Dynamic Selection and the New Gains from Trade with Heterogeneous Firms*

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Abstract

This paper develops an open economy growth model in which firm heterogeneity increases the gains from trade. Technology spillovers from incumbent firms to entrants cause the productivity threshold for firm survival to grow over time as competition becomes tougher. By raising the profits of exporters, trade increases the entry rate and generates a dynamic selection effect that leads to higher growth. The gains from trade can be decomposed into: static gains that equal the total gains from trade in an economy without technology spillovers, and; dynamic gains that are strictly positive. Since trade raises growth through selection, not scale effects, the positive growth effect of trade vanishes when firms are homogeneous. Thus, firm heterogeneity creates a new source of dynamic gains from trade. Calibrating the model using U.S. data implies that dynamic selection approximately triples the gains from trade.

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1 Introduction

Does firm level heterogeneity matter for the aggregate gains from trade? In models with cross-firm productivity differences that follow in the tradition of Melitz (2003) trade liberalization causes the least productive firms to exit and leads to a reallocation of resources towards more productive firms. By increasing average firm productivity, this selection effect generates a new source of gains from trade that is absent from both neoclassical trade theory and Helpman and Krumgan (1985) models of intra-industry trade with homogeneous firms.

However, recent work by Atkeson and Burstein (2010) and Arkolakis, Costinot and Rodríguez-Clare (2012) (henceforth ACRC) argues that in general equilibrium the existence of firm heterogeneity makes little difference to the aggregate gains from trade because the welfare gains from selection are offset by changes in entry and innovation. In particular, ACRC show that in both Krugman (1980) and a version of Melitz (2003) with a Pareto productivity distribution, the gains from trade can be expressed as the same function of two observables: the import penetration ratio and the elasticity of trade with respect to variable trade costs (the trade elasticity). These findings lead the authors to conclude that firm heterogeneity is not important for quantifying the gains from trade.

This paper shows that if we move beyond static steady state economies[1] and incorporate cross-firm productivity differences into a dynamic model of growth through technology diffusion, firm heterogeneity leads to a new dynamic source of gains from trade. Suppose that when new firms are created, innovators learn from incumbent firms. Then selection on firm productivity not only increases the average productivity of existing firms, but causes technology spillovers to entrants. By strengthening the selection effect, trade stimulates technology spillovers and raises the growth rate. The paper shows that these dynamic gains from trade are not offset by countervailing general equilibrium effects and increase the aggregate gains relative to those found in either an equivalent dynamic model with homogeneous firms or the static steady state economies considered by ACRC.

To formalize this argument I consider an economy in which growth is driven by technology diffusion between heterogeneous firms. Most growth theory studies frontier innovation that creates new varieties (Romer 1990) or increases the efficiency with which goods are produced (Aghion and Howitt 1992). However, the

[1]In this paper I use the term "static steady state economies" to refer to both static models and papers such as Melitz (2003) and Atkeson and Burstein (2010) that incorporate dynamics, but do not allow for growth and, consequently, have a steady state that is constant over time.

existence of substantial and persistent productivity differences between firms producing very similar products (Syverson 2011) implies most firms do not use frontier technologies and suggests that technology diffusion plays an important role in the growth process. To capture technology diffusion across firms I extend Melitz (2003) by allowing for entrants to learn from incumbent firms. In particular, I assume that the productivity distribution of entrants is endogenous to the productivity distribution of existing firms.[2] Consequently, selection on firm productivity leads to spillovers that raise the average productivity of entrants and this is sufficient to generate endogenous growth. On the balanced growth path the productivity cut-off below which firms exit grows over time and this dynamic selection mechanism leads to growth in average firm productivity.

In an open economy, only high productivity firms export and the resulting reallocation of resources across firms raises the level of the exit cut-off as in Melitz (2003). However, in addition to the usual static selection effect, trade also impacts dynamic selection by raising the growth rate of the exit cut-off and, consequently, of average productivity and consumption per capita. The key to understanding why trade increases the rate of dynamic selection is the free entry condition. For a given exit cut-off, trade increases average profits. In a static steady state economy this induces a rise in the exit cut-off, which lowers the probability of successful entry and ensures the free entry condition is satisfied. However, with productivity spillovers a higher exit cut-off does not affect the probability of successful entry. Therefore, in order to satisfy free entry, the dynamic selection rate must rise implying more rapid creative destruction and a fall in entrants' expected lifespan.[3]

Dynamic selection is a new channel through which trade can lead to welfare gains when firms are heterogeneous. However, given the findings of Atkeson and Burstein (2010) and ACRC it is natural to ask whether the benefits from an increase in the dynamic selection rate are offset by other general equilibrium effects. To rule out this possibility, the paper shows the welfare effects of trade can be decomposed into two terms. First, a static term that is identical to the gains from trade in Melitz (2003) (assuming a Pareto productivity distribution) and can be written as the same function of the import penetration ratio and the trade elasticity that gives the gains from trade in ACRC. Second, a dynamic term that depends on trade only through the growth rate of per capita consumption. The dynamic term is strictly increasing in the growth

[2] Luttmer (2007) uses an equivalent assumption to develop a closed economy growth model with heterogeneous firms. Unlike Luttmer (2007) I abstract from post-entry productivity dynamics and focus instead on the implications of trade.

[3] Atkeson and Burstein (2010) also highlight the role played by the free entry condition in determining the general equilibrium gains from trade. However, while in a static steady state economy the free entry condition limits the gains from static selection, in this paper free entry is critical in ensuring dynamic gains from trade.

rate since when trade causes dynamic selection it has a positive externality on the productivity of future entrants.

Since trade raises growth, the welfare decomposition implies that the gains from trade in this paper are strictly higher than in the Melitz (2003) model with a Pareto productivity distribution. Conditional on the observed import penetration ratio and trade elasticity, the gains from trade are also strictly higher than in the class of static steady state economies analyzed by ACRC.[4] Moreover, when firms are homogeneous there is no dynamic selection and trade does not affect growth. It follows that in dynamic economies firm heterogeneity matters for the gains from trade.

To assess the magnitude of the gains from trade-induced dynamic selection I calibrate the model using U.S. data. As in ACRC the import penetration ratio is a sufficient statistic for the level of trade integration and the welfare effects of trade can be calculated in terms of a small number of observables and parameters. In addition to the import penetration ratio and the trade elasticity, the calibration uses the rate at which new firms are created, the population growth rate, the intertemporal elasticity of substitution, the discount rate and the elasticity of substitution between goods. The baseline calibration implies that U.S. growth is 11 percent higher than it would be under autarky. More importantly, the increase in the dynamic selection rate triples the gains from trade relative to either a dynamic economy with homogeneous firms or the static steady state economies considered by ACRC. The finding that dynamic selection is quantitatively important for the gains from trade is extremely robust. For plausible parameter variations the dynamic selection effect always at least doubles the gains from trade.

In addition to contributing to the debate over the gains from trade, this paper is closely related to the endogenous growth literature. It develops a tractable growth model with heterogeneous firms in which technology diffusion causes the productivity distribution to evolve over time. Of particular note is that the equilibrium growth rate does not depend on population size – there are no scale effects. Thus, growth driven by selection on firm productivity implies neither the counterfactual prediction that larger economies grow faster (Jones 1995a) nor the semi-endogenous growth prediction that population growth is the only source of long-run growth (Jones 1995b). Scale effects are absent from this paper because both the productivity distribution and the mass of varieties produced are endogenous. In equilibrium a larger population leads to a proportional increase in the mass of varieties produced (unlike in quality ladders growth models), but

[4]The important distinction to note here is that the predicted import penetration ratio and trade elasticity in this paper are the same as in the Pareto productivity version of Melitz (2003), but differ from the predictions made by other models considered by ACRC.

since the creation of new goods does not reduce the cost of future innovations (unlike in expanding varieties growth models) the growth rate is unaffected.

The lack of scale effects is related to the logic of Young (1998) who develops an endogenous growth model without scale effects by merging the quality ladders and expanding varieties frameworks while only allowing knowledge spillovers along the vertical dimension. However, in Young (1998) trade does not affect growth because there is no selection on productivity and trade is simply equivalent to an increase in scale. By contrast, in this paper the selection effect of trade raises growth. Similarly, the paper shows that growth is increasing in the fixed cost of production because a higher fixed cost leads to tougher selection and generates productivity spillovers.

By arguing that trade affects growth because of selection, this paper stands in stark contrast to the previous open economy endogenous growth literature which finds that the implications of trade for growth in a single sector economy depend on scale effects and international knowledge spillovers.[5] For example, Baldwin and Robert-Nicoud (2008) develop an expanding varieties version of Melitz (2003), but do not allow for technology diffusion. Consequently, the firm productivity distribution is constant on the balanced growth path, the relationship between trade and growth is mediated through a scale effect and trade only increases growth if international knowledge spillovers are sufficiently strong. In this paper the effects of trade do not depend on whether productivity spillovers are national or international in scope.

Independently of this paper, a selection based growth model with heterogeneous firms has also been developed in recent work by Perla and Tonetti (2012) who study technology diffusion between incumbent firms. Perla, Tonetti and Waugh (2012) extend this model to an open economy and find that trade can raise or lower growth depending on how the costs of searching for a better technology are specified. However, since these papers hold fixed the mass of varieties produced, they do not include the free entry condition which is critical in shaping the relationship between trade, growth and welfare identified below. This paper is also related to work that seeks to quantify the gains from trade in economies that are not covered by ACRC. Ossa (2012) shows that cross-sectoral heterogeneity in trade elasticities increase the gains from trade relative to ACRC's estimates, but his argument applies regardless of whether or not there is firm level heterogeneity. Edmond, Midrigan and Xu (2012) and Impullitti and Licandro (2012) find that when there are variable mark-ups pro-competitive effects can substantially increase the gains from trade, although Arkolakis et al.

[5]Rivera-Batiz and Romer (1991) and Grossman and Helpman (1991) consider the implications of trade in first generation endogenous growth models. More recent work incorporates heterogeneous firms into expanding varieties (Baldwin and Robert-Nicoud 2008) and quality ladders (Haruyama and Zhao 2008) models.

(2012) show that this will not always be the case. By contrast, this paper focuses on understanding whether firm heterogeneity matters for the gains from trade in a single sector economy with constant mark-ups.

The remainder of the paper is organized as follows. Section 2 introduces the model, while Section 3 solves for the balanced growth path equilibrium and discusses the effects of trade on growth. In Section 4 I characterize household welfare on the balanced growth path and then Section 5 calibrates the model and quantifies the gains from trade. Finally, Section 6 demonstrates the robustness of the paper's results to two extensions of the baseline model, before Section 7 concludes.

2 Technology diffusion model

Consider a world comprised of $J+1$ symmetric economies. When $J=0$ there is a single autarkic economy, while for $J>0$ we have an open economy model. Time t is continuous and the preferences and technological possibilities of each economy are as follows.

2.1 Preferences

Each economy consists of a set of identical households with dynastic preferences and discount rate ρ. The population L_t at time t grows at rate $n \geq 0$ where n is assumed to be constant and exogenously fixed. Each household has constant intertemporal elasticity of substitution preferences and seeks to maximize:

$$U=\int_{t=0}^{\infty} e^{-\rho t} e^{nt} \frac{c_t^{1-\frac{1}{\gamma}}-1}{1-\frac{1}{\gamma}} dt, \tag{1}$$

where c_t denotes consumption per capita and $\gamma>0$ is the intertemporal elasticity of substitution. The numeraire is chosen so that the price of the consumption good is unity. Households can lend or borrow at interest rate r_t and a_t denotes assets per capita. Consequently, the household's budget constraint expressed in per capita terms is:

$$\dot{a}_t = w_t + r_t a_t - c_t - n a_t, \tag{2}$$

where w_t denotes the wage. Note that households do not face any uncertainty.

