

A wake-up call theory of contagion*

Toni Ahnert[†] and Christoph Bertsch[‡]

August 2014

Abstract

We propose a novel theory of financial contagion. We study global coordination games of regime change with an initially uncertain correlation of regional fundamentals. A crisis in one region is a wake-up call to investors in another region that induces a re-assessment of local fundamentals. Contagion after a wake-up call can occur even if investors learn that fundamentals are uncorrelated and common lender effects or balance sheet linkages are absent. Applicable to currency attacks, bank runs, and debt crises, our theory of contagion is supported by existing evidence and generates new testable implications for empirical and experimental work. (JEL D82, F3, G01)

Keywords: contagion, financial crisis, fundamental re-assessment, wake-up call, global games, heterogeneous priors, information choice, disagreement.

*We wish to thank Viral Acharya, Jason Allen, Kartik Anand, Ana Babus (discussant), Jose Berrospide, Elena Carletti, Amil Dasgupta, Michael Ehrmann, Douglas Gale, Piero Gottardi, Antonio Guarino, Ralf Meisenzahl, Kristoffer Nimark (discussant), Alessandro Pavan, Morten Ravn, Wolfram Richter, Jean-Charles Rochet, Nikita Roketskiy, Michal Szkup, Dimitri Vayanos, Laura Veldkamp (discussant) and seminar participants at NBER Summer Institute 2014 in International Finance and Macroeconomics, Barcelona GSE Summer Forum 2014 (Information and Market Frictions; Experimental and Theoretical Macroeconomics), Bank of Canada, CEA 2014, Federal Reserve Bank St. Louis, RES 2014, NORMAC 2013, EEA 2013, Dortmund, London FIT network at LBS, EUI Florence, UCL London, and Board of Governors. An earlier version was circulated as “A wake-up call: information contagion and strategic uncertainty” (Sveriges Riksbank WP 282). First version: June 2012. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of either the Bank of Canada or Sveriges Riksbank.

[†]Financial Studies Division, Bank of Canada, 234 Laurier Avenue W, Ottawa, ON K1A 0G9, Canada. The author conducted part of this research during his PhD at the London School of Economics and Political Science and while visiting New York University and the Federal Reserve Board of Governors. E-mail: tahnert@bankofcanada.ca.

[‡]Research Division, Sveriges Riksbank, SE-103 37 Stockholm, Sweden. The author conducted part of this research during a PostDoc at the University College London, United Kingdom. An earlier version of this paper was part of his Ph.D. thesis at the European University Institute, Florence, Italy. E-mail: christoph.bertsch@riksbank.se.

Wake-up calls are a common explanation for financial contagion (Forbes (2012)).¹ A crisis in one region is a wake-up call to investors in other regions that triggers a re-appraisal of risks, whereby investors re-assess local fundamentals. Weaker fundamentals – possibly due to the exposure to the initial crisis region – or greater uncertainty about fundamentals lead to a financial crisis in other regions.

There is empirical support for contagion due to wake-up calls across markets and over time. Studying equity markets during the global financial crisis of 2007–09, Bekaert et al. (2014) identify wake-up calls as the key driver of contagion. Analyzing eurozone sovereign bond markets, Giordano et al. (2013) find empirical evidence for contagion based on the wake-up call of the Greek crisis of 2009–10. Studying bond markets during the Asian crisis in 1997, Basu (2002) finds evidence for contagion based on the re-assessment of risks in some countries. Van Rijckeghem and Weder (2003) view the Russian crisis in 1998 as the outcome of a wake-up call in emerging markets. From a historical perspective, Ramirez and Zandbergen (2013) document evidence for contagion based on the wake-up call of newspaper reports about distant bank runs in the Panic of 1893.

Despite the empirical evidence on wake-up call contagion, there has been little theoretical work. Our paper closes this gap by proposing a wake-up call theory of contagion. We develop a model of two regions with initial uncertainty about the correlation of regional fundamentals. Regional investors play a standard global coordination game of regime change with incomplete information about the regional fundamental (Morris and Shin (2003)). Contagion is defined as an increase in the probability of a financial crisis in the second region after a crisis in the first region.

Our main contribution is that contagion occurs even if regional fundamentals are uncorrelated and common lenders or balance sheet links are absent. Therefore, our theory of contagion explains how a wake-up call transmits financial crises.

Observing a crisis in region 1 is a wake-up call to investors in region 2 who re-assess the local fundamental. Learning that fundamentals are uncorrelated leads

¹Goldstein (1998) introduced the notion of a wake-up call. Forbes (2012) distinguishes four mutually non-exclusive channels of contagion: trade, banks, portfolio investors, and wake-up calls.

to contagion for two reasons. First, the mean of the fundamental in region 2 is lower after the wake-up call, given that not observing a crisis in region 1 would have been good news for investors since fundamentals may be positively correlated. This mean effect increases the probability of a crisis in region 2 (Vives (2005)).

Second, the variance of the fundamental in region 2 is higher after the wake-up call. When fundamentals are uncorrelated, observing a crisis in region 1 is uninformative for investors in region 2. Hence, there is greater disagreement among informed investors. This variance effect can increase the probability of a crisis in region 2 (Metz (2002)).² Investors attack the regime more aggressively, for example by selling short a currency, withdrawing from a bank, or not rolling over debt.

We further explore the effect of greater disagreement among investors on contagion. We show that the extent of contagion can increase in the proportion of informed investors – even when fundamentals are uncorrelated, so that there is no exposure to the crisis in region 1. This result on the enhanced perception of risk hinges on a large variance effect. Specifically, for the variance effect to outweigh the mean effect, a lower bound on the fundamental in region 1 is required.

Our contagion results prevail with endogenous information. For instance, investors can acquire costly and publicly available information about the correlation of fundamentals. We describe a strategic complementarity in information choices. For sufficiently low information costs, there exists a unique equilibrium in which all investors acquire information after a wake-up call.³ Information acquisition, in turn, can fuel disagreement. While uninformed investors play an invariant strategy, informed investors tailor their strategy to the observed correlation, attacking the regime more aggressively when fundamentals are uncorrelated.

²Greater disagreement after a wake-up call is consistent with “an enhanced perception of risk” after the Russian crisis (Van Rijckeghem and Weder (2001), p. 294). Therefore, our theory can explain, for example, the unexpected spread of the Russian crisis to Brazil in 1998 (Bordo and Murshid (2000) and Forbes (2012)) and similar instances during the Asian crisis in 1997 (Radelet and Sachs (1998) and Corsetti et al. (1999)). See also Pavlova and Rigobon (2008).

³Contagion arises in Calvo and Mendoza (2000) since globalization shifts the incentives of investors from costly information acquisition to imitation and detrimental herding. By contrast, financial contagion arises in our paper because investors acquire information after a wake-up call.

Our theory of contagion has new implications for the empirical literature on banking and currency crises.⁴ Our theory suggests that the likelihood of contagion depends non-linearly on the characteristics of the first region.⁵ In particular, after controlling for the fundamentals of the second region, a crisis in the first region due to extremely low fundamentals is less likely to spread if fundamentals are uncorrelated. Conversely, a crisis in the first region due to moderately low fundamentals is more likely to spread if fundamentals are uncorrelated.

The wake-up call theory of contagion has testable implications for experimental work. Building on Heinemann et al. (2004, 2009), examining contagion within the global games framework in the laboratory is a promising – yet little explored – avenue for future work. We derive three specific testable implications. Since the information choice of investors after a wake-up call is observed in the laboratory, experiments are particularly suitable for testing our predictions about information choice. This allows to test for the enhanced perception of risk after a wake-up call and the non-linear role of fundamentals in the first region.

