

Technological change in resource extraction and endogenous growth

Martin Stuermer *

Gregor Schwerhoff †‡

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Abstract

We add an extractive sector to an endogenous growth model of expanding varieties and directed technological change. Extractive firms reduce the stock of non-renewable resources through extraction, but also increase the stock through R&D investment in extraction technology. Our model accommodates long-term trends in non-renewable resource markets, namely stable prices and exponentially increasing extraction, for which we present data from 1792 to 2009. The model suggests that the development of new extraction technologies neutralizes the increasing demand for non-renewable resources in industrializing countries like China in the long term.

JEL classification: O30, O41, Q30

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*Institute for International Economic Policy (IIW), University of Bonn, Lennéstraße 37, 53113 Bonn, Germany. Email: martin.stuermer@uni-bonn.de

†Potsdam Institute for Climate Impact Research, PO Box 601203, 14412 Potsdam, Germany. Email: schwerhoff@pik-potsdam.de

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1 Introduction

This paper contributes to resolving a contradiction between theoretical predictions and empirical evidence regarding non-renewable resources. According to theory, economic growth is not limited by non-renewable resources because of three factors: technological change in the use of resources, substitution of non-renewable resources by capital, and returns to scale. Given these factors, growth models with a non-renewable resource typically predict growth in output, decreased non-renewable resource extraction, and an increase in price (see Groth, 2007; Aghion and Howitt, 1998).

However, it is a well-established fact that these predictions are not in line with the empirical evidence from the historical evolution of production and prices of non-renewable resources. The extraction of non-renewable resources has increased over time, and there is no persistent increase in the real prices of most non-renewable resources over the long run (see Krautkraemer, 1998; Livernois, 2009; Von Hagen, 1989).

We modify the standard endogenous growth model of expanding varieties and directed technological change by Acemoglu (2002, 2009). We add an extractive sector to the model such that aggregate output is produced from a non-renewable resource and intermediate goods. In the extractive sector, firms can reduce their resource stocks through extraction, but also increase stocks through R&D investment in extraction technology. Given technological change, the non-renewable resource is inexhaustible and there is no scarcity rent. This assumption is in line with evidence that technological change has offset the depletion of the stock of non-renewable resources in the past (Simpson, 1999, and others), and that non-renewable resources are so abundant in the earth's crust that given technological change, "the future will not be limited by sheer availability of important materials" (Nordhaus, 1974, p. 23).

We point out the main differences between the extractive sector and the intermediate goods sector in our model. First, it is necessary to innovate in the extractive sector as resources are extracted from mineral occurrences of decreasing grades. Once the present resource stock is depleted, new R&D investment in extraction technology is necessary to make mineral occurrences of lower grades extractable thus continuing production. A specific extraction technology is only applicable to a mineral occurrence of a certain grade. This is in contrast to the intermediate goods sector where a certain technology can be used infinitely.

Second, we show that under reasonable assumptions the resource stock increases linearly with R&D in extraction technology as two effects offset each other. R&D expenditure has to increase exponentially in order to make mineral occurrences of lower grades extractable. At the same time, the quantity of non-renewable mineral resources in the earth's crust increases exponentially as the grade of its occurrences decreases. It follows that there are constant returns from investment in R&D extraction technology.

Third, non-renewable resources are traded as homogeneous goods such that monopolistic competition is not taking place as in the intermediate goods sector, where the variety of intermediate goods increases. The extractive sector is fully competitive in the market for extraction technologies.

We illustrate the different evolutions of technology based on the characteristics of the two sectors. In order to keep the level of production of the non-renewable resource proportionate to aggregate output, the growth rate of technology in the extractive sector needs to increase over time. This is in contrast to the intermediate goods sector, where the growth rate of technology is constant. The difference is due to the necessity of innovation in the extractive sector, as extraction from lower grades requires new technology.

Our model replicates historical trends in the prices and production of major non-renewable resources, as well as world output for which we present data from the period 1792 to 2009. Exponential aggregate output growth triggers R&D investment in extraction technology. The extraction and use of non-renewable resources increase exponentially whereas its price stays constant over the long term.

Our paper suggests that the increasing demand for non-renewable resources in industrializing countries like China is neutralized by R&D investment in extraction technology. This makes extraction from mineral occurrences of lower grades possible. If historical trends continue, R&D in extraction technology might offset the depletion of today's resources. Even if non-renewable resource use and production increase exponentially, resource prices might stay constant in the long term.

Nordhaus (1974), Simon (1981), Simon (1998), Tilton (2002), and others stress technological change in the extraction and processing of non-renewable resources as an argument against limits to growth. However, efforts to model this aspect typically take technological change in the extraction technology as a given and do not include growth of aggregate output. Heal (1976) introduces a non-renewable resource, which is inexhaustible, but extractable at

different grades and costs in the seminal Hotelling (1931) optimal depletion model. Extraction costs increase with cumulative extraction, but then remain constant when a “backstop technology” (Heal, 1976, p. 371) is reached. Slade (1982) adds exogenous technological change in extraction technology to the Hotelling (1931) model and predicts a U-shaped relative price curve. Cynthia-Lin and Wagner (2007) use a similar model with an inexhaustible non-renewable resource and exogenous technological change. They obtain a constant relative price with increasing extraction.

There are three papers, to our knowledge, that are similar to ours in that they include technological change in the extraction of a non-renewable resource in an endogenous growth model. Fourgeaud et al. (1982) focuses on explaining sudden fluctuations in the development of non-renewable resource prices by allowing the resource stock to grow in a stepwise manner through technological change. Tahvonen and Salo (2001) model the transition from a non-renewable energy resource to a renewable energy resource. Their model follows a learning-by-doing approach as technical change is linearly related to the level of extraction and the level of productive capital. It explains decreasing prices and the increasing use of a non-renewable energy resource over a particular time period before prices increase in the long term. Hart (2012) models resource extraction and demand in a growth model with directed technological change. The key element in his model is the depth of the resource. After a temporary “frontier phase” with a constant resource price and consumption rising at a rate only close to aggregate output, the economy needs to extract resources from greater depths. After this phase, a long-run balanced growth path with constant resource consumption and prices that rise in line with wages is reached.

Our model is, to our knowledge, the first to combine technological change in the extractive sector and mineral occurrences of different grades in an endogenous growth model that explicitly models R&D investment in extraction technology. It also contributes to the literature by pointing out the necessity of innovation in the extractive sector due to its specific characteristics, and their effects on R&D development in comparison to other economic sectors in an endogenous growth model.

To focus on the main argument, we do not take into account of externalities, uncertainty, recycling, substitution, short-run price fluctuations, population growth, and exploration in our model. In particular, recycling will probably become more important for non-fuel, non-renewable resources in the future due to an increasing stock of recyclable materials and

its comparatively low energy requirements (see Steinbach and Wellmer, 2010; Wellmer and Dalheimer, 2012). As recycling adds to the resource stock, this would further strengthen our argument.

In Section 2, we document stylized facts on the long-term development of non-renewable resource prices, production, and world GDP. We also provide geological evidence for the major assumptions of our model regarding technological change. Section 3 describes how we model technological change in the extractive sector. Section 4 presents the setup of the growth model and discusses its theoretical results. Section 5 is where we will draw conclusions.

2 Stylized facts

2.1 Prices, production, and output over the long term

Annual data for major non-renewable resource markets from 1792 to 2009 indicates that real prices are roughly trend-less and that worldwide primary production as well as world GDP grow roughly exponentially.

Figure 1 presents data on the real prices of five major base metals and crude oil. Real prices exhibit strong short-term fluctuations. At the same time, the growth rates of all prices are not significantly different from zero (see Table 2 in the Appendix). The real prices are hence trend-less from 1792 to 2009. This is in line with evidence over shorter time periods provided by Krautkraemer (1998), Von Hagen (1989), Cynthia-Lin and Wagner (2007), and references therein. The real price for crude oil exhibits structural breaks, as shown in Dvir and Rogoff (2010). Overall, the literature is certainly not conclusive (see Pindyck, 1999; Lee et al., 2006; Slade, 1982), but we believe the evidence is sufficient to take trend-less prices as a motivation for our model.

Figure 2 shows that the world primary production of the examined non-renewable resources and world GDP approximately exhibit exponential growth since 1792. A closer statistical examination reveals that the production of non-renewable resources exhibits significantly positive growth rates in the long term. The growth rates of the production of copper, lead, tin, and zinc do not exhibit a statistically significant trend over the long term. Hence, the levels of production of these non-renewable resources grow exponentially over time.

The level of crude oil production follows this exponential pattern up to 1975. Including the time period from 1975 until 2009 reveals a statistically significant negative trend and

therefore, declining growth rates over time due to a structural break in the oil market (Dvir and Rogoff, 2010; Hamilton, 2009). In the case of the production of primary aluminum, we also find declining growth rates over time and hence, no exponential growth of the production level. This might be due to the fact that recycling has become important in the production of aluminum over time (see data by U.S. Geological Survey, 2011a). Recycling is not included in our model nor is it in the data. The growth rates of world GDP exhibit an increasing trend over the long term, hinting at an underlying explosive growth process. As our model does not include population growth, we run the same tests for the per capita data of the respective time series as a robustness check. We find slightly weaker results as Table 4 in the Appendix shows. Overall, we take these stylized facts as motivation to build a model that exhibits trendless resource prices and exponentially increasing worldwide production of non-renewable resources, as well as exponentially increasing aggregate output.

Insert Figure 1 about here.

Insert Figure 2 about here.

2.2 Technological change in the extractive sector

Technological change offsets the depletion of a non-renewable resource stock (Simpson, 1999, and others). Hence, the resource stock is drawn down by extraction, but it increases by technological change in extraction technology. The reason for this phenomenon is that non-renewable resources such as copper, aluminum, or hydrocarbons are extractable at different costs from the earth's crust due to varying grades, thickness, depths, and other characteristics of mineral occurrences. Technological change makes mineral occurrences extractable that, due to high costs, have not been extractable before (see Simpson, 1999; Nordhaus, 1974, and others).

The definition of resources by the U.S.-Geological Survey reflects this fact. It defines resources as “a concentration of naturally occurring solid, liquid, or gaseous material in or on the earth's crust in such form and amount that economic extraction (...) is currently or potentially feasible” (U.S. Geological Survey, 2011b, p. 193). The term economic “implies that

profitable extraction (...) under defined investment assumptions has been established” (U.S. Geological Survey, 2011b, p. 194). The “boundary” between resources and “other occurrences is obviously uncertain, but limits may be specified in terms of grade, quality, thickness, depth, percent extractable, or other economic-feasibility variables” (U.S. Geological Survey, 2011b, p. 194).

Insert Figure 3 about here.

Over time, R&D in extraction technology, namely in prospection and mining equipment, as well as metallurgy and processing, have increased the stock of the resource by making the extraction of materials from mineral occurrences of lower grades or greater depths economically feasible (see Wellmer, 2008; Mudd, 2007). For example, Radetzki (2009) describes how technological change has gradually made possible the extraction of copper from mineral occurrences of decreasing grades. 7000 years ago, human beings used copper in a pure nugget form. Today, humanity extracts copper from mineral occurrences of a low 0.2 to 0.3 percent grade.¹ In line with this narrative evidence, Figure 3 illustrates that the ore grades of U.S. copper mines have steadily decreased over the long term. Mudd (2007) presents similar evidence for the mining of different base-metals in Australia. Overall, history suggests that R&D costs in the extractive sector have increased exponentially in pushing the boundary between mineral occurrences and resources in terms of grades. Developing technologies to make mineral occurrences of 49 percent grade instead of 50 percent grade extractable, has probably required a far smaller investment than developing technologies to make the extraction from mineral occurrences of 0.2 percent grade instead of 1.2 percent grade economically feasible.

