Economic Insights Required for Using Lifecycle Analysis for Policy Decisions

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We develop an analytic and numerical model that integrates land, food and fuel markets and is linked with a sectoral emissions model to examine how the amount of biofuel in the economy impacts the lifecycle emissions of a biofuel under different policies. Our central finding is that the change in GHG emissions due to a unit expansion in biofuel will vary dramatically in the amount of biofuel in the economy and with the policy driving the expansion. The emissions from a unit expansion in corn ethanol due to a blend mandate fall from 12 gCO_2e/MJ to 3 gCO_2e/MJ, as the quantity of ethanol in the economy increases from 6 to 15 billion gallons. For an input subsidy, emissions due to a unit of ethanol increase from 15 gCO_2e/MJ to 26 gCO_2e/MJ over the same increase in ethanol. We discuss the implications of these results for lifecycle analysis.

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Recent literature on the economics and lifecycle emissions of biofuels has established that multimarket adjustments in land and fuel markets can drastically alter the greenhouse gas (GHG) emissions implications of biofuel policies. We contribute to this literature by examining three related questions. First, how are emissions from land and fuel market adjustments linked to each other, and the policy driving the expansion in biofuel? Second, how do land and fuel market adjustments change with the amount of biofuel in the economy? Third, how does the change in lifecycle emissions from a unit of biofuel vary as the amount of biofuel in the economy changes, and how does this differ across policies?

To answer these questions, we extend our earlier work (Bento, Klotz, and Landry forthcoming) and develop an analytic and numerical economic model that integrates land, food and fuel markets, and link it with a sectoral emissions model. The purpose of the analytical model is to provide a tractable formula that decomposes the change in GHG emissions per unit of additional biofuel into land and fuel market channels, and to highlight how the channels of adjustment depend on underlying economic conditions in affected input and output markets—such as the share of land devoted to biofuel production or the share of biofuel in blended fuel, which are both increasing in the amount of biofuel in the economy. We focus on a unit expansion in biofuel so that our results are directly comparable to lifecycle analysis (LCA) measures of GHG savings used by policymakers, and to facilitate a comparison of the GHG consequences of two instruments that reflect past and current US ethanol policies: an input subsidy and a blend mandate.[1] In the numerical analysis, we examine the empirical implications of the analytical formula for the case of corn ethanol, although the insights from our analysis are broadly applicable to any crop-based biofuel.

Previous literature has shown that adjustments in both land and fuel markets have important implications for both the GHG (Huang et al. 2013; Bento, Klotz, and Landry forthcoming) and welfare (Chen et al. 2011; Lapan and Moschini 2012; Landry and Bento 2013) impacts of biofuel policies.[2] The importance of market adjustments due to biofuels has also been recognized by the related literature on the use of LCA for biofuels (National Research Council 2011; Creutzig et al. 2012; Rajagopal and Zilberman 2013). Accounting for adjustments in land (Searchinger et al. 2008; EPA 2010; Plevin et al. 2010) and fuel markets (Rajagopal, Hochman, and Zilberman 2011; Rajagopal 2013) in an LCA framework has been an important focal point of this research area. Likewise, Bento and Klotz (2013) have shown that the policy driving the change in biofuels has important implications for lifecycle emissions.[3]

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[1] LCA metrics of GHGs and GHG savings are used as means for categorizing biofuels (US Renewable Fuel Standard and EU's Renewable Energy Directive) and for rating the emissions intensities of alternative fuels (low carbon fuel standards in California and elsewhere). The blend mandate we examine is similar in spirit to the Renewable Fuel Standard (RFS) for corn ethanol established by the Energy Independence and Security Act of 2007. The input subsidy we examine is similar to the Volumetric Ethanol Excise Tax Credit (VEETC) which was in place for several decades before being allowed to expire at the end of 2011. We consider a range of both policies for illustrative purposes which do not necessarily reflect the magnitudes of either policy established by statute.

[2] There are also a number of studies that separately study the impacts of biofuel policies on either land (Keeney and Hertel 2009; Hertel, Tyner, and Birur 2010) or fuel markets (Khanna, Ando, and Taheripour 2008; de Gorter and Just 2009; Hochman, Rajagopal, and Zilberman 2011; Thompson, Whistance, and Meyer 2011; Rajagopal and Plevin 2013).

[3] Our work here extends also the analysis of Bento and Klotz (2013) by providing an analytical decomposition of each emissions channel, and conducting the numerical analysis for incremental changes in the policies.

Our analysis uncovers three important results that advance previous work in these areas. First, we establish both analytically and numerically that emissions from land markets depend on the economic conditions in land markets, such as the share of land producing corn for ethanol, but are unaffected by the economic conditions in fuel markets, such as the share of ethanol in blended fuel, and the policy driving the expansion. In sharp contrast, depending upon the policy, fuel market emissions can be affected by economic conditions in land markets, along with economic conditions in fuel markets.

Second, changes in the economic conditions in land markets resulting from expansions in ethanol, particularly the increasing share of land going to corn for ethanol, cause land market emissions to grow in magnitude. Given the first result, this expansion in land market emissions will be the same for either policy. In contrast, emissions reductions from fuel markets are increasing in the amount of ethanol in the economy for the mandate, due to changing economic conditions in both land and fuel markets, but decreasing in the amount of ethanol in the economy for the subsidy.

Finally, given the first two results, we find that the change in GHG emissions from a policy induced unit expansion in ethanol can vary dramatically in the amount of ethanol in the economy and with the policy driving expansion. At 6 billion gallons, an additional unit of ethanol due to the mandate increases emissions by 12 gCO_2e/MJ, but at 15 billion gallons an additional unit of ethanol increases emissions by only 3 gCO_2e/MJ. Beyond 16 billion gallons, a unit increase in ethanol due to the mandate will actually reduce emissions. In contrast, the emissions resulting from an expansion in ethanol due to the subsidy increase from 15 gCO_2e/MJ at 6 billion gallons to 26 gCO_2e/MJ at 15 billion gallons.

These three results have first-order implications for the calculation and use of LCA measures of GHG savings. Specifically, the validity of average estimates of lifecycle emissions savings will depend upon the baseline economy and the change in technology under consideration. In addition, analyses that assume constant emissions savings per unit of technology may lead to misleading estimates of total emissions changes. Overall, our results suggest that analysts should use caution when applying the lifecycle analysis metrics from a particular economic and policy context to a different economic and policy context, and great care should be taken to align the lifecycle emissions savings estimates with the policy under consideration.

The rest of this article is organized as follows. We first develop an analytical model with which we decompose the change in emissions from land and fuel market adjustments due to a unit expansion in ethanol, given underlying economic conditions. We then present simulation results that explore how emissions from land and fuel market adjustments change with the quantity of ethanol in the economy. Finally, we discuss the broader implications of the numerical results for the implementation and use lifecycle analysis measures of GHG savings.

Analytical Model

In this section we develop an analytical model that integrates fuel, land and food markets to examine the change in lifecycle emissions resulting from a unit expansion in ethanol due to either a blend mandate or a subsidy.[4] The blend mandate, akin to the Renewable Fuel Standard, establishes that a minimum share of ethanol must be included in blended fuel. The input subsidy provides a payment to fuel blenders for mixing ethanol into blended fuel, similar to the recently expired Volumetric Ethanol Excise Tax Credit.

General Environment

Our framework is a static model of two countries, D and W, which are both open economies. D denotes the United States. W represents the rest of the world (ROW), a collection of open economies that trade with the US. The countries freely trade agricultural crops and crude oil.[5] All other goods are assumed to be immobile. Therefore, the prices of crops and crude oil are determined on the world market, while all other prices are determined domestically. We model explicitly the behavior of economic agents in the US, and treat adjustments in the rest of the world more simply.

Consumer Demand

In the US, a representative household is endowed with land (A) and labor (L). This household receives utility from consuming blended fuel (F), food (X) and a composite consumption good (C), given the following preferences:[6]

$$U(F, X, C) \quad (1)$$

[4] This framework is also used in Bento and Landry (2013) to analyze the welfare implications of the RFS and in Bento, Klotz and Landry (forthcoming) to analyze the carbon leakage resulting from the RFS.

[5] We abstract from the trade of gasoline.