We make a technical contribution to the literature on information choice in global coordination games. Because of the initially uncertain correlation, the priors about the regional fundamental are heterogeneous across investors. Specifically, the prior of uninformed investors follows a mixture distribution.⁶ We analyze the information choice about the correlation of fundamentals under heterogeneous priors and establish strategic complementarity in information choices.

Hellwig and Veldkamp (2009) were the first to study the optimal information choice in strategic models. They show that the information choices of investors inherit the strategic motive of the underlying beauty contest game. Szkup and Trevino

⁴See Glick and Rose (1999) for the role of trade links, Van Rijckeghem and Weder (2001, 2003) for financial links, and Dasgupta et al. (2011) for institutional similarities.

⁵The importance of non-linearities have been examined by Forbes and Rigobon (2002) and Bekaert et al. (2014) in the context of financial market returns and the transmission of information. Favero and Giavazzi (2002) contrast contagion with “flight-to-quality” episodes.

⁶Another global games paper using mixture distributions is Chen et al. (2012), who develop a theory of rumors during political regime change. However, they abstract from both contagion and information acquisition.

(2012b) study a model with a common prior, continuous private information choice and convex information cost. They extend the inheritance result to global games of regime change. In contrast to these papers, we study the acquisition of publicly available information about the correlation of fundamentals and allow for heterogeneous priors. Such information can increase or decrease the precision of the prior about the local fundamental. Hence, there can be greater disagreement among informed investors after a wake-up call, which contributes to contagion.

The wake-up call theory of contagion has several applications. For currency crises, speculators observe a currency attack and are uncertain about the magnitude of trade or financial links or institutional similarity.⁷ For rollover risk and bank runs, wholesale investors observe a run elsewhere and are uncertain about interbank exposures.⁸ For sovereign debt crises, bond holders observe a sovereign default elsewhere and are uncertain about the macroeconomic links, the commitment of the international lender of last resort, or the resources of multilateral bail-out funds.⁹

Alternative theories of financial contagion have been proposed. Regarding direct links, see Allen and Gale (2000) and Dasgupta (2004) for interbank links and Kiyotaki and Moore (2002) for balance-sheet contagion. For a common discount factor channel, see Ammer and Mei (1996) and Kodres and Pritsker (2002). Regarding a common investor base, see Goldstein and Puzner (2004) for risk aversion, Pavlova and Rigobon (2008) for portfolio constraints, Taketa (2004) and Oh (2013) for learning about other investors. In terms of ex-post correlated fundamentals, see Basu (1998) for a common risk factor, and Acharya and Yorulmazer (2008) and Allen et al. (2012) for asset commonality among banks and information contagion.¹⁰ By contrast, we provide a novel and complementary theory of contagion based on the re-assessment of local fundamentals after a wake-up call.

⁷See also Morris and Shin (1998) and Corsetti et al. (2004) for a one-regional global game that builds on the earlier works of Krugman (1979), Flood and Garber (1984), and Obstfeld (1986).

⁸See also Rochet and Vives (2004) and Goldstein and Puzner (2005) for a one-regional global game that builds on the earlier work of Diamond and Dybvig (1983).

⁹See also Corsetti et al. (2006). See Drazen (1999) for membership contagion.

¹⁰See Chen (1999) for a model with information contagion and uninformed junior claimants. See Chen and Suen (2013) for a model of information contagion in the context of model uncertainty.

This paper proceeds as follows. We describe our global games model with initial uncertainty about the correlation of fundamentals in section 1. Using mixture distributions, we obtain the unique equilibrium under exogenous information in section 2. We establish our results on contagion after a wake-up call in section 3. We show in section 4 that these results prevail under endogenous information. Section 5 contains robustness checks and extensions, including private information choice about the local fundamental. In section 6, we link our results to the empirical literature and derive new implications for empirical and experimental work. Section 7 concludes. Derivations and proofs are in the Appendix.

1 Model

We study a sequence of global coordination games of regime change.¹¹ There are two dates and two regions, both indexed by $t \in \{1, 2\}$, because investors in region t only move at date t . Each region is inhabited by a unit continuum of risk-neutral investors indexed by $i \in [0, 1]$.¹²

Actions and payoffs At each date, investors simultaneously decide whether to attack the regime, $a_{it} = 1$, or not, $a_{it} = 0$. The outcome of the attack depends on both the aggregate attack size, $A_t \equiv \int_0^1 a_{it} di$, and a fundamental $\theta_t \in \mathbb{R}$ that measures the strength of the regime. A regime change occurs if sufficiently many investors attack, $A_t > \theta_t$. Following Vives (2005), an investor's payoff from attacking is a benefit $b_t > 0$ if a regime change occurs and a loss $l_t > 0$ otherwise:

$$u(a_{it} = 1, A_t, \theta_t) = b_t \mathbf{1}_{\{A_t > \theta_t\}} - l_t \mathbf{1}_{\{A_t \leq \theta_t\}}. \quad (1)$$

¹¹One-regional global coordination games of regime change are studied in Morris and Shin (1998), Angeletos et al. (2006), Dasgupta (2007). See also Carlsson and van Damme (1993), Frankel et al. (2003), and the review articles of Morris and Shin (2003) and Vives (2005).

¹²Our contagion theory does not require a common investor base, signalling, or herding. A large, and potentially asymmetrically informed, currency speculator is studied by Corsetti et al. (2004).

The payoff from not attacking is normalized to zero. Thus, the differential payoff from attacking increases in the attack size A_t and decreases in the fundamental θ_t . Hence, the attack decisions of investors exhibit global strategic complementarity.

A regime change can be interpreted as a financial crisis. Currency crises, banking crises and sovereign debt crises are natural applications.¹³ The fundamental θ_t can be interpreted as the ability of a monetary authority to defend its currency (Morris and Shin (1998) and Corsetti et al. (2004)), as the measure of investment profitability (Rochet and Vives (2004), Goldstein and Pauzner (2005), Corsetti et al. (2006)) or a sovereign's taxation power or willingness to repay. Investors can be interpreted as currency speculators, as retail or wholesale bank creditors who withdraw funds, or as sovereign debt holders who refuse to roll over.

Information The key feature of our model is the initial uncertainty about the correlation between regional fundamentals, $\rho \equiv \text{corr}(\theta_1, \theta_2)$. This correlation is zero with probability $p \in (0, 1)$ or takes the positive value $\rho_H > 0$:¹⁴

$$\rho = \begin{cases} 0 & \text{w.p. } p \\ \rho_H & \text{w.p. } 1 - p. \end{cases} \quad (2)$$

The initial uncertainty about the correlation of regional fundamentals is motivated by our applications to financial crises. In the context of currency attacks, the ex-ante uncertain correlation reflects the unknown magnitude of trade or financial links or the unknown institutional similarity. In the context of bank runs, it reflects the uncertainty about interbank exposures. In the context of sovereign debt crises, the uncertain correlation reflects the uncertainty about the macroeconomic and financial links across countries. It could also reflect the uncertainty about the resources and commitment of multilateral bail-out funds or the international lender of last resort.

¹³A non-financial application is political regime change. As in the Arab spring, political activists observe a revolution in a neighboring country and are uncertain about its effect on their government's ability to stay in power. See Edmond (2013) for a one-regional global game with endogenous information manipulation (propaganda).

¹⁴We consider the case of negative correlation in section 5.

Regional fundamentals are commonly known to follow a bivariate normal distribution with mean $\mu_t \equiv \mu$, precision $\alpha_t \equiv \alpha \in (0, \infty)$, and realized correlation ρ . There is incomplete information about the regional fundamental θ_t (Carlsson and van Damme (1993)).¹⁵ Each investor receives noisy private information x_{it} before the attack decision (Morris and Shin (2003)):

$$x_{it} \equiv \theta_t + \varepsilon_{it} \tag{3}$$

where idiosyncratic noise ε_{it} is identically and independently normally distributed across investors and regions with zero mean and precision $\gamma > 0$. The random variables for regional fundamentals, the correlation, and the sequences of idiosyncratic noise terms are independent. The information structure is common knowledge.

