SECOND MORTGAGES: VALUATION AND IMPLICATIONS FOR THE PERFORMANCE OF STRUCTURED FINANCIAL PRODUCTS.

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1. Introduction

During the U.S. housing boom, house prices, as measured by the Case-Shiller Composite-20 index, increased by an average annualized rate of 11% between the first quarter of 2001 and the fourth quarter of 2005. Over this same time period, U.S. homeowners extracted an average of slightly under $700 billion of equity *each year* relying on cash-out refinancing, home equity lines-of-credit, and second mortgages (Greenspan and Kennedy [2007]). Given the prominent role played by home equity extraction, it is important to understand its implications for the valuation of residential mortgages and, in turn, the properties of structured financial products, like collateralized debt obligations (CDOs), based on these mortgages.

This paper investigates the valuation and properties of second mortgages while explicitly recognizing that taking on additional mortgage debt to extract equity during periods of increasing house prices synchronizes the financing decisions of homeowners. Khandani, Lo, and Merton [2009] argue that doing so increases the default risk of these homeowners and effectively correlates their default decisions. As anticipated by Baker [2002], a precipitous drop in house prices of the magnitude experienced in the U.S. beginning in 2006 can now result in many homeowners defaulting together.[1] In this case, even the most senior CDO tranches may no longer be protected from default losses.

To better understand these issues, we provide, in the spirit of Black and Cox [1976], a closed-form structural model to value first-lien and subordinated mortgages when homeowners can take on additional mortgage debt to extract equity from their appreciated houses. Interest rate driven refinancings are excluded by fixing the rate of interest at a constant level. This allows us to concentrate on the arguably more important role of house prices, not the behavior of interest rates, in the performance of structured financial products collateralized by mortgages subsequent to the bursting of the U.S. housing bubble. However, unlike extant

[1] Khandani et al. [2009] conclude that the dramatic increase in cash-out refinancing activity and its resultant synchronization of U.S. household's leverage was alone responsible for last decade's dramatic increase in the U.S. housing market's systematic risk. In particular, their historical simulations of this market over the June 2006 to December 2008 time period generated loss estimates of $1.5 trillion with equity extraction as compared to only $280 billion in its absence.

risky mortgage valuation models, we do not exogenously specify house prices and their dynamics. Rather, we take a house's service flow, that is, the rent foregone by owning rather than renting, as our state variable and endogenously derive house prices as well as both first- and junior-lien mortgage values. The role of a house's service flow in our model is analogous to that of a firm's *EBIT* in dynamic capital structure models (see, for example, Goldstein, Ju, and Leland [2001]). As a result, not only can we investigate how house prices affect the performance of mortgages but we can also investigate how mortgage features impact house prices.

We find that house values are lowered when homeowners are afforded the opportunity to extract equity. The more often homeowners can extract equity, the lower house values. These results reflect the increased likelihood of default and so larger bankruptcy costs incurred when additional junior financing is relied upon. We also investigate the risk characteristics of first-lien mortgages in the presence of subordinated mortgages. For example, the waiting time to default is, on average, shorter when homeowners have the option to cash-out refinance. Furthermore, the more often homeowners extract equity, the shorter the waiting time to default reflecting homeowners' increased debt burden. A homeowner's bankruptcy boundary is also affected as default is triggered at a lower service flow when the homeowner can cash-out refinance. This lower bankruptcy boundary reflects the possibility that even when house prices are low, they may still subsequently rebound and the option to cash-out refinance can then be profitably exercised. As expected, the rate of interest charged on subordinate mortgages exceeds that charged on first-lien mortgages. This spread is seen to be sensitive to the dynamics of the housing service-flow process as well as the homeowners' desired loan-to-value ratio.

The bursting of the U.S. housing bubble saw the unraveling of many CDOs. For example, Barnett-Hart [2009] reports that by early 2009 *Aaa* certificates of CDOs originated in 2005 were downgraded, on average, eight notches to *Baa*2, just slightly above investment-grade. Some observers have argued that these large downgrades reflected the fact that credit rating agencies were blind to the possibility that first-lien borrowers could subsequently obtain second loans, so-called “secret seconds” and, as a result, did not recognize the consequences of cash-out refinancing on the performance of CDOs. The likelihood that these second loans could have impacted CDO performance is supported by Goodman, Ashworth, Landy, and Yin [2010]'s calculation that more than 50% of first liens in private label securitizations over the 2000 to 2007 time period had a second lien behind them, obtained either subsequently as a consequence of cash-out refinancing or simultaneously in the form of piggy-back financing. Our structural valuation framework allows us to investigate how CDO performance is affected by cash-out refinancing. To do so, we posit a naïve credit rating agency which rates a CDO while oblivious to the possibility that first-lien mortgage borrowers can obtain second or even third liens to optimally extract equity from their appreciated houses. When the resultant CDO structure is confronted by data generated by homeowners who optimally refinance as well as default, we find that the CDO's resultant performance is consistent with the magnitude of downgrades observed subsequent to the bursting of the U.S. housing bubble. By contrast, our results do not support the argument that the downgrades observed in practice occurred only because the severity of the U.S. housing market downturn was simply underestimated.

The plan of this paper is as follows. Section 2 puts forward and details the properties of a closed-form structural model to value risky residential mortgages. We begin by allowing homeowners to only optimally default. In particular, homeowners pursue a *static* optimal financing policy in which they rely on an exogenously specified optimal loan-to-value ratio when originally purchasing their house. With subsequent house price appreciation, however, homeowners cannot extract equity by obtaining a second mortgage. Next we allow homeowners to optimally extract equity in addition to optimally defaulting. Under an optimal *dynamic* financing policy, homeowners now attempt to maintain their desired loan-to-value ratio over time by obtaining a second- or more junior-lien mortgage when house prices appreciate sufficiently. Section 3 investigates the extent to which the unraveling of CDOs in the aftermath of the bursting of the U.S. housing bubble can be attributed to naïve credit rating agencies who ignored the presence of second loans. We consider a cash CDO collateralized by a pool of first-lien mortgages. The ratings of the CDO's certificates are based on the assumption that homeowners follow a static optimal financing policy and do not extract equity from their appreciated houses. We demonstrate that if homeowners actually follow a dynamic optimal financing policy and obtain second and third mortgages to optimally extract equity, then the presence of these secret seconds and thirds can degrade the performance of the first liens so much so that significant downgrades of the CDO certificates result. Section 4 concludes the paper.

2. A Closed-Form Structural Model of Risky Residential Mortgages

Our underlying state variable is the service flow from a unit of housing, denoted by δ, which represents the cost per unit time of renting the residential property. The role of δ in our model is analogous to that of a firm's *EBIT* in dynamic capital structure models (see, for example, Goldstein et al. [2001]). This is in contrast to the traditional approach of valuing risky mortgages, for example, Kau, Keenan, Mueller, and Epperson [1995], which takes an unlevered house price as a state variable. Our approach views residential real estate itself as a contingent claim on δ which can then be valued alongside the risky mortgage. The effects of changing mortgage features[2] on house prices can be easily explored within this framework.

The dynamics of δ are assumed given by

$$d\delta_t = \delta_t \mu dt + \delta_t \sigma dW_t \tag{1}$$

and, without loss of generality, we fix $\delta_0 = 1$. Here μ denotes the (instantaneous) drift of the housing service-flow process while σ is its (instantaneous) volatility. Modeling housing's

[2]For example, changes in maximum permitted loan-to-value ratios, higher default costs, the ability of homeowners to cash-out refinance, the imposition of transaction costs to dissuade cash-out refinancing, etc.

service-flow dynamics as log-normal will endow our model with a scaling feature which makes it particularly tractable.[3]

A number of simplifying assumptions will be made in valuing claims contingent on δ. First, the owner finances an *exogenously* determined fraction ℓ of the residential property's purchase price by obtaining an *infinite* maturity mortgage requiring a fixed coupon payment rate of c. The reliance on mortgage financing reflects, for example, a tax advantage to debt which is not explicitly modeled. The assumption of infinite maturity is for analytic tractability only. Second, we assume the prevailing risk free interest rate, r, is constant. Therefore interest rate driven refinancing is excluded. Finally, the drift of the service-flow process, μ is less than the risk free rate r. Otherwise the value of an infinite stream of service flow will be infinitely large.[4]

2.1. **Debt and Equity.** We denote the value of the mortgage by $D(\delta_t)$. The owner's residual claim on the house will be referred to as *equity* and denoted by $E(\delta_t)$. Assuming the owner

[3]It is a common feature of derivative pricing models based on a geometric Brownian motion that the derived price function is positive homogenous of degree one in all monetary units. That is, if we denote the price of the residential property at the optimal refinancing point δ as $A(\delta)$ then the model has the feature that $A(\lambda\delta) = \lambda A(\delta)$ for any positive constant λ. Intuitively, this means that if we change the units in which δ is measured from, say, US$ to €, then the value $A(\delta)$ will also change from being denominated in US$ to being denominated in €. As a result, it is without loss of any generality that we fix δ_0 to one.

[4]Note that whatever the service flow's drift μ is, the expected rate of return on the claims we price must always equal the risk free rate r. For example, in the case where we permit default but not prepayment, the corresponding value function for the homeowner's equity claim in the residential property, denoted by $E(\delta_t)$, is determined by the ordinary differential equation given by expression (3). By Itô's Lemma, the (instantaneous) capital gains rate per unit of time is

$$\frac{1}{2}\sigma^2\delta_t^2 E''(\delta_t) + \mu\delta_t E'(\delta_t)$$

while the (instantaneous) dividend rate per unit of time is

$$\delta_t - c.$$

The sum of the (instantaneous) capital gains and dividend rates is

$$\frac{1}{2}\sigma^2\delta_t^2 E''(\delta_t) + \mu\delta_t E'(\delta_t) + \delta_t - c,$$

which, according to expression (3), equals $rE(\delta_t)$. Hence, regardless of the value of μ, equity's (instantaneous) total expected rate of return per unit of time equals the risk free rate r. Since the (instantaneous) capital gains rate of return on the value of an optimally financed unit of residential property is identical to the service flow's drift μ but μ is strictly less than r, this implies that there is an implicit convenience yield of $r - \mu$ associated with owning the house.

never defaults then

$$E(\delta_t) = \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}(\delta_s - c)ds\right] = \frac{\delta_t}{r-\mu} - \frac{c}{r} \tag{2a}$$

$$D(\delta_t) = \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}c ds\right] = \frac{c}{r}. \tag{2b}$$

In this case, the value of the house is simply the sum of the values of the mortgage and equity

$$D(\delta_t) + E(\delta_t) = \frac{\delta_t}{r-\mu}$$

and corresponds to the value obtained from a simplified version of a user cost of housing model.[5]

2.2. **Permitting Default Only.** Suppose now that the fixed rate mortgage is contractually defaultable but is *not* prepayable. This means that once the homeowner has chosen the optimal debt-equity mix when financing the original purchase of the house, the amount of debt outstanding cannot be subsequently altered. We will refer to this as a static optimal financing policy. It will serve as a benchmark against the later case of a dynamic optimal financing policy in which homeowners can subsequently adjust the amount of debt outstanding to extract equity from their appreciated houses.

