Project selection and risk taking under credit constraints

Felipe S. Iachan*

FGV/EPGE

October 6, 2014

Abstract

Credit constraints generate a hedging motive that extends beyond purely financial decisions by also distorting the selection and operation of real investments. We study these distortions through a dynamic model in which collateral constraints emerge endogenously. The hedging motive can be broken down into three components: expected future productivity, leverage capacity, and current net worth. While constrained firms behave as if averse to transitory fluctuations in net worth, additional exposure to factors related to persistent productivity innovations or credit capacity fluctuations increases their value. The most constrained firms abstain from financial hedging while still distorting real decisions to reflect the hedging motive. Firm-level volatility is influenced by project selection and our theory contributes to a potential explanation for the higher volatility of lower net-worth firms.

Keywords: capital budgeting, credit constraints, project selection, risk exposure.

1 Introduction

When external financing is limited, a hedging motive emerges in firms' decisions. Aligning investment opportunities and the availability of internal resources can increase firm value.[1] Sometimes this is reflected on the costly use of financial hedging instruments, such as futures and derivatives. Even when these are not observed, the hedging motive still induces distortions on the selection and operation of real investments. Firms have incentives to select investment projects taking into account the distribution of the value of internal funds in the future. In its turn, the value of internal funds is shaped by the interaction of the firm's

*Praia de Botafogo, 190/1115, Rio de Janeiro, RJ, Brazil. Telephone: (+55)(21)3799-5673; e-mail: felipe.iachan@fgv.br. I would like to thank Heitor Almeida, António Antunes, Eduardo Azevedo, Carlos Carvalho, Leandro Gorno, Humberto Moreira, Adriano Rampini, Alp Simsek, and seminar participants at BI, the Minneapolis Fed, FGV\EESP, the 2013 editions of the Lubramacro, WB/CEMLA, and SBE meetings and the 2014 E.S. Meeting of the Econometric Society.

[1] See Froot, Scharfstein, and Stein (1993) for a canonical reference.

financial policy and exogenous factors, such as the evolution of productivity or credit conditions. Project selection, attitudes towards risk exposure, and financial planning become intertwined. To shed light on these interactions, we study a dynamic model of project selection.

When contemplating alternative capital investments, firms face projects with different exposures to risk factors, correlations with their core business, and financing possibilities. To make matters less abstract, let us illustrate these features with a stylized airline industry example. An airline might decide to expand into a specific route. By doing that, it exposes its revenues to demand factors that shape the fares it can charge and the occupancy rates it can achieve. A choice of operating a route between Boston and New York exposes cash-flows in a specific way to the underlying risks in the industry, such as fluctuations in business travel or regional economic downturns. Other decisions involve the type of capital goods used: for example, more fuel efficient planes reduce an airline's exposure to fuel price shocks. However a plane that is more efficient for a specific route might be less redeployable, facing a thinner secondary market. As consequence, it can be less useful as collateral and expose the firm to more risk in its ability to secure financing.

To formally study the interaction proposed, we analyze a model in which neoclassical firms choose investment projects understanding that each mix of projects requires a specific financial plan. This plan describes investment levels, borrowing, and hedging policies. Projects are allowed to differ on the revenues they generate, including their exposure to particular shocks, and also on the type of capital goods they use. The key set of financial constraints comes from limited enforcement of repayment promises and can be rewritten as simple collateral constraints, as in Rampini and Viswanathan (2010).

A brief summary of our results is the following. Firms that find themselves constrained distort both real investment and financial decisions to reflect the value of internal funds across time and states of the world. When contemplating alternative projects, they also go beyond the evaluation of cash flows from operations and place a premium on a project's ability to attract cheaper collateralized financing. The most constrained firms borrow as much as possible and abstain from financial hedging, but still distort their real decisions to reflect the hedging motive. Additionally, whenever shocks to their revenues are sufficiently persistent, even a transitory increase in exposure to these shocks raises the value of the firm, making smaller firms more willing to take on these risks.

To understand the distortions in project evaluation, it is useful to characterize what drives the variation of the marginal value of internal funds. This value can be described as a forward looking product of marginal returns until a moment in which the firm is sufficiently close to being unconstrained and pays out dividends. Importantly, it is not a standard return on investment that matters but a levered return on internal funds, for which credit capacity plays a central role. Overall, we can identify three determinants of the marginal value of internal funds: expected productivity, leverage capacity, and current net worth. We illustrate the operation of each one of these separately.

We first study the effects of productivity shocks with different degrees of persistence. Productivity here can be understood to broadly encompass the total factor productivity, input costs, and demand fluctuations. Persistent shocks make firms value increased exposure to risk in the present. The origin of this effect lies in the need to partially self-finance. A persistent productivity improvement has two consequences. First it increases cash-flows, which can be used to fund investment and mitigate the effects of credit constraints. Second, it also increases the desired investment levels and, as a consequence, the marginal value of investment.

Whenever a shock is sufficiently persistent, the second effect dominates. The firm's investment needs increase by more than cash-flows can cover. As a consequence it finds itself relatively more constrained after a positive productivity innovations. Conversely, when the innovation is negative, investment drops by more than cash-flows contract and downsizing actually frees up resources. As firms face tighter constraints during growth than during downsizing, they are willing to pay a premium for funds that correlate positively with their own productivity process. An investment project that additionally loads on any determinant of future productivity, even if in a transitory and unpredictable way, becomes effectively more valuable for constrained firms. The reason is that it generates cash-flows that are better aligned with future investment. Therefore, financially-constrained firms are more willing to take any risk that correlates sufficiently with their future productivity.

This effect is exactly reversed regarding transitory shocks. These have little to no effect on the value of future investment but change cash-flows. As such they only create a mismatch between available funds and optimal investment. In the limit case, productivity shocks without any persistence behave as exogenous fluctuations in net worth. While constrained firms might still fail to use financial instruments to hedge against those risks, they are willing to distort their real decisions whenever that reduces the loading on transitory factors.

We also evaluate attitudes towards a possible tightening of credit constraints. Constrained firms are concerned about levered returns on their own funds, which rise when more credit can be obtained. As a consequence, they see a higher premium on resources which are available in states with slacker credit conditions, as more leverage becomes feasible. As such, they do not have incentives to ensure resources for situations in which credit conditions deteriorate and do not ensure against a possible credit cycle. Indeed, projects that show more exposure to credit conditions increase the value of the firm: a project that is more exposed generates more funds exactly when these can be more leveraged through additional borrowing. This result illustrate how fluctuations in credit conditions might induce additional risk-taking and why firms might opt not to hedge against states where credit becomes scarce.

While the results are initially studied analytically and illustrated through simple two-period examples, an infinite-horizon version is evaluated quantitatively in the last section of the paper. For the parameter values obtained in the structural work of Li, Whited, and Wu (2014), more constrained firms place a non-negligible premium on riskier projects. The adoption of an alternative investment project that involves more dispersed returns for a single period can increase the value of the firm by a few dozen basis points. The effect is stronger for smaller, more financially constrained firms, and weaker for larger, dividend-paying firms. Firms that resort to this alternative project also scale down any financial hedging.

We can also quantitatively evaluate another experiment, based on the use of capital goods that serve as better collateral. This would be the case for items that are more redeployable or tangible. In this setting, a modest 10% increase in the degree of collateralization, even for a single period, can generate more than a 3% increase in firm value.

Relation to the literature - In its approach towards financial contracts, this paper follows Rampini and Viswanathan (2010, 2013) that propose a model in which enforcement constraints can be reduced to collateral constraints, similar to Kiyotaki and Moore (1997), but allowing for the trading of state-contingent assets. The focus of those papers is in analyzing financing and risk management. Capital budgeting, as in most of literature studying financial frictions, is reduced to the choice of the scale of investment[2]. They have had success in explaining some empirical regularities and previous puzzles, such as the absence of risk management for the firms typically understood to be the most constrained, and the cross-sectional profile of leasing decisions.

The contribution of the current paper relative to this literature is twofold. Most importantly, we analyze project selection, showing that a hedging motive is displayed in real decisions even for firms that abstain from financial hedging. As in these previous contributions, the most constrained firms choose to borrow as much as possible and do not leave any slack for financial hedging. Once project selection is taken into account, it is particularly for these firms that distorting real decisions becomes a useful tool, acting as a substitute for financial hedging. Additionally, in empirically reasonable cases, these distortions actually make constrained firms favor riskier projects.

As an intermediate step towards studying project selection, we also have another look at the determinants of the hedging motive. While the risk management literature has particularly emphasized the role of variations in net worth in shaping this motive, it has devoted less attention to state-contingent factors behind the marginal value of funds. We shed light on its dependence on both the expected productivity of a marginal investment, which is intrinsically related to the persistence of shocks, and on leverage possibilities, which are related to credit conditions exogenous to the firm.

The current paper is also related to a literature on capital budgeting in environments with frictions, which has two main strands. A first strand studies allocative distortions and efficiency losses that originate from conflicts of interest between owners and privately-informed self-interested managers. [3]In this paper, we study how capital market distortions might feed into distortions in capital budgeting, even in the absence of any such internal conflicts.

A second strand features a macroeconomic perspective. Stylized examples of project selection have appeared in a literature concerned with the aggregate consequences of financial frictions.[4] This paper contributes to that strand by providing a more thorough analysis of the incentives in investment selection, risk

[2]Consider for instance Albuquerque and Hopenhayn (2004); Bolton, Chen, and Wang (2011); Clementi and Hopenhayn (2006); DeMarzo, Fishman, He, and Wang (2012); He and Krishnamurthy (2012); Holmström and Tirole (1998); or Krishnamurthy (2003).

[3]For instance, Harris and Raviv (1996, 1998); Rajan, Servaes, and Zingales (2000); Stein (2002) study difficulties in the allocation of resources to a manager or multiple divisions with conflicting interests. A great survey of work prior to the last decade is available in Stein (2003).

[4]For instance, Aghion, Angeletos, Banerjee, and Manova (2010), Greenwood and Jovanovic (1990), and Matsuyama (2007, 2008). Eisfeldt and Rampini (2007) discuss how credit constraints affect the composition of investment across used and new vintages of capital and provide empirical evidence that more constrained firms favor cheaper, used capital.

taking, and risk management among financially constrained firms. The simple assumption of decreasing returns to scale also adds predictions for the behavior along the cross section of firms that are not present in the previous literature.

The paper also speaks to a recent literature on endogenous volatility, which has attempted to better understand how trade-offs faced by firms help account for the empirical pattern of volatility across countries (Koren and Tenreyro (2013)) and along the business cycle (D'Erasmo and Moscoso Boedo (2013)). This paper adds to that discussion by illustrating first that financially constrained firms have incentives to load on persistent risks to facilitate self-financing. This not only helps account for some empirical regularities in higher volatility of smaller firms[5], but also points out that increases in risk exposure can actually be an optimal response to the limited access to external funds.

Some particularly related papers deserve a longer discussion. Vereshchagina and Hopenhayn (2009) study entrepreneurial risk taking in the presence of borrowing constraints. They show that given that entrepreneurs have a real option of stopping their projects and becoming employees in other firms, they effectively become risk loving for sufficiently low wealth. As a consequence, they are willing to choose riskier projects, even in the absence of a premium, which helps account for the surprising low returns found in empirical studies of entrepreneurship. Their result originates from a non-convexity in the value function, which is induced by the occupational choice. As consequence, entrepreneurs are willing to hold more of any risks, even risk that is uncorrelated with the productivity of their activity. The risk-taking studied in this paper does not rely on such non-convexity. The value function is concave in net worth for each state, but the marginal value of funds is state contingent. Therefore, constrained firms exhibit a hedging motive in their decisions and evaluate risks differently depending on how correlated they are with that value.