Under these assumptions and a no Ponzi game condition the household's utility maximization problem is standard[6] and solving gives the Euler equation:

$$\frac{\dot{c}_t}{c_t} = \gamma(r_t - \rho), \tag{3}$$

together with the transversality condition:

$$\lim_{t \to \infty} \left\{ a_t \exp\left[- \int_0^t (r_s - n) ds \right] \right\} = 0. \tag{4}$$

2.2 Production and trade

Output is produced by monopolistically competitive firms each of which produces a differentiated good. Labor is the only factor of production and all workers are homogeneous and supply one unit of labor per period. There is heterogeneity across firms in labor productivity θ. A firm with productivity θ at time t has marginal cost of production $\frac{w_t}{\theta}$ and must also pay a fixed cost f per period in order to produce. The fixed cost is denominated in units of labor. The firm does not face an investment decision and firm productivity remains constant over time. The final consumption good is produced under perfect competition as a constant elasticity of substitution aggregate of all available goods with elasticity of substitution $\sigma > 1$ and is non-tradable.[7]

Differentiated good producers can sell their output both at home and abroad. However, as in Melitz (2003) firms that select into exporting face both fixed and variable costs of trade. Exporters incur a fixed cost f_x denominated in units of domestic labor, per export market per period, while variable trade costs take the iceberg form. In order to deliver one unit of its product to a foreign market a firm must ship τ units. I assume $\tau^{\sigma-1} f_x > f$ which is a necessary and sufficient condition to ensure that in equilibrium not all firms export. Since I consider a symmetric equilibrium, all parameters and endogenous variables are constant across countries.

For a given distribution of firm productivity levels, the structure of output production and demand in this economy at time t is equivalent to that in Melitz (2003) and characterizing the solutions to firms' static profit maximization problems is straightforward. Firms that produce face isoelastic demand and set factory gate prices as a constant mark-up over marginal costs. Firms only choose to produce if their total variable profits from domestic and foreign markets are sufficient to cover their fixed production costs and firms only export

[6] See, for example, Chapter 2 of Barro and Sala-i-Martin (2004).

[7] This is equivalent to assuming households have constant elasticity of substitution preferences over differentiated goods.

to a given market if their variable profits in that market are sufficient to cover the fixed export cost. Variable profits in each market are strictly increasing in productivity and and since $\tau^{\sigma-1}f_x > f$ the productivity above which firms export exceeds the minimum productivity for entering the domestic market. In particular, there is a cut-off productivity θ_t^* such that firms choose to produce at time t if and only if their productivity is at least θ_t^*. This exit cut-off is given by:

$$\theta_t^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left(\frac{fw_t^\sigma}{c_tL_t}\right)^{\frac{1}{\sigma-1}}. \tag{5}$$

In addition, there is a threshold $\tilde{\theta}_t > \theta_t^*$ such that firms choose to export at time t if and only if their productivity is at least $\tilde{\theta}_t$. The export threshold is:

$$\tilde{\theta}_t = \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\tau\theta_t^*. \tag{6}$$

Firms can lend or borrow at interest rate r_t and the market value $V_t(\theta)$ of a firm with productivity θ is given by the present discounted value of future profits:

$$V_t(\theta) = \int_t^\infty \pi_\tau(\theta)\exp\left(-\int_t^\tau r_s ds\right)d\tau, \tag{7}$$

where π_t denotes the profit flow net of fixed costs at time t from both domestic and export sales and $\pi_t(\theta) = 0$ if the firm does not produce.

In what follows, it will often be convenient to use the change of variables $\phi_t \equiv \frac{\theta}{\theta_t^*}$, where ϕ_t is firm productivity relative to the exit cut-off. I will refer to ϕ_t as a firm's relative productivity. Let $W_t(\phi_t)$ be the value of a firm with relative productivity ϕ_t at time t. Obviously, only firms with $\phi_t \geq 1$ will choose to produce and only firms with $\phi_t \geq \tilde{\phi} \equiv \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\tau$ will choose to export. For these firms prices, employment and profits in the domestic and export markets are given by:

$$p_t^d(\phi_t) = \frac{\sigma}{\sigma-1}\frac{w_t}{\phi_t\theta_t^*}, \quad p_t^x(\phi_t) = \tau p_t^d(\phi_t),$$

$$l^d(\phi_t) = f\left[(\sigma-1)\phi_t^{\sigma-1}+1\right], \quad l^x(\phi_t) = f\tau^{1-\sigma}\left[(\sigma-1)\phi_t^{\sigma-1}+\tilde{\phi}^{\sigma-1}\right], \tag{8}$$

$$\pi_t^d(\phi_t) = f w_t \left[\phi_t^{\sigma-1} - 1\right], \quad \pi_t^x(\phi_t) = f\tau^{1-\sigma} w_t \left[\phi_t^{\sigma-1} - \tilde{\phi}^{\sigma-1}\right], \tag{9}$$

where I have used d and x superscripts to denote the domestic and export markets, respectively. Observe that employment is a stationary function of relative productivity and that, conditional on relative productivity ϕ_t, domestic profits are proportional to the fixed cost of production. Since there are J export markets, total firm employment is given by $l(\phi_t) = l^d(\phi_t) + Jl^x(\phi_t)$ and total firm profits are $\pi_t(\phi_t) = \pi_t^d(\phi_t) + J\pi_t^x(\phi_t)$.

2.3 Entry

To invent a new good, entrants must hire workers to perform research and development (R&D). The R&D technology is such that employing $R_t f_e$ workers to undertake R&D generates a flow R_t of innovations. Each innovation generates an idea for a new good (product innovation). Once equipped with an idea, the innovator chooses how to produce the good by studying the production techniques (technologies, managerial methods, organizational forms, etc.) of existing firms. Consequently, entrants learn about the process technologies of incumbents and this leads to technology diffusion across firms. To formalize this idea I assume that innovators draw their productivity levels from a distribution that depends on the distribution of θ at the time of innovation. In particular, I assume that the productivity distribution of innovators is a scaled version of the productivity distribution of existing producers where the scaling parameter λ measures the strength of spillovers from incumbents to innovators. Thus, if $G_t(\theta)$ is the cumulative productivity distribution function for firms that produce at time t, then innovators receive a productivity draw from a distribution with cumulative distribution function $\tilde{G}_t$ defined by $\tilde{G}_t(\theta) = G_t(\theta/\lambda)$ where $\lambda \in (0, 1]$.[8] This structure of productivity spillovers could be rationalized by assuming that each innovator searches for a process technology to use and is randomly matched with an incumbent firm whose technology she imperfectly imitates.

There is free entry into R&D, implying that in equilibrium the expected cost of innovating equals the expected value of creating a new firm:

$$f_e w_t = \int_\theta V_t(\theta) d\tilde{G}_t(\theta). \tag{10}$$

[8]Given symmetry across countries, the productivity distribution is the same in all countries and it is irrelevant whether spillovers are national or international in scope. Consequently, in this model the effects of trade on growth and welfare do not depend on the extent of international knowledge spillovers.

Entry is financed by a competitive and costless financial intermediation sector which owns the firms and, thereby, enables investors to pool the risk faced by innovators. Consequently, each household effectively owns a balanced portfolio of all firms and R&D projects.[9]

In Melitz (2003) and much of the subsequent literature on firm heterogeneity entrants receive a productivity draw from an exogenously fixed distribution and there is no long run growth. By contrast, the model developed in this paper introduces productivity spillovers from existing producers to innovators and endogenizes the productivity distribution of entrants. These spillovers are sufficient to generate steady state growth as technology diffuses across firms and the productivity distribution of incumbents shifts upwards over time. By allowing for both the creation of new goods and productivity spillovers, the R&D technology in this paper integrates technology diffusion with the expanding varieties growth paradigm.

While most endogenous growth models focus on the role played by R&D in expanding the technology frontier, a recent literature has started to consider how technology diffusion influences growth when firms are heterogeneous. Building on the work of Kortum (1997) who considers an economy in which researchers search for ideas from an exogenously fixed distribution of production techniques, Alvarez, Buera and Lucas (2008, 2011) study the evolution of the production cost distribution when there is no entry, but producers learn from encounters with their lower cost peers. Perla and Tonetti (2012) further develop this approach by endogenizing the firm's search decision and obtain a model where growth is driven by technology upgrading by the least productive firms. However, since these papers hold constant the number of goods produced, they do not allow for free entry into R&D. I show below that, in this paper, the free entry condition plays a crucial role in determining the effects of trade. The structure of technology diffusion assumed above is also used by Luttmer (2007). However, Luttmer (2007) studies a closed economy and focuses not on understanding what determines the equilibrium growth rate, but on characterizing the shape of the upper tail of the productivity distribution when firms are subject to productivity shocks.

How does the relative productivity distribution evolve over time? Let H_t and $\tilde{H}_t$ be the cumulative distribution functions of relative productivity ϕ for existing firms and entrants, respectively. Given the structure of productivity spillovers we must have $\tilde{H}_t(\phi) = H_t(\phi/\lambda)$. Note also that since θ_t^* is the exit cut-off, $\tilde{H}_t(\lambda) = H_t(1) = 0 \,\forall\, t$. To characterize the intertemporal evolution of H_t I will first formulate a law of motion for $H_t(\phi)$ between t and $t + \Delta$ and then take the continuous time limit. Let M_t be the mass

[9] Again, since countries are symmetric it is irrelevant whether asset markets operate at the national or global level.

of producers in the economy at time t and assume the exit cut-off is strictly increasing over time.[10] Then the mass of firms with relative productivity less than ϕ at time $t+\Delta$ is:

$$M_{t+\Delta}H_{t+\Delta}(\phi) = M_t\left[H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\phi\right) - H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right)\right] + \Delta R_t\left[H_t\left(\frac{\phi}{\lambda}\right) - H_t\left(\frac{1}{\lambda}\right)\right]. \qquad (11)$$

Since $\phi_{t+\Delta} \leq \phi \Leftrightarrow \phi_t \leq \frac{\theta^*_{t+\Delta}}{\theta^*_t}\phi$ the first term on the right hand side is the mass of time t incumbents that have relative productivity less than ϕ, but greater than one, at time $t+\Delta$. $M_tH_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\phi\right)$ gives the mass of time t producers with relative productivity less than ϕ at time $t+\Delta$, while $M_tH_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right)$ is the mass of time t incumbents that exit between t and $t+\Delta$ because their productivity falls below the exit cut-off. The second term on the right hand side gives the mass of entrants between t and $t+\Delta$ whose relative productivity falls between one and ϕ.

Letting $\phi \to \infty$ in (11) implies:

$$M_{t+\Delta} = M_t\left[1 - H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right)\right] + \Delta R_t\left[1 - H_t\left(\frac{1}{\lambda}\right)\right], \qquad (12)$$

and taking the limit as $\Delta \to 0$ gives:[11]

$$\frac{\dot{M}_t}{M_t} = -H'_t(1)\frac{\dot{\theta}^*_t}{\theta^*_t} + \left[1 - H_t\left(\frac{1}{\lambda}\right)\right]\frac{R_t}{M_t}. \qquad (13)$$

This expression shows the two channels that affect the number of firms in the economy. R&D generates a flow R_t of innovations, but a fraction $H_t\left(\frac{1}{\lambda}\right)$ of innovators receive a productivity draw below the exit cut-off and choose not to produce. In addition, as the exit cut-off increases firms' relative productivity levels decline and a firm exits when its relative productivity falls below one. The rate at which firms exit due to growth in the exit cut-off depends on the density of the relative productivity distribution at the exit cut-off $H'_t(1)$.