As a result, technological change has offset the higher cost of obtaining resources from mineral occurrences of lower grades. Figure 4 shows that the reserves² of copper have increased by more than 600 percent over the last 60 years. One reason is the introduction of the solvent extraction and electrowinning technology. This two-stage process has made the extraction of copper from mineral occurrences of lower grades economically feasible (Bartos, 2002). There are also the strong effects of innovation on returns-to-scale as larger equipment

¹The Aitik copper mine in Sweden is the mine that extracts copper from the lowest deposits of 0.27 percent in the world (personal communication with F.-W. Wellmer).

²Reserves are those resources for which extraction is considered economically feasible (U.S. Geological Survey, 2011c).

in mining operations becomes feasible.³ Case studies for other minerals also find that technological change has offset cost-increasing degradation of resources (see for example Lasserre and Ouellette, 1991; Mudd, 2007; Simpson, 1999).

Insert Figure 4 about here.

We observe similar developments in the case of hydrocarbons. Using the example of the offshore oil industry, Managi et al. (2004) show that technological change has offset the cost-increasing degradation of resources. Crude oil has been extracted from ever deeper sources in the Gulf of Mexico as Figure 8 in the Appendix shows. Furthermore, technological change and high prices have made it profitable to also extract liquid hydrocarbons from unconventional sources, such as light tight oil, oil sands, and liquid natural gas (International Energy Agency, 2012). As a result, oil reserves have doubled since the 1980s (see Figure 7 in the Appendix).

Overall, empirical evidence suggests that technological change offsets resource depletion by renewing the resource stock from mineral occurrences that had been considered impossible to extract. Furthermore, it is a reasonable assumption that R&D costs in the extractive sector have increased exponentially in terms of making mineral occurrences from lower grades extractable.

2.3 Geological abundance and distribution of the elements in the earth's crust

Computing the total abundance (or quantity) of each element in the earth's crust leads to enormous quantities (see Nordhaus, 1974; Perman et al., 2003). Table 1 shows the respective ratios of the quantities of reserves, resources, and abundance in the earth's crust with respect to annual mine production for several important non-renewable resources. It provides evidence that even non-renewable resources, which are commonly thought to be the most scarce such as gold, are abundant in the earth's crust, and that there is evidence "that the future will not be limited by sheer availability of important materials"(Nordhaus, 1974, p. 23). In addition, most metals are recyclable, which means that the extractable stock in the techno-sphere increases (Wellmer and Dalheimer, 2012).

³Personal communication with F.-W. Wellmer.

The sediments of the earth's crust are also rich in hydrocarbons. Even though conventional oil resources may be exhausted someday, resources of unconventional oil, natural gas, and coal are abundant. Aguilera et al. (2012) conclude that conventional and unconventional resources "are likely to last far longer than many now expect" (p. 59). Overall, Rogner (1997) states about world hydrocarbon resources that "fossil energy appears almost unlimited" (p. 249) given a continuation of historical technological trends.

Insert table 1 about here.

The elements of the earth's crust are not uniformly distributed. Geochemical processes have decreased or increased their local abundance throughout history. Unfortunately, geologists do not agree on the distribution of elements in the earth's crust. Ahrens (1953, 1954) states in his fundamental law of geochemistry that the elements in the earth's crust exhibit a log-normal grade-quantity distribution. Skinner (1979) and Gordon et al. (2007) propose a discontinuity in this distribution due to the so-called "mineralogical barrier" (Skinner, 1979), the approximate point below which metal atoms are trapped by atomic substitution. Due to a lack of geological data, both parties acknowledge that an empirical proof is still needed. In a recent empirical study, Gerst (2008) concludes that he can neither confirm nor refute these two hypotheses. Based on worldwide data on copper deposits over the past 200 years, he finds evidence for a log-normal relationship between copper production and average ore grades. Mudd (2007) analyzes the historical evolution of extraction and grades of mineral occurrences for different base metals in Australia. He comes to the conclusion that production has been continually increasing, partly verging on exponentially, while grades have consistently declined.

The distribution of hydrocarbons in the earth's crust might also differ from the fundamental laws of geochemistry by Ahrens (1953, 1954) due to the distinct formation processes. For example, oil begins to form in the source rock due to the thermogenic breakdown of organic matter (kerogen) at about 60 to 120 degrees Celsius, which is found approximately two to four kilometers of depth. However, Farrell and Brandt (2006) and Aguilera et al. (2012) suggest that a log-normal relationship is also true for liquid hydrocarbon production. Aguilera et al. (2012) also point out that there is no huge break between the average total production costs of conventional and unconventional oil resources.

To conclude, with respect to inference about future supply, we acknowledge that there is uncertainty about the distribution of the elements in the earth’s crust. However, we believe that it is reasonable to assume that the elements are distributed according to a log-normal relationship between the grade of its mineral occurrences and its quantity in the earth’s crust.

3 Modeling technological change in resource extraction

We point out that introducing change in the extraction technology of a non-renewable resource extends the traditional classification of non-renewable and renewable resources by a non-renewable resource that is de facto inexhaustible. We explain why the return to R&D investment in extraction technology is constant in terms of the quantity of the extractable resource.

The typical way of modeling non-renewable resources following Hotelling (1931) assumes that there is a fixed stock of the resource S . This implies that the change in the stock is equal to the resource that is used in time period t .

$$\dot{S}_t = -R_t . \tag{1}$$

As the resource is used, the resource stock decreases over time and the use of the resource R goes to zero, $\lim_{t \rightarrow \infty} R_t = 0$.

Renewable resources such as fish or wood from forests are primarily modeled in a way that the stock increases through a natural regeneration process $G(S_t)$ (see Tahvonen and Kuuluvainen, 1993, for a macroeconomic example). The change in the stock S_t is given as the difference between natural regeneration and the use (or harvesting) of the resource:

$$\dot{S}_t = G(S_t) - R_t . \tag{2}$$

A basic assumption of these models is that natural regeneration never occurs beyond a certain maximum of the stock, $F(\bar{X}_t) = 0$, owing to a limited carrying capacity, which causes decreasing returns to scale in regeneration from a certain point. The sustainable rate of use of the renewable resource is therefore also limited.

Augmenting technological change overcomes “limits to growth” from renewable and non-

renewable resources. A simple production function with a natural resource is

$$Y = F(AR_t) . \quad (3)$$

The use of the natural resource in period t is multiplied by factor augmenting technological change A . The idea is that technological change makes the use of the natural resource more efficient. This compensates for the declining (or non-increasing) input from the resource.

We propose to take a different perspective on non-renewable resources and introduce technological change in extraction technology. The idea is, as presented in the stylized facts before, that R&D in extraction technology increases the stock of the resource. In contrast to factor augmenting technological change, R&D in extraction technology physically increases the stock and not the efficiency of the use of the resource. The evolution of the stock follows:

$$\dot{S}_t = X_t - R_t , \quad (4)$$

The change in the stock \dot{S}_t is the difference between the increase in the stock X_t , owing to R&D in the extraction technology and extraction, and the use of the resource R_t . In contrast to the renewable resource described above, the increase in the stock is not dependent on the actual stock at period t . There is also no regeneration process that imposes limits on the increase in the stock. Overall, there are no ultimate stock constraints, for this type of non-renewable resource given that there is investment in R&D in the extraction technology.

We combine two functions to show that there are constant returns from investment in R&D in terms of the quantity of the extractable resource. The first function describes the mineral occurrences that are extractable for a given state of technology. The second function shows the distribution of the quantity of the resource over grades. Combining these two functions gives the quantity of the resource that becomes extractable from one unit of R&D investment in extraction technology.

Let N_{Rt} be the accumulated extraction technology at time t . We drop the time index to simplify notation. Let d be the grade of the respective mineral occurrences. We define the extraction cost function as a function mapping grades into extraction costs depending on the state of technology:

$$\phi_{N_R} : [0, 1] \times \mathbb{R}_+ \rightarrow \bar{\mathbb{R}}_+, (d, N_R) \mapsto \phi_{N_R}(d) . \quad (5)$$

At technology level $N_R \in \mathbb{R}_+$ the cost of extracting the non-renewable resource from occurrences of grade $d \in [0, 1]$ is $\phi_{N_R}(d) \in \bar{\mathbb{R}}_+ = \mathbb{R}_+ \cup \infty$. There are decreasing returns to scale from R&D investment in extraction technology in terms of grades. This implies that for a given level of technology N_R , ϕ_{N_R} is non-increasing in d :

$$\forall N_R: \quad d > d' \quad \Rightarrow \quad \phi_{N_R}(d) \leq \phi_{N_R}(d'). \quad (6)$$

We assume that R&D increases the productivity of the extraction technology for mineral occurrences of all grades. Therefore, an increase in N_R decreases extraction costs for any given grade:

$$\forall d: \quad \frac{\partial \phi_{N_R}(d)}{\partial N_R} \leq 0. \quad (7)$$

At time t , extraction technology increases by $\frac{\partial N_{Rt}}{\partial t}$ and reduces extraction costs. Firms choose between extracting resources at a higher cost or investing in extraction technology. To simplify this optimization problem, we assume a simple form of the technology function.

Figure 5 panel (a) shows the general form of the extraction cost function. The extraction of the resource from mineral occurrences of lower grades generates higher costs, but due to increasing R&D, the function moves downward.

Insert Figure 5 about here.

Figure 5 panel (b) illustrates a simplified version of the extraction cost function, which we use in the following. A certain grade d_N is associated with a unique level of R&D investment, above which the resource can be extracted at cost $\phi_{N_R} = E$. The function h maps the state of the extraction technology into a value for the grade of the mineral occurrence, which is extractable at cost ϕ_{N_R} :

$$h: \mathbb{R}_+ \rightarrow [0, 1], N_R \mapsto d_{N_R}. \quad (8)$$

At grades lower than d_N extraction is impossible, because the cost is infinite. The technology function takes the degenerate form of

$$\phi_{N_R}(d) = \begin{cases} E, & \text{if } d \geq d_{N_R}, \\ \infty, & \text{if } d < d_{N_R}. \end{cases} \quad (9)$$

This simplifies the optimization. If occurrences with a grade larger than d_{N_R} exist, they are extractable without any additional R&D. Otherwise, R&D is needed to increase the resource stock and make extraction possible.

In order to determine the cost of R&D we specify a functional form for the extraction technology function h :

$$h(N_R) = e^{-\delta_1 N_R t}, \quad \delta_1 \in \mathbb{R}_+, \quad (10)$$

with δ_1 denoting a parameter that determines the shape of the function. Panel (a) in Figure 6 illustrates the shape of $h(N_R)$. The marginal effect of the extraction technology on the extractable occurrences declines as the grade decreases. This follows the suggestion in the stylized facts that R&D costs have increased exponentially in pushing the boundary between mineral occurrences and resources in terms of grades.

Insert Figure 6 about here.

