Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 1121

October 2014

Banks, Capital Flows and Financial Crises

Ozge Akinci and Albert Queralto

Banks, Capital Flows and Financial Crises*

Ozge Akinci and Albert Queralto†

November 7, 2014

Abstract

This paper proposes a macroeconomic model with financial intermediaries (banks), in which banks face occasionally binding leverage constraints and may endogenously affect the strength of their balance sheets by issuing new equity. The model can account for occasional financial crises as a result of the nonlinearity induced by the constraint. Banks' precautionary equity issuance makes financial crises infrequent events occurring along with "regular" business cycle fluctuations. We show that an episode of capital inflows and rapid credit expansion, triggered by low country interest rates, leads banks to endogenously decrease the rate of equity issuance, contributing to a higher likelihood of future crises. Macroprudential policies directed at strengthening banks' balance sheets, such as capital requirements, are shown to lower the probability of financial crises and to enhance welfare.

Keywords: Financial Intermediation; Sudden Stops; Leverage Constraints; Occasionally Binding Constraints.

JEL classification: E32; F41; F44; G15

*Thanks to Dario Caldara, Luca Guerrieri, Matteo Iacoviello, Jesper Linde and Chris Erceg as well as seminar participants in the Federal Reserve Board International Finance Workshop, Georgetown Center for Economic Research Conference, NYU Alumni Conference, SED 2013 Conference, Federal Reserve Bank of Kansas City, Koc University Winter Workshop, SCIEA meeting at the Federal Reserve Bank of Boston, and the CEPR European Summer Symposium in International Macroeconomics for helpful comments and suggestions. The views expressed in this paper are solely the responsibility of the authors, and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

†Board of Governors of the Federal Reserve System, Division of International Finance, 20th St and Constitution Ave NW, Washington, DC, 20551, USA. Contact at ozge.akinci@frb.gov, albert.queralto@frb.gov.

1 Introduction

The recent wave of financial crises across the globe has put financial stability at the forefront of policy discussions. At the same time, it has led to a renewed interest in macroeconomic models that can account for financial crises in a manner consistent with the evidence, and which can be used to analyze the desirability of "macroprudential" policies aimed at preventing systemic financial instability.

Financial crises are infrequent events associated with a significant disruption in financial intermediation. As illustrated in Figure 1, both the recent crises in the U.S. and Europe and the South Korean crisis of 1997 featured unprecedented increases in credit spreads, which reached levels not seen before or since. Accordingly, an important feature of models for financial stability analysis is that they be able to capture the non-linear nature of financial crises, i.e. the fact that they are infrequent events associated with extreme disruption.[1] At the same time, it has often been the case that a highly vulnerable financial sector played a key role in precipitating the crises. In turn, this points to the importance of modeling the incentives for financial intermediaries in non-crisis times to adopt more or less risky balance sheet positions.

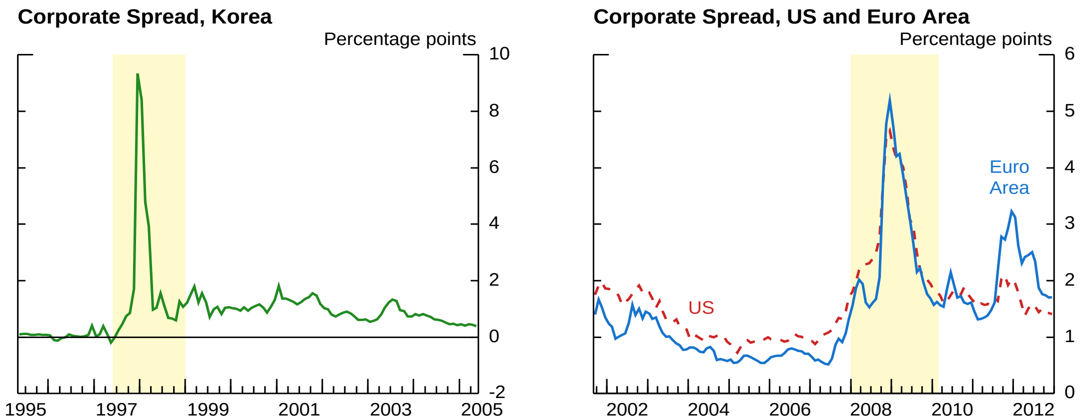
This paper addresses these issues by developing a quantitative macroeconomic model in which financial intermediaries (banks) face occasionally binding leverage constraints, and in which banks can issue equity as well as short term debt. By allowing banks' constraints to be occasionally binding, we can capture infrequent financial crises occurring along with "regular" business cycle fluctuations. We can also use the model to calculate the likelihood of future crises, and to analyze how this likelihood changes with economic conditions and with government policy. In turn, by allowing banks to issue equity we can capture how banks may endogenously affect the evolution of their net worth, a key determinant of the likelihood of crises. We can also analyze whether there is a role for government policy to manipulate the privately optimal equity issuance choice.

Our model is a small open economy with banks, driven by exogenous stochastic disturbances to capital quality (which works to induce fluctuations in banks' net worth) and to the country interest rate. As in Gertler and Kiyotaki (2010) and others, in the model an agency problem may limit banks' ability to borrow in the short-term debt market.[2] Unlike in Gertler and Kiyotaki (2010), in our framework the constraint is occasionally binding; it only binds when banks' net worth is sufficiently low. When this happens, the economy enters into finan-

[1] Stein (2014) also emphasizes the nonlinearity in the relationship between credit spreads and economic activity, with increases in spreads leading to much stronger effects on the economy than decreases. Merton (2009), Kenny and Morgan (2011), Hubrich et al. (2013) also highlight the non-linear nature of financial crises.

[2] See also Gertler and Karadi (2011) and Gertler et al. (2012).

Figure 1: Credit Spreads in Selected Countries



Note: *For Korea, the corporate spread is calculated as the difference between 3-year AA-rated corporate bond and Treasury bond yields of the same maturity. For the U.S. and the Euro Area, corporate spreads are calculated as the difference between 5-year BBB-rated corporate bond and Treasury bond yields of the same maturity. Data source: Haver and Bloomberg.*

cial crisis mode: credit spreads rise sharply due to banks' inability to extend financing, raising the cost of credit for nonfinancial firms. Along the way, the financial accelerator mechanism –*á la* Bernanke et al. (1999) and Kiyotaki and Moore (1997)– operates: declines in bank net worth, by making banks' constraints tighter, lead to declines in investment, which trigger drops in asset prices, leading net worth to drop further. In normal times, when the constraint is not binding, banks anticipate that future shocks may put them against their constraint. This leads them to issue equity at a positive rate, which contributes to a stronger net worth position and thereby reduces the probability of a financial crisis. As a consequence, the economy spends most of the time in the unconstrained region, in which it exhibits fluctuations similar to those of a standard neoclassical small open economy.

We conduct several numerical experiments to illustrate the quantitative properties of the model. We first analyze the economy's response to a decrease in the country interest rate. Low interest rates induce a capital inflow and a credit boom at home, consistent with the evidence that rapid credit expansions are typically financed by borrowing from foreign investors.[3] At the same time, they lead banks to endogenously decrease the pace of new equity issuance. This is because low interest rates induce a persistent decline in the inside value of net worth, as they make it cheaper for banks to raise funds in the short term debt market (via deposits or external borrowing). As a consequence, equity issuance becomes less attractive to banks. We find that this endogenously exposes the economy to a higher risk of a financial crisis in the

[3]See, among others, Mendoza and Terrones (2012) and Bruno and Shin (2012).

future, compared to a case where the equity issuance rate is kept constant. This is consistent with evidence suggesting enhanced vulnerability of the banking sector after credit booms.[4]

In a second experiment, we illustrate the nonlinearity and state dependence induced by the occasionally binding constraint, by showing that a shock to banks' asset quality may generate very different dynamics depending on the initial state of the economy when the shock arrives. In particular, we compare the effects of the shock in "normal times," to the effects when the economy has a stock of foreign debt substantially larger than the average. We find that with high foreign debt, the economy is much more vulnerable to an adverse realization in capital quality: in this case, the shock leads to a binding borrowing constraint, and to significant drops in domestic investment, asset prices and credit, as well as to a sudden stop in capital inflows. By contrast, in normal times the economy proves to be more resilient to an adverse capital quality shock: the economy's state is considerably far from the constrained region, and the shock is not enough to trigger the constraint. As a consequence, the shock induces far less severe effects compared to the case with high foreign debt.

Next, we explore the characteristics of the financial crisis episodes produced by the model, by conducting a long simulation and computing averages across all the events that we identify. We find that the model produces infrequent financial crises qualitatively and quantitatively consistent with the evidence. In particular, crisis periods feature severe disruption in financial intermediation, exemplified by large increases in credit spreads, as well as plunges in domestic investment, consumption and output, occurring along with sudden stops in capital inflows. Crises in the model are triggered not by unusually large adverse realizations of the shocks, but by a moderately adverse sequence of capital quality and country interest rate shocks which push the economy toward the constrained region and eventually trigger the constraint.

Finally, we use the model to assess the desirability of government policy directed at enhancing financial stability. Within our framework, when the constraint binds, banks' ability to borrow is affected by asset prices, since the latter affect bank net worth. This may introduce a pecuniary externality in banks' decision to issue equity in normal times (when the constraint is not binding), as banks do not internalize the consequences for asset price movements of their individual balance sheet position. The existence of the pecuniary externality creates a rationale for government intervention. In particular, we study a government subsidy to new equity issuance, financed by a tax on bank assets. The policy tilts banks' incentives in favor of raising equity, thus strengthening their balance sheet positions. The policy is meant to

[4] Mendoza and Terrones (2012), for example, show that credit booms are systematically related to a boom-bust cycle in production and absorption, asset prices, capital inflows and external deficits. They find that credit boom events often end with financial crises. This holds true for emerging and industrialized countries alike. See also, among others, Gourinchas and Obstfeld (2012) and Schularick and Taylor (2012).

capture the spirit of actual policies such as the Basel III proposal for countercyclical capital buffers for banks, already activated in some countries.[5]

We show that the government subsidy increases the stochastic steady state level of new equity issuance; in this respect, it encourages banks to build a capital buffer which has the potential to increase the resilience of the system to future realizations of adverse shocks. Indeed, we find that the subsidy policy is successful at reducing the probability of occurrence of financial crises. For example, the one-year-ahead likelihood of financial crises is cut by half (from 6 to 3 percent) with a subsidy of 3 percent per unit of equity issued. We also show that this type of policy can increase welfare: the welfare gain peaks at 0.02 percent of steady-state consumption with a 3 percent subsidy. However, a constant subsidy policy can result in welfare losses if the government increases the subsidy level beyond 4.5 percent per unit of equity issued. In this respect, our results point to the importance of implementing a time-varying capital requirement instead of a fixed one.[6]

This paper is related to several strands in the literature. The model economy proposed in this paper endogenously switches between normal times and financial crisis times, as in the recent Nonlinear Dynamic Stochastic General Equilibrium (NDSGE) models proposed by Bianchi (2010), Mendoza (2010), and others.[7] However, in our model, borrowing constraints arise endogenously as a result of an explicit agency problem, as in Gertler and Kiyotaki (2010). This is in contrast with the NDSGE literature, which imposes exogenous collateral constraints to try to capture nonlinear financial crisis dynamics. Novel features of our setup relative to Gertler and Kiyotaki (2010), on the other hand, are twofold. First, Gertler and Kiyotaki (2010) analyze the model's local behavior around a steady state in which the constraint always binds. We instead focus on the global implications when the constraint binds only occasionally. Second, we allow banks to raise new equity. In Gertler and Kiyotaki (2010) and related frameworks, banks' net worth reflects the mechanical evolution of retained earnings, and therefore any explicit precautionary behavior by banks is ruled out by assumption. By allowing this new choice margin for banks, then, we can analyze whether government policies may improve on *laissez-faire* by manipulating that margin.

This paper is also related to recent work by Cespedes et al. (2012), who incorporate

[5]For instance, Switzerland recently used such a tool in the aftermath of financial crises to address financial stability concerns. Faced with signs of overheating in housing sector, and constrained from using monetary policy for both exchange-rate and inflation-targeting reasons, in early 2013 Switzerland activated a countercyclical capital buffer that adds one percentage point of capital requirement for direct and indirect mortgage backed positions secured by Swiss residential property, and in early 2014 the Swiss National Bank recommended an increase to two percentage points.

[6]In the extension of this paper, we are working on the impact of time-varying capital buffers that are explicitly contingent on credit boom episodes.

