Investment Waves under Cross Learning*

Shiyang Huang
London School of Economics

Yao Zeng
Harvard University

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Abstract

We investigate how firms' cross learning amplifies industry-wide investment waves. Firms' technologies are subject to idiosyncratic shocks and a common shock, and their asset prices aggregate speculators' private information about the two types of shocks. In investing, each firm learns from other firms' prices (in addition to its own) to make better inference about the common shock, leading to higher investment sensitivity to the common shock. To respond, speculators put a higher weight on the common shock in trading, making prices even more informative about the common shock. This spiral results in higher investment and price comovements in investment waves. Moreover, cross learning imposes a new pecuniary externality on other firms, because it makes their prices less informative about their idiosyncratic shocks thanks to speculators' endogenous overweighting on the common shock. This externality increases in the number of firms, suggesting that more competitive industries may exhibit more inefficient investment waves.

KEYWORDS: cross learning, investment waves, systematic risks, pecuniary externality, inefficiency, competition

JEL: D62, D83, G14, G31

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1 Introduction

Industry-wide investment waves are commonly observed in history, especially after the arrival of major technology or financial innovations that involve high market uncertainty.[1] However, existing theories usually ignore one of their defining features: high systematic risks associated with many firms.[2] Specifically, during investment waves, a firm's real investment and asset price co-move greatly with those of other firms. (see Rhodes-Kropf, Robinson and Viswanathan, 2005, Pastor and Veronesi, 2009, Hoberg and Phillips, 2010, Bhattacharyya and Purnanandam, 2011, Patton and Verardo, 2012, Greenwood and Hanson, 2014, for recent empirical evidence). Also, both primary and secondary financial market participants overweight industry-wide common news while underweight their idiosyncratic news in making investment decisions (see Peng, Xiong, and Bollerslev, 2007 and more broadly Rhodes-Kropf, Robinson and Viswanathan, 2005, Hoberg and Phillips, 2010, and Bhattacharyya and Purnanandam, 2011). Even more surprisingly, more competitive industries exhibit more inefficient investment waves with higher systematic risks (Hoberg and Phillips, 2010, Greenwood and Hanson, 2014). Our paper provides a new rational theory that helps unify these facts of industry-wide investment waves, which are hard to be synthesized in the previous literature.

Our theory features firms' cross learning, which means that firms learn from other firms' asset prices (in addition to their own asset prices) in making investment decisions, a natural but overlooked empirical fact in the theoretical literature.[3] It has been explicitly identified by recent empirical work (Foucault and Fresard, 2014, Ozoguz and Rebello, 2013). Our model builds on the burgeoning literature that highlights the feedback from secondary market asset prices to primary market investment decisions (see Bond, Edmans and Goldstein, 2012, for a survey). Specifically, since secondary market participants may have incremental information that is unavailable to firms or primary market participants, firms or their capital providers may learn from the asset prices in the secondary markets for making investment decisions, and this in turn affects the asset prices (see Luo, 2005, Chen, Goldstein, and Jiang, 2007, Edmans, Goldstein, and Jiang, 2012, Foucault and Fresard, 2012, for the empirical evidence). The feedback literature, however, has not explored the

[1] The most typical examples include the "railway mania" of the UK in the 1840s, the rapid development of automobiles and radio in the 1920s, and most recently the surge of the Internet in the 1990s, among many others. In addition to technological progress, other notable examples include major financial innovations like asset-backed securities (ABS) and credit default swaps (CDS), as well as the Mississippi Scheme and the South Sea Bubble, in which market structures experienced dramatic changes.

[2] The most popular explanation of investment waves comes from the literature of bubbles (see Brunnermeier and Oehmke, 2013, Xiong, 2013, for surveys of various models and evidence). These theories have focused on the over-investment or over-valuation of one single firm, and have often referred to behavioral aspects. The modern literature of macro-finance (see Brunnermeier, Eisenbach and Sannikov, 2013, for an extensive survey) also generates various forms of over-investment, over-borrowing, and over-lending, by highlighting agency or financial frictions. Also see He and Kondor (2013) for a most recent treatment of two-sided pecuniary externality in generating inefficient investment cycles. This literature focuses more on the systemic implications of over-investment, such as fire sales and financial crises, rather than on the microeconomic anatomy of multi-firm investment waves as we tend to emphasize.

[3] In the seminal field survey by Graham and Harvey (2001), CFOs of firms report that they tend to rely on other firms' prices in making capital budgeting decisions, and this in turn affects CEOs' investment decisions. As far as we know, this point has not been formally taken in the existing corporate finance models.

multi-firm context and the cross-learning mechanism we emphasize which generate industry-wide investment waves.

To address industry-wide investment waves, we extend the classical feedback framework to admit multiple firms with two different types of shocks and cross learning. We postulate that investment opportunities in an industry or an economy are correlated, so that firms have the incentives to learn from each other's asset prices. To illustrate the idea, Figure 1 depicts the classical feedback models without this consideration, even if they can literally accommodate many firms. Although these firms can take advantage of their respective feedback channel for making better investment decisions, they are essentially separated in segmented economies, and hence others' asset prices are irrelevant. This is the reason why the existing feedback models usually feature one representative firm or one single asset.[4]

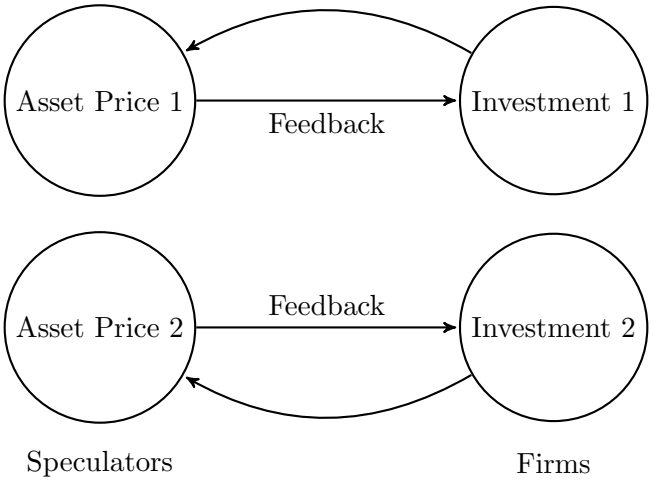

Figure 1: Self-Feedback Benchmark

Instead, the novelty of our work is developing a tractable model featuring two-way cross learning of firms from other firms' asset prices (in addition to their own), and identifying a new pecuniary externality involved. We explicitly model correlated investment opportunities by incorporating two different types of shocks, which necessitate firms' cross learning. When the fundamental of each firm's asset is subject to both a common productivity shock (an industry shock)[5] and an idiosyncratic shock (a firm-specific shock), other firms' asset prices become noisy but informative

[4]One exception is Subrahmanyam and Titman (2013), in which a private firm learns from the stock price of another public firm to make investment decision. The private firm's investment affects the profitability of the public firm through competition, which further generates interesting macroeconomic implications. But the public firm does not invest by itself and the private firm also does not have its own asset price. Hence, their model still features the standard feedback channel as shown in Figure 1. Their formal model also admits two private firms, which introduces an additional externality in terms of investment complementarity that amplifies their feedback effect. But as the authors have claimed, the introduction of two private firms is not essential for most of their results.

[5]In a broader sense, our common shock can also be interpreted as a shock to the entire economy. Hence, our model speaks to not only industry-wide investment waves but also more broadly economy-wide investment waves.

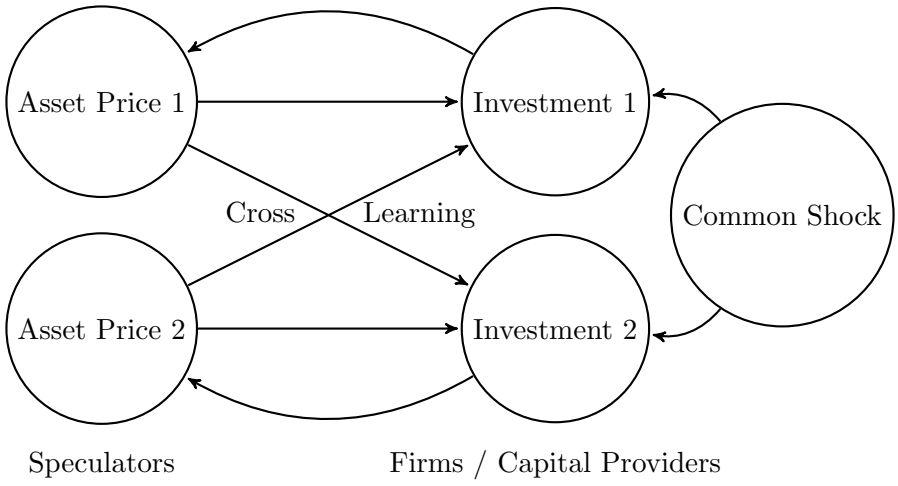

Figure 2: Firm / Capital Provider Cross Learning

signals about the common shock. Thus, the firm in question uses other firms' asset prices (and its own) to know more about the common shock for making better investment decisions, and other firms do the same. Such cross learning makes firms' investments more sensitive to the common shock, encouraging secondary market speculators to weight information about the common shock more in trading. This in turn makes firms' asset prices more informative about the common shock, resulting in even higher investment sensitivity to the common shock. As a consequence, a tiny common shock can be amplified significantly. This mechanism resembles classical rational herding (see Scharfstein and Stein, 1990, Froot, Scharfstein and Stein, 1992, Welch, 1992 and most recently Aghion and Stein, 2008, for a two-task corporate investment setting closer to ours), but we explicitly consider the feedback from financial markets to real economy and our cross-learning mechanism does not rely on any forms of short-termism. Moreover, in our framework, when one firm learns from other firms' asset prices, it does not internalize a negative pecuniary externality that, those prices become less informative about other firms' idiosyncratic shocks thanks to the speculators' endogenous overweighting on the common shock. This externality leads to higher investment inefficiency. Interestingly, the new pecuniary externality takes effect through the informativeness rather than the level of prices. Figure 2 illustrates the idea of cross learning and contrasts it to the standard feedback framework. Empirically, firms' cross learning has been documented by recent studies like Foucault and Fresard (2014)[6] and Ozoguz and Rebello (2013), and the magnitude is shown to be considerable, serving as a strong support to our theory.

The predictions of our model are consistent with many empirical regularities. Compared with

[6]For the purpose of developing empirical hypotheses, Foucault and Fresard (2014) build a suggestive model, featuring one-way learning: one focal firm may learn from its peer firm's price while not the other way around, and the peer firm does not invest. That model lays out a nice foundation for their empirical analysis. However, it does not generate inefficient multi-firm investment waves or comovements with the common productivity shock as emphasized in our paper. The setup and mechanisms of their model are also different from ours.

a benchmark in which firms cannot learn from others' asset prices, cross learning generates a higher weight of speculators on the information of the common shock in trading (as documented by Peng, Xiong, and Bollerslev, 2007 and more broadly by Rhodes-Kropf, Robinson and Viswanathan, 2005, Hoberg and Phillips, 2010, and Bhattacharyya and Purnanandam, 2011) and firms' higher investment sensitivity to the common shock.[7] We further show that under cross learning, a firm's investment and price co-move more greatly with 1) other firms' investments and prices, and with 2) the common productivity shock, fitting in line with the evidence in Rhodes-Kropf, Robinson and Viswanathan (2005), Pastor and Veronesi (2009), Hoberg and Phillips (2010), Bhattacharyya and Purnanandam (2011) and Patton and Verardo (2012). We interpret these patterns as higher systematic risks in industry-wide investment waves. Since the cross-learning mechanism relies on observable prices in public financial markets, it is further supported by Maksimovic, Phillips and Yang (2013)'s empirical findings that systematic risks are stronger among public firms than among private ones. To the best of our knowledge, our work is the first to demonstrate that firms' two-way cross learning has significant effect on investment, prices, and systematic risks.

Highlighting the cross-learning mechanism, we investigate many circumstances in which varying economic conditions generate higher systematic risks in investment waves. First, an increasing uncertainty on the common productivity shock, most typically induced by the introduction of major technological and financial innovations, leads to stronger weighting on the information of the common shock and higher systematic risks. This is consistent with the empirical facts in Brunnermeier and Nagel (2004) and Pastor and Veronesi (2006, 2009) and the more broadly documented evidence in the bubble literature (Brunnermeier and Oehmke, 2013, Xiong, 2013). Our new mechanism contributes to the existing rational learning mechanisms (Pastor and Veronesi, 2009, Johnson, 2007), by featuring both multi-firm investment waves and inefficiency. Second, an improvement of the firms' knowledge about the common productivity shock leads to higher systematic risks, consistent with the facts in Greenwood and Nagel (2009). Last, lower market liquidity or higher variance of idiosyncratic noisy supply also leads to higher systematic risks. These empirical regularities have been frequently ascribed to separate behavioral explanations in the previous literature, whereas our work provides a unified rational explanation.

Our framework allows for a clear welfare analysis, offering a new perspective to look at the relationship between inefficient investment waves and industrial competition. Due to the unaligned interests of firms and speculators in feedback and the new pecuniary externality associated with cross learning, the investment waves are inefficient. In particular, we show that, as the number of firms in an industry increases, cross learning becomes stronger, leading to a more severe pecuniary externality. This suggests a rationale for the puzzling facts identified in Hoberg and Phillips (2010) and Greenwood and Hanson (2014) that more competitive industries exhibit more predictable

[7] Complementary to the theoretical literature that highlights investors' attention allocation to the common shock (see Peng and Xiong, 2006, Veldkamp, 2006, Veldkamp and Wolfers, 2007), our work speaks to its endogenous origin from firms' cross learning as well as its impacts on firms' investment decisions.

financial and real boom-bust cycles as well as greater market and real inefficiencies. According to Hoberg and Phillips (2010), no single existing theory can accommodate their findings. Our cross-learning mechanism with the new pecuniary externality implies that, firms and investors in more competitive industries are more likely to overweight common shock whereas to underweight idiosyncratic shocks, leading to more inefficient investment waves with higher systematic risks, consistent with Hoberg and Phillips (2010) and Greenwood and Hanson (2014)'s findings. Ozoguz and Rebello (2013) have also explicitly identified that the firms in more competitive industries have higher investment sensitivity to stock prices of their peers, which supports our predictions.

Fundamentally, the amplification effect of cross learning stems from a series of endogenous strategic complementarities and a spiral, which are absent in the previous literature. At the beginning, the dependence of investment on asset price results in an endogenous complementarity between each firm's investment sensitivity to and speculators' weight on the common shock. When multiple firms' cross learning is introduced, a new spiral comes out. Cross learning first makes different firms' investments more correlated with the common shock. As a result, speculators find it more profitable to put a higher weight on the information about the common shock. Since asset prices become relatively more informative about the common shock, firms' investment sensitivity to the common shock increases even more. This spiral further implies two new complementarities in the multi-firm setting. The first is among speculators' weights on the information about the common shock in each asset market, and the second is among different firms' investment sensitivity to the common shock. The interaction of the spiral and these endogenous complementarities is again seen in Figure 2, which leads to a strong amplification effect from a fundamental shock to systematic risks. Unlike the existing literature involving complementarities in financial markets (see Veldkamp, 2011, for an extensive review), our mechanism does not posit any exogenous complementarities (for example, higher-order beliefs or coordination in actions) but a well-documented fact that firms learn from their own and other firms' asset prices. In particular, since trading in financial markets exhibits natural strategic substitutability, it is usually hard in the previous literature to attain such endogenous complementarities as we do.

Related Literature. Our work contributes to the literature of rational models on investment booms and busts.[8] Early literature has focused on the role of industrial organizations (for example, Reinganum, 1989, Jovanovic and McDonald, 1994) or self-fulfilling expectations (for example, Shleifer, 1986) in generating investment waves, but financial markets are generally absent in these classic papers. The modern literature has been paying increasing attention to the role of learning in financial markets.[9] In the rational learning model of Rhodes-Kropf and Viswanathan (2004), which

[8]This literature is more broadly related to the bubble literature and the modern macro-finance literature, as discussed above. The focuses of those literatures are however different from ours. Our model is not intended as a general dynamic theory of booms and busts either.

[9]Other notable rational models on investment waves include DeMarzo, Kaniel, and Kremer (2007, 2008), featuring endogenous relative wealth concerns, Carlson, Fisher and Giammarino (2004) and Aguerrevere (2009), featuring investments as real options, and Ozdenoren and Yuan (2013), featuring contractual externalities.

shares a similar signal extraction problem with ours, managers cannot distinguish between common misvaluation and possible idiosyncratic synergies, leading to merger and acquisition waves. Pastor and Veronesi (2009) propose a more explicit learning model, in which the uncertain productivity of a new technology is subject to learning. Learning and the ensuing technology adoption make the uncertainty from idiosyncratic to systematic, generating investment waves. In this spirit, Johnson (2007) argues that firms may learn about uncertain investment opportunities in the form of experimenting, which also generates investment waves. Our contribution to this literature is threefold. First, our model features multiple firms and their cross learning explicitly, which allows us to study industry-wide investment waves directly rather than to look at them from the perspective of single-firm investment cycles. Second, we cast the microstructure of public asset markets explicitly by an adapted Kyle (1985) model, ensuring us to reflect the indispensable role of public financial markets as suggested by Maksimovic, Phillips and Yang (2013). Lastly, our model identifies a new externality regarding the use of information about common shocks and idiosyncratic shocks in making investment decisions.

Our framework also contributes to the burgeoning feedback literature (see Bond, Edmans and Goldstein, 2012, for a survey), for instance, Dow and Gorton (1997) on market and real efficiencies, Subrahmanyam and Titman (2001, 2013) on information cascades and shock propagation, Hirshleifer, Subrahmanyam and Titman (2006) on irrational trading, Foucault and Gehrig (2008) on cross listing, Goldstein and Guembel (2008) on manipulative short selling, Ozdenoren and Yuan (2008) on excess volatility, Edmans (2009) on blockholders trading, Bond, Goldstein and Prescott (2010) on corrective managerial actions, among many others. Closer to the present approach are Goldstein, Ozdenoren and Yuan (2013), Goldstein and Yang (2014a,b), and Sockin and Xiong (2014a,b), all of which highlight the feedback from secondary market speculators' information aggregation to primary market capital providers' scale-varying real investment. Our contribution is to investigate multi-firm feedback by introducing a tractable two-way cross-learning framework with two types of shocks, which generates the new pecuniary externality and various new implications.