Since the mortgage has infinite maturity, we can find $E(\delta)$ and $D(\delta)$ by solving the standard no arbitrage ordinary differential equations.[6] For example, the standard no arbitrage ordinary differential equation (ODE) for equity is

$$\frac{1}{2}\sigma^2\delta^2 E''(\delta) + \mu\delta E'(\delta) - rE(\delta) + \delta - c = 0. \tag{3}$$

[5]See, for example, Poterba [1984]. The simplification stems from excluding depreciation, taxes, and maintenance costs. Under these simplifying assumptions as well as perfect markets, the user cost is the opportunity cost of using the house less any increase in its value and corresponds to the rent paid; see, for example, Dougherty and Van Order [1982].

[6]See, for example, Goldstein et al. [2001].

The general solutions for equity and debt are given by

$$E(\delta) = e\delta^x + \frac{\delta_t}{r-\mu} - \frac{c}{r} \tag{4a}$$

$$D(\delta) = d\delta^x + \frac{c}{r}, \tag{4b}$$

where

$$x = \frac{(\frac{1}{2}\sigma^2 - \mu) - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} < 0$$

is the negative root of expression (3)'s associated quadratic equation while e and d are constants to be determined by initial and boundary conditions which characterize this valuation problem.[7]

The initial conditions describing the mortgage and equity at origination, $\delta_0 = \$1$, are given by

$$D(1) = P \tag{5a}$$

$$E(1) = A - P. \tag{5b}$$

Here P is the mortgage's principal and A is the value at origination of the underlying house financed by the mortgage.

The boundary conditions at the default boundary, $\delta = \delta_B$, are given by

$$E(\delta_B) = 0 \tag{6a}$$

$$E'(\delta_B) = 0 \tag{6b}$$

$$D(\delta_B) = (1-\alpha)\delta_B A. \tag{6c}$$

The first boundary condition states that at default the homeowner's equity stake in the property is worthless. The corresponding smooth-pasting condition is given by the second

[7]We can exclude the term with a positive power greater than one in the general solutions, expression (4), because we know that as δ approaches infinity they must converge to the corresponding values calculated when default is not permitted, expression (2).

boundary condition. The final boundary condition captures the fact that at default the lender receives the then prevailing value of the property $\delta_B A$, that is, the property value at origination scaled by the service-flow level at default, all net of bankruptcy costs where α is the exogenously specified percentage bankruptcy loss.[8]

Because the property is infinitely lived, our valuation framework must make assumptions about its disposition subsequent to a default. In each such case, we assume that foreclosure is immediate and the lender then sells the property for its prevailing value net of bankruptcy costs to a buyer who again finances at a loan-to-value ratio of ℓ using a fixed rate infinite maturity mortgage.

Solving the no arbitrage ordinary differential equations subject to these initial and boundary conditions determines the constants e and d as well the default boundary δ_B, the mortgage principal P and house value at origination A. Finally, the mortgage's fixed coupon payment rate c is implicitly determined by solving

$$\frac{P}{A} = \ell. \tag{7}$$

2.2.1. *Pricing Properties under the Static Optimal Financing Policy.* We investigate the properties of the model for a base-case specification of underlying parameter values. We then sequentially perturb a particular parameter value, holding all other parameter values unchanged, to gauge the model's resultant sensitivities. The results are tabulated in Table 1.

The base case sets the instantaneous drift of the house's service-flow process at $\mu = 2\%$ and an instantaneous volatility of $\sigma = 15\%$ while the prevailing instantaneous risk free rate is fixed at $r = 5\%$. The homeowner's desired loan-to-value ratio is assumed to be $\ell = 80\%$ and a bankruptcy cost of $\alpha = 10\%$ of the then prevailing home value is incurred in the event of default. We solve for the initial value of the home, A, the infinite maturity mortgage's principal, P, as well as its corresponding fixed coupon payment rate c. This results in an

[8]Implicit here and throughout this paper is the assumption that mortgage loans are non-recourse thereby limiting a lender's recovery to the property itself.

implied mortgage rate $y = c/P$. The level of the service-flow variable which triggers default by the homeowner, δ_B, is also determined. To gain additional insight into the likelihood of defaults or, alternatively, the expected length of time until default[9] occurs, we also present the resultant equivalent fixed waiting time to default, $\mathrm{EFWT}(\delta_0)$, as well as the value of an Arrow-Debreu security contingent on default, $\mathrm{ADD}(\delta_0)$, which pays off \$1 only at default. From Table 1 we see that for the base-case specification, the initial value of the house is $A = \$32.52$ while $P = \$26.02$ is borrowed at a mortgage rate of $y = 5.66\%$ to obtain the desired 80% loan-to-value ratio. The homeowner subsequently finds it optimal to default when the house's service flow falls from $\delta_0 = \$1$ to $\delta_B = \$0.64$ which gives an equivalent fixed waiting time to default of EFWT = 23.14 years and an Arrow-Debreu security value contingent on default of ADD = \$0.31.

The initial value of the house A can be seen to be extremely sensitive to the prevailing risk free rate r largely reflecting the fact that it is the discounted value of an infinite stream of service flows. The resultant amount borrowed to maintain the $\ell = 80\%$ loan-to-value ratio varies correspondingly as does the mortgage rate. All else equal, default occurs sooner at a higher risk free rate ($\delta_B = \$0.66$ for $r = 10\%$) as opposed to a lower risk free rate ($\delta_B = \$0.62$ for $r = 3\%$). This reflects the basic property that American options are exercised sooner when interest rates are higher.

As the volatility of the housing service-flow process increases, the mortgage rate increases. For example, the mortgage rate increases from 5.20% at $\sigma = 10\%$ to 6.30% at $\sigma = 20\%$.

[9] The expected waiting time until default is infinite for a geometric Brownian motion with positive drift ($\mu > 0$). In order to calculate a quantifiable measure of the waiting time until default, we use the value of an Arrow-Debreu security contingent on default defined by

$$\mathrm{ADD}(\delta_t) = \mathbb{E}_t\left[e^{-r(\tau_B - t)}\right]$$

where τ_B is the (stochastic) default time. We then define the *equivalent fixed waiting time to default* as the fixed waiting time into the future such that the value of receiving \$1 with certainty after this waiting time would be the same as the value of the Arrow-Debreu security contingent on default. That is, the equivalent fixed waiting time to default, $\mathrm{EFWT}(\delta)$, satisfies

$$\mathrm{ADD}(\delta) = e^{-r\,\mathrm{EFWT}(\delta)}$$

or

$$\mathrm{EFWT}(\delta) = -r \ln(\mathrm{ADD}(\delta)).$$

Default occurs sooner at a higher volatility but is triggered at a lower value of δ_B reflecting the greater likelihood of a rebound in the home's service flow when volatility is higher. Since bankruptcy costs are capitalized in house values, higher bankruptcy costs, α, result in a lower initial house value A and a higher mortgage rate. A higher loan-to-value ratio, ℓ, means that default will occur sooner, also giving rise to a lower initial house value A and a higher mortgage rate.

2.3. **Permitting Default and Cash-Out Refinancing.** We now permit homeowners to cash-out refinance as well as to default. Homeowners follow a dynamic optimal financing policy allowing them the option to extract equity by increasing their mortgage indebtedness in the event that house prices rise. We focus on the effects of cash-out refinancing *per se* by continuing to fix the homeowner's optimal loan-to-value ratio at ℓ. To cash-out refinance and extract equity, the homeowner obtains a second mortgage in an amount incremental to the previous financing so as to give a *combined* loan-to-value ratio of ℓ in light of the new higher house value. Like a closed-end second lien in the U.S., this incremental financing is assumed to be junior to all previous financing. We also assume that absolute priority applies in the event of bankruptcy.

Homeowners, however, cannot decrease their mortgage indebtedness if house prices fall. As pointed out by Khandani et al. [2009], this "ratchet" effect reflects the indivisible nature of housing so that a homeowner cannot simply reduce leverage by selling a portion of a house and use the proceeds to reduce mortgage indebtedness. Also, mortgage loan modification is difficult to accomplish in practice. For example, bankruptcy judges in the U.S. cannot alter the terms of residential mortgages including the principal amount owed.

2.3.1. *Discrete Cash-Out Refinancing.* We assume that a homeowner can cash-out refinance up to n times over the course of owning a property. Given only a finite number of opportunities, the homeowner must determine the service flow at which to optimally cash-out refinance. Analogous to the optimal exercise of an American option, the homeowner trades-off locking in a certain gain from cash-out refinancing today versus waiting for an even larger gain at some future date. Lenders are aware of the homeowner's optimal strategy, including the number of cash-out refinancing opportunities available, and price mortgages accordingly.

We solve this problem by dynamic programming. We start by assuming that no cash-out refinancing options remain and introduce additional homeowner refinancing opportunities as we work backwards in time. For these and other details, see Figure 1. The individual steps of the dynamic programming solution are referred to as regimes and in regime j the homeowner has j of the original n cash-out refinancing options remaining.

The homeowner begins in regime n with a first-lien mortgage used to purchase a house at a loan-to-value ratio of ℓ and all n cash-out refinancing options remaining. With each cash-out refinancing, the homeowner resets the house's combined loan-to-value ratio to ℓ and enters the next regime with one less refinancing opportunity. Given the model's scaling feature, to ease computation and without loss of any generality, we normalize the house's service flow δ to one at the beginning of each regime. Also, as Figure 1 makes clear, the homeowner has the option to default in each regime.