Panel (b) in Figure 6 shows the distribution of the non-renewable resource in the earth's crust. It maps a certain grade onto the total quantity of extractable resources at different grades of the occurrences between d and one, where one corresponds to a 100 percent ore grade or pure metal.

$$D: (0, 1] \rightarrow \mathbb{R}_+, d \mapsto D(d) \quad (11)$$

Note that $D(1) = 0$ means that the resource is not found in 100 percent pure form. Figure 6 panel (b) illustrates the relationship between the two variables. The total quantity of the non-renewable resource is inversely proportional to the grade: as the grade decreases, the extractable quantity of the non-renewable resource increases.

We formulate the relationship in a general way:

$$D(d) = -\delta_2 \ln(d), \quad \delta_2 \in \mathbb{R}_+, \quad (12)$$

where δ_2 determines the steepness of the function.

We combine the two functions and obtain the following proposition. A dot over a variable denotes the time derivative.

Proposition 1 *The total quantity of the resource which has been made extractable over time due to technological change is proportional to N_{Rt} :*

$$D(h(N_{Rt})) = \delta_1 \delta_2 N_{Rt} . \quad (13)$$

Consequently, the newly extractable resource from a marginal investment in R&D is

$$X_t = \frac{\partial D(h(N_{Rt}))}{\partial t} = \delta_1 \delta_2 \dot{N}_{Rt} . \quad (14)$$

According to this result, the quantity of the resource, which is made extractable by a given R&D investment in extraction technology, is independent of past investments or time. An extractive firm invests an amount of \dot{N}_{Rt} in R&D. This gives a smaller return on investment in terms of making occurrences of lower grades extractable. However, this smaller advancement in terms of grade makes the same quantity of the resource extractable, as the firm reaches a grade with a higher extractable amount of resources than before.

4 The growth model

To illustrate the macroeconomic effect of the analysis in Section 3, we build a growth model which allows an endogenous allocation of resources between an intermediate goods sector and an extractive sector based on the framework of directed technological change by Acemoglu (2002).

4.1 The setup

We consider an economy with a representative consumer that has constant relative risk aversion preferences:

$$\int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt . \quad (15)$$

The variable C_t denotes the consumption of aggregate output at time t , ρ is the discount rate, and θ is the coefficient of relative risk aversion. The budget constraint of the consumer is

$$C_t + I_t + M_t \leq Y_t \equiv \left[\gamma Z_t^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) R_t^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} , \quad (16)$$

where I_t is aggregate investment in machines by the two sectors, and M_t denotes aggregate R&D investment in developing new varieties of machines. The usual no-Ponzi game condition applies. According to the right hand side of Equation 16, aggregate output production uses two inputs, intermediate goods Z_t and the non-renewable resource R_t . There are two sectors in the economy that produce the inputs to aggregate output production: the intermediate goods sector and the extractive sector. The distribution parameter γ indicates the respective importance in producing aggregate output Y_t . The R&D expenditure is the sum of R&D expenditure in the intermediate sector and in the extractive sector: $M_t = M_{Z_t} + M_{R_t}$.

The elasticity of substitution is $\varepsilon > 0$. Inputs Z_t and R_t are substitutes for $\varepsilon > 1$. In this case, the resource is not essential for aggregate production (see Dasgupta and Heal, 1980). The Cobb-Douglas case is $\varepsilon = 1$. For $0 < \varepsilon < 1$ the two inputs are complements.

The production function of the intermediate goods sector

The intermediate goods sector follows the basic setup of Acemoglu (2002). It produces intermediate goods Z_t according to the following production function⁴:

$$Z_t = \frac{1}{1-\beta} \left(\int_0^{N_{zt}} x_{zt}(j)^{1-\beta} dj \right) L^\beta, \quad (17)$$

where $\beta \in (0, 1)$. The intermediate goods sector uses labor L_t , which has a fixed supply, and machines as inputs to production. $x_{zt}(j)$ refers to the number of machines that are used for each machine variety j at time t . Machines depreciate fully after use within one period. We denote the number of varieties of machines as N_{zt} . Profits for the firm producing good Z are simply the difference between revenues and the expenses for labor, as well as for the intermediates $x_Z(j)$,

$$\pi_Z = p_Z Z - w_Z L - \int_0^{N_Z} \chi_Z(j) x_Z(j) dj. \quad (18)$$

Sector-specific technology firms invent new technologies for which they hold a fully enforceable patent. They exploit the patent by producing a machine type which corresponds uniquely to their technology. The uniqueness gives them market power which they can use to set a price $\chi_{Z_t}(j)$ above marginal cost. The marginal cost of production in terms of the final good is the same for all machines. Machines depreciate fully after each period, so that the technology owner has to produce the corresponding machines each period.

⁴Like Acemoglu (2002) we assume that the firm level production functions of the two sectors exhibit constant returns to scale, so there is no loss of generality in focusing on the aggregate production functions.

The range of machines expands through R&D expenditure by

$$\dot{N}_{Zt} = \eta_Z M_{Zt} , \quad (19)$$

where M_{Zt} is R&D investment by the technology firms for machines in the intermediate goods sector in terms of the final product, and η_Z is a cost parameter. One unit of final good spent for R&D will generate η_Z new varieties of machines. A technology firm that discovers a new machine receives a patent and becomes its sole supplier.

The production function of the extractive sector

The extractive sector differs from the intermediate goods sector in the production function and in the way technological change takes place. Whereas the intermediate goods sector is labor intensive, the extractive sector is resource intensive, so that we do not model labor input explicitly.

The extractive sector faces stock constraints which the intermediate goods sector does not. The stock of the non-renewable resource at time t is noted $S_t \geq 0$. R_t notifies the quantity of the non-renewable resource that is sold for aggregate output production. Investing in new machines makes occurrences of lower grades extractable and expands the resource stock by X_t . The evolution of the stock follows:

$$\dot{S}_t = X_t - R_t , \quad S_t \geq 0, X_t \geq 0, R_t \geq 0 , \quad (20)$$

where \dot{S}_t is the change in the stock in period t , X_t is the inflow through investment of new machines, and R_t is the outflow by extracting and selling the resource. Note that for $X_t = 0$, this formulation is the standard Hotelling (1931) setup.

Extractive firms increase the resource stock by

$$X_t = \delta_1 \delta_2 \dot{N}_{Rt} x_R(j) , \quad (21)$$

which is equal to Equation 14 in Proposition 1, with the number of machines $x_R(j)$ that extractive firms purchase from sector specific technology firms added. Each machine $x_R(j)$ makes a specific additional mineral occurrence with a lower grade extractable. In contrast to the intermediate goods sector, the use of the machine variety j is bound to a specific deposit.

Each mineral occurrence has the same quantity of the resource, but each at a different grade. Once a firm has extracted the resource from a specific mineral occurrence by use of machine variety j , the next deposit - with a lower grade - is not extractable any more by machine variety j . A new machine of a new variety needs to be bought from the sector specific technology firms. As a result, each variety of machines in the extractive sector can only be used once, whereas in the intermediate goods sector, each machine variety is used infinitely often. We normalize the size of R&D investment to one, $xR(j) = 1$. This is mathematically not exactly the same as in the intermediate goods sector, but it provides a comparable micro-foundation by subdividing the technology growth into units. In the intermediate goods sector, a machine is an infinitesimally small variety, whereas in the extractive sector it is a normalized fraction of R&D investment.

The term \dot{N}_{Rt} denotes the range of the new machine varieties invented by the sector specific technology firms. The extractive sector is consistently under pressure to buy newly developed machines as once developed machines are not able to extract the resource from declining grades. This is in contrast to the intermediate goods sector (see Equation 17), which produces from all machine varieties that have been developed.

The sector specific technology firms develop \dot{N}_{Rt} new patents for machines of the extractive sector analogously to the intermediate goods sector according to:

$$\dot{N}_{Rt} = \eta_R M_{Rt} , \quad (22)$$

where M_{Rt} is spending on R&D in the extractive sector in terms of the final product, and η_R is a cost parameter.

Once the patent has been developed, the technology firms produce the new machine variety j at a unit cost of Ψ in terms of the final good. Technology firms can only produce one machine for each patent. They sell machines to the extractive sector in perfect competition, because the machines are perfect substitutes for producing the resource. This implies that the firm that buys the machines from the technology firms is entirely indifferent between the machines. Since sector specific technology firms have no market power, they obtain a price of the machine above marginal cost.

As the extractive firms can only use each machine variety once, the price of each machine

is

$$\chi_R(j) = \frac{1}{\eta_R} + \Psi, \quad (23)$$

The first term on the right hand side, $\frac{1}{\eta_R}$, is the marginal R&D expenditure for developing one patent. This results from the equation $\eta_R M_R = \dot{N}_R$. Setting $\dot{N}_R = 1$ and solving for M_R , yields $M_R = \frac{1}{\eta_R}$. The second term, Ψ , notifies the cost of producing the machine in terms of aggregate output.

Profits for resource firms are thus given by revenues from selling the resources less the amount of $M_R = \frac{1}{\eta_R} \dot{N}_R = \frac{1}{\delta_1 \delta_2} X_t$ at the price from equation (23):

$$\pi_{Rt} = p_R R_t - \frac{1 + \psi \eta_R}{\eta_R \delta_1 \delta_2} X_t. \quad (24)$$

The production function of the extractive sector is equal to the outflows from the resource stock R_t :

$$R_t = \delta_1 \delta_2 \dot{N}_{Rt} x_R(j) - \dot{S}_t. \quad (25)$$

It illustrates the fundamental difference between the intermediate goods sector and the extractive sector in the relationship between technological change and the respective production. If technology firms stop investing in R&D in the intermediate goods sector, the intermediate goods sector will still be able to produce the good Z_t by buying machines based on the existing patents. However, if investment in R&D of the extraction technology stops at time T , the quantity of the resource that will still be extractable with the machines from the existing technology is limited to the existing stock.

4.2 Results

We begin the formal analysis with the optimization of the extractive firms.⁵ Firms have full control over inflows and outflows from their resource stock. Inflows X_t depend on R&D investment in the extractive sector, and outflows R_t are the sales of the resource to the final good producer. Since the marginal cost for R&D is constant, we obtain the typical result of stock management: inflows and outflows have to balance over time.

⁵Proofs for this section are in the Appendix.

Proposition 2 *The quantity of the resource used in aggregate production equals the quantity of newly acquired resources through R&D: $R_t = X_t$.*

When the resource stock is zero, $S_t = 0$, it is not possible to extract the non-renewable resource without additional R&D in the extractive sector. An extractive firm needs to buy a new machine and hence, trigger investment in R&D by the technology firms. The resulting resource stock can then be extracted and sold to the final goods producer. However, another extractive firm may also invest in R&D, and also extract and sell the resulting resource. This situation of perfect competition means that resource prices are equal to marginal costs, which is the cost of extraction. This also highlights why the case $S_t > 0$ never occurs under the assumption of no uncertainty: An extractive firm investing in R&D will always extract and sell the newly available resource stock, because the selling price will remain constant.

The result is of course affected by the assumption of no uncertainty. Following the standard in growth models, we have assumed in equation 22 that patents for new machines result in a deterministic way from the respective R&D investments. This reflects a long-term perspective. The model could be made more sophisticated by assuming that R&D is stochastic. Extractive firms would then keep a positive stock of the resource S_t to be on the safe side in the case of a series of bad draws in R&D. This stock would grow over time as the economy grows. But in essence, the result above would remain the same: In the long term, resources used in aggregate production equal those added to the resource stock through R&D.