Identifying the externality associated with cross learning contributes to the large pecuniary externality literature.[10] The classical pecuniary externality takes effect through the level of prices: under various frictions, agents do not internalize the impacts of their actions on equilibrium price levels, leading to a welfare loss. In our multi-firm cross-learning framework, instead, firms that make real investment decisions do not fully internalize the impacts of cross learning on equilibrium price informativeness. This leads to a "tragedy of the commons" regarding the use of the price system as an information source. In this sense, our pecuniary externality is reminiscent of the learning externality in the early dynamic learning and herding literature (for example, Vives, 1997) that an agent, when responding to his private information, does not take into account the benefit

[10] See Stiglitz (1982), Greenwald and Stiglitz (1986), Geanakoplos and Polemarchakis (1985), and Farhi and Werning (2013), He and Kondor (2013), Davila (2014) for recent theoretical developments.

of increased informativeness of public information in the future.[11]

Our work is also related to the literature on the interaction across different asset markets, in particular the models that highlight learning. This literature has focused on speculators' learning rather than firms' cross learning as we model. Cespa and Foucault (2014) consider the contagion of illiquidity across segmented markets by introducing a concept of cross-asset learning. By cross-asset learning, speculators trading in one market can potentially learn from the asset price in another market, which generates propagation. Relatedly, Goldstein and Yang (2014c) model an environment in which different speculators are informed of different fundamentals affecting one single asset. Trading on information about the two fundamentals exhibits complementarity, suggesting that greater diversity of information improves price informativeness. Our model complements to those papers by focusing on the implications of firms' cross learning on both real investments and asset prices, in contrast to their exchange economy setting that focuses on trading.

Finally, our framework is broadly related to a large macroeconomic literature focusing on dispersed information. Closely related are Angeletos, Lorenzoni and Pavan (2012) on the role of beauty contest and Amador and Weill (2010) on the crowding-out effect of exogenous public information provision to the use of private information.[12] Compared with Angeletos, Lorenzoni and Pavan (2012), which highlights information spillover from real to financial markets, our cross-learning framework with more detailed financial market structures arguments it with the opposite learning channel explicitly. By modeling cross learning, we can generate the new strategic complementarities and the new spiral towards the common shock. Our externality is also different from theirs that features beauty contest in signaling and higher-order uncertainties. Complementary to Amador and Weill (2010), the externality in our model derives from a different mechanism and suggests a new crowding-out effect: endogenous overweighting on the common shock crowds out the use of information about the idiosyncratic shocks. Two new papers, Liu (2014) and Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014), have embedded explicit feedback mechanism into macroeconomic settings, with focuses on aggregate demand and uncertainty, respectively. None of those papers has distinguished between common and idiosyncratic shocks or considered cross learning.

The rest of the paper is organized as follows. Section 2 lays out the model, featuring correlated investment opportunities and cross learning. Section 3 characterizes the cross-learning equilibrium and benchmarks it to the self-feedback case. Section 4 investigates important implications of cross learning with a focus on systematic risks in investment waves. Section 5 explores the externality and the relationship between investment inefficiency and competition. Section 6 discusses some extensions of the model. All the proofs are delegated to Appendix 8.1 unless otherwise noted.

[11] The recent study of Vives (2014) further combines the classical pecuniary externality (through the level of prices) and the learning externality associated with exogenous public information in an industrial competition context. This is different from our new pecuniary externality through endogenous price informativeness on the two shocks. Its focus is also on the strategic interaction in product markets instead of our endogenous cross learning in financial markets.

[12] Amador and Weill (2010) also relies on the earlier idea in Vives (1993) that the more informative prices are, the less agents rely on private information, with the consequence that less information will be incorporated into prices.

2 The Model

2.1 The Economy

The model extends the feedback framework of Goldstein, Ozdenoren and Yuan (2013) for a different focus on capital provider cross learning.[13] We consider a continuum of 1 of firms, $i \in [0,1)$, each having an asset traded in a secondary market. Each firm i's corresponding asset market is occupied by a mass 1 of informed risk-neutral speculators, respectively. We index speculators for firm i by a couple (i,j), with $j \in [0,1)$.[14] Each firm i's corresponding secondary market is occupied by noise traders. Each firm also has an exclusive capital provider i in a primary market who decides how much capital to provide to the firm for investment purpose.

There are three dates, $t = 0, 1, 2$. At date 0, the speculators trade in their corresponding asset market with their private information, and the asset price aggregates their information. At date 1, the capital providers observe the asset prices of both their own firm and all the other firms. Having observed all the prices and received their private information, the capital providers decide the amount of capital to provide for their corresponding firms and the firms undertake investment accordingly. All the cash flows are realized at date 2.

2.2 Capital Providers and Investment

All the firms in the economy have an identical linear production technology: $Q(I_i) = AF_iI_i$, where I_i is the amount of capital provided by capital provider i to firm i, and A and F_i are two stochastic productivity shocks. Specifically, shock A captures an industry-wide common productivity shock, and shock F_i captures the idiosyncratic productivity shock for firm i only.[15] Denote by a and f_i the natural logs of these shocks, and assume that they are normal and mutually independent:

$$a \sim N(0, 1/\tau_a)\,, \text{ and } f_i \sim N(0, 1/\tau_f)\,,$$

where τ_a and τ_f are positive and $i \in [0,1)$.

The introduction of multiple firms and the two fundamentally different productivity shocks plays an important role in necessitating firms' cross learning. Specifically, if the investment opportunities are uncorrelated, cross learning makes no sense. On the other hand, however, if the investment opportunities are perfectly correlated, all assets become identical and thus there is no need to learn from other's prices as well. To flesh our cross-learning mechanism out, we abstract away from possible industrial organization of the firms' product market.

[13] For related papers building on this framework or sharing a similar mathematical foundation in modeling, see Sockin and Xiong (2014a,b) and Goldstein and Yang (2014a,b). These papers do not consider fundamentally different productivity shocks or multiple firms' cross learning as we do.

[14] Since the speculators do not have a diversification motive, our results are unaffected if we assume that they can trade all assets. In other words, market segmentation in terms of trading plays no roles in our model.

[15] In what follows, we omit the term productivity for brevity at times when there is no confusion.

At date 1, all the capital providers choose the amount of capital I_i simultaneously in their respective primary markets. Capital provider i captures a proportion $\kappa \in (0,1)$ of the output $Q(I_i)$ by providing I_i, which incurs a private quadratic adjustment cost, $C(I_i) = \frac{1}{2}cI_i^2$. Thus, capital provider i's problem at $t = 1$ is

$$\max_{I_i} \mathbb{E}\left[\kappa A F_i I_i - \frac{1}{2}cI_i^2 | \Gamma_i\right], \tag{2.1}$$

where Γ_i is the information set of capital provider i at $t = 1$. It consists of the price of firm i's own asset, P_i, and those of all the other firms' assets, denoted by the set $\{P_{-i}\}$ for brevity,[16] formed at date 0 as endogenous public signals, as well as their private signals about the (log) productivity shocks a and f_i. Specifically, we assume that each capital provider i gets a private noisy and independent signal $s_{a,i}$ about the (log) common productivity shock a with precision τ_s, and another private noisy and independent signal $s_{f,i}$ about its own (log) idiosyncratic productivity shock f_i with precision τ_f:

$$s_{a,i} = a + \varepsilon_{a,i}\,, \text{ where } \varepsilon_{a,i} \sim N(0, 1/\tau_{sa})\,, \text{ and}$$

$$s_{f,i} = f_i + \varepsilon_{f,i}\,, \text{ where } \varepsilon_{f,i} \sim N(0, 1/\tau_{sf})\,.$$

That is, for capital provider i, the information set is $\Gamma_i = \{P_i, \{P_{-i}\}, s_{a,i}, s_{f,i}\}$.

Different from existing literature, one major novelty of our setup is to allow capital providers to learn from other firms' asset prices as well as own firms' prices, which we formally call cross learning. As will be highlighted later, although the capital providers only care about their own firms, they use the prices of other firms' assets for making better investment decisions.

2.3 Speculators and Secondary Market Trading

At date 0, the remaining cash flow $(1-\kappa)Q(I_i)$, as an asset, is traded in a separate competitive secondary market for each firm i. For firm i, denote the price of this asset by P_i. To focus on capital providers' cross learning, we do not consider any possible monetary transfers from the secondary market to the firm, but highlight the information revealed in the secondary market trading.[17] In the asset market of firm i, each speculator (i,j) has two private and independent signals about the common shock and the respective idiosyncratic shock. Specifically, the first signal is about the common shock:

$$x_{ij} = a + \varepsilon_{x,ij}, \text{ where } \varepsilon_{x,ij} \sim N(0, 1/\tau_x)\,,$$

[16] As will be elaborated later, we focus on symmetric equilibria in which the firm in question i always puts the same weight on each of other firms' asset prices in cross learning. Thus, it is unnecessary for us to distinguish between those asset prices in analyzing cross learning.

[17] Hence, this asset can be interpreted as either equity of the firm or a derivative on the return from the firm's investment. See a more detailed justification of this point in Goldstein, Ozdenoren and Yuan (2013).

and the second signal is about the firm-specific idiosyncratic shock:

$$y_{ij} = f_i + \varepsilon_{y,ij}, \text{where } \varepsilon_{y,ij} \sim N(0, 1/\tau_y)\,.$$

Thus, the information set of speculator (i, j) is $\Gamma_{ij} = \{x_{ij}, y_{ij}\}$.[18]

Based on their private information, the speculators submit limited orders in a similar manner of Kyle (1985), with an additional constraint that each speculator can buy or sell up to a unit of the asset.[19] Formally, the speculators maximize their expected trading profit, taking the asset price as given.[20] Their problems at $t = 0$ are

$$\max_{d_{ij}\in[-1,1]} d_{ij}\mathbb{E}\left[(1-\kappa)AF_iI_i - P_i|\,\Gamma_{ij}\right], \tag{2.2}$$

where d_{ij} is speculator (i, j)'s demand. The aggregate demand from the speculators in market i is given by $D_i = \int_0^1 d_{ij}\mathrm{d}j$.

We assume that the noisy supply in asset market i takes the following form:

$$\Delta(\zeta, \xi_i, P_i) = 1 - 2\Phi(\zeta + \xi_i - \lambda \log P_i)\,,$$

where

$$\zeta \sim N(0, \tau_\zeta^{-1})\,, \text{ and } \xi_i \sim N(0, \tau_\xi^{-1})\,.$$

We elaborate the noisy supply. $\Phi(\cdot)$ denotes the cumulative standard normal distribution function. The first shock ζ captures a common noisy supply shock that can be viewed as industry-wide sentiment or industry-wide fund flow. The second shock ξ_i captures the idiosyncratic noisy supply shock in market i that can be viewed as styled trading or uninformed investors' unobserved preferences. The presence of a common noisy supply not only makes our framework more general, but more importantly prevents the aggregate price from fully revealing the common productivity shock. Both noisy supply shocks ζ and ξ_i are independent and also independent of other shocks in the economy. Meanwhile, λ in the noisy supply function captures price elasticity and can be viewed as market liquidity. When λ is high, the demand from speculators can be easily absorbed and thus their aggregate demand has little impact on the asset prices.

Finally, in equilibrium, the prices will clear each asset market by equalizing the aggregate

[18] The fact that the speculators' information set does not consist of the asset prices is not essential. For any firm i, even if its speculators can learn from its asset price P_i, as long as they do not cross learn from other firms' asset prices $\{P_{-i}\}$, all of our results are unaffected.

[19] As discussed in Goldstein, Ozdenoren and Yuan (2013), the specific size of this position limit is inessential for the results, as long as speculators cannot take unlimited positions, otherwise the prices will be fully revealing. This constraint can be easily justified by their capital or borrowing constraints.

[20] The asset price can be viewed as set by an unmodeled market maker, as that in Kyle (1985).

speculator demand to the noisy supply in each asset market i:

$$D_i = \Delta(\zeta, \xi_i, P_i)\,. \tag{2.3}$$

2.4 Discussion

Before proceeding, we discuss some important differences of our settings from the existing feedback literature, in particular, Goldstein, Ozdenoren and Yuan (2013), Foucault and Fresard (2014), and the contemporaneous study by Goldstein and Yang (2014a,b). There are, of course, more differences between our work and the existing literature than what we discuss below, but the following ones are crucial for our mechanism and thus help stand out our contribution.

First, to lay out a foundation for cross learning, our model features a continuum of many firms. To accommodate multiple firms and two-way cross learning imposes new technical challenges in terms of finding closed form solutions. Despite this difficulty, our model provides a tractable approach not only suited for our purpose but potentially useful for future work in other directions.

Second, built upon the multiple-firm setup, our economy features two fundamentally different productivity shocks: one is common to all firms while the other is firm-specific. In most existing papers, there is only one productivity shock. One exception is Goldstein and Yang (2014a,b) who consider two shocks to the cash flow. However, their two shocks are fundamentally symmetric. Specifically, their two shocks differ in an exogenous informational sense that the capital providers perfectly observe one but not the other. Instead, our model allows us to explicitly recover how cross learning affects the endogenous sensitivities of firms' investment on the specific common shock and the idiosyncratic shocks. The presence of the two fundamentally different shocks plays an important role in generating investment waves as well as delivering welfare implications on inefficient investment waves and competition.

Third, to highlight the interaction of the two fundamentally different shocks under cross learning, our model does not feature any public information of the speculators as often seen in the literature. Our efficiency implications come endogenously from a new pecuniary externality absent in previous literature that focuses on coordination failure or higher-order beliefs.

Finally, in contrast to the hypotheses development in Foucault and Fresard (2014), our framework features fully two-way cross learning instead of one-way learning by a focal firm from its peer firm. The one-way learning channel in Foucault and Fresard (2014) gives clear predictions on how the peer firm's stock price may affect the focal firm's investment, but the peer firm itself does not invest or learn. Our framework with two-way cross learning as well as more detailed real and financial market structures captures the new strategic complementarities and spiral toward the common productivity shock. The fundamental difference between the common shock and the idiosyncratic shocks matters only when the fully two-way cross learning is introduced. This eventually generates industry-wide inefficient investment waves consistent with empirical regularities.

3 Cross-Learning Equilibrium

3.1 Equilibrium Definition

We formally introduce the equilibrium concept. We focus on symmetric linear equilibria that are standard in the literature. Specifically, the speculators in market i long one share of the corresponding asset when $\phi_i x_{ij} + y_{ij} > \mu_i$, and short one share otherwise, where ϕ_i and μ_i are two constants that will be determined in equilibrium. Since agents are risk neutral and firms are symmetric in our framework, symmetry further implies that $\phi_i = \phi$ and $\mu_i = \mu$, which mean that all the speculators use symmetric trading strategies in all asset markets, and the information contents of all asset prices are also symmetric.

Definition **1.** *A (symmetric) cross-learning equilibrium is defined as a collection of a price function for each firm i, $P_i(a, f_i, \zeta, \xi_i)$: $\mathbb{R}^4 \to \mathbb{R}$, an investment policy for each capital provider i, $I_i(s_{a,i}, s_{f,i}, P_i, \{P_{-i}\})$: $\mathbb{R}^2 \times \mathbb{R}^\infty \to \mathbb{R}$, and a linear monotone trading strategy for each speculator (i,j), $d_{ij}(x_{ij}, y_{ij}) = \mathbf{1}(\phi_i x_{ij} + y_{ij} > \mu_i) - \mathbf{1}(\phi_i x_{ij} + y_{ij} \leqslant \mu_i)$, such that*

i) each capital provider i's investment policy $I_i(s_{a,i}, s_{f,i}, P_i, \{P_{-i}\})$ solves problem (2.1),

ii) each speculator (i,j)'s trading strategy $d_{ij}(x_{ij}, y_{ij})$ is identical and solves problem (2.2), and

iii) market clearing condition (2.3) is satisfied for each market i.

3.2 Equilibrium Characterization

We characterize the equilibrium, featuring the capital providers' cross learning. The equilibrium is hard to solve because it involves many fixed-point problems, so we follow a step-by-step approach.

Step 1. We first solve for the price functions, which helps characterize the information contents of prices from the capital providers' perspective. We have the following lemma:

Lemma **1.** *The speculators' trading leads to the following equilibrium price of each asset i:*

$$P_i = \exp\left(\frac{\phi_i}{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}a + \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}f_i + \frac{\zeta + \xi_i}{\lambda} - \frac{\mu_i}{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}\right). \tag{3.1}$$

Hence, from any capital provider i's perspective, the price for its own firm i's asset is equivalent to the following signal in predicting the common shock a:

$$z_a(P_i) = \frac{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}\log P_i + \mu_i}{\phi_i} = a + \frac{1}{\phi_i}f_i + \frac{\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}{\phi_i}(\zeta + \xi_i)\,, \tag{3.2}$$

and is equivalent to the following signal in predicting the corresponding idiosyncratic shock f_i:

$$z_f(P_i) = \lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}\log P_i + \mu_i = f_i + \phi_i a + \sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}(\zeta + \xi_i)\,. \tag{3.3}$$

Lemma 1 not only helps specify the information contents of a firm's asset price to its own capital provider, but also hints those to other firms' capital providers. Thus, it suggests the presence of capital providers' cross learning when feasible. The next step formulates the idea.

