

Heterogeneous Countries in a Financial Union*

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Abstract

A financial union is a group of countries, each with its own nontradable goods sector, which can freely exchange tradable goods and debt contracts. In this paper, we establish the effects of shocks in a stylized financial union with heterogeneous regions—a lender North and a borrower South—and constraints on borrowing. We derive positive and normative results. First, when the degree of heterogeneity is high before the shock, the South is disproportionately hurt by the shock, no matter whether the shock strikes in the North or the South. Second, for a given value of the shock, when borrowing constraints bind in the South, the welfare of the North decreases while the welfare of the South may increase. Third, we characterize which policy interventions are able to generate Pareto improvements. Unconditional debt relief for the South fails to do so. Subsidized governmental loans succeed when Southern governments can commit to repay additional debt. Finally, whether or not Southern governments can commit to repay anything, a Pareto improvement is possible using a combination of conditional debt relief and a tax/subsidy package in the South.

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1. Introduction

How do shocks affect the stability of economically and financially integrated groups of countries which are arranged in a core-periphery structure? And what policies are needed to deal with shocks that strike such country groupings?

These questions have come to the fore as European integration has deepened into a European Union and then a Eurozone. While early integration took the form of trade liberalization coupled with limited financial flows (the environment which stimulated Mundell 1961 and other theorists of optimal currency areas), the subsequent removal of barriers to private and public sector debt transactions brought forth a wave of large international flows which reshaped the net asset positions of member countries (documented in Giavazzi and Spaventa 2010 and Chen, Milesi-Ferretti and Tressel 2012). The Eurozone began to feature a Northern “core” supplying and/or intermediating funds to a Southern “periphery,” echoing similar structures found within other groupings of heterogeneous countries and regions—such as East-West capital flows in the United States during the nineteenth century, flows between the United Kingdom and its colonies during the same period, and flows between the United States and the rest of the world today.

What is more, with the recent financial and sovereign debt crisis, the Eurozone is also echoing the distinctive and persistent fragilities of other heterogeneous-country groupings. The pattern of large debt flows from the core to the periphery, followed by a financial crisis with retrenchment in the core and deleveraging in the periphery, seems to prevail in the historical data no matter whether the crisis is triggered by a malfeasance in banks in the core, bursting bubbles in the periphery, or both. The periphery suffers current account reversals and investment collapses, while its effective borrowing rates spike well above lending rates in the core. In fact, the severity of the crisis-era reversal of private capital inflows into Eurozone periphery economies, documented by Merler and Pisani-Ferry (2012), brings to mind a separate literature on emerging market sudden stops initiated by Dornbusch, Goldfajn and Valdés (1995) and Calvo (1998).

Our objective in this paper is to explore the effects of shocks and policy interventions in a simple model of a “financial union”—a collection of countries which enjoy in every period a single union-wide market in tradable goods and noncontingent debt contracts. Our financial union contains heterogeneous countries, grouped into two regions—North and South—which begin with different levels of initial endowments. Each country consists of households, firms and an associated nontradable good, and all prices are flexible.

Our model includes an initial period where consumption occurs, which can deplete assets in some countries or saddle them with noncontingent debt, and then there are two further cycles of debt transactions. The Northern countries endogenously emerge as lenders and the Southern countries as borrowers. In the first cycle, firms invest to produce nontradable goods, and a regional “financial shock” may strike and make some

goods unavailable for investment purposes in one of the regions. In the second cycle, households smooth their income to finance the consumption of tradable and nontradable goods, subject to a borrowing constraint related to the future value of tradable and nontradable endowments.

We are attempting to answer questions similar in flavor to those posed by the literature on optimal currency areas and its modern-day descendants. At the same time, we propose a model displaying features such as borrowing constraints which are more common in the literature on sudden stops in individual economies, because these features appear to be relevant in understanding the Eurozone's recent history as described above. Given this, we hope to offer a novel perspective on the issues we tackle. The single currency of the Eurozone matters to us because it simplifies financial arbitrage, not because it causes a loss of nominal exchange rate flexibility: since all prices are flexible in our model, our results are not based on the role of exchange rate adjustment in a world of sticky prices. Moreover, we focus on heterogeneity of countries in advance of shocks, rather than heterogeneity induced by asymmetric shocks. Finally, we borrow our market imperfections from the sudden stops literature, but we analyze groups of interacting economies instead of individual countries, and we solve for the endogenous general equilibrium interest rates.

Our first result is that the distributional effect of a shock depends greatly on the degree of heterogeneity that prevailed prior to the shock, and not just on which region is actually hit by the shock. When heterogeneity is low, so that regions have small net asset positions before the shock strikes, the region which is hit by the shock suffers most strongly. This outcome echoes the result derived in the optimal currency area literature.

On the other hand, when heterogeneity is high, the outcome changes starkly, and the Southern countries are always disproportionately hurt in terms of consumption, investment and welfare—whether the shock strikes the North or the South. The reason is that the shock has both a direct effect on the region where it strikes by making goods unavailable for investment, and an indirect effect by increasing the interest rate in the first cycle. When heterogeneity is high, the latter effect hurts the South hardest, because the Southern countries either have inherited debt or need to borrow in future periods. By contrast, the lender Northern countries benefit from the increase in the interest rate. The inclusion of investment to produce nontradable goods means that the Southern countries may be disproportionately hurt even when they finish the first period not with debt, but with a small level of assets.

The second result is that when shocks strike anywhere in a heterogeneous union, borrowing constraints may bind for households in the South, and when they do so, it is the Northern countries' welfare which decreases. The reason is that the two cycles of borrowing and lending interact. A shock anywhere in the union which increases the interest rate in the first cycle also causes the Southern countries' debt burden to spiral upward by the second cycle. With more to roll over, the Southern countries are more

likely to face binding borrowing constraints. It does not help that the market value of the Southern countries' future nontradable goods falls. When constraints bind, there emerges a wedge between the lending rate and the implied rate of return calculated from the consumption profile over time in the South.

The North's welfare decreases because as a lender region, it suffers on the infra-marginal dimension from the decrease in the union-wide interest rate in the period when the borrowing constraint binds in the Southern countries. The inclusion of investment to produce nontradable goods can generate a reverse feedback between cycles of borrowing and lending. Because such investment is tied to the future demand for nontradable goods in the same country, a binding constraint on households in the South (reflecting credit supply constraints) in the second cycle causes a collapse in Southern investment (reflecting credit demand) in the first cycle. Therefore, the interest rate may decrease in periods even before the constraint actually binds. This hurts the Northern countries. If the constraint is moderately binding, the Southern countries benefit in net terms from the decrease in interest rates; if the constraint is very binding, the Southern countries' welfare decreases.

The third set of results relates to policy interventions: we outline a series of interventions and establish whether Pareto improvements are possible with each of them. For each intervention, there is a collective action problem. The governments of individual countries in the North have an incentive to free-ride on the actions of others, which means that the intervention will be under-provided. At the same time, governments of individual countries in the South ignore the impact of their actions on the interest rates, and are individually willing to undertake policies that in the aggregate make all the South worse off. Therefore, we will restrict attention in what follows to countries joining two large regional coalitions (i.e. North and South), which then negotiate with each other.

An unconditional gift from the North to the South, which can be interpreted as unconditional debt relief, cannot achieve a Pareto improvement. If the gift is made in the period when the borrowing constraint is binding in the South, it does succeed in relaxing the South's borrowing constraint and increasing the interest rates in all periods. The South benefits from this policy but the North is hurt, because the increase in interest rates is not enough to compensate the North for the cost associated with the gift.

A subsidized governmental loan from the North to the South (i.e. a loan between governments with partial repayment), within a context where the Southern government possesses some capacity to repay loans on its own, can achieve a Pareto improvement. Southern governments borrow from Northern governments and implement transfers within their own countries which in equilibrium help alleviate the borrowing constraints of households in the South. The consumption of Southern households increases in the period when the borrowing constraint is binding, but it decreases in the following period as the loan repayment is made. This loan repayment is essential to make the North better off, which

is why an unconditional gift equal to the subsidy amount does not suffice to generate a Pareto improvement.

At first glance, the subsidy on the loan may appear to run afoul of the transfer problem discussed by Keynes (1929) and Ohlin (1929), which posits that the donor country suffers both directly from the cost of the transfer it provides, and indirectly through a terms of trade deterioration. Therefore, a subsidized loan threatens to reduce the welfare of the North and make a Pareto improvement impossible. However, the original transfer problem does not consider inter-country transfers in a context where the private sector of the donor country has lent, or expects to lend, to the recipient country, and where borrowing constraints bind. The North benefits from providing a subsidy because by inducing the South to accept the governmental loans, it achieves increases in market interest rates, so that households in the North receive a higher return on their “exports” of international loans. Counter-intuitively, the larger is the nontradable goods sector, the larger may be the impact of the loan on Southern investment and on interest rates in general equilibrium, so the larger the subsidy the North may be willing to provide.

Debt relief for the South which is made conditional on a future tax on tradable consumption and a future subsidy on nontradable consumption can achieve a Pareto improvement. This admittedly unorthodox policy package is based on the existence of the nontradable goods sector in the final period of our model. When governments in the South cannot commit to repay additional loans, they must try instead to directly increase the value of the borrowing limit of the South. The value of the tradable endowment is fixed, so the only remaining option is to increase the price of future nontradable goods. To ensure that the North is sufficiently compensated for the debt relief, the higher future nontradable goods price must be achieved without increasing the future tradable consumption of Southern households. Therefore, debt relief for the South must be conditioned on the creation of a future price wedge between tradable and nontradable consumption.

Our model directs future subsidies to be provided for nontradable goods which are collateralizable—such as housing and fixed capital—rather than for all nontradable goods, because the primary objective of the policy intervention is to increase the borrowing capacity of the South. The policy recommendation is counter-intuitive: while the tax on tradable consumption has some echoes of the narrative of fiscal austerity, the subsidy on nontradable consumption does not. Moreover, it raises fears of future asset price bubbles. Our model suggests that notwithstanding excessive asset price valuations in the Eurozone periphery before the shock of the global financial crisis, downward price flexibility in the nontradable sector is prolonging the Eurozone crisis.

For both the governmental loan and the conditional debt relief and fiscal policy package, the set of interventions which regional coalitions negotiate to improve their welfares after the shock is realized (ex post Pareto improvements) may not overlap entirely with the set of interventions which would improve the welfares of both regional coalitions from

a pre-shock perspective (ex ante Pareto improvements). Therefore, both regional coalitions may wish to meet before the shock to negotiate institutions which help shape the bargaining process after the shock. We offer a formal analysis to identify some Pareto-improving policies to be implemented at the ex ante stage. With governmental loans, one can imagine designing institutions like the IMF and the ESM which offer special loans in times of crisis. With the conditional debt relief and fiscal policy package, some degree of institutionalization may be possible, but additional care needs to be taken because the elements of the action space are more disparate, and the coalitions that need to be assembled are more diverse.

The remainder of this paper is organized as follows. Section 2 provides a literature review to complement the discussion above. Section 3 outlines the building blocks of our model and the definition of equilibrium. Section 4 collects the positive results of our environment without government intervention. Section 5 outlines the three policy interventions described above and characterizes the relevant Pareto sets where they exist. Section 6 concludes with some thoughts on policy institutionalization, on comparisons of the interventions to each other and to some actual crisis-era policy measures, and finally on the ongoing discussion about fiscal and banking union.

2. Related Literature

First, the question of how shocks affect the stability of integrated groups of countries was popularized by the optimal currency area literature. Given the post-War environment of expanding trade integration coupled with limited financial integration, this literature focused on the former. Mundell (1961) argued that countries which are hit by asymmetric shocks to export demand should not share membership of a common currency area. McKinnon (1963) and Kenen (1969) identified openness and output diversification as additional membership criteria. This fit into contemporary debates on the role of exchange rate adjustment, for example Friedman (1953). We ask some similar questions to this literature, but we depart from their (explicit and implicit) assumptions that prices are sticky and that countries are identical before shocks strike. Instead of asymmetric shocks, we focus on the heterogeneity of countries before shocks strike—the core-periphery structure—as a primary determinant of post-shock outcomes.

After asymmetric shocks, Kenen (1969) calls for fiscal stimulus financed by inter-country transfers within the currency area. Beetsma and Jensen (2005) and Galí and Monacelli (2008) demonstrate that with sticky prices, country-level fiscal stimulus after shocks is necessary for macroeconomic stabilization. Farhi and Werning (2013a) develop a model with nontradable goods which nests results from the optimal currency area literature while identifying a novel externality: with sticky prices, private insurance is Pareto inefficient, and fiscal transfers may become necessary after asymmetric shocks. In our

paper, we expand the set of policy interventions to include debt relief and budget-neutral tax/subsidy packages, and we attempt to generate Pareto improvements.

Second, we use borrowing constraints from the sudden stops literature for emerging markets. Dornbusch, Goldfajn and Valdés (1995) and Calvo (1998) characterize sudden stops and relate them to debt repayment problems. Caballero and Krishnamurthy (2001) develop a model of sudden stops with domestic and international collateral constraints, both based on tradable output. In order to match the output of calibrated models to the empirics of sudden stops, and to conduct policy analysis, Mendoza (2002, 2006), Mendoza and Smith (2006), Bianchi (2011), and Korinek (2011) have introduced a variety of borrowing constraints based on the value of tradable output, nontradable sector output and/or capital. We use in this paper a by-now standard borrowing constraint based on the market value of future tradable and nontradable endowments.

While these papers typically focus on taxes on capital inflows to limit debt before shocks, Jeanne and Korinek (2013) and a recent paper by Benigno, Chen, Otrok, Rebucci, and Young (2014) explore policy interventions both before and after shocks. In this paper, we mostly analyze policy interventions which are implemented after shocks. At a more fundamental level, while the sudden stop literature analyzes the impact of a binding constraint for a single borrower country, we develop a model with multiple borrower and lender countries with endogenous general equilibrium interest rates. The tax/subsidy packages we consider can be related to the fiscal instruments analyzed by Benigno et al. (2014), but the welfare results are turned around in our setting because interest rates adjust. Nontradable sector subsidies in borrower countries benefit lender countries instead, and borrower countries must be compensated through debt relief.

Third, our results on Pareto-improving governmental loans and conditional debt relief can be related to the literatures on debt overhang and on the multi-period version of Keynes (1929) and Ohlin's (1929) transfer problem. In a seminal paper on debt overhang, Krugman (1988) argued that debt relief may be optimal for lender countries when such relief increases future investment effort by the debtor countries. In our setting, debt relief is the inducement that lender countries must give borrower countries: in exchange, borrower countries implement tax/subsidy packages which benefit lender countries by raising general equilibrium interest rates. The subsidized governmental loans we consider also offer partial debt relief in exchange for a similar effect on interest rates.

On the multi-period transfer problem, Djajić, Lahiri and Raimondos-Møller (1998) and Cremers and Sen (2009) show that inter-country transfers affect interest rates when discount rates and intertemporal substitution elasticities vary across countries. In our paper, preferences are identical across countries, and transfers from lender countries to borrower countries increase general equilibrium interest rates after large shocks which cause borrowing constraints to bind.

Finally, our paper is related to the literature on the evolution of capital flows in the

Eurozone. Giavazzi and Spaventa (2010) present evidence that the Eurozone crisis was caused by the build-up of unsustainable external imbalances in the periphery. Decomposing net foreign asset flows during 2000-08 into transactions within the Eurozone and between the Eurozone and the rest of the world, Chen, Milesi-Ferretti and Tressel (2012) reveal the central role of intra-Eurozone debt flows: large net lending from the core to the periphery financed the current account deficits of the periphery with respect to the rest of the world. Merler and Pisani-Ferry (2012) document the reversal in private capital inflows into individual periphery economies and convincingly argue that they look much like sudden stops in emerging market economies. We present a model consistent with these observations, where a financial union endogenously assumes a core-periphery structure, and where shocks can force any financially integrated country with high levels of inherited debt and future borrowing needs into the inefficiently rapid deleveraging associated with a sudden stop.

Giavazzi and Spaventa (2010) were among the first to argue that Eurozone concerns should be broadened out from sovereign debt to external indebtedness in general. Shambaugh (2012) documents that overall external debt on the eve of the crisis is a better predictor of subsequent problems than just the public debt. Martin and Philippon (2014) calibrate a model to assess the effects on the dynamics of the crisis of private leverage and spreads in addition to fiscal policy, so as to derive appropriate policy interventions. For simplicity, in this paper we have only one variable—the external indebtedness of the private sector—to represent the net asset position of the entire economy, before policy interventions are considered. We include nontradable goods prices in the external borrowing constraint to deliver the requisite positive and normative results.

3. Heterogeneous Countries in a Financial Union

3.1. Model

Countries, regions and the financial union. Each country i contains a unit measure of households which consume tradable and nontradable goods, and a measure μ of firms which invest tradable goods in order to produce nontradable goods. Households in country i have an initial endowment e^i of tradable goods. There are two regions $j \in \{N, S\}$ (North and South), each containing a unit measure of countries. Within each region, the initial endowments of households in each country are identical: $e^i = e^j$ for all $i \in j$. Regions are heterogeneous:

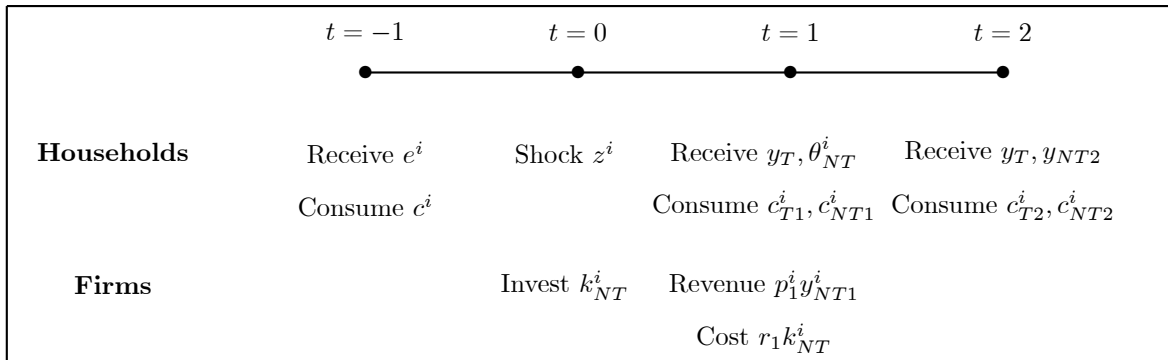
$$e^N \geq e^S, \text{ with } \sum_{j=N,S} e^j = e^T.$$

A financial union is a union of these two regions such that between each period and the next, individuals in all countries competitively trade noncontingent, one-period debt

contracts in a single union-wide market.