For the same reason, despite the presence of an enforcement problem, distortions in risk taking do not originate from the same mechanisms as in the risk shifting and asset substitution literature.[6] Since contracts properly account for possible deviations and assets are observable to competitive lenders, all investment distortions originate from the dispersion in marginal value of funds to the firm and not from a conflict of interest between equity and debt holders.

Another paper, Almeida, Campello, and Weisbach (2011), studies capital budgeting distortions induced by costly access to external funds. It relies on a reduced-form approach describing the choices across a small number of pre-specified projects which differ in liquidity and riskiness. It makes a key assumption that projects are uncorrelated. As a consequence, more constrained firms should do more of both financial and operational hedging and end up being less volatile, a result that is at odds with the empirical evidence both

[5]Consider the evidence for cross-country comparisons in Koren and Tenreyro (2007) which show that firms in less developed countries concentrate on more volatile sectors and on D'Erasmo and Moscoso Boedo (2013) which draw a comparison of mean volatility across the larger-firm COMPUSTAT database versus smaller firms for the Kauffman Firm Survey. Similarly, Davis, Haltiwanger, Jarmin, and Miranda (2007) point out correlations between measures of growth-volatility and typical proxies of financial constraints such as size, age, and publicly-traded status.

[6]Jensen and Meckling (1976) is the seminal reference of this literature. Landier, Sraer, and Thesmar (2011) study investment selection in the presence of risk-shifting incentives and provide some empirical evidence based on mortgage origination.

across countries with different degrees of financial development and across firm types[7]. The current paper generalizes and qualifies their conclusions by illustrating formally how project changes can be evaluated and how firms react in different ways to shocks which are more or less informative about future opportunities.

Organization - The remainder of the paper is organized as follows. Section 2 reviews the model of financing and risk management which takes a project as given, towards a discussion of the key variable behind firm's decisions, the marginal value of internal funds. Project selection is then introduced in two ways: through the evaluation of a small-scale marginal project in Section 2.2 and then through the effects induced by a changing project mix in Section 2.3. These general results are then specialized in Section 3, by imposing more structure on specific elements of the model, with examples that illustrate how firms evaluate exposure to productivity shocks with different degrees of persistence and credit capacity shocks. Section 4 provides quantitative evaluations in a steady state and the final section concludes.

2 Model

We start by introducing the model of a firm's financial decisions, taking its projects as given. This initial set-up draws upon the results from Rampini and Viswanathan (2010), a risk management model in which state-contingent borrowing is limited by endogenous collateral constraints. We first use this baseline model to analyze how limited credit, productivity innovations, and leverage possibilities shape the value the firm places on funds across states of the world. This marginal value of internal funds is the key variable driving distortions in corporate assessment of risky projects.

Guided by that discussion, we then tackle project selection. Sections 2.2 and 2.3 provide some general results on how potentially constrained firms evaluate the adoption of a new project and a deviation towards a different mix of projects, respectively. These results are later specialized through examples in the following part of the paper.

The benchmark set up is the following. Time is discrete and indexed by $t = 0, 1, ..., T$, with $T \leq +\infty$. Uncertainty is described by an exogenous event tree. The initial state s^0 is a singleton and $s^t \in S^t$ denotes the history known at time t. We define the transition probabilities between state s^t and its successors s^{t+1}, $\pi\left(s^{t+1}|s^t\right)$, in the usual way and let $\pi\left(s^t\right)$ denote the unconditional probability of state $s^t \in S^t$.

The economy is populated by two types of risk neutral agents. One has access to production technologies and we call them firms. The other group is composed of lenders who, without direct access to a production technology, provide external funding to firms.

A firm maximizes an expected discounted dividend stream according to

$$E\left[\sum_{t=0}^{T} \beta^t d_t\right],$$

[7]The cross-sectional evidence is reviewed in footnote 5. Additionally, in the empirical financial development literature, better creditor protection is linked to lower firm level volatility in Claessens, Djankov, and Nenova (2001), which study cross-country firm level evidence.

where $\beta \leq 1$.

Different investment projects entail different exposures of cash-flows to the most relevant risk factors, such as input and output prices as well as productivity shocks, both idiosyncratic and aggregate. They might also differ in other relevant ways such as by involving capital goods that are more or less redeployable, serve as better collateral, have different exposure to price fluctuations, or different depreciation rates. In our airline example from the introduction, these were embedded in the decisions of which routes to explore and which aircraft to choose.

Abstractly, we represent a mix of projects as $j \in \mathcal{J}$. We assume that project selection is a once-for-all decision, which is observable to lenders and can be contracted on. As such, we can think of j as an observable firm type.

For concreteness, we allow projects to differ along these three dimensions: how much output is generated in each contingency given capital investment, the price of the capital goods used by the project, and their recovery rate[8]. We can think of the first as the exposure of cash-flows to risks, of the second as the fluctuations in the relevant cost of investment/divestment, and of the third as sensitivity to variations in credit conditions. Thus the project mix determines the evolution of the production function, capital prices, and credit constraints as functions of the exogenous uncertainty embedded in s^t.

Formally a firm running a project mix j uses capital, which is traded at a price $q\left(j, s^t\right)$. Capital k_{t+1} purchased and installed in state s^t generates $F\left(k_{t+1}, j, s^{t+1}\right)$ and $(1-\delta)$ units of depreciated s^{t+1} capital. Here $F\left(\cdot, j, s^{t+1}\right)$ is a standard concave neoclassical production function.

Lenders have a discount factor of $R^{-1} \geq \beta$, are deep-pocketed and not subject to commitment problems, so they are willing to buy and sell contingent claims at an expected rate of return of R. Markets are complete in the sense that assets based on all contingencies can be traded, i.e., a full spanning notion. However, the firm's ability to issue claims on its output is limited by commitment problems.

At date t, after production takes place, a firm can renege on any of its outstanding debt. If that happens, lenders can only recoup a fraction $\theta\left(j, s^t\right)$ of the firm's capital stock after depreciation. They would obtain a total value of $\theta\left(j, s^t\right) q\left(s^t\right)(1-\delta) k_t$[9]. We will refer to $\theta\left(j, s^t\right)$ as a recovery rate. After reneging on its debt, the firm can go back to capital markets with net worth equal to all of the cash-flows it absconded with plus the fraction $\left(1-\theta\left(j, s^t\right)\right)$ of the depreciated capital stock.

Rampini and Viswanathan (2010) show that in a setting like this one, the enforcement constraints to be imposed on the firm's problem greatly simplify: the outstanding level of debt in any state cannot exceed how much a lender would recover if the firm chose to default. Additionally, without actually imposing any restrictions on the maturity structure of repayments, the financial contract can be implemented with state-contingent short-term debt. We use these results and write the firm's recursive problem as

[8] Input and output price changes can be thought of as comprising part of the fluctuations in the productivity of capital. Changes in depreciation rates and depreciation shocks represent just a small departure from the consequences of capital price changes and will not be discussed.

[9] Allowing for recovery of a fraction of output would not lead to any major departure from the results presented later.

$$V\left(w_t, j, s^t\right) \equiv \sup_{d_t, k_{t+1} \geq 0, \left\{b_{t+1}\left(s^{t+1}\right)\right\}} d_t + \beta E\left[V\left(w_{t+1}, j, s^{t+1}\right)\right] \tag{1}$$

subject to resource flow constraints,

$$w_t + R^{-1} E\left[b_{t+1}\left(s^{t+1}\right)\right] \geq d_t + q\left(j, s^t\right) k_{t+1}, \tag{2}$$

$$w_{t+1}\left(s^{t+1}\right) = F\left(k_{t+1}, j, s^t\right) + q\left(j, s^t\right)(1-\delta) k_{t+1} - b_{t+1}\left(s^{t+1}\right), \tag{3}$$

for each $s^{t+1}|s^t$, as well as state-contingent collateral constraints

$$b_{t+1}\left(s^{t+1}\right) \leq \theta\left(j, s^{t+1}\right) q\left(j, s^{t+1}\right)(1-\delta) k_{t+1}. \tag{4}$$

Here, w_t is the firm's net worth and $b_{t+1}\left(s^{t+1}\right)$ is the outstanding amount of debt in state s^{t+1}. We allow the problem to be non-stationary; the dependence on time is implicit in its dependence on the node s^t. This flexibility is useful for the simple finite-horizon examples used later. Equation 2 describes the origins of resources, with net worth and debt on the left-hand side, and their uses on dividend payments and capital purchases, on the right-hand side. Equation 3 describes the evolution of net worth taking into account cash-flow generated, the value of the capital stock, and promised debt repayments. The last set of constraints, in the form of equation 4, represents the endogenous borrowing constraints.

There is an even simpler formulation of the firm's problem. We can define the downpayment required per unit of capital as

$$\wp\left(j, s^t\right) \equiv q\left(j, s^t\right) - E\left[R^{-1}\theta\left(j, s^{t+1}\right)(1-\delta) q\left(j, s^{t+1}\right)|s^t\right] \tag{5}$$

and financial slack, or unused borrowing capacity as,

$$h\left(s^{t+1}\right) \equiv \theta\left(j, s^{t+1}\right)(1-\delta) q\left(j, s^{t+1}\right) k_{t+1} - b_{t+1}\left(s^{t+1}\right). \tag{6}$$

In expression (5), the downpayment requirement is defined as the minimum a firm needs to pay in order to deploy a unit of capital, i.e., how much it spends when it finances a purchase at a unit price $q\left(j, s^t\right)$ by borrowing all that lenders are willing to lend against that collateral. In expression (6), financial slack is the difference between how much the collateral value of the firm's capital is in state s^t, i.e., the borrowing capacity of the firm against that state, and how much the firm is actually pledging to pay from that state onwards. That is, a firm that borrows less than the maximum it could is said to be saving financial slack.

The firm's recursive problem can then be rewritten as

$$V\left(w_t, j, s^t\right) = \max_{d_t, k_{t+1}, h_{t+1} \geq 0} d_t + \beta E_t\left[V\left(w_{t+1}, j, s^{t+1}\right)\right] \tag{7}$$

s.t.

$$w_t \geq d_t + E\left[R^{-1} h_{t+1}\left(s^{t+1}\right)\right] + \wp\left(j, s^t\right) k_{t+1}\left(s^t\right) \tag{8}$$

and

$$w_{t+1}\left(s^{t+1}\right) = F\left(k_{t+1}, j, s^{t+1}\right) + \left(1-\theta\left(s^{t+1}\right)\right) q\left(j, s^{t+1}\right)(1-\delta) k_{t+1} + h_{t+1}\left(s^{t+1}\right). \tag{9}$$

The Envelope Theorem ensures that the multiplier on the first constraint, $\lambda\left(s^t\right)$, equals the shadow value of net worth to the firm, $\frac{\partial V\left(w_t, j, s^t\right)}{\partial w_t}$, which with some abuse of notation will be denoted by $\frac{\partial V\left(s^t\right)}{\partial w_t}$. We will also call it the value of internal funds, interchangeably.

The solution to the recursive maximization problem is then characterized by the following set of first-order conditions:

$$k_{t+1}: \ \beta E_t\left[\frac{\partial V_{t+1}\left(s^{t+1}\right)}{\partial w_{t+1}}\left(\frac{\partial F\left(k_{t+1}, s^{t+1}\right)}{\partial k_{t+1}} + \left(1-\theta\left(j, s^{t+1}\right)\right) q\left(s^{t+1}\right)(1-\delta)\right)\right] \leq \wp\left(j, s^t\right) \frac{\partial V\left(s^t\right)}{\partial w_t}, \tag{10}$$

$$d_t: \ 1 \leq \frac{\partial V\left(s^t\right)}{\partial w_t}, \tag{11}$$

and

$$h_{t+1}\left(s^{t+1}\right): \ \beta R \frac{\partial V\left(s^{t+1}\right)}{\partial w_{t+1}} \leq \frac{\partial V\left(s^t\right)}{\partial w_t}, \tag{12}$$

each of which holds as an equality if the relevant choice variable is strictly positive.

