

# Just-in-Time Inventories, Business Cycles, and the Great Moderation

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## Abstract

This paper examines the implications of a DSGE model regarding the role of inventories in the business cycle. Two questions are examined. We study the extent to which inventories are a cause of cyclical fluctuations and we ask how those fluctuations are affected if the economy adopts just-in-time inventories. In our simulations, we are able to find parameter values associated with inventory holding costs that cause the economy to display cyclical fluctuations but these values are implausibly large. But with these parameters in place, we also find that the use of just-in-time inventories completely eliminates cyclical fluctuations, thus confirming conjectures in the literature that this innovation stabilizes economic activity. Thus we conclude that inventories are unlikely to be the cause of business cycles but just-in-time inventories do indeed provide a stabilizing influence on cycles that might otherwise exist.

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# 1 Introduction

Inventory research historically identified two possible roles for inventories in business cycles. As discussed by Blinder and Maccini (1991, p. 293), early research on inventories established the view that inventories were a destabilizing force in the aggregate economy. Inventory movements were thought to induce business cycles that otherwise would not exist and so inventories were assigned a causal role in the existence of business cycles. A second viewpoint emerged in subsequent research. This line of research viewed inventories as part of the propagation mechanism associated with business cycles but, in this line of research, inventories were not a cause of cycles (Blinder and Maccini 1991, p. 315).<sup>1</sup> More recently, a third possible role for inventories in business cycles has been advanced. It has been suggested that structural changes in inventory management have induced stability in the aggregate economy.

Kahn, McConnell, and Perez-Quiros (2002) argue that the IT revolution in the 1980s improved information flows to firms with implications for the stocks of inventories held by those firms and the stability of the aggregate economy. With better information about expected sales, firms may hold smaller stocks of inventories since output demand can be more accurately estimated. One aspect of this innovation in the holding of inventories by firms is the development of just-in-time inventories. This change in economic structure means that the users of inventories of intermediate materials no longer hold inventories of these intermediate goods. Rather they take deliveries of materials at the moment that they are needed in the production of final output. Inventories of materials may continue to be held

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<sup>1</sup>Blinder and Maccini (1991) and Ramey and West (1999) provide comprehensive surveys of research on inventory investment.

in the aggregate economy but they are held by the producers of intermediate goods rather than by the buyers of these goods. The adoption of just-in-time inventories may therefore be a cause of the Great Moderation, a period of apparent increased stability of the U.S. economy that began in the 1980s.<sup>2</sup> In contrast, in the context of a DSGE model, Iacoviello, Schiantarelli and Schuh (2011) estimate their model and find that changes in the volatility of input and output inventory shocks or in the structural parameters associated with these inventories played a small role in explaining the Great Moderation.

This paper examines the connection between inventories and the business cycle and it does so from two points of view. The first aspect of our analysis is that we are able to examine the role of inventories as a source of business cycle fluctuations. Building on this aspect of our work, we are then able to address the role of just-in-time inventories in reducing economic fluctuations. We are able to do this by constructing two DSGE models of an aggregate economy. In our baseline version of this economy, inventories of finished goods are held by the suppliers of final output to households and these firms are assumed to use intermediate materials in their production process, holding inventories of intermediate materials that they use in production. In the alternative version of our model, just-in-time inventories are used by final goods producers, using intermediate materials purchased from their supplier and it is the suppliers of intermediate materials who hold inventories of intermediate goods. By the choice of parameters in these models, we are able to observe if business cycles arise in the economy without just-in-time inventories and, using the same parameterization, we

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<sup>2</sup>Stock and Watson (2003) discuss and document the Great Moderation although earlier research by a number of authors, including McConnell and Perez-Quiros (2000), suggest there are other possible explanations for its existence in addition to just-in-time inventories. Improvements in the conduct of economic policy (see, e.g., Clarida, Galí and Gertler 2000; Boivin and Giannoni 2006) may cause reduced aggregate economic volatility or there may just be fewer shocks buffeting the economy (see, e.g., Ahmed, Levin and Wilson 2004).

can see if cycles exist in the just-in-time inventory economy. Thus we are able to obtain evidence bearing on the issue of inventories as the cause of business cycles and the ability of just-in-time inventories to stabilize cyclical fluctuations that might otherwise exist.

Using a New Keynesian DSGE model of the economy, we are able to find parameters associated with inventory holding costs that, in response to preference shocks, generate cyclical fluctuations in our model economy. However these parameter values are extremely large, leading us to conclude that they are empirically implausible. Thus inventories seem an unlikely source of business cycle fluctuations. But using these parameter values, we find that the economy with just-in-time inventories reveals a complete absence of cyclical fluctuations. Thus we find compelling evidence that this innovation in economic structure does indeed stabilize an economy that would otherwise be prone to cyclical fluctuations.

The reader may wonder why our results differ from those in Iacoviello, Schiantarelli and Schuh (2011). As we will see later, the key difference is how inventories are modeled. Their work considers a two sector model where one sector holds inventories (goods) and the other (services) does not. More importantly, the goods sector holds inventories of materials. Instead, in our model, we abstract from the service sector but consider how the introduction of Just-in-Time inventories, affects the transmission of shocks. In particular, we consider the effect of having the material producer hold materials inventories until they are needed by the goods sector.

Our paper is organized as follow. The next section describes the structure of our benchmark model. Section 3 provides the structure of our just-in-time economy. Section 4 provides simulation results for both DSGE models while the last section summarizes results and provides suggestions for future research. An Appendix provides analytical results supporting

the analysis in the paper.

## 2 The Benchmark Model

The benchmark model is a simple New Keynesian model with price rigidity where there are two types of firms: final and material goods producers. The economy is composed of a representative household, a continuum of final good producers, a representative material producer, and a monetary authority. The final good producers are monopolistic competitors in their output market and price-takers in their input market. Factors of production are labor services and intermediate materials. The final good producers hold the inventory stocks of their output and materials. The materials producer is operating in competitive input and output markets. Labor services is the only factor of materials production. Material producers do not hold inventories. To focus on the role of inventories, the model does not consider capital accumulation.

### 2.1 Household

The representative household carries  $B_{t-1}$  units of bonds into period  $t$ . During period  $t$ , the household supplies  $l_t$  and  $h_t$  units of labor services to the final goods producer and the materials producer at real wage rate  $w_{l,t}$  and  $w_{h,t}$ , respectively. In addition, the household receives nominal profits from their participation in the production of final goods ( $D_t$ ) and materials ( $\Pi_t$ ). The household uses its income to purchase a consumption bundle ( $c_t$ ) and bonds ( $B_t$ ). The nominal cost of the bond purchase is  $B_t/r_t$  where  $r_t$  denotes the gross nominal interest rate between period  $t$  and  $t + 1$ . The budget constraint of the household

is summarized as

$$c_t + \frac{B_t/r_t}{p_t} \leq \frac{B_{t-1}}{p_t} + w_{l,t}l_t + w_{h,t}h_t + \frac{D_t}{p_t} + \frac{\Pi_t}{p_t}. \quad (1)$$

The household's consumption bundle is defined by a CES aggregate of differentiated final goods:

$$c_t = \left[ \int_0^1 s_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 0 \quad (2)$$

where  $s_t(i)$  denotes the sales of differentiated final good indexed by  $i \in [0, 1]$ . Cost minimization of the household results in the demand for  $s_t(i)$ , which is given by

$$s_t(i) = \left[ \frac{p_t(i)}{p_t} \right]^{-\theta} c_t, \quad (3)$$

where  $p_t(i)$  is the price of a final good indexed by  $i \in [0, 1]$ . Parameter  $\theta$  represents the elasticity of substitution between the differentiated final goods. Aggregate price level is derived as:

$$p_t = \left[ \int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad (4)$$

which is the Dixit-Stiglitz price index.

Given the budget constraint, the household tries to maximize its lifetime utility:

$$\max \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left\{ z_t \ln c_\tau + b \ln (1 - l_\tau - h_\tau) \right\}, \quad (5)$$

where  $z_t$  is the preference factor for consumption. The marginal utility of consumption is increasing in  $z$ . We assume the evolution of the latter to be given by

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t}, \quad (6)$$

where  $\varepsilon_{z,t}$  is an IID shock to the preference factor.

The first order conditions with respect to  $c_t$ ,  $l_t$ ,  $h_t$ , and  $B_t$  are represented by

$$\frac{z_t}{c_t} = \lambda_t, \quad (7)$$

$$\frac{b}{1 - l_t - h_t} = w_{l,t} \lambda_t, \quad (8)$$

$$\frac{b}{1 - l_t - h_t} = w_{h,t} \lambda_t, \quad (9)$$

and

$$\frac{\lambda_t / r_t}{p_t} = \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}}{p_{t+1}} \right), \quad (10)$$

Equation (7) defines the shadow price  $\lambda_t$ , which is the marginal utility of consumption. Equations (8) and (9) represent the optimal supply of labor to the final goods and materials producers, respectively. In equilibrium, the wage rates  $w_{l,t}$  and  $w_{h,t}$  are equalized as the marginal disutilities of labor services are equalized. Equation (10) represents the typical intertemporal Euler equation.

## 2.2 The Final Goods Producer

The final goods producer indexed by  $i \in [0, 1]$  produces entirely to stock, and its finished goods obey the accounting constraint given by

$$f_{t+1}(i) = f_t(i) + \tilde{y}_t(i) - s_t(i), \quad (11)$$

where  $f_t(i)$  refers to the stock of final goods inventory and  $\tilde{y}_t(i)$  is the net flow of final goods production.<sup>3</sup> The final good producer uses intermediate materials to produce final goods, accumulating materials inventory according to

$$m_{t+1}(i) = m_t(i) + d_t(i) - u_t(i), \quad (12)$$

where  $m_t(i)$  is the stock of material inventory,  $d_t(i)$  represents the deliveries of new materials, and  $u_t(i)$  is the use of materials for the production of final goods. Final goods,  $y_t(i)$ , are produced by the following production technology:

$$y_t(i) = a_t [l_t(i)]^\alpha [u_t(i)]^\omega, \quad (13)$$

where the parameters are restricted by  $0 < \alpha < 1$ ,  $0 < \omega < 1$ , and  $0 < \alpha + \omega < 1$  so that the gross production function has positive and diminishing marginal products and is strictly concave in its arguments. Note that it is the usage of materials in production,  $u_t(i)$ , rather than the stock of materials, that appears in the gross production function. The technology factor  $a_t$  follows an AR(1) process:

$$\ln a_t = \rho_a \ln a_{t-1} + \varepsilon_{a,t}, \quad (14)$$

where the technology shock  $\varepsilon_{a,t}$  is an IID innovation to  $a_t$ .