[7]See also Bianchi and Mendoza (2013), Benigno et al. (2012) and Schmitt-Grohe and Uribe (2011).

financial intermediation subject to an occasionally binding credit constraint into a two-period open economy model. Our focus, instead, is to offer a framework that does not stray too far from the standard quantitative DSGE model used in policy analysis, and that is tractable enough to accommodate the features that are often present in that literature. This focus also differentiates our work from other recent papers introducing financial intermediation within a macroeconomic framework, like Brunnermeier and Sannikov (2014) or Boissay et al. (2013).

The remainder of the paper is organized as follows. Section 2 presents model in detail. Section 3 presents the predictions of the model calibrated to a typical small open economy. It analyzes quantitative behavior of the model economy in both normal times and financial crises times, and also explores the characteristics of the financial crisis episodes produced by the model. Section 4 presents the effect of macroprudential policy on the economy. Section 5 concludes.

2 The Model

The model is a small open economy with frictions in financial intermediation, in the spirit of Gertler and Kiyotaki (2010). There are banks who make risky loans to nonfinancial firms and collect deposits from both households and foreigners. Because of an agency problem, banks may be constrained in their access to external funds. In addition, we allow banks to raise new equity from households, so that the evolution of bank net worth reflects banks' endogenously chosen rate of new equity issuance, as well as the mechanical accumulation of retained earnings.[8]

A second novel feature of our setup is that banks' constraints are not permanently binding, as in much of the related literature, but instead bind only occasionally. In normal or "tranquil" times, banks' constraints are not binding: credit spreads are small and the economy's behavior is similar to a frictionless neoclassical framework. When the constraint binds the economy enters into financial crisis mode: credit spreads rise sharply, and investment and credit collapse, consistent with the evidence.

2.1 Households

Each household is composed of a constant fraction $(1 - f)$ of workers and a fraction f of bankers. Workers supply labor to the firms and return their wages to the household. Each

[8]This approach is related to Gertler et al. (2012), in which banks are allowed to issue outside equity as well as debt. We allow the banks to raise inside equity instead.

banker manages a financial intermediary ("bank") and similarly transfers any net earnings back to the household. Within the family there is perfect consumption insurance.

Households do not hold capital directly. Rather, they deposit funds in banks. The deposits held by each household are in intermediaries other than the one owned by the household. Bank deposits are riskless one period securities. The consumption, C_t, bond holdings, B_t, and labor decisions, L_t, are given by maximizing the discounted expected future flow of utility

$$Max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \tag{1}$$

where

$$U(C_t, L_t) \quad = \quad \frac{\left(C_t - \chi \frac{L_t^{1+\epsilon}}{1+\epsilon}\right)^{1-\gamma} - 1}{1-\gamma} \tag{2}$$

subject to the budget constraint

$$C_t + B_t \quad \leq \quad W_t L_t + R_{t-1} B_{t-1} + \Pi_t \tag{3}$$

$\mathbb{E}_t$ denotes the mathematical expectation operator conditional on information available at time t, $\beta \in (0,1)$ represents a subjective discount factor, γ is the coefficient of relative risk aversion, and ϵ determines the wage elasticity of labor supply, which is given by $1/\epsilon$. Utility is defined as in Greenwood et al. (1988), which implies non-separability between consumption and leisure. This assumption eliminates the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor independent of consumption.

Variable W_t is the real wage, R_t is the real interest rate received from holding one period bond, Π_t is total profits distributed to households from their ownership of both banks and firms. The first order conditions of the household's problem are presented in Appendix A.

2.2 Banks

Banks are owned by the households and operated by the bankers within them. In addition to its own equity capital, a bank can obtain external funds from both households, b_t, and foreign investors, b_t^*, such that total external financing available to the bank is given by $d_t = b_t + b_t^*$. We assume that both domestic deposits and foreign borrowing are one-period non-contingent debt. Thus, by arbitrage their returns need to be equalized in equilibrium, a condition we impose at the onset.

In addition, banks in period t can raise an amount e_t of new equity. The new equity is available in the following period to make loans, together with the equity accumulated via retained earnings and with any external borrowing d_{t+1}. Accordingly, in each period the bank uses its net worth n_t (which includes equity raised in the previous period) and external funds d_t, to purchase securities issued by nonfinancial firms, s_t, at price Q_t. In turn, nonfinancial firms use the proceeds to finance their purchases of physical capital.

We assume that banks are "specialists" who are efficient at evaluating and monitoring nonfinancial firms and also at enforcing contractual obligations with these borrowers. That is why firms rely exclusively on banks to obtain funds, and the contracting between banks and nonfinancial firms is frictionless. However, as in Gertler and Kiyotaki (2010) and related papers, we introduce an agency problem whereby the banker managing the bank may decide to default on its obligations and instead transfer a fraction of assets to his family, in which case it is shut down and its creditors can recover the remaining funds. In recognition of this possibility, creditors potentially limit the funds they lend to banks. In this setup, banks may be credit constrained, depending on whether their desired asset holdings per unit of net worth exceeds the maximum allowed by the incentive constraint.

Figure 2: Period-t Timeline for Bankers

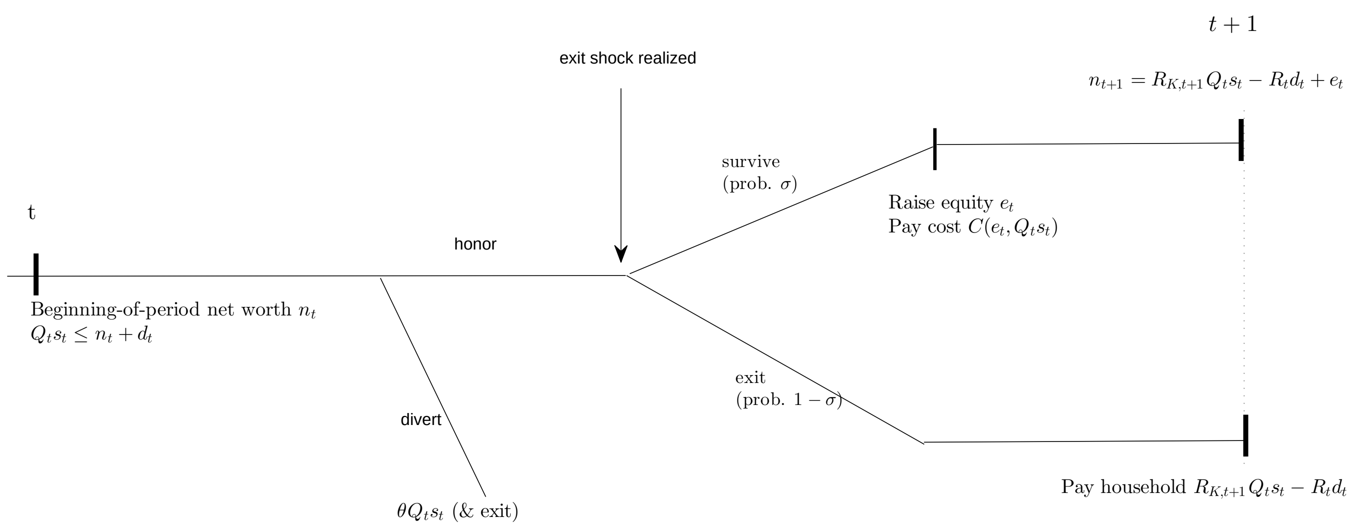


Figure 2 show the timeline for banks. Banks start the period t with net worth n_t.[9] Banks then use their net worth and external funds (issues of short-term bonds) to fund assets $Q_t s_t$,

[9]The newborn ones that replace the banks that exit receive endowment $\xi Q_{t-1} K_{t-1}$.

subject to the balance sheet constraint:

$$Q_t s_t \leq n_t + d_t \tag{4}$$

After purchasing the securities, banks can choose to divert fraction θ of assets funded, in which case they get $\theta Q_t s_t$. The incentive constraint requires that the bank's continuation value be higher than the value of the diverted funds, $V_t \geq \theta Q_t s_t$.

In order to limit bankers ability to save to overcome financial constraints, we assume that with *i.i.d.* probability $1 - \sigma$, a bankers exits, transfers retained earnings to the household and becomes a worker in period $t + 1$. At the *end* of the period t, only surviving banks have the option to raise new equity. In particular, after the bank finds out whether it receives the exit shock, in the case that it continues (with probability σ) it can pay cost $C(e_t, Q_t s_t)$ to raise new equity e_t from the household, which will be available in $t + 1$ to fund assets.[10] The equity issuance cost is meant to capture in a simple way the actual costs and frictions in the process of raising equity that banks face – for example, the costs of finding new investors or the frictions involved in the process of creating and selling new shares.[11]

Accordingly, the total net worth available for surviving banks in $t + 1$ will be given by

$$n_{t+1} = R_{K,t+1} Q_t s_t - R_t d_t + e_t \tag{5}$$

where $R_{K,t+1}$ denotes the gross rate of return on a unit of the bank's assets from t to $t + 1$ and R_t is the rate of return on short-term (risk free) bonds held by the bank's creditors. In the case the bank exits at the end of t (with probability $1 - \sigma$), we assume it does not have the option to issue new equity.[12] It simply pays the household the profits from assets funded in t, net of debt repayments: $R_{K,t+1} Q_t s_t - R_t d_t$.

The objective of the bank is to maximize expected terminal payouts to the household, net of the equity transferred by the household and of the cost of the transfer $C(e_t, Q_t s_t)$. The bank values payoffs in each period and state using $\Lambda_{t,t+i}$, the household's stochastic discount factor.

[10] The cost is allowed to depend on the overall balance sheet size as well as e_t; later we will specialize to $c(x_t) Q_t s_t$, where $x_t \equiv e_t / Q_t s_t$.

[11] An alternative interpretation of $C(e_t, Q_t s_t)$ is that it represents a cost of lowering net dividend payouts. In the model, by assumption banks only pay dividends at a fixed rate $1 - \sigma$ (the exit probability), so that in expectation, before the exit shock is realized, "net dividends" (dividend payouts net of new equity raised) are equal to $(1 - \sigma)[\mathbb{E}_t(R_{K,t+1}) Q_t s_t - R_t d_t] - \sigma e_t$. Then, the cost $C(e_t, Q_t s_t)$ of increasing e_t can be interpreted as a cost of lowering net dividends.

[12] As long as the cost of raising equity is positive, for an exiting bank it would never pay to raise equity, as the new equity would simply be transferred back to the household.

The bank's problem is then to choose state-contingent sequences $\{s_t, d_t, e_t\}_{i=0}^{\infty}$ to maximize

$$\mathbb{E}_t \left[\sum_{i=1}^{\infty} \sigma^{i-1}(1-\sigma)\Lambda_{t,t+i}\overline{n}_{t+i} - \sum_{i=1}^{\infty} \sigma^{i} \left[\Lambda_{t,t+i-1}C(e_{t+i-1}, Q_{t+i-1}s_{t+i-1}) + \Lambda_{t,t+i}e_{t+i-1}\right] \right]$$

subject to (4), (5) and the incentive constraint, where $\overline{n}_t \equiv R_{K,t}Q_{t-1}s_{t-1} - R_{t-1}d_{t-1}$.

Switching to the recursive formulation, we can write a banker's problem as follows:

$$V_t(n_t) = \max_{s_t,d_t,e_t} \; (1-\sigma)\mathbb{E}_t\Lambda_{t,t+1}\left(R_{K,t+1}Q_t s_t - R_t d_t\right) + \sigma\left\{\mathbb{E}_t\Lambda_{t,t+1}\left[V_{t+1}(n_{t+1}) - e_t\right] - C(e_t, Q_t s_t)\right\}$$

subject to

$$Q_t s_t \leq n_t + d_t \quad (6)$$

$$n_{t+1} = R_{K,t+1}Q_t s_t - R_t d_t + e_t \quad (7)$$

$$(1-\sigma)\mathbb{E}_t\Lambda_{t,t+1}\left(R_{K,t+1}Q_t s_t - R_t d_t\right) + \sigma\left[\mathbb{E}_t\Lambda_{t,t+1}\left(V_{t+1}(n_{t+1}) - e_t\right) - C(e_t, Q_t s_t)\right] \geq \theta Q_t s_t \quad (8)$$

Equation (6) is the bank's balance sheet constraint. Equation (7) is the law of motion for the banker's net worth, which includes new equity raised e_t. Equation (8) is the incentive constraint: it states that the banker's continuation value must be greater than the value of diverting funds.

