{\mu\left[D_3' + (C_4 + D_4)C_3'\right]}{1-(C_4+D_4)^2}, \tag{22}$$

where the additional subscript o denotes maximization of total output.

Comparing (12) to (21) and (13) to (22), we know that if $G_2 = 0$, so that there is no negative feedback, then $\mu^{-1}\left[K_{oi} - K_i\right] > 0 \ \ \forall i = 1, 2$.[16] That is, without negative feedback, there is underinvestment in the Nash equilibrium relative to the levels of investment that maximize total output (too little investment in good scenarios with positive μ and too little divestment in bad scenarios with negative μ). But, with negative feedback, for any $\mu > 0$, the planner may choose investment levels lower than the Nash outcome to mitigate the impact of such feedback on the publicly traded firm's cash flow. In this scenario, there will still be underinvestment as long as the feedback is not too negative and the strategic complementarity between the private firms is sufficiently high.[17]

[15] We readily verified that the assumption $C_4 + D_4 < 1$ ensures the negative definiteness of the Hessian matrix, guaranteeing a maximum.

[16] To see this, first note that the denominators in (21) and (22) are smaller than their respective counterparts in (12) and (13). If $G_2 = 0$, then the terms multiplying μ in the numerators of (21) and (22) are greater than the corresponding terms in (12) and (13).

[17] Comparing (12) to (21), the coefficient of μ in the latter is less than that in the former if and only if the inequality $G_2 < \frac{C_3(C_4+D_4)^2 + C_4^2 D_3(C_4+D_4) + G_1(1-C_4D_4) + D_3D_4}{(C_4+D_4)(1-C_4D_4)}$ is satisfied. As the complementarity parameters C_4 and D_4 approach their upper bound of unity from below, the right-hand side becomes ever larger, increasing the tendency for this inequality to hold. An analogous argument holds for the comparison of (13) to (22).

From (1), (2), and (3), the total cash flows of the economy, denoted by π_o, are then given by

$$\begin{aligned} \pi_o &= C_1 + D_1 + \epsilon + k_o\mu + (1 + C_2 + D_2 + C_3K_{o1} + D_3K_{o2})\theta \\ &- 0.5\left[K_{o1}^2 + K_{o2}^2 - 2(C_4 + D_4)K_{o1}K_{o2}\right], \end{aligned}$$

where $k_o \equiv G_1K_{o1} - G_2K_{o2}$. Note that when θ is publicly revealed prior to investment, $\mu = \theta$. Denote the right-hand side of the above expression when $\mu = \theta$ as $\pi_{fo}(\theta)$, where the subscript f denotes full revelation of θ. Further, for a given level of μ and z, denote the final cash flow of the privately held and publicly traded firms at the Nash equilibrium level of capital as $\pi_n(\mu|z)$ and $F_n(\mu|z)$, respectively. We then have the following proposition.

Proposition 5. Suppose that $\theta > 0$ and

$$\pi_{fo}(\theta) > \pi_n(\mu|z = 0) + F_n(\mu|z = 0). \tag{23}$$

Then, relative to a Nash equilibrium with a zero participation shock, an arbitrarily small participation shock $z = z_c > 0$ increases real investment and causes the economy's total cash flow to increase.

The above proposition illustrates a positive element of participation shocks that can inflate stock prices above "fundamental values." In our model, because the private firms' investments complement each other, they tend to underinvest relative to the social optimum. Hence, on the margin, a participation shock that increases their investment expenditures increases total output. This effect is likely to be especially important when the investments of private entrants tend to be more innovative. Indeed, Bill Gates, when asked about the Internet bubble at the 1999 World Economic Forum in Davos, is quoted by Friedman (2005) as having said something to the effect of "Look, you bozos, of course they're a bubble, but you're missing the point. This bubble is attracting so much new capital to this Internet industry, it is going to drive innovation faster and faster."

We now demonstrate via a numerical example how participation shocks can move investment expenditures closer to the levels that maximize total output under full revelation. Consider the parameter set Ω of the previous section with G_1 and G_2 fixed at 1 and 2, respectively, and assume that $\theta = 1$ and $\epsilon = 2$. For this set of parameters, in the full revelation, total optimum case, we have $K_{o1} = 2.1$, $K_{o2} = 1.3$, and $\pi_{fo}(\theta) = 6.39$. In the Nash equilibrium with $z = 0$, however, we have $K_1 = 0.28$, $K_2 = 0.19$, and $\pi_n(\mu|z = 0) + F_n(\mu|z = 0) = 5.33$. In the Nash equilibrium with $z = 0.5$, $K_1 = 0.55$, $K_2 = 0.38$, and $\pi_n(\mu|z = 0.5) + F_n(\mu|z = 0.5) = 5.59$. Thus, a positive participation shock shifts the economy's output higher, and moves real investment closer to the level that maximizes total output under full revelation.[18]

[18] As mentioned at the beginning of this section, too high a participation shock can reverse this result. For example, if $z = 8$, $\pi_n(\mu) + F_n(\mu)$ becomes 0.79, a figure lower than the total optimum of 6.39. Similarly, a large negative participation shock will also drop output below that chosen by the planner under full revelation.

5. The Roles of Risk Aversion and the Variance of Participation Shocks

Up to now, the analysis assumes that the uninformed and informed agents are equally risk averse. In this section we relax this assumption and focus on how changes in the risk aversion of the informed agent, holding the risk aversion of the uninformed agent constant, affects the predictions of our model. This section also considers the effect of changing the variance of participation shocks, which has a similar effect on our model's predictions.

