

# Business-cycle consumption risk and asset prices <sup>\*</sup>

Federico M. Bandi<sup>†</sup> and Andrea Tamoni<sup>‡</sup>

First version: August 2013. This version: December 15, 2014

## Abstract

We disaggregate consumption growth into components with different levels of persistence and show that a single *business-cycle consumption* factor can explain satisfactorily the differences in risk premia across book-to-market and size-sorted portfolios. We argue that accounting for persistence heterogeneity in consumption is important for interpreting *cross-sectional* risk compensations in financial markets but also for capturing the joint *time-series* dynamics of consumption and returns across horizons (for instance, the hump-shaped pricing ability of the covariance between “ultimate consumption” and returns, the hump-shaped structure of long-run risk premia as well as the decaying pattern in consumption growth predictability). Using a novel time/frequency-based data generating process for consumption growth and asset returns, we discuss implications for the asset pricing literature relying on aggregation.

*JEL classification:* C22, C32, E32, E44, G12

*Keywords:* CCAPM, business-cycle consumption, component-wise beta.

---

<sup>\*</sup>The paper was previously circulated under the title: “Scale-specific risk in the consumption CAPM.” We thank Marianne Andries (discussant), Fulvio Ortù, Xiao Qiao, Michela Verardo, Pietro Veronesi and seminar participants at BI Norwegian Business School, Duke Fuqua, Nova School of Business and Economics, the NYU-Stern Macro-Finance Brown Bag, LSE, the 2013 CEPR ESSFM Asset Pricing Week (Gerzensee) and the 2014 Toulouse Financial Econometrics Conference for useful comments and discussions. We are grateful to Xiao Qiao and Tim Kroencke for making their consumption series available to us.

<sup>†</sup>Johns Hopkins University and Edhec-Risk Institute.

<sup>‡</sup>London School of Economics, Finance Department.

# 1 Introduction

It is a basic tenet of economic theory that agents care about their consumption stream. A traditional implication, for valuation in financial markets, of this accepted premise is that the risk of any asset should depend on the covariance of its returns with consumption growth. Assets which pay off in adverse states of nature (i.e., those in which consumption growth is lower) are assets which should be perceived as being less risky and should therefore provide, in equilibrium, lower expected returns. Conversely, those assets whose returns are positively correlated with consumption growth are assets with inferior hedging abilities. Their demand should be lower, thereby justifying lower prices, and higher expected returns.

This logic, grounded in the classical Consumption CAPM (CCAPM) model of Rubinstein (1976), Breeden and Litzenberger (1978), and Breeden (1979), is known not to be supported by economic data. Differences in expected returns across risky securities are empirically not attributable to sheer differences in the variability of the returns of these securities with respect to changes in aggregate consumption.<sup>1</sup> This failure has led to alternative models of economic behavior in which a prominent role is given to suitable modifications of the conventional time-separable utility paradigm.<sup>2</sup>

Is *aggregate* consumption growth the “right” notion of consumption from an asset pricing standpoint? This paper argues that aggregate consumption growth, the subject of much investigation, can be separated into a variety of components, layers, or *details*. These details operate at different frequencies, thereby representing features of the overall consumption stream with cycles of different lengths. While the covariance between *aggregate* consumption growth and individual assets’ returns does not explain the cross-sectional dispersion of expected excess returns, we show that the covariance between specific *details* of the consumption growth process and assets’ returns is an important determinant of risk in financial markets.

In essence, investors may not focus on very high-frequency components of the consumption process. For the purpose of asset pricing, these components may just amount to short-term noise.

---

<sup>1</sup>Campbell (2003) and Cochrane (2001) provide comprehensive discussions.

<sup>2</sup>Fundamental contributions are Abel’s *external habit* as in Abel (1990), Constantinides (1990), Detemple and Zapatero (1991), Campbell and Cochrane (1999) and Mezly, Santos and Veronesi (2004), *inter alia*, Epstein and Zin’s *recursive utility* as in Epstein and Zin (1989, 1991), Bansal and Yaron (2004), and Hansen, Heaton, and Li (2008), among others, and *loss aversion* as in Barberis, Huang and Santos (2001). For an interesting discussion of their unified interpretation as preference theories defined over consumption and a suitable reference factor, as well as for further extensions in an expected utility framework, we refer the interested reader to the work of Garcia, Renault, and Semenov (2004).

Lower frequency components of consumption growth with various degrees of persistence may, however, be important drivers of risk premia. A new methodology for signal extraction, along with the application of suitable pricing techniques, allows us to identify these priced components.

We make three contributions. First, using a data generating process which explicitly separates portfolio returns and consumption growth into uncorrelated components operating at different frequencies, we illustrate the importance for asset pricing of a business-cycle consumption component with periodicity between 2 and 8 years (the usual length of the business cycle according to Burns and Mitchell, 1946; see, also, the more recent, implicit taxonomy in Comin and Gertler, 2006). More explicitly, in the size and book-to-market space represented by the traditional Fama-French (FF) portfolios, we show that a *single-factor* model in which the factor is *business-cycle consumption* (growth) - a component representing consumption fluctuations between 2 and 8 years - yields satisfactory pricing errors and a coefficient of determination around 40%. This result is derived after introducing a novel notion of component-wise beta. We provide evidence for the pricing ability of the beta associated with *business-cycle consumption*.

Second, after recognizing that *short-term* returns are priced by a *long-run* consumption component, we show that reliance on low-frequency consumption features has implications for the asset pricing literature relying on aggregation, be it the aggregation of returns, as in a specification for long-run returns (e.g., Daniel and Marshall, 1997), the aggregation of the factor(s) (e.g., Parker and Julliard, 2005), or both (Bandi et al., 2011). We formalize the mapping between our approach to pricing and aggregation. In doing so, we provide a model-based justification for existing pricing results in the literature.

Third, we emphasize that our proposed component model for returns and consumption growth has *cross-sectional* as well as *time-series* implications. We use the latter to discuss an array of time-series metrics which can be viewed as complementary to the standard cross-sectional metrics. The use of criteria relying on time-series properties of returns and consumption growth offers additional, often-overlooked in the asset pricing literature, dimensions along which pricing models should be evaluated and according to which *business-cycle consumption* appears to fare satisfactorily.

We expand on these three contributions in this Introduction before turning to a formal treatment. Our analysis begins with a model for consumption growth  $g_{t+1} = \log \frac{c_{t+1}}{c_t}$  which expresses it as  $g_t = \sum_{j=1}^J g_t^{(j)} + \pi$ , where the  $g_t^{(j)}$ s are mean-zero frequency-specific consumption details - each detail being uniquely identified by its level of persistence - and  $\pi$  is a mean term. Each

component is associated with fluctuations between  $2^{j-1}$  and  $2^j$  quarters. Importantly, the shocks determining individual components are, in general, *not* aggregates of high-frequency shocks. They are, instead, frequency-specific as well as time-specific. This modeling device represents a departure from classical time-series specifications (see Bandi, Perron, Tamoni and Tebaldi, 2013, BPTT henceforth), one which represents, in our context, the idea that different components of the consumption process may be the result of uncorrelated (across layers) random shocks with different sizes and different half-lives. The separation into  $J+1$  details, with  $J > 1$ , gives us more granularity in the analysis of fluctuations with different cycles that is the case with traditional (2-component) filters of the Beveridge-Nelson type (see Beveridge and Nelson, 1981). This granularity is crucial to evaluate the differential impact of various consumption details on risk premia. Write the generic asset  $i$ 's excess return as  $R_{t,t+1}^{ei} = \sum_{j=1}^J R_{t,t+1}^{ei(j)} + \eta$ , where the symbols have the same interpretation as for  $g_t$ . We show that the size and book-to-market portfolios are suitably priced by  $\text{Cov} \left[ g_{t+1}^{(4)} + g_{t+1}^{(5)}, R_{t,t+1}^{ei(4)} + R_{t,t+1}^{ei(5)} \right]$  (or the corresponding beta) where  $g_{t+1}^{(4)} + g_{t+1}^{(5)}$  is *business-cycle consumption*, the above-mentioned sub-component of the consumption process representing (business-cycle) fluctuations with periodicity between 2 and 8 years.

As stressed, the proposed framework has implications for the asset pricing literature relying on aggregation. An interesting question to ask is whether there are subsets of frequencies over which pricing models, like the traditional CCAPM, fare satisfactorily. This question has been addressed in the time domain (e.g., Bandi et al., 2011) and, as is natural, in the frequency domain (e.g., Berkowitz, 2001, Cogley, 2001, and Yu, 2012). A consistent conclusion of this line of work is that the fit of the model improves as the horizon increases, thereby providing some support for the implication that asset pricing puzzles are largely short-term phenomena having to do with frictions. The focus of the above literature is on *long-run* returns. A related, but different, question is: at which frequency do the shocks that matter to price *short-term* returns of the kind routinely used in asset pricing tests operate? This issue has also been studied in the time domain (e.g., Bansal and Yaron, 2004, Bansal, Dittmar, and Kiku, 2009, and Hansen, Heaton and Li, 2008) as well as in the frequency domain (Dew-Becker and Giglio, 2013). We operate in a time-frequency domain and provide an explicit consumption/return data generating process which expresses all processes as sums of uncorrelated components (and, therefore, shocks) with periodicity of different length. Our decomposition, and the resulting decomposition of the overall covariance between consumption growth and asset returns into sub-covariances (one for each frequency), is explicit about the role of consumption shocks operating

at different scales for the pricing of *short-term* returns. Importantly, since aggregation reveals low-frequency relations (and consumption betas defined on low-frequency components are shown in the paper to capture cross-sectional risk premia), the proposed framework also translates into a price formation mechanism yielding effective cross-sectional pricing for *low-frequency* returns. In this sense, our scale-based decompositions nicely tie low-frequency consumption dynamics to both the pricing of long-run returns (as in, e.g., Daniel and Marshall, 1997) and that of short-run returns (as in, e.g., Bansal and Yaron, 2004).

Ortu, Tamoni, and Tebaldi (2013) also decompose consumption growth into components with heterogeneous levels of persistence. They relate selected consumption components to observable economic proxies and discuss the implications of persistence heterogeneity for the *market risk premium*. The present paper introduces a notion of beta written as a suitable aggregate of component-specific betas and studies the impact of consumption risk defined on details of consumption growth for asset pricing. We discuss the implications for the asset pricing literature relying on various forms of aggregation (e.g., Daniel and Marshall, 1997, and Parker and Julliard, 2005) of a novel data generating process in which cross-sectional variation in risk compensations is determined by scale-specific consumption risk. As a result of this analysis, we show formally that Daniel and Marshall's 2-year factor is readily justified by our *business-cycle consumption* factor.

Koijen, Lustig, and Van Nieuwerburgh (2012) link the excess returns on high minus low book-to-market portfolios to business-cycle risk. To identify the relation between returns and macroeconomic activity, Koijen et al. (2012) adopt an event-time approach which exploits the role of the Cochrane and Piazzesi (2005) factor as a mediating variable. Similarly to Koijen et al. (2012), we emphasize the role played by business-cycle dynamics in cross-sectional pricing. Differently from Koijen et al. (2012), our results stem from a novel notion of component-wise consumption beta which re-affirms the role of (low frequency components of) consumption as a key state variable. Both Koijen et al. (2012) and this paper point to the need for new approaches (such as, investigating windows around low value return events or, in our case, consumption betas over alternative scales and/or aggregation horizons) intended to make the relation between asset prices and macroeconomic growth more transparent.

We find that accounting for persistence heterogeneity in consumption is important for interpreting differences in risk compensation across assets. We also show that it is key for capturing the joint dynamics of consumption growth and returns across different time horizons. To this extent, in

its third contribution, the paper uses a variety of metrics intended to evaluate the pricing ability of a model in which a business-cycle component of the consumption process is the main determinants of the cross-sectional dispersion in risk premia.

First, the model generates positive consumption growth autocorrelations up three lags (quarters) and largely insignificant autocorrelations thereafter. This finding is consistent with data.

Second, following Parker and Julliard (2005) who define risk in terms of covariances with respect to *ultimate consumption* ( $\text{Cov} [g_{t,t+h+1}, R_{t,t+1}^{ei}]$  with  $h$  large), we show that the proposed pricing model closely reproduces their documented hump-shape pattern of  $R^2$ s with a peak corresponding to a time period between 2.5 and 3 years. Averaging, as in the definition of Parker and Julliard's *ultimate consumption*, reveals persistent components by eliminating short-term fluctuations (BPTT, 2013, for a formal treatment). Thus, there is an important conceptual link between *ultimate consumption* and a data generating process, like the one we propose, in which persistent components of the consumption process with business-cycle fluctuations drive risk premia.

Third, we look at predictability of consumption growth as in Piazzesi (2001). In this context, we report the (average, across stocks) covariance between future consumption growth and current excess returns ( $\text{Cov} [g_{t+h,t+h+1}, R_{t,t+1}^{ei}]$ ) to show that, barring short-term seasonal patterns, the component model yields the consumption predictability found in the data.

Finally, as suggested by Cochrane and Hansen (1992), we examine the equity premium at long horizons, namely the (average, across stocks) covariance between long-run consumption growth and long-run excess returns divided by the horizon ( $\frac{1}{h}\text{Cov} [g_{t,t+h}, R_{t,t+h}^{ei}]$ ). In the data, this standardized covariance is typically found to be hump-shaped: it increases up to about 2 years before decreasing monotonically. Our component model for consumption reproduces this pattern satisfactorily.

We view our approach as contributing to a growing literature that seeks to improve the empirical performance of the consumption-based asset pricing model by redefining the relevant measure of consumption. In particular, several recent studies propose alternative notions of consumption to address measurement issues for the purpose of asset pricing. For example, Jagannathan and Wang (2007) show that the CCAPM performs better in annual data when consumption growth is measured over the fourth quarter. Aït-Sahalia et al. (2004) find that luxury goods have greater pricing ability than aggregate consumption. Savov (2011) proxies for consumption using data on municipal solid waste and finds that garbage growth is priced in the cross-section of US and international portfolio returns. Da and Yun (2010) and Chen and Lu (2012) report similar results when using

electricity consumption and carbon dioxide emissions to proxy for consumption. Exploiting the information in micro-level household data, Malloy et al. (2009) use stockholder consumption to price the size and value portfolios. Yogo (2006) studies the role of durable consumption in cross-sectional pricing. Qiao (2013) and Kroencke (2013) use notions of “filtered consumption” from macroeconomic variables and “unfiltered consumption”, respectively, to address the shortcomings of traditional NIPA consumption while supporting the need for a cleaner consumption measure for asset valuation.

Rather than proposing an alternative measure of *aggregate* consumption, this paper relies on the standard “non-durable plus services” consumption series. Starting off with the premise that signal (priced consumption risk) and noise may be frequency-specific, we extract the priced components of the consumption process and identify the frequency over which they operate. While it is natural for us to begin with the most traditional notion of consumption, we emphasize that our suggested frequency/time-based approach, and data generating process, may be applied to any other consumption measure. To this extent, we conclude our discussion by examining alternative consumption series. Importantly, the methods could also be applied to any other factor, other than consumption. For any assumed factor, the proposed approach appears to be ideally suited to shed light on the relative importance of dynamics at different frequencies. In this sense, the paper’s contribution has a general methodological content, one to which we now turn.

## 2 A linear scale-based stochastic discount factor

The most classical utility-based asset pricing formula states that

$$E_t[R_{t,t+1}^i m_{t+1}] = 1, \tag{1}$$

where  $m_{t+1}$  is a stochastic discount factor and  $R_{t,t+1}^i$  is a generic asset  $i$ ’s uncertain return. One implication of Eq. (1) is that, after taking dividends into account, absent risk-neutrality, prices are martingales only once the objective probabilities are suitably modified by the change of measure  $m_{t+1}$ .

In terms of risk compensations in *excess* of the risk-free asset  $R^f$ , one could write

$$E_t[(R_{t,t+1}^i - R^f)m_{t+1}] = E_t[R_{t,t+1}^{ei} m_{t+1}] = 0, \tag{2}$$

or, equivalently, after integrating time  $t$  information out,

$$\mathbb{E}[R_{t,t+1}^{ei} m_{t+1}] = 0,$$

which immediately leads to the unconditional beta-representation

$$\mathbb{E}[R_{t,t+1}^{ei}] = -\lambda \frac{\text{Cov}[m_{t+1}, R_{t,t+1}^{ei}]}{\text{Var}[m_{t+1}]} = -\lambda \beta_i,$$

where  $\lambda = \frac{\text{Var}[m_{t+1}]}{\mathbb{E}[m_{t+1}]}$ .

Is aggregate consumption growth too coarse a measure to deliver meaningful quantities of risk and, consequently, to “imply” meaningful expected returns? We address this issue by working with scale-time decompositions for both consumption growth and the excess returns on test assets and by assuming that  $m_{t+1}$  is driven by consumption details. Write

$$g_{t+1} = \sum_{j=1}^6 g_{t+1}^{(j)} + \pi \tag{3}$$

and

$$R_{t,t+1}^{ei} = \sum_{j=1}^6 R_{t,t+1}^{ei(j)} + \eta. \tag{4}$$

Using Eq. (3), define

$$m_{t+1} = a - \sum_{j=1}^6 b_j g_{t+1}^{(j)}. \tag{5}$$

The details  $\{g_{t+1}^{(j)}, R_{t,t+1}^{ei(j)}\}$  may be thought of as uncorrelated, linear autoregressive processes with a scale-specific autoregressive parameter  $\rho_j$  and scale-specific shocks defined on the dilated time  $t - 2^j$  of each individual scale. Because  $j$  is measured in terms of quarters, each detail is associated with periodic fluctuations between  $2^{j-1}$  and  $2^j$  quarters.