Input and output inventories incur holding costs. Following Maccini and Pagan (2013), the cost structure of inventory holding costs are assumed to be the following. Holding costs

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<sup>3</sup>Because of inventory holding costs and price adjustment costs, net flow of production and gross flow of production are different.



for inventories of final goods output are

$$FHC_t = \kappa_1 \left[ \frac{s_t(i)}{f_t(i)} \right]^{\kappa_2} s_t(i) + \kappa_3 f_t(i), \quad (15)$$

where  $\kappa_1$ ,  $\kappa_2$ , and  $\kappa_3$  are positive parameters. The first term represents the risk of stockouts, implying that the higher level of sales increases this risk while the higher level of output inventory decreases risk. The second term implies that inventory holding incurs costs such as storage or insurance costs which are proportional to the level of inventory. Inventory holding costs for materials have a similar structure,

$$MHC_t = \psi_1 \left[ \frac{y_t(i)}{v_t m_t(i)} \right]^{\psi_2} y_t(i) + \psi_3 v_t m_t(i), \quad (16)$$

where the parameters  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$  are all positive. The relative price of materials is represented by  $v_t$ . The first term implies that the higher level of final goods production increases the likelihood of production disruptions due to the shortage of materials while the increase in the material inventory reduces it. Like output inventory, materials inventory holding incurs cost proportional to the level of inventory, as indicated by the second term.

In addition to the inventory holding costs, the firm has to pay a price adjustment cost which is suggested by Rotemberg (1982):

$$PAC_t(i) = \frac{\phi}{2} \left[ \frac{p_t(i)}{\pi p_{t-1}(i)} - 1 \right]^2 c_t, \quad (17)$$

where  $\pi$  is the steady-state inflation rate. Labor and material input adjustment costs are

specified as Hall (2004) does:

$$LAC_t(i) = \frac{\phi_l}{2} \left[ \frac{l_t(i)}{l_{t-1}(i)} - 1 \right]^2 l_{t-1}(i), \quad (18)$$

and

$$MAC_t(i) = \frac{\phi_u}{2} \left[ \frac{u_t(i)}{u_{t-1}(i)} - 1 \right]^2 u_{t-1}(i). \quad (19)$$

Therefore, the net flow of production is defined to be

$$\tilde{y}_t(i) = y_t(i) - FHC_t(i) - MHC_t(i) - PAC_t(i) - LAC_t(i) - MAC_t(i). \quad (20)$$

The final good producer is assumed to maximize expected profits, represented by

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \lambda_{\tau} \beta^{\tau-t} \left\{ \left[ \frac{p_t(i)}{p_{\tau}} \right] s_t(i) - w_{l,t} l_t(i) - v_t d_t(i) \right\}, \quad (21)$$

subject to the equations (2), (11), (12), (13), and (20). The first order conditions with respect to  $l_t(i)$ ,  $u_t(i)$ ,  $d_t(i)$ ,  $y_t(i)$ ,  $s_t(i)$ ,  $p_t(i)$ ,  $f_{t+1}(i)$ , and  $m_{t+1}(i)$  are

$$\begin{aligned} \lambda_t w_{l,t} &= \alpha \zeta_t(i) \left[ \frac{y_t(i)}{l_t(i)} \right] - \phi_l \eta_t(i) \left[ \frac{l_t(i)}{l_{t-1}(i)} - 1 \right] \\ &\quad - \beta \mathbb{E}_t \eta_{t+1}(i) \left[ \left( \frac{\phi_l}{2} \right) \left[ \frac{l_{t+1}(i)}{l_t(i)} - 1 \right]^2 - \phi_l \left[ \frac{l_{t+1}(i)}{l_t(i)} - 1 \right] \left( \frac{l_{t+1}(i)}{l_t(i)} \right) \right], \end{aligned} \quad (22)$$

$$\begin{aligned} \xi_t(i) &= \omega \zeta_t(i) \left( \frac{y_t(i)}{u_t(i)} \right) - \phi_u \eta_t(i) \left[ \frac{u_t(i)}{u_{t-1}(i)} - 1 \right] \\ &\quad - \beta \mathbb{E}_t \eta_{t+1}(i) \left[ \left( \frac{\phi_u}{2} \right) \left[ \frac{u_{t+1}(i)}{u_t(i)} - 1 \right]^2 - \phi_u \left[ \frac{u_{t+1}(i)}{u_t(i)} - 1 \right] \left( \frac{u_{t+1}(i)}{u_t(i)} \right) \right], \end{aligned} \quad (23)$$

$$\lambda_t x_t = \xi_t(i), \quad (24)$$

$$\eta_t(i) \left\{ 1 - \psi_1 (1 + \psi_2) \left[ \frac{y_t(i)}{x_t m_t(i)} \right]^{\psi_2} \right\} = \zeta_t(i), \quad (25)$$

$$\lambda_t \left( \frac{p_t(i)}{p_t} \right) - \eta_t(i) \left\{ 1 + \kappa_1 (1 + \kappa_2) \left( \frac{s_t(i)}{f_t(i)} \right)^{\kappa_2} \right\} = \mu_t(i) \left( \frac{p_\tau(i)}{p_\tau} \right)^\theta, \quad (26)$$

$$\begin{aligned} 0 = & \lambda_t \left( \frac{s_t(i)}{p_t} \right) - \theta \mu_t(i) \left( \frac{p_t(i)}{p_t} \right)^{\theta-1} \left( \frac{s_t(i)}{p_t} \right) - \eta_t(i) \phi_p \left( \frac{p_t(i)}{\pi p_{t-1}(i)} - 1 \right) \left( \frac{c_t}{\pi p_{t-1}(i)} \right) \\ & + \beta \phi_p \mathbb{E}_t \eta_{t+1}(i) \left( \frac{p_{t+1}(i)}{\pi p_t(i)} - 1 \right) \left( \frac{p_{t+1}(i) c_{t+1}}{\pi p_t(i)^2} \right), \end{aligned} \quad (27)$$

$$\eta_t(i) = \beta \mathbb{E}_t \eta_{t+1}(i) \left\{ 1 + \kappa_1 \kappa_2 \left[ \frac{s_{t+1}(i)}{f_{t+1}(i)} \right]^{\kappa_2+1} - \kappa_3 \right\}, \quad (28)$$

and

$$\xi_t(i) = \beta \mathbb{E}_t \left\{ \xi_{t+1}(i) + \psi_1 \psi_2 \eta_{t+1}(i) \left[ \frac{y_{t+1}(i)}{x_{t+1} m_{t+1}(i)} \right]^{\psi_2+1} x_{t+1} - \psi_3 \eta_{t+1}(i) x_{t+1} \right\}. \quad (29)$$

The Lagrange multipliers  $\mu_t(i)$ ,  $\zeta_t(i)$ ,  $\eta_t(i)$ , and  $\xi_t(i)$  represent the shadow prices of aggregate demand, gross output, output inventory, and materials inventory, respectively. Equations (??) and (??) combined with (??) indicate the typical profit maximization conditions that equate a factor price with the value of its marginal product. Equation (??) shows the optimal choice of production. If the firm chooses to increase the output level marginally, then it has to pay an additional cost of  $\zeta_t(i)$ . At the optimal level of production, cost is equalized to the increased value of output inventory  $\eta_t(i)$  considering the risk of interrupted production ( $\eta_t(i) \psi_1 (1 + \psi_2) [y_t(i) / (v_t m_t(i))]^{\psi_2}$ ). If holding material inventory is not costly ( $\psi_1 = \psi_2 = \psi_3 = 0$ ), then the value of output inventory should be equal to the cost of pro-

duction, in equilibrium. Since the final good producer is monopolistically competitive in its output market, it can control its sales level by adjusting the output price of final goods. Equation (??) indicates that the value of additional sales ( $\mu_t(i) [p_t(i)/p_t]^\theta$ ) should be equal to the value of marginal revenue ( $\lambda_t(p_t(i)/p_t)$ ) net of the value of inventory ( $\eta_t(i)$ ) and the value of marginal output inventory holding cost ( $\eta_t(i) \kappa_1 (1 + \kappa_2) [s_t(i)/f_t(i)]^{\kappa_2}$ ). If output inventory holding is not costly ( $\kappa_1 = \kappa_2 = \kappa_3 = 0$ ), then the value of sales is equalized to the value of real profit. The equation (??) describes the pricing principle of the firm. A marginal increase in the price will generate additional revenue ( $\lambda_t [s_t(i)/p_t]$ ). The firm will consider the value of declined sales due to the price increase and the value of price adjustment cost to decide the new price level. Equations (??) and (??) describe the optimal choice of output and material inventory. When the firm chooses the inventory level for the next period, the firm would equate the cost (the value of inventory in the current period) and the benefit (the the value of the inventory in next period net of the value of marginal inventory holding cost).

## 2.3 The Materials Producer

The material producer uses labor services  $h_t$  to produce its output,  $d_t$ . The technology of materials production is described by the following function:

$$d_t = h_t^\gamma, \tag{30}$$

where  $0 < \gamma < 1$ . The firm does not hold any inventory, thus its production is same as the delivery to (or orders by) the final goods producer. The profit maximizing condition of the

firm is simply

$$w_{h,t} = \gamma v_t \left( \frac{d_t}{h_t} \right). \quad (31)$$

## 2.4 The Monetary Authority

The monetary authority adjusts the nominal interest rate  $r_t$  in response to deviations of detrended output, and inflation rate  $\pi_t = p_t/p_{t-1}$  from their respective steady-state values, according to the policy rule

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + \rho_y \ln(y_t/y) + \rho_\pi \ln(\pi_t/\pi) + \varepsilon_{r,t}, \quad (32)$$

where  $r$ ,  $y$ , and  $\pi$ , are the steady-state values of  $r_t$ ,  $y_t$ , and  $\pi_t$ . A monetary policy shock is represented by  $\varepsilon_{r,t}$ , which is an IID innovation.

## 3 The Just-in-Time Inventory Model

To evaluate the role of Just-in-Time –hereafter JIT– inventories we modify our model in the following way. We still consider a continuum of final goods producers and a continuum of materials producers but the crucial difference between this JIT model and the benchmark is that here materials inventories are held by the materials producer, rather than the final goods producer. As a result, the material producer’s profit maximization problem becomes a dynamic one since it now holds a state variable, the stock of materials. Both producers are monopolistic competitors in their output markets and price-takers in their input markets. The specifications of the household and monetary authority are the same as those in the benchmark model.

### 3.1 The Final Goods Producer

The specification of the final goods producer shares the same structure as the benchmark model, except for two aspects; the firm does not hold materials inventory and differentiated materials should be combined to be used for production.