Equation (10) represents the firm's capital investment Euler equation. Guided by it, we go on to define the levered marginal return on investment as

$$R^{lev}\left(k_{t+1}, j, s^{t+1}\right) \equiv \frac{\frac{\partial F\left(k_{t+1}, j, s^{t+1}\right)}{\partial k_{t+1}} + \left(1-\theta\left(j, s^{t+1}\right)\right) q\left(j, s^{t+1}\right)(1-\delta)}{\wp\left(j, s^t\right)}.$$

This represents the variation in net worth induced by a marginal investment in capital associated to the maximum borrowing possible against that capital as collateral. We rewrite that Euler equation as

$$\frac{\partial V_t\left(w_t, j, s^t\right)}{\partial w_t} \geq \beta E_t\left[\frac{\partial V_{t+1}\left(w_{t+1}, j, s^{t+1}\right)}{\partial w_{t+1}} R^{lev}\left(k_{t+1}, j, s^{t+1}\right)\right], \tag{13}$$

which holds with equality whenever production takes place.

The capital accumulation equation indicates the importance of two endogenous variables: the value of internal funds and the marginal levered return. It also indicates that they are intrinsically related. Their behavior is key for understanding how credit constraints influence the decisions of constrained firms, not only in terms of financial planning, but also their real investment decisions. Therefore, before introducing project selection, we have a deeper look at their behavior.

2.1 The value of net worth

Standard dynamic programming arguments establish that the value function, $V_t\left(w_t, j, s^t\right)$, is concave in w_t, so that the marginal value of net worth is decreasing[10]. Additionally, when the production function is strictly concave, this marginal value reaches one for sufficiently high net worth.

The concavity of the value function has been pointed out as a reason for risk management, along the lines of the argument first put forward by Froot, Scharfstein, and Stein (1993): financially constrained firms become averse to fluctuations in net worth, since they prevent them from deploying adequate levels of capital across states of the world and create dispersion in the value of internal funds across these states. However, there is also state-dependence in the firm's problem. Besides net-worth, leverage possibilities and expected productivity also play central roles, as we illustrate concisely, with a simple finite-time example.

Let us impose Inada conditions to ensure investment in all states and assume that the firm pays dividends surely at a time $\bar{t} \leq T$. This can occur either as its projects involve a finite life or as an outcome that is reached under the optimal policy for the firm. Then,

$$\frac{\partial V\left(s^{\bar{t}}\right)}{\partial w_{\bar{t}}} = 1,\ \forall s^{\bar{t}} \in S^{\bar{t}},$$

and, for all $t < \bar{t}$,

$$\frac{\partial V\left(s^t\right)}{\partial w_t} = \beta^{\bar{t}-t} E_t\left[\prod_{\tau=t+1}^{\bar{t}} R^{lev}\left(k_\tau, j, s^\tau\right)\right]. \tag{14}$$

Therefore, the marginal value of resources within the firm in state s^t depends directly on the composition of the forward levered returns on investment. All else equal, the more constrained, the more levered, and the more productive the firm, the higher these returns are.

Let us the set of projects $\mathcal{J}$ be a singleton and drop the dependence of the notation on projects. Let $t \in \{0, 1, 2\}$, the production function be separable as $F\left(k_{t+1}, s^{t+1}\right) = A\left(s^{t+1}\right) k_{t+1}^{\alpha}$, with $\alpha \in (0,1)$, and capital be fully pledgeable as in Kiyotaki and Moore (1997), $\theta\left(s^t\right) = 1$, for all t, s^t. We focus on $t = 1$, one period before dividends are paid out for sure.

There, whenever $\frac{\partial V}{\partial w_t}\left(s^1\right) > 1$, the firm is effectively constrained in its capital deployment decisions, and uses maximal leverage, investing all its net worth in capital by purchasing $k_2\left(s^1\right) = \frac{w\left(s^1\right)}{\wp\left(s^1\right)}$. In that case, the marginal value of internal funds is

$$\frac{\partial V}{\partial w_t}\left(s^1\right) = \beta\alpha \frac{E\left[A\left(s^2\right)|s^1\right]}{\wp\left(s^1\right)} k^{\alpha-1}\left(s^1\right) = \beta\alpha \frac{E\left[A\left(s^2\right)|s^1\right]}{\wp\left(s^1\right)^{\alpha}} w\left(s^1\right)^{\alpha-1}. \tag{15}$$

We can point out three effects in place. The expected productivity term, embedded in $E\left[A\left(s^2\right)|s^1\right]$, pushes resources towards being more valuable in higher productivity states. The leverage effect, embedded in

[10] A proof is present in Rampini and Viswanathan (2013).

the reciprocal of the downpayment requirement, increases the value of resources when the credit conditions are looser and the downpayment is lower. Notice that decreasing returns to scale dampen this effect, but do not change its sign. Finally, the effect most emphasized in the risk management literature, which originates from the concavity of $f(k_t) \equiv k_{t+1}^{\alpha}$ and makes sure that, ceteris paribus, firms with lower net worth face more severe distortions, deploy less capital, and have higher marginal returns to investment.

We consider next how these three effects interact when firms evaluate alternative investment projects, as well as how they shape the determination of the optimal financial policies.

2.2 Evaluating a marginal project

Suppose the firm faces an alternative short-term project of small scale. Project selection is binary: the firm either undertakes this project ($j = 1$) or not ($j = 0$). The project requires $\epsilon > 0$ units of a given capital good that costs $q^{alt}(s^t)$ per unit today. Investment in this project generates a risky cash-flow of $y^{alt}(s^{t+1})\epsilon$, with which the firm could fully abscond in s^{t+1}. It also reverts some $(1-\delta)\epsilon$ units of depreciated capital, which is valued at $q^{alt}(s^{t+1})$ and has a recovery rate of $\theta^{alt}(s^{t+1})$.

As before, we can define two key objects for describing the firm's capital budgeting decisions. The first one is the downpayment requirement for this new project,

$$\wp^{alt}(s^t) \equiv q^{alt}(s^t) - R^{-1}E\left[(1-\delta)\,\theta(s^{t+1})\,q^{alt}(s^{t+1})\right].$$

The second is its marginal levered return,

$$R^{alt}(s^{t+1}) \equiv \frac{y^{alt}(s^{t+1}) + (1-\delta)\left(1-\theta(s^{t+1})\right)q^{alt}(s^{t+1})}{\wp^{alt}(s^t)}. \tag{16}$$

Then, a first-order approximation to the change in value induced by the adoption of the alternative project is given by the product of the scale ϵ and

$$\beta E_t\left[\frac{\partial V(s^{t+1})}{\partial w_{t+1}}\left\{y^{alt}(s^{t+1}) + (1-\delta)\,\theta(s^{t+1})\right\}|s^t\right] - \frac{\partial V(s^t)}{\partial w_t}\wp^{alt}(s^t). \tag{17}$$

Whenever the capital investment Euler Equation holds with equality, we can substitute for the second term. Then, for any sufficiently small scale ϵ, the first-order effect dominates and the project should be adopted if, and only if,

$$E_t\left[\frac{\partial V(s^{t+1})}{\partial w_{t+1}}\left(R^{alt}(s^{t+1}) - R^{lev}(s^{t+1})\right)\right] \geq 0. \tag{18}$$

We rewrite that condition in a covariance form to establish the following result.

Proposition 1. *Let $\Delta R(s^{t+1}) \equiv R^{alt}(s^{t+1}) - R^{lev}(s^{t+1})$ denote the excess marginal levered return of the alternative project over the baseline investment and $m_{t+1}(s^{t+1}) \equiv \frac{\partial V(s^{t+1})/\partial w_{t+1}}{E_t[\partial V(s^{t+1})/\partial w_{t+1}]}$ denote the normalized* *marginal value of funds. Then, it is optimal to invest in a marginal project if, and only if,*

$$E_t\left[\Delta R\left(s^{t+1}\right)\right]+Cov_t\left(m_{t+1}\left(s^{t+1}\right),\Delta R\left(s^{t+1}\right)\right)\geq 0. \tag{19}$$

A few features call attention. First, given that firms cannot borrow arbitrary amounts, project selection is always comparative: at the margin the main project and any alternative compete for internal funds and become mutually exclusive. Firms that are more constrained have higher leveraged marginal returns, and consequently, face naturally higher hurdle rates.

Second, the relevant return that is taken into account is a leveraged return, not a simple return on investment. A project that is capable of raising more collateralized financing requires a lower downpayment, and as a consequence, less resources to be displaced from other profitable opportunities the firm might have.

Third, firms that are constrained take into account a covariance term: projects that pay out more in the states in which the value of internal resources is higher are preferred. A lower return project might be picked over a higher return project if it pays out more in the states in which the firm is more constrained. The example in Section 3.1 illustrates that when productivity is persistent, firms are actually more constrained after positive, rather than negative, productivity innovations. As a consequence, equation (19) would indicate a positive covariance between $\frac{\partial V\left(s^{t+1}\right)}{\partial w_{t+1}}$ and $R^{lev}\left(s^{t+1}\right)$. It follows that diversification away from the baseline project lowers the value of the firm, even if the alternative project offers higher returns.

Notice also that even in the absence of any technological interactions, such as economies of scope, frictions in access to external funding are capable of generating both substitution and complementarity across projects. Substitution is present when two contemporaneous projects that cannot be fully externally financed compete for the use of the firm's resources. A complementarity arises across time, since projects that offer payouts that covary positively with the marginal value of net worth help finance the firm's most productive investment opportunities. Therefore, although the firm is always maximizing the total net present value of dividends, it is not maximizing NPV project-by-project. A project is evaluated in light of its capital requirements, its ability to attract external funding, and its ability of generating additional funding for the most valuable investment opportunities.

Additionally, the standard net-present value criterion can be recovered as a particular case. If we make the discount factor of lenders and firms the same, by setting $\beta=R^{-1}$, and look at firms that are effectively unconstrained and paying out dividends at s^t, these face $\frac{\partial V\left(s^t\right)}{\partial w_t}=1$. It follows that $\frac{\partial V}{\partial w_{t+j}}\left(s^{t+j}\right)=1$ holds for any node s^{t+j} which is a successor of s^t. Then equation 19 collapses back into the first-best rule of optimal investment: a firm should undertake a project if, and only if, it has a positive net-present value.

2.3 Changes in a firm's project mix

There are two simple approaches towards understanding capital budgeting in a multiple-project environment. The first one is simpler and consists of endowing a firm with a menu of projects into which it allocates internal funds and credit obtained. Let N denote this discrete set. The firm's problem at state s^t becomes one of solving

$$V_t\left(w_t, j, s^t\right) = \max_{d_t, \left\{k_{t+1}^n\right\}, h_{t+1} \geq 0} d_t + \beta E_t\left[V_{t+1}\left(w_{t+1}, j, s^{t+1}\right)\right]$$

with constraints that take into account these multiple alternative projects as

$$w_t \geq d_t + E\left[R^{-1} h_{t+1}\left(s^{t+1}\right)\right] + \sum_{n \in N} \wp^n\left(s^t\right) k_{t+1}^n\left(s^t\right) \tag{20}$$

and

$$w_{t+1}\left(s^{t+1}\right) = \sum_{n \in N}\left[F\left(k_{t+1}^n, j, s^{t+1}\right) + \left(1 - \theta^n\left(s^{t+1}\right)\right) q^n\left(s^{t+1}\right)(1-\delta) k_{t+1}^n\right] + h_{t+1}\left(s^{t+1}\right), \tag{21}$$

where variables with the superscript n are project-specific prices, capital purchases, collateralization, and minimum down-payments.

At the optimum, the investment Euler equation (10) holds now project by project. Indeed, if multiple projects are undertaken, indifference for the use of marginal funds implies equality in a covariance condition identical to the one in Proposition 1.

