Horizon Effects in Average Returns: The Role of Slow Information Diffusion

Oliver Boguth, Murray Carlson, Adlai Fisher, and Mikhail Simutin*

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ABSTRACT

We show that when stocks incorporate information slowly, observed short-horizon portfolio returns are downward-biased. Buy-and-hold strategies amplify the effect when a systematic shock diffuses at heterogeneous speeds across different assets. In contrast, existing theories analyze price noises that are independent of fundamentals, and buy-and-hold portfolio returns are unaffected. Confirming the new predictions, downward bias in average returns reaches 10% annualized in daily and monthly style portfolios and international indices. Slow reaction to market information, identified by gradually declining lagged betas, is an important cause. These findings have natural consequences for performance evaluation.

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Abstract

We show that when stocks incorporate information slowly, observed short-horizon portfolio returns are downward-biased. Buy-and-hold strategies amplify the effect when a systematic shock diffuses at heterogeneous speeds across different assets. In contrast, existing theories analyze price noises that are independent of fundamentals, and buy-and-hold portfolio returns are unaffected. Confirming the new predictions, downward bias in average returns reaches 10% annualized in daily and monthly style portfolios and international indices. Slow reaction to market information, identified by gradually declining lagged betas, is an important cause. These findings have natural consequences for performance evaluation.

1. Introduction

When assessing a trading strategy or an asset pricing model, researchers can choose daily, monthly, annual, or other intervals over which to evaluate returns.[1] Accepted wisdom holds that any return-measurement interval can be empirically valid, with daily returns being most appropriate for investors with a daily horizon, monthly returns matching to investors with a monthly horizon, and so forth.[2]

The microstructure literature offers a caveat: Short-horizon mean returns can be biased. Blume and Stambaugh ("BS", 1983) and Roll (1983) assume measurement errors in prices uncorrelated with fundamental value, and show upward bias in the mean returns of individual stocks and equally-weighted, periodically-rebalanced portfolios. Asparouhova, Bessembinder, and Kalcheva ("ABK", 2010, 2013) demonstrate a related bias in Fama-MacBeth regressions and propose corrections.[3] This literature provides a key takeaway: The mean returns of well-diversified, value-weighted portfolios are unbiased. Indeed, ABK's corrections link portfolio weights to past returns, as occurs naturally under value-weighting. They anticipate (p. 46) that new research considering measurement errors correlated with fundamentals could be important.

We fill this gap by modelling cross-sectional heterogeneity in the speed at which prices adjust to fundamentals. Early analysis of price-adjustment frictions includes Fisher (1966), Scholes and Williams (1977), and Lo and MacKinlay ("LM", 1990), who

[1] Monthly returns are traditional, as in Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), and Fama and French (1992, 1993). Jagannathan and Wang (2007) propose that annual returns more accurately reflect the pricing of fundamental risks. Numerous studies use higher-frequency daily data (Lynch and Musto (2003), Bollen and Busse (2005), Busse and Irvine (2006), Lewellen and Nagel (2006), Barber (2007), Li and Yang (2011), Tetlock (2011), and Ang and Kristensen (2012)).

[2] Fama (1998, p. 294) comments regarding the choice of horizon, *"Long-term investor experience is better captured by compounding short-term returns to obtain long-term buy-and-hold returns... Investor experience is interesting, and long-term [abnormal returns] are thus interesting. But formal tests for abnormal returns should use the return metric called for by the model invoked to estimate expected... returns. The problem, of course, is that discrete-time asset pricing models are silent on the relevant interval for expected returns."*

[3] Other work on noise uncorrelated with fundamentals includes Conrad and Kaul (1993), Canina, Michaely, Thaler, and Womack (1998), Liu and Strong (2008), and Brennan and Wang (2010).

study "nonsynchronous" trade.[4] When securities do not trade simultaneously, observed last transaction prices reflect common information with different delays, generating positive portfolio autocorrelations and cross-serial correlations, impacting beta estimates, and biasing portfolio variance downwards. Empirical effects are however too large, long-lasting, and robust to methodological corrections to be explained entirely by nonsynchronous trade.[5] Broader explanations for slow price reaction include transaction and information-processing costs, dynamic strategies of market-makers and informed traders, and frictions in the movement of capital. Consistent with these explanations, firm size, complexity, and various measures of investor attention influence the speed at which stocks react to news.[6] We refer to these phenomena collectively as "slow information diffusion." In the special case where stocks react to systematic information at different speeds, we refer to "heterogeneous information diffusion."

Slow information diffusion impacts average returns differently than the independent price noise in BS, ABK, and other previous studies. First, slow information diffusion implies a downward- rather than upward-bias in mean returns. Second, buy-and-hold strategies, such as value-weighting, accentuate the downward bias when stocks incorporate systematic information with heterogeneous delay. The corrections suggested by ABK, which depend on mimicking the properties of buy-and-hold portfolios, therefore are not effective remedies.

To understand these theoretical results, consider first a single asset. Slow information diffusion smooths observed returns relative to fundamentals, raising return autocorrelations, and biasing variance downwards. Average logarithmic returns, which are

[4]Campbell, Lo, and MacKinlay (1997) provide a concise overview of the nonsynchronous trade literature.

[5]Fisher (1966) argues that price-adjustment delays cannot be explained by nonsynchronous trade alone. Cohen, Hawawini, Maier, Schwartz, and Whitcomb (1983), Atchison, Butler, and Simonds (1987), Conrad and Kaul (1988), LM, Mech (1993), Boudoukh, Richardson, and Whitelaw (1994), and others make similar points.

[6]See LM, Brennan, Jegadeesh, and Swaminathan (1993), Chan (1993), Badrinath, Kale, and Noe (1995), Sias and Starks (1997), Hong and Stein (1999), Chordia and Swaminathan (2000), Hong, Lim, and Stein (2000), Hirshleifer and Teoh (2003), Hou and Moskowitz (2005), Hou (2007), Cohen and Frazzini (2008), Dellavigna and Pollet (2009), Hirshleifer, Lim, and Teoh (2009, 2011), Menzly and Ozbas (2010), Chordia, Sarkar, and Subrahmanyam (2011), and Cohen and Lou (2012).

additive, are unaffected.[7] Average simple returns, however, require a Jensen's inequality adjustment. Since variance is downward-biased, average simple returns must be downward-biased as well.

Heterogeneous information diffusion causes an additional downward bias in buy-and-hold portfolios. In the wake of a positive systematic shock to fundamentals, lagger stocks become underweighted as they fall behind the news, while their future returns must be relatively high to catch up with the news. Conversely, following a negative systematic shock, laggers become overweighted and will underperform. This negative cross-sectional correlation between portfolio weights and future returns contributes to downward bias in average portfolio returns.

To further illuminate the link between autocorrelations and horizon effects in average returns, we develop an empirically accurate approximation for the relation between average returns at different time scales. The formula builds on the standard Jensen's inequality adjustment. The key drivers of horizon effects in average returns are the level of portfolio volatility and variance ratios of long- to short-horizon volatility. When a variance ratio exceeds one, indicating persistence, the average short-horizon return rescaled linearly or by compounding understates the corresponding longer-horizon return.

Consistent with predictions, we show large apparent downward bias in the short-horizon returns of a variety of style portfolios and international indices. The magnitudes exceed 10% annually in some style portfolios and international indices, with effects strongest for the portfolios suggested by theory: small stocks, low-price stocks, momentum losers, high volatility, and among international indices, emerging and frontier markets. The effects are strongest at short horizons such as one day, but remain

[7] The literature on nonsynchronous trade concludes that implications for mean returns are innocuous based on the properties of logarithmic returns. Scholes and Williams write, *"...expectations of measured returns* [...] *always equal true mean returns"* (p. 113). LM state, *"... nontrading does not affect the mean of observed returns"* (p. 187). In contrast, investors and empirical researchers often draw inferences from simple returns, as for example when comparing simple means, calculating alphas from time-series regressions, or carrying out standard cross-sectional asset pricing tests.

economically meaningful even in monthly returns for some portfolios.

Horizon effects in average returns naturally impact standard performance measures, such as Jensen's (1968) alpha. Controlling for "intervalling" effects in beta (e.g., Dimson, 1979), the alpha of the small-stock portfolio is 65 basis points per quarter and insignificant in daily data, but 1.59 percent and significant in quarterly data. The low-minus-high volatility portfolio generates a significant alpha of 1.75 percent per quarter in daily data, but an insignificant alpha of 68 basis points in quarterly data.

We test whether the effects in short-horizon returns should be interpreted as a downward bias due to slow information diffusion, as opposed to being driven by another source such as positive autocorrelations from time-varying risk premia (e.g., Conrad and Kaul, 1988). For each portfolio, we regress returns on the contemporaneous market index and a set of lagged observations of the index. We use the ratio of the sum of the lagged loadings to the sum of all loadings as a proxy for slow information diffusion, as in Brennan, Jegadeesh, and Swaminathan (1993). Across portfolios, we find a statistically significant relationship between the magnitude of horizon effects in average returns and the importance of lagged market information.

We also structurally estimate the theoretical model of heterogeneous information diffusion, which illuminates the key drivers of horizon effects. Using only five parameters, we closely match portfolio autocorrelations, lagged betas, variance ratios, and horizon effects in average returns. More limited objectives have proven to be a significant challenge in earlier literature (e.g., LM; Boudoukh, Richardson, and Whitelaw, "BRW", 1994). Slow reaction to market information, identified by gradually declining lagged betas, plays a key role in explaining the horizon effects in average returns.

In the process of these investigations, we assess the tradeability of the measured average returns and demonstrate the robustness of horizon effects in subsamples of the data. We also offer advice to empirical researchers on the implications of these findings for methodology, and illustrate potential corrections when using short-horizon returns.

2. A Model of Heterogeneous Information Diffusion

We model a cross-section of N stocks with heterogeneous delays in their price response to news. The "fundamental value" of an individual stock has logarithmic returns

$$r_{it}^{*} = r_f + \sum_{k=1}^{K} \beta_{ik} f_{kt} + \varepsilon_{it}, \tag{1}$$

where r_f is the riskless rate, β_{ik} are loadings, ε_{it} are idiosyncratic shocks, and the realizations f_{kt} represent systematic news. To simplify notation, we rewrite (1) as:

$$r_{it}^{*} = r_f + \sum_{k=1}^{K+N} \beta_{ik} f_{kt}, \tag{2}$$

where the idiosyncratic returns are recast as "factors" by the relation $f_{K+i,t} \equiv \varepsilon_{it}$ for $i \in \{1, ..., N\}$, implying that $\beta_{i,K+j} = 1$ for $i = j$ and zero otherwise. The realizations f_{kt} are independent, normally distributed random variables with mean μ_k and standard deviation σ_k. We normalize the mean of the idiosyncratic shocks to have no effect on simple returns by the Jensen's inequality adjustment $\mu_k = -\frac{1}{2}\sigma_k^2$, $k > K$.

To model heterogeneous information diffusion in returns, we assign each firm i delay parameters $\delta_{ik} \in [0, 1)$ that determine speeds of adjustment to factor news,[8] and we track accumulated information "deficits" using state variables D_{ikt}:

$$\begin{aligned} r_{it} &= r_f + \sum_{k=1}^{K+N} (1 - \delta_{ik}) \left(D_{ik,t-1} + \beta_{ik} f_{kt} \right) && (3) \\ D_{ikt} &= \delta_{ik} \left(D_{ik,t-1} + \beta_{ik} f_{kt} \right). && (4) \end{aligned}$$

When fundamental news arrives, it is added to any existing information deficit $D_{ik,t-1}$. A proportion $(1 - \delta_{ik})$ immediately incorporates into observed returns r_{it}, leaving the proportion δ_{ik} to be revealed in the future.

[8] The model is similar quantitatively, but less tractable, if δ_{ik} follows a Markov chain.

In the remainder of this section, we derive analytical results under the simplifying assumptions of a single common factor ($K = 1$) and two types of securities: "leaders" incorporate fundamentals immediately ($\delta_{ik} = 0$); "laggers" have a common delay $\delta_{ik} = \delta > 0$. For notational convenience, we assume that $r_f = 0$ and $\beta_{i1} = 1$ for all i. The logarithmic returns of leaders then equal their fundamental returns:

$$r_{it}^* = f_{1t} + f_{1+i,t}, \tag{5}$$

while the returns of laggers follow:

$$r_{it} = (1-\delta)\left(D_{i1,t-1} + f_{1t} + D_{i,1+i,t-1} + f_{1+i,t}\right) \tag{6}$$

$$D_{ikt} = \delta(D_{ik,t-1} + f_{kt}). \tag{7}$$

We obtain closed-form expressions for unconditional distributions:

Proposition 1 *Lagger short-run log returns are unconditionally normal with* $\mathbb{E}(r_{it}) = \mu_1 - \frac{1}{2}\sigma_{1+i}^2 = \mathbb{E}(r_{it}^*)$ *and* $\mathrm{Var}(r_{it}) = \frac{1-\delta}{1+\delta}(\sigma_1^2 + \sigma_{1+i}^2) < \sigma_1^2 + \sigma_{1+i}^2 = \mathrm{Var}(r_{it}^*)$. *Mean simple returns are* $\mathbb{E}(e^{r_{it}}) = e^{\mu_1 + \frac{1}{2}\left[\frac{1-\delta}{1+\delta}\right]\sigma_1^2 - \frac{\delta}{1+\delta}\sigma_{1+i}^2} < e^{\mu_1 + \frac{1}{2}\sigma_1^2} = \mathbb{E}(e^{r_{it}^*})$.

Leader and lagger logarithmic return means are equal. However, lagger logarithmic returns are a weighted average of current and past factor realizations, which smooths returns and reduces volatility. By Jensen's inequality, the lagger average simple return is also lower. In contrast, when noise is independent of fundamentals (e.g., BS, ABK), Jensen's inequality biases the mean return upwards.

We next characterize the returns of single-type stock portfolios.

Proposition 2 *The returns* r_t *on a well-diversified, value-weighted portfolio of laggers are unconditionally normally distributed with* $\mathbb{E}(r_t) = \mu_1 = \mathbb{E}(r_t^*)$ *and* $\mathrm{Var}(r_t) = \frac{1-\delta}{1+\delta}\sigma_1^2 < \sigma_1^2 = \mathrm{Var}(r_t^*)$. *Average simple returns are* $\mathbb{E}\left(e^{r_t}\right) = e^{\mu_1 + \frac{1}{2}\left[\frac{1-\delta}{1+\delta}\right]\sigma_1^2} < e^{\mu_1 + \frac{1}{2}\sigma_1^2} = \mathbb{E}\left(e^{r_t^*}\right)$. *The returns on a well-diversified, value-weighted portfolio of leaders are always equal to the fundamental return* r_t^*.

Similar to individual stocks, the lagger-portfolio logarithmic returns are unbiased, while the variance and simple return are downward biased. In BS, ABK, and related studies, value-weighting eliminates the bias in average returns in well-diversified portfolios, but this does not apply to slow price adjustment. Slow reaction to idiosyncratic news diversifies away, but slow reaction to systematic news, which is widely documented in the literature (see footnote 6), does not.

We finally characterize the returns of a heterogeneous-type portfolio.

Proposition 3 *Let R_{Pt} denote the simple return on a well-diversified, value-weighted portfolio of leaders and laggers with a fraction π of leader stocks. The returns follow:*

$$R_{Pt} = (1 - w_{t-1})e^{r_t^*} + w_{t-1}e^{r_t}, \tag{8}$$

$$w_{t-1} = \frac{1-\pi}{\pi e^{D_{1,t-1}} + 1 - \pi}, \tag{9}$$

where D_{1t} is the common systematic delay state for laggers and w_{t-1} is the lagger portfolio weight. The mean return is:

$$\mathbb{E}(R_{Pt}) = \mathbb{E}(1 - w_{t-1})\mathbb{E}\left(e^{r_t^*}\right) + \mathbb{E}(w_{t-1})\mathbb{E}\left(e^{r_t}\right) + \text{Cov}\left(w_{t-1}, e^{r_t}\right), \tag{10}$$

where $\text{Cov}\left(w_{t-1}, e^{r_t}\right) < 0$*.*

The proposition identifies two important sources of bias.[9] First, the downward bias in lagger-portfolio returns (Proposition 2) carries through to the heterogeneous portfolio. Second, the lagger portfolio weights covary negatively with their future expected returns, as captured by the third term of (10). Following a high factor return, the lagger weight is artificially low, while their future returns are expected to be high. Following a negative systematic shock, laggers are overweighted and are expected to underperform. This effect stems from different stocks adjusting at different speeds.

