Wage Rigidity: A Quantitative Solution to Several Asset Pricing Puzzles.*

Jack Favilukis[†]and Xiaoji Lin[‡]

December 22, 2014

Abstract

In standard production models wage volatility is far too high and equity volatility is far too low. A simple modification - sticky wages due to infrequent resetting together with a CES production function - leads to both (i) smoother wages and (ii) higher equity volatility. Furthermore, the model produces several other hard to explain features of financial data: (iii) high Sharpe ratios, (iv) low and smooth interest rates, (v) time-varying equity volatility and premium, (vi) a value premium, and (vii) a downward-sloping equity term structure. Pro-cyclical, volatile wages are a hedge for firms in standard models, smoother wages act like operating leverage, making profits and dividends more risky.

1 Introduction

In most production models used for finance (examples include Jermann (1998), Boldrin et al.

(2001), Kaltenbrunner and Lochstoer (2010)) the volatility of excess equity returns is far too

^{*}We would like to thank Frederico Belo, Gian Luca Clementi, John Cochrane, Max Croce, Bob Goldstein, Vito Gala, Francois Gourio, Jesus Fernandez-Villaverde, Lars Kuehn, Erica Li, Dmitry Livdan, Lars Lochstoer, Stavros Panageas, Monika Piazzesi, Lukas Schmid, Rene Stulz, Stijn Van Nieuwerburgh, Neng Wang, Ivo Welch, Amir Yaron, and Lu Zhang for helpful comments. We thank the seminar participants at the Aarhus University, Beijing University, Cheung Kong Graduate School of Business, Goethe University, Higher School of Economics, London School of Economics, Ohio State University, Renmin University of China, University of Hong Kong, University of Minnesota, CEPR Asset Pricing Week in Gerzensee 2013, Chicago Booth-Deutsche Bank 2011 Symposium, SED 2012 meetings, EFA 2012 meetings, AEA 2013 meetings, WFA 2013 meetings, and China International Finance 2013 Conference. All remaining errors are our own.

[†]Finance Division, Sauder School of Business, University of British Columbia, Henry Angus Building, 2053 Main Mall, Vancouver, BC V6T 1Z2, Canada. Tel: (604) 822-9414 and E-mail: jack.favilukis@sauder.ubc.ca

[‡]Department of Finance, Ohio State University, 2100 Neil Ave, 846 Fisher Hall, Columbus, OH 43210. Tel: 614-292-4318 and E-mail: lin.1376@fisher.osu.edu

low.¹ This low equity volatility is closely related to the equity premium puzzle. Many standard models also fail to match several other important features of financial and accounting data, including conditional variation in the equity volatility and expected return, the value premium, and a downward-sloping equity term structure. A seemingly unrelated characteristic of these models is that wages are too volatile and too highly correlated with output. We show that the failure to match wage dynamics is closely related to the failure to explain financial data.² Introducing sticky wages brings the model quantitatively close to the data for these diverse financial phenomena. To our knowledge, we are the first to quantitatively capture such a wide array of financial moments in a reasonably calibrated general equilibrium model.

In the standard frictionless model, wages are equal to the marginal product of labor, which is perfectly correlated with output and is fairly volatile. This model fails because wages act as a hedge for shareholders. Profits are roughly equal to output minus wages, thus highly volatile and pro-cyclical wages make profits very smooth. Dividends are roughly equal to profits minus investment; because profits are smooth and investment is pro-cyclical, dividends are countercyclical. The firm appears too safe and its equity return is too smooth relative to the data. For example, in standard frictionless models, equity return volatility is between 1% and 5% (depending on the degree of returns to scale), compared to above 20% in the data.

Infrequent wage resetting causes the average wage paid by firms to be equal to the weighted average of historical spot wages. This makes the average wage smoother than the marginal product of labor, and less correlated with output (See, e.g., Shimer (2005), Hall (2006), Gertler and Trigari (2009), and Rudanko (2009) for detailed discussions of wage rigidities in explaining unemployment dynamics). A CES production function, through complementarities between labor and capital, also smoothes wages. When wages are smoother than the marginal product of labor, they are less of a hedge for the firm's shareholders. Profits, which are the residual after wages have been paid, are more volatile; dividends are now pro-cyclical. We refer to this effect

¹In Jermann (1998) and Boldrin et al. (2001) stock returns are volatile; however, most of this volatility is due to extremely volatile risk-free rates.

²This point has also been made by Danthine and Donaldson (2002) and Gourio (2007).

as labor leverage, and it leads to a more volatile return on equity.

Quantitatively, to get an equity volatility close to the data, our model employs other ingredients as well, these include decreasing returns to scale, fixed costs, idiosyncratic productivity shocks, and financial leverage. Our best model has an equity volatility of 15.61%, which explains more than three quarters of the equity volatility in the data. To be fair, this improvement is not due to purely wage rigidity. Table 7 decomposes the total effect into various model ingredients. However, the two channels we highlight - infrequent wage negotiation and CES production - play the crucial role. Their contributions are similar in size and both work through a labor leverage channel; depending on the calibration, they increase equity volatility by a factor of 2 to 4. Without the labor leverage channel, i.e. even with the other ingredients mentioned above, the model produces an equity volatility of only 5.3%.

For the labor leverage channel to be quantitatively important, the labor share (payments to employees as a fraction of output) needs to be large, volatile, and counter-cyclical. Indeed, the labor share in our model behaves like the data and is counter-cyclical; whereas in standard models it is constant. Analogously, for the labor leverage channel to be quantitatively important, the total wage bill needs to be (relatively) smooth and not too pro-cyclical. Again, our model's wage bill behaves as in the data - it is smoother than output and is imperfectly correlated with output; whereas in standard models it is perfectly correlated with output and has the same volatility as output. Labor leverage due to sticky wages acts in a similar way to operating (and even financial) leverage. Because equity is the residual, higher leverage implies riskier equity.

The importance of the labor leverage channel is confirmed by a novel empirical finding. We show that industries with high and counter-cyclical labor share have higher equity volatility and higher CAPM betas. In fact, labor share can explain 38% of all cross-sectional variation in industry volatility, and 54% of the variation in CAPM beta (Figure 1 and Table 1). When we allow firms in our model to differ in their amount of wage rigidity, we find that more rigid firms have more counter-cyclical labor share, higher volatility, and higher CAPM betas.

Cross-sectional variation in labor leverage has additional asset pricing implications. Low

productivity firms that are saddled with high committed wages relative to output are extra risky. These firms are value (low market-to-book) firms. In particular, value firms are loaded with high wages that do not fall as much as output in bad times, which leads to a lower profit to labor expense ratio for value firms and a sizable value premium. Our mechanism in generating value premium is different from Zhang (2005) in that we focus on the endogenous operating leverage effect induced by rigid wages which affects value firms more than growth firms, especially in economic downturns, while Zhang (2005) emphasizes the real frictions on firms' investment. Moreover, the growth rate shocks to aggregate productivity in our model are essentially the long-run risk shocks as in Bansal and Yaron (2004), whereas the pricing kernel in Zhang (2005) is effectively habit persistence as in Campbell and Cochrane (1999).

Similarly, labor leverage is not constant through the business cycle. Because wages are smoother than output, leverage due to wages is higher in recessions than in expansions. Consistent with financial data, this leads to higher equity volatility and a higher expected equity premium during bad times. Favilukis and Lin (2013) explore additional empirical implications of labor leverage and show that consistent with the model in this paper, wage growth negatively forecasts excess stock returns at aggregate, industry, and state level. This happens because during bad (good) times, wages fall (rise) by less than output leading to an increase (decrease) in leverage and therefore equity risk. These finding echo Santos and Veronesi (2006) who show that labor income to consumption ratio is a good predictor of long horizon stock returns.

Finally, our model is able to reproduce the downward-sloping term structure of equity dividends that van Binsburgen et al. (2012) find in the data. The intuition is again due to rigid wages. Wages and output are cointegrated, thus in the long run wages and dividends are expected to be at their normal shares of output. However, in contrast to standard models, in the short run wages can be far from their normal share, making profits and dividends highly pro-cyclical. This results in short term dividends being extra risky. Our channel is distinct from that of Belo et al. (2014a) who show that a mean reverting financial leverage policy can also produce a downward-sloping term structure of equity dividends. Note that most long run risk (LRR) models produce an upward-sloping term structure of equity dividends, while our model produces a downward-sloping term structure despite being a long-run risk model.

Related literature While the macroeconomic literature on wages and labor is quite substantial (for example Pissarides (1979)), there has been surprisingly little work done relating labor frictions to finance; notable exceptions are Uhlig (2007), Merz and Yashiv (2007), and Belo et al. (2014b). The structure of our model is closest to Kaltenbrunner and Lochstoer (2010) and Croce (2014), who build LRR production economies. However, we add labor market frictions and a cross-section of firms.

Our mechanism is similar to Danthine and Donaldson (2002) who (to our knowledge) were the first to emphasize the operating leverage channel through which smoother wages can lead to higher equity volatility. However, our model is quantitatively much closer to the data.³ Our model also generates conditional variation in the time series and the cross-section of expected returns, while Danthine and Donaldson (2002) are silent on these hard to match moments.

Our paper is also complimentary to Gourio (2007), who notes that wages are smoother than output and explores the implications of this for cross-sectional asset pricing. Because wages are smooth, profits should be volatile. He finds that profits are most volatile for low market-tobook (value) firms because they have a smaller gap between output and wages. These firms are therefore more risky. A factor model with the market and wage growth as the two factors does a reasonably good job of explaining the cross-section of asset returns. Unlike Gourio (2007), we explore the asset pricing implications of wage rigidity in a DSGE model.

Recent studies by Petrosky-Nadeau et al. (2013), Li and Palomino (2014), Donangelo (2014) and Berk and Walden (2013) are also closely related to our paper. Petrosky-Nadeau et al. (2013) explore how search frictions in the style of Mortensen and Pissarides (1994) affect asset prices in a general equilibrium setting with production. Like our model, they find that introducing frictions

³The main modeling differences are that (i) the staggered wage contract in our model endogenously implies smooth wages while Danthine and Donaldson (2002) assume an exogenous bargaining and risk sharing between households and the owners of capital, (ii) we use LRR style productivity shock and preferences, (iii) we allow for a non-trivial cross-section of firms. Additionally, as we will discuss below, our model generates high equity volatility without excessively smooth investment.

in the labor market can increase the model's equity volatility. Unlike our model, their channel works mostly through rare events (as in Barro (2006)) during which unemployment spikes up. On the other hand, Li and Palomino (2014) study nominal price and wage rigidity. They find that both types of rigidity increase expected equity returns, but wage rigidities have a larger impact. In Berk and Walden (2013), firms write labor contracts to provide insurance for risk-averse workers; similar to our mechanism, this results in excessively risky equity. In Donangelo (2014), labor mobility is responsible for operating leverage and affects asset prices. Kliem and Uhlig (2013) estimate a DSGE model and find that labor market rigidities are needed to match a high Sharpe ratio. Schmalz (2012) explores the corporate finance implications of modeling labor as an implicit long-term liability and shows that in response to unionization firms hold more cash and set lower financial leverage. Gomes et al. (2013) study the rigidity of nominal debt which creates long-term leverage that works in a similar way to our labor leverage.

Our paper is also related to the literature on long run risk (LRR); however, our contribution is to make LRR viable in a production economy. Bansal and Yaron (2004) have shown that the combination of a high intertemporal elasticity of substitution and a persistent consumption growth rate can deliver a high Sharpe ratio even with a low risk aversion. Croce (2014), Kaltenbrunner and Lochstoer (2010), and Kung and Schmid (2014) have shown that this can work in a production economy, but the excess volatility and time variation of returns remain unresolved. Our model is similar to all of these models but adds infrequent wage resetting. It is important to note that LRR alone cannot produce time-varying excess returns or volatilities. Bansal and Yaron (2004) devote the second half of their paper to adding an exogenous state variable which controls the volatility of equity returns but is orthogonal to LRR. Thus, in addition to a high volatility of equity, an important contribution of our work is to showcase a channel for endogenous conditional variation in equity returns and volatilities in a LRR world.

Finally, our paper is related to the extensive literature on wage rigidities and unemployment dynamics. It has been shown that wage rigidities are crucial to explain U.S. labor market dynamics, e.g., Shimer (2005), Hall (2006), Gertler and Trigari (2009), Pissarides (2009), etc.⁴ For example, Hall wrote, "The incorporation of wage stickiness makes employment realistically sensitive to driving forces." Our paper differs from these macro-papers in that we study asset pricing implications of staggered wage setting, while the models in labor economics fail to match the asset prices observed in the data. This is a problem endemic to most standard models, as observed by Mehra and Prescott (1985).

The rest of the paper is laid out as follows: In section 2 we write down the model. In section 3 we describe the model's calibration. Section 4 compares the results of a standard model to a model with infrequent wage resetting. We show that our baseline model can significantly improve the ability of the standard model to explain the equity volatility and premium, time variation in risk, the value premium, and a downward-sloping term structure of equity. In section 5 we present several extensions of our baseline model which improve its quantitative performance. Section 6 concludes.

2 Model

In this section we describe our model. We begin with the household's problem. We then outline the firm's problem, where the economy's key frictions are described. Finally, we define equilibrium.

In the model financial markets are complete, therefore we consider one representative household who receives labor income, chooses between consumption and saving, and maximizes utility as in Epstein and Zin (1989).

$$U_t = \max\left((1-\beta)C_t^{1-\frac{1}{\psi}} + \beta E_t [U_{t+1}^{1-\theta}]^{\frac{1-\frac{1}{\psi}}{1-\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}}$$
(1)

$$W_{t+1} = (W_t + N_t * \overline{w}_t - C_t) R_{t+1}$$

$$\tag{2}$$

⁴In addition to wage rigidities, search and match frictions are also crucial to capture unemployment dynamics.

where the second equation is the budget constraint of the representative household. The household enters period t with financial wealth W_t , earns labor income $N_t * \overline{w}_t$,⁵ consumes C_t , and invests the remainder in a portfolio of financial assets with return R_{t+1} . Risk aversion is given by θ and the intertemporal elasticity of substitution (IES) by ψ .

The aggregate labor supply is N_t . Labor and leisure do not enter into the household's utility function. However, we allow N_t to be a function of the aggregate stochastic shock. There are several ways to motivate this. For example, this would be the case in a model where all individuals expect to spend 8 hours per day working, searching for jobs, or acquiring new skills, but there is time variation in search frictions or in the fit between employee skills and skills demanded by firms. For example, Andolfatto (1996) shows that time-varying search efficiency is important quantitatively for the standard RBC model to match the comovement between labor and wages. We allow the labor supply to vary because it is important for the validity of our exercise to match both quantities and prices.

An earlier version of the paper had constant aggregate labor supply, we present some of these results in the appendix (Table A5). Although the asset pricing results were quantitatively similar, the model could not match the behavior of the total wage bill or the labor share.⁶

2.1 Firms

The interesting frictions in the model are on the firm's side. We assume a large number of firms (indexed by i and differing in idiosyncratic productivity) choose investment and labor to maximize the present value of future dividend payments where the dividend payments are equal to the firm's output net of investment, wages, operating costs and adjustment costs. Output is

⁵Since at any point in time, different workers may be earning different wages, we assume that the economy is made up of many identical households, or families, each with many individuals. Within each family all resources are pooled so that each family's average wage is equal to the average wage in the economy: \overline{w}_t .

⁶Even though by allowing labor supply to vary, we are able to match both prices and quantities, leaving leisure out of the utility function raises an additional concern. Leisure can provide agents with insurance against consumption fluctuations, as in Uhlig (2007) and Lettau and Uhlig (2000), making it more difficult to match the Sharpe ratio. At the same time, Dittmar et al. (2014) show that adding leisure does not hurt (and may help) to match asset pricing moments in a LRR model. We view our contribution as coming from our model's high volatility and do not believe this channel would be diminished even with a lower Sharpe ratio (and equity premium). Nevertheless, fully incorporating leisure remains an important avenue for future work.

produced from labor and capital. Firms hold beliefs about the discount factor M_{t+1} , which is determined in equilibrium.

2.1.1 Technology

The variable Z_t is an exogenously specified total factor productivity common to all firms. We assume that the growth rate of Z_t is $g_t = ln(\frac{Z_t}{Z_{t-1}})$ and follows a three-state Markov chain. $g_t \in \{g_1, g_2, g_3\}$, where $Pr(g_{t+1} = g_j | g_t = g_i) = \pi_{ij} \ge 0$ and $\sum_{j=1}^3 \pi_{ij} = 1$ for each j.⁷ We assume that the aggregate labor supply is perfectly correlated with aggregate TFP growth and takes the values $N_t = \{N_1, N_2, N_3\}$. Note that labor supply and productivity growth are stationary, while the level of productivity is not.

Idiosyncratic productivity of firm *i* is Z_t^i . This also follows a three-state Markov chain. $Z_t^i \in \{Z_1^i, Z_2^i, Z_3^i\}$, where $Pr(Z_{t+1}^i = Z_j | Z_t^i = Z_k) = \pi_{kj}^Z \ge 0$. The parameters of this process are identical for all firms but the process is independent across firms. Note that unlike aggregate productivity, the level of firm productivity is stationary.

Firm i's output is given by

$$Y_t^i = Z_t^i \left(\alpha(K_t^i)^{\eta} + (1 - \alpha)(Z_t N_t^i)^{\eta \rho} \right)^{\frac{1}{\eta}}.$$
 (3)

Output is produced with CES technology from capital (K_t^i) and labor (N_t^i) , where Z_t is labor augmenting aggregate productivity, and Z_t^i is the firm's idiosyncratic productivity. ρ determines the degree of return to scale (constant return to scale if $\rho = 1$), $\frac{1}{1-\eta}$ is the elasticity of substitution between capital and labor (Cobb-Douglas production if $\eta = 0$), and $(1 - \alpha)\rho$ is related to the share of labor in production.

⁷Although Bansal and Yaron (2004) use a more complicated process to motivate LRR, any process with a persistent growth rate will lead to LRR effects.

2.1.2 The Wage Contract

In standard production models wages are reset each period and employees receive the marginal product of labor. We assume that any employee's wage will be reset in the current period with probability $1 - \mu$.⁸ When $\mu = 0$, our model is identical to models without rigidity: all wages are reset each period, each firm can freely choose the number of its employees, and each firm chooses N_t^i such that its marginal product of labor is equal to the wage. When $\mu > 0$, we must differentiate between the spot wage (w_t) which is paid to all employees resetting wages this period, the economy's average wage (\overline{w}_t) , and the firm's average wage (\overline{w}_t^i) . The firm's choice of employees may no longer make the marginal product equal to either the average or spot wages because firms will also take into account the effect of today's labor choice and today's wage on future obligations.