Step 2. We then characterize the informational consequences of cross learning. Specifically, we show that, when cross learning is feasible, that is, capital provider i's information set includes both P_i and $\{P_{-i}\}$, the capital provider relies on the aggregate price as well as the own asset price (in addition to their own private signals) in inferring the two productivity shocks. We impose the symmetry conditions $\phi_i = \phi$ and $\mu_i = \mu$ to conditions (3.1), (3.2) and (3.3) now as we focus on symmetric equilibria, and we also define the aggregate price as

$$\overline{P} = \int_0^1 P_i \mathrm{d}i \,.^{21}$$

LEMMA **2.** *For capital provider i, when her information set includes both P_i and $\{P_{-i}\}$, these asset prices are informationally equivalent to the following two signals:*

i) a signal based on the aggregate price $\overline{P}$:

$$z_a(\overline{P}) = a + \frac{\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}{\phi}\zeta \tag{3.4}$$

for predicting the common shock a, with the precision

$$\tau_{\overline{pa}} = \frac{\tau_x \tau_y \tau_\zeta \phi^2}{\tau_x + \tau_y \phi^2}\,, \tag{3.5}$$

which is increasing in ϕ, and

ii) a signal based on the own asset price P_i as well as the aggregate price $\overline{P}$:

$$z_{f,i}(\overline{P}) = z_f(P_i) - \phi z_a(\overline{P}) = f_i + \sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}\xi_i \tag{3.6}$$

for predicting the corresponding idiosyncratic shock f_i, with the precision

$$\tau_{\overline{pf}} = \frac{\tau_x \tau_y \tau_\xi}{\tau_x + \tau_y \phi^2}\,, \tag{3.7}$$

which is decreasing in ϕ.

Along with Lemma 1, Lemma 2 implies that cross learning changes the feedback channel in which a capital provider uses asset prices to infer the two productivity shocks: she now uses the aggregate price $\overline{P}$ to infer the common shock a and still uses the own price P_i to infer the idiosyncratic shock f_i. Intuitively, for capital provider i, other firms' asset prices $\{P_{-i}\}$ are uninformative on

[21] The fact that the asset prices are equally weighted in calculating the aggregate price is inessential to their information contents. Our results carry through even if we choose arbitrarily positive weights.

the idiosyncratic shock f_i but informative on the common shock a. Hence, when other firms' asset prices are observable, which is natural in reality, the capital provider of the firm in question uses them to make better inference about the common shock. In particular, in a symmetric equilibrium, the aggregate price is sufficient for this purpose, as all asset prices are symmetric. By the law of large numbers, $\overline{P}$ only aggregates information about the common shock a: the information about idiosyncratic shocks and about the idiosyncratic noisy supply shocks all gets wiped out, while the presence of the common noisy supply shock still prevents the aggregate price from fully revealing. This makes $z_a(\overline{P})$, as characterized in (3.4), the most informative signal about the common shock a the capital provider can get. Moreover, knowing $z_a(\overline{P})$, the capital provider also eliminates the information about the common shock and about the common noisy supply shock when she uses her own price P_i to infer the idiosyncratic shock f_i, as characterized in (3.6).

Step 3. We then solve for the capital providers' optimal investment policy under cross learning. This indicates the real consequences of cross learning. Lemma 2 implies that, under cross learning, capital provider i uses the new signal $z_a(\overline{P})$ and her private signal $s_{a,i}$ to infer the common shock a, and the new signal $z_{f,i}(\overline{P})$ and the private signal $s_{f,i}$ to infer the idiosyncratic shock f_i. Thus, we have the following lemma.

LEMMA **3.** *Observing $s_{a,i}, s_{f,i}, P_i$ and $\{P_{-i}\}$, capital provider i's optimal investment policy is*

$$I_i = \frac{\kappa}{c} \exp\left[\frac{\tau_{sa} s_{a,i} + \tau_{\overline{pa}} z_a(\overline{P})}{\tau_a + \tau_{sa} + \tau_{\overline{pa}}} + \frac{1}{2(\tau_a + \tau_{sa} + \tau_{\overline{pa}})} + \frac{\tau_f s_{f,i} + \tau_{\overline{pf}} z_{f,i}(\overline{P})}{\tau_f + \tau_{sf} + \tau_{\overline{pf}}} + \frac{1}{2(\tau_f + \tau_{sf} + \tau_{\overline{pf}})}\right]. \tag{3.8}$$

The investment policy is intuitive. On the one hand, the optimal amount of investment is higher when the share κ of capital provider is higher while lower when the investment cost c is higher. On the other hand, the capital providers infer the two productivity shocks a and f_i independently but simultaneously in making investment decisions, reflected in the first and third terms in the parenthesis. In particular, the capital providers find it optimal to learn from both the own asset prices as well as other firms' prices, which are summarized in the two new signals $z_a(\overline{P})$ and $z_{f,i}(\overline{P})$. This fits quite in line with the recent empirical facts about firms' and capital providers' cross learning behavior (Foucault and Fresard, 2014, Ozoguz and Rebello, 2013).

According to Lemma 3, we propose the following intuitive concept of investment sensitivity to capture how the capital providers' investment decision responds to the two productivity shocks under cross learning.

DEFINITION **2.** *For capital providers, the investment sensitivity to the common productivity shock and that to the idiosyncratic productivity shock are defined as:*

$$S_a(\tau_{\overline{pa}}) = \frac{\tau_{sa} + \tau_{\overline{pa}}}{\tau_a + \tau_{sa} + \tau_{\overline{pa}}}, \text{ and } S_f(\tau_{\overline{pf}}) = \frac{\tau_{sf} + \tau_{\overline{pf}}}{\tau_f + \tau_{sf} + \tau_{\overline{pf}}},$$

respectively. We call S_a the common investment sensitivity and S_f the idiosyncratic investment sensitivity henceforth.

We highlight that, the investment sensitivity depends on not only the capital providers' private signals about the corresponding shock, but also the new endogenous price signals coming from cross learning as characterized in Lemma 2. In particular, these two notions of investment sensitivity are increasing functions of $\tau_{\overline{pa}}$ and $\tau_{\overline{pf}}$, respectively, which are in turn affected by the speculators' trading strategy. Hence, by Lemma 2, we have the following straightforward lemma that bridges the capital providers' investment sensitivity and the speculators' weight ϕ on the signal of the common productivity shock.

LEMMA **4.** *The common investment sensitivity $S_a(\tau_{\overline{pa}})$ is increasing in ϕ while the idiosyncratic investment sensitivity $S_f(\tau_{\overline{pf}})$ is decreasing in ϕ.*

Lemma 4 is helpful because it offers an intuitive look at the real consequences of learning from asset prices in the economy with two fundamentally different shocks. When speculators' weight ϕ is higher, they put more weight on the information about the common shock, and thus asset prices become more informative about the common shock while less informative about the idiosyncratic shocks. This in turn leads to a more sensitive investment policy in response to the common shock while a less sensitive one in response to the idiosyncratic shock.

Step 4. We finally close the model by solving for the speculators' equilibrium trading strategy, characterized by the weight ϕ and the constant μ. This also pins down other equilibrium outcomes since they are all functions of ϕ.

For speculator (i, j), her expected profit of trading given her available information is

$$\mathbb{E}\left[(1-\kappa)AF_iI_i - P_i | x_{ij}, y_{ij}\right], \tag{3.9}$$

in which I_i and P_i have been characterized by conditions (3.8) and (3.1) respectively.

It is easy to show that, speculators' expected profit (3.9) of trading asset i can be expressed as

$$\mathbb{E}\left[(1-\kappa)AF_iI_i - P_i | x_{ij}, y_{ij}\right] = \frac{\kappa(1-\kappa)}{c}\exp\left(\alpha_0 + \alpha_1 x_{ij} + \alpha_2 y_{ij}\right) - \exp\left(\gamma_0 + \gamma_1 x_{ij} + \gamma_2 y_{ij}\right),$$

where $\alpha_0, \alpha_1, \alpha_2, \gamma_0, \gamma_1$, and γ_2 are all functions of ϕ:

$$\begin{aligned}
\alpha_1 &= (S_a + 1)\frac{\tau_x}{\tau_a + \tau_x}, \\
\alpha_2 &= (S_f + 1)\frac{\tau_y}{\tau_f + \tau_y}, \\
\gamma_1 &= \frac{\phi}{\lambda\sqrt{\tau_x + \tau_y\phi^2}}\frac{\tau_x}{\tau_a + \tau_x}, \\
\gamma_2 &= \frac{1}{\lambda\sqrt{\tau_x + \tau_y\phi^2}}\frac{\tau_y}{\tau_f + \tau_y}.
\end{aligned}$$

By definition, in a symmetric cross-learning equilibrium with cross learning, we have

$$\phi = \frac{\alpha_1 - \gamma_1}{\alpha_2 - \gamma_2}.$$

Plugging in $\alpha_1, \alpha_2, \gamma_1$ and γ_2 yields

$$\phi = \frac{\left(S_a + 1 - \frac{\phi}{\lambda\sqrt{\tau_x + \tau_y\phi^2}}\right)\frac{\tau_x}{\tau_a + \tau_x}}{\left(S_f + 1 - \frac{1}{\lambda\sqrt{\tau_x + \tau_y\phi^2}}\right)\frac{\tau_y}{\tau_f + \tau_y}}. \tag{3.10}$$

Analyzing this equation by further plugging in S_a and S_f, which are both functions of ϕ, we reach a unique cross-learning equilibrium, formally characterized by the following proposition.

PROPOSITION **1.** *For a high enough noisy supply elasticity λ, there exists a cross-learning equilibrium in which the speculators put a positive weight $\phi > 0$ on the signal of the common productivity shock. For a high enough information precision τ_y (of the speculators' signal on the idiosyncratic shock), the equilibrium is unique.*

To establish the existence of a unique equilibrium is essential for our further analysis regarding investment waves, as it allows us to investigate that how changes in economic environment affect investments and prices through the cross-learning mechanism. When ϕ is higher, the speculators put more weight on the information about the common shock in trading, encouraging all the capital providers to respond to the common shock more sensitively through cross learning, which in turn leads to an even higher ϕ. This new spiral gives rise to many implications in line with the empirical phenomena regarding industry-wide investment waves as we explore later.

The conditions to guarantee a unique cross-learning equilibrium are not only standard in the feedback literature (see Goldstein, Ozdenoren and Yuan, 2013, among many others) but empirically plausible. A relatively high noisy supply elasticity λ implies that markets are liquid enough. A relatively high information precision τ_y of the speculators' signal on the idiosyncratic shock suggests that asset market participants understand their target firms better than the whole industry. These two conditions are in particular appropriate when we focus on the contexts leading to investment waves: relatively liquid markets and relatively more uncertain macroeconomic news.[22,23]

3.3 Self-Feedback Benchmark

Having established the existence and uniqueness of a cross-learning equilibrium, we benchmark the cross-learning equilibrium to the corresponding self-feedback equilibrium in a comparable

[22] In the appendix, we explore other sufficient conditions that guarantee a unique cross-learning equilibrium. Our results regarding investment waves and investment inefficiency survive under other sets of sufficient conditions.

[23] We have numerically shown that these conditions are not restrictive. Even for reasonably small λ and τ_y, our model still features a unique cross-learning equilibrium. These numerical results are reported in Section 5.

economy. This self-feedback benchmark helps understand how the presence of cross learning affects the capital providers' investment policy and the speculators' trading strategy, in contrast to the counterfactual where cross learning is absent. In demonstrating these effects, we again focus on the difference of the speculators' weight ϕ on the signal of the common productivity shock in the two respective equilibria, as all equilibrium outcomes are functions of this weight. We still consider unique symmetric equilibria and denote by ϕ' the speculators' weight on the signal of the common productivity shock in the self-feedback benchmark.

Formally, the only difference of the benchmark economy is that, each capital provider i observes its own asset price P_i but not other firms' asset prices $\{P_{-i}\}$. That is, capital provider i's information set is $\Gamma_i = \{P_i, s_{a,i}, s_{f,i}\}$. We have

$$P_i = \exp\left(\frac{\phi'}{\lambda\sqrt{\tau_x^{-1}(\phi')^2+\tau_y^{-1}}}a + \frac{1}{\lambda\sqrt{\tau_x^{-1}(\phi')^2+\tau_y^{-1}}}f_i + \frac{\zeta+\xi_i}{\lambda} - \frac{\mu_i}{\lambda\sqrt{\tau_x^{-1}(\phi')^2+\tau_y^{-1}}}\right),$$

which is equivalent to the following two signals

$$z_a(P_i) = \frac{\lambda\sqrt{\tau_x^{-1}(\phi')^2+\tau_y^{-1}}\log P_i + \mu_i}{\phi'} = a + \frac{1}{\phi'}f_i + \frac{\sqrt{\tau_x^{-1}(\phi')^2+\tau_y^{-1}}}{\phi'}(\zeta+\xi_i)$$

in predicting the common shock a and

$$z_f(P_i) = \lambda\sqrt{\tau_x^{-1}(\phi')^2+\tau_y^{-1}}\log P_i + \mu_i = f_i + \phi' a + \sqrt{\tau_x^{-1}(\phi')^2+\tau_y^{-1}}(\zeta+\xi_i)$$

in predicting the corresponding idiosyncratic shock f_i. The precisions of $z_a(P_i)$ and $z_f(P_i)$ are denoted as τ_{pa} and τ_{pf} where

$$\tau_{pa} = \frac{1}{\frac{1}{(\phi')^2}\tau_f^{-1} + \frac{\tau_x^{-1}(\phi')^2+\tau_y^{-1}}{(\phi')^2}(\tau_\zeta^{-1}+\tau_\xi^{-1})},$$

and

$$\tau_{pf} = \frac{1}{(\phi')^2\tau_a^{-1} + (\tau_x^{-1}(\phi')^2+\tau_y^{-1})(\tau_\zeta^{-1}+\tau_\xi^{-1})}.$$

Following the same definition of investment sensitivity and the same analysis for the capital providers' investment policy and the speculators' trading strategy, we have

$$\begin{aligned}
S_a' &= \frac{\tau_{sa}+\tau_{pa}}{\tau_a+\tau_{sa}+\tau_{pa}} + \frac{\tau_{pf}}{\tau_f+\tau_{sf}+\tau_{pf}}\phi', \\
S_f' &= \frac{\tau_{sf}+\tau_{pf}}{\tau_f+\tau_{sf}+\tau_{pf}} + \frac{\tau_{pa}}{\tau_a+\tau_{sa}+\tau_{pa}}\frac{1}{\phi'}, \\
\alpha_1' &= \left(S_a'+1\right)\frac{\tau_x}{\tau_a+\tau_x},
\end{aligned}$$

$$\begin{aligned}
\alpha_2' &= \left(S_f' + 1\right) \frac{\tau_y}{\tau_f + \tau_y}, \\
\gamma_1' &= \frac{\phi'}{\lambda\sqrt{\tau_x + \tau_y(\phi')^2}} \frac{\tau_x}{\tau_a + \tau_x}, \\
\gamma_2' &= \frac{1}{\lambda\sqrt{\tau_x + \tau_y(\phi')^2}} \frac{\tau_y}{\tau_f + \tau_y}.
\end{aligned}$$

In the self-feedback equilibrium, we also have

$$\phi' = \frac{\alpha_1' - \gamma_1'}{\alpha_2' - \gamma_2'}$$

to pin down the speculators' weight on the information of the common shock. Plugging in $\alpha_1', \alpha_2', \gamma_1'$ and γ_2' yields

$$\phi' = \frac{\left(S_a' + 1 - \frac{\phi'}{\lambda\sqrt{\tau_x + \tau_y(\phi')^2}}\right) \frac{\tau_x}{\tau_a + \tau_x}}{\left(S_f' + 1 - \frac{1}{\lambda\sqrt{\tau_x + \tau_y(\phi')^2}}\right) \frac{\tau_y}{\tau_f + \tau_y}}. \tag{3.11}$$

Therefore, we have the following proposition regarding the comparison between the cross-learning equilibrium and the corresponding self-feedback benchmark. We focus on comparable cases in which a self-feedback equilibrium and its corresponding cross-learning equilibrium are both unique.

PROPOSITION **2.** *For a high enough noisy supply elasticity λ, a low enough idiosyncratic noisy supply shock precision τ_ξ, and a high enough information precision τ_y (of the speculators' signal on the idiosyncratic shock), there exists a unique self-feedback equilibrium in which speculators put a positive weight $\phi' > 0$ on the signal of the common productivity shock. In particular, $\phi' < \phi$, where ϕ is the speculators' weight on the signal of the common productivity shock in the corresponding cross-learning equilibrium.*

The comparison between a cross-learning equilibrium and its corresponding self-feedback equilibrium implies that, the presence of cross-learning may encourage the speculators to put a higher weight ϕ on the information about the common productivity shock. We also have the following straightforward corollary regarding the information precisions of the endogenous price signals and the capital providers' investment sensitivities, all of which are functions of ϕ.

COROLLARY **1.** *Compared to its corresponding self-feedback equilibrium, a cross-learning equilibrium features a higher ratio of the asset price information precision in predicting the common shock to that in prediction the idiosyncratic shock, i.e., $\tau_{\overline{pa}}/\tau_{\overline{pf}} > \tau_{pa}/\tau_{pf}$, and a higher ratio of the investment sensitivity to the common shock to that to the idiosyncratic shock, i.e., $S_a/S_f > S_a'/S_f'$.*

The results in Proposition 2 and Corollary 1 uncover the informational and real consequences of cross learning in equilibrium. Intuitively, when the capital providers are able to cross learn from

each other's asset prices (in addition to their own firms' prices), they indeed do so in equilibrium as other firms' asset prices help them better infer the common shock. This makes firms' investments relatively more correlated with the common shock as well as with each other. Thus, the speculators find it more profitable to put more weight on the information about the common shock. This further makes asset prices becoming relatively more informative about the common shock in guiding investment decisions, and thus the capital providers respond to the common shock even more sensitively in investing. This spiral is absent in existing feedback models, and it indeed plays an important role in amplifying industry-wide investment waves as we fully explore in the next section.

4 Systematic Risks in Investment Waves

The most important implications of cross learning are on the systematic risks in industry-wide investment waves. This comes from the endogenous spiral between the capital providers' investment sensitivity to the common shock and the speculators' weighting on the information about the common shock, as shown in Section 3. In our multi-firm setting, this spiral further leads to two new endogenous strategic complementarities. The new spiral and strategic complementarities help generate empirical implications of systematic risks in many relevant economic environments that seem jointly puzzling otherwise.

