Relaxing competition through speculation: Committing to a negative supply slope*

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Abstract

We demonstrate how commodity producers can take strategic speculative positions in derivatives markets to soften competition in the spot market. In our game, suppliers first choose a portfolio of call options and then compete in supply functions. In equilibrium firms sell forward contracts and buy call options to commit to downward sloping supply functions. Although this strategy is risky, it reduces the elasticity of the residual demand of competitors, who increase their mark-ups in response. We show that this type of strategic speculation increases the level and volatility of commodity prices and decreases welfare.

Keywords: Supply function equilibrium, Option contracts, Strategic commitment, Speculation

JEL codes: C73, D43, D44, G13, L13, L94

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1 Introduction

The trade in commodity derivatives is widespread and trading volumes often surpass that of the underlying commodities.[1] Ideally derivatives markets improve market efficiency as they allow firms to manage risk and facilitate price discovery by aggregating information across market participants.[2] However this paper demonstrates that when producers are strategic, the introduction of a derivatives market increases spot market volatility and harms competition.

In Allaz and Vila's (1993) seminal work on strategic contracting, producers first sell forward contracts and then compete in a Cournot spot market; the introduction of a financial market improves competition, lowers prices and increases total surplus.[3]We generalize this model by (1) considering a larger class of derivatives contracts, (2) by generalizing the form of spot market competition and (3) by introducing uncertainty: In our model producers first choose a portfolio of call option contracts with a range of strike prices and then compete in supply functions in a spot market with uncertain demand as in Klemperer and Meyer (1989), Green and Newbery (1992).[4] The results of Allaz and Vila (1993) are reversed in this more general setting as a new channel is identified through which derivatives markets affects market outcomes.[5]

We show that each producer uses derivatives to commit to a downward sloping supply function, i.e. to produce more when prices are low and less when they

[1]Commodity derivatives markets have seen a 60-fold increase in the value of trade between 1998 and 2008. In 2008 the outstanding value of commodity derivatives equaled $13 trillion. This is twice the worldwide output of commodities, and about 21% of world GDP.

[2]The effect of derivatives trade is a point of debate in the finance literature. Some authors claim that it reduces the variance and level of spot prices and improves price information (Turnovsky, 1983; Cox , 1976; Korniotis, 2009), while other claim the opposite (Hart and Kreps, 1986; Stein, 1987; Figlewski, 1981).

[3]Brandts et al. (2008) confirm Allaz and Vila's (1993) results in economic experiments. In Willems (2005), the Allaz and Vila effect becomes stronger if producers sell a portfolio of financial option contracts and then compete in quantities. Holmberg (2011) shows that contracting is weakly pro-competitive when marginal costs are constant and firms compete in supply functions in the spot market. In Newbery (1998) producers sell contracts to deter entry. Green (1999) shows that forward contracting does not influence competition in markets with linear marginal costs and linear demand if producers coordinate on linear supply function equilibria.

[4]Anderson and Xu (2005; 2006), Anderson and Hu (2012), Aromi (2007), Chao and Wilson (2005) and Niu et al. (2005) have also analyzed how exogenously given forward or option contracts influence supply function competition. But they do not analyze to what extent contracting is strategically driven.

[5]Financial markets also have anti-competitive consequences in Ferreira (2003), who shows that only the monopoly outcome is renegotiation-proof for an infinite number of contracting rounds. In Mahenc and Salanié (2004), Allaz and Vila's result is reversed if firms compete in prices (instead of quantities) on the spot market.

are high.[6] As illustrated in Figure 1 this commitment makes the residual demand curve for each of its competitors steeper (less price-sensitive) and induces competitors to increase mark-ups and reduce total output. In the aggregate, producers commit to a downward sloping supply function, which increases the volatility of the spot price as even a small demand shock will cause large price fluctuations. This *anti-competitive effect* is partly mitigated when demand uncertainty increases. This suggests that option contracts should not be traded near delivery because firms then have a good estimate of demand. Alternatively, the same option contract or supply function should be valid for several delivery periods in order to increase the range of demand levels that contracts need to cover.

For a producer that has sold commodity derivatives, an increase in the spot price will not only increase its spot market revenue; but also its contracted liability, i.e. the value of derivatives contracts sold. Thus after selling derivatives contracts the producer becomes relatively more concerned about volume than mark-ups.[7] In order to commit to a downward sloping supply function a firm sells (or hedges) a large fraction of its output with contracts when the spot price is low, which commits the firm to a high output, and a small fraction when the spot price is high, which commits the firm to a low output. More generally a strategic producer sells a portfolio, for which the liability's sensitivity to the spot price is positive (the *delta* is positive), but decreasing (the *gamma* is negative).[8] When the spot price is low it produces a lot, as a reduction of the production level would increase the price and its contracted liability would soar. On the other hand, when the spot price is high the liability is less sensitive to the spot price and the firm produces less.

A producer can achieve a portfolio with this property by trading option contracts. A call option hedges the buyer against high spot prices; essentially it gives him the right to procure one unit of the good from the seller at a predetermined price, the option's strike price. The contract is only exercised when the spot price

[6] Vives (2011) show that supply function equilibria can be downward sloping, but he considers a different setting with no contracts and private production costs. His mechanism is also different, producers hold back supply at high prices in order to avoid a winner's curse when production costs are sufficiently correlated.

[7] This has been shown for alternative settings by Allaz and Vila (1993), von der Fehr and Harbord (1992), Newbery (1998) and Green (1999). de Frutos and Fabra (2012) show that there are sometimes exceptions to this rule in markets where offers are required to be stepwise. The strategic effect of forward contracts has been shown empirically by Wolak (2000) and Bushnell et al. (2008).

[8] In mathematical finance Greek letters are used to describe the sensitivity of a portfolio to the underlying instrument's price. *Delta* and *Gamma* are the first and second derivative of the portfolio's value with respect to the underlying price.

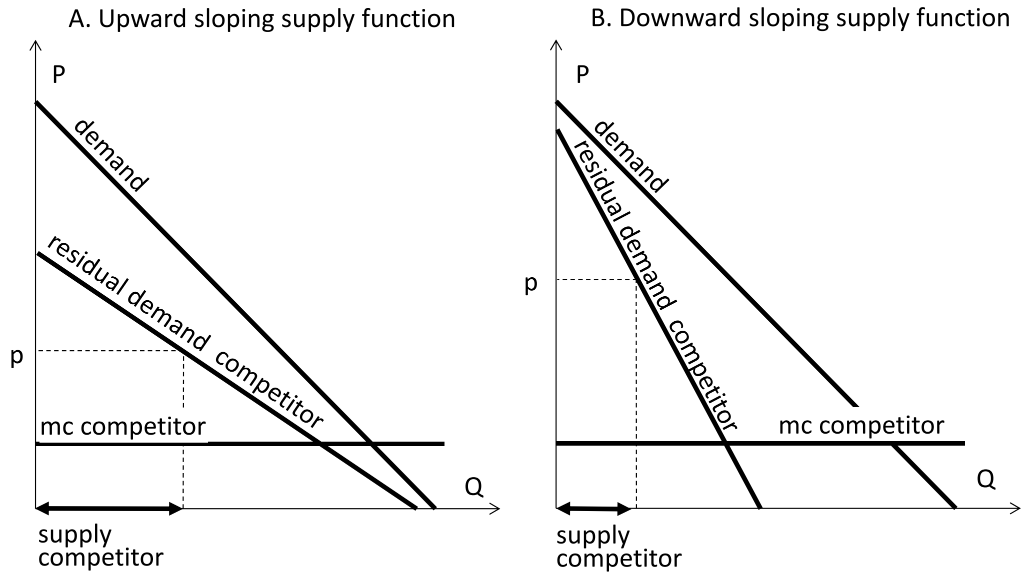

Figure 1: The residual demand function and the production level of a firm's competitor, if the firm bids (A) an upward sloping or (B) a downward sloping supply function.

is above the strike price. By selling a carefully designed portfolio of call-option contracts a producer can fully hedge its risk, that is, ensure that changes in operational profit are perfectly offset by changes in the liability of its contracting portfolio.[9] But we show that a producer instead has incentives to commit to a supply function with a negative slope by *selling* a forward contract and by *buying* a portfolio of call options with a range of strike prices.[10] The forward contract is a promise for future delivery, which creates a liability for the producer. As the spot price increases, the producer will also exercise an increasing amount of call options, and thus partially offset its forward position. So the producer will successively reduce its hedged output (net-sales with contracts) or equivalently the price sensitivity of its liability, as the spot price increases. This is a risky strategy, but the commitment increases the firm's expected profit.

The supply function model is obviously well-suited for spot markets where producers sell their output in a uniform-price auction, as in wholesale electricity markets (Green and Newbery, 1992; Holmberg and Newbery, 2010). This has also been empirically verified.[11] Until recently a handful of electricity producers in the

[9] A forward contract only hedges the price risk, while a portfolio of call options can in addition be used to hedge volume risk (Bessembinder and Lemmon, 2002; Willems and Morbee, 2010).

[10] It follows from the put-call parity that this can also be achieved with a portfolio of forward contracts and put options.

[11] Hortacsu and Puller, (2008), Sioshansi and Oren, (2007) and Wolak (2003) verify that large

Nordic countries regularly made offers that were (partly) downward sloping,[12] and contracting was the main reason for this.[13] In this paper we identify a mechanism that might explain this contracting and bidding behavior. Perhaps producers use contracts to commit to a downward sloping supply in order to soften competition? Related results are found in an empirical study of the German electricity market by Willems et al. (2009). They find no evidence of producers selling call option contracts or equivalent contracts to hedge their output.[14] This is in line with our results as we predict that firms have no incentive to sell call option contracts, but to buy them instead.

Although most markets are not explicitly cleared by uniform-price auctions, Klemperer and Meyer (1989) argue that firms typically face a uniform market price and they need predetermined decision rules for its lower-level managers on how to deal with changing market conditions. Thus firms implicitly commit to supply functions also in the general case. Indeed, Vives (2011) notes that competition in supply functions has been used to model bidding for government procurement contracts, management consulting, airline pricing reservation systems, and provides a reduced form in strategic agency and trade policy models.

The results of our paper have some parallels with delegation games. The main

producers in the electricity market roughly bid as predicted by the SFE model.

[12]Downward sloping supply bids or *illogical bids* were allowed in the Nordic power exchange (Nord Pool) until it introduced a new spot trading algorithm, SESAM, on October 10, 2007 (NPS Exchange Info 15/2007 and 49/2007). This change was motivated among other things by the concern that illogical bids could facilitate market manipulation (NVEs decision dated August 31, 2007, which approved the change). Other concerns were operational difficulty of handling illogical bids and planned market integration with Germany. Statkraft, a Norwegian producer, claimed that illogical bids are sometimes necessary to optimize the operation of mutually owned hydro power plants.