A final good producer indexed by  $i \in [0, 1]$  uses a CES technology to combine the differentiated materials:

$$u_t(i) = \left[ \int_0^1 d_t(j)^{\frac{\tau-1}{\tau}} dj \right]^{\frac{\tau}{\tau-1}}, \quad \tau > 0. \quad (33)$$

where  $d_t(j)$  denote the order of differentiated material indexed by  $j \in [0, 1]$ . Cost minimization of the firm results in the demand for type  $j$  material as

$$d_t(j) = \left[ \frac{\tilde{v}_t(j)}{\tilde{v}_t} \right]^{-\tau} u_t(i), \quad (34)$$

where  $\tilde{v}_t(j)$  is the price of type  $j$  material. Parameter  $\tau$  represents the elasticity of substitution between differentiated materials. The aggregate materials price level is

$$\tilde{v}_t = \left[ \int_0^1 \tilde{v}_t(j)^{1-\tau} dj \right]^{\frac{1}{1-\tau}}. \quad (35)$$

The net flow of final goods production is defined as

$$\tilde{y}_t(i) = y_t(i) - FHC_t(i) - PAC_t(i) - LAC_t(i) - MAC_t(i) \quad (36)$$

where  $FHC_t(i)$  and  $PAC_t(i)$  are defined as in equations (15) and (17). The first order

conditions with respect to  $l_t(i)$ ,  $u_t(i)$ ,  $y_t(i)$ ,  $s_t(i)$ ,  $p_t(i)$ , and  $f_{t+1}(i)$  are as follows:

$$\begin{aligned} \lambda_t w_{l,t} &= \alpha \zeta_t(i) \left( \frac{y_t(i)}{l_t(i)} \right) - \phi_l \eta_t(i) \left[ \frac{l_t(i)}{l_{t-1}(i)} - 1 \right] \\ &\quad - \beta \mathbb{E}_t \eta_{t+1}(i) \left[ \left( \frac{\phi_l}{2} \right) \left[ \frac{l_{t+1}(i)}{l_t(i)} - 1 \right]^2 - \phi_l \left[ \frac{l_{t+1}(i)}{l_t(i)} - 1 \right] \left( \frac{l_{t+1}(i)}{l_t(i)} \right) \right], \end{aligned} \quad (37)$$

$$\begin{aligned} \lambda_t \left( \frac{v_t}{p_t} \right) &= \omega \zeta_t(i) \left( \frac{y_t(i)}{u_t(i)} \right) - \phi_u \eta_t(i) \left[ \frac{u_t(i)}{u_{t-1}(i)} - 1 \right] \\ &\quad - \beta \mathbb{E}_t \eta_{t+1}(i) \left[ \left( \frac{\phi_u}{2} \right) \left[ \frac{u_{t+1}(i)}{u_t(i)} - 1 \right]^2 - \phi_u \left[ \frac{u_{t+1}(i)}{u_t(i)} - 1 \right] \left( \frac{u_{t+1}(i)}{u_t(i)} \right) \right], \end{aligned} \quad (38)$$

$$\eta_t(i) = \zeta_t(i), \quad (39)$$

$$\lambda_t \left( \frac{p_t(i)}{p_t} \right) - \eta_t(i) \left\{ 1 + \kappa_1 (1 + \kappa_2) \left( \frac{s_t(i)}{f_t(i)} \right)^{\kappa_2} \right\} = \mu_t(i) \left( \frac{p_\tau(i)}{p_\tau} \right)^\theta, \quad (40)$$

$$\begin{aligned} 0 &= \lambda_t \left( \frac{s_t(i)}{p_t} \right) - \theta \mu_t(i) \left( \frac{p_t(i)}{p_t} \right)^{\theta-1} \left( \frac{s_t(i)}{p_t} \right) - \eta_t(i) \phi \left( \frac{p_t(i)}{\pi p_{t-1}(i)} - 1 \right) \left( \frac{c_t}{\pi p_{t-1}(i)} \right) \\ &\quad + \beta \phi \mathbb{E}_t \eta_{t+1}(i) \left( \frac{p_{t+1}(i)}{\pi p_t(i)} - 1 \right) \left( \frac{p_{t+1}(i) c_{t+1}}{\pi p_t(i)^2} \right), \end{aligned} \quad (41)$$

and

$$\eta_t(i) = \beta \mathbb{E}_t \eta_{t+1}(i) \left\{ 1 + \kappa_1 \kappa_2 \left( \frac{s_{t+1}(i)}{f_{t+1}(i)} \right)^{\kappa_2+1} - \kappa_3 \right\}. \quad (42)$$

### 3.2 The Materials Producer

A materials producer indexed by  $j \in [0, 1]$  obeys the accounting constraint of material inventory:

$$m_{t+1}(j) = m_t(j) + \tilde{n}_t(j) - d_t(j), \quad (43)$$

where  $m_t(j)$  refers to the stock of materials and  $\tilde{n}_t(j)$  is the net flow of materials production.

The gross production of materials is a function of labor input  $h_t(j)$ :

$$n_t(j) = [h_t(j)]^\gamma, \quad 0 < \gamma < 1. \quad (44)$$

Like the final goods producer, the materials producer has to pay inventory holding costs and a price adjustment cost. These costs are measured in units of materials. Therefore, the net flow of production is defined as

$$\tilde{n}_t(j) = n_t(j) - MHC_t(j) - VAC_t(j) \quad (45)$$

where  $MHC_t(i)$  and  $VAC_t(i)$  represent material inventory holding cost and material price adjustment cost, respectively. The materials inventory holding cost is represented as

$$MHC_t(j) = \psi_1 \left( \frac{d_t(j)}{m_t(j)} \right)^{\psi_2} d_t(j) + \psi_3 m_t(j), \quad (46)$$

where  $\psi_1$ ,  $\psi_2$ , and  $\psi_3$  are positive parameters. The materials price adjustment cost is specified as follows:

$$VAC_t(j) = \frac{\chi}{2} \left[ \frac{\tilde{v}_t(j)}{\pi \tilde{v}_{t-1}(j)} - 1 \right]^2 u_t. \quad (47)$$

The objective of the firm is to maximize its expected real market value, equal to

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \lambda_\tau \left[ \left( \frac{\tilde{v}_\tau(j)}{p_\tau} \right) d_\tau(j) - w_{h,t} h_\tau(j) \right]$$

subject to equations (34), (43), and (44). The first order conditions with respect to  $h_t(j)$ ,



$n_t(j)$ ,  $d_t(j)$ ,  $\tilde{v}_t(j)$ , and  $m_{t+1}(j)$  are

$$\lambda_t w_{h,t} = \gamma \varkappa_t(j) \left( \frac{n_t(j)}{h_t(j)} \right), \quad (48)$$

$$\varkappa_t(j) = \varrho_t(j), \quad (49)$$

$$\lambda_t \left( \frac{v_t(j)}{p_t} \right) - \varrho_t(j) \left\{ 1 + \psi_1 (1 + \psi_2) \left( \frac{d_t(j)}{m_t(j)} \right)^{\psi_2} \right\} = \varsigma_t(j) \left[ \frac{v_t(j)}{v_t} \right]^\tau, \quad (50)$$

$$\begin{aligned} 0 = & \lambda_t \left( \frac{d_t(j)}{p_t} \right) - \tau \varsigma_t(j) \left( \frac{\tilde{v}_t(j)}{\tilde{v}_t} \right)^{\tau-1} \left( \frac{d_t(j)}{\tilde{v}_t} \right) - \varrho_t(j) \chi \left( \frac{\tilde{v}_t(j)}{\pi \tilde{v}_{t-1}(j)} - 1 \right) \left( \frac{u_t}{\pi \tilde{v}_{t-1}(j)} \right) \\ & + \beta \chi \mathbb{E}_t \varrho_{t+1}(j) \left( \frac{\tilde{v}_{t+1}(j)}{\pi \tilde{v}_t(j)} - 1 \right) \left( \frac{\tilde{v}_{t+1}(j) u_{t+1}}{\pi \tilde{v}_t(j)^2} \right), \end{aligned} \quad (51)$$

$$\varrho_t(j) = \beta \mathbb{E}_t \varrho_{t+1}(j) \left\{ 1 + \psi_1 \psi_2 \left( \frac{d_{t+1}(j)}{m_{t+1}(j)} \right)^{\psi_2+1} - \psi_3 \right\}, \quad (52)$$

where the Lagrange multipliers  $\varsigma_t(j)$ ,  $\varkappa_t(i)$ , and  $\varrho_t(i)$  represent the shadow prices of material demand, gross material production, and material inventory, respectively.

## 4 Inventories: Propagation mechanism or the source of business cycles?

Several studies, dating to the work of Holt, Modigliani, Muth, and Simon (1960), have investigated the role of inventories in business cycles. Whereas earlier research (e.g., Metzler (1941)) posited that inventories were a destabilizing force that could induce fluctuations in aggregate output, later research viewed inventories as part of the propagation mechanism, but not the source, of business cycles. (e.g., Wilkinson (1989)). The goal of this section is to

explore, in the context of a New Keynesian model, whether inventories are a source of cycles or whether they only act as a propagation mechanism. Most parameter values are drawn from earlier studies (see Appendix B). The only exceptions are a set of cost parameters used to demonstrate how different adjustment costs and inventory holding costs may lead to infer a different function for inventories. Moreover, in order to evaluate the role that inventories play in the propagation of shocks, we compute the response to three shocks of interest in the business cycle literature: technology, preferences and monetary policy innovations.

#### **4.1 Interactions with labor and material input adjustment costs**

The model in this paper includes several real frictions: labor and input adjustment costs, and monopolistic competition in the market for final goods and materials. As it is common in recent New Keynesian models, these rigidities allow the responses to be smoother so as to better match the behavior observed in the data.

Figures 1-3 depict the response to shocks in the benchmark inventory model where the final goods producers hold the inventories of their final goods and materials. To better grasp the role that different rigidities play in this model, we plot impulse response functions with different adjustment costs turned on. The dashed line represents the response of the variables of interest when there are no labor or material input adjustment costs. The solid line illustrates the response when the labor and material input adjustment costs are turned on. In both scenarios we set the cost of adjusting inventories to the parameter values estimated by Maccini and Pagan (2008).

Notice that the difference between the responses to a productivity shock is negligible for all variables at long horizons (Figure 1). Yet, some differences are evident during the first

two years or so. In particular, when labor and material input adjustment costs are turned on, the reduction in labor and material input is somewhat smaller. In contrast, a positive productivity shock entails a larger accumulation of finished goods inventories and an initial liquidation of material inventories. The difference in the latter becomes insignificant in the long run.<sup>4</sup>

Consider now how the inclusion of adjustment costs affects the response to an innovation in the preference for consumption (Figure 2). Here again differences in the long-run are negligible for all variables. Yet, in the short run the inclusion of adjustment costs smooths the response of labor and materials usage. In addition, the increase in finished goods output induced by the rise in the households' preference for consumption is somewhat smaller. A liquidation of finished goods inventories takes place to fulfill the increase in demand under both scenarios; however, the effect is smaller when there are no adjustment costs. By comparison, materials inventories initially move in the opposite direction with an accumulation taking place when adjustment is costly. From the second quarter until about a year after the shock, material inventories are liquidated at a faster rate in the benchmark scenario.

Lastly, the presence of adjustment costs smooths the short-run effect of a monetary policy shock on output, labor, material input, and material inventories. Instead, it leads to a larger decrease in inventories of finished goods than in the benchmark model. As is the case for the other two shocks, the difference between the two scenarios is negligible in the long run. In brief, quantitative differences are large for the first year, especially in response to a monetary policy shock, however they dissipate about a year and a half after the shocks. Furthermore, the function of inventories in both the benchmark and the model with labor and material

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<sup>4</sup>Impulse responses not reported here but available in the on-line appendix reveal a similar picture when only labor or only material input adjustment costs are turned on.

input adjustment costs is just to propagate business cycles not to generate them.

