

# Synthetic or Real?

## The Equilibrium Effects of Credit Default Swaps on Bond Markets \*

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### Abstract

We develop a model in which credit default swaps (CDSs) are non-redundant securities, based on the observation that they are more liquid than the underlying reference bonds. The introduction of a CDS has an ambiguous effect on bond prices: The CDS market (i) crowds out long bond investors with relatively frequent trading needs, (ii) reduces short selling of the bond, and (iii) leads to the endogenous emergence of basis traders who take levered hedged positions in the bond and the CDS. The model generates testable predictions on the effects of CDS introduction on bond prices, turnover in bond and CDS markets, and the CDS-bond basis. The model can also be used to assess policy interventions. For example, a ban on naked CDSs may raise the issuer's borrowing costs.

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# 1 Introduction

Credit Default Swap (CDS) markets, which are essentially insurance markets for the default risk of corporate or sovereign debt, have grown tremendously over the last decade. However, while there is a relatively large literature on the pricing of these securities, much less work has been done on the economic role of CDS markets. In fact, in most pricing models CDSs are redundant securities, such that the introduction of a CDS market has no effects on the underlying bond market. This irrelevancy feature makes a meaningful analysis of the economic role of CDS markets difficult.

In this paper, we develop a theory of *non-redundant* CDS markets. Our model of bond and CDS markets builds on a simple, well-documented fact: Trading bonds is expensive relative to trading CDSs. Based on this observation, we develop a theory of bond and CDS markets that generates a rich set of predictions regarding the economic role of CDS markets, the effect of CDS markets on the financing cost for bond issuers, the relative pricing of the CDS and the underlying bond (the CDS-bond basis), and trading and turnover in the bond and CDS markets. The model also provides a framework to assess policy interventions, such as the recent E.U. ban of naked CDS positions.

In our model, investors differ across two dimensions, beliefs about default the bond's default probability and investment horizons. First, optimistic investors view the default of the bond as unlikely, while pessimists think that a default is relatively more likely. Second, investors with longer investment horizons (buy-and-hold investors, such as insurance companies) are unlikely to have to sell their position in the future, while investors with shorter investment horizons (active traders and arbitrageurs, such as hedge funds) are more likely to receive liquidity shocks that force them to sell their position.

If only the bond is traded, relatively optimistic investors with sufficiently long trading horizons buy the bond, whereas relatively pessimistic investors with sufficiently long trading horizons take short positions in the bond. Investors with short investment horizons stay out of the market, because for them the transactions costs of trading the bond are too high. In equilibrium, the bond price reflects the average marginal investor's views on default probabilities and the expected future trading costs the average marginal investor incurs over the lifetime of the bond.

When a CDS on the bond is also traded, the equilibrium changes in a number of ways. Specifically, our model shows that the introduction of the CDS has three separate effects on the set of investors that takes positions in the underlying bond: (1) Some investors who previously held a long position in the bond switch to selling CDS protection, putting downward pressure on the bond price. (2) Because of its higher transaction costs, in equilibrium the bond trades at a discount relative to the CDS. Hence, investors who previously shorted the bond switch to buying CDS protection, a more efficient negative bet. The resulting reduction in short selling puts upward pressure on the bond price. (3) Some investors become “negative basis traders” who hold a long position in the bond *and* purchase CDS protection. The model thus endogenously generates the negative basis trade, which has been an immensely popular trading strategy in recent years. If basis traders cannot take leverage, they are price neutral (they do not affect the price of the underlying bond). However, if basis traders can take leverage—a reasonable assumption given that they hold hedged positions—they push up the bond price. In practice, basis trades are often highly levered, although the leverage they can take varies with financial conditions.

Overall, these three effects lead to an ambiguous change in the price of the underlying bond, consistent with the empirical literature, which has found no strong average effect of CDS introduction on bond or loan spreads (Hirtle (2009), Ashcraft and Santos (2009)). Moreover, by characterizing these separate effects of CDS introduction, the model may be helpful in guiding future empirical research on the effect of CDS introduction on the cost of capital.

From an asset pricing perspective, the prediction that the equilibrium price of the bond is (weakly) less than the price of a synthetic bond consisting of a risk-free bond and a short position in the CDS replicates a well-documented empirical phenomenon known as the negative CDS-bond basis (see, e.g., Bai and Collin-Dufresne (2010) and Fontana (2011)). Our model generates a number of predictions regarding both the time-series and cross-sectional variation in the CDS-bond basis: The negative basis decreases in the amount of leverage basis traders can take and increases in bond illiquidity and disagreement about the bond’s default probability.

The endogenous emergence of leveraged basis traders highlights a novel economic role of CDS markets: The introduction of a relatively more liquid derivative market allows buy-and-hold investors who are efficient holders of the illiquid bond to hedge undesired credit risk in the more liquid CDS

market. In the CDS market, the average investor is relatively optimistic about default probabilities, but is not an efficient holder of the bond because of more frequent trading needs. The role of CDS markets is thus similar to liquidity transformation—by repackaging the bond’s default risk into a more liquid security, they allow the transfer of credit risk from efficient holders of the bond to relatively more optimistic shorter-term investors, such as hedge funds. Hence, when bonds are illiquid, a liquid CDS can improve the allocation of credit risk and thus presents an alternative to recent proposals that aim at making the bond market more liquid, for example through standardization.<sup>1</sup> Note that this liquidity view of CDS markets proposed in this paper differs from the traditional view that the main function of CDSs is simply the separation of credit risk and interest rate risk.<sup>2</sup> One reason why the separation of credit and interest rate risk is unlikely to be the full story is that, in this case, the presence of an interest rate swap would make the CDS redundant.

In addition to the predictions on bond and CDS prices, our model generates testable predictions on trading volume in the bond and the CDS markets. First, we show that CDS introduction decreases turnover in the underlying bond. This is the case because investors with more frequent trading needs opt for holding and trading the CDS. This prediction is consistent with recent evidence in [Das, Kalimipalli, and Nayak \(2013\)](#), who document that CDS introduction has resulted in a reduction in turnover in underlying bonds. Second, the model implies that CDS turnover is higher than bond turnover, which is also consistent with empirical evidence. Based on CDS trading data from the DTCC, [Oehmke and Zawadowski \(2013\)](#) report that average monthly CDS turnover is given by 50.8% per month, whereas average monthly turnover in the associated bonds is around 3.5% per month.

Finally, our model provides a framework to study regulatory interventions with respect to CDS markets. For example, a ban on naked CDS positions, as recently imposed by the European Union on sovereign bonds through EU regulation 236/2012, may, in fact, *raise* yields for affected issuers. The reason is that if pessimists cannot take naked CDS positions, they will short the bond, thereby exerting downward price pressure on bond prices. We also explore the effects of banning CDS markets altogether and banning both CDS markets and short positions in the underlying bond. In general, also these interventions have ambiguous effects on bond yields.

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<sup>1</sup>Standardized bonds were recently proposed by the investment management firm BlackRock (“Setting New Standards: The Liquidity Challenge II,” May 2013).

<sup>2</sup>See, e.g., the “Credit Derivatives Handbook” by JPMorgan, Corporate Quantitative Research (2006).

Our paper contributes to the recent theory literature that explores settings in which CDSs (or other derivatives) are non-redundant. [Che and Sethi \(2011\)](#) develop a model in which naked CDSs increase a firm’s cost of capital because they crowd out demand from optimistic bond investors, who use some of their capital to take the other side of bets in the CDS market instead of purchasing the bond. In [Gârleanu and Pedersen \(2011\)](#), derivatives are priced differently from the underlying securities because of exogenous differences in margin requirements. In [Banerjee and Graveline \(2013\)](#), derivatives relax binding short selling constraints when the underlying security is scarce (on “special”). Our paper differs from these papers in its specific focus on CDS markets and the source of non-redundancy, which in our framework comes in the form of trading costs in the spirit of [Amihud and Mendelson \(1986\)](#). In this aspect, our paper is thus related to models in which investors can choose between economically similar markets with differences in transaction costs, such as [Vayanos and Wang \(2007\)](#).

More generally, our paper contributes to a growing literature on the economic effects of CDS markets. [Duffee and Zhou \(2001\)](#), [Morrison \(2005\)](#), [Parlour and Plantin \(2008\)](#), [Parlour and Winton \(2013\)](#), and [Thompson \(2009\)](#) explore how banks can use CDS markets to lay off credit risk. [Allen and Carletti \(2006\)](#) investigate how the availability of CDSs affects financial stability. [Bolton and Oehmke \(2011\)](#) explore the effect of CDSs on the debtor-creditor relationship. [Fostel and Geanakoplos \(2012\)](#) show how the introduction of CDSs can contribute to financial boom and bust cycles. [Zawadowski \(2013\)](#) develops a model in which CDSs are used to hedge exposures in a financial network. [Atkeson, Eisfeldt, and Weill \(2012\)](#) provide a model of the market structure of CDS and, more generally, OTC derivative markets. Finally, general overviews of CDS markets and current policy debates on CDSs can be found in [Stulz \(2010\)](#), [Jarrow \(2011\)](#), and [Bolton and Oehmke \(2013\)](#).<sup>3</sup>

## 2 Model Setup

We consider a financial market with (up to) two assets: a defaultable bond and a CDS that references the bond. The main difference between these two securities lies in their liquidity. As pointed out by

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<sup>3</sup>There is also a growing empirical literature on price discovery in in CDS markets ([Acharya and Johnson \(2007\)](#), [Hilscher, Pollet, and Wilson \(2012\)](#)), the effect of CDSs on loan or credit spreads ([Hirtle \(2009\)](#), [Ashcraft and Santos \(2009\)](#)), the effect of CDSs on leverage and maturity ([Saretto and Tookes \(2013\)](#)), the determinants of CDS market existence and CDS positions ([Oehmke and Zawadowski \(2013\)](#)), the CDS-bond basis ([Blanco, Brennan, and Marsh \(2005\)](#), [Bai and Collin-Dufresne \(2010\)](#), [Fontana \(2011\)](#), [Li, Kim, and Zhang \(2010\)](#)), and the effect of CDS markets on bond market efficiency and liquidity ([Das, Kalimipalli, and Nayak \(2013\)](#)).

Ashcraft and Santos (2009): “Liquidity in the bond market has been limited because many investors hold their bonds until maturity. The secondary market for loans has experienced rapid growth in the recent years, but bank loans remain largely illiquid. Under these circumstances, the development of the CDS market provided banks and investors with a new, less expensive, way to hedge or lay off their risk exposures to firms.”

## 2.1 Bond

We assume that a defaultable bond is traded in positive net supply  $S > 0$ . The bond’s equilibrium price is denoted by  $p$ . The bond matures with Poisson arrival rate  $\lambda$ . As will become clear below, the assumption of Poisson maturity is convenient because it guarantees stationarity. None of our results depend on this assumption. Before maturity the bond pays a continuous coupon rate  $r$ . At maturity, the bond pays back its face value of \$1 with probability  $1 - \pi$ . With probability  $\pi$ , the bond defaults and pays 0.

We capture the widely documented illiquidity of the bond market<sup>4</sup> by assuming that bond investors who have to trade the bond in the secondary market incur trading costs in the spirit of Amihud and Mendelson (1986). We interpret these trading costs broadly as reflecting both the relatively large bid-ask spreads for bonds and the market impact of executing the bond trade. In practice, such trading costs are significant. For example, Bao, Pan, and Wang (2011) estimate effective trading costs for corporate bonds to be between 74 and 221 basis points, depending on the size of the trade, even though their sample only contains the most liquid bonds in the TRACE database. Anecdotally, transactions costs for less liquid bonds are even higher.

To capture bond illiquidity, we assume that investors incur a trading cost  $c_l$  when selling a long position in the bond before maturity (they do not incur this cost if they hold the bond to maturity). Investors who short the bond, on the other hand, incur a trading cost  $c_s$  irrespective of whether they hold the short position until maturity. This reflects that they have to purchase the bond in the secondary market even if they hold the short position to maturity. In addition,  $c_s$  may capture additional costs that short sellers incur, such as lending fees that they may have to pay to the lender of

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<sup>4</sup>See, e.g., Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007), Mahanti, Nashikkar, Subrahmanyam, Chacko, and Mallik (2008), Bao, Pan, and Wang (2011)

the bond. Overall, this setup allows for short bond positions to be more costly than long positions—a common view among bond market participants.