Borrowing and lending. As shown in figure 1, the model has four periods. Goods can be stored one-for-one between $t = -1$ and $t = 0$, but not between the other periods.

Figure 1: Timeline



At $t = -1$, households in all countries receive their initial endowment of tradable goods, and they consume c^i of them. If they have goods left over, they store them; if they consume more goods than they own, they borrow goods to fill the gap. The possibility of storage fixes the union-wide gross interest rate between $t = -1$ and $t = 0$ at unity.

Households enter $t = 0$ with assets or debt. After this, there are two more cycles of borrowing and lending, and this paper is focused on the interaction between these cycles.

At $t = 0$, firms in country i borrow from households at union-wide interest rate r_1 in order to finance investment of k_{NT}^i units of the tradable good. At $t = 1$, this investment produces y_{NT1}^i units of the nontradable good, which is sold at price p_1^i . Households do not consume at $t = 0$ but do consume tradable goods c_{T1}^i and nontradable goods c_{NT1}^i at $t = 1$. Households have two sources of income at $t = 1$. First, households in all countries receive the same tradable endowment y_T . Second, households in each country own an equal share of all firms in the same country, and receive their profits θ_{NT}^i .

At $t = 1$, households can also borrow at union-wide interest rate r_2 in order to finance their consumption. However, there is a borrowing constraint: households can only pledge a fraction $\phi < 1$ of the net present value of the endowments that they receive at $t = 2$. Households in all countries receive identical endowments of y_T units of tradable goods and y_{NT2} units of nontradable goods at $t = 2$. In this period, households consume tradable goods c_{T2}^i and nontradable goods c_{NT2}^i . The price of nontradable goods in country i at $t = 2$ is p_2^i . Therefore, the maximum borrowing at $t = 1$ is:

$$\frac{\phi [y_T + p_2^i y_{NT2}]}{r_2}.$$

Shocks and resource constraints. At $t = 0$, there is a shock $z = (z^N, z^S)$: all households of countries i in region j must purchase $z^i = z^j$ units of the tradable good and burn it.

The shock is designed to replicate a “financial shock” in the real world, where individuals need to put aside goods to compensate for past losses or to raise liquidity buffers. These goods do not contribute to household welfare. The shock has probability distribution:

$$\begin{aligned} z^N &\sim U[0, \bar{z}], z^S = 0 && \text{with probability } \pi \\ z^N = 0, z^S &\sim U[0, \bar{z}] && \text{with probability } \pi \\ z^N = z^S &= 0 && \text{with probability } 1 - 2\pi \end{aligned}$$

Defining $k_{NT}^j = \int_{i \in j} k_{NT}^i di$ and $c^j = \int_{i \in j} c^i di$, the resource constraint at $t = 0$ is:

$$\sum_{j=N,S} k_{NT}^j \leq \sum_{j=N,S} (e^j - c^j - z^j).$$

Defining $c_{Tt}^j = \int_{i \in j} c_{Tt}^i di$, the resource constraints for tradable and nontradable goods at $t = 1$ and $t = 2$ are:

$$\begin{aligned} \sum_{j=N,S} c_{Tt}^j &\leq 2y_T \\ c_{NTt}^i &\leq y_{NTt}^i. \end{aligned}$$

Investment decision. Firms can convert one unit of the tradable good at $t = 0$ into A units of nontradable goods at $t = 1$, earning profits:

$$p_1^i A - r_1.$$

Each country has firms of varying levels of productivity A , and the productivity distribution $A \sim U[0, 1]$ is identical across countries. At $t = 0$, firms choose whether to operate or not. Only firms with sufficiently high productivity $A > \frac{r_1}{p_1^i}$ decide to operate. Investment, output and profits in country i are:

$$\begin{aligned} k_{NT}^i &= \mu \left(1 - \frac{r_1}{p_1^i} \right) \\ y_{NT1}^i &= \frac{\mu}{2} \left[1 - \left(\frac{r_1}{p_1^i} \right)^2 \right] \\ \theta_{NT}^i &= p_1^i y_{NT1}^i - r_1 k_{NT}^i. \end{aligned}$$

Consumption decision. Households in country i maximize expected utility:

$$\log(c^i) + \mathbb{E}_{t=-1} \left\{ \begin{aligned} &\log(c_{T1}^i) + \nu \log(c_{NT1}^i) \\ &+ \log(c_{T2}^i) + \nu \log(c_{NT2}^i) \end{aligned} \right\}$$

subject to the budget constraint:

$$(c^i + z^i) + \frac{c_{T1}^i + p_1^i c_{NT1}^i}{r_1} + \frac{c_{T2}^i + p_2^i c_{NT2}^i}{r_1 r_2} \leq e^i + \frac{y_T + \theta_{NT}^i}{r_1} + \frac{y_T + p_2^i y_{NT2}}{r_1 r_2}$$

and the borrowing constraint:

$$r_1 (c^i + z^i - e^i) + (c_{T1}^i + p_1^i c_{NT1}^i) - (y_T + \theta_{NT}^i) \leq \frac{\phi [y_T + p_2^i y_{NT2}]}{r_2}.$$

Household consumption is given by:

$$\frac{1}{c^i} = \mathbb{E}_{t=-1} \left\{ \frac{r_1}{c_{T1}^i} \right\}$$

$$p_1^i c_{NT1}^i = \nu c_{T1}^i$$

$$p_2^i c_{NT2}^i = \nu c_{T2}^i.$$

When $\nu > 1$, there is *amplification*: spending on nontradable goods is more volatile than spending on tradable goods. When the borrowing constraint is not binding:

$$c_{T2}^i = r_2 c_{T1}^i,$$

and when the borrowing constraint is binding:

$$r_1 (c^i + z^i - e^i) + (c_{T1}^i + p_1^i c_{NT1}^i) - (y_T + \theta_{NT}^i) = \frac{\phi [y_T + p_2^i y_{NT2}]}{r_2}.$$

Model variant without nontradable goods. Our baseline model includes both tradable and nontradable goods. Later on in this paper, for pedagogic purposes, we will state that some of our results hold in a model variant without nontradable goods. To ensure that the model remains solveable when firms and nontradable goods are removed from all periods of it, and only tradable endowments exist, the reader should imagine replacing investment at $t = 0$ with consumption of tradable goods by households, producing additional utility $\log(c_{T0}^i)$ at $t = 0$.

3.2. Equilibrium Definition

Definition 1 *A rational expectations equilibrium for this model is a set of interest rates $\{r_1(z), r_2(z)\}$, prices $\{p_1^i(z), p_2^i(z)\}$ and allocations $\{c^i, k_{NT}^i(z), y_{NT1}^i(z), c_{T1}^i(z), c_{NT1}^i(z), c_{T2}^i(z), c_{NT2}^i(z)\}$ which satisfy the optimality conditions of households and firms, and the financial union's resource constraints for tradable and nontradable goods.*

Lemma 1 *Households and firms of all countries i in the same region j have the same values of all equilibrium variables.*

In particular, all countries i in region j have identical asset positions in all periods. Following Lemma 1, we save on notation in the remainder of this paper by using the region identifier j to index all variables for country $i \in j$.

3.3. Competitive Equilibrium Allocations

Each country in region $j \in \{N, S\}$ is characterized by:

$$\frac{1}{c^j} = \beta \mathbb{E}_{t=-1} \left\{ \frac{r_1}{c_{T1}^j} \right\}$$

$$c_{T1}^j = y_T - r_1 (c^j + z^j + k_{NT}^j - e^j) + \frac{y_T - c_{T2}^j}{r_2}$$

$$p_1^j c_{NT1}^j = \nu c_{T1}^j$$

$$p_2^j c_{NT2}^j = \nu c_{T2}^j$$

$$k_{NT}^j = \mu \left(1 - \frac{r_1}{p_1^j} \right)$$

$$y_{NT1}^j = \frac{\mu}{2} \left[1 - \left(\frac{r_1}{p_1^j} \right)^2 \right]$$

When the borrowing constraint is not binding:

$$c_{T2}^j = r_2 c_{T1}^j,$$

and when the borrowing constraint is binding:

$$c_{T1}^j = y_T - r_1 (c^j + z^j + k_{NT}^j - e^j) + \frac{\phi [y_T + p_2^j y_{NT2}^j]}{r_2}.$$

Notice that in equilibrium, the household's budget and borrowing constraints reduce to equations written purely in terms of tradable goods. Nontradable goods enter the equations only indirectly, in the form of interest payments $r_1 k_{NT}^j$ accrued on the investment k_{NT}^j used at $t = 0$ to produce the nontradable goods y_{NT1}^j at $t = 1$.

Finally, the resource constraints for tradable and nontradable goods are satisfied with equality:

$$\sum_{j=N,S} (c^j + z^j + k_{NT}^j) = \sum_{j=N,S} e^j$$

$$\sum_{j=N,S} c_{T1}^j = 2y_T$$

$$c_{NTt}^i = y_{NTt}^i \text{ for } t = 1 \text{ and } t = 2.$$

Walras' Law allows us to ignore the union-level resource constraint for tradable goods at $t = 2$.

To facilitate the discussion later on in this paper, we define auxiliary variables measuring the equilibrium borrowing of countries in each region j in each period t :

$$b_{-1}^j = c^j - e^j$$

$$b_0^j = c^j + z^j + k_{NT}^j - e^j$$

$$b_1^j = c_{T1}^j + r_1 (c^j + z^j + k_{NT}^j - e^j) - y_T,$$

the equilibrium value of the borrowing limit of countries in each region j at $t = 1$:

$$B_1^j = \frac{\phi [y_T + p_2^j y_{NT2}^j]}{r_2},$$

and the implied rate of return in region j , calculated by comparing the levels of tradable consumption at $t = 1$ and $t = 2$:

$$R_2^j = \frac{c_{T2}^j}{c_{T1}^j}.$$

Lemma 2 $R_2^j > r_2^j$ when the borrowing constraint is binding in region j .

Binding borrowing constraints in region j generate a wedge between the market interest rate r_2^j and the implied rate of return R_2^j .

4. Results

4.1. Heterogeneity and the Impact of Shocks

Lemma 3 *The shock z increases the union-wide interest rate r_1 : $\frac{dr_1}{dz} > 0$.*

In partial equilibrium, the shock z has a direct negative effect on the consumption, investment and welfare of the region that it strikes. In general equilibrium, the shock increases the interest rate r_1 , which has additional indirect effects on the consumption, investment and welfare of both regions. The direction of these indirect effects depends on the asset positions of the regions.

Definition 2 *The degree of heterogeneity H in the financial union is:*

$$H = e^N - e^S \in [0, e^T].$$

Lemma 4 *For all H , $b_{-1}^N < 0$. There exists some $H^* \in (0, e^T)$ independent of the value of the shock z , such that for all $H \in [0, H^*)$, $b_{-1}^S < 0$ and for all $H \in [H^*, e^T]$, $b_{-1}^S \geq 0$.*

Lemma 5 *For $H = z = 0$, $b_0^N = b_0^S = 0$. For $H > 0$ and $z = 0$, $b_0^N < 0$ and $b_0^S > 0$.*

Proposition 1 (Consumption) *A shock $z > 0$ affects consumption of tradable goods at $t = 1$ as follows.*

(i) *Shock in the North, $z^N > 0$. There exists $\hat{H}(z^N) \in (0, H^*)$ such that for all $H \in [0, \hat{H}(z^N))$:*

$$\frac{dc_{T1}^N}{dz^N} < 0 \text{ and } \frac{dc_{T1}^S}{dz^N} > 0,$$

while for all $H \in (\hat{H}(z^N), e^T]$:

$$\frac{dc_{T1}^N}{dz^N} > 0 \text{ and } \frac{dc_{T1}^S}{dz^N} < 0.$$

(ii) *Shock in the South, $z^S > 0$. For all $H \in [0, e^T]$:*

$$\frac{dc_{T1}^N}{dz^S} > 0 \text{ and } \frac{dc_{T1}^S}{dz^S} < 0.$$

Proposition 2 (Investment) *For the region j where the shock $z > 0$ decreases consumption c_{T1}^j , the shock also decreases investment k_{NT}^j . For the region j where the shock $z > 0$ increases consumption c_{T1}^j , the effect on investment is ambiguous. There may exist some subset of $[0, e^T]$ where the increase in consumption c_{T1}^j is sufficiently large that investment k_{NT}^j increases; otherwise, investment decreases.*

Proposition 3 (Welfare) *The higher is H , the more that any shock $z > 0$ hurts the South and the less the shock hurts the North.*

(i) *Shock to the North, $z^N > 0$. There exists $\tilde{H}(z^N) \in [0, \hat{H})$, $\tilde{H}(0) = 0$, $\tilde{H}' > 0$, such that:*

$$\frac{du^S}{dz^N} > 0 \text{ for all } H \in [0, \tilde{H}(z^N)), \text{ and } \frac{du^S}{dz^N} < 0 \text{ for all } H \in (\tilde{H}(z^N), e^T],$$

and there may exist some $\check{H}(z^N) \in (H^, e^T]$ such that:*

$$\frac{du^N}{dz^N} < 0 \text{ for all } H \in [0, \check{H}(z^N)), \text{ and } \frac{du^N}{dz^N} > 0 \text{ for all } H \in (\check{H}(z^N), e^T];$$

otherwise, if such \check{H} does not exist, $\frac{du^N}{dz^N} < 0$ for all $H \in [0, e^T]$.

(ii) Shock to the South, $z^S > 0$. For all $H \in [0, e^T]$:

$$\frac{du^N}{dz^S} > 0 \text{ and } \frac{du^S}{dz^S} < 0.$$

Taken together, these results establish that in a financial union, the distributional effect of a shock depends on the degree of heterogeneity H that prevailed prior to the shock, and not just on which region j is hit by the shock. For a sufficiently high degree of heterogeneity H , the South is always disproportionately hurt in terms of consumption, investment and welfare—whether the shock strikes the North or the South. The shift in emphasis in this paper from the location of the shock to the heterogeneity prior to the shock breaks from the tradition followed in the optimal currency area literature.

We now explain the above results step by step.

Extreme cases. When the degree of heterogeneity $H = 0$, the North and South have identical initial endowments:

$$e^N = e^S,$$

and the location of the shock is the primary determinant of the response to the shock. Figure 2 shows that in this case, a shock in the North $z^N > 0$ decreases welfare u^N in the North and increases welfare u^S in the South.

Households in the North and South consume the same amount $c^N = c^S$ at $t = -1$, and enter $t = 0$ with identical and positive savings (asset positions are $b_{-1}^N = b_{-1}^S < 0$). In the absence of a shock, all the savings are used to finance identical levels of investment $k_{NT}^N = k_{NT}^S$ in both regions.

The shock $z^N > 0$ induces the households of the North to borrow in order to purchase tradable goods to burn at $t = 0$, and at the same time increases the interest rate r_1 on this borrowing. The resulting higher burden of interest payments at $t = 1$ means that the shock decreases c_{T1}^N and increases c_{T1}^S . The South becomes a net lender to the North (net across households and firms).

Within each region, the consumption levels of tradable goods at $t = 1$ and $t = 2$ are identical, unless the borrowing constraint binds for the North. The constraint may bind for large shocks because consumption smoothing motives increase the desired borrowing b_1^N , while the negative income effect decreases the nontradable goods price p_2^N and therefore the borrowing limit B_1^N . If the constraint binds, c_{T1}^N decreases even more steeply, c_{T2}^N remains at a fixed value, and the interest rate r_2 dips below unity.

At $t = 1$, households in the North decrease their spending on tradable and nontradable goods in tandem (if $\nu > 1$, the decrease in spending on nontradable goods is an *amplification* of the decrease in spending on tradable goods). In anticipation of this, and because of the higher interest rate r_1 , firms in the North reduce investment k_{NT}^N at $t = 0$. Therefore, the shock decreases both c_{T1}^N and c_{NT1}^N . Both demand and supply of the nontradable good

decrease, so the effect on p_1^N is ambiguous; for the chosen parameters, p_1^N increases. If the borrowing constraint binds, k_{NT}^N decreases even more steeply. For the chosen parameters, firms in the South also decrease investment k_{NT}^S as a result of the higher interest rate, but the decline is more gentle than in the North.

For $H = 0$, the effect of a shock in the South $z^S > 0$ is the same in magnitude as a shock in the North, but with the region labels reversed. We do not repeat the analysis for that case.

When the degree of heterogeneity $H = e^T$, the North and South have very different initial endowments:

$$e^N = e^T \text{ and } e^S = 0,$$

and the degree of heterogeneity that prevailed prior to the shock is the primary determinant of the response to the shock. For both $z^N > 0$ and $z^S > 0$, the South is disproportionately hurt.

Figure 3 shows that a shock in the North $z^N > 0$ decreases welfare in the South u^S steeply, and welfare in the North u^N gradually.

At $t = -1$, $c^N > c^S$ and households in the South enter $t = 0$ with debt $b_{-1}^S > 0$ while households in the North enter with assets $b_{-1}^N < 0$. Whichever value of the shock $z^N > 0$ is realized, the South must borrow more at $t = 0$: $b_0^S > b_{-1}^S$. In the absence of a shock, $k_{NT}^N > k_{NT}^S$.

The shock $z^N > 0$ induces the households of the North to use up some savings in order to purchase tradable goods to burn at $t = 0$. The interest rate r_1 also increases. However, from this point on, the dynamics are different when $H = e^T$ relative to $H = 0$. This time, it is the South who bears the brunt of the shock—because the increase in r_1 has a positive effect on the North, who is a net lender, and a negative effect on the South, who is a net borrower. The South must now offer a higher interest rate r_1 at $t = 0$ to finance its inherited debt b_{-1}^S and its new borrowing $b_0^S - b_{-1}^S > 0$. At $t = 1$, the shock actually reduces the borrowing of the North b_1^N and increases the borrowing of the South b_1^S .

