Wage Dynamics with Private Learning-by-doing and On-the-job Search

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Abstract

This paper develops an equilibrium job search model in which the employed worker privately accumulates human capital and continually searches for a better paying job. Firms encourage production and discourage turnover by rewarding with bonus payments and long service allowances, respectively, workers with better performance and longer job tenure. Wage growth attends human capital accumulation (productive promotion) and job tenure (non-productive promotion) as well as job-to-job transition. The model is estimated using indirect inference to investigate the effect of human capital accumulation on individual wage growth. In NLSY79 data, the average wage of white male high school graduates after 20 years of market experience is 1.88 times higher than the average of the first full-time wages. A counterfactual experiment using the structural parameter estimates shows that the wage of a typical worker unable to accumulate human capital would grow by 41.8%.

Key Words: Wage Dynamics, Human Capital Accumulation, Wage-Tenure Contract
JEL Classification: D82, E24, J31, J41

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1 Introduction

In the U.S. labor market, the wage of a typical male[1] worker doubles over the course of a 40-year career.[2] Wage growth is generally understood to be the outcome of human capital accumulation subsequent to entering the labor market. But human capital accumulation being neither a sufficient nor necessary condition for wage growth in the frictional labor market, it is necessary to develop a structural model to identify and quantify the relevant sources of wage growth. This paper builds and estimates an equilibrium job search model by focusing on firms' strategic responses to workers' private learning-by-doing and on-the-job search behavior.

Burdett and Mortensen (1998) develop a wage-posting model with on-the-job search in which some firms post high wages to attract workers from other firms and workers ascend the wage ladder only through job-to-job transition. Burdett and Coles (2003) depart from the aforementioned model by building an equilibrium in which firms optimally back-load some portion of wage payments to discourage job turnover and extract more surplus from early leavers. All firms post, and choose different starting points on, a common wage-tenure schedule. In equilibrium, wages grow with the back-loading schedule and job turnover. Burdett, Carrillo-Tudela, and Coles (2009) extend the Burdett and Mortensen (1998) framework by adding human capital accumulation. They assume that each worker accumulates human capital through the deterministic, on the job, learning-by-doing process, and is paid following a single wage-output ratio (the piece rate sharing rule), and that wages grow consequent to job turnover and learning-by-doing. Why firms stick to the piece rate sharing rule is unclear, however.

In fact, each worker privately accumulates human capital on the job through learning-by-doing, while continually searching for a better paying job. Incentives offered by firms unable to monitor the learning-by-doing process or job search outcomes include bonus payments and long-service allowances for superior performance and longer tenure, respectively. This motivates the present model's incorporation of private learning-by-doing into the equilibrium wage-tenure contract framework of Burdett and Coles (2003) and Stevens (2004). The resulting wage in a job rises with worker performance and job tenure. Market equilibrium implies multiple wage-ladders, each associated with workers identified with a particular level of human capital. Workers gradually climb their respective wage ladders along the back-loading wage schedule (non-productive promotion), jump to a higher rung through job-to-job transition, and switch ladders through human capital accumulation or depreciation (productive promotion).

The private learning-by-doing process originates two types of upward pressure in workers' compensation, (i) to induce truthful revelation, a commitment to higher value for better output (internal pressure), and (ii) productive workers being more attractive to poaching firms, a commitment to pay more to retain workers (external pressure). The latter, if it dominates the former, pushes the wage payment beyond the incentive compatible wage level. The incentive compatibility constraint should then be slack and

[1] The masculine pronoun is used throughout the paper because a male sample of workers is used in the empirical analysis.

[2] Topel and Ward (1992) report this fact in cross sectional data. The average wage in the focal sample of white male high school graduates in the 1979 National Longitudinal Survey of Youth after 5, 10, and 20 years of market experience is, respectively, 1.43, 1.61, and 1.88 times higher than the average of the first full-time wages.

information asymmetry no longer matter. Because, given the convexity of the offer and earning distributions, the optimal productive promotion schedule post human capital accumulation may not be well defined, another wage determination mechanism, such as the piece rate sharing rule, is required.[3] If, on the other hand, internal dominates external pressure, the optimal productive promotion schedule post human capital accumulation is determined by the (least cost) incentive compatibility constraints, which should be binding. The learning-by-doing process, practically speaking, being unobservable by firms, the present paper focuses on the case in which internal dominates external pressure.

The empirical analysis estimates the model and performs some counterfactual experiments to investigate returns to both human capital and tenure. A sample of white male high school graduates from the 1979 National Longitudinal Survey of Youth is constructed and tracked with respect to employment and wage history. The structural parameters of the model are estimated using indirect inference. The model implies human capital accumulation to have a permanent effect on wage growth and the effect of job tenure to be reset when a worker experiences unemployment. The difference between usual wage growth and re-employment wage[4] growth is exploited to capture the effect of human capital accumulation (productive promotion) independently of that of job tenure.

In the sample, the average wage after 20 years of market experience is 88% higher than the average of first full-time wages. The counterfactual analysis, which finds that wages would grow by 41.8% without human capital accumulation, implies that the return to human capital is, at best, the other 46% wage growth in the first 20 years. Considering the interactions, this might be construed to be an upper bound on returns to human capital. The limited effect of human capital accumulation is consistent with the sample's estimated slope coefficient in the re-employment wage-experience regression, being almost half the coefficient in the usual wage-experience regression.

Altonji and Shakotko (1987) attribute the lion's share of individual wage growth to returns to experience (general human capital), identified by 'within job wage growth transferred to the next job', and only limited effects to returns to tenure (job specific human capital), identified by 'within job wage growth non-transferable to the subsequent job'. Although not at odds with the empirical findings in Altonji and Shakotko (1987) and Altonji and Williams (2005) in the sense that 'within job wage growth transferred to the next job' is significant in wage growth, the present paper argues that care be exercised in interpreting 'within job wage growth transferred to the next job' as returns to human capital; because non-productive promotion is also transferred to subsequent jobs in the model used here, wage growth through productive promotion is redefined as returns to human capital, and wage growth through non-productive promotion as returns to tenure.

Bagger, Fontaine, Postel-Vinay, and Robin (2006), who study wage dynamics by combining learning-by-doing on the job with the ex post offer matching framework proposed by Postel-Vinay and Robin (2002), assume firm and worker to sign on to a

[3]Also required is anti-discrimination legislation, as in Burdett, Carrillo-Tudela, and Coles (2009), which, in assuming payment of exactly the same piece rate to all workers regardless of job tenure or performance, forestalls back-loading subsequent to, and extraction of all surplus prior to, human capital accumulation. The present paper assumes that two workers with the same job tenure and responsible for the same output cannot be paid differentially.

[4]By re-employment wage is meant the first wage after unemployment.

particular piece rate. An employed worker who finds a recruiting firm is bid new piece rates by the existing and recruiting firms, and accepts the offer with the higher lifetime value. Wages grow through human capital accumulation, that is, 'search and stay' and 'search and switch'. Estimating the model finds wage growth through 'search and stay' to have, consistent with the present paper, a substantial effect. The key difference between the two papers lies in the way returns to tenure are modeled; whereas the earlier paper assumes an ex post offer matching process,[5] the present paper relies on an ex ante, preemptive back-loading scheme.

The paper proceeds as follows. The theoretical model is built and the equilibrium of interest characterized in section 2. The sample is constructed and relevant variables are defined in section 3. The estimation protocol and results are presented in section 4. Section 5 concludes. All proofs and data construction are in the Appendix.

2 The Model

2.1 Environment

Consider a labor market populated by a unit measure of infinitely-lived and homogeneous risk-neutral firms, and mortal and heterogeneous, in terms of the level of human capital, risk-averse workers $y_i \in \mathcal{Y} := \{y_1, y_2, \cdots, y_n\}$. For expositional convenience, a worker having y_i units of human capital is designated a 'y_i-type worker'. Human capital is y_1 units upon a newly-born worker's entry to the labor market, and accumulates throughout his career. A worker who stochastically retires (or dies) is replaced immediately by another newly-born worker. The model is set in continuous time, and all firms and workers discount the future at rate r.