## 4.2 High stock-out avoidance costs

Let us now consider what happens when the final goods producer faces a very high cost of incurring material stockouts. The dotted-dashed lines in Figures 1-3 depict the responses when in addition to turning on the labor and material input adjustment costs, the cost of incurring in material stock-out ( $\psi_2$ ) is very high.

Notice that, regardless of the shock considered, inventories not only propagate business cycles but they generate large fluctuations in output. Consider first the response to a productivity shock (Figure 1). Notice that when we increase the cost of avoiding stock-outs, the response of output is significantly larger on impact and then fluctuates around the benchmark. Similar behavior is observed for sales, labor, material input, wages, the interest rate, and the real price of materials. In contrast, materials inventories fluctuate much less than in the benchmark or in the scenario with only adjustment costs turned on. That is, a rise in the cost of facing stock-outs leads to an initial increase in material inventories as production accelerates, which is followed by a slow decline to the initial level as production returns to the steady state. A similar pattern is observed in response to a preference shock. High stock-out avoidance costs lead to large fluctuations in all variables (Figure 2). These cycles take over two years to dissipate.

As for the monetary policy shock, the effect on the interest rate is very similar to the response in the benchmark and the adjustment cost scenarios: a decline in the first quarter with a return to the steady state level about a year and a half after the shock (Figure 3). Yet also in this case, inventories induce cycles in most variables, which disappear about two

years after the shock.

Summarizing, when material inventories are held by the final goods producer and the cost of facing stock-outs is very high, inventories not only propagate shocks but they amplify fluctuations. The conclusion here is that models estimated with a large degree of stock-out avoidance costs may result in cycles induced by inventory management practices.

## 5 Implications of Just-in-Time Inventories

Since the 1980s, firms started implementing important changes in inventory management. "Just-in-time" techniques have been purportedly a main source of reductions in the inventory sales ratios across U.S. industries. Yet, the extent to which Just-in-Time –hereafter JIT– inventories contributed to changes in the business cycle and, in particular, the Great Moderation is still a question of debate. On the one hand, earlier work by McConnell and Perez-Quiros (2000) and Kahn, McConnell and Perez-Quiros (2002) suggests changes in inventory management practices contributed to the Great Moderation. On the other hand, in the framework of a DSGE model, Iacoviello, Schiantarelli and Schuh (2011) find that input and output inventories contributed very little to the Great Moderation.

This section explores, in the context of a DSGE model, how different assumptions regarding labor, input, and inventory adjustment costs alter the model's predictions regarding the effect of technology, preferences, and monetary policy shocks. Recall that our JIT model differs from the benchmark model in a crucial aspect: material inventories are held by the material producer rather than the final goods producer. This assumption corresponds to the inventory strategy where the final goods producer receives materials only as they are needed

in the production process, which in turn reduces inventory holding costs. In other words, inventories of materials are held by the materials goods producer until they are required downstream. Notice that our specification differs from that in Iacoviello, Schiantarelli and Schuh (2011) –hereafter ISS– in a key feature. Their model considers two distinct sectors, goods and services, which differ in that the former holds inventories while the latter does not. Such specification allows ISS to better match the model with the data. Instead, the two sectors in our economy are two goods’ producing sectors that differ in the type of good they produce (final or materials). Although this distinction might not capture the behavior of the service sector, it has the advantage of enabling us to better model the introduction of JIT inventories, which is the goal of this paper.

## **5.1 JIT inventories with and without labor and material adjustment costs**

The aim of this section is to evaluate whether the role played by labor and material adjustment costs in smoothing the impulse responses is affected by the introduction of JIT inventories. Figures 4, 5, and 6 illustrate the responses to productivity, preferences and monetary policy shocks, respectively. The dashed lines represent the responses in the JIT model where the cost of adjusting labor and material inputs are turned off. The solid lines illustrate the responses when the adjustment costs are turned on. To better grasp the implications of introducing JIT inventories, we use the same parameter values for each scenario as in the benchmark model. As earlier, we set the cost of adjusting inventories to the parameter values estimated by Maccini and Pagan (2008) in both scenarios.

Comparing the responses to a productivity shock in the benchmark and the JIT models

(dashed lines in Figures 1 and 4, respectively) reveals almost no differences for the behavior of finished goods. In fact, inventories, labor, wages, output and sales exhibit virtually the same path in both models. Similarly, the differences between the benchmark and the JIT models are mostly insignificant when comparing the responses to preferences (Figures 2 and 5) and monetary policy shocks (Figures 3 and 6).

As it is the case in the benchmark model, adding adjustment costs smooths out the response of labor, material orders, and material output to a productivity shock in the JIT model (Figure 4). Instead, more of the adjustment is done through increases in wages and the real price of materials. As for the response of inventories, the presence of adjustment costs in the JIT model leads to an initial liquidation followed by a smaller accumulation of material inventories and a larger buildup of finished goods inventories. The difference between the response of finished goods output in the JIT model and the JIT with adjustment costs is not significant.

In contrast, adjustment costs do smooth the response of finished goods output to preferences (Figure 5) and monetary policy shocks (Figure 6). On the one hand, a higher preference for consumption induces a smaller increase in the demand for inputs while it results in larger declines in wages and the real price of materials in the JIT model with adjustment costs. In other words, adjustment costs amplify the response of relative prices to a preference shock. On the other hand, expansionary monetary policy has a muted effect on relative prices when adjustment costs are turned on.

## 5.2 JIT inventories as a stabilizing force

Does the introduction of JIT inventories represents a stabilizing force in the economy? And if so, how? To answer these questions let's take a look at what happens in the JIT model when the cost of incurring in material stock-outs,  $\psi_2$ , is high. Recall that in the benchmark model with adjustment costs a high  $\psi_2$  implied that the final goods producer tried to smooth changes in material inventories, which in turn gave rise to cycles.

Consider now what happens when we introduce JIT inventories. The dotted-dashed lines in Figures 4, 5, and 6 represent, respectively, the responses to productivity, preferences, and monetary policy shocks in the JIT model with adjustment costs and high inventory holding costs. First, notice how the introduction of JIT inventories eliminates the cyclicity in the response to the three shocks of interest. Notice how the responses in the JIT model where adjustment costs are turned on are almost identical, regardless of the magnitude of  $\psi_2$ , for interest rates, as well as for output, inventories, and wages in the finished goods sector. In other words, because inventories of materials are held by the materials producer and not by the finished goods producer, it is the former who faces the costs of running into stockouts. Moreover, in this framework the materials producer has an incentive to smooth out changes in material inventories as it is she –and not the final goods producer– who faces the high stock-out avoidance costs. Hence, a high stock-out avoidance cost has the effect of smoothing out the responses of inventories, orders and real prices of materials and amplifying the response of material's output.



## 6 Conclusion

The Great Moderation is a period of reduced macroeconomic volatility whose origin has been the subject of several lines of research. Some studies have suggested that this time period was one with reduced economic shocks (i.e. good luck). Other researchers have argued that it was an improvement in the way monetary policy was conducted which caused the reduction in volatility. Nevertheless it has also been suggested that this time period was caused by innovations in the holding of inventories. Specifically, better information flows to firms may have caused reduced levels of inventories and/or the adoption of just-in-time (JIT) inventories.

In this paper, we examine the role of JIT inventories as an explanation for the Great Moderation in a DSGE model of the aggregate economy. We model inventories in a manner consistent with a large body of previous inventory research. Our benchmark model assumes that finished goods producers hold inventories of finished goods and materials. In our JIT model, materials inventories are held by the producers of materials, rather than finished goods producers. This change in economic structure provides a precise description of structural change in our economy and we find that this alternative inventory management practice completely eliminates the cyclical responses in the economy to the shocks that we impose. Thus we find compelling evidence that when business cycles originate in the inventory sector, the introduction of JIT inventory practices does indeed act as a stabilizing force in the aggregate economy.

There are a number of additional issues that might be pursued in future research on this topic. We omit capital stocks in our analysis as is traditional in the inventory investment literature. This addition to our model introduces an additional propagation mechanism into

the economy that can affect the dynamics displayed by the model in response to shocks. Further it would be of interest to estimate the parameters of our model to see how our model fits aggregate macroeconomic data. We motivate the existence of business cycles due to rapidly rising marginal inventory holding costs for materials. We might then be able to observe if there is any evidence that business cycles originate in the inventory sector of the economy as suggested in early research on inventory investment. But the results in this paper clearly suggest that structural change in inventory management practices may have a substantial impact upon the business cycle properties of an aggregate economy.

## References

- [1] Ahmed, Shaghil, Andrew Levin, and Beth Anne Wilson (2004), “Recent U.S. Macroeconomic Stability: Good Policies, Good Practices, or Good Luck?,” *The Review of Economics and Statistics*, 86 (3), 824–832.
- [2] Bils, Mark, and James A. Kahn (2000), “What Inventory Behavior Tells Us about Business Cycles,” *American Economic Review*, 90 (3), 458–481.
- [3] Blinder, Alan S. and Louis J. Maccini (2006), “The Resurgence of Inventory Research: What Have We Learned?,” *Journal of Economic Surveys* 5 (4), 291–328.
- [4] Boivin, Jean and Marc P. Giannoni (2006), “Has Monetary Policy Become More Effective?,” *The Review of Economics and Statistics*, 88(3), 445–462.
- [5] Christiano, Lawrence J. (1988) “Why Does Inventory Investment Fluctuate So Much?” *Journal of Monetary Economics* 21 (2-3), 247–80.

- [6] Clarida, R. , Jordi Galí, and Mark Gertler (2000), “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory”, *The Quarterly Journal of Economics*, 115 (1), 147-80.
- [7] Hall, Robert E. (2004), “Measuring Factor Adjustment Costs,” *The Quarterly Journal of Economics*, 119 (3), 899-927.
- [8] Hayashi, Fumio (1982), “Tobin’s Marginal q and Average q: A Neoclassical Interpretation,” *Econometrica*, 50 (1), 213-224.
- [9] Holt, C.C., Franco Modigliani, Herbert A. Simon (1955), “Linear decision rule for production and employment scheduling,” *Management Science* 2, 1–30.
- [10] Humphreys, Brad R. & Louis J. Maccini, Scott Schuh (2001), “Input and Output Inventories,” *Journal of Monetary Economics*, 47 (2), 347-375.
- [11] Iacoviello, Matteo, Fabio Schiantarelli and Scott Schuh (2011), "Input and Output Inventories in General Equilibrium," *International Economic Review* 52 (4), 1179-1213.
- [12] Ireland, Peter N. (2001), “Sticky-Price Models of the Business Cycle: Specification and Stability,” *Journal of Monetary Economics*, 47 (1), 3-18.
- [13] Kahn, James A., Margaret M. McConnell, and Gabriel Perez-Quiros (2002), “On the Causes of the Increased Stability of the U.S. Economy,” *Federal Reserve Bank of New York Policy Review* 8 (1), 183-202.
- [14] Kydland, Finn E., and Edward C. Prescott (1982), “Time to Build and Aggregate Fluctuations,” *Econometrica* 50 (6), 1345–70.