[6] When describing the US portion of the model, we omit the subscript D for ease of notation.

where $U(\cdot)$ is continuous and quasi-concave. The household faces the following budget constraint:

$$P_F F + P_X X + C = L + \pi_A \tag{2}$$

where P_F is the price of blended fuel, P_X is the price of food and the wage rate is normalized to unity. π_A is the net returns to the land endowment. The household chooses F, X, and C to maximize (1) subject to (2). The resulting first-order conditions yield the uncompensated demand functions for blended fuel, food and the composite good given by:

$$F(P_F, P_X, \pi_A) \qquad X(P_F, P_X, \pi_A) \qquad C(P_F, P_X, \pi_A). \tag{3}$$

Fuel Production

A representative fuel blender produces blended fuel from gasoline (G) and ethanol (E). Gasoline and ethanol are assumed to be perfect substitutes in the production of fuel ($F = E + G$).[7] The blend mandate sets the minimum share of ethanol that must be included in blended fuel, $\Theta \geq 0$, whereas the input subsidy, $\tau \geq 0$, lowers the price of ethanol, P_E, faced by fuel blenders. The blender chooses the share of ethanol in blended fuel, θ, to minimize per unit input costs, net of the input subsidy, and subject to a blend mandate:

$$\begin{gathered} \min_{\theta} (P_E - \tau)\theta + P_G(1 - \theta) \\ \text{st. } \; \Theta \leq \theta \leq 1 \end{gathered} \tag{4}$$

[7] This appears to be the most common specification used in the literature. We believe this is an appropriate representation because consumers are largely unaware of the share of ethanol in the fuel they are purchasing. Other authors have used somewhat different specifications, including Ando et al. (2010) who consider a constant elasticity of substitution (CES) relationship. We choose the perfect substitutes specification because the CES specification could be too restrictive. While the perfect substitutes production function does not require calibration, the CES specification relies on share parameters that need to be calibrated to a base year share of ethanol in blended fuel, which may be different from the share of ethanol that results during the policy runs.

where P_G is the price of gasoline. When the ethanol share is less than one, the first order conditions of (4) are:

(5) $$P_E - \tau = P_G + \lambda$$ $$\lambda(\theta - \Theta) \geq 0$$

where $\lambda \geq 0$ is the multiplier for the mandate constraint in (4). Denoting the cost minimizing share of ethanol in blended fuel as $\theta(P_E - \tau, P_G)$ the price of blended fuel is:

(6) $$P_F = \theta(\cdot)(P_E - \tau) + (1 - \theta(\cdot))P_G$$

and the demand functions for ethanol and gasoline are:

(7) $$E = \theta(\cdot)F(\cdot) \qquad G = (1 - \theta(\cdot))F(\cdot)$$

where $F(\cdot)$ is the uncompensated demand for blended fuel from (3). When the mandate is binding, Θ replaces $\theta(\cdot)$ in (6) and (7).

Ethanol and gasoline are each produced by representative firms with constant returns to scale production technology, where ethanol is produced from corn, Y, and labor, given $E = E(Y_E, L_E)$, and gasoline from crude oil, R, and labor, according to $G = G(R_G, L_G)$.[8] The price of gasoline is therefore a function of the price of crude oil, $P_G(P_R)$, and the price of ethanol is as a function of the price of corn, $P_E(P_Y)$. The resulting demand functions for ethanol and gasoline production are:[9]

(8) $$Y_E\big(P_Y, E(\cdot)\big) \qquad R_G\big(P_R, G(\cdot)\big)$$

where $E(\cdot)$ and $G(\cdot)$ are from (7).

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[8]Here Y_E is net of co-products of corn ethanol production, which can be used in livestock rations. In the simulation model, co-products are produced jointly with ethanol and substitute for corn and soybeans in the production of food. See appendix for additional details.

[9] Both here and in what follows in equations (10) and (12), we omit the demand for labor for ease of exposition.

Agricultural Production

The representative household maximizes the net returns to its land endowment by allocating land to the production of corn or other crops, Z, or setting land aside in the Conservation Reserve Program (CRP), for which it receives an annual rental payment.[10] Land enrolled in the CRP is indexed by N.[11]

Letting i index the three uses, $\{Y, Z, N\}$, the allocation of the land endowment is determined by:

$$\text{(9)} \quad \begin{gathered} \pi_A(P_Y, P_Z, \mathrm{A}) = \max_{A_i} \sum_i (P_i y_i(A_i) - l_i)A_i \\ \text{subject to:} \\ \sum_i A_i \leq A \end{gathered}$$

where P_Y and P_Z are world crop prices, A_i is the quantity of land allocated to land use i and l_i is the amount of labor required per unit land to produce crop i. The functions $y_Y(A_Y)$ and $y_Z(A_Z)$ represent the yields of corn and other crops respectively. The function $y_N(A_N)$ is treated as the CRP rental payment in dollars per unit land, so P_N is set to one. $y_i(A_i)$ are assumed to be monotonically decreasing and concave to reflect

[10]The CRP is a government funded program, administered by the USDA, which allows farmers to voluntarily take historical cropland out of agricultural production in exchange for an annual rental payment. There are four major CRP programs, with varying contract lengths, payment rates and enrollment qualifications. We assume that only land in general sign-up or continuous non-CREP programs will be available for conversion to cropland, since the Conservation Reserve Enhancement Program (CREP) and the Farmable Wetland Program (FWP) target specific environmental objectives and offer higher rental rates making this land unlikely to be converted.

[11]We abstract from other domestic land uses, such as pastureland, forest land and rangeland. According to the 2007 Natural Resources Inventory, between 2002 and 2007, the transition of land between cropland, forestry and range was small relative to the transition of land between pasture and cropland (U.S. Department of Agriculture 2009).

decreasing returns to expanded agricultural production and decreasing rental payments for land held in CRP.[12]

The first-order conditions of (9) provide the crop supply functions, as well as the optimal allocation of land to CRP:

$$
\begin{gathered}
Y_S(P_Y, P_Z) = y_Y(A_Y(P_Y, P_Z))A_Y(P_Y, P_Z), \\
Z_S(P_Y, P_Z) = y_Z(A_Z(P_Y, P_Z))A_Z(P_Y, P_Z), \\
A_N(P_Y, P_Z).
\end{gathered} \tag{10}
$$

Food Production

Food is produced from corn and other crops by competitive firms with constant returns to scale technology:

$$X = X(Y_X, Z_X, L_X) \tag{11}$$

where Y_X, Z_X and L_X are the quantities of corn, other crops and labor used in food production. The food producer chooses Y_X, Z_X, and L_X to minimize production costs $P_Y Y_X + P_Z Z_X + L_X$ subject to (11). Given the uncompensated demand for food from (3), the food producer's demand for corn, other crops are:

$$Y_X\big(P_Y, P_Z, X(\cdot)\big) \qquad Z_X\big(P_Y, P_Z, X(\cdot)\big) \tag{12}$$

and $P_X(P_Y, P_Z)$ is the price of food.

[12] Although we do not explicitly model the environmental benefits of land held in CRP as a requisite for entry into the program, our modeling decreasing rental payments to CRP reflects the idea that additional cropland entering the program is likely to be of a lower-quality than cropland currently in the program. In practice a portion of total CRP acreage comes up for annual renewal as contracts expire, and land that is not up for renewal may also be converted but at the cost of a sizeable penalty which must be paid by the landowner. We assume that CRP contracts are never broken, and therefore abstract from this mechanism of CRP conversion. Our adjustments in CRP instead reflect the portion of CRP land that comes up for annual renewal and that is removed from the program.

Crop Export Demand

The rest of the world responds to US biofuel policies only through price channels. We consider a simplified model of crop exports and specify the rest of world excess demand for US crop exports:

(13) $$Y_W = Y_{X,W}(P_Y) - Y_{S,W}(P_Y) \qquad Z_W = Z_{X,W}(P_Z) - Z_{S,W}(P_Z)$$

where $Y_{X,W}(\cdot)$ and $Z_{X,W}(\cdot)$ are the rest of world demand for corn and other crops, and $Y_{S,W}(\cdot)$ and $Z_{S,W}(\cdot)$ are the rest of world supply of corn and other crops. To account for land use change in the rest of the world, we assume that a portion of any increases in rest of world crop supply comes from the expansion of cropland (denoted $A_{N,W}(P_Y, P_Z) = \gamma_Y Y_{S,W} + \gamma_Z Z_{S,W}$) at the expense of a composite land use that includes forest, grassland, shrubland and savanna among others.[13]

Crude Oil Supply

Gasoline producers in the US face a rest of world excess supply of crude oil given by:

(14) $$R = R_S(P_R) - R_W(P_R)$$

where $R_S(\cdot)$ is the world supply function for crude oil and $R_W(\cdot)$ is the rest of world demand for crude oil.

Equilibrium

An equilibrium consists of a price vector, P_Y, P_Z, P_R, such that the world markets for crops and crude oil:

[13]Given the uncertainty regarding the mechanisms of land use adjustment (EPA 2010; Searchinger et al. 2008; Hertel, Tyner, and Birur 2010) and the elasticity of the aggregate supply of cropland (Barr et al. 2011), we take a reduced form approach here in order to provide a transparent accounting of emissions arising from rest of world land use change.

$$
\begin{aligned}
Y_{S,D} &= Y_{X,D} + Y_E + Y_W \\
Z_{S,D} &= Z_{X,D} + Z_W \\
R &= R_G
\end{aligned} \tag{15}
$$

and the labor market in the US clear.[14]

Greenhouse Gas Emissions

We link the economic model above with a disaggregated model of greenhouse gas emissions (GHG) given by:

$$GHG = \phi_E E + \phi_Y A_Y + \phi_Z A_Z + \phi_{N,D} A_{N,D} + \phi_{N,W} A_{N,W} + \phi_G G + \phi_R R_W \tag{16}$$

where ϕ_i are GHG emissions released per unit of good or activity i, and all quantities and emissions factors are specific to country D unless indexed with W.[15]

Change in Lifecycle Emissions due to a Biofuel Policy

In order to derive an analytical expression of the first-order mechanisms that determine the change in emissions due to a policy driven unit expansion in ethanol, we make four simplifying assumptions to the framework laid out above. However, our numerical model does not rely on these simplifications. First, we assume that utility is quasi-linear in C and

[14] The share of ethanol in blended fuel (θ), and the multiplier for the blend mandate constraint (λ) must also satisfy the first order conditions for blended fuel production in equation (5). Although not discussed above, the government budget must be balanced in equilibrium. The government finances the ethanol subsidy, CRP payments, and a lump sum transfer to the representative agent from a non-distortionary labor tax. The lump-sum transfer is searched for to satisfy the government's budget constraint.