Workers A worker is either unemployed or employed. A y_i-type unemployed worker collects unemployment benefits b per instant, finds a job offer at rate λ_u, retires at rate ρ, and becomes a y_{i-1}-type by losing human capital at rate η. Denote as U_i the equilibrium asset value of the y_i-type unemployed worker, and let $F_i(\cdot)$ be the cumulative distribution function of lifetime values offered by recruiting firms to y_i-type workers. Equilibrium support and the cumulative distribution function are endogenously determined later. The HJB equation for the y_i-type unemployed worker is given by

$$rU_i = u(b) + \lambda_u \int \max\{x - U_i, 0\} dF_i(x) - \rho U_i + \eta(U_{i-1} - U_i). \tag{1}$$

There being no worker with human capital strictly less than y_1 units, ignoring U_0 or setting $U_0 = U_1$ in equation (1) implies that a y_1-type worker is not subject to depreciation shock. In the asset value equation (1), the left-hand side is interpreted as the opportunity cost of holding the asset y_i-type unemployment. The terms on the right-hand side are interpreted as the benefit flow from holding the asset U_i, which consists of the dividend flow from the asset, potential gains from job finding, potential loss from retirement, and potential loss from human capital depreciation, respectively.

[5] Ex post offer matching is vulnerable to the criticism that it provides the wrong incentives for a worker to search for an outside offer. See Shimer (2006).

A y_i-type employed worker can produce y_i units of output at every instant. Being time-varying private information, firms cannot pay different wages by worker type. It is assumed, instead, that anti-discrimination legislation dictates that a firm offer the same wage to workers with the same job tenure and performance.[6] Let $\phi : [0, \infty) \to \{y_1, y_2, \cdots, y_n\}$ be a mapping from the interval of job tenure to the set of types to which a worker potentially pretends to be. Operating firms determine wages as a function of job tenure and performance, that is, $w(t, \phi(t))$. It is further assumed that the flow disutility of a y_i-type employed worker who mimics a $\phi(t)$-type and produces $\phi(t)$-units of output at time t is given by

$$c_i(\phi(t)) = \begin{cases} \alpha_0 - \alpha_1(y_i - \phi(t)) & \text{if } \phi(t) \le y_i \\ \infty & \text{otherwise} \end{cases}.$$

This implies that the disutility from working is proportional to hours worked. A y_i-type worker who produces y_i units of output incurs the disutility of α_0. A y_i-type worker who elects to produce $\phi(t)(< y_i)$ units can finish the job earlier and expend his disutility as leisure. The private benefit from misreporting is captured by $\alpha_1(y_i - \phi(t))$. Note that in the market equilibrium, no efficiency loss accrues to the information asymmetry in each match.

A y_i-type employed worker finds another job offer at rate $\lambda \in (0, \lambda_u)$[7], privately accumulates human capital at rate μ, loses it at rate η, separates from his job at rate δ, and retires at rate ρ. The expected lifetime value of the y_i-type employed worker at tenure t who chooses production schedule $\phi(\cdot)$ under contract m, $E_i(t; \phi, m)$, is given by

$$E_i(t; \phi, m) = \int_t^\infty e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)} \Big[u(w(s, \phi_i(s); m)) - c_i(\phi(s)) + z(s; m) \Big] ds,$$

where

$$\begin{aligned} z_i(s; m) &= \delta U_i + \lambda \int_{\underline{E}_i}^{\overline{E}_i} \max\{x, \max_{\phi(\cdot)}\{E_i(s; \phi, m)\}\} dF_i(x) + \mu \max_{\phi(\cdot)}\{E_{i+1}(s; \phi, m)\} \\ &\quad + \eta \max_{\phi(\cdot)}\{E_{i-1}(s; \phi, m)\}. \end{aligned}$$

As above, $E_0(s; \phi, m)$ and $E_{n+1}(s; \phi, m)$ are ignored throughout the paper. A y_i-type employed worker under contract m can choose and update his own production schedule to maximize his lifetime value, which is is characterized at time t by

$$\max_{\phi(\cdot)} \{E_i(t; \phi, m)\}.$$

When a y_i-type employed worker truthfully chooses $\phi(\cdot) = y_i$, $E_i(t; m)$ is used instead of $E_i(t; \phi, m)$. The HJB equation for a y_i-type employed worker with a truthful production schedule is given by

$$\begin{aligned} rE_i(t; m) &= u(w(t, y_i; m)) - c_i(y_i) + \dot{E}_i(t; m) + \lambda \int \max\{x - E_i(t; m), 0\} dF_i(x) - \rho E_i(t; m) \\ &\quad + \delta(U_i - E_i(t; m)) + \mu(E_{i+1}(t; m) - E_i(t; m)) + \eta(E_{i-1}(t; m) - E_i(t; m)). \qquad (2) \end{aligned}$$

[6]The anti-discrimination legislation concept is borrowed, with modifications, from Burdett, Carrillo-Tudela, and Coles (2009), whose assumption that a firm should pay exactly the same piece rate to all workers implies that workers who produce the same output should be paid the same wage. The present paper assumes that wages can vary with employee job tenure and performance.

[7]Recruiting firms are assumed to be contacted more frequently by unemployed than by employed workers.

Firms Each firm maintains one vacancy at every instant. A firm recruits a worker by posting a labor contract, m, that specifies the action profile (or 'terms of trade') that stipulates the worker's output schedule and the lifetime value delivered by the firm under the truthful revelation assumption. That is, m is characterized by $\{(y_i(\cdot), E_i(\cdot; m))\}_{i=1}^n$.

Definition *Contract m is* incentive compatible for y_i-type *if*

$$E_i(t;m) \geq \max_{\phi}\{E_i(t;\phi,m)\} \text{ at each } t \in [0,\infty). \tag{3}$$

In particular, when the contract is incentive compatible for all types, it is called incentive compatible.

As a tie-breaking rule, a y_i-type worker who is indifferent is assumed to truthfully produce y_i. The least cost incentive compatibility is defined separately, as follows.

Definition *Contract, m, is* least cost incentive compatible for y_i-type *if the following statements hold;*

(i) Contract m is incentive compatible for y_i-type workers.

(ii) There exists at least one $\phi : [0,\infty] \to \mathcal{Y} \cap \{y_i\}^c$ such that $E_i(t;m) = E_i(t;\phi,m)$ at any $t \in [0,\infty)$.

In particular, when the contract is least cost incentive compatible for all types, it is called least cost incentive compatible.

When a menu of contracts is accepted by a worker, firm and worker together begin producing immediately. If y_i units are produced by an employee with job tenure t, the operating firm earns revenue y_i and makes wage payment $w(t, y_i; m)$. The match is destroyed when the worker leaves the job, whether voluntarily or involuntarily.[8] Denote as $J_i(t;m)$ the expected value of an operating job with a y_i-type worker under contract m. For expositional convenience, let

$$\hat{z}_i(s;m) = \mu J_{i+1}(s;m) + \eta J_{i-1}(s;m).$$

Given the promised value $\{E_i(0;m)\}_{i=1}^n$, the operating firm with a y_i-type worker chooses the schedule of $\{w(\cdot, \phi_i(\cdot); m)\}_{i=1}^n$ to maximize the expected value

$$\int_t^\infty e^{-\int_t^s [r+\rho+\delta+\lambda(1-F_i(E_i(\tau;m)))+\mu+\eta]d\tau}[y_i(s) - w(s, y_i(s); m) + \hat{z}_i(s;m)]ds \tag{4}$$

subject to the sets of least cost incentive compatibility and promise-keeping constraints. The least cost incentive compatibility constraints presume conditions (i) and (ii) above. The promise-keeping constraints, described in (2), imply that a firm that commits $\{E_i(\cdot;m)\}_{i=1}^n$ in terms of the truthful revelation value should deliver same through wage schedules.

Let u_i and $G_i(x)$ be the proportion of y_i-type unemployed and employed workers, respectively, receiving the value less than x. Denote as $\overline{m}$ and $\underline{m}$ the contract offered

[8] Because, in equilibrium, all jobs yield positive expected profit to firms, there is no endogenous firing.

by the most generous and least generous recruiting firm, respectively.[9] Then, for each type,

$$G_i(E_i(0;\underline{m})) = 0, \quad \frac{\partial G_i(\cdot)}{\partial x} \geq 0, \quad \text{and} \quad \sum_{i=1}^{n}[u_i + G_i(E_i(0;\overline{m}))] = 1.$$

Denote as $\mathcal{M}$ the set of equilibrium contracts. The equal profit condition implies that

$$\sum_{i=1}^{n}(\lambda G_i(E_i(0;m)) + \lambda_u u_i)J_i(0;m) \begin{cases} = \pi, & \text{if } \ m \in \mathcal{M}, \\ < \pi & \text{otherwise.} \end{cases} \tag{5}$$

Aggregating all recruiting firms' strategies yields the distribution of lifetime values offered to each type, $\{F_i\}_{i=1}^n$. Given $\{F_i\}_{i=1}^n$, both employed and unemployed workers behave optimally. Given $\{F_i\}_{i=1}^n$ and workers' optimal behaviors, operating firms choose the productive and non-productive promotion schedules that determine the steady state distribution $\{u_i, G_i(\cdot)\}_{i=1}^n$. Then, $\{u_i, G_i(\cdot), J_i, E_i\}_{i=1}^n$ should be consistent with the equal profit condition (5). The equilibrium is consequently defined as follows.

