Research Policy and U.S. Economic Growth

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Abstract

This paper examines quantitatively the effects of R&D subsidy and government-financed basic research on U.S. economic growth and consumer welfare. To achieve this, we develop an endogenous growth model which takes into account both public and private research investment, and the differences between basic and non-basic research. A calibrated version of the model is able to replicate some important features of the U.S. economy over the period 1953-2009. Our model suggests that government spending on basic research is an effective policy instrument to promote economic growth. Subsidizing private R&D, on the other hand, has no effect on economic growth.

Keywords: Research Policy, Basic and Applied Research, R&D Spending, Endogenous Growth

JEL classification: O31, O38, O41.

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1 Introduction

A central implication of modern growth theory is that long-term economic growth is mainly driven by technological improvements. While certain improvements can be achieved through learning-by-doing (Arrow, 1962; Romer, 1986), most of the innovations we see today are the products of purposeful research and development. Over the past several decades, the U.S. government has adopted various policies to spur R&D investment and economic growth. In this study, we focus on two types of research policies, namely tax incentives for private R&D and government spending on basic research.[1] The goal is to evaluate the effects of these policies on U.S. economic growth and consumer welfare. To achieve this, we develop an endogenous growth model with both public and private research investment. A calibrated version of the model is able to replicate some important features of the U.S. economy over the period 1953-2009. We then perform a series of counterfactual experiments to quantify the impact of research policies on economic growth and consumer welfare.

The effect of government policies on private R&D investment has long been a subject of interest among growth theorists. Since the pioneering work of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), a substantial body of research has examined the determinants of private R&D investment and its relation to economic growth.[2] These so-called R&D-based growth models provide a micro-founded framework for analyzing research policies. Most of the existing studies, however, focus exclusively on the effects of tax incentives (such as tax exemptions and tax credits) for private R&D and overlook the importance of direct government spending on innovative activities.[3] Since World War II, the U.S. government has played a crucial role in funding these activities. Although the overall importance of public R&D spending has declined since the 1960s, the government remains the most important source of financial support for one particular type of research, namely basic research.[4]

Basic research is one of the three major categories of research activities, the other two being applied research and development.[5] Basic research refers to studies that are solely intended to advance our

[1] For more information about the U.S. national innovation system and other research policies, see Mowery (1998) and Shapira and Youtie (2010).

[2] For a detailed review of these studies, see Aghion and Howitt (2005) and Jones (2005).

[3] Examples include Grossman and Helpman (1991), Jones (1995), Peretto (1998), Segerstrom (1998, 2000), Young (1998), Howitt (1999), and Impullitti (2010). Notable exceptions are Park (1998), Morales (2004), and Akcigit *et al.* (2012). The latter group of studies will be discussed in greater detail later on.

[4] Between 1953 and 2009, the share of total R&D spending paid by the U.S. government has dropped from 54.7 percent to 32.0 percent. Over the same time period, the government has paid more than 60 percent of all the expenses on basic research, and its importance has not declined. See Section 2 for a more detailed discussion on these trends.

[5] This classification is adopted by the National Science Foundation (NSF) in the United States and the Organization for Economic Co-operation and Development (OECD). For a formal definition of these research activities, see <http://www.nsf.gov/statistics/randdef/ fedgov.cfm> or the OECD *Frascati Manual 2002*, Section 4.2.

knowledge on fundamental principles or fundamental aspects of phenomena (e.g., research on pure science and pure mathematics). In particular, this type of research is not directed towards any specific application or commercial goal. Applied research and development, on the other hand, refer to studies that are targeted towards a specific practical goal or the actual production of new products. Thus, the objectives of basic and "non-basic" research are fundamentally different. Empirical evidence shows that basic research often provides the basis for non-basic research (Mansfield, 1995; Narin *et al.* 1997), and it seems to have a larger contribution to productivity growth than non-basic research (Mansfield, 1980; Griliches, 1986). These two types of research also differ markedly in terms of funding source and performing sector. Basic research is mostly funded by the government and performed outside of the private sector, whereas applied research and development are mostly funded and performed by the private sector. For instance, 78.3 percent of all the expenses on basic research in 2009 were paid by the government (56.7 percent), universities (10.8 percent) and other non-profit institutions (10.8 percent). The rest was paid by private industries. On the contrary, private industries paid 71.1 percent of all the expenses on applied research and development in 2009, while the government paid 26.2 percent.[6] In terms of performing sector, 80.5 percent of basic research was conducted by universities (53.3 percent), other non-profit institutions (12.2 percent), federally funded research and development centers (7.7 percent), and federal agencies (7.2 percent). The rest was performed by private industries. This contrasts sharply with the fact that 82.0 percent of applied research and development was conducted in the private sector.[7] These observations suggest that it is important to distinguish between basic and non-basic research when analyzing the impact of public R&D spending.

In this study, we develop an endogenous growth model which takes into account the major differences between basic and non-basic research, and the importance of direct government spending on basic research. Our analysis also takes into account several empirical trends in the U.S. economy over the period 1953-2009, such as a rapid increase in the employment of researchers (relative to total employment), a persistent decline in corporate income tax, the introduction of R&D tax credit in 1981, and other changes in fiscal policies. In our model framework, basic research is targeted towards the creation of fundamental knowledge, whereas non-basic research is directed at improving the quality of productive inputs. Fundamental knowledge is beneficial to society because it can improve firm productivity and enhance the

[6] Most of the federal obligations for applied research and development are defense-oriented. For instance, the Department of Defense and the defense programs under the National Nuclear Security Administration accounted for 68.6 percent of these obligations in 2009. On the other hand, these federal agencies only accounted for 5.3 percent of all the federal obligations for basic research in the same year.

[7] See Section 2 for more information about the patterns of R&D spending and how they change over time.

efficiency of non-basic research. This type of knowledge is freely available to all parties once it is discovered. Since there is no private ownership of fundamental knowledge, no firm is willing to invest in basic research. Hence, basic research is entirely funded by the government in our framework.[8] On the other hand, innovations created by non-basic research are protected by patents, which establish perpetual monopoly rights over the sale of the improved products. The profit stream generated by these rights provides the sole incentive for conducting non-basic research. In this study, we confine our attention to non-basic research that is funded and conducted by profit-maximizing firms.[9] Both basic and non-basic research require the use of highly trained and specialized workers, or researchers. The supply of these workers is exogenously given and inelastic at any given point of time. The assumption of inelastic supply captures the fact that it requires a considerable amount of time to train someone to do research, hence the total number of researchers cannot respond immediately to changes in market conditions. In each period, an exogenous fraction of researchers is recruited by the government to conduct basic research. This in turn determines the scale of public R&D spending and the growth rate of fundamental knowledge.

The accumulation of fundamental knowledge brought by basic research, and the continuous improvement of input quality made possible by non-basic research are the two driving forces of technological advancement in our model. Similar to the neoclassical growth paradigm, the model developed here displays a rich set of transitional dynamics which is jointly determined by capital accumulation and technological improvements. In the theoretical analysis, we show that the model economy will eventually converge to a unique long-run balanced-growth equilibrium that does not exhibit the scale effect.[10] In terms of policy implications, our model suggests that government-financed basic research is a strong impetus to long-term economic growth. On the contrary, subsidizing private R&D investment (either in the form of tax exemptions or tax credits) has no effect on long-term economic growth. To further explore the growth and welfare implications of these research policies, we construct a parameterized version of the model and solve for the equilibrium time paths of all major economic variables using numerical methods. Under the baseline parameterization, our model is able to replicate the patterns of R&D investment rate and the

[8] A detailed discussion on this assumption is provided in Section 2.

[9] As mentioned in footnote 6, most of the government spending on non-basic research is defense-oriented. As pointed out by Mowery (2010), the unusual institutional setting of defense R&D programs and the specific nature of military inventions make it difficult to evaluate the impact of defense R&D on the entire economy. Consequently, much of our understanding on the effects of defense R&D is derived from case studies on particular industries. While these studies have raised some interesting issues (such as the effects of public procurement on private R&D, and the civilian innovations that are inspired by defense R&D), we do not pursue them here, but rather focus on the effects of government basic research spending.

[10] More precisely, our model predicts that long-term economic growth is independent of the size of population and the total number of researchers. Similar to the models of Jones (1995), Segerstrom (1998), Peretto (1998), Howitt (1999) and Jones and Williams (2000), our model predicts that long-term economic growth depends positively on the long-term population growth rate. However, perpetual growth in per-capita variables is still possible even if the size of population or the total number of researchers ceases to grow in the long run.

increase in real per-worker GDP in the United States over the period 1953-2009.[11] We then perform a series of counterfactual experiments and welfare analyses in order to gauge the effects of research policies. There are four major findings from the numerical analyses. First, subsidizing private R&D investment has no effect on technological progress and economic growth in both the short run and the long run. This is because R&D subsidies can only stimulate the *demand* for innovative activities and the *demand* for researchers, but have no effect on the *supply* of researchers which is inelastic at any point of time. Hence, subsidizing private R&D will only drive up the equilibrium wage rate for researchers without affecting the equilibrium quantity of labor input in innovative activities. Consequently, it has no effect on technological progress and economic growth.[12] This result is consistent with the empirical findings reported by Goolsbee (1998), and the ideas discussed in Romer (2001). Our second major finding is that the rapid increase in the number of researchers (relative to total employment) is an important contributing factor to U.S. economic growth. Under the baseline parameterization, our model suggests that 21.3 percent of TFP growth and 25.8 percent of the growth in real per-worker output over the period 1953-2009 can be attributed to this factor. Third, unlike R&D subsidies, direct government spending on basic research can effectively promote technological progress and economic growth. Our model suggests that 12.5 percent of TFP growth and 14.3 percent of the growth in real per-worker output between 1953 and 2009 can be attributed to the rising share of government basic research spending in total R&D expenditures.[13] Our results also suggest that about two-thirds of the long-term economic growth in the United States can be attributed to the accumulation of fundamental knowledge. Because of these positive growth effects, a permanent increase in government basic research spending can induce significant welfare gains for consumers. Finally, our results point to the significance of general equilibrium effects (or price effects) when evaluating the welfare implications of research policies. As we mentioned earlier, the effects of R&D subsidies are manifested solely on the equilibrium wage rate for researchers. As for government-financed basic research, a permanent increase in this type of spending not only accelerates the pace of technological progress, it also drives up the equilibrium real interest rates. From an innovating firm's perspective, higher interest rates mean that the profit stream generated by an improved product is now discounted at higher rates. This lowers the firm's incentives to invest in non-basic research and their demand for researchers. Thus,

[11] Throughout this paper, R&D investment rate is defined as the share of total R&D spending in GDP.

[12] There are two reasons why this result does not appear in other studies, such as Peretto (1998), Segerstrom (1998) and Impullitti (2010). First, these studies do not distinguish between researchers and non-researchers. Instead, they assume that all workers are identical and freely mobile between research sector and production sector. Hence, the supply of labor for research activities is not inelastic. Second, these studies typically use labor as the numeraire which means wage rate is normalized to one in every period. Thus, R&D subsidies has no effect on the equilibrium wage rate in their models.

[13] According to our measure, the share of government basic research spending in total R&D expenditures has increased from 7.25 percent in 1953 to 13.86 percent in 2009. The time-series data of this share is shown in Figure 8.

following a permanent increase in government basic research spending, the relative wage of researchers to non-researchers declines initially before rising to the new (and higher) long-run level. Ignoring the price effects during the transition period will thus lead to an overestimation of the welfare gains from government-financed basic research.

Other studies that have explored the growth effects of public R&D spending include Park (1998), Morales (2004), and Akcigit *et al.* (2012). The first two studies have generalized the model of Romer (1990) to allow for government-financed research. Both of them find that government R&D policies can have a significant impact on long-term economic growth. These studies, however, do not explore the quantitative implications of their models. The present study is closer in spirit to Akcigit *et al.* (2012) in the sense that both have developed an endogenous growth model with basic and non-basic research, and explored the quantitative implications of the model. However, the theoretical constructs and quantitative analyses in these two studies are very different. In the model of Akcigit *et al.* (2012), private firms can choose to operate in multiple industries and invest in both basic and applied research. Workers are assumed to be homogeneous and freely mobile across industries and across sectors. In their theoretical analysis, these authors focus on characterizing the variations of R&D spending across different types of firms in the long-run stationary equilibrium, and their implications to long-term economic growth. In the quantitative analysis, they estimate the long-run equilibrium solution of their model using firm-level data for France over the period 2000-2006. In the current study, we focus on the temporal patterns of public and private research spending, and analyze the impact of research policies in both the short run and the long run. In the quantitative analysis, we calibrate our model using existing empirical estimates and aggregate data for the United States over the past several decades, and quantify the effects of research policies that are specific to the United States.

The rest of this paper is organized as follows. Section 2 summarizes the empirical trends of R&D spending and the total number of researchers in the United States. Section 3 describes the model economy. Section 4 defines the equilibrium of the model and presents the main theoretical results. Section 5 explains the choices of the baseline parameter values. Section 6 presents the numerical results. This is followed by some concluding remarks in Section 7.

2 Empirical Trends

In this section, we briefly summarize the key patterns of R&D spending in the United States.[14] Figure 1 shows the share of total R&D spending in U.S. GDP over the period 1953-2009. Between 1953 and 1964, R&D investment rate increased rapidly from 1.36 percent to 2.88 percent. Since then, it has been maintained between two and three percent. Figure 2 shows the distribution of total R&D spending among basic research, applied research and development. Over the entire sample period, development accounted for more than 60 percent of total R&D spending in the United States, while applied research accounted for another 20 percent. Despite the importance of non-basic research, the share of basic research spending has been persistently increasing over the years. In 1953, basic research accounted for 8.9 percent of total R&D spending. This increased to 19.0 percent in 2009.

Next, we turn to the importance of government and private industries in funding and performing these research activities. Figure 3 shows the breakdown of basic research spending by funding source. Table 1 shows the distribution of basic research output (measured in terms of scientific publications) by performing sector. Throughout the entire sample period, the U.S. government played a predominant role in funding basic research. In particular, the government has funded about 70 percent of all the basic research performed in academia, which is the primary contributor of scientific publications. Another important observation from Figure 3 and Table 1 is that, while private industries have funded and performed a substantial amount of basic research, they have produced only a small fraction of the output. This seems to suggest that the amount private firms spend on basic research may not truly reflect their importance in the creation of fundamental knowledge. In a well-cited article on industrial R&D, Rosenberg (1990) suggests two possible reasons why private firms would choose to invest in and maintain a basic research capability: First, this type of capability is crucial for planning and evaluating non-basic research. Second, it is also essential for understanding and exploiting the knowledge created by academic research.[15] Thus, according to this view, the main reason why private firms invest in basic research is to enhance their performance in non-basic research, not to create fundamental knowledge. While it is difficult to test this hypothesis rigorously, it does seem to explain the empirical evidence mentioned above. Given the relatively insignificant role of private industries in the creation of fundamental knowledge, we choose to

[14] Unless otherwise stated, all the data reported in this section were taken from *National Patterns of R&D Resources: 2009 Data Update* compiled by the NSF.

[15] The same author also pointed out that for industrial R&D the distinction between basic and non-basic research is often not that clear. For instance, in the Survey of Industrial Research and Development conducted by the NSF, industrial basic research is defined as "the pursuit of new scientific knowledge or understanding that does not have specific immediate commercial objectives, *although it may be in fields of present or potential commercial interest.*" This definition can be found in *Research and Development in Industry: 2006–07*, Appendix A.

abstract away from basic research funded by private firms. This assumption helps keep the dynamics of the model tractable and allows us to focus on the growth and welfare effects of government basic research spending.

Figure 4 shows the distribution of non-basic research spending by funding source. It is evident from this diagram that the importance of government spending on applied research and development has been declining since the 1960s. Figure 5 shows that this decline is coincident with the cutbacks in defense-related and space-related R&D programs.

In the present study, we only take into account two types of research activities: (i) government-financed basic research, and (ii) applied research and development performed by private industries. Thus, when comparing the model to data, we use the sum of R&D expenditures incurred by these activities as our measure of total R&D spending (see Appendix C for more details). For the period 1953-2009, our measure of total R&D spending represents 77.3 percent of the U.S. total. The implied R&D investment rates are shown in Figure 1 (labelled as "Our Measure").

Over the past several decades, the U.S. has also witnessed a rapid increase in the employment of researchers. Figure 6 shows the growth factor of the number of full-time R&D scientists and engineers employed by private industries and the growth factor of total employment in the United States.[16] Between 1953 and 2009, the employment of private-sector researchers has increased by a factor of 8.75, while total U.S. employment has increased by a factor of 2.29.

3 The Model

3.1 Demographics

The model economy is populated by two types of households, namely high-skilled (H) and low-skilled (L) households. The number of each type of household is constant over time and is normalized to one. Throughout this paper, an index $s \in \{H, L\}$ is used to indicate household type. Each household comprises a growing number of infinitely-lived consumers. All consumers within the same household are identical. Let $N_{s,t}$ be the number of consumers in type-s household at time t. Let $n_{s,t}$ be the growth rate of type-s household between time t and $t+1$, so that $N_{s,t+1} = (1 + n_{s,t}) N_{s,t}$, for $s \in \{H, L\}$. The growth rates

[16] Data on full-time R&D scientists and engineers employed in the private sector are collected by the Business Research and Development and Innovation Survey (BRDIS), conducted by the NSF. These data are available from <http://www.nsf.gov/statistics/iris/ search_hist.cfm?indx=24>. Note that these data do not include academic researchers and researchers employed by the government. Data on the employment of these researchers are relatively scarce. See Footnote 43 and Appendix C for more information. Data on the number of total employed workers (over age 16) are obtained from the Bureau of Labor Statistics website.

are exogenously given and changing over time in a deterministic manner.[17] Total population at time t is given by $N_t = N_{H,t} + N_{L,t}$, with $N_0 = 1$. The share of high-skilled consumers in the population at time t is denoted by $\theta_t \equiv N_{H,t}/N_t$.

3.2 Commodities

There are two types of commodities in this economy. First, there is a single final good which can be used for consumption and investment. The price of final good is normalized to one in every period. Second, there is a continuum of differentiated intermediate goods which can *only* be used to produce final goods. The set of intermediate goods is fixed over time and is represented by $[0,1]$. At the beginning of time 0, all intermediate goods have the same initial quality which is normalized to one. The quality of these goods can be improved over time, but the occurrence of quality improvement is stochastic and idiosyncratic across products. Each improvement raises quality by a factor of $\lambda > 1$. Hence, after $j \in \{1, 2, ...\}$ improvements, the quality of the improved product is λ^j. The probability of quality improvement is determined by a number of factors, including R&D investment made by profit-maximizing firms. The details of this will be explained later. Let $J_t(\omega)$ be the number of improvements realized before time t for intermediate good $\omega \in [0,1]$. Then there are $J_t(\omega) + 1$ different quality grades of the same intermediate good available at time t. Throughout this section, we will use the pair (j, ω) to indicate an intermediate good that is of type ω and quality λ^j. The price of intermediate good (j, ω) at time t is denoted by $p_t(j, \omega)$. Let $Q_t(\omega)$ be the quality level of product $(J_t(\omega), \omega)$, i.e.,

$$Q_t(\omega) = \underbrace{\lambda \times \lambda \times ... \times \lambda}_{J_t(\omega) \text{ times}}.$$

We will refer to this as the state-of-the-art quality for intermediate good ω. The highest quality level among all intermediate goods at time t is represented by $\overline{q}_t \equiv \max\{Q_t(\omega) : \omega \in [0,1]\}$. We will refer to this as the *leading-edge quality* at time t.

3.3 Final-Good Sector

In the final-good sector, there is a continuum of identical firms which produce the same product. In each period, each final-good producer hires low-skilled workers and intermediate inputs, and produces output

[17] The terms "high-skilled workers" and "researchers" are used interchangeably throughout this paper. In the quantitative analysis, we use the time-varying growth rates of $N_{H,t}$ and $N_{L,t}$ to account for the empirical patterns in Figure 6.

according to

$$Y_t = \Theta_t \mathbb{M}_t^{\alpha} L_{Y,t}^{1-\alpha}, \qquad \text{with } \alpha \in (0,1), \tag{1}$$

where Y_t denotes the quantity of final goods produced at time t, $L_{Y,t}$ denotes the quantity of low-skilled labor employed, $\mathbb{M}_t$ is a composite measure of intermediate inputs, and Θ_t is an economy-wide measure of total factor productivity (TFP). The composite measure $\mathbb{M}_t$ is constructed as follows: Let $M_t(j,\omega)$ be the quantity of intermediate input (j,ω) employed at time t. For each $\omega \in [0,1]$, define a *quality-augmented* measure of intermediate input, $\widetilde{M}_t(\omega)$, according to

$$\widetilde{M}_t(\omega) \equiv \sum_{j=0}^{J_t(\omega)} \lambda^j M_t(j,\omega). \tag{2}$$

This measure is defined as a weighted sum of all quality grades available for a given type of intermediate input. It has two main properties. First, high-quality inputs are weighted more heavily than low-quality ones, which means the former are more productive than the latter. Second, different quality grades of the same input are treated as perfect substitutes in the production of final goods. These quality-augmented measures are then aggregated by a standard Dixit-Stiglitz aggregator to form the composite measure $\mathbb{M}_t$. Formally, this is given by

$$\mathbb{M}_t \equiv \left\{ \int_0^1 \left[\widetilde{M}_t(\omega) \right]^{\psi} d\omega \right\}^{\frac{1}{\psi}}, \qquad \text{with } \psi \in (0,1). \tag{3}$$

The elasticity of substitution between any two types of intermediate inputs is given by $1/(1-\psi)$.[18]

The productivity factor Θ_t is taken as exogenously given by the firms, and is determined by

$$\Theta_t \equiv X_t^{\eta}, \qquad \text{with } \eta > 0, \tag{4}$$

where X_t is the stock of fundamental knowledge created by basic research available at the beginning of time t. In the model economy, basic research is entirely funded by the government and its outcomes are freely accessible to all firms and consumers. In other words, fundamental knowledge is non-excludable. It is also non-rivalrous, which means the use of fundamental knowledge by one firm does not reduce its availability to others. Thus, all firms can benefit from the existing stock of fundamental knowledge to

[18] Grossman and Helpman (1991), Segerstrom (1998), Dinopoulos and Segerstrom (1999), and Impullitti (2010) use the same quality-augmented measure and Dixit-Stiglitz aggregator to define a composite measure for a continuum of differentiated products. The main difference is that, in their models, the diffierentiated products take the form of consumption goods rather than intermediate inputs in production. In addition, these studies focus on the special case in which $\psi = 0$.

the same extent. Since fundamental knowledge is a public good, we will also refer to it as public R&D capital. Equation (4) states that public R&D capital is beneficial to firm productivity. This assumption is supported by the empirical findings in Adams (1990), Nadiri and Mamuneas (1994), and Luintel and Khan (2011).[19] An isoelastic function is used to ensure the existence of balanced-growth equilibria. The parameter η can be interpreted as the elasticity of TFP with respect to public R&D capital.

Since the production function in (1) is homogeneous of degree one, we can focus on the choices made by a single, price-taking firm. Let $w_{L,t}$ be the wage rate for low-skilled worker at time t. The representative firm's problem is given by

$$\max_{L_{Y,t},M_t(\cdot)} \left\{ \Theta_t \mathbb{M}_t^{\alpha} L_{Y,t}^{1-\alpha} - w_{L,t} L_{Y,t} - \int_0^1 \left[\sum_{j=1}^{J_t(\omega)} p_t\left(j,\omega\right) M_t\left(j,\omega\right) \right] d\omega \right\},$$

subject to (2) and (3), and the non-negativity constraints on intermediate inputs, i.e., $M_t\left(j,\omega\right) \geq 0$ for all $\left(j,\omega\right)$.[20] The first-order condition with respect to $L_{Y,t}$ is

$$w_{L,t} = \left(1-\alpha\right) \Theta_t \left(\frac{\mathbb{M}_t}{L_{Y,t}} \right)^{\alpha}. \tag{5}$$

The first-order condition with respect to $M_t\left(j,\omega\right)$ is

$$p_t\left(j,\omega\right) \geq \alpha \Theta_t \left(\frac{\mathbb{M}_t}{L_{Y,t}} \right)^{\alpha-1} \left[\frac{\widetilde{M}_t\left(\omega\right)}{\mathbb{M}_t} \right]^{\psi-1} \lambda_j, \tag{6}$$

which holds with equality if $M_t\left(j,\omega\right) > 0$.

The expression on the right-hand side of (6) is the marginal product of $M_t\left(j,\omega\right)$. The first-order condition in (6) has two main implications. First, if the price of input $\left(j,\omega\right)$ is strictly greater than its marginal product, then the representative firm will choose to have $M_t\left(j,\omega\right) = 0$. Second, for any given type of intermediate input, say $\omega \in [0,1]$, the firm will only purchase those quality grades that have the lowest *quality-adjusted* price, $p_t\left(j,\omega\right)/\lambda^j$. Let $\widetilde{p}_t\left(\omega\right)$ be the lowest quality-adjusted price for intermediate input ω at time t. Following Segerstrom (1998), we use the following assumption to break ties: If more

[19] Using data over the period 1956-1986, Nadiri and Mamuneas (1994) estimate the effects of publicly financed R&D capital on the productivity of twelve U.S. manufacturing industries. Their results suggest that public R&D capital has significant cost-saving effects in these industries. Using U.S. data over the period 1953-1980, Adams (1990) find that knowledge created by academic research is an important contributor to productivity growth. Luintel and Khan (2011) report similar results for a group of ten OECD countries over the period 1970-2006.

[20] Given a Cobb-Douglas production function, it is never optimal for the firm to choose $L_{Y,t} = 0$. Hence, we can ignore the non-negativity constraint on labor input. On the other hand, since different quality grades of the same intermediate input are treated as perfect substitutes in (2), the firm may choose to have $M_t\left(j,\omega\right) = 0$ for some $\left(j,\omega\right)$. Thus, it is necessary to take into account the non-negativity constraints on intermediate inputs.

than one quality grades of the same type of intermediate input charge the same quality-adjusted price, then the representative firm will only purchase the one with the highest quality. This ensures that only one quality grade of each type of intermediate input will be used in every period.[21] If the representative firm chooses to use input (j,ω), i.e., if $M_t(j,\omega) > 0$, then its demand function is given by

$$M_t(j,\omega) = \left(\lambda^j \mathbb{P}_t\right)^{\frac{\psi}{1-\psi}} \left[p_t(j,\omega)\right]^{-\frac{1}{1-\psi}} \mathbb{E}_t, \tag{7}$$

where $\mathbb{E}_t$ denotes total expenditure on intermediate inputs at time t, i.e.,

$$\mathbb{E}_t \equiv \int_0^1 \left[\sum_{j=1}^{J_t(\omega)} p_t(j,\omega) M_t(j,\omega)\right] d\omega = \mathbb{P}_t \mathbb{M}_t,$$

and $\mathbb{P}_t$ is a quality-adjusted price index defined by

$$\mathbb{P}_t \equiv \left\{\int_0^1 \left[\widetilde{p}_t(\omega)\right]^{-\frac{\psi}{1-\psi}} d\omega\right\}^{-\frac{1-\psi}{\psi}}. \tag{8}$$

A formal derivation of these expressions can be found in Appendix A.

3.4 Intermediate-Goods Sector

In the intermediate-goods sector, there is a continuum of infinitely-lived firms. Each firm is the original inventor of a single variety of intermediate good.[22] We assume that each original inventor has significant technical and cost advantages in improving its own product over any potential competitors so that it is the only researcher that attempts to improve its product.[23] Successful quality improvement is rewarded with a patent, which bestows perpetual monopoly right over the sale of the improved product. These assumptions imply the following market structure of the intermediate-goods sector. All firms in this sector are monopolistic in nature. Each monopolist specializes in the production and development of a single intermediate good. In particular, all available quality grades of the same product are supplied by a single monopolist.

Each monopolistic firm is made up of two divisions: the manufacturing division and the R&D division.

[21] This assumption also simplifies the pricing decisions faced by the suppliers of these inputs. See footnote 27 for more information.

[22] The original invention process is not considered in here.

[23] Barro and Sala-i-Martin (2004, Section 7.3) have formally shown that in a patent race between an incumbent firm and an outsider, if the former has significant cost advantages in quality improvement research over the latter, then it will be able to drive the outsider out of business and become the only researcher. We do not explicitly incorporate this type of patent race in our model to avoid further complicating the analysis.

The former is responsible for producing the product, and the latter is responsible for conducting quality improvement research.

3.4.1 Production and R&D Technologies

All types of intermediate goods, regardless of quality and variety, can be produced by the same production technology which uses physical capital and low-skilled labor as inputs. This technology is given by

$$Y_{M,t} = \Theta_t K_t^{\rho} L_{M,t}^{1-\rho}, \qquad \text{with } \rho \in (0,1), \tag{9}$$

where $Y_{M,t}$ denotes the quantity of intermediate goods produced at time t, K_t and $L_{M,t}$ denote the quantities of physical capital and low-skilled labor employed. The productivity factor Θ_t is the same as in (4). Similar to the final-good producers, the monopolistic firms do not own the factors of production. Thus, they have to hire low-skilled workers and rent physical capital from the competitive factor markets in every period.

As for the R&D process, we assume that high-skilled labor is the only private input employed in quality improvement research.[24] Let $\Lambda_{M,t}$ be the quantity of high-skilled labor employed by an individual firm at time t, and let $\overline{\Lambda}_{M,t}$ be the average level of high-skilled labor input among all innovating firms in the intermediate-goods sector. The probability of progressing from quality q to λq is given by

$$\Phi_t\left(q, \Lambda_{M,t}\right) \equiv \frac{\Lambda_{M,t}^{\vartheta} \overline{\Lambda}_{M,t}^{1-\vartheta}}{\Omega_t\left(q\right)}, \qquad \text{with } \vartheta \in (0,1), \tag{10}$$

where $\Omega_t\left(q\right)$ is a measure of R&D difficulty at time t. This measure is taken as exogenously given by individual firms, and is determined by

$$\Omega_t\left(q\right) \equiv \nu\left(q, \overline{q}_t\right) X_t^{-\varrho}, \qquad \text{with } \varrho > 0. \tag{11}$$

The function $\nu\left(q, \overline{q}_t\right)$ is assumed to be strictly positive, strictly increasing in the quality of the product $\left(q\right)$, and strictly decreasing in the leading-edge quality $\left(\overline{q}_t\right)$.