Table 1 summarizes the two stages in region 2. The usual coordination stage may be preceded by an information stage. We view a financial crisis as a discontinuous event, after which additional information is available, or can be acquired cheaply. This assumption is motivated by the news coverage of crises and public inquiries. Therefore, two pieces of additional information are available after a crisis in region 1. First, the realized fundamental θ_1 becomes public information.¹⁶ Second, a proportion $n \in [0, 1]$ of investors learn the realized correlation ρ .

Investors in region 2 use these pieces of information to re-assess the local fundamental θ_2 . That is, they update the prior about the unknown fundamental in region 2 after a crisis in region 1. We assume $\rho_H < 1$ to ensure that θ_2 is not fully revealed by observing θ_1 . Subsequently, we endogenize the information available to investors in region 2 after a crisis in region 1.¹⁷

¹⁵Complete information about the fundamental leads to multiple equilibria for interim values of the fundamental $\theta_t \in [0, 1)$. Both $A^* = 0$ and $A^* = 1$ are sustained by self-fulfilling expectations. By contrast, a unique equilibrium exists for other values of the fundamental. All investors attack if the fundamental is low, $\theta_t < 0$, and do not attack if the fundamental is high, $\theta_t \geq 1$.

¹⁶Observing the fundamental θ_1 if and only if a crisis occurs in region 1 is our preferred assumption. We show in section 5 that our main results hold when observing θ_1 is symmetric.

¹⁷We study two cases of costly information choice: publicly available information about the correlation (section 4) and, in a robustness exercise, private information about the local fundamental (section 5). In either case, the information helps investors re-assess the local fundamental θ_2 .

- Date 1:**
- The correlation of fundamentals ρ is realized but unobserved.
 - The fundamentals θ_t are drawn but unobserved.

Coordination stage

- Investors receive private information x_{i1} .
- Investors simultaneously decide whether to attack a_{i1} .
- Payoffs are received in region 1.

Date 2: Information stage: fundamental re-assessment in region 2

- After a crisis in region 1, the following information is available:
 - the fundamental θ_1 is observed by investors in region 2 and
 - a proportion n of investors obtains information about ρ .
- Investors re-assess the local fundamental θ_2 .

Coordination stage

- Investors receive private information x_{i2} .
- Investors simultaneously decide whether to attack a_{i2} .
- Payoffs are received in region 2.

Table 1: Timeline

2 Equilibrium

We review briefly the well-known equilibrium in region 1 (e.g., Vives (2005)). Next, we analyze the case of exogenous information in region 2, whereby a known proportion of investors learn the realized correlation of fundamentals.¹⁸ We show that there exists a unique equilibrium in region 2 for any proportion of informed investors if private information is sufficiently precise.

Region 1 A Bayesian equilibrium in region 1 is an attack decision a_{i1} for each investor $i \in [0, 1]$ and an aggregate attack size A_1 that satisfy both individual optimality and aggregation:

$$a_{i1}^* = \arg \max_{a_{i1} \in \{0,1\}} E[u(a_{i1}, A_1, \theta_1) | x_{i1}] \equiv a(x_{i1}), \forall i \quad (4)$$

$$A_1^* = \int_{-\infty}^{+\infty} a(x_{i1}) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_{i1} - \theta_1)) dx_{i1} \equiv A(\theta_1) \quad (5)$$

where $\phi(x)$ and $\Phi(x)$ denote the probability density function (pdf) and cumulative density function (cdf) of the standard Gaussian random variable.

There exists a unique equilibrium if private information is sufficiently precise (Morris and Shin (2003)).¹⁹ There are two equilibrium conditions. First, the critical mass conditions states that the aggregate attack size equals the *fundamental threshold*, $A_1^* = \theta_1^*$. Second, an investor who receives the *signal threshold* $x_{1i} = x_1^*$ is indifferent between attacking and not attacking.

Lemma 1 [Morris and Shin (2003)] *If private information is sufficiently precise, $\gamma > \underline{\gamma}_0 \equiv \frac{\sigma^2}{2\pi} \in (0, \infty)$, then there exists a unique Bayesian equilibrium in region 1. This equilibrium is in threshold strategies, whereby investor i attacks if and only if $x_{i1} < x_1^*$ and a crisis occurs if and only if $\theta_1 < \theta_1^*$, where the threshold of the*

¹⁸Our results are robust to the optimal information choice by investors. See sections 4 and 5.

¹⁹See Appendix A.1 for the associated derivations and comparative statics results.

fundamental θ_1^* is implicitly defined by:

$$F_1(\theta_1^*) \equiv \Phi\left(\frac{\alpha}{\sqrt{\alpha+\gamma}}(\theta_1^* - \mu) - \sqrt{\frac{\gamma}{\alpha+\gamma}}\Phi^{-1}(\theta_1^*)\right) = \frac{1}{1+b_1/l_1}. \quad (6)$$

and the signal threshold x_1^* is defined by equation (30) in Appendix A.1.

The fundamental threshold θ_1^* strictly decreases in the prior mean μ and strictly increases in the relative gain from attacking b_1/l_1 . Therefore, there exists a unique relative gain that ensures $\theta_1^* = \mu$.

Assumption 1 The relative gain from attacking b_1/l_1 is set to ensure $\theta_1^* = \mu$:

$$\mu \equiv \Phi\left(\frac{\sqrt{\alpha+\gamma}}{\gamma}\Phi^{-1}\left(\frac{b_1/l_1}{b_1/l_1+1}\right)\right). \quad (7)$$

Assumption 1 simplifies the exposition since a crisis in region 1 is due to a low fundamental, $\theta_1 < \mu$. As shown in the working paper version (Ahnert and Bertsch (2013)), our key results generalize.

Region 2 Consider a crisis in region 1, $\theta_1 < \theta_1^*$. After this wake-up call, investors observe θ_1 and a known proportion n of investors learn the realized correlation ρ . Investors re-assess the local fundamental by forming an updated prior about θ_2 . Next, investors use their private information x_{i2} to form a posterior about θ_2 .

Bayesian updating Informed investors re-assess the fundamental in region 2, using both θ_1 and ρ to form an updated prior. Normality is preserved, with conditional mean $\mu_2|\rho, \theta_1 = \rho\theta_1 + (1-\rho)\mu \equiv \mu_2(\rho, \theta_1)$, and variance $\alpha_2|\rho = \frac{\alpha}{1-\rho^2} \equiv \alpha_2(\rho)$:

$$\theta_2|\rho = 0 \sim \mathcal{N}\left(\mu, \frac{1}{\alpha}\right) \quad (8)$$

$$\theta_2|\rho = \rho_H, \theta_1 \sim \mathcal{N}\left(\rho_H\theta_1 + (1-\rho_H)\mu, \frac{1-\rho_H^2}{\alpha}\right). \quad (9)$$

Uninformed investors, by contrast, can only use θ_1 to re-assess the fundamental in region 2. Thus they form a mixture distribution between $\theta_2|\rho = 0$ and $\theta_2|\rho = \rho_H, \theta_1$ by using the ex-ante distribution of the correlation as weights:

$$\theta_2|\theta_1 \equiv p \cdot [\theta_2|\rho = 0] + (1 - p) \cdot [\theta_2|\rho = \rho_H, \theta_1]. \quad (10)$$

