Central Bank Liquidity Provision and Collateral Quality

François Koulischer and Daan Struyven*

This draft: September 2013 (First draft: October 2011)

Abstract

Should central banks lend against low quality collateral? We characterize efficient central bank collateral policy in a model where a bank borrows from the interbank market or the central bank. Collateral has favorable incentive effects but is costly to transfer to lenders who value the collateral less because of imperfect collateral quality. We show that a fall in the quantity or the quality of the bank's collateral can increase interest rates in the economy even with a constant policy rate. A looser central bank collateral policy can reduce the spread, alleviate the credit crunch and increase output. (97 words)

Keywords: Collateral policy, collateral requirements, haircuts, asset encumbrance, repo, monetary policy, liquidity requirements.

JEL Codes: E58, G01, G20.

*Koulischer is at Université Libre de Bruxelles (francois.koulischer@ulb.ac.be) and Struyven is at the Department of Economics, Massachusetts Institute of Technology (daan@mit.edu). We thank Adrien Auclert, Nittai Bergman, Estelle Cantillon, Mathias Dewatripont, Patrick Legros, Eric Mengus, James Poterba, Antoinette Schoar, Alp Simsek, Jean Tirole and Lynn Yu and seminar participants at MIT and ULB for useful discussions. Part of this research was conducted while Koulischer was visiting Stanford University.

1 Introduction

The collateral policy of central banks - or the types of assets central banks should require when lending to commercial banks - has traditionally been absent from discussions on monetary policy. The implicit assumption since Bagehot (1873) has been that central banks should lend only against high quality collateral.[1] In line with this principle, the Federal Reserve bought and sold only Treasuries in its open market operations prior to 2007. During the same period more than half of the collateral pledged by banks to the European Central Bank (ECB) were liquid government bonds.

This changed dramatically during the 2007-2013 financial crisis. Not only did central banks expand the range of assets accepted as collateral, but they also adapted collateral requirements to changing market conditions. For example, when the market for asset-backed securities dried up in the United States and banks became unable to use them as collateral, the Fed provided credit to banks against these illiquid assets (see table 1 and appendix A). In Europe, the ECB removed the rating thresholds for distressed government bonds which private lenders refused to accept as collateral. The policy of setting low collateral requirements in the face of falling quantities and qualities of bank collateral was controversial on both sides of the Atlantic (Buiter, 2008; De Grauwe, 2012).[2]

The changes in the collateral policy of central banks raise two questions: (1) Should the central bank tailor its collateral policy to developments in financial markets and if so, how? (2) How does the collateral policy of the central bank interact with its interest rate policy?

In our model, a commercial bank funds projects in the real economy by borrowing against collateral from the interbank market or the central bank. While collateral prevents the bank from shirking, it is costly to use as its value is lower for investors and the central bank than for the bank. We find that when the bank has plenty of high quality collateral, it borrows in the interbank market against low collateral requirements so that the collateral policy of the central bank has no impact on borrowing. However, when the amount or the quality of the available collateral falls below a threshold, the lack of collateral prevents borrowing. In this case, the collateral policy of the central bank can affect lending, and it is optimal for the central bank to relax its collateral requirements to avoid the credit crunch. Our model suggests that interest rate and collateral policy are complements: when the bank faces a collateral crunch, the return required by the bank from firms and households in the real economy increases without changes in the policy rate, set by the central bank. In these cases, a looser collateral policy can alleviate the negative impact of a lack of bank collateral and lower interest rates in the economy.

We develop our results in three steps. We first consider the situation where a commercial

[1] In Bagehot's words: "If it is known that the Bank of England is freely advancing on what in ordinary times is reckoned a good security – on what is then commonly pledged and easily convertible – the alarm of the solvent merchants and bankers will be stayed." — Bagehot (1873), p. 198.

[2] In April 2009, the U.S. Congress required the Federal Reserve to reveal the names of the banks that received financial assistance as well as the collateral used in these transactions.

bank can only borrow from the interbank market to finance its project. We assume that the interbank market is fully competitive so that lenders in the interbank market earn zero profits in equilibrium. This corresponds to the Holmstrom and Tirole (2011) model of collateralized lending in the presence of moral hazard with the addition that the collateral is characterized by its quality. We define the quality of a collateral as the difference between the bank and the investors' value for the collateral.[3] This is an important consideration for thinking about collateral policy, where not only the quantity but also the quality of collateral matters.

This model helps us understand the interaction between the interest rate and collateral in addressing the moral hazard problem. From the perspective of investors, interest payments and collateral transfers are cash flows that pay in different states of the world (the interest rate is paid if the project succeeds while the collateral is seized if the project fails) but are otherwise substitutes: investors would be willing to trade off a higher interest rate for lower collateral. However, interest payments and collateral transfers have different incentive properties for the commercial bank: a high interest rate reduces the profit from a successful investment, thereby making shirking more attractive. In contrast, a high collateral requirement makes shirking more costly as the commercial bank loses the collateral in case of default. The incentive benefits of collateral are similar to those in Holmstrom and Tirole (2011). The introduction of collateral quality - where the investors and the central bank have a different valuation for the collateral - adds a new trade-off. Because collateral has an extra cost (its transfer in case of default destroys value), in equilibrium the bank does not always pledge all the available collateral but minimizes its use. This allows us to define and explain the behavior of collateral requirements. When investment opportunities are attractive relative to the benefits of shirking, collateral requirements in the interbank market are low. When investment opportunities worsen, collateral requirements in the interbank market increase. The extra cost also explains the use of uncollateralized transactions in the interbank market prior to the 2007-2013 crisis, which other models cannot explain. Collateral quality also enables us to derive cross-sectional predictions regarding the equilibrium mix between interest rate and collateral requirements across collateral quality. We find that both interest rates and collateral requirements increase as the quality of collateral decreases, in line with empirical studies of collateralized lending (Gorton and Metrick, 2012).

In the second step, we consider the case where the central bank is the only potential lender to the commercial bank (there are no investors anymore). This case helps us illustrate how the solution to the moral hazard problem between the commercial bank and the lender (the central bank) changes with the objective function of the lender. In our model the central bank is concerned about total output but discounts expected losses heavily. This implies that, unlike interbank market investors, it can tolerate some losses if this increases the efficiency of the investment undertaken by the commercial bank. We find that, in contrast with the

[3]The difference in valuation is in the spirit of Geanakoplos (2010) and Simsek (2013) who model how differences in valuations affect lending. An alternative interpretation is the mechanism of Shleifer and Vishny (1992) through which liquidation values constrain the capacity to borrow.

collateral requirements of the interbank market, the optimal central bank collateral policy sets low collateral requirements in the face of low quantities and qualities of bank collateral and high collateral requirements otherwise. We also find that the central bank should refuse to lend to banks that have too little high quality available collateral and are "too encumbered to save".

Finally, we consider the interbank market, the central bank and the commercial bank together in the third and last step of our analysis. We assume that both the central bank and investors make an exclusive loan offer to the commercial bank, which selects the most attractive loan. While the coexistence of the two types of lenders does not change the contracts offered by these lenders, it changes the source of the commercial bank's funding. We show that when the bank has a high level of quality collateral available, it borrows from the interbank market only. However when the amount or the quality of available collateral falls below a threshold, the commercial bank borrows from the central bank, which replicates the observed shift from interbank markets to the central bank during the 2007-2013 financial crisis.

We then use these results to revisit the optimal design of monetary policy. Several empirical papers (e.g. Kashyap and Stein, 2000; Jimenez et al., 2012) have shown that the transmission from the short term policy rate, set by the central bank, to the interest rate in the real economy varies with the banks' amount of available collateral (measured as the ratio of securities to assets or the equity level). Banks with less collateral available tighten credit more than other banks when the short term interest rate increases.[4] In our model, the short term interest rate corresponds to the return required from the bank, which we normalize to one (break-even). The interest rate in the real economy can be interpreted in our model as the marginal return required by the bank from its customers. In our model, the return required by the bank from its customers is higher than one in cases where the bank borrows with collateral because of the imperfect collateral quality. Consistent with the empirical evidence, the lower the collateral quality, the higher the wedge between the returns required by the bank and the investors. This wedge increases even further when the bank runs out of collateral, e.g. due to a fall in the value of its collateral. Our model implies that collateral policy facilitates the transmission of monetary policy by reducing the spread between the short-term interest rate (the return required from the bank) and the cost of funding of firms and households in the real economy (the return required by the bank from its customers).

Collateral policy may be an alternative to the broken transmission mechanism of traditional monetary policy when banks have little available collateral,[5] but it is ineffective for tightening during booms when they have plenty of collateral available. During booms, collateral requirements in the interbank market are low and banks prefer to borrow in the interbank market as collateral use is costly. This raises the question of dynamic moral hazard, or whether lenient

[4]For an empirical assessment of the importance of the bank lending channel, see Peek and Rosengren (1997) and Khwaja and Mian (2008). The impact of collateral on funding costs and investment are respectively documented in Benmelech and Bergman (2011) and in Chaney, Sraer, and Thesmar (2012).

[5]There are however limits to the loosening of collateral policy as the central bank should refuse collateral from banks that are "too encumbered to save".

collateral policy of central banks may lead to lower equilibrium collateral quality over time. Our model suggests that one way to address this problem where banks have an incentive to hold too little collateral is to require banks to keep sufficient levels of quality collateral during booms, as with the Basel III Liquidity Coverage Ratio (LCR) requirements.

Section 2 sets up our model of commercial and central bank collateralized lending. Sections 3 and 4 consider collateralized lending by the interbank market and the central bank, respectively. In section 5 we consider the case where both the central bank and the interbank market can fund the commercial bank and we revisit the optimal design of monetary policy. Section 6 considers the Basel III Liquidity Coverage Ratio (LCR) requirements as potential solution to the incentive issues associated to loose collateral policy of central banks. Section 7 concludes.

Table 1: Changes in ECB and Fed collateral policy (2007-2013)

Date		**ECB**	**Federal Reserve**
Oct	2007		Term-Auction Facility: Provided up to $500 bn against residential mortgages (25%), asset-backed securities (ABS) (17%) or commercial loans (15%).
Mar	2008		Term Securities Lending Facility and Primary Dealer Credit Facility: primary dealers can exchange (mainly) mortgage backed securities against Treasuries.
Oct	2008	Credit threshold is lowered to BBB- from A- (except for ABS). Bonds traded on certain non-regulated markets become eligible.	Commercial Paper Funding Facility and Money Market Investor Funding Facility: provide liquidity against asset-backed commercial paper
Nov	2008	Foreign-currency denominated assets become eligible (until January 2011).	Term Asset-Backed Securities Loan Facility: provide loans against newly issued ABSs.
May	2010	Suspension of minimum rating threshold of Greek government debt	
Mar	2011	Suspension of minimum rating threshold of Irish government debt	
Jul	2011	Suspension of minimum rating threshold of Portuguese government debt	
Feb	2012	Additional credit claims (e.g. consumer loans, credit card loans) become eligible	
Jul	2013	Broader ABS eligibility criteria	

Sources: ECB and Federal Reserve

2 Setup

There are three types of agents: a commercial bank (from now on called "bank"), investors in the interbank market and the central bank. The bank seeks a loan to fund an investment. The interbank market and the central bank are potential lenders and compete in offering collateralized loan contracts. We now describe each of these agents in detail.

Bank. A cashless bank has an investment opportunity and starts with a balance sheet of size 1. The bank owns collateral of quality v that pays 1 with probability θ (and is thus worth θ). The other assets (worth $1-\theta$) are encumbered: they are used by the bank for other activities and cannot be used as collateral.[6] The quantity of collateral available θ is common knowledge.

The bank may obtain funding q from the interbank market or the central bank, but the process is not frictionless. If the bank properly manages the reinvestment of her loan q, she obtains $R(q)$ with probability p and zero with probability $1-p$. The return function $R(\cdot)$ is increasing and concave $R'(q) > 0$, $R''(q) < 0$ and satisfies the Inada conditions $\lim_{q\to 0} R^{'}(q) = \infty$, $\lim_{q\to\infty} R^{'}(q) = 0$. If the bank mismanages, the project is guaranteed to yield zero but the bank gets a private benefit $A + Bq$, where $A, B > 0$.

A collateralized loan contract specifies a loan size q, a gross interest rate r and a gross haircut $h \geq 0$, which implies an interest payment of rq and a total value of collateral pledged of hq. The collateral is seized if the project fails and the bank is unable to reimburse the loan.

To ensure that the bank properly manages its investment, the payoff of proper management must exceed the payoff from shirking

$$p\left(R\left(q\right) - rq\right) - \left(1-p\right)hq \geq A + Bq - hq.$$

This incentive compatibility (IC) constraint can be simplified to

$$pR\left(q\right) - prq + phq \geq A + Bq. \tag{IC}$$

The interest rate has negative incentive properties because it makes the IC harder to satisfy. The haircut has positive incentive properties since it decreases the payoff of the bank in case of default and makes the IC easier to satisfy.