Now using (12) to substitute for $M_{t+\Delta}$ in (11), rearranging and taking the limit as $\Delta \to 0$ we obtain the following law of motion for $H_t(\phi)$:

[10] When solving the model I will restrict attention to balanced growth paths on which θ^*_t is strictly increasing in t meaning firms will never choose to temporarily cease production. In an economy with a declining exit cut-off, equilibrium would depend on whether exit from production was temporary or irreversible. I abstract from these issues in this paper.

[11] In obtaining both this expression and equation (14) I assume that θ^*_t is differentiable with respect to t and $H_t(\phi)$ is differentiable with respect to ϕ. Both these conditions will hold on the balanced growth path considered below.

$$
\begin{aligned}
\dot{H}_t(\phi) &= \left\{\phi H_t'(\phi) - H_t'(1)\left[1 - H_t(\phi)\right]\right\} \frac{\dot{\theta}_t^*}{\theta_t^*} \\
&\quad + \left\{ H_t\left(\frac{\phi}{\lambda}\right) - H_t\left(\frac{1}{\lambda}\right) - H_t(\phi)\left[1 - H_t\left(\frac{1}{\lambda}\right)\right]\right\} \frac{R_t}{M_t}.
\end{aligned} \tag{14}
$$

Thus, the evolution of the relative productivity distribution is driven by growth in the exit cut-off and the entry of new firms. When $\dot{H}_t(\phi) = 0$ for all $\phi \geq 1$ the relative productivity distribution is stationary.

2.4 Equilibrium

In addition to consumer and producer optimization, equilibrium requires the labor and asset markets to clear in each economy in all periods. Labor market clearing requires:

$$
L_t = M_t \int_\phi l(\phi) dH_t(\phi) + R_t f_e, \tag{15}
$$

while asset market clearing implies that aggregate household assets equal the combined worth of all firms:

$$
a_t L_t = M_t \int_\phi W_t(\phi) dH_t(\phi). \tag{16}
$$

Finally, as an initial condition I assume that at time zero there exists in each economy a mass $\hat{M}_0$ of firms whose productivity θ has cumulative distribution function $\hat{G}(\theta)$ where $\hat{M}_0$ and $\hat{G}$ are such that in equilibrium some firms have productivity below the exit cut-off at time zero and choose to exit immediately. We are now ready to define the equilibrium.

An equilibrium of the world economy is defined by time paths for $t \in [0, \infty)$ of consumption per capita c_t, assets per capita a_t, wages w_t, the interest rate r_t, the exit cut-off θ_t^*, the export threshold $\tilde{\theta}_t$, firm values $W_t(\phi)$, the mass of firms in each economy M_t, the flow of innovations in each economy R_t and the relative productivity distribution $H_t(\phi)$ such that: (i) households choose c_t to maximize utility subject to the budget constraint (2) implying the Euler equation (3) and the transversality condition (4); (ii) producers maximize profits implying the exit cut-off satisfies (5), the export threshold satisfies (6) and firm value is given by (7); (iii) free entry into R&D implies (10); (iv) the exit cut-off is strictly increasing over time and the evolution of M_t and $H_t(\phi)$ are governed by (13) and (14); (v) labor and asset market clearing imply (15) and (16), respectively, and; (vi) at time zero there are $\hat{M}_0$ potential producers in each economy with productivity distribution $\hat{G}(\theta)$.

3 Balanced growth path

I will solve for a balanced growth path equilibrium on which $c_t, a_t, w_t, \theta_t^*, \tilde{\theta}_t, W_t(\phi), M_t$ and R_t grow at constant rates, r_t is constant and the distribution of relative productivity ϕ is stationary, meaning $\dot{H}_t(\phi) = 0 \,\forall\, t, \phi$. First, observe that if ϕ has a Pareto distribution at time t then $\dot{H}_t(\phi) = 0$. Thus, given the structure of productivity spillovers in this economy the Pareto distribution is self-replicating.[12] Therefore, to obtain a balanced growth path with no transition dynamics I assume that the initial productivity distribution is Pareto.

Assumption 1. *The productivity distribution at time zero is Pareto:* $\hat{G}(\theta) = 1 - \theta^{-k}$ *for* $\theta \geq 1$ *with* $k > \max\{1, \sigma - 1\}$.

Note that since some firms choose to exit immediately at time zero there is no loss of generality in assuming the distribution has a scale parameter equal to one. Given Assumption 1, equation (14) implies that the distribution of relative productivity is Pareto with scale parameter one and shape parameter k for all t. Thus, $H(\phi) = 1 - \phi^{-k} \;\forall\, t, \phi \geq 1$. In addition, it immediately follows that the distribution of productivity θ is Pareto, that the employment, revenue and profit distributions converge asymptotically to Pareto distributions in the right tail and that the employment distribution is stationary.[13]

Now, let $\frac{\dot{c}_t}{c_t} = q$ be the growth rate of consumption per capita. Then the household budget constraint (2) implies that assets per capita and wages grow at the same rate as consumption per capita:

$$\frac{\dot{a}_t}{a_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} = q,$$

while the Euler equation (3) gives:

[12]More generally, solving (14) with $\dot{H}_t(\phi) = 0$ implies:

$$H(\phi) = 1 - \phi^{-k} + \phi^{-k} \int_1^{\phi} F(s) s^{k-1} ds,$$

where $k > 0$ and $F(\phi)$ satisfies:

$$F'(\phi)\phi \frac{\dot{\theta}_t^*}{\theta_t^*} = F(\phi)\frac{\dot{M}_t}{M_t} - F\left(\frac{\phi}{\lambda}\right)\frac{R_t}{M_t},$$

with $F(1) = 0$. Obviously, $F(\phi) = 0$ solves this equation and implies ϕ has a Pareto distribution, but it is not known whether other solutions exist.

[13]It is well known that the upper tails of the distributions of firm sales and employment are well approximated by Pareto distributions (Luttmer 2007). Axtell (2001) argues that Pareto distributions provide a good fit to the entire sales and employment distributions in the U.S.

$$q = \gamma(r - \rho), \tag{17}$$

and the transversality condition (4) requires:

$$r > n + q \Leftrightarrow \frac{1-\gamma}{\gamma} q + \rho - n > 0, \tag{18}$$

where the equivalence follows from (17). This inequality is also sufficient to ensure that household utility is well-defined. Since all output is consumed in each period and economies are symmetric, output per capita is always equal to consumption per capita.

Next, differentiating equation (5) which defines the exit cut-off implies:

$$q = g + \frac{n}{\sigma - 1}. \tag{19}$$

where $g = \frac{\dot{\theta}_t^*}{\theta_t^*}$ is the rate of growth of the exit cut-off and, therefore, the rate at which the productivity distribution shifts upwards. From equation (6) the export threshold is proportional to the exit cut-off meaning that g is also the growth rate of the export threshold and since each firm's productivity θ remains constant over time g is the rate at which a firm's relative productivity ϕ_t decreases. Equation (19) makes clear that there are two sources of consumption per capita growth in this economy. First, productivity growth resulting from a dynamic selection effect. As the exit cut-off increases, the least productive firms are forced to exit and this leads to a reallocation of resources to more productive firms raising average labor productivity and output per capita. This effect is the dynamic analogue of the static selection effect that results from changes in the level of the exit cut-off. Henceforth, I will refer to g as the dynamic selection rate. Understanding what determines the dynamic selection rate is the central concern of this paper.

The second source of growth in consumption per capita is population growth. Using the employment function (8), the labor market clearing condition (15) simplifies to:

$$L_t = \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} M_t f \left[1 + J\tau^{-k} \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] + R_t f_e. \tag{20}$$

Consequently, on a balanced growth path we must have that the mass of producers and the flow of innovations grow at the same rate as population:

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{M}_t}{M_t} = \frac{\dot{R}_t}{R_t} = n.$$

Thus, the link between population growth and consumption per capita growth arises because when the population increases the number of varieties produced grows and, since the final good production technology exhibits love of varieties, this raises consumption per capita.

To solve for the dynamic selection rate we can now substitute the profit function (9) and $\phi_t = \frac{\theta}{\theta_t^*}$ into (7) and solve for the firm value function obtaining:

$$\begin{aligned} V_t(\theta) &= W_t(\phi_t), \\ &= f w_t \left[\frac{\phi_t^{\sigma-1}}{(\sigma-1)g + r - q} \left(1 + I\left[\phi_t \geq \tilde{\phi}\right] \frac{J f_x}{f} \tilde{\phi}^{1-\sigma}\right) \right. \\ &\quad + \frac{(\sigma-1)g}{r-q} \frac{\phi_t^{\frac{q-r}{g}}}{(\sigma-1)g + r - q} \left(1 + I\left[\phi_t \geq \tilde{\phi}\right] \frac{J f_x}{f} \tilde{\phi}^{\frac{q-r}{g}}\right) \\ &\quad \left. - \frac{1}{r-q} \left(1 + I\left[\phi_t \geq \tilde{\phi}\right] \frac{J f_x}{f}\right) \right]. \end{aligned} \tag{21}$$

where $I\left[\phi_t \geq \tilde{\phi}\right]$ is an indicator function that takes value one if a firm's relative productivity is greater than or equal to the export threshold and zero otherwise. Thus, the value of a firm with relative productivity ϕ grows at rate q. Substituting (21) into the free entry condition (10), using $\tilde{G}_t(\theta) = \tilde{H}(\phi) = H\left(\frac{\phi}{\lambda}\right)$ and integrating to obtain the expected value of an innovation implies:

$$q = kg + r - \frac{\sigma-1}{k+1-\sigma} \frac{\lambda^k}{f_e} \left(f + J f_x \tilde{\phi}^{-k}\right). \tag{22}$$

Solving equations (17), (19) and (22) for the three unknowns q, g and r then gives:

$$\begin{aligned} q &= \frac{\gamma}{1+\gamma(k-1)} \left[\frac{\sigma-1}{k+1-\sigma} \frac{\lambda^k f}{f_e} \left(1 + J\tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right) + \frac{kn}{\sigma-1} - \rho \right], \\ g &= \frac{\gamma}{1+\gamma(k-1)} \left[\frac{\sigma-1}{k+1-\sigma} \frac{\lambda^k f}{f_e} \left(1 + J\tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right) - \frac{1-\gamma}{\gamma} \frac{n}{\sigma-1} - \rho \right], \\ r &= \frac{\gamma}{1+\gamma(k-1)} \left[\frac{1}{\gamma} \frac{\sigma-1}{k+1-\sigma} \frac{\lambda^k f}{f_e} \left(1 + J\tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right) + \frac{1}{\gamma} \frac{kn}{\sigma-1} + (k-1)\rho \right]. \end{aligned} \tag{23}$$

Finally, recall that I assumed $g > 0$ when characterizing the evolution of the relative productivity distribution in Section 2.3. To ensure this condition is satisfied and the transversality condition (18) holds I impose the following parameter restrictions.

Assumption 2. *The parameters of the world economy satisfy:*

$$\begin{aligned} \frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e} &> \rho + \frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}, \\ \frac{(1-\gamma)(\sigma-1)}{k+1-\sigma}\frac{\lambda^k f}{f_e}\left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right] &> \gamma k(n-\rho) - (1-\gamma)\frac{k+1-\sigma}{\sigma-1}n. \end{aligned}$$

The first inequality ensures that $g > 0$ holds for any $J \geq 0$, while the second inequality is implied by the transversality condition.

This completes the proof that the world economy has a unique balanced growth path. Note that, since there are no transition dynamics, the world economy is always on the balanced growth path and that the proof holds for any non-negative value of J including the closed economy case where $J = 0$.

Proposition 1. *When Assumptions 1 and 2 hold the world economy has a unique balanced growth path equilibrium on which consumption per capita grows at rate:*

$$q = \frac{\gamma}{1+\gamma(k-1)}\left[\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e}\left(1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right) + \frac{kn}{\sigma-1} - \rho\right].$$

Remembering that Assumption 1 ensures $k > \max\{1, \sigma-1\}$, we immediately obtain a corollary of Proposition 1 characterizing the determinants of the growth rate.