Since the data spans a time frame of about 50 years, our chosen lowest frequency component ( $J = 6$ , in the notation used above) strikes a compromise between identifiability (higher frequency details are easier to identify) and richness of the model (the larger the number of details, the richer the decomposition). Such a component captures fluctuations between 8 years and 16 years. We will show that a large percentage of the pricing ability of consumption growth is associated with



scales lower than  $J = 6$  and corresponding with business-cycle periodicities of 2 to 8 years. Through (business cycle-like) fluctuations in the consumption components, we will generate *directly* (business cycle-like) fluctuations in the stochastic discount factor, a feature which was discussed by Parker and Julliard (2005) as being empirically warranted. We will also provide an alternative - but natural, in our view - channel through which hard-to-detect persistent components in the consumption process affect asset prices, the role of persistence in consumption for asset pricing being highlighted in influential, recent work (e.g., Alvarez and Jermann, 2005, Bansal and Yaron, 2004, and Hansen, Heaton, and Li, 2008).

The decompositions in Eqs. (3) and (4), along with autoregressive dynamics for the details, translate into aggregated processes  $\{g_{t+1}, R_{t,t+1}^{ei}\}$  for which a *generalized* Wold representation holds, one in which the time series are linear combinations of shocks that are both time- and frequency-specific. This representation (heavily used in the simulations in Section 5) captures the idea that economic time series may be suitably interpreted as the result of a cascade of shocks occurring at different times and different frequencies. The representation separates us from traditional approaches in time series. It also separates us from existing methods in the multi-resolution literature. As for the later case, we are not *simply* expressing a conventional time series in terms of its details. If we were to do so, the low-frequency shocks would solely be suitable aggregates of high-frequency shocks, thereby leading to a conventional Wold representation. We are modelling data generating processes for which the resulting time series are the sum of details affected by scale-specific shocks (see BPTT, 2013, for details).

Denote now by  $\{g_{k2^j}^{(j)}, R_{k2^j}^{ei(j)}\}$  the details sampled every  $2^j$  times. Ortu et al. (2013) and BPTT (2014) refer to these sub-series of the original details defined on chronological time as *decimated* series. For each scale, the decimated series are designed to capture all *essential* information about dynamics at the corresponding frequency (Appendix A provides a discussion). Returning to Eq. (5), the uncorrelatedness of the mean-zero details suggest that one should be able to write

$$\beta_i = \sum_{j=1}^6 w^{(j)} \beta_i^{(j)}, \quad (6)$$

where  $\beta_i^{(j)} = \text{Cov}[g_{k2^j}^{(j)}, R_{k2^j}^{ei(j)}] / \text{Var}[g_{k2^j}^{(j)}]$  and  $w^{(j)} = -b_j \frac{\text{Var}[g_{k2^j}^{(j)}]}{\text{Var}[m_i]}$ . In other words, we should be able to express  $\beta_i$  as a weighted average of scale-specific consumption betas  $\beta_i^{(j)}$  with weights

$w^{(j)}$  given by the relative contribution of individual consumption details to the overall variance of the stochastic discount factor. Said differently, there should *not* be any allowance for cross-scale covariance terms.

The following proposition shows formally that our assumed filtering method for the components relying on Haar transforms is effective in yielding exactly this result *in-sample*.<sup>3</sup> We refer to Whitcher et al. (2000) for an analogous (auto-)covariance decomposition in the presence of overlapping observations.

**Proposition (Disaggregating the beta into component-wise betas.)** *Should a Haar filter be applied to  $\{g_{t+1}, R_{t,t+1}^{ei}\}$ , the resulting estimated beta would conform with Eq. (6) exactly.*

**Proof.** See Appendix.

Importantly, the Proposition leads to a (theoretical and empirical) beta formulation conveniently expressed in terms of covariances between *contemporaneous* details of the consumption growth and return processes, namely

$$\mathbb{E}[R_{t,t+1}^{ei}] = \sum_{j=1}^6 \lambda_j \beta_i^{(j)}, \quad (7)$$

where  $\lambda_j = R^f b_j \text{Var} [g_{k2^j}^{(j)}]$ . Eq. (7) can be viewed as a linear factor model in which the factors are the consumption details. This model will be evaluated in the next Section.

## 2.1 Interpreting the CCAPM as a restriction

Under a constant relative risk-aversion utility function  $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$  and a simple approximation,<sup>4</sup> Eq. (1) yields the following expected return-beta representation

$$\mathbb{E}_t[R_{t,t+1}^{ei}] \underset{\text{small } \Delta c_{t+1}}{\approx} \gamma \text{Cov}_t \left[ \log \frac{c_{t+1}}{c_t}, R_{t,t+1}^{ei} \right] = \gamma \text{Cov}_t [g_{t+1}, R_{t,t+1}^{ei}] = \lambda_t \beta_{i,t}, \quad (8)$$

<sup>3</sup>In this section, we focus on economic logic and, therefore, do not discuss identification by virtue of Haar filters formally. We refer the reader to Appendix A for additional information.

We, however, point out right from the start that, while alternative filters could have been employed without affecting the empirical results, using Haar filters is convenient. In particular, they allow us to be concise - as well as formal - about the relation between scales and different levels of aggregation (discussed in Section 5), thereby allowing us to focus on economic logic without unnecessary technical complications. Because the evaluation of the implications of our proposed approach for the cross-sectional asset pricing literature relying on aggregation is a core subject of our analysis (and Section 5 is devoted to it), the structure of Haar filters appears particularly convenient for our purposes.

<sup>4</sup>Ignoring the discount factor, write  $m_{t+1} = \frac{u'(c_{t+1})}{u'(c_t)} = \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} = \left(\frac{\Delta c_{t+1}}{c_t} + 1\right)^{-\gamma} \approx (\Delta \log c_{t+1} + 1)^{-\gamma} \approx 1 - \gamma \Delta \log c_{t+1}$  when  $\Delta c_{t+1}$  is small.

where  $\lambda_t = \gamma \text{Var}_t(g_t)$  and  $\beta_{i,t} = \text{Cov}_t [g_{t+1}, R_{t,t+1}^{ei}] / \text{Var}_t [g_{t+1}]$ . Similarly, in unconditional terms, we have

$$\mathbb{E}[R_{t,t+1}^{ei}] \underset{\text{small } \Delta c_{t+1}}{\approx} \lambda \beta_i \quad (9)$$

where  $\lambda = \gamma \text{Var}(g_t)$  and  $\beta_i = \text{Cov} [g_{t+1}, R_{t,t+1}^{ei}] / \text{Var} [g_{t+1}]$ . The interpretation of Eqs. (8) and (9) is standard and leads to the classical CCAPM: the risk of any asset should be measured by the covariance of the asset's return with respect to consumption growth. Assets whose returns are relatively lower in states of nature in which consumption growth is lower are perceived as riskier and should, therefore, require a higher risk compensation, leading to higher expected returns in excess of the risk-free rate.

Hence, the CCAPM can be viewed as a restriction on the model in Eq. (7), one in which  $b_j = \gamma$  for all  $j$ : the pricing impact of different consumption details is assumed to be the same across scales.

The restriction, i.e., the traditional CCAPM, is not validated by data. Regardless of the sampling period, a broad literature has documented that differences in average excess returns across US assets do not appear to be justifiable based on the corresponding differences in the covariance of the assets' returns with respect to *aggregate* consumption growth.

Using data on the 25 FF portfolios sorted on size and book-to-market between 1963:Q1 and 2013:Q4, as well as data on consumption of non-durables and services, seasonally-adjusted in 1996 chain-weighted dollars,<sup>5</sup> we confirm this standard result.

Figure 1 plots the average returns on the portfolios. It shows the typical decreasing pattern across size (from small to large) with the sole (also typical) exception of the first value quintile for which the trend is largely reversed. We will return to the first value quintile, and the first size/value portfolio in particular, in what follows. Similarly, the figure displays an increasing pattern of average portfolio returns along the book-to-market dimension, i.e., the well-known value premium.

[Insert Figure 1 about here.]

Figure 2–Panel A plots average excess portfolio returns (on the vertical axis) against expected excess returns implied by the unconditional specification in Eq. (9). Predicted returns do not align with historical returns, the largest deviations corresponding with portfolios in the first and in the last value quintile.

---

<sup>5</sup>For complete information on the data, we refer the interested reader to Appendix C.

[Insert Figure 2 about here.]

We conclude this section by pointing out that Gençay et al. (2003, 2005) use scale-specific market return betas,  $\beta^{i(j)}$ , as regressors in cross-sectional regressions with return details,  $R_t^{ei(j)}$ , in order to evaluate *individual* (i.e., one for each scale) versions of the CAPM. We are, instead, interested in the pricing ability of alternative consumption components for *aggregate* returns (rather than for *individual* layers of the return process) as needed in asset pricing applications. We are also interested in the mapping - for pricing purposes - between frequency and *aggregation*, something which we formalize below. Our analysis leads to the explicit provision of a data generating process for both consumption and returns capable of reconciling both cross-sectional and time-series dynamics for purposes of asset valuation.

We now turn to the specification in Eq. (7).

### 3 Scale-specific betas and lambdas

In this section, we first quantify the exposures of the returns on the 25 FF portfolios to consumption risk over alternative scales. We then characterize the frequency-wise risk compensations.

As is customary in the literature, to evaluate the relative price of risk ( $\lambda_j$ ) associated with different scale-specific betas ( $\beta^{i(j)}$ ), we run time-series regressions of the type

$$R_{k2^j, k2^j+2^j}^{ei(j)} = \beta_0 + \beta_i^{(j)} g_{k2^j, k2^j+2^j}^{(j)} + \epsilon_{k2^j+2^j}, \quad (10)$$

with  $k \in \mathbb{Z}$ . The difference between the standard time-series approach in asset pricing and our (scale-wise) approach is that the regressions are on details sampled every  $2^j$  times.

Using the first-pass estimates  $\beta_i^{(j)}$ , we then estimate the specification

$$E[R_{t,t+1}^{e,i}] = \lambda_0 + \sum_{j=1}^6 \lambda_j \beta_i^{(j)} + \alpha_i \quad (11)$$

with an intercept and, when appropriate, with restrictions on the  $\lambda_j$ s.

We begin by analyzing the behavior of  $\beta_i^{(j)}$  and  $\lambda_j$  across alternative scales and portfolios. Figure 3 plots the betas of the 25 FF portfolios associated with different scales. Going from the highest frequency scale to our chosen lowest frequency scale ( $j = 6$ ), we witness a rotation from size to

value. In particular, the consumption detail corresponding to scale  $j = 2$  (i.e., 6 months to 1 year) translates into betas which align very effectively with average returns (as reported in Figure 1) in the size dimension (higher betas for small firm portfolios, lower betas for large firm portfolios) while failing to capture the second dimension, i.e., value. The betas corresponding to lower frequency details of the consumption process ( $j = 3, 4,$  and  $5$ ) display improved alignment in the value space without losing their ability to capture cross-sectional variability in risk premia across size. The scale  $j = 5$ , among them, is the one for which the visual alignment of the betas across both dimensions appears to be the best. The value premium, in particular, is captured rather effectively at this low frequency (4 to 8 years). An important size tilt is, however, also present.

[Insert Figure 3 about here.]

While it is not hard to conjecture that the betas corresponding to scale  $j = 5$  may be the most effective in explaining cross-sectional variability in average returns, the preceding scale  $j = 4$  may represent an improvement in the size space. Interestingly, such an improvement is expected to be brought about even by portfolios in the first value quintile. As documented in Figure 1, these are portfolios for which the typical decreasing trend with size increases does not hold. Consistently, at scale  $j = 4$ , the betas associated with the first value quintile have the same hump-shaped pattern which characterizes average historical returns.

Table 1 reports the numerical values of the betas for all portfolios, along with their statistical significance. Importantly, the largest betas correspond to the scales that, visually, seemed to matter the most economically, due to their alignment along the size/value dimension:  $j = 2, j = 4,$  and  $j = 5$ . Also, these betas are generally significantly differently from zero.

[Insert Table 1 about here.]

Table 2 contains multivariate estimates of the model in Eq. (11) with all lambdas set equal to zero with the exception of the lambdas associated with  $j = 2, j = 4,$  and  $j = 5$ . The lambda estimates corresponding with the fourth and the fifth scale are very statistically significant, whereas the lambda estimate corresponding to the second scale appears to be insignificant (and negative, in terms of its point estimate). The intercept is also statistically insignificant. The pricing errors are small (whether they are measured by the root mean squared alpha or by the mean absolute alpha) and the coefficient of determination is a rather high value of 46%. The numerical values

of the prices of risk associated with scale four and scale five are rather similar, 0.997 and 0.982 respectively. This result is interesting. While  $m_{t+1}$  is in reduced-form, one can conjecture that the risk price  $\lambda_j$  should depend on both the relative variance of the corresponding detail  $\text{Var} \left[ g_{k2^j}^{(j)} \right]$  and a notion of *scale-specific risk aversion*, through the reduced-form parameter  $b_j$ . If this were the case, since  $\lambda_4 \approx \lambda_5$  and  $\text{Var} \left[ g_{k2^j}^{(4)} \right] \approx \text{Var} \left[ g_{k2^j}^{(5)} \right]$  in our data, such a notion of risk-aversion would be stable across business-cycle scales.

[Insert Table 2 about here.]

Dropping the (previously insignificant) second component does not modify these findings in any relevant way. The pricing errors increase, as expected, but only mildly. The coefficient of determination decreases (to 39%) but not by much. This is a two-factor specification in which the factors are, again, the fourth and the fifth component of the consumption growth process.

Going from a *two-factor* specification to a *one-factor* model in which the factor is a business-cycle component (dubbed, earlier, *business-cycle consumption*) given by the sum of the fourth and the fifth component is now natural. It is natural since the corresponding beta would be defined as a linear combination - with weights depending on the relative contribution to total variance of the corresponding consumption component - of the betas associated with the two consumption components, i.e.,

$$\begin{aligned}
\beta_{bcc} &= \frac{\text{Cov} \left[ g_t^{(4)} + g_t^{(5)}, R_t^{ei(4)} + R_t^{ei(5)} \right]}{\text{Var} \left( g_t^{(4)} + g_t^{(5)} \right)} \\
&= \frac{\text{Cov} \left[ g_{k2^j}^{(4)}, R_{k2^j}^{ei(4)} \right]}{\text{Var} \left( g_{k2^j}^{(4)} \right) + \text{Var} \left( g_{k2^j}^{(5)} \right)} + \frac{\text{Cov} \left[ g_{k2^j}^{(5)}, R_{k2^j}^{ei(5)} \right]}{\text{Var} \left( g_{k2^j}^{(4)} \right) + \text{Var} \left( g_{k2^j}^{(5)} \right)} \\
&= \frac{\text{Var} \left( g_{k2^j}^{(4)} \right)}{\text{Var} \left( g_{k2^j}^{(4)} \right) + \text{Var} \left( g_{k2^j}^{(5)} \right)} \beta_i^{(4)} + \frac{\text{Var} \left( g_{k2^j}^{(5)} \right)}{\text{Var} \left( g_{k2^j}^{(4)} \right) + \text{Var} \left( g_{k2^j}^{(5)} \right)} \beta_i^{(5)} \\
&= \varpi^{(4)} \beta_i^{(4)} + \varpi^{(5)} \beta_i^{(5)},
\end{aligned}$$

where the second equality derives from the same argument leading to the Proposition in the previous section. The pricing model would then be

$$\text{E} \left[ R_{t,t+1}^{ei} \right] = \lambda_0 + \lambda_{bcc} \beta_{bcc} + \alpha_i. \tag{12}$$

Table 3 reports the results. The pricing errors are only 8 basis point, per year, higher than in the unrestricted case, with a coefficient of determination of 36%, a marginal reduction over the earlier specification which explicitly separated the two components. The constant remains insignificant. The estimated price of risk  $\widehat{\lambda}_{bcc}$  is significantly estimated with a  $t$ -statistic of about 3.5. We note that, since in our data  $\varpi^{(4)} \approx \varpi^{(5)} \approx \frac{1}{2}$ , the one-factor model can be viewed as a restriction on the two-factor model. The restriction resides in  $b_4 = b_5$ . Should the difference in the reduced-form  $b_j$ s be driven structurally by different levels of risk aversion across horizons, then the restriction would imply similar aversion to risk over the short end and the long end of the business cycle. Regardless of structural interpretations, this empirical evidence lends economic and statistical support to a parsimonious one-factor specification.

[Insert Table 3 about here.]

In essence, there are sound economic reasons to place emphasis on both the fourth and the fifth scale with the sum of the two (*business-cycle consumption*) performing virtually as well as a less restricted two-factor specification. These two scales capture size and value effects in different, but complementary, ways. Reflecting the rotation of the scale-specific betas from size to value as we transition from higher frequency details of the consumption process to lower frequency details, the fourth scale is quite effective along the size dimension. While offering an appealing size tilt, the best-performing fifth scale is more effective along the value dimension. As stressed earlier, these two contiguous scales *jointly* span the accepted length of the business cycle capturing fluctuations in consumption growth between 2 and 8 years.