We turn to the solution of the model:

Proposition 3 *The growth rate of the economy is constant and given by*

$$g = \theta^{-1} \left(\beta \eta_Z L \left[1 - \left(\frac{1 - \gamma}{\gamma} \right)^{\frac{\epsilon}{1 - \epsilon}} \frac{1 + \psi \eta_R}{\eta_r \delta_1 \delta_2} \right]^{\frac{1}{\beta}} - \rho \right).$$

A higher rate of return to R&D investment in new machines of the labor sector, η_Z , increases the growth rate of the economy. We discuss the effects of parameters η_R , δ_1 , and δ_2 on the growth rate in Proposition 5.

In order to understand the role of the non-renewable resource in the economy, we determine its relative importance:

Proposition 4 *The resource intensity of the economy is given by*

$$\frac{R}{Y} = \left[(1 - \gamma) \frac{\eta_R \delta_1 \delta_2}{1 + \psi \eta_R} \right]^\varepsilon. \quad (26)$$

It depends positively on the distribution parameter for the resource γ .

The distribution parameter γ indicates the importance of the resource for the economy, as shown in the production function in Equation 16.

Extractive firms face constant marginal costs of extracting the non-renewable resource, since the resource stock can be expanded due to R&D in extraction technology. The price thus remains constant over time as well:

Proposition 5 *The resource price is*

$$p_{Rt} = \frac{1 + \psi \eta_R}{\eta_r \delta_1 \delta_2}.$$

A higher resource price has the following effects: (i) it decreases the resource intensity, and (ii) it decreases the growth rate of the economy.

This proposition shows that the resource price plays a central role in the model. To understand it, we first consider its determinants and then focus on its effects.

The determinants of the price are given by the parameters η_R , δ_1 , and δ_2 . The productivity of R&D in the extractive sector, defined in Equation 22, and given by η_R , determines the number of new machine varieties that are developed by the sector specific technology firms per unit of aggregate output. The higher this parameter, the higher the resource use in the economy. δ_1 , defined in Equation 10, is a productivity parameter for the marginal effect of R&D investment on the extractability of occurrences of lower grades. δ_2 , defined in Equation 12, determines the steepness of the distribution of elements over mineral occurrences of various grades in the earth's crust. If the quantity of the extractable resource strongly increases as the grade of occurrences decreases, the return on investments in R&D for the extraction technology increases, and the economy uses a larger quantity of the resource in proportion to aggregate output.

The resource price is constant, but Proposition 5 shows that the resource price is high when the productivity parameters are low and vice versa. It states quite intuitively that the

selling price of the resource is low, if the productivity parameters are high.

Moreover, Proposition 5 in combination with Propositions 3 and 4 shows the effect of a lower resource price on the growth rate and the resource intensity of the economy. Both depend negatively on the resource price. When the price is low, the non-renewable resource is used intensively and the resource constraint on growth is weak. When the price is high, the economy uses substitutes, but this reduces growth.

We compare the growth rates of technology in the two sectors.

Proposition 6 *The level of technology in the intermediate goods sector is*

$$N_Z = \left(\frac{1-\gamma}{\gamma} \right)^{-\varepsilon} \left(\frac{\eta_R \delta_1 \delta_2}{1 + \psi \eta_R} \right)^\varepsilon \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma} \right)^\varepsilon \frac{\eta_R \delta_1 \delta_2}{1 + \psi \eta_R} \right)^{\left(\frac{1}{1-\varepsilon} \right) \left(-\varepsilon + \frac{1-\beta}{\beta} \right)} (1-\gamma)^\varepsilon L^{-1} Y .$$

The growth rate of technology in the extractive sector is

$$\dot{N}_R = (1-\gamma)^\varepsilon \frac{\eta_R}{1 + \psi \eta_R} Y .$$

There is thus a qualitative difference in the growth rate of the two sectors. While the *level* of technology in the intermediate goods sector is proportional to output, the *growth rate* of technology in the extractive sector is proportional to output. N_Z therefore has the *constant* growth rate g , as given in Proposition 3. N_R has an *increasing* growth rate. It is the second derivative $\frac{\partial^2 N_R}{\partial t^2}$ which is equal to g .

4.3 Market structure in the resource market

Given the central role of the resource price, it is important to analyze the effect of the market structure in the resource sector. To do this, we consider a variant of the model, where we assume a monopoly in the resource sector instead of full competition.

Proposition 7 *Let the resource market be dominated by a single monopolist and let $\varepsilon > 1$. Then the price of the non-renewable resource is*

$$p_{Rt}^{Mon} = \frac{1 + \psi \eta_R}{\eta_R \delta_1 \delta_2} \frac{\varepsilon}{\varepsilon - 1} .$$

If the resource market is dominated by a single monopolist, the pricing strategy depends strongly on the elasticity of substitution. If Z and R are complements ($0 < \varepsilon < 1$), the economy cannot produce aggregate output without the resource. A resource monopolist could demand an arbitrarily high price in this case. We therefore exclude this case. In the case where the two inputs are substitutes ($1 < \varepsilon$), the pricing becomes a typical monopoly pricing problem: The monopolist imposes the markup $\frac{\varepsilon}{\varepsilon-1}$.

As a corollary of Proposition 7, we note that stronger market power in the resource sector results in an economy with lower resource intensity and a lower growth rate. The reason is that market power puts a markup on prices. One consequence of a lower resource intensity is also less R&D expenditure in the extractive sector, since less technology is needed when less of the resource is sold.

This result highlights that the market structure in the resource market affects the level of price, but not its long-term trend. A monopolist simply takes a markup over the competitive price. This allows us to understand the effect of a change in market structure on resource prices. If resource producers form a cartel for example, the price will be higher in the new steady state, and the resource intensity of the economy will be lower. In the new steady state, the price will again be constant, and the quantity supplied will grow at the same rate as the economy.

4.4 The social planner solution

The social planner solution of our model shows that the market power of technology firms in the intermediate goods sector causes an inefficiency.⁶ The growth rate of the economy g^{opt} is not constant, but growing in the optimum as $\frac{x_Z}{N_Z}$ is not constant in:

$$g^{opt} = \frac{1}{\theta} \left(\frac{1}{\eta_Z(1-\beta)\psi} \frac{x_Z}{N_Z} - \rho \right). \quad (27)$$

As a result, there is no balanced growth path in the social planner solution. This is in contrast to the decentralized solution, where the growth rate of the economy is constant (see Equation 3). The reason for this difference is that there are efficiency losses in the decentralized solution due to the monopoly power of the technology firms for machines in the intermediate goods sector. In the decentralized solution, the quantity of machines, which is

⁶Acemoglu (2002) provides only a decentralized solution of his model. The derivation of the central planner solution of our model is in Appendix Appendix 5.

supplied for each variety $x_Z(j)$, is constant as p_Z and $\chi_Z(j)$ are constant (see Equation 36 in the Appendix). In contrast, in the social planner solution x_Z is proportional to aggregate output as $x_Z = z_2 Y$ (see Equation 73). Furthermore, the decentralized solution features constant returns to scale in the production of Z_t (see Equation 17), since firms do not internalize technology in their production technology. For the social planner, however, technology is endogenous so that production has increasing returns in the factors N_Z and x_Z (see Equation 50 in the Appendix).

The comparison between the decentralized and the social planner solution illustrates the difference between our model based on Acemoglu (2002) and the Schumpeterian model with a non-renewable resource presented by Aghion and Howitt (1998). Aghion and Howitt (1998) make the assumption that “succeeding vintages of goods are increasingly capital intensive” (p. 153) in order to explain an exponent smaller than one on technology. This leads to constant returns to scale in their social planner solution. The idea of the model by Acemoglu (2002) is that there are increasing returns to scale, but these are not exploited due to the inefficiency in the decentralized solution.

The extractive sector does not make a difference to the two solutions. Technology firms are not able to obtain a monopoly price for machines, because machines are linked to the extraction of one specific occurrence, while the produced resource is a homogeneous good. There is therefore no efficiency loss in the extractive sector of the decentralized model. The resource production in the decentralized and in the social planner solution functions in the same efficient way. Comparing the respective first order conditions, the first order condition of the decentralized solution is given by the demand of the final good producer for the resource (see Equation 29 in the Appendix). When substituting the price from Equation 32 to Equation 29 in the Appendix, it becomes identical to the respective first order condition in the social planner solution in Equation 62 in the Appendix.

There is no straightforward way to correct for the inefficiency in the decentralized model. Technology firms obtain patents for machines. The property right of the patent ensures that only the respective firm is able to produce the machine. However, the patent also entails market power in the intermediate goods sector, such that the provided quantity of machines is below the social optimum. There is demand for each variety and each variety is supplied by a single firm. A subsidy on the sale of machines in the intermediate goods sector affects the supply of machines, but does not have an impact on the growth rate of

machine supply. To do so, the government needs to apply policy instruments like a subsidy on sales that increases with time, or modify the market structure by disconnecting R&D investment from the market power created by patents. The latter solution would require government compensation to inventors or some other incentive device. Finally, we have to keep in mind that this inefficiency has also been introduced by Acemoglu (2002) to obtain a balanced growth path in the decentralized solution.

4.5 Discussion

We discuss a number of issues that arrive from our model, namely the assumptions made in Section 3, the comparison to the other models with non-renewable resources, and the question of the ultimate finiteness of the resource.

Function D from Equation 11 shows the amount of the non-renewable resource in the earth's crust for a given occurrence of grade d . Geologists cannot give an exact functional form for D , so we used the form given in Equation 12 as a plausible assumption. How would other functional forms affect the predictions of the model? First, the predictions are valid for all parameter values $\delta_2 \in \mathbb{R}_+$. Secondly, if D is discontinuous with a break at d_0 , at which the parameter changes to $\delta'_2 \in \mathbb{R}_+$, there would be two balanced growth paths: one for the period before, and one for the period after the break. Both paths would behave according to the predictions of the model. They would differ in the extraction cost of producing the resource, level of extraction, and use of the resource in the economy. To see this, recall from Proposition 1 that X_t is a function of δ_2 . A non-exponential form of D would produce results that differ from ours. It could feature a scarcity rent as in the Hotelling (1931) model, as a non-exponential form of D could cause a positive trend in resource prices or the extraction from occurrences at a lower ore grade becomes infeasible. In these cases, the extractive firms would consider the opportunity cost of extracting the resource in the future, in addition to extraction and innovation cost.

How does our model compare to other models with non-renewable resources? We do not assume that resources are finite, as their availability is a function of technological change. As a consequence, resource availability does not limit growth. Substitution of non-renewable resources by capital, technological progress in the use of the resource, and increasing returns to scale are therefore not necessary for sustained growth as in Groth (2007) or Aghion and Howitt (1998). Growth depends on technological change as much as it does in standard

growth models without a non-renewable resource. If the resource would be finite in our model, then the extractive sector would behave in the same way as standard models in the tradition of Hotelling (1931). As Dasgupta and Heal (1980) point out, the growth rate of the economy depends in this case strongly on the degree of substitution between the resource and the other inputs in the economy. For $\varepsilon > 1$ the resource is inessential, for $\varepsilon < 1$, the total output which the economy is capable of producing is finite. The production function is therefore only interesting for the Cobb-Douglas case.

