Product Variety and Asset Pricing*

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Abstract

We build a general-equilibrium asset pricing model of a production economy with multiple, imperfectly substitutable products. We derive closed-form analytical characterizations of the unique equilibrium and the corresponding pricing kernel in the basic single-sector and multi-sector versions of the model. The incorporation of heterogeneous products, whose mass can vary over time, has a significant impact on the equity premium and the risk-free rate. We employ the asset Euler equations derived from the representative agent's portfolio choice problem to estimate the risk aversion and discount rate parameters using the Generalized Method of Moments (GMM). The single-sector model reconciles the equity premium and risk-free rate for a relative risk aversion less than 5 and a quarterly discount rate of around 0.85. The more realistic multi-sector model, which incorporates intra- and inter-sector product substitutabilities, generates the observed equity premium and risk-free rate for relative risk aversion levels less than 2 and discount rates exceeding 0.9. Overall, our study highlights the importance of incorporating the multiplicity of imperfectly substitutable consumption goods in asset pricing models.

Key Words: product heterogeneity, imperfect substitutability, equity premium, risk-free rate puzzle
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1 Introduction

Traditional asset pricing models assume that there is a single consumption good in the economy (see Duffie (2001), Ljungvist and Sargent (2004), Cochrane (2005) for surveys). In reality, however, there are multiple consumption goods that are only imperfectly substitutable. We build a general-equilibrium asset pricing model of a production economy that incorporates agents' preferences for product variety. The model is parsimonious and tractable so that we can obtain closed-form analytical characterizations of the unique equilibrium and the corresponding pricing kernel that values assets in the economy. We calibrate the model and study its implications for asset prices. The incorporation of product variety has a significant impact on the equity premium and risk-free rate. We show that the observed equity premium and risk-free rate can be reconciled for moderate levels of risk aversion and empirically plausible values of discount rates. Overall, our results suggest that product variety is an important determinant of asset prices.

We first develop a discrete-time, infinite horizon model of an economy with a single sector in which there are heterogeneous firms producing distinct, imperfectly substitutable products. The single-sector model can also be viewed as a model of an economy with multiple sectors with the products of each sector being perfectly substitutable so that each sector can be described by a representative firm without loss of generality. We later develop the more realistic multi-sector model that incorporates intra-sector and inter-sector product substitutabilities. We view the single-sector model as a building block of the more realistic multi-sector model, and we use it to illustrate the main economic forces that drive our results. The economy has a continuum of identical agents with "constant elasticity of substitution" (CES) preferences for the goods produced by the firms. Agents own the capital stock and rent it to firms in each period. At the beginning of each period, the aggregate "state" of the economy, which affects the individual productivities of firms, is observed by all market participants. To operate during the period, each firm must supply a fixed amount of capital that depends on the aggregate state (e.g., see Comin and Gertler (2006), Jaimovich and Floetotto (2008)). After the capital is supplied, firms experience idiosyncratic productivity shocks that are independently and identically distributed across firms with a distribution that depends on the aggregate state of the economy. Firms make their production decisions for the period after observing their realized productivities.

Firms produce a continuum of differentiated goods in each period and are monopolistically competitive as in Dixit and Stiglitz (1977). Each firm produces a single good in which it enjoys a monopoly. Firms, however, make their output and pricing decisions taking the aggregate price index—a weighted average of the prices charged by all firms—as given. In the basic model, we make the simplifying assumption that capital cannot be augmented or depleted so that the output produced by firms is consumed by agents in each period. If a firm chooses not to produce, its owners can reinvest its capital stock in other active firms in the economy. In an Appendix, we show that our main implications are robust to an extension of the model that incorporates capital depreciation and investment.

We derive the unique equilibrium that satisfies two key conditions. The *free entry* condition ensures that the value of a firm in any period before its idiosyncratic productivity shock is realized must equal the fixed cost. The *market clearing* condition ensures that the aggregate revenue of active firms must equal the aggregate payoffs that agents obtain from renting their capital. The aggregate price index and the mass of active firms are endogenously determined in equilibrium. In particular, the mass of active firms (and, therefore, the mass of goods) in the economy is *endogenous* and varies with the aggregate state of the economy. The fact that the variety of products is endogenous affects the pricing kernel that determines asset prices in the economy.

Next, we derive the unique pricing kernel that values assets in the economy assuming that agents have access to a complete set of "one period ahead" Arrow securities that are contingent on the aggregate state. In contrast with traditional asset pricing models with a single consumption good, the nominal pricing kernel or stochastic discount factor is the product of the marginal utility of the representative agent at the "aggregate" consumption process (i.e. the consumptions of individual goods aggregated using the CES aggregator), the aggregate consumption itself and the aggregate revenue of active firms.

In the traditional consumption-based capital asset pricing model (CCAPM), where there is a single consumption good (or, alternately, all goods are perfect substitutes), the pricing kernel is completely determined by aggregate consumption. The central sources of the equity premium and risk-free rate puzzles are that aggregate consumption is not volatile enough and does not covary sufficiently with the market. Consequently, we need very high relative risk aversion levels to ensure that the volatility of the pricing kernel is high enough to match the observed equity premium. A high risk aversion, however, implies a low expected value of the pricing kernel and, therefore, an unrealistically high risk-free rate.

A key feature of the pricing kernel in our model is that the CES-aggregated consumption index incorporates the endogenous variety of products in the economy as well as the nontrivial elasticity of substitution between them. Consequently, the pricing kernel differs substantially from those derived in models that do not incorporate product variety, where aggregate consumption is simply the sum of consumptions of different goods. Using our characterization of the product market equilibrium, we show that the "one period ahead" pricing kernel in our model can be re-expressed in terms of the growth in the (endogenous) mass of firms or variety of products in the economy, and the growth in the non-centered $(\sigma - 1)$ moment of the productivity distribution of firms, where σ is the elasticity of substitution. In particular, the pricing kernel declines with an increase in product variety or an increase in firms' productivity distribution in the sense of first order stochastic dominance. A number of studies predict that product variety grows over time and, moreover, the growth in product variety is procyclical (Schumpeter (1939), Schmookler (1966), Shleifer (1986)). More recent studies show empirical support for procyclical product variety growth (Axerloglou (2003), Broda and Weinstein (2007)). The decline of the pricing kernel with product variety, therefore, implies a higher equity premium than in the traditional CCAPM. In addition, when compared to aggregate consumption, moments of the productivity distribution of firms are likely to be more volatile and covary more significantly with the market so that the equity premium could be much higher and the risk-free rate much lower than in traditional asset pricing models with a single consumption good.

We use the basic model to conduct a preliminary analysis of the extent to which the incorporation of product variety is able to reconcile the equity premium and risk-free rate puzzles using the generalized method of moments (GMM) as in Hansen and Singleton (1982). We first exploit our characterization of the product market equilibrium to re-express the pricing kernel in terms of the discount rate and the distribution of firms' revenues/sales. As in studies such as Longstaff and Piazzesi (2004) and Jermann (2005), this allows us to use more reliably measured corporate output data to conduct our estimation exercises (see also the discussion in Section 5 of Cochrane (2005)) We obtain quarterly sales data of non-financial public US firms from Compustat over the period 1962–2009. We calibrate the elasticity of substitution to 3.8 as in Bilbiie et al. (2012). Our GMM estimation exercise shows that the equity premium and risk-free rate are consistent with a relative risk aversion of 2.8 and a discount rate of around 0.8. A number of firms in the aggregate sample are producers of "intermediate" rather than "final" consumption goods. Consequently, we redo the estimation using a sub-sample of firms that only produce final "consumption goods" as surveyed in the Consumer Expenditure Surveys (CEX). Our estimation exercise with this subsample generates estimates of 4.9 for the relative risk aversion and 0.84 for the adjusted discount rate. As a benchmark for comparison, the classical CCAPM model generates risk aversion estimates between 16 and 26 in the various specifications using corporate output data. We also show that, for very large values of the product substitutability, in which case products are almost perfectly substitutable, the estimated risk aversion parameters range from 33 to 70, which highlights the quantitative impact of incorporating imperfect product substitutability in generating the observed equity premium and risk-free rate for empirically plausible risk aversion levels In conclusion, the incorporation of product variety even in the basic, single-sector model is able to generate a high equity premium for moderate relative risk aversion levels less than 5 and discount rates that are around 0.8. Although our estimates of the discount rate in the single-sector model are lower than values that might be considered empirically plausible, they are considerably higher than the discount rates obtained by recent studies such as Kocherlakota and Pistaferri (2009) that are approximately 0.5.

The single-sector model assumes a constant elasticity of substitution between products of different firms. As mentioned earlier, it could also be viewed as a model of an economy with multiple sectors in which each sector produces perfectly substitutable products. In reality, the products of firms within a sector are also imperfectly substitutable. The products of firms within a sector are likely to be closer substitutes than those of firms in different sectors. The incorporation of differing inter- and intra-sector product substitutabilities could have a significant impact on the pricing kernel and asset prices. Accordingly, we extend the model to consider an economy with multiple sectors. As in the basic model, we derive the unique product market equilibrium and the pricing kernel from the representative agent's portfolio choice problem. The pricing kernel can be expressed in terms of the "aggregate" consumption process (the consumptions of individual goods aggregated using the intra- and inter-sector CES aggregators) as well as the aggregate consumption process for the goods produced by *any* given sector. The fact that the mass of products produced by different sectors as well as agents' consumption choices of different goods could vary over time cause the pricing kernel to be considerably more volatile than in the single-sector model.

We carry out our GMM estimation exercise using the multi-sector pricing kernel. In particular, in contrast with the single-sector model, the asset Euler equations lead to an overidentified system because we can choose the aggregate consumption process *for any sector* to express the pricing kernel. We employ the moment conditions for market returns and the risk-free rates. In addition, we also use the moment conditions for individual sector returns in our estimation exercises. In the model, the intra-sector product substitutability is determined by the ratio of a firm's revenue to its profit. Accordingly, we calibrate the intra-sector product substitutability parameters for the different sectors using the average revenue to profit ratios for the sectors. Because the inter-sector product substitutability is not pinned down by the theory, we consider different values of the parameter to examine the robustness of our results. As in the single-sector case, we repeat our estimation exercises for the sub-sample of firms producing final consumption goods.

Across all the scenarios, the results of our estimation exercises are rather consistent and much stronger than in the single-sector case. Our estimates of the relative risk aversion are less than 1.5, and those of the discount rate range from about 0.9 to 0.97 that are closer to estimates of agents' discount rates in the experimental literature (Andersen et al. (2008)). Because we have an overidentified system, we are also able to test the model using Hansen's J-test of overidentifying restrictions. In all cases, we find that the model fails to be rejected by the data and our set of moments are valid.