## 4 Temporal aggregation and asset prices

In this section, we show the sense in which our proposed methodology for risk-detection and pricing is linked to temporal aggregation and, as such, has implications for the long-run asset pricing literature.

In order to do so, we use the structure of the Haar filters (used for the identification of the time-series layers) to derive the expressions

$$\frac{1}{2^s} \sum_{i=1}^{2^s} g_{t+i} := g_{t,t+2^s} = \sum_{j=s+1}^6 g_{t+2^s}^{(j)} + \pi_{t+2^s}^{(6)} \quad (13)$$

and

$$\frac{1}{2^s} \sum_{i=1}^{2^s} R_{t+i}^{ei} := R_{t,t+2^s}^{ei} = \sum_{j=s+1}^6 R_{t+2^s}^{ei(j)} + \eta_{t+2^s}^{(6)}, \quad (14)$$

where  $s = 0, 1, \dots, 5$  denotes the aggregation level and  $\{\pi_{t+2^s}^{(6)}, \eta_{t+2^s}^{(6)}\}$  are long-run averages (see Appendix A).<sup>6</sup> Both equations imply that aggregation of the time series of interest over suitable horizons uncovers information at different scales or, more precisely, for scales that are higher than the one corresponding to the aggregation level (BPTT, 2013, for a thorough treatment).

The expressions also make apparent the link between the scale-wise time-series regression in Eq. (10) (reported here, for convenience, again)

$$R_{t+2^s}^{ei(s)} = \beta_0 + \beta_i^{(s)} g_{t+2^s}^{(s)} + \epsilon_{t+2^s},$$

and the long-horizon regressions

$$R_{t,t+h}^{ei} = \beta_{0,h} + \beta_{i,h} g_{t,t+h} + u_{t+h}. \quad (15)$$

Letting  $h = 2^{s-1}$ , and using Eqs. (13) and (14), the long-horizon regressions can be viewed as filtering out noisy high-frequency components at scales  $j < s$ , thereby capturing co-movements between the components  $g_t^{(s)}$  and  $R_t^{ei(s)}$ . In this sense, we expect the long-horizon betas  $\beta_{i,h}$  to behave similarly to the  $\beta_i^{(s)}$  at scale  $s$ . The sole difference between the two is that  $\beta_{i,h}$  should also be influenced by the co-variation between components  $g_t^{(j)}$  and  $R_t^{ei(j)}$  at higher scales  $j > s$ . Hence, it should also reflect lower frequency fluctuations.

In sum, our proposed data generating process and identification framework justify an alternative approach to the evaluation of pricing models: in order to measure risk exposure, one may run long-horizon regressions to filter out the noisy components in the factor(s) (consumption, in our case) and returns. This alternative route, however, requires the choice of an aggregation horizon. Our previous findings document that business-cycle frequencies between 2 and 8 years, i.e., the frequencies captured by the components  $j = 4$  and  $j = 5$  (jointly defining *business-cycle consumption*), are key to explaining cross-sectional dispersion in average returns on portfolios sorted based on size

---

<sup>6</sup>Eqs. (3) and (4) define our assumed data generating processes. Eqs. (13) and (14) are true, by construction, given our employed filter. For  $J = 6$ , the long-run averages  $\pi_{t+2^s}^{(6)}$  and  $\eta_{t+2^s}^{(6)}$  are close to constant values. Hence, when  $s = 0$ , Eqs. (13) and (14) approximately yield Eqs. (3) and (4), as expected of a valid filter.



and value. To this extent, in agreement with Eq. (15), we choose  $s = 4$ . As a consequence, the long-horizon regression in Eq. (15) relies on returns and consumption growth aggregated over two years. In fact,  $h = 2^{s-1} = 8$  quarters.

We argued that, at this horizon, temporal aggregation eliminates higher-frequency components at scale  $j = 1, 2$  and  $3$ . One may now wonder how close is 2-year consumption growth to the sum of the components at scale  $j = 4$  and  $j = 5$ ? Figure 4 displays the aggregated (over 2 years) consumption growth series along with *business-cycle consumption*. We observe that the two series strongly co-move, a fact also confirmed by a correlation of 0.86. More formally, setting  $s = 3$  in Eq.(13), one obtains

$$g_{t,t+2^3} - \left( g_{t+2^3}^{(4)} + g_{t+2^3}^{(5)} \right) = g_{t+2^3}^{(6)} + \pi_{t+2^s}^{(6)},$$

i.e., the difference between consumption growth aggregated over 2 years and *business-cycle consumption* is driven by a detail,  $g_{t+2^3}^{(6)}$ , operating at lower than business-cycle frequencies (scale  $j = 6$ ) and a long-run average,  $\pi_{t+2^s}^{(6)}$ . When  $g_{t+2^3}^{(6)}$  is relatively small in terms of its volatility, *then* we expect aggregation to perform very satisfactorily as a risk-extraction mechanism. This is the case in our data, as we discuss further below.

Both  $g_{t,t+2^3}$  and  $g_{t+2^3}^{(4)} + g_{t+2^3}^{(5)}$  decline around NBER recessions. These dynamics are analogous to those of variables known to be fluctuating with the business-cycle, like the term-spread and credit spreads (see Ortú et al., 2013, for evidence). To illustrate this point, in Figure 5 we report the logarithm of Campbell and Cochrane’s local coefficient of risk aversion, as well as  $g_{t,t+2^3}$ , over two different horizons. The correlation between the two series over the longer horizon 1995:Q4-2013:Q4 is about  $-67\%$ . This outcome is only mildly due to extreme dynamics between 2006:Q4 and 2013:Q4. Excluding this last period only lowers the (absolute value of the) correlation to  $-53\%$ .

[Insert Figures 4 and 5 about here.]

#### 4.1 The *short-term* compensation of *long-run* betas

Motivated by this analysis, one would expect a CCAPM model based on *long-horizon* betas to explain *short-term* returns. This is, as demonstrated above, an implication of our proposed component-based data generating process. To this extent, we test the specification

$$E[R_{t,t+1}^{ei}] = \lambda_0 + \lambda_h \beta_{i,h} + \alpha_i \tag{16}$$

and compare it to Eq. (12).

Figure 6, Panel A, provides a graphical representation. Average returns are rather well explained by the  $\beta_{i,h,s}$ , the first size and value portfolio being a typical exception. The model achieves an  $R^2$  of 26%, with a mean absolute pricing error of 1.75 percent per year (c.f. Table 4-Panel A). The estimated constant  $\lambda_0$  is, as earlier, not significant. These findings are comparable to (but less strong than) the ones in Table 3-Panel B, where the consumption betas were defined on the sum of two business-cycle components only.

[Insert Table 4 about here.]

[Insert Figure 6 about here.]

Inferior performance as compared to a pure component model is not surprising, in light of the discussion in the previous subsection. Aggregation to 2 years preserves the priced components (the fourth and the fifth) as well as the sixth. We find that 6% of the overall consumption variance is explained by the sixth component, a considerably lower number than the 30% figure corresponding to *business-cycle consumption*. Since the contribution of the sixth component to the overall variance of the process is limited, it is expected that this component would affect estimated covariances and pricing somewhat, but only marginally.

The interaction between the sixth component and the small growth portfolio also has an impact. Should we exclude this portfolio, the model with betas on aggregates and the model with component-wise betas would fare similarly both in terms of  $R^2$  (increasing to about 46%) and pricing errors (reducing to about 1.54 percent per year).

It is important to notice that this analysis hinges on *simple long-run returns*. In light of the mapping between aggregation and component-extraction illustrated above, summing returns - as in the computations of simple returns - is natural. Interestingly, however, the use of *compounded long-run returns*, as generated by the re-investment of past returns as well, delivers superior findings. Table 4, Panel B, shows that, in this case, the  $R^2$  reaches a value of 49%, with a mean absolute pricing error of 1.34 percent per year. Figure 6, Panel B, offers a visual representation.

## 4.2 Revisiting (and justifying) the approach in Daniel and Marshall (1997)

We have shown how one can employ - and, importantly, justify - long-horizon betas in the study of short-run premia. We emphasized how the use of these betas can be motivated in the context of

a component model of the type we propose. Since long-run average returns have the same pattern (across test assets) as short-run average returns, the proposed model provides a formal justification for relating long-horizon returns to long-run betas as well (as in the work of Bandi et al., 2011). In this sense, our scale-based consumption decomposition nicely ties low-frequency consumption dynamics to both the pricing of long-run returns (as in, e.g., Daniel and Marshall, 1997) and to that of short-run returns (as in, e.g., Bansal and Yaron, 2004).

In a well-known contribution, Daniel and Marshall (1997) emphasize that both the equity premium and the risk-free rate puzzles may disappear, in the presence of the right preference specification, once returns and consumption growth are computed over a 2-year horizon. We too provide support for a 2-year horizon of aggregation. We justify aggregation over 2 years formally as a way to eliminate consumption cycles higher than what was shown to be the relevant, for pricing, 2 to 8 years frequency. Differently from Daniel and Marshall (1997), however, we focus on cross-sectional pricing.

In their conclusions, Daniel and Marshall (1997) emphasize that frictions may affect the link between returns and consumption at horizons shorter than 2 years, thereby leading to failures of the classical consumption model. Since they also find the validity of this model (for the equity premium and the risk-free rate) to be questionable at frequencies lower than 2 years, they wonder "what economic model would disrupt this linkage at very long horizons" and, effectively, have different implications at different horizons. By allowing for different risk quantities  $\beta^{i(j)}$  across different layers of the consumption stream and, possibly, for different prices of risk  $\gamma_j$ , we explicitly break the link between alternative horizons of aggregation and address this issue directly. In a time-series context closer to the framework in Daniel and Marshall (1997), BPTT (2013) provide further details about the implications of a component model for differential outcomes upon aggregation.

While the asset pricing literature has a tendency to focus on cross-sectional metrics (like pricing errors and coefficients of determination), more structural approaches to pricing imply temporal dynamics as well. Focusing on metrics derived from these dynamics adds much needed dimensions to the evaluation of any proposed approach. In what follows, we apply different time-series metrics to the model and show that some reported stylized facts are, in fact, hard to replicate without a component-based specification.

## 5 The time-series dynamics of returns and consumption growth: *metrics*

What are the implications of the presence of heterogeneous details in consumption growth for the joint dynamic properties of consumption and asset returns? In order to address this issue, we simulate return and consumption processes according to a component model motivated by the previous empirical analysis. We then employ the simulated process to verify the extent to which several findings regarding the joint behavior of asset returns and consumption growth over horizons of different lengths are satisfactorily replicated.

The mean-zero return details relate to the mean-zero consumption details according to the following specification:

$$R_{t,t+1}^{i(j)} = \begin{cases} \beta_i^{(j)} g_{t+1}^{(j)} & \text{for } j = 4 \text{ and } 5, \\ \epsilon_{i,t}^{(j)} & \text{otherwise} \end{cases},$$

where  $\epsilon_{i,t}^{(j)}$  is  $N(0, \sigma_i^{(j)})$  and  $\sigma_i^{(j)}$  is chosen so as to match the variance of the component  $R_{t,t+1}^{i(j)}$  at scale  $j$  for asset  $i$ . In other words, only the fourth and the fifth component of consumption and returns correlate with each other, their relation being based on the previously reported betas in Table 3. All other components are assumed to be white noise shocks with suitable variances. Importantly, all shocks are scale-specific and, contrary to classical time series modeling, they are, in general, *not* sums of high-frequency shocks.

In essence, returns are generated *solely* based on two details of the consumption process along with (noise) components at all other scales. For consistency with a pricing model which includes both the 4<sup>th</sup> and the 5<sup>th</sup> scale through a *business-cycle consumption* factor, we further impose

$$E[R_{t,t+1}^i] = \lambda_{bcc} \beta_{bcc} = \lambda_{bcc} \left( \frac{1}{2} \beta_i^{(4)} + \frac{1}{2} \beta_i^{(5)} \right), \quad (17)$$

using  $\varpi^{(4)} \approx \varpi^{(5)} \approx \frac{1}{2}$ , where the betas and the lambdas are estimated from the data. Since the consumption details are mean zero, the restriction implied by Eq. (17) is imposed by simply adding a constant term to all simulated return series.

In addition to evaluating the full model further, below we provide a rich assessment of the *relative* contribution, to asset pricing, of shorter (2 to 4 year) cycles in consumption (as represented

by  $g_{t+1}^{(4)}$ ) in addition to the 4 to 8 year cycles delivered by the main component, i.e.,  $g_{t+1}^{(5)}$ . We begin with the autocorrelation of consumption growth.

## 5.1 Autocorrelation of consumption growth

In the data, the consumption growth autocorrelation is positive and significant up to the third quarter. The values implied by the model are very plausible. Figure 7 provides the empirical autocorrelations along with 95% confidence bands and their model-implied counterparts. The first and the second quarter autocorrelations are very closely matched. The third quarter autocorrelation implied by the model is slightly lower than that in the data and outside of the corresponding confidence bands. However, it is clearly positive just like in the data. Keeping in mind that the addition of components, or details, of the consumption process may easily reconcile these small differences, a simple specification with two details (the 4<sup>th</sup> and the 5<sup>th</sup>) appears to capture critical first-order phenomena in the consumption dynamics.

[Insert Figure 7 about here.]

## 5.2 Parker and Julliard's effects

Parker and Julliard (2005) find that consumption growth measured over long horizons (dubbed *ultimate consumption*) is an effective driver of risk premia. In other words, it leads to covariances  $\text{Cov}[g_{t,t+h+1}, R_{t,t+1}^{ei}]$  which, for appropriate values of  $h$  (corresponding to about 3 years in their framework), nicely align with historical average returns in the same size/value space investigated here and, ubiquitously, in much of the literature. Parker and Julliard (2005) report a hump-shape in the coefficients of determination of their pricing model as a function of the horizon over which consumption growth is calculated. Their  $R^2$ s are monotonically increasing up to 11 quarters before decreasing steadily thereafter. For their efficient GMM estimates, the reported  $R^2$  at the peak is around 38%.

We apply the same methodology as in Parker and Julliard (2005) to our simulated data (Figure 8). When using only one dominant component (the 5<sup>th</sup>), the hump-shape is reproduced with a peak between 9 and 10 quarters and a corresponding  $R^2$  of about 23% (Figure 8, solid line). Adding only one additional component (the 4<sup>th</sup>) preserves the location of the peak and raises the  $R^2$  to about 29% (Figure 8, solid line with circles). Said differently, with two details *only*, the 4<sup>th</sup> and the 5<sup>th</sup>,

the adopted specification translates into  $R^2$  spikes at horizons around two years and a half and very close to the 11-quarter time frame emphasized by Parker and Julliard (2005).

[Insert Figure 8 about here.]

As illustrated formally by BPTT (2013), averaging is an effective mechanism to bring to light persistent details while eliminating contaminations with more frequent cyclical fluctuations. In this sense, the mapping between Parker and Julliard’s 3-year horizon and components with cycles between 2 and 8 years is not surprising. On the one hand, *ultimate* consumption can be justified, at a fundamental level, by a data generating process, like the one we propose, in which business-cycle components of the consumption process play a dominant role in the determination of risk premia. On the other hand, the averaging implicit in the definition of ultimate consumption may provide information about the frequency at which the relevant (for asset pricing) details of the consumption process operate. Their 3-year horizon is suggestive of the importance of business-cycle fluctuations in consumption. These fluctuations are captured explicitly by our reported 4<sup>th</sup> and 5<sup>th</sup> detail.

### 5.3 Predictability of consumption growth

A related object of interest, focusing on risk rather than on its pricing, is the covariance of the portfolio returns  $R_{t,t+1}^{ei}$  with respect to future consumption growth  $g_{t,t+h+1}$ , namely  $\text{Cov} [g_{t,t+h+1}, R_{t,t+1}^{ei}]$ . Figure 9 plots this quantity over time along with 95% confidence bands constructed using Newey-West standard errors (dotted lines). The contemporaneous covariance ( $h = 0$ ) is non-zero. Its value increases up to 7 quarters. Beyond that time, the numbers decrease slightly with the horizon, but the confidence bands become larger.

The figure shows that the covariance pattern in the data is well replicated by the adopted specification. In a model with only the 5<sup>th</sup> component, the implied covariances are hump-shaped, as in the data, and close to the empirical ones, but with a peak around 10 quarters (Figure 9–Panel A). Remarkably, the addition of the 4<sup>th</sup> component (in Figure 9–Panel B) leads to a close replication of the trend in the empirical covariances up to 16 quarters (including the hump around 7 quarters).

[Insert Figure 9 about here.]

As suggested by Piazzesi (2001), this long-run risk measure can be decomposed into its individual

elements, i.e.,

$$\text{Cov} [g_{t,t+h+1}, R_{t,t+1}^e] = \sum_{i=0}^h \text{Cov} [g_{t+i+1}, R_{t,t+1}^e].$$

Figure 10 plots the individual elements under the summation sign, i.e., the slopes of the cumulative covariance function in Figure 9. In the data, these slopes are positive up to horizon 8. Barring seasonal fluctuations, a model solely inclusive of the 5<sup>th</sup> detail would fare quite well in replicating this pattern of covariances (Figure 10–Panel A). It would, however, yield an excessively flat structure at high frequencies. As earlier, the addition of the 4<sup>th</sup> component is successful in providing a solution to this issue (Figure 10–Panel B). This component raises the value of the simulated covariances precisely where needed (namely, at short horizons), thereby closely replicating the convex pattern of the empirical covariances.