[15] The emissions coefficient for gasoline includes emissions from gasoline combustion, refining and crude oil recovery. However, the emissions coefficient for ethanol includes only emissions from ethanol production because it is standard to assume that the carbon stored in ethanol and released during ethanol combustion, is absorbed from the atmosphere during the growth of corn (IPCC 2007), and the emissions from corn production are accounted for in the corn emissions factor. While changes in crop production the rest of the world may also affect emissions, and could be tracked in our framework, we focus only on rest of world land use change because emissions from this channel are likely to dominate changes in emissions from crop production (EPA 2010).

additively separable in F and X, and that food production is separable in inputs.[16] Consequently, domestic demand for crops and fuel are functions of their own price and independent of income effects. Second, we assume that gasoline and ethanol are produced solely from crude oil and corn respectively, and we normalize the quantities of crude oil and corn to ethanol equivalent units.[17] As a result, the price of ethanol equals the price of corn ($P_E = P_Y$) and the price of gasoline equals the price of crude oil ($P_G = P_R$). Third, we aggregate total US and ROW demand for corn and other crops as $Y_{X,T}(P_Y)$ and $Z_{X,T}(P_Z)$, respectively. Fourth, we track emissions from domestic corn and other crop production according to crop supply, rather than the land allocation: $\phi_Y A_Y = \phi_Y \mathrm{Y}_{S,D}$ and $\phi_Z A_Z = \phi_Z Z_{S,D}$.[18]

Given these simplifications, the change in emissions due to a unit expansion in ethanol driven by policy j is:

$$(17)\quad \frac{dGHG}{dE_j} = \phi_E + \phi_Y \frac{dY_{S,D}}{dE} + \phi_Z \frac{dZ_{S,D}}{dE} + \phi_{N,D} \frac{dA_{N,D}}{dE} + \phi_{N,W} \frac{dA_{N,W}}{dE} - \phi_G DR_j - \phi_R \frac{\eta^{R_W}}{\eta^G} \frac{R_W}{G} DR_j$$

where: $\frac{dY_{S,D}}{dE}$ and $\frac{dZ_{S,D}}{dE}$ are the changes in corn and other crops supplied domestically, and $\frac{dA_{N,D}}{dE}$ and $\frac{dA_{N,W}}{dE}$ are changes in domestic and ROW non-cropland. These changes are per

[16] The assumption that food and fuel demand eliminates a mechanism through which land market adjustments would be tied to fuel market adjustments. If policies have different impacts on the price of fuel, this would lead to different adjustments in food demand that would in turn affect land adjustments. We allow for this mechanism in our numerical analysis although it has only minor implications for emissions.

[17] That is, $Y_E = E$ and $R_G = G$.

[18] To sign each of the land market emissions channels we further assume the cross price elasticities of crop supply in the U.S. are negative and smaller in magnitude than the own price crop supply elasticities.

unit of ethanol added and do not vary by policy. η^{R_W} is the ROW demand elasticity for crude oil and η^{G} is the gasoline supply elasticity.

DR_j is a policy specific gasoline displacement ratio, which we define as the reduction in gasoline per unit of ethanol added. The displacement ratio for the mandate is:[19]

(18) $$DR_\theta = 1 - \left(\frac{\eta^F}{1+\eta^F}\right)\left(\frac{\left(\frac{1+\eta^E}{\eta^E}\right)P_E - \left(\frac{1+\eta^G}{\eta^G}\right)P_G}{P_F - \left(\frac{\eta^F}{1+\eta^F}\right)\left(\frac{1+\eta^G}{\eta^G}\right)P_G}\right)$$

while, the displacement ratio for the input subsidy is:

(19) $$DR_\tau = 1 - \frac{\eta^F}{\eta^F - \eta^G(1-\theta)}$$

where η^F is the demand elasticity for blended fuel and the supply elasticities of ethanol, η^E, and gasoline are:

(20) $$\eta^E = \frac{1}{\theta F}\left(\eta^{Y_{S,D}}Y_{S,D} + \eta^{Y_{S,W}}Y_{S,W} - \eta^{Y_{X,T}}Y_{X,T} - \left(\frac{\eta_{P_Z}^{Y_{S,D}}\eta_{P_Y}^{Z_{S,D}}P_Z}{\eta^{Z_{S,D}}Z_{S,D} - \eta^{Z_{X,T}}Z_{X,T} + \eta^{Z_{S,W}}Z_{S,W}}\right)Z_{S,D}\right)$$
$$\eta^G = (\eta^{R_S} - \eta^{R_W})\frac{R_S}{(1-\theta)F} + \eta^{R_W}.$$

In (20), $\eta^{Y_{S,D}}$ and $\eta^{Z_{S,D}}$ are the domestic own-price supply elasticities for corn and other crops, $\eta^{Y_{S,W}}$ and $\eta^{Z_{S,W}}$ are the same for the rest of the world, and $\eta^{Y_{X,T}}$ and $\eta^{Z_{X,T}}$ are the total demand elasticities for corn and other crops. $\eta_{P_Z}^{Y_{S,D}}$ and $\eta_{P_Y}^{Z_{S,D}}$ are the cross-price supply elasticities for domestic corn and other-crops. Finally, η^{R_S} is the rest of world supply elasticity for crude oil.

[19] See the analytical appendix for a complete derivation of the formulas in equations (17) through (20).

Equations (17) through (20) can be used to understand changes in emissions resulting from a policy driven unit expansion in ethanol, how the changes in emissions differ between policies, and how each emissions channel evolves as the amount of ethanol in the economy expands. While these are general expressions of economic primitives, in what follows we restrict our discussion to several observations that inform our numerical analysis and do not review all possible cases.

Land Market Emissions Channels

The first term in equation (17) is emissions from ethanol production. The next four terms collectively reflect the change in emissions from adjustments in land markets resulting from a unit expansion in ethanol.[20] The full expressions for the total derivatives in these four terms are provided in the analytical appendix.

The change in emissions from domestic agriculture—the sum of the second and third terms in equation (17)—reflect two competing effects: increasing emissions from corn production, and falling emissions from the production of other crops. An expansion in ethanol causes demand for corn to increase and drives crop prices up, with the price of corn increasing most drastically. The resulting increase in corn supply generates emissions, although the expansion in corn supply is partially mitigated by reductions in corn demand due to the elevated corn price. The expansion of corn leads to the displacement of other crops, and a reduction in emissions. The change in emissions from domestic and rest of world land use change (LUC)—the sum of the third and fourth terms

[20] These five terms reflect a typical consequential LCA of corn ethanol, such as EPA (2010).

in equation (17)—hinges upon the change in non-cropland both domestically and in the rest of the world. Emissions from LUC are positive as elevated crop prices result in the conversion of land to crop production.

We offer two observations with regards to changes in emissions from land markets. First, emissions from this channel are the same for both policies. Since corn is an input into ethanol production, and adjustments in other crop production and land use adjustments are co-determined with the corn market adjustments in equilibrium, all land market adjustments are solely contingent on the change in ethanol demand. As both policies induce a unit expansion in ethanol, emissions from land markets will be the same irrespective of policy.[21]

Second, the change in emissions from land market adjustments resulting from a unit expansion in ethanol will likely increase in the amount of ethanol present in the economy. As the amount of ethanol in the economy expands, the change in emissions from both domestic agriculture and land use will grow in magnitude. Intuitively, this is largely a result of the growing share of land devoted to corn for ethanol production, which is increasing in the amount of ethanol in the economy. As corn for ethanol makes up a larger share of crop markets, less corn is substituted away from other end uses to ethanol. As a result, each unit of ethanol added by policy requires a larger expansion in corn production, causing emissions from this expansion to increase. Further, each additional

[21] This result is dependent on our focus on the emissions resulting from a unit increase in ethanol. An analysis of the change in emissions due to a unit reduction in gasoline due to a biofuel policy using our framework would yield agriculture and land market emissions effects dependent on the quantity of ethanol added to the fuel supply in order to induce the unit reduction in gasoline.

unit of ethanol leads to a larger increase in the prices of both corn and other crops, causing more land to be brought into agricultural production and thus greater emissions from land use change.

Fuel Market Emissions Channels

The final two terms in equation (17) reflect the change in emissions resulting from adjustments in fuel markets due to a policy driven unit expansion in ethanol. These two terms represent, respectively, the change in emissions due to adjustments in domestic gasoline and rest of world crude oil markets. We are able to express both terms as functions of a gasoline displacement ratio that depends on the policy driving the change in ethanol.

The displacement ratios in equations (18) and (19) depend on the impact of each policy on overall fuel consumption, which is mediated through the price of blended fuel. If a policy has no impact on the price and therefore the quantity of blended fuel, then each additional unit of ethanol will displace a unit of gasoline ($DR_j = 1$). However, if the price of blended fuel falls with a policy then blended fuel will increase and each unit of ethanol will displace less than a unit of gasoline ($DR_j < 1$). If the price of blended fuel increases, each unit of ethanol will displace more than a unit of gasoline ($DR_j > 1$). The displacement ratio for the mandate in equation (18) depends upon the elasticities of ethanol and gasoline supply, the elasticity of blended fuel demand, and the share of ethanol in blended fuel which effectively determines the relative prices of ethanol, gasoline and blended fuel in equation (18).[22] In contrast, the displacement ratio for the subsidy in equation (19), depends on the same factors except, notably, the ethanol supply elasticity. We offer three observations with regards to the change in emissions from fuel markets.