Any feasible contract must also ensure that the bank has enough collateral to pledge. This is the collateral capacity (CC) constraint

$$hq \leq \theta. \tag{CC}$$

Interbank Market. The private funding market, which can be thought of as the interbank or money market, is perfectly competitive as in Gale and Hellwig (1985) and Holmstrom and

[6] In contrast with Simsek (2013) and Weymuller (2013), the investment opportunity is unpledgeable as the bank invests in real economy projects (e.g. illiquid loans to entrepreneurs).

Tirole (1997). Investors fund any contract that yields a non-negative profit. Investors receive a cash payment rq if the project succeeds. If the project fails, investors seize the collateral worth hq to the bank and hqv to investors. The term v captures the quality of the collateral. When v is close to 1, investors and the bank have a similar value for the collateral. For instance, this could be the case of high-grade government bonds. When $v < 1$, collateral has a smaller value to the investors than to the bank.[7]

Any equilibrium contract in the interbank market (q, r, h) must ensure that the market clears, i.e. that investors make zero profits

$$prq + (1-p)\, hqv - q = 0. \tag{Market clearing}$$

Central bank. The central bank has two goals: to maximize output and to minimize losses.[8] Central banks are concerned about losses, as it exposes them to political pressure and ultimately reduces their ability to pursue their core mission of output and price stability. These objectives correspond to the way central bankers typically define their mission in practice, and are in line with observed monetary policy (Clouse, Henderson, Orphanides, Small, and Tinsley, 2003; Friedman and Schwartz, 2008; Judd and Rudebusch, 1998; Krugman, 1998; Sargent and Wallace, 1981; Stella, 2005).

Output is given by the sum of the profits of the three agents. Let (q, r, h) be the collateralized loan contract taken by the bank. The expected profit of the bank is given by

$$\Pi_b = p\,(R\,(q) - rq) - (1-p)\, hq.$$

When the central bank and the interbank market compete to offer a loan, we assume exclusive loans, i.e. the bank cannot borrow from both lenders. We set $\lambda = 1$ to indicate that the bank borrows from the interbank market and $\lambda = 0$ if the bank borrows from the central bank. If the bank borrows from the interbank market, the expected profit of interbank investors and the central bank (respectively Π_p and Π_{cb}) is the sum of interest payments and collateral payments minus the lent amount

$$\Pi_p, \Pi_{cb} \equiv prq + (1-p)\, hqv - q.$$

Output is given by

$$\Pi_b + \lambda \Pi_p + (1-\lambda)\, \Pi_{cb} = pR\,(q) - q - (1-v)\,(1-p)\, hq.$$

Interest payments cancel out from the expression of output, as they represent simple transfers from borrowers to lenders. In contrast, collateral transfers hurt output as collateral is trans-

[7] The wedge $1 - v$ could capture that the bank is the first-best user of the collateral, as in Shleifer and Vishny (1992) or Kiyotaki and Moore (1997). Alternatively, the bank may hold optimistic beliefs about the collateral value as in Geanakoplos (2010) and Simsek (2013).

[8] In some New Keynesian models, stabilizing output at its potential also stabilizes inflation (Goodfriend and King, 1997).

ferred to second-best users with probability $1-p$. As a result, output is given by the expected value of the investment minus the losses, due to the inefficient allocation of collateral in case of default.

To capture the sensitivity of the central bank to losses, we assume that the central bank puts a weight on its expected losses. This yields the final objective function for the central bank

$$\text{Output} + \omega \min\left\{0, \Pi_{cb}\right\}.$$

The objective function of the central bank is reduced to the standard utilitarian welfare function when $\omega = 0$.

Finally, we assume that there is a maximum amount of losses that the central bank can support, Π_{CB}^{loss}. This reflects the view that the sensitivity of central banks to losses is highly non-linear (Stella, 1997). This implies the additional constraint

$$\Pi_{cb} \geq \Pi_{cb}^{loss}.$$

Discussion. The bank and its potential lenders (interbank market investors or the central bank) face a problem of investment under moral hazard. Figure 1 illustrates the distinct roles of the collateral requirement (or haircut) h and interest rate r for a fixed loan size q. From the perspective of interbank investors, interest payments and collateral transfers are cash flows that pay in different states of the world (the interest rate is paid if the project succeeds, the collateral is seized if the project fails) but are otherwise substitutes: the investor would be willing to trade off a higher interest rate for lower collateral. This is illustrated in figure 1 by the investors' isorevenue line, which shows all contracts that offer the same revenue to the investors, i.e. $\{h, r\}: \ prq + (1-p)\, hqv = K$ where K is a constant.

However, interest payments and collateral transfers have different incentives properties for the commercial bank: a high interest rate reduces the profit from a successful investment, thereby rendering shirking more attractive. In contrast, a high collateral requirement makes shirking more costly since the commercial bank loses the collateral in case of default. This is illustrated in figure 1 by the thick plain line as all contracts above this line (i.e. with a higher haircut or lower interest rate) also satisfy the IC constraint. The horizontal dotted line shows the collateral capacity CC constraint: it is the maximum amount of collateral that can be pledged by the bank. Contracts above the CC are not feasible because the bank does not have enough collateral to meet the requirements. All contracts below the CC are feasible, so the set of feasible contracts is the shaded area above the incentive compatibility constraint and below the collateral capacity constraint.

The use of collateral is costly as it involves an inefficient transfer to low-value users in case of default. We call this the *liquidity wedge*: while a contract costs $prq + (1-p)\, hq$ to the bank, it only yields an expected revenue $prq + (1-p)\, hqv$ to the investors. We illustrate this in figure 1 by drawing the bank's isocost line, i.e. the set of contracts that have the same cost

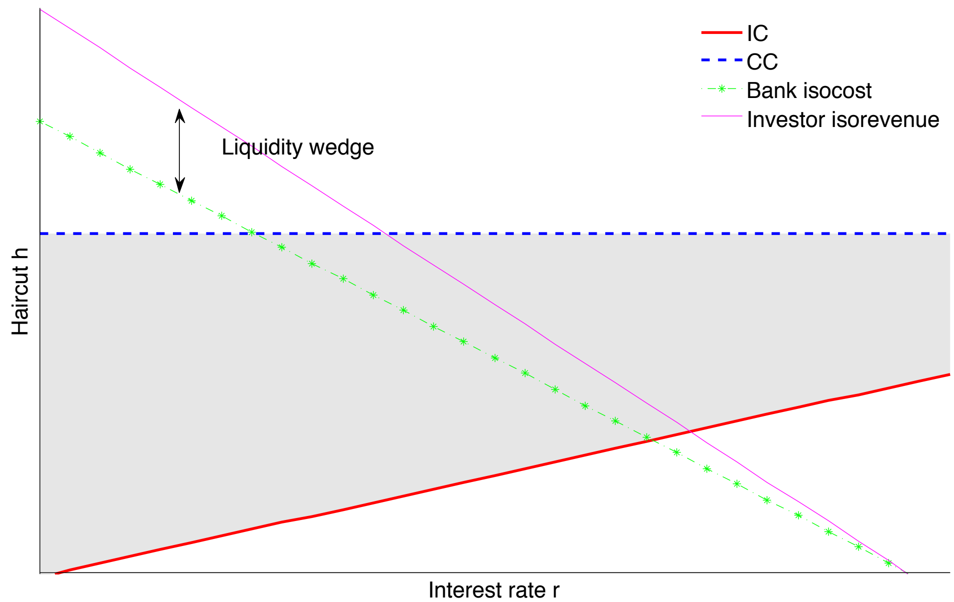

Figure 1: Haircut-Interest rate contract space. This figure shows, for a fixed loan size, the contract space and the incentive- and collateral capacity constraints. The bank's isocost and the investors' isorevenue lines are contracts that have the same cost to the bank and the same return for the investor. The shaded area shows the set of contracts that satisfy the incentive and collateral constraints of the bank.

K to the bank. We choose $prq + (1-p)\,hq = K$ so that when collateral requirements are zero ($h = 0$) the contract has the same cost as the investors' revenue. As the haircut increases, a wedge appears between the isocost and the isorevenue line, as the investors require higher haircuts (or higher interest rates) than the bank to earn a given amount of expected revenue.

First-best Benchmark. Before solving the full model, let us consider the benchmark first-best case where there is no moral hazard so that the IC can be ignored. In a perfectly competitive funding market, the equilibrium contract maximizes:

$$\max_{r,h,q} \pi = p\,(R\,(q) - rq) - (1-p)\,hq$$

such that the expected payoff to investors is zero and that the bank has sufficient collateral available:

$$\begin{aligned} prq + (1-p)\,hqv - q &= 0. \\ hq &\leq \theta. \end{aligned}$$

The first-best contract is uncollateralized ($h = 0$) because of the cost of using collateral of quality $v < 1$. The next lemma characterizes the first-best contract.

Lemma 1. (First-best - private investment) *In the first best, all positive NPV projects are* *undertaken. The loan level q^* solves $R'(q^*) = \frac{1}{p}$, the haircut h is zero and the interest rate is $r = \frac{1}{p}$.*

3 Interbank Market Lending

In this section we consider the situation where the bank can only borrow in the interbank market. The section is divided into three parts. The first part explains in which cases the bank uses collateral to borrow from the interbank market and in which cases it does not. The second and third part discuss properties of respectively the uncollateralized and collateralized borrowing equilibrium.

Collateralized or uncollateralized borrowing? Collateral is used to incentivize the bank as the bank loses the pledged collateral when it shirks. Pledging collateral is unnecessary when the returns from undertaking the project (the investment prospects $R(\cdot)$ and p) are high relative to the private benefits from shirking. The next proposition formalizes this claim using the threshold loan size q^{ec} that solves $R'(q^{ec}) = \frac{1}{p} + \frac{(1-p)(1-v)B}{p}$.

Proposition 1. *[Shift to collateral] The equilibrium contract is uncollateralized when investment prospects $R(\cdot)$ and p are high enough ($h = 0$ if $pR(q^{ec}) - q^{ec} \geq A + Bq^{ec}$). The bank shifts from uncollateralized to collateralized borrowing when investment prospects fall ($h > 0$ else).*

Proof. See appendix B. □

For future reference, we define the condition $C_1 \equiv pR(q^{ec}) - q^{ec} < A + Bq^{ec}$ which corresponds to the case where the bank uses collateral and $\overline{C_1}$ to refer to the other case.

The condition for using collateral is intuitive: when investment prospects are high relative to the private benefits, the moral hazard problem becomes irrelevant as the bank has a high reward if the project succeeds. The uniqueness of the uncollateralized lending contract is arguably less straightforward. The uniqueness is driven by imperfect collateral quality $v < 1$, which imposes an extra cost in case of default so that the bank prefers to borrow without collateral if this is incentive compatible. When $v = 1$ as in Holmstrom and Tirole (2011), the model yields a continuum of equilibria when $\overline{C_1}$ holds, as investors and the bank are indifferent between cash or collateral transfers.[9]

The imperfect collateral quality can explain the use of uncollateralized transactions in the interbank market before the 2007-2013 crisis as well as the ensuing fall in loan sizes, which models without collateral quality cannot account for. Figure 2 shows that the amount of uncollateralized lending in the European interbank market fell by 50% between 2007 and

[9]This suggests a mechanism reminiscent of the Modigliani-Miller theorem where the amount of collateral does not matter when $v = 1$ as long as the bank is properly incentivized (we discuss this in appendix D).

2010, from EUR 160 billion to EUR 80 billion. This is consistent with our model where private benefits from shirking can increase during downturns.[10]

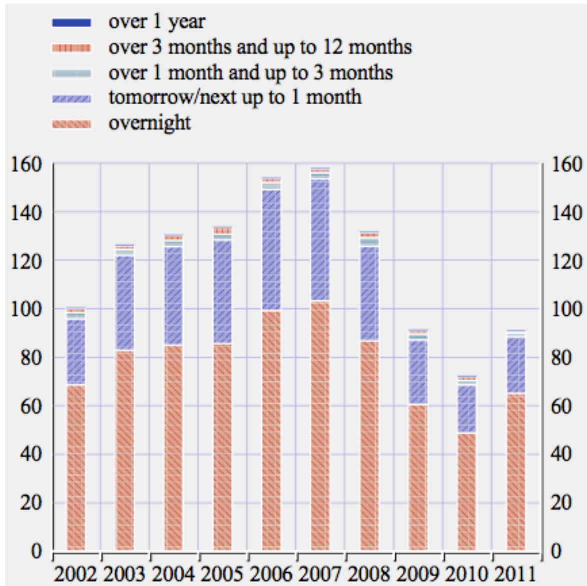

Figure 2: Uncollateralized borrowing volumes and maturities by European banks

This figure plots the total volumes of uncollateralized borrowing by European banks for different maturity structures. The chart is from the ECB April 2012 report "Changes in bank financing patterns" which exploits the annual ECB survey of treasurers from the largest banks of the Eurozone.