We start by relaxing our assumption that the agents are equally risk averse, and we describe the equilibrium in which the risk aversion coefficients of the informed and uninformed differ. Let the risk aversion coefficient of the uninformed agents be denoted by R_U, and let R continue to denote the risk aversion of the informed. The market clearing condition (4) now becomes:

$$m\frac{\theta + k\mu - P}{Rv_\epsilon} + (1-m)\frac{(1+k)\mu - P}{R_U v} + z = 0. \tag{24}$$

We show in the Appendix that in this case, the equilibrium price P can be written as $H_1'\theta + H_2'z$, where the coefficient H_1' is given by

$$H_1' \equiv \frac{A}{B}, \tag{25}$$

where

$$\begin{aligned} A \;\equiv\;& m[kmv_\theta\{m^3 v_\theta(R - R_U) - m^2 R v_\theta + mR^2 v_\epsilon v_z(Rv_\epsilon - R_U(v_\epsilon + v_\theta)) - R^3 v_\epsilon^2 v_z\} \\ +\;& (m^2 v_\theta + R^2 v_\epsilon^2 v_z)\{m^2 v_\theta(R - R_U) - mRv_\theta - R^2 R_U v_\epsilon v_z(v_\epsilon + v_\theta)\}] \end{aligned}$$

and

$$B \equiv (m^2 v_\theta + R^2 v_\epsilon^2 v_z)[m^3 v_\theta (R - R_U) - m^2 R v_\theta + m R^2 v_\epsilon v_z \{R v_\epsilon - R_U (v_\epsilon + v_\theta)\} - R^3 v_\epsilon^2 v_z].$$

Further, the coefficient H_2' is given by

$$H_2' = R v_\epsilon H_1' / m. \tag{26}$$

Interestingly, the difference between H_1 and H_1' does not depend on k. Indeed, suppose that $R_U = \rho R$. Then, from (6) and (25), we have

$$H_1' - H_1 = \frac{(\rho - 1)(1 - m) m R^2 v_\epsilon^2 v_z [m^2 v_\theta + R^2 v_\epsilon^2 v_z (v_\epsilon + v_\theta)]}{B'}, \tag{27}$$

where

$$\begin{aligned} B' \quad &\equiv \quad [m\rho\{m^2 v_\theta + R^2 v_\epsilon v_z (v_\epsilon + v_\theta)\} + (1 - m)(m^2 v_\theta + R^2 v_\epsilon^2 v_z)] \\ &\times \quad [m^2 v_\theta + m R^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z] > 0. \end{aligned}$$

As can be seen, the right-hand side of (27) does not involve k. Thus, because $H_1 - H_1'$ and $H_2 - H_2'$ are both invariant to k,[19] the difference between equilibrium prices when $R_U = R$ and $R_U \neq R$ does not depend on k for given realizations of θ and z. Both cash flows and price involve k, and k affects these quantities and, in turn, the demands of the agents in a way that the price as a function of R_U (for a given R) is invariant to k. Also note from (27) that if $\rho > 1$, that is, if $R < R_U$, $H_1' > H_1$. The intuition is that if the informed agents are less risk averse than the uninformed ones, they trade more aggressively on their information, and this increases the loading of the price on their information θ.

[19]Note that $H_2 = R v_\epsilon H_1 / m$ and $H_2' = R v_\epsilon H_1' / m$, so that if $H_1 - H_1'$ is invariant to k, so is $H_2 - H_2'$.

Using the above results, we now present numerical comparative statics associated with $\text{corr}(F, P)$, the correlation between prices and cash flows of the traded firm. We use the parameter set Ω, for every exogenous parameter, except the one being varied for the comparative static, and assume $R_U = 1$, $G_1 = 1$, and $G_2 = 3$. Figures 2 and 3, respectively, demonstrate that the correlation becomes progressively less negative as the risk aversion of the informed decreases relative to the uninformed, and as the postdate 1 risk of the informed, v_ϵ, decreases. The reason is that the initial price response to a participation shock, which causes the negative correlation via negative feedback, depends on the trading aggressiveness of the informed, which in turn depends inversely on the risk aversion of and the risk borne by the agents.

Figure 4 demonstrates that the correlation between the public firm's stock price and the public firm's cash flow also becomes less negative as the variance of the participation shock increases. An increase in this variance (v_z) decreases the signal-to-noise ratio in the market price, making the price less responsive to a liquidity shock, and thus decreasing the effect of negative feedback on the traded firm's cash flows. Of course, in Figures 2 through 4, the level of feedback is such that $\text{corr}(F, P)$ is negative. If instead we consider $k > -1$, which is a range for k such that $\text{corr}(F, P)$ is positive, then increasing R relative to R_U will make $\text{corr}(F, P)$ less positive. In fact, numerical simulations indicate that $[\text{corr}(F, P)]^2$ goes to zero as R becomes large relative to R_U. This is because as R/R_U becomes unboundedly large, informed traders trade increasingly less aggressively, reducing the extent of learning by the uninformed, so that the price becomes increasingly less informative, and the absolute value of the correlation between cash flows and prices becomes vanishingly small.

6. Amplification of Participation Shocks

Up to this point we have presented a simple model that can be solved in closed form; it generates a number of qualitative results that are consistent with the empirical macrofinance literature. In particular, the model can generate zero correlation between stock returns and subsequent dividend changes but a positive correlation between stock prices and aggregate economic activity. In addition, the model generates negative serial correlation in aggregate stock returns (or equivalently the predictability of returns with price scaled ratios). Within the context of our model, a crucial attribute that determines the strengths of these relations is the participation shock, suggesting that the observed magnitude of predictability in the data requires large and uncertain participation shocks.

In this section we consider channels that might amplify the effect of participation shocks.[20] We first consider the possibility that informed investors are overconfident about the precision of their information. As we show, overconfidence amplifies the participation shock, thereby increasing the strength of the relation between aggregate output and market prices. We then briefly discuss parameter uncertainty, specifically how our results may be altered if the variance of the participation shock is unknown.

[20] Some other recent papers also address amplification of liquidity shocks and signal errors in financial markets. Albagli (2012) argues that because households liquidate assets during crises, intermediaries face tighter financing constraints as a result, and trade less aggressively, lowering the information content of prices. Guerrieri and Lorenzoni (2009) model the notion that in recessions, consumers are liquidity constrained and spend less, thus magnifying the reduction in output due to the recession.

6.1 Overconfident investors

We first consider the possibility that informed investors are overconfident.[21] Specifically, they believe that their information about the technology shock is more precise than it really is, that is, they underestimate v_ϵ to be $v_c < v_\epsilon$. We assume that the overconfidence of the informed is common knowledge to all the noninformed participants in the model. Overconfidence makes the informed agents more aggressive, which in turn makes the price more sensitive to participation shocks. Thus, an increase in the level of overconfidence amplifies the effect of feedback and increases the magnitude of the correlation between the public firm's cash flows and stock prices.[22] The Appendix proves the following proposition.

Proposition 6. Consider a scenario in which informed agents rationally assess all the model's parameters. As one moves away from this setting to one of increasing overconfidence, that is, v_c, the estimate of v_ϵ is progressively lowered, the absolute magnitude of $\text{corr}(F, P)$, the correlation between the traded cash flows and the stock price, increases.