First, while land market emissions are not impacted by the economic conditions in fuel markets (the three fuel elasticities and the share of ethanol in blended fuel), the economic conditions in land markets (the crop supply and demand elasticities and the share of corn going to ethanol relative to other uses) may impact fuel market emissions, depending upon the policy under evaluation. The presence of the elasticity of ethanol supply in the displacement ratio for the mandate in equation (18) and the decomposition of that elasticity in equation (20), demonstrates this point. The elasticity of ethanol supply affects the displacement ratio for the mandate because the mandate simultaneously increases the demand and price of ethanol while reducing the demand and price of gasoline. Therefore, fuel market adjustments due to the mandate depend the elasticity of ethanol supply and, in turn, on the share of land used to produce corn for ethanol. However, the displacement ratio for the subsidy does not depend upon the ethanol supply elasticity and thus fuel market adjustments are independent of the share of land used to produce corn for ethanol. This is because the subsidy more than offsets any increase in the price of ethanol.

Second, as the amount of ethanol in the economy expands, the increasing share of ethanol in blended fuel affects fuel market adjustments for both policies, but the

[22] This follows from the formula for the price of blended fuel given in equation (6).

increasing share of corn going to ethanol relative to other uses only affects the fuel market adjustments due to the mandate. The displacement ratio for the mandate is increasing in the share of ethanol in blended fuel. As ethanol makes up a larger share of blended fuel, the increase in the price of ethanol will have a larger impact on the change in the price of blended fuel due to the mandate, increasing the likelihood that the price of blended fuel will rise and that the displacement ratio will exceed one.[23] In contrast, the displacement ratio for the subsidy is decreasing in the share of ethanol in blended fuel, albeit very slowly. This is because the subsidy, which is increasing in the amount of ethanol in the economy, acts to indirectly subsidize blended fuel.[24]

The increase in the displacement ratio for the mandate due to the rising share of ethanol in blended fuel is further reinforced by the tightening of the elasticity of ethanol supply as the amount of ethanol in the economy expands.[25] As the share of corn going to ethanol relative to other uses increases the ethanol supply elasticity will fall. Consequently, each additional unit expansion in ethanol will cause larger increases the

[23] To link this effect to the formula in equation (18), note that the gap between gasoline and ethanol prices will increase as the quantity of ethanol grows.

[24] Since the quantity of gasoline enters the denominator of the gasoline supply elasticity provided in (20), the gasoline supply elasticity will be smaller when the amount of ethanol present in the economy is greater. However, since the US is a relatively smaller player in international fuel markets than it is in international agricultural markets, the change in the share of U.S. gasoline demanded to total crude oil supplied from a policy induced unit expansion in biofuels is likely to be very small. If the elasticity of gasoline supply is largely constant with respect to the quantity of ethanol in the economy, then the derivative of the displacement ratio for the input subsidy case with respect to θ is negative.

[25] As demonstrated by equation (18), since $(\frac{1+\hat{\eta}^E}{\hat{\eta}^E})$ is increasing in the elasticity of ethanol supply, a fall in the ethanol supply elasticity will thus drive up the numerator in the term on the right.

prices of corn and ethanol, and therefore result in a larger displacement ratio. It follows from our first observation that the displacement ratio for the subsidy is unaffected by the tightening of the ethanol supply elasticity.

Third, the emissions reductions from fuel market adjustments resulting from a unit expansion in ethanol for the two policies will diverge as the amount of ethanol in the economy expands. Emissions due to fuel market adjustments are likely to be increasing in the amount of ethanol in the economy for the mandate but decreasing in the amount of ethanol in the economy for the subsidy. This follows immediately from the previous observation.

Numerical Results

To demonstrate how the land and fuel market emissions resulting from a unit expansion in ethanol outlined in (17) are affected by the quantity of ethanol in the economy, we supplement the analytical model discussed above with a numerical model. The functional forms used in the numerical model, the data sources used to calibrate the model parameters and emissions factors, and the justification of parameter values are discussed in the appendix.

Starting from a baseline with no ethanol policies in place in the year 2015 we increase each policy to expand ethanol quantities from 6 billion gallons to 20 billion gallons over 100 increments.[26] For each increment we approximate the change in

[26] The model is calibrated to 2003 data. To establish the 2015 counterfactual baseline, we solve the model with no ethanol policies in place annually through 2015 while allowing crop yields, ethanol production efficiency, income and the crude oil price to evolve over time. These dynamic assumptions are discussed in the appendix.

lifecycle emissions per unit of ethanol added, which equals the change in emissions over the increment divided by the quantity of ethanol added.[27] We consider a total expansion in ethanol production of 14 billion gallons in order to highlight the differences in lifecycle emissions that could potentially emerge.[28]

Land Market Emissions

Figure 1 decomposes the change in lifecycle emissions (in gCO_2e/MJ) of ethanol added by each policy into land and fuel market emissions channels. The upper two panels show how the emissions from ethanol production and land market adjustments detailed in the first five terms of equation (17) vary as the amount of ethanol in the economy expands. The change in emissions from ethanol production (circle markers), expanded corn production (star markers), and ROW land use change (triangle markers) are the most important determinants of the change in emissions due to ethanol production and land markets. In contrast, the fall in emissions from the displacement of other crops (square markers) and the increase in emissions from domestic land use change (cross markers) are noticeably smaller.

Comparing the upper two panels illustrates that the change in emissions from ethanol production and land market adjustments are virtually the same for the mandate (left

[27] We report emissions in terms of gCO2e/MJ to be consistent with the lifecycle analysis literature.

[28] The range of ethanol quantities that we consider were selected to illustrate the key results of our analysis, and not be taken as reflecting an evaluation of past or current ethanol policies. This is particularly true at the high end of the range because the implicit RFS mandate for corn ethanol reaches only 15 billion gallons in 2015 and because at these quantities the share of ethanol in blended fuel would lie above the current 10% technical limit on the amount of ethanol that is currently permitted to be blended into retail fuel (the so-called 'blend wall').

panel) and subsidy (right panel) at all ethanol quantities. This supports our analytical result that the policy instrument leading to a unit change in ethanol does not have an effect on land market emissions except through the change in demand for ethanol. The cumulative change in emissions from ethanol production and land market adjustments, which is the sum of all of the curves in either panel, steadily increase in the amount of ethanol in the economy, from about 70 gCO_2e/MJ when there are six billion gallons of ethanol in the economy to nearly 80 gCO_2e/MJ when there are 15 billion gallons of ethanol in the economy. The non-constant change in emissions from these channels is largely driven by increased emissions from domestic and rest of world land use change, although the change in emissions from corn production are also slowly increasing. Emissions from land use change increase because each additional unit of ethanol causes ever greater increases in the prices of corn and other crops and thus ever larger expansions in cropland both domestically and in the rest of the world.[29]

Fuel Market Emissions

The two lower panels in figure 1 decompose the change in emissions from fuel market adjustments from a policy induced change in ethanol, which reflects the last two terms in equation (17). With respect to either policy, the displacement of gasoline (circle marker) results in substantial emissions reductions which are only partially offset by expansions

[29] Emissions from ethanol production and other crops are effectively unchanging in the amount of ethanol added. Emissions from ethanol production are constant because the ethanol production technology is assumed to be fixed proportions while emissions reductions from the displacement of other crops remain roughly constant because demand side adjustments which coincide with that reduction are largely equi-proportional with the gradual expansion in prices.

in emissions due to expanded ROW crude oil consumption (star marker). As a result, fuel market adjustments, on net, lead to emissions reductions.

Unlike land market emissions, the change in emissions from fuel market adjustments vary dramatically across policies. Moreover, the fuel market emissions due to the mandate are considerably affected as the quantity of ethanol in the economy expands, but are roughly constant for the subsidy.

For the mandate, the cumulative reduction in emissions from fuel market adjustments increase by about 35%, from roughly 60 gCO_2e/MJ to 80 gCO_2e/MJ, as ethanol quantities increase from 6 billion gallons to 15 billion gallons. Emissions reductions from fuel markets become larger as the amount of ethanol in the economy expands for two reasons. First, the ethanol supply elasticity tightens as the share of land going to produce corn for ethanol increases, reflecting the link between land markets and fuel markets for the case of the mandate. Second, the share of ethanol in blended fuel increases independently of economic adjustments in land markets. Consequently, as the amount of ethanol in the economy expands, the change in the price of ethanol from a unit expansion of ethanol gets ever larger, and that change has a progressively greater impact on the change in the price of blended fuel (see appendix figure 2).

When the amount of ethanol in the economy is small, the price of blended fuel falls in response to the mandate because the reduction in the price of gasoline overwhelms the increase in the price of ethanol (see appendix figure 2). Accordingly, the displacement ratio is less than one. At roughly 15 billion gallons of ethanol in the economy, an increase in ethanol due to the mandate causes no change in the price of blended fuel and thus the displacement ratio equals one. After this point, each unit expansions in ethanol due to the mandate causes the price of blended fuel to increase and the displacement ratio to exceed one.

As the displacement ratio increases in the amount of ethanol in the economy, the reduction in the price of crude oil and the increase in emissions due to expanded ROW crude consumption both become larger. However, the rate at which the change in emissions from expanded ROW crude consumption grows is lower than the rate at which the change in emissions from domestically displaced gasoline falls, so the cumulative reduction in emissions from fuel market adjustments increase in the amount of ethanol in the economy.