Uncollateralized borrowing. The uncollateralized lending contract is similar to the first-best contract: the interest rate is $r = 1/p$ and there are no collateral requirements ($h = 0$). The loan size q is equal to the first-best loan size when the investment prospects p and $R(\cdot)$ are large relative to the private benefits from shirking (A and B) (i.e. when $pR(q^*) - q^* > A + Bq^*$). When this condition does not hold, the IC constraint binds and the loan size is lower than the first-best. In order to keep the bank incentivized, the equilibrium contract reduces the investment so that the average return increases. The next proposition summarizes these findings.

Proposition 2. *[Uncollateralized Efficiency] The loan size q is higher when the bank borrows uncollateralized than when it uses collateral ($q > q^{ec}$ if $\overline{C_1}$ holds). The uncollateralized interest rate and haircut are the same as in the first-best and the loan size is at the first-best level if $pR(q^*) - q^* > A + Bq^*$ else it solves $A + (1+B)q - pR(q) = 0$.*

Proof. See appendix B. ∎

[10]Private benefits may increase in a downturn as firms in distress that do not bear the losses in case of failure shift risks (Landier, Sraer, and Thesmar (2011)). Stein (2013) suggests that banks may "reach for yield" in a low interest rate environment when high leverage levels are required to provide sufficiently high returns.

We emphasize two features of the uncollateralized equilibrium. First, the quantity and quality of available collateral θ and v do not affect investment or interest rate levels as collateral is not used in equilibrium. Therefore banks may borrow extensively in good times even when they have virtually no collateral available. This leaves those banks exposed to the "shifts to collateralized borrowing" from proposition 1, which may explain why banks like Northern Rock in the United Kingdom could rely extensively on interbank borrowing before 2007 but were severely hit by a lack of collateral after August 2007.

A second feature is that the loan size q is higher when the bank borrows without collateral as imperfect collateral quality increases the funding cost of the bank. Therefore the bank reduces investment when using collateral to ensure that the marginal cost remains equal to the marginal return on investment.

Collateralized borrowing. When investment prospects worsen or when private benefits from shirking increase, the equilibrium shifts to collateralized borrowing. Depending on the amount of collateral available and its quality, the bank finds itself in one of three regimes: the enough collateral regime, the collateral crunch or the liquidity dry-up. Each regime corresponds to a specific set of binding constraints. While the initial problem has four constraints (IC, CC, $h \geq 0$ and market clearing), the number of cases to consider can be significantly reduced using two results. First, the haircut constraint ($h \geq 0$) and the IC ($hq \leq \theta$) cannot both bind at the same time. Second, the IC must bind when the bank pledges collateral ($h > 0$) as to minimize the cost associated to the collateral transfer. This leaves us with four cases to consider. The two cases where $h = 0$ binds are the uncollateralized lending regimes discussed above (where the IC binds or slacks). When the bank uses collateral, the IC always binds and the two cases are when the CC is slack (enough collateral) or binds (collateral crunch). When the problem does not have a nonnegative solution, there is a liquidity dry-up.

A key determinant of the collateralized equilibrium is the collateral quality factor $f = p\theta + (1-p)\,\theta v$. This factor f corresponds to the expected social value of a quantity of collateral θ with quality v when it is pledged as collateral: with probability p the project succeeds and the asset keeps its value of θ to the bank and with probability $1-p$ the asset is transferred to the lender who values it at θv. Two thresholds for the collateral quality factor are relevant to distinguish the several regimes. The first one is $\underline{f}_1^I = A + (1+B)\,q^{ec} - pR\left(q^{ec}\right)$ and the second threshold $\underline{f}_2^I$ is the lowest f for which $pR(q) + f - q = A + Bq$ has a real and nonnegative solution in q.

The following proposition characterizes collateralized lending:

Proposition 3. *(<u>Enough Collateral</u>) If $f \geq \underline{f}_1^I$ and C_1, the loan size is $q^{ec} < q^*$, the interest rate is $r = \frac{1-(1-p)hv}{p}$ and the haircut is $h = \frac{A+(1+B)q-pR(q)}{q[(1-p)v+p]}$.*

(<u>Collateral crunch</u>) If $\underline{f}_2^I < f < \underline{f}_1^I$ and C_1, the loan size $q < q^{ec}$ solves $pR(q) + f - q = A + Bq$, the haircut is $h = \theta/q$ and the interest rate is $r = \frac{1-(1-p)hv}{p}$.

(<u>Liquidity dry-up</u>) If $f \leq \underline{f}_2^I$ and C_1, there is a liquidity dry-up ($q = 0$).

Proof. See appendix B. □

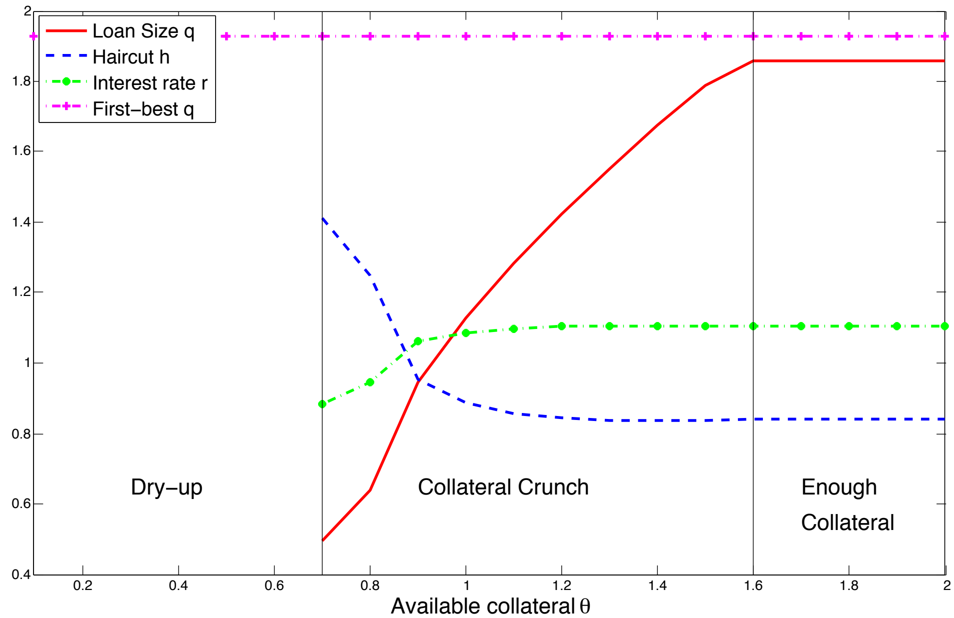

Figure 3: Interbank market collateralized lending regimes

This figure plots the haircut, interest rate and loan size in the three regimes of interbank lending as a function of the amount of available collateral. The three regimes are respectively the dry-up, the collateral crunch and the enough collateral regime. The parameters of the model are given by $A = 0.9$, $B = 0.45$, $v = 0.9$, $p = 0.7$ and the function of investment opportunities for the bank is $R(q) = 2\sqrt{q}$.

Figure 3 illustrates the various collateralized regimes as a function of the amount of available collateral θ for a constant collateral quality v. When the bank has enough collateral, it pledges the amount necessary to satisfy the IC which always binds while the CC is slack. The lending level is lower than that of the first-best as using collateral is costly: with probability $1-p$ the collateral is transferred to investors who only value each unit of the collateral at $v < 1$. The Collateral Crunch regime corresponds to the case where the CC binds: the bank pledges all its available collateral. However, this is insufficient to secure funding at the "enough collateral" level. In this case investment is determined by the amount of available collateral. Finally when the collateral quality factor f falls below $\underline{f}_2^I$, the bank runs out of collateral and liquidity dries up.

In the remainder of this section, we discuss the major comparative statics of the contract terms from proposition 3 and match those comparative statics with the stylized facts of collateralized lending markets.

Proposition 4. *[Irrelevant collateral quantity] The amount of collateral available is irrelevant for contract terms when the bank has enough collateral (If C_1 and $f > \underline{f}_1^I \rightarrow \frac{\partial q}{\partial \theta} = \frac{\partial r}{\partial \theta} = \frac{\partial h}{\partial \theta} =$*

0). *In this regime the loan size falls when collateral quality falls (If* $f > \underline{f}_1^I \rightarrow \frac{\partial q}{\partial v} = \frac{\partial q^{ec}}{\partial v} > 0$*).*

When the bank has enough collateral the loan size, the haircut and the interest rate are independent of the amount of available collateral θ. This differs from models as Shleifer and Vishny (1992) or Holmstrom and Tirole (2011), where the quantity of collateral determines borrowing levels. However collateral quality does influence the loan size even when the bank has enough collateral. The imperfect quality explains why the collateralized loan size (which solves $R'(q) = \frac{1}{p} + \frac{(1-p)(1-v)B}{p} > \frac{1}{p}$) is lower than the first best loan size (given by $R'(q) = 1/p$).

Proposition 5. *[Haircut spikes] When the bank has relatively little collateral available, haircuts spike in response to a negative shock to collateral quality (If* C_1 *and* $\underline{f}_2^I < f < \underline{f}_1^I \rightarrow \frac{\partial h}{\partial v} < 0$*).*

Proof. See appendix C. □

The comparative statics of the haircut match the stylized facts of private repo markets. The predicted negative relationship between collateral quality and haircuts is not only consistent with time series evidence (Gorton and Metrick, 2012) but also with the cross sectional patterns. Table 2 shows the average haircut levels for collateralized loans extended by Fidelity money market funds. From reports on the holdings of funds we extracted the collateral type, interest rate, haircut and counterparties for all repos from 2006 to 2012.[11] The lowest haircuts are for treasuries and commercial paper, arguably the most liquid of the series.

Table 2: Haircuts by asset class for Fidelity money market funds (2004-2011)

	Mean	Min	Max	Std. Dev.
Commercial paper	3.0%	0.4%	4.3%	0.5%
Treasuries	3.2%	1.9%	7.3%	2.3%
Mortgage loan ob.	3.7%	2.0%	7.2%	2.3%
Corporate debt	4.9%	1.9%	7.9%	2.4%
Equities	8.1%	1.1%	17.8%	2.0%
Other	10.7%	7.9%	16.2%	4.8%
Total	6.9%	0.4%	17.8%	2.8%

This table shows the summary statistics of the haircut levels used by Fidelity money market funds in private repos by collateral type. The data are from the SEC N-MFP quarterly filings from July 2004 to August 2011 and include 6 money market funds (MMF). Fidelity is the second largest manager of MMF with a market share of approximately 10% (Weymuller, 2013).

One counterintuitive prediction of our model is that the interest rate falls when banks have little collateral available while interest rates increased during the crisis. This contrast is driven by the moral hazard friction we use: low interest rates reduce the incentives to shirk.[12]

[11] We could extend the dataset to other money market funds (MMF). However, preliminary exploration suggests the patterns in other MMFs are similar to those in the repos of Fidelity.

[12] In reality, other frictions than moral hazard might also play a role and adverse selection for instance might explain why spreads increased during the financial crisis.

One interesting and arguably more realistic prediction concerns the relative order in which the contract terms (q, r, h) respond to negative shocks to the quantity (or quality) of collateral. As visualized in the collateral crunch zone of figure 3, the adjustment to a negative shock to collateral quantity first mainly operates through the loan size q and then later through the haircut h and the interest rate r. The loan size q adjusts first as losses associated to downsizing are initially only second-order, whereas increasing the haircut h leads immediately to first-order losses associated to the liquidity wedge $1 - v$. By focusing on the most valuable projects, the higher average return improves bank incentives. However, as more and more valuable projects are canceled, fixing incentives through downsizing only becomes too costly. Higher haircuts as well as lower interest rates then kick in as a second channel of adjustment.

Proposition 6. *[Liquid first] When the bank pledges collateral, it is more profitable to use high quality collateral (If C_1 and $f > \underline{f}_2^I \rightarrow \frac{\partial \Pi_b}{\partial v} > 0$).*

The liquidity wedge $1 - v > 0$ implies that borrowers have an incentive to use first the most liquid, high quality collateral. Table 3 illustrates the predominance of liquid collateral in repo contracts for Fidelity money market fund repos. Treasuries, arguably the most liquid securities, account for more than half of the collateral used, followed by corporate debt. The European market presents a similar pattern: a survey by the International Capital Market Association (ICMA) suggests that government bonds account for 41 % of the collateral used in European private repos. Corporate bonds and equity follow at 19.1% and 14.7% respectively.

Table 3: Collateral types used by Fidelity money market funds (2004-2011)

	Collateral value ($ trln)	Percent	Cumulative
Treasuries	464.5	51	51
Corporate debt	167.6	18.4	69.4
Mortgage loan obligations	113.0	12.4	81.8
Equities	108.8	12.0	93.8
Other	40.8	4.5	98.2
Commercial paper	14.9	1.6	99.9
Certificates of deposit	0.7	0.1	99.9
Municipals	0.5	0.1	100

This table shows the total value of collateral used by Fidelity money market funds in repos by collateral type. The data are from the SEC N-MFP quarterly filings from July 2004 to August 2011.