To solve the banker's problem, we first guess that the value function is linear in net worth, $V_t(n_t) = \alpha_t n_t$. Define

$$\Omega_{t+1} = (1-\sigma) + \sigma\alpha_{t+1} \quad (9)$$

$$\mu_{K,t} = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{K,t+1} - R_t)] \quad (10)$$

$$\nu_t = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}]R_t \quad (11)$$

$$\nu_{e,t} = \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\alpha_{t+1} - 1\right)\right] \quad (12)$$

Since Ω_{t+1} is the value to the bank of an extra unit of net worth the following period, it acts by "augmenting" banks' stochastic discount factor (SDF), so that their effective SDF is $\Lambda_{t+1}\Omega_{t+1}$. The variable $\nu_{e,t}$ denotes the *net* value today of a transfer by the household that increases bank net worth tomorrow by one unit, conditional on not exiting. In the decision to raise equity, the bank trades off the benefit $\nu_{e,t}$ against the issuing cost.

With these definitions, the problem simplifies to

$$\alpha_t n_t = \max_{s_t, e_t} \quad \mu_{K,t} Q_t s_t + \nu_t n_t + \sigma \left[\nu_{e,t} e_t - C(e_t, Q_t s_t) \right] \tag{13}$$

subject to the incentive constraint:

$$\mu_{K,t} Q_t s_t + \nu_t n_t + \sigma \left[\nu_t e_t - C(e_t, Q_t s_t) \right] \geq \theta Q_t s_t \tag{14}$$

Define $x_t \equiv \frac{e_t}{Q_t s_t}$. We assume that the equity cost takes the following form:

$$C(e_t, Q_t S_t) = c(x_t) Q_t s_t$$

where $c(x_t) = \frac{\kappa}{2} x_t^2$. Then the first order condition for x_t is

$$\begin{aligned} \nu_{e,t} &= c'(x_t) \qquad (15) \\ &= \kappa x_t \qquad (16) \end{aligned}$$

Using this first order condition we can re-write the value function as

$$\alpha_t n_t = \mu_t Q_t s_t + \nu_t n_t$$

where the "total" excess return on assets is defined as $\mu_t \equiv \mu_{K,t} + \sigma \frac{\kappa}{2} x_t^2$.

(Proved in Appendix B.)

Then when the constraint does not bind, $\mu_t = 0$, and thus $\alpha_t = \nu_t$. When the constraint binds, bank asset funding is given by the constraint at equality, $Q_t s_t = \phi_t n_t$ where $\phi_t = \frac{\nu_t}{\theta - \mu_t}$, and $\alpha_t = \nu_t + \mu_t \phi_t$. Therefore, the value of a unit of net worth today α_t is given by

$$\alpha_t = \nu_t + \phi_t \mu_t \tag{17}$$

Since bankers' problem is linear, we can easily aggregate across banks. The law of motion for aggregate net worth is the following:

$$N_t = \sigma \left[(R_{K,t} - R_{t-1} + x_{t-1}) \underbrace{Q_{t-1} K_{t-1}}_{Q_{t-1} S_{t-1}} + R_{t-1} N_{t-1} \right] + (1 - \sigma) \xi Q_{t-1} K_{t-1} \tag{18}$$

2.2.1 The Choice of Equity Issuance

From the first order condition for equity issuance, equation (15),

$$\underbrace{\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\overbrace{\nu_{t+1}+\phi_{t+1}\mu_{t+1}}^{\alpha_{t+1}}-1\right)\right]}_{\nu_{e,t}}=c'(x_t) \tag{19}$$

The left hand side of equation (19) captures the marginal benefit for the bank of issuing one extra unit of equity, while the right hand side captures the marginal cost. Since the banker is ultimately a member of the household, the left hand side of equation (19) captures the benefit of transferring a unit of resources from the household to the bank. Note that if the bank's incentive constraint was never to bind in the future, the benefit of such transfer would be zero: we would then have $\mu_{t+i}=0$ for all $i\geq 1$, which from equations (9), (11) and the Euler equation for riskless debt, $\mathbb{E}_t(\Lambda_{t,t+1}R_t)=1$, implies a solution with $\nu_{t+i}=\Omega_{t+i}=1$ for $i\geq 1$. Therefore, the value of equity issuance $\nu_{e,t}$ would be zero, and the bank would choose to not issue equity.

Conversely, if the constraint is expected to bind in the future (either in $t+1$ or in subsequent periods) we will have $\nu_{e,t}>0$. To the extent that there is a positive probability of the constraint binding in the future (this will be the case in our calibrated model below), the value of issuing equity will always be positive for the bank. In that case, if there were no costs of equity issuance (i.e., if $c(x)=0$ for all x) the net benefit of equity issuance would always be positive, and therefore the bank would choose to raise equity in an infinite amount.

2.3 Nonfinancial Firms

There are two categories of nonfinancial firms: final goods firms and capital producers. In turn, within final goods firms we also distinguish between "capital storage" firms and final goods producers, in order to clarify the role of bank credit used to purchase capital goods.

2.3.1 Final Goods Firms

We assume that there are two types of firms: "capital storage" firms and final goods producers. The first type of firm purchases capital goods from capital good producers, stores them for one period, and then rents them to final goods firms. The latter type of firm combines physical capital rented from capital goods firms with labor to produce final output.[13]

[13] Sargent and Ljungqvist (2004, Chapter 12) present a similar structure with two types of firms. Firms of type I and II in their notation correspond to our final goods producers and capital storage firms, respectively.

Importantly, capital storage firms have to rely on banks to obtain funding, as explained below.

In period $t-1$, a representative capital storage firm purchases K_{t-1} units of physical capital at price Q_{t-1}. It finances these purchases by issuing S_{t-1} securities to banks which pay a state-contingent return $R_{K,t}$ in period t. At the beginning of period t, the realization of the capital quality shock ψ_t determines the effective amount of physical capital in possession of the firm, given by $e^{\psi_t}K_{t-1}$. The firm rents out this capital to final goods firms at price Z_t, and then sells the undepreciated capital $(1-\delta)e^{\psi_t}K_{t-1}$ in the market at price Q_t. The payoff to the firm per unit of physical capital purchased is thus $e^{\psi_t}\left[Z_t+(1-\delta)Q_t\right]$. Since contracting between firms and banks is frictionless, it follows that the return on the securities issued by the firm is given by the following equation:

$$R_{K,t} = e^{\psi_t}\frac{Z_t+(1-\delta)Q_t}{Q_{t-1}} \tag{20}$$

Note that this equation ensures that capital storage firms make zero profits state-by-state.

The capital quality shock $\psi_t \sim N(0,\sigma_\psi)$ is a simple way to introduce an exogenous source of variation in the value of capital, which may be thought of as capturing some form of economic obsolescence.[14] As equation (20) makes clear, the random variable ψ_t provides a source of fluctuations in returns to banks' assets. These fluctuations are enhanced by movements in the endogenous asset price Q_t triggered by fluctuations in ψ_t.

In the aggregate, the law of motion for capital is given by

$$K_t = I_t + (1-\delta)e^{\psi_t}K_{t-1} \tag{21}$$

Final goods firms produce final output Y_t using capital, K_{t-1}, and labor, L_t via the following Cobb-Douglas production function:

$$Y_t = (e^{\psi_t}K_{t-1})^{\alpha}L_t^{1-\alpha} \tag{22}$$

The first order conditions for labor and for physical capital determines are as follows:

$$(1-\alpha)\frac{Y_t}{L_t} = W_t \tag{23}$$

$$\alpha\frac{Y_t}{e^{\psi_t}K_{t-1}} = Z_t \tag{24}$$

[14] Gertler et. al. (2012) provide an explicit microfoundation.

2.3.2 Capital Producers

Capital producers make new capital using input of final output and are subject to adjustment costs. They sell new capital to firms at the price Q_t. The price of capital goods is equal to the marginal cost of investment goods production:

$$Q_t = 1 + \psi_I \left(\frac{I_t}{e^{\psi_t} K_{t-1}} - \delta \right) \tag{25}$$

2.4 International Capital Markets

We follow Schmitt-Grohe and Uribe (2003) and assume that small open economy is subject to debt elastic interest rate premium in the international markets.

$$R_t = \frac{1}{\beta} + \varphi(e^{\frac{B_t^*}{Y} - \bar{b}} - 1) + e^{R_t^* - 1} - 1 \tag{26}$$

where $\bar{b}$ governs the steady state foreign debt to GDP ratio and R_t^* is a shock to the country interest rate, which is assumed to follow an AR(1) process in logs:

$$\begin{aligned} \log(R_t^*) &= \rho_R \log(R_{t-1}^*) + \epsilon_{R,t} \\ \epsilon_{R,t} &\sim N(0, \sigma_R) \end{aligned}$$

2.5 Resource Constraint and Market Clearing

The resource constraint of this economy can be derived from representative household's budget constraint:

$$Y_t = C_t + \left[1 + \frac{1}{2} \psi_I \left(\frac{I_t}{e^{\psi_t} K_{t-1}} - \delta \right)^2 \right] I_t + \sigma \frac{\kappa}{2} x_t^2 Q_t K_t + NX_t \tag{27}$$

The balance of payments equation is

$$R_{t-1} B_{t-1}^* - B_t^* = NX_t \tag{28}$$

where NX stands for the net exports.

(Proved in Appendix C.)

3 Model Analysis

This section illustrates the features of the model via a series of numerical experiments. We solve the model using the Parameterized Expectations Algorithm (PEA), described in Appendix E.[15] Since this method allows us to solve the model fully nonlinearly, we can capture banks' occasionally binding incentive constraint. Moreover, the risk-taking behavior of banks is appropriately captured, as the method fully accounts for shock uncertainty.

3.1 Calibration

Table 1 reports parameter values. The model includes nine conventional preference and technology parameters, for which we choose standard values in the literature.

Table 1: Calibration

Parameter	Symbol	Value	Source/Target
Conventional			
Discount factor	β	0.985	Interest rate (6%, ann.)
Risk aversion	γ	2	Standard RBC value
Inverse Frisch elast.	ϵ	1/3	Frisch lab. sup. elast. (inv)
Labor disutility	χ	2.8125	Steady state labor (30%)
Capital share	α	0.33	Standard RBC value
Capital depreciation	δ	0.025	Mendoza (2010)
Investment adj. cost	Ψ_I	5	BGG (2000)
Debt elast. of interest rate	φ	0.05	
Reference debt/output ratio	$\bar{b}$	0.6	Steady state B/Y of 60%
Financial Intermediaries			
Survival rate	σ	0.95	Expected horizon of 5 yrs, GK (2013)
fraction divertable	θ	0.26	Frequency of crises (2%)
Transfer rate	ξ	0.0001	
Cost of raising equity	κ	5	Stoch. Steady State Leverage of 3.55
Shock Processes			
Persistence of interest rate	ρ_R	0.9	
SD of interest rate innov.	σ_R	0.008	
SD of capital quality	σ_ψ	0.01	

We then assign values to the four parameters relating to financial intermediaries: the survival rate of bankers σ, the fraction of assets that bankers can divert θ, the parameter

[15]See Marcet and Lorenzoni (1998) or Christiano and Fisher (2000). In Appendix H we assess the accuracy of our method by computing Euler equation residuals.

determining the cost of raising equity κ, and the transfer to entering bankers ξ. We calibrate σ to 0.95 as in Gertler and Kiyotaki (2013), implying that bankers survive for about 5 years on average. We set θ to 0.26, to generate a frequency of financial crises of about 2% annually, in line with the data.[16] We set κ to target a leverage ratio of about 3.5 in the stochastic steady state. Finally, we set the transfer rate ξ to a very small number to ensure that this parameter does not alter the results while still allowing the entering bankers to start operations.[17]

We calibrate the exogenous shocks process using data from several small open emerging/advanced economies. The interest rate process is estimated using the sovereign borrowing rate faced by small open economies in international financial markets.[18] Finally, we calibrate the volatility of the capital quality shock (which is *iid*) so that the model delivers a standard deviation of annual output growth of about 1.8 percent, an average across emerging and advanced economies.