In sharp contrast to the mandate, the subsidy leads to emissions reductions from fuel market adjustments that are effectively constant with the quantity of ethanol in the economy. Unlike the mandate case, the ethanol supply elasticity does not impact the displacement ratio for the case of the subsidy and thus the expanding share of land going to produce corn for ethanol has no impact on adjustments in fuel markets. Moreover, as the amount of ethanol in the economy expands, the increasing share of ethanol in blended fuel causes the displacement ratio to fall very slowly. This is negligible in figure 1 since the share of ethanol to blended fuel remains small across the ethanol quantities evaluated.

Lifecycle Emissions Savings

Figure 2 reports the lifecycle emissions savings per MJ of ethanol added by each policy. Lifecycle emissions savings are the negative of the sum of each curve reported in figure 1 for a given policy, or the negative of equation (17). Negative lifecycle emissions savings reflect an increase in emissions. Both the lifecycle emissions savings for the mandate (cross marker) and the lifecycle emissions savings for the subsidy (diamond marker) confirm two key results of this article, namely that the lifecycle emissions savings of corn ethanol are non-constant and vary considerably by policy.

Lifecycle emissions savings per unit of ethanol added by the mandate is monotonically increasing in the amount of ethanol in the economy. In fact, whether a unit of ethanol added by the mandate will increase or decrease emissions depends on the amount of ethanol in the economy. When six billion gallons of ethanol are in the economy, a unit expansion in ethanol increases emissions by 11 gCO_2e/MJ, whereas a unit expansion in ethanol at 15 billion gallons results in an increase in emissions of only 3 gCO_2e/MJ. After 16 billion gallons, each unit expansion of ethanol will actually reduce emissions. Given our earlier decomposition in figure 1, the emissions savings generated by an additional unit of ethanol flips from negative to positive as emissions reductions from fuel market adjustments eventually overcome the increases in emissions from land markets.

In contrast, the lifecycle emission savings per unit of ethanol added by the subsidy is monotonically decreasing in the amount of ethanol in the economy. When there are six billion gallons of ethanol in the economy, a unit expansion in ethanol increases emissions by 15gCO_2e/MJ because the emissions reductions from fuel market adjustments are insufficient to overcome the emissions expansions from land market adjustments. Since emissions from fuel markets are effectively constant with respect to the amount of ethanol in the economy while emissions from land markets are increasing, lifecycle emissions savings decline in the amount of ethanol in the economy. By 15 billion gallons, lifecycle emissions increase by 26 gCO2/MJ per unit expansion of ethanol, or a 60% increase relative to the lifecycle emissions savings at six billion gallons of ethanol.

That the lifecycle emissions savings generated by the two policies move increasingly in opposite directions as the amount of ethanol in the economy expands further highlights the tension between emissions from adjustments in land markets and fuel markets. Expansions in ethanol put increasing pressure on land markets, causing emissions from this channel to increase. If passed through to fuel markets, as is the case for the mandate, the increased pressure on land markets corresponds to larger increases in the price of ethanol and larger emissions reductions from adjustments in fuel markets. The larger emissions reductions in fuel markets lead to smaller increases in lifecycle emissions and eventually to lifecycle emissions savings due to the mandate. However, if the changes in the price of ethanol does not affect fuel market adjustments, as is the case for the subsidy, emissions savings could instead decline with the increasing emissions from land markets as the amount of ethanol in the economy expands.

Implications for Lifecycle Analysis

As the analytical formula and numerical analysis have shown, the lifecycle emissions from a policy induced unit expansion in ethanol are likely to be non-constant in the amount of ethanol in the economy. While our analytical decomposition of emissions channels suggests that it may be possible to generate policy specific adjustment factors to better account for potential non-constant emissions savings, this is beyond the scope of this article. Our findings do, however, lead to two important insights for the methodology and application of lifecycle analysis: 1) the validity of average estimates of lifecycle emissions savings depends upon the baseline amount of technology in the economy and the change in the technology under consideration; and 2) analyses that assume a constant change in emissions per unit of technology added may lead to biased estimates of total emissions changes.

Validity of Average Estimates of Lifecycle Emissions Savings

A common approach to estimating lifecycle emissions savings from biofuels has been to calculate the average emissions per unit biofuel resulting from large-scale changes in the quantity of biofuel (Searchinger et al. 2008; EPA 2010). This approach allows researchers to account for important economic adjustments that arise from large scale changes, and to align the changes in quantities with the anticipated effects of a policy (EPA 2010). However, since lifecycle emissions savings vary with the quantity of a technology in the economy, average estimates of this nature will be affected by the amount of the technology in the baseline economy as well as the magnitude of the expansion being evaluated. Moreover, these average estimates may mask important differences between the emissions resulting from the first and last unit of biofuel added.

To demonstrate these points, table 1 reports the average emissions savings resulting from 2 and 5 billion gallon expansions in ethanol, starting from baselines consisting of 6, 9 and 12 billion gallons of ethanol in the economy, for both the mandate (left panel) and the subsidy (right panel). Comparing the average emissions savings across baselines for a given policy and magnitude expansion demonstrates the importance of the baseline in the calculation of average lifecycle savings. Reflecting the total change in emissions curves given in figure 2, average emissions savings due to the mandate increase with the baseline quantity of ethanol, while average emissions savings due to the subsidy fall with the baseline quantity of ethanol. The average emissions savings for a 2 billion gallon expansion due to the mandate increases from -12.0 gCO_2e/MJ at a baseline of 6 billion gallons to -6.9 gCO_2e/MJ at a baseline of 12 billion gallons. Over the same change in baseline, the average emissions savings for a 2 billion gallon expansion due to the subsidy fall from -16.4 gCO_2e/MJ to -23.8 gCO_2e/MJ. Intuitively, the results for each policy follow Jensen's inequality given the convexity of the lifecycle emissions savings curve for the mandate and the concavity of the lifecycle emissions savings curve for the subsidy (figure 2).

For a given policy and baseline, comparing the average emissions savings resulting from ethanol expansions of different magnitudes highlights the importance of the magnitude change in ethanol for average lifecycle emissions savings calculations. For the mandate, the difference in average emissions savings from a 5 billion gallon expansion relative to a 2 billion gallon expansion range from 0.7 gCO_2e/MJ when the baseline is 6 billion gallons, to a 4.0 gCO_2e/MJ difference when the baseline is 12 billion gallons. For the subsidy, the difference in average emissions savings between the two expansions range from a -1.7 gCO_2e/MJ difference when the baseline is 6 billion gallons, to a -2.4 gCO_2e/MJ difference when the baseline is 12 billion gallons. That the difference both across expansion sizes and baselines is greater for the mandate than the subsidy reflects the fact that the lifecycle emissions savings curve for the mandate is more convex than the lifecycle emissions savings curve for the subsidy is concave.

The calculation of lifecycle emissions savings as the average over a change in technology expansion may also mask how lifecycle emissions vary over the expansion. The 'First' and 'Last' columns in table 1 report the lifecycle emissions savings for the first unit and last unit for each policy, baseline, and expansion combination. The absolute differences between the average emissions savings and the emissions savings from the first and last unit can be dramatic, especially when the amount of ethanol in the baseline is large. For example, the average emissions savings from a 5 billion gallon expansion in ethanol due to a mandate relative to a baseline of 9 billion gallons are -8.8 gCO_2e/MJ, but the first unit of this change generated emissions savings of -11.3 gCO_2e/MJ, while the last unit generated savings of -4.9 gCO_2e/MJ. Likewise, for the same expansion and policy relative to a 12 billion gallon baseline, the first unit increases emissions by 8.5 gCO_2e/MJ whereas the last unit reduces emissions by 5.8 gCO_2e/MJ, although the average emissions savings estimate is -2.9 gCO_2e/MJ. The signs of the average emissions savings and the emissions savings from the last unit of ethanol differ because lifecycle emissions savings due to the mandate become positive after there are 16 billion gallons of ethanol in the economy.

Possible Bias from Constant Emissions Savings Assumption

As figure 2 has shown lifecycle emissions savings are not constant per unit of technology added. Consequently, it is perhaps self-evident that analyses that assume constant emissions savings per unit of biofuel may lead to biased estimates of total emissions changes.[30] However, the extent of this bias is less clear. To show this, table 2 provides the change in emissions from a 5 billion gallon expansion in ethanol calculated using our full numerical model to two alternative estimates of total emissions savings that assume lifecycle emissions are constant either at the emissions savings resulting from the first unit of ethanol added or the last unit of ethanol added. We conduct this analysis for both policies from baselines of 6, 9 and 12 billion gallons of ethanol.

Estimates of emissions savings relying on constant emissions savings from either the first or last unit of ethanol can result in significant bias. Consider a five billion gallon increase in ethanol from a baseline of nine billion gallons. For the mandate, calculations using constant emissions savings from the first unit will overestimate the total change in emissions by 28.8%, while calculations using constant emissions savings from the last unit will underestimate the total change in emissions by 44.2%. For the subsidy, the total change in emissions are underestimated by 15.0% if emissions savings from the first unit are used, but overestimated by 15.7% if emissions savings from the last unit are used.