Note that total supply of a producer consists of its supply in the power exchange plus its supply delivered directly to consumers with bilateral contracts. So even though the power exchange itself does not allow for downward sloping supply bids, total supply could still be downward sloping, as long as bilaterally contracted supply is sufficiently downward sloping.

[13]The meeting of Nord Pool Spot's product group for the physical market (September 15, 2005) discusses partly downward sloping (illogical) bids. According to the minutes "... companies must at times show the combination of production, consumption and *contracts* in hourly bids in a seemingly illogical way" (emphasis added). Note that a vertically integrated firm which is active both in production and retail has some flexibility in setting supply and demand bids, as only its level of net supply (or net demand) will matter. Hence a firm might be able to submit a slightly downward sloping supply bid and a very elastic demand bid, while still having a net supply that increases with the spot price. However, an illogical net-supply can only be explained by the firm's contracts.

[14]Willems et al. (2009) compare two contracting scenarios for the German electricity market: one with standard forward contracts and another with load following contracts. The latter corresponds to firms selling forward contracts and several call option contracts such that the same fraction of output is hedged for each price level in the spot market. They find that the first scenario fits the data best.

differences between our paper and the delegation literature is that we use financial contracts instead of delegation as the commitment device, and that we allow firms to commit to supply functions with any slope. Singh and Vives (1984) and Cheng (1985) analyze a game where the owner of each firm first decides the slope of its supply. The slope is either horizontal (Bertrand) or vertical (Cournot). The implementation of this decision is delegated to the firm's manager, who sets either the firm's price or its output depending on the owner's choice. In equilibrium, each firm commits to play Cournot when demand is certain. As in our model, this makes the residual demand function of competitors less elastic, it softens competition and leads to higher mark-ups. Reisinger and Ressner (2009) show that if demand is sufficiently uncertain, firms commit to play Bertrand. Thus as in our model, uncertainty makes the market more competitive.

The structure of the paper is as follows. The model of strategic option contracting is introduced in Section 2 and its main properties are derived in Section 3. Section 4 presents closed-form results when demand is linear and demand shocks are Pareto distributed of the second-order. Section 5 concludes.

2 Model

We model producers' contracting and supply strategies as a two-stage game. In the first stage, N risk-neutral producers commit by strategically choosing a portfolio of call option contracts with a spectrum of strike prices. Firms' contracting decisions are made simultaneously. Similar to Allaz and Vila (1993), Newbery (1998) and Green (1999), producers disclose their contracting decisions.[15] Risk-neutral, non-strategic counterparties (e.g. consumers or investment banks) with rational expectations ensure that each option price corresponds to the expected value of the contract. This rules out any arbitrage opportunities in the market. In the second stage, firms compete in the spot market. It is a uniform-price auction in which sellers simultaneously submit supply functions. After these offers have been submitted, an additive demand shock is realized. The distribution of the

[15] Contracting is strategic when firms are risk-averse or when contract positions are observable (Hughes and Kao, 1997). Financial trading is anonymous in most markets, and a firm's contract positions are normally not revealed to competitors. Still competitors can get a rough estimate of the firm's forward position by analyzing the turnover in the forward market and the forward price (Ferreira, 2006). Ferreira's theoretical argument is also relevant in practice. Van Eijkel and Moraga-González's (2010) find that firms in the Dutch gas market are able to infer competitors' contract positions and that contracts are used for strategic rather than hedging reasons. Finally, it can be noted that vertical integration with a retailer that is selling the good at a fixed retail price is equivalent to observable contracting.

shock is common knowledge.

A financial call option gives the buyer the right to receive the difference between the spot market price and a predetermined strike price r. The contract is exercised when the spot price is above the strike price. In stage 1, firm $i \in \{1, 2, ..., N\}$ decides how many option contracts to sell (or buy) at each strike price. We assume that 0 and $\overline{p}$ are the lowest and highest realized prices in the market, respectively. The contracting decision is represented by the distribution function $X_i(r) : [0, \overline{p}] \to \mathbb{R}$, the amount of option contracts sold with a strike price less than or equal to r. Firm i can decide to go short ($X_i(r) > 0$) or long ($X_i(r) < 0$). A forward contract corresponds to a call option with strike price zero, as it is always exercised. Thus $X_i(0)$ is the amount of sold forward contracts. We let $X(r) = \sum_{i=1}^{N} X_i(r)$ represent the contracting decision of the industry and $X_{-i}(r) = X(r) - X_i(r)$ those of firm i's competitors.

Let $\sigma(r)$ be the price of an option with strike price r in the contracting market. Producer i's revenue from selling call options in the contracting market is given by: [16]

$$\overline{V}_i = \int_0^{\overline{p}} \sigma(r) \cdot dX_i(r).$$

All call options that are in the money, i.e. for which the strike price is below the spot price, will be exercised. Thus for a given spot price p, the total value $V_i(p)$ of firm i's sold contracts is given by: [17]

$$V_i(p) = \int_0^p (p - r)\, dX_i(r) = \int_0^p X_i(r) dr.$$

Note that the sensitivity of this contract payment with respect to the spot price (the *delta* of the sold portfolio) is exactly equal to $X_i(p)$

$$\frac{dV_i(p)}{dp} = X_i(p). \tag{1}$$

For a given spot price p and output q, firm i's profit from trading in the contract and the spot markets is equal to the revenue from sold contracts $\overline{V}_i$ and spot market sales $p\,q$, minus the cost of exercised contracts $V_i(p)$ and the

[16] We use the Lebesgue-Stieltjes integral, which is standard in probability theory, to integrate over the contract positions.

[17] Note that $P(\varepsilon) - r$ is continuous in r. The second equality follows from the integration by parts formula for the Lebesgue-Stieltjes integral, where one of the factors is continuous at each point (Hewitt and Stromberg, 1965).

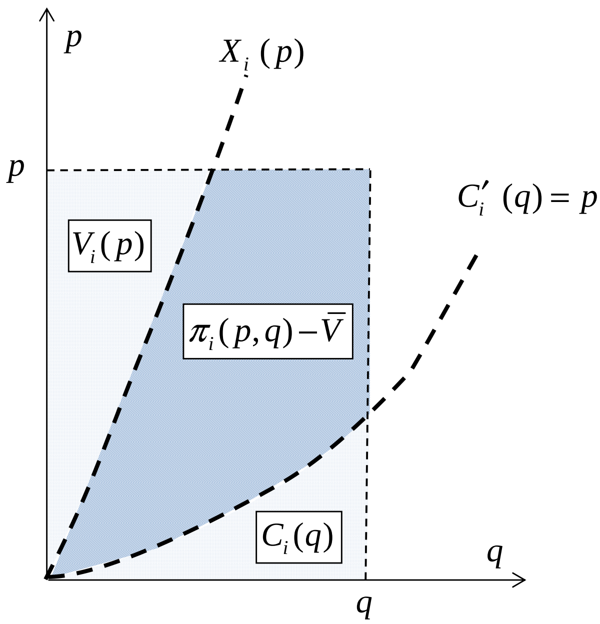

Figure 2: Profit of firm i as a function of p and q.

production cost $C_i(q)$. (See Figure 2)

$$\pi_i(p, q) = \overline{V}_i + pq - V_i(p) - C_i(q). \tag{2}$$

Similar to Green and Newbery (1992), and Klemperer and Meyer (1989) we assume that the cost function $C_i(q)$ is common knowledge, increasing, convex and twice differentiable.

Each producer's supply decision in stage 2, the spot market, is represented by a supply function denoted by $Q_i(p)$. We assume that $Q_i(p)$ and $X_i(p)$ are twice differentiable on $(0, \overline{p})$ and continuous at $\overline{p}$. We let $Q\,(p) = \sum_{i=1}^{N} Q_i(p)$ be the total output of the industry and $Q_{-i}\,(p) = Q\,(p) - Q_i(p)$ that of firm i's competitors.

As in Klemperer and Meyer (1989), demand $D(p, \varepsilon)$ is realized after offers to the spot market have been submitted. The demand function is twice differentiable with respect to the spot price p and is subject to an exogenous additive shock, ε. Hence,

$$D(p, \varepsilon) = D(p) + \varepsilon. \tag{3}$$

The demand function is concave ($D''\,(p) \leq 0$) and downward sloping ($D'\,(0) < 0$). ε is the intercept of the demand function, so $D\,(0) = 0$. We define $D_i(p) := D(p) - Q_{-i}(p)$ as the residual demand of firm i when the demand shock is zero, $\varepsilon = 0$. When firms make their contracting decisions in the first stage, the shock density and its probability distribution are given by $f(\varepsilon)$ and $F(\varepsilon)$, respectively.

The shock density has support on $[0, \overline{\varepsilon}]$ where $\overline{\varepsilon} \in (0, \infty)$, and on this interval $f(\varepsilon)$ is differentiable and positive, $f(\varepsilon) > 0$. The variance of the demand shock is bounded.

For any given demand shock, ε, the spot price is implicitly defined by the market clearing condition: aggregate supply should be equal to total demand. The price function $P(\varepsilon)$ maps the demand shock ε, to the spot price p.

$$P(\varepsilon) : \varepsilon \mapsto p : \; Q(p) = D(p) + \varepsilon.$$

To guarantee existence of an equilibrium price, we assume as in Klemperer and Meyer (1989) and Vives (2011) that all agents' profits will be zero if the market does not clear.

Firm i's expected profit from trading in the contract and the spot markets is:

$$\Pi_i = \int \pi_i \left(P\left(\varepsilon\right), Q_i\left(P\left(\varepsilon\right)\right)\right) dF(\varepsilon) \; . \tag{4}$$

Risk-neutral, price-taking consumers or investment banks trade in the contract market and ensure that the following no-arbitrage condition is satisfied for each strike price r.

$$\forall r : \quad \sigma\left(r\right) = E_{\varepsilon}\left[\max\left(P\left(\varepsilon\right) - r, 0\right)\right]. \tag{5}$$

Hence, the value of the call option is equal to the expected second stage payment from the contract.

3 Analysis

We solve the game by means of backward induction. The properties of Nash equilibria in the second stage spot market are analyzed in Section 3.1. In Section 3.2, we rely on no-arbitrage conditions to derive the expected profit in stage 1 given the contracting position of firms. We derive conditions for optimal contracting in Section 3.3.