Definition *A market equilibrium requires that the following two conditions be met.*

1. *As above, is this redundant with the Arabic numeral (ditto below)? Given* $\{F_i\}_{i=1}^n$, *a* y_i*-type unemployed worker accepts the contract* $\{(y_i, E_i(0;m))\}_{i=1}^n$ *if and only if* *(i)*

$$\max_{\phi}\{E_i(0;\phi,m)\} \geq U_i.$$

2. *(ii) Given* $\{F_i\}_{i=1}^n$, *a* y_i*-type employed worker optimally chooses the production schedule and accepts a new contract* m' *if and only if*

$$\max_{\phi}\{E_i(0;\phi,m')\} \geq \max_{\phi}\{E_i(t;\phi,m)\}.$$

3. *(iii) Given* $\{F_i\}_{i=1}^n$, *an operating firm with contract* m *optimally chooses the wage schedules to deliver* $\{E_i(0;m)\}_{i=1}^n$. *The contracts described by* $\{y_i, E_i(t;m)\}_{i=1}^n$ *are least cost incentive compatible at any* $t \in [0,\infty)$.
4. *(iv) Given* $\{F_i\}_{i=1}^n$, *the optimal behavior of each economic agent determines* $\{u_i, G_i(\cdot)\}_{i=1}^n$.
5. *(v) Given* $\{u_i, G_i(\cdot)\}_{i=1}^n$, *a recruiting firm optimally posts contract* m *given the equal profit condition described in* (5).
6. *(vi) The equilibrium distributions* $\{F_i, G_i(\cdot)\}_{i=1}^n$ *are stationary.*

[9] The argument that some firms make the most generous offer to some types and other firms to other types is rendered moot by least cost incentive compatibility.

2.2 Equilibrium Characterization

The equilibrium in which all firms offer least cost incentive compatible contracts is characterized below. Technically, all least cost incentive compatible constraints are assumed to be binding, and whether firms have incentives to deviate from the least cost incentive compatible contracts is numerically checked. To make the model tractable, the following equilibrium restriction is imposed.

Eq'm Restriction Let $\mathcal{M}$ be the set of contracts offered on equilibrium. For all $E_i(t;m) \in (\underline{E}_i, \overline{E}_i)$,

(*i*) F_i is continuously differentiable and satisfies $F_i'(E_i)$ is bounded away from zero.

(*ii*) $F_1(E_1(t;m)) = F_2(E_2(t;m)) = \cdots = F_n(E_n(t;m))$ for any $m \in \mathcal{M}$ and $t \in [0,\infty)$.

This restriction, which implies that the acceptance (and retention) probability under contract m is same across all types, is based on pre-imposed equilibrium condition that firms have no incentive to screen out any type. Note, too, that the second condition (*ii*) renders the least generous contract $\underline{m}$ and the most generous contact $\overline{m}$ well-defined.

Lemma 1 *Contract m is least cost incentive compatible if and only if*

$$u(w(t,y_i;m)) = u(w(t,y_1;m)) + \alpha_1(y_i - y_1), \quad \textit{at any} \;\; t \in [0,\infty). \tag{6}$$

Lemma 1 characterizes the least cost incentive compatible contracts. The set of least cost incentive compatible contracts, given that no firm has incentives to screen out any particular type, is characterized by

$$\min_i \{E_i(0;\underline{m}) - U_i\} = 0. \tag{7}$$

This implies that the lifetime value of employment delivered by the least generous firm should be not less than the values of unemployment for each type i, and at least one of them should be same to it. The equal profit condition dictates that, given $\underline{m}$,

$$\sum_{i=1}^{n} \lambda_u u_i J_i(0;\underline{m}) = \sum_{i=1}^{n} (\lambda G_i(E_i(0;\overline{m})) + \lambda_u u_i) J_i(0;\overline{m}). \tag{8}$$

There being no contract that dominates contract $\overline{m}$, the operating firm with contract $\overline{m}$ has no incentive to increase a worker's value over $\{E_i(0;\overline{m})\}_{i=1}^n$. That is, for any $i = 1, 2, \cdots, n$,

$$E_i(t;\overline{m}) = E_i(0;\overline{m}), \quad \text{at every} \;\; t \in [0,\infty). \tag{9}$$

This implies that, given i,

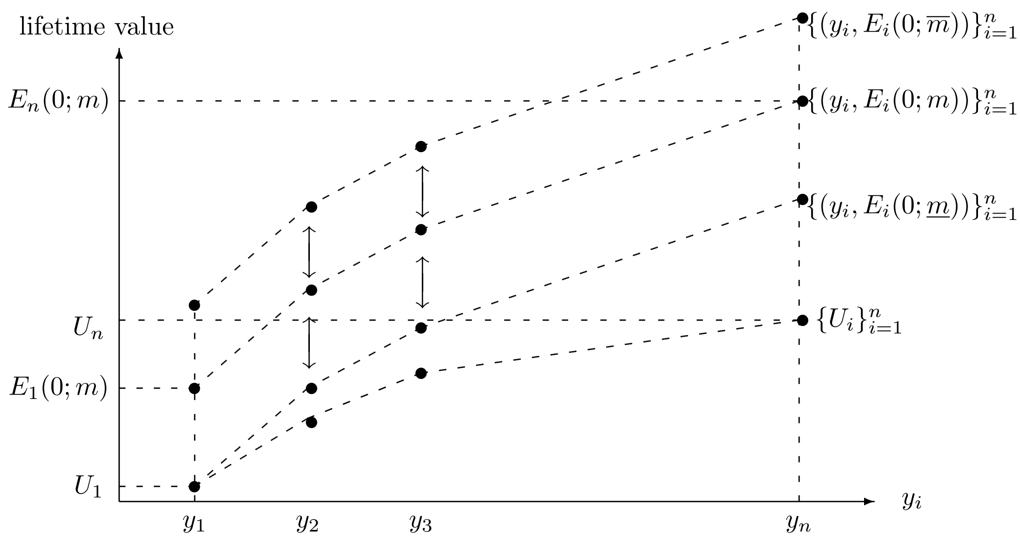
$$E_i(t;\overline{m}) \;\; = \;\; \frac{u(w(t,y_i;\overline{m})) - c_i(y_i) + \delta U_i + \mu E_{i+1}(t;\overline{m}) + \eta E_{i-1}(t;\overline{m})}{r + \rho + \delta + \mu + \eta} = \overline{E}_i, \tag{10}$$

$$J_i(t;\overline{m}) = \frac{y_i - w(t, y_i; \overline{m})}{r+\rho+\delta+\mu+\eta} = \overline{J}_i, \quad \text{and} \tag{11}$$

$$w(t, y_i; \overline{m}) = w(0, y_i; \overline{m}) = \overline{w}_i \quad \text{for any} \quad t \in [0, \infty). \tag{12}$$

The second statement implies that, given $E_i(0;\overline{m})$ by the most generous recruiting firm, no firm has incentives to pay more than the value, which renders the optimal wage schedule for each type by the most generous firm constant. In equilibrium, the wage schedules of all firms are bounded.

The foregoing is summarized in [Figure 1]. If both m and m' are least cost incentive compatible, then $\{(y_i, E_i(0;m))\}_{i=1}^n$ and $\{(y_i, E_i(0;m'))\}_{i=1}^n$ cannot intersect. Lemma 1 determines the size of information rent, which is represented by the slope of the contract curve in [Figure 1]. The gap between $E_i(0;\underline{m})$ and U_i should be zero at at least one y_i.[10] Given $\{F_i(\cdot), G(i,\cdot)\}_{i=1}^n$ and $\underline{m}$, the equal profit condition determines $\overline{m}$.