## 2.2 Credit default swap

In addition to the bond, a CDS that references the bond is available. The CDS is an insurance contract on the bond’s default risk: It pays off \$1 if the bond defaults at maturity and zero otherwise. For simplicity we assume that the CDS is perpetual—once entered, it provides default protection over the entire lifetime of the bond.<sup>5</sup> We denote the CDS’s equilibrium price by  $q$ .<sup>6</sup>

The CDS differs from the bond in two respects. First, because the CDS is a derivative contract, it is in *zero net supply*. Second, the CDS has *lower trading costs* than the bond. Transaction costs in the CDS market are lower for two reasons. The first is standardization: The bonds of a firm are usually fragmented into a number of different issues with different coupons, maturities, covenants, and embedded options, which reduces their liquidity. The CDS market, on the other hand, provides a standardized venue for the firm’s credit risk (see [Stulz \(2010\)](#) and [Oehmke and Zawadowski \(2013\)](#)). Second, a CDS investor who has to terminate an existing CDS position prior to maturity rarely sells his CDS in the secondary market; rather than selling the original contract he simply enters an offsetting CDS contract. Because of these differences, it has become standard in the asset pricing literature to assume that the CDS market is perfectly liquid (see, e.g., [Longstaff, Mithal, and Neis \(2005\)](#)). While this is, of course, not literally true (e.g., [Hilscher, Pollet, and Wilson \(2012\)](#) report bid-ask spreads of 4-6 basis points for credit default swaps on IG bonds), in many cases the CDS market is more liquid than the market on the underlying corporate bond.<sup>7</sup>

To capture the relatively high liquidity of the CDS market, we assume that an investor who has to reverse an existing CDS position before maturity incurs a (relatively low) transaction cost  $c_d$ . A CDS

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<sup>5</sup>In practice, CDS contracts have fixed maturities (tenors), the most common being 1, 5 and 10 years. We use the perpetual setup to accommodate, in the easiest possible way, a CDS on a bond with random maturity. More generally, this setup is comparable to one in which investors match maturities of finite-maturity bonds and CDSs.

<sup>6</sup>In practice, CDS premia are paid over time by the protection buyer, with a potential upfront payment at inception of the contract. The CDS price  $q$  should be interpreted as the present value of future CDS premia and the upfront payment.

<sup>7</sup>This liquidity ranking of CDSs relative to bonds is also consistent with evidence in [Blanco, Brennan, and Marsh \(2005\)](#) and [Acharya and Johnson \(2007\)](#), who document that information is incorporated faster in CDS markets than in bond markets.

position that is held to maturity involves no transaction costs (the holder of the CDS can simply opt for cash settlement).

Overall, we assume that the trading costs in the CDS market are lower than in the bond market, and that shorting the bond is (weakly) more expensive than taking a long position:

$$c_s \geq c_l \geq c_d \geq 0. \tag{1}$$

In much of our analysis, we follow Longstaff, Mithal, and Neis (2005) in assuming, for simplicity, that the CDS market involves no transaction costs, such that  $c_d = 0$ .

### 2.3 Investors

There is a mass of risk-neutral, competitive investors who can trade the bond and the CDS. For simplicity, we set the investors' rate of time preference to zero. Investors are heterogeneous across two dimensions: (i) expected holding periods and (ii) beliefs about default probabilities.

With respect to holding periods, we assume that investors are hit by liquidity shocks with Poisson intensity  $\mu_i \geq 0$ . Investors with low  $\mu_i$  can be interpreted as buy-and-hold investors (for example, insurance companies or pension funds), whereas investors with high  $\mu_i$  are investors subject to more frequent liquidity shocks (for example, hedge funds, mutual funds, or other shorter-term investors). When hit by a liquidity shock, an investor has to liquidate his position and exits the model. To preserve stationarity, we assume that a new investor with the same characteristics enters.

With respect to investor beliefs, we assume investors agree to disagree about the bond's default probability in the sense of Aumann (1976). Specifically, investor  $i$  believes that the bond defaults at maturity with probability  $\pi_i \in [\bar{\pi} - \frac{\Delta}{2}, \bar{\pi} + \frac{\Delta}{2}]$ . To bound probabilities between 0 and 1 we assume that  $1 - \frac{\Delta}{2} \geq \bar{\pi} \geq \frac{\Delta}{2}$ . Taken together, an investor type is thus characterized by an expected trading frequency  $\mu_i$  and a subjective default probability  $\pi_i$ .

Investors can take positions in the bond and the CDS, but are subject to portfolio restrictions that reflect risk management constraints.<sup>8</sup> Specifically, we assume that investors can hold up to one unit of credit risk. Accordingly, each investor can either go long one bond, short one bond, buy one CDS, or

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<sup>8</sup>Given risk neutrality and differences in beliefs among investors, absent portfolio restrictions investors would take infinite positions.

sell one CDS. In addition, investors can buy hedged portfolios. One such option is to buy a bond and insure it by also buying a CDS (a so-called negative basis trade). The other potential hedged trade is to short sell a bond and also sell a CDS (a so-called positive basis trade). Because hedged positions do not involve credit risk, we assume that investors lever up such positions up to a maximum leverage of  $L \geq 1$  ( $L = 1$  implies that hedged investors cannot take leverage). Finally, as an outside option investors can always hold cash, which yields a zero return.

Investor types are distributed according to a joint distribution function  $F(\pi, \mu)$  with associated density  $f(\pi, \mu)$ . Many of our results hold under weak conditions on  $f(\pi, \mu)$ . In particular, we will make two main assumptions on  $f$ :

(A1):  $f$  is symmetric around  $\bar{\pi}$  with respect to the disagreement about the default probability.

(A2):  $f$  satisfies a scaling property with respect to disagreement about the default probabilities: An increase in  $\Delta$  leaves the functional form of the density function  $f$  unchanged.

Assumption (A1) is a simple symmetry assumption. The scaling property (A2) guarantees that an increase in the dispersion parameter  $\Delta$  stretches out the distribution symmetrically on both sides. Jointly, these two assumptions imply that for a given trading horizon  $\mu$  the density  $f$  depends on beliefs only through the absolute distance from the mean belief  $|\pi - \bar{\pi}|$  normalized by the belief dispersion parameter  $\Delta$ :

$$f(\mu, \pi) = f\left(\mu, \frac{|\pi - \bar{\pi}|}{\Delta}\right). \quad (2)$$

Note that these properties are satisfied by many common distributions including the uniform and normal distributions.

While not necessary for most of our results, we make an additional distributional assumption that facilitates the calculation of the joint bond and CDS market equilibrium:

(A3): There is an infinite mass of investors with frequent trading needs  $\mu > \bar{\mu}$ , where  $\bar{\mu}$  is a strictly positive number.

Assumption (A3) implies that there is “abundant short-term capital” in the economy that investors are willing to deploy over short horizons. The supply of long-term buy-and-hold capital, on the other

hand, is limited. As we will see below, this assumption greatly simplifies solving for the equilibrium in the CDS market.

## 2.4 A specific investor distribution: Double uniform

While essentially all of our main results hold under (A1) and (A2), occasionally we will use a particularly simple distribution to calculate closed-form solutions for prices and quantities. For this example distribution, we assume that there is a mass one of “buy-and-hold” investors with infrequent trading needs  $\mu_l \geq 0$ , and a mass  $K_H$  of “speculators” with more frequent trading needs  $\mu_H > \mu_L$ . The parameter  $K_H$  allows us to vary the number of speculators relative to buy-and-hold investors. Both buy-and-hold investors’ and speculators’ beliefs regarding the bond’s default probability are distributed uniformly on  $[\bar{\pi} - \frac{\Delta}{2}, \bar{\pi} + \frac{\Delta}{2}]$ . Accordingly, we will refer to this distribution as the “double-uniform” distribution. The double-uniform distribution is illustrated in Figure 1. Note that the double-uniform distribution satisfies assumptions (A1) and (A2). Assumption (A3) is satisfied for  $K_H \rightarrow \infty$ .

[Figure 1 about here.]

## 3 Benchmark: No CDS Market

Before introducing the CDS, we briefly consider the benchmark case in which only the bond trades. We first consider the case in which only long positions in the bond are possible, then the case in which investors can take both long and short positions in the bond.

### 3.1 Only long positions allowed

Investors maximize their utility given their portfolio constraints. In a setup where investors can either hold a long position in the bond or cash, this implies that investors purchase the bond whenever their valuation of the bond exceeds the outside option of holding cash. Investor  $i$ ’s net valuation of a long position in bond is given by

$$V_{\text{longBOND},i} = -p + \frac{r}{\mu_i + \lambda} + \frac{\mu_i}{\mu_i + \lambda}(p - c_l) + \frac{\lambda}{\mu_i + \lambda}(1 - \pi_i). \quad (3)$$

The interpretation of this expression is as follows. An investor who takes a long position in the bond pays  $p$  today. He then receives the coupon until he either sells the bond or the bond matures, resulting in an expected coupon stream of  $\frac{r}{\mu_i + \lambda}$ . With probability  $\frac{\mu_i}{\mu_i + \lambda}$  the investor has to sell the bond before maturity. Here, the stationarity property of Poisson maturity implies that a non-matured bond at some future liquidation date  $t$  trades at the same price  $p$  as the bond today. Hence, the investor receives  $p - c_l$  when he has to sell the bond before maturity. If the bond matures, the investor receives an expected payoff of  $1 - \pi_i$ . This happens with probability  $\frac{\lambda}{\mu_i + \lambda}$ . Investor  $i$  purchases the bond if his valuation for the bond exceeds the outside option of holding cash,  $V_{\text{longBOND},i} \geq 0$ .

The set of investors that purchase the bond is illustrated by the grey triangle in Figure 2. In equilibrium, the bond price  $p$  adjusts such that the mass of investors in the buying triangle just absorbs the supply of the bond  $S$ . The resulting equilibrium price reflects the valuation of the “average marginal investor” in the buying triangle.

[Figure 2 about here.]

### 3.2 Long and short positions allowed

Now consider the case in which also short sales of the bond are possible. Investors now choose among three portfolios: a long position in the bond, a short position in the bond, or cash. The valuation of a long position in the bond is the same as before and given by equation (3). Investor  $i$ 's valuation of a short position in the bond is given by

$$V_{\text{shortBOND},i} = p - \frac{r}{\mu_i + \lambda} - \frac{\mu_i}{\mu_i + \lambda} p - \frac{\lambda}{\mu_i + \lambda} (1 - \pi_i) - c_s. \quad (4)$$

This valuation reflects that an investor with a short position receives  $p$  today and is required to pay the coupon  $r$  either until the bond matures or the short position is covered in response to a liquidity shock, resulting in an expected coupon obligation of  $\frac{r}{\mu_i + \lambda}$ . If the investor has to cover his short position before maturity, the investor has to purchase the bond for  $p$  (again using the stationarity property), whereas if the bond matures the investor has to cover his short position at an expected cost of  $1 - \pi_i$ . The probabilities of these two events are  $\frac{\mu_i}{\mu_i + \lambda}$  and  $\frac{\lambda}{\mu_i + \lambda}$ , respectively. Finally, the short seller incurs the shorting cost  $c_s$  (recall from above that this cost is incurred irrespective of whether the short

position is held until maturity). Taken together, investor  $i$  takes a short position if  $V_{\text{shortBOND},i} \geq 0$ , which may hold for investors who are sufficiently pessimistic about the bond's default risk.

The resulting demand for long and short positions is illustrated in Figure 3, under the assumption that parameters are such that short positions emerge in equilibrium. The gap between the triangle of long bondholders and short sellers arises because short selling is costly even if held to maturity. This makes it optimal even for some investors who do not face liquidity shocks to stay out of the market. Market clearing requires that the bond price  $p$  adjusts such that the overall amount bought by long investors is equal to the amount shorted plus bond supply  $S$ .

[Figure 3 about here.]

Figure 3 illustrates the impact of allowing short selling. In particular, pessimistic investors with sufficiently long holding horizons now choose to short the bond. Hence, for markets to clear, the triangle of long bondholders has to grow relative to the case in which only long positions are possible, which is illustrated by the dotted line in Figure 3. As a result, the average marginal investor is less optimistic and has a shorter holding horizon than in the case where only long positions are possible, leading to a decrease in the bond price. Note that the reduction in optimism of the marginal investor as a result of short selling mirrors the well-known results of Miller (1977). In addition to the effect on the bond price, short selling leads to an increase in bond turnover, because in the presence of short sellers the average marginal investor has a shorter expected holding horizon for the bond. We summarize this benchmark case in the following proposition.

**Proposition 1. Benchmark: Bond market equilibrium in absence of CDS market.**

*Allowing for short sales (weakly) decreases the bond price and (weakly) increases bond turnover:*

- (i) if  $c_s \geq \bar{c}_s$ , no shorting occurs in equilibrium and price and turnover coincide with the case in which just long positions are allowed*
- (ii) if  $c_s < \bar{c}_s$ , there is positive short interest in equilibrium. The bond price is weakly lower and bond turnover weakly higher than in the absence of shorting.*

## 4 Introducing a CDS Market

We now introduce the CDS contract to the analysis. Analogously to before, we determine the demand for the CDS by calculating the payoffs from long and short CDS positions. Moreover, we calculate the payoff from hedged positions that combine positions in the bond and the CDS. Combining these payoffs with the payoffs to going long or short in the bond, which we derived in equations (3) and (4), we then jointly solve for equilibrium in the bond and the CDS market.