[24] In the existing R&D-based growth models, it is typical to assume that labor is the only input employed in R&D activities. See for instance, Aghion and Howitt (1992), Young (1995), Segerstrom (1998), Dinopoulos and Segerstrom (1999), Peretto (1999), Aghion *et al.* (2001), Impullitti (2010) and Chu *et al.* (2012). Empirical evidence shows that labor compensation is the largest component in private R&D expenditures. According to Dougherty *et al.* (2007, Table 3), labor costs account for 46.5 percent of total manufacturing R&D expenditures in the United States in 1997. The corresponding figures for France, Germany and Japan are 52.8 percent, 61.7 percent and 42.7 percent, respectively. Materials and supplies accounted for less than 20 percent of total manufacturing R&D spending in these countries. MacDonald (1973, Appendix Table V) reports similar findings for a group of ten OECD countries in 1963-1964.

Equations (10) and (11) capture four fundamental aspects of quality improvement research. First, the probability of successful quality improvement is strictly increasing in the firm's own labor input. Since $\vartheta \in (0,1)$, this type of input is subject to diminishing marginal returns. The concavity assumption is consistent with the empirical findings in Pakes and Griliches (1984) and Hall *et al.* (1986). Second, the difficulty of quality improvement research is strictly decreasing in the existing stock of fundamental knowledge. In other words, fundamental knowledge is beneficial to the efficiency of private R&D. This assumption is supported by the empirical findings in Jaffe (1989), Mansfield (1995, 1998), Anselin *et al.* (1997) and Zucker *et al.* (1998). Third, holding other things constant, high-quality products are more difficult to develop than low-quality ones. Thus, it becomes increasingly difficult to move up the quality ladder. This feature is captured by the assumption that $\nu(q, \overline{q}_t)$ is strictly increasing in q for all $\overline{q}_t$.

Finally, research carried out by one firm will also create external benefits for other innovating firms.[25] This type of externality, or knowledge spillover, can occur in two ways: (i) when researchers communicate and interact with each other, and (ii) when individual researchers learn from other people's success (e.g., by reading the codified part of a patent or by reverse engineering). The expression $\overline{\Lambda}_{M,t}^{1-\vartheta}$ in (10) is intended to capture the benefits created by the first channel. The main idea behind this expression is as follows: As the size of the research community expands, more interactions among researchers will be available and so the effects of knowledge spillover will become larger. To ensure the existence of balanced-growth equilibria, we assume that the function in (10) is homogeneous of degree one in $\Lambda_{M,t}$ and $\overline{\Lambda}_{M,t}$. The second channel of knowledge spillover is captured by the assumption that $\nu(q, \overline{q}_t)$ is strictly decreasing in $\overline{q}_t$ for all q. In words, this means all innovating firms can learn and benefit from the leading-edge improvement, so that an increase in $\overline{q}_t$ will increase their research efficiency.[26]

The firm's research outlay at time t is given by $w_{H,t}\Lambda_{M,t}$, where $w_{H,t}$ is the wage rate for high-skilled labor. These expenses are subject to two types of preferential tax treatment which we will explain later. If an improved product is developed at time t, then the firm can start producing it as a monopoly at time $t+1$.

In each period, each monopolistic firm has to make two groups of decisions. First, it has to decide how much to produce and what price to charge for its product. Second, it has to decide how much to

[25] There is now a large body of empirical studies which confirm the existence and significance of knowledge spillover across firms. See, for instance, Jaffe (1986), Audretsch and Feldman (1996, 2004) and references therein.

[26] The variable $\Omega_t(\cdot)$ is similar in spirit to the R&D difficulty index defined in Segerstrom (1998), Dinopoulos and Segerstrom (1999) and Impullitti (2010). In the first study, the difficulty index is determined by the past history of R&D investment in each industry, and is thus different across industries. In Impullitti (2010), the difficulty index is identical for all industries and directly proportional to the size of total population. Dinopoulos and Segerstrom (1999) have considered the effects of trade liberalization under both specifications. These studies, however, do not consider the effects of fundamental knowledge on the difficulty of private R&D.

invest in quality improvement research. Since the monopolists do not own the factors of production, the choices of physical capital and low-skilled labor in the production stage are static in nature and do not interfere with the research investment decisions. Likewise, the choice of high-skilled labor input in the research stage has no direct impact on the production process. Thus, the two groups of decisions can be analyzed separately.

3.4.2 Production and Pricing Decisions

Since all final-good producers use only one quality grade of each type of intermediate input in any period, each intermediate-good producer will only supply one quality grade of its products.[27] In particular, these producers will always choose to produce the best quality grade of their products. To see this, suppose the supplier of intermediate good $\omega \in [0,1]$ chooses to produce quality grade j at time t, for some $j \in \{0,1,...,J_t(\omega)\}$. Given a Cobb-Douglas production function, the minimum cost of producing $\overline{M} \geq 0$ units of this product is $\mathbb{C}_t\left(\overline{M}\right) \equiv \kappa_t \overline{M}$, where κ_t is the unit cost of production. This cost is determined by

$$\kappa_t \equiv \frac{1}{\Theta_t}\left(\frac{R_t}{\rho}\right)^{\rho}\left(\frac{w_{L,t}}{1-\rho}\right)^{1-\rho}, \tag{12}$$

where R_t is the rental price of physical capital at time t. Low-skilled workers are freely mobile across sectors, so there is only one wage rate ($w_{L,t}$) for these workers. Since the production technology is identical for all types of intermediate goods, so is the unit cost of production. The monopoly price of product (j,ω) can be obtained by solving

$$\pi_t(j,\omega) \equiv \max_{p_t(j,\omega)} \left\{\left[p_t(j,\omega) - \kappa_t\right] M_t(j,\omega)\right\},$$

where the market demand function $M_t(j,\omega)$ is stated in (7).[28] The optimal monopoly price involves a proportional mark-up over the unit cost and is identical for all (j,ω). Formally, this is given by

$$p_t(j,\omega) = p_t \equiv \frac{\kappa_t}{\psi}, \qquad \text{for all } (j,\omega). \tag{13}$$

[27] This avoids the intricate issue of how a multiproduct monopolist would set the prices of its products, an issue that is not directly related to the main objective of this study. For more discussion on this type of pricing problem, see Chu *et al.* (2008) and references therein.

[28] Since there is a continuum of firms in the intermediate-goods sector, each firm is infinitesimal when compared to the entire sector. Thus, the aggregate price index $\mathbb{P}_t$ is treated as exogenously given when solving the monopoly pricing problem.

The market demand function and the monopoly profit function are then given by

$$M_t(j,\omega) = \left(\lambda^j \mathbb{P}_t\right)^{\frac{\psi}{1-\psi}} \left(\frac{\kappa_t}{\psi}\right)^{-\frac{1}{1-\psi}} \mathbb{E}_t,$$

$$\pi_t(j,\omega) = (1-\psi)\left(\frac{\psi \lambda^j \mathbb{P}_t}{\kappa_t}\right)^{\frac{\psi}{1-\psi}} \mathbb{E}_t.$$

Note that monopoly profits are strictly increasing in product quality $\left(\lambda^j\right)$. Thus, the monopolist will only produce the best quality grade $J_t(\omega)$ in any period t. Note also that both $M_t(J_t(\omega),\omega)$ and $\pi_t(J_t(\omega),\omega)$ depend on the index ω only indirectly through $J_t(\omega)$. Thus, for an intermediate input with state-of-the-art quality $Q_t(\omega) = q$, the market demand function and the monopoly profit function can be expressed as

$$M_t(q) = (q\mathbb{P}_t)^{\frac{\psi}{1-\psi}} \left(\frac{\kappa_t}{\psi}\right)^{-\frac{1}{1-\psi}} \mathbb{E}_t, \tag{14}$$

$$\pi_t(q) = (1-\psi)\left(\frac{\psi q \mathbb{P}_t}{\kappa_t}\right)^{\frac{\psi}{1-\psi}} \mathbb{E}_t. \tag{15}$$

From this point onward, we will drop the index ω and identify each intermediate-good producer by the state-of-the-art quality of its product.

3.4.3 Research Investment Decisions

Consider a monopolistic firm with state-of-the-art quality q at time t. The firm's profits $\pi_t(q)$ are subject to corporate income tax. The tax rate at time t is denoted by $\tau_{p,t} \in (0,1)$. When deciding how much to invest in quality improvement research, the firm takes into account two types of preferential tax treatment on private R&D expenses. First, these expenses are fully deductible from corporate income tax. Second, these expenses are also eligible for additional subsidies, or R&D tax credits. The rate of subsidy at time t is $\tau_{d,t} \in (0,1)$. Both the corporate income tax rate and the R&D subsidy rate are changing over time in a deterministic manner, and they are fully anticipated by the firm. In each period, the firm distributes its profits (net of taxes and research outlay) as dividends to its owners. Each monopolistic firm has a fixed number of shares outstanding in the equity market. The total number of shares issued by each firm is constant over time and is normalized to one. Each of these shares is a claim to the firm's future dividends. The value of the firm is then given by the present discounted value of its future dividends. Formally, let

$\Delta_t(q)$ be the amount of dividend distributed at time t, which is defined as

$$\begin{aligned}\Delta_t(q) &\equiv (1-\tau_{p,t})\left[\pi_t(q) - w_{H,t}\Lambda_{M,t}\right] + \tau_{d,t}w_{H,t}\Lambda_{M,t} \\ &= (1-\tau_{p,t})\,\pi_t(q) - (1-\tau_{p,t}-\tau_{d,t})\,w_{H,t}\Lambda_{M,t}. \end{aligned} \tag{16}$$

Let $V_t(q)$ be the value of a firm with state-of-the-art quality q at time t. The value function is defined recursively as

$$V_t(q) = \max_{\Lambda_{M,t}} \left\{ \Delta_t(q) + \frac{\Phi_t(q,\Lambda_{M,t})\,V_{t+1}(\lambda q) + [1-\Phi_t(q,\Lambda_{M,t})]\,V_{t+1}(q)}{1+r_{t+1}} \right\}, \tag{17}$$

subject to (10) and (16). The expected future value of the firm is discounted using the market discount rate $(1+r_{t+1})^{-1}$.

Since $\vartheta \in (0,1)$, it is never optimal for the firm to choose $\Lambda_{M,t} = 0$. In other words, all intermediate-good producers will invest in quality improvement research in every period. The first-order condition for $\Lambda_{M,t}$ is given by

$$(1-\tau_{p,t}-\tau_{d,t})\,w_{H,t} = \frac{\vartheta}{\Omega_t(q)}\left(\frac{\Lambda_{M,t}}{\overline{\Lambda}_{M,t}}\right)^{\vartheta-1}\left[\frac{V_{t+1}(\lambda q) - V_{t+1}(q)}{1+r_{t+1}}\right]. \tag{18}$$

Equation (18) states that optimality is achieved when the marginal cost of hiring high-skilled labor equals its marginal benefit. The marginal cost is the subsidized wage rate. The marginal benefit is determined by two factors: (i) the increase in success rate brought by an additional unit of high-skilled labor, and (ii) the present value of the *net* gain from the improved product. Note that a successful improvement from quality q to λq not only creates some new value for the firm [i.e., $V_{t+1}(\lambda q)$], but also destroys the continuation value of the original product [i.e., $V_{t+1}(q)$] which now becomes obsolete. Hence, the benefit of successful research is measured by the net gain in the firm's future value. Holding other things constant, an increase in either the corporate income tax rate ($\tau_{p,t}$) or the R&D tax credit ($\tau_{d,t}$) will lower the marginal cost of hiring high-skilled labor and encourage the firm's demand for research input.

3.5 Distribution of Product Quality

Since the timing and frequency of quality improvement is idiosyncratic across intermediate goods, a non-degenerate distribution of product quality emerges in every period $t \geq 1$. The distribution of product quality is defined over the support $\mathcal{Q} \equiv \left\{1, \lambda, \lambda^2, ...\right\}$. Consider those intermediate goods with state-of-

the-art quality $q \in \mathcal{Q}$ at the beginning of time t. The share of these goods is denoted by $f_t(q) \in [0,1]$, with $\sum_{q\in\mathcal{Q}} f_t(q) = 1$ for all t. The initial distribution is degenerate and is denoted by $f_0(1) = 1$. For future references, define an aggregate quality index $\mathbb{Q}_t$ according to

$$\mathbb{Q}_t = \left\{ \sum_{q\in\mathcal{Q}} f_t(q)\, q^{\frac{\psi}{1-\psi}} \right\}^{\frac{1-\psi}{\psi}}. \tag{19}$$

The initial degenerate distribution then implies $\mathbb{Q}_0 = 1$ and $\overline{q}_0 = 1$.

The evolution of the quality distribution can be characterized as follows. Let $\Lambda_{M,t}(q)$ be the optimal quantity of high-skilled labor employed by an intermediate-good producer with state-of-the-art quality q at time t. The probability of successful improvement is then given by $\widehat{\Phi}_t(q) \equiv \Phi_t[q, \Lambda_{M,t}(q)]$. Starting from the initial condition $f_0(1) = 1$, the quality distribution changes over time according to

$$f_{t+1}(\lambda q) = \widehat{\Phi}_t(q)\, f_t(q) + \left[1 - \widehat{\Phi}_t(\lambda q)\right] f_t(\lambda q), \tag{20}$$

for all $q \in \left\{1, \lambda, \lambda^2, ...\right\}$, and

$$f_{t+1}(1) = \left[1 - \widehat{\Phi}_t(1)\right] f_t(1). \tag{21}$$

At time $t+1$, the set of intermediate goods with state-of-the-art quality λq is the combination of two groups. First, among those with state-of-the-art quality q at time t, a fraction $\widehat{\Phi}_t(q)$ of them will be upgraded to λq at time $t+1$. The size of this group is given by $\widehat{\Phi}_t(q) f_t(q)$. Second, among those with state-of-the-art quality λq at time t, a fraction $\left[1 - \widehat{\Phi}_t(\lambda q)\right]$ of them will fail to improve and remain at the same quality level. The size of this group is given by $\left[1 - \widehat{\Phi}_t(\lambda q)\right] f_t(\lambda q)$. Equation (20) states that $f_{t+1}(\lambda q)$ is given by the sum of these two groups. Equation (21) states that the set of intermediate goods with $q = 1$ are those who fail to improve in every period.

Note that, in every period t, there is always a strictly positive fraction of firms with leading-edge quality $\overline{q}_t$ that succeed in improving their products. Thus, the leading-edge quality level will evolve according to $\overline{q}_{t+1} = \lambda \overline{q}_t$ with probability one.

3.6 Basic Research

In the model economy, a continuum of basic research projects is available in every period. Each project is indexed by a positive scalar z which indicates its quality. The set of research projects available at time t is uniformly distributed over the range $[0, \overline{z}_t]$. The upper boundary $\overline{z}_t$ represents the most advanced

project at time t. This value is assumed to be changing over time in a deterministic manner. The specifics of this will be explained later.

In each period, each member of the high-skilled household draws a single basic research project from the uniform distribution. The draws are independent over time and across individuals. After observing z, a high-skilled consumer then seeks outside funding in order to develop the project. If the project is unfunded, then the consumer will work as a researcher in the intermediate-goods sector (the private sector). Even if outside funding is available, the consumer can still decide whether or not to proceed with the project. If he chooses not to proceed, then again he will work in the private sector. If he chooses to proceed, then he will have to invest his own time endowment (which is normalized to one) on the project. Similar to quality improvement research, basic research only requires high-skilled labor as input and its success is uncertain. If a project of quality z is successfully developed at time t, then it will create $\zeta(z) X_t$ units of new fundamental knowledge. The function $\zeta : [0, \overline{z}_t] \to \mathbb{R}_+$ is continuous and strictly increasing, which means high-quality projects contribute more to the growth rate of fundamental knowledge than low-quality ones. No new knowledge is created if the project fails.

The probability of successfully developing a type-z project at time t is given by

$$\Phi_{x,t}(z) = \frac{1}{\Omega_{x,t}(z)}. \tag{22}$$

In the above expression, the numerator represents individual researcher's own labor input (which is one), and the denominator indicates the difficulty of developing a type-z project at time t. The difficulty index is determined by

$$\Omega_{x,t}(z) \equiv \nu_x(z) \Im(\mathbb{Q}_t) X_t^{-\varrho}, \qquad \text{with } \varrho > 0. \tag{23}$$

Both $\nu_x : \mathbb{R}_+ \to \mathbb{R}_{++}$ and $\Im : \mathbb{R}_+ \to \mathbb{R}_+$ are continuous, strictly increasing functions.

Equation (23) captures three important aspects of basic research. First, the difficulty of basic research in general is strictly decreasing in the available stock of fundamental knowledge. This captures the idea that past discoveries of basic research are beneficial to the productivity of current research.[29] Second, high-quality projects are more difficult to develop than low-quality ones. This assumption is captured by the strictly increasing function $\nu_x(\cdot)$. Third, as the economy becomes more technologically advanced, basic research projects become more difficult for researchers to develop individually. This assumption

[29] This mechanism is often referred to as the *intertemporal knowledge spillover effect*. Note that X_t enters into (11) and (23) in the same way. This assumption is needed to ensure the existence of balanced-growth equilibria.

is captured by the strictly increasing function $\Im(\mathbb{Q}_t)$, where $\mathbb{Q}_t$ is used as a measure of technological development. The main ideas behind this assumption are as follows: A basic research project can be viewed as a collection of tasks. As technology advances, more sophisticated and specialized techniques for conducting basic research are available and adopted. As a result, the tasks involved in basic research projects also become more specialized. This makes it more difficult for a single researcher to complete all the tasks in these projects. This hypothesis provides a potential explanation for the increasing prevalence of collaboration and teamwork among scientists.[30] In particular, it is consistent with the fact that scientific collaboration is more common in fields that require complex instrumentation (or "big science").[31]

The function $\Im(\mathbb{Q}_t)$ represents one of two channels through which innovations brought by quality improvement research can feedback into basic research. The second channel is built upon the idea that major breakthroughs in non-basic research or technology often open up new areas of scientific studies.[32] To capture this mechanism in the most parsimonious manner, we set $\overline{z}_t = \overline{q}_t$ for all t. In words, this means a leading-edge improvement in technology will also broaden the range of scientific research.[33] Under this specification, the upper bound $\overline{z}_t$ will increase deterministically over time according to $\overline{z}_{t+1} = \lambda \overline{z}_t$.

Next, we turn to the supply side of basic research funding. As we mentioned earlier, due to the public-good nature of fundamental knowledge, no firm is willing to invest in basic research. Consequently, basic research projects are solely funded by the government.[34] The amount and allocation of basic research funding is determined by a national research agency. In each period, the government agency chooses a set of research projects so as to maximize the growth rate of fundamental knowledge subject to a policy constraint. Let $\mathcal{D}_t \subseteq [0, \overline{z}_t]$ be a non-empty set of research projects funded by the government. For those research projects with quality $z \in \mathcal{D}_t$, a fraction $\Phi_{x,t}(z)$ of them will succeed and create $\zeta(z) X_t$ units of

[30] For the empirical evidence on the rising trend of scientific collaboration, see Beaver and Rosen (1979) and Stephan (1996, Table 4). Other possible explanations for this phenomenon have been examined by Katz and Martin (1997) and Jones (2009).

[31] For instance, Katz and Martin (1997) and Klein (2005) point out that many projects in high-energy physics, astronomy, oceanography, and life sciences (e.g., the Human Genome Project) require sophisticated and specialized equipments such as particle accelerators, observatories, ocean research vessels, and supercomputers. Consequently, research in these areas typically require collaborations between specialists in different fields. In other words, it is very difficult (if not impossible) for a single researcher to perform all the tasks involved in these research projects.

[32] See, for instance, Rosenberg (1990, p.169) for some specific examples.

[33] Our theoretical analysis can be easily extended to accomodate a more general relationship between $\overline{q}_t$ and $\overline{z}_t$. For instance, one can assume that $\overline{z}_t$ is strictly increasing in $\overline{q}_t$ and $\varkappa_t$, where $\varkappa_t$ represents other exogenous factors that can also affect the range of scientific research. The assumption that $\overline{q}_t$ and $\overline{z}_t$ share the same exogenous growth rate is not essential for our major results.

[34] This also explains why high-skilled workers will not self-finance any unfunded projects: Since there is no market for fundamental knowledge, the gain from a self-financed project is zero, but the opportunity cost ($w_{H,t}$) is always strictly positive.

public R&D capital. The total quantity of public R&D capital produced by the set $\mathcal{D}_t$ is given by

$$\frac{1}{\overline{z}_t}\int_{\mathcal{D}_t}\Phi_{x,t}\left(z\right)\zeta\left(z\right)X_t dz=\left[\frac{1}{\overline{z}_t}\int_{\mathcal{D}_t}\widetilde{\zeta}\left(z\right)dz\right]N_{H,t}X_t^{\varrho+1}\left[\Im\left(\mathbb{Q}_t\right)\right]^{-1},$$

where $\widetilde{\zeta}\left(z\right)\equiv\zeta\left(z\right)/\nu_x\left(z\right)$. The growth of fundamental knowledge is determined by

$$\frac{X_{t+1}}{X_t}=1+\left[\frac{1}{\overline{z}_t}\int_{\mathcal{D}_t}\widetilde{\zeta}\left(z\right)dz\right]N_{H,t}X_t^{\varrho}\left[\Im\left(\mathbb{Q}_t\right)\right]^{-1}, \tag{24}$$

with $X_0>0$ given. In the above expression, the depreciation rate of fundamental knowledge is assumed to be zero.

The national research agency is assumed to follow two types of policies when deciding the size and allocation of basic research funding. First, in order to ensure participation in basic research, the wage rate for basic researchers must be no less than the market wage rate for high-skilled workers. Specifically, the government agency will offer all basic researchers a wage rate $\epsilon_t w_{H,t}$, with $\epsilon_t\geq 1$, regardless of the outcome of their research. If $\epsilon_t=1$, then high-skilled workers will be indifferent between working in the public and the private sector. We assume that, in this case, workers with basic research projects that are qualified for funding will still choose to work in the public sector. Second, the government agency has a targeted scale of basic research in every period. Specifically, this means the government will recruit a fraction $\varpi_t\in(0,1)$ of high-skilled workers into basic research in every period t. This gives rise to a policy constraint on the choice of $\mathcal{D}_t$, which is

$$\left[\frac{1}{\overline{z}_t}\int_{\mathcal{D}_t}dz\right]N_{H,t}=\varpi_t N_{H,t}\Rightarrow\frac{1}{\overline{z}_t}\int_{\mathcal{D}_t}dz=\varpi_t. \tag{25}$$

The amount of government basic research spending ($G_{x,t}$) is then determined by $G_{x,t}=\varpi_t\epsilon_t w_{H,t}N_{H,t}$.

In each period, the government agency's problem is to choose a set of research projects $\mathcal{D}_t\subset[0,\overline{z}_t]$ so as to maximize the growth of public R&D capital in (24) subject to the policy constraint in (25).[35] The solution of this problem depends crucially on the shape of $\widetilde{\zeta}\left(z\right)\equiv\zeta\left(z\right)/\nu_x\left(z\right)$. Note that this function combines two characteristics of basic research projects: (i) the potential contribution of each project as captured by $\zeta\left(\cdot\right)$, and (ii) the difficulty of research project $\nu_x\left(\cdot\right)$ which directly affects the rate of success. Since high-quality projects are also more difficult to develop, the shape of $\widetilde{\zeta}\left(\cdot\right)$ cannot

[35] The sole purpose of this maximization problem is to provide a theoretical foundation for the results in Proposition 1. Alternatively, one can directly assume that the results in Proposition 1 are valid without solving this problem. This will not affect the subsequent analysis. In the present study, we do not consider the socially optimal level of government basic research spending.

be determined *a priori*. In the current study, we focus on the case in which $\widetilde{\zeta}(\cdot)$ is a strictly increasing function, which means the *expected* contribution of a basic research project is strictly increasing in its quality. This assumption is desirable because it gives rise to an intuitive solution which can be obtained in a straightforward way. More specifically, under this assumption, the optimal set of research projects is given by $\mathcal{D}_t = [d_t, \overline{z}_t]$, with $d_t \equiv (1-\varpi_t)\,\overline{z}_t > 0$. This result is summarized in Proposition 1.[36] All proofs can be found in Appendix B.

Proposition 1 *Suppose $\varpi_t \in (0,1)$ for all $t \geq 0$ and $\widetilde{\zeta}(\cdot)$ is a strictly increasing function. Then the optimal set of research projects in any time t is given by $\mathcal{D}_t = [d_t, \overline{z}_t]$, where $d_t \equiv (1-\varpi_t)\,\overline{z}_t > 0$.*

Proposition 1 has two main implications. First, the government will finance those research projects with the highest quality in every period. Second, by recruiting a larger share of high-skilled workers into basic research (i.e., increasing the value of ϖ_t), the government can support a wider range of research projects which in turn enhances the growth rate of fundamental knowledge. In the following analyses, we will adopt the specific form $\widetilde{\zeta}(z) \equiv z^{\varphi}$, with $\varphi > 0$. The expected value of $\widetilde{\zeta}(z)$ over the optimal set $\mathcal{D}_t = [d_t, \overline{z}_t]$ is then given by

$$\frac{1}{\overline{z}_t}\int_{\mathcal{D}_t}\widetilde{\zeta}(z)\,dz = \Xi(\varpi_t)\,\overline{z}_t^{\varphi},$$

where $\Xi(\varpi_t) \equiv \left[1-(1-\varpi_t)^{1+\varphi}\right]/(1+\varphi)$ is strictly increasing in ϖ_t. The maximized growth factor of public R&D capital is now given by

$$\frac{X_{t+1}}{X_t} = 1 + \Xi(\varpi_t)\,\overline{z}_t^{\varphi} N_{H,t} X_t^{\varrho}\left[\Im(\mathbb{Q}_t)\right]^{-1}. \tag{26}$$

3.7 Households

Both high-skilled and low-skilled households have preferences over consumption sequences which can be represented by

$$\sum_{t=0}^{\infty}\beta^t N_{s,t} U(c_{s,t}), \qquad \text{for } s \in \{H, L\}, \tag{27}$$

[36] Two remarks about Proposition 1 are in order. First, similar results can be obtained under a more general distribution of basic research projects. Suppose the distribution of z at time t is given by $\mathcal{H}_t(z)$, where $\mathcal{H}_t : [0, \overline{z}_t] \to [0,1]$ is a continuous, strictly increasing function. If $\widetilde{\zeta}(\cdot)$ is strictly increasing, then the optimal set of research projects at time t is given by $\mathcal{D}_t = [d_t, \overline{z}_t]$, where $d_t \equiv \mathcal{H}_t^{-1}(1-\varpi_t) > 0$. The proof of this statement is available from the author upon request. Second, suppose $\mathcal{H}_t(\cdot)$ is a uniform distribution and $\widetilde{\zeta}(\cdot)$ is strictly decreasing. Then the optimal set of research projects at time t is given by $\mathcal{D}_t = [0, d_t]$, where $d_t = \varpi_t \overline{z}_t > 0$. This result can be obtained by using the same argument as in the proof of Proposition 1. In this case, the government agency will always finance those projects with the lowest quality, which seems unintuitive. Hence, we do not consider the case when $\widetilde{\zeta}(\cdot)$ is strictly decreasing.

where $\beta \in (0,1)$ is the subjective discount factor, $U(\cdot)$ is the (per-period) utility function, and $c_{s,t}$ is the consumption of a type-s consumer at time t. The utility function is given by

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \qquad \text{with } \sigma > 0.$$

All consumers are endowed with one unit of time in every period. Low-skilled consumers supply their time endowment inelastically to the production of final goods and intermediate goods. Their before-tax labor income at time t is $w_{L,t}$. As for high-skilled consumers, if an individual draws a qualified basic research project (i.e., $d_t \leq z \leq \overline{z}_t$), then he will work as a basic researcher and receive $\epsilon_t w_{H,t}$ as before-tax labor income. Otherwise, he will work as a private-sector researcher and receive $w_{H,t}$. Thus, if $\epsilon_t > 1$ for some t, then the high-skilled workers will face idiosyncratic uncertainty in labor income. We assume that the high-skilled household is able to provide complete consumption insurance for its members when necessary. Under this arrangement, all members of the high-skilled household will pool their incomes together and consume the same level of consumption $c_{H,t}$ in every period. The average before-tax wage rate for high-skilled worker is denoted by $\widetilde{w}_{H,t} \equiv [1 + (\epsilon_t - 1)\varpi_t] w_{H,t}$.

The two households can save and borrow through the financial intermediaries in every period. Let $A_{s,t}$ be the asset holdings of type-s household at time t. The before-tax rate of return on these assets is r_t. Households also receive lump-sum transfers from the government in every period. The amount of transfers received by type-s household at time t is $\Upsilon_{s,t}$. Labor income and interest income at time t are taxed at rates $\tau_{w,t} \in (0,1)$ and $\tau_{a,t} \in (0,1)$, respectively. The tax rate on private consumption expenditures is denoted by $\tau_{c,t} > 0$. The budget constraint for type-s household at time t is given by

$$(1+\tau_{c,t}) N_{s,t} c_{s,t} + A_{s,t+1} - A_{s,t} = (1-\tau_{w,t}) \widetilde{w}_{s,t} N_{s,t} + (1-\tau_{a,t}) r_t A_{s,t} + \Upsilon_{s,t}, \tag{28}$$

where $\widetilde{w}_{L,t} = w_{L,t}$.