We also note that bias is substantially increasing in the ethanol baseline for the case of the mandate, but declining slightly in the ethanol baseline for the case of the subsidy. While the absolute bias (the sum of the absolute values of the bias from the first and last units) for the subsidy falls from 31.7% at the 6 billion gallon baseline to 30.0% at the 12

[30] Bias here reflects the idea that estimates of the total change in emissions as equaling a constant emissions factor multiplied by an expansion in technology are likely to differ from the integration of either of the curves in figure 2 over the same technology expansion for any possible baseline. We use hypothetical constant emissions factors inferred from our numerical analysis to demonstrate this point, although another useful comparison would be to a constant emissions factor taken from the literature, which is beyond the scope of this article.

billion gallon baseline, the absolute bias for the mandate jumps from 22.6% to 522.4% for the same change in baseline. This, again, reflects the convexity and concavity of the lifecycle emissions per unit of ethanol curves for the mandate and subsidy, respectively.[31]

Conclusions

In this article we developed an analytic and numerical model that integrates land, food and fuel markets and is linked with a sectoral emissions model to examine how the amount of biofuel in the economy impacts the lifecycle emissions of a biofuel under different policies. This framework allows the lifecycle emissions due to an expansion in biofuel to be determined by the underlying economic conditions in land and fuel markets and for those conditions to be fundamentally intertwined as a result of the policy under evaluation and the amount of biofuel in the economy. These features are critical for determining how the lifecycle emissions per unit of biofuel added by a policy can vary significantly in the amount of biofuel in the economy, which has important implications for the lifecycle analysis of biofuels.

Our central finding is that the change in GHG emissions due to a unit expansion in biofuel will vary dramatically in the amount of biofuel in the economy and with the policy doing the expansion. We demonstrate this result numerically for the case of corn ethanol. When six billion gallons of ethanol are present in the economy, an additional unit of ethanol due to a blend mandate increases emissions by 11 gCO_2e/MJ. At 15 billion gallons however, an additional unit of ethanol due to the blend mandate increases

[31] The growth in absolute bias across baselines for the mandate is also a result of the actual increase in emissions falling as the baseline expands since emissions reductions due to the mandate are only realized at larger quantities of ethanol.

emissions by only 5 gCO_2e/MJ. This reflects that the lifecycle emissions savings from a unit expansion in ethanol due to a mandate are convex and increasing in the amount of ethanol in the economy. In contrast, a unit increase in ethanol due to an input subsidy causes emissions to increase by 15 gCO_2e/MJ when six billion gallons of ethanol are in the economy and 26 gCO_2e/MJ when there are 15 billion gallons. This reflects that the lifecycle emissions savings from a unit expansion in ethanol due to a subsidy are instead concave and decreasing in the amount of ethanol in the economy.

The divergence in lifecycle emissions savings curves with respect to the amount of ethanol in the economy for the two policies reflects that changes to underlying economic conditions in land and fuel markets will affect the two policies differently. We show that emissions from land markets depend on the share of land going to ethanol, which is contingent upon the change in ethanol demand. Therefore, a unit expansion in ethanol due to either policy will have the same impact on emissions from this channel. In contrast, emissions from fuel markets are affected by economic conditions in both land and fuel markets for the mandate, but are affected by only economic conditions in fuel markets for the subsidy. For the mandate, as the amount of ethanol in the economy expands, emission reductions from fuel market adjustments are increasing in the share of ethanol in blended fuel and the share of land used to produce corn for ethanol through the ethanol supply elasticity. However, for the subsidy, fuel market emissions do not depend on the ethanol supply elasticity or the share of land used to produce corn, and consequently emissions savings from fuel markets instead fall in the share of ethanol in blended fuel, albeit very slowly over the quantities of ethanol we evaluate.

That the lifecycle emissions per unit of biofuel vary considerably both between policies and the amount of ethanol in the economy has allowed us to uncover two important insights for the methodology and application of lifecycle analysis. First, average estimates of lifecycle emissions savings depend critically upon the baseline amount of technology in the economy and the change in the technology under consideration. Second, analyses that assume a constant change in emissions per unit of a technology may lead to misleading estimates of total changes in emissions.

While many papers in the LCA literature have focused on the important issue of accounting for uncertainty across emissions and economic parameters (Plevin et al. 2010; Mullins, Griffin, and Matthews 2011; Rajagopal and Plevin 2013), an issue that we abstract from here, our results suggest that equal attention should be placed on understanding the sensitivity of lifecycle savings estimates to the amount of the technology present in the baseline, the change in the technology under consideration, and the policy that drives the change in technology. The sensitivity to underlying policy and economic conditions evident in our analysis highlights a weakness of technology-specific attributional or consequential lifecycle emissions estimates that cannot account for these factors. Our analysis suggests that it is important to delineate the underlying economic and policy context when performing lifecycle analyses that incorporate economic adjustments. Consequently, analysts should use extreme caution when applying the results a lifecycle analysis for particular economic and policy context to a different economic and policy context, and great care should be taken to align lifecycle emissions savings estimates with the appropriate policy under consideration.

Although our numerical analysis brings up considerable issues with the LCA approach, our analytical formula, perhaps, points to a practical solution for aligning lifecycle emissions savings with the economic and policy context that deserves further research. Our analytical decomposition of the change in GHG emissions due to a change in ethanol could possibly be extended to develop a set of adjustment factors that would allow practitioners to obtain first-order and/or second-order approximations of the lifecycle emissions savings of a policy using only technology-specific lifecycle metrics and minimal economic data on elasticities, quantities and prices. Such tractable analytical formulas have proven useful in other applications such as tax policy (Goulder and Williams III 2003), social insurance (Chetty 2006) and welfare analysis (Harberger 1974; Hines 1999). By linking the change in emissions back to key elasticities and market conditions, the adjustment factor approach may also provide analysts with a way to study the sensitivity of the emissions results across many dimensions without relying on computationally expensive economic models.

	Blend Mandate			Subsidy		
	Average	First	Last	Average	First	Last
Baseline, 6 billion gallons						
2 billion gallon increase	-12.0	-12.3	-11.7	-16.4	-15.3	-17.4
5 billion gallon increase	-11.3	-12.3	-9.7	-18.1	-15.3	-21.1
Baseline, 9 billion gallons						
2 billion gallon increase	-10.6	-11.3	-9.7	-19.8	-18.6	-21.1
5 billion gallon increase	-8.8	-11.3	-4.9	-21.8	-18.6	-25.3
Baseline, 12 billion gallons						
2 billion gallon increase	-6.9	-8.5	-4.9	-23.8	-22.4	-25.3
5 billion gallon increase	-2.9	-8.5	5.8	-26.2	-22.4	-30.3

Notes: All emissions reported in gCO_2e/MJ. Average columns report the per unit GHG savings resulting from each policy, baseline, expansion size combination. 'First' and 'Last' columns report, respectively, the emissions savings due to the first and last unit of ethanol for each expansion.

Table 1. Implications of Average Emissions Savings Calculations

	Blend Mandate			Subsidy		
	Total Change in GHG ($TgCO_2e$)	Difference due to constant marginal GHG savings		Total Change in GHG ($TgCO_2e$)	Difference due to constant marginal GHG savings	
		First	Last		First	Last
Baseline, 6 billion gallons	4.5	8.7%	-13.9%	7.3	-15.5%	16.2%
Baseline, 9 billion gallons	3.5	28.8%	-44.2%	8.8	-15.0%	15.7%
Baseline, 12 billion gallons	1.1	210.5%	-311.9%	10.5	-14.6%	15.4%

Notes: Emissions resulting from a 5 billion gallon increase in ethanol. Total change in GHG columns allows emissions per unit ethanol to adjust. Remaining columns report the percent difference between the total change in GHG and the total change in GHG when calculated using constant emissions savings at either the first unit of the expansion, or the last unit of the expansion.