The net payoff to investor  $i$  of purchasing a CDS on the bond is given by

$$V_{\text{buyCDS},i} = -q + \frac{\mu_i}{\mu_i + \lambda} (q - c_d) + \frac{\lambda}{\mu_i + \lambda} \pi_i. \quad (5)$$

This expression reflects the purchase price  $q$  of the CDS, the payoff  $q - c_d$  from early liquidation with probability  $\frac{\mu_i}{\mu_i + \lambda}$  (using stationarity) and the expected CDS payoff of  $\pi_i$  in the case of default at maturity, which happens with probability  $\frac{\lambda}{\mu_i + \lambda}$ . Proceeding analogously, the payoff to investor  $i$  of selling a CDS on the bond is given by

$$V_{\text{sellCDS},i} = q + \frac{\mu_i}{\mu_i + \lambda} (-q - c_d) - \frac{\lambda}{\mu_i + \lambda} \pi_i. \quad (6)$$

The presence of the CDS expands investors' action space. In addition to taking a long or short position in the bond (which yields  $V_{\text{longBOND},i}$  and  $V_{\text{shortBOND},i}$ , respectively), investors can now take a position in the CDS (yielding  $V_{\text{buyCDS},i}$  or  $V_{\text{sellCDS},i} \geq 0$ ), enter a hedged basis trade position (yielding  $L \cdot (V_{\text{buyCDS},i} + V_{\text{longBOND},i})$  in the case of a negative basis trade and  $L \cdot (V_{\text{sellCDS},i} + V_{\text{shortBOND},i})$  for a positive basis trade). Finally, investors can still hold cash as an outside option. Solving for equilibrium in the bond and CDS market requires calculating the demand for bond and CDS positions from the above payoffs and then imposing market clearing to determine the equilibrium prices of the bond and the CDS.

In our main analysis, we will focus on the case in which the CDS market is frictionless ( $c_d = 0$ ). After establishing our main results in the context of this particularly tractable case, we then discuss the more general case in which the CDS market is more liquid than the bond market but not frictionless ( $c_l > c_d > 0$ ) in Section 4.3.

#### 4.1 The effect of CDS introduction on prices and trading in the bond market

The advantage of assuming that CDS markets are frictionless is that the equilibrium in the CDS market becomes particularly simple: When  $c_d = 0$ , equations (5) and (6) imply that all investors for whom  $\pi_i < q$  are willing to sell CDS protection, while all investors with  $\pi_i > q$  are willing to purchase CDS protection. Coupled with assumption (A3) (which states that there is a large mass of investors with frequent trading needs) this implies that the equilibrium price of the CDS is equal to the median investor belief about the bond's default probability:

$$q = \bar{\pi}. \tag{7}$$

To determine the equilibrium bond price, we then investigate how the availability of the CDS priced at  $q = \bar{\pi}$  affects investors' incentives to take long or short positions in the bond. We start with the case in which basis traders cannot take leverage, which is depicted in Figure 4. The figure illustrates that the introduction of the CDS contract has three separate effects on the equilibrium in the bond market. First, when the CDS is available, investors with relatively frequent trading needs, who in absence of the CDS used to purchase the bond, now prefer to sell the CDS. This can be seen in Figure 4 by observing that the triangle of long bondholders has been cut off at the top (for ease of comparison, the triangle of long bond positions in the absence of the CDS is depicted by the dashed line). This crowding out of long bond investors leads to a reduction in demand for the bond, exerting downward pressure on the bond price.

Second, the introduction of the CDS eliminates short selling in the bond. In the figure, the triangle of investors that formerly shorted the bond (depicted by the dotted line on the right) vanishes, because those investors now prefer to purchase CDS protection instead of shorting the bond. The reason why investors prefer to use the CDS market to take bearish bets works through the equilibrium price: Because trading costs for the bond exceed trading costs for the CDS, the bond trades at a discount relative to the CDS. Hence, investors who wish to take a bearish bet on the bond prefer to do this in the CDS market as opposed to taking a short position in the bond. The elimination of short sellers through the introduction of the CDS exerts upward pressure on the bond price.

Third, the introduction of the CDS generates a new class of investors: hedged basis traders. Specifically, investors who in the absence of the CDS would purchase the bond, but whose belief about the bond's default probability is less optimistic than the mean belief  $\bar{\pi}$ , now find it optimal to purchase both the bond *and* the CDS. These investors thus become negative basis traders: They hold a hedged position in the bond and the CDS, thereby locking in the equilibrium price differential between the bond and the CDS. Note that, rather than taking bets on credit risk, these investors act as arbitrageurs.

[Figure 4 about here.]

When basis traders cannot take leverage ( $L = 1$ ), as assumed in Figure 4, their presence does not affect the bond price. The reason is that the investors in the basis trade triangle would have purchased the bond anyway, even in absence of the CDS. When basis traders can take leverage ( $L > 1$ ), on the other hand, the ability to hedge with the CDS allows basis traders to demand more of the bond, such that they exert upward pressure on the bond price. This is depicted in Figure 5, which shows that the ability of basis traders to take leverage raises the equilibrium bond price in two ways. First, holding constant the number of basis traders, their ability to take leverage increases the demand for the bond, thereby putting upward pressure on the bond price. Second, the ability to take leverage makes the basis trade more profitable and thereby increases the number of basis traders. Note that when basis traders can take leverage, some investors to the left of  $\bar{\pi}$  become basis traders. For these investors, the CDS has a negative payoff when seen in isolation. They nonetheless purchase the CDS because it allows them to lever their position in the bond.

The results on basis traders highlight an interesting economic role of CDS markets in our model: The CDS allows buy-and-hold investors to purchase the bond even if they are relatively less optimistic about the bond's default risk, because they can lay off the credit risk to investors who are more optimistic about the bond's default risk (but have more frequent trading needs) via the CDS. Because of their more frequent trading needs, those relatively more optimistic investors would not be efficient holders of the illiquid bond. Hence, by repackaging default risk into a more liquid security and thereby allowing the transfer of credit risk from buy-and-hold investors to speculators in the CDS market, CDSs are vehicles for *liquidity transformation*. This liquidity view of CDSs goes beyond the

traditional view that CDSs simply allow the separation of interest rate risk and default risk (for which a CDS is, in fact, not strictly necessary; an interest rate swap would be sufficient).

The ambiguous effect of CDS introduction on the price of the bond follows directly from the three demand effects discussed above. To see this, assume for simplicity that basis traders cannot take leverage ( $L = 1$ ), such that they do not affect the equilibrium bond price. In this case, the effect of CDS introduction on the bond price depends on whether more investors switch from a long position in the bond to selling CDS, or from a short position in the bond to buying CDS. This, in turn, depends on the distribution of investors on the two-dimensional plane illustrated in Figure 4. Proposition 2 summarizes the effects of CDS introduction on the equilibrium in the bond market.

[Figure 5 about here.]

**Proposition 2.** *The effect of CDS introduction on the bond price. When both the bond and the CDS trade, the equilibrium bond price  $p$  can be higher or lower relative to the benchmark case in which only the bond trades, depending on the relative size of three effects:*

- (i) *CDS introduction crowds out some bond investors, putting downward pressure on the bond price*
- (ii) *CDS introduction eliminates short selling in the bond, putting upward pressure on the bond price*
- (iii) *CDS introduction leads to the endogenous emergence of basis traders who push up the bond price if they can take leverage ( $L > 1$ )*

The prediction that CDS introduction can raise or lower bond yields is in line with the empirical literature on this topic: For example, Ashcraft and Santos (2009), find no evidence that the introduction of CDS contracts has lowered financing costs for the average borrower, but document modest reductions in spreads for safe firms. Hirtle (2009) finds only limited evidence that CDSs have improved firms' access to financing, but finds positive effects of CDSs on access to credit for large term borrowers. By identifying the channels through which the introduction of a CDS affects the bond price, our analysis may help guide empirical research on what kind of bond issuers are more likely to benefit from the introduction of a CDS. We will discuss this in more detail for the case of the double-uniform distribution below.

In addition to the result on the effects of CDS introduction on the bond price, our model also generates implications for turnover in the bond and the CDS market.

**Proposition 3. Turnover in bond and CDS markets.**

- (i) Turnover in the underlying bond decreases when the CDS is introduced.
- (ii) Turnover in the CDS market is higher than turnover in the bond market.

The effect of CDS introduction on bond turnover can be seen directly from Figure 4. As a result of CDS introduction, bond investors with relatively frequent trading needs switch to selling the CDS, thereby reducing bond turnover. In addition, the introduction of the CDS eliminates short sellers, which also leads to a reduction in bond turnover. This prediction is consistent with the empirical evidence in Das, Kalimipalli, and Nayak (2013), who document that CDS introduction is associated with a decrease in trading volume in the underlying bond.

Comparing bond turnover to CDS turnover (which is defined as the amount of CDS trading divided by the notional amount of outstanding CDSs), our model predicts that CDS turnover is larger than bond turnover because CDS investors have higher average trading frequencies than bond investors. (This simple comparison of average trading frequencies of investors suffices because there is no short selling when the CDS trades.) This prediction is consistent with the data: Based on CDS trading data from the DTCC, Oehmke and Zawadowski (2013) report that average monthly CDS turnover is given by 50.8% per month, whereas average turnover in the associated bonds (as reported by TRACE) is around 3.5% per month.<sup>9</sup>

**4.1.1 CDS introduction and the cost of capital: Double-uniform distribution**

Using the double-uniform distribution, we now explicitly calculate the bond price when a CDS is available. Recall that for the double-uniform distribution, assumption (A3) implies that  $K_H \rightarrow \infty$  (i.e., there is a large mass of speculators with more frequent trading needs  $\mu_H$ ). As discussed above, this assumption ensures that the CDS is priced according to the median belief about default probability:  $q = \bar{\pi}$ . We can then solve for the bond price in the presence of the CDS as

$$p_{\text{withCDS}} = \underbrace{1 - \bar{\pi} + \frac{r}{\lambda} - \frac{c_l \mu_L}{\lambda}}_{\text{median buy-hold valuation}} - \frac{(S - \frac{1}{2})^+ \Delta}{1 + 2(L - 1)L}, \tag{8}$$

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<sup>9</sup>CDS turnover is defined as CDS trading divided by the net notional amount of CDS outstanding.

where we assume that  $\mu_H$  is sufficiently large, such that  $\mu_h$  investors do not purchase the bond. Equation (8) shows that we can distinguish two cases. If the supply of the bond is relatively small ( $S \leq 1/2$ ), the bond price is simply given by the valuation of the median buy-and-hold investor ( $\mu = \mu_L$ ). This is the case when the entire bond supply can be held by  $\mu_L$  investors who are (weakly) more optimistic than the median investor. When  $S > 1/2$ , on the other hand, an additional term appears, because some of the bond supply now has to be held by investors who are less optimistic than the median investor. At this point, basis traders become active and absorb the additional bond supply. Hence, the effect of supply shocks beyond  $S = 1/2$  is smaller, the higher the leverage  $L$  that basis traders can take.

Under the double-uniform distribution we can also explicitly calculate the change in the bond yield that results from CDS introduction as

$$dy = \frac{(c_s - S\Delta)\lambda + (c_l + c_s)\mu_L}{2} + \frac{(S - \frac{1}{2})^+ \lambda \Delta}{1 + 2(L - 1)L}. \quad (9)$$

Equation (9) illustrates two of the three effects of CDS introduction discussed above. Recall from above that in this example we assumed that  $\mu_H$ -investors do not purchase the bond, which means that, in the absence of the CDS, the entire supply of the bond is absorbed by “buy-and-hold”  $\mu_L$ -investors. Since none of the investors with frequent trading needs hold the bond to start out with, the introduction of the CDS does not crowd out any bond investors. The two remaining effect are thus the elimination of short selling and the emergence of basis traders, such that the bond yield drops unambiguously. As illustrated by the second term in (9), the decrease in the yield is larger, the more leverage basis traders can take. Yields also decreases by more when disagreement about the default probability ( $\Delta$ ) is large, which reflects the larger increase in the marginal investor’s belief effected through the emergence of basis traders in this case. Finally, equation (9) illustrates the effect of the elimination of short sellers through the introduction of the CDS. Specifically, before the introduction of CDS, short sellers are particularly aggressive when short selling costs are low (low  $c_s$ ) and when the liquidity discount of the bond is low (low  $c_l$ ). Hence, all else equal, the introduction of the CDS lowers bond yields more when  $c_s$  and  $c_l$  are low. We summarize these results in the following corollary.