3.1 The spot market

In the second stage of the game, each firm i observes its competitors' portfolio of option contracts and then chooses its supply function $Q_i(p)$ to maximize the firm's expected profit given the competitors' spot market bids $Q_{-i}(p)$. Our first proposition generalizes the first-order condition in Klemperer and Meyer (1989), so that it applies to a producer holding a portfolio with a range of option contracts:

Proposition 1 (FOC Spot Market) *The necessary first-order conditions (FOC) for a Nash equilibrium in the spot market are given by the following system of ordinary differential equations:*

$$\forall i, \forall p \in (0, \overline{p}) : \quad Q_i(p) - X_i(p) - [p - C_i'(Q_i(p))] \left[Q_{-i}'(p) - D'(p)\right] = 0. \tag{6}$$

Proof. Substituting the market clearing condition

$$Q_i(P(\varepsilon)) = D_i(P(\varepsilon)) + \varepsilon$$

in firm i's objective function (4) we obtain

$$\Pi_i = \int \pi_i(P(\varepsilon), D_i(P(\varepsilon)) + \varepsilon)\, dF(\varepsilon).$$

The first order condition can be found by pointwise differentiation of the integrand with respect to $p = P(\varepsilon)$. Using expression (2) for firm i's profit, we derive the marginal effect of a price increase for a given demand shock ε.

$$\begin{aligned} \frac{d\pi_i(p, D_i(p) + \varepsilon)}{dp} &= D_i(p) + \varepsilon - X_i(p) + (p - C_i'(D_i(p) + \varepsilon))\, D_i'(p) \qquad (7) \\ &= Q_i(p) - X_i(p) + (p - C_i'(Q_i(p)))\, D_i'(p) \end{aligned}$$

The generalized Klemperer and Meyer equations (6) follow from equating this expression to zero ($\frac{d\pi_i}{dp} = 0$) and observing that $D_i'(p) = D'(p) - Q_{-i}'(p)$. ■

Intuitively, we can interpret the first-order condition as follows. A price increase gives a higher spot market revenue for existing quantities, $Q_i(p)$, but it increases the payment firm i needs to make for its contracted obligation, $V_i'(p) = X_i(p)$. Moreover, a price increase will reduce sales volumes, which reduces profits by $p|D_i'(p)|$, but the lower volume will also lead to production cost savings equal to $C_i'(D_i(p))|D_i'(p)|$.

We can now derive a sufficient condition for the solution of the system of first order conditions (FOC) to be a Nash equilibrium.[18]

Proposition 2 (NE Spot Market) *A tuple* $\check{\mathbf{Q}} = \left\{\check{Q}_i(p)\right\}_{i=1}^{N}$ *which satisfies the first order conditions of the second stage game, i.e. the generalized Klemperer and Meyer equations (6) constitutes a Nash equilibrium (NE) in the second-stage if:*

[18] Proposition 2 generalizes previous results for spot markets without contracting by Klemperer and Meyer (1989) and Holmberg et al. (2008). Note that our sufficient conditions are less restrictive than theirs as we do not require monotonic supply functions.

1. *The slope of total supply is larger than the slope of the demand function* $\check{Q}'(p) > D'(p)$ *on the price interval* $(0, \overline{p})$.

2. *Each firm* i *faces a downward sloping residual demand function or has sufficiently flat marginal cost functions. That is* $\check{D}_i'(p)\, C_i''(q) \leq 1$ $\forall q \geq 0$ *and* $\forall p \in (0, \overline{p})$, *where* $\check{D}_i(p) = D(p) - \check{Q}_{-i}(p)$.

Proof. Consider an arbitrary firm i. It takes contract positions $\{X_i(p)\}_{i=1}^N$ as given and assumes that its competitors bid $\check{Q}_{-i}(p)$ as supply. Thus firm i is facing the residual demand $\check{D}_i(p) + \varepsilon$. We prove that bidding $\check{Q}_i(p)$ is profit maximizing for firm i. When demand shock ε^* occurs, the market price is p^* if firm i makes the offer $\check{Q}_i(p^*)$, i.e. $\check{Q}_i(p^*) = \check{D}_i(p^*) + \varepsilon^*$. We will prove that firm i's profit reaches a global maximum at p^* along its residual demand $\check{D}_i(p) + \varepsilon^*$ for every shock outcome ε^*. That is, producing $\check{Q}_i(p^*)$ is ex-post optimal for firm i.

With the offer $\check{Q}_i(p)$, the first-order condition in (6) is satisfied for every price. Subtracting it from (7) yields:

$$\frac{d\pi_i\left(p, \check{D}_i(p) + \varepsilon^*\right)}{dp} = \left[\check{D}_i(p) + \varepsilon^* - \check{Q}_i(p)\right] - \check{D}_i(p)\left[C_i'(\check{D}_i(p) + \varepsilon^*) - C_i'(\check{Q}_i(p))\right] \tag{8}$$

According to the mean value theorem there must exist ξ between $\check{D}_i(p) + \varepsilon^*$ and $\check{Q}_i(p)$ such that $C_i'(\check{D}_i(p) + \varepsilon^*) - C_i'(\check{Q}_i(p)) = C_i''(\xi)\left[\check{D}_i(p) + \varepsilon^* - \check{Q}_i(p)\right]$. So Equation 8 can be rewritten as:

$$\frac{d\pi_i\left(p, \check{D}_i(p) + \varepsilon^*\right)}{dp} = \left[\check{D}_i(p) + \varepsilon^* - \check{Q}_i(p)\right]\left(1 - \check{D}_i(p)\, C_i''(\xi)\right)$$

The second condition of the proposition implies that $\check{D}_i(p) \cdot C_i''(\xi) \leq 1$. So the second factor of the expression is always non-negative. The market clears at price p^* when firm i offers $\check{Q}_i(p)$, so the first factor is zero when $p = p^*$. The first condition of the proposition implies that $\check{Q}_i'(p) > \check{D}_i'(p)$, so the first factor is negative when prices p are above p^* and positive for prices below p^*. Hence we have shown that:

$$\frac{d\pi_i\left(p, \check{D}_i(p) + \varepsilon^*\right)}{dp} \begin{cases} \leq 0 & \text{if } p > p^* \\ = 0 & \text{if } p = p^* \\ \geq 0 & \text{if } p < p^* \end{cases}$$

which is sufficient for a global profit maximum at p^*. We can use the same argument for all shocks ε and all firms i and we can conclude that the tuple $\check{\mathbf{Q}} =$

$\left\{\check{Q}_i(p)\right\}_{i=1}^{N}$ constitutes a Nash equilibrium. ■

Proposition 3 *If firm i sold call options that trace its marginal cost, that is $p = C_i'(X_i(p))\ \forall p \in (0, \overline{p})$, then it bids competitively in the spot market and has constant profits, provided it faces a downward sloping residual demand function or has sufficiently flat marginal cost functions. That is $D_i'(p)\, C_i''(q) < 1\ \forall q \geq 0$ and $\forall p \in (0, \overline{p})$.*

Proof. The call options sold trace the firm's marginal costs, so it is obvious that bidding $Q_i(p) = X_i(p)$ satisfies the necessary first order conditions in Proposition 1. It is also a global optimum for firm i, if the two conditions in Proposition 2 are satisfied. By assumption we have $p = C_i'(X_i(p))$ and $D_i'(p)\, C_i''(q) < 1$ $\forall p \in (0, \overline{p})$, so it follows that $Q_i'(p) = X_i'(p) = \frac{1}{C_i''(X_i(p))} > D_i'(p)$. Hence the first condition is satisfied. The second condition is satisfied by assumption. Given this bidding strategy, firm i's profit is constant:

$$\pi_i(p, Q(p)) = \overline{V}_i + pQ_i(p) - V_i(p) - C_i(Q_i(p)) = \overline{V}_i.$$

This can shown by partial integration or by studying Figure 2. ■

Thus a firm can sell a portfolio that hedges its profit perfectly, but as a result, it will end up selling at marginal cost. If a firm would like to use its market power, it should not hedge all of its capacity.

3.2 The financial market: perfect arbitrage

The no-arbitrage condition (5) is valid for any contracting choice made by the producers. By using this condition and reversing the order of integration, we can rewrite the contracting revenue of firm i:

$$\begin{aligned} \overline{V}_i &= \int_0^{\overline{p}} \sigma(r) \cdot dX_i(r) = \int_0^{\overline{p}} E_\varepsilon \left[\max\left(P(\varepsilon) - r, 0\right)\right] \cdot dX_i(r) \qquad (9) \\ &= E_\varepsilon \left[\int_0^{P(\varepsilon)} \left(P(\varepsilon) - r\right) \cdot dX_i(r)\right] = E_\varepsilon \left[V_i(P(\varepsilon))\right]. \end{aligned}$$

Thus due to perfect arbitrage, the contracting revenue is equal to the expected realized value of the portfolio. We substitute the contract revenue (9) into the pay-off (2). Thus the expected payoff in (4) can be written:

$$\Pi_i = E_\varepsilon\left[\pi_i(\varepsilon)\right] = E_\varepsilon\left[P(\varepsilon) \cdot Q_i(P(\varepsilon)) - C_i(Q_i(P(\varepsilon)))\right]. \qquad (10)$$

Similarly to Allaz and Vila (1993) and Newbery (1998), firm i's expected pay-off does not depend on the contract position directly, but by selling contracts (X_i), it can strategically change the price in the spot market $P(\varepsilon)$.

3.3 The financial market: strategic contracting

In this subsection we solve for equilibrium contracts. In order to simplify our notation we let

$$H(x) := N(N-1)\frac{1-F(x)}{f(x)} \tag{11}$$

be the inverse hazard rate of the probability distribution of the demand shock multiplied by $N(N-1)$, which is a measure of the interaction effect between firms. For simplicity we set cost equal to zero in the remainder of the paper. Firm k's expected profit in the first stage in (10) can then be simplified to $E_\varepsilon\left[P\left(\varepsilon\right)Q_k(P\left(\varepsilon\right))\right]$. We also make a weak assumption on $H(x)$.[19]

Assumption 1 *1. Production costs are zero, i.e.* $C_i(Q_i) \equiv 0$, $\forall i$

2. The inverse hazard rate is decreasing or mildly increasing, $H'(\varepsilon) < 1$ $\forall \varepsilon \in (0, \overline{\varepsilon})$, *i.e. the hazard rate is increasing or mildly decreasing.*

Firm k maximizes profit by trading derivatives in stage 1, taking into account that the spot market outcome should satisfy the generalized Klemperer and Meyer FOC conditions (6), and that clearing of the spot market requires that spot demand must equal spot supply. Hence, firm k's optimal contracting level is determined by the optimal control problem below. We refer to it as firm k's Mathematical Program with Equilibrium Constraints (MPEC). This solution concept is further discussed in Appendix A.

$$\text{MPEC}(k): \qquad \max \int_0^{\overline{p}} p \cdot Q_k(p) \cdot dF\left(\varepsilon\left(p\right)\right). \tag{12}$$

$$s.t. \begin{cases} \forall i: \quad Q_i(p) - X_i(p) = pQ'_{-i}(p) - pD'(p) \\ \qquad\qquad Q\left(p\right) = \varepsilon(p) + D\left(p\right) \end{cases}$$

In order to calculate the contracting levels in equilibrium, we solve all firms' MPEC problems simultaneously. This is sometimes referred to as an Equilibrium Problem with Equilibrium Constraints (EPEC), see Appendix A.