3.2 Stochastic Steady State

The third column of the upper panel of Table 2 contains some moments describing the stochastic steady state of the economy, defined as the point at which the economy settles in the absence of exogenous shocks (but in which agents still expect that shocks might occur in the future).[19] Note that the desired leverage ratio is lower than the maximum leverage allowed by the incentive constraint, implying that the constraint does not bind in the stochastic steady state. In fact, it is considerably far from binding: 2-quarters-ahead and 1-year-ahead probabilities of a financial crisis (defined as any time path in which the constraint binds for at least one period) are 1.15 percent and 5.77 percent, respectively. But because banks anticipate that they may be constrained in the future, they raise equity at a positive rate due to precautionary saving effect (absent the prospect of a binding constraint in the future, banks would prefer to set $x = 0$). This leads them to be better capitalized (i.e., have higher net worth) then they would if their rate of equity issuance was zero, impying that in the aggregate the probability of a financial crisis is lower than it would be if banks chose $x = 0$.

As shown in the lower panel of Table 2, the economy spends most of the time in the unconstrained region: banks are constrained only in 1.98 percent of the quarters. Even though the parameter θ, which governs the strength of financial frictions, is chosen so that the model produces reasonable crisis probabilities, it is a general feature of the model that banks try to

[16] See Schularick and Taylor (2012).

[17] We verified that setting ξ to smaller values has virtually no effect on the results reported.

[18] See, among others, Uribe and Yue (2006) and Akinci (2013).

[19] To calculate the stochastic steady state, we simulate the economy for many periods without any exogenous shocks, until the system converges to a point at which all endogenous variables are constant.

Table 2: Stochastic Steady State

Variables	**Symbol**	**No Policy**	$\tau^s = 0.03$
Output	Y_t	0.8379	0.8395
Consumption	C_t	0.6594	0.6601
Labor	L_t	0.2986	0.2990
Capital	K_t	6.8065	6.8250
Net worth	N_t	1.9162	2.0239
New equity issuance rate	x_t	0.0095	0.0104
Leverage ratio	QK/N	3.55	3.37
Maximum leverage		4.02	3.93
Spread (annualized, %)	$E(R_K) - R$	0.51	0.47
Debt-to-GDP ratio	B/Y	0.58	0.58
Utility	$U(C, L)$	-3.1966	-3.1973
Moments			
Time at the constr. (%)		1.98	0.97
2-qtr-ahead crisis prob. (%)		1.15	0.27
1-yr-ahead crisis prob. (%)		5.77	2.92
SD(annual g_Y) (%)		1.82	1.79
SD(Y)/$\mathbb{E}$(Y) (%)		6.06	6.04
SD(C)/$\mathbb{E}$(C) (%)		5.89	5.88
SD(I)/$\mathbb{E}$(I) (%)		23.30	22.66
SD(NX/Y) (%)		4.88	4.77
Welfare Gain (%)			0.02

avoid hitting the constraint via precautionary issuance of new equity. Accounting for shock uncertainty is of course critical to capture this precautionary behavior: in the deterministic steady state of our model (in which agents do not expect that shocks might occur in the future), banks' new equity issuance, x, and bank net worth, N, are much lower, and the constraint is binding.[20] In the fully nonlinear simulation, however, the constraint only binds roughly 2 percent of the time.

Finally, the business cycle moments of the model are roughly in line with the data. In particular, the model delivers consumption only slightly less volatile than output, investment four times as volatile as output, and a volatility of the net exports-GDP ratio of 4.88 percent, a value in the upper range of its counterpart for emerging economies.[21]

[20]If the constraint was not binding in the deterministic steady state, x would be zero.

[21]See, for example, Aguiar and Gopinath (2007) for business cycle statistics for small open economies.

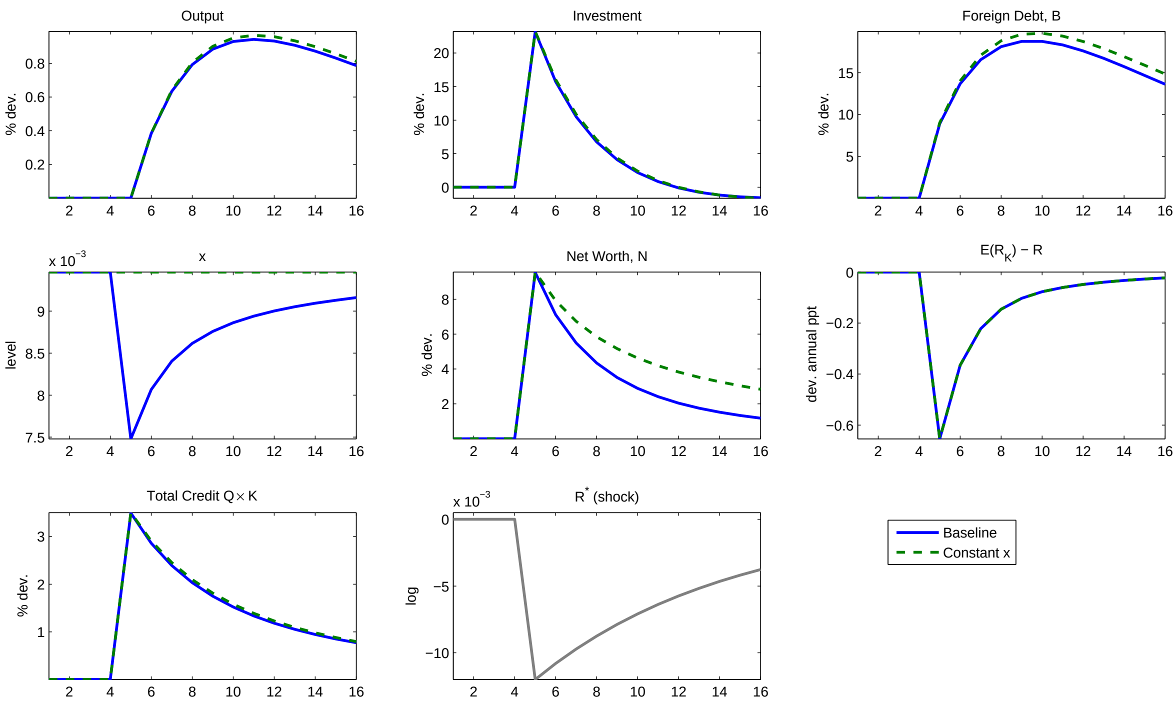
3.3 Credit Booms and Bank Risk-Taking

In the first experiment, shown in Figure 3 and Table 3, we illustrate the consequences in the model of a lower country interest rate, R_t. We consider a one-and-a-half standard deviation decrease in $\epsilon_{R,t}$, the innovation to the interest rate. The blue solid line in Figure 3 shows the responses of the real economy to the shock: as in standard small open economy models, lower country interest rates lead to a boom at home, with rising output, consumption, investment and asset prices. The boom is accompanied by an increase in credit (bottom left panel), and by a compression in the credit spread $\mathbb{E}_t(R_{K,t+1} - R_t)$ (middle right), consistent with the data. The novelty in our framework is that we can assess the implications of the shock for the probability of a financial crisis, as discussed next.

Table 3 shows the probability of a financial crisis occurring within the next two quarters (second column) and within the next year (fourth column), in response to the decline in interest rates in the baseline model. Both probabilities increase as the credit boom progresses. An important factor in accounting for the time path of crisis probabilities is banks' equity issuance choice. In response to the decrease in interest rates, banks *endogenously* decrease the pace at which they raise equity: note from Figure 3 (middle left panel, blue solid line) that x drops substantially (from almost 1 percent to 0.75 percent) and remains below its stochastic steady state value for more than three years. The key force driving the lower rate of equity issuance is that the inside value of net worth, denoted by ν_t, falls persistently in response to the decline of the country interest rate: everything else equal, a lower country interest rate makes deposits cheaper for banks, lowering the value to the bank of a unit of net worth. From equation (19), then, a lower prospective ν_t decreases the attractiveness of raising equity, $\nu_{e,t}$, leading banks to optimally choose a lower x_t. By lowering the rate of equity issuance, the path of bank net worth is lower than it would otherwise have been, which contributes to making a subsequent financial crisis more likely.

To illustrate the latter point clearly, we analyze the following counterfactual scenario: suppose that instead of allowing the banks to endogenously adjust x as they desire in response to the shock, we force x to stay constant at its stochastic steady state level. The green dashed lines in Figure 3 show the path of each variable in the counterfactual experiment with fixed rate of equity raising. Note that banks' net worth now is substantially larger along the boom, as a result of the faster pace of new equity issued: for example, one year and a half after the initiating disturbance (period 10 in the simulation) net worth is 2.9 percent above its stochastic steady state value when banks endogenously reduce x (middle panel in the middle row, blue solid line), whereas it would have been more than 4.6 percent above it steady state value had banks kept x unchanged (green dashed line).

Figure 3: Responses to a Decline in Country Interest Rate



Note: *Responses to a negative innovation to the country interest rate of 2 standard deviations, in the baseline model (blue solid) and in the counterfactual with fixed rate of equity issuance x (green dashed). Variables indicated "% dev." computed as percent deviations from stochastic steady state.*

Table 3: Responses to a Decline in Country Interest Rate, Crises Probabilities

Crises Probabilities (%)

Number of Years After Shock	**2 Quarters Ahead**		**1 year Ahead**	
	Baseline	*Constant x*	*Baseline*	*Constant x*
1	0.6	0.3	4.3	3.9
2	0.9	0.4	5.3	4.2
3	1.0	0.6	5.6	4.5

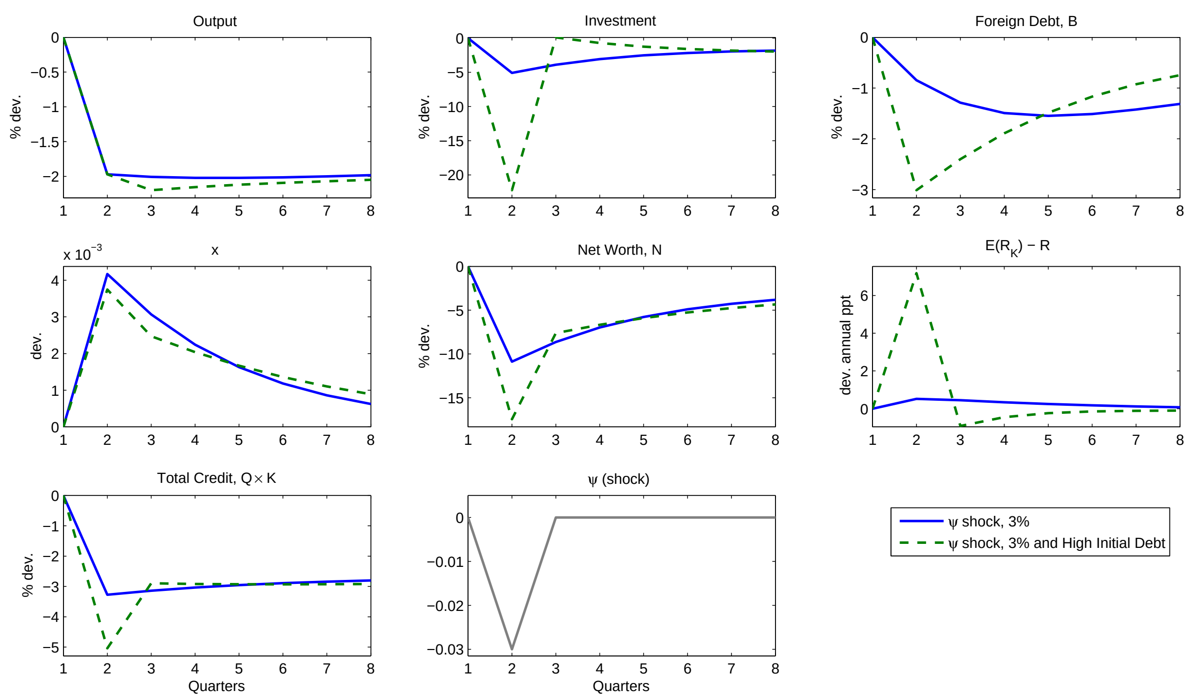
We next analyze the evolution of crisis probabilities in the baseline case (in which banks adjust x endogenously) and in the case with a constant equity issuance rate x. Two quarters ahead crisis probabilities (the third column of Table 3) are cut by about a half when banks keep x constant, and one year ahead crises probabilities (the last column of Table 3) are one third lower one average. The reason is that with stronger equity positions, banks are further away from the constrained region along the credit boom, mitigating the risk of a future financial crisis.

3.4 Nonlinearity and State Dependence

We now perform a simple experiment to illustrate the nonlinearity and state dependence induced by the leverage constraint, as well as the amplification via the financial accelerator mechanism that occurs when banks' incentive constraints bind.