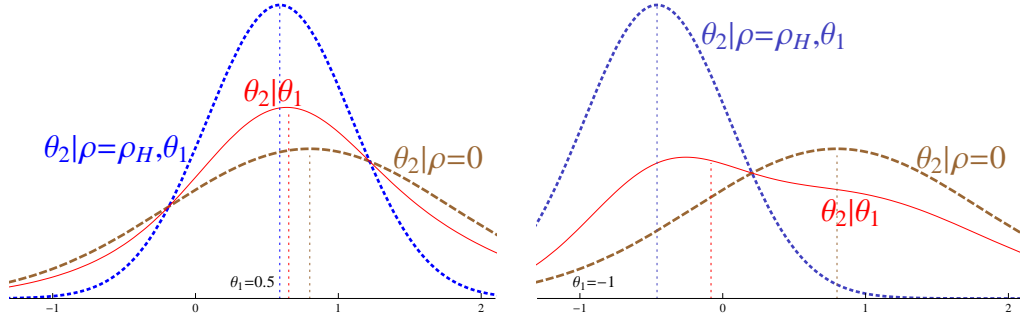


Figure 1: Re-assessment of local fundamentals: The updated prior distributions of informed investors for zero correlation (dashed brown), positive correlation (dotted blue) and of uninformed investors (solid red). Parameters: $\mu = 0.8$, $\alpha = 1$, $p = 0.7$, $\rho_H = 0.7$, $\theta_1 = 0.5$ (left panel), $\theta_1 = -1$ (right panel).

Figure 1 shows the re-assessment of local fundamentals after the initial crisis. It depicts the updated prior distributions for both groups of investors. The updated prior of informed investors, who learn about a zero correlation, has the highest mean and variance. In contrast, learning about positive correlation leads to an updated prior distribution with the lowest mean and variance. The updated prior distribution of uninformed investors can be unimodal, similar to a normal distribution with fat tails (left panel), while it can be bimodal for small values of θ_1 (right panel).

Definition 1 characterizes the strength of the prior about the fundamental.²⁰ We will consider a strong prior when describing our contagion results in section 3.

²⁰As shown in Appendix A.4, a weak prior makes a crisis more likely relative to the prior, $\mu_2(\rho, \theta_1) < \theta_2^*(1, \rho, \theta_1) < 1$, while a strong prior makes a crisis relatively less likely, $0 < \theta_2^*(1, \rho, \theta_1) < \mu_2(\rho, \theta_1)$. These statements hold for each realized correlation.

Definition 1 *The prior about the fundamental is **strong** if, for each realized correlation $\rho \in \{0, \rho_H\}$:*

$$\mu_2(\rho, \theta_1) > \max\{X(\rho), Y(\rho)\}, \quad (11)$$

where:

$$X(\rho) \equiv \Phi\left(-\frac{\sqrt{\alpha_2(\rho) + \gamma}}{\sqrt{\gamma}} \Phi^{-1}\left(\frac{1}{1 + b_2/l_2}\right)\right), \quad (12)$$

$$Y(\rho) \equiv \frac{1}{2} - \frac{\sqrt{\alpha_2(\rho) + \gamma}}{\alpha_2(\rho)} \Phi^{-1}\left(\frac{1}{1 + b_2/l_2}\right). \quad (13)$$

Subsequently, investors use their private information x_{i2} to form a posterior about the fundamental in region 2. Informed investors form a posterior about the fundamental depending on the observed correlation, $\theta_2|\rho = 0, x_{i2}$ and $\theta_2|\rho = \rho_H, x_{i2}$. These posterior distributions are conditionally normally distributed with greater precision and a mean shifted towards the private signal x_{i2} .

Since uninformed investors do not observe the realized correlation, they form a belief using the observed fundamental, θ_1 , and the private signal about the fundamental in region 2, x_{i2} . Let \hat{p} denote this belief about a zero correlation of fundamentals that we derive and analyze in Appendix A.2.2:

$$\hat{p} \equiv \Pr\{\rho = 0|\theta_1, x_{i2}\}. \quad (14)$$

Using the updated belief \hat{p} as weight, the posterior about θ_2 is again an average over the cases of positive and zero correlation, which follows a mixture distribution:

$$\theta_2|\theta_1, x_{i2} \equiv \hat{p} \cdot [\theta_2|\rho = 0, x_{i2}] + (1 - \hat{p}) \cdot [\theta_2|\rho = \rho_H, \theta_1, x_{i2}]. \quad (15)$$

Equilibrium conditions We focus on monotone equilibria. Let $x_{2I}^*(n, \rho, \theta_1)$ and $x_{2U}^*(n, \theta_1)$ denote the signal thresholds below which informed and uninformed investors attack. Likewise, let $\theta_2^*(n, \rho, \theta_1)$ denote the fundamental threshold for each realized correlation. The notation highlights the dependence on the realized funda-

mental, θ_1 , and the proportion of informed investors, n .

The equilibrium in region 2 is characterized by indifference and critical mass conditions. Different to the analysis of region 1, there are now two distinct fundamental thresholds – one for each realized correlation – and thus two critical mass conditions. Similarly, there are now three indifference conditions – one for uninformed investors and one for informed investors for each realized correlation. We derive these conditions in Appendix A.2.

Proposition 1 *Existence of a unique monotone equilibrium.* *Suppose there is a crisis in region 1, $\theta_1 < \theta_1^*$. If private information is sufficiently precise, $\gamma > \underline{\gamma}_1 \in (0, \infty)$, there exists a unique monotone Bayesian equilibrium in region 2 for any proportion of informed investors, $n \in [0, 1]$. For each realized correlation $\rho \in \{0, \rho_H\}$, investors attack if and only if their private signal is sufficiently low: $x_{i2} < x_{2U}^*(n, \theta_1)$ if uninformed and $x_{i2} < x_{2I}^*(n, \rho, \theta_1)$ if informed. A crisis occurs if and only if the fundamental is sufficiently low, $\theta_2 < \theta_2^*(n, \rho, \theta_1)$, for each $\rho \in \{0, \rho_H\}$.*

Proof See Appendix A.3, which also contains the definitions of the thresholds.

The equilibrium analysis in region 2 is more complicated for two reasons. First, the updated priors are heterogeneous across investors since only informed investors observe the realized correlation. Second, the posterior distribution about the fundamental (as well as the updated prior distribution) formed by uninformed investors is a mixture distribution, so normality is lost. Using the results of Milgrom (1981) and Vives (2005), we can show that the best-response function of an individual investor strictly increases in the thresholds used by other investors (see Appendix A.2.3). Hence, the simple and common requirement of precise private information still suffices for uniqueness in monotone equilibrium – despite the heterogeneity in priors and the use of mixture distributions by uninformed investors.

3 Contagion after a wake-up call

In this section we describe our main results. Contagion is defined as an increase in the probability of a crisis in region 2 after a crisis in region 1.

Lemma 2 establishes information contagion in our setup.²¹ A crisis in region 1 is bad news about the fundamental in region 1. Since fundamentals may be correlated, this crisis also is bad news about the fundamental in region 2. Hence, the re-assessment of the local fundamental θ_2 leads to weaker expected fundamentals after a crisis in region 1, increasing the probability of a crisis in region 2.²²

Lemma 2 *Information contagion.* *Suppose private information is sufficiently precise, $\gamma > \underline{\gamma}_3 \in (0, \infty)$, and investors are uninformed about the correlation of fundamentals, $n = 0$. A crisis in region 2 is more likely after a crisis in region 1:*

$$\Pr\{\theta_2 < \theta_2^*(0, \rho, \theta_1) \mid \theta_1 < \theta_1^*\} > \Pr\{\theta_2 < \theta_2^*(0, \rho, \theta_1) \mid \theta_1 \geq \theta_1^*\}. \quad (16)$$

Proof See Appendix A.5.