[19] Most standard probability distributions, such as the normal and uniform distributions, have increasing hazard rates. According to Bulow and Klemperer (2002) it is therefore a weak assumption to only consider probability distributions with increasing hazard rates, i.e. decreasing inverse hazard rates, $H'(\varepsilon) \leq 0$. Note that our assumption is even weaker, as we allow for $H'(\varepsilon) < 1$.

Proposition 4 provides necessary first order conditions for symmetric equilibria and shows that any solution satisfying those equations is on the equilibrium path of a Subgame Perfect NE (SPNE). There can be multiple SPNE when subgames do not have unique equilibria. In such subgames, we use Pareto dominance to refine the set of Nash equilibria. We show that the EPEC solution can weakly be implemented as a Pareto Optimal Subgame Perfect Nash Equilibrium (formal definition in Appendix A). This implies that firms play a Pareto optimal NE in the subgame along the equilibrium path. In subgames off the equilibrium path firms play almost Pareto optimal NE; none of the firms can gain more than an arbitrary small amount ϵ by coordinating on any other equilibrium without making another firm worse off.

Proposition 4 (WPO-SPNE) *Under Assumption 1 any symmetric solution of the set of problems $k = 1...N$ in Equation (12) has to satisfy the following first order conditions:*

$$
\begin{aligned}
H(\varepsilon(p)) &= (pD(p))' + \varepsilon(p) + (N-1)^2 p\varepsilon'(p) \qquad (13)\\
Q(p) &= D(p) + \varepsilon(p) \qquad (14)\\
X(p) &= (pD(p))' + \varepsilon(p) - (N-1)p\varepsilon'(p) \qquad (15)
\end{aligned}
$$

for $\varepsilon(p) \in [\varepsilon_0, \bar{\varepsilon}]$, where $\varepsilon_0 \in [0, \bar{\varepsilon}]$ and $P(\varepsilon_0) = 0$. Moreover, solutions to these equations are weakly implementable as a Pareto Optimal SPNE.

Proof. This follows from Lemmas 1-3 in Appendix B. ■

For a better understanding of the strategic interactions in our game, we take a brief look at firm 1's residual demand function $D_1 + \varepsilon$. It is equal to total demand (term I) minus output of competitors, which is the output that they sell in the contract market (Term II) and in the spot market (Term III).

$$
D_1 + \varepsilon = \underbrace{D + \varepsilon}_{\text{I}} - \underbrace{X_{-1}}_{\text{II}} - \underbrace{(Q_{-1} - X_{-1})}_{\text{III}}
$$

It follows from the generalized Klemperer and Meyer conditions (6) that competitors' net-sales in the spot market are proportional to the slope of their residual demand function. Thus term III can be written as follows

$$\begin{aligned} Q_{-1} - X_{-1} &= \sum_{i \neq 1} p \cdot (Q'_{-i} - D') \\ &= \sum_{i \neq 1} \sum_{j \neq i} Q'_j p - \sum_{i \neq 1} D' p \\ &= (N-1) Q'_1 p + (N-2) Q'_{-1} p - (N-1) D' p, \end{aligned}$$

which allows us to rewrite the residual demand function for firm 1:

$$D_1 + \varepsilon = \underbrace{D + \varepsilon}_{\text{I}} - \underbrace{X_{-1}}_{\text{II}} - \underbrace{(N-1)\,|D'|\;p}_{\text{III.a}} - \underbrace{(N-1)\,Q'_1\;p}_{\text{III.b}} - \underbrace{(N-2) Q'_{-1}\;p}_{\text{III.c}} \tag{16}$$

If demand is more elastic ($|D'|$ larger in term III.a) then the output of its $N-1$ competitors will be larger, and the residual demand that firm 1 faces decreases. Similarly, if firm 1's output is flatter (Q'_1 is large in Term III.b), the output of its competitors' increases, and its residual demand decreases. Term (III.c) is an interaction effect between competitors of firm 1. If one competitor sets a flatter supply function, then the other $(N-2)$ competitors will be more competitive, and the residual demand that firm 1 faces decreases.

In the Allaz and Vila (1993) model firms' production does not depend on prices, $\forall i$, $Q'_i = 0$, and firm 1's residual demand function consist only of the terms I, II, and III.a. Term III.a corresponds to the Stackelberg effect of firm 1: By being a first mover in stage 1, firm 1 can affect its competitors production level in the second stage, as they will react to firm 1's output level.[20] Term II corresponds to the first mover effect of firm 1's competitors. As its competitors sell forward, firm 1 faces a smaller residual demand function. The two additional terms III.b and III.c in our model are a consequence of allowing output to depend on prices.

We can now obtain some intuition on the incentives of firm 1 to make output inelastic or even downward sloping. This can be seen most easily in a duopoly setting ($N = 2$) in which case term (III.c) is zero. It follows from (16) that it is 'costly', either in terms of a reduced quantity or a reduced price, to set a positive slope $Q'_1 > 0$, because it makes its competitor's residual demand curve more elastic, which increases its competitor's output (term III.b becomes larger). Thus we would expect that firm 1 would find it optimal to keep this slope relatively

[20]This term can be understood better by looking at a standard Stackelberg game with one leader and $N-1$ followers. The leader sets output taking into account the subsequent reaction of its followers. Each follower sets its output such that its marginal revenue equals marginal costs, which is zero in our model. Hence, for each follower j, $Q_j + pD' = 0$. The output of one follower is $Q_j = |D'|\,p$, and total output of all followers is given by $(N-1)\,|D'|\,p$.

small or even negative. To achieve this and still sell a significant amount, it will be optimal to produce a relatively large quantity at $p = 0$ and then to keep output fairly inelastic or even backward bending over the whole price range. This is the result that we get in the next section.

4 Closed-form solutions

Relying on Proposition 4, this section derives closed-form solutions of our model, and discusses the welfare effect of derivatives trading.

4.1 Market equilibrium

We make the following simplifying assumptions in order to explicitly solve for an SPNE with the Pareto refinement.

Assumption 2 *Production costs are zero, the demand function $D(p) = -\gamma p$ is linear with $\gamma > 0$ and demand shocks are Pareto distributed of the second-order, so that $f(\varepsilon) = \beta^{1/\alpha}(\alpha\varepsilon + \beta)^{-1/\alpha - 1}$ for $\varepsilon > 0$, where $\beta > 0$ and $\alpha \in (-\infty, \frac{1}{N(N-1)})$.*

The Pareto distribution of the second-order is a family of probability distributions with a wide range of properties (Johnson et al., 1994). For example, for $\alpha = 0$ it gives the exponential distribution and for $\alpha = -1$, the uniform distribution.

Proposition 5 (Closed-Form) *Under Assumption 2 the unique symmetric EPEC solution is given by:*

$$\varepsilon(p) = \varepsilon_0 + \frac{2}{1 - \alpha(N-1)N + (N-1)^2}\gamma p \tag{17}$$

$$Q(p) = \varepsilon_0 + \frac{\alpha(N-1)N - (N-2)N}{1 - \alpha(N-1)N + (N-1)^2}\gamma p \tag{18}$$

$$X(p) = \varepsilon_0 + \frac{2\alpha(N-1)N - 2N^2 + 2N}{1 - \alpha(N-1)N + (N-1)^2}\gamma p, \tag{19}$$

where $\varepsilon_0 = \frac{\beta N(N-1)}{1-\alpha(N-1)N}$. This solution is weakly implementable as a PO-SPNE.

Proof. Lemma 4 in Appendix C shows that under Assumption 2 the unique solution of the set of differential equations (22-24) is given by the linear equations (17-19). It follows from Johnson et al. (1994) that the Pareto distribution of the second-order has a finite variance when $\alpha < \frac{1}{N(N-1)}$. We also note that under

Assumption 2 we have $H'(\varepsilon) = N(N-1)\alpha < 1$. It follows from Proposition 4 that the solution is weakly implementable as a PO SPNE. ■

Figure 3 illustrates the results of Proposition 5 for the special case where $N = 2$ and the demand shock is uniformly distributed.

We notice that with linear demand, the contracting and output functions are linear for a Pareto distribution of the second order. The contracting function $X(p)$ is downward sloping; producers sell forward contracts and buy call options for strike prices above zero. The supply function $Q(p)$ is also downward sloping for $N > 2$ or when $\alpha < 0$. Hence firms produce less, although the demand shock increases. As a result prices increase steeply. Even in the alternative case where the supply function is upward sloping (duopoly $N = 2$ and $\alpha \geq 0$), the curve is still very steep. Indeed, the slope of the total output as a function of price is less than $|D'|$, the slope of monopoly output.

As demand shocks become more uncertain (α increases),[21] the anti-competitive consequences of contracts are mitigated: $Q(0)$ increases and $Q'(p)$ becomes less negative (for $N > 2$), or eventually positive but small (for $N = 2$).

It is also straightforward to verify that total forward sales, $X(0)$, increase with the number of firms. This ensures that the market becomes more competitive for low shock outcomes. However, the total output function will bend backwards more, when the number of firms increases; $Q'(p)$ decreases with more firms in the market. If the support of the shock density is unbounded, ($\alpha \geq 0$) then the second effect will eventually outweigh the first effect for large shock outcomes. Hence in this case, increasing the numbers of firms makes the market less competitive for the highest shock. We attribute this to the interaction effect between competitors, term III.c in (16).

4.2 Welfare effects

Proposition 6 (Deadweight Loss) *The expected deadweight loss Λ for the equilibrium in Proposition 5 is:*

$$\Lambda = \frac{\beta^2 \left(1 - \alpha (N-1) N + (N-1)^2\right)^2}{4\gamma (1-\alpha)(1-2\alpha)\left(1-\alpha (N-1) N\right)^{2-1/\alpha}}. \tag{20}$$

[21] For $\alpha \leq 0$, the support of the demand shock ε is $\left[0, \frac{\beta}{|\alpha|}\right]$, so a less negative α increases the range of demand shocks. For $\alpha \geq 0$, a larger α increases the thickness of the tail of the demand density (Holmberg, 2009).