4 Central Bank Lending

We now consider the case where the central bank is the only source of funding, for instance because it uses its regulatory authority to impose a compulsory scheme. The central bank has the bargaining power and maximizes output while minimizing its own expected losses. Its objective function is

$$W_{cb} = \text{Output} + \omega \text{ Losses}.$$

Plugging in the expressions for output and central bank losses, the central bank solves

$$\max_{r,q,h} W_{cb} = pR\left(q\right) - q - \left(1-p\right)hq\left(1-v\right) + \omega\left(prq + \left(1-p\right)hqv - q\right)1_{\left[\Pi_{cb}\leq 0\right]}$$

under the constraints that the bank is incentivized not to shirk, but to make non-negative profits and ensure sufficient collateral, and that the central bank's profit is higher than its maximum loss, Π_{CB}^{loss}:

$$\begin{aligned} pR\left(q\right) - prq + phq &\geq A + Bq \\ p\left[R\left(q\right) - rq\right] - \left(1-p\right)hq &\geq 0 \\ hq &\leq \theta \\ prq + \left(1-p\right)hqv &\geq \Pi_{CB}^{loss} \end{aligned}$$

When the profit of the central bank $\Pi_{cb} = prq + \left(1-p\right)hqv$ is positive, the objective function becomes:

$$W_{cb} = pR\left(q\right) - q - \left(1-p\right)qh\left(1-v\right).$$

For a given loan size q and neglecting the constraints, the central bank prefers low haircuts. When the profit of the central bank is negative, the objective function is:

$$W_{cb} = pR\left(q\right) - q\left(1+\omega\right) - \left(1-p\right)qh\left(1-v\left(1+\omega\right)\right) + \omega prq.$$

The central bank has now an ambiguous attitude towards collateral. On the one hand, all else equal, higher haircuts lower its losses. On the other hand, haircuts increase the output loss from the liquidity wedge. The net effect depends on the weight given to losses ω. To simplify the presentation of the results we focus on the most interesting case where ω is sufficiently high and exceeds a threshold, $\omega > \underline{\omega} = \frac{(1-p)(1-v)}{v(1-p)+p}$.[13] We also focus on collateralized lending as the uncollateralized contract is identical to the one in the interbank market.

As in the previous section, optimal collateral policy depends on several thresholds for the quality factor $f = p\theta + \left(1-p\right)\theta v$. The thresholds $\underline{f}_1^{CB}$ and $\underline{f}_2^{CB}$ equal

$$\underline{f}_i^{CB} = A + \left(B+1\right)q - pR\left(q\right).$$

where q is the loan size for respectively the equilibrium $f > \underline{f}_1^{CB}$ for $i = 1$ and the loan size of the equilibrium $\underline{f}_1^{CB} > f > \underline{f}_2^{CB}$ for $i = 2$. The collateral threshold $\underline{f}_3^{CB}$ is

$$\underline{f}_3^{CB} = \Pi_{CB}^{loss} + q - pR\left(q\right) + A + Bq,$$

[13] See appendix E for a discussion of the case when the importance of losses relative to output is lower than the threshold (i.e. when $\omega < \underline{\omega}$).

where q is the loan size for the equilibrium $f > \underline{f}_3^{CB}$. The collateral threshold $\underline{f}_4^{CB}$ is the lowest f for which

$$pR(q) + f - A - Bq - \Pi_{CB}^{loss} - q = 0 \tag{1}$$

has a solution.

The following proposition presents the different regimes of optimal collateral policy as a function of f, which are also summarized in Figure 4.

Proposition 7. *(Private Contract) If $\underline{f}_2^{CB} < f$ the central bank offers the same loan contracts as the interbank market does.*

(Lending Floor) If $\underline{f}_3^{CB} < f < \underline{f}_2^{CB}$ the loan size solves $R'(q) = \frac{1+\omega(1+B)}{p(1+\omega)}$, the haircut is $h = \theta/q$ and the interest rate is $prq = pR(q) + p\theta - A - Bq$.

(Loss Limit) If $\underline{f}_4^{CB} < f < \underline{f}_3^{CB}$ the central bank makes the maximum losses allowed Π_{CB}^{loss}. The loan size solves (1), the haircut is $h = \theta/q$ and the interest rate is $r = \frac{\Pi_{CB}^{loss}+q-(1-p)\theta v}{pq}$.

(Too encumbered to save) If $f < \underline{f}_4^{CB}$ the bank is unfunded ($q = 0$).

Proof. See appendix E. □

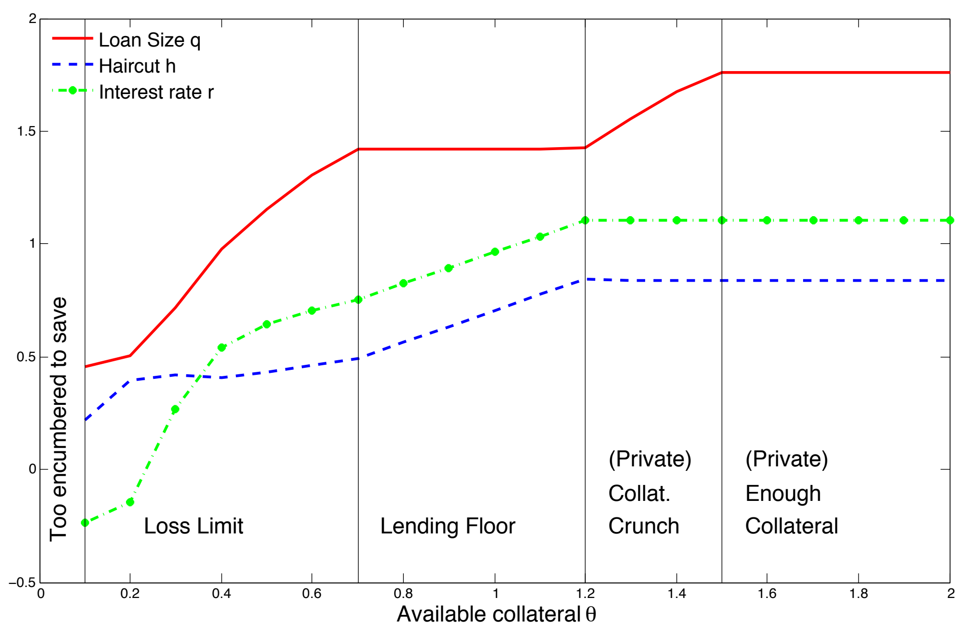

Figure 4: Central bank lending regimes

This figure plots the haircut, interest rate and loan size in the five regimes of optimal central bank lending as a function of the amount of available collateral. The five regimes are listed as the too encumbered to save, the loss limit, the lending floor, the collateral crunch and the enough collateral respectively. The parameters of the model are given by $A = 0.9$, $B = 0.45$, $v = 0.9$, $p = 0.7$, $\Pi_{CB}^{loss} = -0.25$, $\omega = 0.2$ and the production function is $R(q) = 2\sqrt{q}$.

Figure 4 illustrates the different regimes in the central bank only case. Contract terms in the enough collateral case and the collateral crunch are identical for the central bank and for the interbank market. In the collateral crunch case, borrowing is cut when the amount of available collateral θ drops and the central bank breaks even. The following proposition emphasizes the most interesting feature of the lending floor regime.

Proposition 8. *[Haircuts and Collateral Quality] When the amount of available collateral reaches an intermediate level, central bank liquidity provision does not depend on collateral quality ($\partial q/\partial v = 0$ if C_1 and $\underline{f}_3^{CB} < f < \underline{f}_2^{CB}$).*

When the collateral quality factor falls below $\underline{f}_2^{CB}$ and $\underline{f}_3^{CB} < f < \underline{f}_2^{CB}$, the cost to the economy in terms of lost output becomes larger than the welfare cost of central bank losses. The central bank then becomes ready to incur losses in the third regime, the so called lending floor. In this regime, the central bank keeps the lending level constant. As lending is constant and as the CC binds, haircuts decrease when the amount or the quality of the available collateral drops. This matches a key stylized fact: central banks accepted collateral of lower quality during the crisis. The Fed for instance significantly broadened the range of collateral eligible to obtain credit whereas it only buys and sells treasuries to selected counterparties in normal times. Figure 5 shows the total value of collateral pledged to the Fed in the Term Auction Facility (TAF), Primary Dealer Credit Facility (PDCF) and the Term Securities Lending Facility (TSLF) repo operations. In contrast with private markets (table 3), the use of treasuries was limited and banks mostly pledged agency-guaranteed MBS, ABS and corporate bonds. The European Central Bank (ECB) also broadened the range of collateral eligible for repos. It lowered its minimum quality threshold from A- to BBB- after the Lehman bankruptcy. In our model, this implies that assets with a net haircut of $h = 100\%$ subsequently became eligible, with finite h. Figure 6 shows that the collateral pool of the ECB became less liquid as the share of government bonds decreased while the share of ABS, uncovered bank bonds and non-marketable assets increased.

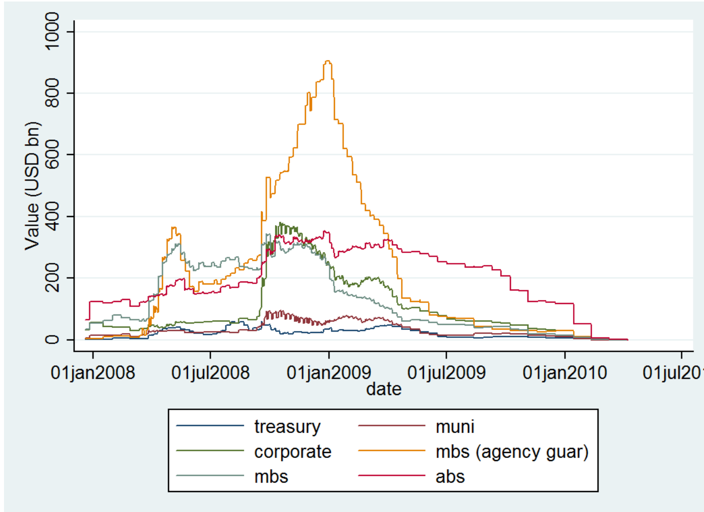

Figure 5: Value of collateral pledged to the Fed

This figure shows the total value of assets pledged to the Federal Reserve as collateral in its Term Auction Facility, Primary Dealer Credit Facility and Term Securities Lending Facility operations. The values are net values following subtractions of the haircut. The different asset types are defined as follows by the Fed: *Treasury*: Unsecured debt issued by the U.S. Department of the Treasury and government-sponsored enterprises. *Muni*: Securities issued by state and local governments and agencies. *Corporate*: Unsecured securities issued by private corporations. *MBS (agency guar)*: Mortgage-backed securities (MBS) and collateralized mortgage obligations (CMO) issued by government-sponsored enterprises. *MBS*: Mortgage-backed securities (MBS) and collateralized mortgage obligations (CMO) issued by private corporations. *ABS*: Securities collateralized by assets other than first-lien mortgages. Includes collateralized debt obligations (CDOs). The data are from the Federal Reserve.

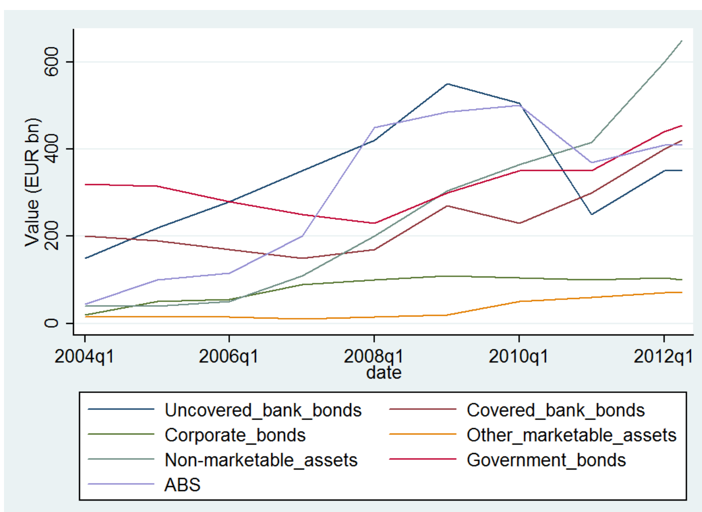

Figure 6: Value of collateral pledged to the ECB

This figure plots the total value of collateral pledged to the ECB by asset type. The values are net values after subtraction of the haircut (Source: ECB).

Finally, the following proposition highlights the refusal of some collateral types as main feature of the too encumbered to save regime.

Proposition 9. *[Too encumbered to save] The central bank refuses some types of collateral (If $f < \underline{f}_4^{CB}$, $q = 0$).*

The central bank stops providing liquidity to the bank if the amount or the quality of bank collateral falls too much, i.e. when $f < \underline{f}_4^{CB}$. In this case, the bank is too encumbered to save: the subsidy required to support bank investment generates losses sufficiently large to undermine the credibility of the central bank. This matches another stylized fact of central bank collateral policy: central banks refused some types of collateral. While sovereign bonds remained eligible throughout the Euro crisis (except for Greek government bonds, which were temporarily ineligible during the restructuring of Greek sovereign debt), several assets issued by private issuers became ineligible because of rating downgrades. For instance, the downgrade of Irish government bonds on 2 February by S&P from A- to BBB+ was followed by downgrades of Irish banks and assets. These downgrades brought the rating of these assets below the ECB minimum rating threshold. A comparison of the lists of eligible assets published by the ECB at the beginning of February 2011 and at the beginning of April 2011 suggests that 77 assets (roughly 10% of assets issued in Ireland and eligible to the ECB) were rendered ineligible during these two months.