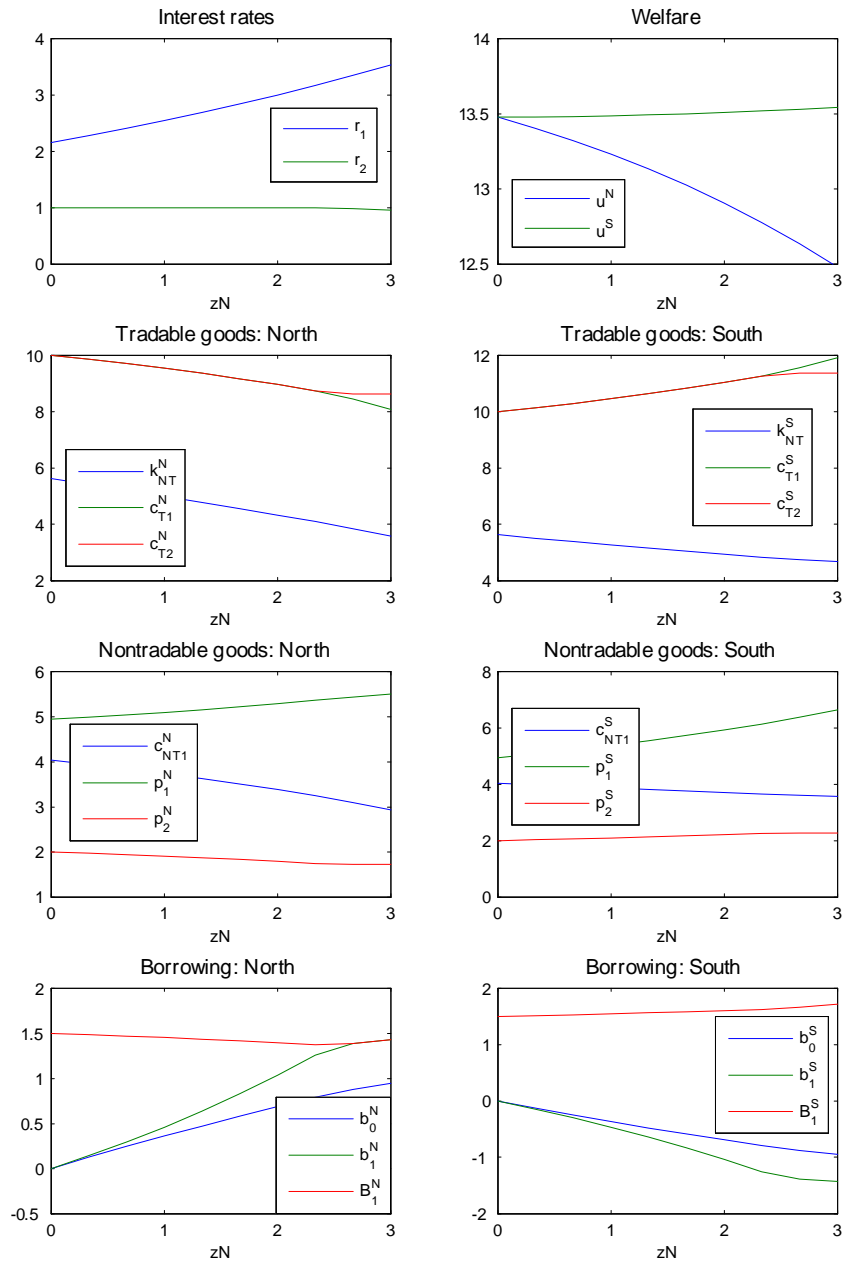
The overall effect is that the shock increases c_{T1}^N and decreases c_{T1}^S . Within each region j , $c_{T1}^j = c_{T2}^j$ except when the borrowing constraint binds. For $H = e^T$, it binds for the South after large shocks, not for the North. If the constraint binds, c_{T1}^S decreases even more steeply, c_{T2}^S remains at a fixed value, and the interest rate r_2 dips below unity.

Notice that for a given level of ϕ , the borrowing constraint is more likely to be binding for $H = e^T$ than for $H = 0$, because in the former case, one of the regions (the South) enters $t = 0$ with a higher level of debt b_0^j . Therefore, for presentational purposes, we set a lower value of ϕ for $H = e^T$.

Firms decrease investment k_{NT}^j in both regions j , but this time more steeply in the South. If the borrowing constraint binds, k_{NT}^S decreases even more steeply.

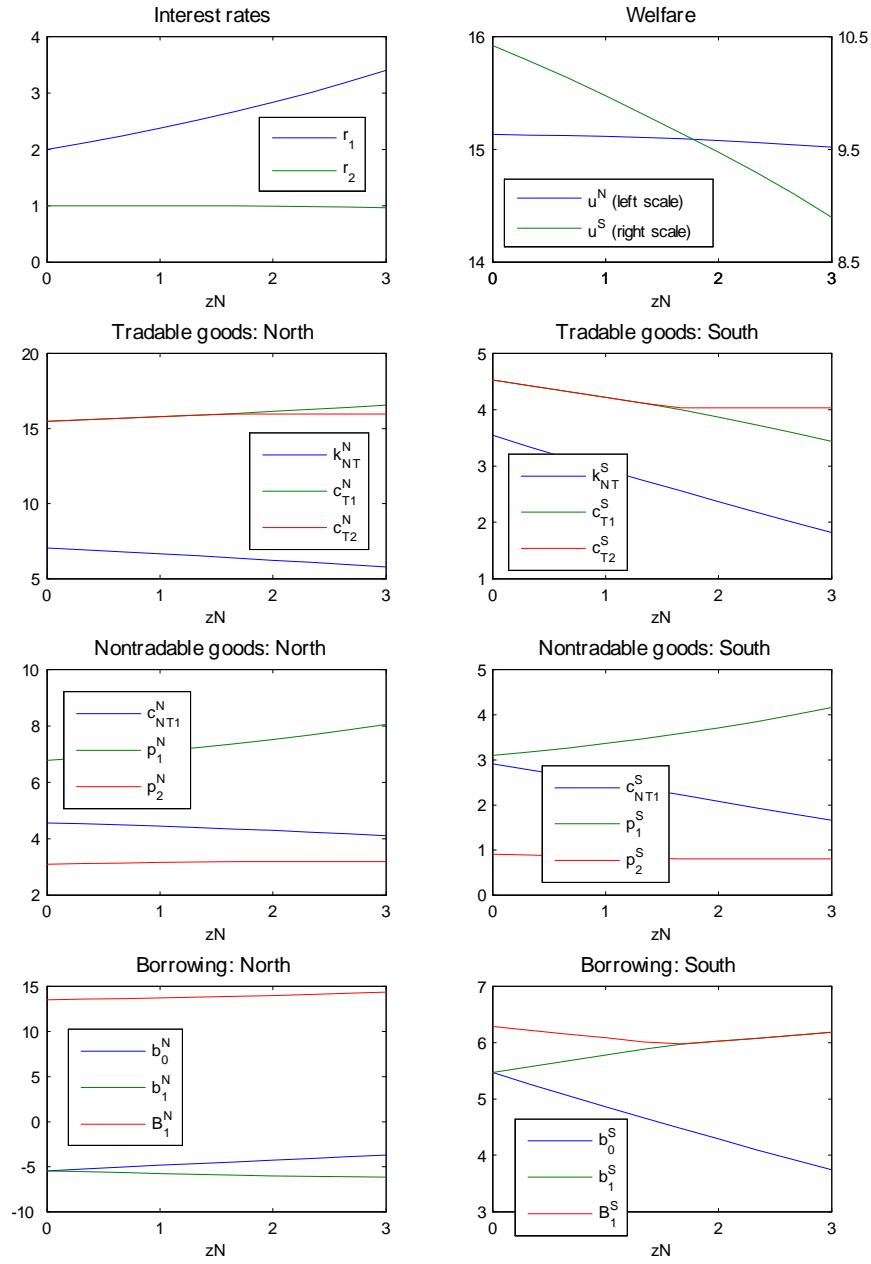
Figure 4 shows the effect of a shock in the South $z^S > 0$ for $H = e^T$. The direct and indirect effects of the shock $z^S > 0$ now both fall on the South. The welfare of the South

Figure 2: $H = 0$, Shock in the North z^N



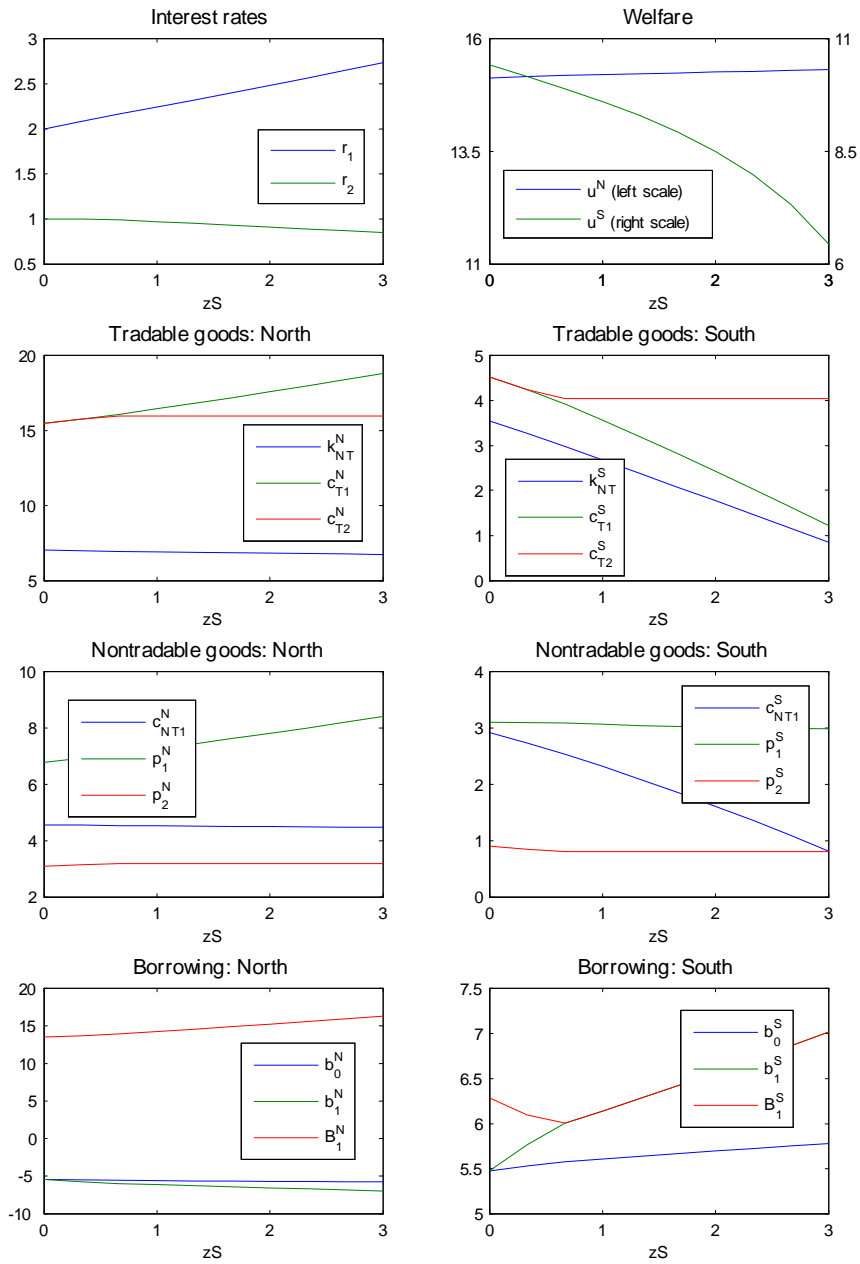
$e^N = e^S = 10$, $v = 2$, $\phi = 0.05$, $\pi = 0.1$, $\mu = 10$, $z^{\text{bar}} = 3$, $y_T = y_{NT2} = 10$
 Period $t=1$: $c^N = c^S = 4.4$, $b_{-1}^N = b_{-1}^S = -5.6$

Figure 3: $H = e^T$, Shock in the North z^N



$e^N = 20$, $e^S = 0$, $v = 2$, $\phi = 0.33$, $\pi = 0.1$, $\mu = 10$, $z^{\text{bar}} = 3$, $y_T = y_{NT2} = 10$
 Period $t = -1$: $c^N = 7.5$, $b_{-1}^N = -12.5$, $c^S = 1.9$, $b_{-1}^S = 1.9$

Figure 4: $H = e^T$, Shock in the South z^S

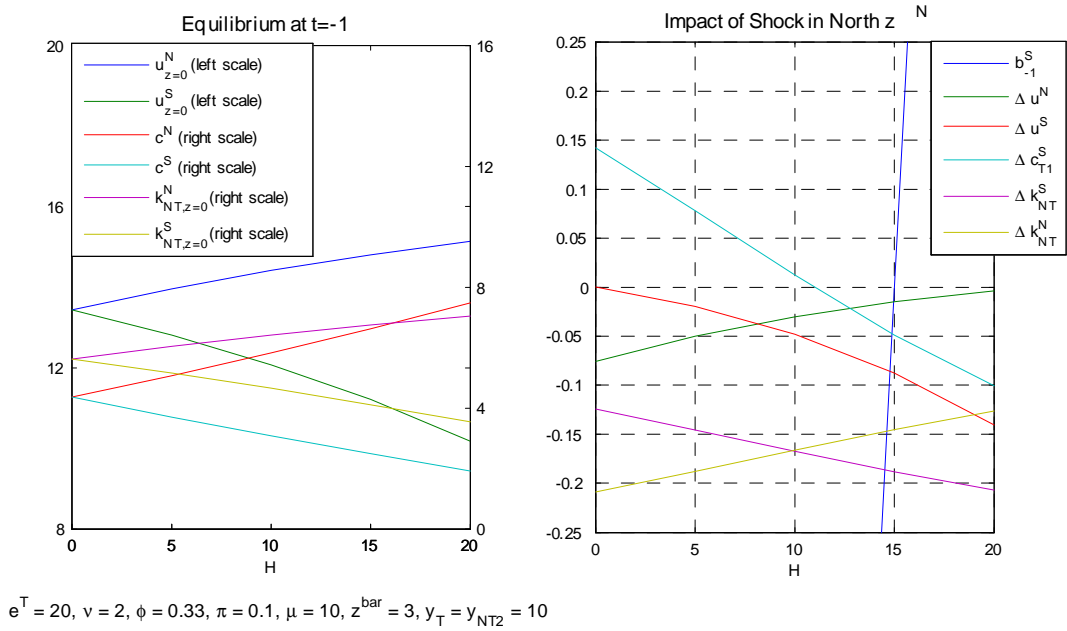


$e^N = 20$, $e^S = 0$, $v = 2$, $\phi = 0.33$, $\pi = 0.1$, $\mu = 10$, $z^{\text{bar}} = 3$, $y_T = y_{NT2} = 10$
 Period $t = -1$: $c^N = 7.5$, $b_{-1}^N = -12.5$, $c^S = 1.9$, $b_{-1}^S = 1.9$

u^S decreases while the welfare of the North u^N increases. The effects on consumption and investment in the South are qualitatively similar to the case of $z^N > 0$ and $H = e^T$, but all the effects are quantitatively more negative. In the North, consumption and investment also behave similarly in qualitative terms, but the effects are quantitatively more positive, and the asset position of households does not deteriorate at $t = 0$.

Intermediate cases. For a sufficiently high degree of heterogeneity $H \in [0, e^T]$, the distributional dynamics of a financial union in response to a shock z switch qualitatively from the kind of response recorded above for $H = 0$, to the kind of response recorded above for $H = e^T$. Figure 5 illustrates propositions 1, 2 and 3.

Figure 5: $H \in [0, e^T]$, Shock in the North z^N



The left panel of figure 5 shows for different degrees of heterogeneity $H \in [0, e^T]$, the levels of consumption c^j and for $z = 0$, the levels of investment k_{NT}^j and welfare u^j . The higher is H , the higher are consumption, investment and welfare in the North and the lower they are in the South.

The right panel of figure 5 illustrates how the qualitative switch from the shock response for $H = 0$ to the shock response for $H = e^T$ occurs. We consider a marginal shock to the North $z^N > 0$ in a financial union with $e^T = 20$. The debt of households in the South b_{-1}^S is shown as a function of H . In addition, the effect of the shock on a range of variables is illustrated, using the Δ prefix, for each value of H .

First, the higher is H , the more that any shock $z^N > 0$ hurts the South and the less the shock hurts the North:

$$\frac{d}{dH} (\Delta u^S) < 0 \text{ and } \frac{d}{dH} (\Delta u^N) > 0.$$

Second, we turn to the thresholds for $H \in [0, e^T]$ described in the propositions. For $H > H^* = 15.0$, which means $e^N > 17.5$ and $e^S < 2.5$, households in the South enter $t = 0$ with debt $b_{-1}^S > 0$. For $H > \hat{H} = 11.0$, which means $e^N > 15.5$ and $e^S < 4.5$, the shock in the North at $t = 0$ decreases c_{T1}^S and increases c_{T1}^N at $t = 1$. Investment k_{NT}^j , and therefore the consumption of nontradable goods c_{NT1}^j , decrease in both North and South for all $H \in [0, e^T]$. Since we consider a marginal shock $z^N > 0$, and we know that $\tilde{H}(0) = 0$, $\Delta u^S < 0$ for all $H \in (0, e^T]$. Finally, for the chosen parameters, \check{H} does not exist so $\Delta u^N < 0$ for all $H \in [0, e^T]$.

Noncontingent debt. When the borrowing constraint is not binding, $r_2 = 1$ and the levels of tradable consumption at $t = 1$ and $t = 2$ are:

$$c_{T1}^j = c_{T2}^j = \frac{1}{2} [r_1 (e^j - c^j - z^j - k_{NT}^j) + 2y_T].$$

Then the main results in this subsection can be illustrated using a collapsed version of the model between $t = 0$ and $t = 1$, taking b_{-1}^j as given. A shock in the North $z^N > 0$ decreases $c_{T1}^S = c_{T2}^S$ if and only if:

$$b_{-1}^S = c^S - e^S > -k_{NT}^S + r_1 \left| \frac{dk_{NT}^S}{dr_1} \right|,$$

where the derivative $\frac{dk_{NT}^S}{dr_1}$ takes into account that changes in the interest rate r_1 affect k_{NT}^S both directly through changes in repayments and indirectly through changes in p_1^S . This expression defines the set $(\hat{H}(z^N), e^T]$. The right hand side of the expression is negative. The value of z^N may determine \hat{H} by affecting r_1 and k_{NT}^S .

The assumptions that debt at $t = -1$ is noncontingent, and that households in the South do not receive an additional endowment at $t = 0$ large enough that they can fully repay all their inherited debt, are sufficient to ensure that $\hat{H} < e^T$ and $H^* < e^T$. On the other hand, the relationship between the degree of heterogeneity $H \in [0, e^T]$ and the distributional dynamics of a financial union in response to a shock z is not much altered by the presence of the borrowing constraint.