Taking prices and government policies as given, the household's problem is to choose a sequence of consumption and asset holdings so as to maximize the discounted lifetime utility in (27), subject to the sequential budget constraint in (28), the no-Ponzi-game condition

$$\lim_{T\to\infty} \left\{ \left[\prod_{t=1}^{T} (1 + (1-\tau_{a,t}) r_t) \right]^{-1} A_{s,T+1} \right\} \geq 0,$$

and the initial condition: $A_{s,0} \geq 0$. The solution of the household's problem is completely characterized

by the sequential budget constraint, the Euler equation for consumption

$$\frac{c_{s,t+1}}{c_{s,t}} = \left\{ \beta \left(\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}} \right) \left[1 + (1-\tau_{a,t+1}) \, r_{t+1} \right] \right\}^{\frac{1}{\sigma}}, \tag{29}$$

and the transversality condition

$$\lim_{T \to \infty} \left[\beta^T \left(\frac{c_{s,T}^{-\sigma}}{1+\tau_{c,T}} \right) A_{s,T+1} \right] = 0. \tag{30}$$

Conditions (29) and (30) together imply that the no-Ponzi-game condition is satisfied.

3.8 Government Expenditures and Financing

The government in this economy performs a number of functions. These include (i) providing lump-sum transfers to the households, (ii) maintaining the patent system, (iii) subsidizing private R&D expenses, and (iv) determining the size and allocation of basic research funding. In each period, the government incurs three types of expenses, namely (i) transfer payments, (ii) subsidy on private R&D expenses, and (iii) basic research spending. These expenses are financed by four types of taxes, which are taxes on labor income, interest income, corporate income and private consumption. Formally, the total amount of tax revenue collected at time t is determined by

$$\begin{aligned} \mathbb{T}_t \equiv \; & \tau_{w,t} \left[\sum_{s \in \{H,L\}} N_{s,t} \widetilde{w}_{s,t} \right] + \tau_{a,t} \left[\sum_{s \in \{H,L\}} r_t A_{s,t} \right] + \tau_{c,t} \left[\sum_{s \in \{H,L\}} N_{s,t} c_{s,t} \right] \\ & + \tau_{p,t} \left\{ \sum_{q \in \mathcal{Q}} f_t(q) \left[\pi_t(q) - w_{H,t} \Lambda_{M,t}(q) \right] \right\}, \end{aligned}$$

where $\sum_{q \in \mathcal{Q}} f_t(q) \, \pi_t(q)$ is the aggregate level of corporate income at time t, and $w_{H,t} \sum_{q \in \mathcal{Q}} f_t(q) \, \Lambda_{M,t}(q)$ is the aggregate level of private R&D investment. The government is required to maintain a balanced budget in every period, so that total government expenses and total tax revenues must be equal at all times. Formally, this means

$$\sum_{s \in \{H,L\}} \Upsilon_{s,t} + \tau_{d,t} w_{H,t} \left[\sum_{q \in \mathcal{Q}} f_t(q) \, \Lambda_{M,t}(q) \right] + G_{x,t} = \mathbb{T}_t, \qquad \text{for all } t \geq 0. \tag{31}$$

All policy instruments, except government transfers, are treated as exogenously given. These include all the tax rates and R&D subsidy rates, $\{\tau_{c,t}, \tau_{w,t}, \tau_{a,t}, \tau_{p,t}, \tau_{d,t}\}_{t=0}^{\infty}$, the shares of high-skilled workers

employed by the government, $\{\varpi_t\}_{t=0}^{\infty}$, and the markup over market wage rate for basic researchers, $\{\epsilon_t\}_{t=0}^{\infty}$. The set of exogenous policy instruments is summarized by $\mathbb{G} \equiv \{\tau_{c,t}, \tau_{w,t}, \tau_{a,t}, \tau_{p,t}, \tau_{d,t}, \varpi_t, \epsilon_t\}_{t=0}^{\infty}$. The values of $\{\Upsilon_{H,t}, \Upsilon_{L,t}\}_{t=0}^{\infty}$ are endogenously determined in equilibrium.

3.9 Financial Sector

In the financial sector, there is a large number of identical, infinitely-lived, risk-neutral financial intermediaries or banks. The banks are the owners of physical capital and equity issued by the firms in the intermediate-goods sector. Let $\overline{K}_t$ be the quantity of physical capital owned by the banks at the beginning of time t. In each period, the banks rent out the stock of physical capital to the producers of intermediate goods at rate R_t. The effective rate of return from physical capital is given by $r_t = R_t - \delta$, where $\delta \in (0,1)$ is the depreciation rate of physical capital.

As for equity, we assume that only banks have access to the equity market. Thus, in equilibrium, all the equity shares issued by the monopolistic firms are held by the banks. Let $\mathcal{S}_t(q)$ be the shares of firms with state-of-the-art quality q that are owned by the banks at time t, and $\mathcal{P}_t(q)$ be the price of each share. The value of all the shares owned by the banks at time t is given by $\sum_{q\in\mathcal{Q}} f_t(q)\,\mathcal{P}_t(q)\,\mathcal{S}_t(q)$.[37] The amount of dividend income generated by this portfolio is $\sum_{q\in\mathcal{Q}} f_t(q)\,\Delta_t(q)\,\mathcal{S}_t(q)$.

In each period, the banks receive rental incomes from physical capital, and dividend incomes from equity shares. They also solicit deposits from the households in each period. The interest rate paid on these deposits is also r_t. All the proceeds received by the banks are used to finance their asset holdings in the next period and offset their deposit liabilities. Since anyone can set up a financial intermediary at zero cost, the banks' profits are driven to zero in every period. The zero-profit condition for the entire financial sector is given by

$$\begin{aligned} & \overline{K}_{t+1} + \sum_{q\in\mathcal{Q}} f_t(q)\,\mathcal{P}_t(q)\,\mathcal{S}_{t+1}(q) + (1+r_t)\sum_{s\in\{H,L\}} A_{s,t} \\ = \;& (1+r_t)\,\overline{K}_t + \sum_{q\in\mathcal{Q}} f_t(q)\,[\mathcal{P}_t(q) + \Delta_t(q)]\,\mathcal{S}_t(q) + \sum_{s\in\{H,L\}} A_{s,t+1}. \end{aligned} \tag{32}$$

[37] Since there is a large number of firms in the intermediate-goods sector, the idiosyncratic risks regarding the success and failure of quality improvement research faced by each monopolist is completely diversified in this portfolio. Thus, there is no aggregate uncertainty in this model.

4 Equilibrium

4.1 Definition

Given a set of exogenous policy instruments $\mathbb{G}$, an equilibrium of this economy consists of (i) sequences of allocations $\{c_{H,t}, c_{L,t}, A_{H,t}, A_{L,t}\}_{t=0}^{\infty}$ for the two households, (ii) sequences of factor inputs employed by the representative final-good producer $\{L_{Y,t}, M_t(\cdot)\}_{t=0}^{\infty}$, where each $M_t(\cdot)$ is a real-valued function defined on $\mathcal{Q}$; (iii) sequences of functions $\{L_{M,t}(\cdot), K_t(\cdot), \Lambda_{M,t}(\cdot), V_t(\cdot), \Delta_t(\cdot)\}_{t=0}^{\infty}$ defined on $\mathcal{Q}$ which specify the intermediate-good producers' input demands, equity value and dividend payments; (iv) sequences of policy variables and public R&D capital $\{G_{x,t}, d_t, X_t\}_{t=0}^{\infty}$; (v) sequences of assets owned by the banks $\left\{\overline{K}_t, \mathcal{S}_t(\cdot)\right\}_{t=0}^{\infty}$, where each $\mathcal{S}_t(\cdot)$ is a real-valued function defined on $\mathcal{Q}$, (vi) a sequence of success rates for private R&D, $\left\{\widehat{\Phi}_t(\cdot)\right\}_{t=0}^{\infty}$, where $\widehat{\Phi}_t : \mathcal{Q} \to [0,1]$ for all t, and a sequence of quality distributions $\{f_t(\cdot)\}_{t=0}^{\infty}$, where $f_t : \mathcal{Q} \to [0,1]$ for all t, and (vii) sequences of prices $\{w_{L,t}, w_{H,t}, R_t, r_t, p_t, \mathcal{P}_t(\cdot)\}_{t=0}^{\infty}$, where $\mathcal{P}_t : \mathcal{Q} \to \mathbb{R}_+$ for all t, such that

(a) Given prices and government policies, $\{c_{s,t}, A_{s,t}\}_{t=0}^{\infty}$ solves type-s household's problem.

(b) Given prices, $\{L_{Y,t}, M_t(\cdot)\}_{t=0}^{\infty}$ solves the final-good producer's problem in every period.

(c) Given factor prices and government policies, $\{L_{M,t}(\cdot), K_t(\cdot), \Lambda_{M,t}(\cdot), V_t(\cdot), \Delta_t(\cdot), p_t\}_{t=0}^{\infty}$ solves the monopolists' problems in the intermediate-goods sector.

(d) Public R&D spending is determined by $G_{x,t} = \varpi_t \epsilon_t w_{H,t} N_{H,t}$ in every period. Public R&D capital accumulates according to (26) with $\mathcal{D}_t = [d_t, \overline{z}_t]$ and $d_t \equiv (1 - \varpi_t)\overline{z}_t$ for all t. The government's budget constraint in (31) is satisfied in every period.

(e) The zero-profit condition for the financial sector holds in every period.

(g) The distribution of product quality evolves according to (20) and (21) in every period.

(h) All markets clear in every period, so that for all $t \geq 0$,

$$L_{Y,t} + \sum_{q \in \mathcal{Q}} f_t(q) L_{M,t}(q) = N_{L,t}, \tag{33}$$

$$\overline{\Lambda}_{M,t} \equiv \sum_{q \in \mathcal{Q}} f_t(q) \Lambda_{M,t}(q) = (1 - \varpi_t) N_{H,t}, \tag{34}$$

$$\sum_{q \in \mathcal{Q}} f_t(q) K_t(q) = \overline{K}_t, \tag{35}$$

$$\mathcal{S}_t(q) = 1, \quad \text{for all } q \in \mathcal{Q}. \tag{36}$$

Equations (33)-(35) state that in equilibrium the supply and demand for low-skilled labor, private-sector researchers and physical capital are equated in every period. Equation (36) states the market-clearing condition for equity.

4.2 Analysis

In this section, we will present and discuss some important features of an equilibrium in this model. A formal derivation of these results can be found in Appendix A. In equilibrium, aggregate output in every period $t \geq 0$ is determined by

$$Y_t = \widetilde{\mathbb{A}} \overline{K}_t^{\rho\alpha} \left(\Psi_t N_{L,t}\right)^{1-\rho\alpha}, \tag{37}$$

where $\rho\alpha \in (0,1)$ and $\widetilde{\mathbb{A}} > 0$ is a constant. The variable Ψ_t is defined as

$$\Psi_t \equiv X_t^{\frac{\eta(1+\alpha)}{1-\rho\alpha}} \mathbb{Q}_t^{\frac{\alpha}{1-\rho\alpha}}. \tag{38}$$

Equation (37) is obtained after combining the production technologies in the final-good sector and the intermediate-goods sector. Similar to the neoclassical production function, the aggregate production function in (37) exhibits diminishing returns with respect to physical capital. In the standard neoclassical growth model, the diminishing marginal product of capital is counteracted by an exogenous growth in labor-augmenting technology. Without this type of technological progress, per-capita output will eventually cease to grow. In the current framework, Ψ_t plays the same role as labor-augmenting technology in the neoclassical framework. The source of growth in our model, however, is endogenously determined. In particular, it is driven by the accumulation of fundamental knowledge created by basic research and the continuous improvement in productive inputs made possible by non-basic research. Formally, the growth factor of Ψ_t can be expressed as

$$\frac{\Psi_{t+1}}{\Psi_t} = \left(\frac{X_{t+1}}{X_t}\right)^{\frac{\eta(1+\alpha)}{1-\rho\alpha}} \left(\frac{\mathbb{Q}_{t+1}}{\mathbb{Q}_t}\right)^{\frac{\alpha}{1-\rho\alpha}}. \tag{39}$$

The accumulation of fundamental knowledge is determined by (26). The growth rate of $\mathbb{Q}_t$ is determined by the dynamics of the quality distribution $\{f_t(q) : q \in \mathcal{Q}\}$, which is in turn determined by the optimal quantity of input employed in quality improvement research $\Lambda_{M,t}(\cdot)$, and the value of the im-

proved products $V_t(\cdot)$. Both $\{V_t(\cdot)\}_{t=0}^{\infty}$ and $\{\Lambda_{M,t}(\cdot)\}_{t=0}^{\infty}$ are sequences of unknown functions that need to be determined in equilibrium. In general, it is very difficult to analyze a non-stationary equilibrium with sequences of unknown functions. For this reason, we seek conditions under which exact analytical solutions for $V_t(\cdot)$ and $\Lambda_{M,t}(\cdot)$ are available. These conditions are stated in Proposition 2.

Proposition 2 *Suppose* $\nu(q,\overline{q}_t) = \xi(\overline{q}_t)\, q^{\frac{\vartheta\psi}{1-\psi}}$ *for all* $q \in \mathcal{Q}$, *where* $\xi : \mathcal{Q} \to \mathbb{R}_+$ *is a strictly decreasing function. Then a solution for* $V_t(\cdot)$ *and* $\Lambda_{M,t}(\cdot)$ *is*

$$V_t(q) = \Gamma_t q^{\frac{\psi}{1-\psi}}, \qquad \textit{and} \qquad \Lambda_{M,t}(q) = (1-\varpi_t) N_{H,t} \left(\frac{q}{\mathbb{Q}_t}\right)^{\frac{\psi}{1-\psi}},$$

where $\Gamma_t > 0$ *is an unknown coefficient. The values of* $\{\Gamma_t\}_{t=0}^{\infty}$ *are determined by the dynamic equation*

$$\frac{\Gamma_{t+1}}{1+r_{t+1}} = \frac{\Gamma_t - (1-\tau_{p,t})(1-\psi)\,\mathbb{E}_t \mathbb{Q}_t^{-\frac{\psi}{1-\psi}}}{1-\vartheta(1-\vartheta)(1-\varpi_t)\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)[\xi(\overline{q}_t)]^{-1} N_{H,t} X_t^{\varrho} \mathbb{Q}_t^{-\frac{\vartheta\psi}{1-\psi}}}. \tag{40}$$

The results in Proposition 2 are extremely useful in simplifying the equilibrium analysis. Instead of solving for two sequences of infinite-dimensional objects (the unknown functions), now we only need to determine a sequence of real numbers $\{\Gamma_t\}_{t=0}^{\infty}$. Note that the analytical solution for $V_t(\cdot)$ and $\Lambda_{M,t}(\cdot)$ are both directly proportional to $q^{\frac{\psi}{1-\psi}}$ in every period $t \geq 0$. This property is intuitive because, in equilibrium, the monopolists' input demand functions, $L_{M,t}(q)$ and $K_t(q)$, and profit function $\pi_t(q)$, are all directly proportional to $q^{\frac{\psi}{1-\psi}}$ in every period.

Using the results in Proposition 2, we can show that $\mathbb{Q}_t$ will evolve over time according to

$$\frac{\mathbb{Q}_{t+1}}{\mathbb{Q}_t} = \left[1 + \frac{1}{\xi(\overline{q}_t)}\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)(1-\varpi_t) N_{H,t} X_t^{\varrho} \mathbb{Q}_t^{-\frac{\vartheta\psi}{1-\psi}}\right]^{\frac{1-\psi}{\psi}}, \tag{41}$$

with initial value $\mathbb{Q}_0 = 1$.[38] Equation (41) states that the growth rate of aggregate quality increases with an increase in (i) the stock of fundamental knowledge, (ii) the number of high-skilled workers available for private R&D, (iii) the size of quality improvement λ, and (iv) the leading-edge quality $\overline{q}_t$. The growth rate of $\mathbb{Q}_t$ is also inversely related to its own level. Notice that the equilibrium dynamics of $\mathbb{Q}_t$ are not affected by the corporate income tax rate $(\tau_{p,t})$ nor the R&D subsidy rate $(\tau_{d,t})$. This follows from the fact that both the equilibrium quantity of labor input employed in quality improvement research [i.e.,

[38] In the equilibrium analysis, there is no need to keep track of the dynamics of $f_t(\cdot)$. This is because the distribution of product quality will affect the aggregate variables only through the aggregate quality index $\mathbb{Q}_t$. In other words, this index serves as a sufficient statistic for the quality distribution. Hence, it suffice to keep track of the dynamics of $\mathbb{Q}_t$.

$(1-\varpi_t)\,N_{H,t}]$ and the distribution of this input [as captured by $\Lambda_{M,t}(\cdot)$] are independent of these policy instruments. The equilibrium dynamics of $\mathbb{Q}_t$ and X_t are now completely characterized by (26) and (41). Once the values of $\{\mathbb{Q}_t, X_t\}_{t=0}^{\infty}$ are determined, we can derive the values of $\{\Psi_t\}_{t=0}^{\infty}$ using (38).

In equilibrium, the economy-wide resources constraint in period t is given by

$$\begin{aligned} C_t + \overline{K}_{t+1} - (1-\delta)\,\overline{K}_t &= \widetilde{\mathbb{A}}\overline{K}_t^{\rho\alpha}\left(\Psi_t N_{L,t}\right)^{1-\rho\alpha} \\ &= \mathbb{A}_t\overline{K}_t^{\rho\alpha}\left(\Psi_t N_t\right)^{1-\rho\alpha}, \end{aligned}$$

where $\mathbb{A}_t \equiv \widetilde{\mathbb{A}}\,(1-\theta_t)^{1-\rho\alpha}$. The second line is obtained by using $N_{L,t} = (1-\theta_t)\,N_t$. The notation $C_t \equiv \sum_{s\in\{H,L\}} N_{s,t}c_{s,t}$ represents aggregate consumption at time t. The Euler equation in (29) states that the growth rates of $c_{H,t}$ and $c_{L,t}$ are identical in every period. Thus, there exists a positive real number μ^* such that $c_{H,t} = \mu^* c_{L,t}$ for all $t \geq 0$. The value of μ^* is endogenously determined in equilibrium (see Appendix A for more details). Based on this observation, we can express aggregate consumption as

$$C_t = \left(\frac{N_{H,t}}{N_t}\frac{c_{H,t}}{c_{L,t}} + \frac{N_{L,t}}{N_t}\right) N_t c_{L,t} = \left[1 + (\mu^* - 1)\,\theta_t\right] N_t c_{L,t}.$$

The growth of aggregate consumption is then determined by

$$\frac{C_{t+1}}{C_t} = \left[\frac{1+(\mu^*-1)\,\theta_{t+1}}{1+(\mu^*-1)\,\theta_t}\right]\left(\frac{N_{t+1}}{N_t}\right)\left\{\beta\left(\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}}\right)\left[1+(1-\tau_{a,t+1})\,r_{t+1}\right]\right\}^{\frac{1}{\sigma}}.$$

Note that changes in the composition of the labor force (as captured by θ_t) and the tax rate on consumption will also affect the growth rate of aggregate consumption.

To characterize the transition paths in this model, we formulate a dynamical system in two transformed variables: $\widehat{c}_t \equiv C_t/\left(\Psi_t N_t\right)$ and $\widehat{k}_t \equiv K_t/\left(\Psi_t N_t\right)$. This system is given by

$$\widehat{k}_{t+1} = \left(\frac{\Psi_{t+1}}{\Psi_t}\right)^{-1}\left(\frac{N_{t+1}}{N_t}\right)^{-1}\left[\mathbb{A}_t\widehat{k}_t^{\rho\alpha} + (1-\delta)\,\widehat{k}_t - \widehat{c}_t\right], \tag{42}$$

$$\frac{\widehat{c}_{t+1}}{\widehat{c}_t} = \left(\frac{\Psi_{t+1}}{\Psi_t}\right)^{-1}\left[\frac{1+(\mu^*-1)\,\theta_{t+1}}{1+(\mu^*-1)\,\theta_t}\right]\left\{\beta\left(\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}}\right)\left[1+(1-\tau_{a,t+1})\,r_{t+1}\right]\right\}^{\frac{1}{\sigma}}, \tag{43}$$

where $r_{t+1} = \psi\rho\alpha\mathbb{A}_{t+1}\widehat{k}_{t+1}^{\rho\alpha-1} - \delta$. The value of $\widehat{k}_0 > 0$ is exogenously given, whereas $\widehat{c}_0$ is endogenously determined as in the standard neoclassical growth model.

4.3 Long-Run Balanced-Growth Equilibrium

In this study, we are interested in both the long-run and short-run effects of research policies. In order to define a long-run balanced-growth equilibrium, all model parameters (including the population growth rates and the exogenous policy instruments) must be time-invariant in the long run. The following assumption is used to ensure this.

Assumption S There exists an integer $T^* > 0$ such that for all $t \geq T^*$, $n_{H,t} = n_{L,t} = n^* > 0$; $\tau_{i,t} = \tau_i^* \in (0,1)$ for $i \in \{w, a, c, p, d\}$; $\varpi_t = \varpi^* \in (0,1)$; and $\epsilon_t = \epsilon^* \geq 1$.

Under this assumption, the size of the two households will grow at the *same* constant rate from period T^* onward.[39] As a result, the share of high-skilled consumers in the population θ_t and the coefficient $\mathbb{A}_t$ in the aggregate production function will remain constant after period T^*, i.e., $\theta_t = \theta^*$ and $\mathbb{A}_t = \mathbb{A}^*$ for all $t \geq T^*$. Given Assumption S, a long-run balanced-growth equilibrium is an equilibrium that satisfies two additional properties: (i) the growth rates of $\mathbb{Q}_t$ and X_t are constant over time, and (ii) the transformed variables $\widehat{c}_t$ and $\widehat{k}_t$ are constant over time. In the remainder of this section, we first establish the existence, uniqueness and stability of long-run balanced-growth equilibrium. We then discuss the factors that determine long-term economic growth.

4.3.1 Existence, Uniqueness and Stability

Obviously, a balanced-growth equilibrium (if exists) will only emerge after period T^*. Thus, we will focus on the dynamics of the economy from period T^* onward, taking as given the values of $\widehat{k}_t$, $\mathbb{Q}_t$ and X_t at $t = T^*$. The transitional dynamics can be separated into two parts. First, the time paths of $\mathbb{Q}_t$ and X_t are completely determined by

$$\frac{X_{t+1}}{X_t} = 1 + \Xi\left(\varpi^*\right) \overline{z}_t^{\varphi} N_{H,t} X_t^{\varrho} \left[\Im\left(\mathbb{Q}_t\right)\right]^{-1},$$

$$\frac{\mathbb{Q}_{t+1}}{\mathbb{Q}_t} = \left[1 + \frac{1}{\xi\left(\overline{q}_t\right)} \left(\lambda^{\frac{\psi}{1-\psi}} - 1\right) \left(1 - \varpi^*\right) N_{H,t} X_t^{\varrho} \mathbb{Q}_t^{-\frac{\vartheta\psi}{1-\psi}}\right]^{\frac{1-\psi}{\psi}},$$

with $\mathbb{Q}_{T^*} > 0$ and $X_{T^*} > 0$ given. We will refer to this as the *subsystem for* $\mathbb{Q}_t$ *and* X_t. Once the values of $\{\mathbb{Q}_t, X_t\}_{t=T^*}^{\infty}$ are determined, we can construct a unique sequence $\{\Psi_t\}_{t=T^*}^{\infty}$ using (38). The values of

[39] If the size of the two households grow at different constant rates in the long run, then the population share of one group will eventually diminish to zero.

$\left\{\widehat{c}_t, \widehat{k}_{t+1}\right\}_{t=T^*}^{\infty}$ are then determined by

$$\widehat{k}_{t+1} = \left(\frac{\Psi_{t+1}}{\Psi_t}\right)^{-1} (1+n^*)^{-1} \left[\mathbb{A}^* \widehat{k}_t^{\rho\alpha} + (1-\delta)\,\widehat{k}_t - \widehat{c}_t\right],$$

$$\frac{\widehat{c}_{t+1}}{\widehat{c}_t} = \left(\frac{\Psi_{t+1}}{\Psi_t}\right)^{-1} \left\{\beta\left[1 + (1-\tau_a^*)\left(\psi\rho\alpha\mathbb{A}^* \widehat{k}_{t+1}^{\rho\alpha-1} - \delta\right)\right]\right\}^{\frac{1}{\sigma}},$$

under a given value of $\widehat{k}_{T^*} > 0$. We will refer to this as the *subsystem for* $\widehat{c}_t$ *and* $\widehat{k}_t$. Note that this is a *non-autonomous* system as it depends on the time-varying growth factor of Ψ_t which is independent of $\widehat{c}_t$ and $\widehat{k}_t$.

A solution to the subsystem for $\mathbb{Q}_t$ and X_t is called a balanced-growth path (BGP) if the growth rates of these variables are constant over time. A BGP is called *globally stable* if it exhibits the following property: Starting from any positive values of X_{T^*} and $\mathbb{Q}_{T^*}$, any solution to the subsystem will eventually converge to the BGP. If the subsystem for $\mathbb{Q}_t$ and X_t is globally stable, then the growth rates of $\mathbb{Q}_t$ and X_t (and hence Ψ_t) will converge to some constants when t is sufficiently large. Let γ^* be the long-term growth rate of Ψ_t. As the growth rate of Ψ_t approaches γ^*, the subsystem for $\widehat{c}_t$ and $\widehat{k}_t$ will converge to an *autonomous* system,

$$\widehat{k}_{t+1} = \frac{\mathbb{A}^* \widehat{k}_t^{\rho\alpha} + (1-\delta)\,\widehat{k}_t - \widehat{c}_t}{(1+\gamma^*)(1+n^*)},$$

$$\frac{\widehat{c}_{t+1}}{\widehat{c}_t} = \frac{\beta^{\frac{1}{\sigma}}}{1+\gamma^*}\left[1 + (1-\tau_a^*)\left(\psi\rho\alpha\mathbb{A}^* \widehat{k}_{t+1}^{\rho\alpha-1} - \delta\right)\right]^{\frac{1}{\sigma}}.$$

Note that this autonomous system is essentially the same as the one in the standard neoclassical growth model. Thus, we know that it has a unique non-trivial steady state $\left(\widehat{c}^*, \widehat{k}^*\right)$ which is saddle-path stable. If the subsystem for $\mathbb{Q}_t$ and X_t has a *unique* globally stable BGP, then the long-run growth rate γ^* is unique and the model economy will eventually converge to a unique long-run balanced-growth equilibrium. Our next proposition provides a set of conditions under which the subsystem for $\mathbb{Q}_t$ and X_t has a unique globally stable BGP.

Proposition 3 *Suppose Assumption S is satisfied. Suppose* $\varrho < \vartheta$, $\Im(\mathbb{Q}_t) \equiv \overline{\Im}\mathbb{Q}_t^{\frac{\vartheta\psi}{1-\psi}}$, *and* $\xi(\overline{q}_t) \equiv \overline{\xi}\overline{q}_t^{-\varphi}$, *where* $\overline{\Im} > 0$ *and* $\overline{\xi} > 0$ *are two arbitrary constants. Then the subsystem for* $\mathbb{Q}_t$ *and* X_t *has a unique globally stable balanced-growth path.*

Both $\overline{\Im}$ and $\overline{\xi}$ are arbitrary constants that have no significance for our subsequent analyses, hence we

set them equal to one. Along the unique globally stable BGP, we have

$$\frac{X_{t+1}}{X_t} = 1 + \Xi(\varpi^*)\,\widetilde{\mathbb{X}}^*, \tag{44}$$

$$\frac{\mathbb{Q}_{t+1}}{\mathbb{Q}_t} = \left[1 + \left(\lambda^{\frac{\psi}{1-\psi}} - 1\right)(1-\varpi^*)\,\widetilde{\mathbb{X}}^*\right]^{\frac{1-\psi}{\psi}}, \tag{45}$$

where $\widetilde{\mathbb{X}}^*$ is the long-run stationary value of the transformed variable $\widetilde{\mathbb{X}}_t \equiv \overline{z}_t^{\varphi} N_{H,t} X_t^{\varrho} \mathbb{Q}_t^{-\frac{\vartheta\psi}{1-\psi}}$. In the proof of Proposition 3, we show that this transformed variable plays a key role in characterizing the subsystem for $\mathbb{Q}_t$ and X_t. The value of $\widetilde{\mathbb{X}}^*$ is uniquely determined by

$$\lambda^{\varphi}(1+n^*)\left[1 + \Xi(\varpi^*)\,\widetilde{\mathbb{X}}^*\right]^{\varrho} = \left[1 + \left(\lambda^{\frac{\psi}{1-\psi}} - 1\right)(1-\varpi^*)\,\widetilde{\mathbb{X}}^*\right]^{\vartheta}. \tag{46}$$

Substituting (44)-(46) into (39) gives

$$\frac{\Psi_{t+1}}{\Psi_t} = 1 + \gamma^* = (1+n^*)^{\widetilde{a}_1}\lambda^{\widetilde{a}_2}\left[1 + \Xi(\varpi^*)\,\widetilde{\mathbb{X}}^*\right]^{\widetilde{a}_3}, \tag{47}$$

with

$$\widetilde{a}_1 = \frac{(1-\psi)\,\alpha}{(1-\rho\alpha)\,\psi\vartheta} > 0, \quad \widetilde{a}_2 = \varphi\widetilde{a}_1 > 0, \quad \text{and} \quad \widetilde{a}_3 = \frac{\vartheta\eta\psi(1+\alpha) + (1-\psi)\,\alpha\varrho}{(1-\rho\alpha)\,\psi\vartheta} > 0.$$

In a balanced-growth equilibrium, all per-capita variables will grow at the same constant rate γ^* in every period, and all major aggregate variables, such as $\left\{Y_t, C_t, \mathbb{E}_t, G_{x,t}, \overline{K}_t\right\}$, will grow by the same constant factor $(1+n^*)(1+\gamma^*)$. What remains is to ensure that the transversality condition in the household's problem is satisfied. This can be achieved by imposing some mild additional conditions.[40] Our existence and uniqueness results are summarized in Proposition 4.