To fix matters, assume the homeowner is in regime j in which j of the original n cash-out refinancing options remain. This means that the homeowner has already cash-out refinanced in each of the previous regimes $i = j+1, \ldots, n$. Therefore, we can take as given all of these previously obtained mortgage loans. To keep track of these earlier loans, we denote by D_{ij} the *cumulative* value in regime j of the mortgage loans obtained by the homeowner prior to and including an earlier regime i, $i = j+1, \ldots, n$.[10] So, for example, D_{nj} represents the value of a property's first lien in regime j while $D_{n-1,j}$ is the corresponding value of *both* the property's first and second liens. We also take as given the default and refinancing triggers, δ_{B_i} and δ_{F_i}, as well as the total coupon payment rates c_i in the earlier regimes i, $i = j+1, \ldots, n$.

[10]It is more convenient to work with cumulative as opposed to individual mortgage loans. Firstly, the homeowner cash-out refinances to achieve a *cumulative* loan-to-value ratio of ℓ. Second, the homeowner only cares about the total coupon payments on the *cumulative* mortgage loans when deciding whether or not to default.

The corresponding initial conditions as well as value-matching and smooth-pasting conditions characterizing the homeowner's optimal default and cash-out refinancing decisions for the general case in which the homeowner has n cash-out refinancing options are detailed in the Appendix. Here we restrict our attention to the simpler case in which the homeowner can only cash-out refinance $n = 2$ times. This simpler case allows us to clearly convey our solution method while stressing the economic intuition underlying the homeowner's decision-making within our valuation framework.

Given $n = 2$ cash-out refinancing opportunities, we follow our dynamic programming approach and begin with regime 0 in which the homeowner can no longer refinance. The value-matching and smooth-pasting conditions for default are given by:

$$E_0(\delta_{B_0}) = 0 \tag{8a}$$

$$E_0'(\delta_{B_0}) = 0 \tag{8b}$$

$$D_{00}(\delta_{B_0}) = (1-\alpha)A_2\delta_{B_0} \tag{8c}$$

$$D_{10}(\delta_{B_0}\delta_{F_1}) = \min\left\{(1-\alpha)A_2\delta_{B_0}\delta_{F_1}, \frac{c_1}{r}\right\} \tag{8d}$$

$$D_{20}(\delta_{B_0}\delta_{F_1}\delta_{F_2}) = \min\left\{(1-\alpha)A_2\delta_{B_0}\delta_{F_1}\delta_{F_2}, \frac{c_2}{r}\right\}. \tag{8e}$$

The homeowner's limited liability is reflected in expressions (8a) and (8b). Conditional on being in regime 0, the homeowner has outstanding not only a first-lien but also second- and third-lien mortgages. As such, this now requires three corresponding value-matching conditions. Espression (8c) captures the fact that at default[11] the total cumulative mortgage debt outstanding will receive the then prevailing value of the property, $A_2\delta_{B_0}$, net of bankruptcy costs. We use here the property's value with two remaining cash-out refinancing options,

[11]In the event of default, the homeowner defaults on all mortgages and lenders are assumed to foreclose instantaneously thereafter. In other words, there is no scope for strategic default behavior by the homeowner, say, defaulting on a first lien but not on a second lien. However, there is little evidence of such behavior during the recent financial crisis by U.S. homeowners with both first and second liens. For example, Jagtiani and Lang [2010] document that among those who defaulted on their second liens, about eighty percent also defaulted on their first liens.

A_2, to capture the fact that at default creditors sell the property to a new homeowner who will once again have two refinancing options. These proceeds will be allocated between the various creditors according to absolute priority. To do so, we also need the value-matching conditions for the more senior mortgages.[12] The initial conditions are:

$$E_0(1) = A_0 - P_0 \tag{9a}$$

$$D_{00}(1) = P_0. \tag{9b}$$

Finally, we find the optimal total coupon payment rate, c_0, in regime 0 by solving

$$\frac{P_0}{A_0} = \ell. \tag{9c}$$

We use this system of eight equations to solve for eight unknowns: the value functions[13] E_0, D_{20}, D_{10}, and D_{00}, the initial property value A_0, the mortgage principal P_0, the optimal default trigger δ_{B_0}, and the optimal total coupon payment rate c_0. In solving this system, we take as given various values determined in the earlier regimes 1 and 2.[14]

[12]Note that the claims D_{10} and D_{20} are not denominated in terms of the house's service flow in regime 0: D_{10} is denominated in terms of the service flow in regime 1 while D_{20} is denominated in terms of the service flow in regime 2. The reason these claims are denominated in terms of regime 1 and 2 service flows is that they are issued and therefore need to be valued in these regimes.

[13]In regime 0 the four value functions, E_0, D_{20}, D_{10}, and D_{00}, each have a single unknown coefficient since in regime 0 there is no refinancing trigger level, cf. expression (4).

[14]In particular, the optimal triggers for default, δ_{B_1} and δ_{B_2}, the optimal triggers for refinancing, δ_{F_1} and δ_{F_2}, the initial property values, A_1 and A_2, and the optimal coupon payment rates, c_1 and c_2.

With this system solved, we proceed to regime 1 in which the homeowner has one refinancing option remaining. In this regime we have the following value-matching and smooth-pasting conditions for default:

$$E_1(\delta_{B_1}) = 0 \tag{10a}$$

$$E_1'(\delta_{B_1}) = 0 \tag{10b}$$

$$D_{11}(\delta_{B_1}) = (1-\alpha)A_2\delta_{B_1} \tag{10c}$$

$$D_{21}(\delta_{B_1}\delta_{F_2}) = \min\left\{(1-\alpha)A_2\delta_{B_1}\delta_{F_2}, \frac{c_2}{r}\right\} \tag{10d}$$

and the following value-matching and smooth-pasting conditions for cash-out refinancing:

$$D_{21}(\delta_{F_1}\delta_{F_2}) = D_{20}(\delta_{F_1}\delta_{F_2}) \tag{11a}$$

$$D_{11}(\delta_{F_1}) = D_{10}(\delta_{F_1}) \tag{11b}$$

$$E_1(\delta_{F_1}) = \delta_{F_1}A_0 - D_{10}(\delta_{F_1}) \tag{11c}$$

$$E_1'(\delta_{F_1}) = A_0 - D_{10}'(\delta_{F_1}). \tag{11d}$$

Expressions (11a) and (11b) reflect the fact that by refinancing and issuing a third lien, the previously issued first and second liens each become senior to the third lien as we enter regime 0. Expression (11c) recognizes that with a cash-out refinancing, the homeowner's equity is now made up of the new equity value of the house when entering regime 0, $\delta_{F_1}(A_0 - P_0))$, and the amount of cash extracted in the refinancing, $\delta_{F_1}P_0 - D_{10}(\delta_{F_1})$. The initial conditions are given by:

$$E_1(1) = A_1 - P_1 \tag{12a}$$

$$D_{11}(1) = P_1. \tag{12b}$$

Finally, we find the optimal total coupon payment rate, c_1, by solving

$$\frac{P_1}{A_1} = \ell. \tag{12c}$$

We use this system of eleven equations to solve for eleven unknowns: the value functions[15] E_1, D_{21}, and D_{11}, the initial property value A_1, the mortgage principal P_1, the optimal triggers for the homeowner to default δ_{B_1}, and refinance δ_{F_1}, and the total coupon payment rate c_1. Again, we solve this system taking as given various values determined in the earlier regime 2.[16]

Having solved this system, we then proceed to regime 2 in which the homeowner has two cash-out refinancing opportunities remaining. Here we have the following value-matching and smooth-pasting conditions for default:

$$E_2(\delta_{B_2}) = 0 \tag{13a}$$

$$E_2'(\delta_{B_2}) = 0 \tag{13b}$$

$$D_{22}(\delta_{B_2}) = (1-\alpha)A_2\delta_{B_2} \tag{13c}$$

and value-matching and smooth-pasting conditions for refinancing:

$$D_{22}(\delta_{F_2}) = D_{21}(\delta_{F_2}) \tag{14a}$$

$$E_2(\delta_{F_2}) = \delta_{F_2}A_1 - D_{21}(\delta_{F_2}) \tag{14b}$$

$$E_2'(\delta_{F_2}) = A_1 - D_{21}'(\delta_{F_2}). \tag{14c}$$

[15]In regime 1 the three value functions, E_1, D_{21}, and D_{11}, each have two unknown coefficients since there are both bankruptcy and refinancing trigger levels.
[16]In particular, the optimal triggers for default, δ_{B_2}, and refinancing, δ_{F_2}, the initial property value, A_2, and the optimal coupon payment rate, c_2.

The initial conditions are given by:

$$E_2(1) = A_2 - P_2 \tag{15a}$$

$$D_{22}(1) = P_2 \tag{15b}$$

and we find the optimal coupon payment rate, c_2, by solving

$$\frac{P_2}{A_2} = \ell. \tag{15c}$$

We use this system of nine equations to solve for nine unknowns: the two value functions, E_2 and D_{22}, each having two unknown coefficients, the initial property value A_2, the mortgage principal P_2, the optimal triggers for the homeowner to default, δ_{B_2}, and refinance, δ_{F_2}, and the optimal coupon payment rate c_2.

2.3.2. *Pricing Properties under the Dynamic Optimal Financing Policy.* Table 2 summarizes the effects of cash-out refinancing for the base-case specification of underlying parameter values. We allow the homeowner the opportunity to cash-out refinance either once ($n = 1$) or twice ($n = 2$). For comparison purposes, we also provide corresponding values for the static optimal financing case previously analyzed in which cash-out refinancing is prohibited.

House values are seen to be lower when homeowners are afforded the opportunity to cash-out refinance. For example, when cash-out refinancing is prohibited, $A = \$32.52$. However, permitting homeowners to cash-out refinance up to $n = 2$ times results in a house value of only $A_2 = \$31.63$, all else being equal.[17] Intuitively, house values are lower in the presence of cash-out refinancing opportunities because the likelihood of future defaults increases. For example, while the value of an Arrow-Debreu security contingent on default in the absence of cash-out refinancing is ADD = \$.31, its value given $n = 2$ cash-out refinancing opportunities

[17]By way of notation, a variable with a subscript denotes the variable's value when the subscripted number of cash-out refinancing opportunities remain. When a variable is presented without a subscript this corresponds to the case where cash-out refinancing is prohibited.

increases to $\text{ADD}_2 = \$.44$. The resultant increase in bankruptcy costs is capitalized in house values.