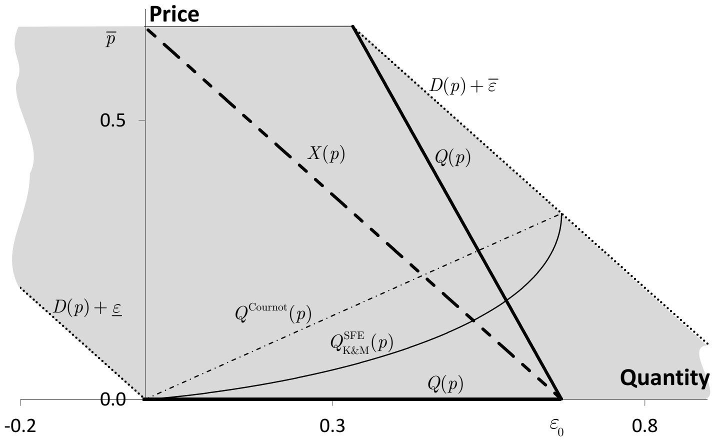

Figure 3: Total contracting $X(p)$ and output $Q(p)$ for the equilibrium described by Proposition 5 for $\alpha = -1$, $\beta = 1$ and $\gamma = 1$. Also shown are the Cournot outcome $Q^{\text{Cournot}}(p)$ and the symmetric monotonic SFE equilibrium of Klemperer and Meyer (1989) $Q^{\text{SFE}}_{\text{K\&M}}(p)$.

Proof. It follows from (17) that:

$$P(\varepsilon) = \begin{cases} A(\varepsilon - \varepsilon_0) & \text{if } \varepsilon > \varepsilon_0 \\ 0 & \text{if } \varepsilon \leq \varepsilon_0 \end{cases}$$

with $A = \frac{1-\alpha(N-1)N+(N-1)^2}{2\gamma}$. As in Holmberg and Newbery (2010), the welfare loss for a given demand shock ε is the deadweight-loss:

$$\lambda(\varepsilon) = \frac{P^2(\varepsilon)\gamma}{2} = \begin{cases} A^2 \frac{\gamma}{2} (\varepsilon - \varepsilon_0)^2 & \text{if } \varepsilon > \varepsilon_0 \\ 0 & \text{otherwise} \end{cases}$$

while the expected welfare loss is given by

$$\Lambda = E_\varepsilon[\lambda(\varepsilon)] = \int_{\varepsilon_0}^{\overline{\varepsilon}} \lambda(\varepsilon) f(\varepsilon) d\varepsilon.$$

Define $T(x)$ as the third-order integral of $f(x)$, so that $T(x) = \frac{-\beta^2}{(\alpha-1)(2\alpha-1)}(\frac{\alpha x+\beta}{\beta})^{2-\frac{1}{\alpha}}$ and $T'''(x) = f(x)$. We can now evaluate the expected loss by twice integrating by parts.

$$\begin{aligned}\Lambda &= \underbrace{\left[\lambda(\varepsilon)T''(\varepsilon)\right]_{\varepsilon_0}^{\overline{\varepsilon}} - \left[\lambda'(\varepsilon)T'(\varepsilon)\right]_{\varepsilon_0}^{\overline{\varepsilon}}}_{=0} + \lambda''(\varepsilon)\left[T(\varepsilon)\right]_{\varepsilon_0}^{\overline{\varepsilon}} \\ &= -A^2\gamma T(\varepsilon_0) = A^2\gamma\beta^2\frac{(1-\alpha N(N-1))^{1/\alpha-2}}{(1-\alpha)(1-2\alpha)} \end{aligned} \tag{21}$$

Note that the first two terms are zero as long as $\alpha < \frac{1}{2}$, because $\overline{\varepsilon} = \frac{-\beta}{\alpha}$ if $\alpha < 0$ and $\overline{\varepsilon} = \infty$ otherwise. Welfare losses are quadratic in ε, therefore $\Lambda''^2\gamma$ is a constant. We get (20) by substituting $A = \frac{1-\alpha(N-1)N+(N-1)^2}{2\gamma}$ into (21). ∎

We now discuss the effect of the number of firms on the market's competitiveness.

Proposition 7 *Under Assumption 2, the expected deadweight loss for the equilibrium in Proposition 5 decreases with the number of symmetric firms.*

Proof. The expected welfare loss is given by (20). Lemma 5 in Appendix C shows that $\frac{1-\alpha(N-1)N+(N-1)^2}{(1-\alpha(N-1)N)^{1-(1/2\alpha)}}$ is decreasing with respect to N which proves the result. ∎

It follows from (20) that the market becomes perfectly competitive (no welfare losses) if the number of firms N goes to infinity and $\alpha \leq 0$.[22]

Proposition 8 (Welfare Comparison) *Expected welfare is lower for the equilibrium in Proposition 5 than for a standard Cournot model without contracting where demand shocks are realized before firms choose production, provided that $N = 2$ and Assumption 2 is satisfied.*

Proof. From the first-order condition of the Cournot market with certain demand it follows that the total duopoly output is: $Q = 2\gamma p$. The market clears when $Q = \varepsilon - \gamma p$, so

$$P_{Cournot}(\varepsilon) = \frac{\varepsilon}{3\gamma}.$$

As before the deadweight loss for a given ε is:

$$\lambda_{Cournot}(\varepsilon) = \frac{P^2_{Cournot}(\varepsilon)\,\gamma}{2} = \frac{\varepsilon^2}{18\gamma}.$$

[22]Note that Assumption 2 does not allow for $\alpha > 0$ when $N \to \infty$.

We can calculate the expected welfare losses $\Lambda_{Cournot} = E_{\varepsilon}\left[\lambda_{Cournot}(\varepsilon)\right]$ as in Proposition 6 by twice integrating by parts:

$$\Lambda_{Cournot} = \lambda''_{Cournot}(\varepsilon)\left[T(\varepsilon)\right]_0^{\bar{\varepsilon}} = -\frac{T(0)}{9\gamma} = \frac{1}{9\gamma}\frac{\beta^2}{(1-\alpha)(1-2\alpha)}.$$

Comparing the welfare losses in the Cournot game above and the welfare losses in Proposition 6 for $N = 2$, we find that:

$$\frac{\Lambda}{\Lambda_{Cournot}} = 9\frac{(1-\alpha)^2}{(1-2\alpha)^{2-1/\alpha}},$$

which is larger than 1 according to Lemma 6 in Appendix C. ∎

Prices for symmetric monotonic SFE without contracts are below the Cournot schedule (Klemperer and Meyer, 1989; Green and Newbery, 1992), i.e. prices in a Cournot equilibrium where demand shocks are realized before firms set production quantity. Thus Proposition 8 has the following implication:[23]

Corollary 1 *The introduction of a derivatives market will lower welfare, provided that $N = 2$, Assumption 2 is satisfied, firms play symmetric monotonic SFE before the reform, and the SPNE in Proposition 5 after the reform.*

This result contrasts with Holmberg's (2011) two-stage model with forward contracting and supply function competition, where the introduction of a forward market weakly improved competition for cases when marginal costs are constant up to a capacity constraint. Similarly, forward markets improved welfare in Allaz and Vila's (1993).

5 Conclusion

Commodity derivatives such as forwards and call options are very useful hedging instruments. However, in an oligopoly market they will also be used strategically. In Allaz and Vila's (1993) seminal study strategic contracting is pro-competitive. However, it is limited in that firms cannot use contracts to commit to a downward sloping supply. In our study, which has a less restrictive strategy space, strategic contracting has anti-competitive consequences.

[23] The standard supply function model without contracts could also have symmetric SFE that are partly downward sloping. Those cases were however not considered by Klemperer and Meyer (1989) nor Green and Newbery (1992). Numerical simulations show that if we allow for those equilibria, financial contracts will still decrease welfare when $\alpha \in (-6, 0)$.

Solving for an equilibrium of a two-stage game with derivative trade followed by spot market competition, we show that risk-neutral producers sell forward contracts and buy call option contracts. This contracting strategy commits them to a fairly inelastic or even downward sloping supply function in the spot market. This is profitable as it will give competitors incentives to increase their mark-ups. The forward sales improve competition for low demand realizations, but the option contracts reduce competition for high demand realizations. Hence commodity derivatives are pro-competitive for low demand, but anti-competitive during high demand. In a duopoly market, the second effect outweighs the first and total surplus decreases when the financial market is introduced. Total forward sales increase in a less concentrated market (more firms), which improves the pro-competitive effect under low demand. In expectation having more firms in the market also reduces welfare losses, even if the anti-competitive effect at high demand becomes more pronounced.

We show that the anti-competitive effects are reduced with more demand uncertainty. It is then optimal for firms to offer supply functions that have a less negative slope, as this allows them to benefit more from both high and low demand realizations. Thus to avoid the anti-competitive effect of speculation, our model suggests that option contracts should not be traded near delivery because firms then have a good estimate of demand. Alternatively, the same option contract or supply function should be valid for several delivery periods in order to increase demand variation.[24] Moreover, market monitors should carefully scrutinize incidents where producers use contracts in a speculative manner.

In our model producers are risk neutral and arbitrage in the financial market is perfect. Therefore, commitment by financial derivatives is costless. As this is not the case in practice, our results should be seen as a limiting case. With risk aversion, firms are expected to reduce tail risk and to hold contracting portfolios that are closer to their actual output, and therefore to offer supply functions that are more upward sloping. Thus contracting should be pro-competitive with sufficient risk-aversion. Also transaction costs in the financial market are likely to reduce the profitability of speculative positions. Considering such imperfections in contracting is likely to reduce the anti-competitive effect for high demand and the pro-competitive effect for low demand realizations.

In our study firms use call options and forward contracts to commit to down-

[24]This is for example the case in the PJM market, where producers' offers are fixed during the whole day to meet a wide range of demand outcomes. PJM is the largest deregulated wholesale electricity market, covering all or parts of 13 U.S. states and the District of Columbia.

ward sloping supply curves. Unlike in spot markets with Cournot competition (Willems, 2005), our results do not depend on whether option contracts are financial or physical. It follows from the call-put parity that firm's could make the same commitment by put options and forward contracts. In practice firms could also use commitment tactics other than financial contracts, for instance by delegating decisions to managers, merging with downstream firms, and making irreversible investments. We believe that the main intuition of our paper, that firms would like to commit to downward sloping supply functions, remains valid in those settings. In this sense our result has some parallels in Zöttl (2010), who models the strategic (irreversible) investments of firms, where firms compete in quantities in a spot market with random demand. He shows that firms will over-invest in technology with low marginal costs (base-load), but choose total investment capacities that are too low from a welfare viewpoint.