[9] Additionally, $\mathbb{E}(1 - w_{t-1}) \neq \pi$, but this effect is small.

3. Horizon Effects in Average Returns

To further clarify the link between autocorrelations, variances, and horizon effects in average returns, we develop an approximation for the relation between average returns at different time scales. Let $\bar{R}_{in} \equiv \mathbb{E}(R_{i,t+1} \dots R_{i,t+n})$ denote the mean gross return of asset i over n periods. We consider geometrically and linearly rescaled means:

$$\bar{R}_{in}^{RS} \equiv \left[\bar{R}_{i1}\right]^n, \tag{11}$$

$$\bar{R}_{in}^{RSl} \equiv 1 + n\left(\bar{R}_{i1} - 1\right), \tag{12}$$

as well as the associated ratios:

$$\nu_{in} \equiv \bar{R}_{in}^{RS}/\bar{R}_{in}, \quad \nu_{in}^{net} \equiv \frac{\bar{R}_{in}^{RS} - 1}{\bar{R}_{in} - 1} = \nu_{in} + \frac{\nu_{in} - 1}{\bar{R}_{in} - 1}, \tag{13}$$

$$\nu_{in}^{l} \equiv \bar{R}_{in}^{RSl}/\bar{R}_{in}, \quad \nu_{in}^{l,net} \equiv \frac{\bar{R}_{in}^{RSl} - 1}{\bar{R}_{in} - 1} = \nu_{in}^{l} + \frac{\nu_{in}^{l} - 1}{\bar{R}_{in} - 1}. \tag{14}$$

Linearly rescaled returns are implicitly relevant whenever an empiricist calculates abnormal performance from a factor-model regression (see Section 4.1.1). Geometric rescaling differs from linear rescaling by the effects of compounding.

The approximation is based on normally distributed single- and n-period logarithmic returns: $r_{i1t} \sim \mathcal{N}(\mu_i, \sigma_i^2)$, $r_{int} \sim \mathcal{N}(n\mu_i, \sigma_{in}^2)$. We show:

Proposition 4 *The ratio of rescaled to buy-and-hold mean returns satisfies*

$$\nu_{in} = e^{n\sigma_i^2(1-VR_{in})/2}, \tag{15}$$

where $VR_{in} \equiv \sigma_{in}^2/\left(n\sigma_i^2\right)$ *is the variance ratio.*

The return ratio ν_{in} is determined by the short-horizon variance σ_i^2 and the variance ratio VR_{in}. Empirically, individual stock returns at short horizons tend to be negatively autocorrelated and $VR_{in} < 1$, while portfolios have positive autocorrelations and $VR_{in} > 1$ (Lo and MacKinlay, 1988). Proposition 4 connects variance ratios to mean returns: In individual stocks, independent measurement errors cause negative autocorrelations and bias short-horizon means upwards (BS); in portfolios, systematic slow information diffusion raises autocorrelations, causing downward bias in short-horizon means (footnote 6 and Propositions 1-3).

For linearly rescaled mean returns, an analytical approximation analogous to Proposition (4) is not available. However, for the range of horizons in this paper, when returns are close to iid the effects of compounding should be small and $\nu_{in}^l \approx \nu_{in}$.[10]

Empirically, we can rescale an n-period return to a horizon of m periods:

$$\bar{R}_{inm} \equiv [\mathbb{E}(R_{i,t+1} \ldots R_{i,t+n})]^{m/n} = \bar{R}_{in}^{m/n}. \tag{16}$$

If returns are iid, then $\bar{R}_{inm} = \bar{R}_{imm} = \bar{R}_{im}$ for all $n, m > 0$. This suggests plotting $\bar{R}_{inm}$ versus the return-period length n, for a fixed rescaling horizon m, as a diagnostic. When such a plot is approximately flat, horizon effects are small.

4. Empirical Evidence

We investigate the magnitude and causes of horizon effects in average returns using U.S. style portfolios and international indices. Appendix B describes the data. Following ABK and earlier authors, we use "initially equal-weighted" ("IEW") and "initially value-weighted" ("IVW") portfolios. These weight at an initial date equally or by value, and rebalance only due to turnover.[11] Minimizing rebalancing reduces biases caused by independent price noise.

4.1. Horizon Effects in U.S. Portfolios

Table 1 shows average returns $\bar{R}_{inm}$ of horizon n geometrically rescaled to horizon $m \geq n$. The values of $n, m \in \{1, 21, 63, 126, 252\}$ correspond to daily, monthly, quarterly,

[10] For example, assuming a one percent average monthly return and independent observations, $\bar{R}_{i12}^{RS} = 1.1268$ and $\bar{R}_{i12}^{RSl} = 1.12$, implying $\nu_{in}^l = 1.12/1.1268 = 0.9939$.

[11] IVW and standard value-weighting differ only due to corporate share issuances and repurchases, which impact standard value weights, but do not impact IVW weights.

semi-annual, and annual horizons. The table shows rescaled average returns for the CRSP market index and top- and bottom-decile portfolios of stocks sorted by market equity, book-to-market ratio, and momentum.

Following from Sections 2 and 3, we expect rescaled daily returns to be smaller than buy-and-hold returns of longer horizons, with larger differences for portfolios that are more volatile and more subject to slow information diffusion. The data support these predictions. For the IEW small stock portfolio, the geometrically rescaled daily means are below the buy-and-hold averages for monthly returns (1.41 percent versus 1.63), quarterly returns (4.30 versus 5.73), and annual returns (18.34 versus 25.87). The magnitudes are even larger for IEW momentum losers: rescaled daily versus buy-and-hold returns are 0.67 versus 1.00 at a monthly horizon, and 8.34 versus 14.97 for an annual horizon. Similar results hold for IVW portfolios.

The apparent downward bias remains quantitatively important in monthly returns for several portfolios. For example, the annualized monthly return of the IEW momentum loser portfolio is 12.72, while the corresponding measures of quarterly, semi-annual, and annual returns are all similar between 14.64 and 14.97.

To better visualize horizon effects, Figure 1 plots the average returns $\bar{R}_{inm}$ versus the measurement lengths $n = 1, ..., 252$ for fixed rescaling horizon $m = 252$ using equation (16). Plots are shown for portfolios based on firm size, book-to-market, momentum, price level, short-term reversal, volatility, illiquidity, and Z-score. All plots slope upwards at short horizons, and flatten out for larger n. In a number of portfolios (small stocks, momentum losers, high volatility, high inverse price, low reversal), the difference between $\bar{R}_{inm}$ for $n = 1$ (daily) and the flat part of the graph is 5-6 percent annually, or more. Many plots show strong upward slope even in monthly returns ($n = 21$); only at a quarterly horizon ($n = 63$) do the plots reliably flatten out.[12]

[12]Throughout the paper we focus on geometric rescaling as in equation (16) since it accounts for compounding. The Internet Appendix reproduces results using linear rescaling following equation (12), which is more

In Table 2, we test the significance of the average return difference for horizons $n < m$. We refer to the shorter-horizon average $\bar{R}_{inm}$ as "rescaled" ("RS"), and to the longer-horizon average $\bar{R}_{imm}$ as "buy-and-hold" ("BH"). Panel A sets the shorter interval to one day ($n = 1$, $RS = \bar{R}_{i,1,63}$). Panel B sets the shorter interval to one month ($n = 21$, $RS = \bar{R}_{i,21,63}$). Following the evidence from Figure 1, we set the longer interval to $m = 63$, i.e., $BH = \bar{R}_{i,63,63}$.[13] The difference $RS - BH$ is significantly negative for all long-only portfolios in daily returns (Panel A), and almost all long-only portfolios in monthly returns (Panel B).

Table 2 also provides the moments necessary to calculate the analytical approximation ν_{in}^{net} of RS/BH, given in Proposition 4. The formula is accurate, with a maximum difference of 0.04 between the empirical value and the approximation.

Consistent with Proposition 4, portfolios with the highest autocorrelation do not necessarily have the largest differences between rescaled and buy-and-hold returns. The high-illiquidity portfolio has large autocorrelations (0.20 in daily returns), but the low standard deviation constrains $RS - BH$ to -0.36 quarterly. The low-illiquidity portfolio exhibits similar $RS - BH$, achieved through a higher standard deviation paired with lower autocorrelation. In contrast, the high- and low-volatility portfolios have comparable autocorrelations (0.20 versus 0.17), but the standard deviation of the high-volatility portfolio is about three times larger than that of the low-volatility portfolio. As a result, an apparently small difference in rescaled daily means ($3.08 - 2.65 = 0.43$) translates to a large difference in actual quarterly returns: $4.52 - 2.79 = 1.73$.

prevalent in empirical work. As anticipated earlier, the differences between the two approaches are very small. The Internet Appendix further shows horizon effects in other attribute-sorted and factor portfolios.

[13]The choice of m is material, driven by two effects. First, some plots in Figure 1 show declines in rescaled averages from one quarter to one year, but this effect is small. The larger issue is that a one-year horizon $m = 252$ has an effective sample size four times smaller than $m = 63$, cutting standard errors approximately in half. In Panel A, the effects are strong and changing to $m = 252$ would not impact conclusions. In Panel B the statistical, but not economic, significance of the results is sensitive to the choice of m.

4.1.1. Horizon Effects in Alphas

Horizon effects in mean returns naturally have implications for the calculation of alphas at different horizons. Consider Jensen's (1968) alpha from n-period returns:

$$\alpha_{in} = \bar{R}_{in} - \left[\bar{R}_{fn} + \beta_{in}\left(\bar{R}_{Mn} - \bar{R}_{fn}\right)\right], \tag{17}$$

A common practice for relating alphas at different horizons is linear rescaling: $\alpha_{in}^{RS} \equiv n\alpha_{i1}$.[14] We can decompose the difference of the two alphas:

Proposition 5 *The difference between linearly-rescaled and buy-and-hold alphas is:*

$$\alpha_{in}^{RS} - \alpha_{in} \approx (\nu_{in}^{l} - 1)\bar{R}_{in} - \beta_{i1}(\nu_{Mn}^{l} - 1)\bar{R}_{Mn} - (\beta_{i1} - \beta_{in})\left(\bar{R}_{Mn} - \bar{R}_{fn}\right). \tag{18}$$

Three components affect the difference: 1) horizon effects in the mean returns of portfolio i, 2) horizon effects in the factor M, and 3) horizon effects in beta.

Table 3 shows daily, monthly, and quarterly rescaled alphas, and decomposes the differences into the components of Proposition 5.[15] The alpha differences are large in many portfolios. For small stocks, the alpha is 0.65 percent per quarter based on daily returns and 0.92 per quarter based on monthly returns, in both cases statistically insignificant. In quarterly returns, the alpha is a much larger 1.59 per quarter, statistically significant at the 10% level.

In long-short portfolios, horizon effects sometimes offset. For example, high book-to-market stocks have an alpha of 0.97 per quarter based on daily returns (Panel A), and 1.87 based on quarterly returns (Panel C). Low book-to-market stocks have an alpha of -1.72 per quarter based on daily returns, and -1.17 based on quarterly returns. The long-short alpha is large and significant in either case (2.69 in daily data and 3.04 in quarterly data). However, daily data suggest that profitability stems largely from the short side, whereas the long side drives profits in quarterly data.

[14] See, e.g., Lewellen and Nagel (2006), Barber (2007), Li and Yang (2011), and Ang and Kristensen (2012).

[15] To make the beta estimates more comparable, we use "sum" betas as suggested by Dimson (1979), with 63 lags for daily-return regressions and three lags for monthly-return regressions.

In other cases, horizon effects do not offset in long-short portfolios leading to different inferences about net alpha. For example, the portfolio that is long low-volatility stocks and short high-volatility stocks generates a statistically significant alpha of 1.75 per quarter based on daily data. The alpha falls to 1.03 per quarter in monthly data and 0.68 in quarterly data, insignificantly different from zero in both cases.

The decompositions in Panels D and E show that alpha differences are primarily driven by component (1) of equation (18), horizon effects in portfolio means. These reach as large as 2.43 per quarter. Component (2), due to horizon effects in the market-portfolio mean, peaks at the smaller level of 0.26 per quarter. Component (3), due to beta differences, reaches 0.71 per quarter. While a substantial prior literature has focused on beta differences (e.g., Scholes and Williams, 1977; Dimson, 1979; Gilbert, Hrdlicka, Kalodimos, and Siegel, 2014), our work is the first to demonstrate the importance of differences in mean returns across horizons.

4.2. Horizon Effects in International Portfolios

We conjecture that international indices based on country, region, and style might exhibit significant horizon effects, driven by slow information diffusion. From Datastream, we obtain US-dollar-denominated MSCI indices for 56 countries with at least ten years of valid monthly data. Details are provided in the Internet Appendix.

We are interested in two questions. First, how large are horizon effects in international indices? Second, what causes the horizon effects? To help address these questions, we sort countries according to their categorization by MSCI as a developing, emerging, or frontier market. If slow information diffusion causes horizon effects and the MSCI categories proxy for market efficiency, we expect the largest rescaled-return differences in frontier markets, moderate effects in emerging markets, and small effects in developed markets.

Figure 2 plots the average return $\bar{R}_{inm}$ against the horizon n, averaging across all countries in each MSCI category. In developed markets, the plot is approximately flat across all horizons. In emerging markets, 1-month returns appear to be downward biased by about 1% relative to the flat part of the graph, and flattening occurs at about 3-4 months. For the frontier markets, the apparent bias in 1-month returns is about 3% annually, and leveling occurs at approximately 6-12 months. These findings are consistent with slower information diffusion in less developed markets.

The distribution of horizon effects within MSCI categories is characterized in Table 4. For each country, we calculate a 12-month buy-and-hold return (BH), and a 1-month return rescaled to an annual horizon (RS). We report means and quartiles of the ratio RS/BH (Panel A) and the difference $RS - BH$ (Panel B). The median of RS/BH is 0.92 in developed markets, 0.82 in emerging markets, and 0.78 in frontier countries. The median of $RS - BH$ is 1.16 percent annually in developed markets, 2.22 in emerging markets, and 3.70 in frontier markets. Consistent with theory, the effects are stronger in less developed markets where information is likely to diffuse slower.

To broaden our international sample, we use 49 MSCI regional indices and six developed-market style portfolios from Ken French's website.[16] Table 5 confirms that horizon effects are stronger in emerging-market regions. For example, in the small stock portfolio the difference $RS - BH$ is substantially larger in Asia Pacific excluding Japan (4.24 percent per year) than in North America (0.08). Figure 3 shows histograms of RS/BH for the international samples. The ratio is almost always below 1 and in many cases substantially lower, confirming pervasive rescaling effects in international indices.

[16] These are formed on 1) size and book-to-market and 2) size and momentum in Asia Pacific excluding Japan, Europe, Global, Japan, and North America. We compute returns following Fama and French (2012).

4.3. Interpretation

We now address questions related to interpretation. First, are the seemingly low average returns for some portfolios at short horizons tradeable? Second, should low short-horizon mean returns be attributed to bias caused by slow reaction to fundamentals, or could the effects be caused by an alternative mechanism such as fundamentals with time-varying risk premia? The two questions are related, but not identical. ABK discuss that measured short-horizon mean returns might deviate from the mean of fundamentals due to "noise," yet still be tradeable to some market participants.[17]

4.3.1. Are the Measured Returns Tradeable?

To address the tradeability of short-horizon returns, we compare horizon effects in the non-investable international indices with a matched sample of investable exchange-traded funds ("ETFs") based on the indices. The approach of comparing properties of a non-investable index with an investible counterpart follows tests in BRW.

Table 6 compares rescaled to buy-and-hold returns for indices and ETFs. In every country, the difference $RS - BH$ is smaller in the ETF than the index. Further, the difference is statistically significant in more than half of the countries, indicating that random chance is an unlikely explanation. Averaging across countries, the difference $RS - BH$ is -1.15 percent per year for the indices, and -0.54 per year for the ETFs. In other words, approximately half of the horizon effects in the index returns with matched ETFs can be attributed to a non-tradeable bias.

[17] Deviations of observed short-horizon means from fundamentals could represent zero-sum gains, available to some investors with low trading costs, at the expense of others. The low short-horizon means we document would require shorting thinly traded securities over short intervals. A more direct strategy would trade on the short-run predictability in laggers.