We begin by plotting the responses to a 3 percent capital quality shock which hits when the economy is at the stochastic steady state (blue solid line in Figure 4). The shock leads net worth to drop about 10 percent on impact (roughly the size of the shock times banks' leverage). The decline in net worth, however, is not large enough to trigger the borrowing constraint implied by the agency friction. As a consequence, the shock has only modest effects on investment, asset prices, foreign debt and credit spreads (note that the shock does induce a sizable decline in output, purely due to the physical destruction of capital).

Figure 4: Responses to Capital Quality Shock



Note: *Responses to a 3% capital quality shock when the economy is at the stochastic steady state (blue solid line) and when its initial foreign debt is 60% higher than it stochastic steady state value (green dashed line). Variables indicated "% dev." computed as percent deviations relative to their no-shock path.*

We next perform a similar experiment, i.e. we hit the model economy with a 3 percent capital quality shock, but we now assume that the economy is in a state of high foreign debt when the shock arrives. In particular, the economy's "initial state" (the state at $t = 1$) features a stock of foreign debt that is 60 percent higher than its stochastic steady state value, while the rest of state variables are at their stochastic steady state values. In this high initial debt state, the ratio of foreign debt to (quarterly) GDP is 93 percent (as opposed to just 58 percent in the stochastic steady state). We then hit the economy with a 3 percent capital quality shock at time $t = 2$.

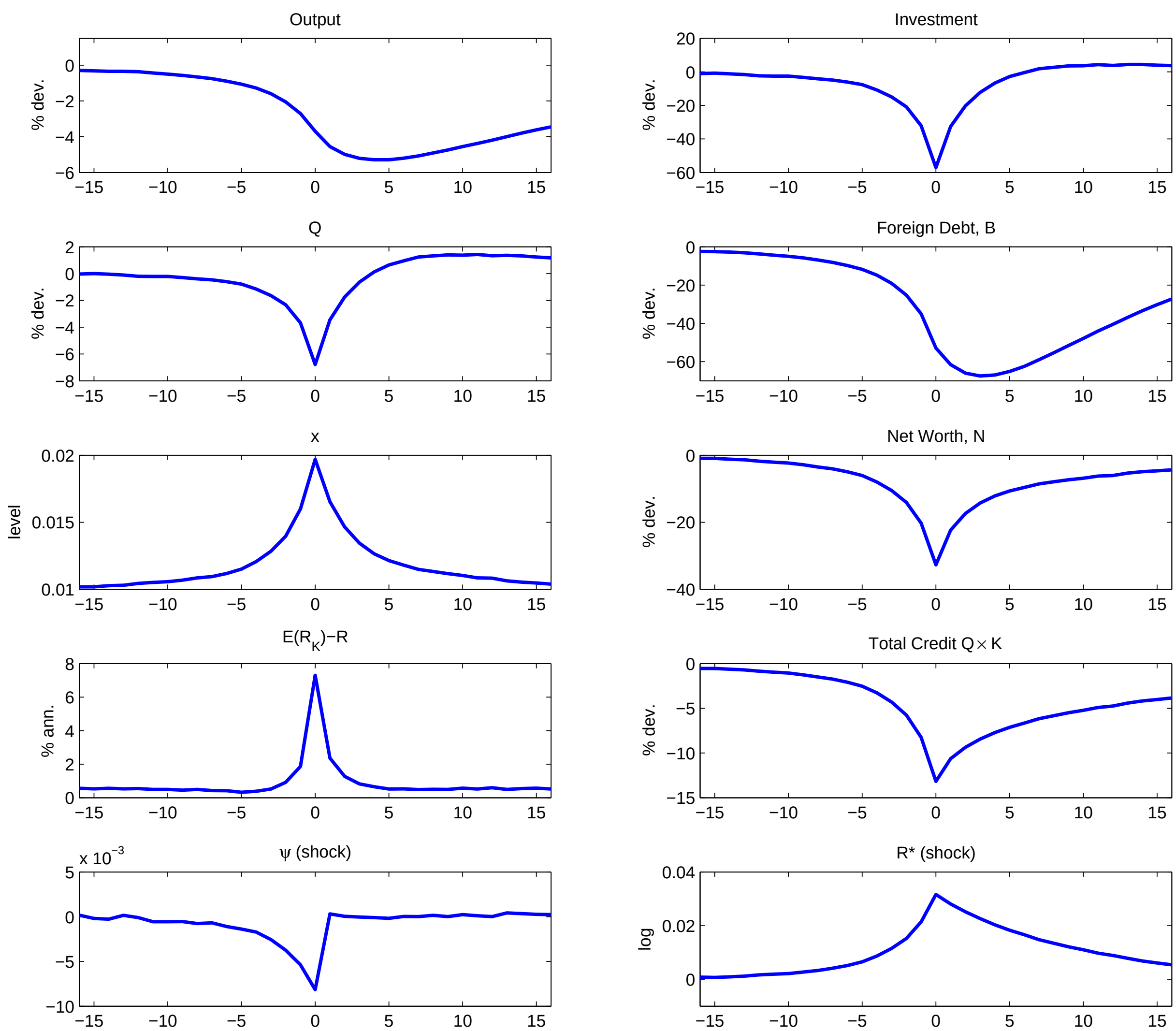
The green dashed line in Figure 4 shows the dynamic effects of the capital quality shock when the economy starts in a high debt state. The decline in bank net worth is now large enough to bring banks up against their constraints. As a consequence, the spread $\mathbb{E}_t(R_{K,t+1} - R_t)$ jumps by about 600 basis points annually. The decline in net worth is roughly 20 percent on impact, almost twice as much as the decline that occurs with a capital quality shock of the same size but with a lower initial foreign debt stock. The sharp decline in net worth is explained by the financial accelerator mechanism that operates when the constraint binds: falling net worth leads investment to drop, which drives asset prices down, leading net worth to drop further. As a consequence, there is a severe drop in investment, of about 20 percent – much more than the decline of 5 percent that occurs with the same size capital quality shock and a lower initial foreign debt stock.

This exercise illustrates that responses of the economy to a shock can be very different depending on the underlying state – in the example, the stock of foreign debt. A high initial foreign debt stock (possibly caused by low country interest rates) might make the economy more vulnerable to future adverse realization of shocks: an adverse shock might lead to a binding leverage constraint, causing a significant drop in investment, asset prices and credit. The financial crisis that arises from the adverse effect of the shock on banks' balance sheets induces a sudden stop in capital inflows: the stock of foreign debt, shown in the third panel in the first row, drops by about 3 percent in the high initial debt case, while it moves much less when the capital quality shock hits when the economy is at the stochastic steady state.

3.5 Average Financial Crisis

We now turn to describing what an average financial crisis looks like in our framework. In particular, we simulate the economy for 150,000 periods and then identify all financial crisis events. We define a crisis event simply as any instance in which banks' incentive constraints bind for one or more consecutive periods. We then consider a window that begins 16 quarters before the crisis and ends 16 quarters after. For each variable, we compute the average period-by-period across all the crisis events that we identify. We normalize the date of the crisis (the quarter in which the constraint first binds) to $t = 0$.

Figure 5: Average Financial Crisis



Note: *We simulate the economy for 150,000 periods and compute average across financial crisis events. A crisis event is defined as a period in which banks' incentive constraint binds. Variables indicated "% dev." computed as percent deviations from their average value in the simulation.*

Figure 5 displays the dynamics around the typical financial crisis episode. In the quarters leading up to the crisis, aggregate bank net worth (third panel, right column) deteriorates sharply, by more than 30 percent in about two years. These equity losses eventually put banks up against their borrowing constraints, leading credit spreads (fourth panel, left column) to jump sharply: the annual spread increases from just 1 percent annually to more than 7 percent in only two quarters. Along the way, with a binding constraint, the financial accelerator mechanism operates, with the drops in net worth, investment and Q reinforcing each other. All told, investment at the trough is about 50 percent below its mean in the simulation. Output drops by almost 6 percent around time 0 and remains persistently depressed thereafter. Finally, note that the economy displays a capital outflow, as reflected in the sharp reduction in the stock of foreign debt (second panel, right column). Therefore, the financial crisis coincides with a sudden stop in capital inflows.

Notice that the rate of equity issuance, x, increases significantly as bank net worth deteriorates: x doubles from just 1 percent before the crisis to 2 percent at the height of the crisis. The deterioration in balance sheets, and the corresponding increase in the likelihood of a crisis, makes inside equity more valuable, prompting banks to increase their rate of issuance. However, this precautionary behavior by banks is ultimately not enough to avoid hitting the constraint, which leads to a sharp and sudden rise in the spread and contributes to the collapse in net worth and investment.

From the bottom two panels in Figure 5, note that the crisis is ultimately the result of adverse realizations of both exogenous shocks: crisis events are triggered by a negative sequence of capital quality shocks, together with an increase in country interest rates. Note however that the realizations of the shocks that trigger the crisis are not abnormally large: at time 0, capital quality is down by a little more than 0.8 percent (less than one standard deviation) and the (log of) R^* is up by about 0.03, a little more than one-and-a-half unconditional standard deviations (recall that capital quality is *iid*, but the log of R^* follows an $AR(1)$ with a 0.9 coefficient).[22]

4 Macroprudential Policy

This section analyzes the effectiveness of macroprudential policy. Within our framework, when the constraint binds, banks' ability to borrow is affected by asset prices, since the latter affect bank net worth. This may introduce a pecuniary externality in banks' decision to issue equity in "normal times" (when the constraint is not binding), as banks do not internalize the consequences for asset price movements of their individual balance sheet position. Thus, there may be a role for the government to introduce a financial policy which tilts banks' incentives in favor of raising more equity in normal times. Several papers have analyzed the role of this pecuniary externality in motivating some form of government intervention. Examples include

[22] Of course, the probability of obtaining repeated realizations of adverse shocks as in Figure 5 is much lower than the probability of a single adverse realization of 1 or 1.5 standard deviations.

Lorenzoni (2008), Korinek (2011), Stein (2012) or Bianchi and Mendoza (2013).

We consider a regulatory subsidy on equity issuance, which the government finances by levying a tax on bank assets. The goal of the regulatory tax/subsidy scheme is to induce banks to raise more equity, thereby contributing to strengthening their balance sheet positions. In particular, the government is assumed to set a subsidy on equity issuance equal to τ^s, financed by a tax on the value of banks' assets equal to τ_t. The subsidy works to reduce the net cost of issuing equity for banks. With the subsidy, the banks' first order condition for equity issues x_t becomes

$$\nu_{e,t} + \tau^s = c'(x_t) \tag{29}$$

Everything else equal, the subsidy induces banks to choose a higher x. Thus, the policy has the flavor of a capital requirement, as it distorts banks' decision in favor of raising more equity.

With the tax τ_t, the banks' balance sheet constraint now becomes

$$(1 + \tau_t)Q_t s_t \leq n_t + d_t \tag{30}$$

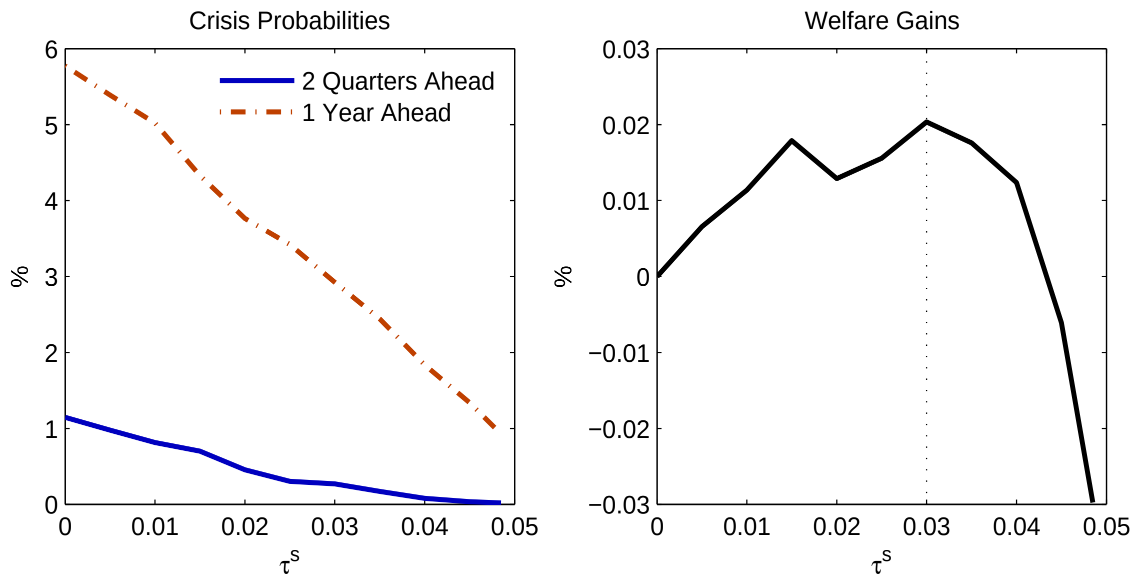
Thus, the tax τ_t increases the cost of financing a balance sheet of a given size. We assume that the government sets τ_t to balance the budget period-by-period, which amounts to setting $\tau_t = \sigma\tau^s x_t$.[23]

We restrict attention to a constant subsidy τ^s. In particular, we study its impact on the stochastic steady state of the model economy, including crisis probabilities, and also on the average time that the economy spends at the constraint. We also compute the welfare effect of the policy. Our criterion is welfare at the stochastic steady state – since in the stochastic steady state agents still expect shocks to hit in the future, the criterion incorporates the welfare impact of reducing the probability of future crises. We express the welfare gains of policy in consumption equivalents by calculating the required permanent increase in the representative household's steady state consumption without policy so that its welfare is the same as with macroprudential policy.