[Insert Figure 10 about here.]

#### 5.4 The equity premium at long horizons

Following Cochrane and Hansen (1992), we conclude this analysis by focusing on the long-run covariance between consumption growth and stock returns divided by the horizon, i.e.,

$$\frac{1}{h} \text{Cov} [g_{t,t+h}, R_{t,t+h}^e].$$

For our data, this normalized covariance is hump-shaped with a peak around 2 years (Figure 11). Using only the 5<sup>th</sup> detail gives us a hump at 3.5 years (Figure 11, Panel A). Introducing the 4<sup>th</sup> detail improves, as before, matters, especially at short horizons (Figure 11, Panel B). In particular, it relocates the hump around the correct time frame. Not only does the model capture the location of the peak, it also does not predict a somewhat counterfactual high covariance of consumption growth and stock returns at long horizons, an implication of the long-run risk model or models entailing monitoring costs and heterogeneous agents in which only a fraction of households adjusts consumption over discrete intervals. We return to this result below.

[Insert Figure 11 about here.]

This last observation leads to a broader point. The above criteria are stringent. They are stringent in the sense that influential and successful, along a variety of important dimensions,

asset pricing models may find them hard to meet. To this extent, we now turn to the analogous implications for a model with external habit (Campbell and Cochrane, 1999) and one with long-run risk (Bansal and Yaron, 2004).

## 5.5 Time series metrics and pricing models

Using the classical external habit model of Campbell and Cochrane (1999) with model parameters provided therein, Figure 12 plots the terms  $\frac{1}{h}\text{Cov}[g_{t+h+1}, R_{t,t+1}^e]$  (Panel A) and the terms  $\frac{1}{h}\text{Cov}[g_{t,t+h}, R_{t,t+h}^e]$  (Panel B) for various horizons  $h$ . The implied covariances are substantially larger than those in the data, particularly at high frequencies.

Figure 13 provides the same quantities for the long-run model of Bansal and Yaron (2004) using parameters provided in Bansal, Kiku, and Yaron (2012). The outcome is even more striking. The implied covariances are considerably larger than what is found in the data. Their pattern is also monotonically increasing rather than hump-shaped, a pattern which is consistent with data and is, as shown earlier, yielded by a component-model for consumption of the kind suggested in this paper.

[Insert Figures 12 and 13 about here.]

Importantly, we do not view these metrics as lessening the relevance of models which have added crucially to our understanding of price formation in financial markets. In light of their stringency, we view them instead as providing support for an alternative dimension to asset pricing, one which explicitly emphasizes the importance of explicitly separating - as in this work - dynamics at different frequencies.

While our proposed methods have been introduced in a *reduced-form* linear specification for the stochastic discount factor, they can be employed to enrich existing *structural* models in order to create a separation between dynamics at alternative scales. One extension, in particular, that we view as important is to justify structurally the potential presence of different levels of risk aversion across frequencies, something that we allowed implicitly through the  $b_j$ s and called earlier *scale-specific risk aversion*. Important progress in this area is being made by Andries et al. (2014).



## 6 Alternative measures of consumption

If business-cycle components matter for asset pricing, measures of consumption intended to address the shortcomings of NIPA consumption should reveal similar effects as those reported above. If these measures are less contaminated by short-term noise than NIPA consumption, identification of the business-cycle components should also be more effective. This section focuses on Qiao’s “filtered” consumption (Qiao, 2013), Kroenke’s “unfiltered” consumption (Kroenke, 2013), and total consumption (e.g., Daniel and Marshall, 1997).

“Filtered” consumption is the result of principal component analysis on suitable macro variables and a lasso regression of NIPA consumption growth on the principal components. Table 5 provides results for a single factor model with “filtered” consumption as the factor (Panel A) and a two-factor model built on extracted (two-to-four years and four-to-eight years) business-cycle components (Panel B). The sum of the two business-cycle components has a 70% correlation with the aggregate “filtered” series. As shown in the original paper, the betas with respect to “filtered” consumption align satisfactorily with average returns, thereby yielding small pricing errors and a high R-squared value of around 55%.<sup>7</sup>

Turning to a pricing model on the two business-cycle components (Panel B), the fit improves with a reduction in the pricing errors and an increase in R-squared. What is, however, surprising is that, contrary to NIPA consumption and the two additional series we examine below, the betas associated with the 4-to-8 year “filtered” consumption component align negatively in the value dimension: high value portfolios have a relatively lower covariation with the lower frequency business-cycle component. This effect results into a precisely estimated, but negative, price of risk. We emphasize that, while both business-cycle components continue to lead to an improved fit, the nature of the betas with respect to 4-to-8 year business-cycle consumption (and the resulting negative price of risk) is specific to “filtered” consumption. What is also specific to “filtered” consumption is the relative variance weight of the 2-to-4 year and the 4-to-8 year components. While in all other consumption series examined in this paper the contribution of the two components to the overall variance of consumption is similar (thereby leading to similar weights in our proposed one-factor specification), for “filtered” consumption the relative contribution of the higher (2-to-4 years) frequency component is drastically larger. The behavior of the betas at scale  $j = 5$  for the “filtered” consumption series

---

<sup>7</sup>Numerical differences between our results and Qiao’s results are the outcome of differences in portfolio choice (he focuses on the ten size and value portfolios separately).

might also be the outcome of differences in sample length.

[Insert Table 5 about here.]

“Unfiltered” consumption is the result of a process which unveils the filter inherent in NIPA consumption. Table 6 reports our findings. Coherently with the results in Kroenke (2013), aggregate “unfiltered” consumption prices the cross-section of the 25 FF portfolios very satisfactorily (with an R-squared of 59%). The correlation between the sum of the two business-cycle components and the aggregate series is 77% in this case. When regressing average returns on the betas associated with the two business-cycle components, we find evidence of a substantial improvement in the fit (Panel B). The resulting R-squared is a remarkable 71%. Importantly for our purposes, the prices of risk associated with the two components are very similar. Consistent with this result, turning to a single factor model (Panel C) in which the two components are weighed by the corresponding contribution to relative variance, as earlier, does not affect the fit relative to a two (business-cycle) factor specification. In other words, the restriction of equal lambdas across components is supported by the data. As for NIPA consumption, a single business-cycle factor performs very satisfactorily relative to raw consumption.

[Insert Table 6 about here.]

Total consumption adds purchases of durables to nondurables and services (Table 7). In agreement with the reported usefulness of durables in cross-sectional pricing (Yogo, 2006), the aggregate series prices the size and value portfolios better than aggregate NIPA consumption (Panel A). Focusing on the two business-cycle components improves matters along a variety of metrics: the intercept becomes insignificant, the pricing errors decrease by 10% to 15%, the coefficient of determination goes from 28% to 44%. Even in this case, the restriction of equal prices of risk across components is supported by the data (Panel C versus Panel B). Again, the empirical performance of one business-cycle factor is economically and statistically indistinguishable from that of a two-factor specification.

[Insert Table 7 about here.]

## 7 Further discussion and conclusions

The economic purity of the CCAPM has led to a variety of approaches intended to reconcile the appeal of consumption-based explanations of the pricing of risky assets with well-known empirical regularities. The use of economically-motivated scaling factors in the definition of a stochastic discount factor defined with respect to consumption (Lettau and Ludvigson, 2001) or the emphasis on alternative utility specifications (and consumption dynamics) capable of suitably modifying a stochastic discount factor, again, defined over consumption (Campbell and Cochrane, 1999, Bansal and Yaron, 2004, and Hansen, Heaton, and Li, 2008, *inter alia*) are successful examples of this reconciliation in the literature.

In this paper we step back a little and take an alternative view of the same issue. We suspect that certain features (components) of the consumption process may matter for the purpose of asset pricing, the relative impact of other components being economically lower. If we separate the covariance between consumption growth and asset returns into sub-covariances (one for each component of the consumption process and each component of the return process), it may be the case that sub-components (i.e., sub-covariances) of the typical object of interest (the overall covariance between aggregate consumption growth and asset returns) explain the observed cross-sectional dispersion in average returns, whereas the overall covariance does not (the latter being a typical finding). In other words, consumption cycles of different length may affect the pricing of risky assets differently. In particular, high-frequency consumption cycles may solely represent short-term noise attenuating the explanatory power of the classical consumption betas.

Consistent with this logic, after careful separation of the consumption components and using portfolios sorted based on traditional dimensions like size and book-to-market, we find that priced consumption risk may be defined in terms of the covariance of (business-cycle) asset return components with a single *business-cycle consumption* factor with a periodicity between 2 and 8 years. Said differently, the cross-sectional dispersion of the risk premia of common test assets depends crucially on business-cycle fluctuations in consumption.

We show that, by zooming in onto the relevant (for pricing) layers of the consumption process, explicit separation of heterogeneous (in terms of their persistence and periodicity) consumption components leads to satisfactory quantities of risk, prices of risk, and pricing errors. While these are ubiquitous, for good economic reasons, *cross-sectional* metrics, they may miss important time-

series dimensions. To address this issue, we focus on suitable *time-series* criteria, namely consumption growth autocorrelation, the hump-shaped pricing ability of the covariance between *ultimate consumption* (as defined in Parker and Julliard, 2005) and returns, the hump-shaped structure of long-run risk premia and the decaying pattern in consumption growth predictability. According to all of these metrics, a heterogeneous-component model for consumption growth fares very satisfactorily in addressing stylized facts about the joint dynamics of consumption and asset returns over time. It does so while remaining within the appealing confines of a consumption-based linear factor model yielding the classical CCAPM as a restriction, a model in which investors solely weigh *different* layers of the consumption process *differently* for the purpose of asset pricing.

The separation of the stochastic discount factors into details operating over different scales is, in our view, of separate methodological interest and can be applied to any factor, our focus being on the standard “non-durable plus services” consumption series employed here in the context of a reduced-form expression for the stochastic discount factor. We leave extensions of the methods to alternative factors as well as to structural (component-based) approaches to asset pricing for future work.

## A Decomposing Time Series along the Persistence Dimension

This section shows how to decompose a time series into components with different levels of persistence.<sup>8</sup>

Given a time series  $\{g_t\}_{t \in \mathbb{Z}}$  we begin by constructing moving averages  $\pi_t^{(j)}$  of size  $2^j$ :

$$\pi_t^{(j)} = \frac{1}{2^j} \sum_{p=0}^{2^j-1} g_{t-p}, \quad (\text{A.1})$$

where  $\pi_t^{(0)} \equiv g_t$ . Given the choice of sample size, it is readily observed that these moving averages satisfy the iterative relation:

$$\pi_t^{(j)} = \frac{\pi_t^{(j-1)} + \pi_{t-2^{j-1}}^{(j-1)}}{2}. \quad (\text{A.2})$$

In words each element  $\pi_t^{(j)}$  is the  $h$ -period moving average with  $h = 2^j$  and time is consistently scaled by a factor  $2^j$ . Next, we denote by  $g_t^{(j)}$  the difference between moving averages of sizes  $2^{j-1}$  and  $2^j$ , i.e. :

$$g_t^{(j)} = \pi_t^{(j-1)} - \pi_t^{(j)}. \quad (\text{A.3})$$

Intuitively,  $g_t^{(j)}$  captures fluctuations that survive to averaging over  $2^{j-1}$  terms but disappear when the average involves  $2^j$  terms, i.e. fluctuations with half-life in the interval  $[2^{j-1}, 2^j)$ . Accordingly, the moving average  $\pi_t^{(j)}$  includes fluctuations whose half-life exceeds  $2^j$  periods. From now on, we refer to the derived time series  $\{g_t^{(j)}\}_{t \in \mathbb{Z}}$  as to the *component* of the original time series  $\{g_t\}_{t \in \mathbb{Z}}$  with level of persistence  $j$ . Since  $\pi_t^{(0)} \equiv g_t$ , by summing up over  $j$  it follows immediately from (A.3) that:

$$g_t = \sum_{j=1}^J g_t^{(j)} + \pi_t^{(J)}, \quad (\text{A.4})$$

for any  $J \geq 1$ . In words, equation (A.4) decomposes the time series  $g_t$  into a sum of components with half-life belonging to a specific interval, plus a residual term that represents a long-run average.

Due to the overlap of the moving averages that define  $g_t^{(j)}$ , the decomposition (A.4) can lead to a biased evaluation of the persistence of the time series  $g_t$ . To address this issue we select the information in the components  $g_t^{(j)}$  and  $\pi_t^{(j)}$  in a suitable manner. In particular, since by definition each component  $g_t^{(j)}$  is a linear combination of the realizations  $g_t, g_{t-1}, \dots, g_{t-2^j+1}$ , to remove any spurious serial correlation introduced by the overlapping of the moving averages we restrict our attention to the sub-series:

$$\{g_t^{(j)}, t = k2^j, k \in \mathbb{Z}\} \quad (\text{A.5})$$

$$\{\pi_t^{(j)}, t = k2^j, k \in \mathbb{Z}\}^9 \quad (\text{A.6})$$

---

<sup>8</sup>This part of the appendix is drawn from Ortu, Tamoni, and Tebaldi (2013).

We refer to these sub-series as to the *decimated components* at level of persistence  $j$  of the original time series. Clearly, persistence in a decimated component is not an artifact; rather, it represents an actual fluctuation of the original series with an half-life in the interval  $[2^{j-1}, 2^j)$ .

The process of decimation controls for spurious persistence by deleting from the components  $g_t^{(j)}$  and  $\pi_t^{(j)}$  *all and only* the information irrelevant to reconstruct the original time series  $g_t$ . Formally, this follows from observing that for any  $J \geq 1$  one can define a linear, invertible operator  $\mathcal{T}^{(J)}$  that maps the decimated components  $\{g_t^{(j)}, t = k2^j, k \in \mathbb{Z}\}$ ,  $j = 1, \dots, J$  and  $\{\pi_t^{(j)}, t = k2^j, k \in \mathbb{Z}\}$  into the time series  $\{g_t\}_{t \in \mathbb{Z}}$ . To illustrate how this works for  $J = 2$  we first observe that in this case (A.1) yields:

$$\pi_t^{(2)} = \frac{g_t + g_{t-1} + g_{t-2} + g_{t-3}}{4}. \quad (\text{A.7})$$

Next we substitute (A.2) into (A.3) and let  $j = 1, 2$  to obtain:

$$\begin{aligned} g_t^{(2)} &= \frac{\pi_t^{(1)} - \pi_{t-1}^{(1)}}{2} = \frac{1}{2} \left( \frac{g_t + g_{t-1}}{2} - \frac{g_{t-2} + g_{t-3}}{2} \right) \\ g_t^{(1)} &= \frac{\pi_t^{(0)} - \pi_{t-1}^{(0)}}{2} = \left( \frac{g_t - g_{t-1}}{2} \right) \\ g_{t-2}^{(1)} &= \frac{\pi_{t-2}^{(0)} - \pi_{t-3}^{(0)}}{2} = \left( \frac{g_{t-2} - g_{t-3}}{2} \right). \end{aligned} \quad (\text{A.8})$$

We then consider the system obtained by stacking (A.7) on top of (A.8), which in matrix notation becomes:

$$\begin{pmatrix} \pi_t^{(2)} \\ g_t^{(2)} \\ g_t^{(1)} \\ g_{t-2}^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} g_t \\ g_{t-1} \\ g_{t-2} \\ g_{t-3} \end{pmatrix}. \quad (\text{A.9})$$

Denoting by  $\mathcal{T}^{(2)}$  the  $(4 \times 4)$  matrix in (A.9), we notice that  $\mathcal{T}^{(2)}$  is orthogonal, that is  $\Lambda^{(2)} \equiv \mathcal{T}^{(2)} (\mathcal{T}^{(2)})^\top$  is diagonal. Moreover, the diagonal elements of  $\Lambda^{(2)}$  are non-vanishing so that  $(\mathcal{T}^{(2)})^{-1} = (\mathcal{T}^{(2)})^\top (\Lambda^{(2)})^{-1}$  is well-defined, and hence:

$$\begin{pmatrix} g_t \\ g_{t-1} \\ g_{t-2} \\ g_{t-3} \end{pmatrix} = (\mathcal{T}^{(2)})^{-1} \begin{pmatrix} \pi_t^{(2)} \\ g_t^{(2)} \\ g_t^{(1)} \\ g_{t-2}^{(1)} \end{pmatrix}. \quad (\text{A.10})$$

By letting  $t$  vary in the set  $\{t = k2^2, k \in \mathbb{Z}\}$ , equation (A.10) shows how to reconstruct uniquely the entire

time series  $\{g_t\}_{t \in \mathbb{Z}}$  from the decimated components  $\{g_t^{(j)}, t = k2^j, k \in \mathbb{Z}\}$ ,  $j = 1, 2$  and  $\{\pi_t^{(2)}, t = k2^2, k \in \mathbb{Z}\}$ .<sup>10</sup>

## B $\beta$ decomposition

We denote by  $\left(M_T^{(J)}\right)$  and  $\left(R_T^{(J)}\right)$  the vectors collecting the  $T = 2^J$  observations of the series  $\{M_t\}$  and  $\{R_t\}$ , respectively:

$$\begin{aligned} M_T^{(J)} &= [M_T, M_{T-1}, \dots, M_1]^\top \\ R_T^{(J)} &= [R_T, R_{T-1}, \dots, R_1]^\top . \end{aligned}$$

We use the transformation (Haar) matrix  $\mathcal{T}^{(J)}$  defined in the previous section, to express the sample covariance between  $M_t$  and  $R_t$  as the sum of the second moments of the decimated components  $\mathbf{M}^{(j)} = \left[M_{2^j}^{(j)}, \dots, M_{k \cdot 2^j}^{(j)}, \dots, M_T^{(j)}\right]^\top$  and  $\mathbf{R}^{(j)} = \left[R_{2^j}^{(j)}, \dots, R_{k \cdot 2^j}^{(j)}, \dots, R_T^{(j)}\right]^\top$ :

$$\begin{aligned} \frac{\sum_{t=1}^T M_t R_t}{T} - \frac{\sum_{t=1}^T M_t}{T} \frac{\sum_{t=1}^T R_t}{T} &= \frac{\left(M_T^{(J)}\right)^\top \left(R_T^{(J)}\right)}{T} - \frac{\sum_{t=1}^T M_t}{T} \frac{\sum_{t=1}^T R_t}{T} \\ &= \frac{\left((\Lambda^{(J)})^{-1/2} \mathcal{T}^{(J)} M_T^{(J)}\right)^\top \left((\Lambda^{(J)})^{-1/2} \mathcal{T}^{(J)} R_T^{(J)}\right)}{T} - \frac{\sum_{t=1}^T M_t}{T} \frac{\sum_{t=1}^T R_t}{T} \\ &= \frac{\sum_{j=1}^J 2^j (\mathbf{M}^{(j)})^\top \mathbf{R}^{(j)}}{T} + 2^J \frac{\pi_M^{(J)} \pi_R^{(J)}}{T} - \frac{\sum_{t=1}^T M_t}{T} \frac{\sum_{t=1}^T R_t}{T} \\ &= \sum_{j=1}^J \frac{(\mathbf{M}^{(j)})^\top \mathbf{R}^{(j)}}{\frac{T}{2^j}} , \end{aligned}$$

where in the second equality we exploit the fact that the matrix  $(\Lambda^{(J)})^{-1/2} \mathcal{T}^{(J)}$  is orthonormal, in the third we exploit the fact that the diagonal elements of the matrix  $\Lambda^{(J)} \equiv \mathcal{T}^{(J)} (\mathcal{T}^{(J)})^\top$  are  $\lambda_1 = \lambda_2 = 1/2^J$ ,  $\lambda_k = 1/2^{J-j+1}$ ,  $k = 2^{j-1} + 1, \dots, 2^j$ ,  $j = 2, \dots, J$ , and in the last row we exploit the definition of the scaling component  $\pi_M^{(J)}$  and  $\pi_R^{(J)}$ .