We now separate this information contagion channel from wake-up call contagion. In Proposition 2 we show that contagion can occur even if investors learn that fundamentals are uncorrelated ex post. That is, the probability of a crisis in region 2 is higher after observing a crisis in region 1 and learning about zero correlation than after observing no crisis. In sum, contagion can occur after a wake-up call without a common investor base, balance sheet links, or ex-post correlated fundamentals.

²¹Information contagion has been established by Acharya and Yorulmazer (2008) and Allen et al. (2012). Acharya and Yorulmazer (2008) show that the funding cost of one bank increases after bad news about another bank when the banks' loan portfolio returns have a common factor. To avoid information contagion ex post, banks herd their investment ex ante. Allen et al. (2012) compare the impact of information contagion on systemic risk across asset structures. Adverse news about the solvency of the banking system leads to runs on multiple banks.

²²This result is consistent with the empirical findings of Eichengreen et al. (1996), whereby a currency crisis elsewhere increases the probability “of a speculative attack by an economically and statistically significant amount” (p. 2).

Proposition 2 *Wake-up call contagion.* *Suppose private information is sufficiently precise, $\gamma > \underline{\gamma}_4 \in (0, \infty)$, public information is sufficiently imprecise, $\alpha < \bar{\alpha} \in (0, \infty)$, and the prior is strong. For any proportion of informed investors, $n \in [0, 1]$, a crisis in region 2 is more likely after a crisis in region 1 – even if investors in region 2 learn that fundamentals are uncorrelated, $\rho = 0$:*

$$\Pr\{\theta_2 < \theta_2^*(n, \rho, \theta_1) | \rho = 0, \theta_1 < \theta_1^*\} > \Pr\{\theta_2 < \theta_2^*(0, \rho, \theta_1) | \theta_1 \geq \theta_1^*\} \forall n. \quad (17)$$

Proof See Appendix A.6.

Proposition 2 states our main result on contagion after a wake-up call. The right-hand side of inequality (17) is unchanged relative to Lemma 2. It conditions on no crisis in region 1, $\theta_1 \geq \theta_1^*$, and allows for any realization of the correlation $\rho \in \{0, \rho_H\}$.²³ By contrast, the left-hand side conditions on uncorrelated fundamentals and allows for any proportion of informed investors after the crisis in region 1.

Intuition If fundamentals are uncorrelated, a crisis in region 1 does not affect the probability of a crisis in region 2. If fundamentals are correlated, however, a crisis in region 1 has consequences for contagion. Specifically, the conditional probabilities on both sides of condition (17) differ for two reasons, each associated with the re-assessment of the local fundamental θ_2 .

First, the mean of the local fundamental matters. Learning that no crisis occurred in region 1 improves the mean of the updated prior on the right-hand side, relative to the case of a crisis in region 1 and no correlation. Hence, this *mean effect* works towards the inequality stated in Proposition 2.

Second, the variance of the local fundamental matters. On the left-hand side, the public information about the local fundamental θ_2 is less precise after learning that fundamentals are uncorrelated. Consequently, private information becomes

²³This conditional probability uses that the realized fundamental θ_1 is publicly observed only after a crisis in region 1. Furthermore, using the events $E_1 = \theta_2 < \theta_2^*$ and $E_2 = \theta_1 \geq \theta_1^*$, the ex-ante probability of a crisis in region 2 after not observing a crisis in region 1 is decomposed by the law of total probability: $\Pr\{E_1|E_2\} = p\Pr\{E_1|\rho = 0, E_2\} + (1 - p)\Pr\{E_1|\rho = \rho_H, E_2\}$.

relatively more precise, which results in greater disagreement among informed investors, who learn that fundamentals are uncorrelated. If the prior is strong, greater disagreement translates into more aggressive attacks and a larger probability of a crisis (Metz (2002)). This *variance effect* works towards the inequality stated in Proposition 2.²⁴ In sum, both the mean and the variance effects are aligned and generate the result of contagion after a wake-up call.

We further explore the variance effect and its implications for contagion due to greater disagreement among investors who learn that fundamentals are uncorrelated. Therefore, we continue by examining the left-hand side of inequality (17). In Proposition 3, we show that the extent of contagion after a wake-up call can increase in the proportion of informed investors – even when fundamentals are uncorrelated.

Proposition 3 *Enhanced perception of risk after a wake-up call.* Suppose private information is sufficiently precise, $\gamma > \underline{\gamma}_2 \in (0, \infty)$, public information is sufficiently imprecise, $\alpha < \bar{\alpha} \in (0, \infty)$, and the prior is strong. Consider a crisis in region 1 triggered by an intermediate realized fundamental $\theta_1 \in (\underline{\theta}_1, \mu)$. If fundamentals are uncorrelated, $\rho = 0$, then a crisis in region 2 is more likely, the more investors are informed about the zero correlation of fundamentals:

$$\frac{d}{dn} \left(\Pr\{\theta_2 < \theta_2^*(n, \rho, \theta_1) | \rho = 0, \theta_1\} \right) > 0, \quad \forall \theta_1 \in (\underline{\theta}_1, \mu], \quad (18)$$

where the lower bound $\underline{\theta}_1$ is defined by:

$$\underline{\theta}_1 \equiv \mu + \frac{1}{\rho_H} \left(\begin{aligned} &(\theta_2^*(1, 0, \theta_1) - \mu) \left[1 - \frac{\alpha}{\alpha_2(\rho_H)} \sqrt{\frac{\alpha_2(\rho_H) + \gamma}{\alpha + \gamma}} \right] \\ &+ \frac{\sqrt{\gamma}}{\alpha_2(\rho_H)} \Phi^{-1}(\theta_2^*(1, 0, \theta_1)) \left[\sqrt{\frac{\alpha_2(\rho_H) + \gamma}{\alpha + \gamma}} - 1 \right] \end{aligned} \right) < \mu. \quad (19)$$

Proof See Appendix A.7. The lower bound $\underline{\theta}_1$ is derived in the proof of Lemma 5.

Figure 1 is key to understanding Proposition 3. Since fundamentals may be positively correlated, a crisis in region 1 reduces both the mean and the variance of

²⁴The variance is zero if no investor is informed, $n = 0$. If some investors are informed after a wake-up call, however, the variance effect contributes to the inequality stated in Proposition 2.

the updated prior about the fundamental in region 2. Likewise, if informed investors learn that fundamentals are uncorrelated, both the mean and the variance of θ_2 are revised upwards, enhancing disagreement. The overall effect of the re-assessment of the local fundamental therefore depends on the relative size of the *mean* and *variance effect*, since these effects move in opposite directions for a strong prior.²⁵

Mean effect If more investors are informed, more investors re-assess the mean of the local fundamental upwards. Better public information – a higher mean of the updated prior $\mu_2(\rho, \theta_1)$ – reduces the fundamental threshold (Vives (2005); Manz (2010)). Consequently, $\theta_2^*(1, 0, \theta_1)$ is *lower* relative to $\theta_2^*(1, \rho_H, \theta_1)$. This mean effect works against the result of enhanced perception of risk after a wake-up call.

Variance effect If more investors are informed, more investors re-assess the precision of the local fundamental downwards. More dispersed public information – a higher variance of the updated prior $\alpha_2(\rho, \theta_1)$ – leads to relatively more precise private information. This induces greater disagreement among informed investors. The fundamental threshold increases in the degree of disagreement if the prior about the fundamental is strong (Metz (2002)).²⁶ Investors attack more aggressively, so $\theta_2^*(1, 0, \theta_1)$ is *higher* relative to $\theta_2^*(1, \rho_H, \theta_1)$. If the prior is strong, this variance effect works in favor of the enhanced perception of risk after a wake-up call.