4.3.2. The Role of Slow Information Diffusion

To confirm the role of slow information diffusion, for each portfolio i we estimate:

$$r_{it} = \alpha_i + \sum_{\tau=0}^{L} \beta_{i\tau} r_{M,t-\tau} + e_{it}. \tag{19}$$

Following Brennan, Jegadeesh, and Swaminathan (1993), we calculate delay measures $\text{DELAY}_i \equiv \sum_{\tau=1}^{L} \hat{\beta}_{i\tau} / \sum_{\tau=0}^{L} \hat{\beta}_{i\tau}$, where we set $L = 63$ days or $L = 3$ months for U.S. style portfolios, and $L = 12$ months for the international indices and ETF returns. We then estimate cross-sectional regressions:

$$(\text{RS/BH})_i = c_0 + c_1 \text{DELAY}_i + \eta_i. \tag{20}$$

If slow reaction to market information contributes to downward bias in short-horizon returns, then the coefficient on DELAY_i is negative.[18]

Table 7 shows results for different samples. The coefficient estimates on DELAY_i are all negative. The results for the index samples (1 to 3) are significant at conventional levels and the ETF sample (4) has a p-value of 0.053. These results support that slow information diffusion contributes to horizon effects.

4.3.3. Horizon Effects Over Time

One might speculate that if horizon effects in average returns are caused by slow information diffusion, the effects should diminish in more recent data. After all, recent technological advances have increased the speed with which market participants can respond to new information, and market liquidity has generally increased.[19]

On the other hand, the amount of information available to market participants, the number of securities, and the complexity of the markets have all increased over time. These offsetting forces could permit slow information diffusion to remain important

[18] Since the variable DELAY_i is itself estimated, the standard errors-in-variables problem attenuates the coefficient estimate towards zero, which should make it more difficult to detect a significant relationship.

[19] The cross-sectional average Amihud (2002) illiquidity measure has fallen significantly since the 1980s.

even in recent data. Indeed, the references listed in footnote 6 all show that slow information diffusion remains important in recent data.

To document any changes in the importance of horizon effects in average returns over time, we carry out a sub-sample analysis. Figure 4 shows horizon effects in four approximately twenty-year sub-periods for the portfolio of small stocks. The effects are largest in the 1926-1949 period, where the difference between buy-and-hold annual average return and rescaled daily mean is 8% per year. However, in the two most-recent twenty-year periods, 1970-1989 and 1990-2009, the magnitude of the horizon effects are also large, at 6% annually. Horizon effects have thus remained empirically important throughout the sample, and are not simply an artifact of earlier trading technologies.

4.4. Recommendations and Corrections

If the low short-horizon average returns in some portfolios represent an econometric bias, then how should empiricists proceed? Since fixed or exogenous portfolio weights eliminate negative covariation between portfolio weights and future returns (third term of equation (10)), one might anticipate that we would recommend corrections based on pre-determined weights, including popular alternatives such as fundamental weighting. To the contrary, no single weighting scheme can solve the problems caused by both independent measurement errors and slow information diffusion. Frequent rebalancing to fixed weights exacerbates the problem from independent measurement errors (BS, ABK), while our results show that IEW, IVW, and other infrequently-rebalanced portfolios can have biased means due to the effects of slow information diffusion.

One useful result is that the effects of slow information diffusion concentrate in returns calculated at short horizons. We therefore recommend infrequent rebalancing, consistent with BS and ABK, combined with the simple diagnostic of plotting the return-measurement interval n versus the rescaled average $\bar{R}_{inm}$, as in Figure 1. If this plot is approximately flat, then horizon effects are not an issue. If the plot shows a strong upward slope at short-horizons, then choosing a return-measurement interval on the flat portion of the plot is a simple way to avoid potential concerns.

In some cases researchers may be specifically interested in short-horizon returns. In terms of simple adjustments,[20] Proposition 4 and and Table 2 show that one can obtain a close approximation of long-run returns by adjusting the short-horizon variance:

$$\bar{R}_{in} \approx \bar{R}_{in}^{RS} / e^{n\sigma_i^2(1-VR_{in})/2}. \tag{21}$$

One issue is that this calculation requires long-horizon returns. If these are available it is likely simpler to work directly with the average long-horizon return.

Researchers who are skeptical that horizon effects in mean returns are entirely due to slow information diffusion may desire a more limited adjustment. In this case, we suggest a natural adjustment that builds on methods researchers already use to account for "intervalling" effects in beta estimation (e.g., Scholes and Williams, 1977). Specifically, following estimation of (19), one can create the artificial return series that would have been generated if all response to market information were instantaneous:

$$\hat{r}_{it} = \hat{\alpha}_i + \left(\sum_{\tau=0}^{L} \hat{\beta}_{i\tau} \right) r_{M,t} + \hat{e}_{it}. \tag{22}$$

By construction, the intercept and beta match the intercept and sum beta of (19). The synthetic returns will however generally have higher variance than the observed returns, because of the elimination of the smoothed response to market information.[21] Through Jensen's inequality, the variance change increases the means and alphas of the simple return series $e^{\hat{r}_{it}} - 1$. This correction exactly adjusts for slow reaction to market information, and is the logical consequence for mean returns and alphas of already accepted methods of beta adjustment.

[20] A structural approach to modelling frictions such as trading costs, price impacts, and slow information diffusion is possible, but challenging.

[21] The synthetic returns have higher variance than the observed returns when $(\sum_{\tau=0}^{L} \hat{\beta}_{i\tau})^2 > (\sum_{\tau=0}^{L} \hat{\beta}_{i\tau}^2)$.

To demonstrate these corrections, we calculate adjusted average returns for small-capitalization and high-volatility portfolios. The variance ratio adjustment (21) increases the average rescaled daily return from 4.30 to 5.38 percent per quarter in the small-capitalization portfolio, and from 3.08 to 4.63 percent per quarter in the high-volatility portfolio, in both cases nearly eliminating horizon effects relative to the buy-and-hold return. The beta adjustment (22) gives a rescaled return of 5.11 percent per quarter for the small capitalization portfolio, and 3.96 percent per quarter for the high volatility portfolio, in both cases eliminating over half of the horizon effects.

5. Structural Estimation

To complete our analysis, we structurally estimate the model in Section 2 using only five parameters. The model matches closely portfolio lagged betas, autocorrelations, variance ratios, and average returns at a range of horizons. Narrower objectives have received considerable attention, with limited success (e.g., LM, BRW).

5.1. Model Specification and Identification Strategy

In the model of Section 2, individual stock returns follow (2) to (4). For tractability, we now assume that each stock i in a given style portfolio has a delay type $\theta(i)$, taking one of Θ integer values. Stocks of the same type have identical delay parameters: $\delta_{ik} = \delta_{jk}$ if $\theta(i) = \theta(j)$. Delay parameters may also vary across factors k, providing empirical flexibility and permitting different speeds of reaction to different types of news. Given that type determines the delay parameters, we henceforth index by θ, i.e., $\delta_{\theta k}$.

Stocks are assigned types according to independent draws from a discrete distribution with probabilities $0 < \pi_\theta \leq 1$, $\sum_\theta \pi_\theta = 1$. To simplify other aspects of the cross-section, stocks are otherwise identical. In particular, in a given style portfolio stocks have the same fundamental loadings: $\beta_{ik} = \beta_k$ for all i. We assume many stocks, and form well-diversified sub-portfolios, each containing stocks of a single type.

Following Lemma 2 in the Appendix, the returns of each type-θ sub-portfolio follow:

$$r_{\theta t} = r_{ft} + \sum_{k=1}^{K} (1 - \delta_{\theta k}) (D_{\theta k,t-1} + \beta_k f_{kt}) \tag{23}$$

$$D_{\theta kt} = \delta_{\theta k} (D_{\theta k,t-1} + \beta_k f_{kt}) . \tag{24}$$

We assume $K = 2$ sources of systematic fundamentals for each style portfolio. The first factor ($k = 1$) represents market information. The second factor ($k = 2$) captures portfolio-specific information that is orthogonal to the market. Without loss of generality, we normalize the loading on the second factor to one, i.e., $\beta_2 = 1$.

The case of a single type, $\Theta = 1$, is naturally parsimonious. After calibrating the market parameters μ_1 and σ_1 to an observable index, only four parameters remain: a market loading β, the standard deviation of the second factor, σ_2, and delay parameters for market and non-market information, δ_1 and δ_2, where the arbitrary index for θ in $\delta_{\theta k}$ is suppressed.[22] When $\delta_1 = \delta_2$ the model collapses to a single factor, matching the "lagger" portfolio in Proposition 2. Information diffusion is slow but not heterogeneous across firms or factors, as in LM and earlier models that imply a fast geometric decline in portfolio autocorrelations.[23] Our model similarly implies a geometric decline in autocorrelations when $\Theta = 1$ and $\delta_1 = \delta_2$, as shown in Lemmas 1 and 2 of the Appendix. When $\Theta = 1$ but $\delta_1 \neq \delta_2$, the model permits a richer autocorrelation structure.

The separate roles of δ_1 and δ_2 can be empirically identified by their effects on lagged market betas. Slow reaction to market information, $0 < \delta_1 < 1$, generates positive portfolio autocorrelations through positive loadings on the lagged market. In contrast, slow reaction to non-market information generates positive portfolio autocorrelations without impacting loadings on the lagged market.

Differentiating between the two channels helps to reveal economic mechanisms. In

[22] The mean of the second factor is calibrated by matching the model-implied logarithmic return of the style portfolio to its empirical value, which can be accomplished analytically given the other parameters.

[23] LM permit random occurrence of nontrading, but all firms have the same nontrading probability. For the geometric decline of autocorrelations, see their equation 2.26.

general, portfolio autocorrelations might be caused by slow reaction to information, or alternatively by time-varying risk premia, e.g., Conrad and Kaul (1988). Existing literature gives evidence of slow price reactions that cannot be explained by time-varying risk premia (see footnote 6). We similarly conjecture that time-varying risk premia are not a likely explanation for horizon effects in average returns: The observed horizon effects are most pronounced in the range of daily to quarterly frequencies, whereas movements in risk-premia are more important at lower frequencies (e.g., Campbell, Lo, and MacKinlay (1997)). Decomposing portfolio autocorrelations into market versus non-market contributions provides direct evidence. Market returns are observable, and already embed time variation in market risk. Autocorrelations caused by lagged loadings on the market strengthen evidence in favor of the "slow reaction" channel.

With two delay types, $\Theta = 2$, we associate $\theta = 0$ with stocks that incorporate information immediately ($\delta_{01} = \delta_{02} = 0$). The model then has five parameters, $\beta, \delta_{11}, \pi_0, \sigma_2, \delta_{12}$, and when $\delta_{11} = \delta_{12}$ matches the analytically tractable case with "leaders" and "laggers" in Proposition 3.

For $\Theta > 2$, to obtain parsimony we draw on the literature on long-memory and power laws in financial economics (Baillie, 1996; Calvet and Fisher, 2008, 2013; Gabaix, 2009). Observing Figure 1, a slow power-law decay may provide a good approximation of portfolio autocorrelations. We again associate the type $\theta = 0$ with immediate information incorporation ($\delta_{01} = \delta_{02} = 0$). Borrowing from Calvet and Fisher (2001, 2004, 2007), the remaining types $\theta = 1, ..., \Theta - 1$ have equal probability, $\pi_1 = ... = \pi_{\Theta-1} = (1 - \pi_0)/(\Theta - 1)$, and the delay parameters are transformed into half-lives with a geometric progression:

$$h_{\theta k} \equiv \frac{\ln 0.5}{\ln \delta_{\theta k}} = 2^{\theta-1} h_{1k}. \tag{25}$$

Following Calvet and Fisher (2008), building on Granger (1980) and Ding and Granger (1996), the equal-weighted aggregation of components with a geometric series of half-lives mimics a power-law decline in autocorrelations.[24] While these are strong assumptions, our empirical work can determine their validity. Independent of the number of delay types, the model has five parameters: $\Psi \equiv (\beta, h_{11}, \pi_0, \sigma_2, h_{12})$. With these limited degrees of freedom, we match a broad set of empirical moments.

5.2. Simulated Method of Moments Estimation

Our empirical approach is a standard simulated method of moments estimation (Lee and Ingram, 1991; Duffie and Singleton, 1993; Gourieroux, Monfort, and Renault, 1993). For a given style portfolio and parameter vector Ψ, we simulate from the model 10 portfolio returns of the same length as the data.[25] For a given set of moments, detailed below, we calculate the differences between the empirical moments and the average moments over model simulations. The differences are squared and summed to form an objective function, which is minimized by varying the parameter vector Ψ.[26]

Regarding motivation for moments, LM show that a model of the first-order autocorrelation of returns has implications for higher-order autocorrelations and variance ratios. It is natural to use such moments to estimate and evaluate the model. Additionally, we decompose autocorrelations into market and non-market channels, using loadings on the market and its lags for identification. Finally, the ultimate goal is to better understand horizon effects in average returns. We do not use these in estimation, but evaluate the ability of the fitted model to match the observed horizon effects.

One issue is that structural estimation can sometimes appear as a "black box" with

[24] The base of the geometric progression is not essential, and is set to two for simplicity. In other related work, BRW model multiple classes of firms with different nontrading probabilities and show enhanced first-order autocorrelations relative to LM, but do not match the empirical level or consider higher order autocorrelations. Their approach requires many parameters. By constraining parameters as suggested by the literature on power laws, we obtain a parsimonious specification that can be structurally estimated across a range of horizons.

[25] This ensures that simulation error plays a negligible role in our results. Moment condition differences are $(1 + 1/nsim)$ times larger than the variance of a single draw, hence standard errors are only $\sqrt{1 + 1/nsim}$ larger due to simulation error versus the case where analytical formulas for the moments are available.

[26] The objective function uses an identity weighting matrix, as in Cochrane (2001). This permits meaningful comparisons across versions of the model, since weights are constant, and has a simple economic interpretation. The results are not sensitive to the exact weighting matrix, consistent with the close fit of the moments.

the source of identification for individual parameters difficult to pin down. To avoid this, we use a two-step procedure. Moment conditions from portfolio loadings on the market and its lags contain information about only three parameters: the fundamental market loading β, the delay parameter for market information, h_{11}, and the probability π_0.[27] We estimate these three parameters in the first stage, using only market loadings as instruments. In the second stage, we estimate σ_2 and h_{21} using style portfolio variances and autocorrelations. This approach clarifies the source of identification for each of the parameters in the model but is not critical to our results; in untabulated results we obtain nearly identical outcomes estimating all parameters in a single step.

We briefly describe the exact moments. In the first stage, we estimate (19) with $L = 63$ lags. Moment conditions are the individual slope coefficients up to five lags, $\beta_{i0}, \cdots, \beta_{i5}$, and "sum" betas $\sum_{\tau=0}^{J} \beta_{i\tau}$ for the values $J = 10, 21, 63$. The second-stage moment conditions are the one-day return variance, the first ten individual portfolio autocorrelations, and sums of autocorrelations for lags $11-20, ..., 41-50, 51-63$.[28]

5.3. Structural Model Estimation Results

Table 8 gives estimation results for small and large stocks. Rows under the heading "Simulated Market Returns" follow the specification of the model exactly. The first-stage objective function ω_1 shows that the fit of the beta regression (19) improves monotonically with the number of types Θ for small stocks. The improvement can be seen in Figure 5. With a single type $\Theta = 1$ the geometric decline of the lagged betas does not match the empirical values, but with $\Theta = 8$ the model and empirical values fit closely. The estimate of h_{11} is 0.46 when $\Theta = 8$, implying that the longest delay has a half life of approximately 30 days. For large stocks h_{11} is approximately zero, implying

[27] OLS estimates are unaffected by σ_2 and h_{12}, which determine residual variance and autocorrelations.

[28] The one-day variance is multiplied by 100, which ensures an almost exact match. The summed autocorrelations are divided by the square root of the number of addends, which gives variability similar to the individual autocorrelations. Variance ratios play a similar role to autocorrelations, and similar results can be obtained using variance ratios rather than autocorrelations.

that all types incorporate market information quickly.

The second-stage results, in both Table 8 and Figure 6, show that versions of the model with $\Theta > 1$ type are able to fit the autocorrelations of the small-stock portfolio well. In all cases the persistence h_{21} of the second factor is less than h_{11} confirming the importance of slow reaction to market information. The last two columns of Table 8 summarize horizon effects, giving the exact value of $RS - BH$ and an approximation based on Proposition 4. Since the model assumes normal distributions, the exact and approximate values are identical (-3.15% annually for $\Theta = 8$), whereas in the data the approximate value is slightly closer to zero than the exact value (-4.91% versus -5.58%). The model captures more than half of the horizon effects in the data.