5 Central Bank and Interbank Market

We now consider the case where both the central bank and the interbank market are a potential funding source for the bank as the collateralized lending scheme offered by the central bank is now voluntary rather than compulsory. The bank first chooses a funding source (the interbank market or the central bank as we assume exclusive loans) and then obtains funds as in section 3 and 4. If the bank chooses to go to the central bank, it must accept the conditions set forth by the central bank. If it goes on the private market, it must offer private investors on the interbank market a contract that yields them nonnegative profits, i.e. there is perfect competition among investors as in section 3.

The bank thus chooses the contract that yields the highest profit. This simplifies the problem and rules out competition between the central bank and investors.[14] The contracts offered by the central bank and the interbank market are the same as those in respectively sections 3 and 4, as recorded in the next lemma.

Lemma 2. *The equilibrium collateralized loan contracts offered by the interbank market and the central bank are the same as those offered when each lender is the only possible source of funding for the bank (propositions 3 and 7 respectively).*

If we assume that the value of the collateral for the central bank is marginally lower than the value for interbank market investors , i.e. $v_{cb} = v - \epsilon$ for $\epsilon \to 0$, we have:

[14] See Tirole (2012) for a model where the offer of the public investor interacts with the private market and Bernheim and Whinston (1986) for a seminal common agency model.

Proposition 10. *(Source, level and structure of bank funding)*

If $f \geq \underline{f}_2^{CB}$, the bank is financed by the interbank market through a loan contract identical to the contract offered in proposition 3.

If $f < \underline{f}_2^{CB}$, the bank is funded by the central bank and the equilibrium contract corresponds to the contract from proposition 7.

Proof. See appendix F. $\square$

Proposition 10 matches the stylized fact that the volume of collateralized loans provided by central banks spiked during the 2007-2013 financial crisis. Major central banks used repos or asset swaps (which are similar to repos) when the quality and quality of bank collateral fell.[15] Figures 7, 8 and 9 illustrate this point by using equity levels of banks as a proxy for available collateral θ. These figures plot the evolution of the average equity levels of the largest banks in the Eurozone, the UK and the US together with the total amount of collateralized lending by the ECB, the Bank of England and the Federal Reserve. In the three cases, low levels of bank equity coincide with high amounts of central bank repos.

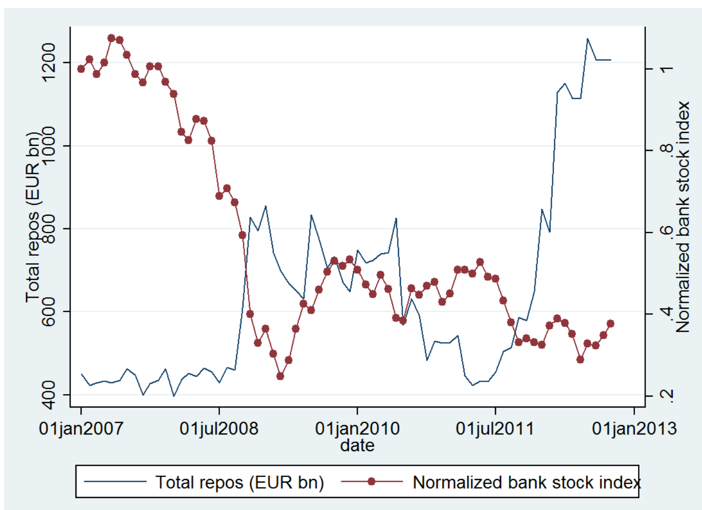

Figure 7: ECB repo outstanding and European banks' equity

This figure shows the total amount of repos allocated by the ECB through its main (1 week) and long-term (3 months or higher) refinancing operations and the Dow Jones European Financials Index. The index is normalized to 1 on January 2007. The repo data are from the ECB.

[15]In an asset swap, the borrower swaps illiquid securities against for instance treasuries and agrees to swap the securities back later on predetermined conditions.

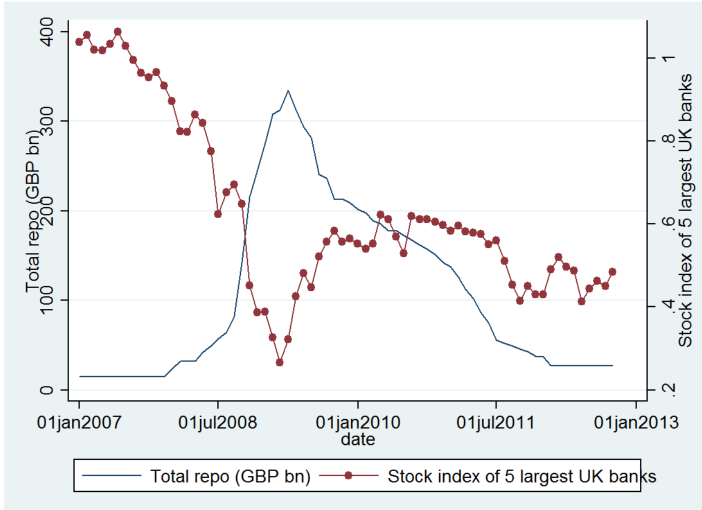

Figure 8: Bank of England repo outstanding and UK banks' equity

This figure shows the total amount of repos allocated by the Bank of England through its term repo operations and the special lending scheme against a non-weighted average of equity levels of U.K. banks. The banks included are HSBC, RBS, Lloyds, Barclays, Standard Chartered plc. The equity index level is normalized to 1 on January 2007. The repo data are from the Bank of England.

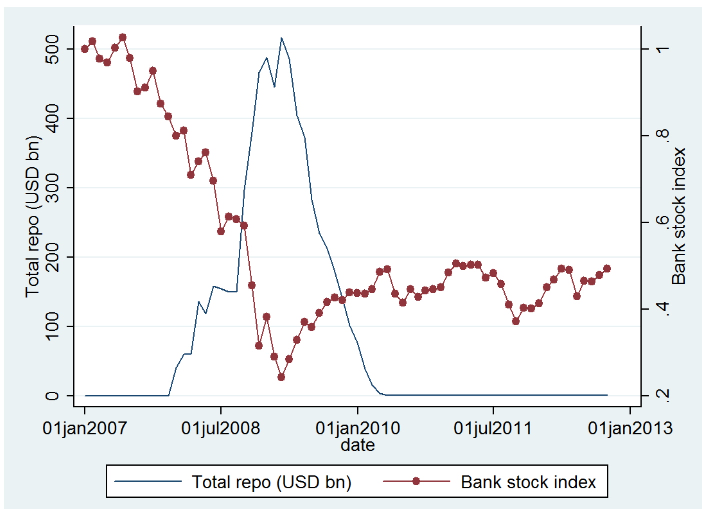

Figure 9: Fed repo outstanding and US banks' equity

This figure shows the total amount of repos allocated by the Federal Reserve through its Term Auction Facility, Primary Dealer Credit Facility and Term Securities Lending Facility operations. The bank stock index is the Dow Jones Financials index, normalized to 1 on January 2007. Data are from the Federal Reserve.

Revisiting optimal monetary policy. We now take a step back and interpret our results in the framework of optimal monetary policy. The traditional view of monetary policy is that the central bank influences short-term interest rates or the monetary base which in turn influences rates in the financial markets and ultimately the availability of credit for firms and households (Bernanke and Blinder, 1992). Several empirical contributions have shed light on the frictions in this transmission process. For example, Kashyap and Stein (2000) use quarterly data on US banks from 1976 to 1993 and show that the effect of monetary policy on lending is stronger for banks with less liquid balance sheets, measured by the ratio of securities to assets. Jimenez, Ongena, Peydro, and Saurina (2012) use loan-level data on credits in Spain and similarly conclude that higher short-term interest rates or lower GDP growth reduce the supply of credit and that this effect is stronger for banks with low capital.

Our paper provides a micro-foundation for the finding that the transmission of traditional monetary policy depends on the quality of bank collateral. Moreover, our paper suggests the use of collateral policy as an alternative to broken transmission mechanisms. The policy rate set by the central bank in its traditional policy can be interpreted in our model as the return required from the bank by the investors, which is normalized to one in our baseline model (break-even). The policy rate could correspond to the interest rate of the central bank in its deposit facility, where investors may lend to the central bank without risk. One might thus extend our model by assuming that the interbank market investors require a return equal to the gross policy rate r_p . Therefore, the bank borrows on the interbank market from investors at an interest rate equal to $\frac{1-(1-p)hv}{p}$ in our baseline model, which corresponds to $\frac{r_p-(1-p)hv}{p}$ in the extended model where we introduce the policy rate r_p .

At the other end of the spectrum, the key monetary policy variable for the central bank is the marginal return $pR'(q)$. This is the return required by the bank from its customers wishing to obtain a loan. When the bank borrows without collateral, the marginal return $pR'(q)$ is equal to r_p, e.g. there is a perfect transmission of monetary policy to the real economy. When the bank shifts to collateralized funding, a spread appears between the policy rate and the return that the bank requires from its customers. This spread is due to the cost of using collateral whose value is different for the bank than for investors. The spread increases as the quality of collateral falls, which typically occurs in downturns when assets are downgraded. We record this in the next proposition.

Proposition 11. *For a given policy rate r_p, negative shocks to the amount or to the quality of bank collateral may reduce the amount of credit in the economy and increase interest rates in the real economy, e.g. the required return by the bank from its customers ($\partial\left(pR'(q)\right)/\partial\theta \leq 0$, $\partial\left(pR'(q)\right)/\partial v \leq 0$).*

We show that if banks have too little collateral available, they may suffer from a collateral crunch. This equilibrium is particularly damaging from a welfare point of view, as the amount of credit allocated to the economy depends on the amount of available bank collateral, and is more disconnected from investment opportunities in the real economy. By relaxing its collateral policy, the central bank can reduce the spread between the policy rate and the funding cost in the real economy. At the zero lower bound ($r_p = 1$), reducing the spread becomes the only option available to central bankers wishing to bring down interest rates in the real economy. As an alternative to the broken transmission of traditional monetary policy, collateral policy in our model can reduce this spread and increase welfare at the cost of imposing losses on the central bank.

6 Liquidity Coverage Ratios

Our paper demonstrates the potential of collateral policy relaxation to increase welfare by reducing the spread between the policy rate and returns in the real economy. However, we abstract from the important incentive issues triggered by collateral policy loosening as both the amount of available collateral θ and whose quality v are treated as exogenous. In reality, lenient central bank collateral policy might lead to lower equilibrium values of collateral, as banks do not internalize the full consequences of disposing of low quality collateral as they would face in the absence of central bank intervention.

To endogenize for instance the quantity of collateral, one could imagine that banks choose their level of due diligence effort e - influencing positively the amount of available collateral θ.[16] The chosen due diligence effort e would balance the expected cost of and return on effort given by an increase in the expected value of collateral. An additional unit of collateral increases wealth mechanically and creates additional value in the collateral crunch case, by relaxing the CC constraint. Therefore, banks have certain incentives to hold liquid collateral.[17] Nevertheless, lenient central bank collateral policy dilutes the return on due diligence effort, which may lead to lower equilibrium values of collateral. Lower collateral values may then increase the expected subsidy and welfare costs of central bank intervention.

How to tackle this moral hazard problem associated with lenient collateral policy in periods when the quality and/or quantity of bank collateral are low? Our model suggests that tightening collateral policy in periods when the quality and/or quality of bank collateral are high is not a panacea, as banks switch to private market funding precisely at these moments. However, "Liquidity Coverage Ratio" (LCR) rules that require banks to hold an adequate stock of available High-Quality Liquid Assets (HQLA) can complement flexible collateral policy more effectively, as they mitigate the associated moral hazard problem.

The introduction of LCR rules would have both costs and benefits. The benefit is that banks would hold more collateral to avoid penalties for non-LCR compliance, when collateral values are endogenous. In turn, higher quantities and qualities of bank collateral would reduce the need to appeal to the balance sheet capacity of central banks and reduce central bank losses as well as the associated welfare costs. On the cost side, LCR rules could immobilize assets that would otherwise be used more efficiently. For a bank with available collateral

[16] In a simple binary set-up the amount of available collateral could be high, θ_H, or low, θ_L while the return on effort would be characterized by $P(\theta_H|e) = e$ and $P(\theta_L|e) = 1 - e$. This effort could then cost the bank $\psi(e)$ where $\psi'(e) > 0$ and $\psi''(e) > 0$. The amount of collateral θ would then depend on due diligence effort and on (idiosyncratic or macro-economic) shocks.

[17] The repo market does not only affect the incentives to hold old, unencumbered assets but also the incentives to invest in the new project. In our model, banks have an incentive to invest in relatively safe, new investment assets- which have a low probability of default $1 - p$ - as the repo cost of the liquidity wedge is proportionate to $(1 - p)(1 - v)$.