**Corollary 1. *CDS introduction and the cost of capital.*** *For the double-uniform distribution with  $K_H \rightarrow \infty$ , the introduction of the CDS reduces the bond yield by more if*

- (i) basis traders can take high leverage  $L$*
- (ii) disagreement  $\Delta$  about default probability is high*
- (iii) bond trading costs  $c_l$  and  $c_s$  are low*

Corollary 1 may be useful in guiding empirical research that looks at the effects of CDS introduction on the cost of capital. For example, CDS introduction is likely to reduce the cost of capital for firms with high levels of disagreement. To the extent that there is more disagreement about the default prospects of lower-rated firms, our model predicts that these firms may particularly gain from CDS introduction. At the same time, CDS introduction is likely to raise bond yields for firms with bonds that are hard to short. For those firms, the CDS market creates an opportunity for pessimistic investors to express their views. In doing so, they crowd out some long bond investors, thereby reducing the bond price. Finally, note that CDS introduction is also more likely to lower financing costs at times when basis traders can take substantial leverage.

## 4.2 The CDS-bond basis

We now turn to the CDS-bond basis, which has attracted considerable attention in the wake of the financial crisis of 2007-2009. The CDS-bond basis is defined as the difference between the spread of a synthetic bond (composed of a long position in a risk-free bond of the same maturity and coupon as the risky bond and a short position in the CDS) and the spread of the actual risky bond. Intuitively speaking, when the CDS-bond basis is negative, the CDS spread is smaller than the bond spread, which means that the physical bond is cheaper than the payoff-equivalent synthetic bond.

Absent frictions, the CDS-bond basis should be approximately zero. The reason is that a portfolio consisting of a long bond position and a CDS that insures the default risk of the bond should yield the risk-free rate.<sup>10</sup> Since the financial crisis, the CDS-bond basis has been consistently negative for most reference entities as documented, for example, by [Bai and Collin-Dufresne \(2010\)](#) and [Fontana](#)

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<sup>10</sup>[Duffie \(1999\)](#) shows that this arbitrage relationship holds exactly if the bond takes the form of a floating-rate note.

(2011). These papers show that empirically the negative CDS-bond basis is correlated with funding conditions of arbitrageurs (i.e., basis traders) and the liquidity of the underlying bond.

In our framework, the negative basis between bonds and CDSs arises endogenously from the difference in trading costs of the bond and the CDS. To calculate the basis, note that in our setting the price of a risk-free bond with the same coupon and maturity as the risky bond is given by  $p_f = 1 + \frac{r}{\lambda}$ . We can then calculate the spread of the risky bond above the risk-free bond by taking the price differential and dividing it by the expected lifetime of the bond  $1/\lambda$ , which yields  $\text{spread}_{\text{bond}} = \lambda(p_f - p)$ . Analogously, given price the CDS price  $q$  we can calculate flow cost of CDS protection (the CDS spread) to be  $\text{spread}_{\text{CDS}} = \lambda q$ . Hence, the CDS-bond basis can be expressed as:

$$\text{basis} = \text{spread}_{\text{CDS}} - \text{spread}_{\text{bond}} = \lambda \left( q + p - 1 - \frac{r}{\lambda} \right) \quad (10)$$

Based on the bond and CDS market equilibrium derived above, the CDS-bond basis satisfies the following properties.

**Proposition 4. *Negative CDS-bond basis.*** *When trading costs for the bond are higher than trading costs for the CDS ( $c_l > c_d$ ), the CDS-bond basis is (weakly) negative. The CDS-bond basis is more negative when:*

- (i) *bond supply  $S$  is larger*
- (ii) *the ability of basis traders to take leverage  $L$  is restricted,*
- (iii) *bond trading costs  $c_l$  are higher,*
- (iv) *disagreement about the bond's default probability  $\Delta$  is higher.*

The source of the negative basis in our model is straightforward: Because trading costs for the bond are higher than those of the CDS, the bond trades at a discount relative to the CDS. This prediction is in line with the growing empirical literature documenting a negative CDS-bond basis. In addition, Proposition 4 makes a number of time-series and cross-sectional predictions on the basis. First, the bases becomes more negative in response to supply shocks in the bond market. Such supply shocks may capture situations in which investors subject to regulatory constraints are forced to sell some of their holdings (as documented in [Ellul, Jotikasthira, and Lundblad \(2011\)](#)). Second, higher

basis trader leverage compresses the negative basis. This is because higher leverage for basis traders increases the demand for the bond, pushing up the bond price and thereby reducing the negative basis. In the time series, this implies that at times when basis traders can take substantial leverage, the basis should be less negative and closer to zero. In contrast, during times of tough financing conditions for basis traders, the equilibrium basis will be driven away from zero and becomes more negative. Empirical evidence suggests that this mechanism was indeed a major contributor to the widening of the negative basis during the financial crisis, when the leverage of basis traders fell from around 25 to 5 (see [Gârleanu and Pedersen \(2011\)](#), [Fontana \(2011\)](#), and [Mitchell and Pulvino \(2012\)](#)).

The remaining two comparative statics in Proposition 4 make cross-sectional predictions on the CDS-bond basis. First, bonds with high trading costs (relative to trading costs in the associated CDS) are predicted to have more negative CDS-bond bases. This is consistent with the empirical evidence in [Bai and Collin-Dufresne \(2010\)](#). Second, the model predicts that bonds for which investors disagree significantly on default probabilities have more negative bases. This can be seen directly from Figure 4: Holding prices fixed, increasing  $\Delta$  leaves the number of investors that take a long position in the bond unchanged, but reduces the number of investors in the basis-trade triangle. Thus, for markets to clear the bond price must drop. In practice, bonds with higher default risk (e.g., high-yield bonds) tend to be characterized by more disagreement about default probabilities. Those bonds also tend to have higher trading costs. For both of these reasons, our model implies that lower-rated bonds should have a more negative basis, as is also documented by [Bai and Collin-Dufresne \(2010\)](#) and [Gârleanu and Pedersen \(2011\)](#).

#### 4.2.1 The CDS-bond basis: Double-uniform distribution

The double-uniform distribution combined with frictionless CDS provides a particularly convenient way to calculate closed-form expressions for the CDS-bond basis. Under the double-uniform distribution, the CDS-bond basis is given by

$$\text{basis} = -c_l \mu_L - \lambda \Delta \frac{(S - \frac{1}{2})^+}{1 + 2(L - 1)L}. \quad (11)$$

The first term in (11) captures the difference in expected trading costs between the bond and the CDS and is independent of the supply of the bond. It thus reflects the component of the CDS-bond basis that is driven directly by bond trading costs. The second term captures how supply shocks, fluctuations in basis trader leverage, and the dispersion of beliefs about the bond’s default probability affect the CDS-bond basis. As the supply of the bond increases, the marginal investor holding the bond becomes less optimistic, leading to a more negative CDS-bond basis. This effect is attenuated by basis traders, who step in to purchase the bond and lay off the credit risk in the CDS market. The more leverage basis traders can take, the smaller is the effect of supply shocks on the basis. Finally, (11) illustrates the effect of disagreement on the basis: More disagreement (higher  $\Delta$ ) increases the second term, leading to a more negative basis. Intuitively speaking, when investor beliefs are more dispersed, for the marginal basis trader holding the bond is less optimistic.

### 4.3 CDS market with frictions

In the analysis above, we focused on the particularly tractable case in which CDSs involve no trading costs ( $c_d = 0$ ) and there is an infinite mass of traders with relatively frequent trading needs. These assumptions make solving for the equilibrium in the CDS market particularly easy because they ensure that  $q = \bar{\pi}$ . We now relax both assumptions in order to analyze the behavior of  $q$ , the size of the CDS market and also to verify the robustness of our previous results. Relaxing Assumption (A3), (which stated that there is an infinite mass of investors with frequent trading needs), means that, in general,  $q \neq \bar{\pi}$ . Hence, the CDS price no longer simply reflect the average belief about the bond’s default probability. Furthermore, if the CDS market is more liquid than the bond market but not frictionless ( $c_l > c_d > 0$ ), the CDS price will reflect trading frictions in the CDS market.

The main differences that result from introducing trading costs also in the CDS market are illustrated in Figures 6 and 7. Most significantly, in contrast to the frictionless CDS case, it is now no longer the case that all investors for which  $\pi_i < \bar{\pi}$  are willing to sell CDS protection and that all investors with  $\pi_i > \bar{\pi}$  are willing to purchase CDS protection. Rather, the “sell CDS” and “buy CDS” regions are now also triangles (rather than rectangles as in Figures 4 and 5), because investors with moderate beliefs and frequent trading needs now prefer to stay out of the market altogether and hold cash (the white regions in Figures 6 and 7).

Despite these differences, the main qualitative results derived in the frictionless CDS setting above remain valid also in the case where the CDS market involves trading frictions. For example, the introduction of a CDS affects the bond market through exactly the same three effects as in the frictionless CDS case: (i) the CDS crowds out some long bondholders, (ii) the CDS eliminates short sellers, and (iii) the CDS leads to the emergence of hedged, potentially leveraged basis traders. As in the frictionless model, basis traders are price neutral when  $L = 1$  (Figure 6), whereas basis traders exert downward pressure on the bond yield when they can take leverage  $L > 1$  (Figure 7).

[Figure 6 about here.]

[Figure 7 about here.]

## 5 Policy Implications

We now apply our framework to analyze a number of policy interventions. Specifically, we consider the effects of (i) banning naked CDS positions (as recently implemented by the European Union with respect to sovereign bonds), (ii) banning CDS markets altogether, and (iii) banning both CDSs and short positions in bonds. The motivation for such interventions is usually to achieve a reduction in bond yields (and thereby borrowing costs) for issuers. While our simple framework is clearly not rich enough to yield detailed policy prescriptions, it does suggest that the effects of such interventions on bond yields are non-trivial and can potentially go in the wrong direction—they may *increase* borrowing costs for issuers.<sup>11</sup>

### 5.1 Banning naked CDS positions

EU regulation No 236/2012, in effect since November 1, 2012, bans purchasing CDS protection as a means of speculation on sovereign bonds. Specifically, the regulation allows CDS purchases for market participants who own the underlying bond or have other significant exposure to the sovereign, but restrict so-called naked CDS positions. Short selling of the bond is allowed under this regulation as long as the short seller is able to borrow the bond before executing the short sale.

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<sup>11</sup>Note that while our framework allows us to investigate the positive effects of certain policy interventions, it does not allow us to determine whether a policy objective (such as a reduction in yields) is desirable in the first place. This reflects the well-known difficulty of making welfare statements in models with heterogeneous beliefs.

The crucial question in assessing the effect of a ban on naked CDS positions is what investors who were previously holding naked CDS protection choose to do instead. Our framework suggests that there are two main effects, which are illustrated in Figure 8: First, some of these investors switch from a naked CDS position to a short position in the bond. Hence, as a result of a ban on naked CDSs, short sellers reappear, putting downward pressure on the bond price. This effect is larger, the lower the cost of short selling the bond. Second, some investors who formerly held a naked CDS position become basis traders and hold the bond and the CDS, up to the maximum leverage  $L$ . This second effect increases demand for the bond, resulting in upward pressure on the bond price.

Hence, the effect of banning naked CDS positions on the cost of borrowing can go in either direction, depending on the relative size of the two effects described above. Contrary to its goal, the current EU regulation that bans naked CDS positions on sovereigns may thus have *raised* financing costs for affected sovereigns. Moreover, the effect of a ban on naked CDS positions cannot be determined simply by its effect on CDS spreads. The reason is that borrowing costs for issuers may rise even if CDS spreads drop. For example, in the appendix we provide an example where, under the double-uniform distribution, the introduction of the CDS ban decreases the CDS spread by 50 basis points but increases the bond spread by 21 basis points.

[Figure 8 about here.]

## 5.2 Banning the CDS market altogether

The second policy intervention we consider is an outright ban of the CDS market. This amounts to a comparison of the equilibrium with a CDS market to the equilibrium without CDS. From Proposition 2, we know that the effect of CDS introduction on the bond yield is ambiguous. Hence, banning CDS markets altogether may either increase or decrease borrowing costs for issuers, depending on the size of the three effects described in Section 4.1. Accordingly, banning CDSs is most likely to decrease yields if the CDS ban does not result in a large increase in short selling. This could be the case either because short selling is costly (high  $c_s$ ) or if the distribution of investors is such that there are not too many pessimistic buy-and-hold investors. On the other hand, banning CDS markets is likely to increase yields when basis traders can take significant leverage ( $L$  is large). Because the CDS ban eliminates basis traders, it would in this case lead to a large reduction in the demand for the bond.