If homeowners can cash-out refinance, the equivalent fixed waiting time to default is shorter as compared to when homeowners are prohibited from cash-out refinancing. Given the opportunity to cash-out refinance $n = 2$ times gives $\text{EFWT}_2 = 16.22$ years, while $\text{EFWT} = 23.14$ years in the absence of cash-out refinancing. Cash-out refinancing increases the homeowner's mortgage indebtedness and so, all else being equal, triggers an earlier default. Similarly, for a given number of refinancing opportunities, the equivalent fixed waiting time to default increases as fewer refinancing opportunities remain. For example, given the opportunity to cash-out refinance up to $n = 2$ times, $\text{EFWT}_2 = 16.22$ years, $\text{EFWT}_1 = 18.28$ years while $\text{EFWT}_0 = 23.04$ years.

From Table 2 we see that the interest rate charged on the first lien varies across the different cash-out refinancing assumptions. For example, for $n = 1$ we have $y_1 = 5.63\%$ while for $n = 2$ we have $y_2 = 5.57\%$. That is, under absolute priority the value of a first lien depends on whether or not a homeowner has the option to cash-out refinance, reflecting the interaction of refinancing and default in the pricing of first-lien mortgages.

Nevertheless, because of the priority afforded to the first lien, the interest rate charged on the senior mortgage is lower than the rates charged on the more junior loans. Interestingly, if the homeowner cash-out refinances twice ($n = 2$), we see in Table 2 that the rate charged on the third lien ($y_0 = 7.31\%$) can actually be lower than that charged on the second lien ($y_1 = 7.54\%$). This result reflects the fact that, unlike the real-world, the lender in our model knows exactly how many cash-out refinancing opportunities the homeowner has remaining. If the homeowner has exhausted all refinancing opportunities, the lender can charge a lower rate on the third lien knowing that now interest income is expected to be received longer. However, the cumulative interest rate paid by the homeowner, calculated across all mortgages taken out, increases with cash-out refinancings. For example, in the case of $n = 2$ refinancing opportunities, a cumulative interest rate of $\bar{y}_1 = 5.64\%$ is paid after the homeowner cash-out refinances once, but increase to a cumulative interest rate of $\bar{y}_0 = 5.70\%$ after the homeowner cash-out refinances twice. Intuitively, the higher cumulative interest rate paid by the homeowner reflects the fact that the homeowner's total mortgage indebtedness is greater after cash-out refinancing twice as opposed to once.

The bankruptcy boundary is also affected by the homeowner's ability to cash-out refinance. As can be seen from Table 2, the service flow at which bankruptcy is triggered is lower when the homeowner has the opportunity to cash-out refinance. For example, in the absence of cash-out refinancing, the homeowner defaults when the service flow hits $\delta_B = \$0.64$ as compared to first defaulting at a service flow of $\delta_{B_2} = \$0.63$ when given $n = 2$ refinancing opportunities. The lower bankruptcy boundary when the homeowner can refinance reflects the possibility that even when home prices are low, they may still subsequently increase and the option to cash-out refinance can be exercised. Intuitively, analogous to the competing risks nature of defaulting and rate refinancing[18], defaulting here eliminates the potentially valuable option to cash-out refinance. Likewise, the bankruptcy boundary increases as fewer refinancing opportunities remain. For example, given the opportunity to cash-out refinance up to $n = 2$ times, bankruptcy is triggered at a service flow of $\delta_{B_1} = \$0.63$ after the first cash-out refinancing but increases to a service flow of $\delta_{B_0} = \$0.64$ after the second cash-out refinancing.

Finally, from Table 2 we see that refinancing is triggered at a higher service flow when the homeowner has only one opportunity to cash-out refinance, $\delta_{F_1} = \$1.33$, as compared to the service flow at which refinancing is first triggered given two opportunities to cash-out refinance, $\delta_{F_2} = \$1.30$. Similarly, given the opportunity to cash-out refinance up to $n = 2$ times, the refinancing boundary increases after the homeowner's first cash-out refinancing, from $\delta_{F_2} = \$1.30$ to $\delta_{F_1} = \$1.35$.

[18]See, for example, Deng, Quigley, and Van Order [2000]

The model's sensitivities to changes in its underlying parameters are tabulated in Table 3. While these changes impact many of the model's properties, our discussion concentrates on their effects on the homeowner's default and refinancing decisions.

When the prevailing risk free rate of interest r is increased, the homeowner is seen to refinance and to default sooner. Focusing on the case in which the homeowner can refinance once, the homeowner does so at a service flow of $\delta_{F_1} = \$1.38$ for $r = 3\%$, but only at a service flow of $\delta_{F_1} = \$1.27$ for $r = 10\%$. In this case, the homeowner defaults at a service flow of $\delta_{B_1} = \$0.61$ for $r = 3\%$ but the bankruptcy boundary increases to a service flow of $\delta_{B_1} = \$0.65$ for $r = 10\%$. As expected, after taking advantage of the only opportunity to cash-out refinance, the equivalent fixed waiting time to default is much shorter when $r = 10\%$, $\text{EFWT}_0 = 14.27$ years, than when $r = 3\%$, $\text{EFWT}_0 = 33.14$ years. These results are consistent with the properties of American options.

The properties of American options also imply that when the service-flow volatility increases, the homeowner sets trigger points consistent with waiting longer to refinance and to default. For example, for $n = 1$ we see that the service flow at which the homeowner refinances when $\sigma = 10\%$ is $\delta_{F_1} = \$1.25$, which increases to a trigger service flow of $\delta_{F_1} = \$1.40$ when $\sigma = 20\%$. Default, on the other hand, is triggered at a service flow of $\delta_{B_1} = \$0.68$ for $\sigma = 10\%$ but falls to a service flow of $\delta_{B_1} = \$0.59$ for $\sigma = 20\%$. The equivalent fixed waiting times to default are shorter in the presence of more volatile housing service flows.

The refinancing boundary is extremely sensitive to prevailing bankruptcy costs α. For example, for $n = 1$ the homeowner cash-out refinances at a service flow of $\delta_{F_1} = \$1.21$ when $\alpha = 5\%$. However, for bankruptcy costs of $\alpha = 20\%$, refinancing is triggered much later at a higher service flow of $\delta_{F_1} = \$1.57$.

The pricing properties are also sensitive to the homeowner's assumed loan-to-value ratio. For example, from Table 3 we see that by increasing the loan-to-value ratio from $\ell = 70\%$ to $\ell = 90\%$, the service flow at which bankruptcy is triggered increases throughout. A higher loan-to-value ratio means that homeowners are more indebted and default sooner.

The equivalent fixed waiting time to default is much shorter and the corresponding value of the Arrow-Debreu security contingent on default is much higher for $\ell = 90\%$ than for $\ell = 70\%$. All yields paid by the homeowner are also higher for the higher loan-to-value ratio reflecting the greater risk of the homeowner defaulting.

3. "Secret Seconds" and the Unraveling of CDOs

The bursting of the U.S. housing bubble saw the unraveling of many CDOs. For example, Barnett-Hart [2009] reports that by early 2009 *Aaa* certificates of CDOs originated in 2005 had been downgraded an average of eight notches[19] to a rating of only *Baa*2, just slightly above investment-grade.

Some critics have argued that these large downgrades reflected the fact that credit rating agencies simply underestimated the severity of the U.S. housing market downturn which caused a sharp increase both in the level of defaults as well as in the correlation of defaults across homeowners.[20] Others have suggested that credit rating agencies were blind to the fact that first-lien borrowers could subsequently obtain second loans and, as a result, ignored the consequences of cash-out refinancing on the performance of CDOs.[21] These so-called "secret seconds" increased the likelihood that a homeowner would default in the event of a downturn in house prices. Moreover, the fact that so many U.S. homeowners relied on second mortgages to extract equity from their homes during the run-up in house prices through 2006 meant that they were more likely to default *en masse* when house prices subsequently fell.

We now investigate the extent to which the unraveling of CDOs in the aftermath of the bursting of the U.S. housing bubble can be attributed to credit rating agencies ignoring the presence of second loans. We also shed light on the role of credit rating agencies underestimating the severity of the U.S. housing downturn on the subsequent performance of CDOs.

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[19] Based on a numerical scale that assigns 1 to a *Aaa* rating and, at the other extreme, 22 to a *D* rating.
[20] See, for example, BIS Committee on the Global Financial System [2008].
[21] See, for example, the discussion in Lewis [2010], page 100.

3.1. **A Hypothetical Cash CDO.** We consider a lender who only originates first-lien mortgages. In particular, we assume that by date t the lender has originated 1,000 first-lien mortgages with their dates of origination being uniformly distributed over the preceding four years[22]. Consistent with our valuation framework, each mortgage is an infinite-maturity loan characterized by the base-case loan-to-value ratio of $\ell = 80\%$ and bankruptcy costs of $\alpha = 10\%$. Each underlying property's service flow is (instantaneously) log normally distributed with the base-case (instantaneous) drift of $\mu = 2\%$ and base-case (instantaneous) volatility of $\sigma = 15\%$. To model correlation between the underlying houses, we follow Downing, Stanton, and Wallace [2007] and split the service-flow shocks into two components: a common component shared across all houses and a house-specific component which is unique to the property. By fixing both the house service-flow volatility and its systematic component, the latter set equal to 2%[23] in the base case, we implicitly specify the correlation between individual houses' service flows.[24] Finally, the risk free rate of interest is fixed at the base-case value of $r = 5\%$.

At date t the loan originator deposits the 1,000 first-lien mortgages[25] in a trust and receives, in turn, the prevailing value of the loans. Relying on this pool of first-lien mortgages as collateral, the trust issues a CDO[26] consisting of two interest-bearing certificates, one senior and the other mezzanine, together with a non-interest bearing residual claim on the mortgage pool's cash flows. The CDO is assumed to have a maturity of ten (10) years.

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[22]The choice of four years is without loss of generality. By originating mortgages over an extended period of time we attempt to ensure that the loans in the pool are heterogenous in their seasoning.

[23]Systematic housing volatility is estimated by the volatility of a diversified portfolio of U.S. residential real estate. To do so, we form a portfolio invested in housing in each of the U.S. states and measure a particular state's housing return by quarterly returns to its FHFA purchase-only non-seasonally adjusted repeat sales index. The portfolio is value-weighted where we form weights based on a state's share of total U.S. mortgage originations as of 2004, as tabulated by HMDA. We choose 2004 because the hypothetical CDO which we subsequently analyze is assumed to be originated in that year. Over the 1999:I to 2003:IV sample period, the volatility of this portfolio is estimated to be 1.2% while over the longer sample period from 1991:I to 2003:IV, which includes the early 1990s downturn in U.S.house prices, the volatility is estimated to be 2.01%.