Corollary 1. *The growth rate of consumption per capita is strictly increasing in the fixed production cost f, the strength of productivity spillovers λ, the intertemporal elasticity of substitution γ, the population growth rate n and the number of trading partners J, but is strictly decreasing in the entry cost f_e, the fixed export cost f_x, the variable trade cost τ and the discount rate ρ.*

To understand Proposition 1 and Corollary 1 it is useful to start by setting $J = 0$ and considering a closed economy. Two features of the autarky equilibrium are particularly noteworthy relative to previous endogenous growth models. First, growth is increasing in the fixed production cost[14] and second, growth is

[14]Luttmer (2007) also finds that the consumption growth rate is increasing in $\frac{f}{f_e}$ when there are productivity spillovers from incumbents to entrants.

independent of population size meaning there are no scale effects. Let us consider each of these findings in turn.

To see why a higher fixed production cost increases the growth rate, start by observing from the profit function (9) that, for a given relative productivity ϕ and wage w_t, profits are proportional to f. Since on the balanced growth path innovators draw ϕ from a stationary distribution it follows that the expected initial profit flow received by a new entrant (relative to the wage) is increasing in f. However, the free entry condition (10) implies that in equilibrium the expected value of innovating (relative to the wage) is independent of f. Therefore, to satisfy the free entry condition the increase in an entrant's expected initial profits generated by a rise in f must be offset by a fall in the entrant's expected future profits which requires that relative productivity ϕ declines at a faster rate and the firm's expected lifespan falls. Thus, higher f increases the rate of dynamic selection g which raises the growth rate q.

When new entrants receive a productivity draw from an exogenously fixed distribution and there are no productivity spillovers as in Melitz (2003), an increase in f still raises profits conditional on ϕ, but it also lowers entrants' expected ϕ by raising the level of the exit cut-off. By contrast, in the growth model considered in this paper variation in the level of the exit cut-off does not change the expected value of R&D because the productivity spillovers are such that entrants draw ϕ from a stationary distribution. Consequently, variation in f must have implications for growth.

The channel through which the free entry condition leads to a positive relationship between the fixed production cost and growth can be isolated by considering the allocation of resources between production and R&D. Since M_t grows at rate n, the exit cut-off θ_t^* grows at rate g, $H_t'(1) = k$ and $H_t\left(\frac{1}{\lambda}\right) = \lambda^k$, equation (13) implies that on a balanced growth path:

$$\frac{R_t}{M_t} = \frac{n + gk}{\lambda^k}, \tag{24}$$

and substituting this expression back into the labor market clearing condition we obtain:

$$M_t = \left[\frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} f \left(1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right) + (n + gk)\frac{f_e}{\lambda^k}\right]^{-1} L_t. \tag{25}$$

From (25) we see that raising f reduces the mass of goods produced and it is this reduction in competition among incumbents that leads to higher profits conditional on ϕ. In addition, (24) shows that higher f raises the flow of innovations relative to the mass of incumbent firms meaning that incumbents face greater competition from entrants. Consequently, the dynamic selection rate rises and growth increases.

Now let us consider the absence of scale effects. Scale effects are a ubiquitous feature of the first generation of endogenous growth models (Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992) where growth depends on the size of the R&D sector which, on a balanced growth path, is proportional to population. However, Jones (1995a) documents that despite continuous growth in both population and the R&D labor force, growth rates in developed countries have been remarkably stable since the second world war.[15] Such concerns prompted Jones (1995b) to pioneer the development of semi-endogenous growth models in which the allocation of resources to R&D remains endogenous, but there are no scale effects because diminishing returns to knowledge creation mean that population growth is the only source of long-run growth. Semi-endogenous growth models have in turn been criticized for attributing long-run growth to a purely exogenous factor and understating the role of incentives to perform R&D in driving growth.

To understand why there are no scale effects in the technology diffusion model observe first that the productivity spillovers from incumbents to entrants that drive growth in this paper are related to the knowledge spillovers found in quality ladders growth models where innovators improve the output quality of incumbents by some fixed proportion (Grossman and Helpman 1991; Aghion and Howitt 1992). In quality ladders models the number of goods produced is constant and, consequently, the profit flow received by innovators is increasing in population, which generates the scale effect. However, in this model the number of goods is endogenous and grows at the same rate as population. Thus, in larger economies producers face more competitors and the incentive to innovate does not depend on market size. Moreover, unlike in expanding varieties growth models (Romer 1990), the creation of new goods does not reduce the cost of R&D for future innovators implying that there are no knowledge spillovers along the horizontal dimension of the model. As equation (24) makes clear, the equilibrium growth rate depends not on the innovation rate, which is proportional to population, but on the innovation rate relative to the mass of producers which is scale independent. A related model that features endogenous growth without scale effects is developed by Young (1998) who allows for R&D to raise both the quality and the number of goods produced, but assumes that knowledge spillovers only occur along the vertical dimension of production. However, in Young (1998) there is no selection on productivity, implying that the dynamic selection effect analyzed in this paper is missing and trade does not affect growth because it is equivalent to an increase in scale.

The effects of parameters other than the fixed production cost on the autarky growth rate are unsurpris-

[15] Although, see Kremer (1993) for evidence that scale effects may be present in the very long run.

ing. Increasing the entry cost by raising f_e must, in equilibrium, lead to an increase in the expected value of innovating and this is achieved through lower growth which increases firms' expected lifespans. Similarly, growth is strictly increasing in the learning parameter λ because when productivity spillovers are stronger an entrant's expected initial relative productivity is higher. Consequently, to ensure the free entry condition (10) holds the dynamic selection rate must increase to offset the rise in initial profits. A higher intertemporal elasticity of substitution or a lower discount rate raise growth by making households more willing to invest now and consume later, while, as discussed above, population growth raises consumption per capita growth through its impact on the growth rate of the mass of producers M_t.

Now let us return to an open economy setting with $J > 0$ and analyze how trade integration affects growth. Relative to autarky, trade is equivalent to increasing f by a factor $1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}$ and, consequently, the equilibrium growth rate is higher in the open economy than in autarky. Moreover, either increasing the number of countries J in the world economy, reducing the variable trade cost τ or reducing the fixed export cost f_x raises growth. The effect of trade on growth operates through the same mechanism as an increase in the fixed production cost. To understand why, start by considering the domestic and export profit functions given in (9). Conditional on a firm's relative productivity and the wage level, domestic profits are independent of the extent of trade integration, while trade increases the profits of firms whose productivity exceeds the export threshold. Thus, since entrants draw relative productivity from a stationary distribution, trade liberalization increases entrants' expected initial profits relative to the wage. The free entry condition (10) then implies that following trade integration the dynamic selection rate must increase in order to keep the expected value of innovating relative to the wage constant. Thus, trade raises growth through a dynamic selection effect.[16] From (24) and (25) we see that the exit cut-off grows more quickly because trade reduces the mass of domestic producers and increases the ratio of entrants to incumbent firms.

The dynamic selection effect of trade identified in this paper is analogous to the static selection effect found in heterogeneous firm models without long run growth such as Melitz (2003). In both cases export profits mandate an increase in the exit cut-off to satisfy free entry. However, while static selection generates a one-off increase in the level of the exit cut-off, when there are productivity spillovers free entry induces growth effects.

Early work on the effects of trade in endogenous growth models found that global integration increases

[16]Note that this analysis holds both for comparisons of the open economy with autarky and for the consequences of a partial trade liberalization resulting from an increase in J or a reduction in either τ or f_x.

growth via the scale effect provided knowledge spillovers are sufficiently international in scope (Rivera-Batiz and Romer 1991; Grossman and Helpman 1991).[17] More recent papers have shown that if firm heterogeneity is included in standard expanding variety (Baldwin and Robert-Nicoud 2008) or quality ladder (Haruyama and Zhao 2008) models the relationship between trade and growth is still mediated through the scale effect. It is unsurprising then that in models without scale effects such as Young (1998) and the semi-endogenous growth model of Dinopoulos and Segerstrom (1999) the long run growth rate is independent of an economy's trade status. However, in this paper growth is driven by selection, not scale, and the dynamic selection mechanism through which trade affects growth does not require the existence of scale effects. Instead, it requires the combination of firm heterogeneity and productivity spillovers.

Both static and dynamic selection create new sources of gains from trade that do not exist when firms are homogeneous. However, as pointed out by Atkeson and Burstein (2010) and ACRC, in general equilibrium the welfare gains generated by the static selection effect are offset by lower entry. In particular, ACRC show that, conditional on the import penetration ratio and the trade elasticity, the gains from trade in Melitz (2003) are the same as in the homogeneous firms model of Krugman (1980). Therefore, incorporating Melitz style firm heterogeneity into static trade models does not increase the calibrated gains from trade. Can the same reasoning be applied to the dynamic selection effect? To answer this question we must move beyond simply considering the equilibrium growth rate to solving for the welfare effects of trade. This is the goal of the next section.

4 Welfare

Proposition 1 gives the consumption growth rate, but household welfare also depends on the level of consumption. This section solves for the consumption level and considers how trade affects welfare.

Substituting $c_t = c_0 e^{qt}$ into the household welfare function (1) and integrating implies:

$$U = \frac{\gamma}{\gamma - 1}\left[\frac{\gamma c_0^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)q + \gamma(\rho - n)} - \frac{1}{\rho - n}\right]. \tag{26}$$

From the household budget constraint (2), the Euler equation (3) and the transversality condition (18) we

[17] A complementary line of research examines how trade integration affects the incentives of asymmetric countries with multiple production sectors to undertake R&D (Grossman and Helpman 1991).

can write the initial level of consumption per capita c_0 in terms of initial wages and assets as:[18]

$$c_0 = w_0 + \left(\frac{1-\gamma}{\gamma} q + \rho - n \right) a_0, \tag{27}$$

where $\frac{1-\gamma}{\gamma} q + \rho - n$ is the marginal propensity to consume out of wealth, which is positive by the transversality condition.

Now using (21) to substitute for $W_t(\phi)$ in the asset market clearing condition (16), integrating the right hand side to obtain average firm value and using (22) gives:

$$a_t L_t = \frac{f_e}{\lambda^k} w_t M_t, \tag{28}$$

which has the intuitive interpretation that the value of the economy's assets at any given time equals the expected R&D cost of replacing all active firms.

Next, to obtain the initial value of the exit cut-off θ_0^* apply Assumption 1, which states that at time zero there are $\hat{M}_0$ potential producers whose productivity is distributed Pareto with shape parameter k and scale parameter one. Therefore, it follows that:

$$\theta_0^* = \left(\frac{\hat{M}_0}{M_0} \right)^{\frac{1}{k}}. \tag{29}$$

We can now solve for initial consumption per capita by combining this expression with equations (5), (19), (23), (25), (27) and (28) to give:

$$c_0 = A_1 f^{-\frac{k+1-\sigma}{k(\sigma-1)}} \left[1 + J\tau^{-k} \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right]^{\frac{1}{k}} \left[1 + \frac{\sigma-1}{k\sigma+1-\sigma} \frac{n+gk}{n+gk+\frac{1-\gamma}{\gamma}q+\rho-n} \right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}, \tag{30}$$

where:

$$A_1 \equiv (\sigma-1) \left(\frac{k}{k+1-\sigma} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{k+1-\sigma}{k\sigma+1-\sigma} \right)^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \hat{M}_0^{\frac{1}{k}} L_0^{\frac{k+1-\sigma}{k(\sigma-1)}} > 0.$$

Remember that Assumption 2 ensures $g > 0$ and $\frac{1-\gamma}{\gamma} q + \rho - n > 0$. Thus, both the numerator and the denominator of the final term in (30) are positive.