### 5.3 Banning CDSs and short selling in the bond

Finally, let us briefly consider the effect of banning the CDS market *and* short positions in the bond. This amounts to a comparison of the long-only case discussed in Proposition 1 to the equilibrium with a CDS market described in Proposition 2. Even this intervention is not guaranteed to lower bond yields for issuers. While restricting short positions prevents the reemergence of short sellers in response to a ban on CDS positions, a trade-off now emerges from the countervailing effects of (i) increased demand for the bond from investors who formerly sold the CDS but now purchase the bond and (ii) the reduction in demand for the bond that results from the elimination of basis traders. Because basis traders are price neutral when they cannot take leverage, in this case a joint ban on CDSs and short selling leads to an unambiguous decrease in the bond yield. When basis traders can take leverage, on the other hand, bond yields may increase or decrease, depending on the relative size of the two effects. The intervention is more likely to raise bond yields the higher the leverage of basis traders  $L$  and the larger the supply of the bond  $S$ .

## 6 Extensions

We now briefly look at two extensions of our model. First, we discuss the case where the CDS is less liquid than the underlying bond. Second, we extend the model to allow for multiple bond issues with different levels of liquidity. A more detailed analysis and proofs can be found in the online appendix.

### 6.1 Illiquid CDS market

The main assumption of our analysis has been that trading costs in the CDS market are lower than in the bond market, reflecting the relative magnitudes of trading costs for most corporate (and at least some sovereign) bonds. However, there are cases in which the reverse assumption makes sense, most notably U.S. Treasuries, where trading costs in the bond are generally *lower* than in the associated CDS. In our model, reversing the assumption on transaction costs would lead to two main changes. First, the CDS-bond basis becomes positive because bonds now trade at a liquidity premium. Second, even though the basis is positive, there are no positive basis traders in equilibrium because the equilibrium basis is not large enough. These results are consistent with the observation that large

sovereigns with liquid bond markets tend to have positive CDS-bond bases, but that these are rarely large enough to be exploited by basis traders.<sup>12</sup>

## 6.2 Two bond issues

Another interesting extension of the model is to consider the situation where an issuer has two types of bonds outstanding: a liquid issue (e.g., a recently issued “on-the-run” bond) and a less liquid issue (e.g., a illiquid “off-the-run” bond). We continue to assume that the CDS is more liquid than either of the two bonds (although the model could be adapted to have the CDS more liquid than the off-the-run bond, but less liquid than the on-the-run bond). Compared to the liquid bond, the less liquid bond will be held by investors with less frequent trading needs and has a larger illiquidity discount and hence a more negative CDS-bond basis. Interestingly, CDS introduction can affect the prices of the two bonds in differently: it may raise the price of the illiquid bond while simultaneously decreasing the price of the liquid bond. Intuitively, the illiquid bond benefits disproportionately from an increase in demand from basis traders, while the liquid bond suffers disproportionately from the crowding out effect of long bond investors into the CDS market. This result provides a potential explanation of the finding that CDS introduction is associated with an increase in the average maturity at which firms borrow ([Saretto and Tookes \(2013\)](#)): Because longer maturity bonds are typically less liquid, a larger positive price effect of CDS introduction on more illiquid bonds makes issuing long-term bonds more attractive.

## 7 Conclusion

This paper provides a liquidity-based model of CDS markets, bond markets and their interaction. In our framework, CDSs are non-redundant because they have lower transaction costs than the underlying bonds, reflecting the liquidity benefits of standardization. Our model shows that when investors are heterogeneous in their trading horizon and beliefs regarding the bond’s default probability, the introduction of a CDS affects the underlying bond market in a non-trivial way. The effect of CDS introduction on the bond price is ambiguous and depends on the amount of short selling absent the

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<sup>12</sup>See, for example, [“Insurers urged to explore positive basis trades”](#) on Risk.net, February 27, 2013, last accessed October 2, 2013.

CDS, the crowding out of long bond investors through the CDS, and the ability of basis traders (who emerge endogenously) to take leverage. Beyond characterizing the impact of CDS introduction on the pricing of the underlying bond, the model also generates testable empirical predictions regarding trading volume in bond and CDS markets, as well as the cross-sectional and time-series properties of the CDS-bond basis. Finally, the model can be used to assess a number of policy measures related to CDS markets, such as the recent E.U. ban on naked CDS positions.

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## A The double-uniform distribution

This appendix contains additional closed-form solutions under the double-uniform distribution that were left out of the main part of the paper for brevity.

### A.1 Double-uniform Benchmark: No CDS Market

This section calculates the equilibrium bond price for the benchmark case without a CDS market under the double-uniform distribution. For brevity, we focus on the case where parameters are such that investors with frequent trading needs  $\mu_H$  do not purchase the bond. This amounts to assuming that  $\mu_H$  is sufficiently high:  $\mu_H > \frac{S\Delta\lambda}{c_l} + \mu_L$ . When only long positions are possible, the bond price is given by

$$p_{\text{long}} = \underbrace{1 - \bar{\pi} + \frac{\Delta}{2} + \frac{r}{\lambda} - \frac{c_l\mu_L}{\lambda}}_{\text{most optimistic buy-hold valuation}} - S\Delta. \quad (12)$$

The equilibrium bond price (12) reflects the valuation of the most optimistic buy-and-hold investor ( $\mu = \mu_L$ ) minus a term that captures the reduction in the marginal buyer's valuation as supply increases, since the bond supply has to be absorbed by moving to increasingly less optimistic buyers.

When both long and short positions are possible,<sup>13</sup> the bond price is given by

$$p_{\text{long\&short}} = \underbrace{1 - \bar{\pi} + \frac{r}{\lambda} - \frac{c_l\mu_L}{\lambda}}_{\text{median buy-hold valuation}} - \frac{S\Delta}{2} + \frac{c_s}{2} + \frac{c_s + c_l}{2} \frac{\mu_L}{\lambda}. \quad (13)$$

The first three terms in equation (13) reflect the median buy-and-hold valuation. The remaining terms reflect the effects of trading costs and supply. First, the larger the outstanding supply, the lower the valuation of the marginal investor, reflected by the term  $-\frac{S\Delta}{2}$ . Note that in comparison to (12) the slope of this effect has halved. This is because an increase in supply lowers the equilibrium price, which drives out some short sellers, thereby reducing the effect of variations in supply on the price. Hence, the presence of short sellers reduces the price impact of supply shocks. The second term,  $\frac{c_s}{2}$  reflects the “fixed costs” of short selling relative to taking long positions. In Figure 3, this effect is captured by the gap between the “buy” triangle and the “short” triangle. The larger this fixed cost of short selling, the higher the equilibrium bond price (the distance between the two triangles is exactly  $c_s$ , which means that the gap increases the valuation of the effective marginal investor by  $\frac{c_s}{2}$ ). Finally, the term  $\frac{c_s + c_l}{2} \frac{\mu_L}{\lambda}$  captures short sellers' attenuation of the bond's lifetime trading

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<sup>13</sup>We again assume that investors with frequent trading needs do not purchase the bond, which requires that  $\mu_H > \frac{(1+S)\Delta\lambda + c_l\mu_L - c_s(\lambda + \mu_L)}{2c_l}$ . We also assume that  $c_s < \frac{(1-S)\Delta\lambda - c_l\mu_L}{\lambda + \mu_L}$ , such that short selling occurs in equilibrium. If this condition is violated, the equilibrium is equivalent to the long-only case above.

costs. When  $c_l$  increases, the price of the bond drops because investors become more reluctant to purchase the bond. However, as the bond price drops, short selling becomes less attractive, such that part of the effect is offset by a drop in short selling. Similarly, an increase in  $c_s$  makes short selling less attractive, thereby raising the price of the bond. However, part of this effect is offset since, as the bond price rises, some long investors drop out of the market.

## A.2 CDS Market with Frictions under the Double-Uniform

When also the CDS market involves trading frictions ( $c_d > 0$ ) and  $K_H < \infty$ , one needs to solve for the bond price  $p$  and the CDS price  $q$  simultaneously. For brevity, we focus on the interesting case, in which (i) only  $\mu_L$ -investors buy bonds, (ii) at least some  $\mu_H$  investors buy and sell CDS, and (iii) negative basis traders appear in equilibrium. Intuitively, this requires that  $\mu_H$  has to be high enough that only the  $\mu_L$  investors buy bonds,  $c_d$  has to be low enough such that there are both CDS sellers and buyers with trading frequency  $\mu_H$ , and the bond supply  $S$  is large enough that basis traders appear. The effects of CDS introduction are qualitatively similar to the frictionless case:

**Corollary 2. *CDS introduction and the cost of capital.*** *For the double-uniform distribution (under the parameter restrictions discussed above), all statements made in Corollary 1 for infinitely liquid CDS markets continue to hold. In addition, when  $L > 1$ , the introduction of the CDS reduces the bond yield by more if  $c_d$  is low.*

The intuition for the additional result is that  $c_d$  only effects the bond price through basis traders. basis traders are more effective at reducing the price difference between CDS and bonds if the CDS market is more liquid. As discussed in the main text, another main difference when  $c_d > 0$  is that the CDS price  $q$  is generally different from the median default probability  $\bar{\pi}$ :

**Corollary 3. *CDS spread.*** *For the double-uniform distribution (under the parameter restrictions discussed above), the equilibrium CDS spread  $\lambda q$*

- (i) increases in disagreement  $\Delta$ ,*
- (ii) increases in leverage  $L$  if basis traders are present in equilibrium,*
- (iii) decreases in the CDS trading cost  $c_d$*
- (iv) is independent of of the bond trading cost  $c_l$ .*

Hence, when  $c_d > 0$  CDS spreads are no longer necessarily an unbiased measure of the average beliefs about the firm's default probability. Moreover, the result that the CDS price increases in disagreement is in line with some commentators' view that that speculation can increase CDS spreads.

The market equilibrium also pins down the size of the CDS market. We define the size of the CDS market  $Q_{CDS}$ , as the total amount of long or, equivalently, short positions.

**Proposition 5. *The size of the CDS market.*** *For the double-uniform distribution (under the parameter restrictions discussed above), the size of the CDS market is*

$$Q_{CDS} = \frac{K_H}{2} \left( 1 - 2 \cdot \frac{c_d \mu_H}{\lambda \Delta} + \frac{(1 + 4 \cdot S \cdot L \cdot (L - 1)) - 2 \cdot \frac{c_d \mu_L}{\lambda \Delta} \cdot (2L - 1)^2}{(2L - 1)^2 + 2(1 + 2(L - 1)L) \cdot K_H} \right). \quad (14)$$

*The size of the CDS market*

- (i) *increases in disagreement  $\Delta$ ,*
- (ii) *increases in basis trader leverage  $L$ ,*
- (iii) *decreases in the CDS trading cost  $c_d$ ,*
- (iv) *is independent of the bond trading costs  $c_l$  and  $c_s$ .*

Note that in both corollaries, the result that the CDS spread  $\lambda q$  and the quantity of CDS is independent of bond trading costs is specific to the double-uniform distribution. The reason for this is that, under the double uniform, there is no intensive margin of adjustment along the trading frequency dimension  $\mu$ . Numerically, one can show that when such adjustment is possible (for example, assuming a two-dimensional uniform distribution) both the CDS spread and the quantity of CDS is increasing in bond trading costs. These predictions are in line with the empirical results in [Oehmke and Zawadowski \(2013\)](#), who show that the equilibrium amount of CDS is increasing disagreement and in the illiquidity of the bond market.

### A.3 Naked CDS Ban under the Double Uniform

For the double-uniform distribution, we can calculate the effect of a ban on naked CDS positions on bond prices explicitly and analyze the determinants of when the policy is likely to reduce borrowing costs. For tractability we focus on the case where there is a large mass of investors with frequent trading needs, such that assumption (A3) is satisfied.

**Proposition 6. *Banning naked CDS positions.*** *Assume that investors are distributed according to the double-uniform distribution with  $K_H \rightarrow \infty$ . Under reasonable simplifying assumptions (see proof), a ban on naked CDSs has an ambiguous effect on the yield of the underlying bond. A naked CDS ban is more effective reducing the bond yield and thus the issuer's borrowing cost if*

- (i) *the bond trading costs  $c_l$  and (especially)  $c_s$  are high.*

(ii) disagreement about the bond's default probability  $\Delta$  is high.

(iii) the CDS trading cost  $c_d$  is low.

(iv) the basis trader leverage  $L$  is high.