When a firm hires a new employee in a period with spot wage w_t , with probability μ it must pay this employee the same wage next period; on average this employee will keep the same wage for $\frac{1}{1-\mu}$ periods. All resetting employees come to the same labor market and the spot wage is selected to clear markets. The firm chooses its total labor force N_t^i each period. These conditions lead to a natural formulation of the firm's average wage as the weighted average of the previous average wage and the spot wage:

$$\overline{w}_{t}^{i} N_{t}^{i} = w_{t} (N_{t}^{i} - \mu N_{t-1}^{i}) + \overline{w}_{t-1}^{i} \mu N_{t-1}^{i}$$
(4)

Here $N_t^i - \mu N_{t-1}^i$ is the number of new employees the firm hires at the spot wage and μN_{t-1}^i is the number of tenured employees with average wage \overline{w}_{t-1}^i .⁹

⁸Note that this is independent of the length of employment. This allows us to keep track of only the number of employees and the average wage as state variables, as opposed to keeping track of the number of employees and the wage of each tenure. This way of modeling wage rigidity is similar in spirit to Gertler and Trigari (2009), but for tractability reasons, we do not model search and match frictions.

⁹It is possible that $N_t^i < \mu N_{t-1}^i$, in which case μN_{t-1}^i cannot be interpreted as tenured employees. In this case we would interpret the total wage bill as including payments to prematurely laid-off employees. Note that the wage bill can be rewritten as $\overline{w}_t^i N_t^i = \overline{w}_{t-1}^i N_t^i + (\mu N_{t-1}^i - N_t^i)(\overline{w}_t^i - w_t)$. Here the first term on the right is the wage paid to current employees and the second term represents the payments to prematurely laid off employees. An earlier version of the paper included the constraint $N_t^i \ge \mu N_{t-1}^i$; the results were largely similar.

Note that the rigidity in our model is a real wage rigidity, although our channel could in principle work through nominal rigidities as well. There is evidence for the importance of both real and nominal rigidities. Micro-level studies of panel data sets comparing actual and notional wage distributions show that nominal wage changes cluster both at zero and at the current inflation rate, with sharp decreases in density to the left of the two mass points. Barwell and Schweitzer (2007), Devicenti et al. (2007), and Bauer et al. (2007) find that downward real wage rigidity is substantial in Great Britain, Germany, and Italy, and that the fraction of real wage cuts prevented by downward real wage rigidity. In a recent international wage flexibility study, Dickens et al. (2007) find that the relative importance of downward real wage rigidity and downward nominal wage rigidity varies greatly across countries while the incidences of both types of wage rigidity are roughly the same. We will later discuss the implications of a model which allows for variations in rigidity.

2.1.3 Accounting

The equation for profit is

$$\Pi(K_t^i) = Y_t^i - \overline{w}_t^i N_t^i - \Psi_t^i \tag{5}$$

 $\Pi(K_t^i)$ is profit, given by output less labor and operating costs.¹⁰ Operating costs are defined as $\Psi_t^i = f * K_t$; they depend on aggregate (but not firm specific) capital.¹¹ Labor costs are $\overline{w}_t^i N_t^i$. The total dividend paid by the firm is

$$D_t^i = \Pi(K_t^i) - I_t^i - \Phi(I_t^i, K_t^i),$$
(6)

¹⁰As there are no taxes or explicit interest expenses we do not differentiate between operating profit and net income, simply calling it profit.

¹¹Because productivity is non-stationary, all model quantities are non-stationary and we cannot allow a constant operating cost as it would grow infinitely large or infinitely small relative to other quantities. All quantities in the model must be scaled by something that is co-integrated with the productivity level, such as aggregate capital. We have also experimented with using the spot wage (w_t) or aggregate productivity (Z_t) and the results appear insensitive to this.

which is profit less investment and capital adjustment costs. Capital adjustment costs are given by $\Phi(I_t^i, K_t^i) = \nu \left(\frac{I_t^i}{K_t^i}\right)^2 K_t^i$.

2.1.4 The Firm's Problem

We will now formally write down firm i's problem. The firm maximizes the present discounted value of future dividends

$$V_t^i = \max_{I_t^i, N_t^i} E_t[\sum_{j=0,\infty} M_{t+j} D_{t+j}^i],$$
(7)

subject to the standard capital accumulation equation

$$K_{t+1}^{i} = (1 - \delta)K_{t}^{i} + I_{t}^{i}, \tag{8}$$

as well as equations (3), (4), (5), and (6). Equation 7 can be rewritten as a Bellman equation: $V_t^i = \max_{I_t^i, N_t^i} D_t + E_t[M_{t+1}V_{t+1}^i].$

The firm's optimal investment choice satisfies the familiar Euler equation in the q-theory of investment:

$$1 + 2\nu \frac{I_t^i}{K_t^i} = E_t [M_{t+1} \frac{\partial V_{t+1}^i}{\partial K_{t+1}^i}].$$
(9)

The firm's optimal labor choice is given by:

$$w_t = \frac{\partial Y_t^i}{\partial N_t^i} + E_t [M_{t+1} \frac{\partial V_{t+1}^i}{\partial N_t^i}]. \tag{10}$$

It is straightforward to see that when $\mu = 0$, then $\frac{\partial V_{t+1}^i}{\partial N_t^i} = 0$ and labor is chosen such that the wage is equal to the marginal product of labor. However, when $\mu > 0$ the next period's value function depends on this period's wage and labor choice; therefore, the wage is generally not equal to the marginal product of labor as the firm considers the effect of today's labor choice on future labor expenses. Depending on how the spot wage compares to the average wage, labor may be higher or lower than the marginal product of labor.

2.2 Equilibrium

We assume that there exists some underlying set of aggregate state variables S_t which is sufficient for this problem. Each firm's individual state variables are given by the vector $S_t^i = [Z_t^i, K_t^i, N_{t-1}^i, \overline{w}_{t-1}^i]$. Because the household is a representative agent, we are able to avoid explicitly solving the household's maximization problem and simply use the first order conditions to find M_{t+1} as an analytic function of consumption or expectations of future consumption. For instance, with CRRA utility, $M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\theta}$ while for Epstein-Zin utility $M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t[U_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{\frac{1}{\psi}-\theta}$.¹²

Equilibrium consists of:

- Beliefs about the transition function of the aggregate state variable and the shocks: $S_{t+1} = \Gamma(S_t, Z_{t+1})$
- Beliefs about the realized stochastic discount factor as a function of the aggregate state variable and the realized shocks: $M(S_t, Z_{t+1})$
- Beliefs about the aggregate spot wage as a function of the aggregate state variable: $w(S_t)$
- Firm policy functions for labor demand N_t^i and investment I_t^i (functions of S_t and S_t^i)

It must also be the case that given the above policy functions all markets clear and the beliefs are consistent with simulated data, and therefore rational:

- The firm's policy functions maximize the firm's problem (satisfy 9 and 10) given beliefs about the wages, the discount factor, and the aggregate state variable.
- The labor market clears: $\sum N_t^i = N_t$. Recall that N_t is a function of the exogenous shock, which is part of the aggregate state S_t .

¹²Given a process for C_t we can recursively solve for all the necessary expectations to calculate M_{t+1} . The appendix provides more details.

- The goods market clears: $C_t = \sum \left(\prod_t^i + \Psi_t^i + \overline{w}_t^i N_t^i I_t^i \right) = \sum D_t^i + \overline{w}_t^i N_t^i + \Phi_t^i + \Psi_t^i$. Note that here we are assuming that all costs are paid by firms to individuals and are therefore consumed. The results look very similar if all costs are instead wasted.
- The beliefs about M_{t+1} are consistent with goods market clearing through the household's Euler Equation.¹³
- Beliefs about the transition of the state variables are correct. For instance if aggregate capital is part of the aggregate state vector S_t , then it must be that $K_{t+1} = (1-\delta)K_t + \sum I_t^i$ where I_t^i is each firm's optimal policy.

3 Calibration

We solve the model at a quarterly frequency using a variation of the Krusell and Smith (1998) algorithm. We discuss the solution method in the appendix. The model requires us to choose the preference parameters: β (time discount factor), θ (risk aversion), ψ (IES); the technology parameters: α and ρ (jointly determine labor share in output and degree of return to scale), $\frac{1}{1-\eta}$ (elasticity of substitution between labor and capital), δ (depreciation), f (operating cost), ν (capital adjustment cost). Finally, we must choose our key parameter μ , which determines the frequency of wage resetting. Additionally, we must choose a process for labor supply, aggregate productivity shocks, and idiosyncratic productivity shocks. Below we justify our choices of these parameters.

Table 2 presents parameter choices for four models of interest: (i) a standard model with Cobb-Douglas technology where all wages are reset each period ($\mu = 0$); (ii) a model with Cobb-Douglas technology where wages are reset once every ten quarters on average ($\mu = 0.9$); (iii) a model with a calibrated elasticity of substitution between labor and capital but where all wages are reset each period; (iv) a model with a calibrated elasticity of substitution between labor and capital where wages are reset once every ten quarters on average.

¹³For example, with CRRA $M_{t+1} = \beta \left(\frac{\sum D_{t+1}^i + \overline{w}_t^i N_{t+1}^i + \Phi_{t+1}^i + \Psi_{t+1}^i}{\sum D_t^i + \overline{w}_t^i N_t^i + \Phi_t^i + \Psi_t^i} \right)^{-\theta}$

Preferences We set $\beta = 0.994$ per quarter; this parameter directly impacts the level of the risk-free rate and is also related to the average investment to output ratio. We set $\theta = 6.5$ to get a reasonably high Sharpe ratio, while keeping risk aversion within the range recommended by Mehra and Prescott (1985). The IES ψ also helps with the Sharpe ratio, it is set to 2 which is consistent with the LRR literature; Bansal and Yaron (2004) show that values above 1 are required for the LRR channel to match asset pricing moments as this ensures that the compensation for long-run expected growth risk is positive.

Technology We set $\delta = 0.025$ to match quarterly depreciation. Our production function has constant elasticity of substitution (CES), which includes Cobb-Douglas production as a special case. We solve our economy with either $\eta = 0$, which implies Cobb-Douglas production, or with $\eta = -1$, which matches empirical estimates of the elasticity of substitution between labor and capital.¹⁴

For the Cobb-Douglas production function $(\eta = 0)$, the parameters α and ρ have clear interpretations. α is the share of capital in production. ρ is the curvature of the production function (equivalently the degree of return to scale). Together, α and ρ determine the labor share $(1 - \alpha)\rho$ and the profit share $(1 - \alpha)(1 - \rho)$.¹⁵ We set $\rho = 0.77$ and $\alpha = 0.23$ so that labor share is $(1 - \alpha)\rho = 0.59$; this matches payments to labor as a share of output (for the private sector) of 0.59 in the data. The profit share is 0.177, which is consistent with numbers used in other models.¹⁶ Additionally, the combination of δ , α , ρ , and β determine the investment-tooutput ratio; in the U.S. this is between 20% and 25% and our model matches this.

¹⁴In our model the elasticity of substitution between labor and capital is $\frac{1}{1-\eta} = 0.5$. In a survey article based on a multitude of studies, Chirinko (2008) argues that this elasticity is between 0.4 and 0.6. One concern may be that the estimate of η would itself be affected by frictions, such as the one in our paper. However, several of the studies cited by Chirinko (2008) are done with micro-level data specifically to account for frictions, therefore η is the exact model analog of their estimate. Furthermore, we have repeated the exercise in Caballero (1994) to estimate η from aggregate quantities in our model by regressing ln(Y/K) on the cost of capital. The estimate of $\frac{1}{1-\eta}$ is 0.51 if the cost of capital is defined as the return on capital, and 0.67 if it is the interest rate plus the log of Tobin's Q.

¹⁵In our model, capital share and profit share are both paid to the owners of capital because the firm owns its capital. However, this may not be true in cases when the firm rents capital. In this case, capital share is paid to the owners of capital, while profit share is paid to the owners of the firm.

¹⁶Gomes (2001) uses 0.05 citing estimates of just above zero by Burnside (1996). Burnside et al. (1995) estimate it to be between 0.1 and 0.2. Khan and Thomas (2008) use 0.104, justifying it by matching the capital to output ratio. Bachmann et al. (2013) use 0.18, justifying it by matching the revenue elasticity of capital.

For the more general CES production function $(\eta = -1)$, α and ρ are still related to labor share, profit share, and the investment to capital ratio, however the relationship is no longer characterized by the simple analytic formulas as above. In particular, if we were to choose the same parameters as for Cobb-Douglas, then we may get quite different labor share and capital share. We set $\rho = 0.77$ and $\alpha = 0.45$, these allow the $\eta = -1$ model to have roughly the same profit share, labor share, and investment-to-output ratio as the Cobb-Douglas model, and as the U.S. economy.

Operating Cost $\Psi_t = f * K_t$ is a fixed cost from the perspective of the firm, however it depends on the aggregate state of the economy, in particular on aggregate capital. In each model, we choose f to match the average market-to-book ratio in the economy, which we estimate to be 1.33 (the actual values of f are in Table 2, details estimating the market-to-book ratio are in the appendix). While we think it is realistic for this cost to increase when aggregate capital is higher (during expansions), the results are not sensitive to this assumption. The results look very similar when Ψ_t is simply growing at the same rate as the economy.¹⁷

Note that fixed costs in the production process also effectively constitute a form of operating leverage. However, its quantitative effect on equity volatility and on the equity premium is minimal. This can be seen by comparing the standard frictionless model to our baseline model (Table 6). All of the models contain fixed costs, however the standard model has a small equity volatility and a negative (unlevered) value premium. What matters is not just the presence of operating leverage, but that operating leverage is less pro-cyclical than output. Leverage due to wage rigidities achieves this, while fixed costs alone do not.

Capital Adjustment Cost Within each model, we choose the capital adjustment cost ν to match the volatility of aggregate investment. Models with different μ or η may require a different adjustment cost for investment volatility to match the data, therefore the level of adjustment cost is different across models. Higher adjustment costs always help to increase equity volatility and the value premium, but they decrease aggregate investment volatility. This restriction on

¹⁷Recall that the model is non-stationary, therefore Ψ_t cannot be a constant and must be scaled by something that is cointegrated with the size of the economy.

matching aggregate investment volatility limits how much work capital adjustment costs can do in helping to match financial moments.

Productivity and Labor Shocks In order for the LRR channel to produce high Sharpe ratios, aggregate productivity must be non-stationary with a stationary growth rate. We specify the growth rate of aggregate productivity $(g_t = ln(\frac{Z_t}{Z_{t-1}}))$ to be a symmetric three-state Markov process with quarterly autocorrelation 0.80, unconditional mean of 0.005, and unconditional standard deviation of 0.019. We choose these numbers to roughly match the autocorrelation, growth rate, and standard deviation of output. Aggregate productivity is then $ln(Z_{t+1}) = ln(Z_t) + g_{t+1}$ which is consistent with LRR.¹⁸

We assume that aggregate labor supply is perfectly correlated with aggregate productivity so that $N_t = N_i$ whenever $g_t = g_i$ where $i \in \{1, 2, 3\}$. We set $N_1 = 0.93$, $N_2 = 1.0$, and $N_3 = 1.07$ to best match the behavior of aggregate employment, as can be seen in Table 3. Of course we are somewhat limited because of the perfect correlation with productivity growth; in principle labor supply could follow an independent process, however this would increase computational time.

The volatility of idiosyncratic productivity shocks Z_t^i depends on the model's scale, that is, which real world production unit (firm, plant) is analogous to the model's production unit. There is no consensus on the right scale to use. For example, the annual autocorrelation and unconditional standard deviation are 0.69 and 0.40 in Zhang (2005), 0.62 and 0.19 in Gomes (2001), 0.86 and 0.04 in Khan and Thomas (2008), while Pastor and Veronesi (2003) estimate that the volatility of firm-level profitability rose from 10% per year in the early 1960s to 45% in the late 1990s. We have experimented with various idiosyncratic shocks and find that our aggregate results are not significantly affected by the size of these shocks. In our model, Z_t^i is a three-state Markov process with quarterly autocorrelation and unconditional standard deviation of 0.9 and 0.1 respectively. We also set the mean of Z^i to be 0.25 so that the average capital in our model is roughly the same as in a model solved annually with the same production function

¹⁸Our process for TFP growth is analogous to a discretized AR(1) process. The actual values are $g_1 = -0.022$, $g_2 = 0.005$, $g_3 = 0.032$ and the transition probabilities are $\pi_{11} = 0.8$, $\pi_{12} = 0.2$, $\pi_{13} = 0$, $\pi_{21} = 0.1$, $\pi_{22} = 0.8$, $\pi_{23} = 0.1$, $\pi_{31} = 0$, $\pi_{32} = 0.2$, $\pi_{33} = 0.8$.

and so that our model's results can be compared to a model solved annually.¹⁹

Frequency of wage resetting In standard models wages are reset once per period and employees receive the marginal product of labor as compensation. This corresponds to the $\mu = 0$ case. However, wages are far too volatile in these models relative to the data. We choose the frequency of resetting to roughly match the volatility of wages in the data. This results in $\mu = 0.9$ or an average resetting frequency of ten quarters. This also implies realistic behavior for the model's labor share and total compensation.

Although μ is chosen to match macroeconomic moments, its value seems consistent with micro evidence on wage rigidity. Given the importance of this parameter for our results, it is useful to add some discussion. We believe that this number is realistic and may even be on the low side (choosing a higher number would strengthen our results). For example, Rich and Tracy (2004) estimate that a majority of labor contracts last between two and five years with a mean of three years, and cite several major renewals (United Auto Workers, United Steel Workers) that are at the top of the range. Anecdotal evidence suggests that assistant professors, investment bankers, and corporate lawyers all wait even longer to be promoted.

Even if explicit contracts are written for a shorter period than our calibration (or not written at all), we believe that ten quarters is a reasonable estimate of how long the real wage of many employees remains unchanged. Campbell III and Kamlani (1997) conducted a survey of 184 firms and find that implicit contracts are an important explanation for wage rigidity of U.S. manufacturing workers, especially of blue-collar workers. For example, if the costs of replacing employees (for employers) and the costs of finding a new job (for employees) are high, the status quo will remain, keeping wages the same without an explicit contract. Another example are workers who receive small raises every year, keeping their real wage constant or growing slowly; indeed, Barwell and Schweitzer (2007) show that this type of rigidity is quite common. Such workers do not experience major changes to their income until they are promoted, or let go, or

¹⁹The Cobb-Douglas production function is insensitive to rescaling, however scaling does matter for certain quantities in the CES production function. The actual values are $Z_L = 0.20$, $Z_M = 0.25$, $Z_H = 0.30$ and the transition probabilities are $\pi_{11}^Z = 0.965$, $\pi_{12}^Z = 0.035$, $\pi_{13}^Z = 0$, $\pi_{21}^Z = 0.0175$, $\pi_{22}^Z = 0.965$, $\pi_{23}^Z = 0.0175$, $\pi_{31}^Z = 0$, $\pi_{32}^Z = 0.035$, $\pi_{33}^Z = 0.965$.

move to another job. Hall (1982) estimates an average job duration of eight years for American workers, Abraham and Farber (1987) estimate similar numbers just for non-unionized workers (presumably unionized workers have even longer durations).²⁰

Financial Leverage We define the unlevered return as $R_{t+1}^{K} = \frac{V_{t+1}}{V_t - D_t}$. However, real world firms are financed both by debt and by equity, with equity being the riskier, residual claim. To compare the model's equity return and dividend to their empirical counterparts, we must make an assumption about financial leverage. Note that the Modigliani and Miller (1958) propositions hold in our model, therefore all assumptions about financial leverage are completely orthogonal to our model's solution; these assumptions only affect the way we report returns and dividends. The appendix describes the financial leverage policy. All tables below report the return on equity, or equivalently, the levered return.