Our model suggests that the non-renewable resource can be thought of as a form of capital: If the extractive firms invest in new machines and trigger R&D in extraction technology, the resource is extractable without limits as an input to aggregate production. This feature marks a distinctive difference from models such as the one of Bretschger and Smulders (2012). They investigate the effect of various assumptions on substitutability and a decentralized market on long-run growth, but keep the assumption of a finite non-renewable resource. Without this assumption, the elasticity of substitution between the non-renewable resource and other input factors is not central to the analysis of limits to growth anymore.

Some might argue that the relationship described in Proposition 1 cannot continue to hold in the future as the amount of non-renewable resources in the earth's crust is ultimately finite. Scarcity will become increasingly important, and the scarcity rent will be positive even in the present. However, for understanding current prices and consumption patterns, current expectations about future developments are important. Given that the quantities of available resources indicated in Table (1) are very large, their ultimate end far in the future does not affect behavior today. Furthermore, when resources in the earth's crust are exhausted, so much time will have passed that technology might have developed to a point where the earth's crust, which makes up one percent of the Earth's mass, is no longer a limit to resource extraction. Deeper parts of the planet or even extraterrestrial sources might be explored. These speculative considerations are not crucial for our model. What is important is that the relation from Proposition 1 has held in the past and looks likely to hold for the foreseeable future. Since in the long term, extracted resources equal the resources added to the resource stock due to R&D in extraction technology, the price for a unit of the resource will equal the extraction cost plus the per-unit cost of R&D and hence, stay constant in the long term. This also explains why scarcity rents cannot be found empirically as shown in Hart and Spiro (2011).

5 Conclusion

This paper examines the long-term evolution of prices and production of major non-renewable resources from a theoretical and empirical perspective. We argue that economic growth causes the production and use of a non-renewable resource to increase exponentially, and its production costs to stay constant in the long term. Economic growth enables firms to invest in R&D in extraction technology, which makes resources from mineral occurrences of lower grades extractable. We explain the long-term evolution of non-renewable resource prices and world production for more than 200 years. If historical trends in technological progress continue, it is possible that non-renewable resources are, within a time frame relevant for humanity, *de facto* inexhaustible.

Our model makes four major simplifications, which should be examined in more detail in future extensions. First, there is no uncertainty in R&D development and therefore, no need to keep a positive stock of the resource. When R&D development is stochastic (as in Dasgupta and Stiglitz (1981)), there would be a need for firms to keep stocks. Second, our model features full competition in the extractive sector. We could obtain a model with monopolistic competition in the extractive sector by introducing privately-owned mineral occurrences. A firm would need to pay a certain upfront cost or exploration cost in order to acquire a mineral occurrence (see, e.g., Cairns and Quyen (1998) and Slade (1988)). This upfront cost would give technology firms a certain monopoly power, as they develop machines that are specific to single mineral occurrences. Third, extractive firms could face a trade-off between accepting high extraction costs due to a lower technology level and investing in R&D to lower extraction costs. The general extraction technology function in Equation 5 provides the basis to generalize this assumption. Farzin et al. (1998) and Doraszelski (2004) treat similar problems. Finally, our model does not include recycling. Recycling will likely become more important for metal production due to the increasing abundance of recyclable materials and the comparatively low energy requirements to recycle the items (see Steinbach and Wellmer, 2010; Wellmer and Dalheimer, 2012). Introducing recycling into our model would further strengthen our argument, as it increases the available stock of the non-renewable resource.

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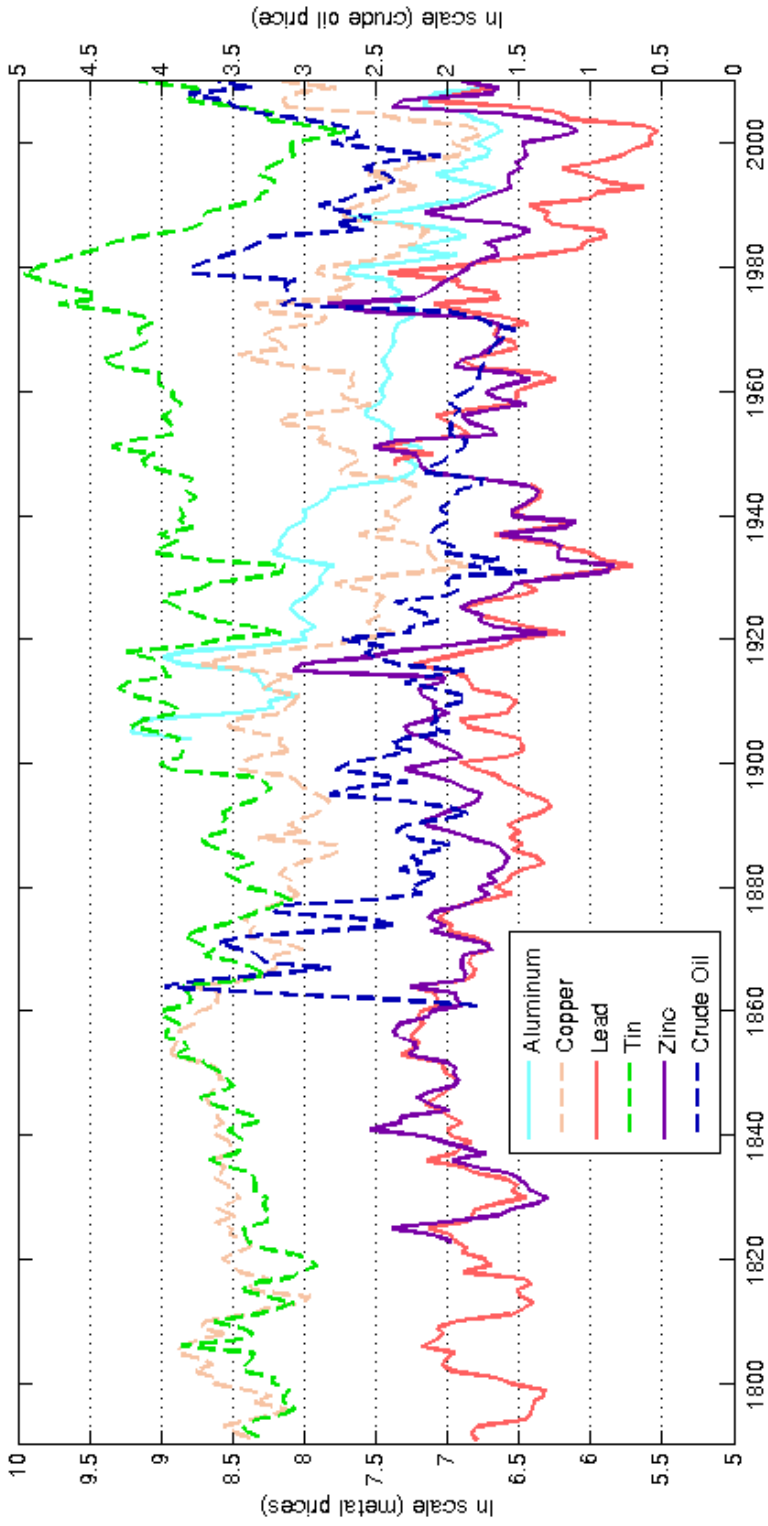
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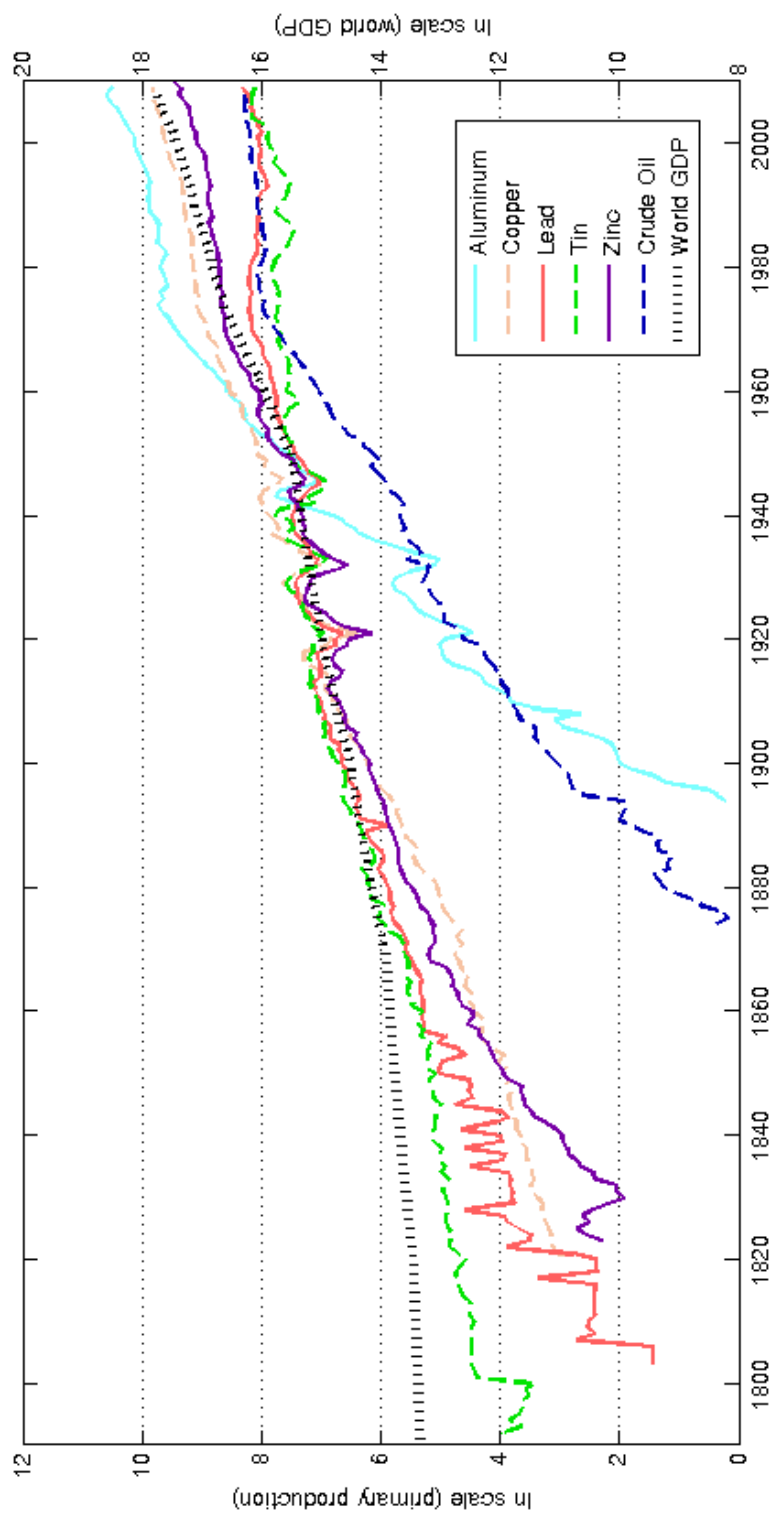
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Appendix 1 Figures and Tables