For the special case of  $L = 1$ :

$$p_{no\ naked\ CDS} - p_{with\ CDS} = \frac{2}{3} \cdot \left[ \left( S - \frac{1}{2} \right) \Delta + (c_l + c_s) \frac{\mu_L}{\lambda} + c_s - c_d \frac{\mu_H + \mu_L}{2\lambda} \right] \quad (15)$$

$$q_{no\ naked\ CDS} - q_{with\ CDS} = -\frac{\Delta}{2} + \frac{c_d \mu_H}{\lambda} < 0 \quad (16)$$

Equation (15) shows that the effect of the naked CDS ban on the bond price  $p$  is ambiguous. While this expression assumes that  $L = 1$ , the net effect is generally ambiguous also when  $L > 1$ . Equation (16), on the other hand, shows that the price of CDS protection  $q$  unambiguously decreases in response to a ban on naked CDS (given the parameter restriction that is needed for CDSs to be traded in equilibrium). Hence, bond spreads and CDS spreads can move in opposite directions in response to a policy intervention that restricts naked CDS positions, which means that a decrease in CDS spreads following a ban on naked CDS positions does not imply that borrowing costs for issuers have come down. For example, if parameters are given by  $S = 0.55, c_d = 0.003, c_l = 0.01, c_s = 0.012, \bar{\pi} = 0.1, \Delta = 0.2, \lambda = 0.2, r = 0, \mu_L = 0, \mu_H = 5, L = 1$ , equations (15) and (16) imply the introduction of the CDS ban decreases the CDS spread by 50 basis points but at the same time increases the bond spread by 21 basis points.

Equation (15) provides a number of insights as to the conditions under which a ban on naked CDS positions is most likely to decrease the issuer's borrowing cost. In particular, we see that less liquid bonds (high  $c_l$  and  $c_s$ ) are more likely to experience a decrease in yields in response to a ban on naked CDS positions. For example, when the cost of shorting  $c_s$  is high, relatively few investors that used to hold a naked CDS position will switch to short selling the bond, such that the bond yield is more likely to decrease after a ban on naked CDSs. The same is true for bonds for which disagreement on default probabilities is higher (high  $\Delta$ ). Finally, a ban on naked CDS positions is more likely to reduce borrowing costs if the CDS market associated with the bond is relatively liquid (low  $c_d$ ). A liquid CDS market lowers the cost of the CDS and thus makes switching to a basis trade more attractive for investors that formerly held a naked CDS position. In the more general case, in which basis traders can take leverage  $L > 1$ , banning naked CDS is more likely to lower borrowing costs if  $L$  is higher. The intuition is that CDS buyers are more likely to switch to the basis trade if  $L$  is high and thus exert an upward pressure on bond prices.

## Appendix B: Proofs

**Proof of Proposition 1.** *Only long positions:* If only long position in the bond are possible, investor  $i$  buys the bond if  $V_{\text{longBOND},i} \geq 0$ , which can be rewritten as

$$\pi_i \leq 1 - p + \frac{r}{\lambda} - \frac{c_l}{\lambda} \mu_i. \quad (\text{A1})$$

Hence, investors must be sufficiently optimistic and have sufficiently long trading horizons. The demand implied by (A1) is illustrated in Figure 2. Market clearing implies that the bond price  $p$  has to adjust such that total demand is equal to the outstanding bond supply  $S$ :

$$\int_0^{\frac{\lambda}{c_l}(1+\frac{r}{\lambda}-(\bar{\pi}-\frac{\Delta}{2})-p)} \int_{\bar{\pi}-\frac{\Delta}{2}}^{1-p+\frac{r}{\lambda}-\frac{c_l\mu}{\lambda}} F(\mu, \pi) d\pi d\mu = S. \quad (\text{A2})$$

*Long and short positions:* Investor  $i$  takes a short position in the bond if  $V_{\text{shortBOND},i} \geq 0$ , which implies

$$\pi_i \geq 1 - p + \frac{r}{\lambda} + c_s + \frac{c_s}{\lambda} \mu_i. \quad (\text{A3})$$

Thus, pessimistic investors with sufficiently long trading horizons short the bond (see Figure 3). For the short selling triangle to appear, it has to be the case that the cost of shorting is sufficiently low:

$$c_s < \bar{c}_s = -1 + p + \frac{\Delta}{2} - \frac{r}{\lambda} + \bar{\pi}. \quad (\text{A4})$$

Note that this is a necessary but not a sufficient condition for positive short interest: Even if the shorting triangle has a non-zero area, the distribution  $F$  of investors still may be such that a zero measure of investors are located in the shorting triangle. If there is no short selling in equilibrium, the bond price  $p$  is the same as in the case where only long positions are allowed and is characterized by (A2). If  $c_s < \bar{c}_s$ , the bond price is characterized by:

$$\int_0^{\frac{\lambda}{c_l}(1+\frac{r}{\lambda}-(\bar{\pi}-\frac{\Delta}{2})-p)} \int_{\bar{\pi}-\frac{\Delta}{2}}^{1-p+\frac{r}{\lambda}-\frac{c_l\mu}{\lambda}} F(\mu, \pi) d\pi d\mu = S + \int_0^{\frac{\lambda}{c_s}(\bar{\pi}+\frac{\Delta}{2}-1+p-\frac{r}{\lambda}-c_s)} \int_{1-p+\frac{r}{\lambda}+c_s+\frac{c_s\mu}{\lambda}}^{\bar{\pi}+\frac{\Delta}{2}} F(\mu, \pi) d\pi d\mu \quad (\text{A5})$$

If there is a strictly positive amount of short selling in equilibrium, the price of the bond  $p$  has to be lower than absent short selling. If  $p$  remained unchanged, short sellers create an excess supply of bond and the bond market would not clear. The bond price  $p$  thus has to decrease. Hence, when  $c_s < \bar{c}_s$ , the bond price  $p$  is weakly lower when short selling is allowed. The statement is weak because there may be a zero measure of traders in the shorting triangle.

The result that bond turnover weakly increases the bond turnover comes from the following observation. If no short selling emerges in equilibrium, turnover is unchanged. If there is a strictly positive amount of short selling, the original holders of the bonds (absent short selling) are still holding the bond and trade it at frequency  $\mu_i$ . Short sellers and the additional buyers who absorb the effective increase in supply increase trading. Because the amount of bonds outstanding remains unchanged, turnover weakly increases when shorting is allowed.  $\square$

**Proof of Proposition 2.** *CDS positions:* We first determine the investor regions illustrated in Figure 4. To determine whether investors will take positions in the CDS, we now compare payoffs of CDS positions to the payoffs from taking positions directly in the underlying bond. Optimistic investors (low  $\pi_i$ ) switch to selling CDS instead of buying the bond if  $V_{\text{sellCDS},i} > V_{\text{longBOND},i}$ , which holds for optimistic investors that are sufficiently likely to be hit by a liquidity shock:

$$\mu_i \geq \frac{\lambda}{c_l - c_d} \left( \frac{r}{\lambda} + 1 - p - q \right). \quad (\text{A6})$$

This condition defines a horizontal line in the  $(\mu, \pi)$  plane. Because the bond is in strictly positive supply ( $S > 0$ ), bond buyers have to be present for bond markets to clear. Because a long position in the bond has higher trading costs than the a short CDS position ( $c_l > c_d$ ), for an investor to buy the bond instead of selling the CDS it has to be the case that  $\frac{r}{\lambda} + 1 - p - q \geq 0$ . Note that equality can only be sufficient if there is a mass  $S$  of investors located at  $\mu_i = 0$ . In (A10), we will show that this condition is also necessary for the emergence of negative basis traders. Similarly, pessimistic investors (high  $\pi_i$ ) switch from shorting the bond to purchasing a CDS if  $V_{\text{buyCDS},i} > V_{\text{longBOND},i}$ . This condition holds for investors who are sufficiently likely to be hit by a liquidity shock:

$$\mu_i \geq \frac{\lambda}{c_s - c_d} \left( -\frac{r}{\lambda} + q - 1 + p - c_s \right). \quad (\text{A7})$$

The previous condition  $\frac{r}{\lambda} + 1 - p - q \geq 0$  in conjunction with  $c_s > c_d > 0$  implies that the right hand side of this inequality is always negative. Hence, there are no short sellers in equilibrium because  $\mu_i \geq 0$  by definition.

*Basis traders:* Investors who become basis traders must prefer the basis trade to buying the bond if  $\pi_i < \bar{\pi}$  and have to prefer it over buying the CDS if  $\pi_i > \bar{\pi}$ . Investors switch from a long position in the bond to a negative basis trade if

$$\pi_i \geq Lq - (L-1) \left( \frac{r}{\lambda} + 1 - p \right) + \frac{\mu_i}{\lambda} (Lc_d + (L-1)c_l), \quad (\text{A8})$$

and switch from buying a CDS to a negative basis trade if

$$\pi_i \leq L \left( \frac{r}{\lambda} + 1 - p \right) - (L-1)q - \frac{\mu_i}{\lambda} ((L-1)c_d + Lc_l). \quad (\text{A9})$$

Thus basis traders form a triangle in the  $(\mu, \pi)$ -plane. The height of this triangle is given by

$$\frac{L(r - (-1 + p + q)\lambda) + \lambda(q - \bar{\pi})}{(-1 + L)c_d + Lc_l} = \frac{\lambda}{c_l} \left(1 + \frac{r}{\lambda} - p - q\right), \quad (\text{A10})$$

which simplifies to the expression on the right hand side given that  $q = \bar{\pi}$  and  $c_d = 0$ . Thus the height of the basis triangle for  $c_d = 0$  is the same as the height of the bond buying region expressed in (A6).

*Effect of basis trader leverage:* Finally, we analyze the effect of increasing basis trader leverage  $L$ . To do this, consider fixing the bond and CDS prices  $p$  and  $q$ . An increase in leverage has two effects. First, higher  $L$  increases the number of bonds each basis trader buys. Second, increasing  $L$  broadens the base of the basis triangle, without affecting its (A10). From (A8), if  $\frac{r}{\lambda} + 1 - p - q > 0$ , the left side of the basis triangle expands to the left as  $L$  increases (the critical value of  $\pi_i$  decreases for any given  $\mu_i$ ). From (A9), if  $\frac{r}{\lambda} + 1 - p - q > 0$ , the right side of the basis triangle expands to the right as  $L$  increases (the critical value of  $\pi_i$  increases for any given  $\mu_i$ ). This second effect implies that more investors become basis traders as  $L$  increases. Both effects increase the demand for bonds, thus the bond market can only clear if the bond price  $p$  increases (note that  $q = \bar{\pi}$  continues to hold).  $\square$

**Proof of Proposition 3.** The introduction of a CDS market changes the bond holding regions in three ways, all of which lead to lower bond turnover (see Figure 4). The elimination of the shorting triangle unambiguously decreases bond trading. Also, of the remaining bond buyers (including basis traders) even those with the highest trading frequency have a lower trading frequency than the bond buyers that have been eliminated through introduction of the CDS. Because the overall required number of bond buyers decreases (the CDS eliminates short selling), the mass of low turnover investors added to the bond buyers (if any), is smaller than the mass of former bond buyers who are crowded out into the CDS market. Since these new bond buyers all have a lower trading frequency than the bond investors crowded out by the CDS market, the amount of equilibrium trading diminishes. Because the bond supply  $S$  is unchanged, turnover in the bond market decreases.

The trading frequency of all investors selling the CDS is higher than the trading frequency of any investor buying the bond either through a long only trade or a basis trade (see Figure 4). Thus, turnover of the CDS is unambiguously higher than that of the bond. (Trades from CDS buyers further add to the higher turnover of CDS.)  $\square$

**Proof of Corollary 1.** Bond yields are reduced by CDS introduction if bond prices increase. All results follow from taking derivatives of  $dy$  w.r.t. the variables of interest (see Equation (9)).  $\square$

**Proof of Proposition 4.** The necessary condition for equilibrium in the proof of Proposition 2 is that  $\frac{r}{\lambda} + 1 - p - q \geq 0$ . Since by definition,  $basis = \lambda(q + p - 1 - \frac{r}{\lambda})$  and  $\lambda > 0$ , the condition for equilibrium implies  $basis \leq 0$ .

*Effect of disagreement:* Assume that disagreement  $\Delta$  increases but prices  $p, q$  remain unchanged. The bond buying rectangle will be stretched horizontally, but given Assumption (A2) (the scaling property of  $f$ ), this does not change the mass of investors who take a long position in the bond. If  $p, q$  are fixed, the size of the basis triangle does not change when the density  $f$  is stretched horizontally. However, by the scaling property of  $f$ , less investors will be distributed in any fixed area of the  $(\mu, \pi)$  plane as  $\Delta$  increases. Hence, there are now fewer basis traders and thereby less demand for the bond, meaning the bond market can only clear if  $p$  drops, leading to a more negative CDS-bond basis.

*Effect of trading costs:* Assume that bond trading cost  $c_l$  increases but prices  $p, q$  remain unchanged. This decreases the height of the bond buying region, which is given by

$$\frac{\lambda}{c_l - c_d} \left( \frac{r}{\lambda} + 1 - p - q \right). \quad (\text{A11})$$

The same is true for the height of the basis triangle. The width of the bond buying region and the basis triangle does not change. Thus there is a smaller mass of investors willing to buy bonds and  $p$  must decrease for markets to clear. Hence, the CDS-bond basis becomes more negative.