Modeling decisions. To allow a “core-periphery” structure to endogenously emerge within the union before shocks strike, the shock strikes at $t = 0$ after some consumption and debt accumulation decisions have already been made. This allows us to break the implicit assumption in the optimal currency area literature that countries are identical before shocks strike. Therefore, for the financial shocks that we are interested in, instead of solely considering asymmetries after shocks strike, we can show how the effects of shocks are related to the degree of heterogeneity that existed in advance of the financial shocks being realized.

Our inclusion of investment in the model enriches the dynamics. In a model variant

without nontradable goods and the associated investment, $\hat{H} = H^*$. The right hand side of the expression above becomes zero, so a shock in the North $z^N > 0$ decreases $c_{T1}^S = c_{T2}^S$ if and only if the households in the South enter $t = 0$ with debt $b_{-1}^S > 0$. In a model with investment to produce nontradable goods, the production function for nontradable goods determines the range of asset positions for which the South is most vulnerable. For the knife-edge Cobb-Douglas production function, again $\hat{H} = H^*$. With the investment specification described in section 3, a shock in the North $z^N > 0$ decreases $c_{T1}^S = c_{T2}^S$ even when households in the South enter $t = 0$ with $b_{-1}^S < 0$, as long as their asset position is small. Therefore, $\hat{H} < H^*$.

Our model is designed to produce results which are robust to more general environments. Introducing consumption by households at $t = 0$, to make the periods more symmetric, adds an additional variable without changing the qualitative results. Similarly, introducing a tradable endowment at $t = 0$ does not alter the results as long as all the prior debt b_{-1}^S cannot always be fully repaid for all $H \in [0, e^T]$. The results are also qualitatively robust to replacing tradable and nontradable endowments with intra-period production, and adding investment in each period to produce tradable and nontradable goods in the next. Finally, introducing heterogeneity in the tradable endowments for $t \geq 1$ changes the precise values of \check{H} , $\tilde{H}(z^N)$, \hat{H} and H^* , but not their relative ordering.

4.2. Impact of Binding Borrowing Constraints

First, we hold the degree of heterogeneity H fixed and outline the effect when the borrowing constraint binds in a given region j . Then, we establish how the degree of heterogeneity H determines in which region j the borrowing constraint binds, after any shock $z \geq 0$.

Proposition 4 (Consumption and Investment) *For a given value of the shock $z \geq 0$, when the borrowing constraint binds in region j :*

$$\begin{aligned} k_{NT}^j &< k_{NT}^{j*}, \\ c_{T1}^j &< c_{T1}^{j*}, c_{NT1}^j < c_{NT1}^{j*}, \\ c_{T2}^j &> c_{T2}^{j*}, \end{aligned}$$

where a starred variable denotes the value of the variable for the same value of the shock z but without the borrowing constraint. The opposite inequalities hold for the other region $-j$.

Proposition 5 (Interest Rates) *For a given value of the shock $z \geq 0$, when the borrowing constraint binds in any region $j \in \{N, S\}$:*

$$r_1 < r_1^*,$$

$$r_2 < r_2^* = 1,$$

$$R_2^j > R_2^{j*} = 1 \text{ and } R_2^{-j} = r_2 < R_2^{-j*} = 1.$$

where a starred variable denotes the value of the variable for the same value of the shock z but without the borrowing constraint.

Proposition 6 (Welfare) *For all shocks $z \geq 0$, when the borrowing constraint binds in region j :*

$$u^{-j} < u^{-j*}.$$

There may exist some $\hat{z} \in [0, \bar{z})$ such that:

$$u^j > u^{j*} \text{ for all } z \in (0, \hat{z}), \text{ and } u^j < u^{j*} \text{ for all } z \in (\hat{z}, \bar{z}];$$

otherwise, if such \hat{z} does not exist:

$$u^j > u^{j*} \text{ for all } z \in [0, \bar{z}].$$

Lemma 6 *For any degree of heterogeneity H , there exists $\tilde{z} \in [0, e^T]$ such that $b_{-1}^N + \tilde{z} = b_{-1}^S$. Define $\check{z} = \min\{\tilde{z}, \bar{z}\}$.*

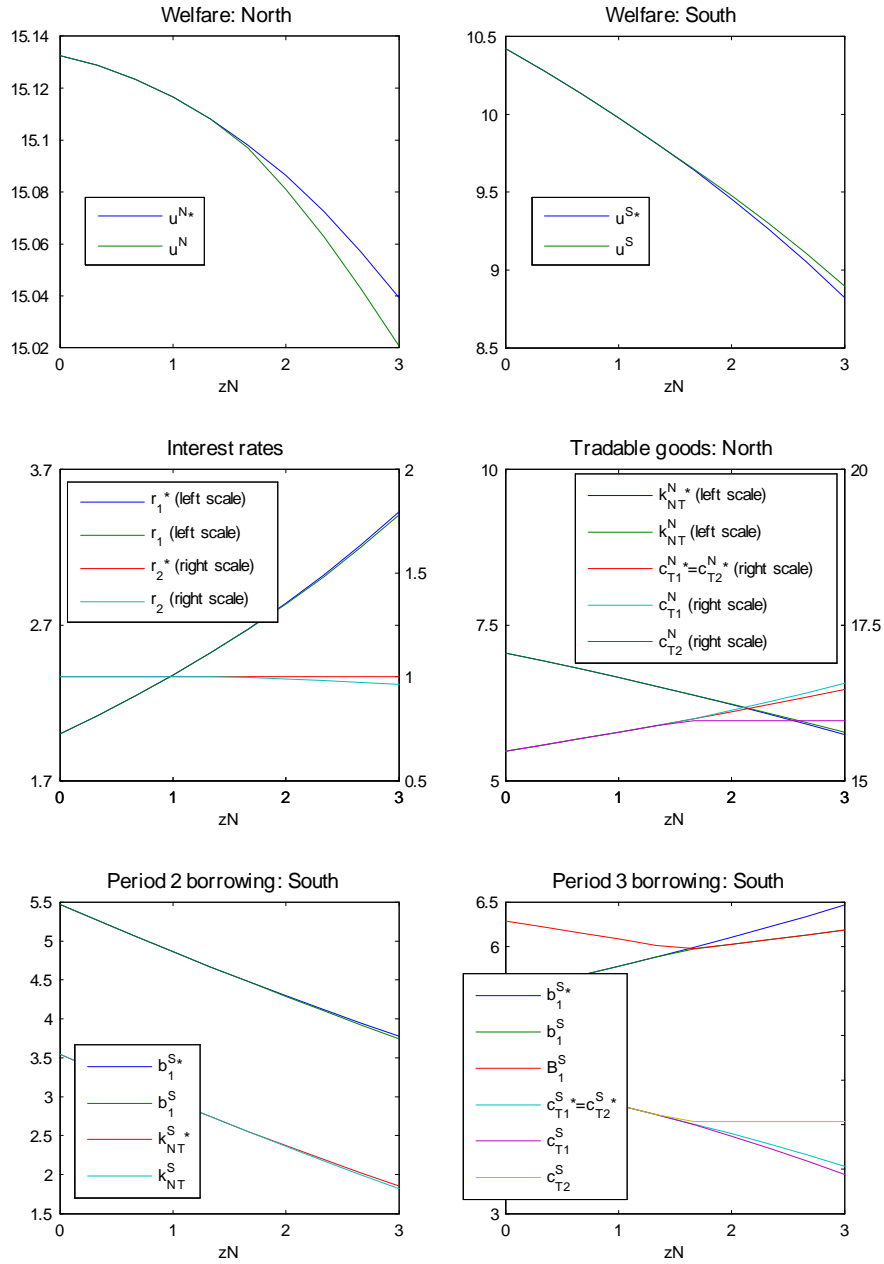
Proposition 7 (Heterogeneity) *For all $H \in [0, \hat{H})$, a shock $z^j \in [0, \bar{z}]$ in region j causes the borrowing constraint to bind in the same region j if ϕ is sufficiently low. For all $H \in (\hat{H}, e^T]$, a shock in any region, $z^N \in [0, \check{z}]$ or $z^S \in [0, \bar{z}]$, causes the borrowing constraint to bind in the South if ϕ is sufficiently low; the shock does not cause the borrowing constraint to bind in the North.*

Figures 6 and 7 are the analogs of figures 4 and 5. Instead of comparing the response to a shock for $H = 0$ and $H = e^T$, now we fix $H = e^T$. For any given value of the shock $z \geq 0$, we compare the equilibria with and without a binding borrowing constraint.

Figures 6 and 7 show that consistent with proposition 7, large shocks in both North and South cause the borrowing constraint to bind in the South. Consistent with propositions 4 and 5, and with the discussion in the previous subsection, the borrowing constraint decreases the levels of investment k_{NT}^S at $t = 0$, and tradable and nontradable consumption levels c_{T1}^S and c_{NT1}^S at $t = 1$ in the South, relative to the equilibrium without the constraint. It also increases the level of tradable consumption c_{T2}^S at $t = 2$ in the South. The interest rates r_1 and r_2 decrease in both periods—the latter below unity—inducing investment and consumption to move in the opposite directions in the North relative to the equilibrium without the constraint. A wedge opens up between the market interest rate $r_2^j < 1$ and the implied rate of return in the South $R_2^j > 1$.

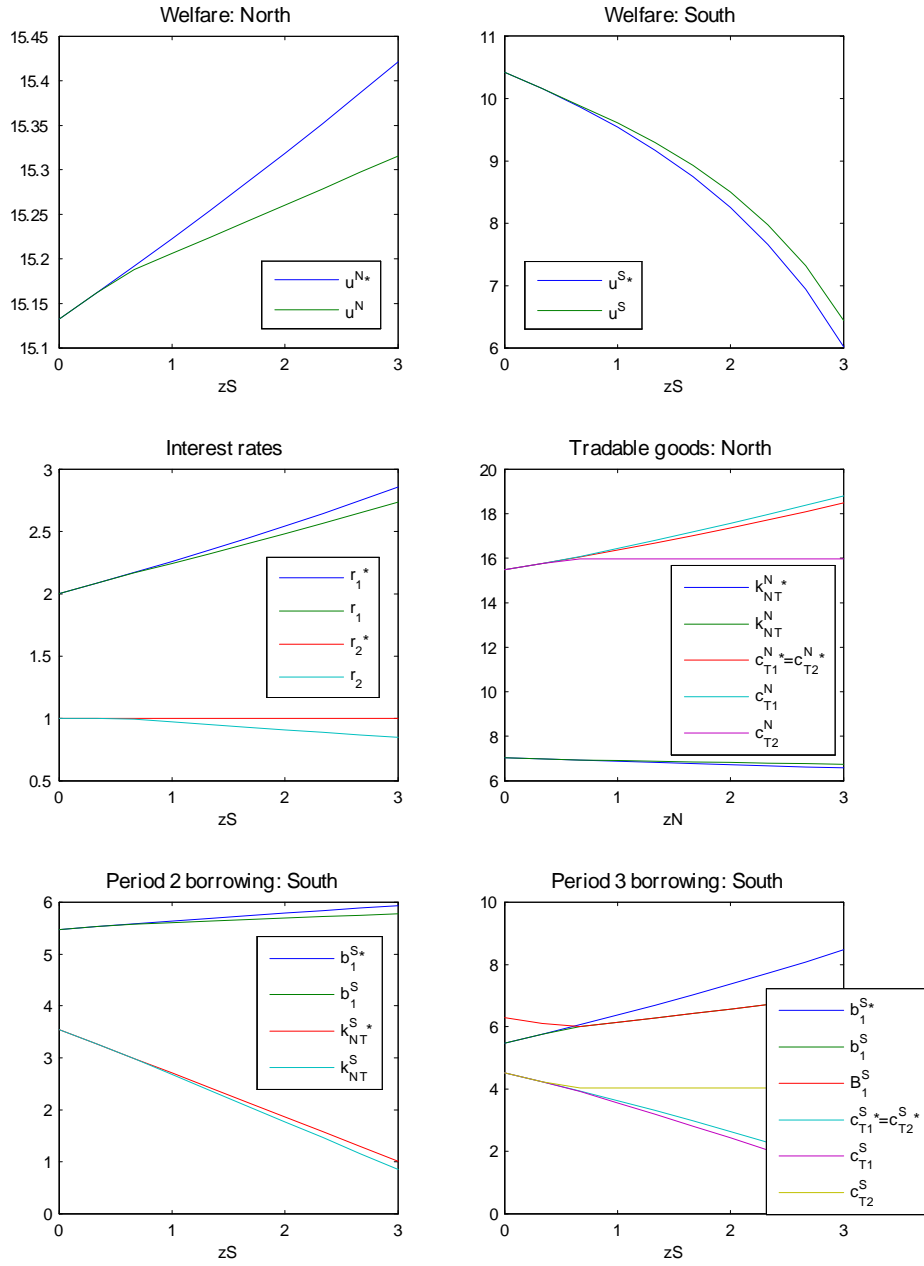
Consistent with proposition 6, the binding borrowing constraint in the South makes the North worse off and the South better off.

Figure 6: Binding Constraint with $H = e^T$, Shock in the North z^N



$e^N = 20, e^S = 0, v = 2, \phi = 0.33, \pi = 0.1, \mu = 10, z^{\text{bar}} = 3, y_T = y_{NT2} = 10$
 Period $t = -1$: $c^N = 7.5, b_{-1}^N = -12.5, c^S = 1.9, b_{-1}^S = 1.9$

Figure 7: Binding Constraint with $H = e^T$, Shock in the South z^S



$e^N = 20, e^S = 0, v = 2, \phi = 0.33, \pi = 0.1, \mu = 10, z^{\text{bar}} = 3, y_T = y_{NT2} = 10$
 Period $t = -1$: $c^N = 7.5, b_{-1}^N = -12.5, c^S = 1.9, b_{-1}^S = 1.9$

Intertemporal substitution margin. Taking the shock $z \geq 0$, the degree of heterogeneity $H \in (\hat{H}, e^T]$ and debt level b_{-1}^j as given, the main results in this subsection can be illustrated using a collapsed version of the model between $t = 1$ and $t = 2$:

$$c_{T1}^S = \min \left\{ \frac{1}{2} [r_1 (e^S - c^S - z^S - k_{NT}^S) + 2y_T], y_T - r_1 (c^S + z^S + k_{NT}^S - e^S) + \frac{\phi [y_T + p_2^S y_{NT2}]}{r_2} \right\},$$

$$c_{T2}^S = \max \left\{ \frac{1}{2} [r_1 (e^S - c^S - z^S - k_{NT}^S) + 2y], \frac{1 - \phi}{1 + \nu} [y_T + p_2^S y_{NT2}] \right\},$$

$$R_2^S = \frac{c_{T2}^S}{c_{T1}^S},$$

such that when the constraint is binding, $r_2 < 1$, $R_2^S > 1$ and the variables c_{Tt}^S take the values on the right sides of the above brackets. Holding the endowments fixed, the constraint binds in the South when r_1 , z^N and z^S are high, and ϕ and p_2^S are low.

The two cycles of borrowing and lending interact. A shock z at $t = 0$ increases the interest rate r_1 between $t = 0$ and $t = 1$, which increases the cost of borrowing for the South between $t = 0$ and $t = 1$ and increases the desired borrowing of the South between $t = 1$ and $t = 2$. This in turn makes the borrowing constraint more likely to bind. Relative to the equilibrium without the constraint, a binding constraint decreases both the interest rate r_2 between $t = 1$ and $t = 2$, and the interest rate r_1 between $t = 0$ and $t = 1$. The constraint distorts the intertemporal substitution margin of the South R_2^S between $t = 1$ and $t = 2$ relative to the interest rate r_2 , while $R_2^N = r_2$ continues to hold in the North.

When the value of ϕ is such that the borrowing constraint is just binding, the decrease in consumption c_{T1}^S has a second-order negative effect on welfare in the South. At the same time, the decrease in r_2 reduces the cost of all infra-marginal borrowing by the South, which has a first-order positive effect on welfare in the South. Therefore, in net terms, welfare in the South u^S increases. Intuitively, each country in the South is hurt by its own decrease in consumption at $t = 1$, but benefits from binding constraints in all other countries in the South, which reduce the interest rate r_2 and therefore repayments at $t = 2$. If the value of ϕ is such that the borrowing constraint is strongly binding, the marginal effect of the decrease in consumption at $t = 1$ may dominate the infra-marginal effect. Therefore, if the constraint is strongly binding, the welfare of the South u^S may decrease relative to the equilibrium without the constraint.

The North suffers both from a change in the time profile of consumption between $t = 1$ and $t = 2$, and from the decrease in the returns on its saving, r_1 and r_2 . Therefore, the binding constraint in the South decreases the welfare of the North u^N relative to the equilibrium without the constraint.

Modeling decisions. The above propositions hold in a model variant without nontradable goods, except that for this variant, $r_1 = r_1^*$ even when the constraint binds. We

add nontradable goods in order to derive two additional results. First, for the production function of nontradable goods we have specified, the binding constraint between $t = 1$ and $t = 2$ decreases not just r_2 but also r_1 , which means that when the constraint binds in the South, it hurts the North in all prior periods, even in the absence of borrowing constraints in those prior periods. Second, a shock $z > 0$ now not only increases the desired borrowing of the South b_1^S , but also decreases the price of nontradable goods p_2^S at $t = 2$. Therefore, with nontradable goods, the shock decreases the borrowing limit B_1^S through a *pecuniary externality*.

Our orchestration of the precise sequence of investment and constraints is not accidental: we have tried to design a parsimonious model to produce results which are robust to more general environments. Our results are qualitatively unaltered even if—as is more plausible—there is a limited endowment in every period, consumption of tradable and nontradable goods in every period subject to constraints, and investment in each period subject to constraints to produce both tradable and nontradable goods in the next. As in the previous subsection, our results are also qualitatively robust to introducing some heterogeneity in the tradable endowments for $t \geq 1$.

Nevertheless, we choose to reduce this more general model to a simpler setup, with investment to produce nontradable goods followed by consumption subject to borrowing constraints, in order to highlight some key relationships from the general model itself. First, relative to investment to produce tradable goods, investment to produce nontradable goods in country i is more sensitive to a constraint-induced collapse in country i 's consumption demand, because nontradable output cannot be exported. Second, making investment also subject to constraints would mix the negative impacts of the constraints on credit demand and supply within every period. Instead, we highlight the impact on credit supply at $t = 1$; we ignore investment in this period because it would respond in a similar manner to consumption. And we highlight the impact on credit demand at $t = 0$, when investment and the interest rate r_1 decrease even if the constraint binds and reduces output only in the next period $t = 1$.