6 References

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A Equilibrium concept and solution method

Klemperer and Meyer (1989) show that multiple Nash Equilibria (NE) may exist in supply function games such as the one played in the second stage of our model.

This complicates the equilibrium analysis of our game. Below we discuss in detail how we refine the Sub-game Perfect Nash Equilibrium (SPNE) concept, and the solution method we use to find such an equilibrium.

A.1 Equilibrium concept

We start with some definitions. Each tuple of contracting actions $\mathbf{X}=\{X_i(p)\}_{i=1}^N$ defines a different subgame in the second stage spot market, which we denote as *subgame* $\mathbf{X}$. The set of subgames will be denoted Ξ. Firm i's *strategy* $\{X_i(p), Q_i(p,\mathbf{X})\}$ specifies its action in the first stage (the contract market) and in each second stage subgame (the spot market). The *strategy profile*, the set of all firms' strategies, is given by $\{\mathbf{X}, \mathbf{Q}(\mathbf{X})\}$, where $\mathbf{Q}(\mathbf{X}) = \{Q_i(p,\mathbf{X})\}_{i=1}^N$ specifies a tuple of supply functions for each subgame $\mathbf{X}$. We let $SFE(\mathbf{X})$ be the *set of Supply Function Equilibria in subgame* $\mathbf{X}$. To rule out non-credible threats in the subgames, we solve for a Subgame Perfect Nash Equilibrium in the two-stage game.

Definition 1 (SPNE) *A strategy profile* $\{\mathbf{X}^*, \mathbf{Q}^*(\mathbf{X})\}$ *constitutes a Subgame Perfect Nash Equilibrium (SPNE) iff:*

$$\begin{array}{lll} (i) & \forall i, \forall X_i & \Pi_i(\mathbf{Q}^*(\mathbf{X}^*), \mathbf{X}^*) \geq \Pi_i(\mathbf{Q}^*(\{X_i, \mathbf{X}^*_{-\mathbf{i}}\}), \{X_i, \mathbf{X}^*_{-i}\}) \\ (ii) & \forall \mathbf{X} \in \Xi & \mathbf{Q}^*(\mathbf{X}) \in SFE(\mathbf{X}). \end{array}$$

The first equation specifies that firms do not have an incentive to change their contracting decisions in the first stage, given competitors' contracting strategies $\mathbf{X}^*_{-i}$ and the equilibria which will be played in the second stage, as described by $\mathbf{Q}^*(\mathbf{X})$. The second equation states that the function $\mathbf{Q}^*(\mathbf{X})$ has to be an equilibrium in each subgame.

There can be multiple SPNE when subgames do not have unique equilibria. In this case, we use Pareto dominance to refine the set of Nash equilibria in subgames with multiple Nash Equilibria. A problem with this refinement is that it can only be applied to subgames where Pareto Optimal NE exists. Although the set $SFE(X)$ is typically non-empty, it might not be closed, so a Pareto optimal SFE might not exist in some subgames $\mathbf{X}$. In our application it is especially problematic to prove existence of a Pareto optimal SFE in subgames off the equilibrium path. To deal with this we allow firms to play SFE which are "almost" Pareto Optimal off the equilibrium path.

Definition 2 (ϵ-Pareto Optimality) *A Supply Function Equilibrium* $\mathbf{Q}^* \in SFE(\mathbf{X})$ *in subgame* $\mathbf{X}$ *is* $\epsilon-$*Pareto optimal if no alternative equilibrium* $\mathbf{Q} \in SFE(\mathbf{X})$ *ex-*

ists which is weakly preferred by all firms $\Pi_i(\mathbf{Q},\mathbf{X}) - \Pi_i(\mathbf{Q}^*,\mathbf{X}) \geq 0$*, and where one firm* j *would gain at least* ϵ, $\pi_j(\mathbf{Q},\mathbf{X}) - \pi_j(\mathbf{Q}^*,\mathbf{X}) > \epsilon$.

Furthermore, the supply function equilibrium is said to be Pareto optimal if the above holds for $\epsilon = 0$.

For a given $\epsilon > 0$, we define $SFE_{\epsilon-PO}(\mathbf{X})$ and $SFE_{PO}(\mathbf{X})$ as the set of ϵ-Pareto Optimal SFE and Pareto Optimal SFE in subgame $\mathbf{X}$. Note that $SFE_{PO}(\mathbf{X}) \subseteq SFE_{\epsilon-PO}(\mathbf{X}) \subseteq SFE(\mathbf{X})$.

Definition 3 (ϵ-PO-SPNE) *For a given* $\epsilon \geq 0$*, a subgame perfect Nash Equilibrium (SPNE)* $\{\mathbf{X}^*, \mathbf{Q}^*(\mathbf{X})\}$ *is an* ϵ*-Pareto Optimal subgame perfect Nash Equilibrium (*ϵ*-PO-SPNE) if:*

$$\begin{aligned} \forall \mathbf{X} \in \Xi \quad & \mathbf{Q}^*(\mathbf{X}) \in SFE_{\epsilon-PO}(\mathbf{X}) \\ & \mathbf{Q}^*(\mathbf{X}^*) \in SFE_{PO}(\mathbf{X}^*) \end{aligned}$$

Furthermore, if those expressions hold for $\epsilon = 0$*, the SPNE is a Pareto Optimal subgame perfect Nash Equilibrium (PO-SPNE).*

This definition requires that firms play a Pareto Optimal Nash equilibrium along the equilibrium path, while off the equilibrium path, they are allowed to play any Nash equilibrium which is not ϵ-dominated by other Nash equilibria in the subgame. So by coordinating on another Nash equilibrium in a subgame off the equilibrium path, no firm can gain more than ϵ without making some other firm worse off.[25] Note that if each subgame has a unique Nash equilibrium, then any SPNE is also a PO-SPNE and an ϵ-PO-SPNE.

A.2 Solution method

In order to find subgame perfect Nash equilibria, the set of supply function equilibria in each sub-game needs to be determined. However, the necessary first order and sufficient second order conditions in Propositions 1 and 2 do not describe all equilibria. In our paper we therefore first solve an Equilibrium Program with Equilibrium Constraints (EPEC). This solution method is often used to compute equilibria of dynamic games, including games with strategic contracting.[26] It is

[25] The Pareto Perfect Equilibrium (PPE) (Bernheim et al., 1987) is related to the PO-SPNE concept, but is a stricter refinement as it imposes that only those PO-SPNE are played which are Pareto Optimal in the first stage. Hence, the set of PPE is a subset of the set of PO-SPNE. In case a PO-SPNE is unique, then it is also a Pareto Perfect Equilibrium (PPE).

[26] Su (2007) and Zhang et al. (2010) use an EPEC to calculate strategic forward contracting in oligopoly markets.

assumed that each firm maximizes its profit subject to the First Order Conditions (the "Equilibrium Constraints"). In the next step we then verify that the solutions of the program can "almost" be implemented as a PO-SPNE. A Pareto optimal SFE is played in the subgame **on** the equilibrium path. In subgames off the equilibrium path, none of the firms can gain more than an arbitrarily small amount ϵ by coordinating on any other equilibrium without making another firm worse off.

We define an *outcome* as a set of actions that firms take along one particular path in the game. Hence we present an outcome as $\{\mathbf{X}^0, \mathbf{Q}^0\}$; the contract curves $\mathbf{X}^0 = \{X_i(p)\}_{i=1}^N$ and supply functions $\mathbf{Q}^0 = \{Q_i(p)\}_{i=1}^N$ that firms offer in the first and second stage, respectively. Let $FOC(\mathbf{X})$ be the set of tuples $\mathbf{Q}(\mathbf{X})$ in subgame $\mathbf{X}$ that satisfy the necessary first order conditions of an SFE as specified in Proposition 1. Thus $SFE(\mathbf{X}) \subseteq FOC(\mathbf{X})$.

A Mathematical Program with Equilibrium Constraints (MPEC, Luo et al., 1996) is an optimization program where a firm maximizes its profit, subject to a set of first order conditions (the equilibrium constraints).

Definition 4 (MPEC Outcome) *An outcome $\{\mathbf{X}^*, \mathbf{Q}^*\}$ is an MPEC outcome for firm i iff it is a solution of firm i's Mathematical Program with Equilibrium constraints (MPEC)*

$$MPEC(i) \quad \forall X_i, \forall \mathbf{Q} \in FOC(\{X_i, \mathbf{X}^*_{-i}\}) \qquad \Pi_i(\mathbf{Q}^*, \mathbf{X}^*) \geq \Pi_i(\mathbf{Q}, \{X_i, \mathbf{X}^*_{-i}\}).$$

An Equilibrium Program with Equilibrium Constraints (EPEC) is the system of MPECs, one for each firm i.

Definition 5 (EPEC Equilibrium) *An outcome $\{\mathbf{X}^*, \mathbf{Q}^*\}$ is an EPEC outcome iff $\forall i$, $\{\mathbf{X}^*, \mathbf{Q}^*\}$ is an MPEC outcome for firm i. We say that $\{\mathbf{X}^*, \mathbf{Q}^*\}$ is an EPEC equilibrium if in addition $\mathbf{Q}^* \in SFE(\mathbf{X}^*)$.*

It is now verified that the EPEC outcome can be reached along the equilibrium path of an ϵ-PO-SPNE for and arbitrarily small ϵ. Formally:

Definition 6 *The outcome $\{\mathbf{X}^*, \mathbf{Q}^*\}$ is weakly implementable as a Pareto Optimal subgame perfect Nash Equilibrium if for any $\epsilon > 0$ there exist ϵ-PO-SPNE, for which $\{\mathbf{X}^*, \mathbf{Q}^*\}$ is on the equilibrium path.*

Proposition 9 *If there exists an SFE in every sub game, i.e. $\forall \mathbf{X} : SFE(\mathbf{X}) \neq \emptyset$, and the monopoly profit is bounded in every subgame, then an EPEC equilibrium*

$\{\mathbf{X}^*, \mathbf{Q}^*\}$ *is weakly implementable as a Pareto Optimal subgame perfect Nash Equilibrium.*

Proof. Step 1: We first prove that the EPEC equilibrium implies that a Pareto Optimal SFE is played along the equilibrium path, i.e. $\mathbf{Q}^*(\mathbf{X}^*) \in SFE_{PO}(\mathbf{X}^*)$. We will use a proof by contradiction. Suppose that there is a Nash equilibrium $\tilde{\mathbf{Q}} \in SFE(\mathbf{X}^*)$ such that

$$\forall i,\ \Pi_i(\tilde{\mathbf{Q}}, \mathbf{X}^*) \geq \Pi_i(\mathbf{Q}^*, \mathbf{X}^*)$$

where the inequality is strict for one of the firms. Without loss of generality assume that for firm 1: $\Pi_1(\tilde{\mathbf{Q}}, \mathbf{X}^*) > \Pi_1(\mathbf{Q}^*, \mathbf{X}^*)$. The definition of the EPEC outcome requires that:

$$\Pi_1(\mathbf{Q}^*, \mathbf{X}^*) \geq \Pi_1(\mathbf{Q}, \mathbf{X}^*)\ \forall\, \mathbf{Q} \in FOC(\mathbf{X}^*\}.$$

As the first order conditions are necessary conditions for an equilibrium, we must have that $\tilde{\mathbf{Q}} \in FOC(\mathbf{X}^*)$. Hence,

$$\Pi_1(\mathbf{Q}^*, \mathbf{X}^*) \geq \Pi_1(\tilde{\mathbf{Q}}, \mathbf{X}^*),$$

which is a contradiction.