More interestingly, under many circumstances, a firm faces decisions that are mutually exclusive, since they involve different ways of conducting a single operation. For instance, an airline might expand the the business class in a given route at the cost of having less seats available for the economy class. This is a decision that changes the behavior of its revenues. Other decisions can change the type of capital goods used, hence their costs or how useful they are as collateral. We model these decisions in a abstract way, by allowing the firm to choose its type, or its project mix, $j \in \mathcal{J}$, where $\mathcal{J}$ is a closed interval in $\mathbb{R}$. We abstain from the N-project notation for simplicity.

Now, the firm's problem involving project selection can be solved in two steps: for each project mix j the optimal financial policy describing borrowing and hedging can be obtained. Let $V\left(w_0, j, s^0\right)$ denote the value achieved when project mix j is executed with net worth w_0. Then, the optimal project choice is the solution to $\max_{j \in \mathcal{J}} V\left(w_0, j, s^0\right)$.

We will proceed by characterizing the effects of small changes in j around a specific mix. We therefore evaluate locally the joint impact of changes in the cash-flow process, the prices of capital goods, and of different recovery rates. The final impact of a different project selection on the value of the firm is a composition of the effects through these three channels.

Towards that end, let us define the net investment at s^t in the standard way as $i\left(s^t\right) \equiv k_{t+1}\left(s^t\right) - (1-\delta) k_t$. Additionally, we write the endogenous borrowing capacity at s^{t+1} as

$$BC\left(k_t, j, s^t\right) \equiv \theta\left(j, s^t\right)(1-\delta) q\left(j, s^t\right) k_t \tag{22}$$

and the firm's net cash-flow as

$$NCF\left(k_t, i_t, j, s^t\right) \equiv F\left(k_t, j, s^t\right) - q\left(j, s^t\right) i_t. \tag{23}$$

Given those two definitions, we can study the consequences of a project selection decision which changes both cash-flows and the firm's financing conditions. Expression 24 below can be seen as specializing and qualifying, in this limited enforcement environment, the results from Myers (1974), which derived general properties of the interactions between investment and financing decisions in a model with reduced-form frictions.

Proposition 2. *A marginal change in the project mix is evaluated according to*

$$\frac{\partial V_0}{\partial j} = E_0\left[\sum_{t=1}^{T} \beta^t \frac{\partial V\left(w_t^*, j, s^t\right)}{\partial w_t}\left(\frac{\partial NCF\left(k_t^*, i_t^*, j, s^t\right)}{\partial j} + \mu\left(s^t\right)\frac{\partial BC\left(k_t^*, j, s^t\right)}{\partial j}\right)\right], \tag{24}$$

where the value $\mu\left(s^t\right) \equiv \frac{\partial V\left(s^{t-1}\right)/\partial w_{t-1}}{\beta R\left(\partial V\left(s^t\right)/\partial w_t\right)} - 1$ *is a normalization of the multiplier on the collateral constraint that limits borrowing between* s^{t-1} *and* s^t.

Equation (24) indicates that we can think of project selection as involving two terms: an adjusted discounted cash-flow term and a borrowing capacity change. The former is analogous to the standard evaluation of cash-flows from projects, but adjusts properly for the shadow value of internal funds. An additional effect emerges in the latter.

A different project mix might change a firm's ability to raise external financing. Since borrowing is limited by commitment problems, these funds are possibly cheaper than internal funds as priced by their shadow value. As a consequence, a premium on borrowing capacity emerges for firms that find themselves against their borrowing constraint. Therefore, projects are not only evaluated according to the net cash-flows from their operation, but also by their ability to attract cheaper collateralized funding.

Going back to the firm's choice among all possible alternatives, we can also use equation (24) to describe the firm's project selection in the following way. Any interior solution needs to satisfy $\frac{\partial V_0}{\partial j} = 0$. This becomes also a sufficient condition for an interior optimum whenever (22)-(23) define net-cash flows and borrowing capacity that are concave in j. In this case, we can think in terms of a firm that takes as given the value of internal funds and the premium on borrowing capacity obtained from the operation of the optimal project and acts as if maximizing the sum of discounted cash-flows plus premium-adjusted borrowing capacity.

Even in more general cases, equation (24) sheds light on which projects can never be optimal and how different decisions, such as favoring a project with riskier or safer cash flows, can change the value of the firm. We use it for characterizing the examples in the next section.

3 Project selection and risk taking

After the general description of the environment and of criteria for evaluating project selection, we now study which qualitative consequences are induced by the hedging motive. For that purpose, we analyze two simple examples, which illustrate how limited access to external finance changes risk-taking incentives of firms. In both examples, there are three dates, $t \in \{0, 1, 2\}$. Discount factors are the same for firms and lenders, $\beta = R^{-1} = 1$. Additionally, in both environments, technology is described by a separable single-factor neoclassical production function $F\left(k_t, j, s^t\right) = A\left(j, s^t\right) f\left(k_t\right)$, with a smooth and strictly concave $f\left(k_t\right) = \frac{k_t^\alpha}{\alpha}$ for $\alpha \in (0, 1)$.

3.1 Persistent Productivity

Uncertainty is described by a random variable ϵ which has finite support and is fully learned at $t = 1$.[11] As consequences, states at $t = 1$ can be related one-to-one with the realization of ϵ and each state s^1 of these has a unique successor s^2. For simplicity, let capital be fully collateralizable, so that $\theta\left(j, s^t\right) = 1$ always, and let $\delta > 0$. Then, the downpayment requirement is also constant across time and states of the world and we simply call it $\wp$.

We will study project selection around a benchmark project, which we call $j = 0$. At the benchmark project, revenue TFP is described by

$$A\left(0, s^1\right) = \overline{A}\left(1 + \epsilon\right)$$

and

$$A\left(0, s^2\right) = \overline{A}\left(1 + \epsilon\right)^\rho .$$

This process is chosen to incorporate, in a concise way, the consequences of innovations to the natural logarithm of TFP which decay at a rate ρ, as in the autoregressive process of order 1 commonly used in the empirical literature.[12]

The following result is in the spirit of Rampini and Viswanathan (2010, 2013) and is useful for guiding the analysis which follows.

Proposition 3 (Rampini and Viswanathan, 2013)**.** *At the baseline project, firms with sufficiently small net worth $w_0 < \underline{w}$ do not engage in financial risk management. That is, they set $h\left(s^1\right) = 0$ for all $s^1 \in S^1$.*

Proof. Appendix. □

The collateral constraints create a trade-off between scale and risk management. A firm that uses financial hedging needs to increase promised repayments in some states of the world. This consumes its borrowing capacity, forcing it to deploy less capital and reach a lower scale. Firms with lower net worth face

[11] Any residual uncertainty would be inconsequential, as the firms pay out all its resources at $t = 2$, so we abstract from it.

[12] Assuming that $E\left[\ln\left(1 + \epsilon\right)\right] = 0$ the unconditional mean of log-TFP is $\ln \overline{A}$, for every s^t, and deviations from that mean have an autocorrelation coefficient of ρ.

higher marginal returns on capital investment. As a consequence, their investment needs outweigh their risk management needs and they refrain from leaving financial slack towards any states at date $t = 1$.

Firms that have moderate net worth might choose to leave financial slack into the next period. They can leave slack either to fund growth prospects, when their productivity is higher than average, or to cover cash-flow shortfalls, when revenues are lower than average. There are two opposing forces at play when productivity shocks are persistent. Whether internal funds are more valuable after a positive innovation in productivity or after a negative one depends on which of these two dominates.

On the one hand, good news about productivity increase the firm's optimal investment. The first-best scale, as well as the marginal value of funds, increase in expected productivity. Both increase by more when returns to scale are closer to constant ($\alpha = 1$), since profit maximization requires larger investment responses to expected productivity. Also, this force is stronger the higher persistence in TFP, as current productivity becomes more informative about future productivity, also making desired investment increase by more in response to a current improvement in conditions.

On the other hand, cash-flows also increase upon a positive productivity shock. Since the revenue function is concave and scale is constrained by net worth, this force pushes the value of funds towards being decreasing in net worth and productivity innovations. The higher the returns to scale the less concavity is displayed by the value function and the weaker is this second force.

Therefore, which of these two forces dominates depends centrally on whether the optimal investment response to a productivity shock is higher or lower than the current cash-flow response. Indeed, a very simple condition emerges, relating the shape of the marginal value of internal funds to both the persistence of shocks and the elasticity of revenue function, a key measure of returns to scale, as indicated below.

Proposition 4. *The marginal value of resources is monotonically increasing (decreasing) in the state $s^1 = \epsilon$ whenever*

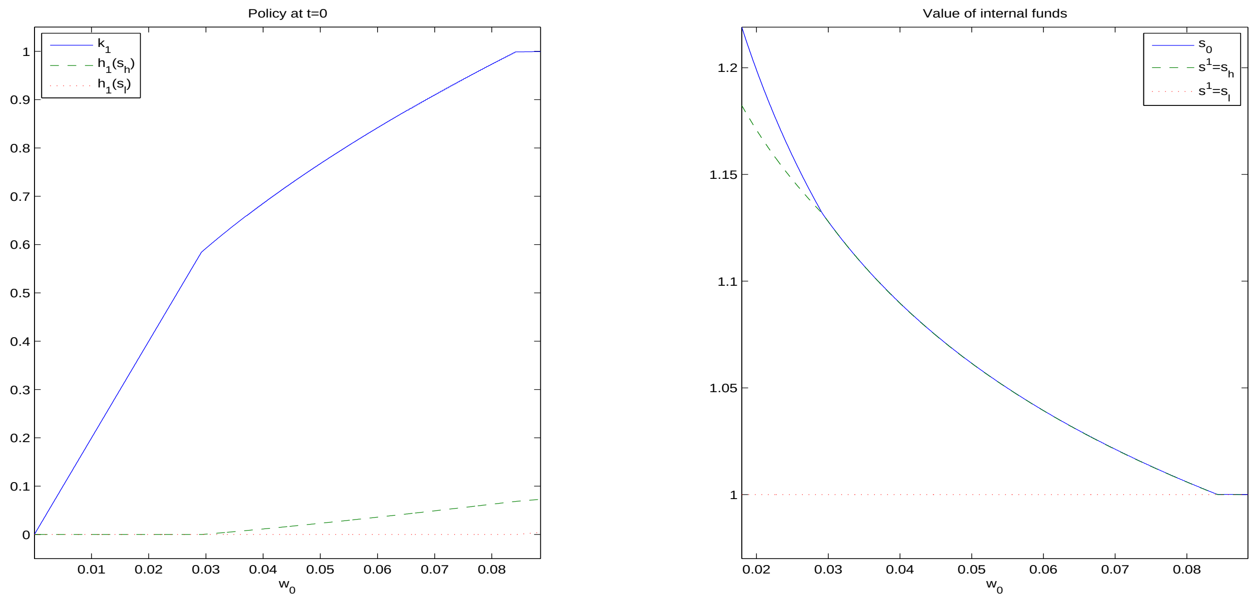
$$\alpha + \rho > (<) 1.$$

Proof. Appendix. □

Proposition 4 shows that, for this environment, whenever the sum of the elasticity of the revenues function and the persistence of log-TFP exceeds unity, investment needs respond to shocks by more than cash-flows, making the marginal value of funds pro-cyclical with respect to the firms fundamentals, which can be measured by TFP, output, or cash-flows. Parameter estimates from the empirical literature typically satisfy this condition, indicating that firms value the conservation of debt capacity for growth and not for dealing with cash-flow shortages.

Notice however that, in the case of a pure cash-flow shock that is irrelevant for predicting future productivity or output, $\rho = 0$ and the condition fails. In that case, resources are more valuable in lower cash-flow states.