Given the good fit of the model to autocorrelations, it seems surprising that horizon effects are not closer to the data. To reconcile this, we estimate a second version of the model, replacing the simulated iid market returns in the model with actual market returns, while continuing to simulate the returns of the second factor. The results, shown in Table 8 and Figures 5 and 6, give similar parameter estimates but a much better match to variance ratios. The fit of this model to horizon effects in average returns is summarized in Figure 7. With $\Theta = 1$ delay type, about half of the horizon effects are captured, and most of the horizon effects counterfactually concentrate at very short horizons. With $\Theta = 8$ types, the model fits the data very closely, especially if one focuses on the normal approximations to both the model and the data.[29]

Empirical results for other US style portfolios are summarized in Table 9. Wherever horizon effects are strong, the model with $\Theta = 8$ components matches portfolio lagged betas, autocorrelations, variance ratios, and horizon effects in mean returns. In these cases, slow reaction to market information, $h_{11} > 0$, is necessary to match lagged betas, and also drives portfolio autocorrelations and horizon effects in average returns. These

[29]Empirical differences caused by nonnormality are interesting but beyond the scope of this paper.

results further support that slow information diffusion is an important contributor to horizon effects in average returns.

6. Conclusion

When stocks adjust to new information slowly, average short-horizon portfolio returns are biased. In contrast to existing literature assuming independent price noise, the bias under slow information diffusion is downward rather than upward, and value weighting does not eliminate the bias.

Consistent with theory, a broad range of style and international portfolios show rescaling effects in daily and monthly average returns. The effects are large, up to 10% annually, statistically significant, and impact inferences about the performance of investment strategies. Comparing horizon effects in non-investable indices versus comparable investable ETF's, we find that approximately half of the horizon effects in indices are non-tradeable. We also show a statistically significant cross-sectional link between exposure to lagged market information and the magnitude of a portfolio's horizon effects. We propose a simple diagnostic for determining the horizon at which the impact of slow reaction becomes negligible, and corrections to average short-horizon returns that account for slow reaction.

A parsimonious version of the model, using only five parameters, permits heterogeneous reaction to market and non-market factors. The model matches empirical lagged betas, autocorrelations, variance ratios, and average returns across a range of horizons. Narrower sets of these moments have proven challenging to understand in earlier literature (LM, BRW). We anticipate growing interest in the precise mechanisms driving information diffusion over time and across stocks.

Appendix A: Proofs

Lemma 1 *The information deficits D_{ikt} follow AR(1) processes and have unconditional normal distributions with* $\mathbb{E}(D_{ikt}) = \frac{\delta}{1-\delta}\mu_k$ *and* $\mathrm{Var}(D_{ikt}) = \frac{\delta^2}{1-\delta^2}\sigma_k^2$.

Proof of Lemma 1: Let $f_{kt} = \mu_k + \sigma_k \xi_{kt}$ where the ξ_{kt} are independent standard normals. Since $0 \leq \delta < 1$, substituting f_{kt} in equation (4) gives the result. □

Lemma 2 *Denote by D_{1t} the systematic information deficit of the lagger portfolio. Returns of a well-diversified, value-weighted portfolio of laggers are given by*

$$r_t = (1-\delta)(D_{1,t-1} + f_{1t}). \tag{26}$$

Proof of Lemma 2: We assume a large number N of stocks i with identically distributed returns and initial price $e^{p_{i0}} = 1$. The value-weighted portfolio return at any date t is:

$$e^{r_t} = \frac{\sum_{i=1}^N e^{p_{it}}}{\sum_{i=1}^N e^{p_{i,t-1}}} = \frac{\frac{1}{N}\sum_{i=1}^N e^{p_{it}}}{\frac{1}{N}\sum_{i=1}^N e^{p_{i,t-1}}}. \tag{27}$$

For each stock, applications of equation (6), backward iteration using equation (7), and the geometric series imply

$$\begin{aligned} p_{it} &= (1-\delta)\sum_{\tau=1}^{t}(f_{1\tau} + f_{1+i,\tau} + D_{i,1,\tau-1} + D_{i,1+i,\tau-1}), \qquad (28)\\ &= (1-\delta)\left(\sum_{\tau=0}^{t-1}\sum_{s=0}^{\tau}\delta^s f_{1,t-\tau} + \sum_{\tau=0}^{t-1}\sum_{s=0}^{\tau}\delta^s f_{1+i,t-\tau} + \sum_{\tau=0}^{t-1}\delta^\tau D_{i,1,0} + \sum_{\tau=0}^{t-1}\delta^\tau D_{i,1+i,0}\right)\\ &= \sum_{\tau=0}^{t-1}(1-\delta^{\tau+1})f_{1,t-\tau} + \sum_{\tau=0}^{t-1}(1-\delta^{\tau+1})f_{1+i,t-\tau} + (1-\delta^t)D_{i,1,0} + (1-\delta^t)D_{i,1+i,0}. \end{aligned}$$

We assume date-0 systematic delay states $D_{i,1,0} = \frac{\delta}{1-\delta}\mu_1$ and that idiosyncratic delay states $D_{i,1+i,0}$ are drawn from their unconditional distributions. The first term of (28) is independent of i and all random variables are independent. As $N \to \infty$,

$$\frac{1}{N}\sum_{i=1}^N e^{p_{it}} \to e^{\sum_{\tau=0}^{t-1}(1-\delta^{\tau+1})f_{1,t-\tau}}\mathbb{E}(e^{\sum_{\tau=0}^{t-1}(1-\delta^{\tau+1})f_{1+i,t-\tau}})e^{\frac{\delta(1-\delta^t)}{1-\delta}\mu_1}\mathbb{E}(e^{(1-\delta^t)D_{i,1+i,0}}), \tag{29}$$

where the expectation is taken at $t=0$. Normal factors and delay states imply:

$$\begin{aligned}\mathbb{E}(e^{\sum_{\tau=0}^{t-1}(1-\delta^{\tau+1})f_{1+i,t-\tau}}) &= e^{-\frac{1}{2}\sigma_i^2\sum_{\tau=0}^{t-1}(1-\delta^{\tau+1})+\frac{1}{2}\sigma_i^2\sum_{\tau=0}^{t-1}(1-\delta^{\tau+1})^2} = e^{-\frac{1}{2}\sigma_i^2\sum_{\tau=0}^{t-1}\delta^{\tau+1}(1-\delta^{\tau+1})} \\ \mathbb{E}(e^{(1-\delta^t)D_{i,1+i,0}}) &= e^{(1-\delta^t)\frac{\delta}{1-\delta}(-\frac{1}{2}\sigma_i^2)+\frac{1}{2}(1-\delta^t)^2\frac{\delta^2}{1-\delta^2}\sigma_i^2}. \qquad (30)\end{aligned}$$

Substituting into (29), calculating the limit of (27), and assuming large t confirms that slow diffusion of idiosyncratic news diversifies away:

$$\begin{aligned} r_t &= \sum_{\tau=0}^{t-1}(1-\delta^{\tau+1})f_{1,t-\tau} - \sum_{\tau=0}^{t-2}(1-\delta^{\tau+1})f_{1,t-1-\tau} \\ &= (1-\delta)f_{1,t} + \sum_{\tau=1}^{t-1}(1-\delta^{\tau+1})f_{1,t-\tau} - \sum_{\tau=1}^{t-1}(1-\delta^{\tau})f_{1,t-\tau} \\ &= (1-\delta)\sum_{\tau=0}^{t-1}\delta^{\tau}f_{1,t-\tau} = (1-\delta)\left(D_{1,t-1}+f_{1t}\right). \qquad \square \qquad (31)\end{aligned}$$

Lemma 3 *Denote by p_t^* and p_t the log prices of the well-diversified portfolios of leaders and laggers, respectively, and by D_{1t} the information deficit of the lagger portfolio. Leader and lagger portfolio logarithmic prices are cointegrated: $p_t^* = p_t + D_{1t}$.*

Proof of Lemma 3: The proof is by induction. As before, $p_0^* = p_0$ and $D_{1,0}=0$, so that $p_0^* = p_0 + D_{1,0} = 0$. Assuming $p_{t-1}^* = p_{t-1} + D_{1,t-1}$, equations (26) and (7) give:

$$\begin{aligned} p_t + D_{1t} &= p_{t-1} + (1-\delta)(D_{1,t-1}+f_{1t}) + \delta(D_{1,t-1}+f_{1t}) \\ &= p_{t-1} + D_{1,t-1} + f_{1t} = p_{t-1}^* + f_{1t} = p_t^*. \qquad \square \qquad (32)\end{aligned}$$

Proof of Proposition 1: The unconditional mean and variance of lagger returns follows by applying expectation and variance operator to both sides of equation (6) and then substituting the expressions for $\mathbb{E}\left(D_{ikt}\right)$ and $\mathrm{Var}\left(D_{ikt}\right)$ from Lemma 1. The lagger variance is bounded by the leader variance since $0<\delta<1$ implies that $0<\frac{1-\delta}{1+\delta}<1$. $\square$

Proof of Proposition 2: The unconditional mean and variance of lagger portfolio returns follow by applying expectation and variance operator to both sides of equation (26) and substituting the expressions for $\mathbb{E}\left(D_{ikt}\right)$ and $\mathrm{Var}\left(D_{ikt}\right)$ from Lemma 1. $\square$

Proof of Proposition 3: Using Lemma 3, the weight w_{t-1} in equation (9) is:

$$\frac{(1-\pi)e^{p_{t-1}}}{\pi e^{p^*_{t-1}}+(1-\pi)e^{p_{t-1}}} = \frac{(1-\pi)e^{p_{t-1}}}{\pi e^{p_{t-1}+D_{1,t-1}}+(1-\pi)e^{p_{t-1}}} = \frac{(1-\pi)}{\pi e^{D_{1,t-1}}+(1-\pi)}. \tag{33}$$

Taking the expectation of (8) and applying the definition of covariance gives (10). □

Proof of Proposition 4: Normality of the returns at all horizons implies $\bar{R}^{RS}_{in} = e^{n\left(\mu_i+\sigma_i^2/2\right)}$ and $\bar{R}_{in} = e^{n\mu_i+\sigma_{in}^2/2}$. The ratio of these expressions gives equation (15). □

Proof of Proposition 5: First calculate

$$\begin{aligned}
\alpha^{RS}_{in} \equiv\ & n\alpha_{i1} = n\bar{R}_{i1} - \left[n\bar{R}_{f1} + \beta_1\left(n\bar{R}_{M1} - n\bar{R}_{f1}\right)\right] = (1+n(\bar{R}_{i1}-1)) \\
& - \left[(1+n(\bar{R}_{f1}-1)) + \beta_1\left((1+n(\bar{R}_{M1}-1)) - (1+n(\bar{R}_{f1}-1))\right)\right] \\
=\ & \nu^l_{in}\bar{R}_{in} - \left[\nu^l_{fn}\bar{R}_{fn} + \beta_1\left(\nu^l_{Mn}\bar{R}_{Mn} - \nu^l_{fn}\bar{R}_{fn}\right)\right]. \qquad (34)
\end{aligned}$$

The difference $\alpha^{RS}_{in} - \alpha_{in}$ is then:

$$\begin{aligned}
&\alpha^{RS}_{in} - \left(\bar{R}_{in} - \left[\bar{R}_{fn} + \beta_n\left(\bar{R}_{Mn} - \bar{R}_{fn}\right)\right]\right) \qquad (35) \\
&= (\nu^l_{in}-1)\bar{R}_{in} - \beta_1(\nu^l_{Mn}-1)\bar{R}_{Mn} + (\beta_n-\beta_1)\left(\bar{R}_{Mn} - \bar{R}_{fn}\right) - (1-\beta_1)(\nu^l_{fn}-1)\bar{R}_{nf}.
\end{aligned}$$

The approximation (18) follows if rescaling has an insignificant impact on the average return of the risk-free asset, $\nu^l_{fn} \approx 1$. □

Appendix B: Descriptions of U.S. Style Portfolios

Using data from CRSP and Compustat, we form decile portfolios on characteristics, and calculate returns for the top and bottom deciles. IEW returns invest $1 in each stock at the beginning of each rebalancing period and holding the resulting portfolio until the next rebalancing date. IVW returns invest in each stock an amount proportional to its most recent market capitalization and hold until the next rebalancing date. The characteristics and their formation details are:

Market Equity: Following Fama and French (1993), we sort on July 1 of year τ by the market value of equity at the end of $\tau - 1$. Portfolios are held for 12 months, and the sample period is 1926-2009.

Book-to-Market: Following Fama and French (1993), we sort on July 1 of year τ based on the ratio of book value of equity to the end-of-calendar-year $\tau - 1$ market value of equity. Book equity is stockholders' book equity plus balance sheet deferred taxes plus investment tax credit less redemption value of preferred stock. If redemption value of preferred stock is unavailable, we use liquidation value. If stockholders' equity value is unavailable, we compute it as the sum of book value of common equity and value of preferred stock. If these items are not available, stockholders' equity is the difference between total assets and total liabilities. Portfolios are held for 12 months, and the sample period is 1964-2009.

Momentum: At the beginning of each month t, we sort by cumulative returns in months $t - 12$ to $t - 2$. Portfolios are held for one month, and the sample period is 1927-2009.

Price: At the beginning of each month t, we sort by price per share at the end of month $t - 2$. Portfolios are held for one month, and the sample period is 1926-2009.

Short-term reversal: At the beginning of each month t, we sort by return in month $t - 1$. Portfolios are held for one month, and the sample period is 1926-2009.

Volatility: At the beginning of each month t, we sort by their standard deviation of daily stock returns estimated from months $t - 13$ to $t - 2$. Portfolios are held for one month, and the sample period is 1927-2009.

Illiquidity: We sort on January 1 of year τ based on the Amihud (2002) price impact measure estimated in year $\tau - 1$. The Amihud price impact measure is the average ratio of absolute daily returns to daily dollar volume. Portfolios are held for 12 months, and the sample period is 1927-2009.

Z-Score: We sort on July 1 of year τ based on the Altman (1968) bankruptcy predictor computed in the fiscal year ending in calendar year $\tau - 1$. Portfolios are held for 12 months, and the sample period is 1964-2009.

Appendix C: Derivation of Standard Errors

We use the delta method to derive standard errors for tests involving $\bar{R}_n^{RS}$, in Tables 2, 5, and 6. For any function of variables $h(\cdot)$, the delta method gives $\text{Var}\,(h(\cdot)) = (\nabla h)'\Sigma(\nabla h)$, where ∇h is the vector of partial derivatives and Σ is the covariance matrix of the variables. Given a series of T gross returns, define:

$$X_{BH} \equiv \begin{bmatrix} R_1 \times ... \times R_n \\ R_{n+1} \times ... \times R_{2n} \\ \vdots \\ R_{T-n+1} \times ... \times R_T \end{bmatrix} \quad X_{RSl} \equiv \begin{bmatrix} (R_1 - 1) + ... + (R_n - 1) + 1 \\ (R_{n+1} - 1) + ... + (R_{2n} - 1) + 1 \\ \vdots \\ (R_{T-n+1} - 1) + ... + (R_T - 1) + 1 \end{bmatrix} \tag{36}$$

The means of X_{BH} and X_{RSl} are respectively $\bar{R}_n$ and $\bar{R}_n^{RSl}$ in equation (12). Denote the variances and covariance by V_{BH}, V_{RSl}, and $V_{RSl,BH}$.