Our implications are robust to a number of modifications and extensions of the model. In Appendix A, we extend the model to incorporate capital depreciation and investment. In Appendix B, we modify the model to allow for long-lived firms that incur sunk costs upon entry and no fixed costs in each period. Further, we allow for firms to experience "death" shocks that force them to exit the market as in Bilbiie et al (2012).

2 Related Literature

We contribute to the literature by building a general equilibrium model that incorporates product variety and studying its implications for asset prices. Scanlon (2008) extends the CCAPM to incorporate multiple products. He calibrates his model and shows that cyclical product variety growth can influence asset prices, generate risk-free rates in line with observed historical averages, and equity premia of about one-third of the observed values (thus a substantial improvement over the baseline CCAPM). In contrast with our study, he exogenously specifies the pricing kernel and further assumes that all relative good prices are identical. In addition, Scanlon's model relies on the calibration of more than 20 parameters (including the coefficient of relative risk aversion of the representative agent) for many of which only crude estimates are available. In contrast, we derive the pricing kernel within a general equilibrium model in which the relative prices of goods are distinct and determined endogenously. Apart from being consistent with data, the variation in relative good prices causes the pricing kernel to differ significantly from the one that Scanlon assumes because it also depends on the aggregate price index; a weighted average of the relative prices of different goods (see Dixit and Stiglitz (1977)). We show how to relate the pricing kernel to sales data and use GMM to structurally estimate the relative risk aversion and discount rate that can rationalize the observed historical equity premia and risk free rates.

A few earlier studies analyze the CCAPM in a multi-good framework. Piazzesi, Schneider, and Tuzel (2007) incorporate a consumption bundle of non-housing consumption and housing services into the CCAPM. Assuming both components are nonseparable in utility, they show that cyclical variation in the housing share raises the expected equity premium, while long-run trends and volatility in the housing share reduce the risk-free rate. Pakoš (2004) and Yogo (2006) focus on a consumption bundle comprising nondurable and durable components, and show how the interaction of both components can increase aggregate consumption risk. Chetty and Szeidl (2007) demonstrate that "consumption commitments" increase the variability of discretionary consumption, and in turn raise the level of consumption risk. Aït-Sahalia, Parker, and Yogo (2004) introduce luxury goods into the CCAPM, and show how the covariance of the consumption of luxury goods with equity returns raises the equity premium. Our approach is different from this literature in that we do not single out particular types of consumption goods that covary more with equity returns. The imperfect product substitutability among all goods and sectors, instead, is the key that drives the endogenous product variety changes and our results.

From a methodological standpoint, our model is related to that of Bilbiie et al (2012) that, in turn, builds on Melitz (2003). They examine the propagation role of expanding product variety on business cycles, whereas our focus is on the asset pricing implications of product variety. Further, they assume homogeneous firms and log utility of consumption for the representative consumer. Because firms are heterogeneous in our model, we allow for non-symmetric equilibria where firms' productivity distribution is non-degenerate. The non-degeneracy of firms' productivity distribution has a significant impact on the pricing kernel and, thereby, asset prices.

We use our model to represent the pricing kernel in terms of firms' revenues and, therefore, use corporate output data to examine the equity premium and risk-free rate puzzles (e.g., see

Kocherlakota (1996) and Campbell (1999) for surveys of the vast literature on these puzzles). In this respect, our approach is similar to that of studies such as Longstaff and Piazzesi (2004) that use corporate output data to examine these puzzles. They show that the incorporation of the sensitivity of corporate cash flows to aggregate shocks leads to an equity premium of about 2.3% and levels of equity volatility that are consistent with those observed in the data. They cannot, however, resolve the risk-free rate puzzle. Jermann (2010) also uses output data to address the equity premium puzzle. In Section 5.2 of his survey, Cochrane (2007) too advocates the use of corporate output data that is more reliably measured than consumption data.

A number of studies modify the traditional CCAPM by altering the preferences of the representative agent to incorporate recursive preferences (Epstein and Zin (1991)) or habit formation (Campbell and Cochrane (1999)). We complement these studies by developing a general-equilibrium model of an economy with multiple, imperfectly substitutable products. We show that the explicit incorporation of preferences for (endogenous) product variety has a quantitatively significant impact on asset prices even with otherwise standard CRRA utility functions for agents. It is also worth mentioning that, because we have a representative agent, we do not rely on the presence of idiosyncratic risk with countercyclical variance and incomplete markets to generate high equity premia (e.g., see Constantinides and Duffie (1996)).

3 Single-Sector Economy

We first build a model of a "single sector" economy with heterogeneous firms producing imperfectly substitutable products. The model can also be viewed as a description of an economy with multiple sectors where firms in each sector produce perfectly substitutable goods. We later develop a more realistic "multi-sector" model that incorporates intra-sector and inter-sector product substitutabilities. The analysis of the simple, single-sector model serves to clarify the main economic forces that drive our results. The economy has an infinite horizon with discrete dates, $0, 1, 2, 3, ...$ We alternately refer to the period $[t, t+1]$ as period t. The economy is populated by a continuum of identical agents in each period with measure one At date 0, each agent is endowed with K units of a single "capital" good. The capital good cannot be consumed so that agents derive no *direct* utility from it. The capital good can, however, be used to produce multiple "consumption" goods.

In each period, firms rent capital from agents. At the beginning of each period, the aggregate "state" of the economy, which determines the individual productivities of firms, is observed by all market participants. To operate during the period, each firm is required to supply a *fixed* amount of capital that depends on the aggregate state as in studies such as Comin and Gertler (2006) and Jaimovich and Floetotto (2008). After the capital is supplied, firms experience idiosyncratic productivity shocks that are independently and identically distributed across firms with a distribution that depends on the aggregate state of the economy. After observing their realized productivities, firms make their production decisions for the period. Production requires additional *variable* capital that firms rent from agents.

Firms produce a continuum of differentiated goods in each period and are monopolistically competitive. Each firm produces a single good in which it enjoys a monopoly. Firms, however, make their output and pricing decisions taking the aggregate price index—a weighted average of the prices charged by all firms—as given. In the basic model, we assume for simplicity that capital cannot be augmented or depleted so that the output produced by firms is consumed by agents in each period. Hence, the total amount of capital in the economy is K through time. In the basic model, therefore, economic growth arises from growth in the variety of products in the economy (see Melitz (2003)). In Appendix A, we modify the model to allow for capital depreciation and investment, and demonstrate that our key implications are unaltered.

Note that the fixed capital is supplied at the beginning of *each* period *after* the aggregate state of the economy is realized, but before firms' idiosyncratic productivity shocks are observed. Firms can choose whether to supply the capital after observing the aggregate state. Further, a firm can choose whether or not to produce for the period after observing its idiosyncratic productivity shock. If a firm chooses not to produce, its owners can reinvest its capital stock in other active firms in the economy. A key aspect of the model is that the mass of active firms (and, therefore, the mass of goods) in the economy is *endogenous* and varies with the aggregate state of the economy. The endogenous mass of firms, in turn, affects the pricing kernel that determines asset prices in the economy. In Appendix 8, we present a modified model with long-lived firms who incur sunk entry costs rather than fixed costs in each period. Further, firms can experience "death" shocks in any period that force them to exit the market (Melitz (2003)). The modified model does not alter the insights gleaned from the model we develop in the main body of the paper.

Figure 1 shows the timeline of events in the model. Note that the aggregate and idiosyncratic shocks occur at the beginning of the period as in the standard "real business cycle" (RBC) model (see

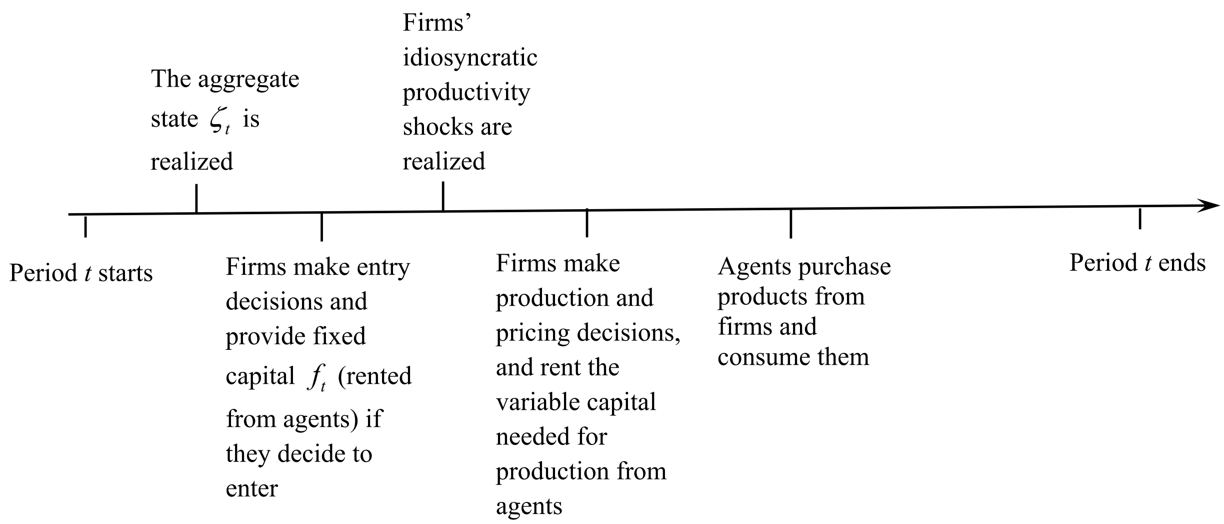

Figure 1: **Timeline of the Model**

Ljungqvist and Sargent (2004)). Consequently, although the timeline shows the sequence of events, firms' entry and production decisions as well as agents' consumption decisions are determined at the beginning of the period based on the aggregate and idiosyncratic productivity shocks. We now describe the elements of the model in more detail in the subsequent sub-sections.

3.1 Preferences

At any date s, the representative consumer has preferences for consumption of the continuum of goods produced by the economy in future periods that are described by

$$\mathcal{U} = E_s \sum_{t=s}^{\infty} \beta^{t-s} U(\Lambda_t), \tag{1}$$

where $E_s[.]$ denotes the expectation with respect to firms' future productivity distributions conditional on the information available at date s. The function U is strictly increasing and concave and satisfies the Inada conditions, $U'(0) = \infty$ and $U'(\infty) = 0$. The consumer's time discount rate is $\beta \in (0, 1)$. In (1),

$$\Lambda_t = \left[\int_{\Omega_t} q_t(\omega_t)^\rho d\omega_t \right]^{\frac{1}{\rho}} ; 0 < \rho < 1. \tag{2}$$

In (2), Ω_t is the set of available goods in the economy in period t, and ω_t is a finite measure on the Borel σ-algebras of Ω_t. It is important to note that the set of available goods, Ω_t in period t is *endogenous* and could, in general, vary over time.