[24]Also, by varying the systematic housing volatility while holding individual houses' service-flow volatility at 15%, we vary the correlation between the individual houses' service flows.

[25]Or however many loans are still performing at date t.

[26]In practice a CDO's collateral is composed of securitized mortgages not whole loans as we assume.

A CDO prioritizes payments to its constituent securities. In our case, the first priority is interest payments to the senior certificate. The second priority is interest payments to the mezzanine certificate. These interest payments are paid currently. Next are principal payments to the senior certificate, followed by principal payments to the mezzanine certificate. Any remaining cash flows are then allocated to the residual certificate. Principal payments are paid on an accrued basis on the CDO's finite maturity date.[27]

If a default occurs, the underlying property is assumed to be immediately foreclosed and the resultant sale proceeds, net of bankruptcy costs, are deposited by the trust in a risk free rate bearing account.[28] Losses are allocated first to the residual class, then to the mezzanine certificate, and finally to the senior certificate.

At the CDO's maturity, the first-lien mortgages remaining in the pool are sold at their prevailing prices. These proceeds together with the liquidation of any accounts in the trust arising from previous foreclosures are used to make principal payments according to the CDO's priority structure. The trust is then terminated.

3.2. **Sizing CDO certificates.** Apart from subordination, we assume that the CDO has no other credit enhancements. Therefore the credit rating assigned to a particular certificate depends solely on the degree of protection afforded the certificate by other certificates subordinate to it. The more subordination provided a particular certificate, the smaller the certificate's expected losses and so the higher its credit rating. Moody's, for example, assigns ratings for both corporate and structured bonds based on the "idealized expected loss rates" given in Table 4. We rely on these loss rates in determining the ratings assigned to the interest-bearing certificates of our hypothetical CDO.[29]

To attain a particular credit rating requires us to determine the size of a certificate's principal so that the desired level of expected losses can be achieved given the underlying

[27]This payout convention is required because we assume infinite maturity mortgages are backing a finite maturity CDO.

[28]We assume that the CDO's pooling and servicing agreement does not require the replacement of any defaulted loan in the pool regardless of how soon the default occurs.

[29]As in practice, the residual certificate is not rated.

collateral's risk characteristics. To do so, we first increase the fraction of the CDO's principal allocated to the senior certificate until across all of our simulations of the underlying correlated collateral the resultant fraction experiences an average loss rate equal to that allowed by the senior certificate's desired rating, for example, *Aaa*. Given we have sized the senior certificate, we then proceed in a similar fashion to size the mezzanine certificate so that its fraction has an average loss rate across all of our simulations equaling that allowed by its desired rating, for example, $Baa3$. The remaining fraction of the CDO's principal is then allocated to the residual certificate.

3.3. **Simulation Results.** In the analysis that follows the credit rating agency is assumed to be naïve meaning that when rating the CDO it does not allow for the possibility that first-lien borrowers may subsequently cash-out refinance.

To emulate this naïve credit rating agency, we simulate, through the CDO's maturity date, the correlated service-flow processes underlying each first-lien mortgage included in the pool. Relying on our static optimal financing policy framework, in which homeowners cannot refinance but can optimally default, we then calculate the losses incurred across the pool for each simulation. We repeat this simulation exercise 1,000,000 times and size the CDO so that the naïve credit rating agency rates the senior certificate as *Aaa* and the mezzanine certificate as $Baa3$. From Table 5, we see that for the assumed base-case parameters, the senior certificate accounts for approximately 93% of the CDO's principal while the mezzanine certificate's size is approximately 4%.

These calculations assume that the homeowners in the pool are confronted with a wide variety of house price paths across the 1,000,000 simulations. However, we can also determine the losses incurred by homeowners in the pool, and therefore the losses passed on to the CDO certificate investors, if house prices behaved similarly to the actual path that U.S. house prices followed subsequent to the year 2000. To do so we measure U.S. home prices by the monthly $FHFA$ purchase only non-seasonally adjusted repeat sales index for the U.S. For purposes of our subsequent analysis, the CDO's issuance date is assumed to be January 2004, meaning that the underlying first-lien mortgages in our simulations are assumed originated between January 2000 and December 2003.

To investigate the performance of this CDO over the actual path of U.S. home prices, we now restrict our attention to the 50,000 simulation paths, out of the total 1,000,000 paths, for which the corresponding simulated quarterly house returns, averaged across the pool, are closest in a mean-squared error sense to the actual quarterly returns of the $FHFA$ index beginning in January 2000. To begin with, we calculate losses across these particular paths assuming that investors optimally default but cannot refinance. This allows us to determine how the naïve credit rating agency would have rated the CDO if, alternatively, it knew with perfect foresight the subsequent severe downturn in U.S. house prices. From Table 5, we see that relative to their original ratings, the senior certificate would be downgraded by only one notch, to $Aa1$, while the mezzanine tranche would be downgraded three notches, to the non-investment grade rating of $Ba3$. These results suggest that even if the naïve credit rating agency had explicitly acknowledged the severe downturn in U.S. house prices, the resultant CDO downgrades would be minimal when compared to the actual downgrades observed.

Next we allow homeowners to not only optimally default but to also optimally refinance up to two times in order to extract equity from their appreciated house values. However, if a homeowner does take out a second mortgage, it is assumed to be "secret" and ignored by the naïve credit rating agency when it initially rates the CDO. To assess the implications of these "secret seconds", we take the resultant CDO and recalculate each certificate's expected losses assuming that homeowners optimally refinance in addition to defaulting over all 1,000,000 simulated house price paths as well as over the 50,000 simulated house price paths most similar to the U.S. experience during the 2000s.

For the base-case parameter assumptions, we see in Table 5 that across all 1,000,000 simulated house price paths the senior certificate would now be downgraded four notches to $A1$ while the mezzanine certificate would be downgraded eight notches to $Caa2$. These downgrades reflect the fact that once homeowners are allowed to cash-out refinance and increase their indebtedness when house prices appreciate, they are more likely to default together when house prices subsequently fall.

This effect is amplified and the resultant downgrades are even larger for the base-case parameter assumptions when we restrict our attention to simulated house price paths which resemble the U.S. experience during the 2000s which saw significant house price appreciation through 2006 followed by an unprecedented decline in house prices. Given this particular house price experience, the senior certificate would have been downgraded by six notches to $A3$ and the mezzanine certificate by eleven notches to a rating of only C had the possibility that homeowners optimally refinance as well as default been explicitly taken into account.

To see this more clearly, Figure 2 displays for the base-case parameter assumptions the frequency distributions of the number of first-lien mortgage defaults in the underlying pool by the CDO's maturity date as functions of the assumed simulated house price paths and whether or not homeowners are assumed to be able to cash-out refinance. The effects of the actual U.S. house price experience and the ability to cash-out refinance on first-lien mortgagors' default behavior are clearly evident. Explicitly acknowledging the severe downturn in U.S. house prices, all else being equal, increases the number of defaults on average. However, allowing homeowners to extract equity, even across all possible simulated house price paths, increases defaults, on average, by more. As expected, allowing homeowners the opportunity to optimally extract equity during the run-up in U.S. house prices during the early 2000s and then the opportunity to optimally default during their subsequent severe decline results in the largest number of defaults on average.

Table 6 investigates the sensitivity of these results to changes in the assumed underlying parameters. As before, given a particular set of parameters, the CDO is sized by the naïve credit rating agency so the the senior certificate is Aaa rated and the mezzanine certificate is Baa rated. As before, the naïve credit rating agency assumes homeowners optimally default but do not cash-out refinance, and relies on 1,000,000 simulated house price paths to assess housing's risk characteristics. We then take the given CDO and recalculate each certificate's expected losses assuming that homeowners can refinance as well as default over all 1,000,000 simulated house price paths and over the 50,000 simulated house price paths most similar to the U.S. experience during the 2000s.

Notice that, as compared to the base case, the size of the *Aaa* rated senior certificate decreases as the riskiness of the underlying collateral increases. In other words, the naïve credit rating agency requires more subordination for the senior certificate to achieve a *Aaa* rating when the collateral's risk increases. For example, for a service-flow (instantaneous) volatility of only 10%, all else being equal, the size of the *Aaa* rated senior certificate is over 97% of the CDO's principal, but represents only 91% of the CDO's principal when the volatility is assumed to be 20%. Similarly, when homeowners' assumed loan-to-value ratio is 90%, all else being equal, the naïve credit rating agency sizes the *Aaa* rated senior certificate at only 92% of the CDO's principal but increases to almost 96% when the assumed loan-to-value ratio is only 70%. In addition, the naïve credit rating agency requires more subordination in order for the senior certificate to be *Aaa* rated if interest rates are high and when bankruptcy costs are high.

From Table 6 we also see that, as before, had the naïve credit rating agency known with perfect foresight the subsequent severe downturn in U.S. house prices, the resultant downgrades would be minimal. The downgrade of the senior certificate is at most one notch, while the mezzanine certificate would be downgraded by at most four notches.

Once again, it is recognizing "secret seconds", neglected by the naïve credit rating agency, which results in the largest downgrades. Even if we calculate expected losses across all 1,000,000 simulated house price paths assuming homeowners optimally cash-out refinance as well as optimally default, the senior certificate is downgraded by up to ten notches, while the mezzanine certificate is downgraded by up to eleven notches. The smallest downgrades correspond to the case in which the homeowners' assumed loan-to-value ratio is 70% in which case the senior certificate would be downgraded to only $Aa2$ and the mezzanine certificate would be downgraded to $B2$. At the other extreme, the largest downgrades result when homeowners' loan-to-value ratio is assumed to be 90%. Here the senior certificate would be downgraded to the non-investment grade $Ba1$ and the mezzanine certificate would be only C rated.

The downgrades remain large if we restrict our attention to simulated house price paths which resemble the U.S. experience during the 2000s. Compared to when all possible simulated house price paths are considered, the CDO certificates experience larger expected losses for all the cases tabulated in Table 6 once homeowners are allowed to optimally cash-out refinance during the run-up in house prices and optimally default during their subsequent severe decline.

4. Summary and Conclusions

Given the prominent role played by home equity extraction during the recent run-up in U.S. house prices, this paper explores its implications for the pricing and properties of residential mortgages, both first liens as well as junior liens, and, in turn, structured financial products based on the first-lien mortgages with junior liens behind them.