To obtain $\text{Var}(\bar{R}_n^{RS})$, set $h(\bar{R}_n^{RSl}) = ((\bar{R}_n^{RSl} - 1)/n) + 1)^n$ and $\Sigma = V_{RSl}$. The expression simplifies to $\text{Var}\left(\bar{R}_n^{RS}\right) = V_{RSl}(((\bar{R}_n^{RSl} - 1)/n) + 1)^{2(n-1)}$. To obtain $\text{Var}(\bar{R}_n^{RS} - \bar{R}_n)$, set $h(\bar{R}_n^{RSl}, \bar{R}_n) = (\frac{\bar{R}_n^{RSl} - 1}{n} + 1)^n - \bar{R}_n$ and

$$\Sigma = \begin{bmatrix} V_{RSl} & V_{RSl,BH} \\ V_{RSl,BH} & V_{BH} \end{bmatrix}. \tag{37}$$

For two series i and j, calculate $\text{Var}([\bar{R}_{in}^{RS} - \bar{R}_{in}] - [\bar{R}_{jn}^{RS} - \bar{R}_{jn}])$ by setting:

$$h\left(\bar{R}_{in}^{RSl}, \bar{R}_{in}, \bar{R}_{jn}^{RSl}, \bar{R}_{jn}\right) = \left(\frac{\bar{R}_{in}^{RSl} - 1}{n} + 1\right)^n - \bar{R}_{in} - \left(\frac{\bar{R}_{jn}^{RSl} - 1}{n} + 1\right)^n + \bar{R}_{jn}, \tag{38}$$

$$\Sigma = \begin{bmatrix} V_{iRSl} & V_{iRSl,iBH} & V_{iRSl,jRSl} & V_{iRSl,jBH} \\ V_{iRSl,iBH} & V_{iBH} & V_{iBHl,jRSl} & V_{iBH,jBH} \\ V_{iRSl,jRSl} & V_{iBH,jRSl} & V_{jRSl} & V_{jRSl,jBH} \\ V_{iRSl,jBH} & V_{iBH,jBH} & V_{jRSl,jBH} & V_{jBH} \end{bmatrix}. \tag{39}$$

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Table 1. Horizon Effects in Style Portfolios

Performance Metric	Holding Horizon 1 day	1 mo	3 mo	6 mo	1 year	Holding Horizon 1 day	1 mo	3 mo	6 mo	1 year
A. CRSP Value-Weighted Index										
Daily	0.04	0.84	2.55	5.17	10.61					
Monthly		0.86	2.62	5.30	10.89					
Quarterly			2.62	5.31	10.90					
Semi-Annual				5.33	10.94					
Annual					10.97					
B. Market Capitalization, Initially Value-Weighted										
	Big					Small				
Daily	0.04	0.80	2.43	4.92	10.09	0.06	1.26	3.83	7.80	16.22
Monthly		0.81	2.45	4.95	10.14		1.46	4.44	9.07	18.97
Quarterly			2.49	5.04	10.32			5.12	10.49	22.09
Semi-Annual				5.06	10.37				10.43	21.95
Annual					10.49					22.65
C. Market Capitalization, Initially Equal-Weighted										
	Big					Small				
Daily	0.04	0.81	2.46	4.97	10.20	0.07	1.41	4.30	8.78	18.34
Monthly		0.83	2.52	5.10	10.46		1.63	4.98	10.21	21.46
Quarterly			2.58	5.22	10.71			5.73	11.78	24.94
Semi-Annual				5.19	10.64				11.78	24.94
Annual					10.69					25.87
D. Book-to-Market, Initially Value-Weighted										
	Value					Growth				
Daily	0.05	1.16	3.52	7.17	14.85	0.04	0.77	2.33	4.72	9.65
Monthly		1.20	3.63	7.39	15.33		0.78	2.37	4.79	9.80
Quarterly			3.64	7.42	15.38			2.42	4.90	10.04
Semi-Annual				7.31	15.16				4.96	10.16
Annual					15.37					10.52
E. Book-to-Market, Initially Equal-Weighted										
	Value					Growth				
Daily	0.06	1.36	4.13	8.42	17.55	0.02	0.48	1.44	2.90	5.89
Monthly		1.49	4.52	9.25	19.36		0.59	1.79	3.62	7.37
Quarterly			4.70	9.62	20.16			1.97	3.98	8.11
Semi-Annual				9.65	20.24				3.82	7.79
Annual					19.91					7.82
F. Momentum, Initially Value-Weighted										
	Winners					Losers				
Daily	0.07	1.51	4.60	9.41	19.71	-0.01	-0.15	-0.46	-0.91	-1.81
Monthly		1.51	4.61	9.44	19.77		0.03	0.09	0.18	0.37
Quarterly			4.72	9.67	20.27			0.37	0.75	1.51
Semi-Annual				9.76	20.47				0.73	1.47
Annual					20.26					1.82
G. Momentum, Initially Equal-Weighted										
	Winners					Losers				
Daily	0.08	1.71	5.23	10.74	22.62	0.03	0.67	2.02	4.09	8.34
Monthly		1.80	5.51	11.32	23.93		1.00	3.04	6.17	12.72
Quarterly			5.74	11.82	25.04			3.47	7.07	14.64
Semi-Annual				12.05	25.56				7.21	14.93
Annual					24.69					14.97

Notes: This table reports average buy-and-hold returns (BH, on the diagonal) as well as geometrically rescaled short horizon returns (RS, above the diagonal). Returns are calculated using periods of $n = 1, 21, 63, 126,$ and 252 days corresponding to Daily, Monthly, Quarterly, Semi-Annual, and Annual frequencies, and are scaled to the corresponding Holding Horizon. Panel A shows averages of the value-weighted CRSP index, and the remaining Panels show either initially equal-weighed (IEW) or initially value-weighted (IVW) averages of size, value, and momentum portfolios. The Appendix provides a detailed description of the portfolios.

Table 2. Horizon Effects: Significance and Decomposition

A. Daily Performance Metric, Quarterly Holding Horizon

	High	Low	HL	High	Low	HL	High	Low	HL	High	Low	HL
	Market Capitalization			Book-to-Market			Momentum			Inverse Price		
RS	2.46	4.30	-1.84	4.13	1.44	2.69	5.23	2.02	3.21	5.54	2.91	2.62
BH	2.58	5.73	-3.15	4.70	1.97	2.73	5.74	3.47	2.27	7.83	3.07	4.76
RS-BH	-0.12	-1.43	1.31	-0.57	-0.53	-0.04	-0.51	-1.45	0.94	-2.29	-0.16	-2.14
	[-2.25]	[-3.13]	[3.13]	[-6.67]	[-4.67]	[-0.45]	[-4.18]	[-4.14]	[3.00]	[-3.93]	[-4.32]	[-3.77]
σ_{RS}	1.09	1.45		0.88	1.22		1.35	1.47		1.47	0.96	
ρ_{RS}	0.08	0.13		0.25	0.19		0.15	0.25		0.23	0.13	
VR	1.33	2.76		3.30	2.14		1.88	2.90		3.63	1.57	
RS/BH	0.95	0.75		0.88	0.73		0.91	0.58		0.71	0.95	
ν_{in}^{net}	0.95	0.79		0.88	0.73		0.91	0.62		0.75	0.94	
	Short-Term Reversal			Volatility			Illiquidity			Z-Score		
RS	3.07	3.06	0.01	3.08	2.65	0.43	3.18	2.94	0.25	3.54	1.30	2.25
BH	3.80	4.31	-0.50	4.52	2.79	1.73	3.54	3.31	0.23	3.82	2.25	1.57
RS-BH	-0.73	-1.24	0.51	-1.44	-0.14	-1.30	-0.36	-0.37	0.01	-0.28	-0.96	0.68
	[-5.14]	[-4.12]	[2.72]	[-4.65]	[-4.20]	[-4.50]	[-4.66]	[-3.27]	[0.20]	[-4.85]	[-5.08]	[4.21]
σ_{RS}	1.30	1.41		1.73	0.66		0.92	1.48		0.87	1.16	
ρ_{RS}	0.18	0.21		0.20	0.17		0.20	0.08		0.17	0.27	
VR	2.30	2.77		2.40	2.06		2.29	1.52		2.17	3.20	
RS/BH	0.81	0.71		0.68	0.95		0.90	0.89		0.93	0.58	
ν_{in}^{net}	0.81	0.73		0.70	0.95		0.90	0.89		0.92	0.58	

B. Monthly Performance Metric, Quarterly Holding Horizon

	Market Capitalization			Book-to-Market			Momentum			Inverse Price		
RS	2.52	4.98	-2.46	4.52	1.79	2.73	5.51	3.04	2.47	6.77	3.02	3.75
BH	2.58	5.73	-3.15	4.70	1.97	2.73	5.74	3.47	2.27	7.83	3.07	4.76
RS-BH	-0.06	-0.74	0.69	-0.17	-0.18	0.00	-0.23	-0.44	0.20	-1.06	-0.05	-1.01
	[-1.35]	[-2.22]	[2.25]	[-2.39]	[-1.81]	[0.01]	[-2.56]	[-1.87]	[0.96]	[-2.56]	[-1.71]	[-2.52]
σ_{RS}	5.39	9.18		6.49	7.42		7.53	10.41		10.84	5.12	
ρ_{RS}	0.08	0.22		0.29	0.14		0.15	0.20		0.23	0.10	
VR	1.14	1.44		1.27	1.21		1.26	1.21		1.41	1.15	
RS/BH	0.98	0.87		0.96	0.91		0.96	0.87		0.86	0.98	
ν_{in}^{net}	0.98	0.90		0.96	0.91		0.96	0.90		0.90	0.98	
	Short-Term Reversal			Volatility			Illiquidity			Z-Score		
RS	3.53	3.81	-0.29	4.12	2.75	1.36	3.41	3.15	0.26	3.74	1.99	1.75
BH	3.80	4.31	-0.50	4.52	2.79	1.73	3.54	3.31	0.23	3.82	2.25	1.57
RS-BH	-0.28	-0.49	0.22	-0.41	-0.04	-0.37	-0.13	-0.16	0.03	-0.08	-0.26	0.18
	[-2.75]	[-2.32]	[1.56]	[-1.54]	[-1.41]	[-1.49]	[-2.60]	[-2.16]	[0.61]	[-1.97]	[-1.45]	[1.15]
σ_{RS}	8.03	9.41		11.31	4.01		5.72	7.73		5.37	8.64	
ρ_{RS}	0.23	0.19		0.19	0.08		0.17	0.12		0.13	0.23	
VR	1.26	1.30		1.17	1.18		1.25	1.17		1.19	1.21	
RS/BH	0.93	0.89		0.91	0.99		0.96	0.95		0.98	0.89	
ν_{in}^{net}	0.93	0.90		0.92	0.98		0.96	0.95		0.98	0.89	

Notes: This table reports and decomposes the horizon effects in initially equally weighted style portfolios. In Panel A, the rescaled (RS) performance measures are obtained from daily returns and compared to quarterly buy-and-hold (BH) returns. Panel B compares monthly RS returns to quarterly BH returns. RS returns are computed by geometrically rescaling the daily (Panel A) or monthly (Panel B) portfolio average returns to a quarterly frequency. BH returns are computed as average of 63-day (quarterly) returns. t-statistics for the difference between RS and BH returns are shown in square brackets and are calculated using the Delta method described in the Appendix. Also reported are the standard deviation σ_{RS} and first-order autocorrelation ρ_{RS} of daily (Panel A) and monthly (Panel B) returns, the relevant variance ratios (VR), as well as the empirically measured bias (RS/BH) and its analytical approximation ν_{in}^{net} from Proposition 4. The Appendix provides a detailed description of the portfolios.

Table 3. Horizon Effects in Alphas

A. Daily Alphas and Betas

	High	Low	HL	High	Low	HL	High	Low	HL	High	Low	HL
	Market Capitalization			Book-to-Market			Momentum			Inverse Price		
Alpha	0.02	0.65	-0.63	0.97	-1.72	2.69	1.95	-1.81	3.76	1.46	0.42	1.04
t (Alpha)	[0.27]	[1.28]	[-1.21]	[2.83]	[-5.01]	[6.75]	[6.30]	[-4.25]	[7.91]	[2.91]	[3.25]	[1.97]
Beta	0.99	1.72	-0.73	1.54	1.60	-0.06	1.38	1.77	-0.39	1.86	0.96	0.90
	Short-Term Reversal			Volatility			Illiquidity			Z-Score		
Alpha	-0.42	-0.62	0.20	-1.11	0.63	-1.75	0.48	-0.22	0.71	0.84	-2.10	2.94
t (Alpha)	[-1.33]	[-1.61]	[0.49]	[-2.20]	[5.55]	[-3.42]	[2.33]	[-0.86]	[2.45]	[3.83]	[-4.57]	[7.14]
Beta	1.55	1.66	-0.11	1.97	0.67	1.30	1.08	1.35	-0.28	1.14	1.82	-0.68

B. Monthly Alphas and Betas

	Market Capitalization			Book-to-Market			Momentum			Inverse Price		
Alpha	0.03	0.92	-0.90	1.42	-1.40	2.82	2.18	-1.02	3.21	2.26	0.48	1.78
t (Alpha)	[0.33]	[1.41]	[-1.34]	[2.60]	[-2.93]	[5.20]	[5.68]	[-1.54]	[4.69]	[2.77]	[3.42]	[2.10]
Beta	0.99	1.90	-0.91	1.47	1.59	-0.12	1.36	1.82	-0.46	2.03	0.95	1.08
	Short-Term Reversal			Volatility			Illiquidity			Z-Score		
Alpha	-0.13	-0.09	-0.04	-0.34	0.69	-1.03	0.67	-0.10	0.76	1.04	-1.36	2.40
t (Alpha)	[-0.33]	[-0.17]	[-0.09]	[-0.49]	[4.57]	[-1.43]	[3.03]	[-0.29]	[2.22]	[3.50]	[-1.76]	[3.68]
Beta	1.58	1.72	-0.14	2.01	0.67	1.34	1.06	1.35	-0.29	1.12	1.73	-0.61

C. Quarterly Alphas and Betas

	Market Capitalization			Book-to-Market			Momentum			Inverse Price		
Alpha	-0.02	1.59	-1.61	1.87	-1.17	3.04	2.46	-0.95	3.41	2.92	0.59	2.33
t (Alpha)	[-0.21]	[1.73]	[-1.72]	[3.06]	[-2.31]	[4.87]	[5.71]	[-1.27]	[4.40]	[2.45]	[3.55]	[1.90]
Beta	1.01	1.91	-0.90	1.22	1.49	-0.26	1.33	1.96	-0.63	2.25	0.90	1.35
	Short-Term Reversal			Volatility			Illiquidity			Z-Score		
Alpha	0.19	0.06	0.13	0.01	0.68	-0.68	0.73	-0.06	0.80	1.12	-1.00	2.11
t (Alpha)	[0.44]	[0.09]	[0.26]	[0.01]	[3.78]	[-0.79]	[3.00]	[-0.18]	[2.04]	[3.60]	[-1.21]	[2.89]
Beta	1.51	1.86	-0.35	2.04	0.69	1.34	1.07	1.38	-0.31	1.12	1.58	-0.46

D. Decomposition of the Difference Between Rescaled Daily and Quarterly Alphas

	Market Capitalization			Book-to-Market			Momentum			Inverse Price		
Component 1	-0.14	-1.51	1.37	-0.64	-0.53	-0.12	-0.64	-1.47	0.83	-2.43	-0.19	-2.24
Component 2	0.14	0.24	-0.10	0.09	0.10	0.00	0.20	0.25	-0.06	0.26	0.13	0.12
Component 3	0.04	0.33	-0.29	-0.37	-0.13	-0.24	-0.08	0.34	-0.42	0.71	-0.10	0.81
Alpha Bias	0.04	-0.94	0.98	-0.90	-0.55	-0.35	-0.52	-0.86	0.35	-1.46	-0.16	-1.29
	Short-Term Reversal			Volatility			Illiquidity			Z-Score		
Component 1	-0.77	-1.28	0.51	-1.48	-0.17	-1.31	-0.40	-0.40	0.01	-0.33	-0.95	0.63
Component 2	0.22	0.24	-0.02	0.24	0.08	0.16	0.15	0.19	-0.04	0.07	0.11	-0.04
Component 3	-0.07	0.36	-0.43	0.12	0.04	0.09	-0.01	0.05	-0.06	-0.03	-0.29	0.26
Alpha Bias	-0.61	-0.68	0.07	-1.12	-0.05	-1.07	-0.25	-0.16	-0.09	-0.28	-1.10	0.82

E. Decomposition of the Difference Between Rescaled Monthly and Quarterly Alphas

	Market Capitalization			Book-to-Market			Momentum			Inverse Price		
Component 1	-0.08	-0.82	0.75	-0.23	-0.18	-0.06	-0.33	-0.46	0.13	-1.21	-0.08	-1.13
Component 2	0.08	0.15	-0.07	0.05	0.06	0.00	0.10	0.14	-0.03	0.14	0.07	0.08
Component 3	0.04	0.02	0.02	-0.29	-0.12	-0.17	-0.06	0.25	-0.30	0.39	-0.09	0.48
Alpha Bias	0.04	-0.67	0.72	-0.45	-0.23	-0.22	-0.28	-0.08	-0.20	-0.66	-0.10	-0.55
	Short-Term Reversal			Volatility			Illiquidity			Z-Score		
Component 1	-0.31	-0.54	0.22	-0.46	-0.06	-0.40	-0.16	-0.19	0.02	-0.12	-0.26	0.15
Component 2	0.12	0.13	-0.01	0.06	0.02	0.04	0.08	0.10	-0.02	0.04	0.06	-0.02
Component 3	-0.13	0.25	-0.39	0.05	0.04	0.00	0.02	0.05	-0.04	-0.01	-0.18	0.17
Alpha Bias	-0.32	-0.15	-0.17	-0.34	0.01	-0.35	-0.07	-0.03	-0.03	-0.08	-0.36	0.28

Notes: This table reports in Panels A and B daily and monthly alphas linearly rescaled to a quarterly horizon. Buy-and-hold quarterly alphas are shown in Panel C. Panels D and E decompose the differences between rescaled and buy-and-hold alphas into three components following Equation (18). Component 1 is the bias in portfolio returns, $(\nu_{in}^{net}-1)(\bar{R}_{in}^{BH}-1)$, component 2 is the bias in factor returns, $-\beta_i(\nu_{Mn}^{net}-1)(\bar{R}_{Mn}^{BH}-1)$, and component 3 is the beta difference, $-(\beta_i-\beta_{in})(\bar{R}_{Mn}^{BH}-1)$. Daily and monthly betas are estimated with 63 and 3 Dimson (1979) lags, respectively. The Appendix provides a detailed description of the portfolios.