5. Policy Interventions

5.1. Social Planner, Individual Countries and Coalitions

Our model features two market imperfections. First, debt between $t = -1$ and $t = 0$ is noncontingent. Second, there is a borrowing constraint, with an associated pecuniary externality, between $t = 1$ and $t = 2$.

Definition 3 *The laissez-faire equilibrium is the competitive equilibrium of subsection 3.3—without any additional intervention.*

The constrained social planner serves as an omnipotent benchmark in this paper. This planner maximizes a weighted sum of welfares of all countries i in both regions j , by making transfers of tradable goods between households in all countries in all periods contingent on the shock z ; households and firms take the transfers as given and maximize utility and profits as in subsection 3.2. Households in country i receive x_{SPt}^i in period t .

Lemma 7 *Households in all countries i in the same region j receive the same transfers: $x_{SPt}^i = x_{SPt}^j$ for all $i \in j$ and for all periods t .*

Proposition 8 (Social Planner) *The social planner fully mitigates both market imperfections through the following actions after a shock $z > 0$.*

(i) *Noncontingent debt. If the shock z decreases c_{T1}^j in region j in the laissez-faire equilibrium:*

$$\frac{dx_{SPt}^j}{dz} > 0, \frac{dx_{SPt}^{-j}}{dz} < 0 \text{ for some } t \geq 0.$$

(ii) *Borrowing constraint. If the shock z causes the constraint to bind in region j in the laissez-faire equilibrium:*

$$\frac{dx_{SPt}^j}{dz} > 0, \frac{dx_{SPt}^{-j}}{dz} < 0 \text{ for } t = 0 \text{ and/or } t = 1.$$

Corollary 1 (Heterogeneity) *For all $H \in [0, \hat{H})$, $\frac{dx_{SPt}^j}{dz} > 0$, $\frac{dx_{SPt}^{-j}}{dz} < 0$ for some $t \geq 0$ if region j is the region hit by the shock. For all $H \in (\hat{H}, e^T]$, $\frac{dx_{SPt}^S}{dz} > 0$, $\frac{dx_{SPt}^N}{dz} < 0$ for some $t \geq 0$ whichever region j is hit by the shock.*

Next, we turn to what happens in a financial union without a social planner.

Each country i has a government, who can act individually or together with other governments from any subset of countries. The government's feasible actions vary in the following subsections, but in all cases, the action space faces a common limitation which ensures that governments cannot easily get around the noncontingent feature of debt.

The government of country i maximizes the welfare of the households in country i . It cannot commit at $t = -1$ to make its actions at $t \geq 0$ contingent on the shock z .

Lemma 8 *If borrowing constraints are not binding in any region j , governments do not undertake any actions at $t \geq 0$ in response to the shock z .*

Lemma 9 *If borrowing constraints are binding in any region j , governments may undertake actions at $t \geq 0$ in response to the shock z .*

Lemmas 8 and 9 outline, in broad terms, under which circumstances governments may wish to act. Conditional on the shock $z \geq 0$ already being realized, the equilibrium

from $t = 0$ onward is Pareto efficient if no constraints bind. Unlike the social planner, governments do not individually extend assistance to countries in different regions in order to offset the income effects of a shock, because any such assistance can only make one country/region better off by making the other country/region worse off.

On the other hand, if the borrowing constraint binds, then there may exist feasible actions which make both countries/regions better off. The existence of such actions depends on the precise feasible space considered, which will vary in the following subsections. Since the borrowing constraint is central, we will consider interventions in the following sections which can alter the profile of consumption between periods $t = 1$ and $t = 2$.

We can make one observation immediately.

Proposition 9 (Individual Governments) *Suppose that borrowing constraints bind in region j after a shock $z \geq 0$ and that governments in the union possess a non-empty action space. Consider a policy combination which makes a single country $i \in j$ marginally better off and increases its consumption c_{T1}^i relative to the laissez-faire equilibrium, when the policy combination is implemented by either the government of country i on its own, or together with a single country in region $-j$. Then if all countries in region j undertake the same policy, all countries in region j become worse off than in the laissez-faire equilibrium.*

Each country i is small relative to the financial union. Therefore, acting individually, each government undertakes actions which improve the welfare of households in their country under the correct assumption that the interest rates r_1 and r_2 remain unchanged. But if a positive measure of the countries within the financial union undertakes these actions, and if borrowing constraints bind in some region j , then r_1 and r_2 do indeed change. In particular, actions which relax the constraint in region j also cause an increase in r_1 and r_2 . The net welfare gain for region j aggregates the welfare gain of the government's actions under the fixed interest rate assumption, and the first-order welfare loss from the higher interest burden. If the former welfare gain is marginal, it is more than offset by the latter welfare loss.

If governments from a positive measure of countries act together, they internalize the impact of their actions on the interest rates r_1 and r_2 . In the following subsections, we restrict attention to policy interventions whereby governments act within coalitions—in particular, regional coalitions of North and South.

Definition 4 *A coalition is a set of governments who undertake the same actions and who negotiate as a group with other governments.*

Definition 5 *A regional coalition j is a coalition of the governments from all countries in the same region j .*

From this point on, we also restrict attention to degrees of heterogeneity $H \in (\hat{H}, e^T]$, such that in the absence of any interventions, if any region j suffers a binding borrowing constraint, it is the South.

5.2. Unconditional Gifts

In this subsection, the government of country i is allowed to undertake the following actions at $t = 1$: lump-sum taxes and transfers on the tradable endowments of households in country i , and making and receiving transfers of tradable goods to and from governments in other countries $-i$, subject to the budget constraint that the total net transfer by each government is zero. All actions must be announced at $t = 0$. The transfers allowed between countries are unconditional gifts: they cannot be made contingent on any future actions, since no future actions are available.

The unconditional gift from the Northern coalition to the Southern coalition at $t = 1$ is x_{NS} . The lump-sum tax on households in the North and the lump-sum transfers to households in the South are both equal to x_{NS} . Households and firms take the transfers as given and maximize utility and profits as in subsection 3.2.

Lemma 10 *The unconditional gift x_{NS} changes the equilibrium equations in the North:*

$$c_{T1}^N = y_T - x_{NS} - r_1 (c^N + z^N + k_{NT}^N - e^N) + \frac{y_T - c_{T2}^N}{r_2},$$

$$b_1^N = c_{T1}^N + r_1 (c^N + z^N + k_{NT}^N - e^N) - y_T + x_{NS},$$

and in the South:

$$c_{T1}^S = y_T + x_{NS} - r_1 (c^S + z^S + k_{NT}^S - e^S) + \frac{y_T - c_{T2}^S}{r_2},$$

$$b_1^S = c_{T1}^S + r_1 (c^S + z^S + k_{NT}^S - e^S) - y_T - x_{NS} \text{ and } B_1^S = \frac{\phi [p_2^S y_{NT2} + y_T]}{r_2}.$$

As a benchmark, we allow the gift to affect equilibrium variables for $t \geq 0$, but we ignore the impact of the gift at $t = 1$ on equilibrium variables at $t = -1$, before the shock is realized.

Definition 6 *The ex post welfare u_{012}^i of country i is the welfare of country i allowing the values of equilibrium variables to vary for periods $t \geq 0$, while keeping all variables at $t = -1$ fixed at the values they take in the laissez-faire equilibrium.*

Lemma 11 *Suppose that in the laissez-faire equilibrium, the constraint binds in the South for $z \in [\underline{z}, \bar{z}]$. Then for $z \in (\underline{z}, \bar{z}]$, an unconditional gift $x_{NS} > 0$ from the Northern*

coalition to the Southern coalition at $t = 1$ has the following effects:

$$\begin{aligned} \frac{dc_{T1}^N}{dx_{NS}} &< 0, \frac{dc_{NT1}^N}{dx_{NS}} < 0, \frac{dp_1^N}{dx_{NS}} < 0, \frac{dk_{NT}^N}{dx_{NS}} < 0, \\ \frac{dc_{T1}^S}{dx_{NS}} &> 0, \frac{dc_{NT1}^S}{dx_{NS}} > 0, \frac{dp_1^S}{dx_{NS}} > 0, \frac{dk_{NT}^S}{dx_{NS}} > 0, \\ \frac{dc_{T2}^N}{dx_{NS}} = \frac{dc_{T2}^S}{dx_{NS}} &= 0, \text{ and } \frac{dr_1}{dx_{NS}} > 0, \frac{dr_2}{dx_{NS}} > 0, \frac{dR_2^S}{dx_{NS}} < 0. \end{aligned}$$

Proposition 10 (Unconditional Gift) *An unconditional gift x_{NS} cannot generate a Pareto improvement. Whether or not the constraint binds in the South, the gift generates:*

$$\frac{du_{012}^N}{dx_{NS}} < 0 \text{ and } \frac{du_{012}^S}{dx_{NS}} > 0.$$

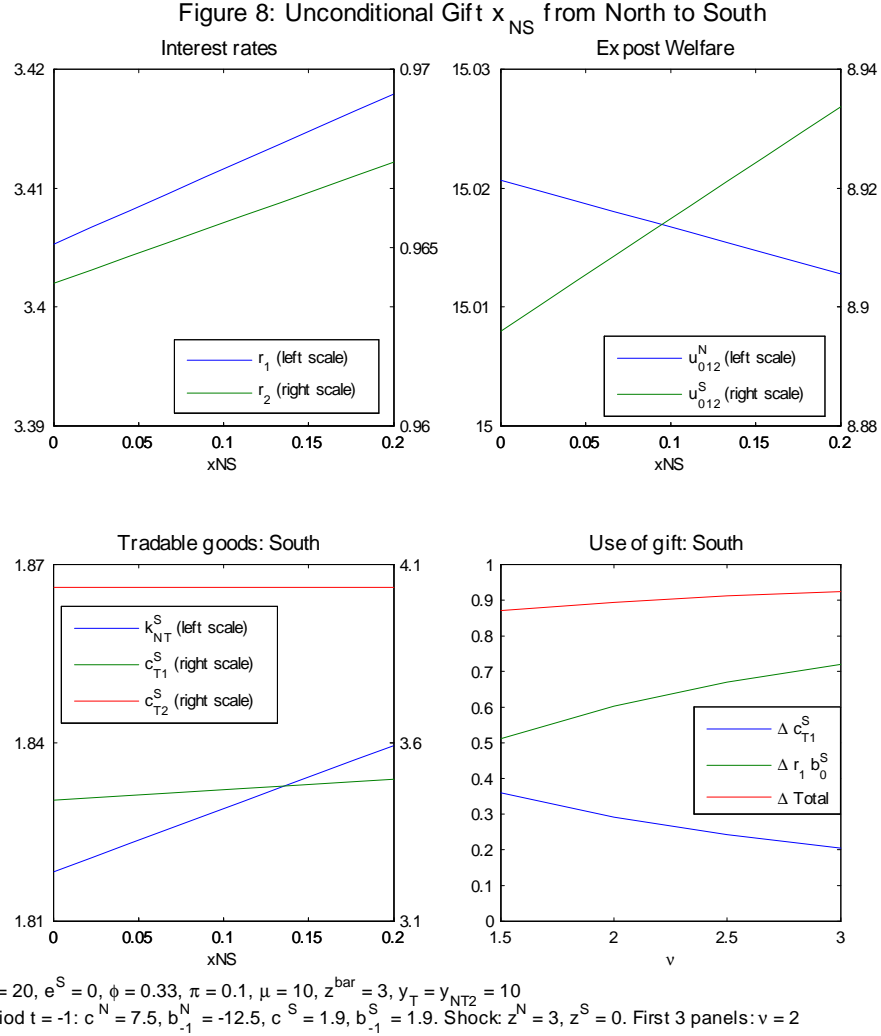
Figure 8 fixes the value of the shock to ($z^N = 3, z^S = 0$), such that the constraint is binding in the South, and illustrates the impact of an unconditional gift $x_{NS} > 0$.

No Pareto improvement is possible using an unconditional gift $x_{NS} > 0$. The ex post welfare of the North decreases and the ex post welfare of the South increases. The transfer of tradable goods at $t = 1$ generates a positive income effect in the South, and the level of tradable consumption c_{T1}^S in the South increases as its constraint is relaxed: the borrowing limit B_1^S actually decreases owing to the increase in r_2 , but the South's desired borrowing b_1^S decreases as well. The resulting *amplified* increase in spending on nontradable goods generates an increase in the nontradable goods price p_1^S , which induces higher investment k_{NT}^S . Consumption and investment in the North both decrease. Interest rates r_1 and r_2 both increase.

The bottom right panel of figure 8 shows that the increase in the consumption of tradable goods Δc_{T1}^S in the South at $t = 1$ is lower than the amount of the gift x_{NS} . There are two reasons for this result. First, the decline in the borrowing limit B_1^S keeps the entire increase in borrowing by the South $\Delta Total$ at $t = 1$ lower than the amount of the gift x_{NS} . Second, the increase in the interest rate r_1 increases the interest repayments of the South at $t = 1$. The increase in interest repayments $\Delta r_1 b_0^S$ represents the portion of the gift x_{NS} which is endogenously recouped by the North.

For a given size of the unconditional gift x_{NS} , the North suffers in net terms, but it suffers less from providing the gift when the size of the nontradable sector is larger. This is because the higher is ν , the larger is the portion of the gift x_{NS} which is endogenously recouped by the North. The higher is ν , the larger is the *amplification* from an increase in spending on tradable goods in the South to an increase in spending on nontradable goods at $t = 1$. Therefore, the larger is the induced increase in investment k_{NT}^S in the South at $t = 0$, and the larger is the resulting increase in the interest rate r_1 . Therefore,

the larger is the accumulated interest burden $r_1 b_0^S$ which has to be repaid by the South to the North at $t = 1$.



5.3. Subsidized Governmental Loans

In this subsection, the government of country i is allowed to undertake all the actions from the previous subsection at $t = 1$. It is also allowed to undertake the same actions at $t = 2$, up to a maximum lump-sum tax equal to $\bar{\tau}_{LS} \in (0, y_T)$. The budget constraint is that the total net transfer by each government is zero within each period. All actions must be announced at $t = 0$. In this subsection, the transfers between countries at $t = 1$ can be conditioned on transfers between countries at $t = 2$.

With this feasible action space, the government is now able to commit to repay loans up to a maximum debt limit. The government decides at $t = 0$ to contract its own loans between $t = 1$ and $t = 2$ if such an action would increase their country's welfare.

A governmental loan from the Northern coalition to the Southern coalition at interest rate r_{NS} entails a transfer from the North to the South of x_{NS} at $t = 1$ and a transfer in the opposite direction of $r_{NS}x_{NS}$ at $t = 2$, subject to $r_{NS}x_{NS} \leq \bar{\tau}_{LS}$. Households and firms take the transfers as given and maximize utility and profits as in subsection 3.2.

Lemma 12 *The governmental loan x_{NS} at interest rate r_{NS} changes the equilibrium equations in the North:*

$$c_{T1}^N = y_T - x_{NS} - r_1 (c^N + z^N + k_{NT}^N - e^N) + \frac{y_T + r_{NS}x_{NS} - c_{T2}^N}{r_2},$$

$$b_1^N = c_{T1}^N + r_1 (c^N + z^N + k_{NT}^N - e^N) - y_T + x_{NS},$$

and in the South:

$$c_{T1}^S = y_T + x_{NS} - r_1 (c^S + z^S + k_{NT}^S - e^S) + \frac{y_T - r_{NS}x_{NS} - c_{T2}^S}{r_2},$$

$$b_1^S = c_{T1}^S + r_1 (c^S + z^S + k_{NT}^S - e^S) - y_T - x_{NS},$$

$$B_1^S = \frac{\phi [p_2^S y_{NT2} + y_T - r_{NS}x_{NS}]}{r_2}.$$

Lemma 13 *For sufficiently low $\bar{\tau}_{LS}$, governmental loans from the Northern to the Southern coalitions are not able to fully relax the constraints of households in the South.*

From the above equations, the equilibrium effect of governmental loans is to relax the borrowing constraints of households in the South. We assume that $\bar{\tau}_{LS}$ is sufficiently low that the constraints cannot be fully relaxed.

In this subsection, we address the major policy questions in this environment. We characterize under which conditions the Northern and Southern coalitions decide to contract governmental loans, whether the interest rate on the loan must be subsidized, and the impact of the loans on consumption, investment and welfare. First, we restrict attention to the equilibrium from $t = 0$ onward and identify conditions for Pareto improvements. Second, we outline the impact of the loan on equilibrium decisions at $t = -1$, and we state conditions for Pareto improvements when all four periods are taken into account. This latter issue relates to how governmental loans, if available, should be institutionalized.

Ex post solution. Notice that the governmental loan possesses some contingency, because it is announced after the shock $t = 0$.

Definition 7 *The ex post Pareto set is written in reduced form as the set of interest rates r_{NS} on a marginal governmental loan between $t = 1$ and $t = 2$ such that both the North and the South have weakly higher ex post welfare u_{012}^j —and at least one region has strictly higher ex post welfare u_{012}^j —in the equilibrium with the governmental loan than in the laissez-faire equilibrium.*

Proposition 11 *Suppose that in the laissez-faire equilibrium, the constraint binds in the South for $z \in [\underline{z}, \bar{z}]$. For $z \in [0, \underline{z}]$, the ex post Pareto set is empty. For each $z \in (\underline{z}, \bar{z}]$, there exists a non-empty ex post Pareto set $[r_{\min}^N(z), r_{\max}^S(z)]$ with the following characteristics:*

$$\begin{aligned} \lim_{z \rightarrow \underline{z}^+} \{r_{\min}^N(z)\} &= \lim_{z \rightarrow \underline{z}^+} \{r_{\max}^S(z)\} < r_2 < 1, \\ \frac{dr_{\min}^N(z)}{dz} &< 0 \text{ and } \frac{dr_{\max}^S(z)}{dz} > 0, \\ r_{\min}^N(z) &< r_2 \text{ for all } z \in (\underline{z}, \bar{z}]. \end{aligned}$$

There may exist $\acute{z} \in [\underline{z}, \bar{z}]$ such that:

$$r_{\max}^S(z) < r_2 \text{ for all } z \in (\underline{z}, \acute{z}), \text{ and } r_{\max}^S(z) \geq r_2 \text{ for all } z \in [\acute{z}, \bar{z}];$$

otherwise, if such \acute{z} does not exist:

$$r_{\max}^S(z) < r_2 \text{ for all } z \in (\underline{z}, \bar{z}].$$

Lemma 14 *The following conditions hold within the Pareto set $[r_{\min}^N(z), r_{\max}^S(z)]$ for all $z \in (\underline{z}, \bar{z}]$:*

$$\begin{aligned} \Delta c_{T1}^N &< 0, \Delta c_{NT1}^N < 0, \Delta p_1^N < 0, \Delta k_{NT}^N < 0, \\ \Delta c_{T2}^N &> 0, \Delta p_2^N > 0 \\ \Delta c_{T1}^S &> 0, \Delta c_{NT1}^S > 0, \Delta p_1^S > 0, \Delta k_{NT}^S > 0, \\ \Delta c_{T2}^S &< 0, \Delta p_2^S < 0, \Delta B_1^S < 0, \\ \Delta r_1 &> 0, \Delta r_2 > 0, \Delta R_2^S < 0, \end{aligned}$$

where the Δ prefix denotes the change in each variable in the equilibrium with the governmental loan relative to the laissez-faire equilibrium.