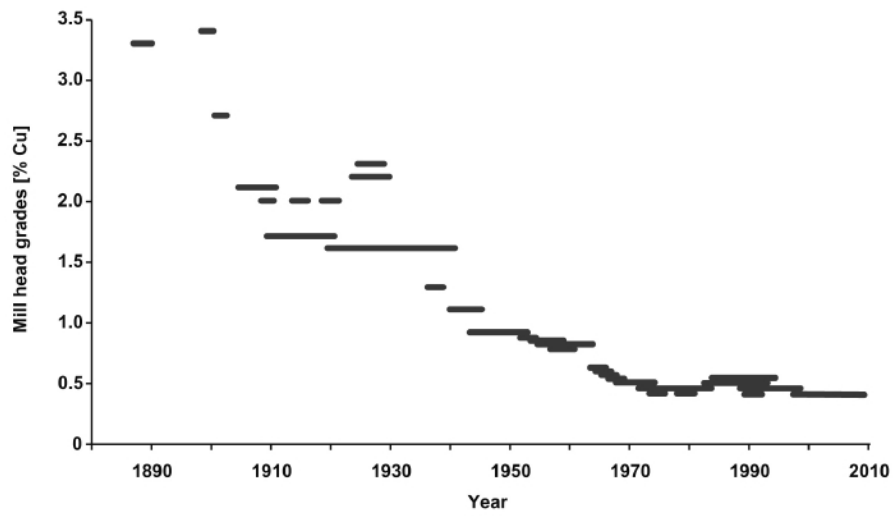
Notes: All prices, except for the price of crude oil, are prices of the London Metal Exchange and its predecessors. As the price of the London Metal Exchange used to be denominated in Sterling in earlier times, we have converted these prices to U.S.-Dollar by using historical exchange rates from Officer (2011). We use the U.S.-Consumer Price Index provided by Officer and Williamson (2011) and the U.S. Bureau of Labor Statistics (2010) for deflating prices with the base year 1980-82. The secondary y-axis relates to the price of crude oil. For data sources and description see Stuermer (2013).

Figure 1: Real prices of major mineral commodities from 1790 to 2009 in natural logs.



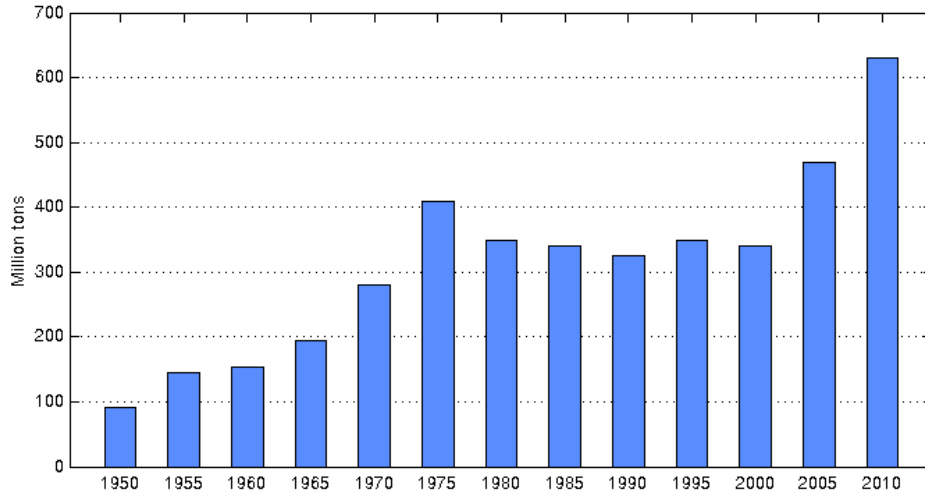
For data sources and description see Stuermer (2013).

Figure 2: World primary production of non-renewable resources and world GDP from 1790 to 2009 in logs.



Source: Scholz and Wellmer (2012).

Figure 3: The historical development of mining of various grades of copper in the U.S.



Sources: Tilton and Lagos C.C. (2007), U.S. Geological Survey (2011b).

Figure 4: Historical evolution of world copper reserves from 1950 to 2010.

	Reserves/ Annual production (Years)	Resources/ Annual production (Years)	Crustal abundance/ Annual production (Years)
Aluminum	139 ^{1a}	263,000 ^{1a}	48,800,000,000 ^{bc}
Copper	43 ^a	189 ^a	95,000,000 ^{ab}
Iron	78 ^a	223 ^a	1,350,000,000 ^{ab}
Lead	21 ^a	362 ^a	70,000,000 ^{ab}
Tin	17 ^a	“Sufficient” ^a	144,000 ^{ab}
Zinc	21 ^a	158 ^a	187,500,000 ^{ab}
Gold	20 ^d	13 ^d	27,160,000 ^{ef}
Rare earth elements ²	827 ^a	“Very large” ^a	n.a.
Coal ³	129 ^g	2,900 ^g	
Oil ⁴	55 ^g	76 ^g	} 1,400,000 ⁶ⁱ
Gas ⁵	59 ^g	410 ^g	

Notes: Reserves include all material which can currently be extracted. The definition of resources can be found in Section 2.2. Sources: ^aU.S. Geological Survey (2012), ^bPerman et al. (2003), ^cU.S. Geological Survey (2011c), ^dU.S. Geological Survey (2011b), ^eNordhaus (1974), ^fU.S. Geological Survey (2010), ^gBGR = Federal Institute for Geosciences and Natural Resources (2011) ^hLittke and Welte (1992). Notes: ¹ data for bauxite, ² rare earth oxide, ³ includes lignite and hard coal, ⁴ includes conventional and unconventional oil, ⁵ includes conventional and unconventional gas, ⁶ all organic carbon in the earth’s crust.

Table 1: Availability of selected non-renewable resources in years of production left in the reserve, resource and crustal mass at the current mine production rate.

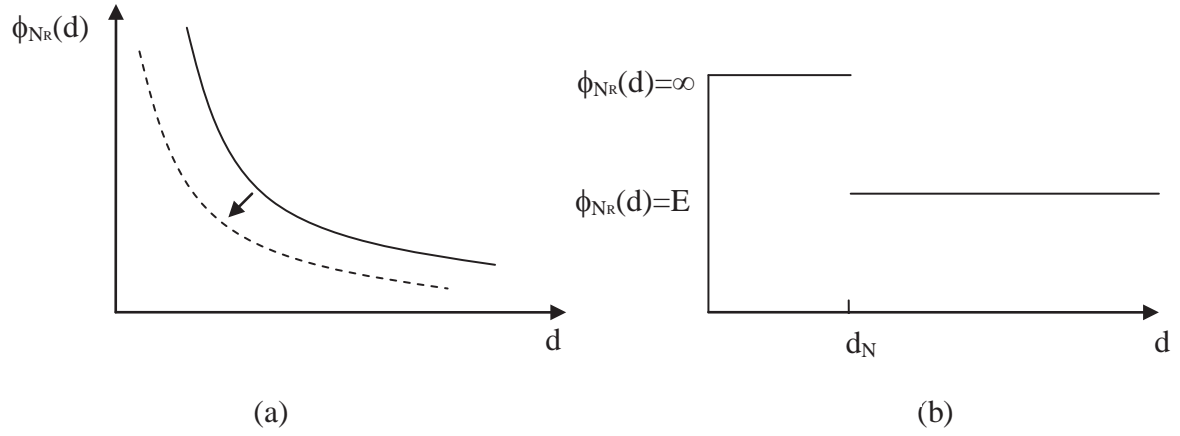


Figure 5: Extraction costs ϕ_{N_R} as a function of deposits of different grades d . General and simplified form.

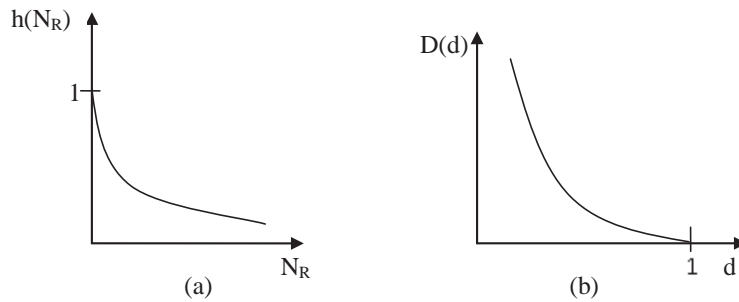
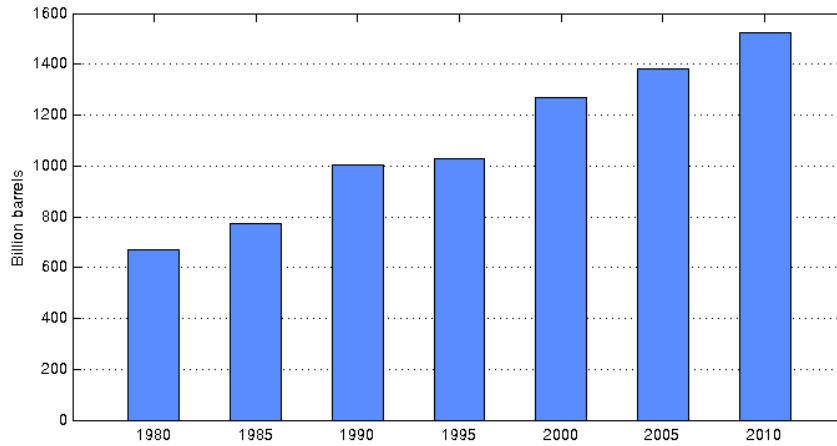


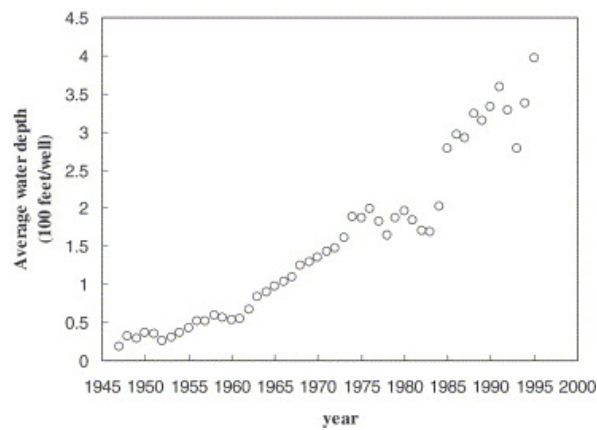
Figure 6: (a) Extractable mineral occurrences of grade $h(N_R)$ as a function of the state of technology N_R . (b) The extractable amount of the non-renewable resource in the earth's crust $D(d)$ at a given grade d of the mineral occurrences.

Appendix 2 Additional figures



Source: British Petroleum (2011).

Figure 7: Historical evolution of oil reserves, including Canadian oil sands from 1980 to 2010.



Source: Managi et al. (2004).

Figure 8: Average water depth of wells drilled in the Gulf of Mexico.