Table 4. Horizon Effects in Country Index Portfolios

	Developed	Emerging	Frontier
A. Ratio of rescaled and buy-and-hold average returns (RS/BH)			
25th percentile	0.85	0.80	0.74
Median	0.92	0.82	0.78
75th percentile	0.97	0.95	0.91
Mean	0.90	0.86	0.81
B. Difference between rescaled and buy-and-hold average returns (RS-BH)			
25th percentile	-1.97	-4.45	-4.54
Median	-1.16	-2.22	-3.70
75th percentile	-0.52	-0.88	-2.19
Mean	-1.26	-4.00	-3.81

Notes: This table reports quartiles and the mean of horizon effects in returns of MSCI country indices within MSCI development classifications. Horizon effects are the ratio (Panel A) or the difference (percent annual, Panel B) between geometrically rescaled average monthly returns compounded to annual frequency (RS) and average annual buy-and-hold returns (BH). Countries are classified by MSCI as Developed, Emerging, or Frontier. Developed countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, and United States. Emerging countries are Brazil, Chile, China, Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Morocco, Peru, Philippines, Poland, Russia, South Africa, South Korea, Taiwan, Thailand, and Turkey. Frontier countries are Argentina, Croatia, Estonia, Jordan, Kenya, Lebanon, Mauritius, Nigeria, Pakistan, Slovenia, and Sri Lanka. The full set of results is available in the Internet Appendix.

Table 5. Horizon Effects in Developed Market Factor Portfolios

	Small	Big	SMB	High	Low	HML	Losers	Winners	WML
				A. Asia Pacific Excluding Japan					
RS	11.80	14.93	-3.13	17.63	9.32	8.31	19.12	9.30	9.83
BH	16.04	16.72	-0.68	20.46	12.82	7.64	22.34	12.66	9.68
RS-BH	-4.24	-1.79	-2.45	-2.83	-3.50	0.66	-3.22	-3.37	0.15
	[-2.34]	[-1.55]	[-2.97]	[-1.76]	[-2.32]	[0.99]	[-2.10]	[-1.50]	[0.08]
RS/BH	0.74	0.89		0.86	0.73		0.86	0.73	
				B. Europe					
RS	9.42	10.32	-0.90	12.45	7.03	5.42	16.48	4.48	12.00
BH	10.92	10.86	0.07	13.48	8.50	4.97	17.62	6.18	11.43
RS-BH	-1.51	-0.54	-0.97	-1.02	-1.47	0.45	-1.14	-1.71	0.57
	[-1.59]	[-0.73]	[-2.91]	[-1.00]	[-1.88]	[0.77]	[-1.27]	[-1.52]	[0.87]
RS/BH	0.86	0.95		0.92	0.83		0.94	0.72	
				C. Global					
RS	9.84	8.81	1.03	11.78	6.58	5.20	14.09	5.97	8.13
BH	10.64	9.21	1.43	12.20	7.73	4.47	14.85	7.00	7.85
RS-BH	-0.80	-0.41	-0.40	-0.42	-1.15	0.73	-0.76	-1.03	0.27
	[-1.19]	[-0.70]	[-1.50]	[-0.60]	[-1.78]	[1.65]	[-1.01]	[-1.17]	[0.42]
RS/BH	0.92	0.96		0.97	0.85		0.95	0.85	
				D. Japan					
RS	2.94	3.15	-0.21	6.13	0.29	5.83	4.15	2.86	1.29
BH	4.21	3.83	0.39	6.56	2.51	4.05	6.20	3.44	2.76
RS-BH	-1.27	-0.68	-0.59	-0.43	-2.21	1.78	-2.06	-0.58	-1.47
	[-1.18]	[-0.87]	[-0.71]	[-0.50]	[-1.40]	[1.18]	[-1.29]	[-0.51]	[-0.81]
RS/BH	0.70	0.82		0.93	0.12		0.67	0.83	
				E. North America					
RS	13.44	10.79	2.65	14.05	9.94	4.11	17.77	9.36	8.42
BH	13.52	11.25	2.27	14.41	10.58	3.83	18.04	10.07	7.98
RS-BH	-0.08	-0.46	0.38	-0.36	-0.64	0.28	-0.27	-0.71	0.44
	[-0.11]	[-0.75]	[1.16]	[-0.50]	[-0.90]	[0.64]	[-0.30]	[-0.76]	[0.58]
RS/BH	0.99	0.96		0.97	0.94		0.99	0.93	

Notes: This table reports horizon effects in returns (percent annual) of the developed market factor portfolios. Geometrically rescaled returns (RS) are average monthly returns compounded to annual frequency. Buy-and-hold returns (BH) are average annual returns. t-statistics for the difference between RS and BH returns are shown in square brackets and are calculated using the Delta method described in the Appendix. The sample period is 1991-2012.

Table 6. Horizon Effects in Non-Investable and Investable Country Portfolios

Country	MSCI Index RS	BH	RS/BH	RS-BH		iShares ETF RS	BH	RS/BH	RS-BH		Index(RS-BH) vs ETF(RS-BH)		Years
Australia	13.07	14.11	0.93	-1.04	[-0.72]	12.38	12.79	0.97	-0.41	[-0.32]	-0.63	[-1.99]	16
Brazil	24.00	28.73	0.84	-4.73	[-1.39]	21.91	26.39	0.83	-4.47	[-1.46]	-0.26	[-0.38]	12
Canada	12.86	13.85	0.93	-0.98	[-0.90]	12.10	12.66	0.96	-0.56	[-0.57]	-0.43	[-2.36]	16
France	9.01	9.48	0.95	-0.47	[-0.56]	8.35	8.51	0.98	-0.17	[-0.21]	-0.30	[-2.10]	16
Germany	10.24	10.94	0.94	-0.70	[-0.64]	9.90	10.27	0.96	-0.37	[-0.35]	-0.32	[-1.65]	16
Hong Kong	9.47	10.60	0.89	-1.13	[-0.74]	8.48	9.23	0.92	-0.75	[-0.49]	-0.38	[-1.70]	16
Italy	7.56	8.11	0.93	-0.55	[-0.45]	6.89	7.15	0.96	-0.25	[-0.22]	-0.30	[-1.46]	16
Japan	1.64	2.69	0.61	-1.05	[-1.33]	1.16	1.96	0.59	-0.81	[-0.92]	-0.24	[-1.36]	16
Malaysia	8.51	12.06	0.71	-3.56	[-1.10]	10.58	10.85	0.98	-0.27	[-0.07]	-3.29	[-1.04]	16
Singapore	10.43	12.85	0.81	-2.42	[-1.41]	8.48	9.74	0.87	-1.25	[-0.74]	-1.17	[-0.93]	16
South Africa	19.65	18.97	1.04	0.68	[0.48]	18.96	17.77	1.07	1.19	[0.82]	-0.51	[-1.64]	9
South Korea	23.05	22.81	1.01	0.24	[0.08]	21.79	21.23	1.03	0.56	[0.19]	-0.32	[-1.63]	12
Spain	11.48	11.66	0.98	-0.18	[-0.19]	10.83	10.61	1.02	0.22	[0.24]	-0.40	[-1.69]	16
Sweden	13.81	15.71	0.88	-1.90	[-1.35]	12.10	13.30	0.91	-1.21	[-1.03]	-0.69	[-1.67]	16
Switzerland	10.07	10.29	0.98	-0.22	[-0.36]	8.99	8.86	1.01	0.13	[0.21]	-0.35	[-1.74]	16
Taiwan	10.41	10.86	0.96	-0.45	[-0.24]	8.35	8.40	0.99	-0.06	[-0.03]	-0.39	[-1.14]	12
United Kingdom	6.79	7.92	0.86	-1.13	[-1.01]	6.18	6.96	0.89	-0.78	[-0.77]	-0.35	[-1.77]	16

Notes: This table reports horizon effects in returns (percent annual) of MSCI country indices and the corresponding exchange-traded funds (ETFs), as well as the difference in effects between indices and ETFs. Geometrically rescaled returns (RS) are average monthly returns compounded to annual frequency. Buy-and-hold returns (BH) are average annual returns. t-statistics for the difference between RS and BH returns are shown in square brackets and are calculated using the Delta method described in the Appendix. The length of the sample periods is shown in the last column, and all sample periods end in 2012.

Table 7. HORIZON EFFECTS AND REACTION TO LAGGED MARKET INFORMATION

Reg	Portfolio	Short Horizon	Long Horizon	L	DELAY		R^2	Obs
(1)	U.S. style	day	quarter	63	-0.470	[-2.75]	0.305	16
(2)	U.S. style	month	quarter	3	-0.335	[-2.65]	0.286	16
(3)	MSCI country	month	year	12	-0.104	[-1.87]	0.044	56
(4)	Country ETFs	month	year	12	-0.134	[-1.62]	0.064	17

Notes: This table reports coefficients, corresponding t-statistics in square brackets, and adjusted R^2 values from the following cross-sectional regressions:

$$(\text{RS/BH})_i = c_0 + c_1 \text{DELAY}_i + \eta_i.$$

RS/BH_i is the ratio of the geometrically rescaled short-horizon average return on portfolio i and the long-horizon buy-and-hold counterpart. The proxy for information delay, $\text{DELAY}_i \equiv \sum_{\tau=1}^{L} \hat{\beta}_{i\tau} / \sum_{\tau=0}^{L} \hat{\beta}_{i\tau}$, is the ratio of the sum of coefficients on the lagged market return to the sum of all market coefficients from regressions of portfolio i logarithmic excess returns on market logarithmic excess returns and its L lags. Regressions (1) and (2) analyze the U.S. style portfolios from Table 2, specification (3) studies MSCI country indices from Table 4, and regression (4) conducts the analysis on the exchange-traded funds (ETFs) from Table 6.

Table 8. Structural Estimation, Small and Large Stock Portfolios

	Step One				Step Two					
	parameters				parameters				$RS - BH$	
Θ	β	h_{11}	p_0	ω_1	σ_2	h_{21}	ω_2	ω	exact	approx
A. Small Stocks									**-5.58**	**-4.91**
Simulated Market Returns										
1	1.45	0.96	-	11.24	18.7	0.02	2.67	13.90	-2.27	-2.27
2	1.64	6.73	0.36	5.72	21.1	0.34	0.62	6.34	-3.51	-3.51
3	1.64	4.60	0.35	5.37	21.1	0.22	0.67	6.04	-3.48	-3.48
4	1.65	3.07	0.33	4.88	20.9	0.13	0.72	5.60	-3.42	-3.42
6	1.66	1.17	0.27	3.66	20.2	0.03	0.84	4.50	-3.21	-3.21
8	1.71	0.46	0.16	2.57	19.8	0.01	0.83	3.40	-3.15	-3.15
Actual Market Returns										
1	1.45	0.96	-	11.24	18.6	0.01	2.49	13.72	-3.25	-3.31
2	1.64	6.74	0.36	5.69	20.4	0.27	0.38	6.07	-4.58	-4.66
3	1.65	4.61	0.36	5.35	19.9	0.15	0.40	5.75	-4.48	-4.55
4	1.65	3.06	0.34	4.87	20.1	0.09	0.42	5.29	-4.47	-4.54
6	1.66	1.16	0.28	3.67	19.6	0.02	0.48	4.14	-4.29	-4.36
8	1.71	0.45	0.17	2.60	19.3	0.00	0.45	3.05	-4.25	-4.33
B. Large Stocks									**-0.48**	**-0.49**
Simulated Market Returns										
1	1.01	0.01	-	0.03	8.8	0.82	0.84	0.88	-0.24	-0.24
2	1.01	0.04	0.58	0.03	7.0	1.22	1.04	1.07	-0.10	-0.10
3	1.01	0.01	0.49	0.03	7.8	0.98	1.00	1.03	-0.15	-0.15
4	1.01	0.01	0.46	0.03	8.3	0.86	1.00	1.03	-0.19	-0.19
6	1.01	0.00	0.39	0.03	9.6	0.71	0.98	1.01	-0.28	-0.28
8	1.01	0.00	0.39	0.03	10.1	0.58	0.99	1.03	-0.28	-0.28
Actual Market Returns										
1	1.01	0.01	-	0.03	3.6	0.36	0.01	0.05	-0.49	-0.51
2	1.01	0.04	0.58	0.03	4.0	1.26	0.01	0.04	-0.51	-0.53
3	1.01	0.01	0.49	0.03	4.0	0.65	0.01	0.04	-0.51	-0.52
4	1.01	0.01	0.46	0.03	3.9	0.40	0.01	0.04	-0.50	-0.52
6	1.01	0.00	0.39	0.03	4.0	0.16	0.01	0.04	-0.50	-0.52
8	1.01	0.00	0.44	0.03	4.0	0.08	0.01	0.04	-0.50	-0.52

Notes: This table reports parameters for structural models with $\Theta \in \{1, 2, 3, 4, 6, 8\}$ estimated using moments produced by small stocks (Panel A) and large stocks (Panel B). Step One moment conditions are the individual slope coefficients, $\beta_{i0}, \cdots, \beta_{i5}$, from regression (19) with $L = 63$ lags and "sum" betas $\sum_{\tau=0}^{J} \beta_{i\tau}$ for $J = 10, 21, 63$. Step Two moment conditions are the one-day return variance, the first ten individual portfolio autocorrelations, and sums of autocorrelations for lags $11 - 20, ..., 41 - 50, 51 - 63$. The minimized SMM objective function values for each stage, ω_1 and ω_2, along with the overall objective function value ω are reported to provide an indication of the goodness of fit across models. The difference between the rescaled and buy-and-hold returns at a horizon of 252 days, $RS - BH$, summarizes the horizon effects produced by the best fitting parameterizations. "*Simulated Market Returns*" draws factor realizations f_{1t} according to the iid data generating process of the model. We set the mean and standard deviation, μ_1 and σ_1, to the sample moments of daily logarithmic CRSP Index returns during 1927 - 2009. "*Actual Market Returns*" sets f_{1t} to the historical realized daily logarithmic CRSP index returns, while continuing to simulate the returns of f_{2t}.