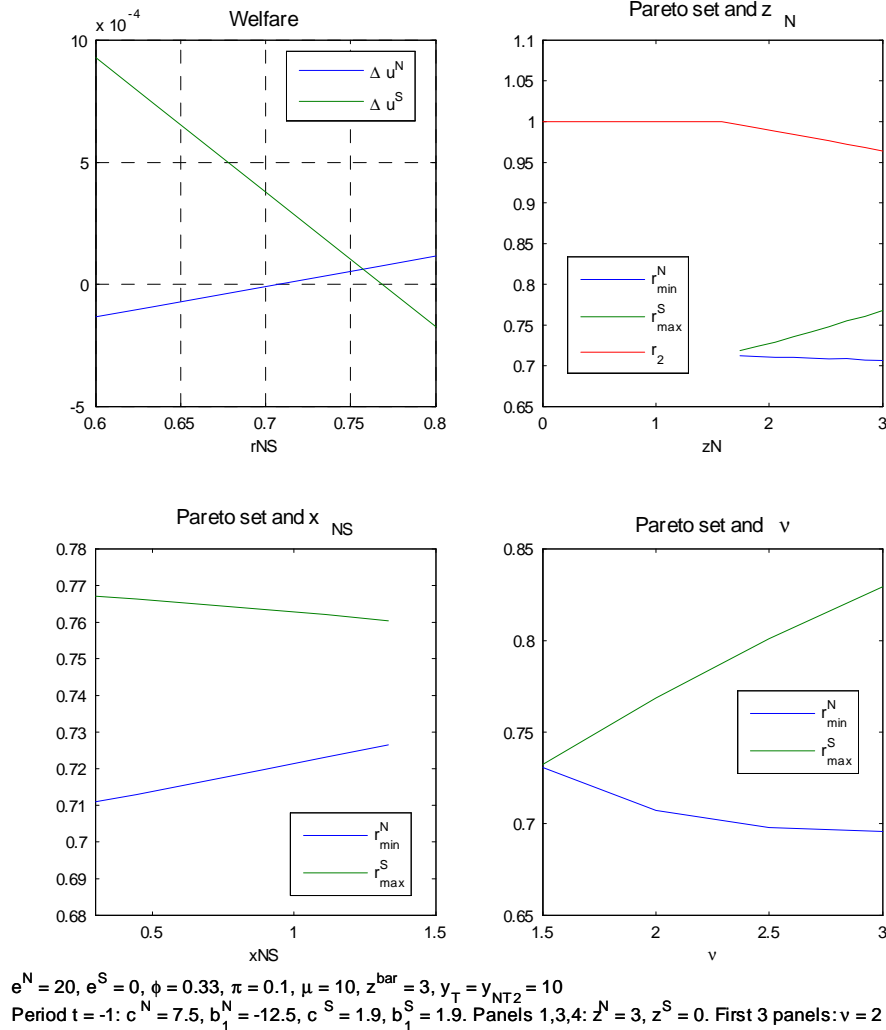
Figure 9 illustrates the impact of a governmental loan $x_{NS} > 0$ at various interest rates r_{NS} , after shocks to the North $z^N > 0$.

The top left panel fixes the value of the shock at $(z^N = 3, z^S = 0)$, such that the constraint is binding in the South, and then considers various interest rates r_{NS} on a marginal governmental loan $x_{NS} > 0$. The ex post welfare of the North is increasing in the interest rate and the ex post welfare of the South is decreasing in the interest rate. For $r_{NS} \geq r_{\min}^N = 0.71$, the ex post welfare of the North is higher in the equilibrium with the loan than in the laissez-faire equilibrium, so $r_{\min}^N = 0.71$ is the lowest interest rate which the North is willing to accept on a governmental loan. For $r_{NS} \leq r_{\max}^S = 0.77$, the ex post welfare of the South is higher in the equilibrium with the loan than in the laissez-faire equilibrium, so $r_{\max}^S = 0.77$ is the highest interest rate which the South is willing to pay on

a governmental loan. Therefore, for $r_{NS} \in [0.71, 0.77]$, both the North and the South have weakly higher ex post welfare, and at least one region has strictly higher ex post welfare, in the equilibrium with the governmental loan relative to the laissez-faire equilibrium.

The ex post Pareto set lies below the market interest rate $r_2 = 0.96$, which means that for the chosen value of the shock z , the governmental loan can only make an ex post Pareto improvement if r_{NS} is subsidized relative to r_2 .

Figure 9: Loan from North to South



The top right panel plots the ex post Pareto set for different values of the shock $z^N \in [0, \bar{z}]$. Consistent with proposition 11, there do not exist ex post Pareto-improving governmental loans for small shocks when the borrowing constraint is not binding. For large shocks when the borrowing constraint binds in the South, a non-empty Pareto set exists. When the constraint is just binding, $\lim_{z \rightarrow \bar{z}^+} \{r_{\min}^N(z^N)\} = \lim_{z \rightarrow \bar{z}^+} \{r_{\max}^S(z^N)\} = 0.72$, so the lowest interest rate which the North is willing to accept is equal to the highest interest rate which the South is willing to pay, and both are below the market interest

rate r_2 . Therefore, the Pareto-improving governmental loan must feature a subsidy for nearby values of the shock z^N .

Then as the shock value z^N increases, $r_{\min}^N(z^N)$ decreases and $r_{\max}^S(z^N)$ increases. For all parameters, $r_{\min}^N(z^N)$ always remains below r_2 . For the chosen parameters, the position of the ex post Pareto set indicates that $r_{\max}^S(z^N)$ is also always below r_2 , which means that ex post Pareto improvements are only possible when the interest rate on the governmental loan r_{NS} is subsidized relative to the market rate r_2 . There exist some other parameter choices such that the South is willing to pay more than the market interest rate on governmental loans after large shocks: $r_{\max}^S(z^N) \geq r_2$ for large z^N .

The bottom left panel shows that as the size of the governmental loan x_{NS} increases, the ex post Pareto set shrinks because the constraint becomes less binding. We show the Pareto set up to a loan size of $x_{NS} = 1.4$ because above this size, the constraint is not binding and further increments in the loan size produce no additional Pareto gains.

For an ex post Pareto improvement, a governmental loan succeeds where an unconditional gift fails. An unconditional gift from the North to the South provides an income effect to the South at the expense of the North, and the North has no incentive to provide this according to Lemma 8. On the other hand, a governmental loan helps get around the borrowing constraint, not only by increasing the after-tax endowment of households in the South at $t = 1$ as the gift also achieves, but crucially by reducing the after-tax endowment of households in the South at $t = 2$. This latter effect is needed for the North to gain from the intervention, and to satisfy Lemma 9.

The reason for a subsidy follows directly from the propositions in subsection 4.2. When the North eases the borrowing constraint of the South by providing the transfer $x_{NS} > 0$ at $t = 1$, the interest rates r_1 and r_2 increase, which transfers welfare from the South to the North. The North is willing to offer a subsidized rate r_{NS} on the governmental loan because it anticipates that the governmental loan will increase the interest rates on private loans made by households in the North to the South. There is a strictly positive gap between $r_{\min}^N(z)$ and $r_{\max}^S(z)$ for $z \in (\underline{z}, \bar{z}]$ because when the constraint binds in the South, households in the South value the marginal unit of consumption at $t = 1$ more than do households in the North, which creates some scope for welfare gains.

Modeling decisions. Again, we include nontradable goods to enrich the model dynamics. The above propositions hold in a model variant without nontradable goods, except that for this variant, $r_1 = r_1^*$ even when the constraint binds. The inclusion of nontradable goods with our chosen production function means that the governmental loan increases not just r_2 through a credit supply effect, but also r_1 through a credit demand effect. These interest rate changes transfer ex post welfare from the South to the North in both periods. If the welfare transfer is larger than in the model variant without nontradable goods, then the North is willing to provide a higher subsidy. The inclusion of nontradable goods also allows us to perform an additional comparative static exercise.

Proposition 12 (Nontradable Sector) *The higher is ν , the larger is the ex post Pareto set $[r_{\min}^N(z), r_{\max}^S(z)]$ for any given value of the shock z :*

$$\frac{dr_{\min}^N(z)}{d\nu} < 0 \text{ and } \frac{dr_{\max}^S(z)}{d\nu} > 0.$$

This result is illustrated in the bottom right panel of figure 9. As in the previous subsection, the higher is ν , the larger is the *amplification* effect, and therefore the larger is the increase in the interest rate r_1 . Therefore, the more willing is the North to subsidize governmental loans to the South. In addition, the higher is ν , the more binding is the constraint in the South, so the higher the interest rate that the South is willing to pay.

Keynes-Ohlin transfer problem. Our results above represent a twist on the time-honored transfer problem of international economics. Keynes feared that inter-country transfers hurt the donor country both directly, as goods leave the country, and indirectly, as the price of the donor country's exports decrease. Following the logic of the original transfer problem, one might imagine that similar effects would occur for the Northern coalition because it provides a subsidized loan.

However, the original transfer problem does not consider inter-country transfers in a context where the private sector of the donor country has lent, or expects to lend, to the recipient country, and where borrowing constraints may bind. The single tradable good in our model means that there are no intratemporal terms of trade effects. In fact, the North even benefits from providing the subsidy, because when borrowing constraints bind in the South, subsidized governmental loans from the North to the South increase the market interest rates r_1 and r_2 . The North actually receives a higher return on its "export good"—a net supply of international loans. And far from exacerbating a terms of trade deterioration, a higher ν amplifies the impact of the governmental loan on market interest rates.

Ex ante solution. Next, we turn to the impact on equilibrium variables at $t = -1$ when both the North and the South expect at $t = -1$ that governmental loans will be announced at $t = 0$. For the remainder of this subsection, all variables at $t = -1$ are no longer fixed at the values they take in the laissez-faire equilibrium, but are allowed to adjust to take into account households' expectations at $t = -1$ regarding future governmental loans. In this vein, we also shift focus from the ex post welfare u_{012}^i to the total welfare u^i of each country i and region j .

Lemma 15 *Suppose that households expect at $t = -1$ that governmental loans will be announced at $t = 0$ after large shocks z which make the constraint binding in the South. Then:*

$$\Delta c^N < 0, \Delta c^S > 0,$$

and there is an ambiguous effect on the interest rate r_1 .

In partial equilibrium, the changes in consumption levels at $t = -1$ would hurt the North and benefit the South, which means that from the perspective of $t = -1$, the North would be willing to offer a lower subsidy on the governmental loan, while the South would be willing to pay a higher interest rate, relative to the ex post problem already analyzed above. However, the general equilibrium change in the interest rate r_1 complicates this reasoning. If r_1 increases, then the North may be willing to offer a higher subsidy and the South may be willing to pay less on the governmental loan.

At $t = -1$, which governmental loans are expected to be announced at $t = 0$? With non-empty Pareto sets, this is potentially a multi-dimensional problem because for each value of the shock z which makes the borrowing constraint of the South binding, there exist a continuum of ex post Pareto-improving loans. We can collapse the dimensionality of the problem by assuming that after any such shock, the interest rate r_{NS} on the governmental loan is determined according to a Nash bargaining game. We refine the notion of the Pareto set accordingly.

Definition 8 $\gamma \in [0, 1]$ is the weight on the welfare of the North, and $1 - \gamma$ is the weight on the welfare of the South, in a Nash bargaining game that takes place at $t = 0$ after the shock z is realized:

$$r_{NS}(x_{NS}) = \arg \max_{r_{NS}} \left\{ \Delta u^N(r_{NS}, x_{NS}) \right\}^\gamma \left\{ \Delta u^S(r_{NS}, x_{NS}) \right\}^{1-\gamma}.$$

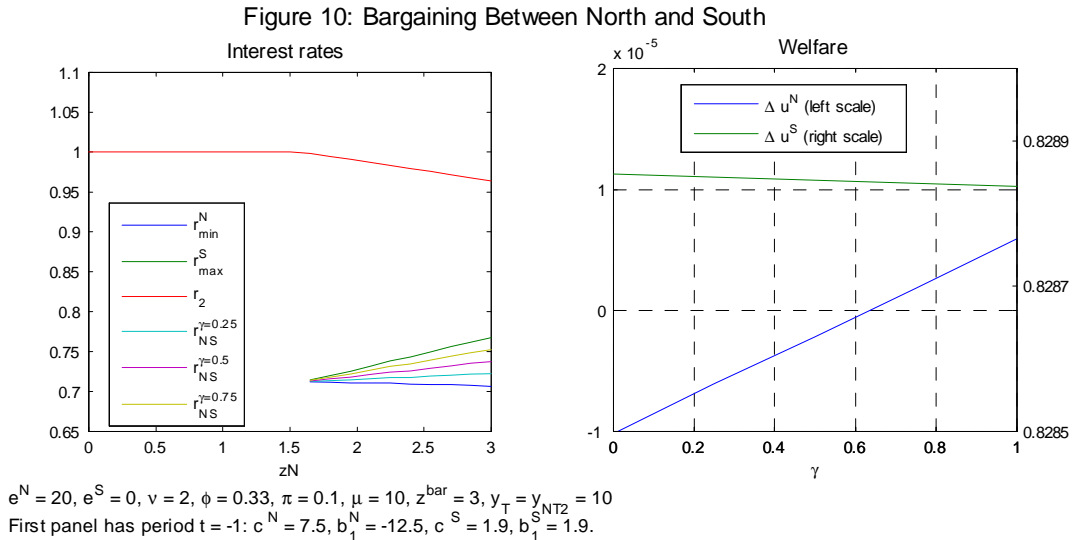
Definition 9 The ex ante Pareto set is written in reduced form as the set of bargaining weights γ for a governmental loan of marginal size, such that both the North and the South have weakly higher welfare w^j —and at least one region has strictly higher welfare w^j —in the equilibrium with the loan than in the laissez-faire equilibrium.

Proposition 13 (Bargaining) For some choices of parameters, there exists a Pareto set $[\hat{\gamma}, 1]$ with $\hat{\gamma} \in [0, 1]$, such that the equilibrium with the governmental loan is a Pareto improvement on the laissez-faire equilibrium.

Figure 10 illustrates these results for a marginal governmental loan $x_{NS} > 0$. The higher is γ , the closer is the interest rate r_{NS} of the governmental loan announced at $t = 0$ to the upper bound of the ex post Pareto set identified in proposition 11. Moreover, from the perspective of $t = -1$, the welfare of the North u^N is increasing in γ , while the welfare of the South u^S is decreasing in it. For the chosen parameters, there exists an ex ante Pareto set with bargaining: $\hat{\gamma} = 0.64 < 1$.

There are two ways to interpret this result: positive and normative. The positive interpretation is that for some choices of parameters and bargaining weights γ , both the North and the South have an improvement in welfare from the perspective of $t = -1$ when they expect that subsidized governmental loans will be negotiated after the shock is

realized at $t = 0$. The normative interpretation is that if the action space of Northern and Southern governments at $t = -1$ includes the possibility of setting the bargaining weight γ , they will negotiate at $t = -1$ to set the weight within the set $[\hat{\gamma}, 1]$ shown above.



Fixing the interest rate and loan size ex ante. The above Nash bargaining game approach is our preferred perspective on the difference between ex ante and ex post welfares. Nevertheless, we recognize that in implementation terms, designing loan institutions such as the IMF or ESM requires general agreements on interest rates and available loan programs before any shocks strike. Therefore, a separate question is whether there exist Pareto-improving contracts when interest rates and loan sizes are fixed ex ante, although the actual loan disbursement is only made if the shock is binding (consistent with proposition 11). We define such a contract as a limited-contingency governmental loan.

Definition 10 *A limited-contingency governmental loan is a governmental loan between $t = 1$ and $t = 2$ such that the loan is provided from the North to the South for all shocks $z \in [\underline{z}, \bar{z}]$ that make the borrowing constraint binding in the South, but such that neither x_{NS} nor r_{NS} are contingent on the shock z .*

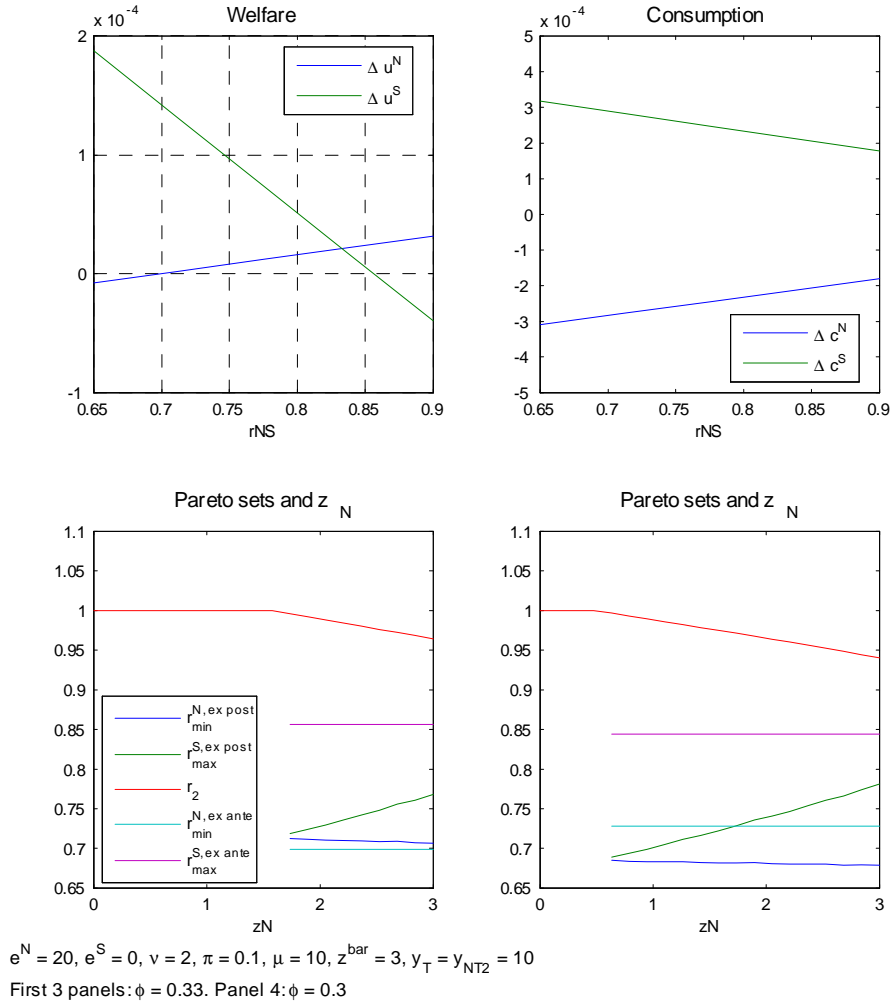
Definition 11 *The ex ante Pareto set is written in reduced form as the set of interest rates r_{NS} on a limited-contingency governmental loan of marginal size, such that both the North and the South have weakly higher welfare u^j —and at least one region has strictly higher welfare u^j —in the equilibrium with the loan than in the laissez-faire equilibrium.*

Proposition 14 (Ex Ante Loan) *If the borrowing constraint in the South binds for some values of the shock z , then there exists a non-empty ex ante Pareto set $[\hat{r}_{\min}^N, \hat{r}_{\max}^S]$.*

Figure 11 illustrates the ex ante Pareto set.