6.2 Parameter uncertainty

As we show in the previous subsection, participation shocks can be amplified if informed investors are overconfident. In this section we describe how uncertainty about

[21]See Odean (1998) for an excellent review of the extensive literature that documents the pervasive overconfidence bias.

[22]Ko and Huang (2007) derive an analogous result by showing that overconfidence causes informed agents to acquire information more aggressively, leading to greater pricing efficiency.

the expected magnitudes of participation and technology shocks can also amplify participation shocks. Because the model cannot be solved in closed form without common knowledge about the magnitude of these shocks, we provide intuition, that is, conjectures, in a stylized setting.

Consider, for example, a setting in which the true volatility of the participation shock is drawn from a distribution of possible volatilities. Although this distribution is common knowledge, the actual volatility that is drawn is not observed by market participants. We conjecture that in this setting, the volatility of prices increases with the volatility of the participation shock that is drawn.

Unfortunately, solving a model in which agents draw parameter values from a prior distribution is quite challenging because in this case, the linearity of conditional expectations formed by the uninformed would be lost (this linearity holds only when variances and unconditional means are nonstochastic). Our intuition comes from the following proposition.

Proposition 7. If investors irrationally believe that the variance of the participation shock is lower than its actual value, then the volatility of the equilibrium price will be greater relative to the rational setting.

Intuitively, prices are more volatile when investors underestimate the volatility of the participation shock because they tend to underweight the possibility that price changes reflect changes in risk premia rather than changes in expected cash flows, and hence reduce the extent to which they take actions that offset the participation shocks, that is, buying the stock when the supply is higher and vice versa.[23] The above proposition should be viewed as "suggestive;" because it considers an investor prior that is a point estimate but also is wrong, and such a belief is inconsistent with rationality.[24]

7. Conclusion

Motivated initially by the equity premium puzzle described by Mehra and Prescott (1985), researchers have struggled with a number of features of the data that relate financial market prices to the macroeconomy. In addition to the magnitude of the equity premium, researchers have developed models to address the time-series properties of default-free interest rates and expected equity returns and to understand how they relate to dividends and aggregate consumption. To a large extent, the goal of this research has been to identify a plausible characterization of preferences that are consistent with the data.

This paper takes an alternative approach to previous research by exploring a subset of the issues considered in the macrofinance literature with a very different type

[23] This result is analogous to the result in Jacklin, Kleidon, and Pfleiderer (1992) and Gennotte and Leland (1990), who find that if agents underestimate demand because of hedging or portfolio insurance, prices become more susceptible to sudden crashes.

[24] Solving an equilibrium in which the degree of uncertainty about participation shocks are drawn from a distribution creates considerable challenges within our setting because prices in this setting are no longer normally distributed. But it is likely that the above intuition can be captured numerically in a simpler setting with specific numerical values. For example, consider the case in which the investor prior distribution is that the standard deviation of the participation shock is 1 with a probability of 0.99 and 10 with a probability of 0.01 but for which the actual standard deviation of the participation shock is 10. We conjecture that the equilibrium price in this setting is very close to that in the ad hoc equilibrium described in Proposition 7.

of model: noisy rational expectations with asymmetric information. In particular, we extend Grossman and Stiglitz's work (1980) to evaluate how stock price movements caused by uninformed participation shocks can influence the profits of public companies as well as overall economic activity. Within the context of this model, we endogenously generate the positive correlations between stock returns and aggregate economic activity, and stock returns and investment, which are observed in the data, as well as the lack of correlation (or even negative correlation) between stock returns and dividends.

Our key contribution is showing that the equilibrium relation between cash flows and prices can be strong or weak, depending on the strategic environment of firms. Specifically, investment by private firms allows private firms to better compete with publicly traded firms. Because these investments tend to increase with the stock price of the traded firm, the investments reduce the cash flows of publicly traded firms, generating a weak or even negative correlation between the public firms' cash flows and public firms' stock prices. In this setting the correlation between aggregate output and stock prices remains positive, because the nontraded sector's cash flows are positively related to the technology shocks, which are also reflected in stock prices. Our results are thus consistent with the documented insignificant relation between cash flows and dividends discussed, for example, in Fama and French (1988), Campbell and Shiller (1988) and Cochrane (2011), as well as a significantly positive relation between aggregate macroeconomic production and stock prices. We develop additional implications that relate proxies for informational efficiency to the strength of the relation between stock prices and total output.

At this point, our research agenda is much less ambitious than the preference-based macrofinance literature. In particular, we address fewer facts, and our focus is on qualitative results based on a closed-form model with CARA preferences. As a result, we only provide a preliminary evaluation of economic magnitudes. Nonetheless, our analysis looks promising, and our results identify various levers that may help future researchers who are interested in exploring the quantitative relation between stock returns and aggregate economic activity.

In addition to the parameters that we explicitly consider in our model, our analysis suggests potential adaptations of the model that may help future researchers match the observed moments in the data. For example, our analysis suggests that over-confident informed investors react more aggressively to information, increasing the correlation between stock prices and economic output. In addition, the volatility of stock prices is influenced by the beliefs of the uninformed investors about the volatility of participation shocks. Thus, when uninformed investors believe that participation shocks are less volatile than they really are, stock prices will be more volatile. Finally, the volatility and serial correlation of returns are likely to be amplified if investors have preferences that make them more risk averse when they are less wealthy.[25] Whether or not deviations from rational expectations or changes in preferences can be exploited to yield economic moments that match the moments observed in the data is a challenge that warrants future research.

[25]Unfortunately, a closed-form solution of our model requires CARA preferences.

References

Admati, A., and P. Pfleiderer. 1986. A monopolistic market for information. *Journal of Economic Theory* 39:400–38.

Albagli, E. 2012. Amplification of uncertainty in illiquid markets. Working Paper, University of Southern California.

Amador, M., and P. Weill. 2012. Learning from private and public observations of others' actions. *Journal of Economic Theory* 147:910–40.

Angeletos, G., G. Lorenzoni, and A. Pavan. 2010. Beauty contests and irrational exuberance: A neoclassical approach. Working Paper, Northwestern University.

Bansal, R., and A. Yaron. 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance* 59:1481–509.

Barro, R. 1990. The stock market and investment. *Review of Financial Studies* 3:115–31.

Campbell, J. 1991. A variance decomposition for stock returns. *Economic Journal* 101:157–79.

Campbell, J., and R. Shiller. 1988. Stock prices, earnings, and expected dividends. *Journal of Finance* 43:661–76.