[18]This is a textbook derivation. See, for example, Barro and Sala-i-Martin (2004), pp.93-94.

Armed with the equilibrium growth rate (23) and the initial consumption level (30) we can now analyze the welfare implications of trade integration. Since there are no transition dynamics, we can compare welfare under different equilibria by considering household welfare on the balanced growth path. Observe that trade affects both growth and the consumption level only through the value of $T \equiv J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}$. T measures the extent of trade integration between countries. T is strictly increasing in the number of countries J in the world economy and the fixed production cost f, but strictly decreasing in the variable trade cost τ and the fixed export cost f_x. When calibrating the model in Section 5 I show that the import penetration ratio is a sufficient statistic for T and that T is monotonically increasing in the import penetration ratio.

Trade affects welfare through two channels. First, trade raises welfare by increasing the level of consumption for a constant growth rate. These static gains from trade z^s are given by the term:

$$z^s = \left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}} = (1+T)^{\frac{1}{k}}$$

in (30). The static gains from trade result from a combination of increased access to imported goods, a reduction in the number of goods produced domestically and reallocation gains caused by an increase in the level of the exit cut-off. Most importantly, the static gains equal the total gains from trade in the absence of trade-induced dynamic selection. Thus, both in static steady state economies such as the variant of Melitz (2003) considered by ACRC where entrants draw productivity from a Pareto distribution and in a version of the model above where innovators draw productivity from a time invariant Pareto distribution (in this case the exit cut-off is constant on the balanced growth path and trade does not affect the consumption growth rate) the gains from trade equal z^s.

Second, trade raises the growth rate through the dynamic selection effect. I will refer to the change in welfare caused by trade-induced variation in the growth rate as the dynamic gains from trade. From (26) we see that increased growth has a direct positive effect on welfare, but (30) shows that it also affects the level of consumption. The level effect is made up of two components. First, there is the increase in $n + gk$ which from (24) occurs because trade raises the innovation rate relative to the mass of producers. This requires a reallocation of labor between production and R&D that decreases the consumption level. Second, variation in q changes households' marginal propensity to consume out of wealth $\frac{1-\gamma}{\gamma}q + \rho - n$. The sign of this effect on c_0 depends on the intertemporal elasticity of substitution γ, but it is positive when $\gamma < 1$. In general, the net effect of higher growth on the consumption level can be either positive or negative and substituting

$g = q - \frac{n}{\sigma-1}$ into (30) and differentiating with respect to q shows that higher growth increases c_0 if and only if:

$$n\left(1 - \frac{1}{k}\frac{1-\gamma}{\gamma}\frac{k+1-\sigma}{\sigma-1}\right) > \rho.$$

However, regardless of the sign of the level effect, substituting for c_0 using (30) and then differentiating (26) with respect to growth shows that the dynamic gains from trade are positive.[19] Thus, the direct positive effect of growth on welfare always outweighs any indirect negative effect resulting from a decline in c_0. Proposition 2 summarizes the welfare effects of trade. The proposition is proved in Appendix A.

Proposition 2. *Trade integration resulting from an increase in the number of trading partners J, a reduction in the fixed export cost f_x or a reduction in the variable trade cost τ increases welfare through two channels: (i) by raising the level of consumption for any given growth rate (static gains), and; (ii) by raising the growth rate of consumption per capita (dynamic gains). The static gains equal the total gains from trade in Melitz (2003) with a Pareto productivity distribution.*

Two observations follow immediately from Proposition 2. First, since both the static and dynamic gains from trade are positive, trade is welfare improving. Second, by raising the growth rate, the dynamic selection effect strictly increases the gains from trade relative to either a static steady state version of the model or a dynamic model without technology diffusion. Since firm heterogeneity is a necessary condition for the existence of the dynamic selection effect, this shows that including heterogeneous firms in a dynamic model with productivity spillovers leads to a new source of gains from trade that is not offset by other general equilibrium effects. In contrast to the findings of Atkeson and Burstein (2010) and ACRC, in this paper firm heterogeneity matters for the gains from trade. Quantifying the magnitude of the dynamic gains from trade is the goal of Section 5.

To understand why the higher growth resulting from trade liberalization is welfare improving consider the efficiency properties of the decentralized equilibrium. By equation (24) the dynamic selection rate is increasing in $\frac{R_t}{M_t}$. As the exit cut-off increases knowledge spillovers cause the productivity distribution of entrants to shift upwards, but innovators cannot appropriate the social value of these spillovers. Thus, there is a positive externality from R&D investment and the flow of innovations relative to the mass of existing producers is inefficiently low in the decentralized equilibrium. A benevolent government can raise welfare

[19] See the proof of Proposition 2 for details.

by introducing either a R&D subsidy or a tax on the fixed production costs since both policies incentivize R&D relative to production and raise the dynamic selection rate.[20] Since the effect of trade on growth is equivalent to an increase in the fixed production cost as discussed in Section 3, trade exploits the productivity spillovers externality to generate dynamic welfare gains.

5 Quantifying the gains from trade

Let us define the gains from trade z in equivalent variation terms as the proportional increase in the autarky level of consumption required to obtain the open economy welfare level. Thus, z satisfies $U\left(zc_0^A, q^A\right) = U\left(c_0, q\right)$ where U, q and c_0 are defined by (26), (23) and (30), respectively, and A superscripts denote autarky values.[21] From (26) we have:

$$z = \frac{c_0}{c_0^A}\left[\frac{(1-\gamma)q^A + \gamma(\rho - n)}{(1-\gamma)q + \gamma(\rho - n)}\right]^{\frac{\gamma}{\gamma-1}}.$$

Observe that if $q = q^A$ the gains from trade are given by the increase in the initial consumption level, which from (30) equals the static gains from trade z^s. The dynamic gains from trade z^d are defined by $z^d = \frac{z}{z^s}$.

To calibrate the gains from trade I will start by using U.S. data and then perform robustness checks against this baseline, but it should be remembered when interpreting the calibration results that the theory assumes symmetry across countries. The key to the calibration is showing that z can be expressed in terms of a small number of observables and commonly used parameters. In particular, it is not necessary to specify values of J, f, f_x, f_e or λ to obtain z.[22]

The static gains from trade depend only on the import penetration ratio (IPR) and the trade elasticity (TE). To see this first calculate import expenditure in each country (IMP) which is given by:

$$IMP_t = \frac{k\sigma}{k+1-\sigma} M_t w_t f J \tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}. \tag{31}$$

Equation (31) shows that k equals the trade elasticity (the elasticity of imports with respect to variable trade costs). Now divide (31) by total domestic sales $c_t L_t$ to obtain:

[20] I assume that the policies are financed by lump sum transfers to households. See Appendix B for details.

[21] The goal of this section is to compare welfare at observed levels of trade with autarky welfare. However, the same techniques can be used to quantify how varying the extent of trade integration affects welfare.

[22] This exercise is similar in spirit to the calibration of the gains from trade in ACRC, although in the dynamic setting considered here more information is required than just the import penetration ratio and the trade elasticity.

$$z^s = \left(\frac{1}{1 - IPR}\right)^{\frac{1}{TE}}. \tag{32}$$

This expression is identical to the formula for calibrating the gains from trade obtained by ACRC. It follows that the calibrated static gains from trade in the technology diffusion model developed in this paper equal the calibrated total gains from trade in the class of static steady economies considered by ACRC.[23] Equation (32) also gives the calibrated total gains from trade in the model above if there are no productivity differences across firms.[24] The U.S. import penetration ratio for 2000, defined as imports of goods and services divided by gross output, was 0.081.[25] Anderson and Van Wincoop (2004) conclude based on available estimates that the trade elasticity is likely to lie between five to ten. I set $k = 7.5$ for the baseline calibration, while in the robustness checks I allow k to vary between two and ten. This interval includes the trade elasticity of four estimated by Simonovska and Waugh (2011).

Next, we can express $\frac{\lambda^k f}{f_e}$, which enters the expression for the growth rate q, as a function of n, k, σ, γ, ρ, IPR and the rate at which new firms are created relative to the mass of existing firms (NF). Since a fraction λ^k of innovations lead to the creation of new firms we have $NF = \lambda^k \frac{R_t}{M_t}$ and using (19), (23) and (24) gives:

$$\frac{\lambda^k f}{f_e} = \frac{k+1-\sigma}{\gamma k(\sigma-1)}(1 - IPR)\left\{\left[1+\gamma(k-1)\right](NF - n) + \frac{k(1-\gamma)}{\sigma-1}n + \gamma k\rho\right\}.$$

Luttmer (2007) reports that U.S. Small Business Administration data shows an entry rate of 11.6% per annum in 2002. Therefore, I set $NF = 0.116$. The last observable needed for the calibration is the population growth rate. According to the World Development Indicators average annual U.S. population growth from 1980-2000 was 1.1%, so I let $n = 0.011$.

Finally, there are three parameters to calibrate: σ, γ and ρ. To calibrate σ observe that the firm employment distribution converges asymptotically to a power function with index $\frac{-k}{\sigma-1}$. Luttmer (2007) shows that for U.S. firms in 2002 the right tail index of the employment distribution equals -1.06. Therefore,

[23] Models covered by ACRC include Anderson (1979), Krugman (1980) and Eaton and Kortum (2002) in addition to the variant of Melitz (2003) with a Pareto productivity distribution.

[24] To see this assume all firms have unit productivity and there are no fixed costs of exporting. Then it is straightforward to show that $q = \frac{n}{\sigma-1}$, meaning there are no dynamic gains from trade, and that the static gains from trade equal $\left(1 + J\tau^{1-\sigma}\right)^{\frac{1}{\sigma-1}} = \left(\frac{1}{1-IPR}\right)^{\frac{1}{TE}}$.

[25] Imports of goods and services are from the World Development Indicators (Edition: April 2012) and gross output is from the OECD STAN Database for Structural Analysis (Vol. 2009).

I let the elasticity of substitution $\sigma = k/1.06 + 1$ implying $\sigma = 8.1$. Note that $k > \max\{1, \sigma - 1\}$ as required by Assumption 1. Helpman, Melitz and Yeaple (2004) use European firm sales data to estimate $k+1-\sigma$ at the industry level, obtaining estimates that mostly lie in the interval between 0.5 and 1 implying $\sigma \in [k, k+1/2]$. In the robustness checks I allow σ to vary over a range that includes this interval.

Although controversy exists over the value of the intertemporal elasticity of substitution, estimates typically lie in the range $(1/4, 1)$. Following García-Peñalosa and Turnovsky (2005) I let $\gamma = 1/3$ in the baseline calibration. A low intertemporal elasticity of substitution will tend to reduce the dynamic gains from trade by making consumers less willing to substitute consumption over time. I also follow García-Peñalosa and Turnovsky (2005) in choosing the discount rate and set $\rho = 0.04$. In the robustness checks I allow γ to vary between 0.25 and 1 and ρ to vary between 0.01 and 0.15. Table 1 summarizes the data and parameter values used for the baseline calibration. Assumption 2 is satisfied both for the baseline calibration and in all the robustness checks.

Table 2 shows the calibration results. The model predicts that consumption per capita growth is 10.7% higher at observed U.S. trade levels than in a counterfactual autarkic economy. Due to the dynamic welfare gains resulting from higher growth, the total calibrated gains from trade are 3.2 times higher than the static gains. Thus, dynamic selection has a quantitatively important effect on the gains from trade.