We find that house values are lowered when homeowners are afforded the opportunity to cash-out refinance reflecting the increased likelihood of default and so larger bankruptcy costs incurred when additional junior financing is relied upon. The risk characteristics of first-lien mortgages are also found to be systematically altered when subordinated mortgages are behind them. For example, the waiting time to default is, on average, shorter when homeowners have the option to cash-out refinance. The more often homeowners do extract equity, the shorter the waiting time to default reflecting homeowners' increased debt burden. A homeowner's bankruptcy boundary is also affected as default is triggered at a lower bankruptcy boundary when the homeowner can cash-out refinance. As expected, the rate of interest charged on subordinate mortgages exceeds that charged on first-lien mortgages with the spread between these rates depending on the characteristics of the underlying housing service-flow process.

While cash-out refinancing activity increases the default risk exposure of homeowners, it also effectively correlates their default decisions. A precipitous drop in house prices, like that subsequently experienced in the U.S., could now result in almost all homeowners defaulting together. When a CDO structured under the assumption that homeowners cannot cash-out refinance is confronted by data in which it is explicitly recognized that homeowners optimally cash-out refinance as well as default, we find that the CDO's resultant performance is broadly consistent with the magnitude of CDO downgrades observed subsequent to the bursting of the U.S. housing bubble. Interestingly, although the sizing of the CDO certificates changes with our parameter choices, the resulting downgrades are fairly stable, indicating that our conclusions are robust to the choice of parameters. By contrast, our results do not support the argument that the downgrades observed in practice occurred only because the severity of the U.S. housing market downturn was simply underestimated.

Taken together, our results call attention to the critical role played by second mortgages in the recent U.S. financial crisis. Second mortgages were the means by which many homeowners extracted equity from their appreciated properties, thereby increasing their leverage and default risk exposure. Moreover, by doing so in concert, these homeowners coordinated their default decisions so that when house prices did fall, the resultant defaults would cluster. Ignoring this possibility had ruinous implications for the performance of the many CDOs and other structured financial products collateralized by first-lien mortgages with second mortgages behind them.

5. Appendix

In this Appendix we detail the corresponding initial conditions as well as value-matching and smooth-pasting conditions characterizing the homeowner's optimal default and cash-out refinancing decisions for the general case in which the homeowner has n cash-out refinancing options.

Assume the homeowner is in regime j in which j of the original n cash-out refinancing options remain. This means that the homeowner has already cash-out refinanced at each of the previous regimes $i = j+1, \ldots, n$. At the beginning of regime j we have the initial conditions:

$$D_{jj}(1) = P_j$$

$$E_j(1) = A_j - P_j$$

where P_j denotes the cumulative principal borrowed after the homeowner's jth refinancing and A_j denotes the then prevailing value of the underlying property. The total coupon payment rate the homeowner will pay during regime j, denoted c_j, is determined so that

$$\frac{P_j}{A_j} = \ell.$$

The default value-matching and smooth-pasting conditions in regime j are given by

$$E_j(\delta_{B_j}) = 0$$

$$E_j'(\delta_{B_j}) = 0$$

$$D_{ij}(\delta_{B_j} \prod_{k=j+1}^{i} \delta_{F_k}) = \min\left\{(1-\alpha)A_n \delta_{B_j} \prod_{k=j+1}^{i} \delta_{F_k}, \frac{c_i}{r}\right\}$$

for $i = j, \ldots, n$. The homeowner defaults when the house's service flow is sufficiently low relative to the total coupon payment rate, c_j, to all the mortgage loans issued. In the event of default, the homeowner defaults on all mortgages and lenders are assumed to foreclose instantaneously thereafter and allocate the available proceeds amongst the existing liens according to absolute priority. To keep track of this, we have $n - j + 1$ value-matching conditions for the cumulative mortgage values. In particular, cumulatively all the mortgages issued in all regimes up to and including regime j, this value being denoted by D_{jj}, will receive $(1 - \alpha)A_n\delta_{B_j}$ in case of default. This reflects the fact that the creditors receive the property value net of bankruptcy costs, α, and that the property can be sold to a new homeowner who again will have exactly n refinancing options.

Similarly, for $j \geq 1$, the refinancing value-matching and smooth-pasting conditions in regime j are given by[30]

$$E_j(\delta_{F_j}) = \delta_{F_j} A_{j-1} - D_{j,j-1}(\delta_{F_j})$$

$$E'_j(\delta_{F_j}) = A_{j-1} - D'_{j,j-1}(\delta_{F_j})$$

$$D_{ij}(\delta_{F_j} \prod_{k=j+1}^{i} \delta_{F_k}) = D_{i,j-1}(\delta_{F_j} \prod_{k=j+1}^{i} \delta_{F_k})$$

for $i = j, \ldots, n$.

Since D_{ij} is the *cumulative* value of all the mortgages issued to the homeowner in regime i and all previous regimes (with higher indices, $i+1, \ldots, n$), we can determine the value (as of regime j) of *just* the mortgage issued in regime i by calculating

$$D_{ij}(\delta_i) - \frac{1}{\delta_{F_{i+1}}} D_{i+1,j}(\delta_{F_{i+1}}\delta_i)$$

for $i = 0, \ldots, n-1$ and $j = 0, \ldots, i$. Similarly, the coupon payment rate of the mortgage *just* issued in regime i is calculated as

$$c_i - \frac{c_{i+1}}{\delta_{F_{i+1}}},$$

for $i = 0, \ldots, n-1$.

[30]Note that for the case $j = 0$ there are no cash-out refinancing opportunities remaining and so these value-matching and smooth-pasting conditions do not apply.

References

D. Baker. The run-up in house prices: Is it real or is it another bubble? Briefing Paper, Center for Economic Policy Research, Washington D.C., 2002.

A. Barnett-Hart. The story of the CDO market meltdown: An empirical analysis. Unpublished Thesis, Harvard College, Cambridge, MA., 2009.

BIS Committee on the Global Financial System. Ratings on structured finance: What went wrong and what can be done to address shortcomings. Working Paper, Bank for International Settlements, Basel, Switzerland, 2008.

F. Black and J. Cox. Valuing corporate securities: Some effects of bond indenture provisions. *The Journal of Finance*, 31:351–367, 1976.

Y. Deng, J. Quigley, and R. Van Order. Mortgage terminations, heterogeneity and the exercise of mortgage options. *Econometrica*, 68:275–308, 2000.

A. Dougherty and R. Van Order. Inflation, housing costs and the cosumer price index. *The American Economic Review*, 72:154–164, 1982.

C. Downing, R. Stanton, and N. Wallace. Volatility, mortgage default and CMBS subordination. Working Paper, Haas School of Business, University of California, Berkeley, CA., 2007.

R. Goldstein, N. Ju, and H. Leland. An EBIT-based model of dynamic capital structure. *The Journal of Business*, 74:483–512, 2001.

L. Goodman, R. Ashworth, B. Landy, and K. Yin. Second liens: How important? *The Journal of Fixed Income*, 20:19–30, 2010.

A. Greenspan and J. Kennedy. Sources and uses of equity extracted from homes. *Finance and Economics Discussion Series, Federal Reserve Board*, 2007.

J. Jagtiani and W. Lang. Strategic default on first and second lien mortgages during the financial crisis. Working Paper, Federal Reserve Bank of Philadelphia, Philadelphia, PA., 2010.

J. Kau, D. Keenan, W. Mueller, and J. Epperson. The valuation at origination of fixed rate mortgages with default and prepayment. *The Journal of Real Estate Finance and Economics*, 11:5–39, 1995.

A. Khandani, A. Lo, and R. Merton. Systemic risk and the refinancing ratchet effect. Working Paper, Sloan School of Management, MIT, Cambridge MA., 2009.

M. Lewis. *The Big Short*. W. W. Norton & Co., New York, N.Y., 2010.

J. Poterba. Tax subsidies to owner-occupied housing: An asset market approach. *The Quarterly Journal of Economics*, 99:729–752, 1984.

Figure 1

A Homeowner's Options to Default and Cash-Out Refinance n Times

This figure depicts our solution algorithm as a function of a house's service flow δ. It begins when the homeowner has n cash-out refinancing opportunities available (Regime n) and concludes when 0 remain (Regime 0).

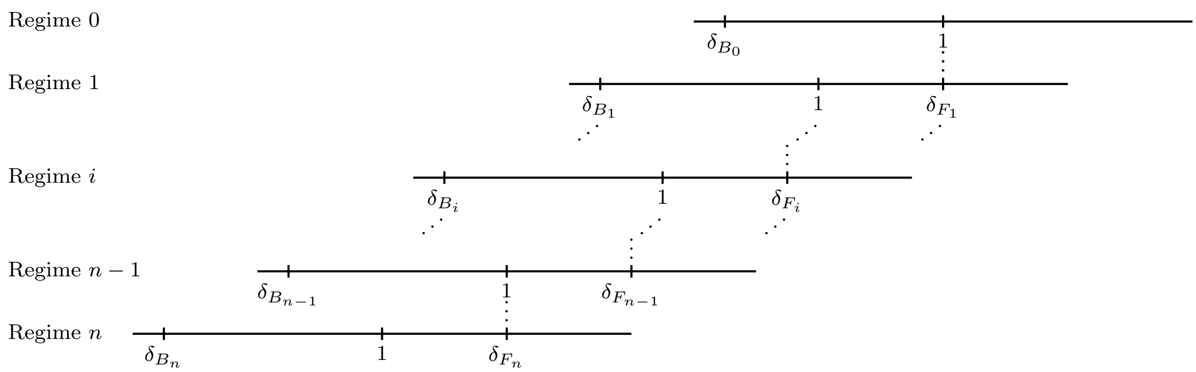

Figure 2

Frequency of First-Lien Mortgage Defaults

This figure displays for the base-case of parameter values the proportion of simulations (vertical axis) in which the corresponding number of loans in the underlying pool of 1,000 first-lien mortgages (horizontal axis) defaulted by the maturity date of our hypothetical cash CDO. The solid line is for the case in which all 1,000,000 simulation paths are considered and homeowners cannot cash-out refinance. The dashed line is for the case in which the 50,000 simulation paths closest in a mean squared error sense to the U.S. housing experience since 2000 are considered and homeowners cannot cash-out refinance. The dotted line is the case in which all 1,000,000 simulation paths are considered and homeowners can cash-out refinance up to $n = 2$ times. The dash-dotted line is the case in which the 50,000 simulation paths closest in a mean squared error sense to the U.S. housing experience since 2000 are considered and homeowners can cash-out refinance up to $n = 2$ times.