- [15] Maccini, Louis J., Bartholomew J. Moore, and Huntley Schaller (2004), "The Interest Rate, Learning, and Inventory Investment," *American Economic Review* 94 (5), 1303-1327.
- [16] Maccini, Louis J. and Adrian Pagan (2013), "Inventories, Fluctuations, and Goods Sector Cycles," *Macroeconomic Dynamics* 17 (1), 89-122.
- [17] Mack, Ruth (1967), *Information, Expectations, and Inventory Fluctuations: A Study of Materials Stock on Hand and on Order*, Columbia University Press, New York.
- [18] McConnell, Margaret M. and Gabriel Perez-Quiros (2000), "Output Fluctuations in the United States: What Has Changed Since the Early 1980's?," *American Economic Review* 90 (5), 1464-1476.
- [19] Metzler, Lloyd A. (1941), "The Nature and Stability of Inventory Cycles," *Review of Economics and Statistics* 23 (3), 113-29.
- [20] Ramey, Valerie A. (1989), "Inventories as Factors of Production and Economic Fluctuations," *American Economic Review* 79 (3), 338-54.
- [21] Ramey, Valerie A. and Kenneth D. West (1999), "Inventories," in *Handbook of Macroeconomics*, Vol. 1B, John B. Taylor and Michael Woodford eds., Elsevier Science B.V., 863-923.
- [22] Rotemberg, Julio (1982), "Sticky Prices in the United States," *Journal of Political Economy*, 90 (6), 1187-1211.
- [23] Stock, James H. and Mark W. Watson (2002), "Has the Business Cycle Changed and Why?" *NBER Macroeconomics Annual* 17, 159-218.

- [24] Wilkinson, Maurice (1989), "Aggregate Inventory Behavior in Large European Economies," *European Economic Review* 33 (1), 181-94.

## 7 Appendix

In this Appendix, a model of a stock-producing firm is set out which reveals that, for a firm holding inventories of finished goods and materials, transition equations that arise from optimizing behavior may involve oscillations in state variables as they approach the steady state. Once the model is described and its optimality criteria given, we derive the characteristic roots that arise in the model and show that those roots may be complex numbers. Further, we are also able to show how we can induce complex roots by the choice of parameters.

The firm is assumed to produce entirely to stock. It produces its output into a stock of finished goods, using intermediate materials in production and so it also holds a stock of intermediate materials inventories. Inventories obey the accounting constraints

$$\dot{f}(t) = y(t) - s, f(0) = f_0 \quad (53a)$$

$$\dot{m}(t) = d(t) - u(t), m(0) = m_0 \quad (53b)$$

where  $f$  refers to finished goods,  $y$  is the flow of output produced,  $s$  denotes sales,  $m$  is the stock of intermediate materials,  $d$  is deliveries of new intermediate materials,  $u$  is the rate at which materials are withdrawn from the stock of materials and used up in production.

Net output is produced according to the production function

$$y(t) = y(\ell(t), u(t)) - h(m(t), s) \quad (54)$$

where  $\ell$  measures labor services in production. As in much of the inventory investment

literature, the firm's capital stock is assumed to be fixed. The gross production function,  $y(\ell(t), u(t))$ , has positive ( $y_\ell > 0, y_u > 0$ ) and diminishing ( $y_{\ell\ell} < 0, y_{uu} < 0$ ) marginal products and it is assumed to be strictly concave in its arguments ( $y_{\ell\ell}y_{uu} - y_{\ell u}^2 > 0$ ). It will also be assumed that  $y_{\ell u} > 0$ . Notice that gross output produced depends upon withdrawals from the stock of materials as in Humphreys, Maccini and Schuh (2001); it does not depend upon the stock of materials. Since output is a flow, withdrawals from the stock of materials (or utilized materials), rather than the stock of materials, should appear as an argument of the production function.<sup>5</sup> Labor services and withdrawals from the stock of materials are always positive.

The gross production function has subtracted from it a holding cost term designed to capture the benefits and costs attached to holding a stock of materials. The reason is that Mack (1967) suggested that there are benefits in production that accrue to the firm by holding a stock of intermediate materials inventories. Production can occur more efficiently when an inventory of intermediate goods is held but there are also costs attached to materials inventories, such as the insurance, maintenance, obsolescence, labor, and physical capital costs incurred by the firm in holding an inventory of intermediate materials for use in production. Thus it will be assumed that the holding cost term is U-shaped in the stock of materials. At low levels of materials stocks, holding costs fall as the stock of materials rises. This is the range where the benefits in production dominate the inventory holding costs of the firm but, eventually, materials holding costs rise as the stock of materials grows, growing larger than the efficiency gains in production. These holding costs are also assumed to be convex in

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<sup>5</sup>Ramey (1989) assumes that a constant fraction of the stock of materials is used up in production. Here we allow the firm to choose, in effect, the rate at which it withdraws materials as well as the stock of materials that it wishes to hold.

the stock of materials inventories ( $h_{mm} > 0$ ). Mack (1967) also suggested that materials are held with an eye towards the level of sales and so the holding cost term subtracted from the gross production function also includes the level of sales (more will be said below about the role of sales in this holding cost function). There is a given initial stock of finished goods.

Equation (53b) is an accounting relationship for the stock of materials. Materials rise with deliveries of new materials and, for simplicity, there are no delivery lags associated with orders of new intermediate materials. A new order for a unit of materials is therefore identical to the delivery of a new unit of materials (see the concluding section of this paper for further discussion of this issue). Deliveries are unrestricted in sign; if  $d < 0$ , the firm is selling off excess materials in second-hand markets. Materials used up in production reduce the stock of materials. Utilized materials must obey the restriction  $0 < u < m$ ; this restriction on utilized materials is assumed to hold without formally imposing the constraint in the optimization problem just as will be done for the restriction that labor services must be positive. The stock of materials could also decline due to depreciation, breakage, or obsolescence that could be captured by subtracting exponential decay in the stock of materials. This possibility is also ignored for the sake of simplicity. Finally, there is a given initial stock of materials.

The firm wishes to maximize

$$J(t) = \int_0^{\infty} R(t)e^{-rt} dt \quad (55a)$$

$$R(t) = s - w\ell(t) - c(f(t), s) - v[d(t) + i(d(t) - u(t))] \quad (55b)$$

where  $R(t)$  is the firm's real cash flow,  $w$  is the real wage (the firm's output price is normalized to unity),  $v$  is the real purchase price of materials, and  $r$  is the discount rate ( $r > 0$ ). Cash



flow is given by the difference between real sales and costs where the latter is comprised of payments for labor services and the costs attached to inventories.

The firm operates in perfect input markets so that factor input prices are parametric to the firm. The wage bill is given by the product of the real wage and labor services used in production. The firm pays for intermediate materials at the time of delivery and there are installation (adjustment) costs attached to net changes in the stock of intermediate materials, costs measured in units of materials. These costs have the standard curvature properties used in the adjustment cost literature.<sup>6</sup>

$$i'(\dot{m}) \gtrless 0, \dot{m} \gtrless 0, i''(\dot{m}) > 0$$

If the firm is to hold finished goods inventories, there must be benefits as well as costs attached to doing so. Inventories could be productive for sales as in Bils and Kahn (2000) but a more commonly used assumption, and one that is used here, is that these benefits are embodied in a cost function such as the one in (55b). For example, it is frequently assumed that inventory holding costs contain a component that is linear in the stock of inventories and that costs also depend upon the gap between actual and desired inventories where the latter is proportional to the level of sales (see Moore, Maccini and Schaller (2004) as an example). The cost function in the cash flow equation embodies these ideas. It is also possible, and we assume this to be the case, that this cost function is U-shaped so that the firm's costs decline initially as the stock of finished goods inventories rises, then increasing after reaching

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<sup>6</sup>Note that there would still be adjustment costs attached to the stock of finished goods even without this assumption. With a strictly concave gross production function, the cost of adding current production to the stock of finished goods would rise at the margin as output increases and so there would still be adjustment costs attached to the stock of finished goods.

a minimum point. At low levels of finished goods inventories, there are benefits to the firm, such as the avoidance of stock-outs, that cause the firm's planned cash flow to rise with the level of inventories. Eventually these benefits are exhausted and the firm's holding costs begin to rise due to the insurance, maintenance and other costs of holding finished goods inventories. It is further assumed that this cost function is convex in the level of inventories ( $c_{ff} > 0$ ). Higher sales provide benefits to the firm by reducing the marginal holding costs of finished goods inventories ( $c_{fs} < 0$ ), a traditional assumption in the inventory investment literature.<sup>7</sup>

## 7.1 Optimality Conditions

The firm's optimality criteria may be found by using (53a-53b), (54) and (55b) to form the Hamiltonian

$$H = s - w\ell - c(f, s) - v[d + i(d - u)] + \lambda[y(\ell, u) - h(m, s) - s] + \pi[d - u]$$

where the time notation has been suppressed. In this expression,  $\lambda$  and  $\pi$  are adjoint variables measuring the values, imputed by the firm, to inventory stock accumulation. The

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<sup>7</sup>This is the implication of the standard quadratic approximation where inventory costs depend upon the square of the gap between inventories and their desired level where the latter is proportional to the level of sales.

following conditions describe optimal behavior by the firm.<sup>8</sup>

$$w = \lambda y_\ell(\ell, u) \quad (56a)$$

$$\pi = \lambda y_u(\ell, u) + v i'(d - u) \quad (56b)$$

$$\pi = v[1 + i'(d - u)] \quad (56c)$$

$$\dot{\lambda} = c_f(f, s) + r\lambda \quad (56d)$$

$$\dot{\pi} = \lambda h_m(m, s) + r\pi \quad (56e)$$

$$\dot{f} = y(\ell, u) - h(m, s) - s \quad (56f)$$

$$\dot{m} = d - u \quad (56g)$$

$$0 = \lim_{t \rightarrow \infty} \lambda(t)e^{-rt} f(t) = \lim_{t \rightarrow \infty} \pi(t)e^{-rt} m(t) \quad (56h)$$

Each of these optimality criteria may be readily interpreted.

Labor services and materials withdrawals are variable factor inputs in production (there are no adjustment costs attached to either of them) so standard static conditions describe the optimal choices of these inputs. Expression (56a) is such a marginal productivity condition for the optimal choice of labor. Combine (56b) and (56c) to give

$$v = \lambda y_u(\ell, u)$$

which is a conventional marginal productivity condition for the materials withdrawn from the stock of materials and used in production. As in the ordinary static theory of the firm,

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<sup>8</sup>The maximized Hamiltonian is easily shown to be strictly concave in the state variables given the maintained assumptions regarding functional forms. An optimal path will exist in this case and it will be unique. These optimality criteria are thus sufficient to determine an optimal path.

the ratio of the marginal products of variable factor inputs is equal to the factor input price ratio. The firm uses the shadow value of finished goods inventory accumulation in evaluating the marginal productivities of its variable factor inputs because it produces its output into a stock of finished goods inventories rather than selling its output directly to final consumers.