4 Results

4.1 The Standard Model

In this section we will discuss the model where production is Cobb-Douglas and wages are reset once per quarter ($\mu = 0$). This is a standard real business cycle model with the addition of LRR. This model is most similar to Croce (2014) and Kaltenbrunner and Lochstoer (2010). We will refer to this model throughout the text as the standard, frictionless model. Like other RBC models (starting with Prescott (1986)), this model is able to match most standard macroeconomic moments, this can be seen in Panel C of Table 3.²¹ Note that although the model is solved at a

 $^{^{20}}$ An alternative way to think about job length is the separation rate or the probability of a worker separating from her job in any particular period. Estimates of separation rates for the US are around 3%/month (Hobijn and Sahin (2009), Shimer (2005)). If separations were equally likely for all workers, this would imply an average job length of around 2.8 years. Although this is similar to our contract length, it is actually far smaller than the average job length. This is because a small number of workers frequently transition between jobs, while a majority of workers stay in their jobs for a long time; a more in depth discussion is provided in Section 5.2.

²¹For both model and data, we report the volatility of all HP-filtered statistics relative to the volatility of HP-filtered GDP, and the volatility of all growth rates relative to the volatility of the growth rate of GDP. We often just refer to "volatility" or "correlation" in the text, without specifying HP-filtered or growth because the point being made is true for both. In the data the annual volatilities of HP-filtered GDP and of GDP growth are respectively 3.98% and 5.12%. We calibrate the productivity process so that the model counterparts are virtually identical.

quarterly frequency, we aggregate model data and report annual moments.

One important exception to this model's success in reproducing macroeconomic moments is wage behavior (rows 5, 6, and 7). In this model the wage is equal to the marginal product of labor, which leads to a constant labor share. This is highly counterfactual; in the data, the volatility of labor share in the private sector is more than half of the volatility of output. Additionally labor share is quite counter-cyclical. The reason for a volatile and counter-cyclical labor share in the data is that the wage is relatively smooth, its volatility is less than half that of output. Thus, after a positive (negative) shock to the economy, wages do not rise (fall) by as much as output, resulting in labor share falling (rising). In the standard model, the wage is nearly twice as volatile as in the data. Similarly, in the data the total wage bill is less volatile than output and is imperfectly correlated with output; in the standard model the total wage bill has exactly the same volatility as output and the two are perfectly correlated. Because labor income comprises such a large fraction of output, this flaw is quantitatively very significant and is responsible for many of this model's failures at matching financial data as discussed below. Our goal is to fix this flaw.

Unlike the first generation of RBC models, which did a poor job at matching financial moments (the well-known Equity Premium Puzzle), this model can produce a high Sharpe ratio through the LRR channel.²² Another common shortcoming of production models that were able to produce high equity volatilities has been that they also implied volatile risk-free rates (for example Jermann (1998) and Boldrin et al. (2001)), however, this was also resolved by LRR (see Kaltenbrunner and Lochstoer (2010) and Croce (2014)). As in Kaltenbrunner and Lochstoer (2010), the risk-free rate is sufficiently smooth in our model.

As shown in Tables 3 and 6, wherever the frictionless model is successful - consumption,

²²First proposed by Bansal and Yaron (2004) in an endowment economy and later incorporated into a production economy by Kaltenbrunner and Lochstoer (2010) and Croce (2014), the LRR channel makes the economy appear risky to households because (i) a high IES makes households care not only about instantaneous shocks to consumption growth but also shocks to expectations of future consumption growth, (ii) shocks to the growth rate (as opposed to the level in standard models) of productivity are persistent causing expectations of future consumption growth to vary over time.

investment, the risk-free rate, the Sharpe ratio - all other models we solve are successful as well.²³ However, in addition to not matching labor market variables, the standard frictionless model has several important flaws. Most evident among these is the volatility of equity returns, which is above 20% in the data but only 5.33% in the model (row 3 in Panel A of Table 6). Because the volatility of equity is so low, the equity premium (which is the Sharpe ratio multiplied by the volatility of equity) is also quite low.

Most models of production economies with financial markets struggle to reproduce the high equity volatility observed in the data. Several have addressed it by increasing the capital adjustment cost; this channel has been used to explain the value premium as well. Figure 3 shows the equity volatility in frictionless models which differ from each other in adjustment cost only. For example, by raising the adjustment cost parameter ν to 15 in the frictionless Cobb-Douglas model, it is possible to increase the unlevered equity volatility and value premium to 10.27% and 2.10% respectively, and with financial leverage to 16.89% and 5.25%! However, this will lead to investment being less than half as volatile as in the data. Note that we do not take this route here, instead, adjustment costs are chosen so that both aggregate and firm level investment volatility in our model match the data.

Another criticism of LRR models, as well as standard real business cycle models, is that they cannot endogenously produce variations in risk premia or equity volatility across the business cycle. As can be seen in Panels B and C of Table 6, in the data (first row) the expected equity premium and equity volatility during bad times are higher than during good times. The standard model (second row) produces virtually no variation in either of these quantities. To fix this, Bansal and Yaron (2004) introduce stochastic volatility into their model through an

²³As discussed earlier, we calibrate the productivity process to roughly match the short term autocorrelation of output, this implies that the autocorrelation of consumption growth (which is endogenous), is somewhat higher than the data - though still within error bounds. Figure A1 of the appendix, plots autocorrelations of output and consumption growth at horizons of 1-10 years. At medium term horizons (3-6 years), both output and consumption growth autocorrelations produced by the model are too high - this is common to many LRR models. We have also solved a model with much lower autocorrelation of output. This model is closer to the data at longer horizons but has far too little autocorrelation at shorter horizons; the equity volatility in this model is 11.66% (compared to 13.15% in our baseline model). Note that our process for aggregate productivity is a simple 3-state Markov process, we believe a more complicated process is necessary to capture both short and long term autocorrelation in output and consumption growth.

additional state variable. Our model produces time-variation in risk premia and equity volatility endogenously.

Additionally, the standard model performs poorly when we consider cross-sectional asset pricing. The well-known value premium puzzle is that low market-to-book (value) stocks have higher average returns than high market-to-book (growth) stocks. However, the opposite is true in the standard model: growth stocks have higher average unlevered returns while the value premium measured on levered returns is very small. These results are in Panel A of Table 6 and in Table 8.

Finally, the standard model performs poorly on several important accounting moments, as can be seen in Panels A and B of Table 5. Profit growth volatility is only 57% that of the data (71% for HP-filtered) and profits are too pro-cyclical. Even more problematic, dividends in the data are pro-cyclical but in this model they are highly counter-cyclical!

As will be shown below, a model with labor market frictions can improve on the standard model along all of these dimensions.

4.2 Infrequent resetting of wages

In this section we will discuss a model which combines two key features: infrequent wage renegotiation and a calibrated CES production function (as opposed to Cobb-Douglas production in the standard model). We refer to this model as our baseline model. For intuition we will also present intermediate models with just CES production, or just infrequent wage renegotiation. We will show that the addition of these two features improves on all of the problems with the standard model discussed above.

Because wages are set infrequently, the average wage is no longer equal to the marginal product of labor but rather to a weighted average of past spot wages. This results in the average wage being smoother than the marginal product of labor (row 5 in Panels D and F of Table 3). Smoother wages mean that the total wage bill (the average wage multiplied by total labor) is less volatile than output, and no longer perfectly correlated with output (row 6). Finally, labor share

(the total wage bill divided by output) is no longer constant, it is relatively volatile and, crucially, counter-cyclical as in the data (row 7). We believe that most models with similar features will have results qualitatively similar to those discussed below. We view infrequent wage setting as one of several mechanisms responsible for the relatively smooth wages in the data.²⁴

The parameter controlling sticky wages is μ , the fraction of employees who renegotiate their wage in a given period. When $\mu > 0$ there are two relevant wages. The average employee who did not renegotiate receives the average wage from the previous period \overline{w}_{t-1}^i . All resetting employees receive the current spot wage w_t , which clears the labor market. The average wage is a weighted average between last period's average wage and this period's spot wage. Consistent with the empirical findings of Pissarides (2009), in our baseline model the spot wage is more volatile than the average wage. Despite this, the average wage is still smoother than the marginal product of labor. Table 3 shows that unlike the standard model, models with wage rigidity (Panels D and F) not only do well at replicating the behavior of standard macroeconomic quantities such as investment and consumption volatility, but also that of wages, the total wage bill, and labor share.

4.2.1 Elasticity of substitution between capital and labor

As discussed above, infrequent renegotiation of wages leads to wages being smoother than in a standard model. This helps improve the model's performance on various financial moments. A properly calibrated elasticity of substitution between capital and labor, even without other frictions, also makes wages smoother and will therefore have a similar effect on financial moments.

Traditionally, production based models have used the Cobb-Douglas production function $(\eta = 0)$. This implies that the elasticity of substitution between labor and capital $(\frac{1}{1-\eta})$ is one. It also implies that (in the absence of additional frictions), the labor share is constant, which is counterfactual. Empirical estimates of this elasticity are below one $(\eta < 0)$. A survey article by Chirinko (2008) discusses various efforts to estimate this elasticity and argues that it is between

²⁴For example, search frictions can lead to sticky wages as in Petrosky-Nadeau et al. (2013).

0.4 and 0.6; we follow this by setting it to 0.5 in our baseline model. A lower elasticity strengthens complimentarity between labor and capital while a higher elasticity makes them substitutes.²⁵

Suppose that wages are frequently reset ($\mu = 0$). In this case, the wage is still equal to the marginal product of labor, regardless of the elasticity of substitution between labor and capital. Ignoring the idiosyncratic component, the marginal product of labor is:

$$\frac{\partial Y_t}{\partial N_t} = \rho(1-\alpha) Y_t^{1-\eta} Z_t^{\rho\eta} N_t^{\rho\eta-1}$$
(11)

To gain understanding for how η affects the wage we perform a simple experiment. Holding constant N_t and K_t while letting Z_t vary, we compute the volatility of output and of the marginal product of labor. We present a ratio of the two volatilities and their correlation in Table 4. As the complimentarity between capital and labor gets stronger, the marginal product of labor (and equivalently the wage) become less volatile relative to output. Nevertheless, the marginal product of capital and output are nearly perfectly correlated even for extreme values of η . Indeed, consistent with this logic, when we compare a Cobb-Douglas, frictionless model to a frictionless model with calibrated elasticity, wage growth volatility falls by 3% and wage bill growth volatility falls by around 20%; results are even stronger for HP-filtered volatility (Table 3, Panels C and E).

4.2.2 Unconditional Financial Moments

Profits and dividends are presented in Panels A and B of Table 5. Profits are approximately equal to output minus the total wage bill. In a standard frictionless model, the average wage and the total wage bill are highly volatile and highly pro-cyclical (the wage bill is perfectly correlated with output). This results in profits being too smooth. Dividends are approximately equal to profits minus investment. Because profits are relatively small in magnitude while investment is highly pro-cyclical, dividends in a standard model are counter-cyclical, exactly the opposite of

²⁵For output defined as in our model, $\lim_{\eta\to-\infty} Y_t = \min(K_t, Z_t N_t)$ making labor and capital perfect compliments and $\lim_{\eta\to\infty} Y_t = \max(K_t, Z_t N_t)$ making them perfect substitutes.

what we observe in the data. Because profits are smooth and dividends are counter-cyclical, the firm's equity return is also too smooth in standard models. In other words, pro-cyclical wages act as a hedge for the firm's shareholders, making equity seem safe.

When the average wage and the total wage bill become smoother and less correlated with the marginal product of labor, profits become more volatile relative to the standard model. Volatile and pro-cyclical profits lead to pro-cyclical dividends. Complimentarity between labor and capital alone (row 4), and infrequent wage setting alone (row 5) each increase the volatility of profits (Panel A) and make dividends more pro-cyclical (Panel B). Our baseline model combines the two (row 6), here, profit and dividend behavior is quite similar to the data.

The relationship between wages, profits, and dividends is also evident in Figure 2, which plots the impulse responses to a positive productivity shock lasting for four consecutive quarters for a standard frictionless model (solid line), a CES calibrated frictionless model (dot-dashed line), and our baseline model (dashed line). In panel A, wages respond much more slowly to a productivity shock in our baseline model, needing around ten years to fully catch up to the standard frictionless model. On the other hand, due to smooth wages, the short term jump in profits (Panel B) in our baseline model is twice that of the standard frictionless model. Finally, dividends (Panel C) actually fall in response to a positive productivity shock in the standard frictionless model because profit is relatively small and smooth while investment is pro-cyclical. In our baseline model, dividends respond positively to a positive productivity shock. Note that a frictionless model with a calibrated CES improves on the standard model due to smoother wages caused by the complimentarity of labor and capital; however, the improvement is far short of our baseline model.

With smoother wages no longer being as strong of a hedge, equity volatility in our baseline model is closer to the data as well. Panel A of Table 6 shows that our baseline model (row 6) has an equity volatility of 13.15%, compared to 20.26% in the data; it is only 5.33% in the standard model. Since the equity volatility and Sharpe ratio are both closer to the data, it is not surprising that our baseline model has an equity premium of 5.57% compared to only 2.27%

in the standard frictionless model. Note that this is all possible without extremely high risk aversion, excessively smooth investment, or an unrealistically volatile risk-free rate.

Our baseline model differs from the standard model in two ways: the CES parameter and infrequent renegotiation. It is useful to explore each of these in isolation: infrequent renegotiation alone raises equity volatility from 5.33% to 9.85% while a CES production function alone raises it to 9.21%. This is because smoother wages lead to more volatile profits and to more pro-cyclical dividends.

Figure 4 displays the quantitative effect on equity volatility for different degrees of wage stickiness. The solid line is for models with Cobb-Douglas production, while the dashed line is for calibrated CES production. In our baseline model, renegotiations happen once every 10 quarters, on average; this corresponds to $\mu = 0.9$. However, the effects on equity volatility are significant even with shorter contract length.

Comparing the frictionless, calibrated CES model to the sticky wage, calibrated CES model also makes evident a subtle but important point. Note that the volatility of the total wage bill in both models is similar to the data (row 6, Panels A, E, F of Table 3). However, as discussed above, the sticky wage model produces much more equity volatility. This highlights the importance of not just the volatility of the total wage bill, but also its correlation with output: in the frictionless model the two are perfectly correlated but in the sticky wage model the correlation is less than perfect, as in the data. Using the same logic as above to go from wages to profits and from profits to dividends, the frictionless, calibrated CES model performs better than the standard frictionless model, but its profits are still far too smooth, and its dividends are still counter-cyclical. This is because despite having the right amount of wage bill volatility, it still has too much correlation. The lower correlation between output and the total wage bill in the sticky model allows for more pro-cyclical dividends.

4.2.3 Decomposing the Quantitative Effect

Although our focus is on wage rigidity, relative to previous models in the literature (i.e. Kaltenbrunner and Lochstoer (2010)), we have also added decreasing returns to scale, fixed costs, CES production, and idiosyncratic productivity risk. Thus, what we refer to as our standard model, is still not quite standard. In this section we decompose the total rise in equity volatility into various components, Table 7 presents these results.

A useful starting point is Model Long-Run Risk II in Kaltenbrunner and Lochstoer (2010), which is the production economy version of Bansal and Yaron (2004). This model is similar to what we refer to as the standard model in our paper, the quantitatively important differences being constant returns to scale compared to decreasing returns to scale ($\rho = 0.77$) in our model, and fixed costs in our model. The model in Kaltenbrunner and Lochstoer (2010) has an unlevered equity return volatility of 0.66% per year. For comparison, we solve our model with constant returns to scale ($\rho = 1$), Cobb-Douglas production ($\alpha = 0.36$), no wage rigidity ($\mu = 0$), no fixed costs (f = 0), and no idiosyncratic productivity risk ($\sigma(Z^i) = 0$), similar to Kaltenbrunner and Lochstoer (2010), the unlevered equity return volatility is 0.47%,²⁶ this model is in the first column of Table 7.

Next, we set $\rho = 0.77$ to decrease the return to scale, the volatility of unlevered equity rises to 2.33% in column 2. Next, we also increase the fixed cost to match the average market to book ratio; this is the model we refer to as the standard model (for example Panel C of Table 3). In this model (column 3) the volatility of unlevered equity is 3.32% (financial leverage raises this number to 5.33%, as can be seen in Table 6).

Next, one at a time, we add CES production ($\eta = -1$, column 4) or wage rigidity ($\mu = 0.9$, column 5), these raise the unlevered equity volatility to 5.38% and 5.18% respectively. Recall that as we explained earlier, although CES production is not actually a friction, the channel through which it increases equity volatility is very similar to actual wage rigidity. The two channels together lead to an unlevered equity volatility of 7.18% (column 6). Note that this

²⁶We believe the difference is due to a different adjustment function in Kaltenbrunner and Lochstoer (2010).

model is identical to our baseline model, except for there is no idiosyncratic risk. Finally, in column 7, we present our baseline model, this has an unlevered equity volatility of 7.48%.

At this stage, a discussion of the idiosyncratic is in order. Note that without idiosyncratic productivity shocks, the First Welfare Theorem holds and wage rigidity has no effect on macroeconomic quantities. This is because with a representative firm, market clearing will imply that the firm always hires N_t workers, and its marginal product of capital is unaffected; labor supply being exogenous is crucial here. Therefore the Euler equation for investment is unaffected. Wage rigidity affects equity returns only through a shifting of risk from wages to dividends.

When firms experience idiosyncratic productivity shocks, the First Welfare Theorem no longer holds - this can be clearly seen in Table 2, which shows that models with higher μ require higher adjustment costs to match the volatility of investment in the data. This is because despite the aggregate labor market clearing, the past wage enters each firm's value function and results in some firms choosing a different number of workers than they would in a frictionless model. Thus, labor is inefficiently allocated across firms with different productivity. Naturally, this affects each firms' marginal product of capital, its Euler equation for investment, and, through aggregation, aggregate consumption and investment. The additional effect of idiosyncratic shocks for equity return is relatively small (7.48% compared to 7.18%), however bigger idiosyncratic shocks result in a larger misallocation a larger increase in equity volatility.

Although the paper focuses on labor market frictions, it may appear that much of the effect is coming from decreasing returns to scale. This is not the quite true. Indeed, in a model with no wage rigidity, decreasing returns to scale (lower ρ) increase the volatility of equity returns. However, as will be explained below, the effects of wage rigidity (higher μ) are actually stronger in a model with constant return to scale. In other words, when comparing models, $\frac{\partial \sigma(R^E)}{\partial \rho^{-1}} > 0$ and $\frac{\partial \sigma(R^E)}{\partial \mu} > 0$, however $\frac{\partial \sigma(R^E)}{\partial \rho^{-1} \partial \mu} < 0$. This can be seen in columns 8 and 9 of Table 7, where we solve a model that is closer to constant returns to scale²⁷ with and without wage rigidity.