[Figure 1]

It remains to examine the strategy of the least generous firm. Given $\underline{m}$, the firm chooses an optimal wage schedule to deliver the committed value $\{E_i(0;\underline{m})\}_{i=1}^n$. For expositional convenience, denote as w_i the wage schedule of the y_i-type worker who produces truthfully under the least cost incentive compatible contract $\underline{m}$. Let

$$\hat{\psi}(t) = \exp[\int_0^t (r+\rho+\delta+\lambda(1-F_1(E_1(s;m))))ds].$$

Lemma 2 *Given $\{F_i\}_{i=1}^n$ and $E_1(0;\underline{m})$, the optimal wage-tenure schedules solve for*

$$\begin{aligned} \dot{J}_i &= -(y_i - w_i + \eta J_{i-1} + \mu J_{i+1}) + [r+\rho+\delta+\lambda(1-F_i(E_i)) + \mu + \eta] J_i, \\ \dot{E}_i &= -u(w_i) + \alpha_0 + (r+\rho+\delta+\lambda(1-F_i(E_i)) + \mu+\eta) E_i \\ &\quad -\delta U_i - \lambda \int_{E_i}^{\overline{E}} x dF_i(x) - \mu E_{i+1} - \eta E_{i-1}, \end{aligned}$$

[10] In most numerical experiments, $E_1(0;\underline{m}) = U_1$ and $E_{-1}(0;\underline{m}) > U_{-1}$.

$$
\begin{aligned}
\dot{w}_1 &= \Big[\frac{\psi_1\hat{\psi}_1 u''(w_1)}{[u'(w_1)]^2} - \sum_{i=2}^{n}\frac{x_{ji}\hat{\psi}_1 u''(w_i)}{[u'(w_i)]^2}\cdot\frac{u'(w_1)}{u'(w_i)}\Big]^{-1} \\
&\quad \Big[\hat{\psi}(t)\sum_{i=1}^{n} x_{ji}\lambda F_i'(E_i)J_i + \frac{\psi_1\dot{\hat{\psi}}}{u'(w_1)} - \sum_{i=2}^{n}\frac{x_{ji}\dot{\hat{\psi}}}{u'(w_i)} + \frac{\dot{\psi}_1\hat{\psi}_1}{u'(w_1)} - \sum_{i=2}^{n}\frac{\dot{x}_{ji}\hat{\psi}_1}{u'(w_i)}\Big] \\
w_i &= u^{-1}(u(w_{i-1}) + \alpha_1\Delta), \quad \textit{and} \\
\dot{x}_i &= x_{i-1}\mu + x_{i+1}\eta - x_i[r + \rho + \delta + \lambda(1 - F_i(E_i)) + \mu + \eta]
\end{aligned}
$$

subject to the boundary conditions:

$$\lim_{t\to\infty}\{J_i, E_i, w_i, x_i\}_{i=1}^{n} = \{\overline{J}_i, \overline{E}_i, \overline{w}_i, 0\}_{i=1}^{n}.$$

The general optimal wage-tenure contract $m(\neq \underline{m})$ of other recruiting firms is now considered. The optimal paths represented by the system of differential equations and their boundary conditions in Lemma 2 are uniquely determined by the baseline salary scale borrowed from Burdett and Coles (2003), which is important because it can be extended to prescribe the wage schedules offered by all firms in a steady state. That the initial value $E_1(0;\underline{m})$ establishes only a starting point means that firms that post $E_1(0;m) > E_1(0;\underline{m})$ move along the same path but from different starting points. That is, recruiting firms choose different starting points on the baseline salary scale,

$$E_i(0;m) = E_i(t;\underline{m}) = E_i(t), \quad J_i(0;m) = J_i(t;\underline{m}) = J_i(t), \quad \text{and}$$
$$w_i(0;m) = w_i(t;\underline{m}) = w_i(t).$$

Given the baseline property, $F : [0,\infty] \to [0,1]$ is defined as the distribution of starting points on the baseline. Let $G_i(t)$ be the proportion of y_i-type employed workers that receives less than $E_i(t;\underline{m})$. Then,

$$F_i(E_i(0;m)) = F_i(E_i(t;\underline{m})) = F(t) \quad \text{and} \quad G(i, E_i(0;m)) = G(i, E_i(t;\underline{m})) = G_i(t).$$

The y_i-type employed workers retire, are laid off, and accumulate or lose human capital at rate $\rho + \delta + \mu + \eta$ such that the outflow from $\overline{G}_i$ is $(\rho + \delta + \mu + \eta)\overline{G}_i$, where $\overline{G}_i = G_i(\infty)$, y_i-type unemployed workers get jobs at rate λ_u, andy_{i-1}-type and y_{i+1}-type employed workers enter into $\overline{G}_i$ at rate μ and η, respectively. The outflow and inflow at steady state are equated to solve for $\overline{G}_i$ and u_i, and, by the same reasoning, for $\dot{G}_i(t)$. The baseline property presents steady state $\{(u_i, G_i)\}_{i=1}^{n}$ in the following.

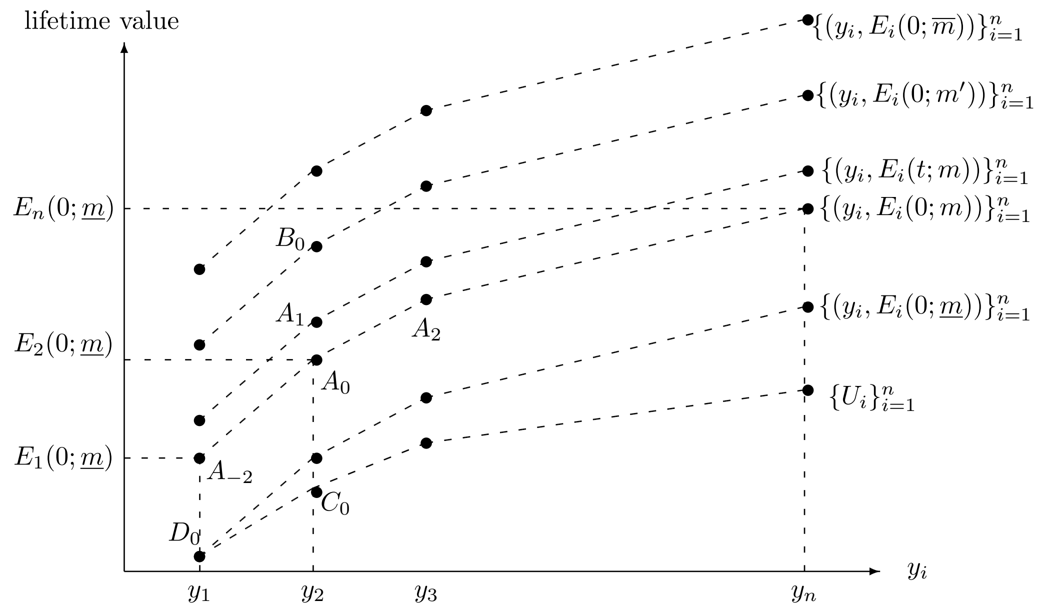
Lemma 3 *In the steady state equilibrium,*

$$\dot{G}_i(t) = \lambda_u F(t) u_i + \mu G_{i-1}(t) + \eta G_{i+1}(t) - (\rho + \delta + \lambda(1 - F(t)) + \mu + \eta) G_i(t),$$

where

$$
\begin{aligned}
u_1 &= \frac{\delta\overline{G}_1 + \rho + \eta_2 u_2}{\lambda_u + \rho}, \\
u_i &= \frac{\delta\overline{G}_i + \eta u_{i+1}}{\lambda_u + \rho + \eta}, \quad i = 2, 3, \cdots, n \quad \textit{and}
\end{aligned}
$$

$$\overline{G}_i = \frac{u_i\lambda_u + \mu\overline{G}_{i-1} + \eta\overline{G}_{i+1}}{\delta+\rho+\mu+\eta}, \quad i = 1, 2, \cdots, n.$$



[Figure 2]

[Figure 2] summarizes Lemma 1 through Lemma 3. A y_2-type employed worker under contract m, denoted by A_0 in the figure, who switches to a better paying job with contract m' jumps to point B_0 on his value ladder. If he is able to remain there without any shocks and is gradually promoted, he will be found, after some time, at A_1. Accumulating (or losing) Δ units of human capital will result in his moving to point A_2 (A_{-2}) on a neighboring ladder. Becoming unemployed will find him moved down to C_0. Upon retiring, he is replaced by a newly-born worker whose career starts at point D_0. Lemma 1 and 2 determine the gains from human capital accumulation and speed of non-productive promotion, respectively. Lemma 3 presents the earnings distribution over the type-value space.

Proposition 1 *The sufficient conditions for a market equilibrium are as follow.*

(i) Given F, Lemma 1-3 are satisfied.

(ii) F is stationary and satisfies the equilibrium restriction.

Because providing a fixed point algorithm to find a market equilibrium does not guarantee the existence of an equilibrium, and it is unclear, moreover, when the least cost incentive compatibility constraints are binding, in lieu of a theoretical proof, the model is solved numerically and whether the implied equilibrium outcome satisfies the sufficient condition is checked. For a broad range of model parameters, a unique fixed point is obtained with all constraints binding. In particular, when μ is small such that a relatively large mass is of the y_1-type, the constraints are binding.[11]

[11] The estimates yield $\mu = 0.023$.