We write all prices in nominal terms, and money (dollars) are simply a unit of account, playing no role in the economy. Our focus throughout the paper is on real variables. If $p_t(\omega_t)$ is the price of good ω_t in period t then, as shown by Dixit and Stiglitz (1977), the optimal consumption and expenditure decisions for individual goods are

$$q_t(\omega_t) = \Lambda_t \left[\frac{p_t(\omega_t)}{P_t}\right]^{-\sigma}; \tag{3}$$

$$\xi_t(\omega_t) = \mathcal{R}_t \left[\frac{p_t(\omega_t)}{P_t}\right]^{1-\sigma}, \ t \in \{0, 1, 2, 3, ...\}, \tag{4}$$

where $\mathcal{R}_t$ is the aggregate expenditure of the representative consumer in period t, and

$$P_t = \left[\int_{\Omega_t} p_t(\omega_t)^{1-\sigma} d\omega_t\right]^{\frac{1}{1-\sigma}}. \tag{5}$$

is the *aggregate price index* in period t—a weighted average of the prices charged by the firms—that determines the consumption and expenditure decisions by (3) and (4).

In (5), the elasticity of substitution is

$$\sigma = \frac{1}{1-\rho} > 1. \tag{6}$$

Following Dixit and Stiglitz (1977), each active firm produces a single product (that is consumed by the representative consumer) in which the firm has a monopoly. However, the firms compete monopolistically in the sense that they take the aggregate price index P_t as given in making the output and pricing decisions for their individual products. Each active firm in a period faces the price elasticity of demand σ. Given that there is a continuum of firms, no single firm perceives itself as having an impact on aggregate equilibrium outcomes.

3.2 Technology

In each period, each agent rents his endowment of capital to firms. At the beginning of period $[t, t+1]$, with $t \geq 0$, the "aggregate state" of the economy, which is observed by all agents, is $\zeta_t \in (0, \infty)$. At date 0, the aggregate state, ζ_0, is non-stochastic. Let the history of aggregate states up to date t be denoted as $\zeta^t \equiv [\zeta_t, \zeta_{t-1}, ..., \zeta_0]$. The unconditional probability density of observing the sequence of aggregate states, ζ^t, is $\mathcal{G}_t(\zeta^t)$. We denote the conditional probability density of the aggregate state ζ_{t+1} in period $t+1$ given the history ζ^t up to period t as $\mathcal{G}_{t+1}(\zeta_{t+1}|\zeta^t)$.

Production requires *fixed* and *variable* amounts of capital, where the variable amount of capital depends on the quantity of a firm's output. Specifically, after observing the aggregate state ζ_t, each firm rents a *fixed* amount of capital f_t that could vary with the aggregate state. Firms subsequently experience idiosyncratic productivity shocks that are i.i.d. across firms and drawn from a continuous distribution with density $\mathfrak{g}_t$. The distribution, $\mathfrak{g}_t$, could also depend on the aggregate state. We avoid explicitly indicating the dependence of f_t, $\mathfrak{g}_t$ and other variables on the aggregate state to simplify the notation.

Consider a firm with productivity α. If the firm chooses to produce q units of its good, the amount of *variable* capital it requires is

$$k = \frac{q}{\alpha}. \tag{7}$$

In making its output and pricing decisions, the firm anticipates the demand schedule (3). Further, each firm takes the aggregate price index P_t and the consumption index Λ_t of the representative consumer as given. If the price of the firm's product is p, let $q(p)$ denote the demand for the product as given by (3). Let r_t be the capital rental rate in period $[t, t+1]$.

The price $p_t(\alpha)$ set by the firm in period t maximizes its *expected profits*—revenue net of capital rental costs—that is, it solves

$$p_t(\alpha) = \arg\max_p [pq(p) - r_t \frac{q(p)}{\alpha}]. \tag{8}$$

Note that there are *multiple* (in fact, a continuum of) firms with the *same* productivity, α. These firms produce distinct goods that have the same price, $p_t(\alpha)$. To simplify the exposition, we slightly abuse the notation to have the productivity, α, as the argument of the price set by the firm.

By (3) and (8), the optimal price set by the firm is

$$p_t(\alpha) = \frac{r_t}{\rho\alpha}, \tag{9}$$

and its output is

$$q_t(\alpha) = \mathcal{R}_t \left(P_t\right)^{\sigma-1} \left(\frac{\rho\alpha}{r_t}\right)^{\sigma}. \tag{10}$$

The firm's *revenue* is

$$\xi_t(\alpha) = p_t(\alpha)q_t(\alpha) = \mathcal{R}_t(P_t\rho\alpha)^{\sigma-1}r_t^{1-\sigma}. \tag{11}$$

By (7) and (10), the total capital the firm rents for production (excluding the initial fixed capital f_t) is

$$k_t(\alpha) = \mathcal{R}_t\,(P_t)^{\sigma-1}\,\rho^{\sigma}\alpha^{\sigma-1}r_t^{-\sigma}. \tag{12}$$

By (11) and (12), and using (6), the firm's *profit*—revenue less capital rental costs—is

$$\pi_t(\alpha) = \xi_t(\alpha) - r_t k_t(\alpha) = \frac{\xi_t(\alpha)}{\sigma}. \tag{13}$$

4 Product Market Equilibrium

Before proceeding to specify the traded assets in the economy, we derive the product market equilibrium because it does not depend on the asset market structure.

4.1 Equilibrium Conditions

First, similar to Comin and Gertler (2006), because there is free entry of firms in each period, and each firm is required to supply the fixed capital f_t at the beginning of period t, the expected profit of an active firm firm must equal the cost of renting the fixed amount of capital f_t. Given that a fixed amount of capital f_t is supplied at the beginning of each period, we have the *free entry equilibrium condition*

$$r_t f_t = \int_0^{\infty} \pi_t(\alpha)\mathfrak{g}_t(\alpha)d\alpha. \tag{14}$$

Note that the above condition must hold for each possible realization of the aggregate state ζ_t in period t.

Second, product markets must clear, which requires that the aggregate revenue of producing firms in each period equal the aggregate expenditure by consumers. By (11) and (13), we have

$$\mathcal{R}_t = M_t \int_0^{\infty} \xi_t(\alpha)\mathfrak{g}_t(\alpha)d\alpha = \frac{1}{\sigma}M_t \int_0^{\infty} \pi_t(\alpha)\mathfrak{g}_t(\alpha)d\alpha, \tag{15}$$

where M_t is the mass of producing firms that is endogenously determined by the above condition. Further, because consumers uses their entire income for consumption of the available goods in the economy, the aggregate expenditure of consumers must equal the aggregate income. Since the total capital stock is K and the rental rate of capital in period t is r_t, we have

$$\mathcal{R}_t = r_t K. \tag{16}$$

4.2 Equilibrium Characterization

Dividing (14) by (15) and using (16),

$$M_t = \frac{K}{f_t \sigma}. \tag{17}$$

From (17), we see that the mass of firms (and, therefore, the mass of goods in the economy) declines with the fixed capital f_t and the elasticity of substitution σ. In particular, because f_t varies with the aggregate state of the economy, ζ_t, the mass of firms also varies. It is reasonable to assume that, as the aggregate state of the economy improves, the required fixed capital investment declines. It follows from (17) that the mass of firms then increases with the aggregate state, that is, better aggregate states are associated with a larger mass of firms and a greater variety of products that is consistent with empirical evidence for procyclical product variety growth (Axerloglou (2003), Broda and Weinstein (2007)). Note from (17) that, consistent with the fact that the equilibrium mass of firms, M_t, is a real variable, it is determined by the real variables, K, f_t and σ.

By (13) and (14), we have

$$r_t f_t = \int_0^\infty \frac{\mathcal{R}_t \left(P_t \rho \alpha\right)^{\sigma-1} r_t^{1-\sigma}}{\sigma} \mathfrak{g}_t(\alpha) d\alpha. \tag{18}$$

Substituting (16) in (18), using (17) and re-arranging terms, we get

$$\frac{r_t}{P_t} = \rho M_t^{\frac{1}{\sigma-1}} \bar{\alpha}_t, \tag{19}$$

where $\bar{\alpha}_t$ represents the average productivity,

$$\bar{\alpha}_t := \left(\int \alpha^{\sigma-1} \mathfrak{g}_t(\alpha) d\alpha \right)^{\frac{1}{\sigma-1}}. \tag{20}$$

The quantity, $\frac{r_t}{P_t}$, is the *real* rental rate of capital. Equivalently, from (9) and (19), the real price of the product of the firm with average productivity is

$$\frac{p_t(\bar{\alpha}_t)}{P_t} = M_t^{\frac{1}{\sigma-1}}. \tag{21}$$

It is also reasonable to assume that the productivity distribution $\mathfrak{g}_t$ increases with the aggregate state of the economy in the sense of first order stochastic dominance, that is, the firm productivity distribution "shifts to the right" as the aggregate state improves so that higher firm productivities are more likely. Thus by (19), the real rental rate of capital increases with the aggregate state.

5 Asset Markets and Asset Prices

We now describe the asset market structure. The economy in any period t is determined by the aggregate state history ζ^t and the distribution of firms. In what follows, all period t variables (carrying a subscript t) depend on the history of aggregate states ζ^t and are viewed as stochastic processes adapted to the filtration generated by (ζ_t). Similarly, expectations conditional with respect to aggregate information at t (that is, conditional on ζ^t) are denoted by $E_t(\cdot)$. We assume that, at each date, there is a complete set of "one period ahead" Arrow securities promising one dollar next period contingent on the future aggregate state. The pricing kernel (state price density) associated to these Arrow securities is denoted by the process (Q_t). Equivalently, the price at date 0 of a dollar at t in history ζ^t is $Q_t(\zeta^t)\mathcal{G}_t(\zeta^t)$ and the price at t of the Arrow security promising a dollar tomorrow in history $\zeta^{t+1} = (\zeta_{t+1}, \zeta^t)$ is

$$\frac{Q_{t+1}(\zeta^{t+1})\mathcal{G}_{t+1}(\zeta^{t+1})}{Q_t(\zeta^t)\mathcal{G}_t(\zeta^t)} = \frac{Q_{t+1}(\zeta^{t+1})}{Q_t(\zeta^t)}\mathcal{G}_{t+1}(\zeta_{t+1}|\zeta^t).$$

5.1 Arrow Security Prices

Since all agents are identical, we can focus on the representative agent who holds capital K. The agent enters period t with holdings of $\widetilde{a}_t$ units of the Arrow security purchased in period $t-1$. The agent's rental income for the period is $r_t K$. In addition to consuming, the agent acquires a portfolio $\widetilde{a}_{t+1}$ of one period Arrow securities traded at t. Note that the agent purchases Arrow securities corresponding to every possible aggregate state in the next period.