Proposition 4 *Suppose the conditions in Proposition 3 are satisfied. In addition, suppose* $\beta(1+n^*) < 1$ *and* $\sigma \geq 1$. *Then there exists a unique balanced-growth equilibrium in which all per-capita variables grow at the same rate* $\gamma^* > 0$ *in every period.*

[40] The transversality condition is satisfied if $\beta(1+\gamma^*)^{1-\sigma}(1+n^*) < 1$. This condition also ensures that household's lifetime utility is finite in the long-run equilibrium. Since $\gamma^* > 0$, this condition is satisfied if $\beta(1+n^*) < 1$ and $\sigma \geq 1$. The assumption $\beta(1+n^*) < 1$ is commonly used in models with population growth. The assumption $\sigma \geq 1$ is commonly used in quantitative macroeconomic studies.

4.3.2 Determinants of Long-Term Economic Growth

As we mentioned earlier, long-term economic growth in this model is driven by the accumulation of fundamental knowledge and the continuous improvement of input quality. These two processes are determined by factors such as the long-term population growth rate (n^*), the share of high-skilled workers participating in basic research (ϖ^*), and the size of quality improvement (λ). The effects of these factors are made clear in the following proposition.

Proposition 5 *Suppose the conditions in Proposition 4 are satisfied. Then the following results hold.*

(i) An increase in n^* will increase the long-term growth rate of $\mathbb{Q}_t$ and X_t.

(ii) An increase in ϖ^* will increase the long-term growth rate of $\mathbb{Q}_t$ and X_t.

(iii) An increase in λ has an ambiguous effect on the long-term growth rate of $\mathbb{Q}_t$ and X_t.

The intuitions behind these results are as follows. First, an increase in n^* means that the total supply of researchers will grow at a faster rate in the long run. This increases the equilibrium quantity of labor input in both basic and non-basic research, and thereby raises the growth rate of fundamental knowledge and input quality. Second, by recruiting a larger fraction of researchers into basic research, the government can support a wider range of research projects and promote the accumulation of fundamental knowledge. This effect is captured by the strictly increasing function $\Xi(\varpi^*)$. Such an increase also means that fewer high-skilled workers will be available for quality improvement research. Holding other things constant, this lowers the long-term growth rate of $\mathbb{Q}_t$ relative to X_t and raises the value of $\widetilde{\mathbb{X}}^*$. The increase in both $\Xi(\varpi^*)$ and $\widetilde{\mathbb{X}}^*$ then leads to a higher long-term growth rate of X_t. As for quality improvement research, our result suggests that the increase in $\widetilde{\mathbb{X}}^*$ will counteract the decline in $(1-\varpi^*)$ so that an increase in ϖ^* will also raise the long-term growth rate of $\mathbb{Q}_t$. Finally, an increase in λ will generate two opposing effects on $\widetilde{\mathbb{X}}^*$. On one hand, such an increase will speed up the long-term growth rate of $\mathbb{Q}_t$ (relative to that of X_t) and lead to a reduction in $\widetilde{\mathbb{X}}^*$. On the other hand, an increase in λ means that the frontiers of science and technology (as captured by $\overline{z}_t$ and $\overline{q}_t$) will advance at a faster pace. This will in turn raise the value of $\widetilde{\mathbb{X}}^*$. Thus, the overall effect on $\widetilde{\mathbb{X}}^*$ is ambiguous. The long-run growth effects of n^* and ϖ^* are summarized in Corollary 6, which follows immediately from Proposition 5.

Corollary 6 *Suppose the conditions in Proposition 4 are satisfied. Then the long-term growth rate γ^* depends positively on the long-term population growth rate (n^*) and the share of high-skilled workers participating in basic research (ϖ^*).*

Our model has three other important predictions regarding long-term economic growth. The first one concerns the so-called *scale effect* (i.e., the effect of population size on long-term economic growth). Unlike the models of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), our model predicts that long-term economic growth is independent of the size of population. Similar to the models of Jones (1995, 2002), Segerstrom (1998), Peretto (1998), Howitt (1999) and Jones and Williams (2000), our model predicts that long-term economic growth depends positively on the population growth rate. This, however, does not mean that population growth is indispensable for economic growth. Our next proposition shows that perpetual growth in per-capita variables is still possible even when there is no population growth in the long run.[41] In this case, perpetual growth is sustained by the continuous advancement of the frontiers of science and technology.

Proposition 7 *Suppose the conditions in Proposition 4 are satisfied. Then the long-term growth rate* γ^* *is strictly positive even when there is no population growth in the long run, i.e.,* $n^* = 0$.

Second, our model suggests that neither corporate income tax $\left(\tau_p^*\right)$ nor R&D subsidy (τ_d^*) has any effect on the long-term growth of $\mathbb{Q}_t$ and X_t. Consequently, providing tax incentives for private R&D has no effect on long-term economic growth. Other theoretical studies, such as Jones (1995), Segerstrom (1998) and Young (1998), also make the same prediction.

Third, our model predicts that both the patent-researcher ratio and the patent-R&D ratio are decreasing over time in the long-run balanced-growth equilibrium. This follows from the facts that both the number of researchers and the amount of total R&D spending are growing at some positive rates in the long-run equilibrium, while the total number of patents granted in each period [given by $\sum_{q\in\mathcal{Q}} f_t(q)\,\widehat{\Phi}_t(q) = (1-\varpi^*)\,\widetilde{\mathbb{X}}^*$] is time-invariant. Hence, the patent-researcher ratio and the patent-R&D ratio are both decreasing over time. This result is consistent with the empirical evidence reported in Kortum (1993) and Segerstrom (1998).

5 Calibration

The main purpose of the numerical analysis is to quantify the effects of R&D subsidies and government basic research spending based on the model developed above. To this end, we first construct a parameterized version of the model and show that it is able to replicate certain features of the U.S. economy over the period 1953-2009. Our benchmark economy is constructed as follows. First, the model is calibrated

[41] This is not true in the models of Jones (1995, 2002), Segerstrom (1998), Howitt (1999) and Jones and Williams (2000).

so that it matches the actual capital-output ratio and R&D investment rate in 1953. Then, we input the actual time series data on employment growth rates and various policy variables over the period 1953-2009, and solve for the equilibrium time paths of all major economic variables. Finally, we impose the assumption that all model parameters are constant from 2009 onward, and compute the unique long-run equilibrium.[42] Since the length of a period in the model is one year and the initial period is 1953, we set $T^* = 56$. Our benchmark model is able to generate patterns of R&D investment and economic growth that are similar to those observed in the United States. We then perform a number of counterfactual experiments and welfare analyses in order to determine the effects of research policies. In this section, we explain the rationale behind our choices of the baseline parameter values. All the numerical results are presented in Section 6.

In the benchmark scenario, all the parameters listed in Table 2 are assumed to be time-invariant. Most of these parameter values are chosen based on actual data or existing empirical estimates. Others are calibrated so that the model can match a number of real-world statistics. The details of this procedure are explained below.

5.1 Parameter Values Based on Empirical Evidence

Households In the present study, high-skilled workers correspond to those researchers who are engaged in government-financed basic research and non-basic research performed in the private sector. Thus, the values of $\{n_{H,t}, \theta_t\}_{t=0}^{T^*}$ should match the annual growth rates of the employment of these researchers, and their share in total U.S. employment over the period 1953-2009. Existing data on these values, however, are very limited.[43] Thus, we have to develop our own proxies for these values. The details of this procedure are described in Appendix C. Regarding the share of researchers in total employment, our estimates show that this share has increased from 0.79 percent in 1953 to 2.97 percent in 2009. Since high-skilled workers only account for a small fraction of total U.S. employment, we equate the values of $\{n_{L,t}\}_{t=0}^{T^*}$ to the annual growth rates of total U.S. employment. Our estimates also show that most of the researchers in the United States are employed in the private sector. Hence, we equate the values of $\{n_{H,t}\}_{t=0}^{T^*}$ to the growth rates of employment of R&D scientists and engineers in the private sector. These two sets of values thus capture the employment growth patterns depicted in Figure 6. The long-run population growth rate (n^*) is set

[42] This computation procedure is similar to the one adopted in Chen *et al.* (2006).

[43] There are two major limitations in existing data sources. First, consistent time series data on the number of researchers employed in different sectors (industry, academia and government) are not available before the 1990s. Second, the number of researchers engaged in different types of research (basic, applied and development) are also not available in the existing data sets.

to 1.50 percent, which matches the average annual growth rate of total employment between 1953 and 2009. Once we impose the restriction $n_{H,t} = n_{L,t} = n^*$ for all $t \geq T^*$, the population share of high-skilled workers (θ_t) will remain constant at its 2009 value which is 2.97 percent (based on our estimates).

As for household preferences, the inverse of elasticity of intertemporal substitution (σ) is set equal to two. This falls within the range of estimates obtained by several empirical studies.[44] It also falls within the range of values that are commonly used in quantitative studies (which is between one and two). The value of β is determined by the calibration procedure described in Section 5.2.

Production Four parameters appear in the production functions of final good and intermediate goods, namely $\{\alpha, \psi, \eta, \rho\}$. Given the Cobb-Douglas specification in (1), the parameter α represents the ratio between expenditures on intermediate inputs and the value of final goods. For the period 1987-2009, the average value of this ratio is 0.7348 in the U.S. data.[45] Hence, we set $\alpha = 0.7348$. As shown in (13), the parameter ψ captures the extent of markup over the marginal cost of producing intermediate inputs. In the baseline parameterization, we set $\psi = 0.9091$ which implies a ten percent markup over the marginal cost. This value falls within the range of estimates obtained by Norrbin (1993) and Basu and Fernald (1997).[46] The value of η corresponds to the elasticity of TFP with respect to public R&D capital. Using (4) and (12), we can show that $-\eta$ is also the elasticity of unit cost (κ_t) with respect to public R&D capital. Empirical estimates of these elasticities are relatively scarce. Nadiri and Mamuneas (1994) find that the cost elasticities of public R&D capital for twelve U.S. industries range from -0.009 to -0.056. In a recent study, Luintel and Khan (2011) estimate that the elasticity of TFP with respect to basic knowledge stock is 0.149. In the baseline parameterization, we set $\eta = 0.10$ which falls between these estimates. In the sensitivity analysis, we will examine the effects of changing ψ and η on the benchmark results. The value of ρ is chosen based on the calibration procedure described in Section 5.2.

[44] Using time series data on aggregate consumption in the United States, Atkeson and Ogaki (1996) estimate that the elasticity of intertemporal substitution (EIS) is about 0.40, which implies $\sigma = 2.50$. Using data from the Consumption Expenditure Survey, Vissing-Jørgensen (2002) estimates that the EIS for bondholders is about 0.80 (which means $\sigma = 1.25$) and that for stockholders is about 0.40, whereas Parker and Preston (2005) estimate that the EIS for U.S. households in general is about 0.61 (which means $\sigma = 1.64$). Our choice of σ thus falls within this range of estimates.

[45] To compute this value, we first collect annual data on intermediate input expenditures from the GDP-by-industry accounts available on the Bureau of Economic Analysis (BEA) website. We then compute the ratio between these expenditures and U.S. GDP for each year in the period 1987-2009. The average value over this period is 0.7348. Data on intermediate input expenditures for all industries are not available prior 1987.

[46] Using data on twenty-one U.S. industries over the period 1950-1984, Norrbin (1993) estimates that the average markup in these industries is about five to fifteen percent. Using data over a similar time period, Basu and Fernald (1997) estimate that the returns to scale of production in the U.S. manufacturing sector is about 1.09 [see their Table 1 Panel (A)]. Based on a pure profit rate of three percent, this implies a 12.4 percent markup in the manufacturing sector. For more discussion on this calculation, see Basu and Fernald (1997, p.253).

Quality Improvement Research Two parameters are involved in the quality improvement research process. The first one, ϑ, captures the degree of diminishing returns in private R&D. More specifically, this parameter can be interpreted as the elasticity of private R&D output with respect to the firm's own input in the research process. The second parameter, ϱ, captures the strength of knowledge spillovers from basic research to quality improvement research. This parameter can be interpreted as the elasticity of private R&D output with respect to public R&D capital. In the existing empirical literature, private R&D output is typically measured by the number of patents generated by individual firms within a certain time period. Hall *et al.* (1986) estimate that the long-run elasticity of corporate patent with respect to firms' own R&D investment is about 0.39 to 0.66, depending on the choice of estimation model. Blundell *et al.* (2002) report an estimate of about 0.50 for the same long-run elasticity. Jaffe (1989) examines the effects of both industrial R&D spending and university R&D spending on corporate patents. For industrial R&D spending, the estimated elasticity ranges from 0.6 to 0.9. For university R&D spending, the estimated elasticity ranges from 0.04 to 0.28. Based on these findings, we use $\vartheta = 0.50$ and $\varrho = 0.20$ as our benchmark values.

Government Policies When constructing the benchmark economy, we take into account changes in a number of policy variables. These include the tax rates on incomes and consumption, $\{\tau_{w,t}, \tau_{a,t}, \tau_{c,t}, \tau_{p,t}\}_{t=0}^{T^*}$, and the policy instruments that are related to research activities, $\{\tau_{d,t}, \varpi_t, \epsilon_t\}_{t=0}^{T^*}$. For the tax rates on labor income ($\tau_{w,t}$), we use the time series of overall marginal income tax rate reported in Barro and Redlick (2011, Table 1). This overall tax rate is the sum of federal income tax, state income tax and Social Security tax in the United States. For the tax rate on interest income ($\tau_{a,t}$), we use the same time series but exclude the Social Security tax which is a payroll tax. Since the data reported in Barro and Redlick (2011) only cover up to the year 2006, we assume that these two tax rates are constant from 2006 onward. The long-run tax rates τ_w^* and τ_a^* are 35.3 percent and 26.0 percent, respectively. The tax rates on private consumption expenditures ($\tau_{c,t}$) are approximated using data from the National Income and Product Accounts.[47] The long-run tax rate τ_c^* is 4.29 percent, which is the value in 2009. The corporate income tax rates $\{\tau_{p,t}\}_{t=0}^{T^*}$ are chosen based on the statutory tax brackets and rates listed in Statistics of Income (SOI) Historical Table 24. These values are depicted in Figure 7. The U.S. corporate income tax rate has been declining throughout the sample period. The most significant development is the reduction

[47] First, we obtain annual data on total sales taxes collected by state and local governments over the period 1953-2009. Then, we compute the ratio between these tax revenues and total private consumption expenditures for each year in this period. The resulting values are used as our proxies for the consumption tax rates.

in 1987 brought by the Tax Reform Act of 1986. The long-run tax rate τ_p^* is set equal to 35 percent, which is the tax rate for the highest corporate income tax bracket in 2009.

As for R&D policies, we assume that government-employed basic researchers receive the same wage rate as private-sector researchers, so that $\epsilon_t = 1$ for all $t \geq 0$. Under this assumption, ϖ_t coincides with the share of government basic research spending in our measure of total R&D expenditures. Thus, we use the actual time series data on this share as the values of $\{\varpi_t\}_{t=0}^{T^*}$. The inputted values are shown in Figure 8. The long-run value ϖ^* is 13.86 percent, which is the value in 2009. Finally, the values of $\{\tau_{d,t}\}_{t=0}^{T^*}$ are computed using the method described in Bloom *et al.* (2002). The details of this procedure are reported in Appendix C. The time series of the *combined* subsidy rate for private R&D (i.e., the sum of $\tau_{p,t}$ and $\tau_{d,t}$) is shown in Figure 7. Note that there are two major changes in R&D tax credit, one in 1981 and the other in 1990. The long-run R&D tax credit (τ_d^*) is 12.22 percent, which equals the value in the last year of our sample.

5.2 Calibrated Parameters

Seven parameters remain undetermined up to this point. These include household's subjective discount factor β, the depreciation rate of physical capital δ, the size of quality improvement λ, one parameter in the production function of intermediate goods ρ, one parameter in the basic research process φ, the initial value of physical capital per effective unit of labor $\widehat{k}_0$, and the initial value of public R&D capital X_0. The first five parameters are chosen so that the model's unique long-run balanced-growth equilibrium matches five long-run statistics. The two initial values are chosen (under the calibrated values of the first five parameters) so that the benchmark economy can match two statistics in 1953. These targeted statistics are summarized in Table 3.

We begin with the five target statistics for the long-run equilibrium. The first target is the average annual growth rate of real per-worker GDP in the United States over the period 1953-2009, which is 1.56 percent. The second target is the share of labor income in U.S. GDP over the same time period, which is 0.5342.[48] The third target is the before-tax rate of return for physical capital. Using the procedure described in Cooley and Prescott (1995), we find that the average rate of return over the period 1953-2009

[48] This value is computed using data on compensation of employees and proprietors' income over the period 1953-2009. We assume that the share of labor income in proprietors' income is identical to that of the entire economy, so that

$$\text{Labor's share of income} = \frac{\text{Compensation of Employees}}{\text{GDP - Proprietors' Income}}.$$

The same procedure is also used in Gomme and Rupert (2007, Section 4.2).

is 8 percent (see Appendix C for more details). The fourth target is the average value of the capital-output ratio over the period 1953-2009, which is 2.4272. To compute this value, we use the sum of private fixed assets and the end-of-year stock of private inventories as our measure of aggregate physical capital. Our fifth target is intended to capture the long-run level of R&D investment rate in the United States. As shown in Figure 1, R&D investment rate has been fluctuating significantly during the 1960s and the 1970s, but has been stabilized since 1983. For this reason, we use the average value of R&D investment rate over the period 1983-2009 as our target statistic. The value is 2.06 percent.[49]

After the value of $\{\beta, \delta, \lambda, \rho, \varphi\}$ are determined, we then choose the value of $\widehat{k}_0$ and X_0 so that the model can match (i) the capital-output ratio in 1953, which is 2.4097, and (ii) the R&D investment rate in 1953, which is 1.0 percent.

6 Numerical Results

6.1 Benchmark Economy

Major Economic Variables Table 3 summarizes the main properties of the long-run equilibrium in our benchmark model. Apart from the five targeted statistics, our model is also able to match quite closely two other statistics that are not targeted. The first one is the average value of the consumption-output ratio in the United States over the period 1953-2009. The second one is the average value of the ratio of total tax receipts to U.S. GDP.[50] The last result shows that the size of government in our model is similar to its real-world counterpart. Table 4 shows the predicted values of four "great ratios" over the period 1953-2009. Overall, our benchmark model is able to generate reasonable values for these ratios. The model, however, cannot capture the secular movements of the capital-output ratio and the downward trend of labor income share.

Patterns of R&D Investment Rate Figure 9 shows the time series of R&D investment rate generated by our benchmark model along with the actual data. In the year 1953, total R&D spending accounted for about one percent of GDP in the United States. This increased sharply during the subsequent years, peaked at 2.20 percent in 1963, and declined during the second half of the 1960s. Our model is able to

[49] For the period 1983-2009, the average annual growth rate of real GDP per-worker is 1.60 percent, the share of labor income in U.S. GDP is 0.5050, the average capital-output ratio is 2.4235, and the average (before-tax) rate of return from physical capital is again 8 percent. All these values are quite close to their counterparts for the period 1953-2009. Thus, our fifth target is not inconsistent with the first four targets.

[50] Total tax receipts is defined as the sum of revenues collected from (i) personal income taxes, (ii) taxes on production and imports, (iii) taxes on corporate incomes, and (iv) contributions for government social insurance. These data are obtained from the National Income and Product Accounts Table 3.1.

mimic this hump-shaped pattern, though the peak magnitude of the model-generated series is lower than that in the data. In our benchmark model, R&D investment rate increases from 0.93 percent to 1.97 percent over the period 1953-1963. Thus, the model is able to explain about 86 percent of the observed increase in R&D investment rate during this period. Our benchmark model is also able to replicate the U-shaped pattern over the period 1969-1985.

The sharp fluctuations in the model-generated series during 1981-1990 are mainly driven by the changes in the combined subsidy rate for private R&D. For instance, the sharp increase in 1981 and 1990 are driven by the increase in R&D tax credit in these two years. Similarly, the abrupt fall in 1987 is due to the reduction in corporate income tax which lowers the combined subsidy rate. The effects of these policies will be further explored in Section 6.2.

Growth Accounting Between 1953 and 2009, real per-worker GDP in the United States has increased by a factor of 2.384, which is equivalent to an average annual growth rate of 1.56 percent. In our benchmark model, real per-worker output has increased by a factor of 2.473 during the transition period. This implies an average annual growth rate of 1.63 percent. Using the standard growth accounting technique, we can decompose this growth rate into (i) TFP growth and (ii) the growth of physical capital per worker. Formally, let $y_t \equiv Y_t/N_t$ and $k_t \equiv \overline{K}_t/N_t$ denote per-worker output and per-worker capital at time t. Using the equations in Section 4.2, we can express the growth of y_t as

$$\frac{y_{t+1}}{y_t} = \underbrace{\left[\frac{\mathbb{A}_{t+1}}{\mathbb{A}_t}\left(\frac{\Psi_{t+1}}{\Psi_t}\right)^{1-\rho\alpha}\right]}_{\text{TFP Growth}} \times \underbrace{\left(\frac{k_{t+1}}{k_t}\right)^{\rho\alpha}}_{\text{Capital Deepening}} .$$

The first component in the above expression mainly captures the contribution of TFP growth, and the second component captures the contribution of capital deepening.[51] The value of $\rho\alpha$ is 0.4615 under the baseline parameterization. Using equation (39), we can further decompose TFP growth into two components: (i) growth due to fundamental knowledge accumulation, and (ii) growth due to quality improvement. The results of this exercise are summarized in the first column of Table 5. Between 1953 and 2009, the average annual growth rate of TFP in the benchmark model is 0.94 percent. This accounts for about 58 percent of the average annual growth rate of per-worker output in the model. The remaining 42 percent is due to capital deepening. These results are consistent with the empirical findings

[51] Changes in $\mathbb{A}_t$ are entirely driven by the changes in the employment share of high-skilled workers (θ_t). For the period 1953-2009, the value of $\mathbb{A}_t$ has only reduced by 1.2 percent, while the value of Ψ_t has increased by 270 percent. Thus, the growth of the first component is essentially driven by the growth of Ψ_t.

in Aghion and Howitt (2007, Table 1). In our benchmark model, 56 percent of TFP growth is due to the accumulation of fundamental knowledge. The remaining 44 percent is due to quality improvement.

6.2 Counterfactual Experiments

In the benchmark model, we have taken into account changes in several exogenous factors, such as (i) corporate income tax, (ii) R&D tax credit, (iii) the share of government basic research spending in total R&D expenditures, and (iv) the growth rates of high-skilled and low-skilled employment. In this subsection, we examine the significance of each of these factors in explaining R&D investment and economic growth over the period 1953-2009. This is achieved by considering a series of counterfactual experiments. In the first experiment, the corporate income tax rate is fixed at its 1953 value, so that $\tau_{p,t} = 0.52$, for all $t \geq 0$. All other parameters are the same as in the benchmark economy. In the second experiment, the R&D subsidy rate is fixed at its 1953 value, so that $\tau_{d,t} = 0.0007$, for all $t \geq 0$. The time paths of the combined subsidy rate in these two experiments are shown in Figure 10. In the third experiment, the share of government basic research spending in total R&D expenditures is fixed at its 1953 value, so that $\varpi_t = 0.0725$, for all $t \geq 0$. In the fourth experiment, we assume that high-skilled and low-skilled employment have been growing at the same rate since 1953. As a result, the employment share of high-skilled workers has remained constant since 1953, i.e., $\theta_t = 0.0079$, for all $t \geq 0$. We have also performed three other experiments in which the tax rates on consumption, labor income and capital income are fixed at their 1953 values. In terms of R&D investment and economic growth, these counterfactual scenarios produce essentially the same results as the benchmark model.[52] Hence, we do not report these results here.

Figure 11 shows the R&D investment rates generated by the economy in Experiment 1, along with the benchmark results and the actual data. Fixing the corporate income tax rate at 52 percent has two opposing effects on R&D investment. On one hand, higher tax rates lower the (after-tax) profits that can be extracted from an improved product, and thereby lower the firm's incentive to innovate. Holding other factors constant, this will lower the R&D investment rate in every period. On the other hand, the combined subsidy rates after 1986 are now substantially higher than those in the benchmark model (see Figure 10). This induces a strong increase in R&D investment from 1986 onward which more than offsets

[52] In the time-series data that we inputted into the model, the capital income tax rates have not changed much over time. Thus, fixing this tax rate at its 1953 value has very little impact on our benchmark results. However, this does not mean that capital income tax is unimportant in our model. As we will see in Section 6.4, large changes in the long-run value of this tax rate will have significant impact on the long-run interest rate, which will in turn affect private R&D investment and consumer welfare.

the reduction brought by the first effect. Thus, for the period 1986-2009, the R&D investment rates in Experiment 1 are actually higher than in the benchmark model. Note that without the tax cut in 1987, the model no longer predicts an abrupt fall in R&D investment rate in this year.

Figure 12 shows the R&D investment rates generated by the economy in Experiment 2. For the period prior 1981, the combined subsidy rates in this experiment are the same as in the benchmark model. Thus, the two sets of results coincide before 1981. For the period 1981-2009, the combined subsidy rates in Experiment 2 are lower than in the benchmark model. As a result, the R&D investment rates in Experiment 2 are also lower than in the benchmark model over this time period. Note that without the increase in R&D tax credit in 1981 and 1990, the model no longer predicts a sharp increase in R&D investment rate in these two years.

While changes in corporate income tax and R&D subsidy have direct impact on the firms' incentive to innovate and hence their demand for high-skilled workers, these policy changes have no effect on the supply of these workers. In the current framework, the equilibrium quantity of labor input in private R&D is determined by $(1-\varpi_t) N_{H,t}$ in every period t. The allocation of this input across firms is determined by the function $\Lambda_{M,t}(\cdot)$ as specified in Proposition 2. Neither $(1-\varpi_t) N_{H,t}$ nor $\Lambda_{M,t}(\cdot)$ is affected by the corporate income tax rate and the R&D subsidy rate. Consequently, these policy instruments have no effect on the equilibrium dynamics of X_t and $\mathbb{Q}_t$, and no effect on economic growth in both the short run and the long run. These findings are summarized in the second and third columns of Table 5. This also means that the results in Figures 11 and 12 are merely reflecting the effects of these policy instruments on the wage rate of high-skilled workers.

In the third experiment, the share of government basic research spending in total R&D expenditures (ϖ_t) is fixed at its 1953 value. As shown in Figure 8, this share has increased substantially since 1960. Thus, for most part of the transition period, the economy in Experiment 3 has a much smaller scale of basic research than in the benchmark economy. As a result of this, the average growth rate of X_t in Experiment 3 is about 44 percent lower than in the benchmark model (see the fourth column of Table 5). This in turn reduces the growth rates of TFP and per-worker output by 25.5 percent and 14.7 percent, respectively. Figure 13 summarizes the changes in the *level* of private R&D spending during the transition period. To better understand these changes, we have performed two additional steps: First, we have separated the

changes in private R&D spending into *quantity changes* and *price changes* according to

$$\text{Private R\&D Spending} = \underbrace{(1-\varpi_t)\,N_{H,t}}_{\text{Quantity}} \cdot \underbrace{w_{H,t}}_{\text{Price}}.$$

Second, we have expressed all the values in Figure 13 as ratios to the benchmark results.[53]

Since the values of $N_{H,t}$ in this experiment are the same as in the benchmark model, the quantity changes depicted in Figure 13 are entirely due to changes in $(1-\varpi_t)$. A reduction in ϖ_t means that a larger fraction of high-skilled workers are now allocated to quality improvement research. Thus, holding other things constant, these quantity changes will lead to higher levels of private R&D spending and lower growth rates of fundamental knowledge. The effects on $w_{H,t}$ are threefold: First, a reduction in ϖ_t increases the supply of high-skilled workers in the private sector, which pushes down the equilibrium wage rate. Second, since fundamental knowledge is conducive to the efficiency of private R&D, a reduction in the growth rate of X_t will lower the net gain from an improved product, and thereby lower the firms' demand for high-skilled workers. Finally, a reduction in ϖ_t will also affect private R&D through the equilibrium interest rate. This happens because fundamental knowledge is also conducive to firm productivity. Thus, a slowdown in the accumulation of fundamental knowledge will have a negative impact on the marginal product of physical capital. For this reason, the equilibrium interest rates in Experiment 3 are much lower than in the benchmark model (see Figure 14). From an innovating firm's perspective, this means the profit stream generated by an improved product is now discounted at lower rates. This leads to a higher net gain from quality improvement research, and thereby encourages the demand for researchers. The third effect is the reason why the values of $w_{H,t}$ in Experiment 3 are higher than in the benchmark model during the early stage of the transition period. Eventually, the first two effects dominate and depress the value of $w_{H,t}$.

In the fourth experiment, we assume that high-skilled and low-skilled employment have been growing at the same rate since 1953. As suggested by Figure 6, this means the economy in Experiment 4 has a much smaller number of researchers than in the benchmark economy. As shown in the fifth column of Table 5, the reduction in high-skilled labor has a detrimental impact on research activities and technological progress. Specifically, the average growth rates of X_t and $\mathbb{Q}_t$ in Experiment 4 are 52.7 percent and 54.4

[53] In Experiments 1 and 2, changes in $\tau_{p,t}$ and $\tau_{d,t}$ have no effect on the equilibrium time path of aggregate output. In other words, these changes do not affect the denominator of R&D investment rate. Hence, it makes little difference whether we report the R&D investment rates or the levels of R&D spending in Figures 11 and 12. The changes that we considered in Experiments 3 and 4, however, will change both R&D spending and aggregate output. The effects on aggregate output are already summarized in Table 5, so we choose to focus on the level of R&D spending in Figures 13 and 15.

percent lower than in the benchmark model. These together reduce the growth rates of TFP and per-worker output by 45.7 percent and 33.1 percent, respectively. Figure 15 shows that reducing the number of researchers will have a strong positive impact on their wage rate. This is mainly due to a combination of reduced supply and lower values of interest rate (see Figure 14). These price changes partly offset the reduction in private R&D spending brought by the quantity changes.