4.1 Impacts of Speculators' Weight on Systematic Risks

It is instructive to first investigate the impacts of the speculators' weight ϕ (on the information of the common shock) on systematic risks, taking the weight as given. Along the way, we also introduce our measures of systematic risks in investment waves.

DEFINITION **3.** *The correlation coefficients between the investments of two firms and between the asset prices of two firms are defined as:*

$$\beta_I = \frac{Cov(\log I_i, \log I_j)}{\sqrt{Var(\log I_i)}\sqrt{Var(\log I_j)}}, \text{ and } \beta_P = \frac{Cov(\log P_i, \log P_j)}{\sqrt{Var(\log P_i)}\sqrt{Var(\log P_j)}},$$

respectively. We call β_I the investment beta and β_P the price beta henceforth.

We take the investment beta β_I and the price beta β_P as two major measures of systematic risks in investment waves, on both the real and financial aspects, respectively. Typically, stronger investment waves are associated with a higher β_I and a higher β_P. However, as the recent study by Hong and Sraer (2013) argues, some investment waves only exhibit a higher investment beta β_I but not a higher price beta β_P. Hence, it is helpful to us to distinguish between these two betas in characterizing different types of investment waves.

We have the following intuitive result on the impacts of the speculators' weight ϕ on the two betas. When the speculators put a higher weight on the information of the common shock, the capital

providers' investment sensitivities to the common shock increases, which makes their investments more correlated. Moreover, this in turn encourages the speculators to put a higher weight on the common productivity shock, which results in a higher correlation between asset prices. With the comparison between a cross-learning equilibrium and its corresponding self-feedback equilibrium in Section 3, these predictions shed lights on the empirical regularities in papers such as Rhodes-Kropf, Robinson and Viswanathan (2005), Pastor and Veronesi (2006, 2009), Hoberg and Phillips (2010), Bhattacharyya and Purnanandam (2011) and Patton and Verardo (2012).

LEMMA **5.** *Both the investment beta β_I and the price beta β_P are increasing in ϕ when $\phi > 0$.*

Similarly, we also look at the correlations between investment and the two productivity shocks, respectively. As a complement to Definition 2 of investment sensitivity and the associated Lemma 4, the following definition shoots a closer look at the equilibrium investments' correlation with the two shocks.

DEFINITION **4.** *The correlation coefficient between investment and the common productivity shock and that between investment and the idiosyncratic productivity shock are defined as:*

$$\beta_A = \frac{Cov(\log I_i, \log A)}{\sqrt{Var(\log I_i)}\sqrt{Var(\log A)}}, \text{ and } \beta_F = \frac{Cov(\log I_i, \log F_i)}{\sqrt{Var(\log I_i)}\sqrt{Var(\log F_i)}},$$

respectively. We call β_A the common investment correlation and β_F the idiosyncratic investment correlation henceforth.

Intuitively, when the speculators put a higher weight on the information of the common shock, both investments and prices become more correlated with the common productivity shock instead of the idiosyncratic shocks. This is because the asset prices become more informative in predicting the common shock but less informative in predicting the idiosyncratic shock.

LEMMA **6.** *The common investment correlation β_A is increasing in ϕ while the idiosyncratic investment correlation β_F is decreasing in ϕ when $\phi > 0$.*

In what follows, we focus on the investment beta β_I and the price beta β_P in exploring the full equilibrium dynamics, highlighting the speculators' endogenous weight and equilibrium systematic risks under cross learning. The investigation on the common investment correlation β_A and the idiosyncratic investment correlation β_F yields the same insights.

4.2 Endogenous Cross Learning and Systematic Risks

Having established the impacts of the speculators' weight ϕ (on the information about the common shock) on systematic risks, we turn to one of the most interesting parts of the paper, which investigates how the changes of economic environments affect equilibrium systematic risks through

the cross-learning mechanism. This unifies several empirical regularities that are otherwise hard to reconcile without taking the capital providers' cross learning into account. Mathematically, we perform formal comparative statics of the equilibrium betas with respect to exogenous parameters. We elaborate the first comparative statics (with respect to the common uncertainty) in more detail to explore the underlying mechanism, and the other comparative statics will follow the same intuition.

4.2.1 Common Uncertainty

We first focus on the effects of common uncertainty, which is captured by the prior precision τ_a of the common productivity shock. We view the change of common uncertainty as an important case, because a majority of industry-wide and economy-wide investment waves is associated with an increasing common uncertainty at the first place. The most typical driver for an increasing common uncertainty is the arrival of all-purpose technology or financial innovations, as documented in Brunnermeier and Nagel (2004), Pastor and Veronesi (2006, 2009), and more broadly the literature of bubbles. Our predictions help deliver a new perspective to look at the impacts of innovations and the accompanying increasing common uncertainty on the systematic risks in investment waves, highlighting the cross-learning mechanism.

We use the following assumption (only valid in this subsection on common uncertainly) to flesh out the cross-learning mechanism.

Assumption **1.** *The ratios τ_{sa}/τ_a (of the capital providers' signal precision on the common shock and the prior precision on the common shock) and τ_x/τ_a (of the speculators' signal precision on the common shock and the prior precision on the common shock) are kept as constants when τ_a changes.*

Assumption 1 not only helps shut down a direct information channel that confounds the cross-learning mechanism (only in this case about common uncertainty) but also captures the reality better. By keeping the two ratios constant, both the capital providers and the speculators do not find their private information more valuable in predicting the common productivity shock. This is actually closer to the reality that, when the common uncertainty increases, no market participant naturally has an advantage in resolving the common uncertainty. In this case, our cross-learning mechanism plays an important amplification role that is impossible otherwise. Assumption 1 is also completely benign; our results are only stronger without it.

Lemma **7.** *Increasing the common uncertainty leads to a higher weight of the speculators on the information about the common shock. Specifically, the speculators' weight ϕ is decreasing in τ_a.*

From Lemma 7, we understand that an increasing in the common uncertainty leads to a stronger cross-learning spiral towards the common shock, despite that both the capital providers and

speculators experience equally increasing uncertainty in their private information on the common shock. The following proposition further establishes the impacts on the equilibrium systematic risks.

PROPOSITION **3.** *Increasing the common uncertainty leads to both a higher investment beta and a higher price beta in equilibrium. Specifically, β_I and β_P are both decreasing in τ_a. We further decompose the effects into two negative components (the same for β_I and β_P):*

$$\frac{d\beta(\tau_a,\phi)}{d\tau_a} = \underbrace{\frac{\partial\beta(\tau_a,\phi)}{\partial\tau_a}}_{\textit{Mechanical Effect}<0} + \underbrace{\frac{\partial\beta(\tau_a,\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_a}}_{\textit{Cross-Learning Effect}<0} < 0\,.$$

Proposition 3 indicates two effects contributing to the higher systematic risks associated with an increasing common uncertainty. The first is a mechanical effect that does not depend on the endogenous interaction between the capital providers and the speculators under cross learning. Intuitively, when the common uncertainty increases, speculators' investment sensitivity to the common shock increases as well. This immediately results in a higher correlation among firms' investments and prices. Figure 3 illustrates this mechanical effect in a two-firm example.

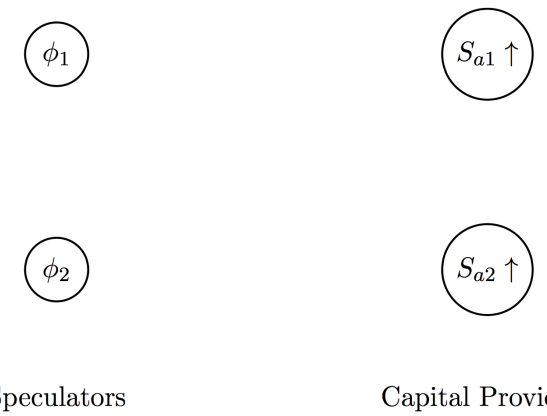

Figure 3: Mechanical Effect on Systematic Risks

The second effect, the cross-learning effect, is more interesting and only at play in our multi-firm cross-learning framework with two types of shocks. It reflects the new spiral between the capital providers' investment sensitivity to the common shock and the speculators' weight on the signal of the common shock. Interestingly, it takes place even when only some (not all) firms in the economy perceive the increasing common uncertainty.[24] Figure 4 illustrates this cross-learning effect in a two-firm example. Suppose, without loss of generality, firm 1's capital provider perceives the increasing common uncertainty. As in the upper-left panel, firm 1's investment sensitivity to the common shock S_{a1} first increases (along with a decreasing investment sensitivity to its idiosyncratic shock),

[24] Technically, this requires some non-measure-zero firms to perceive the increasing common shock.

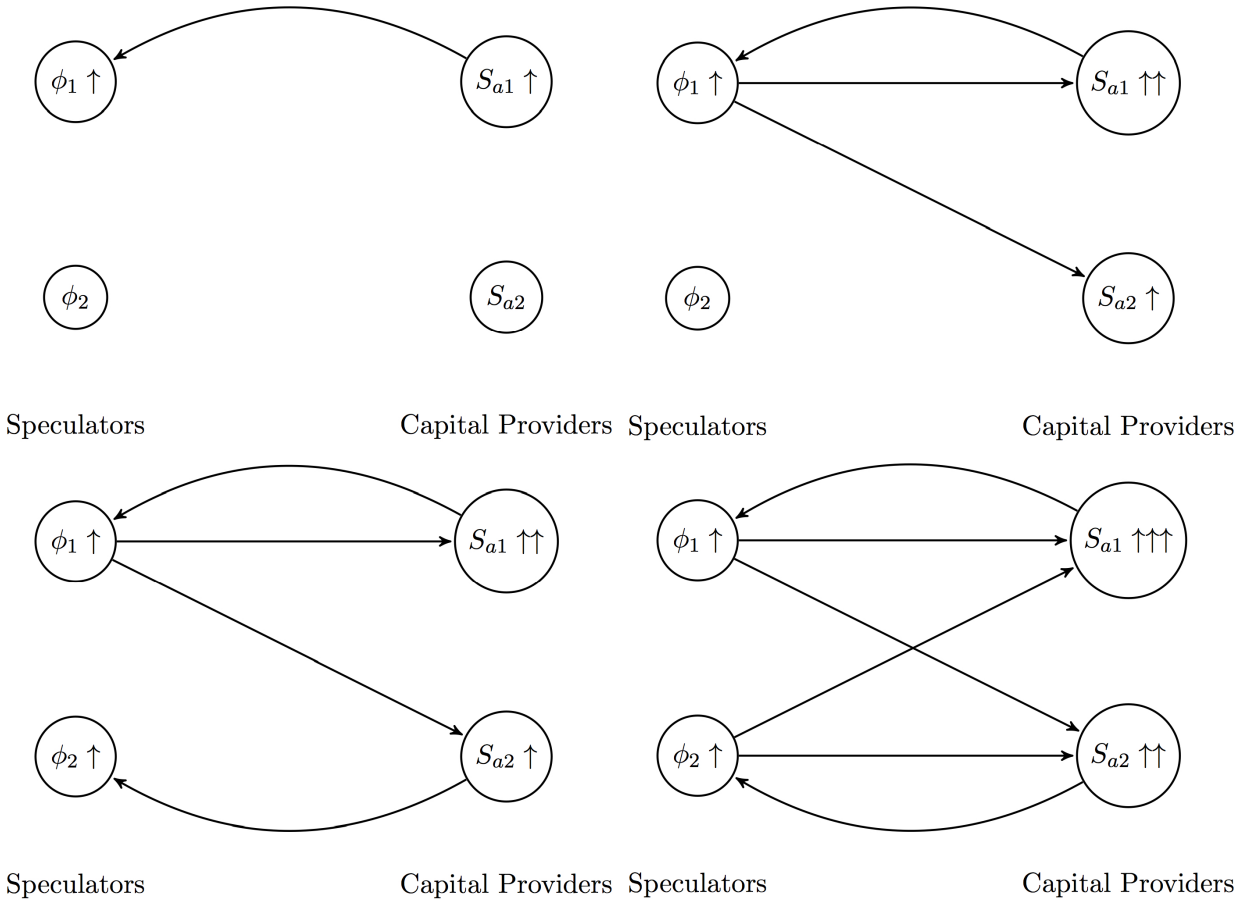

Figure 4: Learning Effect on Systematic Risks

Order: Upper-Left, Upper-Right, Lower-Left, Lower-Right

leading to a higher weight ϕ_1 on the information of the common shock by its speculators. Then, as in the upper-right panel, a higher ϕ_1 results in an even higher S_{a1} since firm 1 learns from its own price. More importantly, because of cross learning, firm 2's investment sensitivity to the common shock S_{a2} also increases, since firm 2 finds firm 1's price more informative about the common shock and thus understands the common shock better. It then naturally leads to a higher weight ϕ_2 on the information of the common shock by firm 2's speculators, as in the lower-left panel. Finally, the increase of ϕ_2 results in even higher S_{a1} and S_{a2} by cross learning, as in the lower-right panel. The entire process suggests two new strategic complementarities only under cross learning: the first is among speculators' weights on the information about the common shock in each market, and the second is among different firms' relative investment sensitivities to the common shock. With the two strategic complementarities, the spiral goes on and on and eventually pushes the economy to a new equilibrium with much higher systematic risks.

Our predictions on systematic risks after an increasing common uncertainty are consistent with the literature (Brunnermeier and Nagel, 2004, Pastor and Veronesi, 2006, 2009) that documents the increasing systematic risks after major technological innovations, as these innovations often

come with industry-wide uncertain market prospects. In particular, the cross-learning effect sheds lights on the huge magnitude of systematic risks in these investment waves that are often ascribed to behavioral biases (see Brunnermeier and Oehmke, 2013, Xiong, 2013, for surveys).

4.2.2 Capital Providers' Access to Information

We then turn to the capital providers' access to private information, captured by the two precisions τ_{sa} and τ_{sf} regarding the two productivity shocks, respectively. Again, we have the following lemma pertaining to the speculators' endogenous weight.

LEMMA **8.** *Increasing the capital providers' information precision on the common shock leads to a higher weight of the speculators on the information about the common shock, while increasing the capital providers' information precision on the idiosyncratic shock leads to a lower weight. Specifically, the speculators' weight ϕ is increasing in τ_{sa} while decreasing in τ_{sf}.*

Lemma 8 prescribes that, when the capital providers have better information on the common shock, the equilibrium cross-learning spiral towards the common shock is also stronger; while better information on the idiosyncratic shock pushes the cross-learning spiral towards the idiosyncratic shocks. This further leads to the following proposition. Similar to Proposition 3, we have the mechanical effect and the cross-learning effect, both in the same direction.

PROPOSITION **4.** *For the capital providers' access to private information, we have the following results.*

i) Increasing the precision on the common shock leads to a higher investment beta when the precision is not large, and always a higher price beta; specifically, β_I is increasing in τ_{sa} when $\tau_{sa} > \tau_a + \tau_x\tau_\zeta$ and β_P is always increasing in τ_{sa}.

ii) Increasing the precision on the idiosyncratic shock leads to both a lower investment beta and a lower price beta; specifically, β_I and β_P are always decreasing in τ_{sf}.

The predictions here are broadly supported by the empirical evidence in Greenwood and Nagel (2009). It suggests that younger and more confident capital providers, who tend to have better knowledge about the industry-wide common shock compared to that on their idiosyncratic shocks, tilt their investments more towards the common shock, leading to higher investment and price correlations. Greenwood and Nagel (2009) admit that the magnitude of systematic risks they have observed is obviously larger than any existing rational models can accommodate and thus refer to behavioral explanations. In this sense, our predictions provide a new angle to investigate such effects from a rational perspective, highlighting the potential of strong cross learning.

4.2.3 Liquidity Trading

We also investigate the effects of liquidity trading, captured by the market liquidity λ and the two precisions of noisy supplies τ_ζ and τ_ξ. Similarly, we have the following intuitive lemma on the

speculators' endogenous weight.

LEMMA **9.** *For liquidity trading, a higher weight of the speculators on the information about the common shock results from a lower market liquidity, a lower variance of common noisy supply, or a higher variance of idiosyncratic noisy supply. Specifically, the speculators' weight ϕ is decreasing in λ, increasing in τ_ζ, and decreasing in τ_ξ.*

The predictions along the three dimensions are all intuitive. When the market liquidity is higher, it is easier for the noisy traders to absorb speculators' demand, so that the cross-learning spiral towards the common shock is weaker. When the variance of the common noisy supply is lower, speculators are more likely to trade upon the common productivity shock. In contrast, when the variance of the idiosyncratic noisy supply is lower, speculators are less likely to trade upon the common shock, which results in a weaker spiral towards the common shock,

These predictions are further reflected in the following proposition, speaking to the overall effects of liquidity trading on investment waves. Again, similar to Proposition 3, we have the mechanical effect and the cross-learning effect in the same direction.

PROPOSITION **5.** *For liquidity trading, we have the following results.*

i) A higher investment beta β_I results from a lower market liquidity, a lower variance of common noisy supply, or a higher variance of idiosyncratic noisy supply. Specifically, β_I is decreasing in λ, increasing in τ_ζ, and decreasing in τ_ξ.

ii) A higher price beta β_P results from a lower market liquidity, or a higher variance of idiosyncratic noisy supply. Specifically, β_P is decreasing in λ and decreasing in τ_ξ.

5 Investment Inefficiency and Competition

An important question is that how firms' cross learning affects real investment efficiency. On the positive side, cross learning allows capital providers to take advantage of more information that would not be available if they were not able to observe their own and other firms' asset prices. However, the interests between capital providers and speculators in learning the two types of shocks are not perfectly aligned. More importantly, each firm's cross learning further creates a new pecuniary externality on other firms. These frictions result in investment inefficiency. In particular, the pecuniary externality associated with cross learning increases in the number of firms, suggesting that more competitive industries may exhibit more inefficient investment waves.