Proposition 4 can be generalized for an environment with multiple factors shaping uncertainty, where one factor could be a pure cash-flow shock and other factors can be related to productivity innovations with different degrees of persistence. The conclusion is again that the marginal value of internal funds is increasing in the realization of any sufficiently persistent factor, while it is decreasing in transitory ones. Additionally, it is easy to show that whenever the firm is constrained at $t = 1$, the ordering of the marginal value of internal funds is strict for all states for which financial slack is not saved.



(a) Capital investment and financial slack.

(b) Value of internal funds.

Figure 1: Capital purchases, financial hedging, and value of funds as a function of initial net worth. Full persistence, 2-state example.

In figure 1, we illustrate optimal investment, risk management, and the value of internal funds for a 2-state case in which the shocks are fully persistent. In this example, target capital levels are more volatile than cash-flows and funds become more valuable after a positive productivity innovation. Notice, through the panel on the right-hand side, that the marginal value of internal funds is decreasing holding each state fixed, illustrating the concavity of the value function in net worth. The fact that they are dispersed, and more so for low net-worth firms, originates the hedging motive and the rationale for distortions in cash-flow evaluation and risk-taking.

Firms are split into three categories. Low net worth firms abstain from financial risk management. It is worth noting, however, that they face the highest dispersion in the value of internal funds at date $t = 1$. It is not the lack of benefits from risk management that leads them to refrain from it, but the even larger benefits from using as much leverage as possible at $t = 0$ that leads to this decision.

There is also an intermediate range of firms, that save financial slack for growth at $t = 1$. They borrow as much as possible against the worst state of nature and downsize when it occurs. This downsizing is not particularly costly, as their first-best capital level is also reduced, making net worth is less valuable. When the best productivity state happens, the fact that they saved financial slack allows them to grow by more.

Last, firms with sufficiently high net worth at the initial date can always deploy the first-best level of capital. As such, they do not face any dispersion in the internal value of funds.

For firms in the first two groups, the dispersion in the internal value of funds leads to distortions in project selection. One of the particular distortions regards the evaluation of risky projects. Consider a case in which the scalar j indexes projects according to their exposure to the shock ϵ at $t = 1$. In this sense, the index works similarly to a beta coefficient in asset pricing models, describing exposure to the risk embedded in the realization of the shock. Formally, this is represented by

$$A\left(j, s^1\right) = \overline{A}\left(1 + \epsilon\right) + j\epsilon.$$

We focus on the case in which this increased exposure is purely transitory, so that firms cannot exploit the predictability of future productivity which is present from $t = 1$ into $t = 2$. That means that

$$A\left(j, s^2\right) = A\left(0, s^2\right) = \overline{A}\left(1 + \epsilon\right)^{\rho}.$$

Therefore, a project with a higher j index is one that is more exposed to the productivity risk before any information about it is revealed.

In the absence of any credit frictions, a neoclassical firm exhibits a profit function which is neutral to such risks.[13] However, once credit constraints are present, this additional exposure leads to cash-flow risks that change the alignment of internal funds and investment opportunities. When shocks are sufficiently persistent, additional exposure indeed helps better align investment and its funding, as summarized by the proposition below.

Proposition 5. *The effects of a temporary increase in exposure to cash-flow risk on the firm's value are directly proportional to*

$$Cov\left(\frac{\partial V\left(s^1\right)}{\partial w_1}, \epsilon\right) k_1^{\alpha}. \tag{25}$$

Hence, the value of the firm is increasing in risk-exposure, j, whenever $\alpha + \rho > 1$, while it is decreasing whenever $\alpha + \rho < 1$.

Proof. In the appendix. □

Whenever the sum of the curvature parameter of the revenue function, α, and the persistence parameter,

[13] Notice that a change in the index j increases exposure to a shock that is unpredictable at the moment in which capital levels are chosen at $t = 0$. Any change in that index is also irrelevant for productivity $t = 2$. These two conditions ensure that changes in the exposure index are inconsequential for a unconstrained risk-neutral firm.

If, on the other hand, either capital investment decisions at the first date could be made contingent on the realization of ϵ or the index increased the dispersion of productivity at $t = 2$, after the realization was learned, the standard argument that ensures the convexity of the profit function would imply that a unconstrained firm also increases in value with an increase in the exposure index j. It could simply readjust input demands more strongly in response to an increase in exposure and obtain a higher expected value.

ρ, is sufficiently high, firms seem risk-loving in the following sense: the mean preserving spread of cash-flows induced by additional exposure to productivity risk increases the value of any constrained firm, as the covariance in equation 25 is strictly positive. The case is reversed when shocks are mostly transitory: the covariance term becomes negative and additional exposure to cash-flow risk only works towards additionally decreasing the value of the firm.

Proposition 5 can also be easily generalized towards an environment with multiple risk factors. Its main conclusion is then that, for credit-constrained firms, additional exposure to any persistent factor is value increasing, while exposure to transitory fluctuations is value reducing.

Notice also that, from the covariance term in expression 25, the more dispersed the internal value of funds is across the different states of the world, the larger the hedging motive inducing firms to distort their real decisions. The example in Figure 1 emphasizes that this hedging motive tends to be the largest for the firms that actually abstain from financial hedging. This happens for two reasons.

First they are the most distant from the optimal scale. Therefore, marginal distortions get magnified. Second they endogenously abstain from financial hedging, which whenever used would even out variations in the marginal value of a dollar across different periods and states. These firms do not abstain from financial hedging because of a lower benefit from doing so, but because the scale sacrifice required to conserve any financial slack is too costly.

It is also worth noting that when project selection involves only exposure to short-run cash-flow fluctuations, risk only matters through a net worth channel. As the firm's value function is concave in net worth, due to the combination of decreasing returns to scale and credit constraints, that concavity tends to carry over towards project selection. This intuition is formalized in the next proposition, which shows that for any firm that does not engage in risk management, the value function at $t = 0$ is concave in the project selection variable.[14]

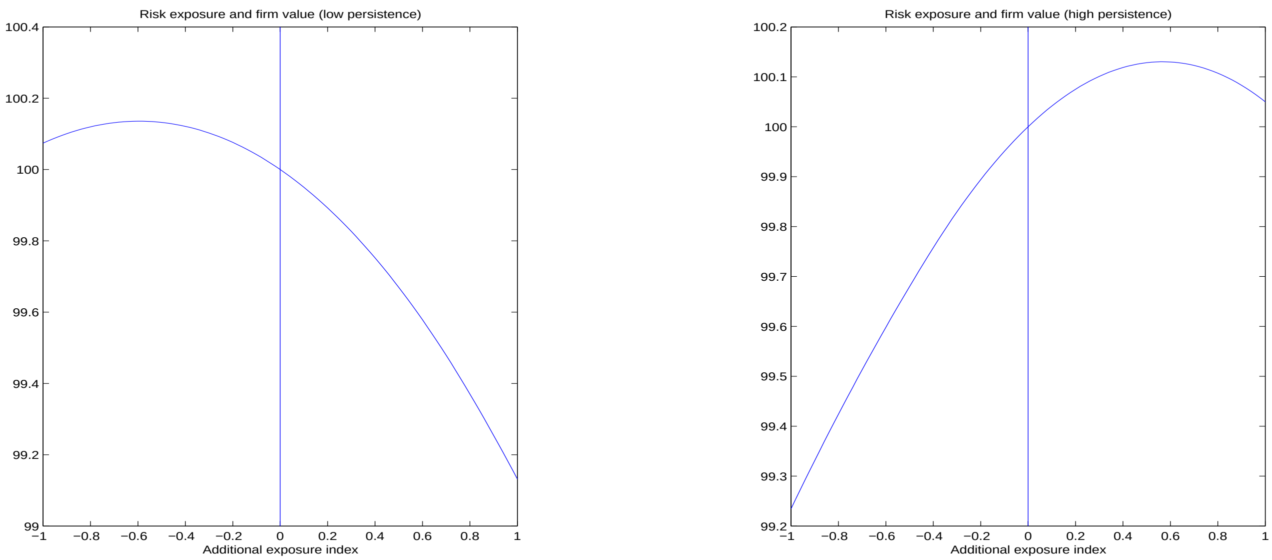
Proposition 6. *Whenever $w_0 < \underline{w}$, $V\left(w_0, j, s^0\right)$ is locally concave in the exposure index j.*

Proof. In the appendix. $\square$

Proposition 6 highlights that even firms that appear risk-loving and are willing to distort their decisions towards increased risk exposure are only guided by a motive to better match investment opportunities and cash-flow. The origin of this motive lies in the dispersion in the marginal value of funds and not in a convexity of the value function. Thus, the channel that pushes constrained firms towards larger exposure to persistent factors is, therefore, clearly distinguishable from real options interpretations or from risk-shifting agency conflicts.

Figure 2 displays the value at the initial date as a function of the risk exposure index j for a firm that is constrained below the efficient scale. In the left-hand side panel, the degree of persistence in the shock is low,

[14] An extension for all firms is complicated by the following mechanism: once a firm uses both financial risk management and real distortions, a distortion in real exposure might lead to a substitution away from financial instruments and an increase in scale, which would increase further the marginal benefit of additional distortions.



(a) $\rho = 0.1$

(b) $\rho = 0.4$

Figure 2: Value as a function of risk exposure(j), for different degrees of persistence ρ.
Both examples feature $\alpha = 0.75$. Additional parameter restrictions as described in the beginning of this section. Firm value is normalized as percentage of the value at the benchmark $j = 0$. Initial net worth is sufficient to finance exactly half of the unconstrained capital level.

so reductions in risk exposure away from $j = 0$ lead to gains in value. On the right-hand side panel, the same situation is plotted for a more persistent productivity shock. Notice now that increases away from the initial exposure lead to increases in firm value. This additional risk taking stems from the need to better match investment opportunities and cash-flows and not from a convexity in the value function, which is concave in both net worth and risk exposure.

3.2 Credit Capacity Shocks

The environment described in the previous section illustrated the connections between the persistence of shocks and the evaluation of cash-flow risk. In the presence of sufficiently persistent shocks, firms do not insure net worth for the lowest productivity states.

Shocks to their credit capacity work much in the same way: they reduce a firm's leverage ability and, as a consequence, the return it can make on internal funds. The example in this section illustrates how firms can sacrifice net worth and investment levels in low credit capacity states in order to invest more when credit conditions are more favorable. Again, they opt not to insure against negative shocks and additional exposure to such shocks can be shown to increase the value of the firm.

To formalize this reasoning, let there be two states that are learned at $t = 1$ which we call $s^1 = s^1_h, s^1_l$ and let us study the situation around a fixed project mix $j = \bar{j}$. From $t = 1$ into $t = 2$ the event tree evolves trivially: there is a singleton as a successor of either s^1. We refer to them as $s^2 = s^2_h, s^2_l$. The states s^1_h, s^1_l imply a one to one mapping with $\theta\left(\bar{j}, s^2\right)$, how much lenders expect to recover if the firm decides to walk away from its debt right after production at $t = 2$. We let

$$0 < \theta\left(j, s^2_l\right) < \theta\left(j, s^2_h\right) < 1.$$

Variation in these recovery rates changes how much credit can be obtained against the same collateral in a way that is orthogonal to any movements that could be happening in collateral prices. It is the simplest way to introduce a credit cycle which is unrelated to the productivity of investment.[15]

All other variables in the environment are constant across projects, time and states. Productivity is constant, $A\left(j, s^t\right) = A$ for all t, $s^t \in S^t$, and $j \in \mathcal{J}$. For simplicity, we also let $\theta_1\left(j, s^1\right) = \theta_1$ for all $s^1 \in S^1$ and $j \in \mathcal{J}$. The depreciation rate is set to zero, $\delta = 0$, implying that all firms find themselves constrained in all states.[16] This assumption is made for simplicity, to directly illustrate how sufficiently constrained firms would evaluate exposure to credit fluctuations, while still avoiding the need to describe several cases.