θ, that faces a liquidity coverage requirement of K, the "new" collateral available to obtain short-term funds, would be $\theta - K$. LCR rules would force banks to keep collateral on their balance sheet. Since these assets cannot be used for funding purposes, they are equivalent to encumbered assets. The rest of the model would hold as such, with $\theta - K$ being the new level of available collateral.

The LCR requirements may prove particularly useful in normal times, in the uncollateralized lending equilibrium. In this case, LCR rules might counterbalance the temptation of some commercial banks to save on due diligence effort and to reduce their amount of high quality collateral, as it does not affect their access to funding. However, changes in the environment may create a swing to collateralized lending, with a potential credit crunch for banks that have too little or low quality collateral. Our model shows that in time of financial distress, the LCR requirements may potentially worsen a credit crunch. In this case, it may be desirable to relax the requirements and use the asset buffer.

The idea that the LCR requirements $K(s)$ should depend on the state of the economy s is included in the January 2013 regulations proposed by the Basel III Committee. The LCR rules prescribe the availability of a stock of HQLA assets that can be converted into cash easily and immediately in private markets to meet its liquidity needs for a 30 calendar day liquidity stress scenario (Noked, 2013). The requirements will start at 60% in 2015 and then gradually increase by 10% every year to reach 100% in 2019. HQLA include government bonds and corporate debt rated above BBB-. The haircuts vary across assets. For example, corporate debt rated A+ to BBB- or equity will have a haircut of 50%. To conclude, the combination of relaxing collateral policy in downturns and imposing more stringent LCR rules in booms has the potential to maintain investment levels in downturns while providing banks with proper incentives for collateral and liquidity management.

7 Conclusion

During the 2007-2013 financial crisis, many central banks loosened their lending policies by lending more to commercial banks, against lower quality collateral, than in more normal times. This paper characterizes efficient collateral policies of central banks. In our model, a bank borrows from the interbank market or the central bank to fund projects in the economy. Providing collateral has favorable incentive effects, but it is costly to transfer collateral to lenders who have a lower value for the collateral because of imperfect collateral quality.

When the bank borrows from the interbank market, we find that it uses no collateral when return prospects in the economy are high relative to moral hazard concerns. The equilibrium however shifts to collateralized borrowing in downturns. In this case, both the quality and the quantity of bank collateral determine the loan size. We show that the lack of collateral or a fall in the quality of bank collateral can trigger a credit crunch and increase the spread between the policy rate and interest rates in the real economy. It may then be optimal for the central bank to relax its collateral policy and lend more for a given quantity of collateral in order to lower interest rates in the economy closer to the policy rate.

The policy of low collateral requirements in turbulent times may lower incentives of banks to hold enough high quality collateral. Collateral policy is ineffective in good times to influence bank borrowing as interbank collateral requirements are low and banks borrow from the private markets. Our model suggests that introducing minimum collateral availability requirements may help attenuate this "soft collateral budget constraint". The concept of minimum collateral availability requirements is part of the Basel III reform which includes Liquidity Coverage Ratio (LCR) rules coming into force in the coming years.

Our study suggests several avenues for future research. First, one may build on our stylized facts to empirically identify the role of collateral in the transmission of monetary policy. One could also extend the model to formalize the dynamic moral hazard problem that we sketched, created by the central bank policy of low collateral requirements in downturns, to pin down optimal collateral availability requirements for banks over the business cycle.

References

Bagehot, W. (1873): *Lombard Street: a Description of the Money Market*. London: Henry S. King and Co.

Benmelech, E., and N. K. Bergman (2011): "Bankruptcy and the Collateral Channel," *The Journal of Finance*, 66(2), 337–378.

Bernanke, B. S., and A. S. Blinder (1992): "The Federal Funds Rate and the Channels of Monetary Transmission," *The American Economic Review*, 82(4), 901–921.

Bernheim, B. D., and M. D. Whinston (1986): "Common Agency," *Econometrica*, 54(4), 923–942.

Buiter, W. H. (2008): "Can Central Banks Go Broke?," Discussion Paper 24, CEPR Discussion Paper.

Cassola, N., A. Hortaçsu, and J. Kastl (2013): "The 2007 Subprime Market Crisis Through the Lens of European Central Bank Auctions for Short-Term Funds," *Econometrica*, 81(4), 1309–1345.

Chaney, T., D. Sraer, and D. Thesmar (2012): "The Collateral Channel: How Real Estate Shocks Affect Corporate Investment," *The American Economic Review*, 102(6), 2381–2409.

Clouse, J., D. Henderson, A. Orphanides, D. Small, and P. Tinsley (2003): "Monetary policy when the nominal short-term interest rate is zero," *The B.E. Journal of Macroeconomics*, 3(1), 1–12.

De Grauwe, P. (2012): *The Economics of Monetary Union*. Oxford University Press.

Friedman, M., and A. J. Schwartz (2008): *A monetary history of the United States, 1867-1960*, no. 12. Princeton University Press.

Gale, D., and M. Hellwig (1985): "Incentive-Compatible Debt Contracts: The One-Period Problem," *The Review of Economic Studies*, 52(4), 647–663.

Geanakoplos, J. (2010): "The Leverage Cycle," *NBER Macroeconomics Annual*, 24(1), 1–66.

Goodfriend, M., and R. King (1997): "The New Neoclassical Synthesis and the Role of Monetary Policy," Discussion paper.

Gorton, G., and A. Metrick (2012): "Securitized Banking and the Run on Repo," *Journal of Financial Economics*, 104(3), 425–451.

Holmstrom, B., and J. Tirole (1997): "Financial intermediation, loanable funds, and the real sector," *The Quarterly Journal of Economics*, 112(3), 663–691.

——— (2011): *Inside and Outside Liquidity*. MIT Press.

Jimenez, G., S. Ongena, J.-L. Peydro, and J. Saurina (2012): "Credit Supply and Monetary Policy: Identifying the Bank Balance-Sheet Channel with Loan Applications," *American Economic Review*, 102(5), 2301–2326.

Judd, J. P., and G. D. Rudebusch (1998): "Taylor's Rule and the Fed: 1970-1997," *Economic Review-Federal Reserve Bank of San Francisco*, (3), 3–16.

Kashyap, A. K., and J. C. Stein (2000): "What Do a Million Observations on Banks Say about the Transmission of Monetary Policy?," *American Economic Review*, 90(3), 407–428.

Khwaja, A. I., and A. Mian (2008): "Tracing the impact of bank liquidity shocks: Evidence from an emerging market," *The American Economic Review*, 98(4), 1413–1442.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *The Journal of Political Economy*, 105(2), 211–248.

Klemperer, P. (2010): "The Product-Mix Auction: A New Auction Design for Differentiated Goods," *Journal of the European Economic Association*, 8(2-3), 526–536.

Krugman, P. R. (1998): "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, 3(2), 137–205.

Landier, A., D. Sraer, and D. Thesmar (2011): "The risk-Shifting Hypothesis : Evidence from Subprime Originations," *American Finance Association 2012 Chicago Meetings Paper*, (11-279), 48.

Noked, N. (2013): "Basel Committee Revises Basel III Liquidity Coverage Ratio," Discussion paper, Harvard Law School.

Peek, J., and E. S. Rosengren (1997): "The International Transmission of Financial Shocks: The Case of Japan," *The American Economic Review*, 87(4), 495–505.

Sargent, T. J., and N. Wallace (1981): "Some unpleasant monetarist arithmetic," *Federal Reserve Bank of Minneapolis Quarterly Review*, 5(3), 1 – 17.

Shleifer, A., and R. W. Vishny (1992): "Liquidation Values and Debt Capacity: A Market Equilibrium Approach," *The Journal of Finance*, 47(4), 1343–1366.

Simsek, A. (2013): "Belief Disagreements and Collateral Constraints," *Econometrica*, 81(1), 1–53.

STEIN, J. C. (2013): "Overheating in Credit Markets: Origins, Measurement, and Policy Responses," *Speech at the "Restoring Household Financial Stability after the Great Recession: Why Household Balance Sheets Matter" research symposium*.

STELLA, P. (1997): "Do Central Banks Need Capital?," *Working Paper of the International Monetary Fund*, 97(83).

——— (2005): "Central bank financial strength, transparency, and policy credibility," *Working Paper of the International Monetary Fund*, 52(2), 335–365.

TIROLE, J. (2012): "Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning," *American Economic Review*, 102(1), 29–59.

WEYMULLER, C.-H. (2013): "Haircuts and Relationships," Discussion paper.

A Institutional appendix: Collateral frameworks at the ECB, the BoE and the Fed

This section provides a brief introduction to the European Central Bank's, the Bank of England's and the Federal Reserve's liquidity provision from 2002 to 2013. From 2002 to 2007, the main objective of central banks' liquidity provision was to influence the price of short term loans in money markets by changing the supply of liquidity. Although the goal remained unchanged from 2007 to 2013, Central Banks had to significantly adapt their operational frameworks because of the financial turmoil.

A.1 European Central Bank

The ECB used two types of repo operations from 2002 to 2007. The main refinancing operations (MRO) have a maturity of one week and are sold through weekly auctions.[18] The long term refinancing operations (LTRO) have a maturity of three months and are also sold through weekly auctions. The total liquidity allocated by the ECB fluctuated around EUR 500 billion depending on the market liquidity needs. Approximately 2/3 of this amount was allocated through the MROs.

The range of collateral accepted for these operations has always been relatively broad, partly due to the concatenation of national central banks' collateral frameworks at the time of introduction of the euro. The general rules on collateral eligibility are set by the ECB's Governing Council. National Central Banks then implement these rules on a daily basis. Eligible assets include several type of bonds (corporate bonds, government bonds, uncovered bank bonds, covered bonds). Before 2008, these bonds had to have a credit-rating above or equal to A. This rating must be attributed by recognized Eurosystem credit assessment institutions. Assets without rating may also benefit from an internal rating from the National Central Bank. Collateral is marked to market on a daily basis and non-marketable assets are priced and rated by National Central Banks.

The repo operations remained the ECB's main tool tool during the financial crisis from 2007 to 2013. The repo operations where however modified in several key dimensions. First, the ECB started to sell repos with maturities of six months, one year and up to three years. The ECB also allocated repos in foreign currencies: US dollar, Swiss Frank, British pounds and Japanese yen. These operations where mostly wound down by the end of 2009, although the dollar operations restarted in May 2010. Third, the ECB dropped its auction allocation procedure and adopted a full allotment procedure whereby all banks could take up as much liquidity as needed at a fixed interest rate. Last but not least, the ECB broadened the criteria for collateral eligibility. These changes are summarized in table 1.

[18] The auction format is a discriminatory auction. See Cassola, Hortaçsu, and Kastl (2013) for an empirical analysis of these auctions.

A.2 Federal Reserve

From 2002 to 2007 the Fed used open market operations to control the liquidity available in money markets. By buying or selling high-quality assets (Treasuries), the Federal Reserve provides or withdraws liquidity from the system in order to keep the Fed funds rate in line with the policy rate. These operations are restricted to a limited number of counterparties: the primary dealers.

Repurchase agreements or similar contracts like asset swaps were at the heart of the Fed's most important facilities during the crisis. These facilities include the Term Auction Facility (TAF) which allocated loans with a maturity of one week to three months. The TAF amounts allocated topped USD 500 billion around May 2009. Other repo facilities included the Term Securities Lending Facility (TSLF), whereby the Fed loaned treasuries to primary dealers against less liquid collateral. Finally, the Primary Dealer Credit Facility (PCDF) is similar to the TAF: it allocated repos to primary dealers. The PCDF amount lent reached USD 150 billion in October 2008.

The range of collateral accepted at these operations is broader than the type of securities accepted from 2002 to 2007. As shown in section 4, ABS were the most widely used collateral type.

A.3 Bank of England

The Bank of England also uses repos to implement its monetary policy. In 2006 the BOE changed its reserve requirements system so that banks would only have to meet their requirements over a medium period, instead of having to meet the requirement every day. The bank offered reserves for a maturity of 3, 6, 9 and 12 months. The total stock of long-term operations was around GBP 15 billion.

As a response to the financial crisis, the Bank also extended its collateral framework in the Fall of 2008 to include CMBS and corporate debt as well as RMBS and covered bonds. The Bank increased the size of its operations during the crisis to GBP 180 billion in November 2008, up from around GBP 20 billion a year before.

In order to avoid moral hazard, the Bank charged a higher interest rates on repos using "extended" collateral. The spread with the rates on the repos with high-quality collateral was fixed. In February 2010, the bank introduced the product mix auction. Described by Klemperer (2010), this auction allocates loans with "narrow" and "broad" collateral at different interest rates. These interests rates are set during the auction and vary with the banks' bids and the BOE's allocation policy.

B Equilibrium private lending (proof of propositions 1, 2, 3 and 4)

We first prove that when $pR(q^{ec}) - q^{ec} > A + Bq^{ec}$, uncollateralized lending is optimal (proposition 1). We then characterize uncollateralized lending (proposition 2) and collateralized lending (proposition 3), when $pR(q^{ec}) - q^{ec} < A + Bq^{ec}$.