3 Data

The source of the data used in the study is the 1979 National Longitudinal Survey of Youth (NLSY79), which contains weekly work records from 1978 through 2010. The model implies that workers' wages vary with their (unobserved) productivity and (observed) job tenure. NLSY79 is well suited to the analysis of careers (and human capital accumulation) because it reports weekly labor force status from the high school period, which enables individual workers' entire work histories, from their first jobs, to be examined. The manner of constructing the sample such that five jobs are tracked in each survey round is detailed in Appendix B. The focus here is mainly on defining the adopted variables.

Workers The survey begins with individuals 14-22 years old in 1979. The sample extracts from these individuals white male high school graduates, the largest demographic group in NLSY79, who completed 12th grade or received the equivalent degree (GED) between the ages of 17 and 20 after 1978, and reported no more than 12 years of education until the most recent survey. The age and year restrictions were imposed to exclude individuals with unusual or hidden experience. Receiving a high school diploma earlier than age 17, later than age 20, or before 1978 is presumed to reflect such experience, which is not captured in the survey. By similar reasoning, individuals who entered military service were excluded. Following this selection rule yields a sample of 776 individuals.

Full-time Employment and Non-employment Full-time employment is understood to mean working, on average, more than 30 hours per week. Calculating average hours worked as the weighted (by the number of weeks) mean of hours worked at the match (with the same employer) throughout a worker's career precludes a transition between part-time and full-time work with the same employer, but includes the case of working less than 30 hours per week for a short period as a trainee or intern and more than 30 hours for a long period as a regular worker. Periods reported as out of the labor force, no information, unemployment, and employment with average hours worked less than 30 hours per week are recoded as non-employment, the counterpart of unemployment in the model. When hours worked per week is unavailable, hours worked is calculated as 'hours worked per day' times five working days per week.

Labor Market Entry Following Farber and Gibbons (1996) and Yamaguchi (2009), a worker employed full-time for more than half of three consecutive years for the first time is assumed to have made the transition from school to work. Each individual's work history is tracked from his first transition. By construction, all workers in the sample begin their careers as employed workers. To mitigate any risk of potential bias, the first unemployment period before the first job is ignored.

Tenures Job tenure is defined as the length of a continuous period of work with one employer, employment tenure as the duration of consecutive job spells without non-employment. The source of the difference is job-to-job transition–switching to a new from an old job resets job, but not employment, tenure–but how to determine job-to-job transition in the sample is not clear. To accommodate instances like brief vacations interposed between jobs, non-employment spells of less than three weeks duration, being assumed to more likely be an outcome of on-the-job search than an employment-unemployment-employment transition, are discarded. This selection integrates into subsequent job tenure 1,702 instances of short term non-employment.

Although the model implies that there are neither recalled jobs nor returned workers, NLSY79 includes numerous instances of workers who returned to jobs they left for varying periods. In instances like unpaid vacations or hospitalization, in which the absence is planned in advance by both parties and the worker's return considered in the previous labor contract, the former and recalled job are considered one job, and the new contract, post return, is affected by the previous contract. Instances of unplanned absence are considered two different jobs because the fact of the worker's return affects neither the previous nor the new labor contract. As in Pavan (2008), an intermediate period of sufficiently short duration is naturally presumed to be more likely to have been planned. In the sample, if a worker returned to a previous job within one quarter, the intermediate work history is dropped and the two jobs are considered one continued job. Otherwise, they are considered two different jobs. This provision results in 923 short-term (less than one quarter) non-employment periods, and 84 temporary jobs being classified 'planned return' and dropped. In 555 cases, workers returned to an old job after one quarter. The final sample contains 4,325 employer-employee matches[12] and 4,880 jobs.

Experiences Because the model assumes human capital to be accumulated only on the job, it is necessary to distinguish worker from market experience. Hence, worker experience is defined as the sum of all employment spells, market experience calculated by subtracting age at entry from a worker's current age. In the NLSY79 data set, wages are reported on the interview date and, if it ended, on the job's end date. Beginning with the 1985 survey, the first wage on the job was solicited. Because first wage data might be biased by the reporting of the first wage for jobs started before 1985 and kept until the 1985 survey, the simulation uses the first wage only for jobs started after or continued until the 1985 survey. For the first wages reported, the re-employment wage, a key variable for estimating the effect of human capital accumulation, is defined as the first wage after non-employment. The sample of 13,735 wage observations includes some with potential coding errors.[13] Because of the difficulty of fitting all data points (especially at the ends) using a simple model, the top and bottom 2.5% of wage observations were discarded and the remaining 95% made the focus of analysis.

The final sample contains 665 individuals, 4,325 employer-employee matches, 4,880 jobs, and 14,298 observations. Construction of the data set is detailed in the Appendix.

[12]Because NLSY79 does not distinguish 'job' from 'employer-employee match', all returning cases are considered one job.

[13]For example, $0.03 per hour was the lowest, $862.69 per hour the highest, wage reported (after adjustment by monthly CPI).

4 Estimation

4.1 Estimation Procedure

Maximum likelihood inference not being numerically feasible, indirect inference was used, as in Bagger, Fontaine, Postel-Vinay, and Robin (2006). Indirect inference requires[14] that the structural model replicate the true data generating process in terms of specific target moments given a true value of the structural parameter vector θ_0. Denote as $g(\theta)$ the vector of the target moments simulated by the parameter vector θ, which is estimated by minimizing the distance between the set of sample moments from NLSY79 and set of moments from the simulations. The moment vector is simulated and calculated k times and the average taken. The simulated moments estimator of θ_0 is defined as

$$\hat{\theta} = \arg\min_{\theta} \quad (\overline{g}_k(\theta) - g(\theta_0))^T \widehat{w}_n (\overline{g}_k(\theta) - g(\theta_0)),$$

where $\widehat{w}_n$ is a positive definite matrix that converges in probability to a deterministic positive definite matrix W. The covariance matrix of the auxiliary statistics is estimated by re-sampling 665 individuals with replacement 1,000 times (n=1000) and taking the inverse. The entire wage and employment history of a selected individual i is included in the sample. For each set of simulated moments, the simulation is repeated 200 times and the average of the moments from each simulation (k=200) taken.

In the absence of any theoretical evidence of the uniqueness of the minimum value, the objective function is minimized using both the Nelder-Mead and simulated annealing algorithms. The Nelder-Mead method is used repeatedly. When the distance reaches a local minimum, the size of the simplex is reset and the algorithm restarted from the local minimum. If the program stops at a point sufficiently close to the local minimum, the simulated annealing method is invoked. Although the latter involves heavy computation, the probability of reaching a global minimum can be increased by applying the simulated annealing method repeatedly. In the present paper, the process is repeated with four different starting points. Obtaining the same estimates for the structural parameters is taken to be a global minimizer.

4.2 Estimation Specification

CARA (exponential) utility with risk aversion parameter γ is assumed for the empirical implementation.

$$u(w) = -\exp(-\gamma w)$$

Normalizing $y_{n_j} = 1.0$, the most productive worker produces one unit of output; $y_1 = 0.4$ and $\Delta = 0.1$ are then set so as to have seven types of workers ($n_j = 7$).[15] The latter choice is arbitrary, but absent output data it is difficult to obtain inference from these parameters. The number of equilibrium contracts is fixed at 20 levels[16]

[14] For details on 'indirect inference', see Gourieroux, Monfort, and Renault (1993) and Gourieroux and Monfort (1997).

[15] Level of human capital is discretized into seven levels.

[16] The cases with 19 and 21 equilibrium contracts are reported later.

and $s = 0.01$. The highest being nearly eight times the lowest wage in the sample, the highest wage is eight or nine times greater than the lowest wage depending on parameter values. The interest rate r is fixed at 0.012.

The short history of the NLSY79 data set makes it is difficult to estimate the arrival rate of the retirement shock ρ. Assuming the average worker to remain in the labor market for 40 years fixes ρ at 1/160. To match the actual survival probability, an 'attrition probability' of 2.5%, introduced in each survey round, accounts for the survey's loss, over time, of some workers who would otherwise remain in the labor market.

4.3 Estimation

Seven structural parameters are estimated: four Poisson arrival rates $(\delta, \lambda_u, \lambda, \mu)$, the risk aversion parameter γ, the unemployment benefit b (or w_{max}), and cost function parameter c_1.