Step 2: We will now prove that in every subgame $\mathbf{X}$, and for each $\epsilon > 0$ there exists an ϵ−PO-SFE, i.e. $\forall \mathbf{X}, \forall \epsilon > 0$, $SFE_{\epsilon-PO}(\mathbf{X}) \neq \emptyset$. Define total industry profit as $\Pi_I(\mathbf{Q}, \mathbf{X}) \equiv \sum_i \Pi_i(\mathbf{Q}, \mathbf{X})$. Total industry profit $\Pi_I(\mathbf{Q}, \mathbf{X})$ is bounded above by the monopoly profit Π^M, so that $\Pi_I(\mathbf{Q}, \mathbf{X}) \leq \Pi^M$. It follows from Dedekind completeness that every non-empty set of real numbers having an upper bound must also have a least upper bound. Thus we let $\Pi^{sup}(\mathbf{X})$ denote the least-upper bound (or supremum) of equilibrium industry profits in subgame $\mathbf{X}$. Hence, $\Pi_I(\mathbf{Q}, \mathbf{X}) \leq \Pi^{sup}(\mathbf{X})$. By assumption there exists one equilibrium in every subgame. For any $\epsilon > 0$ one can always find one $\mathbf{Q}_\epsilon \in SFE(\mathbf{X})$, such that $\Pi_I(\mathbf{Q}_{\boldsymbol{\epsilon}}, \mathbf{X}) \geq \Pi^{sup}(\mathbf{X}) - \epsilon$, otherwise $\Pi^{sup}(\mathbf{X})$ would not be the least-upper bound of equilibrium industry profits in subgame $\mathbf{X}$. We now prove by contradiction that the SFE $\mathbf{Q}_{\boldsymbol{\epsilon}}$ is ϵ-Pareto Optimal. Suppose it were not, then there exist an alternative SFE $\tilde{\mathbf{Q}} \in SFE(\mathbf{X})$ such that some firm i improves its profit by at least ϵ, $\Pi_i(\tilde{\mathbf{Q}}, \mathbf{X}) - \Pi_i(\mathbf{Q}_{\boldsymbol{\epsilon}}, \mathbf{X}) > \epsilon$ while other firms $j \neq i$ receive at least as much as before $\Pi_j(\tilde{\mathbf{Q}}, \mathbf{X}) - \Pi_j(\mathbf{Q}_{\boldsymbol{\epsilon}}, \mathbf{X}) \geq 0$. This implies however that $\Pi_I(\tilde{\mathbf{Q}}, \mathbf{X}) > \Pi_I(\mathbf{Q}_{\boldsymbol{\epsilon}}, \mathbf{X}) + \epsilon \geq \Pi^{sup}(\mathbf{X})$, which is impossible given the definition of $\Pi^{sup}(\mathbf{X})$.

Step 3: It follows from step 2 that we can always find an ϵ-Pareto optimal supply function equilibrium $\mathbf{Q}_\epsilon(\mathbf{X}) \in SFE_{\varepsilon-PO}(\mathbf{X})$ in each subgame, while the equilibrium $\mathbf{Q}^*$ is played along the equilibrium path $\mathbf{X}^*$, i.e. $\mathbf{Q}_\epsilon(\mathbf{X}^*) = \mathbf{Q}^*$, according to step 1. In order to prove that $\{\mathbf{X}^*, \mathbf{Q}_\epsilon(\mathbf{X})\}$ is an ϵ-PO-SPNE, it remains to be shown that deviations from $\mathbf{X}^*$ in the first stage are not profitable. As the first order conditions are necessary conditions for an equilibrium, we must have that $\mathbf{Q}_\epsilon(\mathbf{X}) \in FOC(\mathbf{X})$. Definition 5 implies that firm i has no profitable deviation $X_i(p) \neq X_i^*(p)$, such that $\Pi_i(\mathbf{Q}^*, \mathbf{X}^*) < \Pi_i(\mathbf{Q}_\epsilon(\{X_i, \mathbf{X}^*_{-\mathbf{i}}\}), \{X_i, \mathbf{X}^*_{-i}\})$. Thus we can conclude from Definitions 1 and 3 that $\{\mathbf{X}^*, \mathbf{Q}_\epsilon(\mathbf{X})\}$ is an ϵ-PO-SPNE. Such an ϵ-PO-SPNE can be found for any $\epsilon > 0$, so the Proposition now follows from Definition 6. ■

B Strategic contracting

Lemma 1 *Under Assumption 1 any symmetric solution of problems $k = 1...N$ in Equation (12) has to satisfy the following first order conditions:*

$$H(\varepsilon(p)) = (pD(p))' + \varepsilon(p) + (N-1)^2 p\varepsilon'(p) \quad (22)$$

$$Q(p) = D(p) + \varepsilon(p) \quad (23)$$

$$X(p) = (pD(p))' + \varepsilon(p) - (N-1)p\varepsilon'(p) \quad (24)$$

for $\varepsilon(p) \in [\varepsilon_0, \overline{\varepsilon}]$, where $\varepsilon_0 \in [0, \overline{\varepsilon}]$ and $P(\varepsilon_0) = 0$. Solutions to these equations are EPEC outcomes. That is, for each firm i, playing X_i globally solves its MPEC(i) problem.

Proof. Step 1: Without loss of generality we solve for the optimal contract of firm $k = 1$. In the optimal control problem, $X_1(p)$ only appears in the first constraint for $i = 1$. Thus firm 1 is free to choose $X_1(p)$ to satisfy this equation without influencing other constraints or the objective function. Thus this equation just defines $X_1(p)$, and can thus be neglected for now. In the objective function and in the second and third constraints, competitors' total output matters, but not how it is divided between these firms. We can therefore sum up the remaining $(N-1)$ equations (for cases $i \neq 1$) of the first constraint into one single constraint. Using that $F(\varepsilon(\overline{p})) = 1$ and integration by parts we can now rewrite the dynamic optimization problem as follows:

$$\max \int_0^{\overline{p}} (pQ_1)' \left[1 - F(\varepsilon)\right] dp. \tag{25}$$

$$s.t. \qquad (N-1)pQ_1' + (N-2)pQ_{-1}' = (N-1)pD' + Q_{-1} - X_{-1} \tag{26}$$

$$Q_{-1} + Q_1 = D + \varepsilon, \tag{27}$$

where, as before, the subscript $_{-i}$ refers to the sum of a variable over all firms, excluding firm i. Thus firm 1's expected profit is given by the integral of its marginal profit $\frac{\partial}{\partial p}\left(p\,Q_1(p)\right)$ at price p, weighted by $1 - F(\varepsilon(p))$, the probability that the realized price is larger than p, and this also makes sense intuitively.

Step 2: We simplify the dynamic optimization problem by rewriting the constraints and then substituting them into the objective function. By adding $N-1$ times the market equilibrium (27) to the constraint (26) and we get:

$$(N-1)(pQ_1)' = (N-1)\left(pD\right)' - X_{-1} + (N-1)\varepsilon - (N-2)(pQ_{-1})'. \tag{28}$$

We use the market equilibrium identity in (27) to write $(pQ_{-1})'$ as a function of $(pQ_1)'$.

$$(pQ_{-1})' = (pD)' + (p\varepsilon)' - (pQ_1)',$$

which we can substitute into (28), to give an expression for the marginal profit $(pQ_1)'$

$$(pQ_1)' = (pD)' - X_{-1} + \varepsilon - (N-2)\cdot p\cdot \varepsilon'.$$

Substituting this marginal profit into the objective function in (25) gives the following optimization problem:

$$\max \int_0^{\overline{p}} \{\underbrace{(pD)' - X_{-1}}_{h_1(p)} + \varepsilon - (N-2)\cdot p\cdot \varepsilon'\} \left[1 - F(\varepsilon)\right] dp. \tag{29}$$

Step 3: We now derive the first order conditions of the optimization program (29). First we write it as the sum of two integrals:

$$\max \int_0^{\overline{p}} \{h_1(p) + \varepsilon\} \left[1 - F(\varepsilon)\right] dp \; - (N-2) \int_0^{\overline{p}} p\cdot (G(\varepsilon) - G\left(\overline{\varepsilon}\right))' dp,$$

where $G(\varepsilon) = \int_0^{\varepsilon}(1 - F(t))dt$. Note that $\varepsilon\left(\overline{p}\right) = \overline{\varepsilon}$. Thus the second term can be rewritten using integration by parts:

$$\max \int_0^{\overline{p}} \underbrace{\{[h_1(p) + \varepsilon] [1 - F(\varepsilon)] + (N-2) (G(\varepsilon) - G(\overline{\varepsilon}))\}}_{\theta(p,\varepsilon)} dp.$$

The integrand only depends on $\varepsilon(p)$ and p, so we can maximize the integral by maximizing $\theta(p, \varepsilon)$ for each p. Thus for every p we want to find the $\varepsilon(p)$ that maximizes $\theta(p, \varepsilon)$.

$$\frac{\partial \theta(p, \varepsilon)}{\partial \varepsilon} = (N-1)(1 - F(\varepsilon(p))) - (h_1(p) + \varepsilon(p)) f(\varepsilon(p)) \tag{30}$$

Setting $\frac{\partial \theta(p,\varepsilon)}{\partial \varepsilon} = 0$ and multiplying the equation with $\frac{N}{f(\varepsilon(p))}$, the first order condition of this optimization problem can be written as:

$$H(\varepsilon) - Nh_1(p) - N\varepsilon = 0. \tag{31}$$

Step 4: We want to know under what circumstances solutions to this condition globally maximize profits. Let $\widetilde{\varepsilon}(p)$ be a solution to this equation for a given contracting choice of the competitors, $X_{-1}(p)$. We see from (30) that $\frac{\partial \theta(p,\varepsilon)}{\partial \varepsilon}$ has the same sign as $H(\varepsilon(p)) - Nh_1(p) + N\varepsilon(p)$. It follows from Assumption 1.2 that $H'(\varepsilon) < 1$. Thus we realize that for all price levels p:

$$H(\varepsilon) - Nh_1(p) - N\varepsilon \quad \begin{matrix} \leq 0 \text{ if } \varepsilon > \widetilde{\varepsilon}(p) \\ = 0 \text{ if } \varepsilon = \widetilde{\varepsilon}(p) \\ \geq 0 \text{ if } \varepsilon < \widetilde{\varepsilon}(p). \end{matrix}$$

Accordingly, $\widetilde{\varepsilon}(p)$ globally maximizes θ at each price. We can repeat the argument for any firm and thus solutions to (31) are EPEC outcomes according to Definition 5.