Table 2. Implications of Constant Marginal Emissions Savings

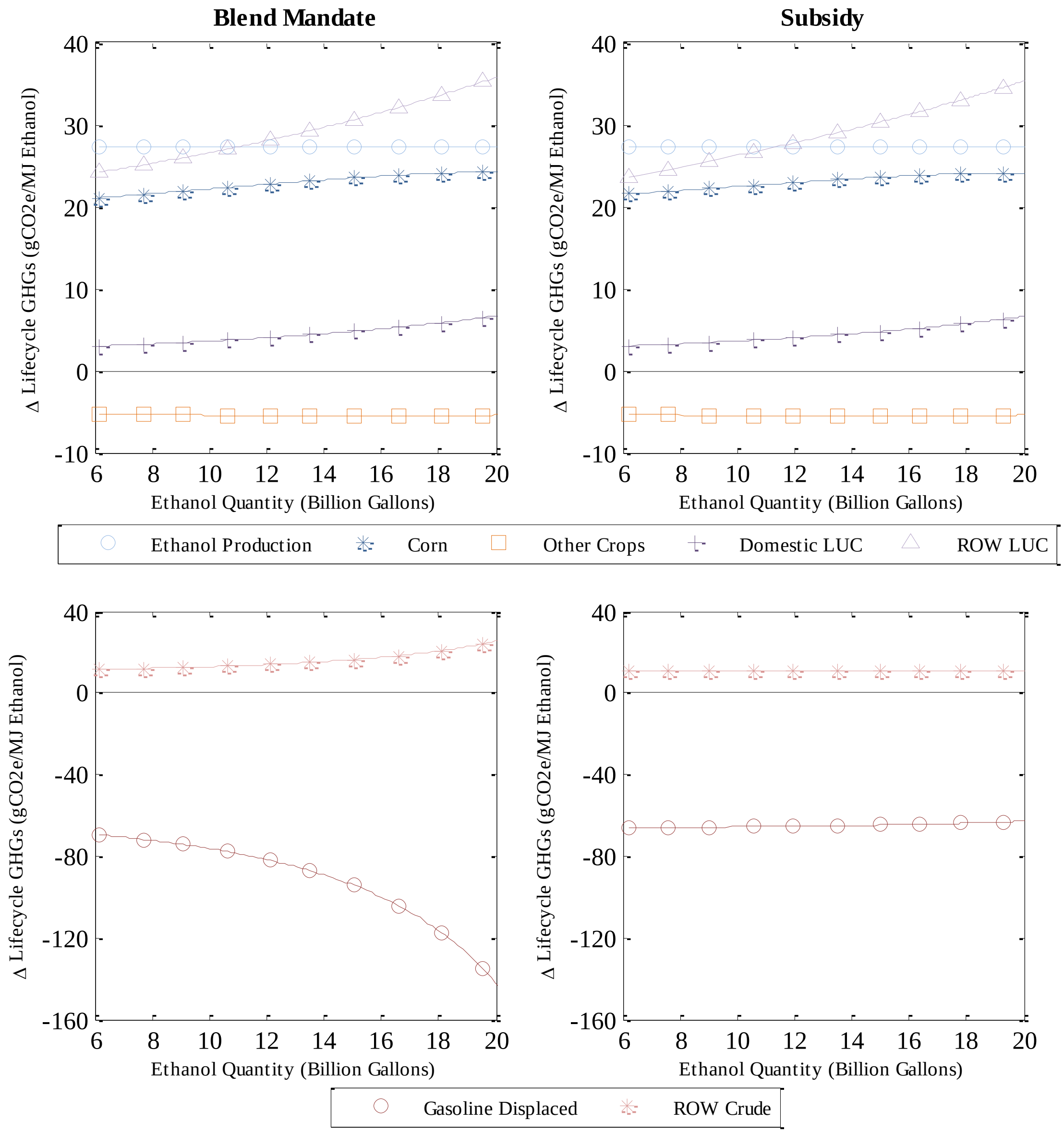

Figure 1. Land and Fuel Market Emissions As Ethanol Quantities Expand

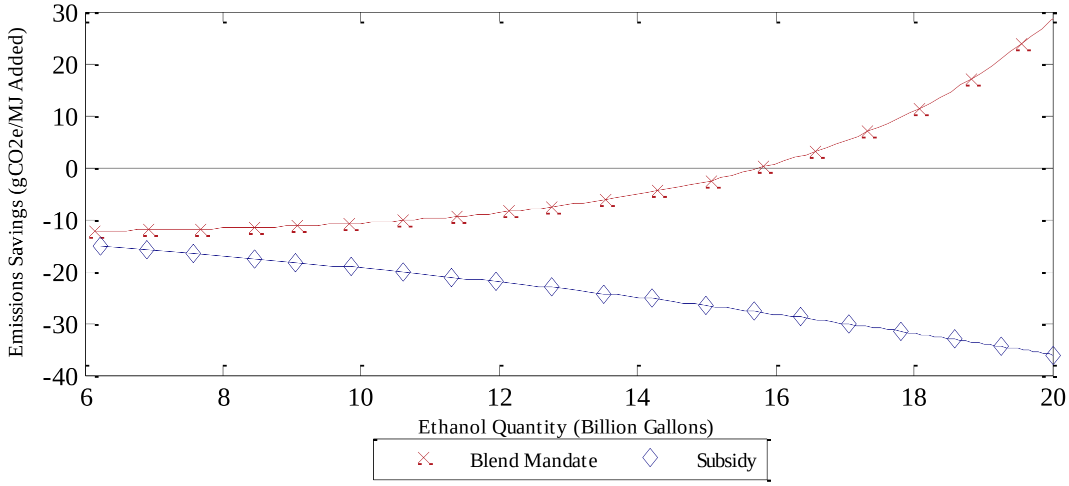

Figure 2. Lifecycle Emissions Savings as Ethanol Quantities Expand

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Analytical Appendix

This appendix includes the complete derivations for equations (17) through (20). We proceed by first decomposing land market emissions and then decomposing fuel market emissions.

Land Market Emissions

The change in emissions from agriculture and land use due to an expansion in ethanol are:

$$\text{(A.1)} \qquad \frac{dGHG_{land}}{dE} = \phi_E + \phi_Y \frac{dY_{S,D}}{dE} + \phi_Z \frac{dZ_{S,D}}{dE} + \phi_{N,D} \frac{dA_{N,D}}{dE} + \phi_{N,W} \frac{dA_{N,W}}{dE}.$$

Each total derivative in equation (A.1) is a function of the change in the prices of corn and other crops. Given the crop market clearing conditions:

$$\text{(A.2)} \qquad \begin{aligned} Y_{S,D}(P_Y, P_Z) &= Y_{X,T}(P_Y) + E - Y_{S,W}(P_Y) \\ Z_{S,D}(P_Y, P_Z) &= Z_{X,T}(P_Z) - Z_{S,W}(P_Z) \end{aligned}$$

the changes in crop prices can be obtained by totally differentiating crop market clearing conditions with respect to E. This will provide two linear equations in $\frac{dP_Y}{dE}$ and $\frac{dP_Z}{dE}$:

$$\text{(A.3)} \qquad \begin{aligned} \left(\frac{\partial Y_{S,D}}{\partial P_Y} - \frac{dY_{X,T}}{dP_Y} + \frac{dY_{S,W}}{dP_Y}\right)\frac{dP_Y}{dE} + \left(\frac{\partial Y_{S,D}}{\partial P_Z}\right)\frac{dP_Z}{dE} &= 1 \\ \left(\frac{\partial Z_{S,D}}{\partial P_Y}\right)\frac{dP_Y}{dE} + \left(\frac{\partial Z_{S,D}}{\partial P_Z} - \frac{dZ_{X,T}}{dP_Z} + \frac{dZ_{S,W}}{dP_Z}\right)\frac{dP_Z}{dE} &= 0. \end{aligned}$$

from which the changes in crop prices can be obtained using Cramer's Rule. The change in crop prices, in terms of elasticities and initial quantities are:

$$\text{(A.4)} \qquad \begin{aligned} \frac{dP_Y}{dE} &= \frac{P_Y\left(\eta^{Z_{S,D}} Z_{S,D} - \eta^{Z_{X,T}} Z_{X,T} + \eta^{Z_{S,W}} Z_{S,W}\right)}{\left(\eta^{Y_{S,D}} Y_{S,D} - \eta^{Y_{X,T}} Y_{X,T} + \eta^{Y_{S,W}} Y_{S,W}\right)\left(\eta^{Z_{S,D}} Z_{S,D} - \eta^{Z_{X,T}} Z_{X,T} + \eta^{Z_{S,W}} Z_{S,W}\right) - \eta_{P_Z}^{Y_{S,D}} Y_{S,D} \eta_{P_Y}^{Z_{S,D}} Z_{S,D}} \\ \frac{dP_Z}{dE} &= \frac{-P_Z \eta_{P_Y}^{Z_{S,D}} Z_{S,D}}{\left(\eta^{Y_{S,D}} Y_{S,D} - \eta^{Y_{X,T}} Y_{X,T} + \eta^{Y_{S,W}} Y_{S,W}\right)\left(\eta^{Z_{S,D}} Z_{S,D} - \eta^{Z_{X,T}} Z_{X,T} + \eta^{Z_{S,W}} Z_{S,W}\right) - \eta_{P_Z}^{Y_{S,D}} Y_{S,D} \eta_{P_Y}^{Z_{S,D}} Z_{S,D}}. \end{aligned}$$

The change crop supplies can be obtained by rearranging the total derivatives of the crop market clearing conditions and the changes in land use can be obtained by totally differentiating the land use equations $A_{N,D}$ and $A_{N,W}$:

$$\text{(A.5)} \qquad \begin{aligned} \frac{dY_{S,D}}{dE} &= 1 + \left(\frac{dY_{X,T}}{dP_Y} - \frac{dY_{S,W}}{dP_Y}\right)\frac{dP_Y}{dE} \\ \frac{dZ_{S,D}}{dE} &= \left(\frac{dZ_{X,T}}{dP_Z} + \frac{dZ_{S,W}}{dP_Z}\right)\frac{dP_Z}{dE} \\ \frac{dA_{N,D}}{dE} &= \frac{dA_{N,D}}{dP_Y}\frac{dP_Y}{dE} + \frac{dA_{N,D}}{dP_Z}\frac{dP_Z}{dE} \\ \frac{dA_{N,W}}{dE} &= \gamma_Y \frac{dY_{S,W}}{dP_Y}\frac{dP_Y}{dE} + \gamma_Z \frac{dZ_{S,W}}{dP_Z}\frac{dP_Z}{dE} \end{aligned}$$

and plugging in equation (A.4) for the total derivatives of crop prices:

$$\text{(A.6)} \qquad \begin{aligned} \frac{dY_{S,D}}{dE} &= 1 - \frac{\left(\eta^{Y_{S,W}}Y_{S,W} - \eta^{Y_{X,T}}Y_{X,T}\right)\left(\eta^{Z_{S,D}}Z_{S,D} - \eta^{Z_{X,T}}Z_{X,T} + \eta^{Z_{S,W}}Z_{S,W}\right)}{\Phi} \\ \frac{dZ_{S,D}}{dE} &= \frac{-\left(\eta^{Z_{X,T}}Z_{X,T} - \eta^{Z_{S,W}}Z_{S,W}\right)\eta_{P_Y}^{Z_{S,D}}Z_{S,D}}{\Phi} \\ \frac{dA_{N,D}}{dE} &= \frac{\left(\eta_{P_Y}^{A_{N,D}}\left(\eta^{Y_{X,T}}Y_{X,T} - \eta^{Y_{S,W}}Y_{S,W}\right) - \eta_{P_Z}^{A_{N,D}}\eta_{P_Y}^{Z_{S,D}}Z_{S,D}\right)A_{N,D}}{\Phi} \\ \frac{dA_{N,W}}{dE} &= \frac{\gamma_Y\eta^{Y_{S,W}}Y_{S,W}\left(\eta^{Z_{S,D}}Z_{S,D} - \eta^{Z_{X,T}}Z_{X,T} + \eta^{Z_{S,W}}Z_{S,W}\right) - \gamma_Z\eta^{Z_{S,W}}\eta_{P_Y}^{Z_{S,D}}Z_{S,W}Z_{S,D}}{\Phi} \\ \Phi &= \left(\eta^{Y_{S,D}}Y_{S,D} - \eta^{Y_{X,T}}Y_{X,T} + \eta^{Y_{S,W}}Y_{S,W}\right)\left(\eta^{Z_{S,D}}Z_{S,D} - \eta^{Z_{X,T}}Z_{X,T} + \eta^{Z_{S,W}}Z_{S,W}\right) - \eta_{P_Z}^{Y_{S,D}}Y_{S,D}\eta_{P_Y}^{Z_{S,D}}Z_{S,D}. \end{aligned}$$