²⁷For technical reasons our algorithm is unable to solve models where ρ is too close to 1. Because the First Welfare Theorem holds, any frictionless model (without idiosyncratic risk) can be solved using a simple planner's problem algorithm. This is how we solve the $\rho = 1$ case. For the frictionless case where $\rho < 1$, we can use either the planner's problem algorithm, or our algorithm; we have confirmed that the two lead to identical results.

Relative to the $\rho = 0.77$ case, the equity volatility in the frictionless model falls by 41% (compare 8 and 3), while in the rigidity case, only by 7% (compare 9 and 7).

The reason the effect of rigidity is stronger when returns of scale are closer to constant is that with constant returns to scale, even a small increase in expected profitability induces large increases in scale and leads to large increases in equity values. Compared to a frictionless model, rigid wages lead to larger increases in expected profitability after a positive productivity shock. While this is true for both constant and decreasing returns to scale, decreasing returns to scale imply that the firm is unable to fully take advantage of changes in profitability associated with wage rigidity because being far away from optimal capital stock is costly.

4.2.4 Conditional Asset Pricing Moments

It is well-known that financial moments exhibit conditional variation. The volatility of equity returns tends to be autocorrelated; it is also higher in recessions than expansions. For example, in our sample volatility was 20.19% following periods of low GDP growth (bottom 33%) and 18.33% following periods of high GDP growth (top 33%). Average excess equity returns are also higher during recessions (10.57%) than during expansions (8.36%). An extensive literature has documented that expected returns are predictable, with business cycle related variables such as the term spread, the default spread, the dividend yield, and the consumption wealth ratio all having predictive power.

In our model, the effect of sticky wages is much like that of financial or operating leverage: the equity return is the residual after other factors are paid. If these other factor payments are fixed or slow to adjust, the equity return is more risky. Labor leverage makes the equity return more risky on average, however labor leverage is not constant through the business cycle. Because wages adjust slowly, they are relatively high compared to output during bad times, making bad times especially risky. Panel B of Table 6 compares equity volatility and equity premia during bad times. Similar to the data, the model produces higher expected volatility and return during bad times. Panel C of Table 6 regresses long horizon equity return and volatility

on the book-to-market ratio. There is a positive relationship in both the data, and our model, but a negative relationship in the frictionless model. When wage obligations are high, firms' value is low but expected returns are high due to operating leverage caused by the high wages.

In a companion paper Favilukis and Lin (2013) explore additional empirical implications of wage stickiness. They show that consistent with the intuition of this paper, aggregate wage growth negatively forecasts aggregate stock returns. This is because after a negative (positive) productivity shock, output falls (rises) by more than the wage, leading to a rise (fall) in leverage due to labor. Traditional variables known to forecast long-horizon returns (dividend yield, term spread, default spread, and the consumption wealth ratio) do not subsume wage growth when considered simultaneously. Reversing the regression also showcases the importance of wage stickiness: stock returns (which are forward-looking) positively forecast both GDP growth and wage growth, however they negatively forecast wage growth minus GDP growth, suggesting that wages adjust sluggishly to positive economic news. Finally, they show that wage growth negatively forecasts stock returns at the U.S. state and at the industry level, and that more rigid states and industries (defined by the inverse of past wage growth volatility) have more forecastable returns. These empirical findings are all supportive of the mechanism in our model.

4.2.5 Cross-sectional Asset Pricing Moments

The model also has implications for the cross-section. We split firms into market-to-book quintiles in the model using V_t^i from Equation 7 as the market value and K_t^i as the book value similar to Fama and French (1992). Low book-to-market firms are referred to as growth and high book-tomarket firms are value. Compared to other firms, in both the model and data, growth (value) firms invest and hire more (less).

The value premium puzzle is that value firms have higher average returns than growth firms; the difference in returns is the value premium. Just as operating leverage due to wages varies through time and leads to conditional variation in expected return, operating leverage due to wages also varies cross-sectionally and leads to cross-sectional variation in expected return. We present several statistics for each book-to-market portfolio in Panels A (data) and B (baseline model) of Table $8.^{28}$

To summarize operating leverage due to wages we compute the profit to labor expenses ratio for each quintile, as shown in the second column of Table 8. Value firms have much lower profit to labor expenses than growth firms, that is, value firms are burdened with high labor expenses. The profit to labor expenses ratio is 0.45 for value and 0.70 for growth in the data, compared to 0.09 for value and 0.61 for growth in our baseline model. The reason why value stocks are riskier in this environment is that during bad times, wages are relatively high and most firms want to reduce labor. This is especially true for low productivity firms, who also tend to be value firms.²⁹ However, reducing labor is costly as it leaves the firm with an even higher average wage (see Equation 4), therefore low productivity firms suffer disproportionately during recessions.

In Panel A of Table 6 we report the value premium (column six). Because there are differences in leverage between value and growth, we also report the difference in unlevered return (return on capital) between the two portfolios; this allows us to separate the effect of financial leverage from labor market frictions. This table shows that the standard frictionless model produces a negative unlevered value premium; that is, value stocks have lower average returns than growth stocks. Leverage ratios are reported in Table 8. Because value stocks have higher leverage than growth stocks, the levered value premium is positive in the standard model, despite the unlevered value premium being negative. However, in the standard model the levered value premium is only 0.25% compared to 5.37% in the data.

When acting alone, higher complimentarity between labor and capital raises the unlevered value premium slightly, as does wage rigidity. When the two are combined in our baseline model, the unlevered value premium is 0.51%. This is because 1) higher complimentarity between labor and capital makes it hard for value firms to use the less costly input to smooth productivity

²⁸The spread in book-to-market in our model is smaller than in the data in part because our firm level productivity shock contains only three states (high/medium/low) and is therefore too simple. We have experimented with more realistic distributions and the spread in book-to-market becomes larger.

²⁹Applying similar logic to managerial compensation, Lustig et al. (2011) argue that compensation is set in good times, therefore firms who have experienced negative shocks after good times are extra risky because of relatively high CEO pay.

shocks; 2) wage rigidity further increases the cross-section of risk dispersion through relatively higher wage obligations for value firms.³⁰

Combining the effects of labor market frictions together with financial leverage results in a 2.58% value premium in our baseline model. The large difference between the unlevered and levered value premium may seem to imply that financial leverage is doing all of the work. However, it is actually the interaction between financial leverage and labor market frictions that matters. For example, despite all models having financial leverage, the levered value premium in the standard frictionless model is eight times smaller than in our baseline model. Furthermore, the difference in leverage between value and growth in the data is even bigger than in our model (Table 8), therefore the 0.51% unlevered value premium is not necessarily small. Although we are unable to compute the observed unlevered value premium for the full sample, Choi (2013) shows that in 1982-2007, the unlevered (or asset) value premium was less than one third of the realized equity value premium.

Notably Zhang (2005), Carlson et al. (2004), and Cooper (2006) also generate a sizable value premium. However, our model differs from these papers in three important ways. First, the above are partial equilibrium models in which equilibrium prices are exogenously specified, while ours is a full-fledged general equilibrium model.³¹ In general equilibrium, prices may dampen the firms' investment demand, making it harder to generate large cross-sectional risk dispersion. In fact, our frictionless, Cobb-Douglas model is the standard investment-based model with capital adjustment costs in general equilibrium, but the (unlevered) value premium in this model is negative. Second, we use general equilibrium implications to guide the strength of the adjustment cost. As argued earlier, increasing adjustment costs can revive the value premium even in the standard model (Figure 3), however, it would result in unrealistically smooth investment. Third, the channel through which our model delivers a value premium is quite different. Our model hinges on firms'

³⁰The model implied unlevered value premium can be bigger if we introduce an adjustment cost on changes in labor. However, this would further complicate the model mechanism and we choose to leave it out of the baseline model. The results of the model with costly labor hiring are available upon request.

³¹Recent papers by Ai et al. (2013b) and Ai et al. (2013a) also study the value premium in general equilibrium but the channel is through intangible capital and growth options, respectively.

wage expense being rigid, for which we provide direct evidence from Compustat firms; the above models employ a combination of high capital adjustment costs and fixed production costs, which are harder to identify.

Although these results suggest that wage rigidities are important for the value premium, they cannot tell the full story. Note that in our model, the conditional CAPM very nearly holds, and the value premium is due to β only. However, in the data (1929-2013) it is both due to α and β differences;³² this can be seen in columns 4 and 5 of Table 8. Many production models are subject to the same criticism. A potential way forward is to add a second aggregate shock, for example, in addition to a standard TFP shock, Belo et al. (2014b) introduce a shock to the cost of adjusting the labor force and find that the model produces higher α 's for value firms.

4.2.6 Term Structure of the Equity Premium

van Binsburgen et al. (2012) use option prices to compute the price of the dividend on the aggregate stock market at various horizons, which they term "dividend strips." Note that the value of the aggregate stock market is the sum of all dividend strips across all maturities. They compute the term structure of these strips and show that it is downward-sloping. The expected return and expected volatility on short term strips is higher than those of long-term strips.

van Binsburgen et al. (2012) point out that several leading models are unable to produce this downward-sloping term structure of equity. In particular, the standard LRR model, first proposed by Bansal and Yaron (2004), has an upward-sloping term structure because their calibration implies that most of the risk is associated with variations in expectations of long horizon consumption growth. Since consumption and dividends are cointegrated, long horizon dividends load on this risk. However, there is a second, short run shock which matters for short term dividends but is not highly correlated with the long run component. Thus, short horizon dividends are less risky than long horizon dividends.

In our model, the term structure of equity return and volatility is downward-sloping.³³ This

³²Many value premium studies start in 1963, in this case the value premium is due to α only.

 $^{^{33}}$ Ai et al. (2012) show that putting vintage capital into a production economy can also result in a downward-

can be seen on the left-hand side of Figure 5, which presents unlevered expected returns (top) and volatility (bottom). Our baseline model is the dashed line. Note that van Binsburgen et al. (2012)'s criticism applies to the standard, frictionless model (solid line), which produces an upward-sloping term structure of equity with an expected return of -6% (unlevered) at the shortest horizons. The frictionless model with a calibrated CES (dot-dashed line) does slightly better, the term structure is still upward-sloping but nearly flat. Both models with wage rigid-ity (dotted line for Cobb-Douglas and dashed line for CES) produce a downward-sloping term structure even for unlevered returns, with the short end being about 1.7% per year higher than the long end. Adding financial leverage (right side) can raise this difference to nearly 10%.

The reason our model is able to produce a downward-sloping term structure while a standard model cannot is twofold. First, our productivity process is slightly different from Bansal and Yaron (2004). Our productivity shock has the same key feature necessary for LRR - productivity growth is predictable - however our process is simpler in that there is only one source of risk. Our process is equivalent to a discretized version of the process in Bansal and Yaron (2004) where the short run and long run shocks are perfectly correlated. Thus short run dividends are more risky here compared to a more complicated two shock process.

However, having this simpler process is not enough. Note that all of our models have the same productivity process, yet the frictionless model still has an upward-sloping equity term structure. This is because the standard model has counter-cyclical dividends, as discussed above.³⁴ Thus, the standard model's failure at matching the properties of aggregate dividends and equity volatility is closely related to its failure to match a downward-sloping term structure of equity. In our baseline model the term structure of equity is downward-sloping because wage rigidity makes firms very risky in the short run - a negative productivity shock results in big drops to profits and dividends. However, wages and profits are cointegrated and therefore expected to return to their normal shares of output in the long run, thus short run dividends are riskier than

sloping term structure of equity.

³⁴It is interesting to note that short term dividend strips in the frictionless model can be quite volatile. However, this does not necessarily imply volatile equity return because short term dividend strips carry a negative equity premium, while long-term dividend strips carry a positive premium, thus the two hedge each other to some degree.

long run dividends.

It is useful to compare our result to Belo et al. (2014a) who show that a mean-reverting leverage policy generates a downward-sloping term structure. Although both channels work due to cointegration, as described in the previous paragraph, our channel is distinct from theirs. This can be seen by comparing unlevered (left) to levered (right) returns in Figure 5. Note that our effect works even without leverage, although leverage certainly makes it stronger. Furthermore, even with a reasonably calibrated financial leverage, we find that the standard frictionless model has a slightly upward-sloping equity term structure (solid line on the right).

5 Extensions

In this section we present three extensions of our baseline model. The first allows rigidity to differ across firms, the distribution of μ is symmetric. The second allows for an asymmetric distribution. In the third, rigidity is identical across firms, however the economy-wide μ varies through time. All three improve the quantitative performance of our model. Although we discuss some of the quantitative results from these models in this section, for brevity we relegate the full results to the appendix. For these extensions, Tables A2, A3, and A4 present the macroeconomic, accounting, and financial moments analogous to those reported in Tables 3, 5, and 6.

5.1 Symmetric cross-sectional heterogeneity in rigidity μ

In our baseline model all firms have identical μ and differ only by their history of idiosyncratic productivity shocks. In this model we relax the assumption of a constant μ , in particular the fraction of workers who do not renegotiate wages for firm *i* at time *t* is μ_t^i , which follows a twostate Markov process where the probability of switching is 0.02 per quarter.³⁵ In the high state $\mu_t^i = 0.95$ (20 quarters) and in the low state $\mu_t^i = 0.85$ (6.67 quarters). On average 10.3% of employees renegotiate their contract in any quarter, thus there is slightly less aggregate rigidity

³⁵This low switching probability is to make sure firms do not switch industries too often. However, our results are not sensitive to this probability.

than in our baseline model.³⁶ For simplicity, we shut down the idiosyncratic productivity shocks so that $\sigma(Z_t^i) = 0$. As will be discussed below, heterogeneity in rigidity seems consistent with the data and if anything, this calibration understates the heterogeneity in the data.

The macroeconomic and accounting moments in this model are very similar to the baseline model and to the data (Tables A2 and A3). This is because for these quantities it is the average number of employees keeping their past wage that matters. However, the equity return volatility improves from 13.15% in our baseline model to 14.45%. The intuition for this is evident in Figure 4 which shows that across models with different rigidity, aggregate return volatility is a convex function of μ . Since convexity implies that $E[\sigma(\mu)] > \sigma(E[\mu])$, a model with heterogenous μ has more equity volatility.³⁷ Much of the aggregate volatility is being driven by higher volatility in the rigid sector. We will relate this to the data below.

Interestingly, even though there are no idiosyncratic productivity shocks, the unlevered value premium is significantly greater than in the baseline model.³⁸ The reason for the value premium is also slightly different. In the baseline model firms which experienced negative productivity shocks had lower market-to-book ratios and higher expected return; however, all firms had the same rigidity. Here, firms with high rigidity are endogenously the ones with lower market-to-book ratios and higher expected returns.

Heterogeneity in μ allows this model to speak to another feature of the data. Recall that in the introduction, one of the motivations for our work was the empirical finding that the average labor share combined with the covariance of labor share with output can explain 38% of the variation in industry volatility and 54% of the variation in industry CAPM beta. This can be

³⁶Since the transition probability matrix is symmetric, the fraction of each type is always half. 50% of the firms have $\mu_t^i = 0.85$ implying that 15% of their employees renegotiate; the other 50% have $\mu_t^i = 0.95$ implying that 5% of their employees renegotiate. If both firms have the same number of employees, then 10% of all employees renegotiate per quarter. In simulated data, high (low) μ firms have roughly 5.6% less (more) employees than the average firm, thus the fraction of renegotiating employees is 0.5*1.056*0.15+0.5*0.944*0.05=10.28%. Note that the fraction of employees who keep their jobs, 1-0.1028 = 0.8972, is approximately equal to the average μ . On the other hand, the average job durations in the two types of firms are 6.67 and 20, however $1 - \frac{1}{0.5(6.67+20)} = 0.9625$ is much bigger than the fraction of employees who keep their job.

³⁷As explained in footnote 36, when considering heterogeneity the relevant target is the average μ .

³⁸The levered value premium is smaller. This is because much of the levered value premium is due to firms getting closer to default after negative productivity shocks. In this model idiosyncratic productivity shocks are shut down, thus the effect of financial leverage is less important.
seen in Figure 1 and Table 1. Our model's explanation for this is as follows.

In a standard (Cobb-Douglas) frictionless economy, the labor share is constant. The sticky wage channel works by creating operating leverage through a counter-cyclical labor share (compensation of employees as a fraction of output). Thus, the model suggests that a counter-cyclical labor share implies higher volatility. This result should be stronger if average labor share is high, because if labor is a small factor in production, the wage bill will have little effect on operating leverage. This intuition suggests that if there is cross-sectional variation in labor market frictions, then industry average returns, return volatilities, and CAPM betas should all be positively related to average labor share and negatively to the covariance of labor share with output. The above empirical results are exactly consistent with this intuition for volatility and beta, although the relationship is insignificant for average returns.

The intuition above was clear even in the model with constant μ ; however, the heterogenous μ model allows us to compare the model and data directly. We group all high μ firms together as a high rigidity industry and do the same for the low rigidity industry; we then compute return and labor share for each industry portfolio.³⁹ Consistent with the intuition above, we can confirm that the high rigidity industry has higher average returns, higher volatility, and higher CAPM beta. Furthermore, this industry has a more counter-cyclical labor share.⁴⁰

As mentioned above, we believe that our heterogeneity calibration is conservative. A proper calibration exercise would require data on the length of individual wage contracts in each industry, which we do not have. However, we can compare the distribution of $cov(GDP, LS^i)$ across industries. In the data, across 26 industries the mean and standard deviation of this covariance are -0.00074 and 0.00099. In our model they are -0.00052 and 0.00005.

³⁹Since individual firms randomly switch between high and low rigidity, the composition of each industry portfolio is not constant. However, this is no different than the data.

⁴⁰The high rigidity industry has a lower average labor share, which is inconsistent with the data. The reason for this is that high rigidity firms in the model endogenously choose fewer employees. However, we do not view this as a problem for our intuition. Note that the above model has only one type of heterogeneity - in rigidity μ . We have instead solved a model (not reported) with heterogeneity in α (capital share) in which the average labor share pattern is consistent with our intuition. A model with heterogeneity in both μ and α would likely describe the data well; however, solving this model is more costly numerically.

5.2 Asymmetric cross-sectional heterogeneity in rigidity μ

In the previous section the heterogeneity in μ was symmetric. In this section we solve an identical model with asymmetric heterogeneity. In particular, the transition probability for high μ firms is 0.02, but it is 0.06 for low μ firms. This implies that 75% of all firms have a high μ . We choose the high μ to be 0.95 (20 quarters), as in the previous section, and the low μ to be 0.75 (4 quarters) so that the average μ is still 0.9 as in all of our models. On average 10.5% of employees renegotiate their contract in any quarter, thus there is slightly less aggregate rigidity compared to the symmetric model where the number was 10.3%.

Why is this type of asymmetry reasonable? Estimates of separation rates for the US are around 3%/month (Hobijn and Sahin (2009), Shimer (2005)). If separations were equally likely for all workers, this would imply an average job length of around 2.8 years. However Hall (1982) estimates an average job duration of 8 years for American workers, Abraham and Farber (1987) estimate similar numbers just for non-unionized workers (presumably unionized workers have even longer durations). This is because a small number of workers frequently transition between jobs, while a majority of workers stay in their jobs for a long time. For example, Hall (1995) writes "Separation rates are sensitive to the accounting period because a small fraction of jobs but a large fraction of separations come from jobs lasting as little as a day." Hobijn and Sahin (2009) show that 15% of jobs have a tenure below 6 months; Davis et al. (2006) show that if one estimates separations based on workers who have held their job for a full quarter (as opposed to all separations), the quarterly separation rate falls from 24% to 10.7%. Thus we model a smaller segment of the economy with high job turnover, and a larger sector of the economy with low job turnover.