The agent's budget constraint is

$$\overbrace{P_t\Lambda_t}^{\text{expenditure on consumption}} + \overbrace{\int \widetilde{a}_{t+1}(\zeta^{t+1})\frac{Q_{t+1}(\zeta^{t+1})}{Q_t(\zeta^t)}\mathcal{G}_{t+1}(\zeta_{t+1}|\zeta^t)d\,\zeta_{t+1}}^{\text{expenditure on "next period" Arrow securities}} \leq$$

$$\leq \overbrace{\widetilde{a}_t}^{\text{payoff from "previous period" Arrow securities}} + \overbrace{r_t K}^{\text{rental income}} . \tag{22}$$

To rule out Ponzi schemes, the holdings in Arrow securities must be bounded below by an exogenous limit $-A$ where $A > 0$, that is,

$$\widetilde{a}_{t+1} \geq -A \quad \forall t. \tag{23}$$

The agent chooses his consumption plan and purchases of Arrow securities to solve

$$\max_{\{\widetilde{a}_t,\widetilde{c}_t\}} \mathcal{U} = \max_{\{\widetilde{a}_t,\widetilde{c}_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U\left(\Lambda_t\right) \tag{24}$$

subject to the sequence of budget constraints (22) and the borrowing constraints (23). At the optimum, the borrowing constraints (23) are non-binding. The first order conditions of the agent's maximization problem pin down the pricing kernel Q and stochastic discount factor (SDF) for valuing nominal assets,

$$\Phi_{t+1} := \frac{Q_{t+1}}{Q_t} = \beta\frac{U'(\Lambda_{t+1})P_t}{U'(\Lambda_t)P_{t+1}} = \beta\frac{U'(\Lambda_{t+1})\Lambda_{t+1}\mathcal{R}_t}{U'(\Lambda_t)\Lambda_t\mathcal{R}_{t+1}}. \tag{25}$$

The aggregate consumption index Λ_t in the SDF can be expressed in terms of the primitive parameters of the economy and the aggregate sales. Indeed, by (19),

$$\Lambda_t = K\frac{r_t}{P_t} = K\rho\left(\frac{K}{f_t\sigma}\right)^{\frac{1}{\sigma-1}}\bar{\alpha}_t. \tag{26}$$

5.2 Asset Euler Equations

Consider any (nominal) asset in the economy. If R_{t+1} is the realized return of the asset over the period $[t, t+1]$, the following Euler equation must be satisfied:

$$1 = E_t \left[\Phi_{t+1} R_{t+1} \right]. \tag{27}$$

As in the traditional asset pricing literature, assume that

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma}; \gamma > 0. \tag{28}$$

By (25), the SDF is

$$\Phi_{t+1} = \beta \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right)^{1-\gamma} \frac{\mathcal{R}_t}{\mathcal{R}_{t+1}} = \beta \left(\frac{M_{t+1}}{M_t} \cdot \frac{\bar{\alpha}_{t+1}^{\sigma-1}}{\bar{\alpha}_t^{\sigma-1}} \right)^{\frac{1-\gamma}{\sigma-1}} \frac{\mathcal{R}_t}{\mathcal{R}_{t+1}}. \tag{29}$$

Let us now qualitatively explore the potential of the incorporation of product variety to reconcile the "equity premium" and "risk-free" rate puzzles. The central sources of the puzzles in the traditional consumption-based asset pricing model is that aggregate consumption is not volatile enough and does not covary sufficiently with the market. Consequently, we need very high values of γ to ensure that the volatility of the pricing kernel is high enough. This worsens the risk-free rate puzzle as the expectation of the pricing kernel decreases with high γ.

The term, $\left(\frac{M_{t+1}}{M_t}\right)$ is the growth in the mass of firms/products, while $\frac{\bar{\alpha}_{t+1}^{\sigma-1}}{\bar{\alpha}_t^{\sigma-1}}$ is the growth in the non-centered $(\sigma - 1)$ moment of the productivity distribution of firms. Both these terms are raised to the power $\frac{1-\gamma}{\sigma-1}$ in the integrals above. As discussed earlier, if the distribution of firms' productivities increases with the aggregate state, then the number of firms covaries with the market return. Moments of the firms' productivity distribution also covary with the market return. Further, if ρ is significantly less than one and, therefore, σ is small (products are imperfectly substitutable), the exponent of the two terms could be large and negative for even moderate levels of the relative risk aversion γ.

6 Empirical Analysis of Single-Sector Model

In this section, we use the GMM approach to estimate the risk aversion and discount rate using the Euler equation (27) (see Hansen and Singleton (1983)).

6.1 Data and Empirical Methodology

We first re-express the pricing kernel in a more convenient form for our estimation. Denote the productivity of the least productive firm in period t by $\underline{\alpha}_t$. For an arbitrary firm with productivity α, the ratio of sales of this firm to the least productive firm is

$$\frac{\xi_t(\alpha)}{\xi_t(\underline{\alpha}_t)} = \frac{\alpha^{\sigma-1}}{\underline{\alpha}_t^{\sigma-1}}.$$

Consequently,

$$\frac{\int_0^\infty \xi_t(\alpha)\mathfrak{g}_t(\alpha)d\alpha}{\xi_t(\underline{\alpha}_t)} = \frac{\int_0^\infty \alpha^{\sigma-1}\mathfrak{g}_t(\alpha)d\alpha}{\underline{\alpha}_t^{\sigma-1}}$$

Since $\mathcal{R}_t = M_t \int_0^\infty \xi_t(\alpha)\mathfrak{g}_t(\alpha)d\alpha$, it follows from the above and (20) that

$$\frac{\mathcal{R}_t}{\xi_t(\underline{\alpha}_t)} = M_t \cdot \frac{\bar{\alpha}_t^{\sigma-1}}{\underline{\alpha}_t^{\sigma-1}}.$$

By (29), the SDF can be expressed as follows:

$$\Phi_{t+1} = \beta \left(\frac{\underline{\alpha}_{t+1}}{\underline{\alpha}_t}\right)^{1-\gamma} \left(\frac{\mathcal{R}_{t+1}/\xi_{t+1}(\underline{\alpha}_{t+1})}{\mathcal{R}_t/\xi_t(\underline{\alpha}_t)}\right)^{\frac{1-\gamma}{\sigma-1}} \cdot \frac{\mathcal{R}_t}{\mathcal{R}_{t+1}}. \tag{30}$$

We allow for a stochastic growth rate of minimum productivity, $\frac{\underline{\alpha}_{t+1}}{\underline{\alpha}_t}$, but assume that it is independent of the other factors in the SDF above and on asset returns conditional on period t information. Further, assume that

$$E_t\left[\left(\frac{\underline{\alpha}_{t+1}}{\underline{\alpha}_t}\right)^{1-\gamma}\right] = \mu, \tag{31}$$

where μ is a constant.

Let the market return over period $[t, t+1]$ be R_{t+1}^M and the risk-free rate be R_t^{rf}. The asset

Euler equations corresponding to the market return and the risk-free rate are

$$E_t[\Phi_{t+1}(R_{t+1}^M - R_t^{rf})] = 0 \tag{32}$$

and

$$E_t[\Phi_{t+1}(1 + R_t^{rf})] = 1 \tag{33}$$

By (30) and (31), we can rewrite (32) as

$$\beta\mu E_t\left[\left(\frac{\mathcal{R}_{t+1}/\xi_{t+1}(\underline{\alpha}_{t+1})}{\mathcal{R}_t/\xi_t(\underline{\alpha}_t)}\right)^{\frac{1-\gamma}{\sigma-1}} \cdot \frac{\mathcal{R}_t}{\mathcal{R}_{t+1}}(R_{t+1}^M - R_t^{rf})\right] = 0, \tag{34}$$

and (33) as

$$\beta\mu E_t\left[\left(\frac{\mathcal{R}_{t+1}/\xi_{t+1}(\underline{\alpha}_{t+1})}{\mathcal{R}_t/\xi_t(\underline{\alpha}_t)}\right)^{\frac{1-\gamma}{\sigma-1}} \cdot \frac{\mathcal{R}_t}{\mathcal{R}_{t+1}}(1 + R_t^{rf})\right] = 1. \tag{35}$$

If we define the "adjusted" discount factor $\bar{\beta}$ as

$$\bar{\beta} = \beta\mu, \tag{36}$$

then (34) and (35) can be used to identify the relative risk aversion, γ, and the adjusted discount rate, $\bar{\beta}$. The Euler equations (32) and (33) are expressed in terms of the aggregate sales of firms, $\mathcal{R}_{t+1}$ and $\mathcal{R}_t$, and the sales of the smallest/least productive firm, $\xi_{t+1}(\underline{\alpha}_{t+1})$ and $\xi_t(\underline{\alpha}_t)$, in periods $t+1$ and t, respectively.

We use quarterly sales data of non-financial public US firms from Compustat over the period 1962–2009. To map the model to the data, therefore, the length of each period is a quarter. In the model, firms make production decisions and generate sales at the beginning of each period. Consistent with this "beginning-of-quarter" timing convention, and in conformity with previous studies (e.g., see Campbell (1999), Yogo (2006)), we measure sales growth for a quarter as the next quarter's sales divided by the current quarter's sales. [1] We also take $\xi_t(\underline{\alpha}_t)$ to be the minimum sales among all firms. In unreported results, we find similar results when we define $\xi_t(\underline{\alpha}_t)$ to be average revenue of selected lower quantiles (5 % and 1 %) of firms.

[1] Because firms report sales for fiscal quarters which may not coincide with calendar quarters, we use the sales data in the first fiscal quarter that starts after the beginning of calendar quarter t.

Because the products of certain firms are intermediate goods and are not used for final consumption, the aggregate revenue may overestimate the production of consumption goods. Accordingly, we use an additional method to measure the aggregate sales of consumption goods. We classify an industry (in the Fama-French 48 industries) as a "final consumption" industry if it produces categories of the consumption goods surveyed in the Consumer Expenditure Surveys (CEX). The classification is shown in Table A1. We then use the aggregate consumption of all firms in the final consumption industries to proxy for the aggregate revenue $\mathcal{R}_t$. We acknowledge that the classification is not perfect as some industries produce both final and intermediate goods, but incorporating this in a consistent manner is very difficult given the available data.