Next, I consider the robustness of these results. First, with respect to the import penetration ratio. Unsurprisingly, the gains from trade are higher when trade integration is greater (Figure 1). Increasing the import penetration ratio from 0.051 (Japan) to 0.36 (Belgium) raises welfare gains from 2.2% to 19.2%. More importantly, the ratio of the total gains to the static gains, which measures the proportional increase in the gains from trade due to dynamic selection, remains approximately constant as the import penetration ratio varies. Figure 2 plots the growth rate under trade relative to the autarky growth rate on the left hand axis and the total gains from trade relative to the static gains from trade on the right hand axis. The total gains are a little over three times larger than the static gains for all levels of the import penetration ratio between zero and 0.5.

Finally, Figure 3 shows the consequences of allowing the remaining calibration parameters to vary. I plot how the growth increase due to trade and the ratio of the total gains to the static gains depend on each parameter in turn, while holding the other parameters fixed at their baseline values.[26] In all cases the dynamic gains from trade are quantitatively important and the results suggest that dynamic selection at

[26]The sole exception is Figure 3c, where I adjust σ to ensure $\sigma = k/1.06 + 1$ always holds as the trade elasticity varies.

least doubles the gains from trade. For example, either lowering the intertemporal elasticity of substitution or raising the discount rate reduces the dynamic gains from trade because it lowers the value of future consumption growth. However, even if the intertemporal elasticity of substitution is reduced to one quarter, the gains from trade are 2.7 times higher with dynamic selection, while the discount rate must exceed 14% before the total gains from trade are less than double the static gains.

6 Extensions – market structure and R&D technology

This section considers the robustness of the results above to two variants of the baseline model. First, I change the market structure by developing a version of the model in which there is perfect competition between firms producing a single homogeneous output good. In this extension I also relax the assumption that all economies are symmetric by analyzing a small open economy. Second, I consider an alternative specification of the R&D technology which allows for decreasing returns to scale in R&D and congestion in the technology diffusion process. In both cases I show that trade continues to increase growth by raising the dynamic selection rate and, consequently, generates dynamic gains that raise the welfare gains from trade relative to comparable economies without trade-induced dynamic selection.

6.1 Competitive output markets

Firm heterogeneity, productivity spillovers and fixed costs of exporting are necessary for trade to induce dynamic selection, but monopolistic competition is not. To see this point consider the following variant of the model in which output is sold in competitive markets. Assume there is a single homogeneous output good produced by heterogeneous firms that face decreasing returns to scale in production. A firm with productivity θ that employs l workers produces θl^{β} units of output with $\beta \in (0, 1)$. In order to produce all firms must also pay a fixed cost f per period, denominated in units of labor. Consider a small home economy that takes world prices as given and faces segmented export markets. The output good can be freely traded at a price equal to one or firms can choose to pay a fixed export cost f_x, denominated in units of domestic labor, in order to access export markets in which the output price is $p \geq 1$.[27] Otherwise the model is as described in Section 2 above.

Under these assumptions the model can be solved using similar reasoning to that applied in Section 3

[27] Autarky is equivalent to setting $p = 1$ since in this case there is no incentive to trade.

and the basic structure of the home economy's balanced growth path equilibrium is unchanged.[28] There is an exit cut-off $\theta_t^* = \beta^{-\beta}(1-\beta)^{-(1-\beta)} f^{1-\beta} w_t$ below which firms do not produce and an export threshold $\tilde{\theta}_t = \left[\left(p^{\frac{1}{1-\beta}} - 1\right)\frac{f}{f_x}\right]^{-(1-\beta)} \theta_t^*$ above which firms pay the fixed export cost. However, unlike in the baseline model, firms whose productivity exceeds the export threshold export all their output to take advantage of the higher output price. The entry process is unchanged from the baseline model and firm productivity continues to have a Pareto distribution with shape parameter k. I assume $k > \frac{1}{1-\beta}$. Since there is no love of variety both the exit cut-off and consumption per capita grow at the same rate q, which is given by:

$$q = \frac{\gamma}{1+\gamma(k-1)}\left[\frac{1}{k(1-\beta)-1}\frac{\lambda^k f}{f_e}\left(1+\left(p^{\frac{1}{1-\beta}}-1\right)^{k(1-\beta)}\left(\frac{f}{f_x}\right)^{k(1-\beta)-1}\right)-\rho\right]. \tag{33}$$

Thus, growth is higher with trade than in autarky and q is increasing in p and decreasing in f_x. In autarky growth is strictly increasing in $\frac{f}{f_e}$ as in the baseline model. The dynamic selection mechanism that drives these results is the same as that identified in the baseline model.

Solving for the initial consumption level shows that:

$$c_0 \propto f^{\frac{1}{k}-(1-\beta)}\left[1+\left(p^{\frac{1}{1-\beta}}-1\right)^{k(1-\beta)}\left(\frac{f}{f_x}\right)^{k(1-\beta)-1}\right]^{\frac{1}{k}}\left[1+\frac{1}{k-1}\frac{n+qk}{n+qk+\frac{1-\gamma}{\gamma}q+\rho-n}\right]^{-\frac{k-1}{k}}. \tag{34}$$

Since household welfare is still given by (26), equations (33) and (34) imply that trade affects welfare only through the term $T = \left(p^{\frac{1}{1-\beta}}-1\right)^{k(1-\beta)}\left(\frac{f}{f_x}\right)^{k(1-\beta)-1}$ which measures the extent of trade integration.[29] Moreover, the gains from trade can be decomposed into a static term and a dynamic term which depends on the growth rate q. Differentiating welfare with respect to q shows that the dynamic gains from trade are positive implying that, as in the baseline model, dynamic selection generates a new source of gains from trade. Proposition 3 summarizes these results.

Proposition 3. *When output markets are competitive and firms that pay a fixed export cost can sell their output for a higher price abroad, the growth rate of consumption per capita is higher under trade than in*

[28] A detailed characterization of the balanced growth path equilibrium is given in Appendix C.

[29] Assuming balanced trade and that only firms which pay the fixed export cost are exporters (i.e. gross and net imports are equal) T can be written in terms of observables as:

$$1+T = \left(1-PX^{1-\frac{1}{k(1-\beta)}}\right)\frac{1}{1-IPR},$$

where PX denotes the share of firms that export.

autarky. The positive effect of trade on growth increases the welfare gains from trade.

Finally, it is worth noting that the economy considered in this section is formally equivalent to an autarky economy with a competitive output market where firms can increase their productivity by a factor p by paying a fixed cost f_x. Thus, trade has the same implications for growth and welfare as introducing a non-convex investment technology. While the focus of this paper is on using the productivity spillovers model to understand the welfare effects of trade, this observation highlights how the framework developed here may prove useful in other contexts.

6.2 R&D technology

The baseline model features constant returns to scale in R&D. In this section I generalize the R&D technology to allow for congestion in technology diffusion. Assume that when $R_t f_e$ workers are employed in R&D the flow of new innovations is $\Psi(R_t, M_t)$ where Ψ is homogeneous of degree one, strictly increasing in R_t, weakly increasing in M_t and $\Psi(0,0) = 0$.[30] Ψ is a matching function that gives the number of innovators who are matched with incumbents and learn from the incumbents' production techniques. Allowing Ψ to depend on M_t introduces decreasing returns to scale in R&D investment and implies that R&D is more productive when there are more incumbent firms to learn from.

Given this R&D technology we can solve for the balanced growth path equilibrium following the same reasoning applied above. Modifying the R&D technology does not affect households' welfare maximization problem or firms' static profit maximization problem meaning that (17) and (19) continue to hold. However, the free entry condition now implies:

$$q = kg + r - \frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e}\left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]\psi\left(\frac{M_t}{R_t}\right),$$

where $\psi\left(\frac{M_t}{R_t}\right) \equiv \Psi\left(1, \frac{M_t}{R_t}\right) = \frac{1}{R_t}\Psi(R_t, M_t)$. Combining this expression with (17) and (19) gives:

$$q = \frac{\gamma}{1+\gamma(k-1)}\left[\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e}\left(1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right)\psi\left(\frac{M_t}{R_t}\right) + \frac{kn}{\sigma-1} - \rho\right]. \quad (35)$$

Comparing (35) with the baseline economy growth rate given by equation (23), the only difference is the

[30]The baseline model corresponds to the case $\Psi(R_t, M_t) = R_t$. Assuming Ψ is homogeneous of degree one ensures the existence of a balanced growth path equilibrium.

inclusion of $\psi\left(\frac{M_t}{R_t}\right)$. To obtain the equilibrium value of $\frac{R_t}{M_t}$ note that in this version of the model equation (13), which gives the rate at which new firms are created, becomes:

$$\frac{R_t}{M_t}\psi\left(\frac{M_t}{R_t}\right) = \frac{1}{\lambda^k}\left(kq - \frac{k+1-\sigma}{\sigma-1}n\right). \tag{36}$$

Equations (35) and (36) define a system of two equations in the two unknowns q and $\frac{R_t}{M_t}$. It is not possible to solve for q explicitly, but it is straightforward to show that q is higher under trade than in autarky and is strictly increasing in $T = J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}$.[31] Thus, as in the baseline model, trade raises growth by increasing the rate of dynamic selection. Moreover, solving for the initial consumption level shows that c_0 is still given by (30). It follows that even with decreasing returns to scale in R&D there exist dynamic gains resulting from the pro-growth effects of trade that increase the total gains from trade relative both to an equivalent dynamic model without productivity spillovers and to the static steady state version of Melitz (2003) where entrants receive productivity draws from a Pareto distribution. Proposition 4 summarizes these results.

Proposition 4. *When there is congestion in technology diffusion trade integration causes both static welfare gains that increase the consumption level for any given growth rate and dynamic welfare gains that raise the growth rate of consumption per capita. The static gains equal the total gains from trade in Melitz (2003) with a Pareto productivity distribution.*

To calibrate the model with congestion in technology diffusion let $\Psi(R_t, M_t) = R_t^{\alpha} M_t^{1-\alpha}$ where $\alpha \in (0, 1]$ parameterizes the returns to scale in R&D. Figure 4 shows how trade affects growth and welfare as α varies between zero and one with other observables and parameters held constant at their baseline values from Table 1.[32] Reducing the returns to scale in R&D lowers the dynamic gains from trade as reallocating labor from production to R&D has a smaller effect on growth. However, provided the returns to scale exceed one half, the dynamic selection effect of trade at least doubles the gains from trade.

[31] See the proof of Proposition 4 for details.

[32] The balanced growth path equilibrium conditions do not imply the existence of an observable that can be used to calibrate α directly and I am not aware of any empirical work that estimates the returns to scale in R&D when R&D is aimed at learning about existing technologies. Allowing for congestion in technology diffusion does not affect the calibration of the static gains from trade.

7 Conclusions

The firm heterogeneity literature following Melitz (2003) shows that trade induces selection on firm productivity. However, previous work has focused exclusively on the static selection effect of trade on productivity levels. This paper shows that when the productivity of entrants is endogenous to the productivity distribution of incumbent firms trade also has a dynamic selection effect that raises the growth rates of average productivity and consumption per capita. The dynamic selection effect is important for two reasons. First, it identifies a new channel linking trade and growth that does not require the existence of scale effects. Second, it generates a new source of gains from trade that: (i) exists only when firms are heterogeneous, and; (ii) increases the gains from trade relative to those found in static steady state economies. Thus, dynamic selection shows that firm heterogeneity matters for the size of the gains from trade.

To isolate the novel predictions that result from dynamic selection, this paper embeds technology diffusion in an otherwise standard dynamic open economy with heterogeneous firms. In future work it would be interesting to extend the framework by incorporating: frontier innovation to study the interaction between incentives to expand the technology frontier and incentives to invest in learning about existing technologies; international knowledge spillovers between asymmetric countries to study the interaction between within and cross-country technology diffusion, and; technology diffusion between incumbents to allow for within firm technology upgrading. Moreover, comparing this paper with Baldwin and Robert-Nicoud (2008) shows that including firm heterogeneity in an endogenous growth framework is not sufficient to generate dynamic selection. Dynamic selection also requires that knowledge spillovers affect the productivity distribution of entrants, rather than the cost of R&D. More empirical work on knowledge spillovers would be useful to assess the relative importance of these two spillover channels and guide future theoretical work.