Table 9. Structural Estimation, Additional Portfolios

	Step One				Step Two					
	parameters				parameters				RS-BH	
Θ	β	h_{11}	p_0	ω_1	σ_2	h_{21}	ω_2	ω	exact	approx
Momentum, Low									**-6.62**	**-5.90**
1	1.65	0.75	-	4.92	16.4	9.86	9.25	14.18	-4.67	-4.73
8	1.74	0.13	0.03	1.06	21.4	0.05	0.78	1.84	-5.43	-5.50
High									**-2.29**	**-2.26**
1	1.28	0.36	-	1.64	19.2	100.00	1.76	3.41	-1.99	-2.03
8	1.37	0.47	0.67	0.82	12.3	0.02	0.52	1.34	-2.10	-2.15
Volatility, Low									**-0.58**	**-0.60**
1	0.68	0.37	-	0.11	6.0	14.47	0.47	0.58	-0.53	-0.54
8	0.68	0.02	0.00	0.04	1.1	0.80	0.20	0.24	-0.45	-0.45
High									**-7.52**	**-6.83**
1	1.79	0.63	-	6.70	16.6	0.03	1.64	8.34	-4.09	-4.17
8	1.93	0.20	0.26	1.70	22.4	0.03	0.44	2.15	-5.98	-6.08
B/M, Low									**-2.26**	**-2.33**
1	1.44	0.46	-	4.31	18.1	18.78	1.49	5.80	-2.30	-2.40
8	1.60	0.49	0.55	0.31	13.3	0.39	0.50	0.81	-2.16	-2.28
High									**-3.08**	**-3.15**
1	1.31	1.05	-	11.20	10.4	0.03	6.01	17.20	-1.91	-2.00
8	1.63	0.79	0.25	0.09	14.2	0.13	0.91	1.00	-3.13	-3.24
Illiquidity, Low									**-1.67**	**-1.62**
1	1.33	0.19	-	0.33	11.1	5.64	0.55	0.88	-1.57	-1.61
8	1.37	0.80	0.91	0.06	10.2	0.80	0.07	0.13	-1.26	-1.30
High									**-1.57**	**-1.59**
1	1.03	0.55	-	0.92	10.3	78.59	3.00	3.92	-1.39	-1.41
8	1.05	0.06	0.00	0.42	8.1	0.00	0.42	0.84	-1.45	-1.47
Z-score, Low									**-4.53**	**-4.38**
1	1.55	0.86	-	11.95	11.5	0.01	5.41	17.35	-2.22	-2.34
8	1.83	0.53	0.27	0.32	21.2	0.23	1.39	1.71	-4.30	-4.45
High									**-1.26**	**-1.30**
1	1.09	0.49	-	1.76	7.4	0.00	2.05	3.81	-0.83	-0.88
8	1.13	0.13	0.33	0.07	7.9	0.08	0.30	0.38	-1.22	-1.27
Inverse Price, Low									**-0.71**	**-0.75**
1	0.95	0.29	-	0.15	4.2	15.79	0.24	0.40	-0.74	-0.75
8	0.96	0.02	0.31	0.09	4.5	0.00	0.12	0.20	-0.74	-0.76
High									**-10.08**	**-8.46**
1	1.59	1.03	-	9.93	21.8	100.00	23.82	33.75	-4.77	-4.84
8	1.81	0.30	0.08	1.81	24.5	0.04	0.96	2.77	-6.89	-6.98
Short-term Reversal, Low									**-5.26**	**-4.78**
1	1.55	0.66	-	3.60	17.4	24.74	7.37	10.96	-3.93	-4.00
8	1.63	0.12	0.11	0.99	16.6	0.02	0.43	1.42	-4.07	-4.14
High									**-3.54**	**-3.32**
1	1.38	0.50	-	4.01	17.5	25.38	3.42	7.44	-2.97	-3.03
8	1.52	0.35	0.47	0.72	12.7	0.02	0.38	1.11	-2.89	-2.94

Notes: This table reports parameters for structural models with $\Theta \in \{1, 8\}$ estimated using moments produced by US style portfolios. Step One moment conditions are the individual slope coefficients, $\beta_{i0}, \cdots, \beta_{i5}$, from regression (19) with $L = 63$ lags and "sum" betas $\sum_{\tau=0}^{J} \beta_{i\tau}$ for $J = 10, 21, 63$. Step Two moment conditions are the one-day return variance, the first ten individual portfolio autocorrelations, and sums of autocorrelations for lags $11-20, ..., 41-50, 51-63$. The minimized SMM objective function values for each stage, ω_1 and ω_2, along with the overall objective function value ω are reported to provide an indication of the goodness of fit across models. The difference between the rescaled and buy-and-hold returns at a horizon of 252 days, $RS - BH$, summarizes the horizon effects produced by the best fitting parameterizations. The factor realizations f_{1t} are the historical realized daily logarithmic CRSP index returns, and f_{2t} is simulated.

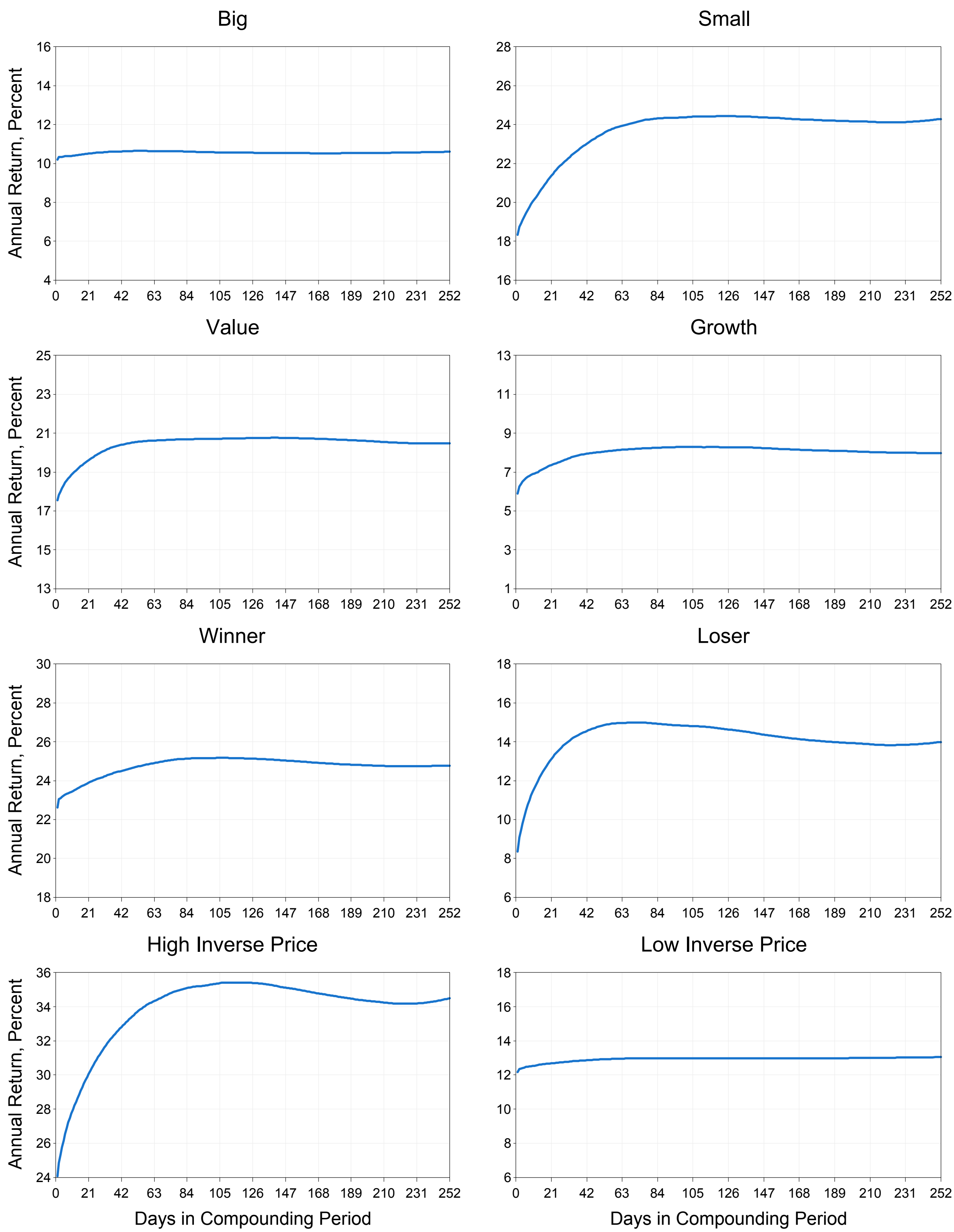

Figure 1. This figure continues on the following page.

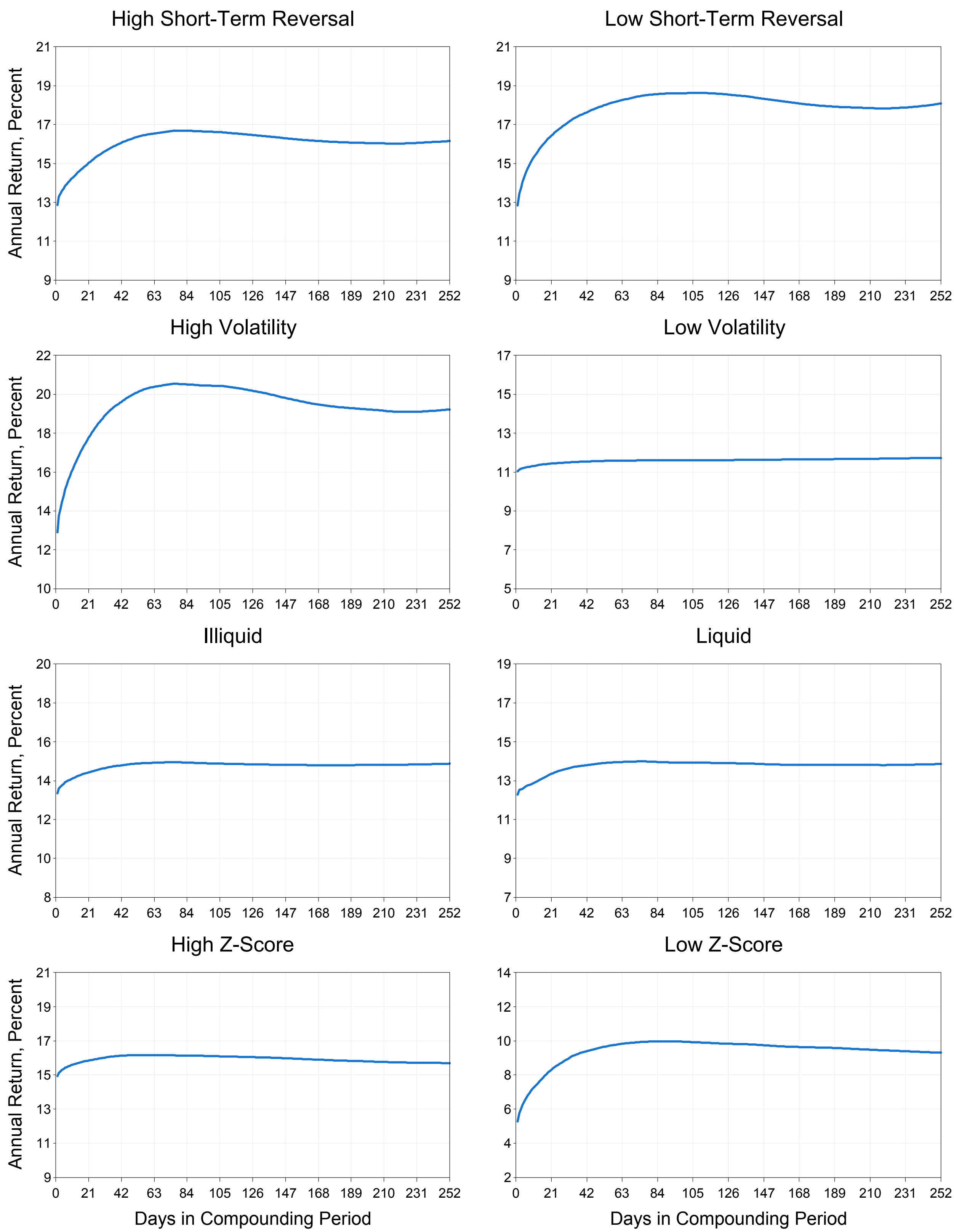

Figure 1. This figure plots for different style portfolios average rolling n-day buy-and-hold returns scaled to an annual equivalent, $\bar{R}_{in,252} - 1 = \left[\mathbb{E}\left(R_t \cdots R_{t+n-1}\right)\right]^{252/n} - 1$ against the buy-and-hold horizon n. A complete description of the test portfolios is provided in the appendix. Computational details are available in the Internet Appendix.

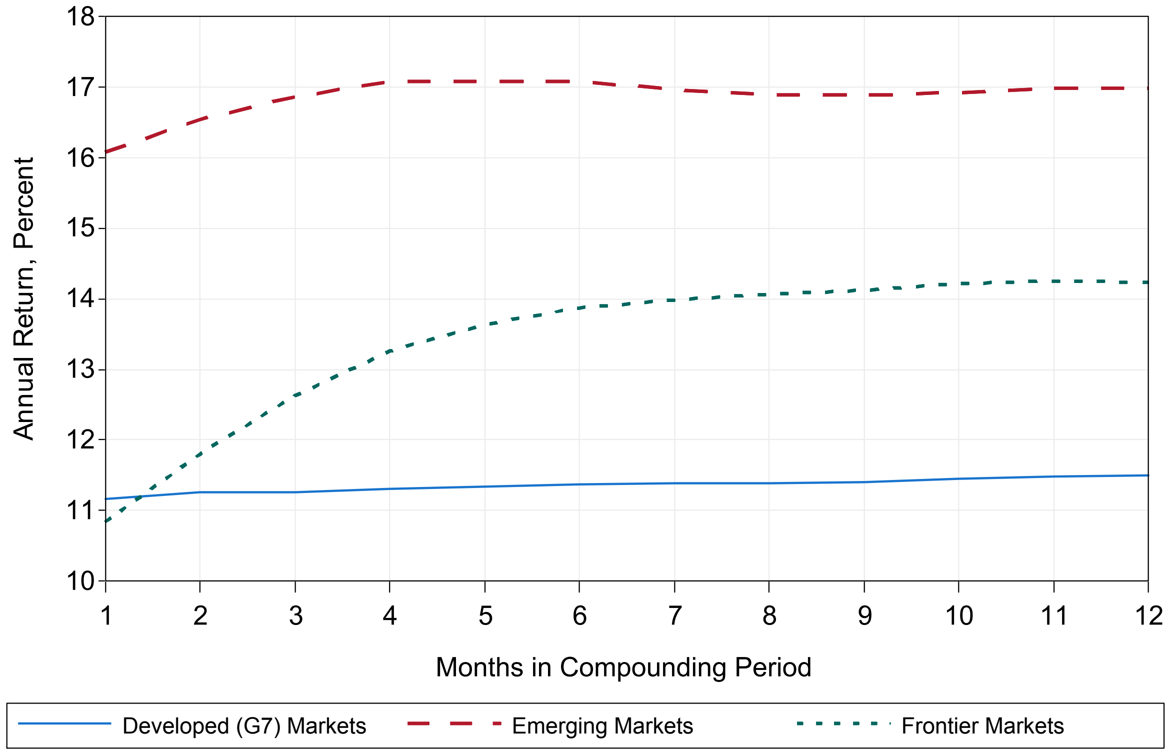

Figure 2. This figure plots for three MSCI regional portfolios average rolling n-month buy-and-hold returns scaled to an annual equivalent, $\bar{R}_{in,12} - 1 = [\mathbb{E}\left(R_t \cdots R_{t+n-1}\right)]^{12/n} - 1$ against the buy-and-hold horizon n. Computational details are available in the Internet Appendix.

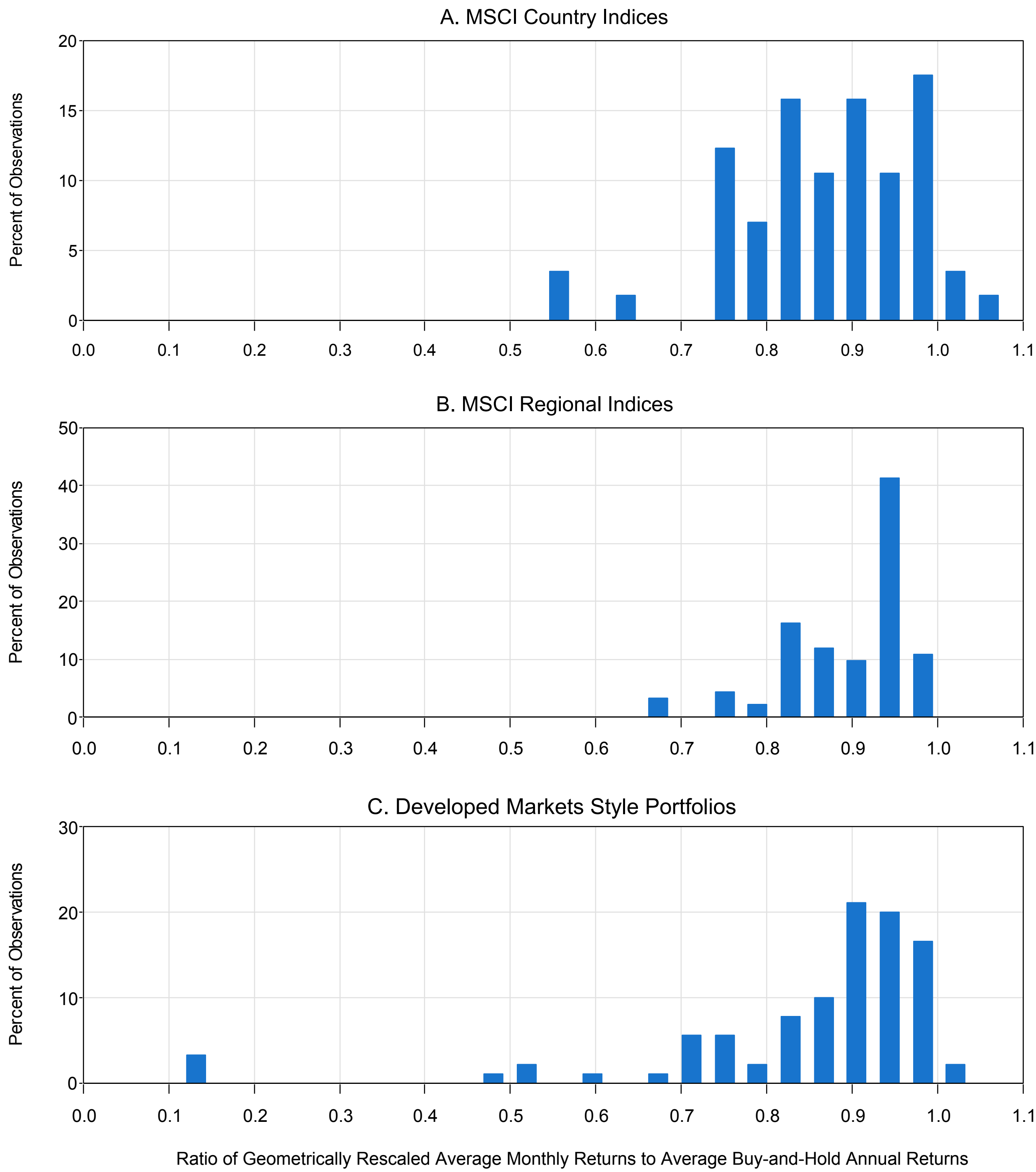

Figure 3. This figure plots histograms of the ratios of average monthly returns compounded 12 months (RS) to average annual buy-and-hold (BH) returns for three sets of portfolios: MSCI country index portfolios (Panel A), MSCI regional index portfolios (Panel B), and developed market style portfolios (Panel C).