Expression (56c) shows that there is a measure of Tobin's marginal  $q$  (Hayashi 1982) associated with the stock of materials. Marginal  $q$  ( $q = \pi/v$ ) lies above or below unity as the firm accumulates or decumulates materials. Equations (56d-56e) can be integrated to show that the imputed values of inventories measure the discounted marginal benefits attached to inventories. (56f) and (56g) are repetitions of accounting relationships while the expressions in (56h) are transversality conditions implying that the imputed values of inventory accumulation recede to zero as time grows arbitrarily large.

These conditions will now be used to describe the behavior of the firm along the path to the steady state.

## 7.2 Dynamics

The transition equations that describe the evolution of the firm's state and costate variables are given below. The transition equations may be derived by eliminating the instruments

using the optimality criteria in (56a)-(56c). These are given by

$$u = \widehat{u}(\lambda, w, v), \ell = \widehat{\ell}(\lambda, w, v), d - u = \widehat{g}(\pi/v) \quad (57a)$$

$$\frac{\partial u}{\partial \lambda} = \frac{y_\ell y_{\ell u} - y_{\ell\ell} y_u}{\lambda (y_{\ell\ell} y_{uu} - y_{\ell u}^2)} > 0, \frac{\partial u}{\partial w} = \frac{-y_{\ell u}}{\lambda (y_{\ell\ell} y_{uu} - y_{\ell u}^2)} < 0 \quad (57b)$$

$$\frac{\partial u}{\partial v} = \frac{y_{\ell\ell}}{\lambda (y_{\ell\ell} y_{uu} - y_{\ell u}^2)} < 0, \frac{\partial \ell}{\partial \lambda} = \frac{y_u y_{\ell u} - y_{uu} y_\ell}{\lambda (y_{\ell\ell} y_{uu} - y_{\ell u}^2)} > 0, \quad (57c)$$

$$\frac{\partial \ell}{\partial w} = \frac{y_{uu}}{\lambda (y_{\ell\ell} y_{uu} - y_{\ell u}^2)} < 0, \frac{\partial \ell}{\partial v} = \frac{-y_{\ell u}}{\lambda (y_{\ell\ell} y_{uu} - y_{\ell u}^2)} < 0, \quad (57d)$$

$$\widehat{g}'(\pi/v) = i''(d - u)^{-1} > 0. \quad (57e)$$

These relationships lead to the transition equations below.

$$\dot{\lambda} = c_f(f, s) + r\lambda \quad (58)$$

$$\dot{\pi} = \lambda h_m(m, s) + r\pi \quad (59)$$

$$\dot{f} = y(\widehat{\ell}(\lambda, w, v), \widehat{u}(\lambda, w, v)) - h(m, s) - s \quad (60)$$

$$\dot{m} = \widehat{g}(\pi/v) \quad (61)$$

Defining  $\widetilde{\lambda}(t) = \lambda(t) - \lambda^*$ ,  $\widetilde{\pi}(t) = \pi(t) - \pi^*$ ,  $\widetilde{f}(t) = f(t) - f^*$ ,  $\widetilde{m}(t) = m(t) - m^*$ , the linear approximation to the transition equations is

$$\begin{bmatrix} \dot{\widetilde{\lambda}}(t) \\ \dot{\widetilde{\pi}}(t) \\ \dot{\widetilde{f}}(t) \\ \dot{\widetilde{m}}(t) \end{bmatrix} = \begin{bmatrix} r & 0 & \Delta_{13} & 0 \\ \Delta_{21} & r & 0 & \Delta_{24} \\ \Delta_{31} & 0 & 0 & \Delta_{34} \\ 0 & \Delta_{42} & 0 & 0 \end{bmatrix} \begin{bmatrix} \widetilde{\lambda}(t) \\ \widetilde{\pi}(t) \\ \widetilde{f}(t) \\ \widetilde{m}(t) \end{bmatrix} \quad (62)$$

where the coefficients in (62), evaluated in the steady state, are defined as

$$\Delta_{13} = c_{ff} > 0, \Delta_{21} = h_m(m, s) < 0, \Delta_{24} = \lambda h_{mm}(m, s) > 0, \quad (63)$$

$$\Delta_{31} = \frac{2y_{\ell u}y_{\ell}y_u - y_{uu}y_{\ell}^2 - y_{\ell\ell}y_u^2}{\lambda(y_{\ell\ell}y_{uu} - y_{\ell u}^2)} > 0, \Delta_{34} = -\Delta_{21}, \Delta_{42} = \frac{1}{v_i''(d-u)} > 0. \quad (64)$$

It can be shown that  $|\Delta| = \Delta_{13}\Delta_{42}(\Delta_{21}^2 + \Delta_{31}\Delta_{24}) = \kappa_1\kappa_2\kappa_3\kappa_4 > 0$  where  $\kappa_i$  denotes the characteristic roots in this system. These characteristic roots are found by forming  $|\Delta - \kappa I_4| = 0$  where  $I_4$  refers to an identity matrix of order four. The roots are assumed to be distinct (a slight perturbation of underlying parameters in the optimization problem can induce distinct roots) and the roots are given by

$$\kappa_i = \frac{r}{2} \pm \sqrt{\frac{r^2}{4} + \frac{\Delta_{13}\Delta_{31} + \Delta_{24}\Delta_{42}}{2} \pm \sqrt{\left(\frac{\Delta_{13}\Delta_{31} - \Delta_{24}\Delta_{42}}{2}\right)^2 - \Delta_{13}\Delta_{42}\Delta_{21}^2}}.$$

The roots may be complex because  $((\Delta_{13}\Delta_{31} - \Delta_{24}\Delta_{42})/2)^2 - \Delta_{13}\Delta_{42}\Delta_{21}^2$  can be negative. If the roots are complex, they will occur in conjugate pairs. The roots are thus symmetric about  $r/2$  with two stable roots having negative real parts and two unstable roots with positive real parts. Whether or not the roots are real or complex, it is true that  $\kappa_1 + \kappa_2 < 0$  and  $\kappa_1\kappa_2 > 0$  with  $\kappa_{1,2}$  denoting the stable roots.

Using the expression that determines the existence of complex roots, it is easily seen that the larger is the term  $\Delta_{21}$ , the more likely it is that complex roots will exist. This term is given by the marginal holding costs for materials which leads us to choose the parameters in the materials holding cost function when attempting to induce business cycles in our DSGE model. But note that holding costs for finished goods also appear in the expression

governing the existence of complex roots so we chose these parameters as well so that we could precisely focus our analysis on inventories and their role in causing business cycles.

In the case of just-in-time inventories, we do not provide any analytical results here because the dynamics arising from the firms holding inventories is straightforward. In each model, there is one inventory state variable that is held by each firm and, with the usual concavity restrictions that are made, maximized Hamiltonians will be strictly concave in the state variables, and saddlepaths arise that describe the path to the steady state. The transition equations will have real characteristic roots so that no oscillations in stocks will arise. All of these features of the just-in-time inventory problems are very standard in macroeconomic research.

Table A.1

Parameter	Value		Description	Source
	Benchmark/JIT	High cost		
$\beta$	0.9900		Discount rate	
$\alpha$	0.2500		Share of labor in final good production	Levinson and Petrin (2003)
$\omega$	0.3750		Share of material in final good production	Levinson and Petrin (2003)
$\gamma$	0.6667		Share of labor in material production	
$\kappa_1$	0.1000	1	Parameter for output inventory	Maccini and Pagan (2008)
$\kappa_2$	0.0260	275	Parameter for output inventory	Maccini and Pagan (2008)
$\kappa_3$	0.0400	1	Parameter for output inventory	Maccini and Pagan (2008)
$\psi_1$	0.1000	1	Parameter for material inventory	Maccini and Pagan (2008)
$\psi_2$	0.0150	250	Parameter for material inventory	Maccini and Pagan (2008)
$\psi_3$	0.0700	1	Parameter for material inventory	Maccini and Pagan (2008)
$\theta$	6.0000		Elasticity of substitution for different goods	Rotemberg and Woodford (1992)
$\phi$	50.0000		Price adjustment cost parameter: final good	Ireland (2001)
$\phi_l$	0	0.075	Labor input adjustment cost	Hall (2004)
$\phi_u$	0	0.050	Material input adjustment cost	Hall (2004)
$\chi$	50		Price adjustment cost parameter: material	Ireland (2001)
$\tau$	6		Elasticity of substitution for different material	Rotemberg and Woodford (1992)
$l$	0.1650		Steady state supply of labor	
$\pi$	1.0060		Steady state inflation rate	
$\rho_r$	0.5000		Taylor rule parameter for $r_{t-1}/r$	
$\rho_y$	0.5000		Taylor rule parameter for $y_t/y$	
$\rho_\pi$	1.5000		Taylor rule parameter for $\pi_t/\pi$	
$a$	1.0000		Steady state level of productivity shock	
$z$	1.0000		Steady state level of preference shock	
$\rho_a$	0.7000		Persistence of final good productivity shock	
$\rho_z$	0.7000		Persistence of preference shock	
$\sigma_a$	0.1000		Magnitude of innovation of productivity shock	
$\sigma_z$	0.1000		Magnitude of innovation of preference shock	
$\sigma_r$	0.1000		Magnitude of innovation of monetary policy shock	

Notes: The parameters  $\chi$  and  $\tau$  only appear in the Just-in-Time (JIT) inventory model.