Now observe that the component  $\mathbf{M}^{(j)}$  at scale  $j$  has exactly  $\frac{T}{2^j}$  observations due to decimation. Under the usual assumptions, the weak law of large numbers gives

$$\frac{(\mathbf{M}^{(j)})^\top \mathbf{R}^{(j)}}{\frac{T}{2^j}} \xrightarrow{p} \mathbb{E} \left[ M_t^{(j)} R_t^{(j)} \right]$$

and, therefore, we have that

$$\text{Cov} [M_t, R_t] = \sum_{j=1}^J \mathbb{E} \left[ M_t^{(j)} R_t^{(j)} \right] .$$

---

<sup>10</sup>For an extension of this procedure to any  $J \geq 2$  and a recursive algorithm for the construction of the matrix  $\mathcal{T}^{(J)}$ , associated to an arbitrary level of persistence  $J$ , see e.g. Mallat (1989).

Transitioning now to the corresponding beta decomposition is natural.

## C Data

Our empirical exercise is conducted on data sampled on a quarterly frequency. The data cover the first quarter of 1963 through fourth quarter of 2012. Following earlier work (e.g. Hansen and Singleton, 1983 and Bansal and Yaron, 2004), we use data on U.S. real nondurables and services consumption per capita from the Bureau of Economic Analysis. We make the standard end-of-period timing assumption that consumption during period  $t$  takes place at the end of the period. Growth rates are constructed by taking the first difference of the corresponding log series.

The portfolios employed in our empirical tests sort firms on dimensions that lead to cross-sectional dispersion in measured risk premia. We first consider a  $5 \times 5$  two-way sort on market capitalization and book-to-market resulting in 25 portfolios (see Fama and French (1993)). We also consider one-way sorted portfolio. The particular characteristics that we consider are firms' market value, book-to-market ratio, and past returns (momentum). Data on returns from these portfolio sorts are obtained from Ken French's web site at Dartmouth college.<sup>11</sup> Portfolios comprise stocks listed on NYSE, AMEX and NASDAQ. Returns on value weighted portfolios are used, but results are very similar when using equal weighted portfolios. The market is the value-weight return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) and the excess returns are with respect to the one-month Treasury bill rate (from Ibbotson Associates). The returns on equity and the risk-free rate are aggregated to a quarterly level by multiplying returns within a quarter.

---

<sup>11</sup>Details on how these portfolios are formed are available from [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).



## References

- [1] Abel, A.B., 1990. "Asset prices under habit formation and catching up with the Joneses." *American Economic Review, Papers and Proceedings*, 38-42.
- [2] Ait-Sahalia, Yacine, Parker, Jonathan A. and Motohiro Yogo, 2004. "Luxury Goods and the Equity Premium," *Journal of Finance*, vol. 59(6), pages 2959-3004.
- [3] Alvarez, F., and U.J. Jermann, 2005. "Using asset prices to measure the persistence of the marginal utility of wealth," *Econometrica*, Vol. 73(6), 1977-2016.
- [4] Andries, Marianne, Thomas M. Eisenbach, and Martin C. Schmalz), 2014. "Asset Pricing with Horizon Dependent Risk Aversion." *Working paper*.
- [5] Asness, C.S., T.J. Moskowitz, and L.H. Pedersen, 2013. "Value and momentum everywhere." *Journal of Finance*, 929-986.
- [6] Bandi, F., B. Perron, A. Tamoni, and C. Tebaldi, 2013. "The scale of predictability." *SSRN eLibrary*.
- [7] Bandi, Federico M., Rene' Garcia, Abraham Lioui and Benoit Perron, 2011. "A Long-Horizon Perspective on the Cross-Section of Expected Returns". *Working paper*.
- [8] Bansal, R., and A. Yaron, 2004. "Risks for the long-run: a potential resolution of asset pricing puzzles." *Journal of Finance*, 1481-1509.
- [9] Bansal, R., R. Dittmar, and D. Kiku, 2009. "Cointegration and Consumption Risks in Asset Returns." *Review of Financial Studies*, 1343-1375.
- [10] Bansal, R., Kiku, D. and Yaron, A., 2012. "Risks for the Long Run: Estimation with Time Aggregation". *SSRN eLibrary*.
- [11] Banz, R.W., 1981. "The relationship between return and market value of common stocks." *Journal of Financial Economics*, 3-18.
- [12] Barberis, N., M. Huang, and T. Santos, 2001. "Prospect theory and asset prices." *Quarterly Journal of Economics*, 1-53.

- [13] Berkowitz, J., 2001. "Generalized spectral estimation of the consumption-based asset pricing model." *Journal of Econometrics*, 269-288.
- [14] Beveridge, Stephen & Nelson, Charles R., 1981. "A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'," *Journal of Monetary Economics*, vol. 7(2), pages 151-174.
- [15] Breeden, D.T., 1979. "An intertemporal asset pricing model with stochastic consumption and investment opportunities." *Journal of Financial Economics*, 265-296.
- [16] Breeden, D.T., and R. H. Litzenberger, 1978. "Prices of State-contingent Claims Implicit in Option Prices." *The Journal of Business*, 621-51.
- [17] Burns, A.F., and W. C. Mitchell, 1946. "Measuring Business Cycles," NBER Books, National Bureau of Economic Research Inc., 46-1, July.
- [18] Campbell, J.Y., 2003. Consumption-based asset pricing. *Handbook of the Economics of Finance*, edited by G.M. Constantinides, M. Harris and R. Stulz. Elsevier.
- [19] Campbell, J.Y., and J.H. Cochrane, 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 205-251.
- [20] Chen, Zhuo and Lu, Andrea, 2013. "Carbon Dioxide Emissions and Asset Pricing". *Northern University working paper*.
- [21] Cochrane, J.H., 2001. Asset pricing. Princeton University Press, Princeton, NJ.
- [22] Cochrane, J.H., and L.P. Hansen, 1992. "Asset pricing lessons for macroeconomics." NBER Macroeconomics Annual 1992, Edited by O.J. Blanchard and S. Fischer.
- [23] Cochrane, John H. and Monika Piazzesi, 2005. "Bond Risk Premia," *American Economic Review*, vol. 95(1), pages 138-160, March.
- [24] Cogley, T., 2001. "A frequency decomposition of approximation errors in stochastic discount factors models." *International Economic Review*, 473-503.
- [25] Comin, D., and M. Gertler, 2006. "Medium-term business cycles." *American Economic Review*, 523-551.

- [26] Constantinides, G., 1990. "Habit formation: a resolution of the equity premium puzzle." *Journal of Political Economy*, 519-543.
- [27] Daniel, K. and D. Marshall, 1997. "The equity premium puzzle and the risk-free rate puzzle at long horizons." *Macroeconomic Dynamics*, 452-484.
- [28] Detemple, J.B., and F. Zapatero, 1991. "Asset prices in an exchange economy with habit formation." *Econometrica*, 1633-1657.
- [29] Da, Zhi and Yun, Hayong, 2011. "Electricity Consumption and Asset Prices." *University of Notre Dame working paper*.
- [30] Dew-Becker, I. and S. Giglio, 2013. "Asset pricing in the frequency domain: theory and empirics." *Working paper*.
- [31] Epstein, L.G. and S. Zin, 1989. "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework." *Econometrica*, 937-969.
- [32] Epstein, L.G. and S. Zin, 1991. "Substitution, risk aversion, and the temporal behavior of consumption and asset returns: an empirical analysis." *Journal of Political Economy*, 263-286.
- [33] Fama, E. and K. French, 1993. "Common risk factors in the returns on stocks and bonds." *Journal of Financial Economics*, 3-56.
- [34] Fama, E. and K. French, 1996. "Multifactor explanations of asset pricing anomalies." *Journal of Finance*, 55-84.
- [35] Garcia, R., E. Renault, and A. Semenov, 2004. "A consumption CAPM with a reference level." *Working paper*.
- [36] Gençay, R., F. Selçuk, and B. Whitcher, 2003. "Multiscale Systematic Risk", *Journal of International Money and Finance*.
- [37] Gençay, R., F. Selçuk, and B. Whitcher, 2005. "Systematic risk and time scales", *Quantitative Finance*.
- [38] Jagannathan, Ravi and Yong Wang, 2007. "Lazy Investors, Discretionary Consumption, and the Cross-Section of Stock Returns," *Journal of Finance*, vol. 62(4), pages 1623-1661, 08.

- [39] Koijen, R., Lustig, H., Van Nieuwerburgh, S., 2012. “The cross-section and time-series of stock and bond returns”. Working paper.
- [40] Hansen, L.P., J. Heaton, and N. Li, 2008. “Consumption strikes back? Measuring long-run risk.” *Journal of Political Economy*, 260-302.
- [41] Kroencke, T. Alexander, 2013. “Asset Pricing without Garbage.” *SSRN eLibrary*.
- [42] Lettau, M., and S. Ludvigson, 2001. “Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying.” *Journal of Political Economy*, 1238-1287.
- [43] Malloy, C., T.J. Moskowitz, and A. Vissing-Jorgensen, 2009. “Long-run stock holder consumption risk and asset returns.” *Journal of Finance*, vol. 64(6), 2427-2479.
- [44] Menzly, L., T. Santos, and P. Veronesi, 2004. “Understanding predictability.” *Journal of Political Economy*, 1-47.
- [45] Ortu, F., A. Tamoni, and C. Tebaldi, 2013. “Long-Run Risk and the Persistence of Consumption Shocks.” *Review of Financial Studies*, 26(11): 2876-2915.
- [46] Parker, J.A., and C. Julliard, 2005. “Consumption risk and the cross section of expected returns.” *Journal of Political Economy*, 185-222.
- [47] Piazzesi, M., 2001. Comment on Gabaix and Laibson “The 6D bias and the equity premium puzzle.” Discussion prepared for the NBER Macroeconomics Annual Conference.
- [48] Qiao, Xiao, 2013. “Cross-Sectional Evidence in Consumption Mismeasurement”. *Working paper*.
- [49] Rosenberg, B., K. Reid, and R. Lanstein, 1985. “Persuasive evidence of market inefficiency.” *Journal of Portfolio Management*, 18-28.
- [50] Rubinstein, M., 1976. “The valuation of uncertain income streams and the pricing of options.” *Bell Journal of Economics* , 407-425.
- [51] Savov, Alexi, 2011. “Asset Pricing with Garbage,” *Journal of Finance*, vol. 66(1), pages 177-201, 02.

- [52] Stattman, D., 1980. "Book values and stock returns." *The Chicago MBA: A Journal of Selected Papers*, 25-45.
- [53] Tamoni, A., 2011. "The multi-horizon dynamics of risk and returns." *SSRN eLibrary*.
- [54] Whitcher, B., P. Guttorp and D. B. Percival, 2000. "Wavelet analysis of covariance with application to atmospheric time series," *Journal of Geophysical Research*, vol. 105(11).
- [55] Yogo, Motohiro, 2006. "A Consumption-Based Explanation of Expected Stock Returns," *Journal of Finance*, vol. 61(2), pages 539-580.
- [56] Yu, Jianfeng, 2012. "Using long-run consumption-return correlations to test asset pricing models," *Review of Economic Dynamics*, vol. 15(3), pages 317-335, October.

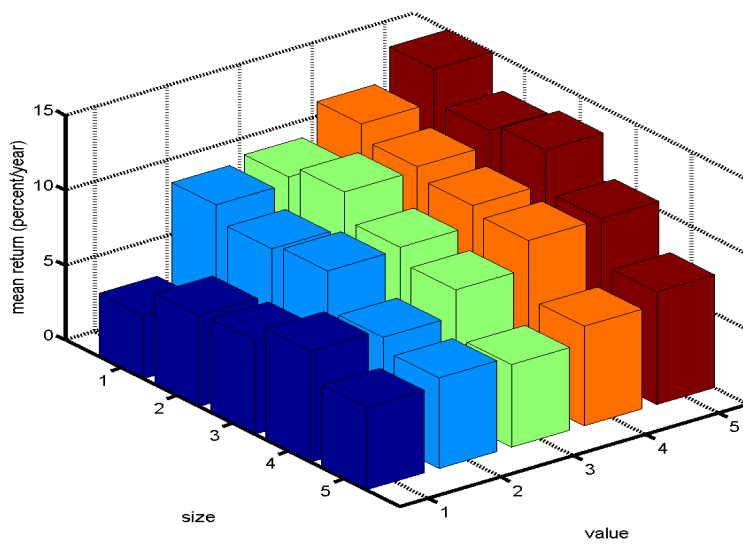
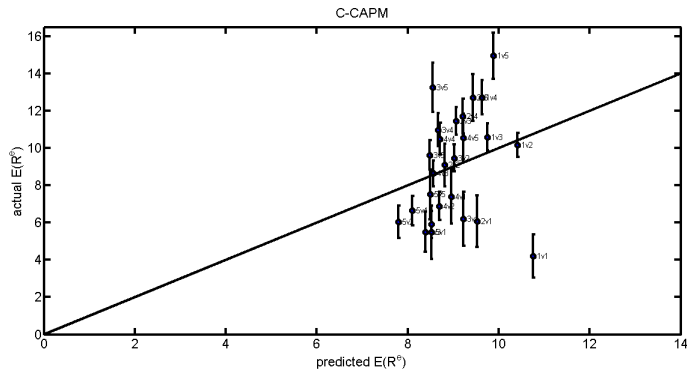
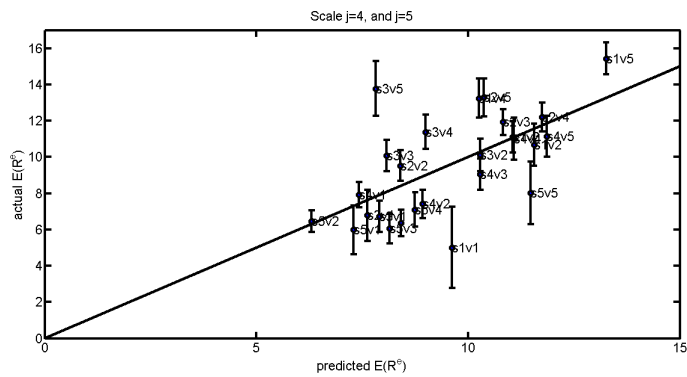


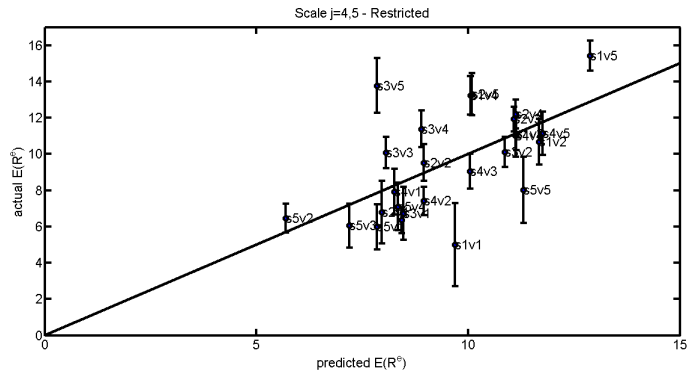
Figure 1: Average realized returns of the 25 Fama-French portfolios sorted on Size and Book-to-Market.