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Appendix A – Proofs

Proof of Proposition 2

To show that the dynamic gains from trade are positive substitute (30) and (19) into (26) and differentiate with respect to q to obtain:

$$\begin{aligned}\frac{dU}{dq} &\propto -(k\sigma+1-\sigma)\gamma D_1\left(kD_1-\frac{1-\gamma}{\gamma}D_2\right)+k\gamma(D_1+D_2)\left[k\sigma(D_1+D_2)-(\sigma-1)D_1\right],\\ &= k^2\gamma\sigma D_2^2+D_1D_2\left[k^2\gamma\sigma+(k\sigma+1-\sigma)(1+\gamma(k-1))\right],\\ &> 0\end{aligned}$$

where $D_1 \equiv \frac{1-\gamma}{\gamma}q+\rho-n$ and $D_2 \equiv n+gk$. In the first line of the above expression, the first term on the right hand side captures the indirect effect of higher growth on welfare through changes in c_0, while the second term captures the direct effect. The final inequality comes from observing that Assumption 2 implies both $D_1 > 0$ and $D_2 > 0$.

To obtain a version of the model developed in this paper without productivity spillovers assume that new entrants receive a productivity draw from a Pareto distribution with scale parameter one and shape parameter k. Thus, $\tilde{G}(\theta) = 1-\theta^{-k}$ is independent of t. Assuming the baseline model is otherwise unchanged, the same reasoning used in Section 2.3 above implies:

$$\frac{\dot{M}_t}{M_t} = -k\frac{\dot{\theta}_t^*}{\theta_t^*}+\frac{R_t}{M_t}\theta_t^{*-k}.$$

It immediately follows that on a balanced growth path the exit cut-off must be constant implying $g = 0$. Consumer optimization and the solution for the exit cut-off (5) then give $q = \frac{n}{\sigma-1}$ meaning that the growth rate is independent of trade integration. With this result in hand it is straightforward to solve the remainder of the model and show $c_0 \propto z^s$.

Proof of Proposition 4

To prove the proposition I need to show that q is strictly increasing in T. The result can be proved by taking the total derivatives of (35) and (36) and rearranging to obtain $\frac{dq}{dT}$, but here is a simpler argument. Suppose

T increases, but q does not. Then (35) implies that $\psi\left(\frac{M_t}{R_t}\right)$ must decrease which requires a fall in $\frac{M_t}{R_t}$. From the definition of ψ we have that $\frac{R_t}{M_t}\psi\left(\frac{M_t}{R_t}\right) = \Psi\left(\frac{R_t}{M_t}, 1\right)$ which increases when $\frac{M_t}{R_t}$ falls. Therefore, we must have that the left hand side of (36) increases, while the right hand side does not giving a contradiction. It follows that an increase in T must lead to an increase in q.

Appendix B – Taxes

A complete optimal policy analysis of the dynamic selection model lies beyond the scope of this paper. However, to better understand its efficiency properties we can analyze the welfare effects of linear taxes on fixed costs and R&D. Consider a single autarkic economy in which the government taxes the fixed cost of production at rate v and subsidizes R&D at rate v_e.[33] Thus, each firm must pay $w_t f(1+v)$ per period in order to produce and employing an R&D worker costs $w_t(1-v_e)$. Also, assume that the government balances its budget through lump sum transfers to households and that entry is governed by the general form of the R&D technology introduced in Section 6.2 which allows for the possibility of congestion in technology diffusion.

Under these assumptions it is straightforward to solve for the balanced growth path equilibrium using reasoning analogous to that applied in Section 3 above. Provided $g > 0$ and the transversality condition holds then there exists a unique balanced growth path equilibrium on which:

$$q = \frac{\gamma}{1+\gamma(k-1)}\left[\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e}\psi\left(\frac{M_t}{R_t}\right)\frac{1+v}{1-v_e}+\frac{kn}{\sigma-1}-\rho\right], \tag{37}$$

$$\begin{aligned} c_0 \;=\; & A_1\left(k\sigma+1-\sigma\right)^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}} f^{-\frac{k+1-\sigma}{k(\sigma-1)}}\left(1+v\right) \\ & * \left[(k+1-\sigma)+k(\sigma-1)(1+v)+(\sigma-1)\frac{1+v}{1-v_e}\frac{n+gk}{n+gk+\frac{1-\gamma}{\gamma}q+\rho-n}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}, \end{aligned} \tag{38}$$

where $\frac{R_t}{M_t}$ satisfies (36) as before. Observe that either taxing the fixed production cost or subsidizing R&D leads to higher growth by increasing the ratio of fixed costs to R&D costs and raising the dynamic selection rate. Also, by comparing (37) with (23) and (38) with (30) we see that while tax policy can mimic the growth effect of trade integration it cannot simultaneously replicate the effect of trade on the level of consumption.

Household welfare on the balanced growth path still depends on q and c_0 through equation (26). Therefore, to analyze the welfare effects of tax policy we can substitute (37) and (38) into (26) and then differentiate with respect to v and v_e while using (36) to account for the endogeneity of $\frac{R_t}{M_t}$. For the sake of brevity the resulting algebra is omitted, but there are two main findings.

[33] The results in this appendix generalize immediately to the open economy model, but only if we abstract from strategic policy interactions across countries by imposing symmetric taxes in all economies.

First, when $v = v_e = 0$ welfare is strictly increasing in v. Moreover, provided $\Psi(R_t, M_t) = R_t$ welfare is also strictly increasing in v_e. This means that in the baseline model with constant returns to scale in R&D either taxing the fixed cost of production or subsidizing R&D raises welfare relative to an economy without taxes. In each case the policy is welfare improving because it increases the firm creation rate $\lambda^k \frac{R_t}{M_t}$ which is inefficiently low in the decentralized equilibrium since innovators do not internalize the productivity spillovers that entry generates.

Second, if the government chooses v and v_e simultaneously to maximize welfare the optimal tax rates satisfy:[34]

$$
\begin{aligned}
v &= \frac{f_e}{\lambda^k f} \frac{n + gk}{\psi\left(\frac{M_t}{R_t}\right)}, \\
v_e &= 1 - \frac{\psi\left(\frac{M_t}{R_t}\right)}{\psi\left(\frac{M_t}{R_t}\right) - \frac{M_t}{R_t}\psi'\left(\frac{M_t}{R_t}\right)} \left[1 + \frac{k+1-\sigma}{\sigma-1} \frac{v}{1+v} \frac{\psi\left(\frac{M_t}{R_t}\right)}{\psi\left(\frac{M_t}{R_t}\right) - \frac{M_t}{R_t}\psi'\left(\frac{M_t}{R_t}\right)}\right]^{-1}.
\end{aligned}
$$

It immediately follows that the government sets $v > 0$ implying a tax on the fixed costs of production. In addition, in the baseline model with constant returns to scale in R&D we have $v_e > 0$ meaning entry is subsidized. However, when there is congestion in technology diffusion R&D may either be subsidized or taxed depending on the shape of Ψ.

[34] These results hold assuming the government's maximization problem is concave. In general concavity is not guaranteed, but a sufficient condition that ensures concavity is $\Psi(R_t, M_t) = R_t^{\alpha} M_t^{1-\alpha}$ with $\alpha \in (0, 1]$ and $(1-\gamma)(k+1-\sigma) > \alpha\gamma k(\sigma - 1)$.

Appendix C – Competitive output markets

This Appendix characterizes in greater detail the balanced growth path equilibrium of the competitive output markets model considered in Section 6.1. First, observe that preferences and the R&D technology are unchanged from the baseline model implying that the free entry condition (10), the Euler equation (17), the transversality condition (18), the entry rate condition (24), the welfare expression (26), the initial consumption level equation (27), the asset value equation (28) and the initial exit cut-off condition (29) continue to hold.

Now consider production and trade. Profit maximization by a firm with productivity θ implies:

$$\begin{aligned} l_t(\theta) &= p^{\frac{\iota_t}{1-\beta}} \left(\frac{\beta\theta}{w_t}\right)^{\frac{1}{1-\beta}} + f + \iota_t f_x, \\ \pi_t(\theta) &= p^{\frac{\iota_t}{1-\beta}} (1-\beta)\beta^{\frac{\beta}{1-\beta}} \theta^{\frac{1}{1-\beta}} w_t^{\frac{-\beta}{1-\beta}} - f w_t - \iota_t f_x w_t, \end{aligned} \tag{39}$$

where ι_t is an indicator variable that takes value one if the firm chooses to pay the fixed export cost at time t and zero otherwise. Since the firm will choose to pay the fixed export cost only if profits are higher when $\iota_t = 1$, it follows from (39) that there exists an exit cut-off $\theta_t^* = \beta^{-\beta}(1-\beta)^{-(1-\beta)} f^{1-\beta} w_t$ and an export threshold $\tilde{\theta}_t = \left[\left(p^{\frac{1}{1-\beta}} - 1\right) \frac{f}{f_x}\right]^{-(1-\beta)} \theta_t^*$ such that at time t only firms with productivity below θ_t^* exit and only firms with productivity above $\tilde{\theta}_t$ pay the fixed export cost. To ensure that some, but not all, firms pay the fixed export cost I assume $p > 1$ and $\left[\left(p^{\frac{1}{1-\beta}} - 1\right) \frac{f}{f_x}\right]^{-(1-\beta)} > 1$. As $p > 1$ all firms that pay the fixed export cost choose to export their entire output.

As in the baseline model, the relative productivity distribution is Pareto with scale parameter one and shape parameter k. Moreover, differentiating the definition of the exit cut-off gives:

$$g = q.$$

Next, we can make the change of variables $\phi_t = \frac{\theta}{\theta_t^*}$ in the profit function (39) and substitute into the free entry condition (10) to obtain:

$$q = kg + r - \frac{1}{k(1-\beta) - 1} \frac{\lambda^k f}{f_e} \left[1 + \frac{f_x}{f} \left(\frac{\tilde{\theta}_t}{\theta_t^*}\right)^{-k}\right].$$

Combining this expression with $g = q$, the Euler equation (17) and the definition of the export threshold gives the equilibrium growth rate (33) given in the main text. Having solved for q it is then straightforward to check that the initial consumption level satisfies (34).