*Effect of basis trader leverage:* This follows immediately from the proof of Proposition 2, which shows that  $p$  is increasing in  $L$ . □

**Proof of Corollary 2.** From market clearing, the equilibrium bond price is given by:

$$p = \frac{2(1-2L)^2 r + \lambda(2+\Delta-2S\Delta-2\bar{\pi}-4(-1+L)L(-2+S\Delta+2\bar{\pi})) - 2(1-2L)^2 c_l \mu_L}{2\lambda((1-2L)^2 + (2+4(-1+L)L)K_H)} + \frac{2K_H((2+4(-1+L)L)r + \lambda(2+\Delta-2S\Delta-4(-1+L)L(-1+\bar{\pi})-2\bar{\pi}) - 2(c_l + 2(-1+L)L(c_d + c_l))\mu_L)}{2\lambda((1-2L)^2 + (2+4(-1+L)L)K_H)} \quad (\text{A12})$$

subtracting Equation (13) from this and taking the respective derivatives yields the results. The derivative w.r.t.  $L$  is positive if and only if  $K_H(\Delta\lambda(2S-1) - 2c_d\mu_L) + \Delta\lambda(S-1) > 0$ , which is exactly the condition for negative basis traders to emerge in equilibrium. The derivative w.r.t.  $c_d$  is  $\frac{4(L-1)LK_H\mu_L}{(4(L-1)L+2)K_H+(2L-1)^2}$ , which is positive if and only if  $L > 1$ . □

**Proof of Corollary 3.** From market clearing, the equilibrium CDS price is given by:

$$q = \frac{(1+4(-1+L)LS)\Delta\lambda + 2(1-2L)^2\lambda\bar{\pi} + 4(1+2(-1+L)L)\lambda\bar{\pi}K_H - 2(1-2L)^2c_d\mu_L}{2\lambda((1-2L)^2 + (2+4(-1+L)L)K_H)}. \quad (\text{A13})$$

The results follow from taking partial derivatives. The derivative w.r.t.  $L$  is positive, if and only if

$$K_H(\Delta\lambda(2S-1) - 2c_d\mu_L) + \Delta\lambda(S-1) > 0, \quad (\text{A14})$$

which is exactly the condition for negative basis traders in equilibrium.  $\square$

**Proof of Proposition 5.** Assuming that the CDS is traded, negative basis traders are present and that only buy-and-hold ( $\mu_L$ ) investors buy bonds,  $Q_{CDS}$  is calculated as the amount of CDSs sold. The result follows using straightforward algebra.  $\square$

**Proof of Proposition 6.** First, we assume that parameters are such that basis traders are present before and after the ban. This requires that  $S > 1/2$  and  $\mu_L \leq \frac{-4Lc_d\mu_H + \lambda(2-3L)c_s + 2\Delta\lambda L(S+1)}{4Lc_d + (3L-2)(c_l + c_s)}$ .

Second, we assume only  $\mu_L$ -investors buy bonds both before and after the ban, which requires that  $\mu_H \geq \max \left[ \frac{\mu_L(2(L-1)L(c_d+c_l)+c_l)+\Delta\lambda(S-\frac{1}{2})}{(2(L-1)L+1)(c_l-c_d)}, \frac{(2L-1)Lc_d\mu_L+(2(L-1)L+1)c_l\mu_L-(L+1)c_s(\lambda+\mu_L)+\Delta\lambda(S+1)}{(2L-1)L(c_l-c_d)+2c_l} \right]$ . Third, we assume that  $\mu_H$ -investors buy and sell CDSs before the ban and that only optimistic  $\mu_H$ -investors sell CDS after the ban (optimistic investors with longer trading horizons  $\mu_L$  hold the bond). This requires that  $\mu_H \leq \frac{\Delta\lambda}{2c_d}$ . Fourth, we assume that  $c_s \leq \frac{-L(2L-1)c_d(\mu_H+\mu_L)-(2(L-1)L+1)c_l\mu_L+\Delta\lambda((2L-1)L-S+1)}{(2(L-1)L+1)(\lambda+\mu_L)}$ , such that short bond positions reappear after the ban.

Closed-form solutions for general  $L$  in case of the double-uniform distribution and  $K_H \rightarrow \infty$  are then given by

$$p_{\text{no naked CDS}} - p_{\text{withCDS}} = \frac{((1+L)S - 1 + L^2 - 3L^3 + 2L^4) \Delta + (1 + 2L(L-1)) \left( (1+L)c_s - (2L-1)Lc_d \frac{\mu_H}{\lambda} \right) + L(2L-3)c_d \frac{\mu_L}{\lambda} + (2L^3 - L + 1)(c_l + c_s) \frac{\mu_L}{\lambda}}{2 - 5L + 8L^2 - 6L^3 + 4L^4} \quad (\text{A15})$$

$$q_{\text{no naked CDS}} - q_{\text{withCDS}} = -\frac{\Delta}{2} + \frac{c_d\mu_H}{\lambda} \quad (\text{A16})$$

One can show that the first derivative of the price difference w.r.t.  $c_s$  and  $c_l$  is positive. If  $L > 1$ , the derivative w.r.t.  $c_d$  is negative.  $\square$

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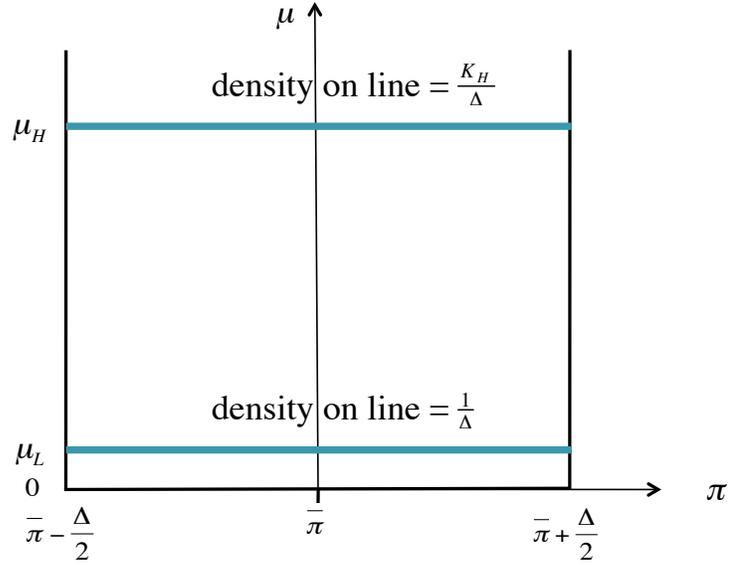


Figure 1: **The double-uniform distribution**

The figure illustrates the double-uniform distribution used to calculate closed-form solutions. There is a mass one of “buy-and-hold” investors with trading frequency  $\mu_L$  and a mass  $K_H$  of investors with more frequent trading needs  $\mu_H$ . For both classes of investors, the beliefs about the bond’s default probability are uniformly distributed on  $[\bar{\pi} - \frac{\Delta}{2}, \bar{\pi} + \frac{\Delta}{2}]$ .

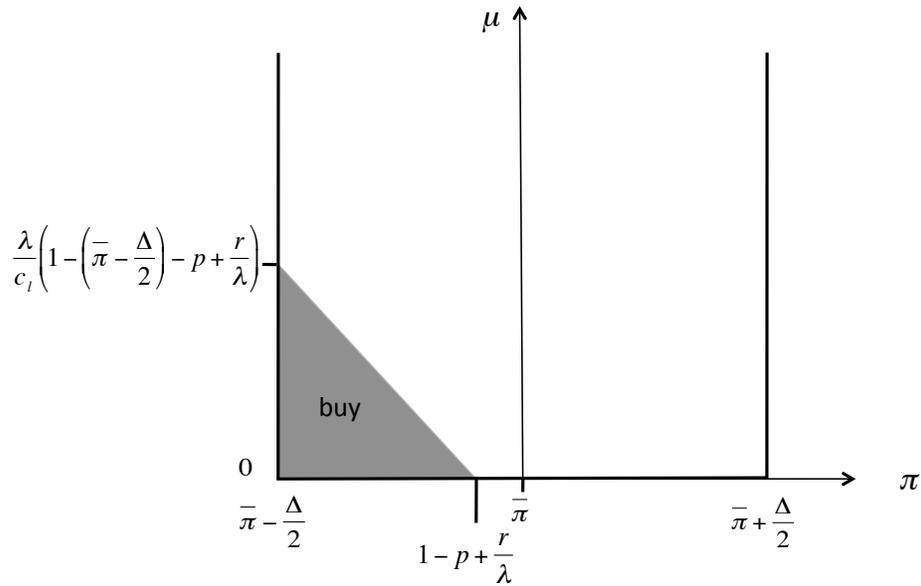


Figure 2: **Bond market equilibrium without short selling**

The figure illustrates the equilibrium when only the bond is trading and only long positions in the bond are allowed. The bond is purchased by investors who are sufficiently optimistic about the bond's default probability and who have long enough holding horizons, as illustrated by the grey triangle.

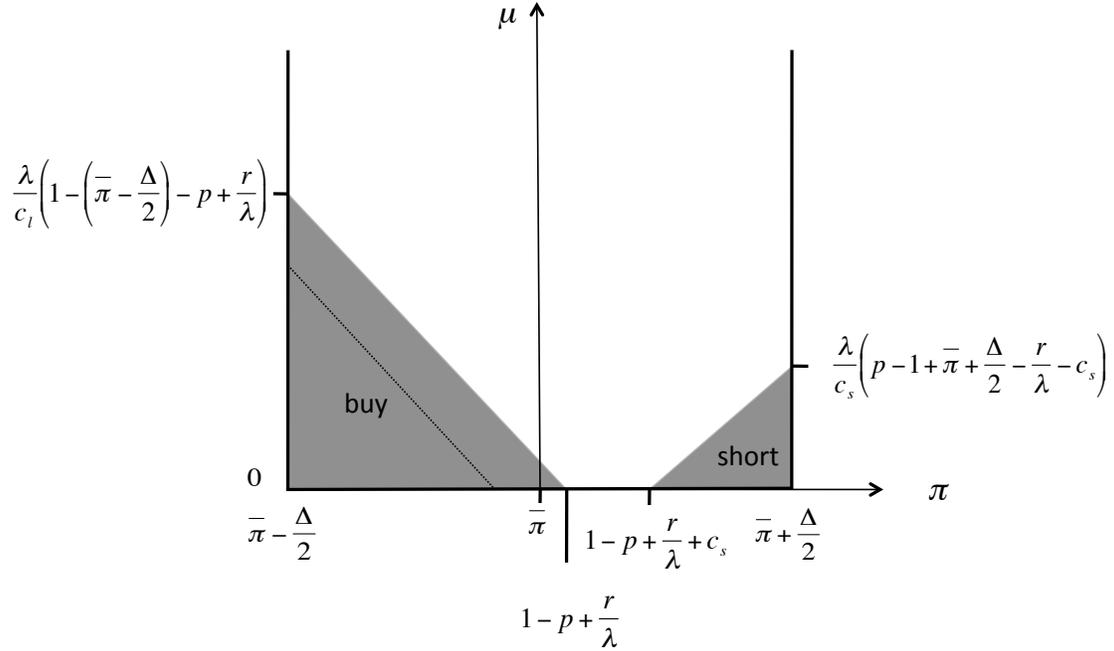


Figure 3: **Bond market equilibrium with short selling**

The figure illustrates the equilibrium when only the bond is trading and both long and short positions in the bond are possible. Given that  $c_s$  is not too large, investors who are pessimistic about the bond's default probability and have sufficiently long holding horizons short the bond. For markets to clear, this means that the triangle of long bondholders has to grow relative to the case in which only long positions are possible (illustrated by the dotted line). The average marginal investor is less optimistic and has a shorter holding horizon than when only long positions are possible, leading to a decrease in the bond price.