The probability of a crisis in region 2 increases in the proportion of informed investors if the variance effect dominates the mean effect. Thus, a sizable variance effect is at the heart of the result on the enhanced perception of risk. This label arises since the result is driven by the enhanced disagreement of informed investors and the associated greater concern for the attacking behavior of other investors (strategic uncertainty). The variance effect outweighs the mean effect under the conditions of Lemma 5, specifically the lower bound $\underline{\theta}_1$ that restricts the size of the mean effect.

²⁵See Appendix A.4 for comparative static results and their dependence on these effects.

²⁶Related to Metz (2002), see also Iachan and Nenov (2014) for an investigation of the sensitivity of the net payoffs to the fundamentals when the relative precision of private information changes.

Figure 2 illustrates this link between the fundamental thresholds and the proportion of informed investors. Proposition 3 implies the ranking of fundamental thresholds $\theta_2^*(1, 0, \theta_1) > \theta_2^*(0, 0, \theta_1)$. For zero realized correlation, there is a one-to-one mapping between the ranking of thresholds and of the probabilities of a crisis. This ranking extends to any proportion of informed investors, $n \in (0, 1)$, whereby more informed investors increase the probability of a crisis in region 2.²⁷

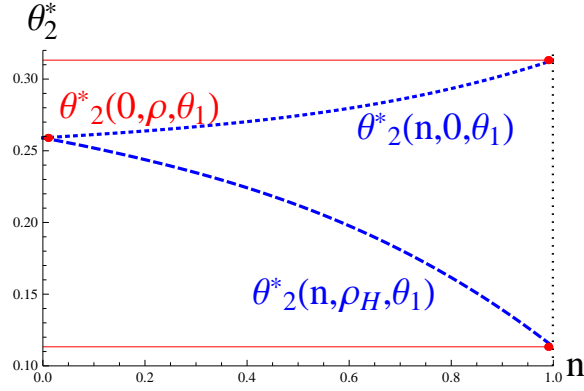


Figure 2: The fundamental thresholds and the proportion of informed investors. Parameters: $\mu = 0.8$, $\alpha = 1$, $\gamma = 1$, $b_2 = l_2 = 1$, $p = 0.7$, $\rho_H = 0.7$, $\theta_1 = 0.7 < \mu$.

Formally, Lemma 6 in Appendix A.4.4 states that the fundamental thresholds evolve continuously and monotonically in the proportion of informed investors, provided sufficiently precise private and sufficiently imprecise public information. In particular, the distance, $|\theta_2^*(n, 0, \theta_1) - \theta_2^*(n, \rho_H, \theta_1)|$, continuously increases in the proportion of informed investors, so the fundamental thresholds for $\rho = 0$ and $\rho = \rho_H$ diverge. Intuitively, informed investors capitalize on their information advantage. While uninformed investors must use the same signal threshold irrespective of the realized correlation, informed investors adjust their signal thresholds.²⁸

²⁷This is an uninformed-is-bliss feature. More information can lead to adverse outcomes in Hirshleifer (1971). Information acquisition can be privately optimal but has a negative public value, since it makes co-insurance for risk-averse agents infeasible. Instead, Morris and Shin (2007) analyze optimal communication and provide a rationale for coarse information, for instance in credit ratings. Dang et al. (2012) provide an "ignorance-is-bliss" argument, whereby information insensitivity is key to security design in the money market. More transparency can also be harmful in an expert model with career concerns (Prat (2005)).

²⁸A larger proportion of informed investors raises the fundamental threshold $\theta_2^*(n, 0, \theta_1)$, as in-

4 Information acquisition

We endogenize the information investors use to re-assess the local fundamental after a wake-up call. We study the costly acquisition of information about the correlation of fundamentals ρ , which helps to improve the forecast about θ_2 . At the information stage, investors simultaneously decide whether to purchase a perfectly revealing and publicly available signal at a cost $c > 0$.²⁹ Each investor can purchase the same signal and observes it privately. In terms of wholesale investors or currency speculators, costly information acquisition could be access to Bloomberg and Datastream terminals or hiring analysts who assess the publicly available data.

This section shows that the contagion results obtained under exogenous information prevail with endogenous information after a wake-up call.³⁰ A unique equilibrium is obtained for a sufficiently low information cost, in which all investors acquire information after a wake-up call.³¹ We also establish a theoretical result about the information choice in a coordination game of regime change. Specifically, the information choices of investors exhibit strategic complementarity similar to Hellwig and Veldkamp (2009), who study a beauty contest coordination game.

We analyze pure-strategy perfect Bayesian equilibrium (PBE) in threshold strategies (Definition 2). Based on the previous analysis, such as Proposition 1 and

vestors attack more aggressively after learning $\rho = 0$, compared with uninformed investors (Part (a) of Lemma 6; see thick dotted line in Figure 2). The opposite holds for positive correlation, $\rho = \rho_H$, when informed investors attack relatively less aggressively, so $\theta_2^*(n, \rho_H, \theta_1)$ decreases in the proportion of informed investors (thick dashed line in Figure 2). Finally, the difference between these thresholds increases in the proportion of informed investors (Part (b) of Lemma 6).

²⁹We discuss noisy signals about the correlation in Appendix 5.

³⁰We argue in section 5 that the key insights on wake-up call contagion prevail when considering an alternative information acquisition game, whereby investors can re-assess fundamentals in region 2 by purchasing more precise private information about the local fundamental θ_2 at a convex cost.

³¹We do not explicitly model the information choice of investors in the absence of a wake-up call. Our view is that crises are discontinuous events, whereby the available information – and, by extension, the information cost – depend on whether a crisis occurred. Consequently, we argued earlier that θ_1 is only observed after a crisis in region 1, for example motivated by the news coverage of crises and public inquiry. Likewise, the information cost is much higher in the absence of a crisis in region 1, so information acquisition does not occur in the absence of a wake-up call.

Lemma 6, we study the incentives to acquire information.³² Let $d_i \in \{I, U\}$ denote the information choice of investor i and let $a_{iI} \equiv a_{i2}(d_i = I)$ and $a_{iU} \equiv a_{i2}(d_i = U)$ denote the corresponding attack rules. First, we analyze the optimal information choice d_i^* and derive a strategic complementarity in information choices (Lemma 3). Second, we show that the fundamental re-assessment after a wake-up call – the heart of our contagion mechanism – arises endogenously in the unique equilibrium, provided the information cost is sufficiently low (Proposition 4).

Definition 2 *A pure-strategy monotone perfect Bayesian equilibrium comprises an information choice $d_i^* \in \{I, U\}$ for each investor $i \in [0, 1]$, an aggregate proportion of informed investors $n^* \in [0, 1]$, an attack rule $a_{i2d}^*(n^*; \theta_1, x_{i2}) \in \{0, 1\}$ for each investor, and an aggregate attack size $A_2^* \in [0, 1]$ such that:*

1. *All investors optimally choose d_i at the information stage.*
2. *The proportion n^* is consistent with the individually optimal information choices $\{d_i^*\}_{i \in [0, 1]}$.*
3. *Uninformed investors have an optimal attack rule $a_{2U}^*(n^*; \theta_1, x_{i2})$. For any given realization of $\rho \in \{0, \rho_H\}$, informed investors have an optimal attack rule $a_{2I}^*(n^*; \theta_1, \rho, x_{i2})$.*
4. *The proportion A_2^* is consistent with the individually optimal attack decisions:*

$$A_2^* \equiv A(n^*; \theta_2, \rho) = n^* \int_{-\infty}^{+\infty} a_{2I}^*(n^*; \theta_1, \rho, x_{i2}) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_{i2} - \theta_2)) dx_{i2} \quad (20)$$

$$+ (1 - n^*) \int_{-\infty}^{+\infty} a_{2U}^*(n^*; \theta_1, x_{i2}) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_{i2} - \theta_2)) dx_{i2}, \quad \forall \rho \in \{0, \rho_H\}.$$

We establish strategic complementarity in information choices: an investor's incentive to acquire information increases in the proportion of informed investors. This property arises from the monotonicity in signal thresholds (Lemma 6 C).