The average nonemployment spell, average job spell, and average length of unemployment over the first five years are used to capture the dynamic flow of workers. The model implies that the rate at which workers are promoted increases with the accumulation of human capital, and that job turnover is more likely among young workers with less human capital. Examining the sample's total non-employment (or employment) period over the first five years reveals an average unemployment duration of 0.471 years, job spell of 2.175 years, and that the average worker keeps a full time job during 88.3% of the first five years.

One of the main objectives of the present empirical study being to estimate the effect of human capital accumulation independently of the effect of strategic promotion and job turnover, the log re-employment wage($\widehat{w}$), re-employment wage being defined as the first wage after unemployment, is regressed on work experience,

$$\hat{w_k} = \beta_0 + \beta_1 \times work\ experience_k + \varepsilon_k,$$

where ε_k is a statistical residual. Adopting β_1 as the auxiliary moment captures wage growth attributable to the accumulation of work experience. The regression coefficient being insufficient to identify the frequency and magnitude of the human capital accumulation shock, human capital accumulation in the model is determined to occur at rate μ and increase workers' wages by a certain amount affected by c_1. Information on the re-employment wage distribution is exploited to capture the frequency and magnitude of individual shocks. The ratios of the 3rd to the 1st and 2nd to the 1st quartile of the distribution are calculated. The auxiliary regression indicates a coefficient β_1 of 0.109, and the quartile ratios are 1.775 and 1.281, respectively.

The slope of the wage-tenure profile is captured by regressing wages reported in the first five years ($\tilde{w}$) on market experience,

$$\tilde{w}_k = \alpha_0 + \alpha_1 \times market\ experience_k + u_k,$$

where α_1 is adopted as one auxiliary moment. The focus on wages reported in the first five years makes it more likely that promotion rates are dependent on the level of human capital, and the focus on a narrow and identical group serves to more accurately capture the slope. In the sample, $\alpha = 0.052$.

Finally, to capture overall wage growth (or wage-age profile), I add some additional auxiliary moments. Denote by w_1 the first wage reported within the first 6 months after the transition to work. Also denote by w_5, w_{10}, and w_{20} the average of wages reported first after 5 years, 10 years, and 20 years of market experience, respectively. I take the ratios w_5/w_1, w_{10}/w_1, and w_{20}/w_1, which are 1.430, 1.616, and 1.881, respectively. The auxiliary moments from the sample and the bootstrapping standard errors are summarized in the second column of [Table 1]. It also reports the estimates of corresponding moments from the simulation based on estimates of structural parameters.[17]

Overall wage growth (or wage-age profile) is captured by adding some auxiliary moments. Denote as w_1 the first wage reported within the first six months after transitioning to work. Denote as w_5, w_{10}, and w_{20}, respectively, the average of wages reported first after 5, 10, and 20 years of market experience. The ratios w_5/w_1, w_{10}/w_1, and w_{20}/w_1 are 1.430, 1.616, and 1.881, respectively. The auxiliary moments from the sample and bootstrapping standard errors are summarized in the second column of [Table 1], which also reports the estimates of corresponding moments from the simulation based on estimates of the structural parameters.

Table 1: Auxiliary Moments

	sample moment	simulated moment
average unemployment duration (yr)	0.471 (0.013)	0.467 (0.052)
average job duration (yr)	2.175 (0.044)	2.185 (0.458)
average unemployment periods in the first 5years	0.117 (0.004)	0.123 (0.025)
$\Delta log(\tilde{w})$ / $\Delta work\ experience$	0.023 (0.002)	0.025 (0.020)
3rd/1st quartile ratio of reemployment wage dist.	1.775 (0.031)	1.774 (0.467)
2nd/1st quartile ratio of reemployment wage dist.	1.281 (0.019)	1.282 (0.091)
$\Delta log(w)$ / $\Delta market\ experience$	0.052 (0.004)	0.054 (0.034)
w_{20}/w_1	1.881 (0.042)	1.913 (0.361)
w_{10}/w_1	1.616 (0.033)	1.645 (1.001)
w_5/w_1	1.430 (0.026)	1.478 (0.070)

*Standard errors in the second column are estimated using bootstrap. The asymptotic standard errors of the estimated moments are reported in parentheses in the third column.

The second column of Table 1 shows auxiliary moments calculated from NLSY79. The numbers in parentheses are the bootstrapping standard errors used to estimate the weight matrix. The third column provides the estimates of the moments from simulation. [Table 2] reports the estimates of the structural parameters.

4.4 Counterfactual Analysis

A counterfactual experiment is performed to explore how human capital accumulation contributes to wage growth. The experiment is designed to show how much would be earned by a representative worker unable to accumulate any human capital. It is necessary to keep all players' strategies unchanged. As before, firms optimally choose

[17] The asymptotic standard errors will be reported soon.

Table 2: Parameter Estimation

Parameter	Estimates
δ (separation shock)	0.091
λ_u (offer finding rate by unemployed workers)	0.580
λ (offer finding rate by employed workers)	0.446
μ (human capital accumulation shock)	0.023
b (unemployment benefit)	0.413
c_1 (cost parameter)	0.302
γ (risk aversion parameter)	0.450

their strategies assuming that workers stochastically accumulate human capital. But here it is further assumed that a worker remains in the same state when hit by the human capital accumulation shock. The experiment is repeated with 665 workers and an artificial data set is constructed.

Table 3: Counter Factual Analysis

		w_5/w_1	w_{10}/w_1	w_{20}/w_1
data		1.430	1.614	1.881
estimation	with human capital accumulation	1.478	1.645	1.913
	without human capital accumulation	1.354	1.416	1.418

[Table 3], which compares the average wage growth of the two groups, shows the average wage to grow by 43%, 61.4%, and 88.1% in the first 5, 10, and 20 years, respectively, and in the present estimation, by 47.8%, 64.5%, and 91.3%, respectively. In the absence of human capital accumulation, the average wage grows partly due to non-productive promotion and partly due to job turnover; the growth rates without productive promotion are 35%, 41.6%, and 41.8% in the first 5, 10, and 20 years, respectively.

The finding that wages grow by 41.8% without human capital accumulation seems surprising, and may be construed to contradict the conclusions of Altonji and Shakotko (1987) and Altonji and Williams (2005), who show 'returns to job tenure' to account for, at most, 11%, and 'returns to experience' for the preponderance, of wage growth, leading them to conclude that human capital accumulation accounts for most wage growth. The source of this seemingly different result is different definitions. The aforementioned authors define the job tenure effect as the wage loss that would be incurred were a worker to move to a new job, with the same values for the error components; they interpret all partial effects of market experience as general human capital accumulation effects. In the model developed here, wage growth with one employer, through both productive and non-productive promotion, is transferred to the next job through the reservation value. Strictly applying the earlier definition to the present model, 'returns to job tenure' is zero because workers lose nothing in job-to-job transition. Instead, the effect of human capital accumulation is overestimated because 'returns to experience' also includes wage growth through non-productive promotion.

That wage growth through non-productive promotion and job-to-job transition exhibits a concave pattern is also of interest. Wages grow by 35% without human capital accumulation in the first five years, after which the growth rate moderates. When back-loading of wage payments is allowed, a recruiting firm has an incentive to post a contract with low contingent values and promote a recruited worker to a higher valued contract later. The longer a worker stays, the higher the wage increases and the more the job turnover and promotion rates decline. This shows faster wage growth in early periods to be explained by the strategic back-loading scheme as well as by a concave learning curve.

5 Conclusion

This paper develops and estimates an equilibrium job search model with unobserved human capital. An equilibrium is built with multiple wage-ladders that a worker can climb or navigate among. A worker climbs gradually through non-productive promotion, jumps to a higher rung through job-to-job transition, or, upon accumulating human capital, switches to a higher-valued ladder through productive promotion. In the empirical study, the model is estimated using indirect inference and the effect of human capital accumulation captured using the re-employment wage after unemployment. A counterfactual experiment, conducted after estimating the model, finds that the wage of a typical worker unable to accumulate human capital would grow by 41.8%.

The sample being composed entirely of white male high school graduates, who can hardly be expected to be homogeneous, an attempt is made to add ex ante heterogeneity on the worker side to the framework developed in the paper. The model proposes that lifetime value is a function not of worker type, but of what a worker actually produces. Adding ex ante heterogeneity at a worker's level of human capital thus does not require any additional state variables. But more careful attention to the sufficient condition is warranted.