Finally, to ensure that the assumption $g > 0$ and the transversality condition are satisfied for all $p \geq 1$ Assumption 2 should be replaced by:

$$\begin{aligned} \frac{1}{k(1-\beta)-1}\frac{\lambda^k f}{f_e} &> \rho, \\ (1-\gamma)\frac{1}{k(1-\beta)-1}\frac{\lambda^k f}{f_e} &> \gamma k(n-\rho)+(1-\gamma)n. \end{aligned}$$

Table 1: Calibration observables and parameters

Observable/parameter		Value	Source
Import penetration ratio	IPR	0.081	U.S. import penetration ratio in 2000
Firm creation rate	NF	0.116	U.S. Small Business Administration 2002
Population growth rate	n	0.011	U.S. average 1980-2000
Trade elasticity	k	7.5	Anderson and Van Wincoop (2004)
Elasticity of substitution across goods	σ	8.1	$\sigma = k/1.06 + 1$ to match right tail index of employment distribution
Intertemporal elasticity of substitution	γ	0.33	García-Peñalosa and Turnovsky (2005)
Discount rate	ρ	0.04	García-Peñalosa and Turnovsky (2005)

Table 2: Calibration results

Outcome		Value
Growth rate - trade	q	0.0156
Growth rate - autarky	q^A	0.0141
Growth (trade vs. autarky)	q/q^A	**1.107**
Consumption level (trade vs. autarky)	c_0/c_0^A	1.010
Static gains from trade	z^s	1.011
Dynamic gains from trade	z^d	1.025
Total gains from trade	z	1.036
Gains from trade (total vs. static)	$(z-1)/(z^s-1)$	**3.2**

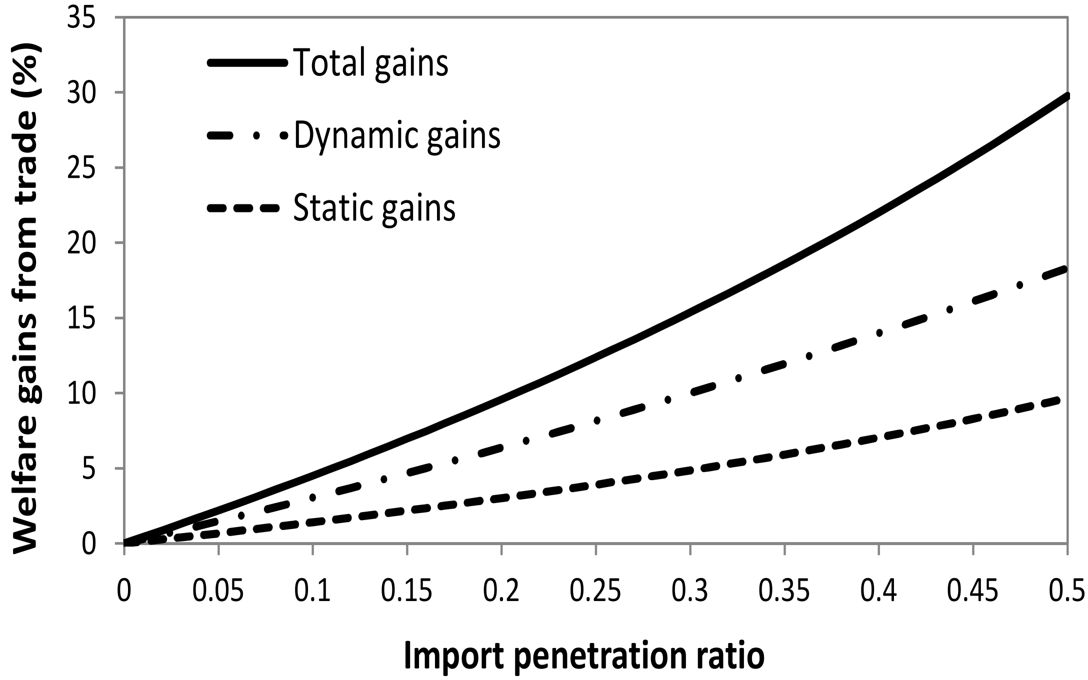

Figure 1: Import Penetration Ratio and the Gains from Trade

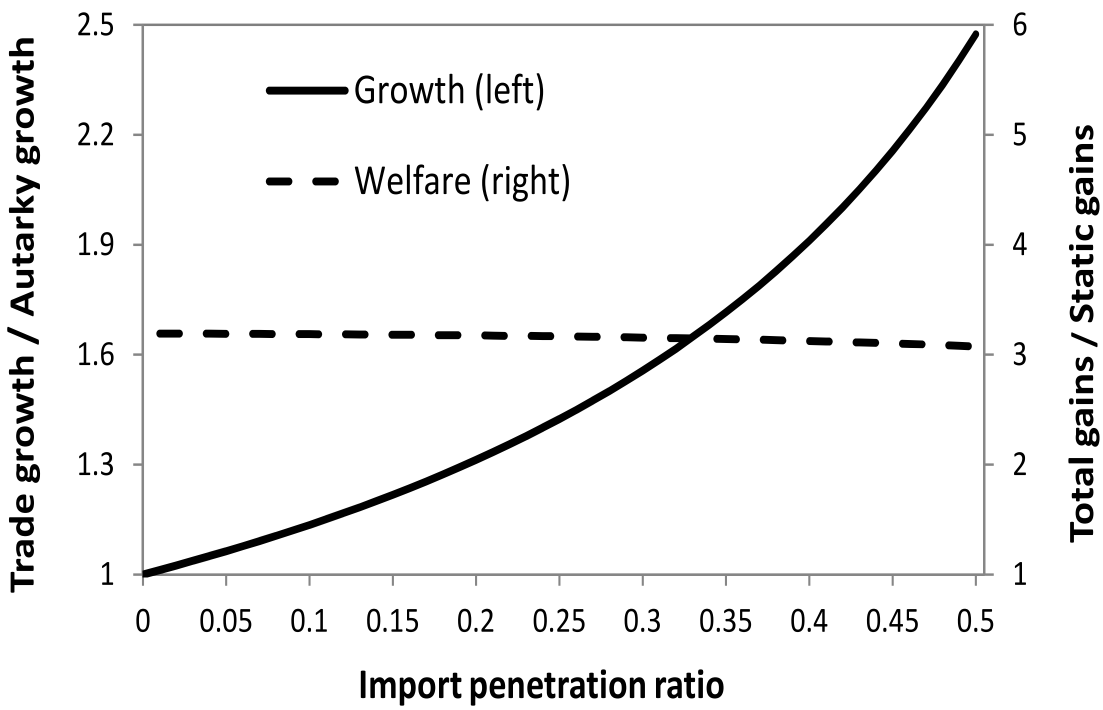

Figure 2: Import Penetration Ratio and the Dynamic Gains from Trade

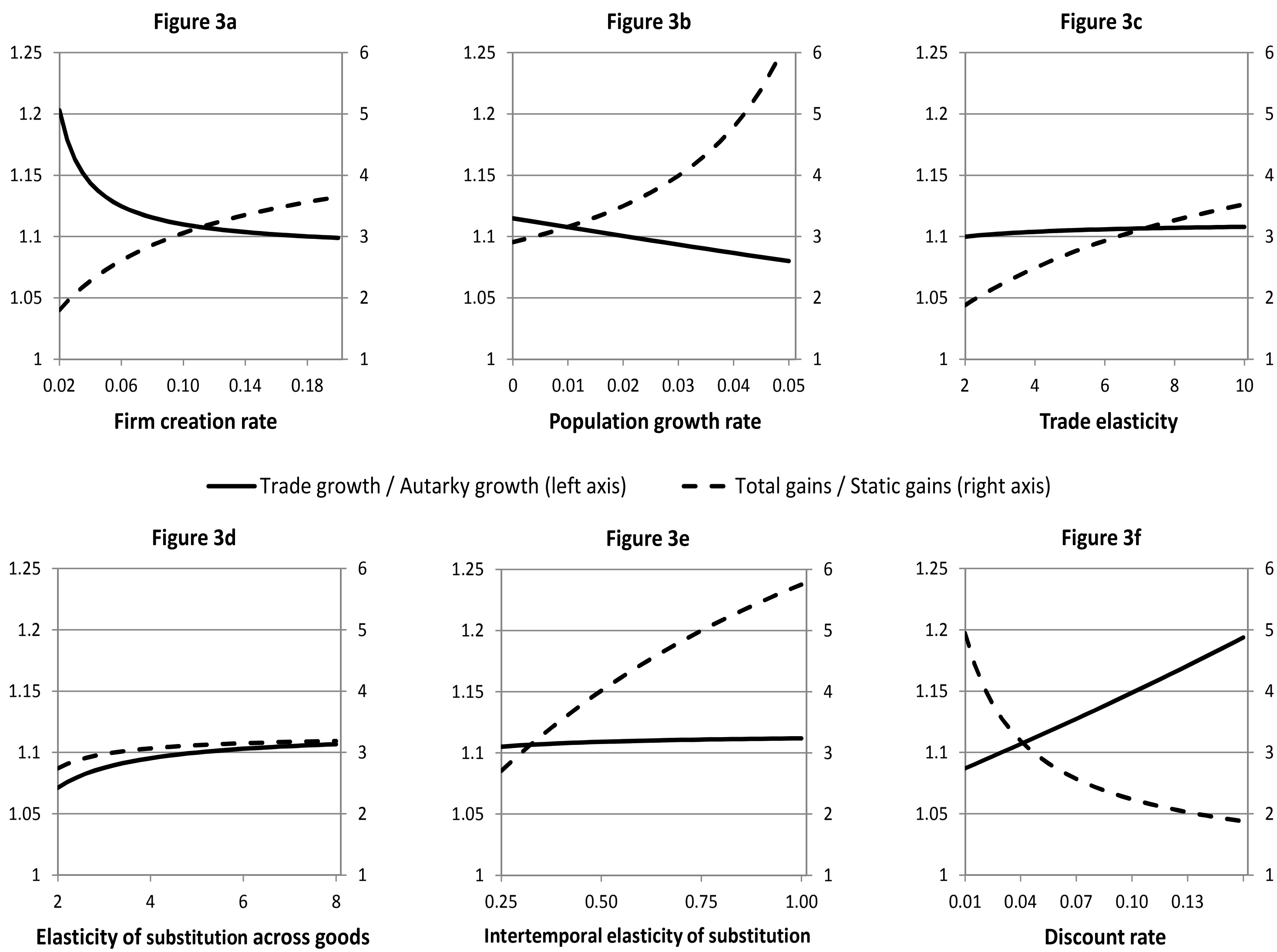

Figure 3: Dynamic Gains from Trade

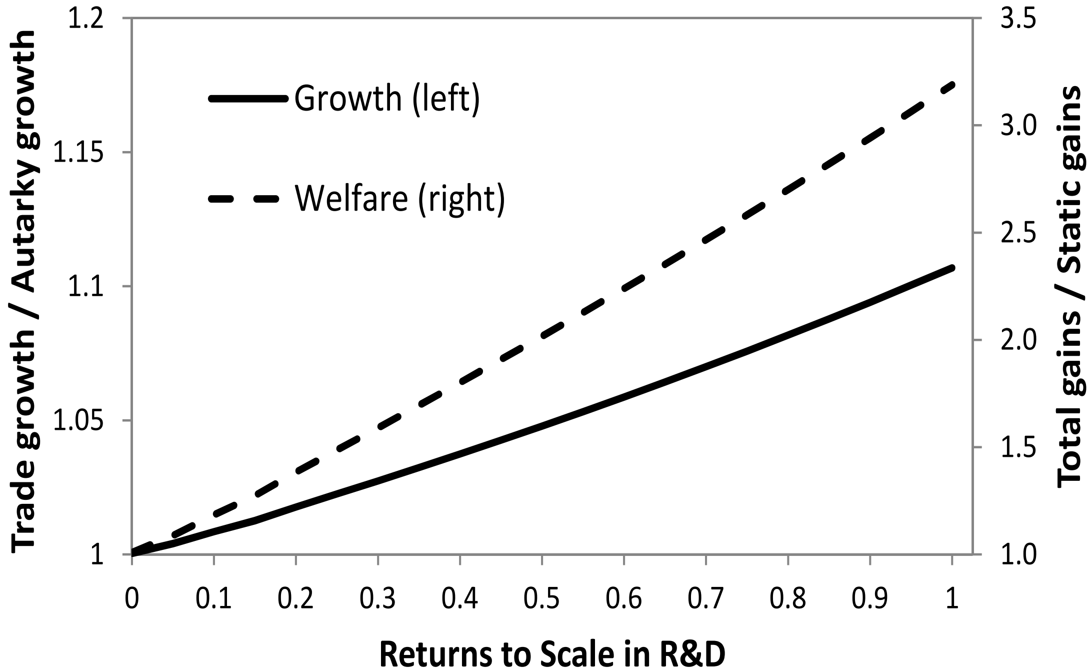

Figure 4: Returns to Scale in R&D and the Dynamic Gains from Trade