Figure 6 reports crisis probabilities and welfare gains for a range of values of the equity subsidy τ^s. From the left panel, note that the policy is effective at reducing the probability of future financial crises: the 2-quarter-ahead crisis probability (blue solid line) drops from 1.15 percent without macroprudential policy, to only 0.27 percent with an equity subsidy of 3 percent ($\tau^s = 0.03$). Similarly, the 1-year-ahead probability of crisis falls from 5.77 percent with no policy to 2.92 percent with a subsidy of 3 percent. Turning to welfare, the right

[23] The government expenditures in the subsidy are $\tau^s\sigma x_t Q_t S_t$, while the revenues from the tax on bank assets are $\tau_t Q_t S_t$. See Appendix 6 for details on how the tax/subsidy scheme affects the banker's problem.

Figure 6: Effects of Macroprudential Policy on Crisis Probabilities and Welfare

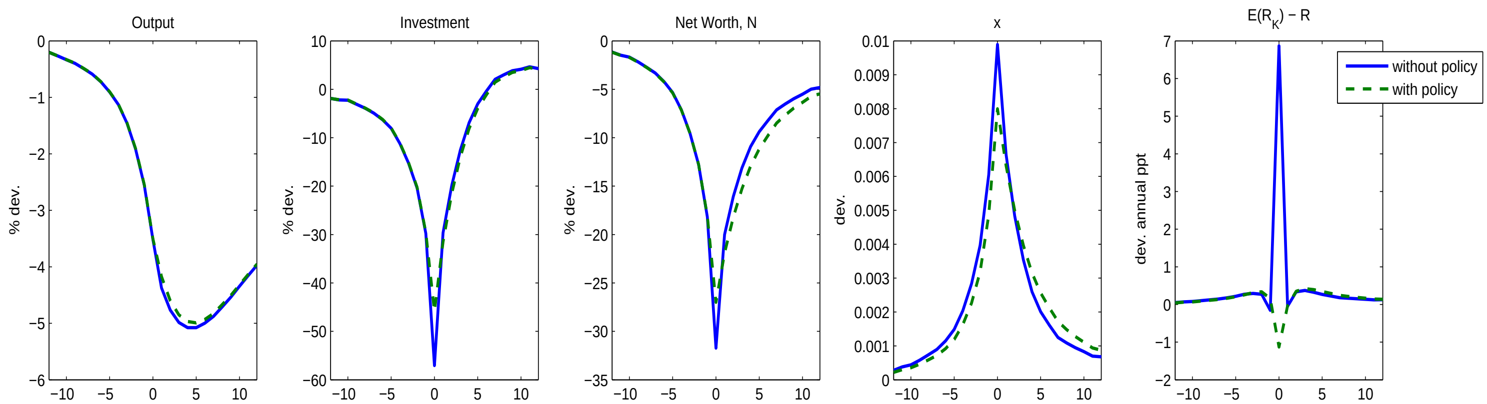


panel of Figure 6 indicates that introducing the tax/subsidy scheme is effective at improving welfare in the stochastic steady state. Subsidy rates τ^s ranging from 0 to about 4.5 percent all increase welfare, with the welfare-maximizing subsidy equal to 3 percent.

The last column in Table 2 reports statistics for the stochastic steady state with the subsidy that maximizes stochastic steady state welfare, $\tau^s = 0.03$. Note that the policy induces a substantial increase in the rate of equity issuance x. As a consequence, bank net worth in the steady state is about 5 percent higher. This helps lower crisis probabilities, as well as the average time spent at the constraint: note that the latter is cut by half to just 1 percent with policy. Finally, note that current period utility tends to be lower with the tax/subsidy scheme in place (consumption is higher, but hours are too). However, total welfare is still higher, reflecting the reduced probability of crises occurring in the future.

Finally, Figure 7 reports the consequences of a "crisis event" with and without macroprudential policy. In particular, we proceed as follows: we first take the (averaged) sequence of values of the exogenous variables (ψ and R^*) that trigger a financial crisis, as identified in the previous section (i.e. the values in the two bottom panels of Figure 5). We then feed these shocks to the stochastic steady state of the economy without macroprudential policy (blue solid line) and with macroprudential policy (green dashed line). Note that with the policy in place, the same shocks induce milder declines in investment and net worth: investment at time 0 is down 45 percent with policy compared to 57 percent without, and net worth drops by 26 percent without policy compared to almost 32 percent with policy. This is despite a

Figure 7: Event Analysis: Model with and without Macroprudential Policy



substantially faster increase in the pace at which banks raise equity in the case with no policy (x increases by 1 percentage point at time 0 with no policy, compared to only 0.8 percentage points without policy). The credit spread is little affected with the policy in place, while it rises sharply by 700 basis points without policy. Finally, the path of output is lower at time 0 and beyond without policy, compared to the case when the tax/subsidy scheme is in place.

5 Conclusion

We have developed a small open economy framework with banks that face occasionally binding leverage constraints. The latter feature implies that the model can generate the type of nonlinear dynamics usually associated with financial crises and sudden stops; i.e., the model produces episodes of financial crises nested within normal business cycle fluctuations, and does not need to rely on unusually large shocks to produce a crisis. A virtue of our approach is that by analyzing a fully nonlinear solution, we can adequately capture the risk-taking behavior of banks. Moreover, by allowing banks to issue equity, we can capture how banks endogenously adjust the strength of their balance sheet in response to economic conditions. We can also examine whether there is a role for policy in manipulating banks' equity issuance decision. We show that an appropriate tax/subsidy scheme by the government is effective in reducing the probability of financial crises and thereby in improving upon *laissez faire*.

Our focus has been to produce a framework that is tractable enough to accommodate easily the features used in the DSGE literature to enhance the quantitative performance and to facilitate policy analysis. In ongoing work, we use the model to analyze time-varying policies that are explicitly contingent on a credit boom episode, such as the countercyclical capital buffers that have been proposed by several policymakers.[24]

Countercyclical capital requirements are not the only tool through which the policymaker can reduce the inefficiencies arising from pecuniary externalities. Our model can also provide useful insight into the relative benefits of capital controls vis-á-vis bank capital requirements. Finally, another interesting avenue of future research would be to augment the model with nominal rigidities, and use it to analyze the implications of financial stability considerations for the conduct of monetary policy. We are working on those extensions.

[24]See Norges Bank (2013) or Basel Committee on Banking Supervision (2010).

References

Aguiar, M., Gopinath, G., 2007. Emerging market business cycles: The cycle is the trend. Journal of Political Economy 115 (1), 69–102. 21

Akinci, O., 2013. Global financial conditions, country spreads and macroeconomic fluctuations in emerging countries. Journal of International Economics. 18

Basel Committee on Banking Supervision, 2010. Guidance for national authorities operating the countercyclical capital buffer. Tech. rep., Bank for International Settlements. 24

Benigno, G. D., Chen, H., Otrok, C., Rebucci, A., Young, E. R., 2012. Optimal policy for macro-financial stability. Cep discussion papers, Centre for Economic Performance, LSE.
URL http://EconPapers.repec.org/RePEc:cep:cepdps:dp1172 7

Bernanke, B. S., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. In: Taylor, J. B., Woodford, M. (Eds.), Handbook of Macroeconomics. Vol. 1 of Handbook of Macroeconomics. Elsevier, Ch. 21, pp. 1341–1393.
URL http://ideas.repec.org/h/eee/macchp/1-21.html 1

Bianchi, J., 2010. Credit externalities: Macroeconomic effects and policy implications. The American Economic Review, 398–402. 1

Bianchi, J., Mendoza, E. G., 2013. Optimal time-consistent macroprudential policy. Tech. rep., National Bureau of Economic Research. 7, 4

Boissay, F., Collard, F., Smets, F., Feb. 2013. Booms and systemic banking crises. Working Paper Series 1514, European Central Bank.
URL http://ideas.repec.org/p/ecb/ecbwps/20131514.html 1

Brunnermeier, M. K., Sannikov, Y., 2014. A macroeconomic model with a financial sector. American Economic Review 104, 379–421. 1

Bruno, V., Shin, H., 2012. Capital flows and the risk-taking channel of monetary policy. In: Paper presented at the 11th BIS Annual Conference. Vol. 22. p. 23. 3

Cespedes, L. F., Chang, R., Velasco, A., October 2012. Financial intermediation, exchange rates, and unconventional policy in an open economy. Working Paper 18431, National Bureau of Economic Research.
URL http://www.nber.org/papers/w18431 1

Christiano, L. J., Fisher, J. D., 2000. Algorithms for solving dynamic models with occasionally binding constraints. Journal of Economic Dynamics and Control 24 (8), 1179–1232. 15

Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. Journal of Monetary Economics 58, 17–34. 2

Gertler, M., Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis. In: Friedman, B. M., Woodford, M. (Eds.), Handbook of Monetary Economics. Vol. 3 of Handbook of Monetary Economics. Elsevier, Ch. 11, pp. 547–599.
URL http://ideas.repec.org/h/eee/monchp/3-11.html 1, 2, 2.2

Gertler, M., Kiyotaki, N., 2013. Banking, liquidity and bank runs in an infinite-horizon economy. Tech. rep., National Bureau of Economic Research. 3.1

Gertler, M., Kiyotaki, N., Queralto, A., 2012. Financial crises, bank risk exposure and government financial policy. Journal of Monetary Economics 59, 17–34. 2, 8

Gourinchas, P.-O., Obstfeld, M., January 2012. Stories of the twentieth century for the twenty-first. American Economic Journal: Macroeconomics 4 (1), 226–65.
URL http://ideas.repec.org/a/aea/aejmac/v4y2012i1p226-65.html 4

Greenwood, J., Hercowitz, Z., Huffman, G. W., June 1988. Investment, capacity utilization, and the real business cycle. American Economic Review 78 (3), 402–17.
URL http://ideas.repec.org/a/aea/aecrev/v78y1988i3p402-17.html 2.1

Guerrieri, L., Iacoviello, M., 2012. A toolkit to solve models with occasionally binding constraints easily. Working paper, Federal Reserve Board. 0

Hubrich, K., DAgostino, A., Cervená, M., Ciccarelli, M., Guarda, P., Haavio, M., Jeanfils, P., Mendicino, C., Ortega, E., Valderrama, M. T., et al., 2013. Financial shocks and the macroeconomy: heterogeneity and non-linearities. Tech. rep., European Central Bank. 1

Judd, K. L., 1992. Projection methods for solving aggregate growth models. Journal of Economic Theory 58 (2), 410–452. H

Kenny, G., Morgan, J., 2011. Some lessons from the financial crisis for the economic analysis. Tech. rep., European Central Bank. 1

Kiyotaki, N., Moore, J., 1997. Credit cycles. The Journal of Political Economy 105 (2), 211–248. 1

Korinek, A., 2011. Systemic risk-taking: amplification effects, externalities, and regulatory responses. Networks Financial Institute Working Paper (2011-WP), 13. 4

Lorenzoni, G., 2008. Inefficient credit booms. The Review of Economic Studies 75 (3), 809–833. 4

Marcet, A., Lorenzoni, G., 1998. Parameterized expectations approach; some practical issues. 15

Mendoza, E., 2010. Sudden Stops, Financial Crises, and Leverage. American Economic Review 100 (5), 1941–66. 1