6.3 Long-run Equilibrium

In this subsection, we focus on the long-run balanced-growth equilibrium of our model and perform two sets of comparative statics exercises. The first exercise is intended to quantify the long-run effects of government basic research spending. This is achieved by computing a series of balanced-growth equilibria under different values of ϖ^* ranging from zero to 0.20. In each of these equilibria, the parameters $\{\rho, \beta, \delta\}$ are recalibrated so that three of the targeted statistics (the share of labor income in total output, real interest rate and capital-output ratio) are the same as in the benchmark model. All other parameters are fixed at their benchmark values. In the second exercise, we examine how changes in $\{\eta, \varrho, \psi\}$ would affect the benchmark results. To achieve this, we compute a series of long-run equilibria, each involves changing the value of one of these parameters. The values of $\{\rho, \beta, \delta\}$ are again recalibrated in each case. All other parameters (including ϖ^*) are fixed at their benchmark values.

The results of the first exercise are shown in Figures 16a and 16b. The main message of our findings is clear: By expanding the scale of publicly financed basic research, the government can effectively promote total R&D investment and long-term economic growth. For instance, increasing ϖ^* from its benchmark value (13.86 percent) to 20 percent will raise the R&D investment rate from 2.08 percent to 2.52 percent and the long-run growth rate from 1.56 percent to 2.31 percent. On the other hand, if we completely eliminate government-financed basic research (by setting $\varpi^* = 0$), then the long-run growth rate will drop to 0.53 percent. This result suggests that about two-thirds of the long-term economic growth in our benchmark model is originated from basic research. Figure 16b shows that private R&D investment also increases as ϖ^* increases. This result, however, depends crucially on our treatment of the long-run interest rate. If the long-run interest rate is allowed to vary, then expanding the scale of government-financed basic research will lower private R&D investment. This happens because an increase in ϖ^* will drive up the long-run interest rate. For reasons explained earlier, such an increase will lower the equilibrium wage rate for high-skilled workers and thereby lower private R&D spending.

The results of the second exercise are reported in Table 6. First, changing the value of η (which is

the elasticity of TFP with respect to fundamental knowledge) has no effect on the growth rates of X_t and $\mathbb{Q}_t$. This happens because this parameter does not appear in equations (26) and (41) which capture the basic and non-basic research processes. Increasing this parameter value, however, will enhance the benefits of fundamental knowledge on firm productivity. Thus, holding other things constant, a higher value of η will increase TFP growth and long-term economic growth. Second, increasing the value of ϱ will enhance the benefits of fundamental knowledge on basic and non-basic research. This has the effect of encouraging private R&D investment and stimulating the growth of X_t and $\mathbb{Q}_t$. Overall, the results in Table 6 suggest that our benchmark model is not very sensitive to changes in η and ϱ.

Finally, we consider the effects of changing the value of ψ. This parameter has two roles in our model. First, it determines the elasticity of substitution between any two types of intermediate inputs in the production of final goods. Second, it determines the extent of markup over the cost of producing intermediate goods. In the baseline parameterization, ψ is set equal to 0.9091 which implies a ten percent markup rate. In the comparative statics exercise, we consider two alternative values: $\psi = 0.9524$ which implies a markup rate of five percent, and $\psi = 0.8696$ which implies a markup rate of fifteen percent. Reducing the value of ψ has two main effects. First, by increasing the markup rate and the monopoly profits from an improved product, such a reduction will encourage private R&D investment. Second, reducing the value of ψ will increase the complementarity between different types of intermediate inputs. Holding other factors constant, this will induce a redistribution of high-skilled labor from high-quality firms to low-quality ones.[54] Such a redistribution will promote the growth rate of $\mathbb{Q}_t$, which will in turn lead to a faster growth rate of TFP and per-worker output. Because of these two effects, our benchmark model is rather sensitive to changes in ψ.

6.4 Welfare Analysis

We now turn to the effects of government basic research spending on consumer welfare. Suppose the model economy is initially in the benchmark long-run balanced-growth equilibrium with $\varpi_0^* = 0.1386$ and $\gamma_0^* = 0.0156$ (the status quo). Suppose now the government wants to achieve a long-term growth target $\gamma_1^* > \gamma_0^*$ by permanently increasing the scale of basic research from ϖ_0^* to ϖ_1^*. After the new policy is announced and implemented, the economy gradually converges to a new long-run balanced-growth

[54] As shown in Proposition 2, the allocation of high-skilled labor across firms is determined by $\Lambda_{M,t}(q) = (1-\varpi_t) N_{H,t} (q/\mathbb{Q}_t)^{\frac{\psi}{1-\psi}}$. Note that the value of $\psi/(1-\psi)$ decreases as ψ decreases. For the values of ψ that we have considered, $\psi/(1-\psi) > 1$. Thus, decreasing the value of ψ will raise the value of $\Lambda_{M,t}(q)$ for $q < \mathbb{Q}_t$ and reduce the value of $\Lambda_{M,t}(q)$ for $q > \mathbb{Q}_t$. In other words, high-skilled workers are redistributed from high-quality firms to low-quality ones.

equilibrium with ϖ_1^* and γ_1^*. The task at hand is to quantify the welfare effects induced by this policy change. This is achieved by constructing a consumption-equivalent measure of welfare changes along the line of Lucas (1987). Formally, let $\{c_{s,t}\}_{t=0}^{\infty}$ be the equilibrium consumption sequence for a type-s consumer in the status quo, and let $\{\widetilde{c}_{s,t}(\varpi_1^*)\}_{t=0}^{\infty}$ be the equilibrium consumption sequence under the new policy. Time 0 in here refers to the period when the new policy is announced and implemented. Our welfare measure $\mathbf{z}_s(\varpi_1^*)$ is defined according to

$$\sum_{t=0}^{\infty}\beta^t N_{s,t}U\left[\mathbf{z}_s(\varpi_1^*)\,c_{s,t}\right]=\sum_{t=0}^{\infty}\beta^t N_{s,t}U\left[\widetilde{c}_{s,t}(\varpi_1^*)\right]. \tag{48}$$

The intuition behind this measure is as follows. Suppose the type-s household is better off under the new policy, so that

$$\sum_{t=0}^{\infty}\beta^t N_{s,t}U\left(c_{s,t}\right)<\sum_{t=0}^{\infty}\beta^t N_{s,t}U\left[\widetilde{c}_{s,t}(\varpi_1^*)\right].$$

In this case, the value of $\mathbf{z}_s(\varpi_1^*)$ is greater than one. Equation (48) states that the welfare gain brought by the new policy is equivalent to increasing the status-quo consumption by a factor of $\mathbf{z}_s(\varpi_1^*)>1$ in every period. Conversely, $\mathbf{z}_s(\varpi_1^*)$ is less than one if the type-s household is worse off under the new policy. In this case, each type-s consumer is willing to give up a fraction $[1-\mathbf{z}_s(\varpi_1^*)]$ of his status-quo consumption in every period so as to revert the policy change.

Following our discussion in the previous sections, we know that changes in the value of ϖ^* will induce significant changes in the equilibrium interest rate, which will in turn affect the equilibrium wage rate for high-skilled workers. To highlight the welfare effects of these changes, we compute our welfare measures under two different scenarios. In the first scenario, all parameters except ϖ^* are fixed at their benchmark values. In particular, the employment growth rates and all other exogenous policy variables are assumed to be constant over time and take the same values as in the benchmark model. In the second scenario, the increase in ϖ^* is accompanied by a simultaneous reduction in the capital income tax rate τ_a^* at time $t=0$. The new value of τ_a^* is chosen so that the real interest rate in the new long-run equilibrium is the same as in the benchmark model (i.e., 8 percent). All other parameters are fixed at their benchmark values. The results of these exercises are reported in Table 7. The first two columns of this table show the long-term growth target and the required value of ϖ_1^*. The third, fourth and fifth columns show some properties of the new long-run equilibrium. The last two columns show the consumption-equivalent measures of welfare changes. Panel A reports the results obtained when τ_a^* is fixed at its benchmark value. Panel B reports the results obtained when τ_a^* is adjusted so that the long-run interest rate is maintained at

8 percent. In both scenarios, increasing the value of ϖ^* creates significant welfare gains for both types of consumers. However, the magnitude of these gains (especially for the high-skilled workers) depend crucially on our treatment of the long-run interest rate. In the first scenario, an increase in ϖ^* will drive up the equilibrium real interest rate which affect negatively the wage rate for high-skilled workers. As a result of this, the ratio between $w_{H,t}$ and $w_{L,t}$ declines initially along the transition path. This explains why the welfare gains for high-skilled consumers are slightly less than those for the low-skilled consumers in Panel A. In the second scenario, the negative price changes due to the increase in interest rate are mitigated. As a result, the ratio between $w_{H,t}$ and $w_{L,t}$ is monotonically increasing along the transition path, and the values of $\mathbf{z}_H(\varpi_1^*)$ are much higher than those of $\mathbf{z}_L(\varpi_1^*)$ in Panel B.

7 Concluding Remarks

In this paper, we have developed an endogenous growth model which takes into account both public and private R&D investment, and the differences between basic and non-basic research. In particular, the present study represents one of the few attempts to explore the significance of basic research in a dynamic general equilibrium model. Our results suggest that the accumulation of fundamental knowledge is an important contributing factor to U.S. economic growth. In terms of policy implications, this means government spending on basic research is a powerful policy instrument to promote economic growth. Two other important findings also emerged from the analysis. First, subsidizing private R&D investment (either through tax exemption or tax credit) has no effect on economic growth in both the short run and the long run. Second, the rapid increase in the number of scientists and engineers in the past several decades has a major contribution to U.S. economic growth. Taken together, these two results suggest that it is more important to stimulate the supply of researchers than to stimulate the demand for them. Since the supply of researchers is often affected by other government policies (such as immigration policies and education policies), our results suggest that these policies may also be important in explaining R&D investment and economic growth. One direction of future research is to examine these issues using the model developed in this study.

Table 1 Distribution of S&E Articles Published by U.S. Institutions (%)

	1995		2000		2005	
Government	8.9	(9.1)	8.1	(8.8)	7.5	(8.0)
Industry	8.1	(18.8)	7.3	(16.5)	6.4	(14.5)
Academia	71.6	(51.2)	72.8	(53.6)	74.6	(57.0)
FFRDCs	2.8	(11.2)	2.7	(11.7)	2.8	(8.6)
Non-Profit	8.0	(9.8)	8.5	(9.4)	8.2	(11.9)

Note: "S&E" stands for science and engineering. "FFRDCs" stands for federally funded research and development centers. Figures in parentheses are the shares of basic research performed by the sectors.

Source: National Science Foundation, Science and Engineering Indicators, 2012.

Table 2 Baseline Parameterization

n^*	Long-run population growth rate	0.0150
θ^*	Long-run population share of high-skilled workers	0.0297
σ	Inverse of elasticity of intertemporal substitution	2.0000
β	Subjective discount factor	0.9738†
α	Share of intermediate input expenses in final output	0.7348
ψ	Inverse of markup over marginal cost	0.9091
η	Elasticity of TFP with respect to public R&D capital	0.1000
ρ	Input share in intermediate good production	0.6281†
ϑ	Degree of diminishing returns to private R&D input	0.5000
ϱ	Parameter in basic and non-basic research	0.1300
φ	Parameter in basic research	0.2020†
δ	Depreciation rate of physical capital	0.0929†
λ	Size of quality improvement	1.0236†
τ_w^*	Long-run tax rate on labor income	0.3530
τ_a^*	Long-run tax rate on interest income	0.2602
τ_c^*	Long-run tax rate on private consumption	0.0429
τ_p^*	Long-run tax rate on corporate income	0.3500
τ_d^*	Long-run subsidy rate on private R&D spending	0.1222
ϖ^*	Long-run share of basic research spending in total R&D	0.1386

Note: Parameters marked with an "†" are determined by the calibration procedure described in Section 5.2. All other parameter values are chosen based on empirical evidence.

Table 3 Long-run Equilibrium in the Benchmark Economy

	Data	Model
Annual growth rate (%)	1.56*	1.56
Before-tax interest rate (%)	8.00*	8.00
Ratios to GDP		
Physical capital	2.4272*	2.4272
Labor income	0.5342*	0.5344
Total R&D spending	0.0206*	0.0208
Private consumption	0.6505	0.6997
Total tax revenues	0.2629	0.3007

Notes: Data marked with an asterisk are the target statistics.

Table 4 Benchmark Economy during the Transition Period

	1953-60	1961-70	1971-80	1981-90	1991-2000	2001-09	**1953-2009**
Ratios to GDP							
Physical capital							
Model	2.49	2.52	2.42	2.33	2.25	2.19	**2.36**
Data	2.44	2.23	2.56	2.56	2.30	2.49	**2.43**
Labor income							
Model	0.53	0.53	0.53	0.53	0.54	0.54	**0.53**
Data	0.58	0.57	0.55	0.51	0.51	0.50	**0.53**
Consumption							
Model	0.69	0.69	0.70	0.71	0.72	0.73	**0.71**
Data	0.63	0.62	0.63	0.65	0.68	0.70	**0.65**
Tax receipts							
Model	0.24	0.26	0.31	0.33	0.33	0.31	**0.30**
Data	0.23	0.25	0.27	0.27	0.28	0.28	**0.26**

Table 5 Counterfactual Experiments

		Counterfactual Experiments			
	Benchmark	(1)	(2)	(3)	(4)
Average growth rates (%)					
Per-worker output	1.63	1.63	1.63	1.35	1.09
Per-worker capital	1.49	1.49	1.49	1.39	1.25
TFP	0.94	0.94	0.94	0.70	0.51
Public R&D capital	3.13	3.13	3.13	1.75	1.65
Aggregate quality	0.57	0.57	0.57	0.58	0.31
Contributions to output growth (%)					
Per-worker capital	0.69	0.69	0.69	0.65	0.58
TFP	0.94	0.94	0.94	0.70	0.51
Contributions to TFP growth (%)					
Public R&D capital	0.54	0.54	0.54	0.30	0.28
Aggregate quality	0.42	0.42	0.42	0.42	0.23

Table 6 Sensitivity of Long-Run Equilibrium to Selected Parameters

	Long-term Growth Rate (%)				% of R&D Spending in Total Output		
	y	TFP	X	$\mathbb{Q}$	Public	Private	Total
Benchmark	**1.56**	**0.84**	**2.88**	**0.47**	**0.29**	**1.79**	**2.08**
$\eta = 0.0500$	1.10	0.59	2.88	0.47	0.28	1.71	1.99
$\eta = 0.1500$	2.03	1.09	2.88	0.47	0.30	1.88	2.18
$\varrho = 0.0400$	1.38	0.74	2.54	0.41	0.27	1.66	1.93
$\varrho = 0.2800$	2.02	1.07	3.70	0.60	0.33	2.07	2.40
$\psi = 0.9524$	0.66	0.35	1.15	0.21	0.13	0.82	0.95
$\psi = 0.8696$	2.70	1.43	5.02	0.78	0.48	3.01	3.49

Note: The variable y stands for per-worker output. The benchmark values are $\eta = 0.1$, $\varrho = 0.13$ and $\psi = 0.9091$.

Table 7 Welfare Analysis

Panel (A) Long-run interest rate is allowed to vary.

			Share of Total Output			
γ^*	ϖ^*	r^*	Private R&D	Total R&D	$\mathbf{z}_L(\varpi_1^*)$	$\mathbf{z}_H(\varpi_1^*)$
0.0156	**0.1386**	**0.0800**	**0.0179**	**0.0208**	**1.0000**	**1.0000**
0.0160	0.1430	0.0813	0.0178	0.0208	1.0097	0.0094
0.0170	0.1520	0.0839	0.0176	0.0208	1.0296	1.0286
0.0180	0.1610	0.0867	0.0175	0.0208	1.0496	1.0480

Panel (B) Long-run interest rate is fixed at benchmark value.

			Share of Total Output			
γ^*	ϖ^*	τ_a^*	Private R&D	Total R&D	$\mathbf{z}_L(\varpi_1^*)$	$\mathbf{z}_H(\varpi_1^*)$
0.0156	**0.1386**	**0.2602**	**0.0179**	**0.0208**	**1.0000**	**1.0000**
0.0160	0.1430	0.2490	0.0180	0.0210	1.0109	1.0166
0.0170	0.1520	0.2240	0.0183	0.0216	1.0335	1.0522
0.0180	0.1610	0.1980	0.0186	0.0222	1.0560	1.0890

Note: Figures in bold are results from the benchmark model (the status quo).

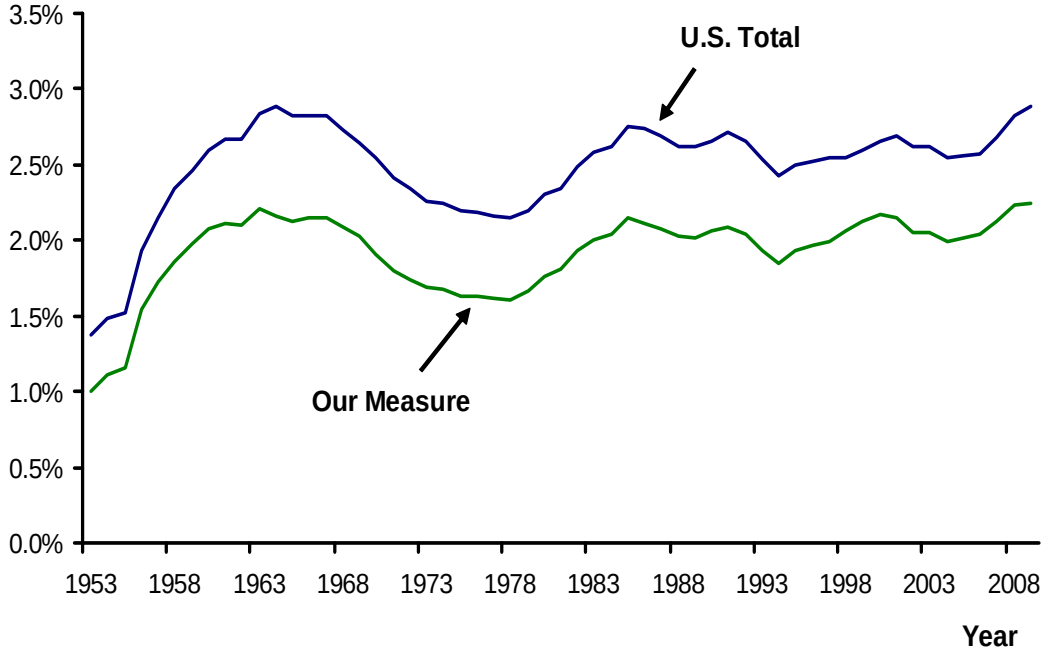

Figure 1: Share of Total R&D Spending in U.S. GDP, 1953-2009.

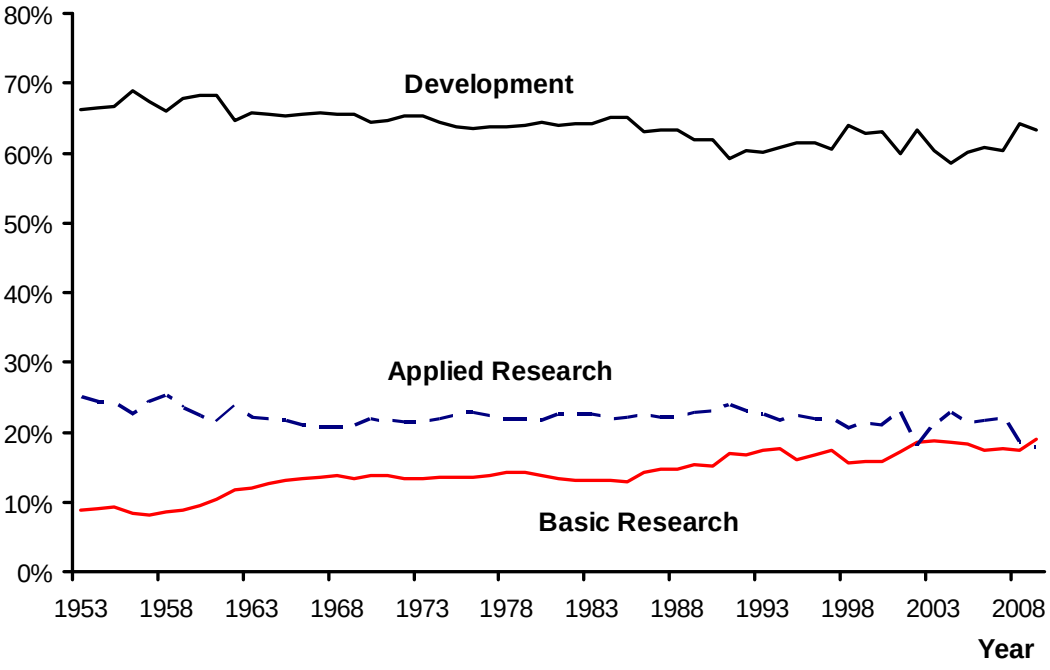

Figure 2: Distribution of Total R&D Spending by Type of Research, 1953-2009.

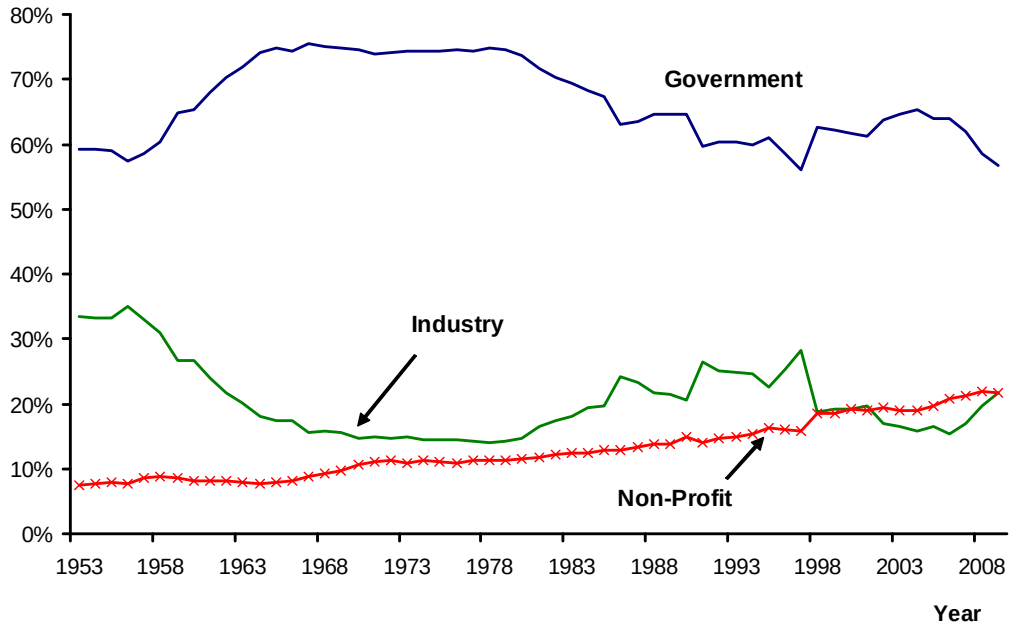

Figure 3: Distribution of Basic Research Expenditures by Source of Funds, 1953-2009.

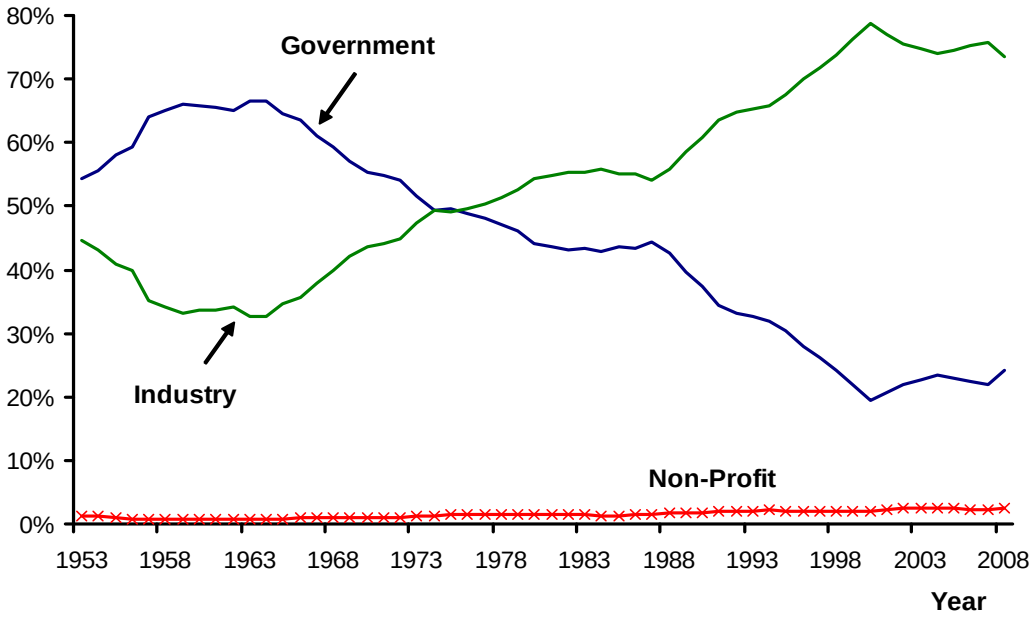

Figure 4: Distribution of Applied Research and Development Spending by Source of Funds, 1953-2009.

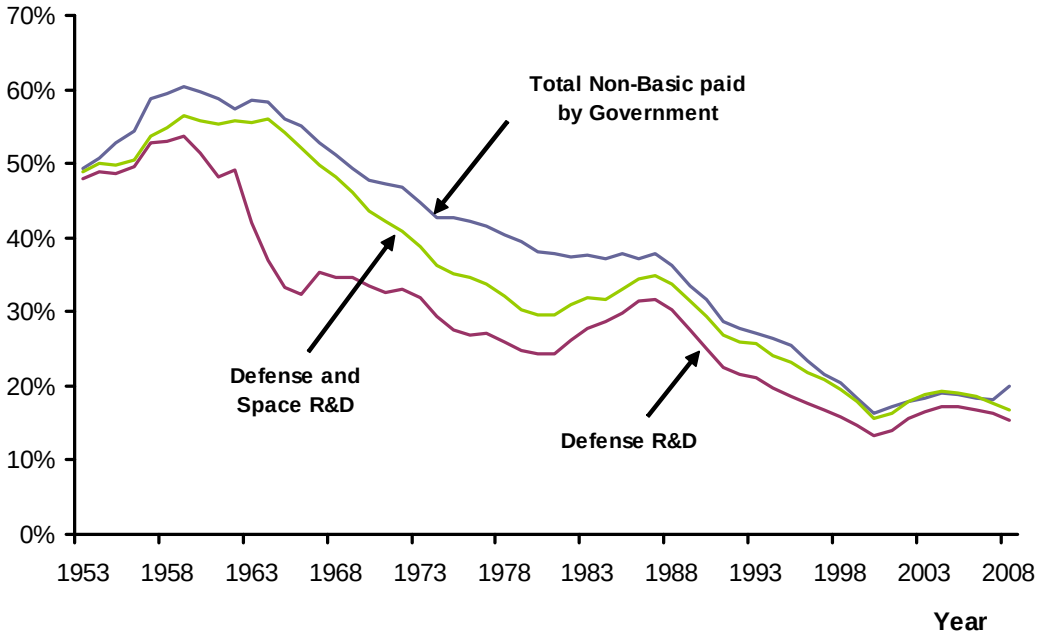

Figure 5: Components of Government R&D Spending as Percentages of Total U.S. R&D Spending.

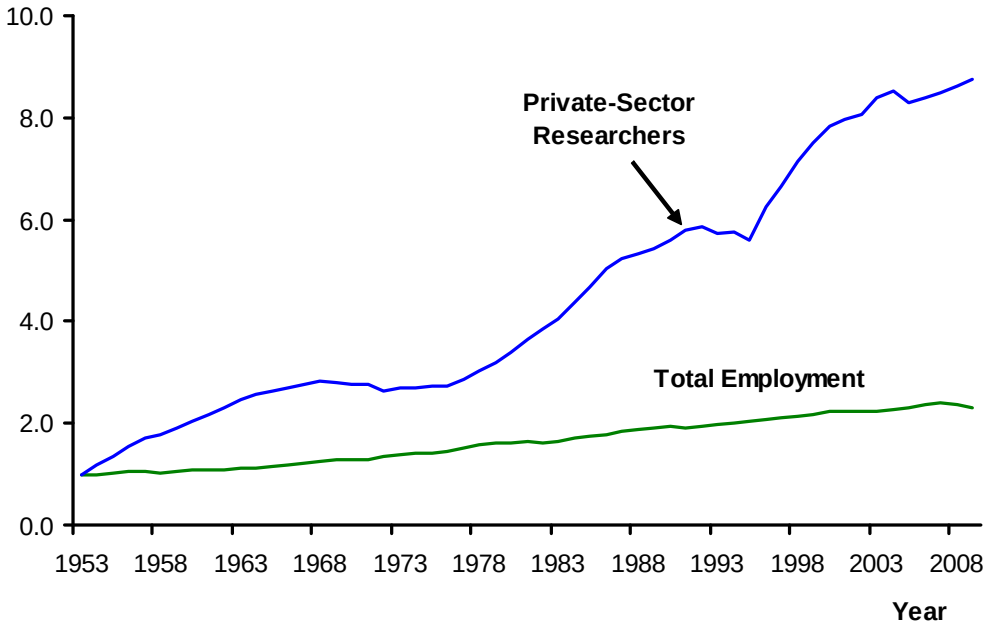

Figure 6: Employment of Private-Sector Researchers and Total U.S. Employment (1953 = 1.0).