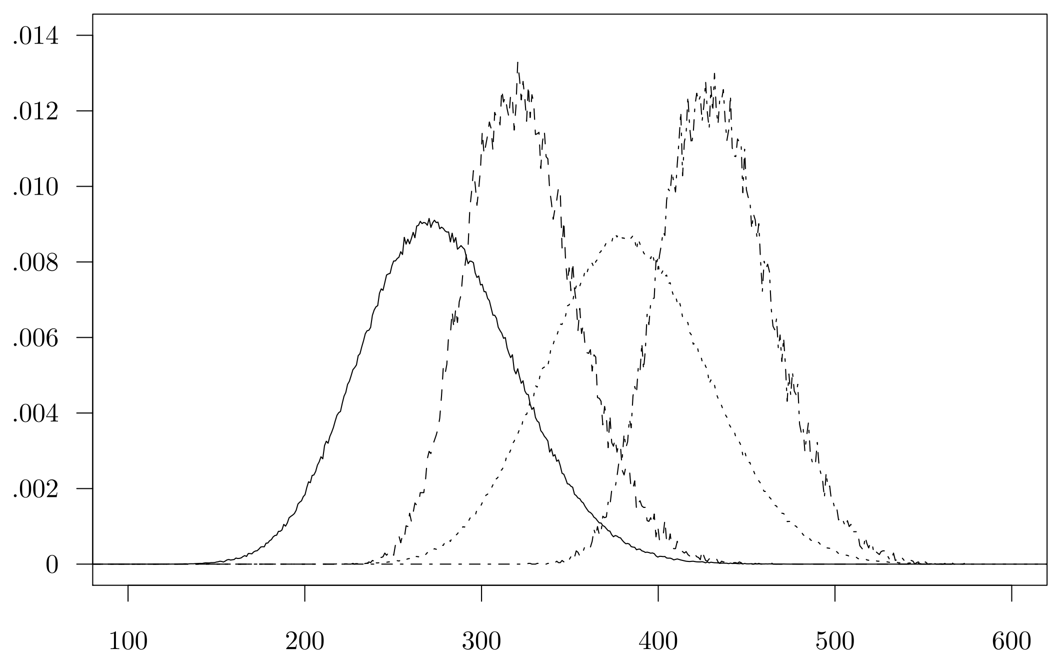

Table 1
Valuation when Homeowners Cannot Cash-Out Refinance

This table provides values of the underlying house (A), first-lien mortgage principal (P) and mortgage rate (y) in addition to the critical service flows (δ_B) at which the homeowner optimally defaults with corresponding equivalent fixed waiting time to default (EFWT) and values of Arrow-Debreu security contingent on default (ADD). We assume a base case of parameter values as well as perturbing the base case by assuming an alternative parameter value as indicated in the Table's column headings.

		r		σ		α		ℓ	
	Base Case	3%	10%	10%	20%	5%	20%	70%	90%
A	\$32.52	\$97.14	\$12.27	\$32.95	\$32.18	\$32.93	\$31.70	\$32.93	\$31.50
P	\$26.02	\$77.71	\$9.82	\$26.36	\$25.75	\$26.34	\$25.36	\$23.05	\$28.35
y	5.66%	3.54%	10.82%	5.20%	6.30%	5.57%	5.86%	5.40%	6.18%
δ_B	\$0.64	\$0.62	\$0.66	\$0.69	\$0.60	\$0.63	\$0.64	\$0.54	\$0.75
EFWT	23.14	33.25	14.27	37.68	16.35	23.33	22.72	31.60	14.37
ADD	\$0.31	\$0.37	\$0.24	\$0.15	\$0.44	\$0.31	\$0.32	\$0.21	\$0.49

Table 2
Valuation when Homeowners Can Cash-Out Refinance: Base-Case Parameters

This table provides values of the underlying house (A), incremental mortgage rate (y) and cumulative mortgage rate ($\bar{y}$) when the homeowner can optimally cash-out refinance either once ($n = 1$) or twice ($n = 2$). We assume the base case of parameter values. The critical service flows at which the homeowner optimally cash-out refinances (δ_F) and optimally defaults (δ_B) with corresponding equivalent fixed waiting times to default (EFWT) and values of Arrow-Debreu security contingent on default (ADD) are also provided. Throughout subscripts denote the number of cash-out refinancing opportunities remaining.

Up to $n = 1$ refinancings

A	y	$\bar{y}$	δ_B	δ_F	EFWT	ADD
A_1=\$32.034	y_1=5.628%	$\bar{y}_1$=5.628%	δ_{B_1}=\$0.630	δ_{F_1}=\$1.329	EFWT_1=18.323	ADD_1=\$0.400
A_0=\$32.433	y_0=7.303%	$\bar{y}_0$=5.681%	δ_{B_0}=\$0.634		EFWT_0=23.081	ADD_0=\$0.315

Up to $n = 2$ refinancings

A	y	$\bar{y}$	δ_B	δ_F	EFWT	ADD
A_2=\$31.626	y_2=5.569%	$\bar{y}_2$=5.569%	δ_{B_2}=\$0.626	δ_{F_2}=\$1.303	EFWT_2=16.225	ADD_2=\$0.444
A_1=\$31.918	y_1=7.547%	$\bar{y}_1$=5.644%	δ_{B_1}=\$0.630	δ_{F_1}=\$1.349	EFWT_1=18.276	ADD_1=\$0.401
A_0=\$32.357	y_0=7.309%	$\bar{y}_0$=5.698%	δ_{B_0}=\$0.634		EFWT_0=23.043	ADD_0=\$0.316

Table 3

Valuation when Homeowners Can Cash-Out Refinance: Comparative Statics

This table provides values of the underlying house (A), incremental mortgage rate (y) and cumulative mortgage rate ($\bar{y}$) when the homeowner can optimally cash-out refinance either once ($n = 1$) or twice ($n = 2$). We perturb the base case of parameter values by assuming an alternative parameter value as indicated. The critical service flows at which the homeowner optimally cash-out refinances (δ_F) and optimally defaults (δ_B) with corresponding equivalent fixed waiting times to default (EFWT) and values of Arrow-Debreu security contingent on default (ADD) are also provided. Throughout subscripts denote the number of cash-out refinancing opportunities remaining.

Comparative statics for $r = 3\%$ with up to n refinancings

n	A	y	$\bar{y}$	δ_B	δ_F	EFWT	ADD
1	A_1=\$95.135	y_1=3.498%	$\bar{y}_1$=3.498%	δ_{B_1}=\$0.614	δ_{F_1}=\$1.382	EFWT_1=25.251	ADD_1=\$0.469
	A_0=\$96.712	y_0=4.761%	$\bar{y}_0$=3.556%	δ_{B_0}=\$0.618		EFWT_0=33.147	ADD_0=\$0.370
2	A_2=\$93.134	y_2=3.440%	$\bar{y}_2$=3.440%	δ_{B_2}=\$0.610	δ_{F_2}=\$1.367	EFWT_2=21.575	ADD_2=\$0.523
	A_1=\$94.452	y_1=4.880%	$\bar{y}_1$=3.515%	δ_{B_1}=\$0.615	δ_{F_1}=\$1.418	EFWT_1=25.081	ADD_1=\$0.471
	A_0=\$96.283	y_0=4.770%	$\bar{y}_0$=3.578%	δ_{B_0}=\$0.619		EFWT_0=33.046	ADD_0=\$0.371

Comparative statics for $r = 10\%$ with up to n refinancings

1	A_1=\$12.156	y_1=10.805%	$\bar{y}_1$=10.805%	δ_{B_1}=\$0.654	δ_{F_1}=\$1.268	EFWT_1=11.904	ADD_1=\$0.304
	A_0=\$12.254	y_0=13.159%	$\bar{y}_0$=10.843%	δ_{B_0}=\$0.657		EFWT_0=14.267	ADD_0=\$0.240
2	A_2=\$12.077	y_2=10.754%	$\bar{y}_2$=10.754%	δ_{B_2}=\$0.651	δ_{F_2}=\$1.238	EFWT_2=10.912	ADD_2=\$0.336
	A_1=\$12.140	y_1=13.574%	$\bar{y}_1$=10.818%	δ_{B_1}=\$0.654	δ_{F_1}=\$1.277	EFWT_1=11.897	ADD_1=\$0.304
	A_0=\$12.243	y_0=13.163%	$\bar{y}_0$=10.856%	δ_{B_0}=\$0.657		EFWT_0=14.259	ADD_0=\$0.240

Comparative statics for $\sigma = 10\%$ with up to n refinancings

n	A	y	$\bar{y}$	δ_B	δ_F	EFWT	ADD
1	A_1=\$32.705	y_1=5.196%	$\bar{y}_1$=5.196%	δ_{B_1}=\$0.684	δ_{F_1}=\$1.251	EFWT_1=31.005	ADD_1=\$0.212
	A_0=\$32.927	y_0=5.896%	$\bar{y}_0$=5.209%	δ_{B_0}=\$0.686		EFWT_0=37.678	ADD_0=\$0.152
2	A_2=\$32.504	y_2=5.190%	$\bar{y}_2$=5.190%	δ_{B_2}=\$0.683	δ_{F_2}=\$1.234	EFWT_2=27.862	ADD_2=\$0.248
	A_1=\$32.674	y_1=5.913%	$\bar{y}_1$=5.200%	δ_{B_1}=\$0.684	δ_{F_1}=\$1.259	EFWT_1=30.986	ADD_1=\$0.212
	A_0=\$32.907	y_0=5.897%	$\bar{y}_0$=5.213%	δ_{B_0}=\$0.686		EFWT_0=37.658	ADD_0=\$0.152

Comparative statics for $\sigma = 20\%$ with up to n refinancings

1	A_1=\$31.492	y_1=6.235%	$\bar{y}_1$=6.235%	δ_{B_1}=\$0.592	δ_{F_1}=\$1.395	EFWT_1=12.662	ADD_1=\$0.531
	A_0=\$32.013	y_0=8.974%	$\bar{y}_0$=6.347%	δ_{B_0}=\$0.597		EFWT_0=16.290	ADD_0=\$0.443
2	A_2=\$30.915	y_2=6.042%	$\bar{y}_2$=6.042%	δ_{B_2}=\$0.582	δ_{F_2}=\$1.359	EFWT_2=11.185	ADD_2=\$0.572
	A_1=\$31.281	y_1=9.837%	$\bar{y}_1$=6.268%	δ_{B_1}=\$0.592	δ_{F_1}=\$1.426	EFWT_1=12.602	ADD_1=\$0.533
	A_0=\$31.870	y_0=8.992%	$\bar{y}_0$=6.385%	δ_{B_0}=\$0.598		EFWT_0=16.241	ADD_0=\$0.444