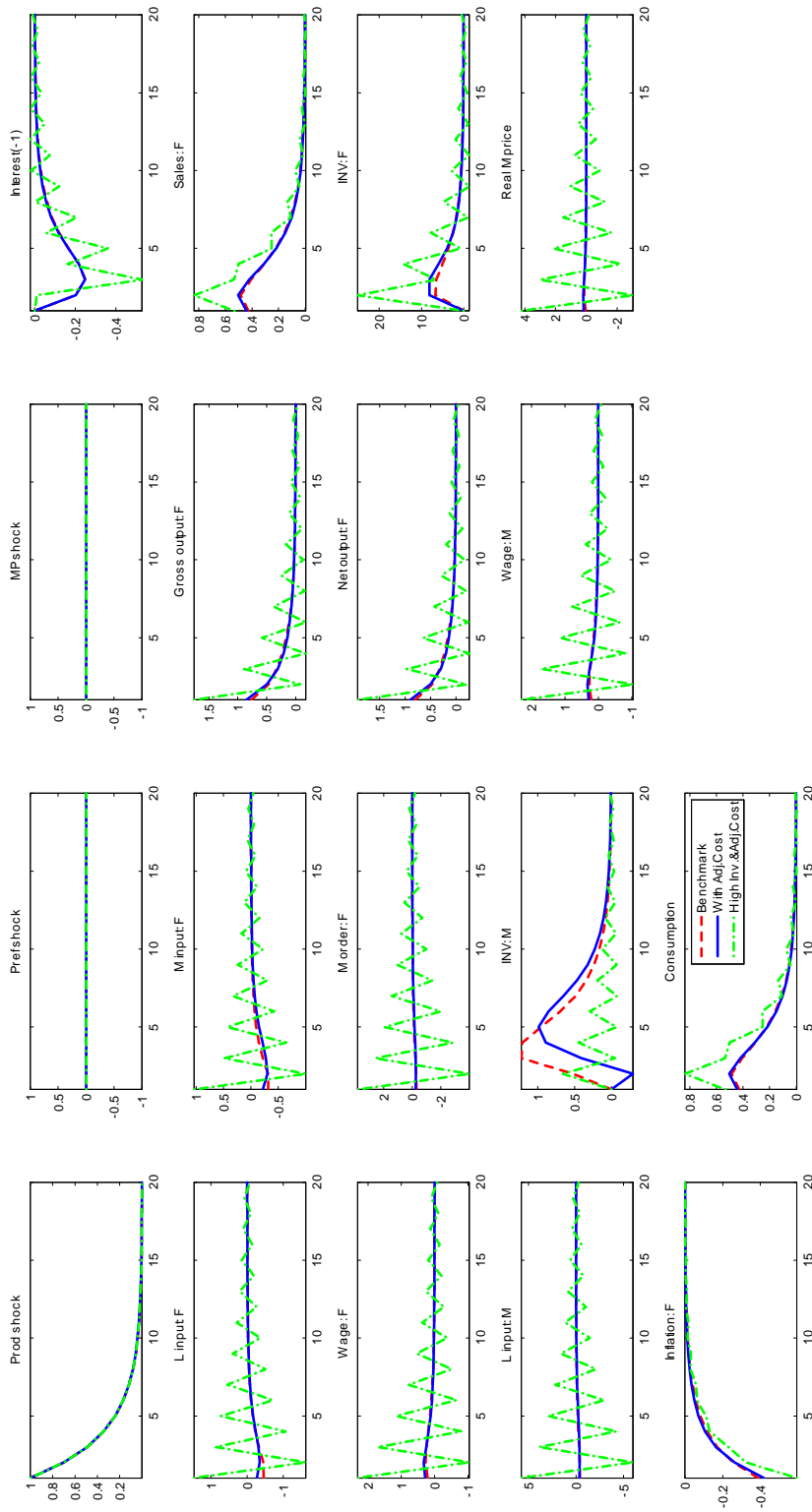


Figure 1. Response to a Productivity Shock in the Benchmark Model

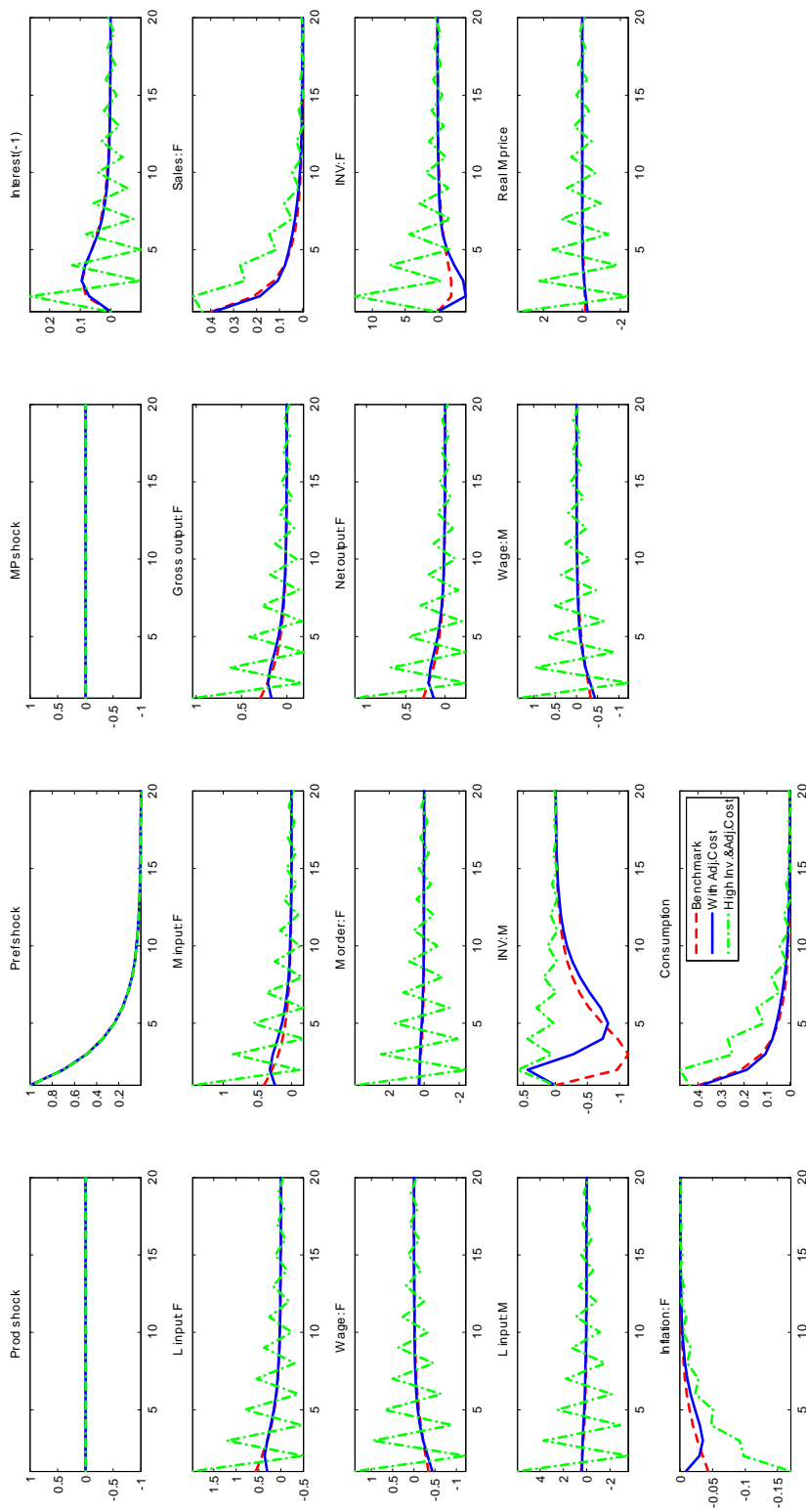


Figure 2. Response to a Preference Shock in the Benchmark Model

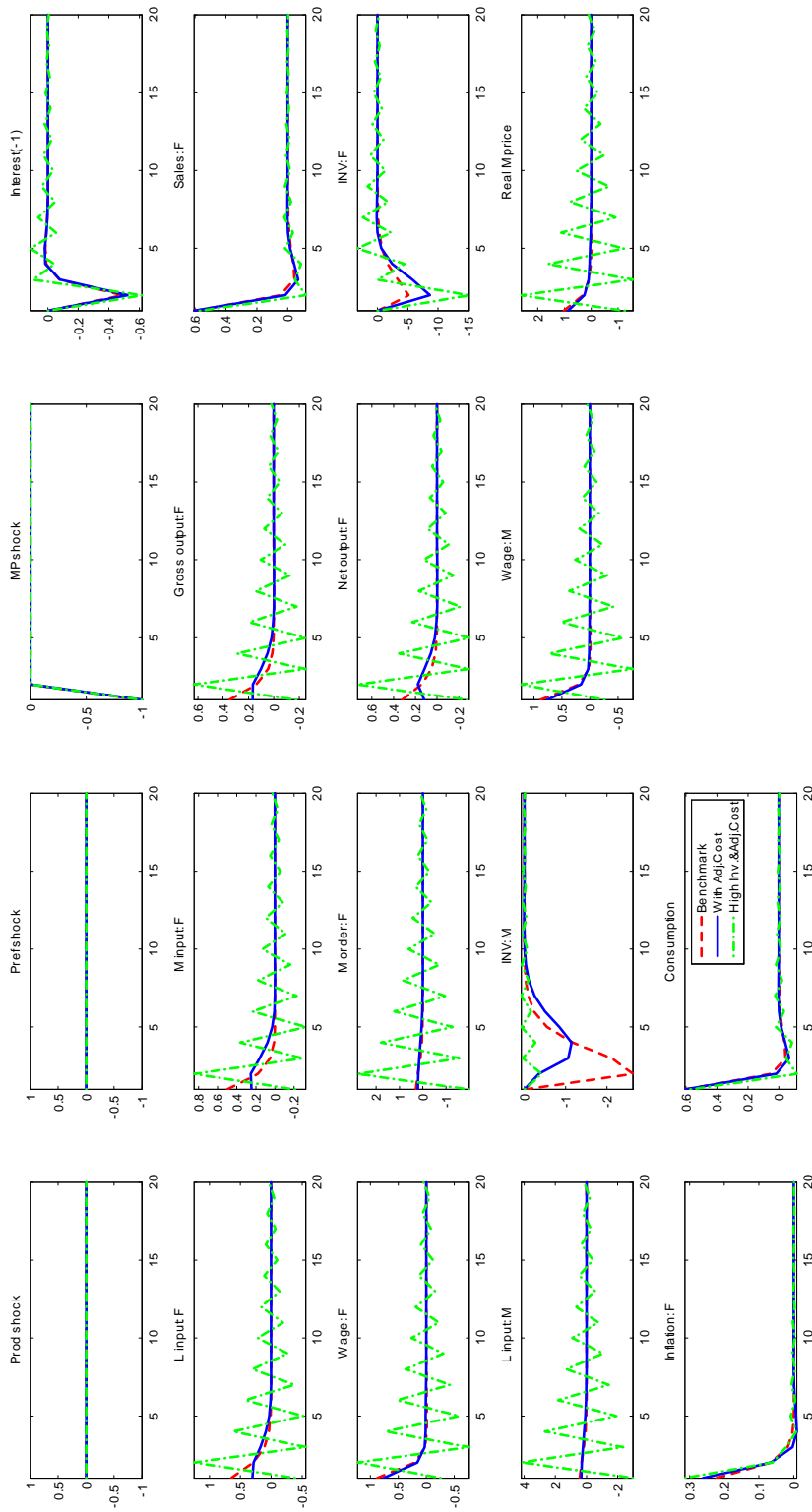


Figure 3. Response to a Monetary Policy Shock in the Benchmark Model