As in the symmetric case, the macroeconomic and accounting moments in this model are very similar to the baseline model and to the data (Tables A2 and A3). However, the equity volatility in this model is 15.61%, compared to 13.15% in our baseline model and 14.31% in the symmetric heterogeneity model. The intuition is again the convexity of volatility as a function of μ .

5.3 Changes over time in rigidity μ

A large body of work has both empirically and theoretically argued that downward wage rigidity is more important than upward wage rigidity.⁴¹ Furthermore, Daly et al. (2012) show that downward rigidity is more important during recessions. To incorporate this into our model we allow μ_t to vary through time. For simplicity, μ_t takes on the values 0.8775, 0.9, or 0.9225 and is perfectly negatively correlated with aggregate productivity growth, which follows a three-state Markov process. Unlike the two extensions with cross-sectional heterogeneity in μ , here we do not shut down the idiosyncratic productivity process; it follows the same three-state Markov chain as in the baseline case.

Again, the macroeconomic and accounting moments in this model are very similar to the baseline model, and to the data (Tables A2 and A3). The aggregate equity volatility is 14.31%, as with the cross-sectional variation in μ , variation over time also raises the volatility relative to the baseline model. Note that the two effects are independent; thus we believe that combining them would raise equity volatility further.

6 Conclusion

In this paper we show that introducing wage rigidity into an otherwise standard model can greatly improve the model's ability to quantitatively match financial data. The problem with standard models is that wages are far too volatile and pro-cyclical relative to the data. Wages therefore act as a hedge for the firm's owners, making profits too smooth and dividends counter-cyclical. As a result, the equity volatility in the data is about four times that of standard models.

In our model, the average wage is smoother due to infrequent wage resetting and to higher complimentarity between labor and capital. As a result, both profit and dividend behavior look very much like the data, and the volatility of equity returns is now 75% that of the data. The same channel brings the model closer to explaining several other unresolved puzzles in

⁴¹For example see Kahneman et al. (1986), Bewley (1999), Barwell and Schweitzer (2007), Dickens et al. (2007), Babecky et al. (2010), and Schmitt-Grohe and Uribe (2013).

financial data. In particular, in our model, as in the data, equity volatility and expected returns are counter-cyclical, there is a significant value premium, and the term structure of equity is downward-sloping.

One shortcoming of our modeling strategy is that we are unable to endogenize the labor supply decision. It is possible that introducing leisure would give households an additional channel for insurance and would bring down the Sharpe ratio, as in Lettau and Uhlig (2000). Another shortcoming of our model is that financial leverage is exogenously specified. Relaxing this assumption would allow researchers to look for links between labor markets and corporate policy, as well as bond prices.

While our model improves significantly on the standard model, the volatility of equity and the value premium are still short of the data. Furthermore, the value premium in our model is due to value firms having higher market risk; in the data the value premium is a combination of higher market risk and higher CAPM alpha.⁴² One possible way to rectify this is by adding a second shock. The labor market may be the right place to look for such a shock, for example the parameters of the production function which control labor share, or the workers' bargaining power, which determines contract length, may be time-varying.

Empirical work exploring the relationship between labor market variables and asset returns may shed light on such a shock. There are additional empirical implications as well. Figure 1 gives a flavor of this. Labor markets frictions are not constant. For example, wage rigidity may vary through time, across firms, or across countries; this has implication for expected returns and for firm behavior. We leave such exploration for future work.

 $^{^{42}}$ We also note that the failure of the unconditional CAPM on value portfolios in the data seems to be sample specific. In the long sample starting 1929, which is what we consider here, the value premium is driven by both a combination of CAPM beta and alpha. The CAPM is unable to explain the returns of book-to-market portfolios in the 1965 to 2013 period.

A Data

Stock returns are from the Ken French's web page:

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, all stock return data is from 1929 to 2012. Accounting information is from the CRSP/Compustat Merged Annual Industrial Files, the sample is from 1950 to 2012. We exclude from the sample any firm-year observation with missing data or for which total assets or the gross capital stock are either zero or negative. In addition, as standard, we omit firms whose primary SIC classification is between 4900 and 4999 (regulated firms) or between 6000 and 6999 (financial firms). Firm profit is net sales (SALE) minus the sum cost of goods sold (COGS) and selling, general and administrative expense (XSGA). We aggregate all firms' profit scaled by investment price deflator to compute total real profit. Corporate payout is from the Flow of Funds; this data is from 1946 to 2012. We define net equity payouts in a similar manner to Jermann and Quadrini (2012); in particular they are net dividends paid (line 3 from Table F.102) minus the change in the book value of corporate equities (line 35 from Table F.101). The net payout is the net equity payout plus net interest (line 29 from Table F.7).

All remaining data is from NIPA, available 1929-2013. GDP, consumption (non-durable consumption plus services), and investment (fixed private investment plus durable goods) are from Table 1.1.5. Employment is full-time and part-time employees from Table 6.4. Although we are calibrating our aggregate economy (output, investment, consumption, employment) to match the aggregate U.S. economy, for the labor market variables we target the private sector only. We believe this is appropriate because we are interested in the effect of labor market frictions on asset prices, which are relevant only for the private sector. Therefore, the frictions in our model should match the labor market frictions in the U.S. private sector. The total wage bill is the sum of compensation of employees in private industries and supplements to wages, both in Table 2.1. The wage is the total wage bill divided by private sector GDP, which we define as GDP minus government expenditures from Table 1.1.5. Because of large temporary movements of people

out of the private sector during World War II, we report the labor market variables for post-war only; consequently when we present scaled statistics, the scaling variable is computed for the analogous period rather than the full sample.

Finally, exclusively for Table 1 and Figure 1, we compute industry level variables. Industry labor share is constructed as industry employee compensation divided by the sum of industry compensation, profit, and capital consumption (investment). This data comes from NIPA Section 6. After matching industries, we are left with 26 industries for the period 1929-2000; a detailed description of this data and the matching procedure is given in Favilukis and Lin (2013).

All nominal variables are scaled by the Consumer Price Index from the BLS.

B Financial Leverage

The firm's return on capital is $R_{t+1}^K = \frac{V_{t+1}}{V_t - D_t}$. However, real world firms are financed both by debt and by equity, with equity being the riskier, residual claim. To compare the model's equity return and dividend to their empirical counterparts, we must make an assumption about financial leverage. Note that the Modigliani and Miller (1958) propositions hold in our model, therefore all assumptions about financial leverage are completely orthogonal to our model's solution; these assumptions only affect the way we report returns and dividends.

It is standard in the literature to assume that all firms keep leverage constant,⁴³ however in the data leverage is quite sticky. We have solved the model for both constant and sticky leverage. All return moments we report look very similar for both, however dividends are too volatile if leverage is constant. Because sticky leverage is both realistic, and improves the model's performance, we assume that the firm's choice of debt is sticky and follows

$$B_{t} = \rho^{B} B_{t-1} + (1 - \rho^{B}) B_{t}^{*}$$

$$B_{t}^{*} = \lambda (V_{t} - D_{t})$$
(12)

⁴³One example is Boldrin et al. (1999) who provide additional discussion.

where B_t is the market value of corporate debt (B_t is borrowed at t, $B_t R_{t+1}^B$ is owed at t+1) and B_t^* is the target debt level. We assume that the target debt level is a constant fraction of the firm's ex-dividend value $V_t - D_t$.

Combining this assumption about the evolution of debt with the Modigliani and Miller (1958) second proposition implies that the equity dividend and the equity return are:

$$D_t^E = D_t + B_t - B_{t-1} R_t^B (13)$$

$$R_t^E = \frac{V_t - B_{t-1} R_t^B}{V_{t-1} - B_{t-1} - D_{t-1}}$$
(14)

where D_t is the total payout to all of the firm's financial stake holders (debt and equity) and R_t^B is the (risky) return on debt. Let R_{t-1}^P be the return promised at t-1 to be repaid at t. Its derivation is discussed below. Default occurs at t if $V_t < B_{t-1}R_{t-1}^P$, since due to limited liability, equity holders can walk away instead of receiving a net return below -100%. When the firm does not default, $R_t^B = R_{t-1}^P$; when the firm does default, the equity return is -100% and creditors take over the firm, so that $R_t^B = \frac{V_t}{B_{t-1}} < R_{t-1}^P$. At this stage the firm's debt is reset to zero and then follows Equation 12. The promised payment R_t^P is set such that

$$1 = E_t[R_{t+1}^B M_{t+1}] = R_t^p E_t[M_{t+1} | d \neq 1] * (1 - p(d = 1)) + E_t[M_{t+1} \frac{V_{t+1}}{B_t} | d = 1] * p(d = 1)$$
(15)

where d = 1 indicates default and p(d = 1) is the probability of default.

If we were to assume that leverage is constant, we would simply set $\rho^B = 0$. In this case the equity return takes the more familiar form: $R_t^E = R_t^B + \frac{1}{1-\lambda}(R_t^K - R_t^B)$.

We estimate the target debt to equity ratio to be 0.59, which implies that $\lambda = 0.37$. We find debt to be quite sticky, with annual estimates of ρ^B between 0.46 and 0.99, depending on the specification. Details of these estimates are below. We set quarterly $\rho^B = 0.9$ in all of our models, this allows us to roughly match the dividend volatility in the data. As noted earlier, we have also experimented with $\rho^B = 0$. With this calibration all of our key results are very similar

to $\rho^B = 0.9$, however the process for equity dividends is too volatile.

B.1 Estimation of market-to-book and debt-to-equity

Our model is solved without financial leverage; financial leverage (defined by ρ^B and λ and described below) is then added once the model is solved, as described in the text. Therefore, the relevant market-to-book ratio to match during the model solution stage is the market-to-book ratio for the entire firm value (the enterprise value). From Compustat we calculate the market-to-book ratio for equity to be 1.64 and the book debt to market equity ratio to be 0.59. Sweeney et al. (1997) find that outside of the Volcker period, aggregate market-to-book values for debt are very close to one. These numbers imply a market-to-book of 1.33 for enterprise value. This also implies that the debt-to-value ratio is $\lambda = \frac{0.59}{1+0.59} = 0.37$; this is the number we use in our calibration. An alternative is to estimate $\lambda = Avg(\frac{B}{B+E})$ where B is the value of debt from the flow of funds and E is the total market value of equity from CRSP (both variables are described in more detail in the next section). This method implies $\lambda = 0.41$. This slightly higher leverage would make the results even stronger.

B.2 Estimation of ρ^B

To estimate the dependence of debt issuance on past issuance, we use levels of debt from the flow of funds. In particular, we aim to estimate $B_{t+1} = \rho^B B_t + (1 - \rho^B) * \lambda V_t$ where B is the market value of corporate debt and V is the enterprise value of the corporate sector (equity plus debt). We define B to be non-financial credit market instrument liabilities (line 28 from Table L.101 in the Flow of Funds).⁴⁴ We define V to be B plus total market value of NYSE/AMEX/NASDAQ from CRSP.⁴⁵ Because the values of debt and equity are non-stationary, we cannot directly

⁴⁴As an alternative, we defined debt to be credit market liabilities minus assets (line 7 from Table L.101) and the estimated ρ^B is very similar because the credit assets of non-financial corporations are very small relative to liabilities.

⁴⁵Since the set of firms in CRSP is not exactly the same firms as the ones for whom debt is defined in the flow of funds, this definition of V is slightly problematic. However, note that this should mostly affect our estimate of λ but not necessarily ρ^B . For this reason we use other sources to estimate λ , as described in the previous section. Nevertheless, as discussed in the previous section, the two methods imply similar values for λ . Furthermore, we

estimate the equation above.

We use several different specifications to estimate ρ^B . In particular, specification 1 simply estimates ρ^B to be the autocorrelation of HP-filtered *B*. In specification 2 we regress $B_{t+1} = a_0 + a_1B_t + a_2V_t$ where both *B* and *V* are HP-filtered; we either define $\rho^B = a_1$ (specification 2a, λ unrestricted) or $\rho^B = 1 - a_2/\lambda$ (specification 2b, $\lambda = 0.37$). However, the HP-filter may cause the estimation to miss out on important low-frequency dependence of debt on past debt. In specifications 3 and 4 we do not HP-filter. In specification 3, we regress $B_{t+1}/B_t = a_0 + a_1V_t/B_t$ and either define $\rho^B = a_0$ (specification 3a, λ unrestricted) or $\rho^B = 1 - a_1/\lambda$ (specification 3b, $\lambda = 0.37$. Finally, in specification 4 we regress $B_{t+1}/B_t - B_t/B_{t-1} = a_0 + a_1(V_t/B_t - V_{t-1}/B_{t-1})$ and define $\rho^B = 1 - a_1/\lambda$ where $\lambda = 0.37$. The different estimates are presented in Table A1.

C Solving the model

Making the Model Stationary

Note that the model is not stationary. In order to solve it numerically, we must first detrend it and rewrite it in terms of stationary quantities. We will describe the detrending procedure in this section, and the actual numerical solution of the stationary model in the next section.

Along the balanced growth path of this economy, output, consumption, investment and capital will all grow with Z_t^{ρ} . To see this, consider the planner's problem analogous to our model but with no frictions:

$$U_{t} = \max\left((1-\beta)C_{t}^{1-\frac{1}{\psi}} + \beta E_{t}[U_{t+1}^{1-\theta}]^{\frac{1-\frac{1}{\psi}}{1-\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}}$$

$$C_{t} = (\alpha K_{t}^{\eta} + (1-\alpha)(Z_{t}N_{t})^{\eta\rho})^{\frac{1}{\eta}} - I_{t}$$

$$K_{t+1} = (1-\delta)K_{t} + I_{t}$$
(16)

where Z_t and N_t follow the stochastic processes described in the main text. Because Z_t is nonstationary, so are many of the other relevant quantities in this problem. Define $k_t = \frac{K_t}{Z_t^p}$, $i_t = \frac{I_t}{Z_t^p}$,

have redone everything with logs, or by defining V to be just market value, without adding the debt. Resultant estimates of ρ^B are very similar.

 $c_t = \frac{C_t}{Z_t^{\rho}}$, and $u_t = \frac{U_t}{Z_t^{\rho}}$. The above equations can be rewritten as:

$$u_{t} = \max\left((1-\beta)c_{t}^{1-\frac{1}{\psi}} + \beta E_{t} \left[\left(\frac{Z_{t+1}}{Z_{t}} \right)^{\rho(1-\theta)} u_{t+1}^{1-\frac{1}{\psi}} \right]^{\frac{1-\frac{1}{\psi}}{1-\theta}} \right)^{\frac{1}{1-\frac{1}{\psi}}} (17)$$

$$c_{t} = (\alpha k_{t}^{\eta} + (1-\alpha)N_{t}^{\eta\rho})^{\frac{1}{\eta}} - i_{t}$$

$$k_{t+1} = \left(\frac{Z_{t+1}}{Z_{t}} \right)^{-\rho} ((1-\delta)k_{t} + i_{t})$$

Note that now there are no non-stationary inputs.

We will use the same trend variable Z_t^{ρ} to rewrite our decentralized problem. The firm's problem is:

$$V(Z_{t}^{i}, K_{t}^{i}, N_{t-1}^{i}, \overline{W}_{t-1}^{i}; \Omega_{t}) = \max_{I_{t}^{i}, N_{t}^{i}} D_{t} + E_{t}[M_{t+1}V(Z_{t+1}^{i}, K_{t+1}^{i}, N_{t}^{i}, \overline{W}_{t}^{i}; \Omega_{t+1})]$$

$$D_{t} = Z_{t}^{i} \left(\alpha(K_{t}^{i})^{\eta} + (1 - \alpha)(Z_{t}N_{t}^{i})^{\rho\eta}\right)^{\frac{1}{\eta}} - \left(\overline{W}_{t-1}^{i}N_{t-1}^{i}\mu + W_{t}(N_{t}^{i} - N_{t-1}^{i}\mu)\right) - K_{t}f - \left(\overline{W}_{t-1}^{i}N_{t-1}^{i}\mu + W_{t}(N_{t}^{i} - N_{t-1}^{i}\mu)\right) - K_{t}f$$

$$-\nu \left(\frac{I_{t}^{i}}{K_{t}^{i}} - \delta\right)^{2} K_{t}^{i}$$

$$K_{t+1}^{i} = (1 - \delta)K_{t}^{i} + I_{t}^{i} - \frac{\overline{W}_{t-1}^{i}N_{t-1}^{i}\mu + W_{t}(N_{t}^{i} - N_{t-1}^{i}\mu)}{N_{t}^{i}}$$
(18)

Where D_t is the net payout of the firm, Z_t^i is the idiosyncratic productivity, K_t^i is the firm's individual capital, N_{t-1}^i is the firm's employment last period, \overline{W}_{t-1}^i is the firm's average wage last period. Ω_t is a multi-dimensional vector of all relevant aggregate state variables. In theory, Ω_t may be infinite-dimensional as it must carry information about the full distribution of capital across firms. In our approximate numerical solution, $\Omega_t = \{Z_t, j_t, j_{t-1}, K_t, \overline{W}_{t-1}\}$ where Z_t is aggregate productivity, j_t is the discrete state of the Markov process governing productivity growth, K_t is the average capital in the economy, and \overline{W}_{t-1} is the average wage at t-1. We will discuss the rationale for including these variables in the aggregate state below. Aggregating across firms, the aggregate quantities in Ω_t evolve as:

$$K_{t+1} = (1 - \delta)K_t + \sum_{t} I_t^i$$

$$\overline{W}_t = \frac{\overline{W}_{t-1}N_{t-1}\mu + W_t(N_t - N_{t-1}\mu)}{N_t}$$
(19)

The household's problem implies that

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{E_t[U_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{\frac{1}{\psi}-\theta}$$

$$C_t = \sum \left(Z_t^i \left(\alpha(K_t^i)^{\eta} + (1-\alpha)(Z_t N_t^i)^{\rho\eta}\right)^{\frac{1}{\eta}} - I_t^i\right)$$

$$U_t = \left((1-\beta)C_t^{1-\frac{1}{\psi}} + \beta E_t[U_{t+1}^{1-\theta}]^{\frac{1-\frac{1}{\psi}}{1-\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}}$$
(20)

Define $k_t^i = \frac{K_t^i}{Z_t^{\rho}}$, $k_t = \frac{K_t}{Z_t^{\rho}}$, $i_t^i = \frac{I_t^i}{Z_t^{\rho}}$, $w_t = \frac{W_t}{Z_t^{\rho}}$, $d_t = \frac{D_t}{Z_t^{\rho}}$, $\overline{w}_t^i = \frac{\overline{W}_t^i}{Z_{t+1}^{\rho}}$, and $\overline{w}_t = \frac{\overline{W}_t}{Z_{t+1}^{\rho}}$ (note that the timing of \overline{w}_t^i and \overline{w}_t differs from the others). Fundamentally, the reason Z_t^{ρ} is the right variable for detrending is that d_t (explicitly written out below, in equation 21) is not a function of Z_t . Also define the vector of detrended aggregate state variables $\Theta_t = \{j_t, j_{t-1}, k_t, \overline{w}_{t-1}\}$.