(a) Standard C-CAPM



(b) Scale  $j = 4, 5$



(c) Scale  $j = 4, 5$  - Restricted

Figure 2: **Cross-Sectional Fit.** **Panel A:** The figure plots fitted versus average actual excess returns (% per year) of standard consumption-capm model for the 25 size and book-to-market portfolios. **Panel B:** The figure plots fitted and average returns when the priced factors are the consumption components at scale  $j = 4, 5$ . **Panel C:** The figure plots fitted and average returns when the priced factor is restricted to be the sum of the consumption components at scale  $j = 4, 5$ .

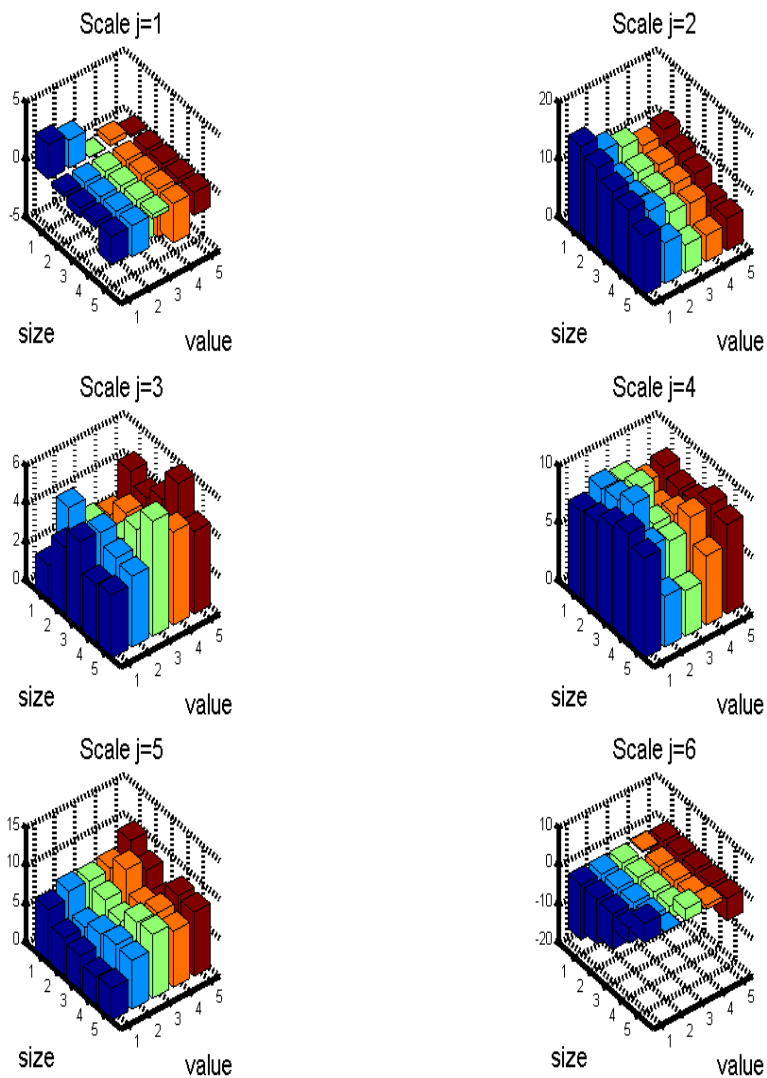


Figure 3: Betas  $\beta_i^{(j)}$  for portfolios  $i = 1, \dots, N$ . Each Panel refers to a scale  $j = 1, \dots, J$  (scale  $j$  capture fluctuations between  $2^{(j-1)}$  and  $2^{jj}$  quarters).



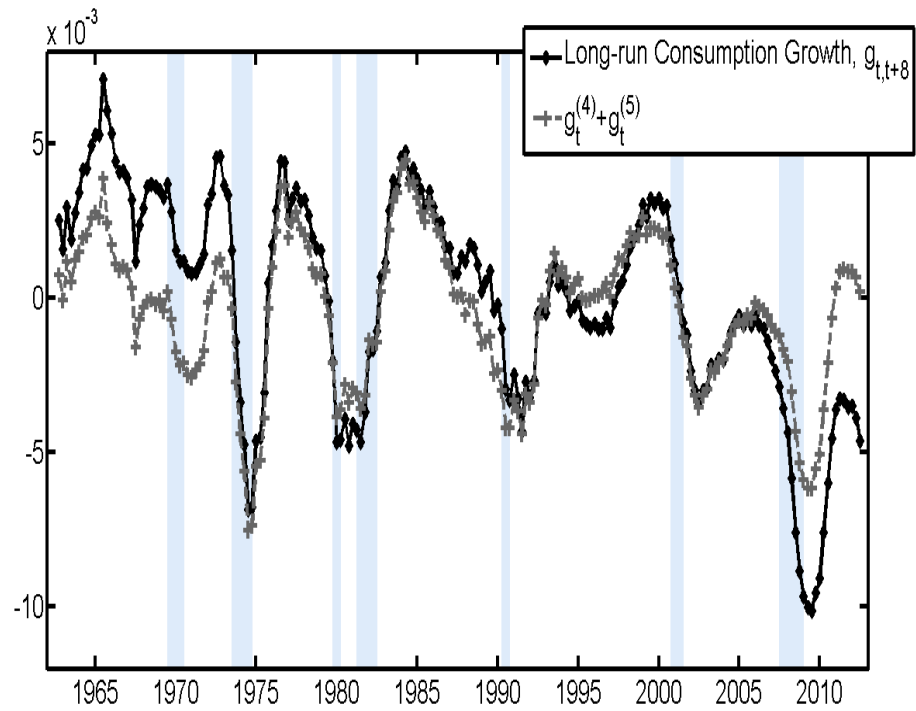
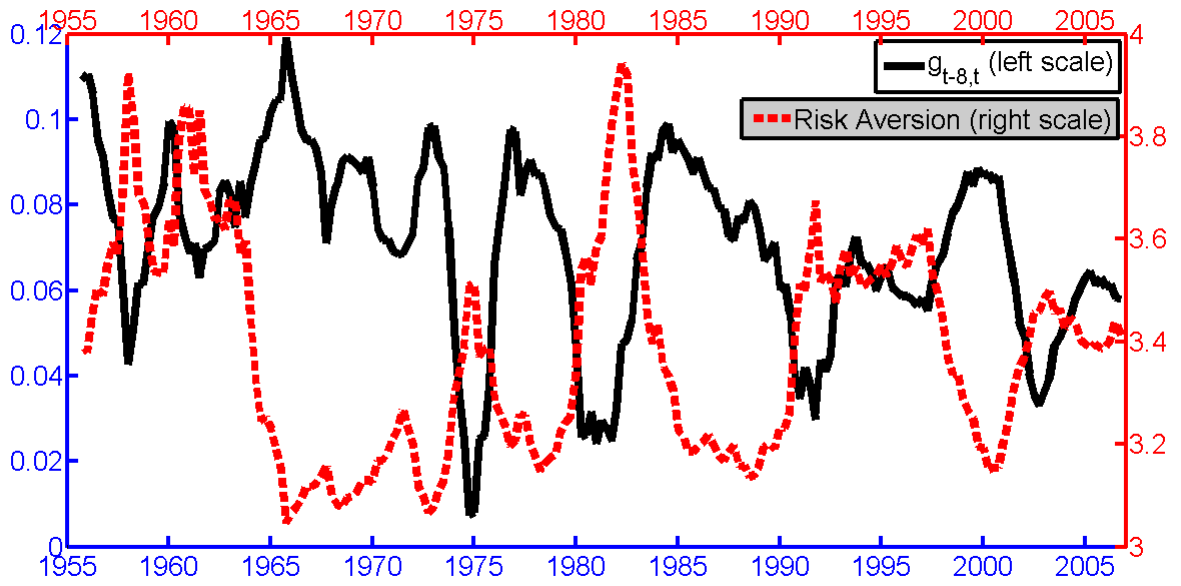
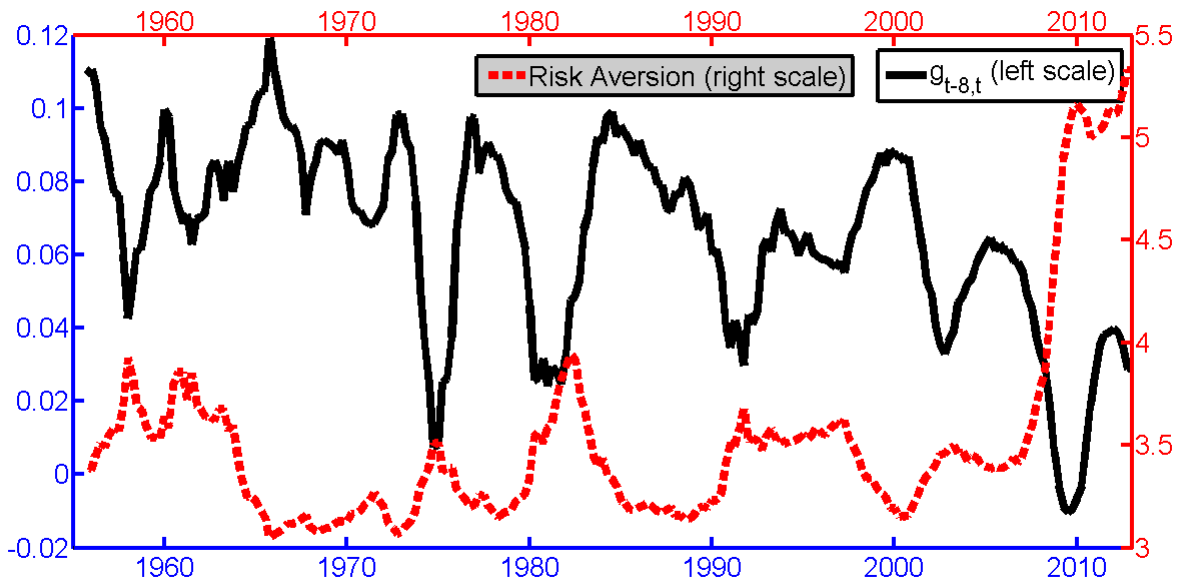


Figure 4: Comparison between consumption growth aggregated over 2 years,  $g_{t-8,t}$ , and the components  $g_t^{(4)}$  and  $g_t^{(5)}$  capturing cycles between 2 and 4 years, and between 4 and 8 years, respectively. The shaded regions are NBER recessions, from peak to trough.

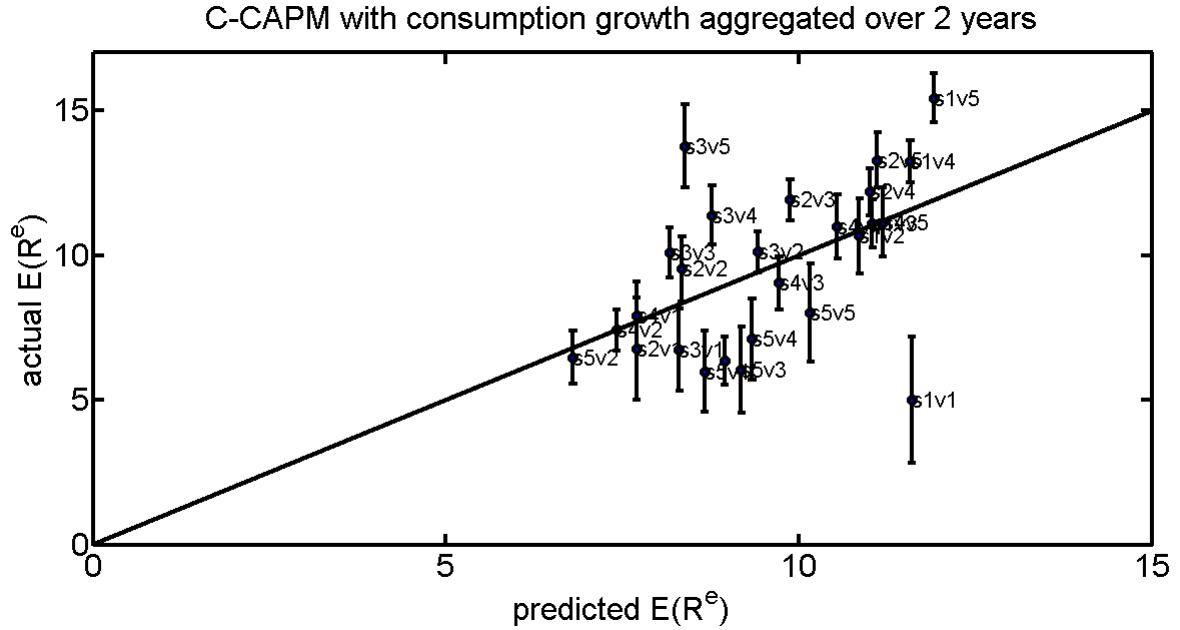


(a) Aggregated consumption growth and Risk aversion - 1955-2006

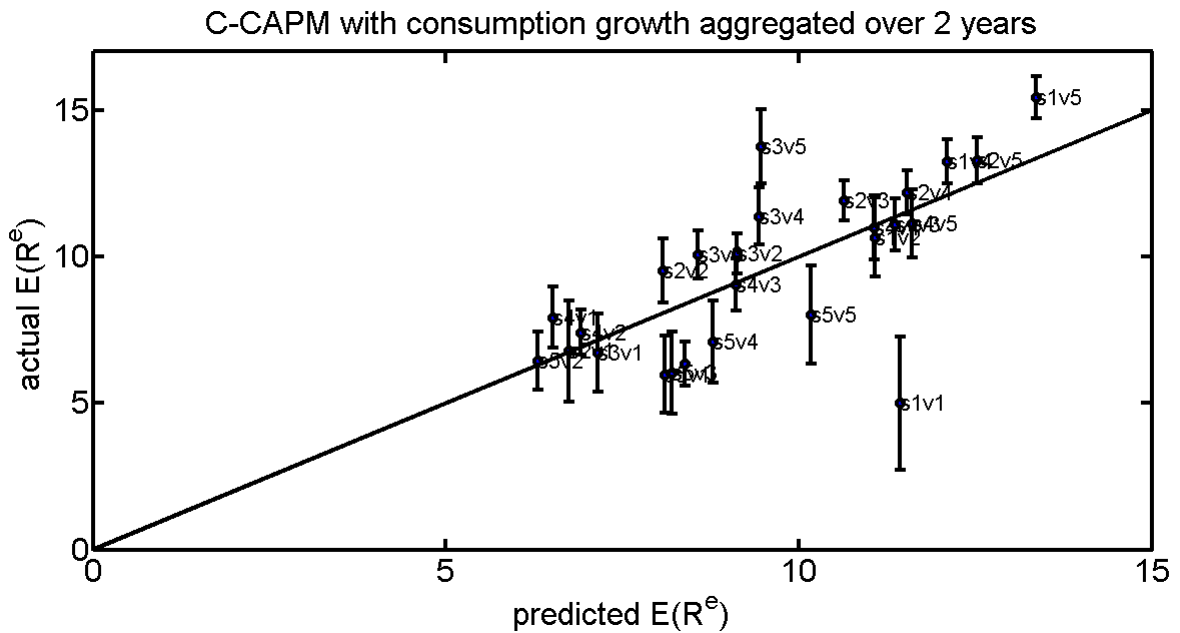


(b) Aggregated consumption growth and Risk aversion - 1955-2013

Figure 5: **Consumption growth aggregated over 2 years,  $g_{t-8,t}$ , and risk aversion. Panel A:** Sample 1955Q4-2006Q4. **Panel B:** Sample 1955Q4-2013Q4. Log of local coefficient of relative risk aversion  $ra_t = \log(\gamma) - s_t$ , where  $s_t = \log\left(\frac{C_t - H_t}{C_t}\right)$  is log surplus consumption ratio and  $\gamma$  is the instantaneous utility curvature parameter. Correlation is 53% in the shorter sample and 67% in the full sample.



(a) Simple long-run returns



(b) Compounded long-run returns

Figure 6: **Cross-Sectional Fit - 2-years Aggregated Consumption.** **Panel A:** Simple long-run returns - we approximate long-horizon excess returns by summing one-period excess returns,  $\sum_{i=1}^h R_{t+i}^{ei}$ . **Panel B:** Compounded long-run returns - we obtain long-horizon excess returns by  $\prod_{i=1}^h R_{t+i}^i - \prod_{i=1}^h R_{t+i}^f$ . The figure plots fitted versus average actual excess returns (% per year) of the long-run consumption-capm model a-la Daniel and Marshall (1999) for the linear compounding (Panel A) and gross compounding (Panel B) case respectively.