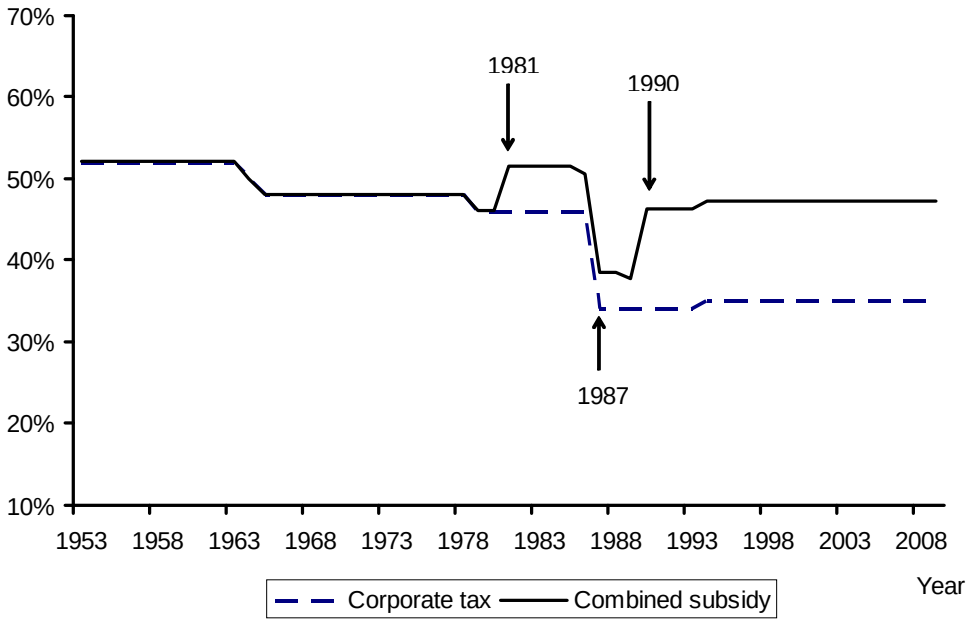

Figure 7: Tax Incentives for Private R&D in the United States.

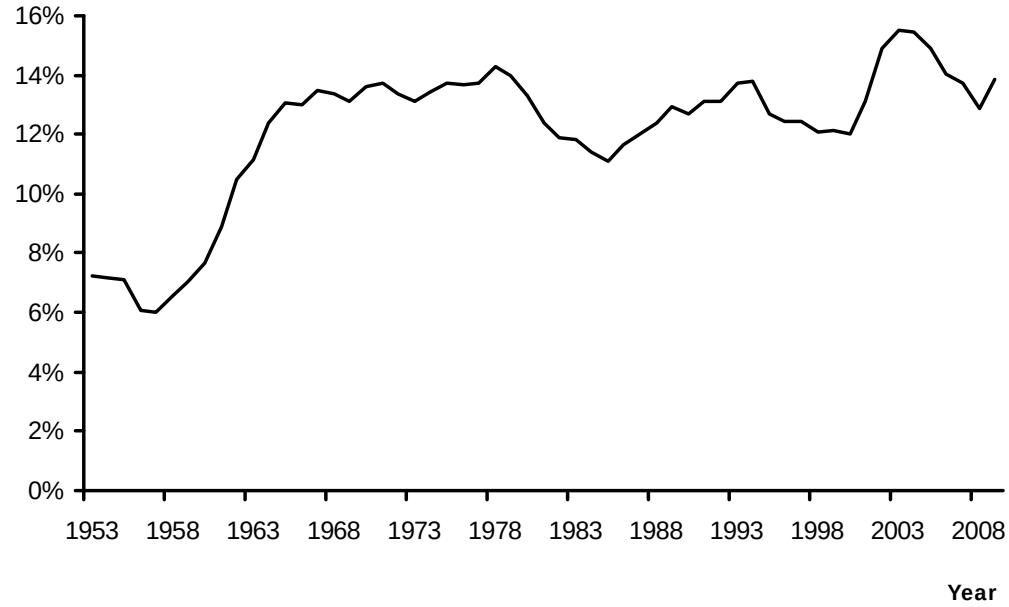

Figure 8: Share of Government Basic Research Spending in Our Measure of Total R&D Expenditures.

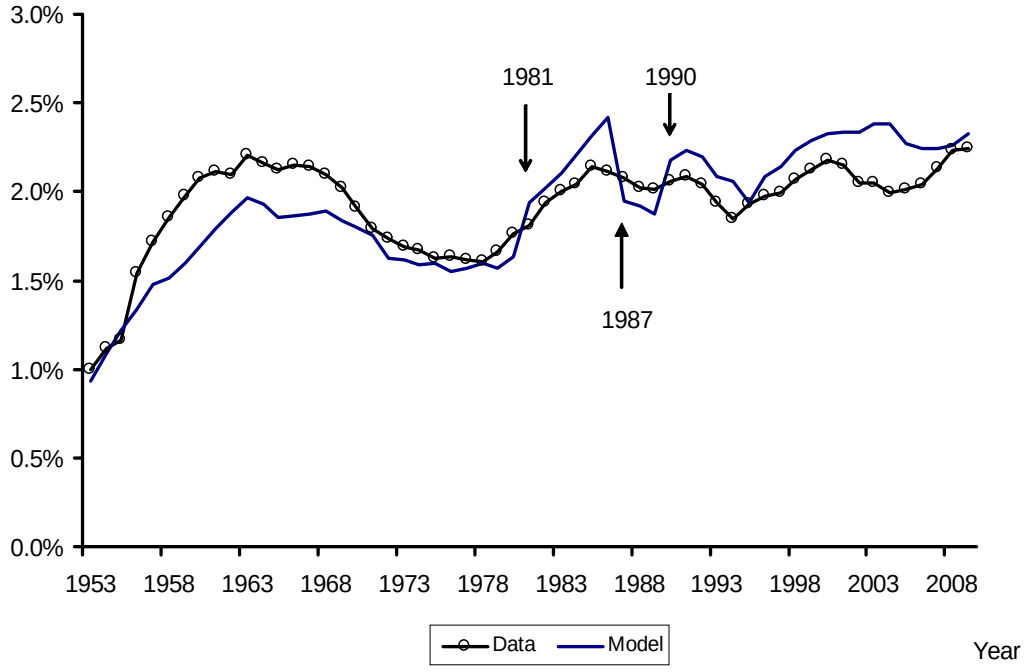

Figure 9: R&D Investment Rates in Benchmark Economy.

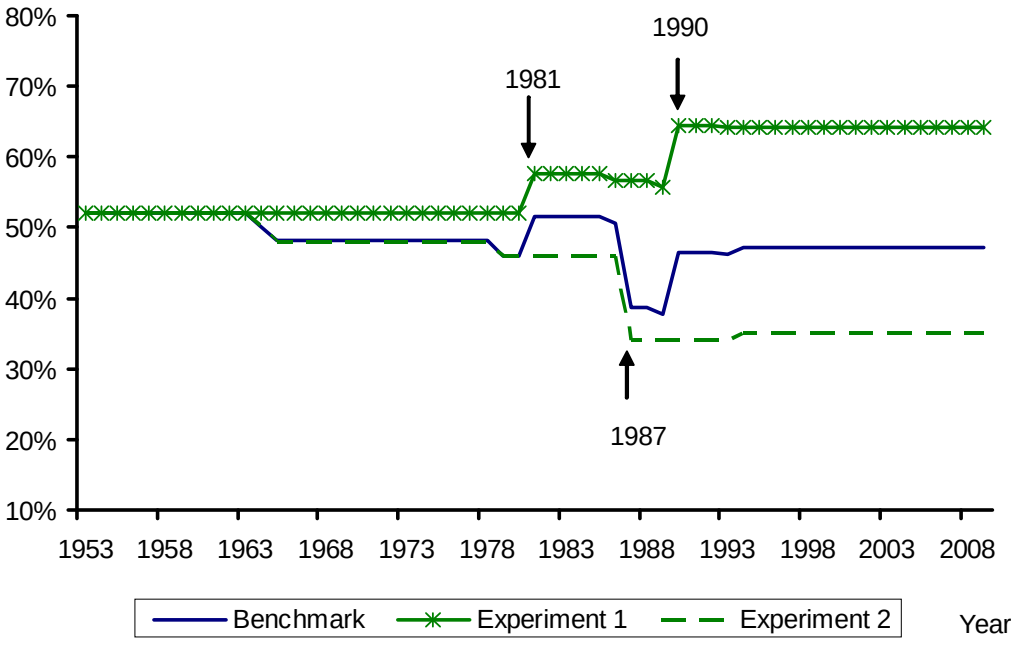

Figure 10: Combined Subsidy Rates for Private R&D in Experiments 1 and 2.

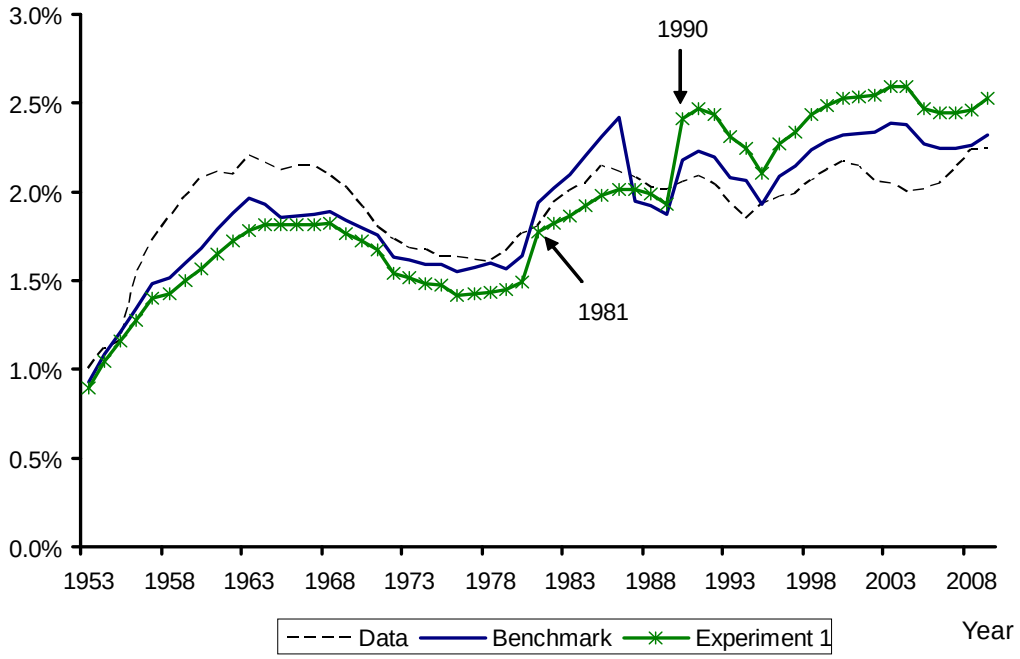

Figure 11: R&D Investment Rates in Experiment 1.

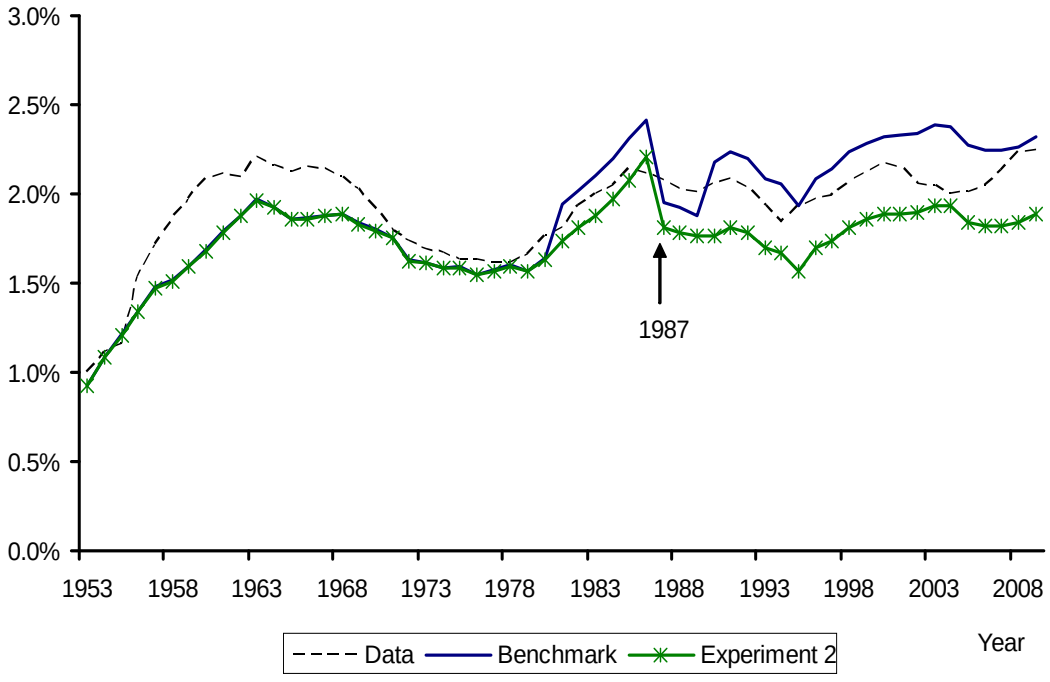

Figure 12: R&D Investment Rates in Experiment 2.

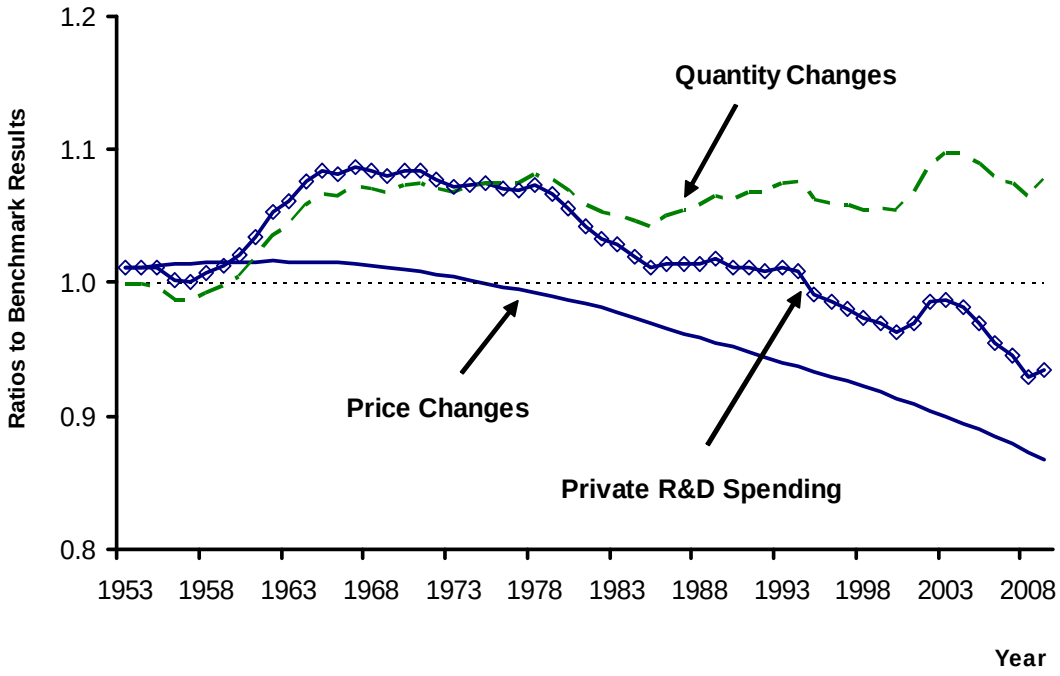

Figure 13: Private R&D Spending in Experiment 3 (Relative to Benchmark Results).

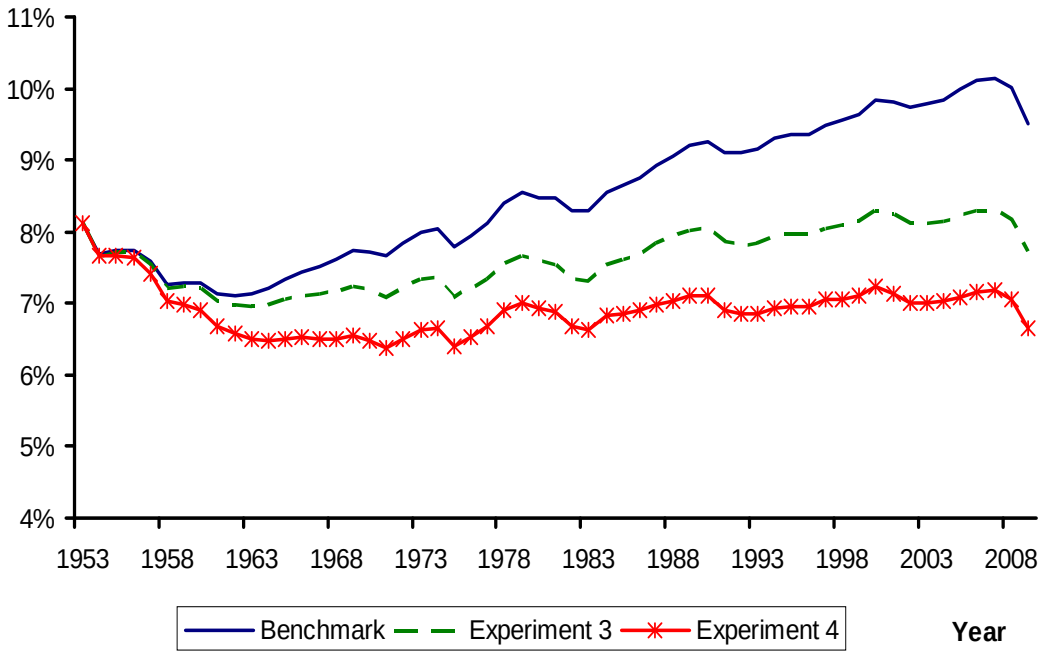

Figure 14: Real Interest Rates in Experiments 3 and 4.

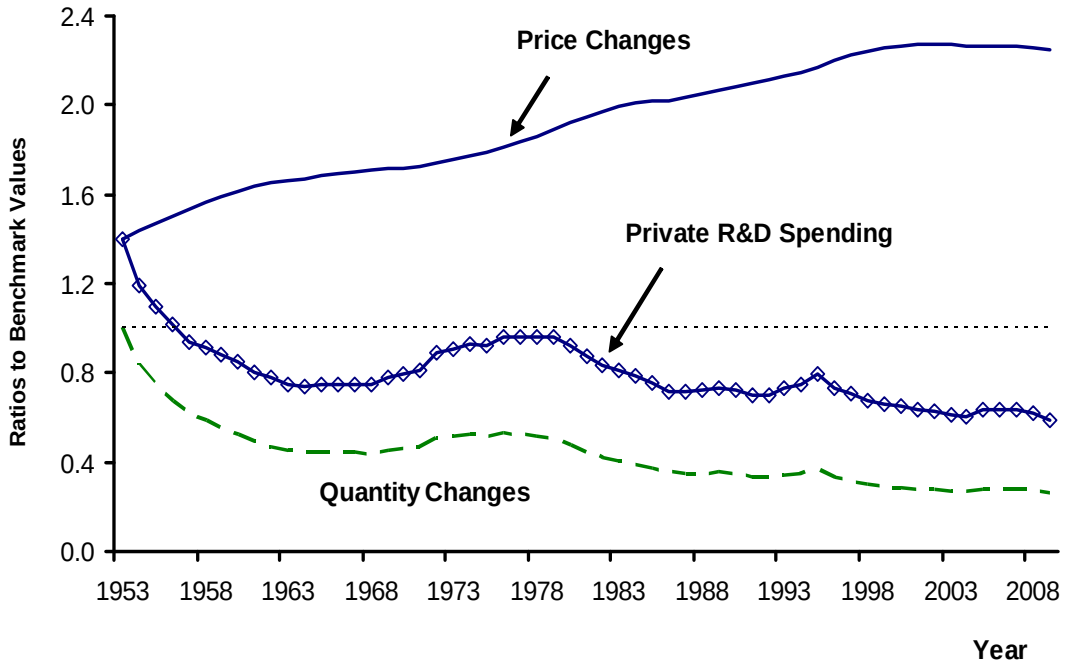

Figure 15: Private R&D Spending in Experiment 4 (Relative to Benchmark Results).

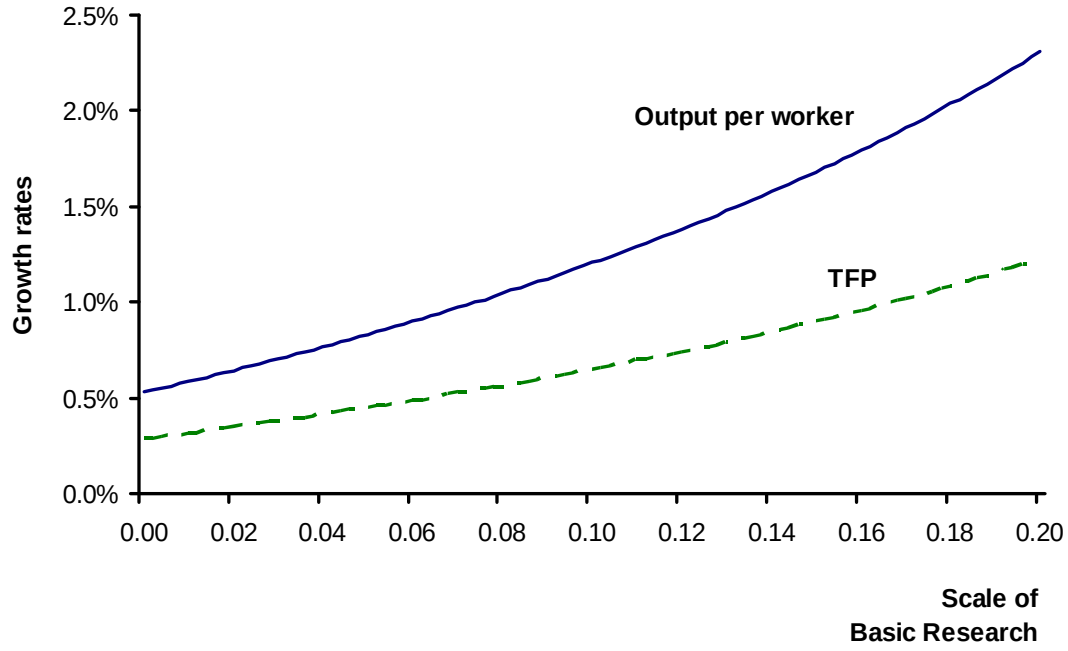

Figure 16a: Relations between Economic Growth and Basic Research in the Long Run.

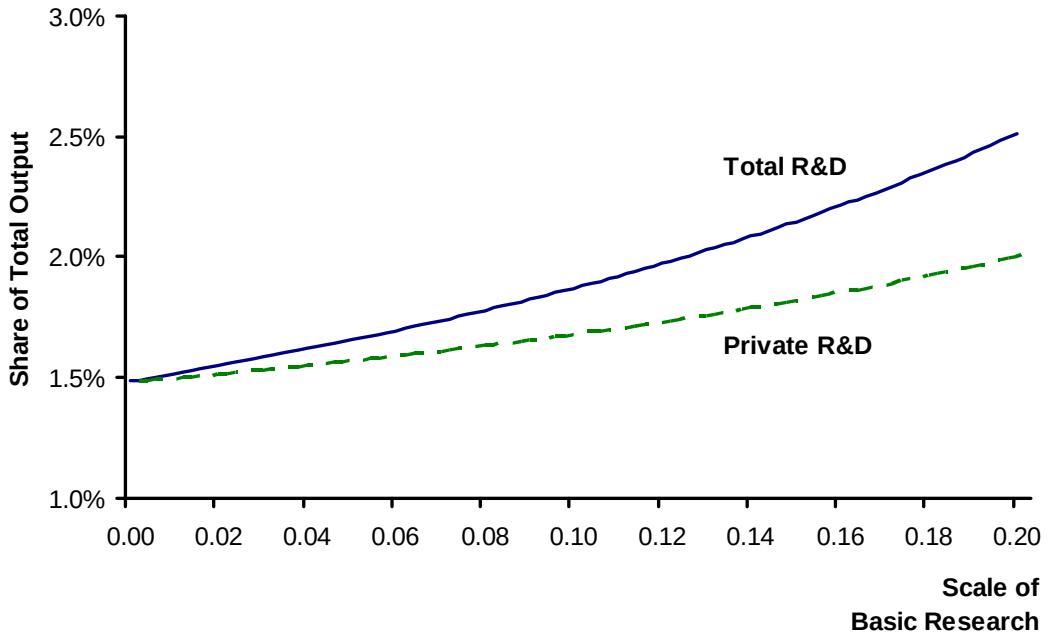

Figure 16b: Relations between R&D Spending and Basic Research in the Long Run.

Appendix A: Mathematical Derivations

Final-Good Producer's Problem

Equation (7) can be derived as follows. First, note that even though the final-good producer can choose to have $M_t(j,\omega) = 0$ for some (j,ω), it must use *at least* one quality grade of each type of intermediate input so that $\widetilde{M}_t(\omega) > 0$ for all $\omega \in [0,1]$. Let $\widetilde{p}_t(\omega)$ be the lowest quality-adjusted price of intermediate good $\omega \in [0,1]$. Equation (6) can be restated as

$$\widetilde{p}_t(\omega) = \alpha\Theta_t \left(\frac{\mathbb{M}_t}{L_{Y,t}}\right)^{\alpha-1} \mathbb{M}_t^{1-\psi} \left[\widetilde{M}_t(\omega)\right]^{\psi-1} = \left(\frac{\alpha Y_t}{\mathbb{M}_t^{\psi}}\right) \left[\widetilde{M}_t(\omega)\right]^{\psi-1}. \tag{49}$$

Let $\mathbb{E}_t$ be the total expenditure on intermediate inputs at time t. Then, we have

$$\begin{aligned} \mathbb{E}_t &\equiv \int_0^1 \left[\sum_{j=1}^{J_t(\omega)} p_t(j,\omega) M_t(j,\omega)\right] d\omega \\ &= \int_0^1 \widetilde{p}_t(\omega) \left[\sum_{j=1}^{J_t(\omega)} \lambda^j M_t(j,\omega)\right] d\omega = \int_0^1 \widetilde{p}_t(\omega) \widetilde{M}_t(\omega)\, d\omega. \end{aligned} \tag{50}$$

Given the Cobb-Douglas production function in (1), the parameter α indicates the share of $\mathbb{E}_t$ in total output, i.e., $\mathbb{E}_t = \alpha Y_t$.[55] Substituting this into (49) and rearranging terms gives

$$\widetilde{M}_t(\omega) = \left(\frac{\mathbb{E}_t}{\mathbb{M}_t^{\psi}}\right)^{\frac{1}{1-\psi}} \left[\widetilde{p}_t(\omega)\right]^{-\frac{1}{1-\psi}}.$$

Substituting this into (50) gives

$$\mathbb{E}_t = \left(\frac{\mathbb{E}_t}{\mathbb{M}_t^{\psi}}\right)^{\frac{1}{1-\psi}} \int_0^1 \left[\widetilde{p}_t(\omega)\right]^{-\frac{\psi}{1-\psi}} d\omega = \left(\frac{\mathbb{E}_t}{\mathbb{M}_t^{\psi}}\right)^{\frac{1}{1-\psi}} (\mathbb{P}_t)^{-\frac{\psi}{1-\psi}}.$$

Combining these two equations gives

$$\widetilde{M}_t(\omega) = \left[\widetilde{p}_t(\omega)\right]^{-\frac{1}{1-\psi}} (\mathbb{P}_t)^{\frac{\psi}{1-\psi}} \mathbb{E}_t. \tag{51}$$

If the final-good producer chooses to employ only intermediate good (j,ω), then we have $\widetilde{M}_t(\omega) = \lambda^j M_t(j,\omega)$ and $\widetilde{p}_t(\omega) = p_t(j,\omega)/\lambda^j$. Equation (7) can be obtained by substituting these into (51).

[55] One can also derive this directly by first multiplying both sides of (49) by $\widetilde{M}_t(\omega)$, and then integrating across all types of intermediate inputs.

Production in Intermediate-Goods Sector

The minimum cost required to produce $\overline{M} \geq 0$ units of intermediate good (j, ω) can be obtained by solving the problem below:

$$\mathbb{C}_t\left(\overline{M}\right) \equiv \min_{L_{M,t}, K_t} \left\{ w_{L,t} L_{M,t} + R_t K_t \right\},$$

subject to $\Theta_t K_t^{\rho} L_{M,t}^{1-\rho} \geq \overline{M}$. The solution of this problem involves a pair of factor demand functions:

$$L_{M,t} = \frac{\overline{M}}{\Theta_t} \left(\frac{w_{L,t}}{1-\rho} \right)^{-\rho} \left(\frac{R_t}{\rho} \right)^{\rho} \qquad \text{and} \qquad K_t = \frac{\overline{M}}{\Theta_t} \left(\frac{w_{L,t}}{1-\rho} \right)^{1-\rho} \left(\frac{R_t}{\rho} \right)^{\rho-1}, \tag{52}$$

and a linear cost function. The derivations of the cost function and the optimal monopoly price are standard and thus omitted.