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Appendices

A Mathematical Appendix

Proof of Lemma 1 i) First, suppose that contract m is least cost incentive compatible. We want to show that condition (6) should be true for all types. Since contract m is least cost incentive compatible, it should be least cost incentive compatible for each type y_i. Consider the case of $i = 2$ first. There should be $\hat{y} = (\hat{y}_1, \hat{y}_2, \cdots, \hat{y}_n)$ with $\hat{y}_2 : [0, \infty] \to \mathcal{Y} \cap \{y_2\}^c$ and $\hat{y}_i : [0, \infty] \to \mathcal{Y}$ for each $i \neq 2$, such that $E_2(\cdot; \hat{y}, m) = E_2(\cdot; m)$. At every $t \in [0, \infty)$,

$$\begin{aligned}
&E_2(t;m) - E_2(t;\hat{y}_1,m) \\
&= \int_t^\infty e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)}\big[u(w(s,y_2;m)) - c_2(y_2) + z_2(s;m)\big]ds \\
&\quad - \int_t^\infty e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)}\big[u(w(s,y_1;m)) - c_2(y_1) + z_2(s;m)\big]ds \qquad \text{(A1)} \\
&= \int_t^\infty e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)}[u(w(s,y_2;m)) - c_2(y_2) - u(w(s,y_1;m)) + c_2(y_1)]ds \\
&= \int_t^\infty e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)}[u(w(s,y_2;m)) - u(w(s,y_1;m)) - \alpha_1(y_2 - y_1)]ds \\
&= 0,
\end{aligned}$$

which implies that

$$u(w(\cdot,y_2;m)) = u(w(\cdot,y_1;m)) + \alpha_1(y_2 - y_1). \qquad \text{(A2)}$$

Now, we want to show that once condition (6) is true for $i \leq k$, it is also true for $i = k+1$. Assume that condition (6) is true for all $i = 1, 2, \cdots, k$. That is,

$$u(w(\cdot,y_i;m)) = u(w(\cdot,y_1;m)) + \alpha_1(y_i - y_1) \quad \text{for any} \quad i = 1, 2, \cdots, k. \qquad \text{(A3)}$$

Let $i = k+1$. Since contract m is least cost incentive compatible for the y_i-type, there exists at least one $\hat{y}' : [0, \infty] \to \mathcal{Y}^n$ with $\hat{y}'_{k+1} : [0, \infty] \to \mathcal{Y} \cap \{y_{k+1}\}^c$ and $\hat{y}'_i : [0, \infty] \to \mathcal{Y}$ for each $i \neq k+1$, such that $E_i(\cdot; m) = E_i(\cdot; \hat{y}', m)$. Define set $T_j \subset [0, \infty)$ such that if $\hat{y}'_{k+1}(t) = y_j$, then $t \in T_j$, where $j = 1, 2, \cdots, k$. Then, for every $t \in [0, \infty)$,

$$\begin{aligned}
&E_i(t;m) - E_i(t;\hat{y}',m) \\
&= \int_t^\infty e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)}\big[u(w(s,y_i;m)) - c_i(y_i) + z_i(s;m)\big]ds \\
&\quad - \int_t^\infty e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)}\big[u(w(s,\hat{y}'_i(s);m)) - c_i(\hat{y}'_i(s)) + z_i(s;m)\big]ds \\
&= \int_t^\infty e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)}[u(w(s,y_i;m)) - c_i(y_i) - u(w(s,\hat{y}'_i(s);m)) + c_i(\hat{y}'_i(s))]ds \\
&= \int_t^\infty e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)}[u(w(s,y_i;m)) - u(w(s,\hat{y}'_i(s);m)) - \alpha_1(y_i - \hat{y}'_i(s))]ds \\
&= \sum_{j=1}^{k} \int_{T_j \cap [t,\infty]} e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)}[u(w(s,y_i;m)) - u(w(s,y_j;m)) - \alpha_1(y_i - y_j)]ds
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{j=1}^{k} \int_{T_j \cap [t,\infty]} e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)} [u(w(s,y_i;m)) - u(w(s,y_1;m)) - \alpha_1(y_i - y_1)] ds \\
&= \int_t^{\infty} e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)} [u(w(s,y_i;m)) - u(w(s,y_1;m)) - \alpha_1(y_i - y_1)] ds \\
&= 0.
\end{aligned}
$$

Thus, we obtain that when $i = k+1$,

$$
u(w(t,y_i;m)) = u(w(t,y_1;m)) + \alpha_1(y_i - y_1) \ \text{ at any } \ t \in [0,\infty). \tag{A4}
$$

The mathematical induction yields condition (6).

ii) Conversely, I want to show that if condition (6) holds under contract m, it should be least cost incentive compatible. It is sufficient to show that the y_i-type worker has no incentive to deviate. Consider an arbitrary downward deviation with

$$
\phi''(t) = \begin{cases} y_1 & \text{if } \ t \in T_{i1} \\ \vdots & \vdots \qquad \vdots \\ y_{i-1} & \text{if } \ t \in T_{ii-1} \end{cases} . \tag{A5}
$$

and $\bigcup_{j=1}^{i-1} T_{ij} = [0,\infty)$. For any $t \in [0,\infty)$ and $i \in \{1,2,\cdots,n\}$,

$$
\begin{aligned}
&E_i(t;m) - E_i(t;\phi'',m) \\
&= \int_t^{\infty} e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)} [u(w(s,y_i;m)) - c_i(y_i) + z_i(s;m)] ds \qquad \text{(A6)} \\
&\quad - \int_t^{\infty} e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)} [u(w(s,\phi_i''(s);m)) - c_i(\phi_i''(s)) + z_i(s;m)] ds \\
&= \int_t^{\infty} e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)} [u(w(s,y_i;m)) - c_i(y_i) - u(w(s,\phi_i''(s);m)) + c_i(\phi_i''(s))] ds \\
&= \sum_{j=1}^{i} \int_{T_{ij}} e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)} [u(w(s,y_i;m)) - c_i(y_i) - u(w(s,y_j;m)) + c_i(y_j)] ds \\
&= \sum_{j=1}^{i-1} \int_{T_{ij}} e^{-(r+\rho+\delta+\lambda+\mu+\eta)(s-t)} [u(w(s,y_i;m)) - u(w(s,y_1;m)) - \alpha_1(y_i - y_1)] ds \\
&= 0.
\end{aligned}
$$

There is no profitable deviation and there exists $\phi'' : [0,\infty] \to \mathcal{Y} \cap \{y_i\}^c$ such that $E_i(\cdot;m) = E_i(\cdot;\phi'',m)$. Since this is true for all $i = 1,2,\cdots,n$, contract m is least cost incentive compatible. *Q.E.D.*

Proof of Lemma 2 Rewrite the operating firm's problem:

$$
\begin{aligned}
&\max_{w_1(\cdot)} \int_0^{\infty} \psi_1 [y_1 - w_1 + \mu J_2] ds \\
&\quad s.t. \quad \dot{\psi}_1 = -[r + \rho + \delta + \lambda(1 - F_1(E_1)) + \mu] \psi_1
\end{aligned}
$$

$$\dot{E}_i = -u(w_i) + \alpha_0 + (r+\rho+\delta+\lambda(1-F_i(E_i))+\mu+\eta)E_i - z_i^*(E_i,m)$$
$$\dot{J}_i = -y_i + w_i + (r+\rho+\delta+\lambda(1-F_i(E_i))+\mu+\eta)J_i - \tilde{\varphi}_i(m)$$
$$u(w_i) = u(w_{i-1}) + \Delta = u(w_1) + (i-1)\Delta$$

The Hamiltonian of the problem is

$$\begin{aligned}
\mathcal{H} &= \psi_1[y_1 - w_1 + \mu J_2] - x_\psi[r+\rho+\delta+\lambda(1-F_1(E_1))+\mu]\psi_1 \\
&+ \sum_{i=1}^n x_{ei}[-u(w_1) + \alpha_0 + \alpha_1(y_i - y_1) + (r+\rho+\delta+\lambda(1-F_i(E_i))+\mu+\eta)E_i - \varphi_i(E_i;m)] \\
&+ \sum_{i=2}^n x_{ji}[-y_i + u^{-1}(u(w_1)+\alpha_1(y_i-y_1)) + (r+\rho+\delta+\lambda(1-F_i(E_i))+\mu+\eta)J_i - \tilde{\varphi}_i(m)]
\end{aligned}$$

Applying the maximum principle yields the following first order condition and differential equations.