³²In contrast to section 3, we no longer need to assume common knowledge about the proportion of informed investors. Furthermore, under the stated conditions on the information cost, the information choice of investors is in dominant actions.

An individual investor i compares the expected utility from acquiring information, $E[u(d_i = I, n)] \equiv EU_I - c$ to the the expected utility from not acquiring information, $E[u(d_i = U, n)] \equiv EU_U$. Both expressions are defined in Appendix A.9. The expected utility of acquiring information comprises the benefit of attacking if a crisis occurs, the cost of attacking if no crisis occurs, and the information cost. This expression also takes into account the possible realizations of the correlation, since these affect the signal threshold of an informed investor, $x_{2I}^*(n, 0, \theta_1)$ and $x_{2I}^*(n, \rho_H, \theta_1)$. By contrast, an uninformed investor cannot tailor the attack strategy and must use the *same* signal threshold $x_{2U}^*(n, \theta_1)$ throughout.

Optimality requires that investors acquire information if and only if the expected utility differential $EU_I - EU_U$ is no smaller than the information cost. In other words, it pays to acquire information if the benefit of using tailored signal thresholds covers at least the information cost:

$$EU_I - EU_U \geq c \Rightarrow d_i^* = I. \quad (21)$$

Let $\bar{c}(n, \theta_1) \equiv EU_I - EU_U$ be the information cost that makes an investor indifferent between the information choices, for a given proportion of informed investors:

$$\begin{aligned} \bar{c}(n, \theta_1) = & p \left(\int_{-\infty}^{\theta_2^*(n, 0, \theta_1)} b_2 \int_{x_{2U}^*(n, \theta_1)}^{x_{2I}^*(n, 0, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | 0, \theta_1) d\theta_2 \right. \\ & \left. - \int_{\theta_2^*(n, 0, \theta_1)}^{+\infty} l_2 \int_{x_{2U}^*(n, \theta_1)}^{x_{2I}^*(n, 0, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | 0, \theta_1) d\theta_2 \right) - \quad (22) \\ & (1 - p) \left(\int_{-\infty}^{\theta_2^*(n, \rho_H, \theta_1)} b_2 \int_{x_{2I}^*(n, \rho_H, \theta_1)}^{x_{2U}^*(n, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | \rho_H, \theta_1) d\theta_2 \right. \\ & \left. - \int_{\theta_2^*(n, \rho_H, \theta_1)}^{+\infty} l_2 \int_{x_{2I}^*(n, \rho_H, \theta_1)}^{x_{2U}^*(n, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | \rho_H, \theta_1) d\theta_2 \right), \end{aligned}$$

where the distribution of the fundamental conditional on the realized correlation, $f(\theta_2 | \rho, \theta_1)$, is normal with mean $\mu_2(\rho, \theta_1)$ and precision $\alpha_2(\rho)$ and the distribution of the private signal conditional on the fundamental, $g(x | \theta_2)$, is normal with mean θ_2 and precision γ . In Appendix A.8, we provide intuition for the benefits of a tailored signal threshold used by informed investors. We also describe the type-I and type-II errors investors make in their attack behavior.

Lemma 3 states how the threshold information cost changes with the proportion of informed investors. Hellwig and Veldkamp (2009) show that information choices inherit the strategic complementarity or substitutability from the underlying beauty contest game.³³ We show that this inheritance result extends to a global coordination game of regime change, particularly in the context of ex-ante uncertainty about the correlation of fundamentals and publicly available information.

Lemma 3 *Strategic complementarity in information choice.* *Suppose the prior about fundamentals in region 2 is strong, private information is precise, $\gamma > \underline{\gamma}_2 < \infty$, and public information is imprecise, $0 < \alpha < \bar{\alpha}$. After a crisis in region 1, the incentives to acquire information increase in the proportion of informed investors:*

$$\frac{d\bar{c}(n, \theta_1)}{dn} \geq 0 \quad \forall \theta_1 < \mu. \quad (23)$$

Furthermore, for any proportion of informed investors, $n \in [0, 1]$, we have:

$$\bar{c}(n, \underline{\theta}_1) = 0; \quad \bar{c}(n, \theta_1) > 0 \quad \forall \theta_1 \neq \underline{\theta}_1. \quad (24)$$

Proof See Appendix A.9.

If $\theta_1 = \underline{\theta}_1$, then the signal thresholds of informed and uninformed investors coincide, $x_{2I}^* = x_{2U}^*$. Therefore, there is no benefit of acquiring information, since no tailored attack strategy can be used by an informed investors. As a result, investors are not willing to acquire costly information.

As stated in Proposition 4, strategic complementarity in information choices implies simple conditions sufficient for the existence of a unique equilibrium. The fundamental re-assessment after a wake-up call entails the acquisition of information in dominant actions, $n^* = 1$, for a small positive cost $c \in (0, \bar{c}(0, \theta_1))$, for any

³³Ahnert and Kakhbod (2014) obtain strategic complementarity in information choices in a one-region global coordination game of regime change with a common prior, a discrete private information choice and heterogeneous information costs. They show that the information choice of investors amplifies the probability of a financial crisis.

$\theta_1 \neq \underline{\theta}_1$. Contagion after a wake-up call arises endogenously – despite zero correlation of fundamentals as observed by informed investors.

Proposition 4 *Existence of a unique equilibrium with wake-up call contagion.* Suppose the prior about the fundamentals in region 2 is strong, private information is precise, $\gamma > \max\{\underline{\gamma}_2, \underline{\gamma}_4\} < \infty$, and public information is imprecise, $\alpha < \bar{\alpha} > 0$. After a crisis in region 1, $\theta_1 < \mu$, there exists a unique monotone pure-strategy PBE if the information cost is sufficiently small, $c < \bar{c}(0, \theta_1)$. All investors acquire information, $n^* = 1$, and use the signal threshold $x_{2I}^*(1, \rho, \theta_1)$ for each $\rho \in \{0, \rho_H\}$. Even if fundamentals are uncorrelated, contagion occurs after a wake-up call.

Proof See Appendix A.10.

Furthermore, Corollary 1 builds on Proposition 4. It states conditions sufficient for the enhanced perception of risk after a wake-up call to arise under endogenous information (see Proposition 3 for exogenous information). It compares two equilibria: one with information acquisition if the information cost is low, and one without information acquisition if the information cost is high.

Corollary 1 *Enhanced perception of risk after a wake-up call.* Consider the sufficient conditions of Proposition 4. For high information costs, $\bar{c}(1, \theta_1) < c$, there exists a unique equilibrium with no information acquisition, $n^* = 0$, and investors use the signal threshold $x_{2U}^*(0, \theta_1)$. Suppose the fundamental in region 1 takes an intermediate value, $\theta_1 \in (\underline{\theta}_1, \mu)$, and the correlation is zero. Then, the probability of a crisis in region 2 is higher in the equilibrium with information acquisition, supported by $c < \bar{c}(0, \theta_1)$, than in the equilibrium without information acquisition, supported by $\bar{c}(1, \theta_1) < c$.

Proof See Appendix A.11.

5 Discussion

We discuss several model extensions and alternative assumptions in this section.

Endogenous precision of private information In section 4, we analyzed endogenous information about the correlation of fundamentals, which helps investors in region 2 re-assess the local fundamental θ_2 . Here we extend our analysis to private information choice about the local fundamental θ_2 .³⁴ We show that wake-up call contagion is further strengthened under private information choice.