Mendoza, E. G., Terrones, M. E., Sep. 2012. An anatomy of credit booms and their demise. NBER Working Papers 18379, National Bureau of Economic Research, Inc.
URL `http://ideas.repec.org/p/nbr/nberwo/18379.html` 3, 4

Merton, R. M., 2009. Observations on the science of finance in the practice of finance. 1

Norges Bank, 2013. Criteria for an appropriate countercyclical capital buffer. Tech. rep., Norges Bank. 24

Schmitt-Grohe, S., Uribe, M., October 2003. Closing small open economy models. Journal of International Economics 61 (1), 163–185.
URL `http://ideas.repec.org/a/eee/inecon/v61y2003i1p163-185.html` 2.4

Schmitt-Grohe, S., Uribe, M., Feb. 2011. Pegs and Pain. CEPR Discussion Papers 8275, C.E.P.R. Discussion Papers.
URL `http://ideas.repec.org/p/cpr/ceprdp/8275.html` 7

Schularick, M., Taylor, A. M., 2012. Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. American Economic Review 102 (2), 1029–61.
URL `http://www.aeaweb.org/articles.php?doi=10.1257/aer.102.2.1029` 4, 16

Stein, J., 2012. Monetary policy as financial stability regulation. The Quarterly Journal of Economics 127, 57–95. 4

Stein, J., 2014. Incorporating financial stability considerations into a monetary policy framework. 1

Uribe, M., Yue, V. Z., 2006. Country spreads and emerging countries: Who drives whom? Journal of international Economics 69 (1), 6–36. 18

Appendix

A Household's Optimality Conditions

$$
\begin{aligned}
\left(C_t - \chi \frac{L_t^{1+\epsilon}}{1+\epsilon}\right)^{-\gamma} &= \lambda_t && (31)\\
\chi L_t^{\epsilon} &= W_t && (32)\\
\mathbb{E}_t(\Lambda_{t,t+1} R_t) &= 1 && (33)
\end{aligned}
$$

Household's stochastic discount factor is defined as

$$
\Lambda_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{34}
$$

where $\lambda_t = U_{c,t}$ is the marginal utility of consumption.

B Banks' Value Function

Using the optimality conditions for new equity issuance, equation (15), we can simplify the second part of the value function, equation (13), as the following:

$$
\begin{aligned}
\nu_{e,t} e_t - C(e_t, Q_t s_t) &= \nu_{e,t} x_t Q_t s_t - c(x_t) Q_t s_t\\
&= (\nu_{e,t} x_t - \frac{\kappa}{2} x_t^2) Q_t s_t\\
&= \frac{\kappa}{2} x_t^2 Q_t s_t
\end{aligned}
$$

So the value function becomes

$$
\alpha_t n_t = \mu_{K,t} Q_t s_t + \nu_t n_t + \sigma \frac{\kappa}{2} x_t^2 Q_t s_t
$$

Defining the "total" excess return on assets μ_t as $\mu_t \equiv \mu_{K,t} + \sigma \frac{\kappa}{2} x_t^2$, we can re-write the value function as $\alpha_t n_t = \mu_t Q_t s_t + \nu_t n_t$.

C Resource Constraint and Balance of Payments

Aggregate the bank's budget constraint across banks and combine with the household's budget constraint and with the market clearing condition for claims on capital ($S_t = K_t$) to obtain

$$Q_tK_t + R_{t-1}B^*_{t-1} + C_t + \sigma\frac{\kappa}{2}x_t^2Q_tK_t \leq W_tL_t + R_{K,t}Q_{t-1}K_{t-1} + B^*_t + \Pi^F_t + \Pi^C_t \tag{35}$$

The last two terms, Π^F_t and Π^C_t, are the profits of final goods firms and capital producers, respectively. They are given by their respective budget constraints:

$$Y_t + Q_t\left(1-\delta\right)e^{\psi_t}K_{t-1} = \Pi^F_t + W_tL_t + R_{K,t}Q_{t-1}K_{t-1} \tag{36}$$

$$\Pi^C_t = Q_tI_t - \left[1 + \frac{1}{2}\psi_I(\frac{I_t}{e^{\psi_t}K_{t-1}} - \delta)^2\right]I_t \tag{37}$$

Using these expressions, we can derive the resource constraint and the balance of payments equation for the economy as the following:

$$Y_t = C_t + \left[1 + \frac{1}{2}\psi_I\left(\frac{I_t}{e^{\psi_t}K_{t-1}} - \delta\right)^2\right]I_t + \sigma\frac{\kappa}{2}x_t^2Q_tK_t + NX_t \tag{38}$$

$$R_{t-1}B^*_{t-1} - B^*_t = NX_t \tag{39}$$

D Model Policy Functions

To further illustrate the nonlinear features of the model, arising due to banks' occasionally binding constraints, Figure 8 report policy functions for the state variables. The effective capital stock $\overline{K}_t$ refers to capital stock at the beginning of the period, after the capital quality shock realizes: $\overline{K}_t \equiv e^{\psi_t}K_{t-1}$. The predetermined amount of net worth refers to the part of time-t net worth that is predetermined in period $t-1$, i.e. that does not depend on the realization of capital quality and asset prices in period t:

$$\overline{N}_{t-1} \equiv \sigma\left[x_{t-1}Q_{t-1}K_{t-1} - R_{t-1}\left(\underbrace{Q_{t-1}K_{t-1} - N_{t-1}}_{\equiv D_{t-1}}\right)\right] + (1-\sigma)\xi Q_{t-1}K_{t-1}$$

$\overline{N}_{t-1}$ can be interpreted as the total equity raised in period $t-1$ (including the equity brought in by the newborn bankers who replace the exiting ones, captured by the last term above) net of the total financial sector debt incurred in period $t-1$ (including interest). $\overline{N}_{t-1}$ is one of the key state variables of the aggregate system.

Total aggregate net worth is then:

$$N_t = \sigma R_{K,t} Q_{t-1} K_{t-1} + \overline{N}_{t-1} = \left[Z_t + (1-\delta) Q_t\right] e^{\psi_t} K_{t-1} + \overline{N}_{t-1}$$

The remaining state variables are the stock of external debt (including interest) $\overline{B}_{t-1} \equiv R_{t-1} B_{t-1}$, and the exogenous shock to the country interest rate, R_t^*. Note that for all state variables, there exists a threshold level after which the constraint starts to bind.

E Complete Model and Solution Method

E.1 The Complete Model

$$Y_t + B_t = C_t + \left[1 + \frac{1}{2}\psi_I \left(\frac{I_t}{e^{\psi_t} K_{t-1}} - \delta\right)^2\right] I_t + \sigma \frac{\kappa}{2} x_t^2 Q_t K_t + R_{t-1} B_{t-1} \tag{40}$$

$$K_t = I_t + (1-\delta) e^{\psi_t} K_{t-1} \tag{41}$$

$$Q_t = 1 + \psi_I \left(\frac{I_t}{e^{\psi_t} K_{t-1}} - \delta\right) \tag{42}$$

$$\mathbb{E}_t \left(\Lambda_{t,t+1}\right) R_t = 1 \tag{43}$$

$$\Lambda_{t-1,t} = \beta \frac{U_{C,t}}{U_{C,t-1}} \tag{44}$$

$$U_{C,t} = \left(C_t - \chi \frac{L_t^{1+\epsilon}}{1+\epsilon}\right)^{-\gamma} \tag{45}$$

$$R_{K,t} = e^{\psi_t} \frac{\alpha \frac{Y_t}{e^{\psi_t} K_{t-1}} + (1-\delta) Q_t}{Q_{t-1}} \tag{46}$$

$$Y_t = \left(e^{\psi_t} K_{t-1}\right)^{\alpha} L_t^{1-\alpha} \tag{47}$$

$$\mu_{K,t} = \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{K,t+1} - R_t\right)\right] \tag{48}$$

$$\mu_t = \mu_{K,t} + \sigma \frac{\kappa}{2} x_t^2 \tag{49}$$

$$\nu_t = \mathbb{E}_t \left(\Lambda_{t,t+1} \Omega_{t+1}\right) R_t \tag{50}$$

$$\Omega_t = 1 - \sigma + \sigma(\nu_t + \phi_t \mu_t) \tag{51}$$

$$\nu_t^e = \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\nu_{t+1} + \mu_{t+1} \phi_{t+1}\right)\right] \tag{52}$$

$$N_t = \sigma R_{K,t} Q_{t-1} K_{t-1} + \overline{N}_{t-1} \tag{53}$$

$$\overline{N}_t = \sigma \left(x_t Q_t K_t - R_t Q_t K_t + R_t N_t\right) + (1-\sigma) \xi_t Q_t K_t \tag{54}$$

$$(1-\alpha) \frac{Y_t}{L_t} = \chi L_t^{1+\epsilon} \tag{55}$$

$$R_t = \frac{1}{\beta} + \varphi\left(e^{\frac{B_t}{Y_t}-\bar{b}} - 1\right) + e^{R_t^*-1} - 1 \tag{56}$$

$$\phi_t = \frac{\nu_t}{\theta - \mu_t} \tag{57}$$

$$x_t = \frac{\nu_t^e}{\kappa} \tag{58}$$

If the constraint does not bind, we must have $\mu_t = 0$. If it binds, then we have $Q_t K_t = \phi_t N_t$ (and $\mu_t > 0$). Define the *effective* amount of capital at the beginning of t as $\overline{K}_t \equiv e^{\psi_t} K_{t-1}$, and the stock of external debt plus interest as $\overline{B}_{t-1} \equiv R_{t-1} B_{t-1}$. The state variables in this economy are $\overline{K}_t$, $\overline{N}_{t-1}$, R_t^* and $\overline{B}_{t-1}$.

E.2 Solution Method

The four state variables at the beginning of period t are the following: the effective amount of capital $\overline{K}_t$, the predetermined part of aggregate net worth $\overline{N}_{t-1}$, the stock of external debt (including interest) $R_{t-1}B_{t-1}$, and the foreign interest rate shock R_t^*. Define the state vector at the beginning of period t: $\mathbf{s}_t \equiv \left\{\overline{K}_t, \overline{N}_{t-1}, R_t^*, \overline{B}_{t-1}\right\}$.

We solve the nonlinear model by Parameterized Expectations. To so, we need to approximate five expectations as a function of the state vector. Let $\varepsilon(\mathbf{s}_t)$ denote the set of the corresponding five functions of the state vector. Knowing the functions $\varepsilon(\mathbf{s}_t)$ and the state $\mathbf{s}_t$, one can solve nonlinearly for all of the model's endogenous variables. We use the following algorithm:

0. Simulate a long time series using OccBin (Guerrieri and Iacoviello (2012)). Use the simulated data to obtain $\varepsilon^0(\mathbf{s}_t)$, by regressing the expectations produced by OccBin on the state variables. At this step we can check on the accuracy of the function used to approximate the expectations, by comparing the expectations produced by OccBin (which captures the kink accurately) with those produced by the parameterized function.

1. Given $\varepsilon^0(\mathbf{s}_t)$, solve the system of equations that characterizes the equilibrium. To do so, at each period t we first solve the system assuming that the constraint does not bind, which implies $\mu_t = 0$. We then check if bank leverage is above the maximum allowed by the constraint. If it is not we proceed; if it is, we again solve the system, this time imposing that the constraint binds. At each period, we obtain one-period-ahead expectations by quadrature.

2. Obtain a new set of expectation functions $\varepsilon^1(\mathbf{s}_t)$ by running regressions using the data simulated in step 1.

3. Compare $\varepsilon^1(\mathbf{s}_t)$ with $\varepsilon^0(\mathbf{s}_t)$. If they are close, stop. If not, update $\varepsilon^0(\mathbf{s}_t)$ and go back to step 1.

F Computation of Crisis Probabilities

At period t_0, we compute the crisis probability at horizon t_0+j, defined as the probability of at least one crisis in periods t_0+1 until t_0+j, as follows. First, obtain draws for the exogenous innovations $\{\epsilon_{R,t}, \psi_t\}_{t_0+1}^{t_0+j}$, together with their associated probabilities. For each history of realizations of shocks h, defined as each possible sequence of realizations of $\{\epsilon_{R,t}, \psi_t\}_{t_0+1}^{t_0+j}$, let the set of histories in which there is at least one crisis be H. Then the probability of a crisis (for a given horizon j) is the sum of the probabilities of each of the histories in H, i.e. $\sum_{h\in H} p(h)$. In the body of the paper we report the results for $j = 2, 4$.