Appendix 3 Regression results

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil
Range		1905-2009	1792-2009	1792-2009	1792-2009	1824-2009	1862-2009
Constant	Coeff.	-1.774	0.572	0.150	1.800	1.072	8.242
	t-stat.	(-0.180)	(0.203)	(0.052)	(0.660)	(0.205)	(0.828)
Lin.Trend	Coeff.	0.008	0.009	0.016	0.001	0.014	-0.021
	t-stat.	(0.137)	(0.428)	(0.714)	(0.069)	(0.357)	(-0.317)
Range		1905-2009	1850-2009	1850-2009	1862-2009	1850-2009	1850-2009
Constant	Coeff.	-1.299	0.109	-0.268	2.439	1.894	7.002
	t-stat.	(-0.200)	(0.030)	(-0.073)	(0.711)	(0.407)	(1.112)
Lin.Trend	Coeff.	0.008	0.020	0.030	-0.004	0.013	-0.021
	t-stat.	(0.137)	(0.518)	(0.755)	(-0.109)	(0.267)	(-0.317)
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	-0.903	-1.428	-0.490	1.068	2.764	-1.974
	t-stat.	(-0.239)	(-0.332)	(-0.102)	(0.269)	(0.443)	(-0.338)
Lin.Trend	Coeff.	0.008	0.055	0.054	0.010	0.010	0.100
	t-stat.	(0.137)	(0.820)	(0.713)	(0.168)	(0.099)	(1.106)
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	2.269	1.556	-3.688	-0.061	-0.515	3.445
	t-stat.	(0.479)	(0.240)	(-0.505)	(-0.011)	(-0.062)	(0.354)
Lin.Trend	Coeff.	-0.055	0.041	0.198	0.049	0.103	0.090
	t-stat.	(-0.411)	(0.225)	(0.958)	(0.307)	(0.441)	(0.326)
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	-0.549	1.323	0.370	3.719	1.136	-1.111
	t-stat.	(-0.088)	(0.266)	(0.081)	(0.812)	(0.176)	(-0.176)
Lin.Trend	Coeff.	-0.003	0.011	0.030	-0.012	0.051	0.094
	t-stat.	(-0.033)	(0.135)	(0.383)	(-0.152)	(0.468)	(0.875)

Notes: The table presents coefficients and *t*-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 2: Tests of the stylized fact that the growth rates of real prices of mineral commodities equal zero and do not follow a statistically significant trend.

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.464	4.86	16.045	4.552	30.801	35.734	0.128
	t-stat.	*** 3.810	*** 2.694	*** 3.275	* 2.231	** 2.58	*** 4.365	0.959
Lin.Trend	Coeff.	-0.221	-0.006	-0.087	-0.016	-0.174	-0.182	0.018
	t-stat.	** -2.568	-0.439	** -2.294	-0.999	* -1.975	*** -3.334	*** 16.583
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.464	5.801	6.032	3.569	5.579	25.198	0.995
	t-stat.	*** 3.810	*** 3.461	***3.371	* 2.185	*** 3.774	*** 4.81	*** 5.49
Lin.Trend	Coeff.	-0.221	-0.018	-0.038	-0.015	-0.021	-0.182	0.019
	t-stat.	** -2.568	-1.007	-1.938	-0.833	-1.308	*** -3.334	*** 9.797
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	19.703	5.965	2.980	2.844	4.44	9.883	2.004
	t-stat.	*** 5.498	*** 2.651	* 2.043	1.361	* 2.225	*** 6.912	*** 7.8
Trend	Coeff.	-0.178	0.035	-0.019	-0.015	-0.018	-0.083	0.018
	t-stat.	*** 3.174	-0.995	-0.853	-0.464	-0.592	*** -3.711	***4.549
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	10.781	5.043	13.205	0.051	5.675	9.897	4.729
	t-stat.	*** 7.169	*** 4.979	*** 2.936	0.028	*** 4.619	*** 9.574	*** 12.89
Lin.Trend	Coeff.	-0.171	-0.057	-0.48	0.04	-0.078	-0.196	-0.028
	t-stat.	*** -3.999	-1.978	-1.553	0.768	* -2.255	*** -6.64	*** -2.724
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.75	6.307	3.851	3.762	4.384	12.272	1.244
	t-stat.	*** 4.846	** 2.543	1.938	1.664	* 2.032	*** 4.060	*** 5.509
Lin.Trend	Coeff.	-0.53	-0.024	-0.018	-0.026	-0.005	-0.072	0.027
	t-stat.	*** -2.974	-0.566	-0.536	-0.66	-1.26	-1.403	***7.045

Notes: The table presents coefficients and t -statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 3: Tests for the stylized facts that growth rates of world primary production and world GDP are equal to zero and trendless.

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.301	5.474	20.57	4.427	30.7	35.689	0.032
	t-stat.	*** 3.824	*** 3.06	*** 3.845	* 2.181	** 2.584	*** 4.379	0.276
Lin.Trend	Coeff.	-0.229	-0.018	-0.125	-0.023	-0.182	-0.19	0.01
	t-stat.	*** -2.677	-1.367	*** -3.025	-1.457	* -2.071	*** -3.499	*** 11.066
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.301	5.399	5.629	3.179	5.18	24.681	0.628
	t-stat.	*** 3.824	*** 3.254	***3.169	1.961	*** 3.541	*** 4.733	*** 4.052
Lin.Trend	Coeff.	-0.229	-0.027	-0.047	-0.024	-0.03	-0.19	0.01
	t-stat.	*** -2.677	-1.523	** -2.442	-1.348	-1.895	*** -3.499	*** 5.876
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	18.595	4.985	2.028	1.903	3.473	8.869	1.071
	t-stat.	*** 5.242	* 2.241	1.41	0.918	1.763	*** 6.306	*** 4.862
Trend	Coeff.	-0.184	-0.042	-0.027	-0.023	-0.026	-0.09	0.01
	t-stat.	*** -3.315	-1.214	-1.186	-0.694	-0.404	*** -4.084	*** 3.01
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	8.583	2.952	1.141	-1.954	3.578	7.716	2.632
	t-stat.	*** 5.742	*** 2.892	1.04	1.086	*** 2.87	*** 7.493	*** 7.444
Lin.Trend	Coeff.	-0.156	-0.044	-0.35	0.051	-0.065	-0.18	-0.016
	t-stat.	*** -3.667	-1.515	-1.129	0.997	-1.819	*** -6.14	-1.551
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.004	5.854	3.413	3.317	3.942	11.789	0.834
	t-stat.	*** 4.81	** 2.386	1.738	1.480	1.851	*** 3.933	*** 4.509
Lin.Trend	Coeff.	-0.542	-0.038	-0.032	-0.039	-0.019	-0.086	0.013
	t-stat.	*** -3.06	-0.908	-0.959	-1.028	-0.517	-1.691	***4.004

Notes: The table presents coefficients and t -statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 4: Tests for the stylized fact that growth rates of world per capita primary production and world per capita GDP are equal to zero and trendless.

Appendix 4 Proofs

Proof of Proposition 1

$$\begin{aligned} D(d_{N_{Rt}}) &= -\delta_2 \ln(d_{N_{Rt}}) \\ &= -\delta_2 \ln(e^{-\delta_1 N_{Rt}}) \\ &= \delta_1 \delta_2 N_{Rt} \end{aligned}$$

□

Proof of Proposition 2

The final good producer demands the resource for aggregate production. The price of the final good is the numeraire. The first order condition with respect to the resource from production (see Equation 16) is

$$Y^{\frac{1}{\varepsilon}}(1-\gamma)R^{-\frac{1}{\varepsilon}} - p_R = 0, \quad (28)$$

so that the demand for the resource is

$$R = \frac{Y(1-\gamma)^\varepsilon}{p_R^\varepsilon}. \quad (29)$$

Assume that initially, the resource stock available to the extractive firms is zero, $S_t = 0$. Revenues are given by $p_R R$ and expenses are given by $M_R = \frac{1}{\eta_R} \dot{N}_R$ in terms of the final good. Given the machine price from Equation 23, the per-unit production cost of the resource is

$$\left(\frac{1}{\eta_R} + \psi\right) \frac{1}{\delta_1 \delta_2} = \frac{1 + \psi \eta_R}{\eta_R \delta_1 \delta_2}. \quad (30)$$

The extractive firms make profits

$$\pi_{Rt} = p_R R_t - \frac{1 + \psi \eta_R}{\eta_R \delta_1 \delta_2} X_t. \quad (31)$$

Since the stock of the resource S cannot be negative, newly acquired resources cannot be less than the resources sold to the final good producer: $X_t \geq R_t$. Newly acquired resources in excess of those sold could be stored. In a world without uncertainty, however, this would not

be profitable. The price therefore must be equal to marginal cost:

$$p_R = \frac{1 + \psi\eta_R}{\eta_R\delta_1\delta_2} . \quad (32)$$

It remains to consider the case of a positive initial stock of the resource, $S_t > 0$. Under perfect competition, this stock is immediately sold off to the final good producer such that the case of $S_t = 0$ returns. \square

Proof of Proposition 3

The first order conditions (FOC) of the final good producer for the optimal input of Z and R are $Y^{\frac{1}{\varepsilon}}\gamma Z^{-\frac{1}{\varepsilon}} - p_Z = 0$ and $Y^{\frac{1}{\varepsilon}}(1 - \gamma)R^{-\frac{1}{\varepsilon}} - p_R = 0$, where the final good is the numeraire. From this the relative price is

$$p = \frac{p_R}{p_Z} = \frac{1 - \gamma}{\gamma} \left(\frac{R}{Z} \right)^{-\frac{1}{\varepsilon}} . \quad (33)$$

Setting the price of the final good as the numeraire gives (for the derivation of the price index see the derivation of Equation (12.11) in Acemoglu (2009)):

$$[\gamma^\varepsilon p_Z^{1-\varepsilon} + (1 - \gamma)^\varepsilon p_R^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = P = 1 . \quad (34)$$

The intermediate goods sector

As in Acemoglu (2009), the maximization problem in the intermediate goods sector is:

$$\max_{L, \{x_Z(j)\}} p_Z Z - w_Z L - \int_0^{N_Z} \chi_Z(j) x_Z(j) dj . \quad (35)$$

The FOC with respect to $x_Z(j)$ is $p_Z x_Z(j)^{-\beta} L^\beta - \chi_Z(j) = 0$ so that

$$x_Z(j) = \left(\frac{p_Z}{\chi_Z(j)} \right)^{\frac{1}{\beta}} L . \quad (36)$$

From the FOC with respect to L we obtain the wage rate

$$w_Z = \frac{\beta}{1 - \beta} p_Z \left(\int_0^{N_Z} x_Z(j)^{-\beta} dj \right) L^{\beta-1} . \quad (37)$$

The profits of the technology firms are:

$$\pi_Z(j) = (\chi_Z(j) - \psi)x_Z(j) . \quad (38)$$

Substituting Equation 36 into Equation 38 we calculate the FOC with respect to the price of a machine $\chi_Z(j)$: $\left(\frac{p_Z}{\chi_Z(j)}\right)^{\frac{1}{\beta}} L - (\chi_Z(j) - \psi)p_Z^{\frac{1}{\beta}} \frac{1}{\beta} \chi_Z(j)^{\frac{1}{\beta}-1} L = 0$. Solving this for $\chi_Z(j)$ yields $\chi_Z(j) = \frac{\psi}{1-\beta}$. Following Acemoglu (2002) we normalize $\psi = 1 - \beta$ so that $\chi_Z(j) = 1$. Combining this result with Equations 36 and 38 we write profits as

$$\pi_Z(j) = \beta p_Z^{\frac{1}{\beta}} L . \quad (39)$$

The present discounted value is:

$$rV_Z - \dot{V}_Z = \pi_Z . \quad (40)$$

The steady state ($\dot{V} = 0$) is:

$$V_Z = \frac{\beta p_Z^{\frac{1}{\beta}} L}{r} . \quad (41)$$

Substituting Equation 36 into Equation 17 yields

$$Z = \frac{1}{1-\beta} p_Z^{\frac{1-\beta}{\beta}} N_Z L . \quad (42)$$

Solving for the variables of the intermediate goods sector

Solving Equation 34 for p_Z yields

$$p_Z = \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} p_R \right)^{\frac{1}{1-\varepsilon}} . \quad (43)$$

This can be used, together with the expression for R from Equation 29 and the expression for p_R from Equation 32 to determine Z as a function of Y from Equation 33. We obtain the range of machines N_Z as a function of Y from Equation 42.