It is straightforward to check that the firm's value function is linear in Z_t^{ρ} , that is:

$$V(Z_t^i, K_t^i, N_{t-1}^i, \overline{W}_{t-1}^i; \Omega_t) = Z_t^{\rho} v(Z_t^i, k_t^i, N_{t-1}^i, \overline{w}_{t-1}^i; \Theta_t)$$

where v does not depend on the non-stationary Z_t .⁴⁶ In this case, all of the above equations can be rewritten without any non-stationary inputs.

We summarize the stationary economy of our model as follows:

⁴⁶This can be accomplished by induction. If it is true at t + 1 then $E_t[M_{t+1}V(Z_{t+1}^i, K_{t+1}^i, N_t^i, \overline{W}_t^i; \Omega_{t+1})] = Z_t^{\rho} E_t[\left(\frac{Z_{t+1}}{Z_t}\right)^{\rho} M_{t+1}v(Z_t^i, k_t^i, N_{t-1}^i, \overline{w}_{t-1}^i; \Theta_t)]$. Additionally, d_t is not a function of Z_t , thus the right-hand side of equation 18 can be written as $Z_t^{\rho} \left(d_t + E_t[\left(\frac{Z_{t+1}}{Z_t}\right)^{\rho} M_{t+1}v(Z_t^i, k_t^i, N_{t-1}^i, \overline{w}_{t-1}^i; \Theta_t)] \right)$ where the term in parentheses does not depend on Z_t . Since Z_t^{ρ} does not affect the maximization problem, it can be pulled outside of the max operator. Because the right-hand side is linear in Z_t^{ρ} , the left-hand side must be as well.

The stationary firm's problem:

$$v(Z_{t}^{i}, k_{t}^{i}, N_{t-1}^{i}, \overline{w}_{t-1}^{i}; \Theta_{t}) = \max_{i_{t}^{i}, N_{t}^{i}} d_{t} + E_{t} \left[\left(\frac{Z_{t+1}}{Z_{t}} \right)^{\rho} M_{t+1} v(Z_{t+1}^{i}, k_{t+1}^{i}, N_{t}^{i}, \overline{w}_{t}^{i}; \Theta_{t+1}) \right] \\ d_{t} = Z_{t}^{i} \left(\alpha(k_{t}^{i})^{\eta} + (1 - \alpha)(N_{t}^{i})^{\rho\eta} \right)^{\frac{1}{\eta}} \\ - \left(\overline{w}_{t-1}^{i} N_{t-1}^{i} \mu + w_{t} (N_{t}^{i} - N_{t-1}^{i} \mu) \right) - k_{t} f \\ - \nu \left(\frac{i_{t}^{i}}{k_{t}^{i}} - \delta \right)^{2} k_{t}^{i} \\ k_{t+1}^{i} = \left((1 - \delta) k_{t}^{i} + i_{t}^{i} \right) \left(\frac{Z_{t+1}}{Z_{t}} \right)^{-\rho} \\ \overline{w}_{t}^{i} = \left(\frac{\overline{w}_{t-1}^{i} N_{t-1}^{i} \mu + (N_{t}^{i} - N_{t-1}^{i} \mu) w_{t}}{N_{t}^{i}} \right) \left(\frac{Z_{t+1}}{Z_{t}} \right)^{-\rho}$$

$$(21)$$

The aggregate state variables:

$$k_{t+1} = \left((1-\delta)k_t + \sum i_t^i \right) \left(\frac{Z_{t+1}}{Z_t} \right)^{-\rho}$$

$$\overline{w}_t = \left(\frac{\overline{w}_{t-1}N_{t-1}\mu + (N_t - N_{t-1}\mu)w_t}{N_t} \right) \left(\frac{Z_{t+1}}{Z_t} \right)^{-\rho}$$
(22)

Additionally, the spot wage w_t must be such that $\sum N_t^i = N_t$.

The household's stationary equilibrium conditions:

$$M_{t+1} = \beta \left(\frac{Z_{t+1}}{Z_t}\right)^{-\rho\theta} \left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\psi}} \left(\frac{u_{t+1}}{E_t [\left(\frac{Z_{t+1}}{Z_t}\right)^{\rho(1-\theta)} u_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{\frac{1}{\psi}-\theta}$$

$$c_t = \sum \left(Z_t^i \left(\alpha(k_t^i)^{\eta} + (1-\alpha)(N_t^i)^{\rho\eta}\right)^{\frac{1}{\eta}} - \sum i_t^i\right)$$

$$u_t = \left((1-\beta)c_t^{1-\frac{1}{\psi}} + \beta E_t \left[\left(\frac{Z_{t+1}}{Z_t}\right)^{\rho(1-\theta)} u_{t+1}^{1-\theta}\right]^{\frac{1-\frac{1}{\psi}}{1-\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}}$$
(23)

Numerical Solution

We will now explain how we numerically solve the economy described by equations 21, 22, and 23. We will also justify our choice of the state space. The algorithm is a variation of the algorithm in Krusell and Smith (1998). Generally there is no proof that an equilibrium exists, this solution method is referred to as an approximate bounded rational equilibrium. The solution method consists of performing two steps and then repeating them until convergence. The first step solves the firm's problem given a particular set of beliefs; the inputs are beliefs and the outputs are policy functions. The second step updates these beliefs from simulating the economy; the inputs are policy functions and the outputs are beliefs. These steps are repeated until the beliefs have converged and are consistent with simulated data. We will refer to the sequence of step one, followed by step two as an iteration.

The problem is solved using Fortran 77, and parallelized using OpenMP. It then runs on eight parallel processors. The full model takes about four hours per iteration and requires 50 -100 iterations to converge. We are appreciative to The Ohio State University high-performance computing center for the computational resources.

Step 1 We begin this step with beliefs about aggregate investment, the spot wage, aggregate consumption, and the stochastic discount factor as functions of the aggregate state. These beliefs are $\sum i_t^i = \iota^n(\Theta_t)$, $w_t = \omega^n(\Theta_t)$, $c_t = \zeta^n(\Theta_t)$, and $M_{t+1} = \mathbb{M}^n(\Theta_t, j_{t+1})$ where *n* is the number of the iteration.⁴⁷ These beliefs, together with equations 21 and 22 specify a well-defined (and relatively standard) partial equilibrium firm problem. We solve this problem using value function iteration.

For numerical reasons, we find it better to rescale some of the state variables. In particular instead of $(\overline{w}_{t-1}^i, N_{t-1}^i)$ we use $(N_{t-1}^i \overline{w}_{t-1}^i, N_{t-1}^i)$. This rescaling is innocuous since one can easily go back and forth. The reason for the scaling is that it can be shown that when there are no adjustment costs, the firm's problem is linear in $N_{t-1}^i \overline{w}_{t-1}^i$ and N_{t-1}^i , thus we believe the rescaling would lead to a more efficient algorithm. Furthermore, it allows us to check the accuracy of our solution in the special case of $\nu = 0$. Instead of \overline{w}_{t-1} we use \overline{w}_{t-1} scaled by the aggregate marginal product of labor. The reason for this is that \overline{w}_{t-1} is highly correlated with k_t , which makes formation of beliefs more difficult. Furthermore, when two state variables are correlated, large areas of the grid are left unused, which wastes computing power. The scaling reduces this correlation.

Our grid sizes are 40 for k_t^i , 19 for \overline{w}_{t-1}^i , 12 for N_{t-1}^i , 20 for k_t , and 6 for \overline{w}_{t-1} . Recall that we

⁴⁷Note that the stochastic discount factor depends on the aggregate state today, and the realized aggregate shock next period.

also have grids of size 3 for the aggregate Markov chain at t, the aggregate Markov chain at t-1, and the individual Markov chain at t. We have experimented with grid sizes extensively and set them large enough that our results are not affected by any further increases. It is important to set the grid edges some distance away from where typical variables reside, despite these values being "off-equilibrium." At the same setting the edges too far away from model equilibrium will require a very large number of grid points, which is numerically infeasible; therefore, some experimentation is in order. We find that the results are more sensitive to the sizes of firm level grids than to the aggregate grids.

Once the value function iteration is complete, we have two policy functions for the firm: investment $i^i(Z_t^i, k_t^i, N_{t-1}^i, \overline{w}_{t-1}^i; \Theta_t)$ and employment $N^i(Z_t^i, k_t^i, N_{t-1}^i, \overline{w}_{t-1}^i; \Theta_t)$.

Step 2 In this step we use the policy functions to simulate the economy and then use simulated data to update the beliefs. We simulate the economy for 10,000 firms, a higher number does not affect any of our results. Before beginning the simulation we must specify an initial distribution of idiosyncratic productivity, capital, past wages, and past labor. We simulate the economy for 3,500 periods and throw away the first 500 periods so as to let the simulation settle into its normal behavior; this also assures us that the initial distribution has no effect on our results.

One complication during the simulation is that we must clear the labor market each period. The difficulty is that each firm's choice of labor is a function of the state variables only. The state variables are fixed and known at the beginning of each period. Thus labor is determined at the start of the period. The firms have beliefs about the spot wage as a function of the state, however prior to convergence these beliefs may be incorrect and therefore labor demand may not equal labor supply. The actual market clearing spot wage (as opposed to the belief) is undefined because at this stage in the simulation, nothing can change the state variables or firms' labor demand. To deal with this problem we use the following workaround: During the simulation, we assume that each firm's labor demand is $N_t^i = N^i(Z_t^i, k_t^i, N_{t-1}^i, \overline{w}_{t-1}^i; \Theta_t) \frac{\omega^n(\Theta_t)}{w_t}$ where $\omega^n(\Theta_t)$ is the belief about the spot wage used during the value function iteration step. Note that once our algorithm has converged, the belief is consistent with the spot wage. However, before convergence we are able to pick the spot wage in any period so as to clear markets, that is $w_t = \omega^n(\Theta_t) \frac{\sum N^i(Z_t^i, k_t^i, N_{t-1}^i; \Theta_t)}{N_t}$.

Once the simulation is complete we have a time series for all relevant aggregate variables. We use these time series to update the beliefs. Krusell and Smith (1998) suggest regressing the relevant variables on the state variables, however we find this problematic. This is because linear regressions imply strange behavior "off-equilibrium," which then leads to problems in the value function iteration step. Adding higher-ordered terms does not help as it leads to overfitting.

We propose an alternative, non-parametric approach. We will define a belief separately for each point in the state space. There are two types of grid points in the state space: those that are near where the simulated data resides ("on-equilibrium"), and those that are not ("offequilibrium").

We define a point as "on-equilibrium" if there are more than 20 simulated periods in which the root mean square distance between the state variables in that period, and the grid point is smaller than a fixed bound. We then run a local regression of our variables of interest (consumption, investment, spot wage) on the state variables near the grid point. The predicted value of our variable of interest computed at the grid point is then our updated belief at this grid point.

For the remaining "off-equilibrium" grid points we use root mean square distance to find the closest simulated period. We then shift the distribution of capital and past labor from that period. For example, suppose the grid point has capital and past wage (k_a, \overline{w}_a) . Suppose the nearest simulated period has distributions of capital and past wages with average values (k_b, \overline{w}_b) . Then each firm's capital and average wage are shifted by $k_b - k_a$ and $\overline{w}_b - \overline{w}_a$. We then take the shifted distribution as an initial distribution⁴⁸ and simulate for one period. The result of this simulation is then assigned as the updated belief to this "off-equilibrium" grid point.

There is one additional caveat. It is important to put a weight on old beliefs during updating; without it the procedure may not converge. We have found that the lower the capital adjustment cost, the higher the required weight. For zero adjustment cost, the weight may sometimes need

⁴⁸Some care must be taken to prevent firms from having negative capital and average wage.

to be as high as 0.998. For our baseline model the weight we use is 0.85 and likely an even lower weight would have sufficed.⁴⁹

The above procedure describes how to form updated beliefs for investment, spot wages, and consumption as a function of the state. These updated beliefs are $\sum i_t^i = \iota^{n+1}(\Theta_t)$, $w_t = \omega^{n+1}(\Theta_t)$, and $c_t = \zeta^{n+1}(\Theta_t)$. It still remains to update the belief for the stochastic discount factor. Note that with CRRA utility this would be straightforward: $\mathbb{M}^{n+1}(\Theta_t, j_{t+1}) = \beta \left(\frac{Z_{t+1}}{Z_t}\right)^{-\rho\theta} \left(\frac{\zeta^{n+1}(\Theta_{t+1})}{\zeta^{n+1}(\Theta_t)}\right)^{-\frac{1}{\psi}}$. For the more general case we instead set

$$\mathbb{M}^{n+1}(\Theta_t, j_{t+1}) = \beta \left(\frac{Z_{t+1}}{Z_t}\right)^{-\rho\theta} \left(\frac{\zeta^{n+1}(\Theta_{t+1})}{\zeta^{n+1}(\Theta_t)}\right)^{-\frac{1}{\psi}} \left(\frac{u(\Theta_{t+1})}{E_t[\left(\frac{Z_{t+1}}{Z_t}\right)^{\rho(1-\theta)}u(\Theta_{t+1})^{1-\theta}]^{\frac{1}{1-\theta}}}\right)^{\frac{1}{\psi}-\theta}$$

where $u(\Theta_t)$ comes from separately solving the recursion:

$$u(\Theta_t) = \left((1-\beta)\zeta^{n+1}(\Theta_t)^{1-\frac{1}{\psi}} + \beta E_t \left[\left(\frac{Z_{t+1}}{Z_t} \right)^{\rho(1-\theta)} u(\Theta_{t+1})^{1-\theta} \right]^{\frac{1-\frac{1}{\psi}}{1-\theta}} \right)^{\frac{1}{1-\frac{1}{\psi}}}$$

This recursion is also solved with value function iteration using the same aggregate grids as the firm's problem. However, it typically takes less than a second because the state space is much smaller (there are no individual firm variables), and because there are no choice variables.

Once steps one and two are complete we check whether the algorithm has converged; if it has not we restart step one with updated beliefs. Convergence means that the absolute distance between $\iota^{n+1}(\Theta_t)$ and $\iota^n(\Theta_t)$ is sufficiently small (same for $\zeta^{n+1}(\Theta_t)$ and $\omega^{n+1}(\Theta_t)$).

In addition to confirming that the beliefs have converged, it is standard to perform other checks. This solution method is referred to as an approximate bounded rational equilibrium. It is rational because the beliefs of the firms and agents are exactly equal to the best forecast an econometrician could achieve with in simulated data using the state defined variables. However,

 $^{^{49}}$ This is because even if rational equilibria exist, they are only weakly stable in the sense described by Marcet and Sargent (1989).

it is bounded because the forecast may still not be very good (low R^2), or because there may be additional variables that may either improve the forecast, or whose inclusion may change the equilibrium allocations or prices.

The lowest R^2 in our forecasting equations is 0.9996; this compares well to other models solved with the Krusell and Smith (1998) approach and leads us to think that the forecast is fairly accurate.⁵⁰ We have also, one at a time, experimented with additional state variables: the second moment of the capital distribution, the second moment of the past wage distribution, and the second moment of the past labor demand distribution. Unfortunately, due to numerical constraints, we could not add these as continuous state variables but only as a discrete high or low signal. Adding these moments did not affect our results.

Finally, it may be helpful to discuss why we included the variables we did in the state space. Generally, the whole distribution of capital, labor, and wages across firms may be relevant. Krusell and Smith (1998) suggest summarizing it by the first moment, and possibly higher moments. Our frictionless economy is similar to the economy solved by Krusell and Smith (1998), the difference being heterogenous households versus heterogenous firms. They found that the first moment of the capital distribution is enough to summarize the state space because it is this that is most relevant for the output and consumption that the economy will produce, as well as the return on capital. Indeed, in our model we find that average capital contributes most to R^2 .

In our model, firms also must form a belief about the spot wage they will face. When $\mu = 0$, there is a tight (though not exact) relationship between average capital and the spot wage and so average capital is a sufficient state variable. However, when $\mu > 0$ we find that average capital alone does not produce a sufficiently high R^2 . It is then natural to add the average of last period's wage to the state space and indeed this results in a high R^2 .

The reason why Θ_t contains the current state of the exogenous Markov process j_t is obvious, however there is a subtle reason why j_{t-1} must be included as well. Because \overline{w}_{t-1} is part of

⁵⁰Since we apply a non-parametric approach, we define the $R^2 = 1 - \frac{\sum (x_t - E[x_t|\Theta_t])^2}{\sum (x_t - E[x_t])^2}$ where $E[x_t]$ is the unconditional mean and $E[x_t|\Theta_t]$ is our forecast.

the aggregate state, firms must be able to forecast \overline{w}_t . Equation 22 shows that to forecast \overline{w}_t we need both today's aggregate labor supply N_t and last period's N_{t-1} . Since, by assumption, labor supply is a function of the Markov process, j_{t-1} is sufficient for knowing N_{t-1} . This is also the reason why adding labor in the utility function is a much more difficult problem to solve numerically. With labor in the utility function, last period's labor is endogenous and we would need to keep track of it as an additional state variable as opposed to just keeping track of j_{t-1} . Unlike the discrete Markov state, this is a continuous variable and would require a finer grid.

References

- Abraham, Katherine, and Henry Farber, 1987, Job duration, seniority, and earnings, American Economic Review 77, 278–297.
- Ai, Hengjie, Mariano Croce, Anthony Diercks, and Kai Li, 2012, Production-based term structure of equity returns, *Working Paper*.
- Ai, Hengjie, Mariano Croce, and Kai Li, 2013a, Growth to value: option exercise and the crosssection of equity returns, *Journal of Financial Economics* 107, 325–349.
- Ai, Hengjie, Mariano Croce, and Kai Li, 2013b, Toward a quantitative general equilibrium asset pricing model with intangible capital, *Review of Financial Studies* 26, 491–530.
- Andolfatto, David, 1996, Business cycles and labor-market search, *American Economic Review* 86, 112–132.
- Babecky, Jan, Philip Du Caju, Theodora Kosma, Martina Lawless, Julian Messina, and Tairi Room, 2010, Downward nominal and real wage rigidity survey evidence from European firms, *Scandinavian Journal of Economics* 112, 884–910.
- Bachmann, Ruediger, Ricardo Caballero, and Eduardo Engel, 2013, Aggregate implications of lumpy investment: New evidence and a DSGE model, American Economic Journal: Macroeconomics 5, 29–67.

- Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Barro, Robert, 2006, Rare disasters and asset markets in the twentieth century, *Quarterly Journal* of Economics 121, 823–866.
- Barwell, Richard, and Mark Schweitzer, 2007, The incidence of nominal and real wage rigidities in Great Britain: 1978-98, *Economic Journal* 117, 553–569.
- Bauer, Thomas, Lorenz Goette, and Uwe Sunde, 2007, Wage rigidity: Measurement, causes and consequences, *Economic Journal* 117, 499–507.
- Belo, Frederico, Pierre Collin-Dufresne, and Robert Goldstein, 2014a, Dividend dynamics and the term structure of dividend strips, *Journal of Finance* forthcoming.
- Belo, Frederico, Xiaoji Lin, and Santiago Bazdresch, 2014b, Labor hiring, investment and stock return predictability in the cross section, *Journal of Political Economy* 122, 129–177.
- Berk, Jonathan B., and Johan Walden, 2013, Limited capital market participation and human capital risk, *Review of Asset Pricing Studies* 3, 1–37.
- Bewley, Truman, 1999, Why Wages Don't Fall During a Recession (Harvard University Press, Cambridge, Massachusetts, and London, England).
- Boldrin, M., L. Christiano, and J. Fisher, 1999, Habit persistence, asset returns, and the business cycle, *Federal Reserve Bank of Chicago Working Paper No. 99-14*.
- Boldrin, M., L. Christiano, and J. Fisher, 2001, Habit persistence, asset returns, and the business cycle, American Economic Review 91, 149–166.
- Burnside, Craig, 1996, Production function regressions, returns to scale, and externalities, *Jour*nal of Monetary Economics 37, 177–201.

- Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo, 1995, Capital utilization and returns to scale, NBER Macroeconomics Annual 10, 67–110.
- Caballero, R., 1994, Small sample bias and adjustment costs, The Review of Economics and Statistics 76, 52–58.
- Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.
- Campbell III, Carl M., and Kunal S. Kamlani, 1997, The reasons for wage rigidity: Evidence from a survey of firms, *The Quarterly Journal of Economics* 112, 759–789.
- Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross section of returns, *Journal of Finance* 59, 2577–2603.
- Chirinko, Robert, 2008, [sigma]: The long and short of it, *Journal of Macroeconomics* 30, 671–686.
- Choi, Jaewon, 2013, What drives the value premium: The role of asset risk and leverage, *Review* of *Financial Studies* 26, 2845–2875.
- Cooper, Ilan, 2006, Asset pricing implications of non-convex adjustment costs and irreversibility of investment, *Journal of Finance* 61, 139–170.
- Croce, Max, 2014, Long-run productivity risk: A new hope for production-based asset pricing?, Journal of Monetary Economics 66, 13–31.
- Daly, Mary, Bart Hobijn, and Brian Lucking, 2012, Why has wage growth stayed strong?, *FRBSF Economic Letter*.
- Danthine, Jean-Pierre, and John B. Donaldson, 2002, Labour relations and asset returns, *Review* of *Economic Studies* 69, 41–64.

- Davis, Steven, Jason Faberman, and John Haltiwanger, 2006, The flow approach to labor markets: New data sources and micro-macro links, *Journal of Economic Perspectives* 20, 3–26.
- Devicenti, Francesco, Agata Maida, and Paolo Sestito, 2007, Downward wage rigidity in Italy: Micro-based measures and implications, *Economic Journal* 117, 530–552.
- Dickens, William, Lorenz Goette, Erica Groshen, Steinar Holden, Julian Messina, Mark Schweitzer, Jarkko Turunen, and Melanie Ward, 2007, How wages change: Micro evidence from the international wage flexibility project, *Journal of Economic Perspectives* 21, 195–214.
- Dittmar, Robert, Francisco Palomino, and Wei Yang, 2014, Leisure preferences, long run risks, and human capital returns, *Working paper*.
- Donangelo, Andres, 2014, Labor mobility: implications for asset pricing, *Journal of Finance* 68, 1321–1346.
- Epstein, Larry, and Stanley Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework, *Econometrica* 57, 937–969.
- Fama, Eugene, and Kenneth French, 1992, The cross-section of expected stock returns, Journal of Finance 47, 427–465.
- Favilukis, Jack, and Xiaoji Lin, 2013, Does wage rigidity make firms riskier? Evidence from long horizon return predictability, Working Paper .
- Gertler, Mark, and Antonella Trigari, 2009, Unemployment dynamics with Nash wage bargaining, Journal of Political Economy 117, 38–86.
- Gomes, Joao, 2001, Financing investment, American Economic Review 91, 1263–1285.
- Gomes, Joao, Urban Jermann, and Lukas Schmid, 2013, Sticky leverage, Working Paper.
- Gourio, Francois, 2007, Labor leverage, firms heterogeneous sensitivities to the business cycle, and the cross-section of returns, *Working Paper*.

- Hall, Robert, 1982, The importance of lifetime jobs in the U.S. economy, American Economic Review 72, 716–724.
- Hall, Robert, 1995, Lost jobs, Brookings Papers on Economic Activity 1995, 221–273.
- Hall, Robert, 2006, Employment fluctuations with equilibrium wage stickiness, American Economic Review 95, 50–65.
- Hobijn, Bart, and Aysegul Sahin, 2009, Job-finding and separation rates in the OECD, *Economic Letters* 104, 107–111.
- Jermann, Urban J., 1998, Asset pricing in production economies, *Journal of Monetary Economics* 41, 257–275.
- Jermann, Urban J., and Vincenzo Quadrini, 2012, Macroeconomic effects of financial shocks, American Economic Review 102, 238–271.
- Kahneman, Daniel, Jack Knetsch, and Richard Thaler, 1986, Fairness as a constraint on profit seeking: Entitlements in the market, *American Economic Review* 76, 728–741.
- Kaltenbrunner, Georg, and Lars Lochstoer, 2010, Long-run risk through consumption smoothing, *Review of Financial Studies* 23, 3141–3189.
- Khan, Aubhik, and Julia Thomas, 2008, Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics, *Econometrica* 76, 395–436.
- Kliem, Martin, and Harald Uhlig, 2013, Bayesian estimation of a DSGE model with asset prices, Working Paper .
- Krusell, Per, and Anthony A. Smith, Jr., 1998, Income and wealth heterogeneity in the macroeconomy, *The Journal of Political Economy* 106, 867–896.
- Kung, Howard, and Lucas Schmid, 2014, Innovation, growth, and asset prices, *Journal of Finance* forthcoming.

- Lettau, Martin, and Harald Uhlig, 2000, Can habit formation be reconciled with business cycle facts?, *Review of Economic Dynamics* 3, 79–99.
- Li, Erica, and Francisco Palomino, 2014, Nominal rigidities, asset returns, and monetary policy, Journal of Monetary Economics 66, 210–225.
- Lustig, Hanno, Chad Syverson, and Stijn Van Nieuwerburgh, 2011, Technological change and the growing inequality in managerial compensation, *Journal of Financial Economics* 99, 601–627.
- Marcet, Albert, and Thomas Sargent, 1989, Convergence of least squares learning mechanisms in self-referential linear stochastic models, *Journal of Economic Theory* 48, 337–368.
- Mehra, Rajnish, and Edward Prescott, 1985, The equity premium: A puzzle, Journal of Monetary Economics 15, 145–161.
- Merz, Monika, and Eran Yashiv, 2007, Labor and the market value of the firm, American Economic Review 97, 1419–1431.
- Modigliani, F., and M. Miller, 1958, The cost of capital, corporation finance and the theory of investment, American Economic Review 48, 261–297.
- Mortensen, Dale, and Christopher Pissarides, 1994, Job creation and job destruction in the theory of unemployment, *Review of Economic Studies* 61, 397–415.
- Pastor, Lubos, and Pietro Veronesi, 2003, Stock valuation and learning about profitability, *Jour*nal of Finance 58, 1749–1789.
- Petrosky-Nadeau, Nicolas, Lu Zhang, and Lars-Alexander Kuehn, 2013, An equilibrium asset pricing model with labor markets search, *Working Paper*.
- Pissarides, Christopher, 1979, Job matchings with state employment agencies and random search, *Economic Journal* 89, 818–833.

- Pissarides, Christopher, 2009, The unemployment volatility puzzle: Is wage stickiness the answer?, *Econometrica* 77, 1339–1369.
- Prescott, Edward C., 1986, Theory ahead of business cycle measurement, *Federal Reserve Bank* of Minneapolis Staff Report.
- Rich, Robert, and Joseph Tracy, 2004, Uncertainty and labor contract durations, The Review of Economics and Statistics 86, 270–287.
- Rudanko, Leena, 2009, Labor market dynamics under long-term wage contracting, Journal of Monetary Economics 56, 170–183.
- Santos, Tano, and Pietro Veronesi, 2006, Labor income and predictable stock returns, Review of Financial Studies 19, 1–44.
- Schmalz, Martin, 2012, Managing human capital risk, Working Paper.
- Schmitt-Grohe, Stephanie, and Martin Uribe, 2013, Downward nominal rigidity and the case for temporary inflation in the Eurozone, *Journal of Economic Perspectives* 27, 193–212.
- Shimer, Robert, 2005, The cyclical behavior of equilibrium unemployment and vacancies, *American Economic Review* 95, 25–49.
- Sweeney, Richard, Arthur Warga, and Drew Winters, 1997, The market value of debt, market versus book value of debt, and returns to assets, *Financial Management* 26, 5–21.
- Uhlig, Harald, 2007, Explaining asset prices with external habits and wage rigidity in a DSGE model, *American Economic Review* 97, 239–243.
- van Binsburgen, Jules, Michael Brandt, and Ralph Koijen, 2012, On the timing and pricing of dividends, American Economic Review 102, 1596–1618.
- Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.

Figure 1: Labor Market Variables Explain Industry Return, Volatility, Beta

This figure plots the univariate relationships between the average industry return and the average industry labor share (upper left), the average industry return and the covariance of the industry's labor share with aggregate private output (lower left), the industry return volatility and the average industry labor share (upper middle), the industry return volatility and the covariance of the industry's labor share with aggregate private output (lower middle), the industry CAPM beta and the average industry labor share (upper right), and the industry CAPM beta and the covariance of the industry's labor share with aggregate output (lower right).



Figure 2: Impulse responses

This figure plots the impulse responses of the aggregate wage, aggregate profit, and aggregate dividend to a positive productivity shock. The positive productivity shock lasts for one year (four consecutive quarters of high growth). The solid line represents the standard frictionless model (Cobb-Douglas production and no wage rigidity), the dot-dashed line represents the frictionless model with a calibrated CES, and the dashed line represents the calibrated CES production with wage rigidity. The x-axis is years after the shock, the y-axis is model quantity at any time relative to time zero.



Figure 3: Comparative statics of the capital adjustment cost in the frictionless model

This figure compares the volatility of the unlevered equity return (upper panel), and the unlevered value premium (lower panel) in a frictionless model as we vary capital adjustment costs. The x-axis plots the volatility of HP-filtered investment relative to HP-filtered output (this is monotonically related to the adjustment cost). We present both Cobb-Douglas technology (solid line), and a calibrated CES (dashed line).



Figure 4: Comparative statics of μ

This figure compares the volatility of the unlevered equity return as we vary wage rigidity (μ) across different models. All models have capital adjustment costs calibrated to match investment volatility and fixed costs calibrated to match the average market-to-book ratio. On the upper panel we plot volatility (y-axis) against average job duration $\frac{1}{1-\mu}$, on the lower panel we plot volatility (y-axis) against the probability of remaining in a job μ . We present both Cobb-Douglas technology (solid line), and a calibrated CES (dashed line).



Figure 5: Term structure of equity

This figure plots the term structure of expected return and volatility for dividend strips, calculated as in van Binsburgen et al. (2012). The solid line represents the standard frictionless model (Cobb-Douglas production and no wage rigidity), the dotted line represents the Cobb-Douglas model with wage rigidity, the dot-dashed line represents the frictionless model with a calibrated CES and the dashed line represents the model with a calibrated CES and wage rigidity. We present unlevered returns on the left, and levered on the right.

Table 1: Labor Market Variables Explain Industry Return, Volatility, Beta

This table presents results of cross-sectional regressions where the unconditional average return, volatility, or CAPM beta of an industry are regressed on industry characteristics. The characteristics are the average labor share in an industry and the covariance of an industry's labor share with aggregate private GDP. Specification 1 includes just average labor share, specification 2 includes just the covariance, and specification 3 includes both. T-statistics are in parentheses. Results are for 1929-2000.

Panel A: Average Return

Specification	1	2	3
$B_{E[LS]}$	2.57		2.59
	(1.05)		(1.07)
$B_{Cov[LS,GDP]}$		-0.03	0.02
		(0.11)	(0.09)
R^2	0.07	0.00	0.07

Panel B: Volatility

Specification	1	2	3
$B_{E[LS]}$	20.96		18.70
	(3.52)		(2.90)
$B_{Cov[LS,GDP]}$		-2.79	-2.44
		(2.56)	(2.16)
R^2	0.23	0.20	0.38

Panel C: CAPM beta

Specification	1	2	3
$B_{E[LS]}$	0.77		2.59
	(2.52)		(2.76)
$B_{Cov[LS,GDP]}$		-0.16	-0.15
		(3.38)	(3.30)
R^2	0.19	0.42	0.54

Table 2: Calibration

All model parameters are listed in this table. Note that most parameters are shared by all models and only four parameters (η , ν , f, μ) vary across models. The model is solved at a quarterly frequency.

Parameter	Description	Ι	II	III	IV
		Cobb-Douglas	Cobb-Douglas	Calibrated	Calibrated
				CES	CES
		Frequent	Infrequent	Frequent	Infrequent
		Resetting	Resetting	Resetting	Resetting
		Preferenc	es		
β	Time	0.994	0.994	0.994	0.994
	Preference				
θ	Risk Aversion	6.5	6.5	6.5	6.5
ψ	IES	2.0	2.0	2.0	2.0
		Productio	on		
α	Capital Weight	0.23	0.23	0.45	0.45
	in Production				
ρ	Determines Labor	0.77	0.77	0.77	0.77
	and Profit Shares				
δ	Depreciation	0.025	0.025	0.025	0.025
$\frac{1}{1-\eta}$	Labor Capital	1.00	1.00	0.50	0.50
,	Elasticity				
ν	Adjustment Cost	0.3	1.2	3.5	4.5
f	Operating	0.0160	0.0195	0.0187	0.0198
	Cost				
μ	Probability of	0	0.9	0	0.9
	No Resetting				

Table 3: Macroeconomic Moments

This table compares annual macroeconomic moments from the data (1929-2013) to several versions of our model. A variable denoted as x is HP-filtered, a variable denoted as Δx is a growth rate. The variables are GDP (y), consumption (c), investment (i), employment (n), average wage (w), wage bill (w*n), and labor share $(\frac{w*n}{y})$. The labor market variables w, w*n, and $\frac{w*n}{y}$ are for private sector only, and any scaling is by private sector GDP; private sector data is available from 1948 to 2013. In the data $\sigma(y) = 3.98\%$ and $\sigma(\Delta y) = 5.12\%$, productivity shocks are calibrated such that all models (almost exactly) match these numbers. The data moments and bootstrapped standard errors are in Panels A and B; we also present the Cobb-Douglas frictionless model (C), the Cobb-Douglas sticky wage model (D), the CES frictionless model (E), and the CES sticky wage model (F).

x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.57	1.00	1.00	0.54
c	0.51	0.72	0.53	0.53	0.79	0.50
i	3.04	0.07	0.55	2.84	0.27	0.39
n	0.78	0.92	0.55	0.73	0.89	0.47
w	0.44	0.60	0.36	0.49	0.61	0.32
w * n	0.87	0.84	0.33	0.91	0.85	0.19
$\frac{w*n}{y}$	0.56	-0.49	0.36	0.53	-0.43	0.20
Panel	C: Col	ob-Dou	glas, $\mu =$	0		
\overline{x}	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.46	1.00	1.00	0.43
c	0.54	0.92	0.51	0.65	0.91	0.64
i	3.06	0.95	0.46	2.79	0.90	0.28
n	0.84	0.56	0.20	0.82	0.55	-0.08
w	0.88	0.61	0.57	0.88	0.63	0.62
w * n	1.00	1.00	0.46	1.00	1.00	0.43
$\frac{w*n}{y}$	0.00	0.00	0.00	0.00	0.00	0.00
Panel	E: CES	S, $\mu = 0$)			
x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.46	1.00	1.00	0.42
c	0.60	0.95	0.55	0.67	0.93	0.66
i	3.04	0.95	0.42	2.86	0.91	0.24
n	0.83	0.56	0.20	0.81	0.55	-0.08
w	0.83	0.35	0.51	0.85	0.40	0.53
w * n	0.76	0.99	0.49	0.80	0.99	0.52
$\frac{w*n}{y}$	0.27	-0.93	0.42	0.25	-0.85	0.25

Panel B: Data standard errors

x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	0.00	0.00	0.11	0.00	0.00	0.10
c	0.05	0.09	0.12	0.05	0.08	0.12
i	0.26	0.24	0.14	0.45	0.26	0.12
n	0.04	0.03	0.11	0.04	0.03	0.11
w	0.04	0.08	0.12	0.04	0.09	0.12
w * n	0.04	0.06	0.11	0.06	0.04	0.14
$\frac{w*n}{y}$	0.09	0.07	0.15	0.08	0.10	0.20
Panel	D: Col	ob-Doug	glas, $\mu =$	0.9		
x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.47	1.00	1.00	0.43
c	0.60	0.95	0.53	0.68	0.94	0.63
i	2.95	0.95	0.43	2.70	0.91	0.25
n	0.83	0.55	0.20	0.81	0.54	-0.08
w	0.47	0.47	0.70	0.60	0.61	0.86
w * n	0.77	0.88	0.19	0.89	0.90	0.17
$\frac{w*n}{y}$	0.48	-0.66	0.45	0.43	-0.46	0.28
Panel	F: CES	$S, \mu = 0$	0.9 (Base	line mo	del)	
x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.46	1.00	1.00	0.42
c	0.62	0.95	0.55	0.68	0.94	0.64
i	3.00	0.95	0.41	2.83	0.91	0.23
n	0.83	0.56	0.20	0.82	0.55	-0.08
w	0.40	0.38	0.71	0.55	0.54	0.88
w * n	0.74	0.83	0.14	0.86	0.87	0.11
$\frac{w*n}{y}$	0.56	-0.68	0.49	0.50	-0.52	0.35

Table 4: Labor capital elasticity and the wage

Here we set N = 1, K = 3, $\alpha = 0.23$, $\rho = 0.77$, $Z_t \sim N(1, 0.1)$. We set K roughly equal to average capital in our frictionless model. We set the volatility of Z_t higher than in the model only to highlight the effect of η on volatility of wages; qualitatively everything remains the same for lower volatility of Z_t . All other parameters are identical to our calibration. We compute $Y_t = (\alpha K^{\rho} + (1 - \alpha) * (Z_t N)^{\rho \eta})^{\frac{1}{\eta}}$ and $\frac{\partial Y_t}{\partial N} = \rho(1 - \alpha)Y^{1-\eta}Z^{\rho\eta}N^{\rho\eta-1}$ and report the ratios of their volatilities and their correlation. Note that $\eta = 0$ corresponds to Cobb-Douglas production.