$$0 = -\psi_1 - \sum_{i=1}^n x_{ei}u'(w_1) + \sum_{i=2}^n \frac{x_{ji}u'(w_1)}{u'(u^{-1}(u(w_1)+(i-1)\Delta))} \tag{A7}$$
$$\dot{x}_\psi = -[y_1 - w_1 + \mu J_2] + x_\psi[r+\rho+\delta+\lambda(1-F_1(E_1))+\mu] \tag{A8}$$
$$\dot{x}_{ji'} = x_{ji'-1}\mu + x_{ji'+1}\eta - x_{ji'}[r+\rho+\delta+\lambda(1-F_{i'}(E_{i'}))+\mu+\eta] \tag{A9}$$
$$\dot{x}_{ei} = x_{ei-1}\mu + x_{ei+1}\eta - x_{ei}[r+\rho+\delta+\lambda(1-F_i(E_i))+\mu+\eta] + x_{ji}\lambda F_i'(E_i)J_i \tag{A10}$$

where $x_{j1}(t) = -\psi(t)$, $i' = 2,3,\cdots,n$ and $i = 1,2,\cdots,n$. From (A8), I obtain

$$\begin{aligned}
\dot{x}_\psi\psi_1(t) + x_\psi\dot{\psi}_1(t) &= -[y_1 - w_1 + \mu J_2]\psi_1(t) \\
\iff \quad x_\psi &= \int_t^\infty [y_1 - w_1 + \mu J_2]\frac{\psi_1(0,\tau)}{\psi_1(0,t)}d\tau + A_\psi\psi_1^{-1}(t) \\
&= \int_t^\infty \psi_1(t,\tau)[y_1 - w_1 + \mu J_2]d\tau = J_1.
\end{aligned}$$

Plugging (A11) into (A8) yields

$$\dot{J}_1 = -[y_1 - w_1 + \mu J_2] + J_1[r+\rho+\delta+\lambda(1-F_1(E_1))+\mu]. \tag{A11}$$

Summing up all equations in (A10) and reordering yields

$$\sum_{i=1}^n \dot{x}_{ei} = \sum_{i=1}^n x_{ji}\lambda F_i'(E_i)J_i - (r+\rho+\delta+\lambda(1-F_1(E_1)))\sum_{i=1}^n x_{ei}.$$

Let

$$\hat{\psi}(t) = \exp[\int_0^t (r+\rho+\delta+\lambda(1-F_1(E_1(s;m))))ds]. \tag{A12}$$

Multiplying by integrating factor on both sides of equation (A12) yields

$$\hat{\psi}(t)\sum_{i=1}^n \dot{x}_{ei} + (r+\rho+\delta+\lambda(1-F_1(E_1)))\hat{\psi}(t)\sum_{i=1}^n x_{ei} = \hat{\psi}(t)\sum_{i=1}^n x_{ji}\lambda F_i'(E_i)J_i. \tag{A13}$$

Then, multiplying by on both sides of equation (A7), dividing by $u'(w_1)$, taking derivative with respect to t, and combining with equation (A13) yields

$$\begin{aligned}\hat{\psi}(t)\sum_{i=1}^{n} x_{ji}\lambda F_i'(E_i)J_i &= \Big[-\frac{\psi_1}{u'(w_1)}+\sum_{i=2}^{n}\frac{x_{ji}}{u'(u^{-1}(u(w_1)+(i-1)\Delta))}\Big]\dot{\hat{\psi}}-\frac{\dot{\psi}_1}{u'(w_1)}\\ &\quad +\sum_{i=2}^{n}\frac{\dot{x}_{ji}}{u'(w_i)}+\Big[\frac{\psi_1\hat{\psi}_1 u''(w_1)}{[u'(w_1)]^2}-\sum_{i=2}^{n}\frac{x_{ji}\hat{\psi}_1 u''(w_i)}{[u'(w_i)]^2}\cdot\frac{u'(w_1)}{u'(w_i)}\Big]\dot{w}_1\end{aligned}$$

Rewriting this, I obtain

$$\begin{aligned}\dot{w}_1 &= \Big[\frac{\psi_1\hat{\psi}_1 u''(w_1)}{[u'(w_1)]^2}-\sum_{i=2}^{n}\frac{x_{ji}\hat{\psi}_1 u''(w_i)}{[u'(w_i)]^2}\cdot\frac{u'(w_1)}{u'(w_i)}\Big]^{-1}\\ &\quad \Big[\hat{\psi}(t)\sum_{i=1}^{n} x_{ji}\lambda F_i'(E_i)J_i+\frac{\psi_1\dot{\hat{\psi}}}{u'(w_1)}-\sum_{i=2}^{n}\frac{x_{ji}\dot{\hat{\psi}}}{u'(u^{-1}(u(w_1)+(i-1)\Delta))}+\frac{\dot{\psi}_1\hat{\psi}_1}{u'(w_1)}-\sum_{i=2}^{n}\frac{\dot{x}_{ji}\hat{\psi}_1}{u'(w_i)}\Big]\end{aligned}$$

It derives Lemma 2. *Q.E.D*

Proof of Lemma 3 Consider the outflow from and inflow into y_i-type unemployment for any arbitrarily small time interval $dt > 0$. By equating them, I obtain that when $i = 1$,

$$\delta dt\overline{G}_1 + \eta dtu_2 + \rho dt(1-u_1) = (\lambda+\mu)dtu_1 \tag{A14}$$

and when $i > 1$,

$$\delta dt\overline{G}_i + \mu dtu_{i-1} + \eta dtu_{i+1} = (\rho+\lambda+\mu+\eta)dtu_i. \tag{A15}$$

Also, consider the proportion of y_i-type employed workers. Equating the inflow and outflow yields

$$\lambda dtu_i + \mu dt\overline{G}_{i-1} + \eta dt\overline{G}_{i+1} = (\rho+\delta+\mu+\eta)dt\overline{G}_i, \quad i = 1,2,\cdots,n \tag{A16}$$

Sending $dt \to 0$ and combining (A14), (A15) and (A16) all together yields the initial values in Lemma 3. By the similar reasoning, the differential equations in Lemma 3 are obtains. *Q.E.D*

B Construction of the Sample

Appendix B describes the steps used to construct the sample.

1. The analysis is restricted to white male high school graduates, the largest demographic group in the NLSY79 dataset. In the first survey round, 8,736 of 12,686 respondents are identified as 'white'[18] and 6,403 as 'male'. Combining these responses yields an initial sample of 4,393 white males. Of these, 1,990 individuals have completed the 12th grade or received the equivalent degree (GED) without reporting further education until the most recent (2010) survey.

2. Individuals with hidden or unusual experience are excluded by dropping 994 individuals who graduated before January 1, 1978 or before age 17 (the 204th month) or after age 20 (the 240th month). Eliminating individuals who entered military service at least once before the last survey results in another 220 individuals being dropped. These selection rules leave 776 workers and 34,010 work-records in the sample.[19]

3. Full-time employment is defined as the match with weekly hours worked, on average, in excess of 30. Average hours worked is calculated as the weighted (by the number of weeks) mean of hours worked at all matches with the same employer throughout a worker's career. For weekly hours worked less than 10, the maximum number between weekly and daily hours worked is multiplied by five working days. When only daily hours worked are reported, they are multiplied by five working days. Before calculating the average, the hours worked per week are top-coded so that they cannot exceed 96.[20]

4. Following Farber and Gibbons (1996) and Yamaguchi (2009), a worker who works full-time for more than 78 weeks in three consecutive years for the first time is assumed to have made the transition from school to work. Work records before this transition are dropped. This leaves 752 workers, 5,955 full-time employers, 573 part-time employers, and 27,606 work records.

5. All non-full-time employment, such as 'out of labor force', 'no information', 'unemployment', and employment with average weekly hours worked less than 30 are recoded as non-employment, and all consecutive non-employment spells merged. This leaves 752 workers, 5,955 full-time employers, 5,918 non-employment spells, and 24,808 work records in the sample.[21]

[18] The sample of 'white' is obtained using question (R01727.00) rather than question (R02147.00).

[19] In the non-military sample, 20 individuals reported more than one graduation date; assuming these to be coding errors, the graduation dates closest to age 18 (the 222nd month) were selected.

[20] Ninety-eight cases are top-coded.

[21] Although the NLSY79 does not distinguish employers from job, each is defined separately. In particular, a worker who returns to an old employer can be considered a planned return or a random re-match. Returns not planned should be considered a different job with the same employer.

6. Workers after long term non-employment being assumed no longer to be attached to the regular labor market, observations subsequent to three years of non-employment spells are dropped. This leaves 752 workers, 5,801 (full-time) employers, 5,413 non-employment spells, and 24,019 work records.

7. Quitting an old and starting a new job within a span of three weeks is considered a job-to-job transition, in which case the intermediate period in the next job spell is included. A return to an old job within 13 weeks is assumed to be a planned return and recoded as a single continuous job. This leaves the final sample of 665 workers, 4,796 jobs, and 14,298 observations.