The top panels fix the value of the shock at $(z^N = 3, z^S = 0)$, such that the constraint is binding in the South, and then considers various interest rates r_{NS} on a marginal limited-contingency governmental loan $x_{NS} > 0$. The Δ prefix denotes the change in each variable in the equilibrium with the loan relative to the laissez-faire equilibrium. As the interest rate r_{NS} on the limited-contingency governmental loan increases, the consumption levels and welfare u^N of the North increase and the consumption levels and welfare u^S of the South decrease. For $r_{NS} \in [0.70, 0.86]$, both the North and the South have weakly higher welfare, and at least one region has strictly higher welfare, in the equilibrium with the loan relative to the laissez-faire equilibrium.

Figure 11: Ex Ante and Ex Post Pareto Sets



The bottom panels of figure 11 show the next ex ante Pareto sets for different values of ϕ . We have included the corresponding ex post Pareto sets for comparison purposes. Since they do not always overlap, as in the bottom right panel, ex ante agreements may rule out interest rates which turn out to be Pareto-improving in the ex post problem.

5.4. Conditional Gift and Tax/Subsidy Package

The final policy intervention we consider in this paper is a combination of a gift and a fiscal policy package. Just like in subsection 5.2, gifts between countries i and regions j at $t = 1$ are feasible subject to each government's budget constraint on net transfers. In addition, the government of country i is now allowed at $t = 2$ to impose taxes and subsidies on the consumption of both tradable and nontradable goods, subject to a balanced budget constraint within the same period. All actions must be announced at $t = 0$, and the gifts at $t = 1$ can be made conditional on a tax and subsidy package at $t = 2$.

Unlike in the previous subsection, the government is no longer able to commit at $t = 1$ to repay any loans at $t = 2$. By contrast, the government is able to commit at $t = 1$ to undertake a fiscal policy package at $t = 2$. The policy package does not entail governmental transfers between countries at $t = 2$.

The gift from the Northern coalition to the Southern coalition at $t = 1$ is g_{NS} . At $t = 2$, the ad valorem tax on tradable consumption in region j is $\tau^j \geq 0$ and the ad valorem subsidy on nontradable consumption in region j is $\eta^j \in [0, 1]$. Households and firms take the transfers, taxes and subsidies as given and maximize utility and profits as in subsection 3.2.

Lemma 16 *The fiscal policy package (τ^j, η^j) in region j satisfies:*

$$\tau^j c_{T2}^j = \eta^j p_2^j c_{NT2}^j.$$

The package can be indexed by τ^j alone, because $\eta^j = \eta^j(\tau^j)$ follows directly from the balanced budget constraint.

Lemma 17 *The conditional gift g_{NS} and fiscal policy packages τ^j change the equilibrium equations in the North:*

$$c_{T1}^N = y_T - g_{NS} - r_1 (c^N + z^N + k_{NT}^N - e^N) + \frac{y_T - c_{T2}^N}{r_2},$$

$$b_1^N = c_{T1}^N + r_1 (c^N + z^N + k_{NT}^N - e^N) - y_T + g_{NS},$$

and in the South:

$$c_{T1}^S = y_T + g_{NS} - r_1 (c^S + z^S + k_{NT}^S - e^S) + \frac{y_T - c_{T2}^S}{r_2},$$

$$b_1^S = c_{T1}^S + r_1 (c^S + z^S + k_{NT}^S - e^S) - y_T - g_{NS} \text{ and } B_1^S = \frac{\phi [p_2^S y_{NT2} + y_T]}{r_2}.$$

Finally, in both regions j :

$$(1 + \tau^j) c_{T2}^j = r_2 c_{T1}^j,$$

$$(\nu + \tau^j + \nu\tau^j) c_{T2}^j = p_2^j c_{NT2}^j.$$

In equilibrium, the tax and subsidy cancel in the budget constraints of the households, because of the balanced budget constraint of the government. Therefore, only the gift g_{NS} at $t = 1$ is visible in the budget constraints of the households, while the taxes τ^j at $t = 2$ are visible in the intertemporal consumption conditions between $t = 1$ and $t = 2$ and in the intratemporal consumption decisions at $t = 2$.

Ex post solution. As in the previous subsection, we begin by focusing on the equilibrium from $t = 0$ onward, while assuming that all variables at $t = -1$ remain fixed at the values they take in the laissez-faire equilibrium.

We first solve for the fiscal policy package on its own, then we allow for a gift which is made conditional on the implementation of the fiscal policy package.

Fiscal policy package. In the previous subsection, the governmental loan from North to South relaxed the borrowing constraints of households in the South by raising their after-tax endowment at $t = 1$, which decreased their desired borrowing b_1^S . Consumption in the South increased at $t = 1$ despite some tightening of the borrowing limit B_1^S . The latter tightening, recorded in proposition 11, was the result of a decrease in p_2^S as the South repaid the governmental loan at $t = 2$ and an increase in the market interest rate r_2 .

In the current subsection, the government is not able to commit to repay, so the only way to relax the borrowing constraints of households in the South is to increase B_2^S —for example, by increasing p_2^S . This increase allows the South to consume more of the tradable good at $t = 1$. The benefit to the South from higher consumption at $t = 1$ is straightforward. For such a change in the equilibrium to have any chance of also increasing the welfare of the North, it must be that the South consumes less of the tradable good at $t = 2$. This conjecture is consistent with our earlier comparison between the unconditional gift and the loan.

A subsidy η^S on nontradable consumption, financed by the proceeds from a tax τ^S on tradable consumption, introduces a price wedge between nontradable and tradable consumption at $t = 2$, and thereby succeeds in both increasing p_2^S and decreasing c_{T2}^S . By comparison, a transfer from the North to the South at $t = 2$ would not succeed, because it would increase both p_2^S and c_{T2}^S .

Since we are primarily concerned with increasing p_2^S , we consider implementing the policy package only in the South.

Lemma 18 *Whether or not the constraint binds in the South, implementing the fiscal policy package $\tau^S \geq 0$ in the South has the following effects:*

$$\frac{dc_{T1}^N}{d\tau^S} < 0, \frac{dc_{NT1}^N}{d\tau^S} < 0, \frac{dp_1^N}{d\tau^S} < 0, \frac{dk_{NT}^N}{d\tau^S} < 0,$$

$$\begin{aligned} \frac{dc_{T1}^S}{d\tau^S} &> 0, \frac{dc_{NT1}^S}{dx_{NS}} > 0, \frac{dp_1^S}{d\tau^S} > 0, \frac{dk_{NT}^S}{d\tau^S} > 0, \\ \frac{dc_{T2}^N}{d\tau^S} &> 0, \frac{dp_2^N}{d\tau^S} > 0 \text{ and } \frac{dc_{T2}^S}{d\tau^S} < 0, \frac{dp_2^S}{d\tau^S} > 0, \frac{dB_1^S}{d\tau^S} > 0, \\ \frac{dr_1}{dx_{NS}} &> 0, \frac{dr_2}{dx_{NS}} > 0, \frac{dR_2^S}{dx_{NS}} < 0. \end{aligned}$$

Proposition 15 (Fiscal Package) *Suppose that in the laissez-faire equilibrium, the constraint binds in the South for $z \in [z, \bar{z}]$. Implementing the fiscal policy package $\tau^S \geq 0$ in the South has the following effects:*

$$\frac{du_{012}^N}{d\tau^S} > 0 \text{ for all } z \in [0, \bar{z}].$$

There exist some choice of parameters under which the thresholds $\hat{z} \in (z, \bar{z}]$ and $\hat{\tau}^S(\hat{z}) \geq 0$ exist, such that:

$$\begin{aligned} \frac{du_{012}^S}{d\tau^S} &< 0 \text{ for all } z \in [0, \hat{z}), \\ \frac{du_{012}^S}{d\tau^S} &> 0 \text{ for all } z \in (\hat{z}, \bar{z}] \text{ and } \tau^S \in [0, \hat{\tau}^S) \\ \frac{du_{012}^S}{d\tau^S} &< 0 \text{ for all } z \in (\hat{z}, \bar{z}] \text{ and } \tau^S \in (\hat{\tau}^S, 1], \end{aligned}$$

and a fiscal policy package on its own can generate a Pareto improvement. Otherwise, if such \hat{z} and $\hat{\tau}^S(\hat{z})$ does not exist:

$$\frac{du_{012}^S}{d\tau^S} < 0 \text{ for all } z \in [0, \bar{z}],$$

and a fiscal policy package on its own cannot generate a Pareto improvement.

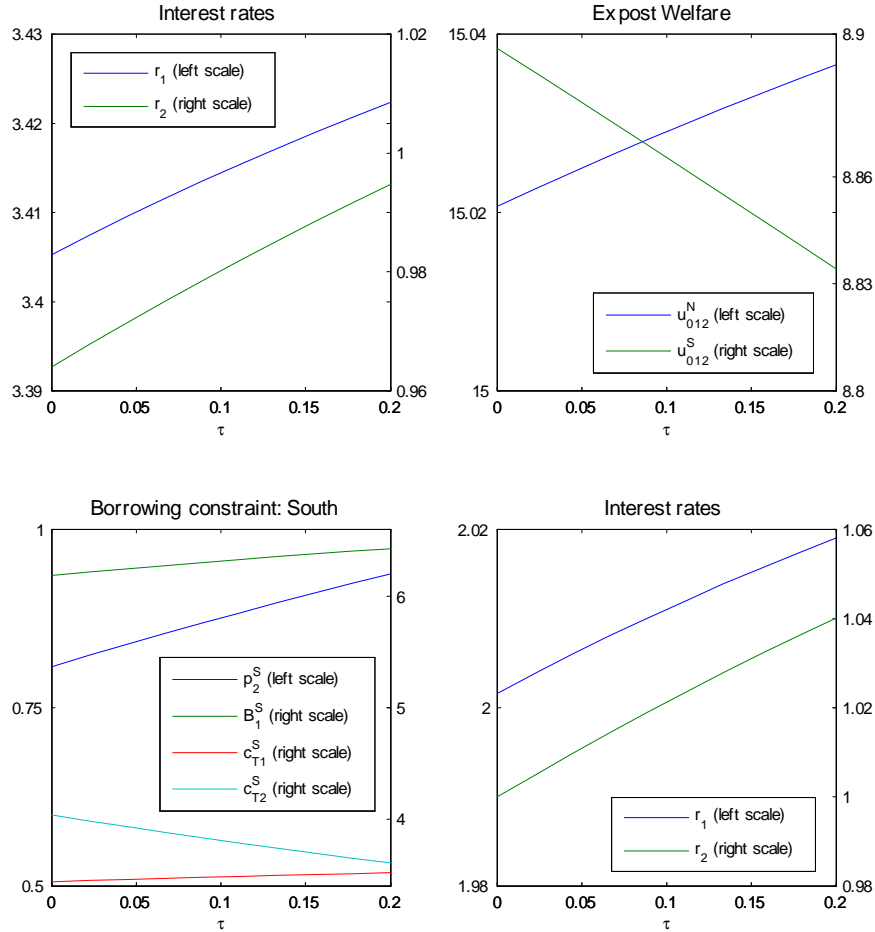
Figure 12 fixes the value of the shock to $(z^N = 3, z^S = 0)$, such that the constraint is binding in the South, and illustrates the impact of the fiscal policy package being implemented in the South.

The fiscal policy package increases the price of nontradable goods in the South p_2^S at $t = 2$, and increases the borrowing limit B_1^S . Tradable and nontradable consumption levels c_{T1}^S and c_{NT1}^S in the South increase at $t = 1$, and the level of tradable consumption c_{T2}^S in the South decreases at $t = 2$. In general equilibrium, the additional borrowing capacity of the South also causes an increase in the market interest rates r_1 and r_2 .

For the chosen parameters, the ex post welfare of the North u_{012}^N increases and the ex post welfare of the South u_{012}^S decreases, so no Pareto improvement is possible. The increase in consumption of the South at $t = 1$ is too small relative to the increase in

the interest burden on the South's previous borrowing—in net terms, hurting the ex post welfare of the South u_{012}^S .

Figure 12: Fiscal Policy Package τ^S in the South



$e^N = 20, e^S = 0, v = 2, \phi = 0.33, \pi = 0.1, \mu = 10, z^{\text{bar}} = 3, y_T = y_{NT2} = 10$
 Period $t = -1$: $c^N = 7.5, b_{-1}^N = -12.5, c^S = 1.9, b_{-1}^S = 1.9$. First 3 panels: $z^N = 3, z^S = 0$. Panel 4: $z^N = z^S = 0$.

The bottom right panel of figure 12 shows the impact of a shock on r_1 and r_2 when the value of the shock is ($z^N = 0, z^S = 0$). Unlike the unconditional gift case described in subsection 5.2, the fiscal policy package increases both interest rates even when the constraint is not binding in the South at the laissez-faire equilibrium. The reason is that the tax τ^S enters the intertemporal decision of households in the South:

$$R_2^S = \frac{c_{T2}^S}{c_{T1}^S} = \frac{r_2}{1 + \tau^S}.$$

The fiscal policy package increases c_{T1}^S relative to c_{T2}^S , generating upward pressure on both interest rates. When the constraint is binding, the fiscal policy package has an additional effect through the higher borrowing limit B_1^S .

Notice that according to the proposition above, there exist some other parameter choices and extremely large shocks such that a fiscal policy package on its own can generate a Pareto improvement. However, this is only possible if the constraint is so strongly binding that the increase in the interest burden of the South generated by the increase in r_1 and r_2 is more than offset, in ex post welfare terms, by a very high marginal value of consumption by the South at $t = 1$. Notice that this result does not violate proposition 9 because each Southern country is more than marginally better off if it undertakes the policy on its own after extreme shocks which cause such strongly binding constraints.

Even for these special parameter choices and extremely large shocks, the policy package cannot generate a Pareto improvement when the constraint is not binding, or when the constraint is moderately binding.

Gift and fiscal policy package combination. Given the extreme conditions that are needed for the fiscal policy package to generate a Pareto improvement on its own, we now turn to the question of whether adding a conditional gift g_{NS} from the North to the South at $t = 1$ could be useful.

Definition 12 *The ex post Pareto set is written in reduced form as the set of sizes of the conditional gift g_{NS} at $t = 1$ associated with a fiscal policy package at $t = 2$ of given size τ^S , such that both the North and the South have weakly higher ex post welfare u_{012}^j —and at least one region has strictly higher ex post welfare u_{012}^j —in the equilibrium with the conditional gift and fiscal policy package than in the laissez-faire equilibrium.*

Proposition 16 (Gift and Fiscal Package) *Suppose that in the laissez-faire equilibrium, the constraint binds in the South for $z \in [\underline{z}, \bar{z}]$. Consider a small fiscal policy package τ^S . Then for $z \in [0, \underline{z}]$, the ex post Pareto set is empty; for each $z \in (\underline{z}, \bar{z}]$, there exists a non-empty ex post Pareto set $[g_{\min}^S(z), g_{\max}^N(z)]$ with the following characteristics:*

$$\lim_{z \rightarrow \bar{z}^+} \{g_{\min}^S(z)\} = \lim_{z \rightarrow \bar{z}^+} \{g_{\max}^N(z)\} > 0,$$

$$\frac{dg_{\min}^S(z)}{dz} < \frac{dg_{\max}^N(z)}{dz} < 0 \text{ for all } z \in (\underline{z}, \bar{z}].$$

$$g_{\max}^N(z) > 0 \text{ for all } z \in (\underline{z}, \bar{z}].$$

For choices of parameters such that \hat{z} and $\hat{\tau}^S(\hat{z})$ as defined in proposition 15 do exist:

$$g_{\min}^S(z) > 0 \text{ for all } z \in (\underline{z}, \hat{z}), \text{ and } g_{\min}^S(z) \leq 0 \text{ for all } z \in [\hat{z}, \bar{z}];$$

otherwise, if such \hat{z} does not exist:

$$g_{\min}^S(z) > 0 \text{ for all } z \in (\underline{z}, \bar{z}].$$

Lemma 19 *The following conditions hold within the Pareto set $[g_{\min}^S(z), g_{\max}^N(z)]$ for all $z \in (\underline{z}, \bar{z}]$:*

$$\begin{aligned} \Delta c_{T1}^N &< 0, \Delta c_{NT1}^N < 0, \Delta p_1^N < 0, \Delta k_{NT}^N < 0, \\ \Delta c_{T2}^N &> 0, \Delta p_2^N > 0 \\ \Delta c_{T1}^S &> 0, \Delta c_{NT1}^S > 0, \Delta p_1^S > 0, \Delta k_{NT}^S > 0, \\ \Delta c_{T2}^S &< 0, \Delta p_2^S > 0, \Delta B_1^S > 0, \\ \Delta r_1 &> 0, \Delta r_2 > 0, \Delta R_2^S < 0, \end{aligned}$$

where the Δ prefix denotes the change in each variable in the equilibrium with the governmental loan relative to the laissez-faire equilibrium.

Figure 13 illustrates the impact of a small fiscal policy package $\tau^S = 0.01$, with various sizes of the conditional gift g_{NS} , after shocks to the North $z^N > 0$.