As a consequence of the variation in the recovery rate, the firm's borrowing capacity given any investment level depends on the underlying state. A firm that invests k_2 can borrow up to

$$BC\left(k_2, j, s^1\right) = \theta\left(j, s^2\right) k_2,$$

implying a downpayment requirement that is reduced when credit conditions improve, since

$$\wp\left(j, s^1\right) = 1 - \theta\left(j, s^2\right).$$

The variation in the downpayment requirement is directly responsible for making the return on internal funds increase as credit conditions improve. As a consequence, firms fail both to insure their investment at s^1_l and prefer projects that have a higher sensitivity to credit conditions[17]. This is formalized in the next two propositions below.

Proposition 7. *Consider the environment described in the last few paragraphs. Then,*

1. *every firm faces shadow values of net worth which are pro-cyclical with respect to the credit fluctuations, i.e.,* $\frac{\partial V\left(s^1_h\right)}{\partial w_1} > \frac{\partial V\left(s^1_l\right)}{\partial w_1}$*. As a consequence, firms never save financial slack towards the low collateralization state.*

[15]Modeling a credit fluctuation through a change in the recovery rate instead of the prices of capital goods has the advantage of generating an effect which is entirely orthogonal to productivity, since investment in capital produces both output and some capital after depreciation at $t+1$. Therefore, a reduction in $q\left(j, s^{t+1}\right)$ directly makes investment less productive.

Although it is hard to motivate a literal change in a recovery rate along the business cycle, we can interpret shocks to this variable as any shocks that affect how much a lender is willing to offer against a given amount of collateral. For instance, a deterioration of adverse selection in credit markets would have similar effects.

[16]With zero depreciation and no discouting, additional capital investment always dominates dividend payments at $t = 0, 1$. Effectively, each firm always finds itself constrained, as it is always below the first-best level of capital investment.

[17]In the sense of having a higher variance of the recovery rate for a given mean.

2. *capital investment is increasing in $s_h^1 > s_l^1$, with $k_2\left(s_h^1\right) > k_2\left(s_l^1\right)$.*

Proposition 8. *When projects differ only in terms of the dispersion of $\theta\left(j, s^2\right)$ around a same mean, with $j \in \mathcal{J}$ indexing this dispersion, then an increase in the dispersion of credit shocks increases the value of the firm, as*

$$\frac{\partial V_0}{\partial j} \propto \left(\frac{\partial V\left(s_h^1\right)}{\partial w_1} - 1\right) k_2\left(s_h\right) - \left(\frac{\partial V\left(s_l^1\right)}{\partial w_1} - 1\right) k_2\left(s_l\right) > 0. \tag{26}$$

There is a simple interpretation of the effects identified in equation (26) above.

When credit constraints are relaxed at s_h^t, the firm can borrow more for every purchased unit of capital. This borrowing generates funds valued at $\frac{\partial V\left(s_h^1\right)}{\partial w_t}$, a value that exceeds the cost of their repayment at $t = 2$, where the marginal value of funds is unitary. The difference between these two values is the premium on borrowing capacity.

There are two reasons for why the relaxation of borrowing constraints in the high leverage state $\left(s_h^1\right)$ more than offsets an equivalent tightening at the low leverage state $\left(s_l^1\right)$. The first one is that the value of being able to borrow more for each unit of capital is higher in the former than in the latter . The second is that the increase in borrowing capacity interacts with more units of capital, since leverage is higher at s_h^1.

4 A Quantitative Assessment

In this section, we quantitatively evaluate how credit constraints distort the selection across alternative projects. In particular, we assess two alternatives to a baseline technology, which is based on estimated parameters. A first alternative proposed involves returns which, although having the same mean as the baseline technology, are more exposed to the fundamentals of the firm; while a second alternative offers a higher degree of collateralization of the capital goods used.

For the baseline technology, we use the parameter estimates from Li, Whited, and Wu (2014) who estimate a baseline Rampini and Viswanathan (2010, 2013) limited-commitment model. Alternative projects are not present. Their parameter estimates, obtained through the use of the simulated method of moments and based on Compustat data, are provided in Table 1 below.

$R^{-1} - \beta$	δ	α	$\rho_{\log A}$	$\sigma_{\log A}$	θ
0.0139	0.077	0.938	0.550	0.158	0.318

Table 1: Parameter estimates from Li, Whited, and Wu (2014).
Capital prices are normalized to one. The discount factor β accounts for a 0.01% probability of firm exit and a separately estimated gap relative to the market discount factor. The parameter α reflects the elasticity of a revenue function of the form $A\left(s^t\right) k_t^{\alpha}$. Additionally, $\rho_{\log A}$ and $\sigma_{\log A}$ represent respectively the persistence and standard deviation of the natural logarithm of $A\left(s^t\right)$. Last, the parameter θ represents the degree of collateralization of capital, meaning that for each dollar of capital deployed 31¢ could be received by a lender upon a potential liquidation.

We additionally set $R = 1.019$ in accordance with the real interest rate in their sample period. In the numerical implementation, we use a Tauchen and Hussey (1991) 3-state Markov-Chain approximation for the productivity process and a 100-point shape-preserving spline approximation to the value function. All evaluations that follow are based on the median state, $s_t = s_{medium}$, from which productivity and the level of the value function might increase or decrease with equal probabilities.

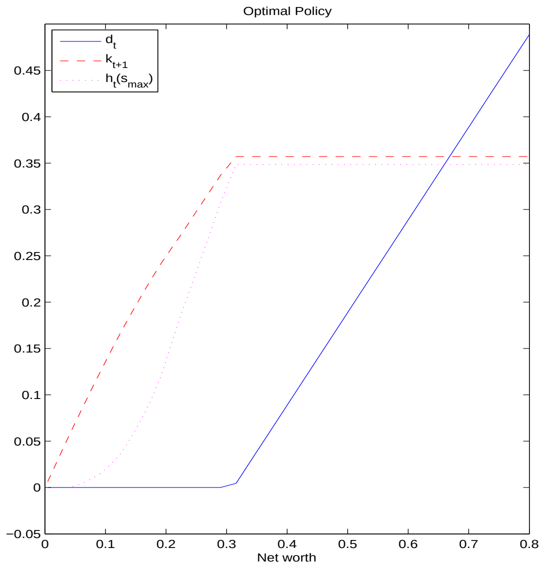
In this environment, policy functions at state s_t consist of five elements: dividends, capital purchases, and financial slack contingent on each of the three possible future states. Each of these five elements is a function of current net worth. Figure 3a plots dividends, capital purchases and financial slack saved for the highest-productivity future state, in which the firm grows faster.[18] These are the three functions that become positive for at least some sufficiently high level of net worth. No firm saves financial slack for the realization of the intermediate or worst cases of productivity. Therefore, firms borrow as much as possible against these two states, while conserving some debt capacity to speed up growth in case productivity improves.

These financial decisions clearly reflect a ranking of the marginal value of funds across future states of nature in which resources are more valuable in higher productivity states. Figure 3b plots the marginal value of internal funds in future states s_{t+1} given the current level of net worth and optimal policies. The left-most (dot-dashed) vertical line separates firms that exhaust their debt capacity, to its left, from firms that use their financial policy to save financial slack for the high productivity state. That optimal financial hedging decision prevents firms from being excessively constrained in the growth state and bounds the marginal value of funds in that state, as illustrated by the plateau reached by the dot-dashed curve in Figure 3b. The right-most (dotted) vertical line separates non-dividend-paying firms, to its left, from dividend-paying firms to its right. Dividend paying firms only differ in the dividend level, sharing a common policy for all other financial decisions and capital investment levels. As such, the marginal value of funds across states is the same for all these firms. Notice that they are still constrained and that some dispersion in the value of funds across states still remains.[19]

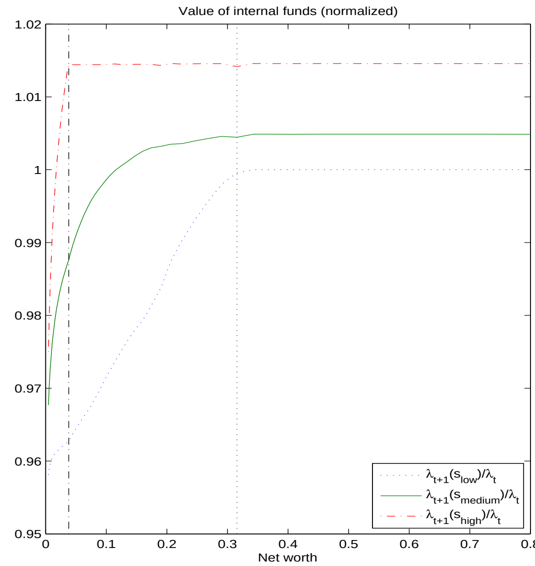
This dispersion in the value of funds is the driver of the hedging motive present in corporate decisions. Figure 4 plots a dispersion measure based on the ratio between the marginal value of funds in the highest productivity state for the next date and the lowest productivity one. The vertical lines again mark the first firm in size to save financial slack for the high productivity state and the first firm to pay out dividends. Notice that this ratio hits a maximum at the point in which hedging is the most valuable at the margin, for a firm which is just indifferent between exhausting debt capacity and saving financial slack, and reaches a plateau strictly above one for dividend paying firms. The most distorted firm sees a premium of about 5.2% on cash-flows that are delivered at the highest continuation state relative to the lowest one. This premium stabilizes at approximately 1.5% for firms that are sufficiently close to being unconstrained and start paying out dividends. While the magnitude of this premium is very small small if compared to risk-premia from

[18] All variables are scaled by the first-best level of capital, $k_{t+1}^{FB}(s_{medium})$, which solves $E\left[A(s_{t+1})|s_{medium}\right]\alpha k_{t+1}^{FB}(s_{medium}) = R + \delta - 1$.

[19] This essentially originates from the difference in discount factors between lenders and borrowers, combined with limited collateralization, which prevents the first-best level of investment from ever being reached.



(a) Optimal policies regarding dividends, capital, and financial slack left into the highest productivity state. Financial slack is uniformly zero for the remaining two states. All variables normalized by the first-best level of capital.



(b) Marginal value of internal funds at s_{t+1}, normalized by marginal value at s_t as a function of net worth.

Figure 3: Optimal Policies and marginal value of internal funds.

consumer-based asset pricing models, it can distort project selection in non-trivial ways. It is also important that is has an apparently reversed-sign in comparison: resources in relatively better, higher output and productivity states, are more valuable to the firm at the margin.

These observations motivate the study of the alternatives we evaluate. They both consist of short-run alternative projects, which can be undertaken for a single period only.[20] Alternative 1 is based on a project with returns that are riskier, but have the same expectation as the baseline technology: for any level of investment, they increase by an additional 10% relative to the median state when the high state is realized and drop by an analogous 10% when the low state is realized[21]. As such, they generate more resources when the constrained firm values them the most at a cost of less resources in the lowest productivity state. Additionally, given that the new project generates a different distribution of internal funds across states of nature, firms that undertake the alternative investment can also reoptimize on their financial decisions.

Figure 5a plots the gain from this alternative in terms of the relative increase in the value function measured in basis points. The maximum a firm gains is approximately 36 basis points, or a 0.36% increase in its value. Notice that the maximal gain is achieved close to the firm that has the highest dispersion in the

[20]There are two reasons for evaluating these short-term exposure changes. The first one is that, as future productivity is unaffected by the current exposure decision and a productivity change is entirely unexpected, the convexity of the profit function does not play any role. The second is that short-run changes naturally offer a lower bound on the gains from more flexible deviations.

[21]We implicitly assume that not only TFP changes, but also the value $t+1$ of capital after depreciation. Prices of new investment and future TFP are unaffected.