Equilibrium Analysis

Income Accounting

First, consider the final-good sector. Given the Cobb-Douglas production function in (1), it is immediate to see that

$$w_{L,t} L_{Y,t} = (1-\alpha)\, Y_t, \qquad \text{and} \qquad \mathbb{E}_t = \alpha Y_t. \tag{53}$$

Next, consider the intermediate-goods sector. Let $Q_t(\omega)$ be the state-of-the-art quality for intermediate good $\omega \in [0,1]$. Then the quality-adjusted price of this product is

$$\widetilde{p}_t(\omega) = \frac{p_t}{Q_t(\omega)} = \frac{\kappa_t}{\psi} \left[Q_t(\omega) \right]^{-1}.$$

Substituting this into (8) gives

$$\mathbb{P}_t = \left(\frac{\kappa_t}{\psi} \right) \left(\mathbb{Q}_t \right)^{-1},$$

where

$$\mathbb{Q}_t \equiv \left\{ \int_0^1 \left[Q_t(\omega) \right]^{\frac{\psi}{1-\psi}} d\omega \right\}^{\frac{1-\psi}{\psi}} = \left[\sum_{q \in \mathcal{Q}} f_t(q)\, q^{\frac{\psi}{1-\psi}} \right]^{\frac{1-\psi}{\psi}}.$$

Substituting the expression for $\mathbb{P}_t$ into (14) and (15) gives

$$M_t(q) = \left(\frac{\kappa_t}{\psi} \right)^{-1} \left(\frac{q}{\mathbb{Q}_t} \right)^{\frac{\psi}{1-\psi}} \mathbb{E}_t, \qquad \text{and} \qquad \pi_t(q) = (1-\psi) \left(\frac{q}{\mathbb{Q}_t} \right)^{\frac{\psi}{1-\psi}} \mathbb{E}_t. \tag{54}$$

Substituting $M_t(q)$ into (52) gives the factor demand functions for firms with state-of-the-art quality q,

$$L_{M,t}(q) = \frac{\psi(1-\rho)\mathbb{E}_t}{w_{L,t}}\left(\frac{q}{\mathbb{Q}_t}\right)^{\frac{\psi}{1-\psi}}, \qquad \text{and} \qquad K_t(q) = \frac{\psi\rho\mathbb{E}_t}{R_t}\left(\frac{q}{\mathbb{Q}_t}\right)^{\frac{\psi}{1-\psi}}.$$

Aggregating these factor demand functions across the quality distribution gives

$$w_{L,t}\sum_{q\in\mathcal{Q}} f_t(q) L_{M,t}(q) = \psi(1-\rho)\mathbb{E}_t = \psi(1-\rho)\alpha Y_t, \tag{55}$$

$$R_t\sum_{q\in\mathcal{Q}} f_t(q) K_t(q) = \psi\rho\mathbb{E}_t = \psi\rho\alpha Y_t. \tag{56}$$

Similarly, aggregating the profit function in (54) across all intermediate-good producers gives

$$\sum_{q\in\mathcal{Q}} f_t(q)\pi_t(q) = (1-\psi)\mathbb{E}_t = (1-\psi)\alpha Y_t.$$

To summarize, final goods produced in each period are distributed according to

$$Y_t = w_{L,t}L_{Y,t} + w_{L,t}\overline{L}_{M,t} + R_t\overline{K}_t + \sum_{q\in\mathcal{Q}} f_t(q)\pi_t(q),$$

where $\overline{L}_{M,t} \equiv \sum_{q\in\mathcal{Q}} f_t(q) L_{M,t}(q)$ is the total number of low-skilled workers employed in the intermediate-goods sector. Equations (53) and (55) imply

$$\frac{L_{Y,t}}{\overline{L}_{M,t}} = \frac{1-\alpha}{\psi(1-\rho)\alpha}.$$

Thus, we can write $L_{Y,t} = \overline{s}_Y N_{L,t}$ and $\overline{L}_{M,t} = (1-\overline{s}_Y) N_{L,t}$, where

$$\overline{s}_Y \equiv \frac{1-\alpha}{1-\alpha+\psi(1-\rho)\alpha} \in (0,1).$$

Recall the definition of distributed dividends in (16). Aggregating this across the quality distribution gives

$$\sum_{q\in\mathcal{Q}} f_t(q)\Delta_t(q) = (1-\tau_{p,t})\sum_{q\in\mathcal{Q}} f_t(q)\pi_t(q) - (1-\tau_{p,t}-\tau_{d,t}) w_{H,t}(1-\varpi_t) N_{H,t}$$

$$\Rightarrow \sum_{q\in\mathcal{Q}} f_t(q)\pi_t(q) = \tau_{p,t}\sum_{q\in\mathcal{Q}} f_t(q)\pi_t(q) + \sum_{q\in\mathcal{Q}} f_t(q)\Delta_t(q) + (1-\tau_{p,t}-\tau_{d,t}) w_{H,t}(1-\varpi_t) N_{H,t}.$$

The last equation shows that monopoly profits in each period are divided among three uses: (i) corporate

income tax, (ii) dividend income to equity holders, and (iii) subsidized wages to private-sector researchers, which is equivalent to private R&D spending paid by the monopolists.

Final Output

We now provide a formal derivation of (37). Substituting the expression for $M_t(q)$ into (3) gives

$$\mathbb{M}_t = \left\{ \sum_{q \in \mathcal{Q}} f_t(q) \left[q M_t(q) \right]^{\psi} \right\}^{\frac{1}{\psi}} = \frac{\psi \mathbb{E}_t \mathbb{Q}_t}{\kappa_t} = \frac{\alpha \psi Y_t \mathbb{Q}_t}{\kappa_t}.$$

Recall the definition of κ_t in (12). Substituting (55) and (56) into this gives

$$\kappa_t \equiv \frac{1}{\Theta_t} \left(\frac{\psi \alpha Y_t}{\overline{K}_t} \right)^{\rho} \left(\frac{\psi \alpha Y_t}{\overline{L}_{M,t}} \right)^{1-\rho} = \frac{\psi \alpha Y_t}{\Theta_t \overline{K}_t^{\rho} \overline{L}_{M,t}^{1-\rho}}.$$

Combining these two equations gives $\mathbb{M}_t = \mathbb{Q}_t \Theta_t \overline{K}_t^{\rho} \overline{L}_{M,t}^{1-\rho}$. Substituting this expression into the production function in (1) gives

$$Y_t = \Theta_t^{1+\alpha} \mathbb{Q}_t^{\alpha} \overline{K}_t^{\rho\alpha} \overline{L}_{M,t}^{(1-\rho)\alpha} L_{Y,t}^{1-\alpha}.$$

Since $L_{Y,t} = \overline{s}_Y N_{L,t}$ and $\overline{L}_{M,t} = (1 - \overline{s}_Y) N_{L,t}$, we can rewrite this as

$$Y_t = \underbrace{\overline{s}_Y^{1-\alpha} (1 - \overline{s}_Y)^{(1-\rho)\alpha}}_{\widetilde{\mathbb{A}}} \Theta_t^{1+\alpha} \mathbb{Q}_t^{\alpha} \overline{K}_t^{\rho\alpha} N_{L,t}^{1-\rho\alpha},$$

where $\rho\alpha \in (0,1)$. This is the aggregate production function in (37).

Dynamics of Aggregate Quality Index

We now provide a formal derivation of (41). Recall the first-order condition for $\Lambda_{M,t}$, i.e.,

$$(1 - \tau_{p,t} - \tau_{d,t}) w_{H,t} = \frac{\vartheta}{\Omega_t(q)} \left[\frac{\Lambda_{M,t}(q)}{\overline{\Lambda}_{M,t}} \right]^{\vartheta - 1} \left[\frac{V_{t+1}(\lambda q) - V_{t+1}(q)}{1 + r_{t+1}} \right]. \tag{57}$$

Multiplying both sides by $\Lambda_{M,t}(q)$ gives

$$(1 - \tau_{p,t} - \tau_{d,t}) w_{H,t} \Lambda_{M,t}(q) = \vartheta \widehat{\Phi}_t(q) \left[\frac{V_{t+1}(\lambda q) - V_{t+1}(q)}{1 + r_{t+1}} \right].$$

Suppose the conditions in Proposition 3 are satisfied. Then we have $V_t(q) = \Gamma_t q^{\frac{\psi}{1-\psi}}$ for all q. After substituting this into the above expression and aggregating across the quality distribution, we can get

$$(1-\tau_{p,t}-\tau_{d,t})\, w_{H,t}\overline{\Lambda}_{M,t} = \vartheta \left[\frac{\Gamma_{t+1}\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)}{1+r_{t+1}}\right]\left[\sum_{q\in\mathcal{Q}} f_t(q)\,\widehat{\Phi}_t(q)\, q^{\frac{\psi}{1-\psi}}\right]. \tag{58}$$

Similarly, substituting the expressions of $\nu(q,\overline{q}_t)$, $V_t(q)$ and $\Lambda_{M,t}(q)$ into (57) gives

$$(1-\tau_{p,t}-\tau_{d,t})\, w_{H,t} = \frac{\vartheta}{\xi(\overline{q}_t)} X_t^{\varrho}\mathbb{Q}_t^{\frac{(1-\vartheta)\psi}{1-\psi}}\left[\frac{\Gamma_{t+1}\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)}{1+r_{t+1}}\right] \tag{59}$$

Combining (58) and (59) gives

$$\sum_{q\in\mathcal{Q}} f_t(q)\,\widehat{\Phi}_t(q)\, q^{\frac{\psi}{1-\psi}} = \frac{1}{\xi(\overline{q}_t)}(1-\varpi_t) N_{H,t} X_t^{\varrho}\mathbb{Q}_t^{\frac{(1-\vartheta)\psi}{1-\psi}}. \tag{60}$$

Next, using (19)-(21), we can get

$$\begin{aligned}
\sum_{q\in\mathcal{Q}} f_{t+1}(q)\, q^{\frac{\psi}{1-\psi}} &= f_{t+1}(1) + f_{t+1}(\lambda)\,\lambda^{\frac{\psi}{1-\psi}} + f_{t+1}\left(\lambda^2\right)\left(\lambda^2\right)^{\frac{\psi}{1-\psi}} + ... \\
&= \left[1-\widehat{\Phi}_t(1)\right] f_t(1) + \left\{\widehat{\Phi}_t(1) f_t(1) + \left[1-\widehat{\Phi}_t(\lambda)\right] f_t(\lambda)\right\}\lambda^{\frac{\psi}{1-\psi}} + ... \\
&= \left[1+\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)\widehat{\Phi}_t(1)\right] f_t(1) + \left[1+\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)\widehat{\Phi}_t(\lambda)\right]\lambda^{\frac{\psi}{1-\psi}} f_t(\lambda) + ... \\
&= \sum_{q\in\mathcal{Q}}\left[1+\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)\widehat{\Phi}_t(q)\right] q^{\frac{\psi}{1-\psi}} f_t(q).
\end{aligned}$$

Hence, we have

$$\sum_{q\in\mathcal{Q}} f_{t+1}(q)\, q^{\frac{\psi}{1-\psi}} = \sum_{q\in\mathcal{Q}} f_t(q)\, q^{\frac{\psi}{1-\psi}} + \left(\lambda^{\frac{\psi}{1-\psi}}-1\right)\sum_{q\in\mathcal{Q}} f_t(q)\,\widehat{\Phi}_t(q)\, q^{\frac{\psi}{1-\psi}}$$

$$\Rightarrow \left(\mathbb{Q}_{t+1}\right)^{\frac{\psi}{1-\psi}} = \left(\mathbb{Q}_t\right)^{\frac{\psi}{1-\psi}} + \frac{1}{\xi(\overline{q}_t)}(1-\varpi_t)\left(\lambda^{\frac{\psi}{1-\psi}}-1\right) N_{H,t} X_t^{\varrho}\mathbb{Q}_t^{\frac{(1-\vartheta)\psi}{1-\psi}}.$$

Equation (41) follows immediately from this equation.

Dynamical System in Transformed Variables

Equations (42) and (43) are derived as follows. Define $\widehat{c}_t \equiv C_t/(\Psi_t N_t)$ and $\widehat{k}_t \equiv K_t/(\Psi_t N_t)$. Dividing both sides of the economy-wide resources constraint by $\Psi_t N_t$ gives

$$\widehat{c}_t + \left(\frac{\Psi_{t+1}N_{t+1}}{\Psi_t N_t}\right)\widehat{k}_{t+1} - (1-\delta)\,\widehat{k}_t = \mathbb{A}_t \widehat{k}_t^{\rho\alpha}.$$

Equation (42) follows immediately from this expression. As shown in the main text, aggregate consumption at time t can be expressed as

$$C_t = [1 + (\mu^* - 1)\,\theta_t]\, N_t c_{L,t} \Rightarrow \widehat{c}_t = \frac{1}{\Psi_t}\,[1 + (\mu^* - 1)\,\theta_t]\, c_{L,t}.$$

The growth of $\widehat{c}_t$ is then determined by

$$\frac{\widehat{c}_{t+1}}{\widehat{c}_t} = \left(\frac{\Psi_{t+1}}{\Psi_t}\right)^{-1}\left[\frac{1 + (\mu^* - 1)\,\theta_{t+1}}{1 + (\mu^* - 1)\,\theta_t}\right]\frac{c_{L,t+1}}{c_{L,t}}.$$

Equation (43) can be obtained by combining this and the Euler equation in (29).

Consumption Ratio between High-Skilled and Low-Skilled Households

The consumption ratio between high-skilled and low-skilled households is constant over time and is denoted by μ^*. The mathematical derivation of μ^* is tedious and does not add much to our understanding of the model. Hence, we only describe the key steps here.[56] First, using the exogenous sequence of tax rates and the equilibrium time paths of prices and transfers, we can construct the consolidated (or lifetime) budget constraint for the two households. In the consolidated budget constraint, the total discounted value of lifetime consumption is equated to the total discounted value of lifetime income plus the quantity of initial assets $(A_{s,0})$. Second, by repeated substitutions using the Euler equation in (29), we can get

$$c_{s,t+1} = \left(\beta \widetilde{R}_{t+1}\right)^{\frac{1}{\sigma}}\left(\frac{1+\tau_{c,t}}{1+\tau_{c,t+1}}\right)^{\frac{1}{\sigma}} c_{s,t} = \left[\prod_{j=1}^{t+1}\left(\beta \widetilde{R}_j\right)^{\frac{1}{\sigma}}\right]\left(\frac{1+\tau_{c,0}}{1+\tau_{c,t+1}}\right)^{\frac{1}{\sigma}} c_{s,0},$$

for $s \in \{H, L\}$, where $\widetilde{R}_t \equiv 1 + (1-\tau_{a,t})\, r_t$. Third, after substituting the above expression into the consolidated budget constraint, we can derive an expression for $c_{s,0}$ in terms of $\{\tau_{a,t}, \tau_{c,t}, \tau_{a,t}, w_{s,t}, r_t, \Upsilon_{s,t}, N_{s,t}\}_{t=0}^{\infty}$ and $A_{s,0}$. Finally, we need to determine the value of $\sum_{s\in\{H,L\}} A_{s,0}$ and how it is allocated between the

[56] A complete derivation of μ^* is available from the author upon request.

two types of households. After solving for the equilibrium time paths of all major aggregate variables, we can derive the value of $\sum_{s\in\{H,L\}} A_{s,0}$ (by backward induction from the unique long-run equilibrium) using the zero-profit condition for the financial intermediaries, i.e.,

$$\overline{K}_{t+1} + (1+r_t) \sum_{s\in\{H,L\}} A_{s,t} = (1+r_t)\,\overline{K}_t + \sum_{q\in\mathcal{Q}} f_t\,(q)\,\Delta_t\,(q) + \sum_{s\in\{H,L\}} A_{s,t+1}.$$

Once the value of $\sum_{s\in\{H,L\}} A_{s,0}$ is known, we then divide it between the two households according to

$$\frac{A_{H,0}}{A_{L,0}} = \frac{N_{H,0}}{N_{L,0}} = \frac{\theta_0}{1-\theta_0}.$$

This is equivalent to assuming that all the initial assets are distributed evenly among all the individuals, regardless of their skill level.

Appendix B: Proofs

Proof of Proposition 1

Fix $\overline{z}_t > 0$ and $\varpi_t \in (0,1)$. A set of basic research projects $\mathcal{D}_t \subset [0, \overline{z}_t]$ is called feasible if it satisfies the policy constraint at time t, i.e., $\int_{\mathcal{D}_t} dz = \varpi_t \overline{z}_t$. The proof of this proposition is built upon a simple observation: Consider an arbitrary interval $\mathcal{D}_0 = \left[\underline{d}, \overline{d}\right]$, with $0 \leq \underline{d} < \overline{d} < \overline{z}_t$ that is feasible under ϖ_t. For any $\varepsilon \in \left(0, \overline{z}_t - \overline{d}\right]$, construct another interval $\mathcal{D}_\varepsilon = \left[\underline{d} + \varepsilon, \overline{d} + \varepsilon\right]$ which is also feasible under ϖ_t. Then we will have $\int_{\mathcal{D}_\varepsilon} \widetilde{\zeta}(z)\, dz > \int_{\mathcal{D}_0} \widetilde{\zeta}(z)\, dz$. This intermediate result has the following implication: For any given values of $\{\overline{z}_t, \varpi_t, X_t, N_{H,t}, \mathbb{Q}_t\}$ and for any feasible interval $\mathcal{D}_0$ with upper boundary $\overline{d} < \overline{z}_t$, we can always increase the growth rate of public R&D capital by shifting the interval $\mathcal{D}_0$ to the right. We will refer to $\mathcal{D}_\varepsilon$ as a rightward shift of $\mathcal{D}_0$. The proof of this claim is straightforward. Since $\widetilde{\zeta}(\cdot)$ is a strictly increasing function, the expected value of $\widetilde{\zeta}(\cdot)$ over the range $\mathcal{D}_\varepsilon$ must be greater than that over the range $\mathcal{D}_0$. Formally, this means

$$\frac{1}{\overline{d} - \underline{d}} \int_{\mathcal{D}_\varepsilon} \widetilde{\zeta}(z)\, dz > \frac{1}{\overline{d} - \underline{d}} \int_{\mathcal{D}_0} \widetilde{\zeta}(z)\, dz \Rightarrow \int_{\mathcal{D}_\varepsilon} \widetilde{\zeta}(z)\, dz > \int_{\mathcal{D}_0} \widetilde{\zeta}(z)\, dz.$$

We now apply this result to show that the interval $\mathcal{D}_t = [d_t, \overline{z}_t]$, with $d_t \equiv (1 - \varpi_t)\, \overline{z}_t$, dominates all other feasible set. Given a continuous distribution function of z, any feasible set of research projects can be expressed as the union of a finite collection of disjoint intervals. Let $\{\mathcal{D}_{1,t}, ..., \mathcal{D}_{I,t}\}$ be an arbitrary finite collection of disjoint intervals so that its union is a feasible set. If $I = 1$, then by the above argument, we can always increase the growth rate of public R&D capital by shifting the interval to the right until the upper boundary is $\overline{z}_t$. Suppose $I > 1$. Let $\mathcal{D}_{i,t} = \left[\underline{d}_{i,t}, \overline{d}_{i,t}\right]$, with $\underline{d}_{i,t} < \overline{d}_{i,t} \leq \overline{z}_t$ for all i. The intervals are arranged so that $\underline{d}_{1,t} < ... < \underline{d}_{I,t}$. Since these are disjoint intervals, we have $\overline{d}_{i,t} < \underline{d}_{i+1,t}$, for $i = 1, 2, ..., I-1$. The growth rate of public R&D capital between period t and $t+1$ is given by

$$\left[\frac{1}{\overline{z}_t} \sum_{i=1}^{I} \int_{\mathcal{D}_{i,t}} \widetilde{\zeta}(z)\, dz\right] N_{H,t} X_t^{\varrho} \left[\Im\left(\mathbb{Q}_t\right)\right]^{-1}.$$

Fix the values of $\{\overline{z}_t, \varpi_t, X_t, N_{H,t}, \mathbb{Q}_t\}$. Now consider another collection of intervals $\left\{\mathcal{D}'_{1,t}, ..., \mathcal{D}'_{I,t}\right\}$, with $\mathcal{D}'_{i,t} = \mathcal{D}_{i,t}$ for $i \in \{1, 2, ..., I-2, I\}$, and $\mathcal{D}'_{I-1,t} = \left[\underline{d}'_{I-1,t}, \overline{d}'_{I-1,t}\right]$ so that

$$\overline{d}'_{I-1,t} = \underline{d}_{I,t}, \qquad \text{and} \qquad \overline{d}'_{I-1,t} - \underline{d}'_{I-1,t} = \overline{d}_{I-1,t} - \underline{d}_{I-1,t}.$$

In words, these mean $\mathcal{D}_{I-1,t}$ and $\mathcal{D}'_{I-1,t}$ have the same mass under the uniform distribution so that the union of $\left\{\mathcal{D}'_{1,t},...,\mathcal{D}'_{I,t}\right\}$ is also a feasible set. In addition, $\mathcal{D}'_{I-1,t}$ is a rightward shift of $\mathcal{D}_{I-1,t}$ which means $\int_{\mathcal{D}'_{I-1,t}}\widetilde{\zeta}(z)\,dz > \int_{\mathcal{D}_{I-1,t}}\widetilde{\zeta}(z)\,dz$. Hence, the growth rate of public R&D capital under $\left\{\mathcal{D}'_{1,t},...,\mathcal{D}'_{I,t}\right\}$ is higher than that under $\{\mathcal{D}_{1,t},...,\mathcal{D}_{I,t}\}$. Note that $\mathcal{D}'_{I-1,t}$ and $\mathcal{D}'_{I,t}$ now form a single interval $\left[\underline{d}'_{I-1,t},\overline{d}_{I,t}\right]$, thus there are only $(I-1)$ disjoint intervals in $\left\{\mathcal{D}'_{1,t},...,\mathcal{D}'_{I,t}\right\}$. In other words, we can increase the growth rate of public R&D capital by collapsing the disjoint intervals. By repeating the same argument, we can show that one single interval with endpoints $\left(\overline{d}_{I,t}-\varpi_t\overline{z}_t\right)$ and $\overline{d}_{I,t}$ dominates any collection of I disjoint intervals with the same upper boundary $\overline{d}_{I,t}$. Then we go back to the case where $I=1$, and the single dominant interval is $\mathcal{D}_t=[d_t,\overline{z}_t]$, with $d_t\equiv(1-\varpi_t)\,\overline{z}_t$. This completes the proof.

Proof of Proposition 2

For any $t\geq 0$, the value function $V_t(\cdot)$ and the optimal quantity of high-skilled labor input $\Lambda_{M,t}(\cdot)$ must satisfy three conditions: (i) the first-order condition for $\Lambda_{M,t}$:

$$(1-\tau_{p,t}-\tau_{d,t})\,w_{H,t}=\frac{\vartheta}{\Omega_t(q)}\left[\frac{\Lambda_{M,t}(q)}{\overline{\Lambda}_{M,t}}\right]^{\vartheta-1}\left[\frac{V_{t+1}(\lambda q)-V_{t+1}(q)}{1+r_{t+1}}\right], \tag{61}$$

(ii) the Bellman equation for the monopolist's problem:

$$\begin{aligned} V_t(q) &= (1-\tau_{p,t})\,\pi_t(q)+(1-\tau_{p,t}-\tau_{d,t})\,w_{H,t}\Lambda_{M,t}(q)+\widehat{\Phi}_t(q)\left[\frac{V_{t+1}(\lambda q)-V_{t+1}(q)}{1+r_{t+1}}\right]+\frac{V_{t+1}(q)}{1+r_{t+1}} \\ &= (1-\tau_{p,t})\,\pi_t(q)+\left(\frac{1}{\vartheta}-1\right)(1-\tau_{p,t}-\tau_{d,t})\,w_{H,t}\Lambda_{M,t}(q)+\frac{V_{t+1}(q)}{1+r_{t+1}}, \end{aligned} \tag{62}$$

where $\pi_t(q)=(1-\psi)\,\mathbb{E}_t\left(q/\mathbb{Q}_t\right)^{\frac{\psi}{1-\psi}}$, and (iii) the market-clearing condition for private-sector researchers:

$$\overline{\Lambda}_{M,t}\equiv\sum_{q\in\mathcal{Q}}f_t(q)\,\Lambda_{M,t}(q)=(1-\varpi_t)\,N_{H,t}. \tag{63}$$

To prove this proposition, it suffice to show that there exists a sequence of positive real numbers $\{\Gamma_t,\mathcal{F}_t\}_{t=0}^{\infty}$ such that the analytical solutions $V_t(q)=\Gamma_t q^{\frac{\psi}{1-\psi}}$ and $\Lambda_{M,t}(q)=\mathcal{F}_t q^{\frac{\psi}{1-\psi}}$ satisfy (61)-(63).

Substituting $\Lambda_{M,t}(q)=\mathcal{F}_t q^{\frac{\psi}{1-\psi}}$ into the market-clearing condition gives

$$\mathcal{F}_t\sum_{q\in\mathcal{Q}}f_t(q)\,q^{\frac{\psi}{1-\psi}}=\mathcal{F}_t\mathbb{Q}_t^{\frac{\psi}{1-\psi}}=\overline{\Lambda}_{M,t}=(1-\varpi_t)\,N_{H,t} \tag{64}$$

$$\Rightarrow \mathcal{F}_t = (1-\varpi_t) N_{H,t} \mathbb{Q}_t^{-\frac{\psi}{1-\psi}}, \qquad \text{for all } t.$$

Next, substitute the expressions for $\nu(q, \overline{q}_t)$, $V_t(q)$ and $\Lambda_{M,t}(q)$ into (61) to get

$$\begin{aligned}(1-\tau_{p,t}-\tau_{d,t}) w_{H,t} &= \frac{\vartheta}{\xi(\overline{q}_t)} X_t^{\varrho} q^{-\frac{\vartheta\psi}{1-\psi}} \left[\frac{\mathcal{F}_t q^{\frac{\psi}{1-\psi}}}{\overline{\Lambda}_{M,t}}\right]^{\vartheta-1} \left[\frac{\Gamma_{t+1}\left(\lambda^{\frac{\psi}{1-\psi}}-1\right) q^{\frac{\psi}{1-\psi}}}{1+r_{t+1}}\right] \\ &= \frac{\vartheta}{\xi(\overline{q}_t)} X_t^{\varrho} \mathbb{Q}_t^{\frac{(1-\vartheta)\psi}{1-\psi}} \left[\frac{\Gamma_{t+1}\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)}{1+r_{t+1}}\right]. \end{aligned} \tag{65}$$

The second line is obtained by using (64). Similarly, substitute the expressions for $\pi_t(q)$, $\nu(q, \overline{q}_t)$, $V_t(q)$ and $\Lambda_{M,t}(q)$ into (62) to get

$$\begin{aligned}\Gamma_t &= (1-\tau_{p,t})(1-\psi)\mathbb{E}_t \mathbb{Q}_t^{-\frac{\psi}{1-\psi}} + \left(\frac{1}{\vartheta}-1\right)(1-\tau_{p,t}-\tau_{d,t}) w_{H,t}\mathcal{F}_t + \frac{\Gamma_{t+1}}{1+r_{t+1}} \qquad (66) \\ &= (1-\tau_{p,t})(1-\psi)\mathbb{E}_t \mathbb{Q}_t^{-\frac{\psi}{1-\psi}} + \left[1+\frac{1}{\xi(\overline{q}_t)}(1-\vartheta)\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)(1-\varpi_t) N_{H,t} X_t^{\varrho} \mathbb{Q}_t^{-\frac{\vartheta\psi}{1-\psi}}\right]\left[\frac{\Gamma_{t+1}}{1+r_{t+1}}\right]. \end{aligned}$$

The second line is obtained by using (64) and (65). Equation (40) follows immediately from the above equation. An alternative expression for Γ_t can be obtained as follows: First, rewrite (65) as

$$\frac{\Gamma_{t+1}}{1+r_{t+1}} = \frac{\xi(\overline{q}_t)(1-\tau_{p,t}-\tau_{d,t})}{\vartheta\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)} w_{H,t} X_t^{-\varrho} \mathbb{Q}_t^{-\frac{(1-\vartheta)\psi}{1-\psi}}.$$

Then, substitute this expression into (66) to get

$$\Gamma_t = (1-\tau_{p,t})(1-\psi)\mathbb{E}_t \mathbb{Q}_t^{-\frac{\psi}{1-\psi}} + \left[(1-\vartheta)\overline{\Lambda}_{M,t} + \frac{\xi(\overline{q}_t)}{\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)} X_t^{-\varrho} \mathbb{Q}_t^{\frac{\vartheta\psi}{1-\psi}}\right]\left[\frac{1}{\vartheta}(1-\tau_{p,t}-\tau_{d,t}) w_{H,t} \mathbb{Q}_t^{-\frac{\psi}{1-\psi}}\right], \tag{67}$$

which is strictly positive for all $t \geq 0$. To summarize, equations (64) and (67) show that there exists a pair of positive real numbers, Γ_t and $\mathcal{F}_t$, such that the proposed solutions $V_t(q) = \Gamma_t q^{\frac{\psi}{1-\psi}}$ and $\Lambda_{M,t}(q) = \mathcal{F}_t q^{\frac{\psi}{1-\psi}}$ satisfy (61)-(63) for all t. This completes the proof of Proposition 2.