We obtain the monthly value-weighted market returns from CRSP and the monthly risk-free returns from Ken French's website. We then calculate quarterly market returns and quarterly risk-free returns by compounding the monthly returns. Table 1 presents the summary statistics of the returns and revenue ratios.

By (34) and (33), the sample analogue of the SDF can be written as

$$\hat{\Phi}_{t+1} = \bar{\beta} \left(\frac{\hat{\mathcal{R}}_{t+1}/\hat{\xi}_{t+1}(\underline{\alpha}_{t+1})}{\hat{\mathcal{R}}_t/\hat{\xi}_t(\underline{\alpha}_t)} \right)^{\frac{1-\gamma}{\sigma-1}} \frac{\hat{\mathcal{R}}_t}{\hat{\mathcal{R}}_{t+1}}, \tag{37}$$

where we use "hats" $(\hat{\ })$ to denote the empirical proxies for model variables. In the GMM estimation, we use the following sample analogue of (34):

$$\frac{1}{T} \sum_{t=1}^{T} \hat{\Phi}_{t+1}(R_{t+1}^M - R_t^{rf}) = 0. \tag{38}$$

The sample analogue of (35) is

$$\frac{1}{T} \sum_{t=1}^{T} \hat{\Phi}_{t+1}(1 + R_t^{rf}) = 1. \tag{39}$$

6.2 Results

Using the sample representation of the pricing kernel (37) and the moment condition (38), we estimate the risk aversion γ. In the estimation we consider two possible values of σ. We choose the first value, $\sigma = 3.8$, following Bilbiie, Ghironi and Melitz (2012). In the second case, we estimate σ

using the equation (13),

$$\sigma = \int_0^\infty \frac{\xi_t(\alpha)}{\pi_t(\alpha)} \mathfrak{g}_t(\alpha) d\alpha. \tag{40}$$

The sample analogue of (40) is the average of sales to profits ratios across all firms in our sample. Using this approach, we set $\hat{\sigma} = 6.2$.

We report the estimation results in Table 2. In the estimation, we adjust standard errors for heteroskedasticity and autocorrelation following Newey and West. The results show that the equity risk premium is consistent with a risk aversion of 2.8, when $\sigma = 3.8$ and sales of all firms are used. Among the different scenarios, the highest estimate is 8.3 that isachieved when we use both $\sigma = 6.2$ and sales normalized by consumption shares. These values are much lower than the typical values of relative risk aversion (>50) required to match the equity risk premium in classical consumption-based asset pricing models (see, for example, Mehra and Prescott (1985)).

We next estimate the relative risk aversion γ and the adjusted discount factor $\bar{\beta}$ simultaneously using the moment conditions (38) and (39). Table 3 reports the results of the GMM estimation. In all the scenarios, the adjusted discount factor $\bar{\beta}$ is estimated to be 0.81. Note from (36), however, that the adjusted discount factor differs from the standard time discount factor β. To come up with an estimate for μ, we use the Bureau of Labor Statistics data on annual growth in multifactor productivity (MFP) for the private business sector in US. The average annual growth in MFP for the period 1962-2009 is around 2%, or 0.5% quarterly. Setting $\gamma = 5$, and ignoring Jensen inequality effects, we get $\mu \approx 0.975$. In Appendix 8, we extend the model to allow for firm exits as in Melitz (2003). As we show in the Appendix, the incorporation of firm exits causes the adjusted discount rate to be below the standard time discount rate by an additional factor $1 - \eta$, with η being the "death" probability of a firm. If we set η to 2.5% in Bilbiie et al (2012), our estimates of the adjusted discount rate $\bar{\beta}$ imply estimates of the standard time discount rate β that are about 4% higher, or about 0.84. It is worth noting that, although the time discount rates are lower than the values that are considered empirically plausible, they are considerably higher than the estimates obtained by recent studies such as Kocherlakota and Pistaferri (2009) that are around 0.5.

To illustrate the importance of product variety in generating the observed equity premium and risk-free rate, we consider the following benchmark cases as points of comparison. First, we consider the classical CCAPM model where all consumption goods are perfectly substitutable. In this case, the classical argument applies and the sample analogue of the pricing kernel is given by

$$\hat{\Phi}_t = \beta \left(\frac{\hat{\mathcal{R}}_{t+1}/CPI_{t+1}}{\hat{\mathcal{R}}_t/CPI_t} \right)^{-\gamma}$$

where CPI_t is the price index at time t. We write down the pricing kernel in terms of output rather than consumption data as in studies such as Longstaff and Piazzesi (2004) and Jermann (2010) to facilitate a direct comparison with our model (see also the discussion in Section 5 of Cochrane (2007)). We again use GMM and the above pricing kernelto estimate the risk aversion and interest rate.

Table 4 Panel A reports the results of the estimation. The estimated values of γ range between 16 and 32. Although these estimates are smaller than the traditional risk aversions required (> 50), they are also much greater than the estimates using our model with imperfectly substitutable goods in Table 3. The estimates of $\bar{\beta}$ in Panel A are far smaller than 1, suggesting another limitation of the CCAPM model in reconciling realized asset returns.

To further highlight the role of product variety in driving observed asset returns, we next consider the cases where $\sigma \gg 0$ in our model, that is, cases where the products are almost perfectly substitutable. Table 4, Panel B reports the results. The estimates of γ in these cases are substantially larger, ranging between 33 and 140. The estimates of $\bar{\beta}$ are unchanged from those in Table 3, because the sample pricing kernel (37) is invariant in $\frac{1-\gamma}{\sigma-1}$. Overall, these comparisons suggest that product heterogeneity in our model plays an important role in explaining asset prices.

The single-sector model is a simplified model of the real economy with many different industries. Therefore, the above results, although encouraging, should be viewed as an initial attempt to reconcile the equity premium and risk-free rate puzzles. As we will see shortly, the multi-sector model that we develop in the next section generates significantly lower estimates of γ and higher estimates of β that are closer to values that are considered empirically plausible.

7 Multi-Sector Economy

7.1 The Model

We now extend the model to incorporate multiple sectors/industries. This extension is important because, in reality, the products of firms within a sector are likely to be closer substitutes than those of firms in different sectors. As mentioned earlier, the single-sector model could also be viewed as a description of an economy with multiple sectors in which the products within each sector are perfectly substitutable. In this respect, the full-fledged multi-sector model we now develop generalizes the single-sector model by incorporating intra-sector and inter-sector product substitutabilities.

Suppose that there are N sectors denoted as $1, 2, ..., N$. As in the basic model, there is a unit mass of identical agents each endowed with capital K. The preferences of the representative consumer are now given by

$$\mathcal{U} = E_s \sum_{t=s}^{\infty} \beta^t U\left(\Lambda_t\right), \ 0 < \delta < 1 \tag{41}$$

where

$$\Lambda_t = \left[\sum_{i=1}^{N} \Lambda_{it}^{\delta}\right]^{\frac{1}{\delta}}, \tag{42}$$

$$\Lambda_{it} = \left[\int_{\Omega_{it}} q_{it}(\omega_{it})^{\rho_i} d\omega_{it}\right]^{\frac{1}{\rho_i}}; 0 < \rho_i < 1; \ 1 \leq i \leq N. \tag{43}$$

In (42) and (44), Λ_{it} is the contribution to the consumer's total consumption index from the goods produced by sector i, δ determines the degree of product substitutability across sectors, while ρ_i determines the degree of product substitutability within sector i.

If $p_{it}(\omega_{it})$ is the price of good ω_{it} produced by sector i in period t, then the optimal consumption and expenditure decisions for individual goods are

$$q_{it}(\omega_{it}) = \Lambda_{it}\left[\frac{p_{it}(\omega_{it})}{P_{it}}\right]^{-\sigma_i}; \tag{44}$$

$$\xi_{it}(\omega_{it}) = \mathcal{R}_{it}\left[\frac{p_{it}(\omega_{it})}{P_{it}}\right]^{1-\sigma_i}, \ t \in \{0, 1, 2, 3, ...\}. \tag{45}$$

In the above $\mathcal{R}_{it} = P_{it}\Lambda_{it}$ is the aggregate expenditure of the representative consumer on sector i in period t, and

$$P_{it} = \left[\int_{\Omega_{it}} p_{it}(\omega_{it})^{1-\sigma_i} d\omega_{it}\right]^{\frac{1}{1-\sigma_i}} \tag{46}$$

is the *aggregate price index* of sector i in period t, where

$$\sigma_i = \frac{1}{1-\rho_i} \tag{47}$$

is the product substitutability of the products in sector i.

The consumer's expenditure $\mathcal{R}_{it}$ on sector i solves the following optimization program

$$\max_{\{\mathcal{R}_{it}\}} \left[\sum_{i=1}^{N} \Lambda_{it}^{\delta}\right]^{1/\delta} = \max_{\{\mathcal{R}_{it}\}} \left[\sum_{i=1}^{N} \left(\frac{\mathcal{R}_{it}}{P_{it}}\right)^{\delta}\right]^{1/\delta}$$

subject to

$$\sum_{i=1}^{N} \mathcal{R}_{it} = \mathcal{R}_t.$$

The solution to the above program is

$$\mathcal{R}_{it} = \mathcal{R}_t \left[\frac{P_{it}}{P_t}\right]^{1-\tau}, \tag{48}$$

where

$$P_t = \left[\sum_{i=1}^{N} P_{it}^{1-\tau}\right]^{\frac{1}{1-\tau}} \tag{49}$$

is the aggregate price index for the entire economy, and

$$\tau = \frac{1}{1-\delta} \tag{50}$$

is the *inter-sector product substitutability.*

The information structure is as described in the single-sector model. The initial fixed capital investment, however, could vary across sectors and is denoted as f_{it} for sector i. The productivity distribution of firms in sector i is $\mathfrak{g}_{it}$. By the same arguments used in Section 3, the price of the good produced by a firm with productivity α in sector i is

$$p_{it}(\alpha) = \frac{r_t}{\rho_i \alpha}, \tag{51}$$

and its output is

$$q_{it}(\alpha) = \mathcal{R}_{it} \left(P_{it}\right)^{\sigma_i - 1} \left(\frac{\rho_i \alpha}{r_t}\right)^{\sigma_i}. \tag{52}$$

The firm's *revenue* is

$$\xi_{it}(\alpha) = p_{it}(\alpha) q_{it}(\alpha) = \mathcal{R}_{it}(P_{it}\rho_i\alpha)^{\sigma_i - 1} r_t^{1-\sigma_i}. \tag{53}$$