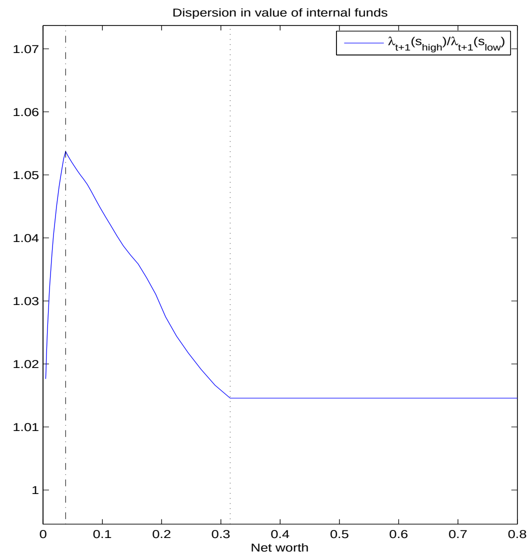
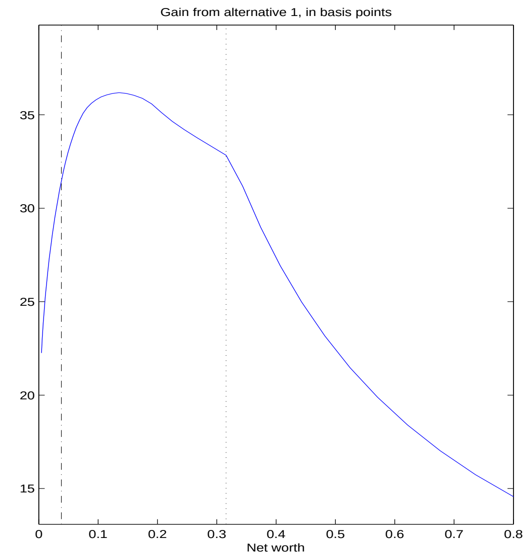


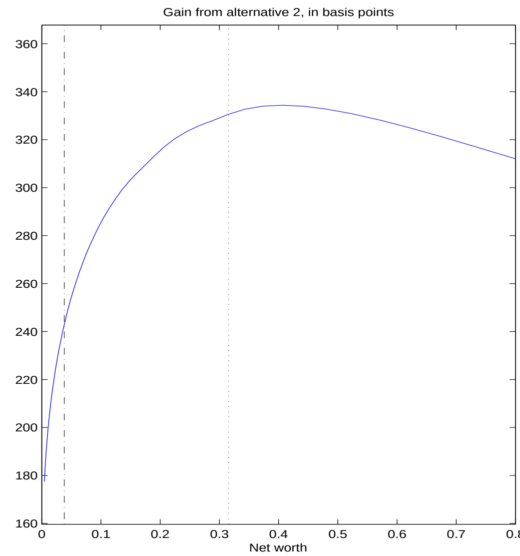
Figure 4: Dispersion in the marginal value of internal funds as measured by the ratio between the maximum value it achieves in a continuation state to its minimum.

marginal value of resources across states, but is not achieved exactly there. The difference originates from the additional gains from the reoptimization of financial decisions. Firms that previously saved debt capacity for the high productivity state can now scale down their use of hedging instruments, since the riskier project better aligns the availability of funds at $t+1$ with the marginal benefits from investment at that date. Thus, the selection of a riskier project acts a natural substitute to financial hedging.

It is also worth noting that even dividend-paying firms have smaller but non-negligible gains from this riskier project. Also, if there were a clear trade-off between increased risk and a lower return, smaller more constrained firms would be more willing to undertake riskier projects even if sacrificing a couple of dozen basis points of expected return.

While the evaluation of alternative 1 focuses on risk-taking in investment, we can also study the gains potentially achieved by a project that directly relaxes financial constraints. Towards that end, alternative 2 consists of the following change: it increases the pledgeability of capital by 10%, making each dollar of capital capable of backing the repayment of an additional 3.18 cents. The gains from the adaption of this alternative for a single period are displayed in Figure 5b, which shows that its adoption can increase the value function from around 1.8% to up 3.3% relative to the baseline value. As expected, since these gains are directly related to the value funds to a credit constrained firm and not to its relatively mild dispersion, they are an order of magnitude larger than the gains obtained from alternative 1.





(a) Relative increase in the value function, measured in basis points, from the adoption of a one period alternative project. Alternative project 1 has more dispersed returns than the baseline technology.

(b) Relative increase in the value function, measured in basis points, from the adoption of a one period alternative project. Alternative project 2 uses capital which can serve as better collateral.

Figure 5: Relative gains from the adoptions of alternative projects 1 and 2.

5 Conclusion

Limited enforcement constrains access to external funds and creates a hedging motive that distorts project selection even for firms that do not use financial instruments for hedging. Two key objects are intrinsically related and are responsible for the distortions illustrated: the shadow value of internal funds and the levered return on these funds.

While the hedging motive creates a desire to smooth out transitory cash-flow fluctuations, empirically reasonable levels of persistence in productivity shocks make constrained firms more willing to bear risks that are correlated with their productivity processes. Therefore, since this form of risk-taking facilitates self-financing, this paper illustrates a channel through which more constrained firms become endogenously more volatile. This increase in volatility is indeed an optimal response to the financial constraints firms are subject to.

We have also emphasized the importance of leverage and illustrated that sufficiently constrained firms are willing to take on more exposure to credit conditions. Leverage makes internal funds complementary to external funds and can make resources more valuable to the firm when credit conditions are slacker. As a consequence, constrained firms might show a risk-taking attitude towards their exposure to external credit conditions.

Appendix

In this appendix, we collect all proofs omitted from the main text. To make notation more concise, we use the multiplier $\lambda\left(s_t\right)=\frac{\partial V\left(w_t,j,s^t\right)}{\partial w_t}$ to denote the marginal value of funds.

Proof of Proposition 2. Marginal changes in j lead to changes in cash-flows, prices, and recovery rates; each of these contributes with some marginal effects into the value function at s^0. These are represented by $\sum_{t=1}^{T} E\left[\beta^t \lambda\left(s^t\right)\frac{\partial F\left(k_t^*,j,s^t\right)}{\partial j}\right]$ for the discounted value of changes in cash-flows, $\frac{\partial V_0}{\partial q\left(s^t\right)}=\beta^t\pi\left(s^t\right)\lambda_t\left(s^t\right)\left\{-i^*\left(s^t\right)+\mu\left(s^t\right)(1-\delta)\,\theta\left(j,s^t\right)k_t^*\left(s^{t-1}\right)\right\}$ for a change in each capital price, and $\frac{\partial V_0}{\partial\theta\left(s^t\right)}=\beta^t\pi\left(s^{t+1}\right)\lambda_t\left(s^t\right)\mu\left(s^t\right)q\left(s^t\right)(1-\delta)\,k_t^*\left(s^t\right)$ for a change in the recovery rate. Combining these terms and using definitions 22 and 23 for borrowing capacity and net cash-flows, we obtain expression 24. □

Proof of Proposition 3. From the first-order conditions in (10)-(12), the marginal value of funds at s^1 is described by

$$\lambda\left(s^1\right)=\begin{cases}1, & \text{if } \frac{A\left(s^2\right)\left[A\left(s^1\right)k_0\right]^{\alpha-1}}{\wp}\le 1,\\ \frac{A\left(s^2\right)\left[A\left(s^1\right)k_0\right]^{\alpha-1}}{\wp}, & \text{if } \lambda\left(s^0\right)>\frac{A\left(s^2\right)\left[A\left(s^1\right)k_0\right]^{\alpha-1}}{\wp}>1,\\ \lambda\left(s^0\right), & \text{if } \frac{A\left(s^2\right)\left[A\left(s^1\right)k_0\right]^{\alpha-1}}{\wp}\ge\lambda\left(s^0\right).\end{cases}\tag{27}$$

Take $\overline{\lambda_1}\equiv\max_{s^1}\lambda\left(s^1\right)$. Notice that $k_0\le\frac{w_0}{\wp}$, so $\lambda\left(s^0\right)\ge E\left[\frac{A\left(s^1\right)}{\wp}\left(\frac{w_0}{\wp}\right)^{\alpha-1}\lambda\left(s^1\right)\right]$ and $\frac{\lambda\left(s^0\right)}{\overline{\lambda_1}}\ge E\left[\frac{A\left(s^1\right)}{\wp}\frac{\lambda\left(s^1\right)}{\overline{\lambda_1}}\right]\left(\frac{w_0}{\wp}\right)^{\alpha-1}$. Notice that $\frac{\lambda\left(s^1\right)}{\overline{\lambda_1}}$ remains bounded from below and $\lim_{w_0\to 0}\left(\frac{w_0}{\wp}\right)^{\alpha-1}=\infty$. Thus, for w_0 sufficiently close to zero $\lambda\left(s^0\right)>\max_{s^1}\lambda\left(s^1\right)$ and $h\left(s^1\right)=0$ for all $s^1\in S^1$. □

Proof of Proposition 4. We use expression 27 and write $A\left(s^2\right)=\overline{A}\left(1+\epsilon\right)^{\rho}$ and $A\left(s^1\right)=\overline{A}\left(1+\epsilon\right)$. Notice then that $\lambda\left(s^1\right)$ is monotonically increasing the product $\left(1+\epsilon\right)^{\rho}\left(1+\epsilon\right)^{\alpha-1}$. This product is increasing (decreasing) in ϵ, whenever $\rho+\alpha>(<)1$. □

Proof of Proposition 5. In this particular case of expression 24, firms evaluate a change in a project cash-flow according to

$$\left.\frac{\partial V_0}{\partial j}\right|_{j=\bar{j}}=\frac{\beta}{\alpha}E\left[\lambda_1\left(s_1\right)\frac{\partial A_1\left(\bar{j},s^1\right)}{\partial j}\right]k_1^{\alpha}=\frac{\beta}{\alpha}E\left[\lambda_1\left(s_1\right)\epsilon\right]k_1^{\alpha}.$$

Given that ϵ has zero mean, $E\left[\lambda_1\left(s_1\right)\epsilon\right]=Cov\left(\lambda\left(s^1\right),\epsilon\right)$. The second part of the proposition follows from combining the covariance condition with Proposition 4. □

Proof of Proposition 6. Whenever $w_0<\underline{w}$, the solution to the firm's problem features $h\left(s^1\right)=0$ for each $s^1\in S^1$ as well as $d=0$ for any initial net worth in a neighborhood of w_0. Therefore, in this neighborhood, the value function is given by $V\left(w_0,j,s^0\right)=E\left[V\left(\frac{A\left(j,s^1\right)\left(\frac{w_0}{\wp}\right)^{\alpha}}{\alpha\wp},0,s^1\right)\right]$. Given that $V\left(\frac{A\left(j,s^1\right)\left(\frac{w_0}{\wp}\right)^{\alpha}}{\alpha\wp},0,s^1\right)$

is concave in its first argument (net worth)[22] and $A\left(j,s^1\right)$ is linear in j, $V\left(w_0,j,s^0\right)$ is concave in j. Additionally, concavity is strict as long as the firm finds itself constrained in at least one (positive-probability) $s^1 \in S^1$. $\square$

Proofs from Section 3.2

We have $\lambda_1\left(s^1\right) = \min\left\{\frac{A\left(\frac{w_1\left(s^1\right)}{\wp\left(s^1\right)}\right)^{\alpha-1}+\left(1-\theta_2\left(s^2\right)\right)}{\wp\left(s^1\right)},1\right\}$ for $s^1 = s_h^1, s_l^1$ and s^2 being its unique successor. Given that $\wp\left(s^1\right) = 1-\theta_2\left(s^2\right)$, it simplifies to $\lambda_1\left(s^1\right) = 1 + A\left[w_1\left(s^1\right)\right]^{\alpha-1}\left[\wp\left(s^1\right)\right]^{-\alpha} > 1$. This expression is decreasing in both $\wp\left(s^1\right)$ and $w_1\left(s^1\right)$. As a consequence, it is increasing in $\theta\left(s^2\right)$.