Proof of Proposition 3

Suppose Assumption S is satisfied. Suppose $\varrho < \vartheta$, $\Im\left(\mathbb{Q}_t\right) = \overline{\Im}\mathbb{Q}_t^{\frac{\vartheta\psi}{1-\psi}}$, with $\overline{\Im} > 0$, and $\xi\left(\overline{q}_t\right) \equiv \overline{\xi}\overline{q}_t^{-\varphi}$, with $\overline{\xi} > 0$. Then the subsystem for X_t and $\mathbb{Q}_t$ can be rewritten as

$$\frac{X_{t+1}}{X_t} = 1 + \frac{1}{\overline{\Im}}\Xi\left(\varpi^*\right)\overline{z}_t^{\varphi}N_{H,t}X_t^{\varrho}\mathbb{Q}_t^{-\frac{\vartheta\psi}{1-\psi}},$$

$$\frac{\mathbb{Q}_{t+1}}{\mathbb{Q}_t} = \left[1 + \frac{1}{\overline{\xi}}\left(\lambda^{\frac{\psi}{1-\psi}} - 1\right)\left(1 - \varpi^*\right)\overline{z}_t^{\varphi}N_{H,t}X_t^{\varrho}\mathbb{Q}_t^{-\frac{\vartheta\psi}{1-\psi}}\right]^{\frac{1-\psi}{\psi}},$$

where we have also used the assumption $\overline{z}_t = \overline{q}_t$ for all t. Define a transformed variable $\widetilde{\mathbb{X}}_t \equiv \overline{z}_t^{\varphi}N_{H,t}X_t^{\varrho}\mathbb{Q}_t^{-\frac{\vartheta\psi}{1-\psi}}$. Using the above equations and $\overline{z}_{t+1} = \lambda\overline{z}_t$, we can derive a single dynamic equation in $\widetilde{\mathbb{X}}_t$, which is

$$\begin{aligned}\frac{\widetilde{\mathbb{X}}_{t+1}}{\widetilde{\mathbb{X}}_t} &= \frac{\lambda^{\varphi}\left(1+n^*\right)\left[1 + \overline{\Im}^{-1}\Xi\left(\varpi^*\right)\widetilde{\mathbb{X}}_t\right]^{\varrho}}{\left[1 + \overline{\xi}^{-1}\left(\lambda^{\frac{\psi}{1-\psi}} - 1\right)\left(1 - \varpi^*\right)\widetilde{\mathbb{X}}_t\right]^{\vartheta}} \\ &= \left\{\frac{\lambda^{\frac{\varphi}{\vartheta}}\left(1+n^*\right)^{\frac{1}{\vartheta}}\left[1 + \overline{\Im}^{-1}\Xi\left(\varpi^*\right)\widetilde{\mathbb{X}}_t\right]^{\frac{\varrho}{\vartheta}}}{1 + \overline{\xi}^{-1}\left(\lambda^{\frac{\psi}{1-\psi}} - 1\right)\left(1 - \varpi^*\right)\widetilde{\mathbb{X}}_t}\right\}^{\vartheta}. \end{aligned} \tag{68}$$

The initial value of $\widetilde{\mathbb{X}}_t$ is predetermined and is given by $\widetilde{\mathbb{X}}_0 = \theta_0 X_0^{\varrho} > 0$ as $N_0 = \mathbb{Q}_0 = \overline{q}_0 = 1$. Along any balanced-growth path for X_t and $\mathbb{Q}_t$, the transformed variable $\widetilde{\mathbb{X}}_t$ must be invariant over time. Conversely, if $\widetilde{\mathbb{X}}_t = \widetilde{\mathbb{X}}^*$ for all t, then the growth rates of X_t and $\mathbb{Q}_t$ must be constant over time. Thus, the existence and uniqueness of BGP for X_t and $\mathbb{Q}_t$ is equivalent to the existence and uniqueness of (non-trivial) steady state for (68). In what follows, we will focus on the dynamic equation in (68).

When $\widetilde{\mathbb{X}}_t = 0$ the growth factor in (68) is $\lambda^{\varphi}\left(1+n^*\right) > 1$. When $\widetilde{\mathbb{X}}_t$ approaches positive infinity, this growth factor will converge to zero as $\varrho < \vartheta$. Thus, by the intermediate value theorem, there exists at least one non-trivial steady state for (68). To establish uniqueness, define a pair of functions:

$$\mathcal{N}_1\left(\widetilde{\mathbb{X}}_t\right) \equiv \lambda^{\frac{\varphi}{\vartheta}}\left(1+n^*\right)^{\frac{1}{\vartheta}}\left[1 + \overline{\Im}^{-1}\Xi\left(\varpi^*\right)\widetilde{\mathbb{X}}_t\right]^{\frac{\varrho}{\vartheta}}, \tag{69}$$

$$\mathcal{N}_2\left(\widetilde{\mathbb{X}}_t\right) \equiv 1 + \overline{\xi}^{-1}\left(\lambda^{\frac{\psi}{1-\psi}} - 1\right)\left(1 - \varpi^*\right)\widetilde{\mathbb{X}}_t. \tag{70}$$

The first function, $\mathcal{N}_1\left(\cdot\right)$, is continuously differentiable, strictly increasing and strictly concave as $0 < \varrho < \vartheta$. The second function, $\mathcal{N}_2\left(\cdot\right)$, is an affine function. Any steady-state value $\widetilde{\mathbb{X}}^*$ must satisfy $\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*\right) = \mathcal{N}_2\left(\widetilde{\mathbb{X}}^*\right)$. Suppose the contrary that there exists more than one non-trivial steady state

for (68). Let $\widetilde{\mathbb{X}}_1^*$ and $\widetilde{\mathbb{X}}_2^*$ be the smallest and second smallest steady-state values, respectively. Since $\mathcal{N}_1(0) = \lambda^{\frac{\varphi}{\vartheta}}(1+n^*)^{\frac{1}{\vartheta}} > \mathcal{N}_2(0) = 1$, $\mathcal{N}_2(\cdot)$ must be cutting $\mathcal{N}_1(\cdot)$ from below at $\widetilde{\mathbb{X}}_1^*$ and $\mathcal{N}_1(\cdot)$ must be cutting $\mathcal{N}_2(\cdot)$ from below at $\widetilde{\mathbb{X}}_2^*$. This means for any $\varkappa \in (0,1)$, we have

$$\begin{aligned}\mathcal{N}_1\left(\varkappa\widetilde{\mathbb{X}}_1^* + (1-\varkappa)\widetilde{\mathbb{X}}_2^*\right) &< \varkappa\mathcal{N}_2\left(\widetilde{\mathbb{X}}_1^*\right) + (1-\varkappa)\mathcal{N}_2\left(\widetilde{\mathbb{X}}_2^*\right) \\ &= \varkappa\mathcal{N}_1\left(\widetilde{\mathbb{X}}_1^*\right) + (1-\varkappa)\mathcal{N}_1\left(\widetilde{\mathbb{X}}_2^*\right),\end{aligned}$$

which contradicts the fact that $\mathcal{N}_1(\cdot)$ is strictly concave. Hence, there exists a unique steady state for (68). In sum, we have shown the existence of a unique $\widetilde{\mathbb{X}}^*$ such that (i) $\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*\right) = \mathcal{N}_2\left(\widetilde{\mathbb{X}}^*\right)$, (ii) $\mathcal{N}_1\left(\widetilde{\mathbb{X}}_t\right) > \mathcal{N}_2\left(\widetilde{\mathbb{X}}_t\right)$, whenever $\widetilde{\mathbb{X}}_t < \widetilde{\mathbb{X}}^*$, (iii) $\mathcal{N}_1\left(\widetilde{\mathbb{X}}_t\right) < \mathcal{N}_2\left(\widetilde{\mathbb{X}}_t\right)$, whenever $\widetilde{\mathbb{X}}_t > \widetilde{\mathbb{X}}^*$, and (iv) $\mathcal{N}_1'\left(\widetilde{\mathbb{X}}^*\right) < \mathcal{N}_2'\left(\widetilde{\mathbb{X}}^*\right)$. In addition, $\mathcal{N}_1(0) > \mathcal{N}_2(0)$.

Finally, we need to show that the unique steady state $\widetilde{\mathbb{X}}^*$ is globally stable. For expositional convenience, rewrite (68) as $\widetilde{\mathbb{X}}_{t+1} = \mathcal{B}\left(\widetilde{\mathbb{X}}_t\right)$, where

$$\mathcal{B}\left(\widetilde{\mathbb{X}}\right) \equiv \left[\frac{\mathcal{N}_1\left(\widetilde{\mathbb{X}}\right)}{\mathcal{N}_2\left(\widetilde{\mathbb{X}}\right)}\right]^{\vartheta}\widetilde{\mathbb{X}}.$$

The above results imply that $\mathcal{B}(0) = 0$, $\mathcal{B}\left(\widetilde{\mathbb{X}}_t\right) > \widetilde{\mathbb{X}}_t$ whenever $\widetilde{\mathbb{X}}_t < \widetilde{\mathbb{X}}^*$, and $\mathcal{B}\left(\widetilde{\mathbb{X}}_t\right) < \widetilde{\mathbb{X}}_t$ whenever $\widetilde{\mathbb{X}}_t > \widetilde{\mathbb{X}}^*$. The unique steady state is globally stable if the slope of $\mathcal{B}(\cdot)$ evaluated at $\widetilde{\mathbb{X}}^*$ is between zero and one. Straightforward differentiation gives

$$\mathcal{B}'\left(\widetilde{\mathbb{X}}\right) = \left[\frac{\mathcal{N}_1\left(\widetilde{\mathbb{X}}\right)}{\mathcal{N}_2\left(\widetilde{\mathbb{X}}\right)}\right]^{\vartheta}\left\{\left[\frac{\mathcal{N}_1'\left(\widetilde{\mathbb{X}}\right)}{\mathcal{N}_1\left(\widetilde{\mathbb{X}}\right)} - \frac{\mathcal{N}_2'\left(\widetilde{\mathbb{X}}\right)}{\mathcal{N}_2\left(\widetilde{\mathbb{X}}\right)}\right]\vartheta\widetilde{\mathbb{X}} + 1\right\},$$

for all $\widetilde{\mathbb{X}} \geq 0$. In particular,

$$\mathcal{B}'\left(\widetilde{\mathbb{X}}^*\right) = \underbrace{\left[\mathcal{N}_1'\left(\widetilde{\mathbb{X}}^*\right) - \mathcal{N}_2'\left(\widetilde{\mathbb{X}}^*\right)\right]}_{(-)}\frac{\vartheta\widetilde{\mathbb{X}}^*}{\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*\right)} + 1 < 1.$$

To show that $\mathcal{B}'\left(\widetilde{\mathbb{X}}^*\right)$ is strictly positive, rewrite the above expression as

$$\mathcal{B}'\left(\widetilde{\mathbb{X}}^*\right) = \frac{1}{\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*\right)}\left\{\vartheta\widetilde{\mathbb{X}}^*\mathcal{N}_1'\left(\widetilde{\mathbb{X}}^*\right) + (1-\vartheta)\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*\right) + \vartheta\left[\mathcal{N}_2\left(\widetilde{\mathbb{X}}^*\right) - \widetilde{\mathbb{X}}^*\mathcal{N}_2'\left(\widetilde{\mathbb{X}}^*\right)\right]\right\},$$

where $\mathcal{N}_2\left(\widetilde{\mathbb{X}}^*\right) - \widetilde{\mathbb{X}}^*\mathcal{N}_2'\left(\widetilde{\mathbb{X}}^*\right) = 1$ due to its linearity property. Hence, $\mathcal{B}'\left(\widetilde{\mathbb{X}}^*\right) > 0$. This completes the proof of Proposition 3.

Proof of Proposition 4

Suppose the conditions in Proposition 3 are satisfied. Then, in the long run, the variable Ψ_t will be growing at the constant rate γ^* specified in (47). In the proof of Proposition 3, we show that the transformed variable $\widetilde{\mathbb{X}}_t \equiv \overline{z}_t^{\varphi} N_{H,t} X_t^{\varrho} \mathbb{Q}_t^{-\frac{\vartheta\psi}{1-\psi}}$ will eventually converge to a unique constant value. This means, in the long run, $\overline{z}_t^{\varphi} N_{H,t} X_t^{\varrho}$ will be growing at the same rate as $\mathbb{Q}_t^{\frac{\vartheta\psi}{1-\psi}}$. Hence, we have

$$\frac{\mathbb{Q}_{t+1}}{\mathbb{Q}_t} = \left[\lambda^{\varphi}\left(1+n^*\right)\left(\frac{X_{t+1}}{X_t}\right)^{\varrho}\right]^{\frac{1-\psi}{\vartheta\psi}}. \tag{71}$$

Substituting (71) into (39) gives (47). In any balanced-growth equilibrium, both $\widehat{c}_t$ and $\widehat{k}_t$ are stationary over time which means both per-capita consumption (i.e., $c_{H,t}$ and $c_{L,t}$) and per-capita physical capital must be growing at the same rate as Ψ_t. Using the Euler equation in (29), we can get

$$1+\gamma^* = \beta^{\frac{1}{\sigma}}\left[1+\left(1-\tau_a^*\right)r^*\right]^{\frac{1}{\sigma}},$$

where $r^* = \psi\rho\alpha\mathbb{A}^*\left(\widehat{k}^*\right)^{\rho\alpha-1} - \delta$. This gives a unique value of $\widehat{k}^*$, which is

$$\widehat{k}^* = \left\{\frac{\psi\rho\alpha\mathbb{A}^*\left(1-\tau_a^*\right)\beta}{\left(1+\gamma^*\right)^{\sigma} - \left[1-\delta\left(1-\tau_a^*\right)\right]\beta}\right\}^{\frac{1}{1-\rho\alpha}} > 0.$$

The value of $\widehat{c}^*$ is uniquely determined by

$$\widehat{c}^* = \mathbb{A}^*\left(\widehat{k}^*\right)^{\rho\alpha} - \left[\left(1+\gamma^*\right)\left(1+n^*\right) - \left(1-\delta\right)\right]\widehat{k}^*.$$

This confirms that there exists a unique pair of values $\left(\widehat{c}^*, \widehat{k}^*\right)$ that satisfies the balanced-growth conditions. Finally, since $c_{s,t}$ and $A_{s,t}$ grow by the factors $\left(1+\gamma^*\right)$ and $\left(1+\gamma^*\right)\left(1+n^*\right)$, respectively, in every period, the transversality condition is satisfied if $\beta\left(1+\gamma^*\right)^{1-\sigma}\left(1+n^*\right) < 1$. For any $\gamma^* > 0$, the assumptions $\sigma \geq 1$ and $\beta\left(1+n^*\right) < 1$ are sufficient to ensure that this condition is satisfied. This establishes the existence and uniqueness of long-run balanced-growth equilibrium.

Proof of Proposition 5

Part (i) To establish this result, it suffice to show that the unique steady-state value $\widetilde{\mathbb{X}}^*$ is strictly increasing in n^*. Recall the two functions, $\mathcal{N}_1(\cdot)$ and $\mathcal{N}_2(\cdot)$, defined in (69) and (70). Set $\overline{\Im} = \overline{\xi} = 1$. Note that only $\mathcal{N}_1(\cdot)$ depends on n^*. To highlight this dependence, we will express this function as $\mathcal{N}_1\left(\widetilde{\mathbb{X}}; n^*\right)$. By the definition of $\widetilde{\mathbb{X}}^*$, we have $\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*; n^*\right) = \mathcal{N}_2\left(\widetilde{\mathbb{X}}^*\right)$. Totally differentiating this with respect to $\widetilde{\mathbb{X}}^*$ and n^* gives

$$\left\{\frac{\partial}{\partial n^*}\left[\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*; n^*\right)\right]\right\} dn^* = \left\{\frac{\partial}{\partial \widetilde{\mathbb{X}}^*}\left[\mathcal{N}_2\left(\widetilde{\mathbb{X}}^*\right) - \mathcal{N}_1\left(\widetilde{\mathbb{X}}^*; n^*\right)\right]\right\} d\widetilde{\mathbb{X}}^*.$$

$$\Rightarrow \frac{d\widetilde{\mathbb{X}}^*}{dn^*} = \frac{\frac{\partial}{\partial n^*}\left[\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*; n^*\right)\right]}{\frac{\partial}{\partial \widetilde{\mathbb{X}}^*}\left[\mathcal{N}_2\left(\widetilde{\mathbb{X}}^*\right) - \mathcal{N}_1\left(\widetilde{\mathbb{X}}^*; n^*\right)\right]}.$$

As explained in the proof of Proposition 3, the slope of $\mathcal{N}_1\left(\widetilde{\mathbb{X}}; n^*\right)$ must be strictly less than that of $\mathcal{N}_2\left(\widetilde{\mathbb{X}}\right)$ at the steady state. Hence, the denominator of the above expression is strictly positive. It is straightforward to show that the numerator is also strictly positive. Hence, an increase in n^* will raise the value of $\widetilde{\mathbb{X}}^*$. This in turn raises the long-term growth rates of $\mathbb{Q}_t$ and X_t.

Part (ii) Rewrite the steady state condition as $\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*; \varpi^*\right) = \mathcal{N}_2\left(\widetilde{\mathbb{X}}^*; \varpi^*\right)$. Totally differentiating this with respect to $\widetilde{\mathbb{X}}^*$ and ϖ^* gives

$$\frac{d\widetilde{\mathbb{X}}^*}{d\varpi^*} = \frac{\frac{\partial}{\partial \varpi^*}\left[\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*; \varpi^*\right) - \mathcal{N}_2\left(\widetilde{\mathbb{X}}^*; \varpi^*\right)\right]}{\frac{\partial}{\partial \widetilde{\mathbb{X}}^*}\left[\mathcal{N}_2\left(\widetilde{\mathbb{X}}^*; \varpi^*\right) - \mathcal{N}_1\left(\widetilde{\mathbb{X}}^*; \varpi^*\right)\right]}.$$

Again, the denominator of this expression is strictly positive. As for the numerator, we have

$$\frac{\partial}{\partial \varpi^*}\left[\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*; \varpi^*\right)\right] = \lambda^{\frac{\varphi}{\vartheta}}\left(1+n^*\right)^{\frac{1}{\vartheta}}\left[1+\Xi\left(\varpi^*\right)\widetilde{\mathbb{X}}^*\right]^{\frac{\varrho}{\vartheta}-1}\frac{\varrho}{\vartheta}\Xi'\left(\varpi^*\right)\widetilde{\mathbb{X}}^* > 0,$$

$$\frac{\partial}{\partial \varpi^*}\left[\mathcal{N}_2\left(\widetilde{\mathbb{X}}^*; \varpi^*\right)\right] = -\left(\lambda^{\frac{\psi}{1-\psi}} - 1\right)\widetilde{\mathbb{X}}^* < 0.$$

Hence, the numerator is also strictly positive. These results show that an increase in ϖ^* will raise the value of $\widetilde{\mathbb{X}}^*$ and the long-run growth rate of X_t. To see that this will also increase the long-run growth rate of $\mathbb{Q}_t$, it suffice to note that (45) is equivalent to (71) when $\widetilde{\mathbb{X}}_t = \widetilde{\mathbb{X}}^*$.

Part (iii) Using the same argument as in part (ii), we can show that an increase in λ will increase the value of $\widetilde{\mathbb{X}}^*$ if and only if $\frac{\partial}{\partial\lambda}\left[\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*;\lambda\right)\right] > \frac{\partial}{\partial\lambda}\left[\mathcal{N}_2\left(\widetilde{\mathbb{X}}^*;\lambda\right)\right]$. From (44) and (45), we can get

$$\frac{\partial}{\partial\lambda}\left[\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*;\lambda\right)\right] = \frac{\varphi}{\vartheta\lambda}\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*;\lambda\right) \qquad \text{and} \qquad \frac{\partial}{\partial\lambda}\left[\mathcal{N}_2\left(\widetilde{\mathbb{X}}^*;\lambda\right)\right] = \frac{\psi}{1-\psi}\lambda^{\frac{\psi}{1-\psi}-1}\left(1-\varpi^*\right)\widetilde{\mathbb{X}}^*.$$

Hence, we have

$$\begin{aligned}
& \frac{\partial}{\partial\lambda}\left[\mathcal{N}_1\left(\widetilde{\mathbb{X}}^*;\lambda\right) - \mathcal{N}_2\left(\widetilde{\mathbb{X}}^*;\lambda\right)\right] \\
= & \frac{1}{\lambda}\left\{\frac{\varphi}{\vartheta}\left[1+\left(\lambda^{\frac{\psi}{1-\psi}}-1\right)\left(1-\varpi^*\right)\widetilde{\mathbb{X}}^*\right] - \frac{\psi}{1-\psi}\lambda^{\frac{\psi}{1-\psi}}\left(1-\varpi^*\right)\widetilde{\mathbb{X}}^*\right\} \\
= & \frac{1}{\lambda}\left\{\frac{\varphi}{\vartheta} + \left[\frac{\varphi}{\vartheta}\left(\lambda^{\frac{\psi}{1-\psi}}-1\right) - \frac{\psi}{1-\psi}\lambda^{\frac{\psi}{1-\psi}}\right]\left(1-\varpi^*\right)\widetilde{\mathbb{X}}^*\right\}.
\end{aligned}$$

This expression is strictly positive if $\frac{\varphi}{\vartheta}\left(\lambda^{\frac{\psi}{1-\psi}}-1\right) \geq \frac{\psi}{1-\psi}\lambda^{\frac{\psi}{1-\psi}}$. But in general this expression can be either positive or negative, hence the result in part (iii) of this proposition.

Proof of Proposition 7

Again consider the two functions $\mathcal{N}_1(\cdot)$ and $\mathcal{N}_2(\cdot)$ as defined in (69) and (70). Set $\overline{\Im} = \overline{\xi} = 1$. When $n^* = 0$, the function $\mathcal{N}_1(\cdot)$ becomes

$$\mathcal{N}_1\left(\widetilde{\mathbb{X}}_t\right) \equiv \lambda^{\frac{\varphi}{\vartheta}}\left[1+\Xi\left(\varpi^*\right)\widetilde{\mathbb{X}}_t\right]^{\frac{\varrho}{\vartheta}}.$$

Note that this does not change the fact that $\mathcal{N}_1(\cdot)$ is continuously differentiable, strictly increasing and strictly concave. Most importantly, since $\lambda^{\frac{\varphi}{\vartheta}} > 1$, it is still the case that $\mathcal{N}_1(0) > \mathcal{N}_2(0) = 1$. Thus, the proof of Proposition 3 remains valid.

Appendix C: Data and Calibration

Total R&D Spending Data on R&D spending are obtained from *National Patterns of R&D Resources: 2009 Data Update*, which is available on the NSF website. As we mentioned in Section 2, we only consider two types of research activities in the current study, namely (i) government-financed basic research, and (ii) applied research and development performed in the private sector. Thus, our measure of total R&D expenditures is defined as the sum of three components in the actual data: (i) basic research funded by the federal government and other governments, (ii) applied research performed by industry and paid by either the government or industry, and (iii) development performed by industry and paid by either the government or industry.

Employment of Researchers Total employment of researchers is defined as the sum of (i) academic and government researchers participating in basic research, and (ii) private-sector researchers participating in applied research and development. The data source for private-sector researchers is described in footnote 16. For the employment of academic researchers, there are three existing data sources. The first one is the National Center for Education Statistics. This source contains data on the number of full-time faculty (instruction, research and public service) employed in degree-granting institutions over the period 1989-2009. The second data source is the Scientists and Engineers Statistical Data System (SESTAT) available on the NSF website. This source contains data on the number of scientists and engineers (whose primary or secondary work activity is research and development) employed in different sectors (business, academia and government). Data are available for the years 1993, 1995, 1997, 1999, 2003 and 2006. A third data source is the Survey of Doctorate Recipients (SDR), which collects data on individuals who have received a doctorate degree in a science, engineering, or health field from a U.S. academic institution. This is the source of the data reported in Stephan (1996, Table 2). We do not use these data because according to the SESTAT, researchers with doctorate degree only accounted for a small fraction of all employed researchers (e.g., 10.2 percent in 2006). Thus, focusing on these researchers alone will significantly understate the total quantity of labor input in research activities. None of these sources classifies researchers by type of research (basic, applied and development).

The following procedure is used to construct our proxies for $\{\theta_t\}_{t=0}^{T^*}$. The first step is to estimate the total number of researchers participating in the two research activities mentioned above for the year 2006. We use this year as our starting value because it is the most recent year in which all necessary data are available. To approximate the number of researchers participating in government-funded basic

research, we multiply the total number of academic and government researchers by the share of basic research expenses in total R&D *performed* by these institutions. According to the SESTAT data, the number of scientists and engineers (whose primary or secondary work activity is research and development) employed in academia (4-year colleges and universities) and government in 2006 were 715,000 and 574,000, respectively. These numbers were taken from Table 13 of *Characteristics of Scientists and Engineers in the United States: 2006.* For the same year, basic research accounted for 71.8 percent of all R&D *performed* by universities and colleges. The corresponding figure for federal and local governments was 11.2 percent. Thus, our estimate for the number of basic researchers in 2006 is $715,000 \times 0.718 + 574,000 \times 0.112 = 577,658$. Similarly, in order to get a proxy for the number of private-sector researchers participating in non-basic research, we multiply the total number of private-sector researchers (3.57 million in 2006) by the percentage of applied research and development spending in total R&D performed by private firms (96.4 percent in 2006). We then take the sum of these two proxies as the total number of researchers in 2006. This number represents 2.78 percent of total U.S. employment in 2006.

The above calculations show two things. First, researchers only account for a small fraction of total employment in the United States. Thus, we equate the values of $\{n_{L,t}\}_{t=0}^{T^*}$ to the annual growth rates of total employment over the period 1953-2009. Second, most of the researchers are employed in the private sector (85.6 percent in 2006). Thus, we equate the values of $\{n_{H,t}\}_{t=0}^{T^*}$ to the annual growth rates of the employment of R&D scientists and engineers in the private sector. Given the values of $\{n_{L,t}, n_{H,t}\}_{t=0}^{T^*}$ and the value of θ_t in 2006, we can derive all other values of θ_t using the following procedure. For any $t > 0$, we have

$$\theta_t \equiv \frac{N_{H,t}}{N_{H,t} + N_{L,t}} \Rightarrow \frac{N_{H,t}}{N_{L,t}} = \frac{\theta_t}{1 - \theta_t}.$$

Combining this and $N_{s,t} = (1 + n_{s,t-1}) N_{s,t-1}$ gives

$$\frac{N_{H,t}}{N_{L,t}} = \frac{(1 + n_{H,t-1}) N_{H,t-1}}{(1 + n_{L,t-1}) N_{L,t-1}} = \frac{\theta_t}{1 - \theta_t}$$

$$\Rightarrow \frac{\theta_t}{1 - \theta_t} = \frac{(1 + n_{H,t-1}) \theta_{t-1}}{(1 + n_{L,t-1}) (1 - \theta_{t-1})}.$$

Note that once we impose the restriction $n_{H,t} = n_{L,t}$ for all $t \geq T^*$, we will have $\theta_t = \theta_{T^*}$ for all $t > T^*$.

Subsidy Rate on Private R&D The actual calculation of corporate R&D tax credit under the U.S. federal tax code is very a complicated process. For this reason, the values of $\{\tau_{d,t}\}_{t=0}^{T^*}$ in our model are

Table A1 Effective R&D Tax Credit

	Bloom *et al.* (2002) (Percent)	Hall (1993) (Percent)		Bloom *et al.* (2002) (Percent)	Hall (1993) (Percent)
1979	0.05	—	1989	3.80	2.19
1980	0.05	0.0	1990	12.40	7.69
1981	5.63	3.04	1991	12.40	7.49
1982	5.63	5.05	1992	12.40	—
1983	5.63	3.64	1993	12.22	—
1984	5.63	4.60	1994	12.22	—
1985	5.63	5.01	1995	12.22	—
1986	4.57	3.11	1996	12.22	—
1987	4.61	2.66	1997	12.22	—
1988	4.61	3.50			

only intended to capture the *effective* subsidy rates provided by the U.S. system. To compute these values, we follow the conceptual framework described in Bloom *et al.* (2002). Let A_t^d be the amount of corporate income tax deduction for each dollar of R&D investment at time t, and let A_t^c be the amount of tax credit for each dollar of R&D investment. Then, the user cost of private R&D investment at time t is determined by

$$\rho_t^d \equiv \left[\frac{1-\left(A_t^d + A_t^c\right)}{1-\tau_{p,t}}\right](r_t+\delta)\,,$$

where $\tau_{p,t}$ is the corporate income tax rate at time t, r_t is the real interest rate and δ is the depreciation rate of firm's assets. Our goal here is to recover the values of A_t^c using the information provided in Bloom *et al.* (2002). First, the estimated values of ρ_t^d for the United States over the period 1979-1997 can be found in their Table A2. These authors assume that the real interest rate is ten percent throughout the sample period. The corporate income tax rates that they used can be found in their Appendix A9. Since they also include state taxes on corporate income, the values that they used are slightly different from the ones in Figure 7. The depreciation rate that they used is a weighted average for three different types of assets (current expenditure on R&D, buildings, and plant and machinery). Based on the information provided on pages 6-7 in Bloom *et al.* (2002), we get a value of 27.94 percent for δ. Since private R&D expenditures are fully deductible from corporate income tax, we set $A_t^d = \tau_{p,t}$. The implied values of A_t^c

are shown in Table A1. We also compare these values to the average effective R&E tax credit reported in Hall (1993, Table 3). For years prior 1979, we assume that the subsidy rate is the same as in 1979, which is 0.05 percent. For years after 1997, we assume that the subsidy rate has remained constant since 1997.

Before-tax Real Interest Rate The before-tax real interest rate is computed using the procedure described in Cooley and Prescott (1995). Let Y_{KP} be the amount of income on fixed private capital, which is defined as

$$Y_{KP} = \text{Unambiguous Capital Income} + \theta_P \times (\text{Ambiguous Capital Income}).$$

"Unambiguous capital income" refers to the sum of rental income, corporate profits and net interest. "Ambiguous capital income" is the sum of proprietors' income and net national product (NNP) minus national income. The variable θ_P is defined according to

$$\theta_P = \frac{\text{Unambiguous Capital Income +DEP}}{\text{DEP - Proprietors' Income + National Income}},$$

where DEP denotes consumption of fixed capital, which is the difference between GNP and NNP. The before-tax rate of return (r) is then given by $r = (Y_{KP} - DEP)/K_P$, where K_P denotes private fixed asset. All the data involved in this computation are available from the NIPAs.

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