In order to sign the changes in agriculture and land use, we first establish that, conditional on our assumptions for the relationship between the own and cross-price elasticities, the denominator for each equation in (A.6), Φ, will be positive. The change in corn production is bounded between zeros and one. One is an upper bound because the second term in $\frac{dY_{S,D}}{dE}$ is always positive, which can be observed directly. The lower bound of zero requires that the second term in $\frac{dY_{S,D}}{dE}$ be less than one, which is the case because the denominator contains two positive terms that are not present in the numerator. With

$\Phi > 0$ established, the change in other crops will be negative, $\frac{dZ_{S,D}}{dE} < 0$, and that the domestic and rest of world land use terms are both positive, $\frac{dA_{N,W}}{dE} > 0, \frac{dA_{N,D}}{dE} > 0$.

Fuel Market Emissions

To simplify the expressions for fuel market emissions, we collapse all land and fuel market adjustments into supply functions for ethanol and gasoline denoted $E_S(P_E)$ and $G_S(P_G)$. The supply function for ethanol is the excess supply function for corn going to ethanol production. The supply elasticity of for ethanol can be derived by totally differentiating the corn market clearing condition with respect to P_Y, and multiplying through by P_Y/E:

(A.7)
$$\frac{dE}{dP_Y} = \frac{\partial Y_{S,D}}{\partial P_Y} - \frac{dY_{X,T}}{dP_Y} + \frac{dY_{S,W}}{dP_Y} + \frac{\partial Y_{S,D}}{\partial P_Z}\frac{dP_Z}{dP_Y}$$
$$\eta^E = \eta^{Y_{S,D}}\frac{Y_{S,D}}{E} + \eta^{Y_{S,W}}\frac{Y_{S,W}}{E} - \eta^{Y_{X,T}}\frac{Y_{X,T}}{E} - \left(\frac{\eta_{P_Z}^{Y_{S,D}}\eta_{P_Y}^{Z_{S,D}}P_Z}{\eta^{Z_{S,D}}Z_{S,D} - \eta^{Z_{X,T}}Z_{X,T} + \eta^{Z_{S,W}}Z_{S,W}}\right)\frac{Z_{S,D}}{E}$$

where $\frac{dP_Z}{dP_Y}$ can be solved for from the total derivatives of the crop prices in equation (A.4):

(A.8)
$$\frac{dP_Z}{dP_Y} = \frac{-\frac{\partial Z_{S,D}}{\partial P_Y}}{\frac{\partial Z_{S,D}}{\partial P_Z} - \frac{dZ_{X,T}}{dP_Z} + \frac{dZ_{S,W}}{dP_Z}}.$$

The supply function for gasoline is the excess supply function for crude oil going to gasoline production. The elasticity of gasoline supply can be derived by rearranging and totally differentiating the crude oil market clearing condition, then dividing through by P_G/G:

(A.9)
$$\frac{dG}{dP_G} = \frac{dR_S}{dP_G} - \frac{dR_W}{dP_G} \rightarrow \eta^G = \eta^{R_S}\frac{R_S}{G} - \eta^{R_W}\frac{R_W}{G}.$$

The change in emissions from fuel markets are:

(A.10)
$$\frac{dGHG_{Fuel}}{dE} = \phi_G\left(\frac{dF}{dP_F}\frac{dP_F}{dE} - 1\right) + \phi_R\frac{dR^W}{dP_G}\frac{dP_G}{dE}$$

and gasoline displacement ratio, the reduction gasoline per unit expansion in ethanol, is:

(A.11)
$$DR = 1 - \frac{dF}{dP_F}\frac{dP_F}{dE}$$

so fuel market emissions can be written as:

(A.12)
$$\frac{dGHG_{Fuel}}{dE} = -\phi_G DR - \phi_R\frac{\eta^{R_W}}{\eta^G}\frac{R_W}{G}DR.$$

The final term is found by solving the ethanol and gasoline market clearing conditions for $\frac{dP_G}{dE}$ as a function of the displacement ratio. The ethanol and gasoline market clearing conditions are:

(A.13)
$$\begin{gathered} E_S(P_E) = \theta F(P_F) \\ G_S(P_G) = (1-\theta)F(P_F). \end{gathered}$$

Totally differentiating with respect to E yields two equations:

(A.14)
$$\begin{gathered} 1 = \frac{d\theta}{dE}F + \frac{dF}{dP_F}\frac{dP_F}{dE}\theta \\ \frac{dG}{dP_G}\frac{dP_G}{dE} = (1-\theta)\frac{dF}{dP_F}\frac{dP_F}{dE} - \frac{d\theta}{dE}F \end{gathered}$$

that can be used to solve for $\frac{dP_G}{dE}$ in terms of $\frac{dP_F}{dE}$ and therefore the displacement ratio:

(A.15)
$$\begin{gathered} \frac{d\theta}{dE}F = 1 - \frac{dF}{dP_F}\frac{dP_F}{dE}\theta \\ \frac{dG}{dP_G}\frac{dP_G}{dE} = (1-\theta)\frac{dF}{dP_F}\frac{dP_F}{dE} - 1 + \frac{dF}{dP_F}\frac{dP_F}{dE}\theta \\ \frac{dP_G}{dE} = \frac{\frac{dF}{dP_F}\frac{dP_F}{dE} - 1}{\frac{dG}{dP_G}} = -\frac{P_G}{G}\frac{DR}{\eta^G} \end{gathered}$$

The displacement ratio depends on the policy driving the change in ethanol and depends on how the total derivative of the blended fuel price. We start with the mandate. The total derivative of the zero profit condition for fuel production,$P_F F = P_E E + P_G G$, with respect to the ethanol quantity is:

$$\frac{dP_F}{dE}\left(F + \frac{dF}{dP_F}P_F\right) = \frac{dP_E}{dE}E + P_E + \frac{dP_G}{dE}\left(G + \frac{dG}{dP_G}P_G\right). \tag{A.16}$$

Substituting in the total derivative of the price of gasoline from (A.15), elasticities and initial quantities, then isolating $\frac{dP_F}{dE}$ yields:

$$\frac{dP_F}{dE} = \frac{\left(\frac{1+\eta^E}{\eta^E}\right)P_E - \left(\frac{1+\eta^G}{\eta^G}\right)P_G}{F\left(1 + \frac{P_G}{P_F}\left(\frac{\eta^F}{1+\eta^F}\right)\left(\frac{1+\eta^G}{\eta^G}\right)\right)} \tag{A.17}$$

which can be substituted into equation (A.11) to yield the displacement ratio for the blend mandate in (18).

For the input subsidy, $P_G = P_F = P_E - \tau$ in equilibrium so the gasoline displacement ratio is:

$$DR_\tau = 1 - \frac{dF}{dP_F}\left(\frac{dP_E}{dE} - \frac{d\tau}{dE}\right) \tag{A.18}$$

The total derivatives of the ethanol and gasoline market clearing conditions:

$$\begin{gathered}\frac{d\theta}{dE}F + \theta\frac{dF}{dP_F}\left(\frac{dP_E}{dE} - \frac{d\tau}{dE}\right) - 1 = 0 \\ (1-\theta)\frac{dF}{dP_F}\left(\frac{dP_E}{dE} - \frac{d\tau}{dE}\right) - \frac{d\theta}{dE}F - \frac{dG}{dP_G}\left(\frac{dP_E}{dE} - \frac{d\tau}{dE}\right) = 0.\end{gathered} \tag{A.19}$$

can be used to solve for the total change in the subsidized price of ethanol:

$$\frac{dP_E}{dE} - \frac{d\tau}{dE} = \frac{1}{\frac{dF}{dP_F} - \frac{dG}{dP_G}}. \tag{A.20}$$

Plugging equation (A.20) into equation (A.18) and substituting in elasticities and initial conditions yields equation (19) in the text.

Note that the second term in equation (18) can be negative, so the displacement ratio for the mandate can lie above one. The displacement ratio for the subsidy lies between zero and one: $0 \leq DR_{\tau} \leq 1$. This requires that the second term in equation (19) lie above zero but below one. The second term in equation (19) is greater than zero because both numerator and denominator are negative. Since the denominator is a larger negative number than the numerator, it must be the case that the displacement ratio is below one.