Chen, Q., I. Goldstein, and W. Jiang. 2007. Price informativeness and information sensitivity to stock price. *Review of Financial Studies* 20:619–50.

Choi, J., S. Hauser, and K. Kopecky. 1999. Does the stock market predict real activity? Time series evidence from the G-7 countries. *Journal of Banking and Finance* 23:1771–92.

Cochrane, J. 1996. A cross-sectional test of a production-based asset pricing model. *Journal of Political Economy* 104:572–621.

——. 2011. Presidential address: Discount rates. *Journal of Finance* 66:1047–108.

Cooper, R., and A. John. 1988. Coordinating coordination failures in Keynesian models. *Quarterly Journal of Economics* 103:441–63.

Fama, E. 1990. Stock returns, expected returns, and real activity. *Journal of Finance* 45:1089–108.

Fama, E., and K. French. 1988. Dividend yields and expected stock returns. *Journal of Financial Economics* 22:3–25.

Fishman, M., and K. Hagerty. 1989. Disclosure decisions by firms and the competition for price efficiency. *Journal of Finance* 44:633–46.

Friedman, T. 2005. The world is flat: A brief history of the twenty-first century. New York: Farrar, Straus, and Giroux.

Gennotte, G., and H. Leland. 1990. Market liquidity, hedging, and crashes. *American*

Economic Review 80:999–1021.

Goldstein, I., and A. Guembel. 2008. Manipulation and the allocational role of prices. *Review of Economic Studies* 75:133–64.

Goldstein, I., E. Ozdenoren, and K. Yuan. 2013. Trading frenzies and their impact on real investment. *Journal of Financial Economics* 109:566–82.

Grossman, S. 1976. On the efficiency of competitive stock markets where trades have diverse information. *Journal of Finance* 31:573–85.

Grossman, S., and J. Stiglitz. 1980. On the impossibility of informationally efficient markets. *American Economic Review* 70:393–408.

Guerrieri, V., and G. Lorenzoni. 2009. Liquidity and trading dynamics. *Econometrica* 77:1751–90.

Hansen, L., J. Heaton, and N. Li. 2008. Consumption strikes back? Measuring long run risk. *Journal of Political Economy* 116:260–302.

Hansen, L., and T. Sargent. 2007. Fragile beliefs and the price of model uncertainty. Working Paper, University of Chicago.

Hassan, T., and T. Mertens. 2011. The social cost of near-rational investment. Working Paper, University of Chicago.

Hellwig, M. 1980. On the aggregation of information in competitive markets. *Journal* *of Economic Theory* 22:477–98.

Holden, C., and A. Subrahmanyam. 1996. Risk aversion, liquidity, and endogenous short horizons. *Review of Financial Studies* 9:691–722.

Jacklin, C., A. Kleidon, and P. Pfleiderer. 1992. Underestimation of portfolio insurance and the crash of October 1987. *Review of Financial Studies* 5:35–63.

Ko, J., and Z. Huang. 2007. Arrogance can be a virtue: Overconfidence, information acquisition, and market efficiency. *Journal of Financial Economics* 84:529–60.

Larrain, B., and M. Yogo. 2008. Does firm value move too much to be justified by subsequent changes in cash flow? *Journal of Financial Economics* 87:200–26.

Leland, H. 1992. Insider trading: Should it be prohibited? *Journal of Political Economy* 100:859–87.

LeRoy, S., and R. Porter. 1981. The present-value relation: Tests based on implied variance bounds. *Econometrica* 49:555–74.

Marakani, S. 2009. Long run consumption risks: Are they there? Working Paper, Northwestern University.

Michelacci, C., and J. Suarez. 2004. Business creation and the stock market. *Review of Economic Studies* 71:459–81.

Mehra, R., and E. Prescott. 1985. The equity premium: A puzzle. *Journal of*

Monetary Economics 15:145–61.

Odean, T. 1998. Volume, volatility, price, and profit when all traders are above average. *Journal of Finance* 53:1887–934.

Ozdenoren, E., and K. Yuan. 2008. Feedback effects and asset prices. *Journal of Finance* 63:1939–75.

Schumpeter, J. 1911. *A theory of economic development.* Cambridge: Harvard University Press.

Schwert, W. 1990. Stock returns and real economic activity: A century of evidence. *Journal of Finance* 45:1237–57.

Subrahmanyam, A., and S. Titman. 2001. Feedback from stock prices to cash flows. *Journal of Finance* 56:2389–413.

Sundaram, A., T. John, and K. John. 1996. An empirical analysis of strategic competition and firm values: The case of R&D competition. *Journal of Financial Economics* 40:459–86.

Appendix

Proof of Lemma 1: As part of their optimization, the uninformed agents solve a filtration problem that infers θ from the price P, which is a linear combination of θ and z. Let

$$\tau \equiv \frac{m\theta}{Rv_\epsilon} + z.$$

Note that (4) can be solved for P and written as

$$P = [mv + (1-m)v_\epsilon]^{-1} [Rvv_\epsilon \tau + \mu\{kmv + (1-m)(1+k)v_\epsilon\}] . \quad \text{(A1)}$$

Because μ and v are nonstochastic from the uninformed's perspective, P is observationally equivalent to τ. Thus, we have

$$\mu = E(\theta|\tau) = \frac{Rmv_\theta v_\epsilon}{m^2 v_\theta + R^2 v_\epsilon^2 v_z}\tau, \quad \text{(A2)}$$

and

$$v = v_\epsilon + v_\theta - \frac{m^2 v_\theta^2}{m^2 v_\theta + R^2 v_\epsilon^2 v_z}. \quad \text{(A3)}$$

Note that μ can be written as

$$\mu = a_1\theta + a_2 z, \quad \text{(A4)}$$

where

$$a_1 = \frac{m^2 v_\theta}{m^2 v_\theta + R^2 v_\epsilon^2 v_z} \quad \text{(A5)}$$

and

$$a_2 = \frac{Rmv_\theta v_\epsilon}{m^2 v_\theta + R^2 v_\epsilon^2 v_z}. \quad \text{(A6)}$$

Note that the bigger is v_θ, the bigger are the coefficients a_1 and a_2. Thus, a given informational or participation shock has a bigger impact on cash flows if the volatility of the information variable is higher.