The total capital the firm uses for production (excluding the initial fixed capital f_{it}) is

$$k_{it}(\alpha) = \mathcal{R}_{it} \left(P_{it}\right)^{\sigma_i - 1} \rho_i^{\sigma_i} \alpha^{\sigma_i - 1} r_t^{-\sigma_i}, \tag{54}$$

and the firm's profit is

$$\pi_{it}(\alpha) = \xi_{it}(\alpha) - r_t k_{it}(\alpha) = \frac{\xi_{it}(\alpha)}{\sigma_i}. \tag{55}$$

Denote the average productivity in sector i by

$$\bar{\alpha}_{it} := \left[\int_0^\infty \alpha^{\sigma_i - 1} \mathfrak{g}_{it}(\alpha) d\alpha\right]^{\frac{1}{\sigma_i - 1}}.$$

Using arguments along the lines of those used in the analysis of the single-sector model, the equilibrium of the product market in each period is determined by

$$M_{it} = \frac{K_{it}}{f_{it}\sigma_i}, \tag{56}$$

where $K_{it} = \mathcal{R}_{it}/r_t$. The aggregate price index for sector i is

$$P_{it} = M_{it}^{-\frac{1}{\sigma_i - 1}} \frac{r_t}{\rho_i \bar{\alpha}_{it}}. \tag{57}$$

The asset market structure is as described in Section 5 except that we now have multiple sectors. The economy in any period t is determined by the aggregate state history ζ^t, the distribution of fixed capital investments across sectors, $\left\{f_{it}(\zeta^t); i = 1, ..., N\right\}$, and the distributions of firm productivities in each sector, $\left\{\mathfrak{g}_{it}(\zeta^t); i = 1, ..., N\right\}$. Extending (22), the budget constraints of the representative agent now become

$$\sum_{i=1}^{N} P_{it}\Lambda_{it} + E_t \frac{Q_{t+1}}{Q_t} a_{t+1} \leq a_t + r_t K. \tag{58}$$

The market for Arrow securities is as described in Section 5.1.

Maximizing 41 subject to the budget constraints (58) and the borrowing constraints (23), and using the fact that the borrowing constraints (23) are non-binding at the optimum, we obtain the SDF

$$\Phi_{t+1} := \frac{Q_{t+1}}{Q_t} = \beta \frac{U'(\Lambda_{t+1})\Lambda_{t+1}^{1-\delta}\Lambda_{i,t+1}^{\delta-1}P_{it}}{U'(\Lambda_t)\Lambda_t^{1-\delta}\Lambda_{it}^{\delta-1}P_{i,t+1}}. \tag{59}$$

Using $\Lambda_{it} = \mathcal{R}_{it}/P_{it}$, we can re-express the SDF as

$$\Phi_{t+1} = \beta \frac{U'(\Lambda_{t+1})\Lambda_{t+1}^{1-\delta}\Lambda_{i,t+1}^{\delta}\mathcal{R}_{it}}{U'(\Lambda_t)\Lambda_t^{1-\delta}\Lambda_{it}^{\delta}\mathcal{R}_{i,t+1}}. \tag{60}$$

Note that the SDF can be expressed using the consumption index, $\Lambda_{i,t+1}$, and size, $\mathcal{R}_{it}$, corresponding to *any* sector i, which (as we see later) leads to overidentifying conditions in our estimation of the risk aversion and discount rate in the multi-sector model.

For CRRA utility with coefficient of relative risk aversion γ, the SDF is

$$\Phi_{t+1} = \beta \frac{\Lambda_{t+1}^{1-\delta-\gamma}\Lambda_{i,t+1}^{\delta}\mathcal{R}_{it}}{\Lambda_t^{1-\delta-\gamma}\Lambda_{it}^{\delta}\mathcal{R}_{i,t+1}}. \tag{61}$$

By (56) and (57),

$$\Lambda_{it} = \frac{\mathcal{R}_{it}}{P_{it}} = K_{it}\frac{r_t}{P_{it}} = M_{it} f_{it}\sigma_i \frac{r_t}{P_{it}} = \rho_i f_{it}\sigma_i M_{it}^{\frac{1}{\rho_i}}\bar{\alpha}_{it}. \tag{62}$$

7.2 Empirical Methodology

Similar to Section 6, we use GMM to estimate the risk aversion and discount rate parameters in the multi-sector model that are compatible with realized asset returns. First, we develop empirical analogues of the pricing kernel in the multi-sector model. In fact, equation (61) suggests that there is an empirical analogue of the pricing kernel for each sector $i = 1, \ldots N$. Denote the productivity of the least productive firm in sector i in period t by $\underline{\alpha}_{it}$, and assume it is sector-independent, $\underline{\alpha}_{it} = \underline{\alpha}_t$. From (57),

$$\Lambda_{it} = \frac{\mathcal{R}_{it}}{P_{it}} = K_{it}\frac{r_t}{P_{it}} = \rho_i K_{it} M_{it}^{\frac{1}{\sigma_i - 1}}\bar{\alpha}_{it}. \tag{63}$$

Using the same argument as in Section 6.1, we obtain

$$\Lambda_{it} = \rho_i K_{it}\left(\frac{\mathcal{R}_{it}}{\xi_{it}(\underline{\alpha}_t)}\right)^{\frac{1}{\sigma_i-1}}\underline{\alpha}_t. \tag{64}$$

Therefore the aggregate consumption index can be expressed as

$$\Lambda_t = \left[\sum_{i=1}^{N}\rho_i^{\delta}K_{it}^{\delta}\left(\frac{\mathcal{R}_{it}}{\xi_{it}(\underline{\alpha}_t)}\right)^{\frac{\delta}{\sigma_i-1}}\right]^{\frac{1}{\delta}}\underline{\alpha}_t.$$

By (61),

$$\bar{\Phi}_{t+1} := \frac{\Phi_{t+1}}{\beta \frac{\underline{\alpha}_{t+1}^{1-\gamma}}{\underline{\alpha}_t^{1-\gamma}}} = \frac{\left[\sum_{i=1}^N \rho_i^\delta K_{i,t+1}^\delta \left(\frac{\mathcal{R}_{i,t+1}}{\xi_{i,t+1}(\underline{\alpha}_{t+1})}\right)^{\frac{\delta}{\sigma_i - 1}}\right]^{\frac{1-\delta-\gamma}{\delta}}}{\left[\sum_{i=1}^N \rho_i^\delta K_{it}^\delta \left(\frac{\mathcal{R}_{it}}{\xi_{it}(\underline{\alpha}_t)}\right)^{\frac{\delta}{\sigma_i - 1}}\right]^{\frac{1-\delta-\gamma}{\delta}}} \frac{K_{i,t+1}\left(\frac{\mathcal{R}_{i,t+1}}{\xi_{i,t+1}(\underline{\alpha}_t)}\right)^{\frac{\delta}{\sigma_i - 1}}}{K_{it}\left(\frac{\mathcal{R}_{it}}{\xi_{it}(\underline{\alpha}_t)}\right)^{\frac{\delta}{\sigma_i - 1}}} \frac{\mathcal{R}_{it}}{\mathcal{R}_{i,t+1}}. \tag{65}$$

As in Section 6.1, we assume that the growth rate of minimum productivity, $\frac{\underline{\alpha}_{t+1}}{\underline{\alpha}_t}$, can vary stochastically but is conditionally independent of returns and of the other factors in the expression for $\bar{\Phi}_{t+1}$ given in (65). Further,

$$E_t\left[\left(\frac{\underline{\alpha}_{t+1}}{\underline{\alpha}_t}\right)^{1-\gamma}\right] = \mu, \tag{66}$$

where μ is a constant.

Recall that the market return over period $[t, t+1]$ is R_{t+1}^M and the risk-free rate is R_t^{rf}. Denote by R_{t+1}^i the return of sector i. We have the following asset Euler equations, with $\bar{\Phi}_{t+1}$ given by (65):

$$\bar{\beta} E\left[\bar{\Phi}_{t+1}(R_{t+1}^M - R_t^{rf})\right] = 0, \tag{67}$$

$$\bar{\beta} E\left[\bar{\Phi}_{t+1}(1 + R_t^{rf})\right] - 1 = 0, \tag{68}$$

$$\bar{\beta} E\left[\bar{\Phi}_{t+1}(R_{t+1}^i - R_t^{rf})\right] = 0. \tag{69}$$

The above set of equations can then be used to identify the relative risk aversion, γ, and the adjusted discount rate, $\bar{\beta}$. As we describe shortly, we directly calibrate the parameters ρ_i using empirical data. Because the parameter δ that determines the inter-sector product substitutability cannot easily be pinned down by the data, we consider a wide range of possible values of δ in our estimation exercises for robustness.

We use the Fama-French 12 industries as the sectors. Since we exclude financial firms, there are 11 sectors. For sector returns, we use quarterly value-weighted equity returns in each sector from CRSP. We use the same market returns and risk-free rates as before. Analogous to Section 6, we use the aggregate sales, minimum sales, and aggregate fixed assets to proxy for $\mathcal{R}_{it}$, $\xi_{it}(\alpha_t)$, and K_{it}.

By (55), we use the average sales to profits ratio in industry i over our sample period as empirical analogues for the intra-sector product substitutability parameters σ_i and set $\rho_i = \frac{\sigma_i - 1}{\sigma_i}$. We report the calibrated values of $\hat{\sigma}_i$ using this procedure in Table A2. Our final sample consists of data for 11 sectors over 154 quarters from 1971 to 2009. By substituting the empirical analogues into the Euler equations (67), (68), and (69) for each sector, we obtain an over-identified system of 33 moment equations that we can use to estimate the parameters $\bar{\beta}$ and γ.

7.3 Results

We present the GMM estimation results of the multi-sector model in Table 5. As mentioned earlier, because the inter-sector product substitutability δ is not directly determined by our model, we consider different values of δ (and therefore of $\tau = \frac{1}{1-\delta}$) to examine the robustness of the results. In Panels B and C of Table 5, we consider $\tau = 3.8$ and 6.2, respectively, to match the product substitutability parameters in the single-sector model. In Panels A and D, we consider more extreme values $\tau = 1.1$ and 10.0. We also consider the case where we use just the moment equations for the market return and risk-free rate ((67) and (68)) and the case where all the moment equations including those that involve sector returns ((67), (68), and (69)) are used. Note that this estimation approach in fact incorporates the possibility that firms in different sectors may produce different proportions of products for intermediate use and for final consumption. Assuming that the proportion of production for final consumption is stable across firms within a sector, the ratio of aggregate sales to minimum sales $\frac{\mathcal{R}_{it}}{\xi_{it}(\underline{\alpha}_t)}$ is equal to the ratio of aggregate sales of final consumption goods to the minimum sales of final consumption goods in that sector. Thus the empirical pricing kernel given by (65) remains exactly the same if we use sales of consumption goods rather than sales of all goods. To further mitigate concerns about the measurement of final consumption, we proceed as in the single-sector model by considering only the sales and fixed assets of firms that produce final consumption goods.