Table 3 (continued)

Comparative statics for $\alpha = 5\%$ with up to n refinancings

n	A	y	$\bar{y}$	δ_B	δ_F	EFWT	ADD
1	A_1=\$32.725 A_0=\$32.892	y_1=5.538% y_0=7.265%	$\bar{y}_1$=5.538% $\bar{y}_0$=5.576%	δ_{B_1}=\$0.629 δ_{B_0}=\$0.631	δ_{F_1}=\$1.205	EFWT_1=18.943 EFWT_0=23.311	ADD_1=\$0.388 ADD_0=\$0.312
2	A_2=\$32.545 A_1=\$32.674 A_0=\$32.857	y_2=5.412% y_1=8.188% y_0=7.267%	$\bar{y}_2$=5.412% $\bar{y}_1$=5.544% $\bar{y}_0$=5.584%	δ_{B_2}=\$0.622 δ_{B_1}=\$0.629 δ_{B_0}=\$0.631	δ_{F_2}=\$1.188 δ_{F_1}=\$1.215	EFWT_2=16.683 EFWT_1=18.866 EFWT_0=23.295	ADD_2=\$0.434 ADD_1=\$0.389 ADD_0=\$0.312

Comparative statics for $\alpha = 20\%$ with up to n refinancings

1	A_1=\$30.666 A_0=\$31.516	y_1=5.841% y_0=7.382%	$\bar{y}_1$=5.841% $\bar{y}_0$=5.899%	δ_{B_1}=\$0.634 δ_{B_0}=\$0.640	δ_{F_1}=\$1.567	EFWT_1=18.157 EFWT_0=22.621	ADD_1=\$0.403 ADD_0=\$0.323
2	A_2=\$29.925 A_1=\$30.476 A_0=\$31.385	y_2=5.831% y_1=7.449% y_0=7.394%	$\bar{y}_2$=5.831% $\bar{y}_1$=5.874% $\bar{y}_0$=5.932%	δ_{B_2}=\$0.630 δ_{B_1}=\$0.634 δ_{B_0}=\$0.641	δ_{F_2}=\$1.510 δ_{F_1}=\$1.603	EFWT_2=16.593 EFWT_1=18.176 EFWT_0=22.556	ADD_2=\$0.436 ADD_1=\$0.403 ADD_0=\$0.324

Comparative statics for $\ell = 70\%$ with up to n refinancings

n	A	y	$\bar{y}$	δ_B	δ_F	EFWT	ADD
1	A_1=\$32.700 A_0=\$32.903	y_1=5.385% y_0=6.297%	$\bar{y}_1$=5.385% $\bar{y}_0$=5.409%	δ_{B_1}=\$0.534 δ_{B_0}=\$0.536	δ_{F_1}=\$1.314	EFWT_1=26.549 EFWT_0=31.602	ADD_1=\$0.265 ADD_0=\$0.206
2	A_2=\$32.516 A_1=\$32.671 A_0=\$32.884	y_2=5.346% y_1=6.464% y_0=6.298%	$\bar{y}_2$=5.346% $\bar{y}_1$=5.390% $\bar{y}_0$=5.414%	δ_{B_2}=\$0.531 δ_{B_1}=\$0.534 δ_{B_0}=\$0.536	δ_{F_2}=\$1.289 δ_{F_1}=\$1.323	EFWT_2=24.156 EFWT_1=26.159 EFWT_0=31.588	ADD_2=\$0.299 ADD_1=\$0.266 ADD_0=\$0.206

Comparative statics for $\ell = 90\%$ with up to n refinancings

1	A_1=\$30.179 A_0=\$31.045	y_1=6.118% y_0=9.822%	$\bar{y}_1$=6.118% $\bar{y}_0$=6.282%	δ_{B_1}=\$0.747 δ_{B_0}=\$0.755	δ_{F_1}=\$1.368	EFWT_1=10.203 EFWT_0=14.230	ADD_1=\$0.600 ADD_0=\$0.491
2	A_2=\$29.081 A_1=\$29.640 A_0=\$30.659	y_2=6.041% y_1=10.133% y_0=9.874%	$\bar{y}_2$=6.041% $\bar{y}_1$=6.196% $\bar{y}_0$=6.375%	δ_{B_2}=\$0.741 δ_{B_1}=\$0.748 δ_{B_0}=\$0.757	δ_{F_2}=\$1.336 δ_{F_1}=\$1.415	EFWT_2=8.739 EFWT_1=10.170 EFWT_0=14.121	ADD_2=\$0.646 ADD_1=\$0.601 ADD_0=\$0.494

Table 4
Moody's Ratings and Their Expected Loss Criteria

This table shows Moody's ratings and their corresponding expected loss rates.

Corporate Rating	Expected Loss Rate
Aaa	0.0010%
Aa1	0.0116%
Aa2	0.0259%
Aa3	0.5560%
A1	0.1040%
A2	0.1898%
A3	0.2870%
Baa1	0.4565%
Baa2	0.6600%
Baa3	1.3090%
Ba1	2.3100%
Ba2	3.7400%
Ba3	5.3845%
B1	7.6175%
B2	9.9715%
B3	13.2220%
Caa1	17.8634%
Caa2	24.1340%
Caa3	36.4331%
Ca	50.0000%
C	80.0000%
D	90.0000%

Table 5
CDOs and "Secret Seconds": Base-Case Parameters

We size a cash CDO and determine its certificates' expected losses under a variety of assumptions. We assume the base case of parameter values. In the Baseline case, a naïve credit rating agency relies on 1,000,000 simulation paths in which homeowners optimally default but cannot cash-out refinance. In the U.S. Experience w/o Refis homeowners cannot cash-out refinance but attention is restricted to the 50,000 simulation paths closest in a mean-squared error sense to the behavior of U.S. house prices since 2000. Homeowners can optimally cash-out refinance up to $n = 2$ times in the remaining two cases: across all 1,000,000 simulation paths in the case All Paths with Refis and across the 50,000 simulation paths closest in a mean-squared error sense to the behavior of U.S. house prices since 2000 in the case U.S. Experience with Refis.

	Baseline		U.S. Experience w/o Refis		All Paths with Refis		U.S. Experience with Refis	
Size	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating
0.934802	0.00100%	Aaa	0.00254%	Aa1	0.08029%	A1	0.26287%	A3
0.036543	1.30899%	Baa3	4.90510%	Ba3	20.6876%	Caa2	50.5956%	C

Table 6
CDOs and "Secret Seconds": Comparative Statics

We size a cash CDO and determine its certificates' expected losses under a variety of assumptions. We perturb the base case of parameter values by assuming an alternative parameter value as indicated. In the Baseline case, a naïve credit rating agency relies on 1,000,000 simulation paths in which homeowners optimally default but cannot cash-out refinance. In the U.S. Experience w/o Refis homeowners cannot cash-out refinance but attention is restricted to the 50,000 simulation paths closest in a mean-squared error sense to the behavior of U.S. house prices since 2000. Homeowners can optimally cash-out refinance up to $n = 2$ times in the remaining two cases: across all 1,000,000 simulation paths in the case All Paths with Refis and across the 50,000 simulation paths closest in a mean-squared error sense to the behavior of U.S. house prices since 2000 in the case U.S. Experience with Refis.

Comparative statics: $r = 3\%$

	Baseline		U.S. Experience w/o Refis		All Paths with Refis		U.S. Experience with Refis	
Size	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating
0.937332	0.00100%	Aaa	0.00289%	Aa1	0.18470%	A2	0.57963%	Baa2
0.035090	1.30899%	Baa3	5.18210%	Ba3	32.0130%	Caa3	71.1710%	C
Comparative statics: $r = 10\%$								
0.915339	0.00100%	Aaa	0.00184%	Aa1	0.05740%	A1	0.15955%	A2
0.046260	1.30895%	Baa3	4.26170%	Ba3	16.4320%	Caa1	37.6440%	Ca

Comparative statics: $\sigma = 10\%$

	Baseline		U.S. Experience w/o Refis		All Paths with Refis		U.S. Experience with Refis	
Size	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating
0.978113	0.00100%	Aaa	0.00113%	Aa1	0.14600%	A2	0.49595%	Baa2
0.009582	1.30899%	Baa3	10.5171%	B3	41.4475%	Caa1	89.9732%	D
Comparative statics: $\sigma = 20\%$								
0.927421	0.00100%	Aaa	0.00206%	Aa1	0.21690%	A3	0.56959%	Baa2
0.040381	1.30899%	Baa3	4.43379%	B3	31.9757%	Caa3	64.1569 %	C

Comparative statics: $\alpha = 10\%$

	Baseline		U.S. Experience w/o Refis		All Paths with Refis		U.S. Experience with Refis	
Size	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating
0.942210	0.00100%	Aaa	0.00265%	Aa1	0.00701%	Aa1	0.02073%	Aa2
0.030699	1.30890%	Baa3	4.93580%	Ba3	6.0508%	B1	17.6870 %	Caa1
Comparative statics: $\alpha = 20\%$								
0.919141	0.00100%	Aaa	0.00237%	Aa1	0.52693%	Baa2	1.33640%	Ba1
0.049153	1.30896%	Baa3	4.85380%	Ba3	48.0470%	Ca	86.4520%	D

Table 6 (continued)

Comparative statics: $\ell = 70\%$

	Baseline		U.S. Experience w/o Refis		All Paths with Refis		U.S. Experience with Refis	
Size	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating	Loss Rate	Rating
0.948891	0.00100%	Aaa	0.00321%	Aa1	0.01357%	Aa2	0.05378%	Aa3
0.027716	1.30899%	Baa3	5.33540%	Ba3	8.0967%	B2	26.5730%	Caa3

Comparative statics: $\ell = 90\%$

0.921512	0.00100%	Aaa	0.00196%	Aa1	1.66690%	Ba1	2.60380%	Ba2
0.045477	1.30894%	Baa3	4.32640%	Ba3	78.3080%	C	98.1560%	D