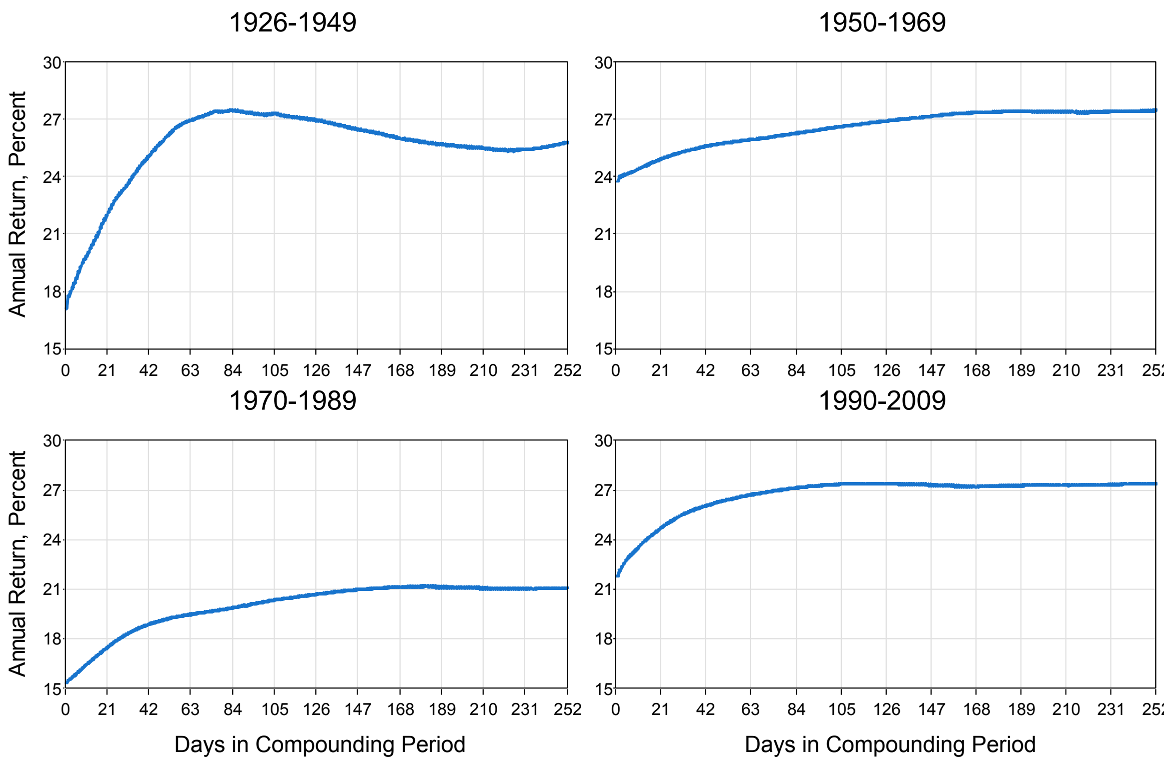

Figure 4. This figure plots for the small-stock portfolio average rolling n-day buy-and-hold returns scaled to an annual equivalent, $\bar{R}_{in,252} - 1 = \left[\mathbb{E}\left(R_t \cdots R_{t+n-1}\right)\right]^{252/n} - 1$ against the buy-and-hold horizon n in four subperiods. Computational details are available in the Internet Appendix.

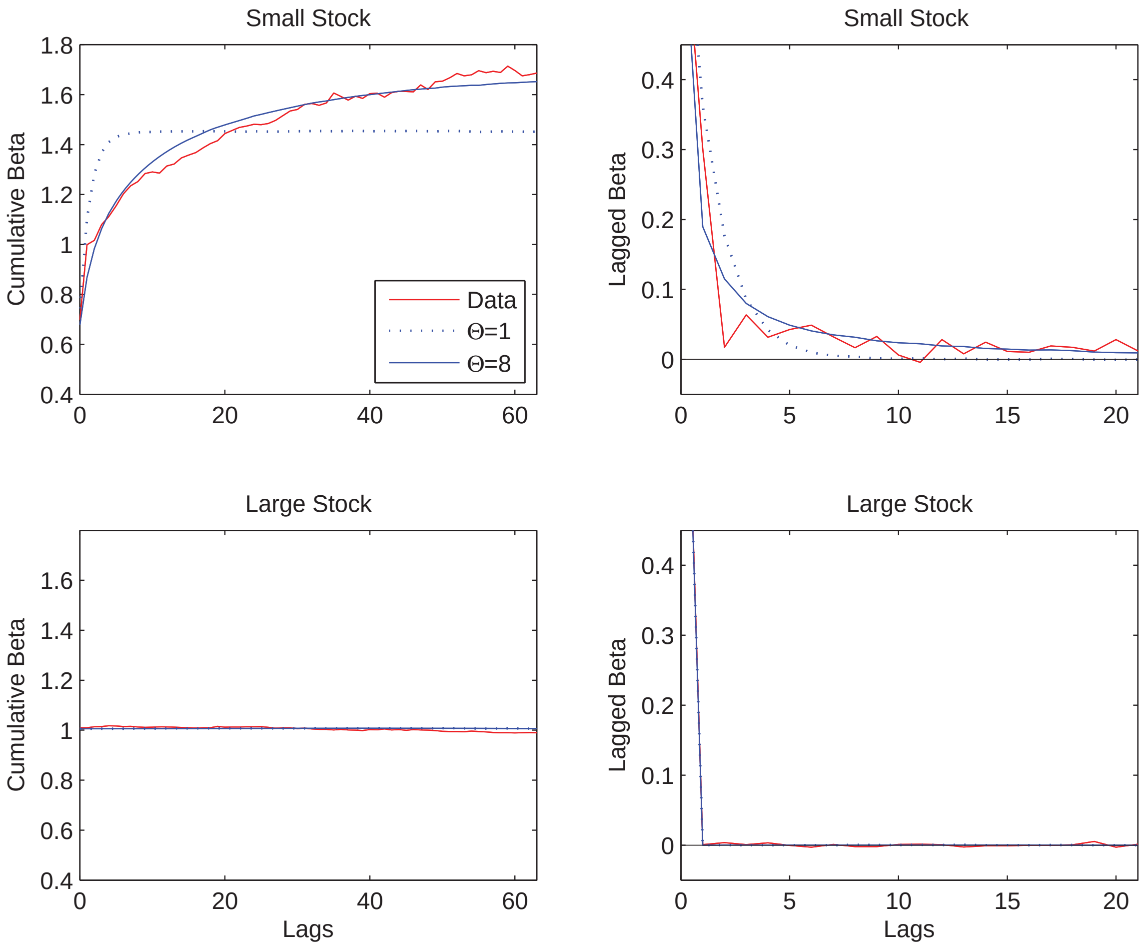

Figure 5. This figure plots cumulative "sum" betas and lagged betas from regression (19), estimated in actual data (red line) and in data simulated from the model with $\Theta = 1$ (dotted line) and $\Theta = 8$ (blue line) delay states. The regressions utilize contemporaneous daily log returns and $L = 63$ lagged daily returns. The top panels report results for small stocks and the bottom panels reports results for large stocks.

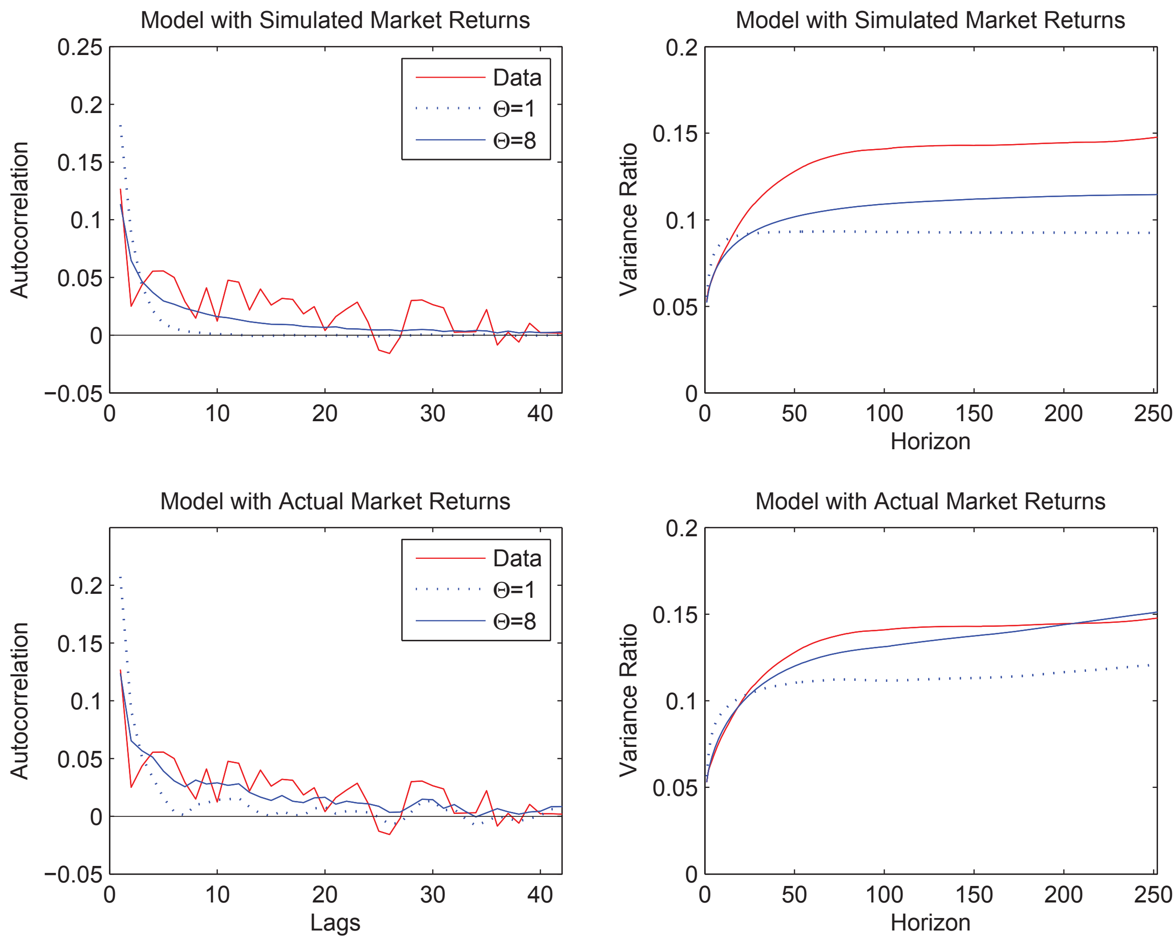

Figure 6. This figure plots return autocorrelations (left panels) and variance ratios (right panels), estimated in actual data (red line) and in data simulated from the model with $\Theta = 1$ (dotted line) and $\Theta = 8$ (blue line) delay states. The top panels are produced using the model estimated with simulated market returns and the bottom panels are produced using the model estimated with actual market returns.

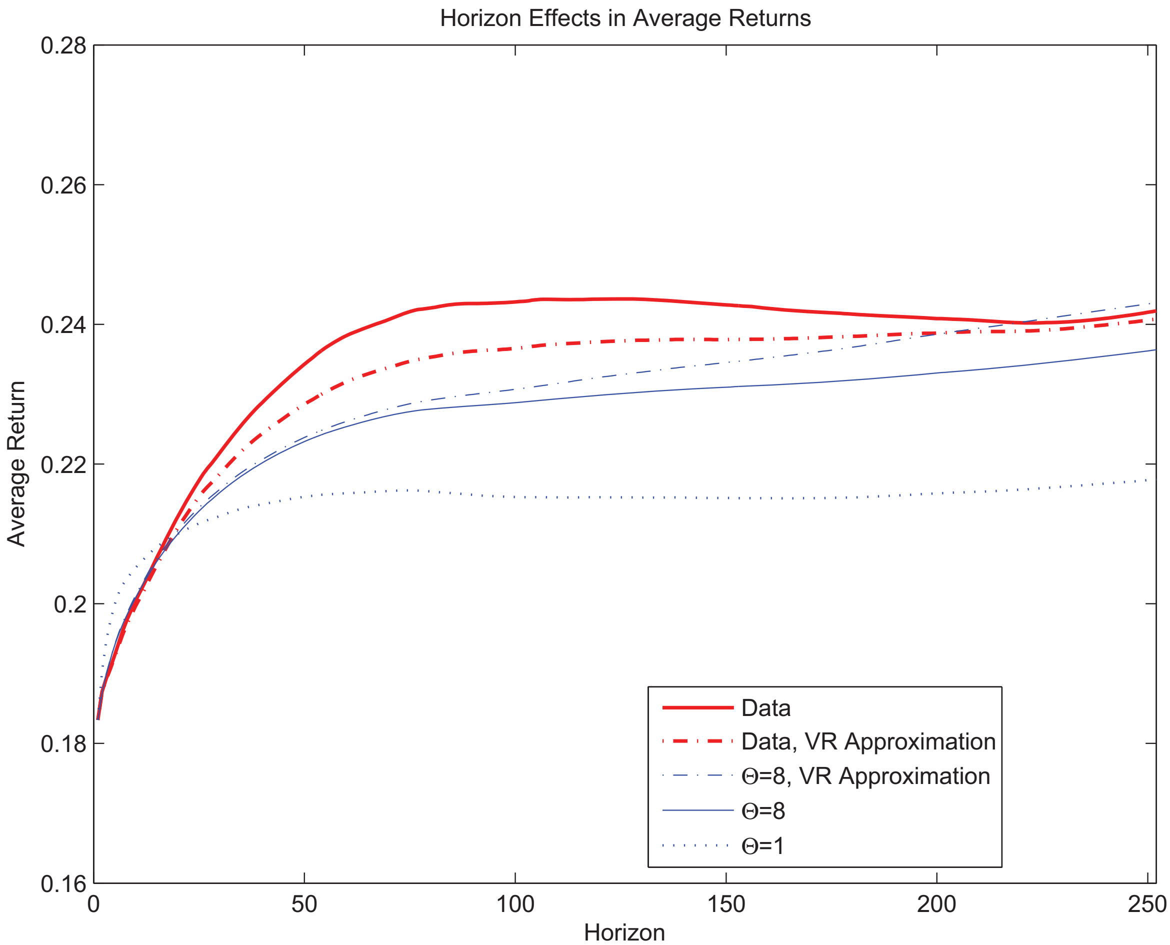

Figure 7. This figure plots average rolling n-day buy-and-hold returns scaled to an annual equivalent, $\bar{R}_{in,252} - 1 = \left[\mathbb{E}\left(R_t \cdots R_{t+n-1}\right)\right]^{252/n} - 1$ against the buy-and-hold horizon n for the small-stock portfolio (red line) as well as for data simulated from the model with $\Theta = 1$ (dotted line) and $\Theta = 8$ (blue solid line) delay states. The parameter estimates are reported in Table 8. The normal approximations are produced using the approximating relationship for horizon effects in Proposition 4 for the data (red dashed line) and the model simulated data with $\Theta = 8$ (blue dashed line).