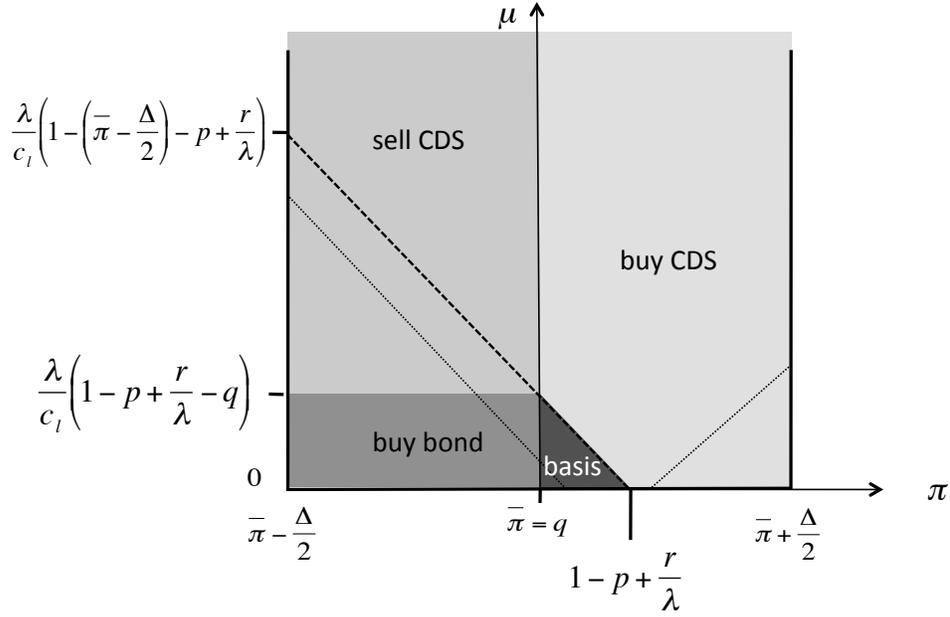


Figure 4: **Bond and CDS market equilibrium (basis traders cannot take leverage)**

This figure illustrates the equilibrium when both the bond and the CDS are trading and basis traders cannot take leverage ( $L = 1$ ). The introduction of the CDS has three effects: (i) Some investors who absent the CDS would purchase the bond now choose to sell CDS protection. (ii) Because the bond trades at a discount relative to the CDS, all former short sellers prefer to purchase CDS protection, which eliminates the shorting triangle. (iii) Investors who formerly bought the bond but whose beliefs about the bond's default probability are below the median belief  $\bar{\pi}$  become basis traders who purchase the bond and buy CDS protection. The dotted lines illustrate the long and short triangles in the absence of the CDS.

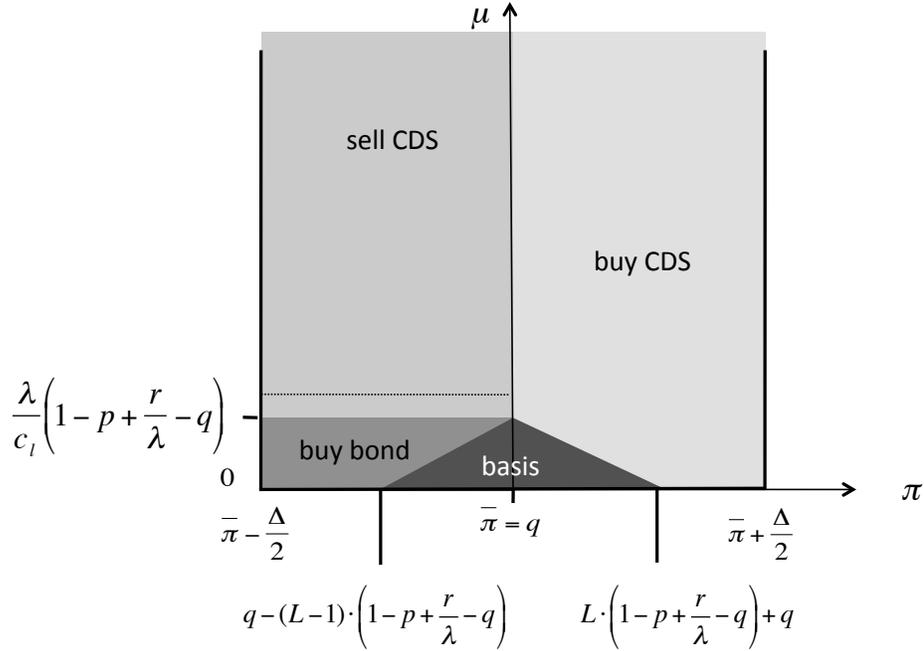


Figure 5: **Bond and CDS market equilibrium (basis traders can take leverage)**

The figure illustrates the equilibrium when both the bond and the CDS are trading and basis traders can take leverage ( $L > 1$ ). The ability to take leverage makes the basis trade more attractive, such that the basis trade triangle expands compared to Figure 4. Because of the increased demand from basis traders, the average marginal investor who buys the bond becomes more optimistic and has a longer holding horizon than when  $L = 1$ . For ease of comparison, the dashed line illustrates the rectangle of investors who purchase the bond when basis traders cannot take leverage ( $L = 1$ ).

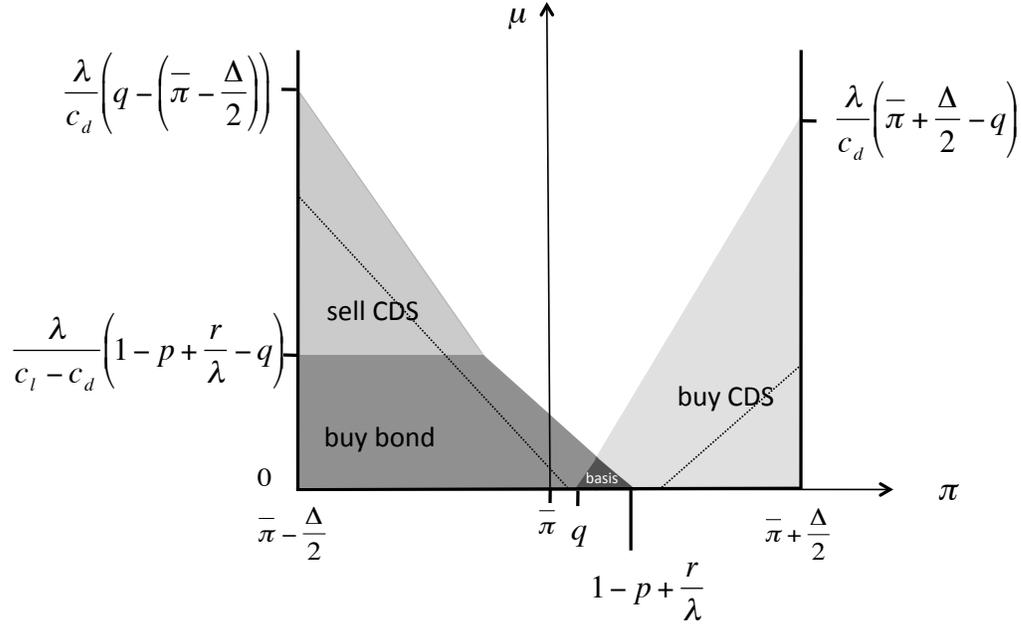


Figure 6: **Equilibrium with CDS when  $c_d > 0$ , basis traders cannot take leverage**

The figure illustrates the equilibrium when also the CDS involves trading frictions ( $c_d > 0$ ) and basis traders cannot take leverage ( $L = 1$ ). Compared to Figure 4, where  $c_d = 0$ , the *sell CDS* and *buy CDS* regions are now triangles, reflecting higher CDS trading costs for investors as the trading frequency  $\mu$  increases. As in the case with frictionless CDS, the introduction of the CDS has three effects: (i) Some investors who absent the CDS would purchase the bond now choose to sell CDS protection. (ii) Because of the negative CDS-bond basis, all former short sellers prefer to purchase a CDS, which eliminates the shorting triangle. (iii) Investors who formerly bought the bond but whose beliefs about the bond's default probability are sufficiently pessimistic such that they buying CDS protection has a positive payoff become basis traders who purchase the bond and buy CDS protection. For ease of comparison, the dotted lines are the *buy bond* and *short bond* triangles when no CDS is available.

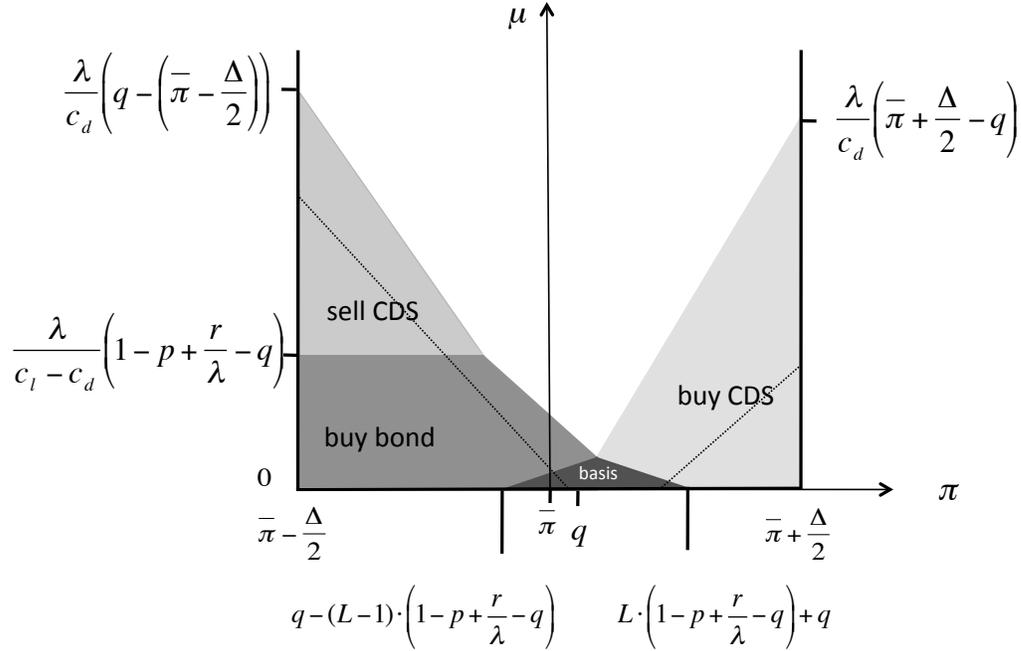


Figure 7: **Equilibrium with CDS when  $c_d > 0$ , basis traders can take leverage**

The figure illustrates the equilibrium when also the CDS involves trading frictions ( $c_d > 0$ ) and basis traders can take leverage ( $L > 1$ ). Relative to Figure 6, the triangle of basis traders expands because the ability to take leverage increases the profitability of the basis trade. The resulting increase in demand for the bond pushes up the bond price, analogously to the arguments in the frictionless CDS ( $c_d = 0$ ) case. For ease of comparison, the dotted lines are the *buy bond* and *short bond* triangles when no CDS is available.

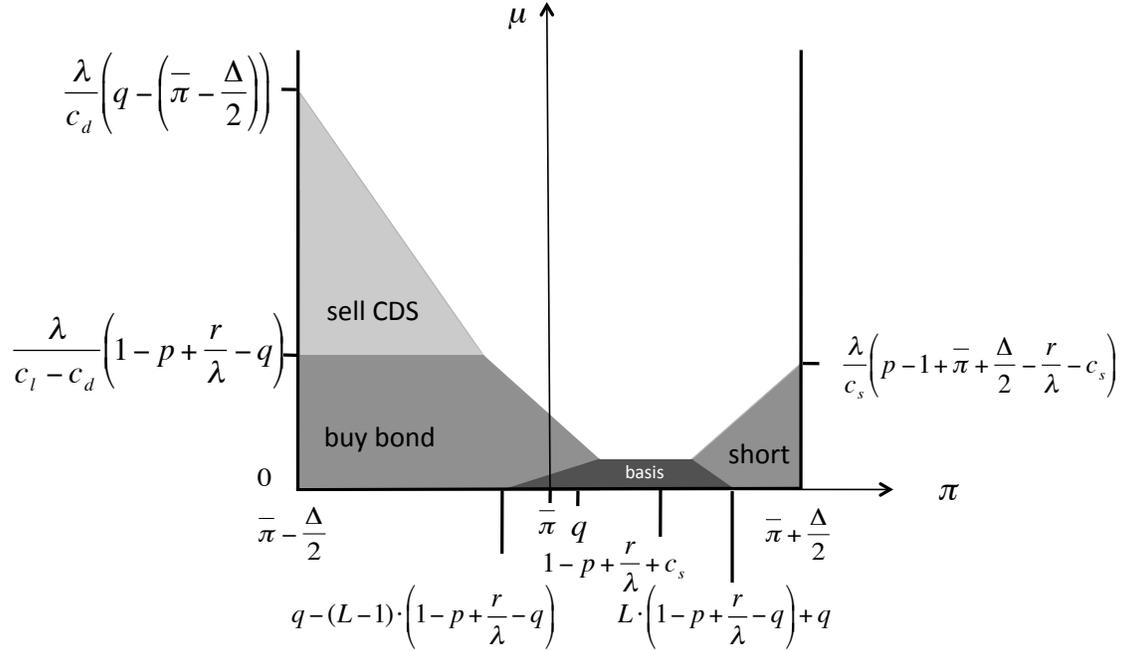


Figure 8: **Banning naked CDS when  $c_d > 0$  and  $L > 1$**

The figure illustrates the equilibrium when naked CDS positions are banned. Compared to Figure 7, which depicts the same setup except that naked CDS positions are allowed, there are two major changes. Some investors who used to purchase naked CDS protection now choose to short the bond, exerting downward pressure on the bond price. Some investors who used to purchase naked CDS protection now become basis traders, which exerts upward pressure on the bond price. The overall effect of banning naked CDS positions on the bond price (and, hence, the borrowing cost for the issuer) is ambiguous.