Solving for P from (4), we have

$$P = \frac{Rvv_\epsilon z + mv\theta + \mu[k\{m(v - v_\epsilon) + v_\epsilon\} + v_\epsilon(1 - m)]}{m(v - v_\epsilon) + v_\epsilon}.$$

Substituting for μ and v from (A2) and (A3), respectively, we have the expressions for H_1 and H_2 in (6) and (7). ‖

Proof of Proposition 2: The covariance of total cash flows with price is $\text{cov}(F + \pi_1 + \pi_2, P)$. Note that the expressions for π_1 and π_2 in (14) and (15), respectively, have some terms that are quadratic forms of normal random variables. These quadratic forms do not affect the covariance, however. To see this first note that μ is a linear function θ and z, so that μ^2 is a quadratic form in θ and z. Then, observe that $\text{cov}(\mu^2, P) = \text{cov}(\mu\theta, P) = 0$, because θ^2, θz, and z^2 are all uncorrelated with P, which is a linear combination of θ and z. For example, $\text{cov}(\theta z, \theta) = E(\theta^2 z) - E(\theta z)E(\theta) = 0$, because θ and z are independent, as are θ^2 and z (functions of independent random variables are also independent), and both θ and z have zero mean. Similarly, $\text{cov}(\theta, \theta^2) = E(\theta^3) - E(\theta)E(\theta^2) = 0$, because the normal distribution has zero skewness and θ has zero mean. Analogous calculations can be done to show that $\text{cov}(\theta z, z) = \text{cov}(z, z^2) = 0$. This shows that the covariances of the quadratic forms of normals in (14) and (15) with P are zero. We thus have

$$\begin{aligned} \text{cov}(F + \pi_1 + \pi_2, P) &= \text{cov}[\theta(1 + C_2 + D_2) + \epsilon + k\mu, H_1\theta + H_2 z] \\ &= (1 + C_2 + D_2 + ka_1)H_1 v_\theta + ka_2 H_2 v_z \\ &= H_1(1 + k + C_2 + D_2)v_\theta. \end{aligned} \tag{A7}$$

The proposition thus follows. ‖

Proof of Proposition 3: Note that $H_1 > 0$ and $k < -1$ imply that $\text{corr}(F, P) < 0$ (from Proposition 1). Also observe from (A7) that if $k > -(1 + C_2 + D_2)$, $\text{corr}(F + \pi_1 + \pi_2, P) > 0$. Then, from (8) and (5) we have that

$$\text{cov}(\mu, P) = a_1 H_1 v_\theta + a_2 H_2 v_z. \tag{A8}$$

From (9), (10), and (6) the right-hand side of (A8) equals

$$\frac{mv_\theta[kmv_\theta(m^2 v_\theta + mR^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z) + \{mv_\theta + R^2 v_\epsilon v_z(v_\epsilon + v_\theta)\}\{m^2 v_\theta + R^2 v_\epsilon^2 v_z\}]}{(m^2 v_\theta + mR^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z)(m^2 v_\theta + R^2 v_\epsilon^2 v_z)},$$

which is positive if $H_1 > 0$ (from (6)). The proposition thus follows. $\|$

Proof of Proposition 4: From Lemma 1, the variance of the price P can be written as

$$\text{var}(P) = H_1^2 v_\theta + H_2^2 v_z = H_1^2 \left[v_\theta + m^{-2} R^2 v_\epsilon^2 v_z\right]. \tag{A9}$$

Now, it follows from (6), (A7), and (A9) that

$$\beta_T = \frac{mv_\theta(1 + k + q)(m^2 v_\theta + mR^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z)}{kmv_\theta(m^2 v_\theta + mR^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z) + [mv_\theta + R^2 v_\epsilon v_z(v_\epsilon + v_\theta)](m^2 v_\theta + R^2 v_\epsilon^2 v_z)}. \tag{A10}$$

Now, let

$$\begin{aligned} L \;\equiv\; & \Big[kmv_\theta(m^2 v_\theta + mR^2 v_\epsilon v_\theta v_z + R^2 v_\epsilon^2 v_z) + m^3 v_\theta^2 + m^2 R^2 v_\epsilon v_\theta v_z(v_\epsilon + v_\theta) \\ + \;& mR^2 v_\epsilon^2 v_\theta v_z + R^4 v_\epsilon^3 v_z^2 (v_\epsilon + v_\theta)\Big]^2. \end{aligned}$$

We then have

$$\begin{aligned} \frac{d\beta_T}{dv_z} \;=\; & -mR^2 v_\epsilon^2 v_\theta (1 + k + q)\Big[m^4 v_\theta^2 + 2m^2 R^2 v_\epsilon v_\theta v_z(v_\epsilon + v_\theta) \\ + \;& mR^4 v_\epsilon^2 v_\theta v_z^2 (v_\epsilon + v_\theta) + R^4 v_\epsilon^3 v_z^2 (v_\epsilon + v_\theta)\Big]/L, \\ \frac{d\beta_T}{dv_\epsilon} \;=\; & -mR^2 v_\epsilon^2 v_\theta v_z (1 + k + q)\Big[2m^4 v_\theta^2 + m^3 R^2 v_\epsilon v_\theta^2 v_z + m^2 R^2 v_\epsilon v_\theta v_z (4v_\epsilon + 3v_\theta) \end{aligned}$$

$$
\begin{aligned}
&+ \quad mR^4 v_\epsilon^2 v_\theta v_z^2 (3v_\epsilon + 2v_\theta) + R^4 v_\epsilon^3 v_z^2 (2v_\epsilon + v_\theta)\Big] /L, \quad \text{and} \\
\frac{d\beta_T}{dm} &= R^2 v_\epsilon^2 v_\theta v_z (1+k+q) \Big[m^4 v_\theta^2 + m^2 R^2 v_\epsilon v_\theta v_z (2v_\epsilon + 3v_\theta) \\
&+ \quad 2mR^4 v_\epsilon^2 v_\theta v_z^2 (v_\epsilon + v_\theta) + R^4 v_\epsilon^3 v_z^2 (v_\epsilon + v_\theta)\Big] /L.
\end{aligned}
$$

Because $L > 0$, and, under the conditions in Proposition 3, $1+k+q > 0$, the first two derivatives are negative, and the third is positive. Also, note that $\beta_T > 0$, because the denominator of the right-hand side of (A10) is identical to the denominator of (6), which is required to be positive for $H_1 > 0$ (a positive H_1 is a premise of Proposition 3).