In evaluating that how these frictions affect investment efficiency, we proceed by two steps. First, we evaluate the overall investment efficiency and show that any cross-learning equilibrium always features investment inefficiency. Then we characterize the new pecuniary externality induced by cross learning to better understand the origin of such inefficiency. By doing this, we

particularly underscore the implications of competition on inefficient investment waves through the new pecuniary externality.

5.1 Overall Investment Efficiency

Formally, we define investment efficiency by the ex-ante expected net benefit of the total investments by all the firms, given that capital providers may learn from all publicly available asset prices:

DEFINITION **5.** *The investment efficiency of the economy is defined as*

$$R = \int_0^1 R_i di \,,$$

where

$$R_i = \mathbb{E}\left[\mathbb{E}\left[AF_iI_i - \frac{c}{2}I_i^2|\Gamma_i\right]\right]$$

denotes each firm i's ex-ante expected net benefit of investment, given its capital provider's information set under cross learning: $\Gamma_i = \{P_i, \{P_{-i}\}, s_{a,i}, s_{f,i}\}$.

We have the following proposition indicating the universal presence of investment inefficiency in a cross-learning equilibrium. We focus on the cases in which a unique cross-learning equilibrium is guaranteed.

PROPOSITION **6.** *There always exists a unique optimal weight $\phi^* \geqslant 0$ of the speculators on the signal of the common shock that maximizes investment efficiency. In particular, for a high enough noisy supply elasticity λ and a high enough information precision τ_y (of the speculators' signal on the idiosyncratic shock), the optimal weight is always smaller than that in the corresponding cross-learning equilibrium, i.e., $\phi^* < \phi$.*

Proposition 6 indicates that when the speculators' signal on the idiosyncratic shock is relatively more precise, they tend to put an inefficiently high weight on the other signal about the common shock. This makes capital providers to respond to the common shock inefficiently too sensitively, leading to inefficient investment waves. This particular inefficiency fits quite in line with what we have observed in typical investment waves (for example, Rhodes-Kropf, Robinson and Viswanathan, 2005, Peng, Xiong, and Bollerslev, 2007, Hoberg and Phillips, 2010, Bhattacharyya and Purnanandam, 2011) that both primary and secondary market investors pay inefficiently too much attention to common shocks or noisy macroeconomics news while ignore informative idiosyncratic news.[25]

To better understand the impacts of cross learning on investment efficiency and potentially shed lights on corrective policies, we perform comparative statistics of investment efficiency with respect

[25] Our framework is in fact general enough to admit the opposite case: when the speculators' signal on the common shock is relatively more precise, they tend to put an inefficiently too high weight on the signal about the idiosyncratic shocks, also leading to generic investment inefficiency. This case is empirically less plausible, but we still explore the theoretical possibilities in the appendix.

to several economic parameters. Again, we focus on unique cross-learning equilibria by assuming that the noisy supply elasticity λ and the information precision τ_y (of speculators' signal on the idiosyncratic shock) are high enough.

PROPOSITION **7.** *In a cross-learning equilibrium, investment efficiency is higher when the market liquidity is higher, or the precision of idiosyncratic noisy supply is higher, or the capital providers' information precision on the idiosyncratic productivity shock is higher. Specifically, R is increasing in λ, τ_ξ, and τ_{sf}.*

The comparative statics regarding the investment efficiency are intuitive. First, a higher market liquidity has a corrective effect on the investment efficiency. That is, in a deeper asset market, the speculators' trading positions can be more easily absorbed. Specifically, when an asset market is more liquid or deeper, it becomes harder for the same amount of informed trading to impact the asset price. This is in particular beneficial when cross learning is strong after the arrival of major innovations or other common news involving high uncertainty, because the inefficient impact from speculators' overuse of information about the common shock can be better absorbed.

Importantly, this corrective effect on real investment efficiency helps justify recent regulatory concerns and practices by the SEC in limiting informed speculators' trading positions but at the same time lifting the participation barrier to less informed market makers and retail investors. These two are hard to be reconciled as approaches to correct investors' irrationality or to sidestep limits to arbitrage. In this sense, our cross-learning mechanism does a better job in delivering policy implications than typical models featuring bubbles.

Second, increasing investment efficiency in an economy with cross learning calls for a better use of information about the idiosyncratic shocks in the economy. Any policies on financial disclosure or government communication failing to keep this point in mind may end up crowding out the idiosyncratic news and resulting in investment inefficiency. This policy implication fits broadly in line with the recent studies that speak to the dark side of financial disclosures or central bank communications (Di Maggio and Pagano, 2013, Kurlat and Veldkamp, 2013). Theoretically, the endogenous overuse of information on the common shock due to multi-firm cross learning results in an inefficient crowding-out effect on the use of information on the idiosyncratic shock. Thus, it also complements the idea on the crowding-out effect of public information provision on the use of private information (see Amador and Weill, 2010).

5.2 Competition and Cross Learning

To help better understand the origin of investment inefficiency, we perform a theoretical exercise to further identify a new pecuniary externality induced by cross learning. In particular, in doing so, we extend our baseline model to admit finite number of firms. This allows us not only to underscore the efficiency change associated with different extent of cross learning but to investigate the relationship

between competition and inefficient investment waves, which has been a well documented puzzle in recent empirical literature (see Hoberg and Phillips, 2010, Greenwood and Hanson, 2014, among others).

We first outline the extended cross-learning framework. A major challenge in identifying the externality associated with cross learning is to deal with the information endowment effect. Specifically, when the actual number of firms increase, the total amount of information in the economy also increases, leading to an efficiency gain to each firm. This information endowment effect confounds the identification of externalities and thus needs to be controlled properly. To achieve this goal, our extended cross-learning framework still features a continuum of 1 of firms being able to learn from all asset prices. However, we assume that the speculators do not fully internalize capital providers' cross learning. Concretely, they believe that each firm only observes and learns from as many as $n \geqslant 1$ asset prices, including its own price. This setting delivers an equilibrium weight (of the speculators on the information about the common shock) identical to that in a corresponding economy with n finite firms operating and the speculators fully internalizing their cross learning, while keeps the total amount of information endowment invariant with n. Hence, we are able to stand out the externality associated with cross learning as the number of firms increases.

We rigorously formulate the idea above as follows. We divide all the firms into $n \geqslant 1$ groups, a continuum of $1/n$ of firms in each group. The firms still observe and learn from all the asset prices as in the baseline model, regardless of the grouping. However, the speculators do not fully internalize firms' cross learning as before. Specifically, let $i \in [0, 1/n)$ denote one firm in the first group. The speculators believe that for any i, the n firms in the set $\{i + k/n | 0 \leqslant k \leqslant n-1, k \in \mathbb{Z}\}$ learn only from the asset prices of each other but not from the asset prices of other firms outside the set. Figure 5 offers an illustration of the case when $n = 3$, in which the speculators believe that the three red firms (i, $i + 1/3$, and $i + 2/3$) cross learn only from each other and the three blue firms (i', $i' + 1/3$, and $i' + 2/3$) cross learn only from each other, similar for other firm triples.

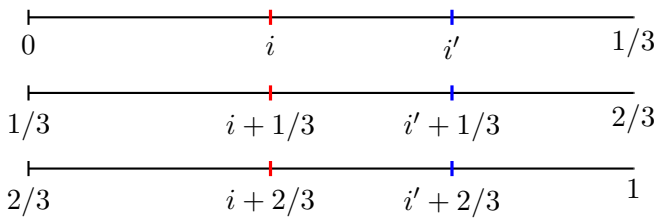

Figure 5: Extended Cross-Learning Framework

This setting has several advantages, both economically and technically. First, it casts industry competition in a straightforward way. Since the speculators are risk neutral, it looks to them as if

there are exactly n firms operating in the economy. Thus, the speculators' weight in equilibrium is identical to that in a corresponding economy with exactly n firms and the speculators fully internalizing their cross learning. Second, it helps identify the pecuniary externality induced by cross learning while keeps the total information endowment fixed. Especially, the efficiency change associated with cross learning takes place only through the speculators' endogenous weighting over the two types of shocks, making it possible to distinguish that from firms' actual information endowment. Last, this setting offers a smooth transition between the baseline model with full cross learning (as n goes to infinity) and the self-feedback benchmark (as n equals to 1). This not only makes our analysis analytically tractable but helps unify all the results and intuitions.

We acknowledge again that we are abstracting away from any possible industrial organization of the firms' product markets. Rather, we make use of the number of firms as a proxy for competition, which we believe is the most relevant measure.[26] This allows us to underscore the cross-learning mechanism by highlighting it as the only interaction among firms. In this sense, our model serves as a benchmark for further research that may take more aspects of industrial competition along with firms' cross learning into account.[27]

We proceed to characterize the equilibrium in the extended framework and the corresponding investment efficiency. We still consider symmetric equilibria, and denote by ϕ_n the speculators' weight on the signal of the common shock, when the speculators believe that each firm only learns from as many as n asset prices, including its own. For convenience, we call the associated equilibrium an n-learning equilibrium.

Formally, each capital provider i still observes its own asset price P_i and all other firms' asset prices $\{P_{-i}\}$. Same as before, the information content of any asset price is characterized by

$$P_i = \exp\left(\frac{\phi_n}{\lambda\sqrt{\tau_x^{-1}\phi_n^2+\tau_y^{-1}}}a + \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi_n^2+\tau_y^{-1}}}f_i + \frac{\zeta+\xi_i}{\lambda} - \frac{\mu_i}{\lambda\sqrt{\tau_x^{-1}\phi_n^2+\tau_y^{-1}}}\right),$$

equivalent to a signal

$$z_n\left(P_i\right) = \phi_n a + f_i + \sqrt{\tau_x^{-1}\phi_n^2+\tau_y^{-1}}\left(\zeta+\xi_i\right).$$

However, in an n-learning equilibrium, the speculators believe that each capital provider only learns from its own price as well as the other $n-1$ firms' asset prices. Specifically, from the speculators' perspective, due to symmetry, each capital provider i has four signals: the own private signals $s_{a,i}$ and $s_{f,i}$, the signal $z_n(P_i)$ from its own asset price, and another signal $z_n(P_{-i})$ coming

[26] To use the number of firms to proxy competition is common in the literature, especially when information is a focus (see Vives, 2010, for a survey).

[27] For example, Peress (2010) offers an interesting analysis on the impacts of monopolistic competition in product markets on stock market efficiency, but does not consider feedback to real investments or cross learning as we do. He does not consider the implications on investment waves as well.

from the other $n-1$ asset prices:[28]

$$z_n\left(P_{-i}\right)=\phi_n a+\frac{\sum_{l\neq i} f_l}{n-1}+\sqrt{\tau_x^{-1}\phi_n^2+\tau_y^{-1}}\left(\zeta+\frac{\sum_{l\neq i}\xi_l}{n-1}\right).$$

From the speculators' perspective, capital provider i uses these four signals to infer the sum of the two (log) productivity shocks, $a+f_i$, in making investment decisions. Concretely, the speculators believe that capital provider i updates beliefs as

$$\mathbb{E}[a+f_i|\Gamma_i]=\mathbf{z}'\mathrm{Var}(\mathbf{z})^{-1}\mathrm{Cov}(a+f_i,\mathbf{z}), \tag{5.12}$$

where $\mathbf{z}=[s_{a,i},s_{f,i},z_n(P_i),z_n(P_{-i})]'$. As a consequence, the speculators' perceived investment sensitivities S_{an} to the common shock and S_{fn} to the idiosyncratic shocks are read off from the conditional expectation (5.12). Following the same approach as before in solving for the speculators' optimal weight in trading, we finally get

$$\phi_n=\frac{\left(S_{an}+1-\frac{\phi_n}{\lambda\sqrt{\tau_x+\tau_y\phi_n^2}}\right)\frac{\tau_x}{\tau_a+\tau_x}}{\left(S_{fn}+1-\frac{1}{\lambda\sqrt{\tau_x+\tau_y\phi_n^2}}\right)\frac{\tau_y}{\tau_f+\tau_y}}. \tag{5.13}$$

Clearly, the equilibrium condition (5.13), with the investment sensitivities prescribed by condition (5.12), is equivalent to that in a corresponding economy with n firms operating and speculators fully internalizing their cross learning, so is the equilibrium weight ϕ_n, while the expression of investment efficiency can be shown to be the same as that in Definition 5. Therefore, we have the following proposition regarding the equilibrium weight ϕ_n and investment efficiency R_n in an n-learning equilibrium. We still focus on comparable cases in which all n-learning equilibria are unique.

PROPOSITION **8.** *For a high enough noisy supply elasticity λ, a low enough idiosyncratic noisy supply shock precision τ_ξ, and a high enough information precision τ_y (of the speculators' signal on the idiosyncratic shock),*

i) for all $n\geqslant 1$, there exists a unique n-learning equilibrium in which the speculators put a positive weight $\phi_n>0$ on the signal of the common productivity shock,

ii) for all $n>1$, $\phi^*<\phi'(=\phi_1)<\phi_n<\phi$, in particular, ϕ_n is increasing in n, where ϕ, ϕ' are the equilibrium weights in the baseline cross-learning equilibrium and in the self-feedback equilibrium, respectively, and ϕ^* is the optimal weight that maximizes investment efficiency, and

iii) for all $n\geqslant 1$, $R<R_n<R^*$, in particular, R_n is decreasing in n, where R is the investment efficiency in the baseline cross-learning equilibrium and R^* is the optimal investment efficiency.

[28] When $n=1$, only the first three signals are relevant and the n-learning equilibrium degenerates to a self-feedback equilibrium.

Proposition 8 offers a clear identification of the externality and efficiency loss associated with cross learning. Under the parameters we are interested, when the number of firms increases, cross learning makes the speculators to put an increasing weight ϕ_n on the signal of the common shock. Along with the established results in Section 4, this suggests stronger investment waves with higher systematic risks. Moreover, since the information endowment is controlled, this leads to decreasing investment efficiency, associated with an increasing extent of cross learning. The key to understand this is a new externality through the speculators' weighting over the two types of shocks in response to the capital providers' cross learning. When each capital provider learns from other firms' asset prices, she only cares about her own investment decision and wants to use other firms' asset prices for better inferring the common shock. This makes her investment more sensitive to the common shock, which in turn encourages the speculators to put a higher weight on the signal of the common shock. However, she does not internalize the endogenous cost on other firms' investment decisions, because her cross learning makes asset prices endogenously less informative on other firms' idiosyncratic productivity shocks, through the speculators' endogenous response in terms of weighting the two shocks. When there are more firms in the economy, the speculators respond more heavily to the capital providers' cross learning and each asset price is also used by more firms, which implies a stronger externality not being internalized by each capital provider in cross learning.

We highlight the externality we have identified as a new pecuniary externality, taking effect through the informativeness of prices instead of price levels. In the classical pecuniary externality literature (see Stiglitz, 1982, Greenwald and Stiglitz, 1986, and Geanakoplos and Polemarchakis, 1985, and for recent theoretical developments see Farhi and Werning, 2013, He and Kondor, 2013, and Davila, 2014 for a comprehensive treatment), agents do not internalize the impacts of their actions on equilibrium price levels, leading to a welfare loss under various frictions. In particular, the classical pecuniary externality generates welfare transfers across agents through the levels of prices. In our framework, instead, the capital providers do not fully internalize the impacts of cross learning on equilibrium price informativeness. This leads to a typical "tragedy of the commons" regarding the use of the price system as an information source under multi-firm cross learning. This tragedy-of-the-commons observation is absent in classical single-firm feedback models. In this sense, our pecuniary externality is also reminiscent of the notion of learning externality in the earlier dynamic learning and herding literature (for example, Vives, 1997) that an agent, when responding to private information, does not take into account the benefit of increased informativeness of public information in the future. This literature, however, does not explicitly consider the roles of financial markets and in particular the feedback from market prices to investments as we do.

Along with the results in Section 4, the new pecuniary externality associated with cross learning offers a new perspective to investigate the puzzling fact that more competitive industries exhibit more inefficient investment waves with higher systematic risks. This fact has been recently docu-

mented in Hoberg and Phillips (2010) and shown to be robust after many relevant controls. As they suggest, however, no single theory in the literature can accommodate their findings. More recently, Greenwood and Hanson (2014) find a similar pattern in the cargo ship industry that also applies to other industries. They estimate a behavioral theory in which firms over-extrapolate exogenous demand shocks and partially neglect the endogenous investment responses of their competitors. Our fully rational cross-learning framework helps reconcile these facts by explicitly identifying the pecuniary externality associated with competition and its impacts on real investment efficiency. Relatedly, Ozoguz and Rebello (2013) have empirically identified that firms in more competitive industries adapt investments more sensitively to stock prices of their peers, which supports our theory.

It is worth noting that, when $n = 1$, that is, the speculators believe that there is only one firm operating, the economy still features investment inefficiency. Under the parameters we are interested, this benchmark investment inefficiency comes from the fact that the capital providers find the information about their idiosyncratic shocks more valuable whereas the speculators still find it profitable to put a considerable weight on the signal of the common shock in trading. This conflict of interests between capital providers (or firms) and speculators is generally present in the feedback literature in different forms (see the survey by Bond, Edmans and Goldstein, 2012), and Goldstein and Yang (2014a) formally identify it as the mismatch channel of feedback. Thus, the contribution of our work is first to extend the mismatch channel to a multi-firm feedback framework with two fundamentally different types of shocks, and then more importantly, to identify the new pecuniary externality associated with cross learning that is absent in classical feedback models.

Although our framework allows for an analytical characterization, we also offer numerical examples to help illustrate the pecuniary externality and efficiency loss associated with different extent of cross learning. We set $\tau_a = \tau_f = \tau_{sa} = \tau_{sf} = \tau_x = \tau_\zeta = 1$, $\tau_y = 10$, $\tau_\xi = 0.1$, $\lambda = 2$, $\kappa = 1$, and $c = 0.5$. The left panel of Figure 6 depicts the equilibrium weight ϕ_n as n increases as the blue solid line. When n becomes larger, the weight gradually approaches that in the baseline cross-learning equilibrium, as depicted by the red dashed line. The right panel of Figure 6 depicts the log of the efficiency loss due to cross learning, measured by $\log(R^*/R_n)$. As shown in the blue solid line, the efficiency loss associated with cross learning increases in n, suggesting a more severe pecuniary externality as competition becomes stronger. In the baseline cross-learning equilibrium, the pecuniary externality is the strongest, as depicted by the red dashed line. These results are robust to a very wide range of parameters once λ and τ_y are relatively large while τ_ξ is relatively small, which are empirically relevant as we discussed above.