Lemma 1. *In the environment described in example 2, $\lambda_1\left(s_h^1\right) > \lambda_1\left(s_l^1\right) > 1$.*

Proof. There are two cases to consider: $h\left(s_h^1\right) = 0$ and $h\left(s_h^1\right) > 0$. In the former, $w_1\left(s_h^1\right) = \frac{A}{\alpha}k_0^\alpha + \left(1-\theta_0\right)k_0 \leq w_1\left(s_l^1\right)$. Then, $\lambda_1\left(s_h^1\right) = 1 + A\left[w_1\left(s_h^1\right)\right]^{\alpha-1}\left[\wp\left(s_h^1\right)\right]^{-\alpha} \geq 1 + A\left[w_1\left(s_l^1\right)\right]^{\alpha-1}\left[\wp\left(s_h^1\right)\right]^{-\alpha} > 1 + A\left[w_1\left(s_l^1\right)\right]^{\alpha-1}\left[\wp\left(s_l^1\right)\right]^{-\alpha} = \lambda_1\left(s_l^1\right)$. In the latter, we need $\lambda_1\left(s_h^1\right) = \lambda_0 \geq \lambda_1\left(s_l^1\right)$. If equality between all three multipliers were to happen, the investment Euler equation would establish that $\lambda_0 = \left(1 + A\left[w_0\right]^{\alpha-1}\left[\varrho_0\right]^{-\alpha}\right)E\left[\lambda_1\left(s^1\right)\right] \implies 1 = 1 + A\left[w_0\right]^{\alpha-1}\left[\varrho_0\right]^{-\alpha} > 1$ reaching a contradiction. $\square$

Proof of Proposition 7. Both statements in Proposition (7) follow from the lemma above. The first one is immediate. For the second one, we argue that since $\lambda\left(s^1\right) > 0$ for $s^1 = s_h^1, s_l^1$, we get that firms resort to maximal leverage at $t = 1$ and $k_1\left(s_h^1\right) \geq \frac{\frac{A}{\alpha}k_0^\alpha+(1-\theta_0)k_0}{\wp\left(s_h^1\right)} > \frac{\frac{A}{\alpha}k_0^\alpha+(1-\theta_0)k_0}{\wp\left(s_l^1\right)} = k_2\left(s_l^1\right)$ where the last equality follows from the fact that $\lambda_0 \geq \lambda_1\left(s_h^1\right) > \lambda_1\left(s_l^1\right)$, which ensures that $h\left(s_l^1\right) = 0$. $\square$

Proof of Proposition 8. When changes occur with respect to $\theta\left(j,s^2\right)$ only, projects are evaluated according to

$$\frac{\partial V_0}{\partial j} = \beta E\left[\left(\lambda\left(s^1\right) - \frac{\beta\lambda\left(s^2\right)}{R}\right)\frac{\partial\theta\left(j,s^2\right)}{\partial j}k_2\left(s^1\right)\right],$$

which simplifies further, given that $\beta = R = \lambda\left(s^2\right) = 1$. Additionally, we describe projects ordered as mean preserving spreads $\theta\left(j,s^2\right)$ with $\theta\left(j,s^2\right) = \theta\left(\bar{j},s^2\right) + \gamma\left(j\right)\Delta\left(s^2\right)$ where $E\left[\Delta\right] = 0$ and $\Delta\left(s^2 = s_h\right) > 0 > \Delta\left(s^2 = s_h\right)$ and $\gamma\left(j\right)$ as a smooth increasing function with a root in $\bar{j}$. Then,

$$\begin{aligned}\frac{\partial V_0}{\partial j}|_{\bar{j}} &= \gamma'\left(j\right)E\left[\left(\lambda\left(s^1\right)-1\right)k_2\left(s^1\right)\Delta\left(s^2\right)\right] \\ &\propto \left(\lambda\left(s_h^1\right)-1\right)k_2\left(s_h^1\right) - \left(\lambda\left(s_l^1\right)-1\right)k_2\left(s_l^1\right).\end{aligned}$$

Given both statements in Proposition (7), we can sign this as a positive term. $\square$

[22] A proof is omitted here, but follows trivially from the s^1 continuation problem, which features a concave production function.

References

Acemoglu, D., and F. Zilibotti (1997): "Was Prometheus unbound by chance? Risk, diversification, and growth," *Journal of political economy*, 105(4), 709–751.

Acharya, V. V., Y. Amihud, and L. Litov (2011): "Creditor rights and corporate risk-taking," *Journal of Financial Economics*, 102(1), 150–166.

Aghion, P., G.-M. Angeletos, A. Banerjee, and K. Manova (2010): "Volatility and growth: Credit constraints and the composition of investment," *Journal of Monetary Economics*, 57(3), 246–265.

Albuquerque, R., and H. A. Hopenhayn (2004): "Optimal lending contracts and firm dynamics," *The Review of Economic Studies*, 71(2), 285–315.

Almeida, H., M. Campello, and M. S. Weisbach (2011): "Corporate financial and investment policies when future financing is not frictionless," *Journal of Corporate Finance*, 17(3), 675–693.

Alvarez, F., and U. J. Jermann (2000): "Efficiency, equilibrium, and asset pricing with risk of default," *Econometrica*, 68(4), 775–797.

Bolton, P., H. Chen, and N. Wang (2011): "A unified theory of Tobin's q, corporate investment, financing, and risk management," *The Journal of Finance*, 66(5), 1545–1578.

Claessens, S., S. Djankov, and T. Nenova (2001): "Corporate growth and risk around the world," *Financial crises in emerging markets*, (2001-2007), 305.

Clementi, G. L., and H. A. Hopenhayn (2006): "A theory of financing constraints and firm dynamics," *The Quarterly Journal of Economics*, 121(1), 229–265.

Collard-Wexler, A., J. Asker, and J. De Loecker (2011): "Productivity volatility and the misallocation of resources in developing economies," Discussion paper, National Bureau of Economic Research.

Cooper, R. W., and J. C. Haltiwanger (2006): "On the nature of capital adjustment costs," *The Review of Economic Studies*, 73(3), 611–633.

Davis, S. J., J. Haltiwanger, R. Jarmin, and J. Miranda (2007): "Volatility and dispersion in business growth rates: Publicly traded versus privately held firms," in *NBER Macroeconomics Annual 2006, Volume 21*, pp. 107–180. MIT Press.

DeMarzo, P. M., M. J. Fishman, Z. He, and N. Wang (2012): "Dynamic agency and the q theory of investment," *The Journal of Finance*, 67(6), 2295–2340.

D'Erasmo, P. N., and H. J. Moscoso Boedo (2013): "Intangibles and Endogenous Firm Volatility over the Business Cycle," *manuscript, University of Virginia.*

Di Tella, S. (2012): "Uncertainty Shocks and Balance Sheet Recessions," Discussion paper, mimeo, MIT.

Eisfeldt, A. L., and A. A. Rampini (2007): "New or used? Investment with credit constraints," *Journal of Monetary Economics*, 54(8), 2656–2681.

Froot, K. A., D. S. Scharfstein, and J. C. Stein (1993): "Risk Managements Coordinating Corporate Investment and Financing Policies," *the Journal of Finance*, 48(5), 1629–1658.

Gomes, J. F. (2001): "Financing investment," *American Economic Review*, pp. 1263–1285.

Greenwood, J., and B. Jovanovic (1990): "Financial Development, Growth, and the Distribution of Income," *Journal of Political Economy*, pp. 1076–1107.

Harris, M., and A. Raviv (1996): "The capital budgeting process: Incentives and information," *The Journal of Finance*, 51(4), 1139–1174.

——— (1998): "Capital budgeting and delegation," *Journal of Financial Economics*, 50(3), 259–289.

He, Z., and A. Krishnamurthy (2012): "A model of capital and crises," *The Review of Economic Studies*, 79(2), 735–777.

Hennessy, C. A., and T. M. Whited (2005): "Debt dynamics," *The Journal of Finance*, 60(3), 1129–1165.

Holmström, B., and J. Tirole (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106(1), 1–40.

——— (2000): "Liquidity and risk management," *Journal of Money, Credit and Banking*, pp. 295–319.

——— (2011): *Inside and outside liquidity.*

Iachan, F. S. (2012): "Essays in macroeconomics: liquidity and taxation," Ph.D. thesis, Massachusetts Institute of Technology.

Jensen, M. C., and W. H. Meckling (1976): "Theory of the firm: Managerial behavior, agency costs and ownership structure," *Journal of financial economics*, 3(4), 305–360.

John, K., L. Litov, and B. Yeung (2008): "Corporate Governance and Risk-Taking," *The Journal of Finance*, 63(4), 1679–1728.

Kehoe, T. J., and D. K. Levine (1993): "Debt-constrained asset markets," *The Review of Economic Studies*, 60(4), 865–888.

Khan, A., and J. K. Thomas (2003): "Nonconvex factor adjustments in equilibrium business cycle models: do nonlinearities matter?," *Journal of monetary economics*, 50(2), 331–360.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105(2).

Koren, M., and S. Tenreyro (2007): "Volatility and development," *The Quarterly Journal of Economics*, 122(1), 243–287.

——— (2013): "Technological diversification," *The American Economic Review*, 103(1), 378–414.

Krishnamurthy, A. (2003): "Collateral constraints and the amplification mechanism," *Journal of Economic Theory*, 111(2), 277–292.

Landier, A., D. Sraer, and D. Thesmar (2011): "The risk-shifting hypothesis: Evidence from subprime originations," *TSE Working Paper*, 11.

Li, S., T. M. Whited, and Y. Wu (2014): "Collateral, Taxes, and Leverage," *Available at SSRN 2360391.*

Matsuyama, K. (2007): "Credit traps and credit cycles," *The American Economic Review*, 97(1), 503–516.

——— (2008): "Aggregate implications of credit market imperfections," in *NBER Macroeconomics Annual 2007, Volume 22*, pp. 1–60. University of Chicago Press.

Mello, A. S., and J. E. Parsons (2000): "Hedging and liquidity," *Review of Financial Studies*, 13(1), 127–153.

Midrigan, V., and D. Y. Xu (2013): "Finance and misallocation: Evidence from plant-level data," Discussion paper.

Myers, S. C. (1974): "Interactions of corporate financing and investment decisions: implications for capital budgeting," *The Journal of finance*, 29(1), 1–25.

Rajan, R., H. Servaes, and L. Zingales (2000): "The cost of diversity: The diversification discount and inefficient investment," *The Journal of Finance*, 55(1), 35–80.

Rampini, A., A. Sufi, and S. Viswanathan (2011): "Dynamic risk management," *Dynamic Risk Management (March 1, 2012).*

Rampini, A., and S. Viswanathan (2010): "Collateral, risk management, and the distribution of debt capacity," *The Journal of Finance*, 65(6), 2293–2322.

——— (2013): "Collateral and capital structure," *Journal of Financial Economics.*

Stein, J. C. (2002): "Information production and capital allocation: Decentralized versus hierarchical firms," *The Journal of Finance*, 57(5), 1891–1921.

——— (2003): "Agency, information and corporate investment," *Handbook of the Economics of Finance*, 1, 111–165.

Tauchen, G., and R. Hussey (1991): "Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models," *Econometrica: Journal of the Econometric Society*, pp. 371–396.

Tufano, P. (1996): "Who manages risk? An empirical examination of risk management practices in the gold mining industry," *The Journal of Finance*, 51(4), 1097–1137.

Vereshchagina, G., and H. A. Hopenhayn (2009): "Risk Taking by Entrepreneurs," *The American Economic Review*, 99(5), 1808–1830.