We now turn to β_K. Let

$$\Delta \equiv \frac{C_3 + D_3 + C_3 D_4 + C_4 D_3}{1 - C_4 D_4}.$$

From (12) and (13), and noting that $P = H_1\theta + H_2 v_z$, and that $\mu = a_1\theta + a_2 z$, we have that

$$\beta_K = \frac{\Delta(a_1 H_1 v_\theta + a_2 H_2 v_z)}{H_1^2 v_\theta + H_2^2 v_z}.$$

Now, observing from Proposition 1 that $H_2 = (Rv_\epsilon/m)H_1$, and substituting for H_1, a_1, and a_2 from (6), (9), and (10), respectively, we find that

$$\beta_K = \frac{\beta_T \Delta}{(1+k+q)}.$$

Now, $\Delta > 0$, and, as noted above, the conditions in Proposition 3 imply that $1+k+q > 0$, indicating that $\beta_K > 0$. Further, Δ and $1+k+q$ do not involve v_z, v_ϵ, or m. So, the signs of the derivatives of β_K with respect to v_z, v_ϵ, and m, are the same as the corresponding ones for β_T. This proves the proposition. $\|$

Proof of Proposition 5: Note that (14) can be written as

$$\pi_1 = C_1 + \theta C_2 + (C_3 + C_4 D_3)\left[\frac{(C_3 + C_4 D_3)[2\mu\theta C_3(1 - C_4 D_4)}{2(1 - C_4 D_4)^2} + \frac{\mu^2\{C_4 D_3 + C_3(2C_4 D_4 - 1)\}]}{2(1 - C_4 D_4)^2}\right]. \tag{A11}$$

Now observe that $\mu = a_1\theta + a_2 z$, and from (9) and (10), a_1 and a_2 are positive. For an arbitrarily small shock $z = z_c$, the term involving $\mu^2 = a_1^2\theta^2 + a_2^2 z_c^2$ can be ignored; further, the coefficient of $\mu\theta = a_1\theta^2 + a_2\theta z_c$ in (A11) is positive. Thus, a small shock $z = z_c > 0$ increases the right-hand side of (A11) relative to the case in which $z = 0$. ‖

Proof of Equations (25) and (26): Solving for the price P from (24), we have

$$P = \frac{RR_U v v_\epsilon z + k\mu[Rv_\epsilon - m(Rv_\epsilon + R_U v)] + m(R_U\theta v - \mu R v_\epsilon) + \mu R v_\epsilon}{Rv_\epsilon - m(Rv_\epsilon - R_U v)}. \tag{A12}$$

Note that since v and τ do not depend on the risk aversion of the uninformed, they remain unchanged (as does μ) relative to their algebraic representations in Section 1 and the proof of Proposition 1. Substituting for v and μ from (A3) and (8), respectively, into (A12) above yields (25) and (26). ‖

Proof of Proposition 6: Under overconfidence, the variable $\tau = \theta + Rv_c z/m$. Thus, the analogs of (9) and (10) become

$$a_1 = \frac{m^2 v_\theta}{m^2 v_\theta + R^2 v_c^2 v_z} \tag{A13}$$

and

$$a_2 = \frac{Rm v_\theta v_c}{m^2 v_\theta + R^2 v_c^2 v_z}. \tag{A14}$$

Using techniques similar to that used for Proposition 1, we find that the price P takes the firm

$$P = H_1^o \theta + H_2^o z$$

where $H_1^o = A_o/B_o$, with

$$\begin{aligned} A_o \equiv\ & m(kmv_\theta(m^3 v_\theta(Rv_c - R_U v_\epsilon) - m^2 Rv_c v_\theta + mR^2 v_\epsilon v_z(Rv_c - R_U(v_\epsilon + v_\theta)) \\ - \ & R^3 v_c v_\epsilon^2 v_z) + m^4 v_\theta^2 (Rv_c - R_U v_\epsilon) - m^3 R v_c v_\theta^2 + m^2 R^2 v_\epsilon v_\theta v_z (Rv_c v_\epsilon \\ - \ & R_U(v_C^2 + v_\epsilon(v_\epsilon + v_\theta))) - mR^3 v_C v_\epsilon^2 v_\theta v_z - R^4 R_U v_C^2 v_\epsilon^2 v_z^2 (v_\epsilon + v_\theta)) \end{aligned}$$

and

$$B_o \equiv (m^2 v_\theta + R^2 v_c^2 v_z)(m^3 v_\theta(Rv_c - R_U v_\epsilon) - m^2 R v_c v_\theta + mR^2 v_\epsilon v_z (Rv_c - R_U(v_\epsilon + v_\theta)) - R^3 v_c v_\epsilon^2 v_z).$$

Further, $H_2^o = (Rv_c H_1^o)/m$. Note that the covariance

$$\text{cov}(F, P) = H_1^o(1 + ka_1)v_\theta + H_2^o k a_2 v_z = H_1^o(1 + k)v_\theta.$$

We also have that

$$\text{var}(P) = H_1^{o2} v_\theta + H_2^{o2} v_z$$

and

$$\text{var}(F) = (1 + ka_1)^2 v_\theta + v_\epsilon + k^2 a_2^2 v_z.$$

All this implies that

$$\text{corr}(F, P) = \frac{m \ \text{sgn}(H_1^o)(1 + k)v_\theta}{[k^2 m^2 v_\theta^2 + 2km^2 v_\theta^2 + (v_\epsilon + v_\theta)(m^2 v_\theta + R^2 v_c^2 v_\theta)]^{0.5}}.$$

The absolute magnitude of the above correlation increases as v_c decreases, because the absolute value of the numerator does not involve v_c and the denominator is increasing in v_c. $\|$

Proof of Proposition 7: Suppose that the true variance of participation shocks is v_z' whereas uninformed agents estimate it to be v_z. The volatility of the price is

$$\text{var}_I(P) = H_1'^2 v_\theta + H_2'^2 v_z',$$

and when the true volatility of z is v_z, the volatility of the price is

$$\text{var}(P) = H_1'^2 v_\theta + H_2'^2 v_z',$$

where the subscripts I denotes irrationality. Now, substituting for H_1' from (25), and using (26), we have

$$\text{var}_I(P) - \text{var}(P) = (v_z' - v_z)\gamma_1/\gamma_2,$$

where

$$\begin{aligned}\gamma_1 &\equiv R^2 v_\epsilon^2 m^2 [kmv_\theta\{m^3 v_\theta(R - R_U) - m^2 R v_\theta + mR^2 v_\epsilon v_z(Rv_\epsilon - R_U(v_\epsilon + v_\theta)) - R^3 v_\epsilon^2 v_z\} \\ &+ (m^2 v_\theta + R^2 v_\epsilon^2 v_z)\{m^2 v_\theta(R - R_U) - mRv_\theta - R^2 R_U v_\epsilon v_z(v_\epsilon + v_\theta)\}]^2\end{aligned}$$

and

$$\begin{aligned}\gamma_2 &\equiv m^2(m^2 v_\theta + R^2 v_\epsilon^2 v_z)^2 [m^3 v_\theta(R - R_U) - m^2 R v_\theta + mR^2 v_\epsilon v_z\{Rv_\epsilon - R_U(v_\epsilon + v_\theta)\} \\ &- R^3 v_\epsilon^2 v_z]^2.\end{aligned}$$