Admittedly, our identification of the pecuniary externality associated with cross learning does not attempt to offer a comprehensive evaluation of the merits of competition. Relatedly, the investment efficiency R_n in an n-learning equilibrium cannot be interpreted as a direct measure of the investment efficiency in an actual competitive industry with n firms. Our point is focused,

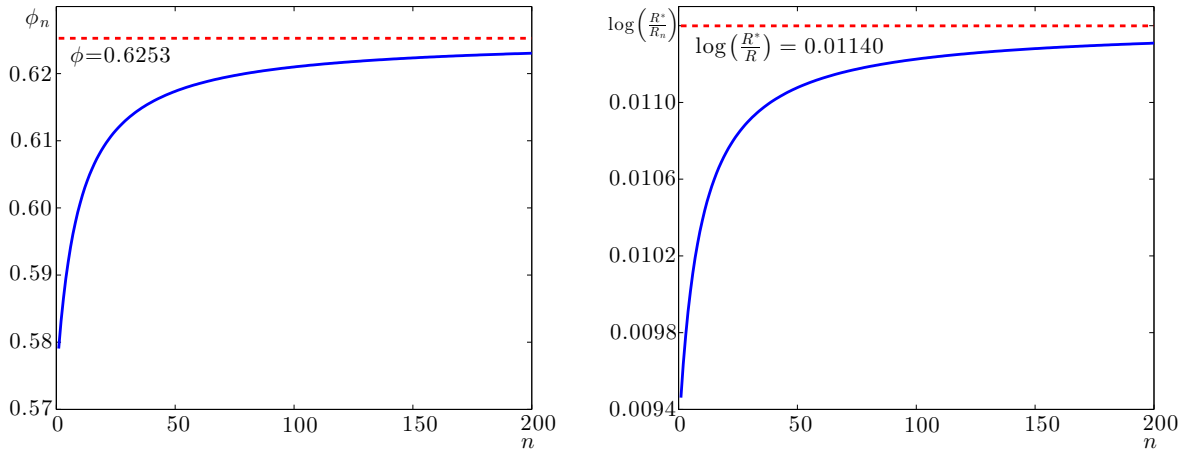

Figure 6: Competition on Cross Learning and Efficiency Loss

however, to suggest a new perspective to look at the relationship between inefficient investment waves and competition, a puzzling fact well documented recently and hard to be reconciled with existing theories. We admit that, despite the fact that competition increases the extent of cross learning, with new adverse implications for investment efficiency, it may well remain desirable when all other social benefits and costs of competition are taken into account.

6 Discussion

Our cross learning framework focuses on the systematic risks and investment inefficiency in investment waves, which we believe are less understood in the literature. It is also natural to rely on our framework to shed lights on some other commonly observed phenomena and to add new insights. This section discusses two directions.

6.1 Over-investment under Cross Learning

Investment waves usually exhibit both high systematic risks (second moment) and over-investment (first moment). Although the latter has been well addressed in the literature, our framework is easily adaptable to generate so. Especially, our cross learning framework offers a new perspective to explain why over-investment happens more often in technologies or industries that are more sensitive to common shocks.

We keep all the settings in our baseline model except for introducing two different investment technologies. Specifically, each firm i now has two mutually exclusive projects, one only subject to the common shock A while the other only subject to the idiosyncratic shock F_i. We call the former common project and the latter idiosyncratic project. Introducing the two types of projects is a parsimonious way to model the cross-section of different technologies or industries according

to their different sensitivity to the common shock. For simplicity, here we only allow each firm to allocate a fixed amount of money between the two projects. Hence, each capital provider's problem is:

$$\max_{I_i \in [0,1]} \left[AI_i + F_i(1 - I_i) | \Gamma_i \right].$$

We again highlight cross learning: $\Gamma_i = \{P_i, \{P_{-i}\}, s_{a,i}, s_{f,i}\}$. This adapted setting is in the similar spirit of Dow, Goldstein and Guembel (2011) but enriches it with both cross learning and the firm's debate between the common project and the idiosyncratic project.

Following the same equilibrium concept as our baseline model, one can show that a cross-learning equilibrium features over-investment in the common project while under-investment in the idiosyncratic project, compared to the first best. The intuition is the same as before. When the capital providers are able to cross learn, the speculators again find it more profitable to put a higher weight on the information about the common shock. This makes the prices more informative about the common shock and thus encourages the capital providers to invest more on the common project while less on their idiosyncratic projects.

Complementary to the existing literature about over-investment, our cross-learning mechanism has two new implications. First, over-investment is more likely to happen in technologies or industries that are more sensitive to common shocks, which is reflected by the common project in our stylized model. This fits quite in line with the recent episodes of over-investment in the IT industry and in housing markets. Second, which is perhaps more subtle and interesting, over-investment in the common project is always accompanied by under-investment in the idiosyncratic projects at the same time. This suggests that over-investment does not necessarily imply an inefficiently large economy scale but rather an inefficient composition of various economic activities.

The comparative statics of the adapted model also offer predictions consistent with the reality. For example, when the common project has higher ex-ante expected productivity, cross learning is stronger and thus the equilibrium features a higher level of over-investment in the common shock. Dow, Goldstein and Guembel (2011) and more recently Fajgelbaum, Schaal, and Taschereau-Dumouchel (2014) provide full-fledged models to demonstrate such a relationship between investment and information provision. They have similar predictions on how beliefs of productivity affect investment decisions. These papers, though featuring self-feedback and speaking to the level of investment directly, do not consider cross learning and the two types of shocks as we do.

6.2 Industry Momentum under Cross Learning

The contemporaneous study by Sockin and Xiong (2014b) uses a feedback model to generate return momentum in a housing cycle context. Although our model does not aim to provide a general dynamic account for investment waves, the introduction of multiple firms and the two types of shocks also help shed lights on the understanding of momentum by further establishing a channel

between cross learning and industry momentum.

Industry momentum, first identified by Moskowitz and Grinblatt (1999), suggests that industry portfolios also exhibit considerable momentum, and it even accounts for much of the individual stock momentum. As discussed by Moskowitz and Grinblatt (1999), individual stock momentum may be explained by a number of behavioral theories focusing on investors' information barrier or risk appetite. But there have been no formal theories that directly speak to the existence and the magnitude of industry momentum. Our framework can potentially offer a consistent rational theory for both individual stock momentum and industry momentum, highlighting firms' investment activities and their cross learning instead.

In our benchmark three-period model, the standard definition of overall individual momentum is

$$M_i = \text{Cov}\left(\log(AF_iI_i) - \log P_i, \log P_i\right) ,$$

and industry momentum can be defined as

$$\overline{M} = \text{Cov}\left(\log \int_0^1 AF_iI_i \text{d}i - \log \overline{P}, \log \overline{P}\right) .$$

It is straightforward to show that both individual stock momentum and industry momentum exist in equilibrium, and their magnitudes increase in the speculators' weight ϕ on the information about the common shock. Specifically, M_i and $\overline{M}$ are always positive when the noisy supply elasticity λ and the information precision τ_y (of speculators' signal on the idiosyncratic shock) are large enough, and they increase in ϕ. Intuitively, when the speculators put a higher weight on the common shock, the asset prices become more informative about the common shock and thus firms' investment also becomes more sensitive to the common shock. Therefore, the common shock plays a more important role in determining both the asset prices and the eventual cash flows of the firms, implying both a stronger individual stock momentum and a stronger industry momentum. Moreover, according to the results regarding the relationship between cross learning and competition in Section 5, both individual stock momentum and industry momentum may be stronger in more competitive industries, also due to a stronger cross learning effect.

It is worth highlighting that our mechanism to generate individual momentum and industry momentum is fundamentally different from the prevailing explanations that highlight overconfidence (Daniel, Hirshleifer, and Subrahmanyam, 1998), sentiment (Barberis, Shleifer, and Vishny, 1998), or slow information diffusion (Hong and Stein, 1999). In those models, investors generally ignore some information content revealed by asset prices. In contrast, in our model, the capital providers' rational cross learning from all available asset prices plays a central role.

7 Conclusion

Firms and capital providers' cross learning behavior is not only empirically important but also theoretically relevant for commonly observed investment waves. We have developed a tractable model to admit cross learning and delivered a series of predictions regarding investment waves. We have illustrated that investment waves comes from new strategic complementarities and a spiral that coordinate capital providers' investment sensitivity and speculators' weight in trading towards the common productivity shock. However, cross learning may lead to higher investment inefficiency, because capital providers do not internalize the new externality that other firms' asset prices become less informative on their idiosyncratic productivity shocks. In more competitive industries, cross learning tends to be stronger, potentially leading to more inefficient investment waves with higher systematic risks. Hence, appropriate policy interventions are called for to correct the inefficiency in industry-investment waves.

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8 Appendix

8.1 Proofs

This appendix provides all proofs omitted above with auxiliary results.

Proof of Lemma 1. According to the equilibrium definition, any speculator (i,j) longs one share of asset i when $\phi_i x_{ij} + y_{ij} > \mu_i$ and shorts one share otherwise. Equivalently, speculator (i,j) longs one share of asset i when

$$\frac{\phi_i x_{ij} + y_{ij}}{\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}} > \frac{\mu_i - (\phi_i a + f_i)}{\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}},$$

and shorts one share otherwise. Thus, in asset market i, all speculators' aggregate demand is

$$D_i = 1 - 2\Phi\left(\frac{\mu_i - (\phi_i a + f_i)}{\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}\right).$$

Hence, in equilibrium, market clearing implies

$$1 - 2\Phi\left(\frac{\mu_i - (\phi_i a + f_i)}{\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}\right) = 1 - 2\Phi(\zeta + \xi_i - \lambda \log P_i),$$

which further implies that the equilibrium price in asset market i is

$$P_i = \exp\left(\frac{\phi_i}{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}a + \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}f_i + \frac{\zeta + \xi_i}{\lambda} - \frac{\mu_i}{\lambda\sqrt{\tau_x^{-1}\phi_i^2 + \tau_y^{-1}}}\right).$$

This concludes the proof. □

Proof of Lemma 2. In a symmetric equilibrium, the capital providers put a same weight ϕ on the information of the common shock in any asset market i. Thus, by Lemma 1, for asset price P_i, its equivalent signal in predicting the common shock a becomes

$$z_a(P_i) = a + \frac{1}{\phi}f_i + \frac{\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}{\phi}(\zeta + \xi_i).$$

Since f_i and ξ_i are both i.i.d. and have zero means, the aggregate price $\overline{P}$ is equivalent to the following signal

$$z_a(\overline{P}) = \int_0^1 \left(a + \frac{1}{\phi}f_i + \frac{\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}{\phi}(\zeta + \xi_i)\right) \mathrm{d}i = a + \frac{\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}{\phi}\zeta$$

in predicting the common shock a. It immediately follows the construction of the other signal

$z_{f,i}(\overline{P})$ in predicting the idiosyncratic shock.

Finally, it is easy to verify that any combination of the asset prices $\{P_i, i \in [0,1]\}$ cannot be more informative in predicting the two productivity shocks. This concludes the proof. □

Proof of Lemma 3. From the capital providers' problem (2.1), the first order condition is

$$\begin{aligned} I_i &= \frac{\kappa}{c}\mathbb{E}[\exp(a+f_i)|\Gamma_i] \\ &= \frac{\kappa}{c}\exp\left(\mathbb{E}[a+f_i|\Gamma_i] + \frac{1}{2}\mathrm{Var}[a+f_i|\Gamma_i]\right). \end{aligned}$$

By Lemma 2, we know that $s_{a,i}$ and $z_a(\overline{P})$ are only informative about the common shock a and $s_{f,i}$ and $z_{f,i}(\overline{P})$ are only informative about the idiosyncratic shock f_i. Applying Bayesian updating immediately leads to the following optimal investment policy

$$I_i = \frac{\kappa}{c}\exp\left[\frac{\tau_{sa}s_{a,i}+\tau_{\overline{pa}}z_a(\overline{P})}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} + \frac{1}{2(\tau_a+\tau_{sa}+\tau_{\overline{pa}})} + \frac{\tau_f s_{f,i}+\tau_{\overline{pf}}z_{f,i}(\overline{P})}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} + \frac{1}{2(\tau_f+\tau_{sf}+\tau_{\overline{pf}})}\right].$$

This concludes the proof. □

Proof of Lemma 4. This is a direct application of Lemma 2 to Definition 2. □

Proof of Proposition 1. We proceed step by step.

Step 1: Proof of the existence of the solution.

Following the equilibrium condition (3.10) for the cross-learning case, let

$$g(\phi) = \phi - \frac{\alpha_1-\gamma_1}{\alpha_2-\gamma_2} = \phi - \frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}+1-\frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}+1-\frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\frac{\tau_y}{\tau_f+\tau_y}},$$

where $\tau_{\overline{pa}}$ is given by (3.5) and $\tau_{\overline{pf}}$ is given by (3.7), both being function of ϕ. It is easy to check that $\lim_{\phi\to-\infty} g(\phi) < 0$ and $\lim_{\phi\to+\infty} g(\phi) > 0$ by the following two equations:

$$\lim_{\phi\to-\infty}\frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}+1-\frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}+1-\frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\frac{\tau_y}{\tau_f+\tau_y}} = \frac{\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_y}{\tau_f+\tau_y}}\frac{\frac{\tau_{sa}+\tau_\zeta\tau_x}{\tau_a+\tau_{sa}+\tau_\zeta\tau_x}+1-\frac{\sqrt{\tau_x}}{\lambda}}{\frac{\tau_{sf}}{\tau_f+\tau_{sf}}+1},$$

and

$$\lim_{\phi\to+\infty}\frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}+1-\frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}+1-\frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\frac{\tau_y}{\tau_f+\tau_y}} = \frac{\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_y}{\tau_f+\tau_y}}\frac{\frac{\tau_{sa}+\tau_\zeta\tau_x}{\tau_a+\tau_{sa}+\tau_\zeta\tau_x}+1-\frac{\sqrt{\tau_x}}{\lambda}}{\frac{\tau_{sf}}{\tau_f+\tau_{sf}}+1}.$$

The analysis above indicates that there always exists a solution of ϕ to the equilibrium condition (3.10), i.e., $g(\phi)=0$, by the intermediate value theorem. Especially, when $\lambda > 1/\sqrt{\tau_x^{-1}}$, we know that

$$f(0) = -\frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}+1}{\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}+1-\frac{1}{\lambda\sqrt{\tau_x^{-1}}}}\frac{\frac{\tau_x}{\tau_a+\tau_x}}{\frac{\tau_y}{\tau_f+\tau_y}} < 0\,.$$

We conclude that there always exists a positive solution $\phi > 0$ as long as λ is large enough.

Step 2: Proof of the uniqueness of the solution when τ_f is large enough.

By simple algebra, the equilibrium condition (3.10) is re-expressed as

$$\left(\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}+1-\frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\right)\frac{\tau_x}{\tau_a+\tau_x} = \phi\left(\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}+1-\frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\right)\frac{\tau_y}{\tau_f+\tau_y}\,. \tag{8.1}$$

Applying Taylor expansion to the terms in equation (8.1) with respect to τ_y^{-1} yields:

$$\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}} = -\frac{\tau_a}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}-\frac{\tau_a}{(\tau_a+\tau_{sa}+\tau_{\overline{pa}})^2}\frac{\tau_\zeta}{\tau_x^{-2}\phi^2}\,\tau_y^{-1}+o(\tau_y^{-1})\,,$$

$$\frac{\tau_{sf}+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}} = 1-\frac{\tau_f}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}-\frac{\tau_f}{(\tau_f+\tau_{sf}+\tau_{\overline{pf}})^2}\frac{\tau_\xi}{\tau_x^{-2}\phi^4}\,\tau_y^{-1}+o(\tau_y^{-1})\,,$$

$$\frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}} = \frac{1}{\lambda\sqrt{\tau_x^{-1}}}-\frac{1}{2\lambda\tau_x^{-\frac{3}{2}}\phi^2}\tau_y^{-1}+o(\tau_y^{-1})\,,$$

and

$$\frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}} = \frac{1}{\lambda\phi\sqrt{\tau_x^{-1}}}-\frac{1}{2\lambda\tau_x^{-\frac{3}{2}}\phi^3}\tau_y^{-1}+o(\tau_y^{-1})\,.$$

Plugging them back into equation (8.1), we have:

$$\begin{aligned}&\frac{\tau_x}{\tau_a+\tau_x}\frac{\tau_f+\tau_y}{\tau_y}\left[2-\frac{\tau_a}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}-\frac{\tau_a}{(\tau_a+\tau_{sa}+\tau_{\overline{pa}})^2}\frac{\tau_\zeta}{\tau_x^{-2}\phi^2}\,\tau_y^{-1}-\frac{1}{\lambda\sqrt{\tau_x^{-1}}}+\frac{1}{2\lambda\tau_x^{-\frac{3}{2}}\phi^2}\tau_y^{-1}+o(\tau_y^{-1})\right]\\ =\;&\phi\left[2-\frac{\tau_f}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}-\frac{\tau_f}{(\tau_f+\tau_{sf}+\tau_{\overline{pf}})^2}\frac{\tau_\xi}{\tau_x^{-2}\phi^4}\,\tau_y^{-1}-\frac{1}{\lambda\phi\sqrt{\tau_x^{-1}}}+\frac{1}{2\lambda\tau_x^{-\frac{3}{2}}\phi^3}\tau_y^{-1}+o(\tau_y^{-1})\right]\,,\end{aligned}$$

which becomes a cubic equation of ϕ when τ_y goes to infinity:

$$\left(\frac{\tau_{sa}+\tau_x\tau_\zeta}{\tau_a+\tau_{sa}+\tau_x\tau_\zeta}+1-\frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a+\tau_x} = \frac{\tau_{sf}\phi^3+\tau_x\tau_\xi\phi}{\tau_f\phi^2+\tau_{sf}\phi^2+\tau_x\tau_\xi}+\phi-\frac{\sqrt{\tau_x}}{\lambda}\,. \tag{8.2}$$