The top left panel fixes the value of the shock at $(z^N = 3, z^S = 0)$, such that the constraint is binding in the South, and then considers various sizes of the conditional gift g_{NS} at $t = 1$ to complement the fiscal policy package of fixed size at $t = 2$. The ex post welfare of the North is decreasing in the gift and the ex post welfare of the South is increasing in the gift. For $g_{NS} \leq g_{\max}^N = 0.022$, the ex post welfare of the North is higher in the equilibrium with the gift and policy package than in the laissez-faire equilibrium, so $g_{\max}^N = 0.022$ is the maximum gift which the North is willing to give. For $g_{NS} \geq g_{\min}^S = 0.016$, the ex post welfare of the South is higher in the equilibrium with the gift and policy package than in the laissez-faire equilibrium, so $g_{\min}^S = 0.016$ is the lowest gift which the South must be given to make the South willing to undertake the fiscal policy package.

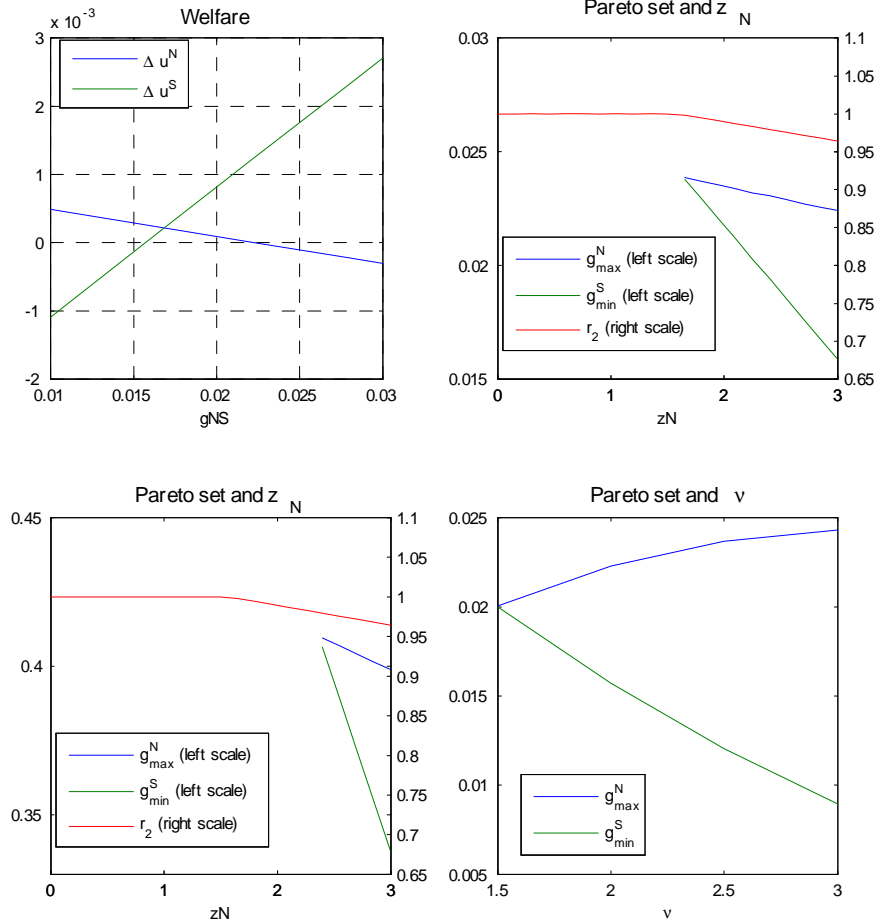
Therefore, for $g_{NS} \in [0.016, 0.022]$, both the North and the South have weakly higher ex post welfare, and at least one region has strictly higher ex post welfare, in the equilibrium with the combined policy intervention relative to the laissez-faire equilibrium. The ex post Pareto set lies above zero, which means that an ex post Pareto improvement requires a positive gift from the North to the South at $t = 1$.

The top right panel plots the ex post Pareto set for different values of the shock $z^N \in [0, \bar{z}]$. Consistent with proposition 16, there do not exist ex post Pareto-improving policy interventions for small shocks when the borrowing constraint is not binding. For large shocks when the borrowing constraint binds in the South, a non-empty Pareto set exists. When the constraint is just binding, $\lim_{z \rightarrow \underline{z}^+} \{r_{\min}^N(z^N)\} = \lim_{z \rightarrow \underline{z}^+} \{r_{\max}^S(z^N)\} = 0.024$, so the maximum gift which the North is willing to give is equal to the lowest gift the South must be given, and both are above zero. Therefore, the Pareto-improving intervention must feature a positive gift for nearby values of the shock z^N .

Then as the shock value z^N increases, both $g_{\min}^S(z)$ and $g_{\max}^N(z)$ decrease. The Pareto

set remains non-empty because the former decreases faster than the latter. For all parameters, $g_{\max}^N(z)$ always remains above zero. For the chosen parameters, the position of the ex post Pareto set indicates that $g_{\min}^S(z)$ is also always above zero, which means that ex post Pareto improvements are only possible with positive sizes of the conditional gift. As in proposition 15, there exist some other parameter choices such that the South is willing to undertake the fiscal policy intervention for free.

Figure 13: Conditional Gift g_{NS} and Fiscal Policy Package τ^S



$e^N = 20$, $e^S = 0$, $\phi = 0.33$, $\pi = 0.1$, $\mu = 10$, $z^{\text{bar}} = 3$, $y_T = y_{NT2} = 10$. Panels 1,2,4: $\tau^S = 0.01$. Panel 3: $\tau^S = 0.1$
 Period $t = -1$: $c^N = 7.5$, $b_1^N = -12.5$, $c^S = 1.9$, $b_1^S = 1.9$. Panels 1,3,4: $z^N = 3$, $z^S = 0$. First 3 panels: $v = 2$

The reason for a positive gift is as follows. For a small fiscal policy package, the distortions to tradable and nontradable consumption c_{T2}^S and c_{NT2}^S in the South at $t = 2$ have a second-order negative effect on the ex post welfare of the South. At $t = 1$, the relaxation of the borrowing constraint generates a higher consumption level c_{T1}^S in the South. Finally, there is a first-order welfare transfer from the South to the North. If the latter effect dominates the others, then the fiscal package cannot generate an ex post Pareto improvement on its own, and a positive gift $g_{NS} > 0$ must be provided to make the South willing to undertake the fiscal policy package.

There is a strictly positive gap between $g_{\min}^S(z)$ and $g_{\max}^N(z)$ for $z \in (z, \bar{z}]$ because when the constraint binds in the South, households in the South value the marginal unit of consumption at $t = 1$ more than do households in the North, which creates some scope for welfare gains.

The bottom left panel of figure 13 shows that for a larger fiscal policy package $\tau^S = 0.2$, the ex post Pareto set $[g_{\min}^S(z), g_{\max}^N(z)]$ shifts: it now only exists for higher values of the shock z^N , and a higher gift must be provided at $t = 1$. The higher gift compensates for the larger distortions to tradable and nontradable consumption c_{T2}^S and c_{NT2}^S in the South at $t = 2$.

Nontradable goods. The existence of the nontradable goods sector at $t = 2$ is absolutely necessary for our result. In a model variant without nontradable goods, there is no way that the borrowing limit of the South B_1^S can be increased. And without raising the borrowing limit, there is no way that consumption levels in the South can be increased at $t = 1$, because in this subsection, the government has no ability to commit to repay.

In addition, as in previous subsections, the size of the ex post Pareto set $[g_{\min}^S(z), g_{\max}^N(z)]$ depends on ν .

Proposition 17 (Nontradable goods) *The higher is ν , the larger is the ex post Pareto set $[g_{\min}^S(z), g_{\max}^N(z)]$ for any given value of the shock z :*

$$\frac{dg_{\min}^S(z)}{d\nu} < 0 \text{ and } \frac{dg_{\max}^N(z)}{d\nu} > 0.$$

There are now two reasons for this result, which is illustrated in the bottom right panel of figure 13. First, as in the previous subsections, the higher is ν , the larger is the *amplification* effect from an increase in tradable spending onto nontradable spending at $t = 1$, and therefore the larger is the increase in the interest rate r_1 . Second, the higher is ν , the larger is the impact of the fiscal policy package on the nontradable goods price at $t = 2$, and therefore the larger the increase in the borrowing limit B_1^S and interest rate r_2 . For both these reasons, the North is willing to provide a larger gift.

Modeling decisions. The conditional gift and fiscal policy package is more likely to generate an ex post Pareto improvement than several alternative nontradable sector interventions which also succeed in increasing p_2^S and decreasing c_{T2}^S . Take, for example, the introduction of wasteful government spending—whereby at $t = 2$, the government purchases and burns nontradable goods financed by a tax on tradable consumption. This policy moves p_2^S and c_{T2}^S in the necessary directions, but the purchased nontradable goods do not contribute to household welfare. Therefore, the set of possible ex post Pareto improvements shrinks. On the other hand, if the government simply transfers its purchases of nontradable goods to households, then there is no impact on p_2^S and c_{T2}^S because the households undo the government's actions.

Policy considerations. Two practical considerations apply when translating the results of this model into policy implementation. First, the conditional gift g_{NS} at $t = 1$ associated with an ex post Pareto improvement can be interpreted as conditional debt relief. It is always smaller than the desired borrowing of the South b_1^S , and it is conditioned on fiscal actions in the next period $t = 2$. Comparing subsection 5.2 to the current subsection, our model establishes that unconditional debt relief cannot generate an ex post Pareto improvement, but conditional debt relief can.

Second, remember that the purpose of the fiscal policy package at $t = 2$ is not just to reduce the South's tradable consumption at $t = 2$, but to increase the borrowing limit of the South between $t = 1$ and $t = 2$. In practice, unlike our model, not all nontradable goods can be used as collateral against borrowing. Therefore, our model recommends that subsidies are targeted to the prices of collateralizable nontradable goods and assets—such as housing and fixed capital—rather than equally on all nontradable goods.

Ex ante solution. To complete this subsection, we allow all variables at $t = -1$ to adjust to take into account households' expectations at $t = -1$ regarding future conditional gifts and fiscal policy packages. We shift focus from the ex post welfare u_{012}^i to the total welfare u^i of each country i and region j . The results in the previous subsection can be extended.

Lemma 20 *Suppose that households expect at $t = -1$ that conditional gifts and fiscal packages will be announced at $t = 0$ after large shocks z which make the constraint binding in the South. Then:*

$$\Delta c^N < 0, \Delta c^S > 0,$$

and there is an ambiguous effect on the interest rate r_1 .

Paralleling the previous subsection, we first assume that after any shock which makes the borrowing constraint of the South binding, the conditional gift g_{NS} at $t = 1$ associated with the fixed-size fiscal policy package τ^S at $t = 2$ is determined according to a Nash bargaining game. We again refine the notion of the Pareto set accordingly.

Definition 13 $\gamma \in [0, 1]$ *is the weight on the welfare of the North, and $1 - \gamma$ is the weight on the welfare of the South, in a Nash bargaining game that takes place at $t = 0$ after the shock z is realized:*

$$g_{NS}(\tau^S) = \arg \max_{g_{NS}} \{ \Delta u^N(g_{NS}, \tau^S) \}^\gamma \{ \Delta u^S(g_{NS}, \tau^S) \}^{1-\gamma}.$$

Definition 14 *The ex ante Pareto set is written in reduced form as the set of bargaining weights γ for a fixed-size fiscal policy package τ^S , such that both the North and the South have weakly higher welfare w^j —and at least one region has strictly higher welfare w^j —in the equilibrium with the loan than in the laissez-faire equilibrium.*

Proposition 18 (Bargaining) *For some choices of parameters, there exists a Pareto set $[\hat{\gamma}, 1]$ with $\hat{\gamma} \in [0, 1]$, such that the equilibrium with the conditional gift and fiscal policy package is a Pareto improvement on the laissez-faire equilibrium.*

Fixing the gift and fiscal package ex ante. Next, for implementation concerns, we allow the Northern and Southern coalitions to commit at $t = -1$ to limited-contingency gift and fiscal policy packages.

Definition 15 *A limited-contingency gift and fiscal policy package is a combination of the conditional gift and tax/subsidy package for all shocks $z \in [\underline{z}, \bar{z}]$ that make the borrowing constraint binding in the South, but such that neither g_{NS} nor τ^S are contingent on the shock z .*

Definition 16 *The ex ante Pareto set is written in reduced form as the set of sizes of the conditional gift g_{NS} at $t = 1$ associated with a fiscal policy package at $t = 2$ of given size τ^S , such that both the North and the South have weakly higher welfare u^j —and at least one region has strictly higher welfare u^j —in the equilibrium with the conditional gift and fiscal policy package than in the laissez-faire equilibrium.*

Proposition 19 (Ex Ante Gift and Package) *If the borrowing constraint in the South binds for some values of the shock z , then there exists a non-empty ex ante Pareto set $[\tilde{g}_{\min}^S(z), \tilde{g}_{\max}^N(z)]$.*

The normative interpretation of our results on the ex ante solution is that in general, the North and/or South would use any power that they possess at $t = -1$ to shape the bargaining between North and South after the shock z at $t = 0$. The North and/or South would try at $t = -1$ to rule out those combinations of gifts and fiscal policy packages that generate ex post but not ex ante Pareto improvements.

This normative interpretation is not without some tension. In the previous subsection, the features of governmental loans that we discussed institutionalizing at $t = -1$ were the interest rate r_{NS} , and the bargaining power γ over the interest rate. We could imagine designing special governmental loans which would be activated specifically for large shocks which cause constraints to bind in the South.

In the current subsection, the features that would need to be institutionalized are the haircuts for debt relief and the precise levels of fiscal taxes and subsidies. All of these governmental policy tools are used to address a wide variety of social ills, and therefore appear more difficult to commit to in advance. Nevertheless, if policymakers are to adopt the recommendations of our model, some work on institutionalization for even these more disparate tools is necessary.

6. Conclusion

Institutions for a financial union. Philosophically, our model and policy recommendations represent a departure from the optimal currency area literature—a literature which has shaped much thinking about supranational integration processes in general, and the European financial and sovereign debt crisis in particular. Instead, we design a model of supranational integration which builds on, and extends, fundamental market imperfections which have traditionally been found in the sudden stop literature for emerging market economies, and which have more recently been adapted in various forms for the analysis of individual economies in the European periphery.

We focus on noncontingent debt and borrowing constraints instead of the rigidity in price adjustment implicit or explicit within the optimal currency area literature. As a result, our conception of a union of countries is different. First, it is defined in a financial sense, with a free flow of debt contracts. Second, instead of dividing the countries within a union into those which have not suffered from shocks and those which have, we divide the countries into those with high assets—the Northern “core”—and those with high debt—the Southern “periphery”. In a heterogeneous union, our policy recommendations are not that countries who have suffered from shocks should obtain support from the rest, but rather that the South should receive support from the North, irrespective of where the shock materialized.

Bargaining between regional coalitions of countries is needed in order to overcome a collective action problem. If the Northern countries do not act in a coalition, then support will be under-provided. The government of each country in the North ignores their own impact on union-wide interest rates, and hopes instead that the borrowing constraint in the South is relaxed by the actions of the rest of the governments of the North. The government of each country in the South also ignores their own impact on union-wide interest rates and is willing to undertake any actions that help relax their borrowing constraint, even if such actions when undertaken by all the Southern countries ends up hurting all of them. Pareto improvements are made possible through bargaining at the regional level.

When governmental actions are available, whether in the form of governmental loans, conditional gifts and/or taxes and subsidies, some degree of institutionalization of such actions is desirable. Actions which all governments will agree to once the shock has been realized may or may not be consistent with Pareto improvements from the perspective of countries before the shock has been realized. In general, before any shocks occur, some restrictions should be placed on the actions available in future regional bargaining.

Comparing policy interventions. We hope that our model can help shed light on past and ongoing policy discussions about the European financial and sovereign debt crisis. In our model, unconditional gifts from North to South cannot alter interest rates sufficiently

to compensate the North, which means that such redistributive measures will be resisted. Governmental loans from North to South can succeed in generating Pareto improvements, even when they appear to carry a subsidy, because the general equilibrium effects of such lending benefit the North. However, notice that governmental loans only have potency if governments in the South have some debt capacity independent of the private sector. This was true in the early days of the crisis, but is false for several Southern governments today.

If fiscal capacity is used up, attention must turn to other governmental actions that are available to raise borrowing limits in the South. In our model, conditional debt relief tied to a tax/subsidy package is feasible without violating any constraints, and can generate Pareto improvements. So we face a puzzle: in positive terms, why have such policies not already been implemented?

We can think of three reasons. First, the coalitions needed to agree on simultaneous debt relief and fiscal policies are highly diverse. In some countries, debt levels were initially high in the private sector, while in others the public sector was first constrained. And taxes and subsidies are fraught with rigidities owing to political economy considerations. Second, different governmental tools are controlled by different institutions. For example, targeted quantitative easing by authorities could help increase the prices of collateralizable nontradable assets in the South, as our model recommends, but will be opposed by the North without coordinated fiscal policies that limit tradable consumption in the South. Third, the policy package is counter-intuitive. While austerity across the board is easily supported by a narrative of fiscal profligacy and asset price bubbles in the South, a proposal to impose taxes on tradable consumption and subsidies on collateralizable nontradable assets is not. Our model suggests that irrespective of past valuations before the shock was realized, excessive downward price flexibility in the nontradable sector is prolonging the crisis.

Fiscal and banking union. There remains to be solved a complex implementation problem to breathe life into any proposed policy interventions. At the moment, the terminology of fiscal and banking union remains in considerable flux. The main lesson from our model is that the borrowing constraints of constrained countries need to be relaxed in ways that also benefit the lender countries. In practice, governmental loans can be helpful, and to the extent that they must be specifically designed and made available in advance, institutionalizing them in the form of a fiscal union and/or IMF membership is recommended. Moreover, governments are best suited to impose taxes and subsidies, and to provide targeted tax cuts to reach borrowing-constrained agents who have no access to the banking system.

On the other hand, banks are best suited to identify which households are in fact constrained, and therefore may be an essential vehicle for Southern governments who are figuring out how to most efficiently use the funds they have received from governments in

the North. The North is willing to offer a higher subsidy if more of the governmental loan is forcibly channeled to constrained households in the South, rather than to generalized spending on public goods—which means that subsidized supranational loans may be easier to operationalize when the loans are earmarked for banking sector resolution. Once fiscal capacity is used up, bank participation is needed to enact debt relief and to provide support to the prices of collateralizable nontradable assets.

7. References

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