Because γ_1 and γ_2 are positive, the volatility of the price is therefore higher when the volatility of the participation shock is underestimated ($v_z < v_z'$). $\|$

Figure 1: Correlation between total cash flows and cash flows of the traded asset, as a function of the feedback parameter, k

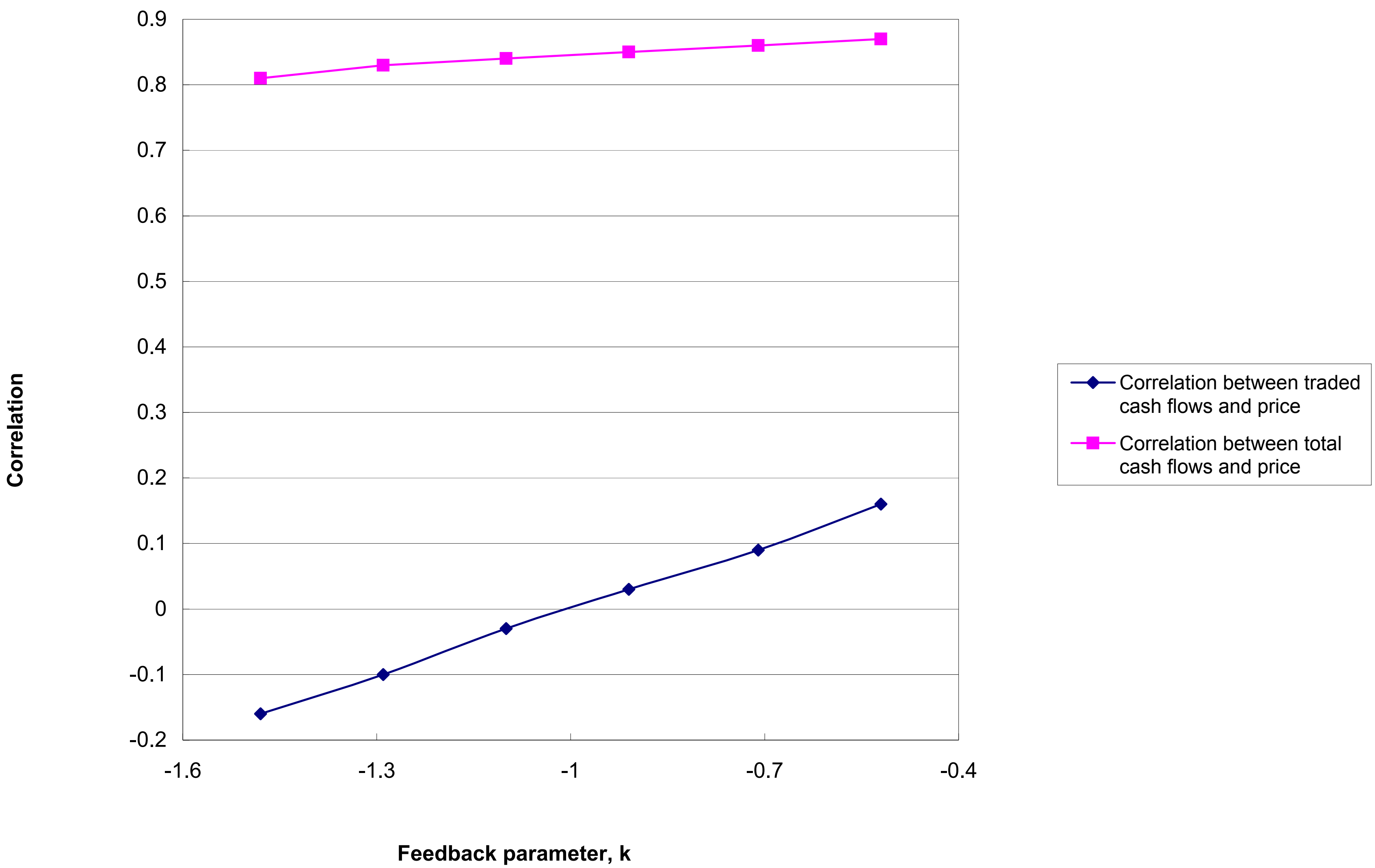

Figure 2: Correlation between cash flows and the market price as a function of the feedback parameter, k