Note that, the left hand side of equation (8.2) does not depends on ϕ. Denote by $h(\phi)$ the right hand side of (8.2), and its first order derivative with respect to ϕ is given by

$$\frac{\partial h(\phi)}{\partial\phi} = 1-\frac{\tau_f\phi^2}{\tau_f\phi^2+\tau_{sf}\phi^2+\tau_x\tau_\xi}+1-\frac{2\tau_f\tau_x\tau_\xi\phi^2}{(\tau_f\phi^2+\tau_{sf}\phi^2+\tau_x\tau_\xi)^2} > 0\,,$$

which indicates that the right hand side of equation (8.2) is increasing in ϕ and thus we have a unique solution to equation (8.2). Therefore, since $g(\phi)$ is a continuous function of τ_y, there always

exists one unique solution to $g(\phi) = 0$, i.e., equation (8.1), when τ_y is large enough. This concludes the proof. □

Proof of Proposition 2. This proof is similar to the proof of Proposition 1. In the benchmark case without cross learning, the equilibrium condition (3.11) is re-expressed as

$$\begin{aligned}
&\left(\frac{\tau_{sa}+\tau_{pa}}{\tau_a+\tau_{sa}+\tau_{pa}}+\frac{\tau_{pf}}{\tau_f+\tau_{sf}+\tau_{pf}}\phi'+1-\frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\right)\frac{\tau_x}{\tau_a+\tau_x}\\
=\;&\phi\left(\frac{\tau_{sf}+\tau_{pf}}{\tau_f+\tau_{sf}+\tau_{pf}}+\frac{\tau_{pa}}{\tau_a+\tau_{sa}+\tau_{pa}}\frac{1}{\phi'}+1-\frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2+\tau_y^{-1}}}\right)\frac{\tau_y}{\tau_f+\tau_y},
\end{aligned}$$

which further reduces to

$$\left(\frac{\tau_{sa}}{\tau_a+\tau_{sa}}+1-\frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a+\tau_x}=\phi\left(\frac{\tau_{sf}}{\tau_f+\tau_{sf}}+1-\frac{\sqrt{\tau_x}}{\lambda\phi}\right), \tag{8.3}$$

when τ_y goes to infinity, τ_ξ goes to zero and $\lambda > \sqrt{\tau_x}$. Since the right hand side of equation (8.3) is increasing inf ϕ, we know that there must exist one unique solution to equation (8.3).

On the other hand, when τ_ξ goes to zero (and when τ_y goes to infinity and $\lambda > \sqrt{\tau_x}$), equation (8.2) in the case with cross learning becomes

$$\left(\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}+1-\frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a+\tau_x}=\phi\left(\frac{\tau_{sf}}{\tau_f+\tau_{sf}}+1-\frac{\sqrt{\tau_x}}{\lambda\phi}\right). \tag{8.4}$$

We compare between the two equations (8.3) and (8.4). It is clear that their right hand sides are the same, while the left hand side of (8.3) (for the benchmark case without cross learning) is smaller than the left hand side of (8.4) (for the case with cross learning). Thanks to the continuity with respect to τ_y and τ_ξ of the two equilibrium conditions (3.11) and (3.10) in the two cases, we conclude that the equilibrium ϕ' in the benchmark case without cross learning is always lower than the equilibrium ϕ is the problem with cross learning, as long as $\lambda > \sqrt{\tau_x}$, τ_y is large enough, and τ_ξ is small enough. □

In the following proofs, we will frequently refer to the two notions of investment sensitivity defined in Definition 2, i.e., common investment sensitivity S_a and idiosyncratic investment sensitivity S_f. They are both functions of ϕ in equilibrium.

Proof of Lemma 5. We first consider the investment beta β_I. Recall the investment policy (3.8), we have

$$\log I_i=\frac{\tau_{sa}s_{a,i}+\tau_{\overline{pa}}z_a(\overline{P})}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}+\frac{1}{2(\tau_a+\tau_{sa}+\tau_{\overline{pa}})}+\frac{\tau_f s_{f,i}+\tau_{\overline{pf}}z_{f,i}(\overline{P})}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}+\frac{1}{2(\tau_f+\tau_{sf}+\tau_{\overline{pf}})}.$$

Following the definition of β_I and after some algebra, we reach

$$\beta_I = \frac{S_a/\tau_a - \frac{\tau_{sa}}{(\tau_a+\tau_{sa}+\tau_{\overline{pa}})^2}}{S_a/\tau_a + S_f/\tau_f}\,.$$

To simplify the analysis, let $g_1 = S_a/\tau_a$, $g_2 = \tau_{sa}/\left(\tau_a + \tau_{sa} + \tau_{\overline{pa}}\right)^2$, and $g_3 = S_f/\tau_f$. By Lemma 4, it is straightforward that g_1 is increasing in ϕ and both g_2 and g_3 are decreasing in ϕ. Thus, as $\phi > 0$, we also have that g_1 is increasing in ϕ^2 and both g_2 and g_3 are decreasing in ϕ^2. Furthermore, we have

$$\begin{aligned}\frac{\partial \beta_I}{\partial \phi^2} &= \frac{g_1' - g_2'}{g_1+g_3} - \frac{(g_1-g_2)(g_1'+g_3')}{(g_1+g_3)^2} \\ &= \frac{[(g_1'-g_2')g_1 - (g_1-g_2)g_1'] + (g_1'-g_2')g_3 - (g_1-g_2)g_3'}{(g_1+g_3)^2}\,,\end{aligned}$$

where g_1', g_2' and g_3' stands for the first order derivative with respect to ϕ^2.

Since we know that $g_1' - g_2' > g_1'$ (due to the fact that g_2 is decreasing in ϕ^2) and $g_1 > g_1 - g_2$, we have $(g_1' - g_2')g_1 - (g_1 - g_2)g_1' > 0$. Meanwhile, we have $g_1' > 0$, $g_2' < 0$, and $g_3 > 0$, so that $(g_1' - g_2')g_3 > 0$. Lastly, since $g_1 > g_2$ and $g_3' < 0$, we also know that $(g_1 - g_2)g_3' < 0$. Therefore, we conclude that $\partial\beta_I/\partial\phi^2 > 0$, i.e., β_I is an increasing function of ϕ when $\phi > 0$.

We then consider the price beta β_P. Recall the pricing function (3.1), we have

$$\log P_i = \frac{\phi}{\lambda\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}a + \frac{1}{\lambda\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}f_i + \frac{\zeta + \xi_i}{\lambda} - \frac{\mu}{\lambda\sqrt{\tau_x^{-1}\phi^2 + \tau_y^{-1}}}\,.$$

Following the definition of β_P and after some algebra, we reach that

$$\beta_P = \frac{\frac{\phi^2}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_a} + \frac{1}{\tau_\zeta}}{\frac{\phi^2}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_a} + \frac{1}{\tau_\zeta} + \frac{1}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_f} + \frac{1}{\tau_\xi}}\,.$$

To simplify, let

$$h_1 = \frac{\phi^2}{\tau_x^{-1}\phi^2 + \tau_y^{-1}}\frac{1}{\tau_a} + \frac{1}{\tau_\zeta} \tag{8.5}$$

and

$$h_2 = \frac{1}{\tau_x^{-1}\phi^2 + \tau_y^{-1}}\frac{1}{\tau_f} + \frac{1}{\tau_\xi}\,. \tag{8.6}$$

It is straightforward that h_1 is increasing in ϕ and h_2 is decreasing in ϕ. Hence, we have

$$\frac{\partial \beta_P}{\partial \phi^2} = \frac{-g_1\frac{\partial g_2}{\partial \phi^2}}{(g_1+g_2)^2} > 0\,,$$

which indicates that β_P is an increasing function of ϕ when $\phi > 0$. □

PROOF OF LEMMA 6. We first consider common investment correlation β_A. Recall the investment policy (3.8) and the definition of β_A, we have

$$\begin{aligned}\beta_A &= \frac{\mathrm{Cov}(\log I_i, \log A)}{\sqrt{\mathrm{Var}(\log I_i)}\sqrt{\mathrm{Var}(\log A)}} \\ &= \frac{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}}{\sqrt{\tau_a}\sqrt{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}\frac{1}{\tau_a}+\frac{\tau_f+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}\frac{1}{\tau_f}}} = \frac{S_a}{\sqrt{\tau_a}\sqrt{S_a/\tau_a+S_f/\tau_f}}.\end{aligned}$$

It is convenient for us to consider instead

$$\frac{1}{\tau_a\beta_A^2} = \frac{S_a/\tau_a+S_f/\tau_f}{S_a^2} = \frac{1}{\tau_a S_a}+\frac{S_f}{\tau_f S_a^2}.$$

By Lemma 4, since S_a is increasing in ϕ^2 and S_f is decreasing in ϕ^2 when $\phi > 0$, it is straightforward that $1/\tau_a\beta_A^2$ is decreasing in ϕ^2. This indicates that β_A is an increasing function of ϕ when $\phi > 0$.

We then consider the idiosyncratic investment correlation β_F. Again, recall the investment policy (3.8) and the definition of β_F, we have

$$\begin{aligned}\beta_F &= \frac{\mathrm{Cov}(\log I_i, \log F_i)}{\sqrt{\mathrm{Var}(\log I_i)}\sqrt{\mathrm{Var}(\log F_i)}} \\ &= \frac{\frac{\tau_f+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}}{\sqrt{\tau_f}\sqrt{\frac{\tau_{sa}+\tau_{\overline{pa}}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}\frac{1}{\tau_a}+\frac{\tau_f+\tau_{\overline{pf}}}{\tau_f+\tau_{sf}+\tau_{\overline{pf}}}\frac{1}{\tau_f}}} = \frac{S_f}{\sqrt{\tau_f}\sqrt{S_a/\tau_a+S_f/\tau_f}}.\end{aligned}$$

Similarly, it is convenient for us to consider instead

$$\frac{1}{\tau_f\beta_F^2} = \frac{S_a/\tau_a+S_f/\tau_f}{S_f^2} = \frac{S_a}{\tau_a S_f^2}+\frac{1}{\tau_f S_f},$$

which is decreasing in ϕ^2, again by Lemma 4. Hence, we conclude that β_F is an increasing function of ϕ as well, when $\phi > 0$. □

PROOF OF LEMMA 7. Following Assumption 1, we keep the ratios τ_{sa}/τ_a and τ_x/τ_a constant when consider the changes of τ_a. We also focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed. Specifically, the reduced equilibrium condition (8.2) (in the proof of Proposition 1) is re-expressed as

$$\frac{\tau_{sa}/\tau_a+\tau_x\tau_\zeta/\tau_a}{\tau_a(1+\tau_{sa}/\tau_a+\tau_x\tau_\zeta/\tau_a)}\frac{\tau_x/\tau_a}{1+\tau_x/\tau_a}+\frac{\tau_x/\tau_a}{1+\tau_x/\tau_a}+\frac{\sqrt{\frac{\tau_x}{\tau_a}\tau_a}}{\lambda}\frac{1}{1+\tau_x/\tau_a} = \frac{\tau_{sf}\phi^3+\frac{\tau_x}{\tau_a}\tau_\xi\phi\tau_a}{\tau_f\phi^2+\tau_{sf}\phi^2+\frac{\tau_x}{\tau_a}\tau_\xi\tau_a}+\phi. \tag{8.7}$$

When λ is high enough, the first order derivative of the left hand side of equation (8.7) with

respect to τ_a is

$$-\frac{\tau_{sa}/\tau_a + \tau_x\tau_\zeta/\tau_a}{\tau_a^2(1+\tau_{sa}/\tau_a+\tau_x\tau_\zeta/\tau_a)}\frac{\tau_x/\tau_a}{1+\tau_x/\tau_a} + \frac{\sqrt{\tau_x/\tau_a}}{\lambda}\frac{1}{1+\tau_x/\tau_a}\frac{1}{2\sqrt{\tau_a}} < 0\,.$$

And it is straightforward that the right hand side of (8.7) is an increasing function of τ_a. Thus, when τ_a increases, the left hand side of (8.7) decreases, which further calls for a decreasing ϕ to make the right hand side of (8.7) to decrease as well. This concludes the proof. □

Proof of Proposition 3. Again, following Assumption 1, we keep the ratios τ_{sa}/τ_a and τ_x/τ_a constant when consider the changes of τ_a. We still focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed.

We first consider the investment beta

$$\beta_I = \frac{S_a - g}{S_a + \tau_a S_f/\tau_f}\,,$$

where

$$g = \left(\frac{\tau_{sa}}{\tau_a+\tau_{sa}+\tau_{\overline{pa}}}\right)^2\frac{\tau_a}{\tau_{sa}}\,. \tag{8.8}$$

It is instructive for us to decompose the total effects of the changing of τ_a on β_I into two parts: the mechanical effect and the cross-learning effect:

$$\frac{d\beta_I(\tau_a,\phi)}{d\tau_a} = \frac{\partial\beta_I(\tau_a,\phi)}{\partial\tau_a} + \frac{\partial\beta_I(\tau_a,\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_a}\,.$$

The sign of the second term, the cross-learning effect, is straightforward by Lemma 5 and Lemma 7. Specifically, Lemma 5 indicates that $\partial\beta_I(\tau_a,\phi)/\partial\phi > 0$ and Lemma 7 indicates that $\partial\phi/\partial\tau_a < 0$, so that the cross-learning effect is negative in this case.

For the first term, the mechanical effect, since we keep τ_{sa}/τ_a and τ_x/τ_a constant and $\tau_{\overline{pa}} = \tau_x\tau_\varsigma$ when τ_y goes to infinity, we know that S_a and g are constant in this case. On the other hand, S_f is increasing in τ_x given $\tau_{\overline{pf}} = \tau_x\tau_\xi/\phi^2$ and thus is also increasing in τ_a given that τ_x/τ_a is constant. This indicates that $\tau_a S_f/\tau_f$ is increasing in τ_a. Hence, we know that $\partial\beta_I(\tau_a,\phi)/\partial\tau_a < 0$, i.e., the mechanical effect is negative as well.

Taking the two effects together, we know that the total effect is also negative, i.e., $d\beta_I(\tau_a,\phi)/d\tau_a < 0$.

We then consider the price beta

$$\beta_P = \frac{\frac{\phi^2}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_a}+\frac{1}{\tau_\zeta}}{\frac{\phi^2}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_a}+\frac{1}{\tau_\zeta}+\frac{1}{\tau_x^{-1}\phi^2+\tau_y^{-1}}\frac{1}{\tau_f}+\frac{1}{\tau_\xi}} = \frac{h_1}{h_1+h_2}\,,$$

where h_1 and h_2 are already defined in (8.5) and (8.6) (in the proof of Lemma 5).

Again, we decompose the total effects of the changing of τ_a on β_P into two parts: the mechanical effect and the cross-learning effect:

$$\frac{d\beta_P(\tau_a,\phi)}{d\tau_a} = \frac{\partial\beta_P(\tau_a,\phi)}{\partial\tau_a} + \frac{\partial\beta_P(\tau_a,\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_a}.$$

Keep in mind that we keep τ_{sa}/τ_a and τ_x/τ_a constant in this case. First, it is straightforward to see that the mechanical effect $\partial\beta_P(\tau_a,\phi)/\partial\tau_a$ is negative, because β_P is an increasing function of h_1 that is in turn decreasing in τ_a at the same time. Furthermore, Lemma 5 indicates that $\partial\beta_P(\tau_a,\phi)/\partial\phi > 0$ and Lemma 7 indicates that $\partial\phi/\partial\tau_a < 0$, which together imply that the cross-learning effect is negative as well. Therefore, we conclude that the total effect is also negative, i.e., $d\beta_P(\tau_a,\phi)/d\tau_a < 0$. □

Proof of Lemma 8. The proof is similar to the proof of Lemma 7. We again focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed. In this case, we recall the reduced equilibrium condition (8.2) (in the proof of Proposition 1):

$$\left(\frac{\tau_{sa}+\tau_x\tau_\zeta}{\tau_a+\tau_{sa}+\tau_x\tau_\zeta}+1-\frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a+\tau_x} = \frac{\tau_{sf}\phi^3+\tau_x\tau_\xi\phi}{\tau_f\phi^2+\tau_{sf}\phi^2+\tau_x\tau_\xi}+\phi-\frac{\sqrt{\tau_x}}{\lambda}.$$

It is clear that the left hand side of (8.2) is increasing in τ_{sa}. Hence, when τ_{sa} increase, the right hand side of (8.2) increases as well. On the other hand, we have already known that the right hand side of (8.2) is increasing in ϕ. Hence, in equilibrium, ϕ increases. This indicates that ϕ is an increasing function of τ_{sa}.

The analysis is similar for τ_{sf}. The right hand side of (8.2) is increasing in τ_{sf}, while the left hand side is independent of τ_{sf}. Thus, when τ_{sf} increase, ϕ decreases to ensure a constant right hand side of (8.2). This indicates that ϕ is a decreasing function of τ_{sf}. □

Proof of Proposition 4. We first consider the comparative statics with respect to τ_{sa}. For the investment beta β_I, we have

$$\beta_I = 1 - \frac{S_f/\tau_f + g/\tau_a}{S_a/\tau_a + S_f/\tau_f},$$

where g is already defined in (8.8) (in the proof of Proposition 3).