The results across all the different scenarios produce consistent parameter estimates. The estimates of the relative risk aversion γ range between 0.9 and 1.4, and the adjusted discount rate $\bar{\beta}$ between 0.85 and 0.97. As in the single sector model, adjusting for the expected growth in minimum productivity and the exit probability of firms implies that β is higher that $\bar{\beta}$ by approximately 4%. Therefore, the multi-sector model is compatible with the observed equity risk premia, risk premia, and even sector return premia, with empirically plausible values of the risk aversion and values of the discount rate that are closer to those estimated in experimental studies (e.g. see Andersen et al (2008)). We also report Hansen's J-statistics for overidentifying restrictions and the associated p-values. In all cases, the J-statistics are not significantly different from zero. Therefore, the model fails to be rejected by the data, and the set of moments we use are valid.

A potential concern with our analysis is that our empirical proxy for the pricing kernel could be distorted by the fact that we normalize the aggregate revenue by the revenue of the smallest firm in (65). We carry out additional robustness tests to mitigate such concerns. Specifically, we replace the terms $\xi_{i,t+1}(\underline{\alpha}_{t+1})$ and $\xi_{it}(\underline{\alpha}_t)$ in (65) by the average revenue of selected lower quantiles of firms, specifically, the bottom 1% and 5% of firms, respectively, in sector i. Table A3 presents the results of our analysis. We see that the results are broadly consistent with those of Table 5.

8 Conclusions

We build a general equilibrium model of a production economy with multiple, imperfectly substitutable products. Our model is parsimonious and tractable enough that we can provide closed-form analytical characterizations of the unique equilibrium and the corresponding pricing kernel that values assets in the economy. We show that the incorporation of product variety and the variation of the endogenous mass of products over time can reconcile the observed equity premium and risk-free rates for empirically reasonable values of agents' risk aversion and values of the discount rate that are much closer to empirically plausible values than the estimates obtained by recent studies. In the multi-sector model, which incorporates intra- and inter-sector product substitutability, our estimates of the relative risk aversion are below 2 and the discount rate estimates exceed 0.9, consistent with estimates of these parameters obtained in experimental studies. Broadly, our analysis shows that product variety is a significant determinant of asset prices.

In future research, it would be interesting to extend the model to incorporate investment. Such an extension would permit a more detailed exploration of asset pricing phenomena. Another fruitful extension would be to allow for (exogenous or endogenous) market incompleteness. We leave these and other extensions for future research.

Appendix A: Capital Depreciation and Investment

In this Appendix, we present a variation of the model in which capital depreciates, and consumers can invest in the accumulation of capital. To simplify the exposition, we consider the one-sector model although the multi-sector model can be analogously modified to incorporate capital depreciation and investment.

As in Section IV and Appendix E of Bilbiie et al. (2012), investment in capital (I_t) requires the same composite of available product varieties as the consumption basket. The aggregate capital accumulation equation is

$$K_{t+1} = (1 - \delta^K)K_t + I_t,$$

where δ^K is the depreciation rate, and I_t is the investment at period t. Denote by a_t the (nominal) Arrow securities holdings the representative agent starts period t with. We assume that the Arrow security holdings are subject to the same "no Ponzi" conditions as before (lower bound large enough so that the constraints do not bind in equilibrium). Capital is nonnegative. These nonnegativity constraints do not bind due to the Inada conditions imposed on U. The budget constraint of the representative agent is

$$P_t\Lambda_t + P_tI_t + E_t\Phi_{t+1}a_{t+1} = a_t + r_tK_t.$$

From the above equation, we see that the total income from renting the capital stock, r_tK_t, and the agent's holdings in Arrow securities, a_t, are used to finance the purchase of the consumption "basket", Λ_t, the investment "basket", I_t, and the next period Arrow securities. The first order conditions for Arrow security holdings give the stochastic discount factor,

$$\Phi_{t+1} = \beta\frac{U'(\Lambda_{t+1})P_t}{U'(\Lambda_t)P_{t+1}},$$

which has the same expression as in (25). The first order conditions with respect to capital investment lead to

$$1 = E_t\Phi_{t+1}\frac{P_{t+1}}{P_t}\left(1 - \delta^K + \frac{r_{t+1}}{P_{t+1}}\right),$$

or equivalently, to

$$1 = \beta E_t\frac{U'(\Lambda_{t+1})}{U'(\Lambda_t)}\left(1 - \delta^K + \frac{r_{t+1}}{P_{t+1}}\right).$$

The market clearing condition $a_t = 0$ implies the aggregate accounting equation

$$P_t\Lambda_t + P_tI_t = r_tK_t.$$

We can use the Euler equations to estimate the risk aversion and discount rate as in the basic model. The difference, however, is that aggregate revenue equals aggregate consumption plus investment. Recall, however, that we also carried out estimation exercises in the main body restricting consideration to the sales of final "consumption goods," and obtained similar estimates for the relative risk aversion and discount rate parameters. Those results are, therefore, directly applicable to the extended model with capital depreciation and investment. In summary, the incorporation of capital depreciation and investment does not significantly alter the results of our analysis of the baseline model (see Section 6).

Appendix B: Sunk Entry Costs and Firm Exit

In this Appendix, we present a variation of the model that allows for firms to incur sunk costs upon entry, but no fixed costs in each period. Further, firms can experience a "death" shock in any period that forces them to exit. Again, we consider the single-sector model for simplicity.

A firm must incur an exogenous sunk entry cost f_t^e (in units of capital) if it decides to enter in period t. There are no fixed production costs, and thus entering firms produce every period until they are hit by a death shock that occurs with probability η in every period. Entrants start producing only next period, while the death shock can occur even at the end of the entry period. The expected value of discounted profits of a new entrant at t (and also the ex-dividend value of a firm at t) at t is

$$v_t := E_t \sum_{s>t} (1-\eta)^{s-t} \frac{Q_s}{Q_t} d_s,$$

where $d_s := E_\alpha \pi_s(\alpha)$ are the average dividends (profits) of a firm active at s and Q is the nominal pricing kernel. Denoting by M_t^e the mass of new entrants at t, the evolution of the number of firms is

$$M_{t+1} = (1-\eta)(M_t + M_t^e).$$

The free entry condition implies

$$v_t \begin{cases} = f_t^e r_t & \text{if } M_t^e > 0, \\ \leq f_t^e r_t & \text{otherwise.} \end{cases}$$

The agent can buy a fraction x_t of the total market index (total firms) priced at a unit price v_t and invest in nominal one-period ahead Arrow securities with face values a_{t+1}, hence his budget is

$$P_t \Lambda_t + E_t \frac{Q_{t+1}}{Q_t} a_{t+1} + v_t (M_t + M_t^e) x_t = r_t K + a_t + (v_t + d_t) M_t x_{t-1}.$$

In equilibrium, $x_t = 1$. The optimality conditions for Arrow securities imply that the SDF is

$$\Phi_{t+1} := \frac{Q_{t+1}}{Q_t} = \beta(1-\eta) \frac{U'(\Lambda_{t+1}) P_t}{U'(\Lambda_t) P_{t+1}} = \beta(1-\eta) \frac{U'(\Lambda_{t+1}) \Lambda_{t+1} \mathcal{R}_t}{U'(\Lambda_t) \Lambda_t \mathcal{R}_{t+1}},$$

while the optimality condition for holding shares in the market index is

$$1 = E_t \Phi_{t+1} \frac{v_{t+1} + d_{t+1}}{v_t}.$$

From the standpoint of our empirical analysis, this model is equivalent to our basic model with fixed costs in each period. The only difference is that the estimate for the discount rate of an agent is now in fact an estimate for $\beta(1-\eta)$, rather than only of β. As we discussed in the main body, incorporating the possibility of firm exits leads to *higher* discount rates than those in the basic model.

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Table 1: Summary Statistics

This table shows the summary statistics of aggregate revenues, market returns, and equity risk premium. The sample period is from 1962 to 2009. The observations are quarterly.

Variable	Mean	Median	Std	No. Obs.
Panel A. Returns				
Market return	0.026	0.036	0.088	192
Risk-free rate	0.013	0.013	0.007	192
Equity Premium	0.013	0.024	0.088	192
Panel B. All Firms				
Number of Firms	4,213	5,053	2,141	192
Aggregate Sales ($M)	1,021,641	742,433	861,465	192
Log(Sales)	13.3	13.5	1.1	192
Log(Sales/Min Sales)	18.4	20.4	3.6	192
Panel C. FF 48 Final Consumption Firms				
Consumption Share (FF48)	0.416	0.392	0.052	192
Log(Sales)	12.7	12.9	1.1	192
Log(Sales/Min Sales)	17.3	19.8	3.9	192

Table 2: GMM Estimates of the Risk Aversion γ

Standard errors in the GMM estimation are calculated following Newey and West.

		Sales of All Firms	Sales of Firms Producing Final Goods
$\sigma = 3.8$	γ	2.8225	4.9230
		(0.6069)	(2.0558)
$\sigma = 6.2$	γ	4.3846	8.2857
		(1.1272)	(3.8180)
No. Quarters		191	191

Table 3: GMM Estimates of the Risk Aversion γ and the Adjusted Discount Rate $\bar{\beta}$

Standard errors in the GMM estimation are calculated following Newey and West.

		All Firms	Firms Producing Final Goods
$\sigma = 3.8$	γ	2.8225	4.9230
		(0.5785)	(1.9859)
	$\bar{\beta}$	0.8161	0.8065
		(0.1114)	(0.2067)
$\sigma = 6.2$	γ	4.3846	8.2857
		(1.0744)	(3.6882)
	$\bar{\beta}$	0.8161	0.8065
		(0.1114)	(0.2067)
No. Quarters		191	191

Table 4: Alternative GMM Estimates of the Risk Aversion γ and the Adjusted Discount Rate $\bar{\beta}$

Standard errors in the GMM estimation are calculated following Newey and West.