Uncollateralized The bank maximizes profits

$$\max_{r,h,q} \pi = p\left[R(q) - rq\right] - (1-p)\, hq \tag{2}$$

such that it is incentivized not to shirk, it has enough collateral and investors participate in the collateralized lending contract,

$$\begin{aligned} pR(q) - prq + phq &\geq A + Bq \\ hq &\leq \theta \\ prq + (1-p)\, hqv - q &= 0 \end{aligned}$$

Suppose $R(q^*) - rq^* > \frac{A+Bq^*}{p}$. For $h \geq 0$, the condition $R(q^*) - rq^* > \frac{A+Bq^*}{p}$ ensures that the bank's IC is satisfied at q^*. We may also ignore the CC since the IC is satisfied even when $h = 0$, so there is always enough collateral to satisfy the IC. The problem then reduces to maximizing (2) subject to the market clearing condition. This is exactly the first-best problem.

Suppose now that $pR(q^{ec}) - q^{ec} > A + Bq^{ec}$ and $R(q^*) - rq^* > \frac{A+Bq^*}{p}$. The solution to the bank's problem in the "enough collateral" regime would yield a negative haircut. To see this one must refer to the solution to the general problem in the next subsection. Equation (5) characterizes the optimal haircut when the bank has enough collateral. It is indeed negative because:

$$h = \frac{A + (1+B)\, q^{ec} - pR(q^{ec})}{q^{ec}\left[(1-p)\, v + p\right]} < 0 \tag{3}$$

The haircut must be nonnegative because the investors have no legacy assets to give to the bank. However, the first-best level with zero haircut would violate the IC constraint since $pR(q^*) - prq^* > A + Bq^*$. The optimal contract therefore specifies the highest loan size such that the haircut is zero, so the loan size is given by the binding IC and solves $A + (1+B)\, q - pR(q) = 0$. The interest rate remains such that the investor breaks even, $r = 1/p$.

Collateralized The proof of the collateralized contract uses the following result:

Lemma 3. *If $pR(q^*) - prq^* < A + Bq^*$, the bank's IC must bind.*

Proof. If $pR(q^*) - prq^* < A + Bq^*$, the IC is violated with an uncollateralized loan $h = 0$ if $q = q^*$. Suppose that the optimal contract, indexed 0, has $h > 0$ and the IC is slack. Reduce the value of collateral pledged hq by ϵ and increase the interest payment rq by $\epsilon v\frac{(1-p)}{p}$. This new contract ensures that the market clearing condition (investors' IR) is respected and offers the bank a higher profit π_1:

$$\begin{aligned}\pi_1 &= p\left[R(q) - rq - \epsilon v\frac{(1-p)}{p}\right] - (1-p)[hq - \epsilon] \\ &= \pi_0 + \epsilon(1-v)(1-p) > 0\end{aligned}$$

so by contradiction the initial contract cannot be optimal. □

The proof of proposition 3 is:

Proof. **Enough Collateral:** Consider the case $f \geq \underline{f}_1$ (where $f = \theta p + \theta(1-p)v$). Suppose, then verify, that the CC is slack. By lemma 3, the bank's IC binds. The problem is:

$$\max_{r,h,q} \pi = p[R(q) - rq] - (1-p)hq$$

such that:

$$pR(q) - prq + phq = A + Bq, \tag{4}$$

$$prq + (1-p)hqv - q = 0.$$

The two constraints give the following expression for the total collateral pledged:

$$hq = \frac{A + (1+B)q - pR(q)}{(1-p)v + p}, \tag{5}$$

Plugging this in the IC (4):

$$prq = pR(q) + \frac{p}{(1-p)v + p}[A + (1+B)q - pR(q)] - A - Bq, \tag{6}$$

Substitute prq and hq in the maximization problem:

$$\max_q \pi = pR(q) - \left[pR(q) + \frac{p}{(1-p)v + p}[A + (1+B)q - pR(q)] - A - Bq\right] ...$$

$$... - (1-p)\frac{A + (1+B)q - pR(q)}{(1-p)v + p}$$

The FOC yields:

$$-(1-p)(1-v)B + pR'(q) - 1 = 0$$

so:

$$pR'(q) = 1 + (1-p)(1-v)B.$$

so the loan size is below first best, and this effect is magnified with low p and low v. The interest rate is given by:

$$prq = pR(q) + \frac{p}{(1-p)v+p}\left[A + (1+B)q - pR(q)\right] - A - Bq,$$

this simplifies to:

$$prq = \frac{(1-p)vpR(q) - (A+Bq)(1-p)v_\theta + pq}{(1-p)v+p}$$

Again:

$$prq = \frac{(1-p)v(pR(q) - A - Bq) + pq}{(1-p)v_\theta + p}$$

and the haircut is given by (5).

Finally we verify that the CC is slack because

$$\theta > \underline{\theta}_1 = \frac{A + (1+B)q - pR(q)}{(1-p)v+p}$$

since $f > \underline{f}_1$.

Collateral crunch

In the collateral crunch case, the CC binds because the available collateral is insufficient to ensure that the bank does not shirk with a loan level as in the "enough collateral" regime.

The bank maximizes profits

$$\max_{r,h,q} \pi = p\left[R(q) - rq\right] - (1-p)hq$$

such that

$$R(q) - rq + hq = \frac{A+Bq}{p}, \tag{7}$$

$$hq = \theta \tag{8}$$

$$prq + (1-p)hqv - q = 0. \tag{9}$$

The equilibrium is determined by the constraints. Plug the CC (8) in the IC (7) and market clearing (9) and sum the 2 constraints:

$$pR(q) + p\theta + (1-p)\theta v - q = A + Bq.$$

The loan level q solves this equation. The haircut is $h = \theta/q$ and the interest rate is determined by

$$r = \frac{q - (1-p)\theta v}{pq}.$$

Dry-up

A liquidity dry-up occurs when the profits generated by the project and the available collateral become insufficient to protect the investor from moral hazard for any given loan level.

The loan level q in the collateral crunch solves

$$pR(q) + f - q = A + Bq. \tag{10}$$

A dry-up may occur in two cases: (1) the loan size q that solves equation (10) is negative and (2) the equation does not have a real solution in q. The first case can be ruled out since we know that when $q = 0$ the function is positive. To see this, rewrite (10) as

$$G(q) + K = 0. \tag{11}$$

where $G(q) = pR(q) - (B+1)q$ and $K = f - A$.[19] The function $G(\cdot)$ crosses the $y = 0$ axis twice, at $q = 0$ and at $q > 0$. The relevant solution for our case is the right-hand solution, i.e. the highest q that solves this equation. Let $\hat{q} = \arg\max(G(q))$. We know that $\hat{q} > 0$ because $G(\cdot)$ is concave. In equation (11) we add a constant K to the function $G(q)$. For all K, the solution to (11) cannot be lower than $\hat{q}$, which is positive. Hence we may rule out case 1 where $q < 0$.

However for low values of θ (case 2), this equation may not have a solution. In fact when

$$\arg\max_{q}\{pR(q) + f - q - A - Bq\} = 0 \tag{12}$$

for any value of f lower than the $\underline{f}_2$ that solves equation(12), there are no solution to the investment equation (10). $\square$

C Comparative statics of haircuts in private markets (proof of proposition 5)

This section formalizes the comparative statics of the haircut h in the collateral crunch case[20] with respect to the asset's quality v.

Proof of comparative statics of v

Proof. We need to show that the haircut decreases in v in the collateral crunch regime. Remember that in this regime the haircut is set as $h = \theta/q$ and q solves

[19] Note that $G(\cdot)$ is a typical concave function, which crosses $(0,0)$, has $\lim_{q\to\infty} G = -\infty$ and has $G'(0) = \infty$.

[20] A proof of the comparative statics for the "enough collateral" regime is available from the authors upon request.

$$pR(q) + (p + (1-p)v)\theta - q = A + Bq, \tag{13}$$

so q unambiguously increases in v. To see this, note that (13) may be rewritten as

$$G(q) + K = 0. \tag{14}$$

where $G(q) = pR(q) - (B+1)q$ and $K = (p + (1-p)v)\theta - A$.[21] We are interested in the highest q that solves this equation. Since it increases when K increases and since K increases in v, then the loan size increases in v. This implies that the haircut $h = \theta/q$ decreases in v. □

D Perfect quality and irrelevance of haircuts

This section shows that multiple equilibria arise when collateral is perfectly liquid, $v = 1$.

Proposition 12. *Suppose that contract* (q_0, r_0, h_0) *is an "enough collateral" equilibrium for* $v < 1$, *and such that* $v = 1 - \epsilon$ *where* $\epsilon \to 0$. *Let* $\alpha = 1 - h_0(1-p)$. *Then any contract* $\left(q_0, \frac{\gamma}{p}, \frac{(1-\gamma)}{(1-p)}\right)$ *where* $\gamma \in \left[1 - \frac{(1-p)\theta}{q_0}, \alpha\right]$ *is also an equilibrium when* $v = 1$.

Proof. In the "enough collateral" regime, the CC is slack. Hence collateral requirements can be increased and interest rates reduced -while keeping the IR binding- until the CC binds. The CC is slack as long as $\frac{(1-\gamma)}{(1-p)}q_0 \leq \theta$. This condition is respected if $1 - \frac{(1-p)\theta}{q_0} < \gamma$. Lower haircuts but higher interest rate payments than in the initial contract are not possible as the IC would then be violated. This is why $\gamma < \alpha$. □

E Optimal central bank lending

There are two cases to consider for the central bank's optimal lending contract. The first one is when the central bank's losses have a large weight in its objective function relative to output: $\omega > \underline{\omega} = \frac{(1-p)(1-v)}{v(1-p)+p}$. This is the case that we consider in the paper and the proof is in subsection E.1. The second case (when $\omega < \underline{\omega} = \frac{(1-p)(1-v)}{v(1-p)+p}$) is developed in subsection E.2. In this case the central bank's focus on output relative to losses is so important that the central bank always makes the maximum losses in order to support output.

[21] $G(\cdot)$ is a typical concave function, which crosses $(0,0)$, has $\lim_{q\to\infty} G = -\infty$ and has $G'(0) = \infty$.

E.1 Optimal central bank lending with large weight on losses (proof of proposition 7)

Proof. The proof of the central bank's optimal repo contract relies on two lemmas. Lemma 4 shows that for a given loan size, the central bank prefers to make zero profits instead of making a loss even if this entails a higher collateral requirement and thus a higher cost of liquidity wedge. We use collateral amount thresholds $\underline{\theta}_1, \underline{\theta}_2, \ldots$ which can be translated into quality factor thresholds $\underline{f}_1, \underline{f}_2, \ldots$. □

Lemma 4. *If the central bank can capture bank surplus to avoid a loss by increasing haircuts, it should do it.*

Proof. Suppose that the CC is such that a break-even contract is attainable without violating the CC. Suppose that the optimal contract is (q, r, h) such that the central bank makes losses, i.e. $\Pi_{CB}^{loss} < \Pi_{CB} < 0$. The IR is slack (else total output would be negative)

Consider a contract $(r + \epsilon, q, h + \epsilon)$. The constraints: (i) the IC is still satisfied; (ii) the IR tightens but if the central bank still makes losses, it is still slack; (iii) the CC is still satisfied (by assumption); (iv) the CB's profit is higher (but still negative). The impact on welfare is:

$$\Delta W_{CB} = q\epsilon \left[\omega \left[p + (1-p)\, v\right] - (1-p)\,(1-v)\right].$$

This is positive since $\omega > \underline{\omega} = \frac{(1-p)(1-v)}{v(1-p)+p}$, so the initial contract wasn't optimal. It is therefore optimal for the central bank to increase the haircut and the interest rate until $\Pi_{CB} = 0$. □

General proof. The proof proceeds in 5 steps. First, we show that if profit is negative and CC is slack (i.e. $\theta \geq \underline{\theta}_1^{CB}$), then the central bank always increases the haircut and the interest rate to the point where it makes zero profit. Second, if $\underline{\theta}_2^{CB} \leq \theta \leq \underline{\theta}_1^{CB}$, then it is optimal to keep the CC binding and reduce loan size. Third, if $\underline{\theta}_3^{CB} \leq \theta \leq \underline{\theta}_2^{CB}$, then the central bank keeps the CC and IC binding but maintains a constant loan size and starts making losses - until it hits the maximum loss constraint Π_{CB}^{loss}. Fourth, we show how for $\underline{\theta}_4^{CB} \leq \theta \leq \underline{\theta}_3^{CB}$ the central bank reduces the loan size while keeping losses constant at π_{CB}^{loss}. Fifth, when $\theta \leq \underline{\theta}_4^{CB}$ the central bank stops lending altogether: the bank is "too encumbered to save".