Step 5: We now solve for symmetric equilibria. Multiplying equation (28) with N, and assuming symmetry we find

$$X = N(pD)' + N\varepsilon - (N-1)(pQ)'$$

Substituting the market clearing identity $Q(p) = D(p) + \varepsilon$ for Q, we obtain equation (24). Reinserting the definition of $h_1(p)$ in (29) into the first order condition (31) and assuming symmetry $(NX_{-i} = (N-1)X)$ we find

$$H(\varepsilon) = N(pD)' + N\varepsilon - (N-1)X$$

Substituting X with equation (24) gives the differential equation in (22). Equation (23) describes the market clearing condition. ■

Lemma 2 *The EPEC outcome in Lemma 1 is an EPEC equilibrium.*

Proof. It follows from Definition 5 that the EPEC outcome is an EPEC equilibrium if the two conditions in Proposition 2 are satisfied. Marginal costs are zero according to Assumption 1.1 so the second condition is satisfied. It remains to show that the first condition $Q' > D'$ or equivalently $\varepsilon'(p) > 0$ is satisfied.

First we want to verify that $\varepsilon'(0) > 0$. We rewrite the first order condition (22) and evaluate it at $p = 0$:

$$(N-1)^2 \varepsilon'(0) = \lim_{p \to 0} \frac{H(\varepsilon(p)) - \varepsilon(p) - D(p)}{p} - D'(0)$$

The limit only exists when $H(\varepsilon(0)) = \varepsilon(0)$. We use l'Hôpital's rule and collect the terms with $\varepsilon'(0)$ to find

$$\varepsilon'(0)\left((N-1)^2 + 1 - H'\right) = -2D'(0).$$

The second factor on the left hand side is positive given Assumption 1.2. The right hand side is also positive, as $D'(0) < 0$, given our assumptions on the demand function in Section 2. Thus we must have that $\varepsilon'(0) > 0$.

In the next step we show that whenever $\varepsilon'(p) = 0$ for a given strictly positive price $p > 0$, it must be that $\varepsilon''(p) > 0$. Differentiating the first order condition (22) with respect to p, we find

$$H'(\varepsilon)\varepsilon' = (N-1)^2(\varepsilon' + p\varepsilon'') + \varepsilon' + (pD)''$$

For price levels where $\varepsilon' = 0$, this expression simplifies to:

$$(N-1)^2 p\varepsilon'' = -(pD)''$$

The right hand side is positive (we assumed downward and concave demand functions in Section 2), and we consider strictly positive prices, so it must be that $\varepsilon''(p) > 0$ when $\varepsilon' = 0$.

In the last step we show that $\varepsilon'(p) > 0$ for all prices. Our proof is by contradiction. Assume that the inequality $\varepsilon'(p) > 0$ is violated for some $p > 0$. Let p_0 be the lowest price above 0 where $\varepsilon'(p) = 0$. Thus our assumptions would imply that $\varepsilon'(p) > 0$ for $p \in [0, p_0)$ and that $\varepsilon'(p_0) = 0$, which requires that $\varepsilon''(p_0) \leq 0$.

However, this is impossible as we have just shown that whenever $\varepsilon'(p_0) = 0$ it must be that $\varepsilon''(p_0) > 0$. Hence, $\varepsilon'(p) > 0$ for $p \in [0, \overline{p}]$. ■

Lemma 3 *The EPEC outcome in Lemma 1 is weakly implementable as a PO SPNE.*

Proof. It follows from Proposition 2 that the EPEC outcome in Lemma 1 is an EPEC equilibrium. We know from Proposition 9 that the EPEC equilibrium is weakly implementable as a PO SPNE if (1) the monopoly payoff is bounded and (2) there exist an SFE in every subgame. We prove that both conditions hold.

Let $\gamma = -D'(0) > 0$. Thus it follows from the assumed properties of the demand function in Section 2 that

$$D(p) \leq -\gamma p.$$

We realize that the monopoly profit for demand $D(p)$ is bounded by the monopoly profit for demand $-\gamma p$. In the latter case, the monopolist would set a monopoly price $P(\varepsilon) = \frac{\varepsilon}{2\gamma}$ and receive a monopoly profit $\pi^M(\varepsilon) = \frac{\varepsilon^2}{4\gamma}$. In expectation this monopoly profit is:

$$\Pi^M = \int_0^{\overline{\varepsilon}} \pi^M(\varepsilon) f(\varepsilon) d\varepsilon = \frac{1}{4\gamma} \int_0^{\overline{\varepsilon}} \varepsilon^2 f(\varepsilon) d\varepsilon.$$

In Section 2 we make the assumption that ε has a bounded variance, so the expected monopoly profit must be bounded, also when $\overline{\varepsilon}$ is arbitrarily large.

Finally we note that Bertrand offers at $p = 0$ constitute a Nash equilibrium in every subgame irrespective of contracting. If competitors' total offers meet maximum demand at $p = 0$, then the profit of a firm is always zero irrespective of its offer, and it might as well choose its supply offer such that its total output meets maximum demand at $p = 0$. Thus the Bertrand outcome is always an equilibrium. ■

C Closed form solutions

Lemma 4 *Under assumption 2 the unique solution of the set of differential equations (22-24) is given by the linear equations (17-19).*

Proof. Under Assumption 2 we have $\frac{1-F(\varepsilon)}{f(\varepsilon)} = \alpha\varepsilon + \beta$ (Holmberg, 2009) and $D(p) = -\gamma p$, which simplifies (22) to

$$N(N-1)\beta + 2\gamma p = (N-1)^2 p\varepsilon' + \varepsilon \left[1 - N(N-1)\alpha\right].$$

This is a first order differential equation in the form:

$$a\varepsilon + p\varepsilon' = g(p) \tag{32}$$

with

$$a = \frac{1-\alpha\,(N-1)\,N}{(N-1)^2} \tag{33}$$

and

$$g(p) = \frac{N}{N-1}\beta + \frac{2}{(N-1)^2}\gamma p. \tag{34}$$

Both sides of (32) can then be multiplied with the integrating factor p^{a-1} and integrated. As long as $a > 0$ or equivalently $\alpha < \frac{1}{(N-1)N}$, we have that $p^a \varepsilon\,(p)$ is zero at $p = 0$, so

$$\varepsilon(p) = p^{-a}\int_0^p g(t)t^{a-1}dt.$$

Substituting a and $g(t)$ by their definitions in (33) and (34) and solving for this integral gives us equation (17). The optimal supply and contracting functions in (18) and (19) then follows from substituting (17) into (23) and (24), respectively. ■

Lemma 5 *Under Assumption 2, it must be that* $\xi(N,\alpha) = \frac{1-\alpha(N-1)N+(N-1)^2}{(1-\alpha(N-1)N)^{1-(1/2\alpha)}}$ *is decreasing with respect to* N.

Proof. The partial derivative of this expression with respect to N is

$$\frac{\partial \xi(N,\alpha)}{\partial N} = -\frac{\varpi(N,\alpha)}{2\,(1-\alpha\,(N-1)\,N)^{\frac{4\alpha-1}{2\alpha}}}$$

with $\varpi(N,\alpha) = (2N - 5N^2 + 2N^3 + 2) - \alpha(N-1)(2N^2 - 3N + 2)$. As the denominator is always positive for $\alpha \in \left(-\infty, \frac{1}{(N-1)N}\right)$, we need to show that ϖ is positive as well. We have

$$\frac{\partial \varpi(N,\alpha)}{\partial \alpha} = -(N-1)(2N^2 - 3N + 2) \leq 0$$

for $N \geq 2$. According to Assumption 2 we have $\alpha < \frac{1}{N(N-1)}$, so $\frac{1}{2}$ is an upper boundary for α and $\varpi(N,\alpha)$ is bounded from below by $\varpi\left(N, \frac{1}{2}\right)$

$$\varpi(N,\alpha) > \varpi(N, \frac{1}{2}) = \frac{1}{2}(N-2)(N+1)(2N-3) \geq 0$$

for $N \geq 2$, which establishes the result. ■

Lemma 6 *For* $\alpha \in \left(-\infty, \frac{1}{2}\right)$ *it must be that* $9\frac{(1-\alpha)^2}{(1-2\alpha)^{2-1/\alpha}} > 1$

Proof. The left hand side of the inequality can be written as $9\exp(z(\alpha))$ with

$$z(\alpha) = [2\ln(1-\alpha) + (\frac{1}{\alpha} - 2)\ln(1-2\alpha)].$$

We now prove that $\alpha = 0$ is a minimum of the function $z(\alpha)$. By differentiation of z with respect to α we obtain that:

$$\alpha^2(1-\alpha)\frac{dz}{d\alpha} = -2\alpha - (1-\alpha)\ln(1-2\alpha)$$

Love (1980) shows that $\frac{x}{1+\frac{1}{2}x} < \ln(1+x)$ if $x > 0$ and $\frac{x}{1+\frac{1}{2}x} > \ln(1+x)$ if $-1 < x < 0$. From this it directly follows that $\frac{dz}{d\alpha} > 0$ if $.5 > \alpha > 0$ and $\frac{dz}{d\alpha} < 0$ if $\alpha < 0$. The minimum of z is therefore achieved at $\alpha = 0$. Using l'Hopital's rule it can be shown that $z(0) = -2$. Thus the minimum of $9e^{z(\alpha)}$ is $9e^{-2}$, which is larger than 1. ■