We also decompose the total effects of the changing of τ_{sa} on β_I into two parts: the mechanical effect and the cross-learning effect:

$$\frac{d\beta_I(\tau_{sa},\phi)}{d\tau_{sa}} = \frac{\partial\beta_I(\tau_{sa},\phi)}{\partial\tau_{sa}} + \frac{\partial\beta_I(\tau_{sa},\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_{sa}}.$$

Clearly, Lemma 5 and Lemma 8 indicate that the second term, the cross-learning effect, is positive. For the first term, the mechanical effect, we first know that S_a is increasing in τ_{sa}, given ϕ fixed. Also, it is easy to show that g/τ_a is increasing in τ_{sa} (given ϕ fixed) when $\tau_{sa} < \tau_a + \tau_x\tau_\zeta$,

and decreasing in τ_{sa} (also given ϕ fixed) when $\tau_{sa} > \tau_a + \tau_x\tau_\zeta$. Hence, we conclude that when $\tau_{sa} > \tau_a + \tau_x\tau_\zeta$, the mechanical effect is positive, and thus total effect $d\beta_I(\tau_{sa}, \phi)/d\tau_{sa}$ is positive as well. When $\tau_{sa} < \tau_a + \tau_x\tau_\zeta$, the mechanical effect is negative and thus the total effect is ambiguous.

For the price beta β_P, there is only cross-learning effect but no mechanical effect. Hence, by Lemma 5 and Lemma 8 we have that

$$\frac{d\beta_P(\tau_{sa}, \phi)}{d\tau_{sa}} = \frac{\partial\beta_P(\tau_{sa}, \phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_{sa}} > 0\,.$$

We then consider the comparative statics with respect to τ_{sf} in a similar manner. For the investment beta β_I, we have

$$\beta_I = \frac{S_a/\tau_a - g/\tau_a}{S_a/\tau_a + S_f/\tau_f}\,.$$

By the similar decomposition and again by Lemma 5 and Lemma 8, we know that both the mechanical effect and the cross-learning effect in this case are negative. So that the total effect is also negative:

$$\frac{d\beta_I(\tau_{sf}, \phi)}{d\tau_{sf}} = \frac{\partial\beta_I(\tau_{sf}, \phi)}{\partial\tau_{sf}} + \frac{\partial\beta_I(\tau_{sf}, \phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_{sf}} < 0\,.$$

For the price β_P, again, there is only cross-learning effect but no mechanical effect. Hence, by Lemma 5 and Lemma 8 we have that

$$\frac{d\beta_P(\tau_{sf}, \phi)}{d\tau_{sf}} = \frac{\partial\beta_P(\tau_{sf}, \phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_{sf}} < 0\,.$$

This concludes the proof. □

Proof of Lemma 9. We focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed. We first consider the effect of λ. Recall the re-expressed reduced equilibrium condition (8.7) (in the proof of Lemma 7):

$$\frac{\tau_{sa}/\tau_a + \tau_x\tau_\zeta/\tau_a}{\tau_a(1 + \tau_{sa}/\tau_a + \tau_x\tau_\zeta/\tau_a)}\frac{\tau_x/\tau_a}{1 + \tau_x/\tau_a} + \frac{\tau_x/\tau_a}{1 + \tau_x/\tau_a} + \frac{\sqrt{\frac{\tau_x}{\tau_a}\tau_a}}{\lambda}\frac{1}{1 + \tau_x/\tau_a} = \frac{\tau_{sf}\phi^3 + \frac{\tau_x}{\tau_a}\tau_\xi\phi\tau_a}{\tau_f\phi^2 + \tau_{sf}\phi^2 + \frac{\tau_x}{\tau_a}\tau_\xi\tau_a} + \phi\,.$$

It is clear that the left hand side of (8.7) is decreasing in λ while the right hand side is independent of λ. Hence, when λ increases, ϕ decreases in equilibrium.

We then consider the effects of τ_ζ and τ_ξ. Recall the reduced equilibrium condition (8.2) (in the proof of Proposition 1):

$$\left(\frac{\tau_{sa} + \tau_x\tau_\zeta}{\tau_a + \tau_{sa} + \tau_x\tau_\zeta} + 1 - \frac{\sqrt{\tau_x}}{\lambda}\right)\frac{\tau_x}{\tau_a + \tau_x} = \frac{\tau_{sf}\phi^3 + \tau_x\tau_\xi\phi}{\tau_f\phi^2 + \tau_{sf}\phi^2 + \tau_x\tau_\xi} + \phi - \frac{\sqrt{\tau_x}}{\lambda}\,.$$

On the one hand, the left hand side of (8.2) is increasing in τ_ζ, while the right hand side is independent of τ_ζ, so that ϕ is increasing in τ_ζ in equilibrium. On the other hand, the right hand

side of (8.2) is increasing in τ_ξ while the left hand side is independent of τ_ξ, so that ϕ is decreasing in τ_ξ in equilibrium. □

Proof of Proposition 5. We first consider the comparative statics with respect to λ. Since λ has no mechanical effect on ether β_I or β_P, we focus on the cross-learning effect along. By Lemma 5 and Lemma 9, we know that both β_P and β_I are decreasing in λ.

We then consider the comparative statics with respect to τ_ζ. For the investment beta β_I, we have

$$\beta_I = \frac{S_a - g}{S_a + \tau_a S_f/\tau_f},$$

where g is defined in (8.8) (in the proof of Proposition 3).

Again, we decompose the total effects of the changing of τ_ζ on β_I into two parts: the mechanical effect and the cross-learning effect:

$$\frac{d\beta_I(\tau_\zeta,\phi)}{d\tau_\zeta} = \frac{\partial\beta_I(\tau_\zeta,\phi)}{\partial\tau_\zeta} + \frac{\partial\beta_I(\tau_\zeta,\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_\zeta}.$$

By Lemma 5 and Lemma 9, we know that the cross-learning effect is positive. For the mechanical effect, when ϕ is fixed, it is easy to show that $\partial\beta_I(\tau_\zeta,\phi)/\partial\tau_{\overline{pa}} > 0$. Since we know that $\partial\tau_{\overline{pa}}/\partial\tau_\zeta > 0$, we get that the mechanical effect is also positive. Hence, the total effect $d\beta_I(\tau_\zeta,\phi)/d\tau_\zeta$ is positive.

However, the total effect on the price beta β_P is ambiguous in this case. We have

$$\beta_P = \frac{h_1}{h_1 + h_2},$$

where h_1 and h_2 are already defined in (8.5) and (8.6) (in the proof of Lemma 5). Decomposition gives

$$\frac{d\beta_P(\tau_\zeta,\phi)}{d\tau_\zeta} = \frac{\partial\beta_P(\tau_\zeta,\phi)}{\partial\tau_\zeta} + \frac{\partial\beta_P(\tau_\zeta,\phi)}{\partial\phi}\frac{\partial\phi}{\partial\tau_\zeta}.$$

Lemma 5 and Lemma 9 give a positive cross-learning effect, i.e., the second term. However, it is easy to show that the first term, the mechanical effect, is negative. Hence, the total effect is ambiguous and will be determined by other model parameters.

We finally consider the comparative statics with respect to τ_ξ. Similarly, we follow the decomposition above. For the investment beta β_I, by Lemma 5 and Lemma 9, we know that the cross-learning effect is negative. For the mechanical effect, when ϕ is fixed, it is easy to show that $\partial\beta_I(\tau_\xi,\phi)/\partial\tau_{\overline{pf}} > 0$. Since we know that $\partial\tau_{\overline{pf}}/\partial\tau_\xi > 0$, we get that the mechanical effect is also negative. Hence, the total effect $d\beta_I(\tau_\xi,\phi)/d\tau_\xi$ is negative. Following similar arguments and again by Lemma 5 and Lemma 9, we know that both the mechanical effect and the cross-learning effect on the price beta β_P are also negative, so that the total effect on β_P is negative as well. □

PROOF OF PROPOSITION 6. Following the definition of real investment efficiency, we know that

$$R = \int_0^1 R_i di \,,$$

where

$$R_i = \mathbb{E}\left[AF_i I_i - \frac{c}{2} I_i^2\right] = \frac{\kappa(2-\kappa)}{2c} \mathbb{E}\left[AF_i \mathbb{E}\left[AF_i | \Gamma_i\right]\right] \,,$$

and

$$\mathbb{E}[AF_i|\Gamma_i] = \exp\left[\frac{\tau_{sa} s_{a,i} + \tau_{\overline{pa}} z_a(\overline{P})}{\tau_a + \tau_{sa} + \tau_{\overline{pa}}} + \frac{1}{2(\tau_a + \tau_{sa} + \tau_{\overline{pa}})} + \frac{\tau_f s_{f,i} + \tau_{\overline{pf}} z_{f,i}(\overline{P})}{\tau_f + \tau_{sf} + \tau_{\overline{pf}}} + \frac{1}{2(\tau_f + \tau_{sf} + \tau_{\overline{pf}})}\right] .$$

Since κ and c are constant, without loss of generality, we set $\kappa = 1$ and $c = 0.5$ to ease the exposition. After some tedious algebra, the investment efficiency R is re-expressed in a much simpler and more intuitive form:

$$R = \exp\left(\frac{1+S_a}{\tau_a} + \frac{1+S_f}{\tau_f}\right) . \tag{8.9}$$

We solve for the socially optimal ϕ^* that maximizes R. Taking the first order condition gives

$$\frac{\partial \log R}{\partial(\phi^2)} = \frac{\tau_x^2 \tau_\zeta \tau_y}{(\tau_a + \tau_{sa} + \tau_{\overline{pa}})^2} \frac{1}{(\tau_x + \tau_y \phi^2)^2} - \frac{\tau_x \tau_y^2 \tau_\xi}{(\tau_f + \tau_{sf} + \tau_{\overline{pf}})^2} \frac{1}{(\tau_x + \tau_y \phi^2)^2} = 0 \,, \tag{8.10}$$

which reduces to

$$\frac{(\tau_a + \tau_{sa} + \tau_{\overline{pa}})^2}{(\tau_f + \tau_{sf} + \tau_{\overline{pf}})^2} = \frac{\tau_x \tau_\zeta}{\tau_y \tau_\xi} \,. \tag{8.11}$$

Since $\tau_{\overline{pa}}$ is increasing in ϕ^2 while $\tau_{\overline{pf}}$ is decreasing in ϕ^2, we know that the left hand side of (8.11) is increasing in ϕ^2. Therefore, there is a unique non-negative solution of ϕ^*.

We further compare between the socially optimal weight ϕ^* and the weight ϕ in the cross-learning equilibrium, focusing on the case in which τ_y and λ are large enough so that there is always a unique positive solution of ϕ. We re-express the first order condition (8.10) as

$$\frac{\partial \log R}{\partial(\phi^2)} = \frac{\tau_x \tau_y^2}{(\tau_a + \tau_{sa} + \tau_{\overline{pa}})^2} \left[\frac{\tau_x \tau_\zeta}{\tau_y} - \frac{(\tau_a + \tau_{sa} + \tau_{\overline{pa}})^2 \tau_\xi}{(\tau_f + \tau_{sf} + \tau_{\overline{pf}})^2}\right] = 0 \,. \tag{8.12}$$

When τ_y goes to infinity, we know that $\tau_x \tau_\zeta / \tau_y$ goes to 0, and we also have

$$\left(\frac{\tau_a + \tau_{sa} + \tau_{\overline{pa}}}{\tau_f + \tau_{sf} + \tau_{\overline{pf}}}\right)^2 = \left(\frac{\tau_a + \tau_{sa} + \tau_x \tau_\zeta}{\tau_f + \tau_{sf} + \tau_x \tau_\zeta / \phi^2}\right)^2 > 0 \,.$$

Hence, when τ_y and λ are large enough, the left hand side of (8.12) is always negative. Therefore, we conclude that the cross-learning equilibrium ϕ is always larger than the socially optimal ϕ^* when

τ_y and λ are large enough. □

PROOF OF PROPOSITION 7. Recall the expression of investment efficiency (8.9):

$$R = \exp\left(\frac{1+S_a}{\tau_a} + \frac{1+S_f}{\tau_f}\right).$$

We again focus on the case when τ_y and λ are large enough so that a unique solution of ϕ is guaranteed. We first consider the comparative statics with respect to λ. Lemma 4 implies that $\partial S_f/\partial\phi < 0$ and Lemma 9 implies that $\partial\phi/\partial\lambda < 0$. Since there is no direct effect of λ on S_f, we know that S_f is increasing in λ in equilibrium. Moreover, because the effect of ϕ on S_a is negligible when τ_y is sufficiently large, we eventually know that that R is an increasing function of λ.

We then consider τ_ξ. Similarly, Lemma 4 implies that $\partial S_f/\partial\phi < 0$ and Lemma 9 implies that $\partial\phi/\partial\tau_\xi < 0$, so that S_f, and thus R is increasing in τ_ξ in equilibrium.

We finally consider τ_{sf}. It is clear that $\partial S_f/\partial\tau_{sf} > 0$, i.e., the mechanical effect is positive. For the cross-learning effect, Lemma 4 implies that $\partial S_f/\partial\phi < 0$ and Lemma 8 implies that $\partial\phi/\partial\tau_\xi < 0$. Hence, the total effect is positive as well, i.e., S_f is increasing in τ_{sf} in equilibrium. Since the effect of ϕ on S_a is negligible when τ_y is sufficiently large, we eventually know that that R is an increasing function of τ_{sf}. □

PROOF OF PROPOSITION 8. Part i) is straightforward following the proofs of Proposition 1 and Proposition 2. For part ii), we make use of the conditional expectation (5.12). Specifically, we have

$$\mathrm{Var}(\mathbf{z}) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{bmatrix},$$

where

$$\begin{aligned}
\sigma_{11} &= \tau_a^{-1} + \tau_{sa}^{-1}, \\
\sigma_{12} &= 0, \\
\sigma_{13} &= \phi_n \tau_a^{-1}, \\
\sigma_{14} &= \phi_n \tau_a^{-1}, \\
\sigma_{22} &= \tau_f^{-1} + \tau_{sf}^{-1}, \\
\sigma_{23} &= \tau_f^{-1}, \\
\sigma_{24} &= 0, \\
\sigma_{33} &= \tau_f^{-1} + \phi_n^2 \tau_a^{-1} + (\tau_x^{-1}\phi_n^2 + \tau_y^{-1})(\tau_\zeta^{-1} + \tau_\xi^{-1}), \\
\sigma_{34} &= \phi_n^2 \tau_a^{-1} + (\tau_x^{-1}\phi_n^2 + \tau_y^{-1})\tau_\zeta^{-1},
\end{aligned}$$

$$\sigma_{44} = \frac{\tau_f^{-1}}{n-1} + \phi_n^2\tau_a^{-1} + (\tau_x^{-1}\phi_n^2 + \tau_y^{-1})\left(\tau_\zeta^{-1} + \frac{\tau_\xi^{-1}}{n-1}\right),$$

and

$$\mathrm{Cov}(\mathbf{z}, a + f_i) = [\tau_a^{-1}, \tau_f^{-1}, \phi_n\tau_a^{-1} + \tau_f^{-1}, \phi_n\tau_a^{-1}].$$

By condition (5.12), we get the expressions of S_{an} and S_{fn} after some tedious algebra and plug them into the equilibrium condition. Denote by RHS the right hand side of the equilibrium condition (5.13) and we get

$$\frac{\partial \lim_{\tau_y\to\infty} RHS(\phi_n, n)}{\partial n} = C_1C_2C_3C_4C_5(C_6 + C_7 + C_8)^2, \tag{8.13}$$

where

$$\begin{aligned}
C_1 &= \phi_n^2(\tau_f + \tau_{sf})\tau_\zeta + \tau_x\tau_\zeta\tau_\xi - \phi_n(\tau_a + \tau_{sa} + \tau_x\tau_\zeta)\tau_\xi, \\
C_2 &= \phi_n^2\tau_a(\tau_f + 2\tau_{sf})\tau_\zeta + 2\tau_a\tau_x\tau_\zeta\tau_\xi + \phi_n\tau_f(\tau_a + 2(\tau_{sa} + \tau_x\tau_\zeta))\tau_\xi, \\
C_3 &= \phi_n^2\tau_f + \tau_x\tau_\xi, \\
C_4 &= \phi_n^2\tau_f\tau_x^2\tau_\xi, \\
C_5 &= \tau_a + \tau_x, \\
C_6 &= \phi_n^3\tau_f^2\tau_x\tau_\zeta\tau_\xi + \phi_n\tau_f\tau_x^2\tau_\zeta\tau_\xi^2 + 2(\tau_a + \tau_{sa})\tau_x^2\tau_\zeta\tau_\xi^2, \\
C_7 &= \phi_n^4\tau_f(\tau_f + 2\tau_{sf})((\tau_a + \tau_{sa})\tau_\zeta + n(\tau_a + \tau_{sa} + \tau_x\tau_\zeta)\tau_\xi), \\
C_8 &= \phi_n^2\tau_x\tau_\xi((\tau_a + \tau_{sa})(3\tau_f + 2\tau_{sf})\tau_\zeta + ((2n-1)\tau_f + 2\tau_{sf})(\tau_a + \tau_{sa} + \tau_x\tau_\zeta)\tau_\xi).
\end{aligned}$$

Note that, only the first term C_1 has a negative component. However, when τ_ξ is small enough, C_1 is always strictly positive, so is the entire derivative (8.13). It implies that when τ_y is large enough and τ_ξ is small enough, the equilibrium ϕ_n is increasing in n. Also, the proof of Proposition 6 directly implies that $\phi'(=\phi_1) > \phi^*$, so that $\phi_n > \phi^*$ for all $n \geqslant 1$.

Finally, for part iii), by the proof of Proposition 6, in particular condition (8.9), we know that

$$R_n = \exp\left(\frac{1 + S_a(\phi_n)}{\tau_a} + \frac{1 + S_f(\phi_n)}{\tau_f}\right),$$

where S_a and S_f are the capital providers' investment sensitivities in the baseline cross-learning case pinned down by however the equilibrium weight in the corresponding n-learning equilibrium. By Lemma 4, it follows that $R < R_n < R^*$ for all $n \geqslant 1$. This concludes the proof. □