Part 1: Central bank's enough collateral case

Proof. The problem with positive profits is:

$$\max_{r,q,h} W_{cb} = pR(q) - q - (1-p)\, qh\,(1-v),$$

s.t.

$$R(q) - rq + hq \geq \frac{A + Bq}{p}$$

$$p\left[R(q) - rq\right] - (1-p)\, hq \geq 0$$

$$hq \leq \theta$$

$$\Pi_{CB} \geq 0$$

All else equal, welfare W_{cb} decreases in haircut so the central bank chooses the lowest haircuts given the constraints. Interest rates are similarly lowered to keep the bank incentivized, and they are lowered until the central bank loss constraint $\pi_{CB} \geq 0$ binds. This implies that the IC binds. Indeed, suppose that the IC is slack, then you can increase r, lower h and increase the objective function.

The proof of this part is then similar to that of proposition 13, with $\Pi_{CB}^{loss} = 0$.

Part 2: Collateral crunch regime

The central bank solves

$$\max_{r,q,h} W_{cb} = pR(q) - q - (1-p)\, qh\,(1-v)\,,$$

such that

$$R(q) - rq + hq = \frac{A + Bq}{p}$$

$$prq + (1-p)\, hqv - q = 0$$

$$hq = \theta$$

The constraints determine the equilibrium: see proof of proposition 13, with $\Pi_{CB}^{loss} = 0$. The optimal loan size solves

$$pR(q) + p\theta + (1-p)\,\theta v - q - A - Bq = 0. \tag{15}$$

The haircut is determined by $hq = \theta$ and the interest rate is determined by $prq = q - (1-p)\,\theta v$.

Part 3: Lending floor regime

The central bank now makes negative profits because $\theta \leq \underline{\theta}_2^{CB}$ so equation (15) does not have a real solution. The problem is:

$$\max_{r,q,h} W_{cb} = pR(q) - q\,[1+\omega] - (1-p)\, qh\,(1 - v\,(1+\omega)) + \omega prq,$$

such that

$$pR(q) - prq + phq = A + Bq$$

$$hq = \theta$$

Plug CC and the IC constraint in the objective function:

$$\max_{q} W_{cb} = pR(q) - q\,[1+\omega] - (1-p)\,\theta\,(1 - v\,(1+\omega)) \ldots$$

$$\ldots + \omega\,(pR(q) + p\theta - A - Bq)\,,$$

The FOC gives:

$$R'(q) = \frac{1+\omega(1+B)}{p(1+\omega)} \tag{16}$$

The haircut is given by $hq = \theta$ and the interest rate by $prq = pR(q) + phq - A - Bq$.

Lastly, the threshold $\underline{\theta}_3$ is the pledgeable asset level θ at which the central bank's maximum loss constraint binds:

$$prq + (1-p)\theta v - q = \Pi_{CB}^{loss}$$

where q is given by (16), i.e.

$$\underline{\theta}_3 = \frac{\Pi_{CB}^{loss} + q - prq}{(1-p)v_\theta}.$$

Part 4: Loss limit regime

The problem is:

$$\max_{r,q,h} W_{cb} = pR(q) - q[1+\omega(1-p)+\omega] - (1-p)qh(1-v(1+\omega)) + \omega prq,$$

s.t.

$$pR(q) - pRq + phq = A + Bq$$

$$prq + (1-p)hqv - q = \Pi_{CB}^{loss}$$

$$hq = \theta$$

The three constraints in 3 unknowns determine the equilibrium. A few computations yield (see proof of proposition 13, with $\pi_{CB}^{loss} = 0$ for more details) the following equation for the loan size:

$$pR(q) + p\theta + (1-p)\theta v - q - A - Bq - \Pi_{CB}^{loss} = 0. \tag{17}$$

The haircut is determined by $hq = \theta$ and the interest rate is determined by

$$prq = q + \Pi_{CB}^{loss} - (1-p)\theta v.$$

Part 5 - Too encumbered to save

If (17) does not have a real and nonnegative a solution, the bank is unfunded. Thus $\underline{\theta}_4^{CB}$ is the lowest θ for which (17) has a real and nonnegative solution. □

E.2 Optimal central bank lending with low weight on losses

The case where the central bank is focused on output is driven by the central bank's desire to avoid costs of inefficient collateral transfer. The central bank provides haircut subsidies to the bank and makes maximum losses Π_{CB}^{loss}. When the bank has few collateral available, i.e. θ is below $\underline{\theta}_L$, the CC is binding: central bank subsidies are insufficient to avoid shirking and the central bank thus reduces lending to keep the bank incentivized.

If the central bank is focused on output ($\omega < \underline{\omega} = \frac{(1-p)(1-v)}{v(1-p)+p}$), we have:

Proposition 13. *If the central bank is focused on output relative to losses (i.e. $\omega < \underline{\omega} = \frac{(1-p)(1-v)}{v(1-p)+p}$), then the policy depends on the amount of collateral available:*

(Minimal inefficient collateral transfer) If $\theta > \underline{\theta}_L$, then the level of bank borrowing solves

$$R'(q) = \frac{(1+\omega)\left((1-p)\,v + p\right) + (B+1)\,(1-p)\,(1-v)}{p}$$

The haircut and interest rates are given by $h = \frac{\Pi_{CB}^{loss} + A + Bq - pR(q) + q}{q[(1-p)v+p]}$ and $prq = \Pi_{CB}^{loss} - (1-p)\,hqv + q$. The IC binds, the CC and IR are slack and the central bank's profit is $\pi_{CB}^{loss} < 0$. The collateral quality threshold $\underline{\theta}_L$ is given by:

$$\underline{\theta}_L = \frac{A + Bq + q + \Pi_{CB}^{loss} - pR(q)}{p(1-v) + v}.$$

(Collateral crunch) If $\theta < \underline{\theta}_L$, the level of borrowing q solves

$$pR(q) + (1-p)\,\theta v + p\theta - A - Bq - \pi_{CB}^{loss} - q = 0,$$

the interest rate is $r = \frac{\Pi_{CB}^{loss} - (1-p)\theta v}{pq}$ while the haircut is $h = \theta/q$.

Proof. The proof proceeds in two steps. First, we prove the minimal inefficient collateral transfer regime by showing that it is optimal for the central bank that weights profits (whether negative or positive) by ω to make losses in order to reduce the collateral requirement and the inefficient collateral transfer. We consider a slightly enlarged objective function for the central bank where not only losses are taken into account for welfare but also the profits. We show that in this case it is optimal for the central bank to make losses. We then show that when the central bank only weights losses (i.e. negative profits) by ω_L, then its welfare with nonnegative profits must be lower (because if not the previous contract was always attainable). Thus is the optimal to make losses.

Second, we show that in when $\theta < \underline{\theta}_L$ the losses required to support the minimal ICT loan size would impose a loss on the central bank larger than π_{CB}^{loss}. The central bank therefore reduces the loan size: this is the collateral crunch regime.

Minimal inefficient collateral transfer regime

(1) Suppose the central bank accounts for profits with weight ω, whether profits are negative or positive. The problem is

$$\max_{r,q,h} W_{cb} = pR(q) - q\,[1+\omega] - (1-p)\,qh\,(1 - v_\theta\,(1+\omega)) + \omega prq,$$

s.t.

$$R(q) - rq + hq \geq \frac{A + Bq}{p}$$

$$p\left[R\left(q\right)-rq\right]-\left(1-p\right)hq\geq 0$$

$$hq\leq\theta.$$

$$\Pi_{CB}\geq\Pi_{CB}^{loss}$$

In this case it is always optimal for the central bank to make losses π_{CB}^{loss} and to reduce collateral requirements until the IC binds. Let us start with a contract (r,q,h) such that the IC binds and the bank's profit is positive (i.e. IR is slack), and such that the central bank's profit is between zero and π_{CB}^{loss}, i.e. $\pi_{CB}^{loss}<\pi_{CB}<0$. Consider a contract $(r-\epsilon,q,h-\epsilon)$. The constraints: (i) the IC is still satisfied; (ii) the IR is relaxed; (iii) the CC is relaxed; (iv) the CB's profit is lower (but still higher than π_{CB}^{loss}). The impact on welfare is:

$$\Delta W_{CB}=q\epsilon\left[\left(1-p\right)\left(1-v_{\theta}\left(1+\omega\right)\right)-\omega p\right]$$

This is positive since $\omega<\underline{\omega}=\frac{(1-p)(1-v_{\theta})}{v_{\theta}(1-p)+p}$. The reasoning may be iterated until $\Pi_{CB}=\Pi_{CB}^{loss}$.

(2) This proves that the allocation where the central bank makes zero profit and has lowest haircuts possible has higher welfare (where positive and negative profits are weighted by ω) than when the central bank makes positive profit with a same quantity. Now consider the welfare function where only negative profits are weighted by ω, non-negative profits are weighted by zero. For any allocation with nonnegative profits, welfare must be higher in the case where profits are weighted than when they are not. To see this, suppose a contract is optimal when (nonnegative) profits are not weighted. Take the same contract when they are weighted. Then welfare must be higher since profits are nonnegative. Thus welfare must be at least higher when profit is weighted by $\omega>0$ and the CB makes nonnegative profits.

(3) These two statements imply that, for any q, the optimal interest rate and haircut policy is the one where haircuts are lowest given that the central bank makes losses Π_{CB}^{loss} and the IC binds.

The problem is

$$\max_{r,q,h}W_{cb}=pR\left(q\right)-q\left[1+\omega\right]-\left(1-p\right)qh\left(1-v\left(1+\omega\right)\right)+\omega prq,$$

s.t. the bank's IC and the central bank's maximum loss constraint bind:

$$pR\left(q\right)-prq+phq=A+Bq$$

$$prq+\left(1-p\right)hqv-q=\Pi_{CB}^{loss}$$

Sum the 2 constraints:

$$pR\left(q\right)+\left(1-p\right)hqv+phq-q=\Pi_{CB}^{loss}+A+Bq$$

this gives the haircut

$$hq = \frac{\Pi_{CB}^{loss} + A + Bq - pR(q) + q}{(1-p)v + p} \tag{18}$$

The interest rate is

$$prq = \Pi_{CB}^{loss} - (1-p)v\frac{\Pi_{CB}^{loss} + A + Bq - pR(q) + q}{(1-p)v + p} + q \tag{19}$$

Plug this in the welfare function:

$$\max_{r,q,h} W_{cb} = pR(q) - q[1+\omega]...$$

$$... - (1-p)(1 - v(1+\omega))\left(\frac{\Pi_{CB}^{loss} + A + Bq - pR(q)}{(1-p)v + p}\right)...$$

$$... + \omega\left(\pi_{CB}^{loss} - (1-p)v\frac{\Pi_{CB}^{loss} + A + Bq - pR(q)}{(1-p)v + p}\right)$$

and take the FOC. The equilibrium contract is therefore:

$$R'(q) = \frac{(1+\omega_L)((1-p)v_\theta + p) + B(1-p)(1-v_\theta)}{p}.$$

The haircut is given by (18) and the interest rate is given by (19). The latter can be simplified to:

$$prq = \frac{p\Pi_{CB}^{loss} - (1-p)v(A - pR(q)) + q[p - (1-p)vB]}{(1-p)v + p}$$

Collateral crunch regime

If $\theta \leq hq = \frac{\Pi_{CB}^{loss}+A+Bq-pR(q)+q}{(1-p)v_\theta+p} = \underline{\theta}_L$, the enough collateral contract is not implementable because it violates the CC. In this case the problem is to maximize welfare given that the CC and IC bind and the CB makes losses π_{CB}^{loss}.

The problem is

$$\max_{r,q,h} W_{cb} = pR(q) - q[1+\omega] - (1-p)qh(1 - v(1+\omega)) + \omega prq,$$

s.t.

$$pR(q) - prq + phq = A + Bq$$

$$prq + (1-p)hqv - q = \Pi_{CB}^{loss}$$

$$hq = \theta. \tag{20}$$

The constraints determine the optimal contract:

$$pR(q) - prq + p\theta = A + Bq$$

$$prq + (1-p)\,\theta v - q = \Pi_{CB}^{loss}$$

Sum the 2 equations. The loan level q solves:

$$pR(q) + p\theta + (1-p)\,\theta v - q - A - Bq - \Pi_{CB}^{loss} = 0.$$

The haircut is determined by (20) and the interest rate is determined by:

$$prq = \Pi_{CB}^{loss} - (1-p)\,\theta v + q.$$

□

F Central bank and the interbank market (proof of proposition 10)

Proof. In our model, the bank compares its profit with the central bank and with the interbank market given its available collateral θ.

Suppose $\underline{f}_2^{CB} < f$. In this case the funding level q is the same with the central bank and with the interbank market and is given by:

$$R'(q) = \frac{1 + (1-p)(1-v)\,B}{p}.$$

If we assume that the asset value is slightly lower for the central bank than for the investors, i.e. the central bank has $v_{cb} = v - \epsilon$ for $\epsilon \to 0$, the quantity lent by the central bank will be lower than that of the private market. The bank thus has a higher surplus in the private market than at the central bank when $\underline{f}_2^{CB} < f$.

If $f < \underline{f}_2^{CB}$, the central bank is willing to make losses to support investment while the investor still requires to break even. Hence the bank's profits are higher at the central bank. □