The growth rate

The consumer earns wages from working in the sector which produces good Z and earns interest on investing in the technology N_Z . The budget constraint thus is $C = w_Z L + rM$.

Maximizing utility in Equation 15 with respect to consumption and investments yields the first order conditions $C^{-\theta}e^{-\rho t} = \lambda$ and $\dot{\lambda} = -r\lambda$ so that the growth rate of consumption is

$$g_c = \theta^{-1}(r - \rho) . \quad (44)$$

This will be equal to output growth on the balanced growth path. We can thus solve for the interest rate and obtain $r = \theta g + \rho$. The free entry condition for the technology firms imposes that profits from investing in patents must be zero. Revenue per unit of R&D investment is given by V_Z , cost is equal to $\frac{1}{\eta_Z}$. Consequently, we have $\eta_Z V_Z = 1$. Substituting Equation 41 into it we obtain $\frac{\eta_Z \beta p_Z^{\frac{1}{\beta}} L}{r} = 1$. Solving this for r and substituting into Equation 44 we obtain

$$g = \theta^{-1}(\beta \eta_Z L p_Z^{\frac{1}{\beta}} - \rho) . \quad (45)$$

Plugging this in Equation 43 yields the growth rate. □

Proof of Proposition 4

Substitute Equation 32 into Equation 29. □

Proof of Proposition 5

The total cost of extracting resources can be split into the price of the new machine and the extraction cost. The technology costs have been derived in Proposition 1 as proportional to R&D in extraction technology. The extraction cost is given by the constant E . Since the extraction cost is constant and this model focusses on the innovation side, we make the simplifying assumption of zero extraction cost, $E = 0$. Therefore the total cost is given by the cost for the new machine.

The extractive firms sell the resource R , to the final good producer at price p_R . Its total revenues are thus $R p_R$. The expenses are given by the price of a machine, $\chi_R(j)$ times the number of machines bought, $\dot{N}_{Rt} x_R(j)$ with $x_R(j) = 1$. Total expenses are thus $\frac{1}{\chi_R(j)} \dot{N}_{Rt}$. The extraction firms are in perfect competition, just like firms in the intermediate goods sector. Therefore profits are zero, its revenues must equal expenses: $R p_R = \chi_R(j) \dot{N}_{Rt}$. Inserting Equation 21 we obtain $\delta_1 \delta_2 \dot{N}_{Rt} x_R(j) p_R = \chi_R(j) \dot{N}_{Rt}$, so that $p_R = \frac{1 + \psi \eta_R}{\eta_R \delta_1 \delta_2}$. □

Proof of Proposition 6

We use Equation 43, together with the expression for R from Equation 29 and the expression for p_R from Equation 32 to determine Z as a function of Y from Equation 33. This can then be used to obtain the range of machines N_Z as a function of Y from Equation 42.

The expression for \dot{N}_R follows from equation 14, Proposition 2 as well as equation 29. \square

Proof of Proposition 7

The resource monopolist maximizes Equation 31. A monopolist will spread a potential endowment of $S_t > 0$ over time, but will drive it down to zero nevertheless. In any case, inflow and outflow will have to be balanced in the long run so that $X_t = R_t$. Substituting the expression for R from Equation 29 into Equation 31, the FOC for p_R is

$$\frac{Y(1-\gamma)^\varepsilon}{p_R^\varepsilon} + \left(p_R - \frac{1 + \psi\eta_R}{\eta_R\delta_1\delta_2} \right) (-\varepsilon p_R^{-\varepsilon-1} Y(1-\gamma)^\varepsilon) = 0, \quad (46)$$

which gives

$$p_R^{Mon} = \frac{1 + \psi\eta_R}{\eta_R\delta_1\delta_2} \frac{\varepsilon}{\varepsilon - 1}. \quad (47)$$

\square

Appendix 5 The social planner solution

For comparison we present a social planner solution of the model. Since production has increasing returns to scale as apparent in Equation 51, there is no constant growth rate. This can be seen in Equation 75 below when considering the two preceding equations.

Preferences and budget

Household preferences are

$$\int_0^\infty \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \quad (48)$$

where ρ is the discount rate and θ is the coefficient of relative risk aversion.

The budget constraint is

$$C + I + M \leq Y = \left[\gamma Z^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) R^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (49)$$

where I is aggregate investment into new machines and M is aggregate R&D expenditure. The R&D expenditure is used for research in the production of Z and R : $M = M_Z + M_R$. Aggregate production uses two inputs, intermediate goods Z and resources R , with elasticity of substitution ε and distribution parameter γ .

The production function of good Z

The production function of Z is

$$Z = \frac{1}{1-\beta} \left(\int_0^{N_Z} x_Z(j)^{1-\beta} dj \right) L^\beta, \quad (50)$$

where L is labor. Production inputs are therefore labor and machines $x_Z(j)$ of variety j . The range of machine varieties is denoted N_Z . The social planner chooses the $x_Z(j)$ identical, so that we can write

$$Z = \frac{1}{1-\beta} N_Z x_Z^{1-\beta} L^\beta. \quad (51)$$

Intermediates x_Z depreciate fully after use and the marginal cost of production is the same for all machine varieties and equal to ψ in terms of the final good. Investment in machines is thus given by

$$x_Z \psi = I. \quad (52)$$

The range of intermediates expands through investment in R&D by the following production function

$$\dot{N}_L = \eta_Z M_Z, \quad (53)$$

where M_Z is spending on R&D and η_Z is a cost parameter. One unit of the final good spent for R&D will generate η_Z new varieties of machines.

Production of the resource

The evolution of the resource stock follows:

$$\dot{S}_t = X_t - R_t, \quad S_t \geq 0, X_t \geq 0, R_t \geq 0. \quad (54)$$

The per unit production cost of the resource is as in equation (30):

$$\frac{1}{\eta_R \delta_1 \delta_2}. \quad (55)$$

The cost for R&D in the extractive sector is analogous to R&D in the intermediate goods sector and follows:

$$\dot{N}_R = \eta_R M_R . \quad (56)$$

The social planner chooses $X = R$ such that

$$R_t = X_t = \delta_1 \delta_2 \dot{N}_{Rt} = \frac{1}{\eta_R \delta_1 \delta_2} M_R . \quad (57)$$

The objective function and first order conditions

The social planner maximizes the intertemporal utility from consumption as defined in equation 48 with respect to the endogenous variables C , N_Z , x_Z , R , and M_Z , and subject to the budget constraint:

$$\left[\gamma Z^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) R^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} - C - I - M_Z - \frac{1}{\eta_R \delta_1 \delta_2} R = 0 , \quad (58)$$

where $M = M_Z + \frac{1+\psi\eta_R}{\eta_R \delta_1 \delta_2} R$ is the aggregate R&D expenditure and $I = \psi x_Z + \psi x_R$ is the aggregate expenditure on machines.

The Hamiltonian to be maximized by the social planner is therefore

$$\begin{aligned} H = & \frac{C(t)^{1-\theta} - 1}{1-\theta} \\ & + \lambda \left(\left[\gamma \left(\frac{1}{1-\beta} N_Z x_Z^{1-\beta} L^\beta \right)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) R^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} - C - \psi x_Z - \psi x_R - \right. \\ & \left. - M_Z - \frac{1}{\eta_R \delta_1 \delta_2} R \right) \\ & + \mu \eta_Z M_Z . \end{aligned}$$

The first order conditions are

$$\frac{\partial H}{\partial C} = C^{-\theta} - \lambda = 0, \quad (59)$$

$$\frac{\partial H}{\partial N_Z} = \lambda Y^{\frac{1}{\varepsilon}} \gamma Z^{\frac{\varepsilon-1}{\varepsilon}} N_Z^{-1} = \mu \rho - \dot{\mu}, \quad (60)$$

$$\frac{\partial H}{\partial x_Z} = \lambda Y^{\frac{1}{\varepsilon}} \gamma Z^{\frac{\varepsilon-1}{\varepsilon}} (1 - \beta) x_Z^{-1} - \lambda \psi = 0, \quad (61)$$

$$\frac{\partial H}{\partial R} = \lambda Y^{\frac{1}{\varepsilon}} (1 - \gamma) R^{-\frac{1}{\varepsilon}} - \lambda \frac{1}{\eta_R \delta_1 \delta_2} = 0, \quad (62)$$

$$\frac{\partial H}{\partial M_Z} = -\lambda + \mu \eta_Z = 0. \quad (63)$$

$$(64)$$

Derivation of the growth rate

The FOC for C in growth rates is

$$g_\lambda = -\theta g_C. \quad (65)$$

Substituting the FOC for M_Z into the FOC for N_Z gives

$$g_\mu = \rho - \eta_Z Y^{\frac{1}{\varepsilon}} \gamma Z^{\frac{\varepsilon-1}{\varepsilon}} N_Z^{-1} \quad (66)$$

and

$$g_\lambda = g_\mu. \quad (67)$$

From the FOC for x_Z we obtain

$$Y^{\frac{1}{\varepsilon}} \gamma Z^{\frac{\varepsilon-1}{\varepsilon}} = \frac{x_Z \psi}{(1 - \beta)}. \quad (68)$$

The FOC for R shows that the ratio of resource use and output is constant

$$\frac{R}{Y} = (\eta_R \delta_1 \delta_2 (1 - \gamma))^\varepsilon \quad (69)$$

and consequently that they have the same growth rate:

$$g_R = g_Y . \quad (70)$$

Substituting (69) into the production function (49) yields

$$\left[1 - (1 - \gamma) (\eta_R \delta_1 \delta_2 (1 - \gamma))^{\varepsilon - 1} \right] Y^{\frac{\varepsilon - 1}{\varepsilon}} = \gamma Z^{\frac{\varepsilon - 1}{\varepsilon}} \quad (71)$$

such that

$$Z = z_1 Y , \quad (72)$$

where $z_1 = \left[\frac{1}{\gamma} (1 - (1 - \gamma) (\eta_R \delta_1 \delta_2 (1 - \gamma))^{\varepsilon - 1}) \right]^{\frac{\varepsilon}{\varepsilon - 1}}$ is a constant.

Substituting this into (68) yields

$$x_Z = z_2 Y , \quad (73)$$

where $z_2 = \gamma z_1^{\frac{\varepsilon - 1}{\varepsilon}} (1 - \beta) \frac{1}{\psi}$ is again a constant.

Substituting this in Equation 51 yields

$$N_Z = z_1 (1 - \beta) L^{-\beta} Y^\beta . \quad (74)$$

Therefore $\frac{x_Z}{N_Z}$ is not constant.

Combining Equations 65, 66, 67, and 68, we obtain the growth rate of the economy

$$g^{opt} = \frac{1}{\theta} \left(\frac{1}{\eta_Z (1 - \beta) \psi} \frac{x_Z}{N_Z} - \rho \right) . \quad (75)$$