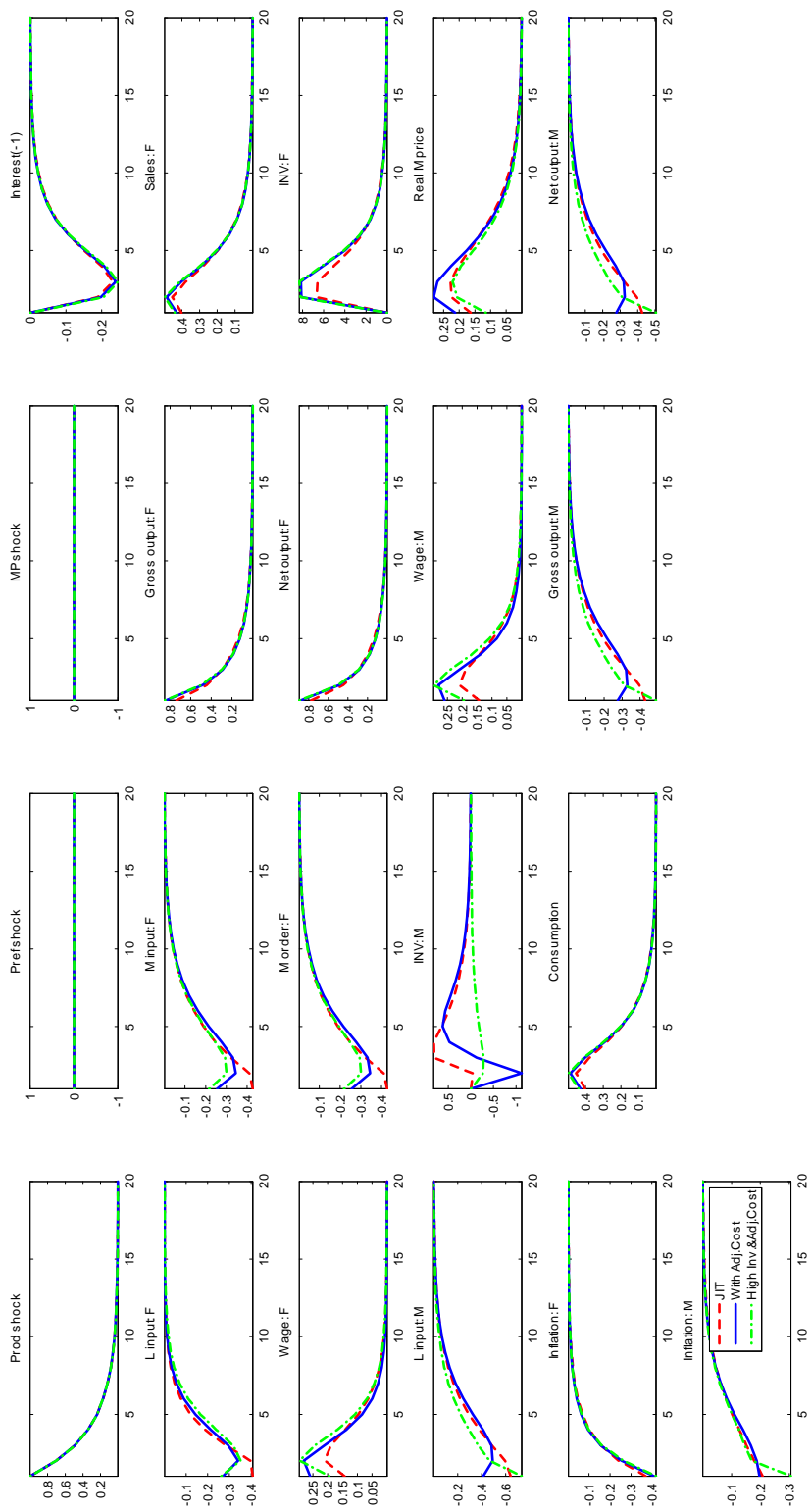


Figure 4. Response to a Productivity Shock in the JIT Model

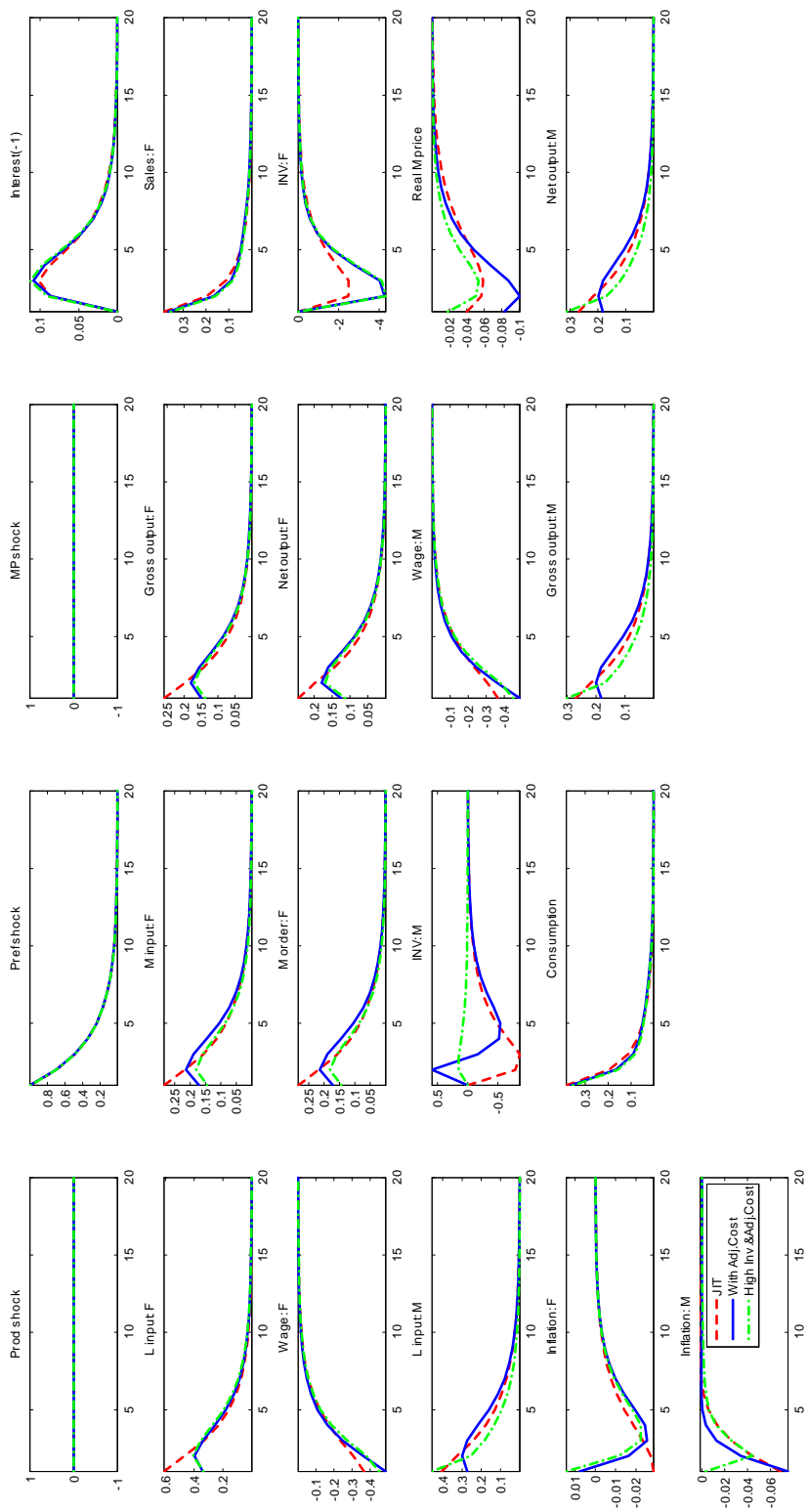


Figure 5. Response to a Preference Shock in the JIT Model

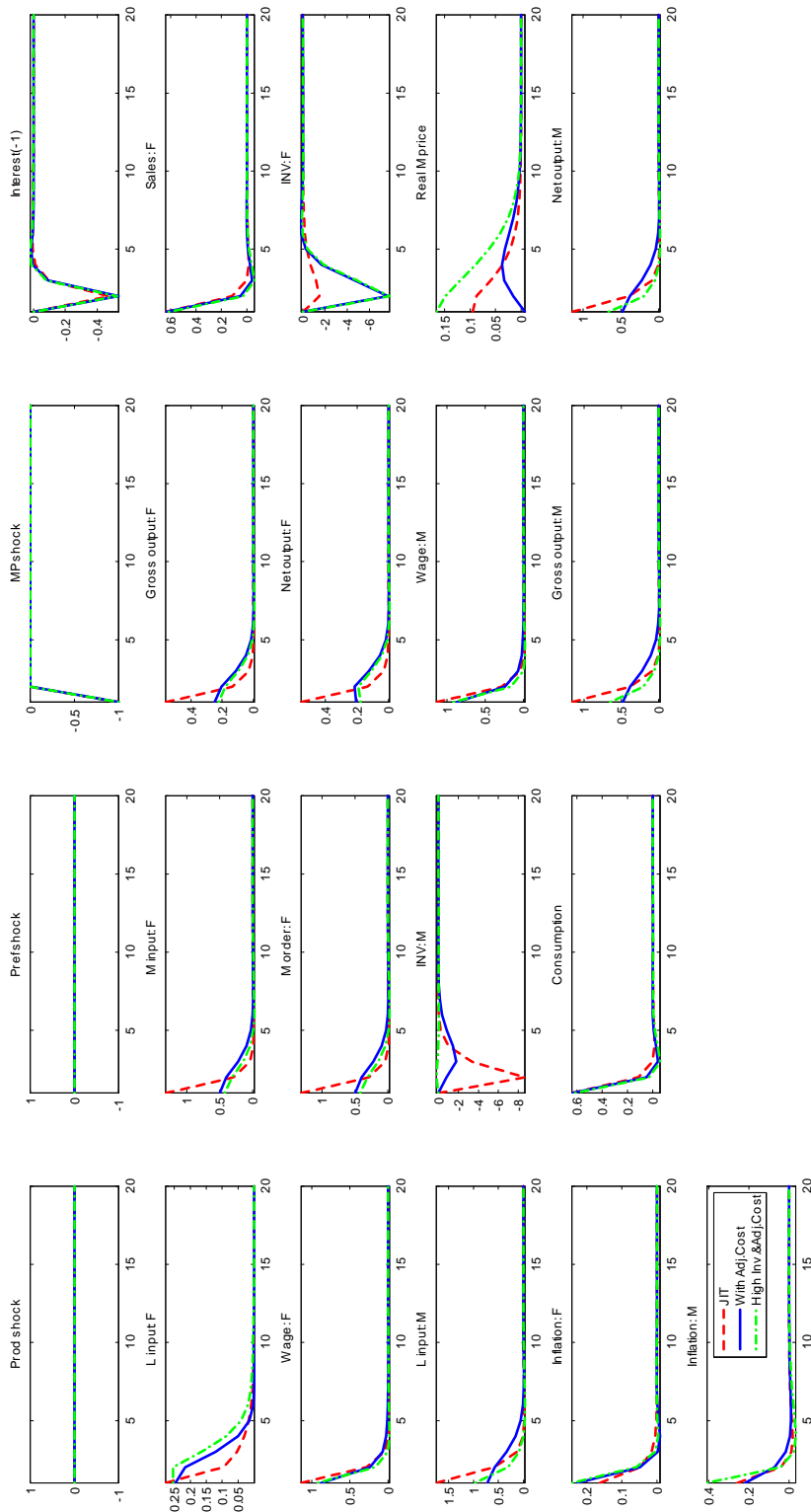


Figure 6. Response to a Monetary Policy Shock in the JIT Model