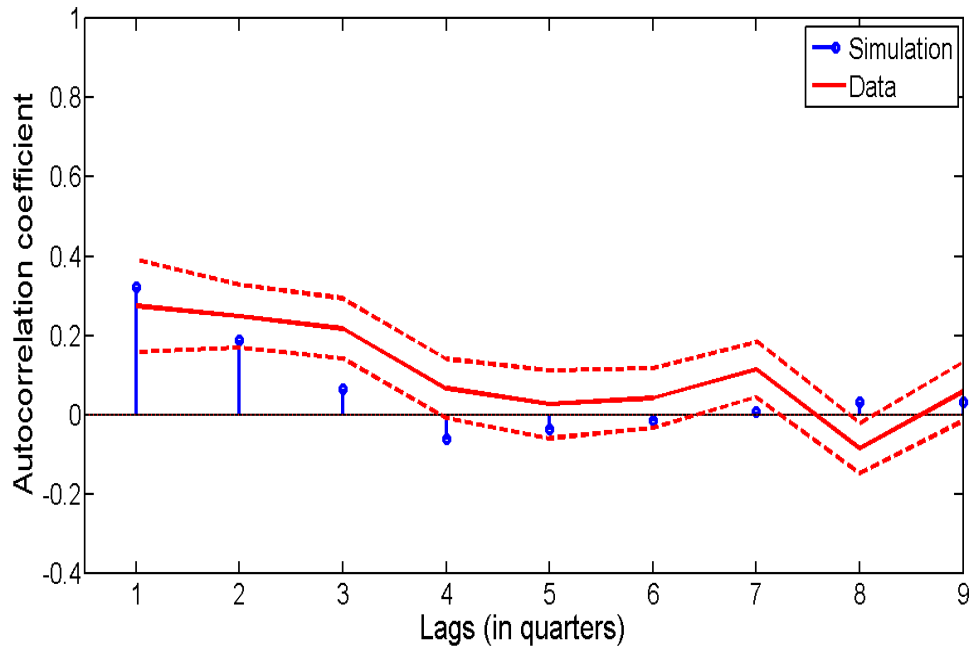


Figure 7: Autocorrelation of consumption growth at different lags in the scale-wise model (vertical bar), and in the data (solid line) together with 95% confidence bounds (dashed lines).

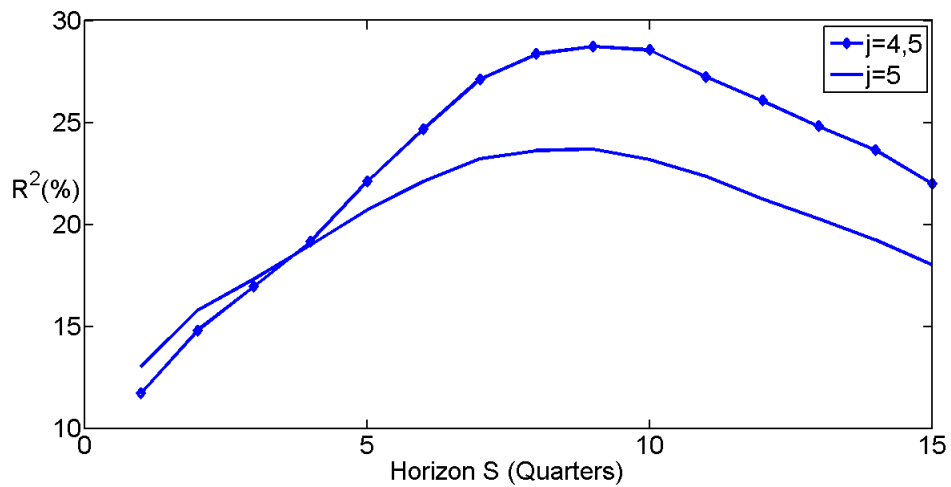


Figure 8: Average (across portfolios) of  $R^2$  obtained from cross-sectional regressions a-la Parker and Julliard.

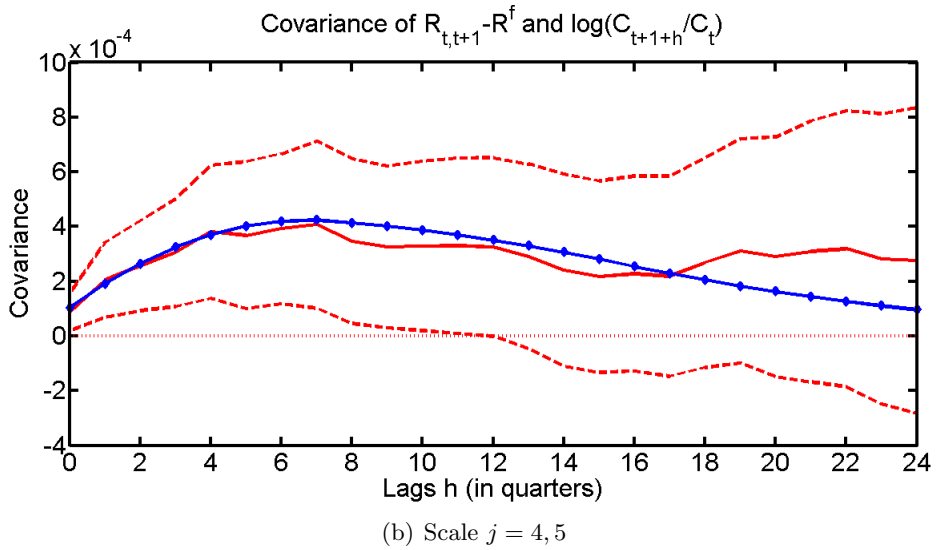
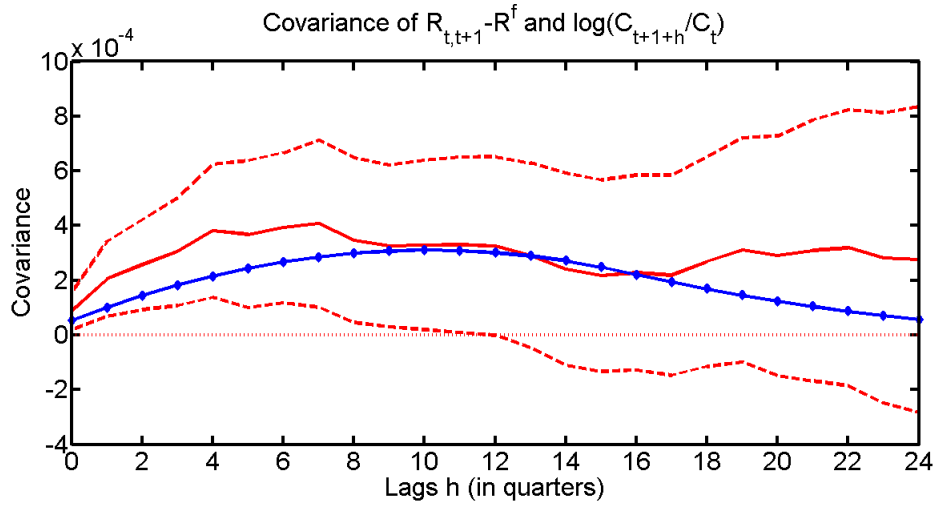
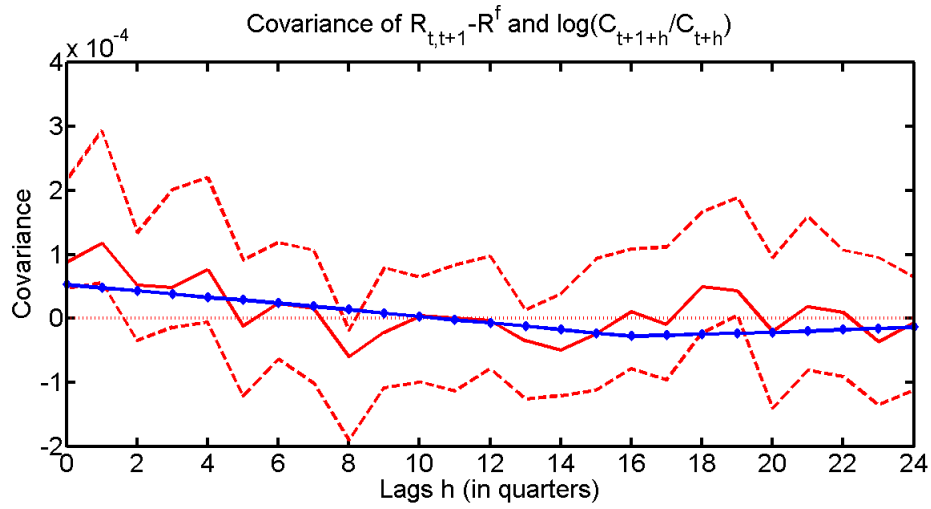
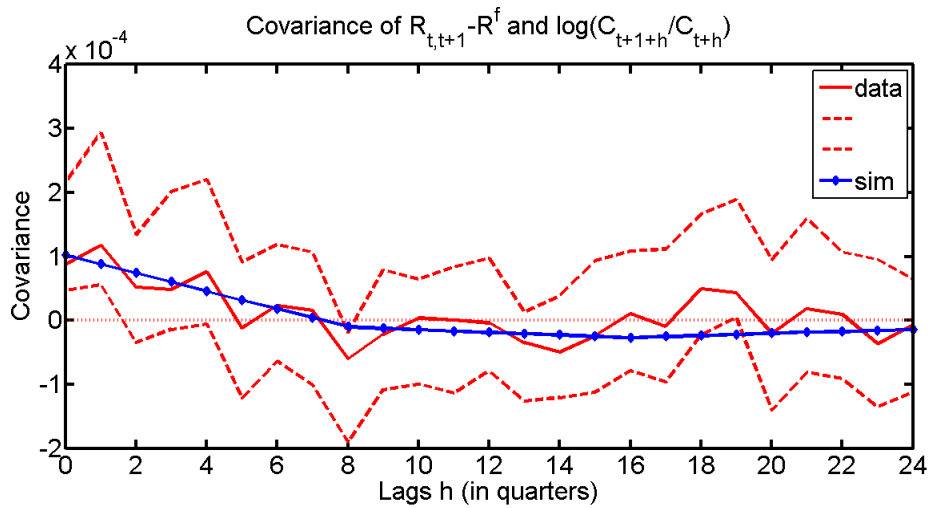


Figure 9: Covariance of returns from time  $t$  to time  $t + 1$ ,  $R_{t,t+1}^{ei}$ , with consumption growth from time  $t$  to time  $t + 1 + h$ ,  $\log(\frac{C_{t+h+1}}{C_t})$ , for different quarterly horizons  $h$ . Scale-wise model with solid diamonds ( $j = 5$  and  $j = 4, 5$ ). The dashed lines are 95% confidence bounds based on Newey-West standard errors.

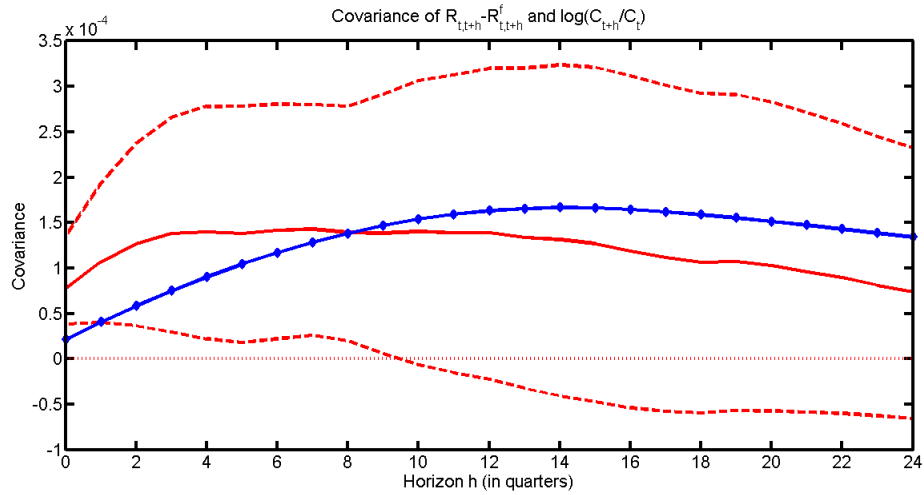


(a) Scale  $j = 5$

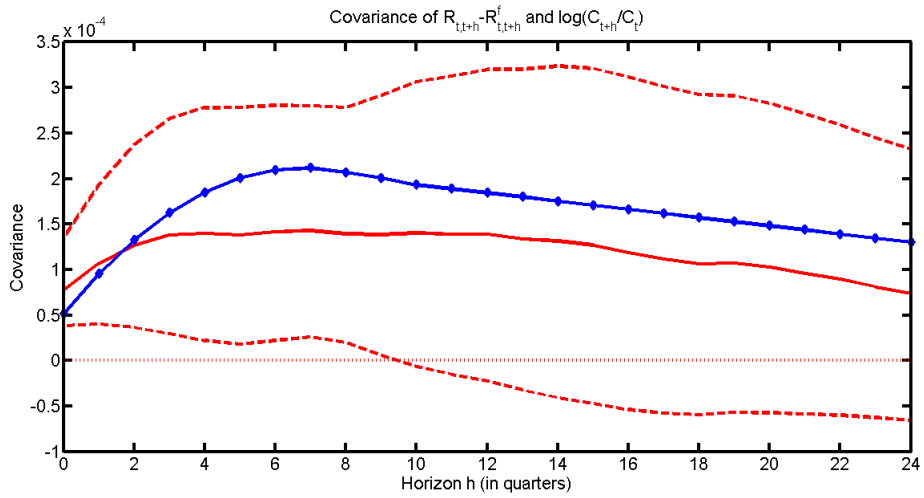


(b) Scale  $j = 4, 5$

Figure 10: Covariance of consumption growth,  $\log(\frac{C_{t+h+1}}{C_{t+h}})$ , and stock returns,  $R_{t,t+1}^{ei}$  from time  $t$  to time  $t + 1$ . Scale-wise model with solid diamonds ( $j = 5$  and  $j = 4, 5$ ). The dashed lines are 95% confidence bounds based on Newey-West standard errors.

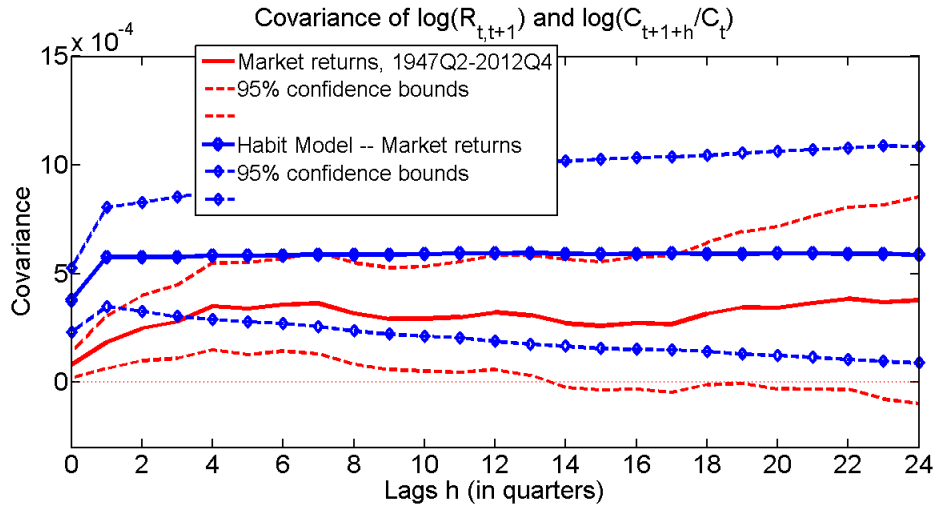


(a) Scale  $j = 5$

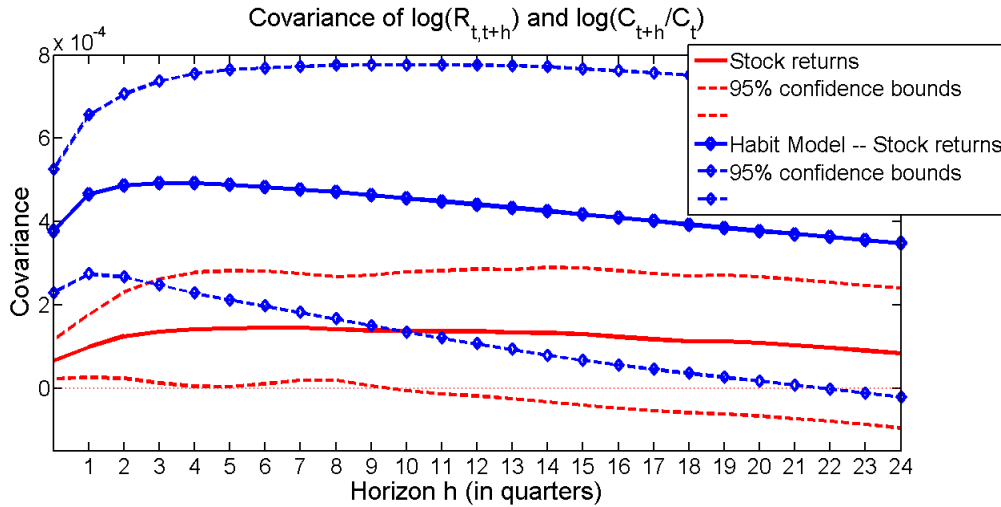


(b) Scale  $j = 4, 5$

Figure 11: Covariance of consumption growth,  $\log(\frac{C_{t+h}}{C_t})$ , and stock returns,  $R_{t,t+h}^{ei}$ , divided by horizon  $h$ . Scale-wise model with solid diamonds. The dashed lines are 95% confidence bounds based on Newey-West standard errors.