Panel A. GMM using the CCAPM Model and Production-Based Pricing Kernel

	All Firms	Firms Producing Final Goods
γ	26.4015	16.7304
	(124.8302)	(10.1441)
$\bar{\beta}$	0.2989	0.6843
	(3.9824)	(0.4162)
No. Quarters	191	191

Panel B. Models with almost substitutable goods (with large σ values)

		All Firms	Firms Producing Final Goods
$\sigma = 50$	γ	32.8937	69.6533
		(10.1246)	(34.7537)
	$\bar{\beta}$	0.8161	0.8065
		(0.1114)	(0.2067)
$\sigma = 100$	γ	65.4382	139.7077
		(20.4559)	(70.2167)
	$\bar{\beta}$	0.8161	0.8065
		(0.1114)	(0.2067)
No. Quarters		191	191

Table 5: GMM Estimates of Risk Aversion γ and Adjusted Discount Rate $\bar{\beta}$

Standard errors in the GMM estimation are calculated following Newey and West. The sample consists of 154 quarters from June 1971 to December 2009. In columns 1 and 3, the set of 22 moments for market returns and risk-free rates are used. In columns 2 and 4, the set of all 33 moments for market returns, risk-free rates, and sector returns are used.

	Sales of All Firms		Sales of Firms Producing Final Goods	
	(1)	(2)	(3)	(4)
Panel A. $\tau = 1.1, \delta = 0.1$				
γ	1.0708	1.1462	1.1937	1.4414
	(0.2142)	(0.1114)	(0.1199)	(0.1268)
$\bar{\beta}$	0.9680	0.9692	0.9684	0.9704
	(0.0075)	(0.0034)	(0.0034)	(0.0027)
Hansen's J-stat.	14.69	20.19	9.76	9.01
J-test p-value	(0.79)	(0.93)	(0.94)	(1.00)
Panel B. $\tau = 3.8, \delta = 0.74$				
γ	1.0139	1.0122	0.9124	0.9394
	(0.0702)	(0.0430)	(0.0950)	(0.0583)
$\bar{\beta}$	0.9164	0.9136	0.9073	0.8880
	(0.0063)	(0.0034)	(0.0073)	(0.0047)
Hansen's J-stat.	13.04	8.69	15.42	10.90
J-test p-value	(0.88)	(1.00)	(0.63)	(1.00)

(Continued)

	Sales of All Firms		Sales of Firms Producing Final Goods	
	(1)	(2)	(3)	(4)
Panel C. $\tau = 6.2, \delta = 0.84$				
γ	0.9978	0.9720	0.9397	0.8953
	(0.0725)	(0.0453)	(0.0912)	(0.0733)
$\bar{\beta}$	0.9038	0.8950	0.9065	0.8637
	(0.0107)	(0.0041)	(0.0086)	(0.0079)
Hansen's J-stat.	16.00	10.16	23.33	13.09
J-test p-value	(0.72)	(1.00)	(0.18)	(0.99)
Panel D. $\tau = 10.0, \delta = 0.9$				
γ	0.9495	0.9464	0.9109	0.8726
	(0.1210)	(0.0483)	(0.0928)	(0.0869)
$\bar{\beta}$	0.8946	0.8827	0.8963	0.8493
	(0.0169)	(0.0057)	(0.0098)	(0.0100)
Hansen's J-stat.	19.92	11.40	24.47	15.09
J-test p-value	(0.46)	(1.00)	(0.14)	(0.98)

Table A1: Final Consumption Goods Classification of Fama-French 48 Industries

This table shows the classification of Fama-French 48 industries into final consumption industries.

Number	Abbreviation	Description	Final Goods
1	Agric	Agriculture	0
2	Food	Food Products	1
3	Soda	Candy & Soda	1
4	Beer	Beer & Liquor	1
5	Smoke	Tobacco Products	1
6	Toys	Recreation	1
8	Books	Printing and Publishing	1
9	Hshld	Consumer Goods	1
10	Clths	Apparel	1
11	Hlth	Healthcare	1
12	MedEq	Medical Equipment	0
13	Drugs	Pharmaceutical Products	1
14	Chems	Chemicals	0
16	Txtls	Textiles	0
17	BldMt	Construction Materials	0
18	Cnstr	Construction	1
19	Steel	Steel Works Etc	0
20	FabPr	Fabricated Products	0
21	Mach	Machinery	0
22	ElcEq	Electrical Equipment	1
23	Autos	Automobiles and Trucks	1
24	Aero	Aircraft	0
25	Ships	Shipbuilding, Railroad Equipment	0
26	Guns	Defense	0
27	Gold	Precious Metals	0
28	Mines	Non-Metallic and Industrial Metal Mining	0
29	Coal	Coal	0
30	Oil	Petroleum and Natural Gas	0
31	Util	Utilities	1
32	Telcm	Communication	1
33	PerSv	Personal Services	1
34	BusSv	Business Services	0
35	Comps	Computers	1
36	Chips	Electronic Equipment	0
37	LabEq	Measuring and Control Equipment	0
38	Paper	Business Supplies	0
39	Boxes	Shipping Containers	1
40	Trans	Transportation	1

(Continued)

Number	Abbreviation	Description	Final Goods
41	Whlsl	Wholesale	0
42	Rtail	Retail	1
43	Meals	Restaurants, Hotels, Motels	1
44	Banks	Banking	0
45	Insur	Insurance	0
46	RlEst	Real Estate	1
48	Other	Almost Nothing	0

Table A2: Intra-sector Product Substitutability Parameters in the Multi-sector Model

This table shows the values of intra-sector product substitutability parameters estimated by the mean sales to profits ratios in the Fama-French 12 industry sectors. The financial industry is excluded.

Fama-French Industry Index	Name	Mean Sales/Profits Ratio ($\hat{\sigma}_i$)
1	Consumer Nondurables	8.65
2	Consumer Durables	8.41
3	Manufacturing	8.16
4	Energy	3.54
5	Chemicals	6.35
6	Technology	4.98
7	Telecommunication	3.07
8	Utilities	5.31
9	Shops	14.30
10	Health care	3.60
12	Other	7.06

Table A3: GMM Estimates of Risk Aversion γ and Adjusted Discount Rate $\bar{\beta}$: Robustness Checks

Standard errors in the GMM estimation are calculated following Newey and West. In columns 1 and 3, the set of 22 moments for market returns and risk-free rates are used. In columns 2 and 4, the set of all 33 moments are used. In Panels A, B, C, and D, we report the results using the pricing kernel with minimum sales in a sector estimated by the average sales in the bottom one percentile of firms in that sector. In Panels E, F, G, and H, we report the results using the pricing kernel with minimum sales in a sector estimated by the average sales in the bottom five percentile of firms in that sector.

	Sales of All Firms		Sales of Firms Producing Final Goods	
	(1)	(2)	(3)	(4)
Panel A. $\tau = 1.1, \delta = 0.1$				
γ	1.0409	1.1091	1.2680	1.5073
	(0.1904)	(0.0911)	(0.1247)	(0.1155)
$\bar{\beta}$	0.9674	0.9682	0.9696	0.9703
	(0.0056)	(0.0033)	(0.0029)	(0.0025)
Hansen's J-stat.	15.42	17.93	7.33	7.56
J-test p-value	(0.75)	(0.97)	(0.99)	(1.00)
Panel B. $\tau = 3.8, \delta = 0.74$				
γ	0.9912	1.0086	1.0505	1.0951
	(0.2595)	(0.0470)	(0.0871)	(0.0522)
$\bar{\beta}$	0.9284	0.9240	0.8959	0.8928
	(0.0140)	(0.0030)	(0.0059)	(0.0032)
Hansen's J-stat.	20.43	10.09	9.41	7.58
J-test p-value	(0.43)	(1.00)	(0.95)	(1.00)
Panel C. $\tau = 6.2, \delta = 0.84$				
γ	0.9614	0.9729	1.0341	1.0976
	(0.1080)	(0.0601)	(0.0981)	(0.0607)
$\bar{\beta}$	0.9163	0.9097	0.8732	0.8647
	(0.0084)	(0.0049)	(0.0088)	(0.0045)
Hansen's J-stat.	19.94	12.70	11.57	7.91
J-test p-value	(0.46)	(1.00)	(0.87)	(1.00)
Panel D. $\tau = 10.0, \delta = 0.9$				
γ	0.9678	0.9549	1.0411	1.1095
	(0.0665)	(0.0625)	(0.1099)	(0.0639)
$\bar{\beta}$	0.9083	0.9001	0.8562	0.8446
	(0.0088)	(0.0063)	(0.0105)	(0.0057)
Hansen's J-stat.	15.28	16.06	12.23	8.99
J-test p-value	(0.76)	(0.99)	(0.84)	(1.00)

(Continued)

	Sales of All Firms		Sales of Firms Producing Final Goods	
	(1)	(2)	(3)	(4)
Panel E. $\tau = 1.1, \delta = 0.1$				
γ	0.7512	0.4083	0.9290	0.9927
	(0.2235)	(0.2918)	(0.2304)	(0.1392)
$\bar{\beta}$	0.9543	0.9266	0.9623	0.9626
	(0.0088)	(0.0148)	(0.0074)	(0.0043)
Hansen's J-stat.	13.18	16.05	26.35	21.87
J-test p-value	(0.87)	(0.99)	(0.09)	(0.79)
Panel F. $\tau = 3.8, \delta = 0.74$				
γ	0.9396	0.9485	1.3429	1.3860
	(0.0598)	(0.0688)	(0.1126)	(0.2888)
$\bar{\beta}$	0.9407	0.9450	0.9371	0.9382
	(0.0033)	(0.0038)	(0.0043)	(0.0082)
Hansen's J-stat.	12.22	35.00	13.12	21.26
J-test p-value	(0.91)	(0.28)	(0.78)	(0.81)
Panel G. $\tau = 6.2, \delta = 0.84$				
γ	0.9145	0.9175	1.4632	1.4535
	(0.0511)	(0.0741)	(0.1252)	(0.1688)
$\bar{\beta}$	0.9310	0.9318	0.9244	0.9294
	(0.0036)	(0.0036)	(0.0055)	(0.0059)
Hansen's J-stat.	11.00	20.39	11.79	22.76
J-test p-value	(0.95)	(0.93)	(0.86)	(0.74)
Panel H. $\tau = 10.0, \delta = 0.9$				
γ	0.8957	0.8908	1.5425	1.5539
	(0.0503)	(0.0652)	(0.1109)	(0.1108)
$\bar{\beta}$	0.9247	0.9246	0.9158	0.9166
	(0.0039)	(0.0035)	(0.0057)	(0.0056)
Hansen's J-stat.	11.02	16.68	11.73	17.30
J-test p-value	(0.95)	(0.98)	(0.86)	(0.94)