η	$\frac{\sigma(\frac{\partial Y}{\partial N})}{\sigma(Y)}$	$\frac{\sigma(ln(\frac{\partial Y}{\partial N}))}{\sigma(ln(Y))}$	$\sigma(\tfrac{\partial Y}{\partial N}, Y)$
-2.0	0.70	0.94	0.999
-1.0	0.63	0.90	0.999
-0.5	0.60	0.91	1.000
0.0	0.59	1.00	1.000
0.5	0.64	1.26	1.000
1.0	0.77	1.91	1.000
2.0	1.33	6.49	0.999

Table 5: Accounting Moments

This table compares annual accounting moments from the data to several versions of our model. The data on profits is for 1950-2012 (Compustat), the data on dividends is 1946-2013 (Flow of Funds). Bootstrapped standard errors are in parentheses. A variable denoted as x is HP-filtered, a variable denoted as Δx is a growth rate. For dividends we consider the change in dividends normalized by output instead of the dividend growth rate because dividends are sometimes very close to zero resulting in very large growth rates. We report both the dividend to equity (total dividends plus share repurchases), and the dividend paid by the corporate sector (dividend to equity plus net interest).

Panel A: Profit

	$\frac{\sigma(\pi)}{\sigma(y)}$	$\rho(\pi,y)$	$AC(\pi)$	$\frac{\sigma(\Delta \pi)}{\sigma(\Delta y)}$	$\rho(\Delta\pi,\Delta y)$	$AC(\Delta \pi)$
Data	3.01	0.64	0.23	3.27	0.60	-0.07
	(0.47)	(0.06)	(0.11)	(0.55)	(0.07)	(0.16)
C-D $\mu = 0$	1.03	0.98	0.46	1.01	0.97	0.40
C-D $\mu = 0.9$	2.14	0.92	0.59	1.86	0.88	0.56
CES $\mu = 0$	1.78	0.98	0.46	1.66	0.96	0.35
CES $\mu = 0.9$	2.57	0.94	0.57	2.27	0.90	0.51

Panel B: Dividend

	$\frac{\sigma(d)}{\sigma(y)}$	ho(d,y)	$\mathrm{AC}(d)$	$\frac{\sigma(\frac{\Delta d}{y})}{\sigma(\Delta y)}$	$\rho(\frac{\Delta d}{y}, \Delta y)$	$\operatorname{AC}(\frac{\Delta d}{y})$	$rac{\sigma(d^e)}{\sigma(y)}$	$\rho(d^e,y)$	$AC(\pi)$	$\frac{\sigma(\frac{\Delta d^e}{y})}{\sigma(\Delta y)}$	$\rho(\tfrac{\Delta d^e}{y},\Delta y)$	$\operatorname{AC}(\frac{\Delta d^e}{y})$
Data	4.08	0.26	0.34	0.58	0.38	0.17	9.55	0.39	0.30	0.49	0.40	0.09
	(0.86)	(0.10)	(0.18)	(0.10)	(0.10)	(0.19)	(2.17)	(0.08)	(0.17)	(0.09)	(0.10)	(0.21)
C-D $\mu = 0$	6.80	-0.91	0.40	0.34	-0.75	0.18	3.57	-0.65	0.21	0.24	-0.16	0.10
C-D $\mu = 0.9$	6.62	-0.01	0.14	0.38	0.01	-0.08	6.84	0.87	0.54	0.43	0.78	0.50
CES $\mu = 0$	1.54	-0.80	0.29	0.07	-0.13	0.26	3.33	0.80	0.36	0.30	0.79	0.25
CES $\mu = 0.9$	4.52	0.09	0.23	0.43	0.17	0.09	5.64	0.90	0.56	0.55	0.82	0.51

Table 6: Financial Moments

This table compares annual financial moments from the data to several versions of our model. The data on returns is 1929-2013 from Ken French's web site. In Panel A, the value premium is defined as the difference in average returns between firms in the top quintile and bottom quintile of a book-to-market sorting. For simulated data, we also present the unlevered value premium, which is computed for the entire firm. In Panel B, we present the expected return and volatility over the next 5 years, conditional on being in a recession or expansion today (the definition of recessions is described in the main text). In panel C we present results from long horizon regressions of equity returns (3, 12, 60, and 120 quarters) on the book-to-market ratio. For the data, in parentheses we present one standard deviation in panels A and B, and t-statistics in panel C.

	$E[R^F]$	$\sigma(R^F)$	$E[R^E]$	$\sigma(R^E)$	SR	$E[R^V-R^G]$	$\beta^V-\beta^G$	$E[R^{V,UL} - R^{G,UL}]$	$\beta^{V,UL}-\beta^{G,UL}$
Data	0.42	3.58	7.68	20.26	0.36	5.37	0.29		
	(0.38)	(0.43)	(2.18)	(1.60)	(0.12)	(2.09)	(0.09)		
C-D $\mu = 0$	1.03	0.90	3.30	5.33	0.46	0.25	0.11	-0.20	-0.09
C-D $\mu = 0.9$	0.98	0.89	4.28	9.85	0.37	0.79	0.23	-0.15	-0.04
CES $\mu = 0$	1.19	1.02	5.10	9.21	0.45	1.41	0.32	0.20	0.06
CES $\mu = 0.9$	1.00	0.99	6.56	13.15	0.44	2.58	0.49	0.51	0.13

Panel A: Unconditional financial moments

Panel B: Conditional financial moments

	$E[R^E - R^F Rec]$	$E[R^E - R^F Exp]$	$\sigma(R^E - R^F Rec)$	$\sigma(R^E - R^F Exp)$
Data	10.57	8.36	20.19	18.33
	(1.33)	(1.37)	(1.88)	(0.87)
C-D $\mu = 0$	2.22	2.34	4.60	4.63
C-D $\mu = 0.9$	3.39	3.25	8.69	8.19
CES $\mu = 0$	3.91	3.82	8.41	7.46
CES $\mu = 0.9$	5.68	5.26	12.65	10.60

Panel C: Predictive regression

	Return				Volatility			
	3	12	60	120	3	12	60	120
Data	-0.15	1.80	1.09	0.79	-	1.51	1.11	0.40
	(0.31)	(2.48)	(2.94)	(2.31)	-	(1.72)	(2.49)	(1.09)
C-D $\mu = 0$	-2.16	-5.51	-2.76	0.26	-	-3.26	-0.48	-4.84
C-D $\mu = 0.9$	-0.79	-2.09	-0.85	0.94	-	-0.21	1.95	-1.87
CES $\mu = 0$	-0.91	0.09	0.33	1.89	-	1.14	3.84	-1.11
CES $\mu = 0.9$	0.30	0.64	0.83	1.85	-	2.49	4.95	0.15

_

Table 7: Decomposing the effect

Column	1	2	3	4	5	6	7	8	9
Production	C-D	C-D	C-D	CES	C-D	CES	CES	C-D	CES
Return to Scale (ρ)	1.00	0.77	0.77	0.77	0.77	0.77	0.77	0.90	0.90
Fixed Cost	No	No	Yes						
Rigidity	No	No	No	No	Yes	Yes	Yes	No	Yes
Idiosyncratic Risk	No	No	No	No	No	No	Yes	No	No
$\sigma(R^K)$	0.47	2.33	3.32	5.38	5.18	7.18	7.48	1.97	6.94

This table presents the unlevered equity volatility from several models.

Table 8: The value premium

This table presents average book-to-market of equity, profit to labor expenses ratio, financial leverage, and mean excess return for five portfolios sorted on book-to-market. The top panel contains data and the bottom panel results from our baseline model, with wage rigidity ($\mu = 0.9$) and a calibrated CES ($\frac{1}{1-\eta} = 0.5$). For simulated data, we also present the unlevered value premium, which is computed for the entire firm. Profits are defined as Revenue-COGS-SGA from Compustat, labor expenses are employees from Compustat multiplied by the average wage for the firm's industry from NIPA.

Panel A: Data

	B/M	$\frac{Profit}{LaborExpense}$	$rac{Debt}{Debt+Equity}$	α	β	$E[R^i - R^f]$
Growth	0.20	0.70	0.19	-0.55	0.97	6.51
2	0.47	0.62	0.33	0.28	0.93	7.01
3	0.70	0.61	0.42	1.28	1.02	8.69
4	0.97	0.54	0.49	1.91	1.13	10.10
Value	2.85	0.45	0.78	2.73	1.26	11.88
V-G	2.65	-0.25	0.59	3.28	0.29	5.37

Panel B: Model

	B/M	$\frac{Profit}{LaborExpense}$	$rac{Debt}{Debt+Equity}$	α	β	$E[R^i - R^f]$	$E[R^{i,UL} - R^f]$
Growth	0.32	0.61	0.35	0.09	0.88	4.98	3.10
2	0.49	0.49	0.37	0.13	0.95	5.41	3.34
3	0.61	0.44	0.41	0.14	1.01	5.78	3.36
4	0.79	0.41	0.49	0.06	1.18	6.62	3.38
Value	1.79	0.09	0.53	-0.06	1.37	7.56	3.61
V-G	1.47	-0.52	0.18	-0.14	0.49	2.58	0.51
Table A1: Estimates of ρ^B

This table presents estimates of ρ^B for several different specifications. The specifications are described in detail in the text.

Specification	λ unrestricted	$\lambda = 0.37$
1	0.61	—
2	0.46	0.49
3	0.99	0.94
4	—	0.81

Table A2: Macroeconomic Moments

This table compares annual macroeconomic moments from the data to several extensions of our model. The variable descriptions are identical to Table 3. The data moments and bootstrapped standard errors are in Panels A and B; we also present our baseline model (C), and models with symmetric cross-sectional heterogeneity in μ (D), asymmetric cross-sectional heterogeneity in μ (E), and time-series variation in μ (F).

Panel A: Data means

x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.57	1.00	1.00	0.54
С	0.51	0.72	0.53	0.53	0.79	0.50
i	3.04	0.07	0.55	2.84	0.27	0.39
n	0.78	0.92	0.55	0.73	0.89	0.47
w	0.44	0.60	0.36	0.49	0.61	0.32
w * n	0.87	0.84	0.33	0.91	0.85	0.19
$\frac{w*n}{y}$	0.56	-0.49	0.36	0.53	-0.43	0.20
Panel	C: Bas	seline m	odel			

x		$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	$\operatorname{AC}(x)$	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x,\Delta y)$	$AC(\Delta x)$
y		1.00	1.00	0.46	1.00	1.00	0.42
c		0.62	0.95	0.55	0.68	0.94	0.64
i		3.00	0.95	0.41	2.83	0.91	0.23
n		0.83	0.56	0.20	0.82	0.55	-0.08
w		0.40	0.38	0.71	0.55	0.54	0.88
w *	n	0.74	0.83	0.14	0.86	0.87	0.11
$\frac{w*n}{u}$	n	0.56	-0.68	0.49	0.50	-0.52	0.35

Panel E: Asymmetric heterogeneity in μ

x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.46	1.00	1.00	0.42
c	0.60	0.95	0.55	0.67	0.94	0.65
i	3.00	0.96	0.41	2.84	0.91	0.23
n	0.83	0.57	0.20	0.81	0.56	-0.08
w	0.35	0.51	0.67	0.49	0.59	0.86
w * n	0.78	0.83	0.16	0.86	0.86	0.07
$\frac{w*n}{y}$	0.56	-0.62	0.50	0.52	-0.51	0.41

Panel B: Data standard errors

x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	0.00	0.00	0.11	0.00	0.00	0.10
c	0.05	0.09	0.12	0.05	0.08	0.12
i	0.26	0.24	0.14	0.45	0.26	0.12
n	0.04	0.03	0.11	0.04	0.03	0.11
w	0.04	0.08	0.12	0.04	0.09	0.12
w * n	0.04	0.06	0.11	0.06	0.04	0.14
$\frac{w*n}{y}$	0.09	0.07	0.15	0.08	0.10	0.20
Panel	D: Syr	nmetric	heteroge	eneity i	n µ	
x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.46	1.00	1.00	0.42
c	0.61	0.95	0.55	0.68	0.94	0.65
i	3.01	0.95	0.41	2.84	0.91	0.22
n	0.83	0.57	0.20	0.81	0.56	-0.08
w	0.38	0.47	0.69	0.51	0.58	0.86
w * n	0.77	0.84	0.16	0.86	0.87	0.09
$\frac{w*n}{y}$	0.54	-0.64	0.49	0.49	-0.51	0.38
Panel	F: Tin	ne varia	tion in μ			
x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.46	1.00	1.00	0.42
c	0.60	0.95	0.55	0.67	0.94	0.65
i	3.00	0.95	0.41	2.84	0.91	0.22
n	0.83	0.57	0.20	0.81	0.56	-0.08
w	0.35	0.51	0.67	0.49	0.59	0.86
w * n	0.78	0.83	0.16	0.86	0.86	0.07
$\frac{w*n}{y}$	0.56	-0.62	0.50	0.52	-0.50	0.41

Table A3: Accounting Moments

This table compares annual accounting moments from the data to several extensions of our model. The variable descriptions are identical to Table 5. The data moments and bootstrapped standard errors are in rows 1 and 2; we also present our baseline model (3), and models with symmetric cross-sectional heterogeneity in μ (4), asymmetric cross-sectional heterogeneity in μ (5), and time-series variation in μ (6).

Panel A: Profit

	$\frac{\sigma(\pi)}{\sigma(y)}$	$ ho(\pi,y)$	$AC(\pi)$	$\frac{\sigma(\Delta \pi)}{\sigma(\Delta y)}$	$\rho(\Delta \pi, \Delta y)$	$AC(\Delta \tau)$	τ)					
Data	3.01	0.64	0.23	3.27	0.60	-0.07						
	(0.47)	(0.06)	(0.11)	(0.55)	(0.07)	(0.16))					
Baseline	2.57	0.94	0.57	2.27	0.90	0.51						
Symmetric μ^i	2.44	0.94	0.56	2.20	0.91	0.52						
Asymmetric μ^i	2.12	0.90	0.60	1.90	0.86	0.60						
Variation in μ_t	2.37	0.89	0.59	2.05	0.85	0.57						
Panel B: Divide:	nd											
	$\frac{\sigma(d)}{\sigma(y)}$	ho(d,y)	$\mathrm{AC}(d)$	$\frac{\sigma(\frac{\Delta d}{y})}{\sigma(\Delta y)}$	$\rho(\frac{\Delta d}{y}, \Delta y)$	$\operatorname{AC}(\frac{\Delta d}{y})$	$\frac{\sigma(d^e)}{\sigma(y)}$	$\rho(d^e,y)$	$AC(\pi)$	$\frac{\sigma(\frac{\Delta d^e}{y})}{\sigma(\Delta y)}$	$\rho(\tfrac{\Delta d^e}{y},\Delta y)$	$\operatorname{AC}(\frac{\Delta d^e}{y})$
Data	4.08	0.26	0.34	0.58	0.38	0.17	9.55	0.39	0.30	0.49	0.40	0.09
	(0.86)	(0.10)	(0.18)	(0.10)	(0.10)	(0.19)	(2.17)	(0.08)	(0.17)	(0.09)	(0.10)	(0.21)
Baseline	4.52	0.09	0.23	0.43	0.17	0.09	5.64	0.90	0.56	0.55	0.82	0.51
Symmetric μ^i	4.12	0.01	0.24	0.43	0.16	0.14	5.21	0.92	0.56	0.58	0.86	0.52
Asymmetric μ^i	3.95	0.00	0.26	0.45	0.17	0.18	5.18	0.93	0.55	0.63	0.87	0.49
Variation in μ_t	4.46	0.14	0.26	0.44	0.21	0.14	6.08	0.89	0.55	0.62	0.82	0.48

Table A4: Financial Moments

This table compares annual financial moments from the data to several extensions of our model. The variable descriptions are identical to Table 6. The data moments and bootstrapped standard errors are in rows 1 and 2; we also present our baseline model (3), and models with symmetric cross-sectional heterogeneity in μ (4), asymmetric cross-sectional heterogeneity in μ (5), and time-series variation in μ (6).

Panel A: Unconditional financial moments

	$E[R^F]$	$\sigma(R^F)$	$E[R^E]$	$\sigma(R^E)$	SR	$E[R^V - R^G]$	$\beta^V-\beta^G$	$E[R^{V,UL} - R^{G,UL}]$	$\beta^{V,UL} - \beta^{G,UL}$
Data	0.42	3.58	7.68	20.26	0.36	5.37	0.29		
	(0.38)	(0.43)	(2.18)	(1.60)	(0.12)	(2.09)	(0.09)		
Baseline	1.17	0.99	6.56	13.15	0.44	2.58	0.49	0.51	0.13
Symmetric μ^i	1.18	0.99	7.34	14.45	0.44	2.71	0.43	1.54	0.26
Asymmetric μ^i	0.98	0.98	7.68	15.61	0.44	2.59	0.39	1.33	0.20
Variation in μ_t	1.00	0.98	6.96	14.31	0.43	1.81	0.41	0.63	0.11
Panel B: Condit	ional fina	ancial mo	oments						
	$E[R^E -$	$-R^F Rec$] $E[R^E]$	$ -R^F E$	xp] $\sigma(x)$	$R^E - R^F R\epsilon$	$ec) \sigma(R)$	$E - R^F Exp)$	
Data	10	0.57		8.36		20.19		18.33	
	(1	.33)		(1.37)		(1.88)		(0.87)	
Baseline	5	.68		5.26		12.65		10.60	
Symmetric μ^i	6	.35		5.82		14.12		11.55	
Asymmetric μ^i	7	.07		6.20		15.59		12.40	
Variation in μ_t	6	.26		5.48		14.28		11.17	

Table A5: Constant labor

This table presents results from a version of the model where $N_t = 1$, and without idiosyncratic risk ($\sigma(Z^i) = 0$). For brevity, we present only the standard model (Cobb-Douglas and $\mu = 0$) and the baseline model (CES and $\mu = 0.9$), and only the macro-economic and financial results.

Panel A: Standard model

x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.60	1.00	1.00	0.67
С	0.54	0.92	0.55	0.69	0.90	0.70
i	2.80	0.96	0.57	2.49	0.89	0.52
n	0.00	0.00	0.00	0.00	0.00	0.00
w	1.00	1.00	0.60	1.00	1.00	0.67
w * n	1.00	1.00	0.60	1.00	1.00	0.67
$\frac{w*n}{y}$	0.00	0.00	0.00	0.00	0.00	0.00

Panel B: Baseline model

x	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x,y)$	AC(x)	$\frac{\sigma(\Delta x)}{\sigma(\Delta y)}$	$\rho(\Delta x, \Delta y)$	$AC(\Delta x)$
y	1.00	1.00	0.60	1.00	1.00	0.67
c	0.70	0.88	0.61	0.78	0.90	0.74
i	3.07	0.88	0.53	2.77	0.82	0.43
n	0.00	0.04	0.00	0.00	-0.00	0.00
w	0.49	0.82	0.71	0.66	0.84	0.88
w * n	0.49	0.82	0.71	0.66	0.84	0.88
$\frac{w*n}{y}$	0.66	-0.91	0.56	0.58	-0.78	0.50

Panel C: Unconditional financial moments

	$E[R^F]$	$\sigma(R^F)$	$E[R^E]$	$\sigma(R^E)$	SR
Standard	0.97	0.86	2.20	5.25	0.42
Baseline	1.23	1.06	6.18	12.23	0.42

Figure A1: Autocorrelation of consumption and output growth

This figure plots the autocorrelation of consumption growth (top panel) and output growth at various horizons (bottom panel). Dashed lines indicate two standard deviation bounds.