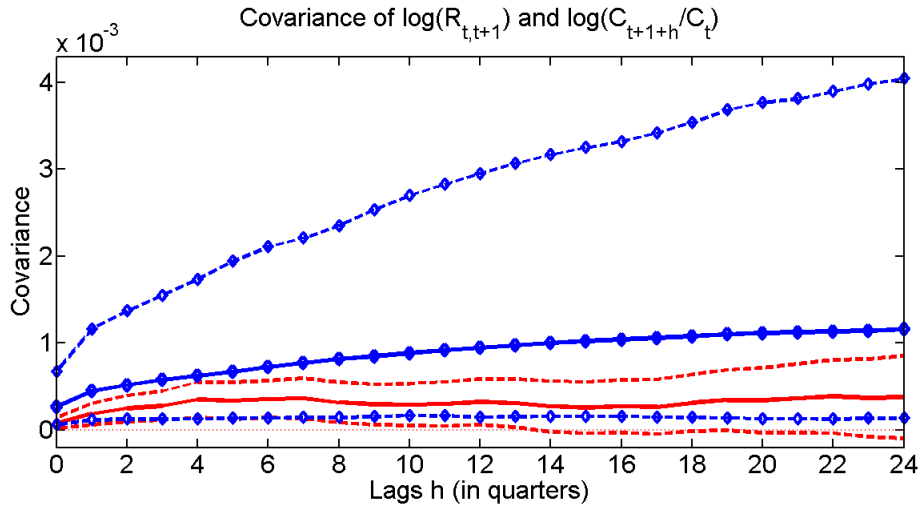
(a) Lagged equity returns and consumption growth at long horizons



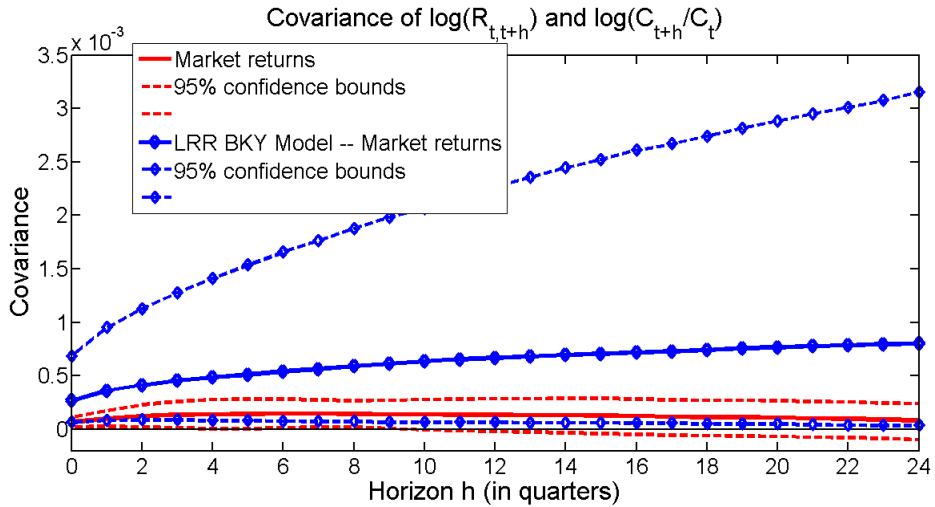
(b) Equity premium at long horizons

Figure 12: **External Habit. Panel A:** Covariance of returns from time  $t$  to time  $t + 1$ ,  $R_{t,t+1}^{ei}$ , with consumption growth from time  $t$  to time  $t + 1 + h$ ,  $\log(\frac{C_{t+1+h}}{C_t})$ , for different quarterly horizons  $h$ . **Panel B:** Covariance of consumption growth,  $\log(\frac{C_{t+h}}{C_t})$ , and stock returns,  $R_{t,t+h}^{ei}$ , divided by horizon  $h$ . Habit model with solid diamonds. Sample 1947Q2-2013Q4. The dashed lines are 95% confidence bounds based on Newey-West standard errors.





(a) Lagged equity returns and consumption growth at long horizons



(b) Equity premium at long horizons

Figure 13: **Long-run Risks. Panel A:** Covariance of returns from time  $t$  to time  $t + 1$ ,  $R_{t,t+1}^{ei}$ , with consumption growth from time  $t$  to time  $t + 1 + h$ ,  $\log(\frac{C_{t+h+1}}{C_t})$ , for different quarterly horizons  $h$ . **Panel B:** Covariance of consumption growth,  $\log(\frac{C_{t+h}}{C_t})$ , and stock returns,  $R_{t,t+h}^{ei}$ , divided by horizon  $h$ . LRR model with solid diamonds. Sample 1947Q2-2013Q4. The dashed lines are 95% confidence bounds based on Newey-West standard errors.

Panel A: Portfolio Risk Measures  $R_{k2^j}^{ei(j)} = \beta_i^{(j)} g_{k2^j} + \epsilon_{k2^j}$

Portfolio	Component $j$				
	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$
11	2.52 ( 0.47)	<b>14.74</b> ( 2.94)	-1.21 ( -0.26)	<b>12.42</b> ( 1.81)	6.97 ( 0.90)
12	1.90 ( 0.41)	<b>12.46</b> ( 3.03)	2.16 ( 0.61)	<b>11.84</b> ( 2.08)	8.09 ( 1.17)
13	-0.48 ( -0.12)	<b>11.05</b> ( 2.97)	1.18 ( 0.37)	<b>10.68</b> ( 2.07)	7.86 ( 1.27)
14	-0.14 ( -0.04)	<b>9.64</b> ( 2.72)	1.51 ( 0.52)	<b>9.43</b> ( 1.99)	8.05 ( 1.42)
15	-1.10 ( -0.25)	<b>10.12</b> ( 2.70)	2.77 ( 0.82)	<b>10.02</b> ( 1.75)	<b>10.10</b> ( 1.66)
21	-0.92 ( -0.19)	<b>14.13</b> ( 3.07)	0.95 ( 0.28)	<b>10.75</b> ( 1.95)	5.17 ( 0.77)
22	-1.61 ( -0.38)	<b>9.79</b> ( 2.54)	0.56 ( 0.19)	<b>10.84</b> ( 2.62)	5.13 ( 0.97)
23	-2.65 ( -0.70)	<b>9.61</b> ( 2.89)	1.36 ( 0.52)	<b>10.87</b> ( 2.52)	<b>7.24</b> ( 1.82)
24	-2.79 ( -0.75)	<b>8.87</b> ( 2.88)	2.67 ( 0.94)	<b>8.11</b> ( 2.17)	<b>9.73</b> ( 1.88)
25	-1.72 ( -0.41)	<b>8.87</b> ( 2.51)	2.95 ( 1.01)	<b>9.56</b> ( 2.37)	<b>8.09</b> ( 1.66)
31	-1.39 ( -0.31)	<b>12.71</b> ( 2.98)	2.20 ( 0.68)	<b>10.62</b> ( 2.21)	4.69 ( 0.92)
32	-3.02 ( -0.81)	<b>9.34</b> ( 2.61)	2.98 ( 1.27)	<b>11.35</b> ( 3.01)	6.15 ( 1.28)
33	-3.01 ( -0.88)	<b>9.02</b> ( 2.81)	2.54 ( 1.13)	<b>8.88</b> ( 2.91)	6.20 ( 1.40)
34	-3.33 ( -0.99)	<b>8.40</b> ( 2.69)	2.16 ( 0.78)	<b>9.16</b> ( 2.21)	<b>7.00</b> ( 1.68)
35	-3.46 ( -0.90)	<b>8.47</b> ( 2.58)	3.68 ( 1.24)	<b>9.05</b> ( 2.44)	<b>6.06</b> ( 1.67)
41	-0.84 ( -0.21)	<b>12.08</b> ( 3.35)	0.91 ( 0.32)	<b>11.05</b> ( 2.60)	3.78 ( 1.06)
42	-1.87 ( -0.54)	<b>10.28</b> ( 3.08)	2.45 ( 1.07)	<b>8.81</b> ( 2.27)	<b>6.45</b> ( 1.65)
43	-4.07 ( -1.27)	<b>7.79</b> ( 2.55)	3.17 ( 1.42)	<b>8.15</b> ( 1.88)	<b>7.82</b> ( 1.99)
44	-4.35 ( -1.35)	<b>7.60</b> ( 2.64)	1.96 ( 0.77)	<b>9.98</b> ( 2.51)	<b>7.70</b> ( 2.38)
45	-0.63 ( -0.17)	<b>5.88</b> ( 1.69)	4.65 ( 1.62)	<b>9.09</b> ( 1.99)	<b>8.37</b> ( 2.61)
51	-2.39 ( -0.81)	<b>10.50</b> ( 3.74)	1.82 ( 0.69)	<b>9.21</b> ( 2.80)	4.05 ( 0.89)
52	-2.75 ( -0.98)	<b>7.57</b> ( 2.86)	2.54 ( 1.39)	<b>6.45</b> ( 1.92)	<b>6.27</b> ( 1.78)
53	-0.20 ( -0.07)	<b>5.24</b> ( 2.24)	<b>5.09</b> ( 2.50)	4.86 ( 1.33)	<b>8.07</b> ( 3.13)
54	-3.40 ( -1.32)	<b>4.91</b> ( 1.83)	3.46 ( 1.39)	<b>6.97</b> ( 2.22)	<b>7.12</b> ( 1.94)
55	-2.03 ( -0.66)	<b>5.94</b> ( 2.18)	3.13 ( 1.04)	<b>8.83</b> ( 2.97)	<b>8.20</b> ( 1.85)

Table 1: **25 Portfolios Formed on Size and Book-to-Market.** The Table reports:

**Panel A:  $\beta_i^{(2)}, \beta_i^{(4)}, \beta_i^{(5)}$  - second-pass regression**

Constant	$\lambda_2$	$\lambda_4$	$\lambda_5$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	-0.457	0.864	0.820	1.92	1.63	66.740	23	0.000	
(-)	( 0.290)	(0.217)	( 0.220)						
-2.686	-0.428	0.997	0.982	1.85	1.54	59.634	22	0.000	0.46
(4.391)	(0.308)	(0.284)	(0.355)						

Table 2: **Multivariate model - 25 Portfolios Formed on Size and Book-to-Market.** Second-pass regressions with a constant. The Table reports: the estimates of the prices of risk on the consumption component and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the  $R^2$  of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year.

**Panel A: Unrestricted C-CAPM,  $E[R_{t,t+1}^{ei}] = \lambda_4\beta_i^{(4)} + \lambda_5\beta_i^{(5)}$  - second-pass regression**

Constant	$\lambda_4$	$\lambda_5$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	0.439	0.908	2.26	1.83	58.078	24	0.000	
(-)	(0.340)	(0.238)						
-5.663	0.860	1.254	2.19	1.68	56.139	23	0.000	0.39
(3.728)	(0.390)	(0.373)						

**Panel B: Restricted C-CAPM,  $E[R_{t,t+1}^{ei}] = \lambda_{restr}(\beta_i^{(4)} * w^{(4)} + \beta_i^{(5)} * w^{(5)})$  - second-pass regression**

Constant	$\lambda_{restr}$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	1.324	2.33	1.90	59.677	25	0.000	
(-)	(0.402)						
-6.428	2.201	2.24	1.76	58.017	24	0.000	0.36
(4.006)	(0.635)						

Table 3: **H0:  $\lambda_4 = \lambda_5$  25 Portfolios Formed on Size and Book-to-Market** Second-pass regressions without and with a constant. The Table reports: the estimates of the prices of risk on the consumption component  $j$  and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the  $R^2$  of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year. In Panel B, we restrict the two price of risk to be the same.

**Panel A:**  $\beta_{DM}$  from simple returns - second-pass regression  $E[R_{t,t+1}^{ei}] = \lambda_{DM}\beta_{DM}$

Constant	$\lambda_{DM}$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	3.028	2.51	1.85	68.032	24	0.000	
(-)	(0.909)						
3.583	1.941	2.38	1.75	58.793	23	0.000	0.26
(2.826)	(0.696)						

**Panel B:**  $\beta_{DM}$  from compounded returns - second-pass regression  $E[R_{t,t+1}^{ei}] = \lambda_{DM}\beta_{DM}$

Constant	$\lambda_2$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	2.402	2.13	1.56	65.463	24	0.000	
(-)	(0.708)						
2.920	1.716	1.97	1.37	57.009	23	0.000	0.49
(2.886)	(0.508)						

Table 4: **One factor model (Daniel and Marshall) - 25 Portfolios Formed on Size and Book-to-Market.** Second-pass regressions with a constant. The Table reports: the estimates of the prices of risk on the consumption component and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the  $R^2$  of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year. Simple returns means we approximate long-horizon excess returns by summing one-period excess returns,  $\sum_{i=1}^h R_{t+i}^{ei}$ . Compounded returns means we obtain long-horizon excess returns by  $\prod_{i=1}^h R_{t+i}^i - \prod_{i=1}^h R_{t+i}^f$ .

**Panel A: Raw C-CAPM,  $E[R_{t,t+1}^e] = \lambda\beta_i$  - second-pass regression**

Constant	$\lambda$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	1.923	2.12	1.68	34.306	24	0.000	
(-)	(0.689)						
-2.352	2.365	2.08	1.65	34.093	23	0.000	0.543
(4.026)	(0.801)						

**Panel B: Unrestricted C-CAPM,  $E[R_{t,t+1}^e] = \lambda_4\beta^{i(4)} + \lambda_5\beta^{i(5)}$  - second-pass regression**

Constant	$\lambda_4$	$\lambda_5$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	1.621	-0.405	1.72	1.46	60.630	23	0.000	
(-)	(0.362)	(0.183)						
1.418	1.465	-0.412	1.70	1.41	52.590	22	0.001	0.65
(3.207)	(0.473)	(0.187)						

Table 5: **Filtered consumption series using principal-components lasso regression (PCLR) - see Qiao (2013)**: Second-pass regressions without and with a constant. The Table reports: the estimates of the prices of risk on the consumption component  $j$  and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the  $R^2$  of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year. In Panel B, we restrict the two price of risk to be the same. The sample is annual, from 1965 to 2007.

**Panel A: Raw C-CAPM,  $E[R_{t,t+1}^{e_i}] = \lambda\beta_i$  - second-pass regression**

Constant	$\lambda$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	7.547	2.52	1.96	42.059	24	0.013	
(-)	(2.011)						
5.095	4.134	1.72	1.44	38.657	23	0.022	0.59
(3.280)	(1.418)						

**Panel B: Unrestricted C-CAPM,  $E[R_{t,t+1}^{e_i}] = \lambda_4\beta^{i(4)} + \lambda_5\beta^{i(5)}$  - second-pass regression**

Constant	$\lambda_4$	$\lambda_5$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	1.002	1.050	2.22	1.78	67.286	23	0.000	
(-)	(0.637)	(0.407)						
-6.213	1.730	1.571	1.50	1.13	47.051	22	0.001	0.71
(3.814)	(0.600)	(0.599)						

**Panel C: Restricted C-CAPM,  $E[R_{t,t+1}^{e_i}] = \lambda_{restr}(\beta^{i(4)} * w^{(4)} + \beta^{i(5)} * w^{(5)})$  - second-pass regression**

Constant	$\lambda_{restr}$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	1.882	2.00	1.56	51.279	24	0.001	
(-)	(0.567)						
-2.277	2.311	1.97	1.56	47.376	23	0.002	0.71
(2.816)	(0.693)						

Table 6: **Unfiltered NIPA consumption - see Kroencke (2013)**: Second-pass regressions without and with a constant. The Table reports: the estimates of the prices of risk on the consumption component  $j$  and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the  $R^2$  of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year. In Panel B, we restrict the two price of risk to be the same.

**Panel A: Raw C-CAPM,  $E[R_{t,t+1}^{e_i}] = \lambda\beta_i$  - second-pass regression**

Constant	$\lambda$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	3.475	3.00	2.14	72.384	25	0.000	
(-)	(1.068)						
5.439	1.597	2.36	1.82	63.632	24	0.000	0.28
(2.273)	(0.788)						

**Panel B: Unrestricted C-CAPM,  $E[R_{t,t+1}^{e_i}] = \lambda_4\beta^{i(4)} + \lambda_5\beta^{i(5)}$  - second-pass regression**

Constant	$\lambda_4$	$\lambda_5$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	0.789	1.111	2.20	1.79	57.155	24	0.000	
(-)	(0.428)	(0.356)						
-5.791	1.311	1.669	2.09	1.64	55.263	23	0.000	0.44
(3.893)	(0.521)	(0.526)						

**Panel C: Restricted C-CAPM,  $E[R_{t,t+1}^{e_i}] = \lambda_{restr}(\beta^{i(4)} * w^{(4)} + \beta^{i(5)} * w^{(5)})$  - second-pass regression**

Constant	$\lambda_{restr}$	$\sqrt{\alpha^2}$	$\ \alpha\ $	$\chi^2$ -stat	DoF	p-value	$R^2$
0	1.802	2.23	1.79	59.859	25	0.000	
(-)	(0.547)						
-5.658	2.848	2.12	1.65	57.795	24	0.000	0.42
(3.868)	(0.869)						

Table 7: **Total consumption expenditures (including purchases of nondurables, services, and durable consumption goods):** Second-pass regressions without and with a constant. The Table reports: the estimates of the prices of risk on the consumption component  $j$  and the constant term; the asymptotic standard errors (in parentheses) for these estimates, ignoring the sampling error in the betas; and asymptotic test statistics for the hypothesis that the alphas are all zero. The last column reports the  $R^2$  of cross-sectional regression. We also report the root mean square alpha and the mean absolute alpha (MAPE) across all securities. They are expressed in percent per year. In Panel B, we restrict the two price of risk to be the same.