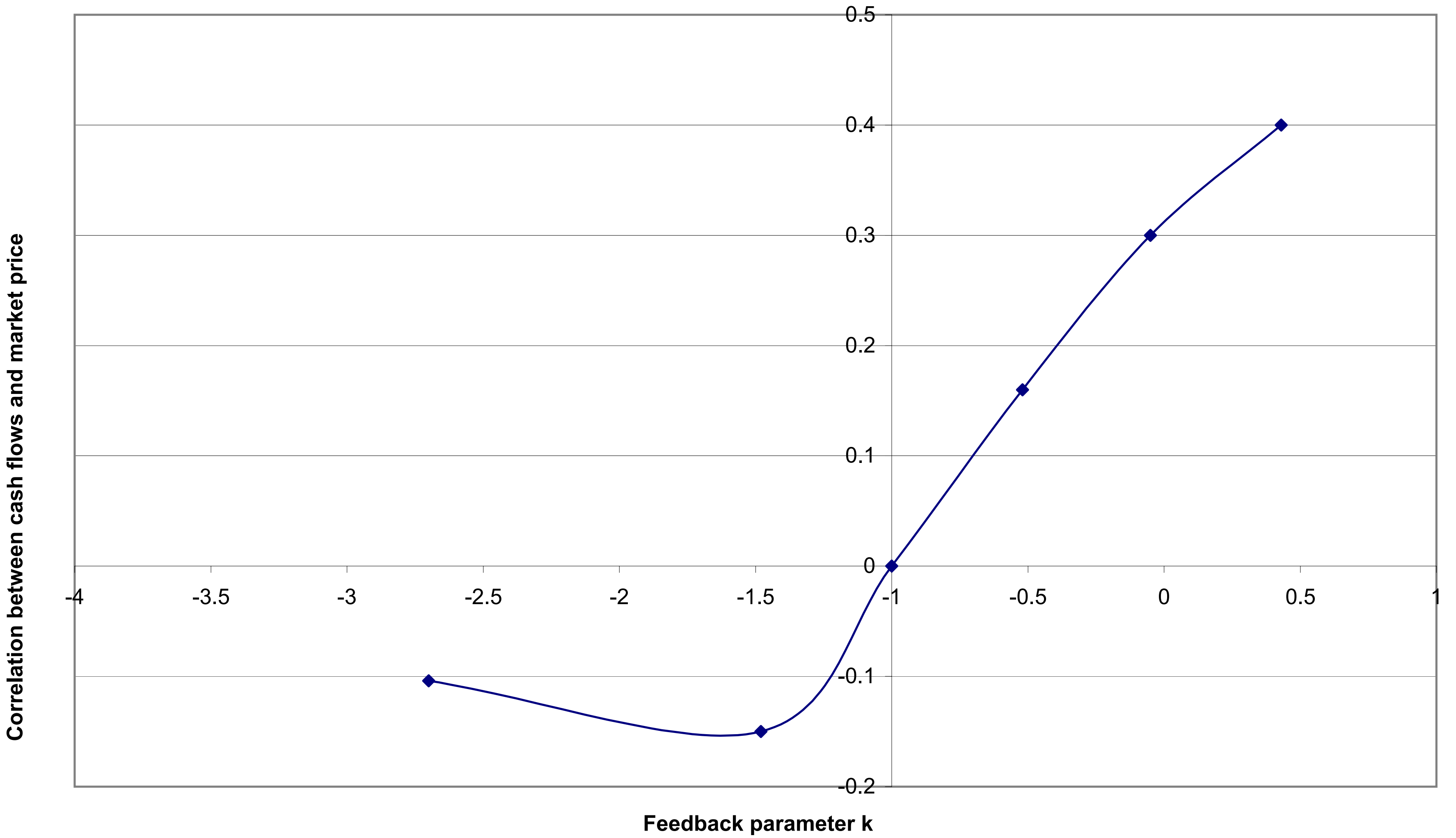

Figure 3: Correlation between cash flows and the market price as a function of the risk borne by the informed agents

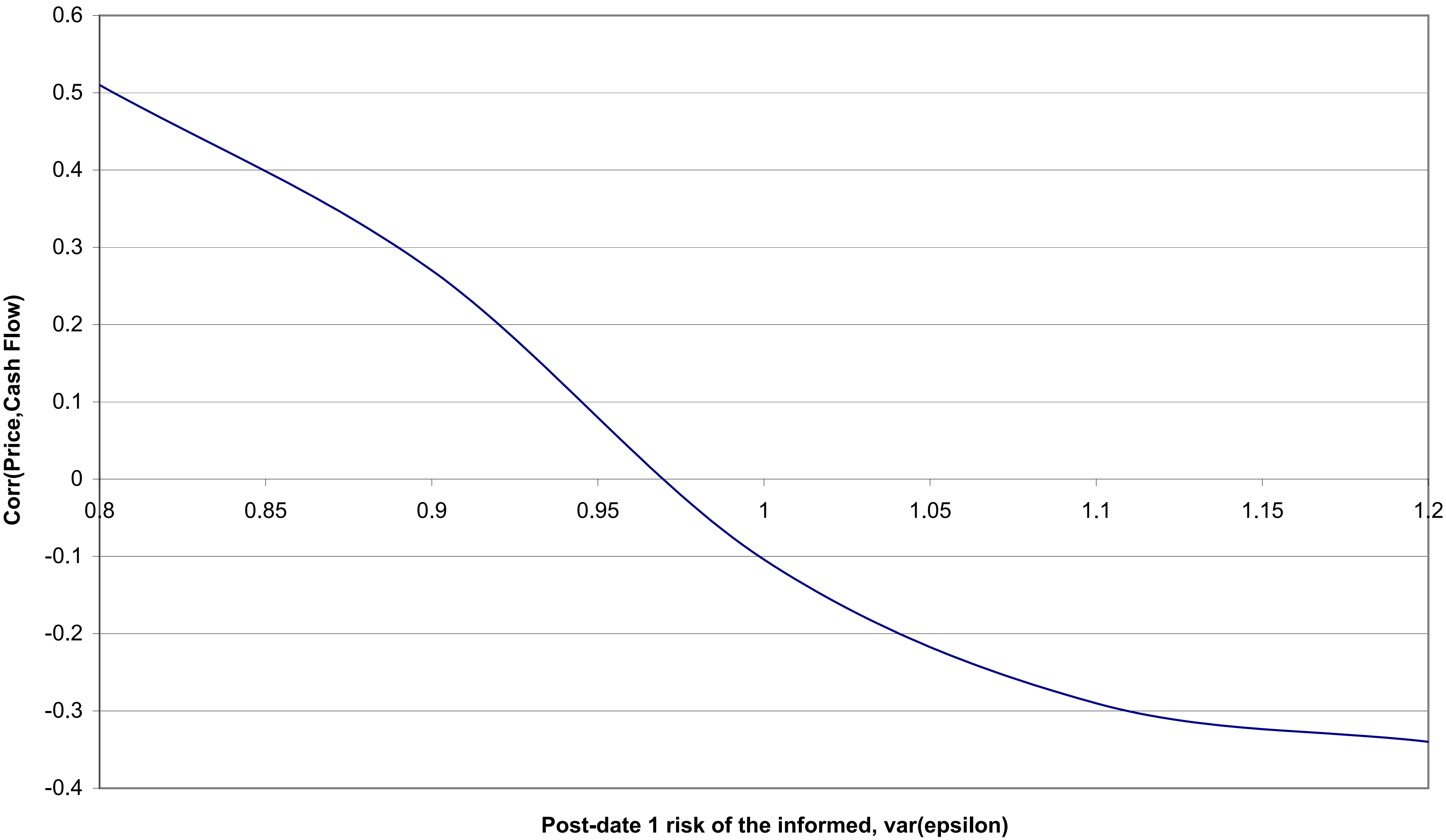

Figure 4: Correlation between cash flows and the market price as a function of the risk aversion of the informed agents