G Banker's Problem with Policy

In this section we lay out the banking problem in the presence of government subsidy which tilts banks' incentive in favor of raising more equity. In particular, we suppose that the government offers banks a fixed subsidy of τ^s per unit of new equity issued and finances the subsidy with a tax τ_t on total assets such that $\tau_t = \sigma\tau^s x_t$ to achieve balanced budget for the government.

Bankers' problem is now given by

$$V_t(n_t) = \max_{s_t,d_t,e_t} \ (1-\sigma)\mathbb{E}_t\Lambda_{t,t+1}\left(R_{K,t+1}Q_ts_t - R_td_t\right) + \sigma\left\{\mathbb{E}_t\Lambda_{t,t+1}\left[V_{t+1}(n_{t+1}) - e_t\right] - C(e_t, Q_ts_t) + \tau^s e_t\right\} \tag{59}$$

subject to

$$(1+\tau_t)Q_ts_t \leq n_t + d_t \tag{60}$$

$$n_{t+1} = R_{K,t+1}Q_ts_t - R_td_t + e_t \tag{61}$$

$$(1-\sigma)\mathbb{E}_t\Lambda_{t,t+1}\left(R_{K,t+1}Q_ts_t - R_td_t\right) + \sigma\left[\mathbb{E}_t\Lambda_{t,t+1}\left(V_{t+1}(n_{t+1}) - e_t\right) - C(e_t, Q_ts_t) + \tau^s e_t\right] \geq \theta Q_ts_t \tag{62}$$

As before, to solve the banker's problem, first guess that the value function is $V_t(n_t) = \alpha_t n_t$.

Define

$$
\begin{aligned}
\Omega_{t+1} &= (1-\sigma)+\sigma\alpha_{t+1} \\
\mu_{K,t}^{\tau} &= \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{K,t+1}-(1+\tau_t)R_t)] \\
\nu_t &= \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}]R_t \\
\nu_{e,t} &= \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\alpha_{t+1}-1\right)\right]
\end{aligned}
$$

With these definitions, the problem simplifies to

$$
\alpha_t n_t = \max_{s_t,e_t} \quad \mu_{K,t}^{\tau}Q_t s_t + \nu_t n_t + \sigma\left[\nu_{e,t}e_t - C(e_t, Q_t s_t) + \tau^s e_t\right] \tag{63}
$$

subject to

$$
\mu_{K,t}^{\tau}Q_t s_t + \nu_t n_t + \sigma\left[\nu_t e_t - C(e_t, Q_t s_t) + \tau^s e_t\right] \geq \theta Q_t s_t \tag{64}
$$

The first order condition for x_t is

$$
\begin{aligned}
\nu_{e,t}+\tau^s &= c'(x_t) \\
&= \kappa x_t
\end{aligned}
$$

Using this first order condition we can simplify the second part of the value function

$$
\begin{aligned}
\nu_{e,t}e_t - C(e_t, Q_t s_t) + \tau^s e_t &= \nu_{e,t}x_t Q_t s_t - c(x_t)Q_t s_t + \tau^s x_t Q_t s_t \\
&= \left[(\nu_{e,t}+\tau^s)x_t - \frac{\kappa}{2}x_t^2\right]Q_t s_t \\
&= \frac{\kappa}{2}x_t^2 Q_t s_t
\end{aligned}
$$

So the value function becomes

$$
\mu_{K,t}^{\tau}Q_t s_t + \nu_t n_t + \sigma\frac{\kappa}{2}x_t^2 Q_t s_t
$$

Define the "total" excess return on assets μ_t as $\mu_t \equiv \mu_{K,t}^{\tau} + \sigma\frac{\kappa}{2}x_t^2$

The law of motion for aggregate net worth is now

$$
N_t = \sigma\left[(R_{K,t}-(1+\tau_{t-1})R_{t-1}+x_{t-1})\underbrace{Q_{t-1}K_{t-1}}_{Q_{t-1}S_{t-1}} + R_{t-1}N_{t-1}\right] + (1-\sigma)\xi Q_{t-1}K_{t-1} \tag{65}
$$

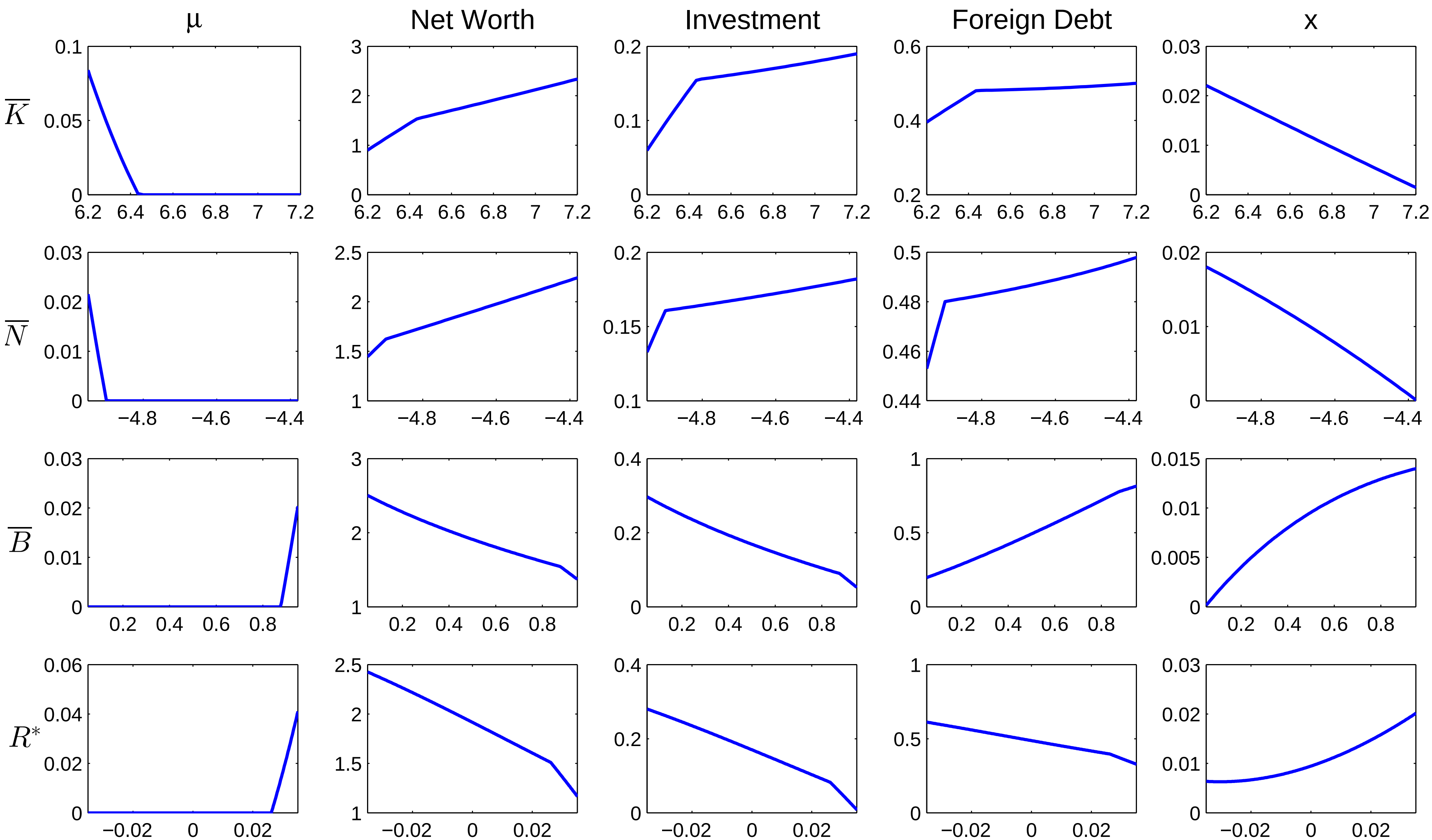
H Euler Residuals

Following Judd (1992), we provide a check on the accuracy of our solution method by computing Euler equation errors. Moving from the Euler equation for consumption, we define the Euler equation error (as a fraction of units of consumption) as

$$err_t = \left| \frac{[\beta \mathbb{E}_t(U_{C,t+1})R_t]^{-\frac{1}{\gamma}} + \frac{\chi}{1+\epsilon} L_t^{1+\epsilon} - C_t}{C_t} \right| \tag{66}$$

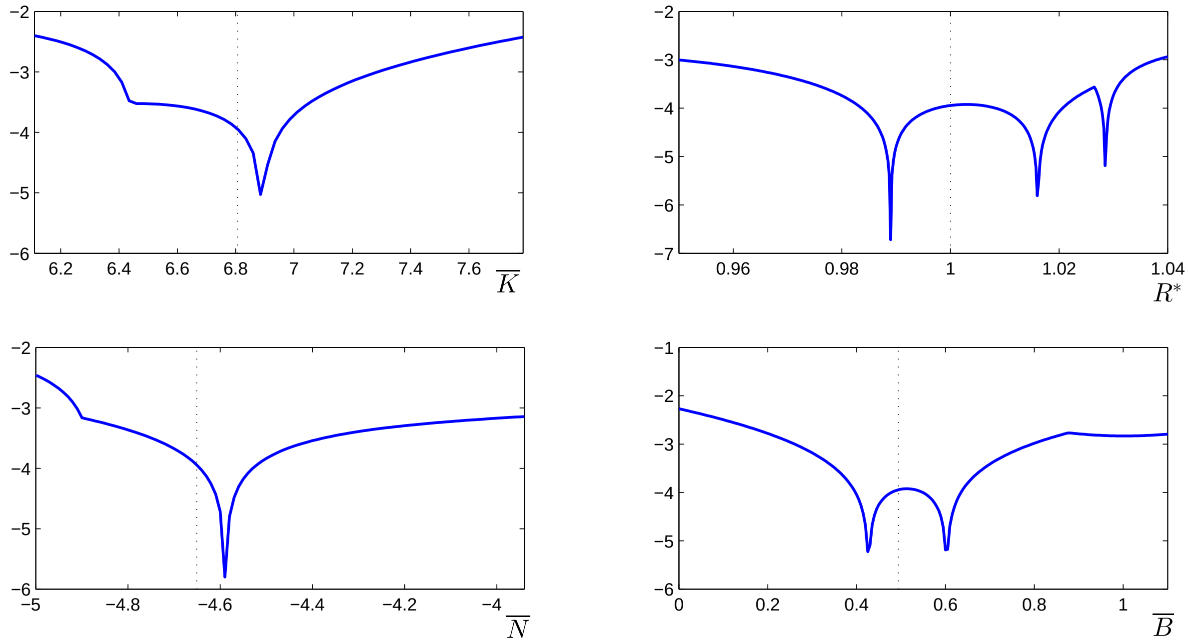
Figures 9 and 10 respectively show the errors as a function of the state variables and a histogram for a given simulation. We express the errors in decimal log scale, as is common in the literature. The Euler errors are reasonably small, and comparable to those found in the literature. The average error is about -3.5. To interpret, recall that under the decimal log scale a value of, say, -4 is that the error is sized at $1 per $10,000 of consumption.

Figure 8: Model Policy Functions



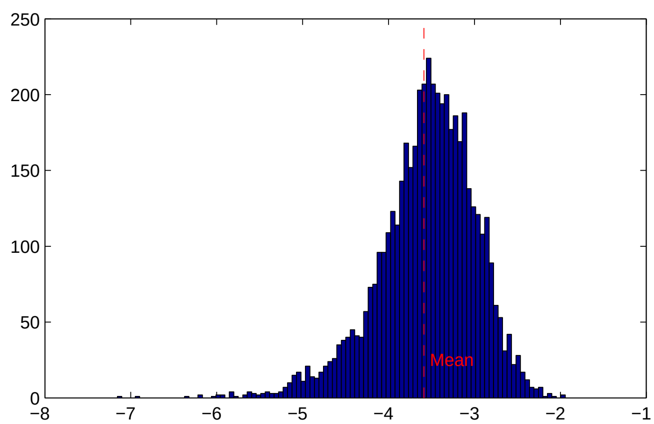
Note: Model policy functions. Each row of the figure moves along the corresponding state variable, while keeping the others at their stochastic steady state value.

Figure 9: Euler Residuals



Note: Euler equation errors as a function of the model's state variables. Each plot moves along the indicated state variable, while keeping the others at stochastic steady state. Vertical lines indicate values in the stochastic steady state.

Figure 10: Histogram of Euler Residuals



Note: Histogram for Euler equation errors in a model simulation.