Our modeling of private information acquisition follows Szkup and Trevino (2012b), who propose a model in which investors choose the precision of their private information subject to convex information costs. After observing a crisis in region 1, investors in region 2 simultaneously choose the precision of their signal about θ_2 . To simplify the exposition, we restrict attention to the case when the information cost for the signal about the correlation is sufficiently low, such that all investors learn the realized correlation ρ after the wake-up call. Szkup and Trevino (2012b) develop a single-region global games model with a related payoff structure:

$$\begin{aligned} u(a_i = 1, A, \theta) &= (1 - T) \mathbf{1}_{\{A > 1 - \theta\}} - T \mathbf{1}_{\{A \leq 1 - \theta\}} \\ u(a_i = 0, A, \theta) &= 0, \end{aligned} \tag{25}$$

where $\theta \sim \mathcal{N}(\mu_\theta, \tau_\theta^{-1})$ is unobserved but investors receive the private signal $x_i | \theta \sim \mathcal{N}(\theta, \tau^{-1})$. For the special case of $T = 1/2$ and $b_2 = l_2 = 1/2$, we have an equivalent formulation, where we just insert the subscript for region 2:

$$\begin{aligned} u(a_{i2} = 1, A_2, \theta_2) &= 1/2 \mathbf{1}_{\{A_2 > 1 - \theta \equiv \theta_2\}} - 1/2 \mathbf{1}_{\{A_2 \leq 1 - \theta \equiv \theta_2\}} \\ u(a_{i2} = 0, A_2, \theta_2) &= 0, \end{aligned} \tag{26}$$

where $\theta_2 \sim \mathcal{N}(\mu_2, \alpha_2^{-1})$, with $\mu_2 = 1 - \mu_\theta$ and $\alpha_2 = \tau_\theta$.

³⁴We thank our discussant Laura Veldkamp for suggesting to analyze this case.

Szkup and Trevino (2012b) show that there exists a unique equilibrium in the information game under certain assumptions on the convex cost function for acquiring more precise private signals. In Appendix A.12, we specify these assumptions and derive the benefit from a higher private signal precision for investors in region 2 of our model who learn about the correlation after a crisis in region 1. We show that this benefit function is identical to the one derived by Szkup and Trevino.

Furthermore, building on the results of Szkup and Trevino (2012b), we find that the marginal benefit of increasing the precision of private information decreases in the precision of public information, provided the prior is sufficiently strong. Extending the analysis of Szkup and Trevino, we show that the marginal benefit of increasing the private signal precision decreases in the mean of public information if the prior is that fundamentals are sufficiently strong.

Formally, for the special case of $b_2 = l_2 = 1/2$, we find that a decrease in α_2 has two effects on the fundamental threshold. Both effects go in the same direction and increase θ_2^* (as well as the probability of a crisis in region 2). First, $d\theta_2^*/d\alpha_2 < 0$ for a given level of γ_2 and, second, $d\gamma_2^*/d\alpha_2 < 0$, which also decreases θ_2^* because $d\theta_2^*/d\gamma_2 > 0$. Furthermore, we find that an increase in μ_2 also has two effects that go in the same direction and both decrease θ_2^* . First, $d\theta_2^*/d\mu_2 < 0$ and, second, $d\gamma_2^*/d\mu_2 < 0$, which also decreases θ_2^* because $d\theta_2^*/d\gamma_2 > 0$.

Taken together, these results imply that the wake-up call contagion result of Proposition 2 can be strengthened if the prior is sufficiently strong. The strengthening of the result is reflected in the endogenous private signal precisions, which further increase the difference in the equilibrium fundamental thresholds, θ_2^* :

$$\begin{aligned} & \Pr\{\theta_2 < \theta_2^*(n = 1, \rho, \theta_1; \gamma_2^*) | \rho = 0, \theta_1 < \theta_1^*\} > \\ & \Pr\{\theta_2 < \theta_2^*(n = 0, \rho, \theta_1; \gamma_2^*) | \theta_1 \geq \theta_1^*\}, \end{aligned} \quad (27)$$

where $[\gamma_2^* | \rho = 0, \theta_1 < \theta_1^*] > [\gamma_2^* | \theta_1 \geq \theta_1^*]$. Intuitively, the private signal precision is relatively higher on the left-hand side for two reasons. First, the zero correlation makes public information more disperse (decrease in α_2) that leads to a relatively

higher θ_2^* on the left-hand side. Second, not observing a crisis in the first region means that fundamentals in region 1 must have been good. This leads to an upward revision in μ_2 and, hence, to a decrease in the optimally chosen private signal precision. This effect is associated with a relatively lower θ_2^* on the right-hand side.

Learning about the fundamental after a crisis We assume that a crisis in region 1 is a discontinuous event, after which θ_1 is commonly known. In contrast, suppose that θ_1 is always learned irrespective of the occurrence of a crisis event in region 1. Our results on wake-up call contagion and information contagion continue to hold. This is demonstrated in case 1 of the proof of Lemma 2, which can be readily extended to show that also the result in Proposition 2 continues to hold.

Next, suppose that θ_1 is never learned. That is, investors in region 2 only observe whether there was a crisis in region 1 ($\theta_1 < \mu$), or not ($\theta_1 \geq \mu$). Again, our results on wake-up call contagion and information contagion continue to hold. This insight is immediate for wake-up call contagion (Proposition 2). Since the realized correlation is zero, the realization of θ_1 does not matter. For information contagion (Lemma 2), the proof (especially, case 2) would need to be modified if θ_1 is unobserved throughout. However, the result of inequality (16) prevails, because the left-hand side is a weighted average over less favorable values of θ_1 .

Noisy signal about the correlation Our key insights hold if learning about the correlation was imperfect. Suppose that the signal about ρ , received or purchased by investors, is noisy. Such noise implies that informed investors also use mixture distributions to form updated priors about the local fundamental in region 2. That is, the signal thresholds of informed investors would become more similar to those of uninformed investors as we move from perfect to imperfect learning about the correlation. While the quantitative difference between the signal thresholds used by informed and uninformed shrinks, our qualitative results are unaffected.

Negative cross-regional correlation of fundamentals We assume that fundamentals may be positively correlated across regions, $\rho_H > 0$. In contrast, suppose for now that fundamentals may be negatively correlated, $\rho_H < 0$. Hence, a low realized fundamental in region 1 shifts the conditional distribution of the fundamental in region 2 after investors learn that $\rho = \rho_H$, implying a higher conditional mean of the prior, $\mu_2(\rho_H, \theta_1) > \mu$. As a result, the fundamental threshold ranking from Lemma 5 is reversed, as can be seen in Table 2 in Appendix A.4.3. Hence, the reverse of the enhanced perception of risk effect in Proposition 3 arises when $\rho_H < 0$. After a crisis in region 1 and if the correlation is non-zero, $\rho = \rho_H < 0$, contagion is more likely if investors are informed about the correlation than when uninformed.

Modeling the ex-ante uncertainty of correlation The ex-ante distribution of the correlation of regional fundamentals is determined by p and ρ_H . A variation in p has a quantitative effect only. In particular, a change in p leaves $\underline{\theta}_1$ unchanged, while an increase (decrease) in p reduces (increases) the difference between $\theta_2^*(0, \rho, \theta_1)$ and $\theta_2^*(1, 0, \theta_1)$. Hence, the effect of an enhanced perception of risk after a wake-up call in Proposition 3 is weakened (strengthened) when p increases (decreases), leaving our qualitative results unaffected. The effect of changing ρ_H is harder to understand because it affects both $\underline{\theta}_1$ and the prior in region 2. However, we can show an *ex-post stability* effect. The result of Proposition 3 holds with opposite inequality if $\rho = \rho_H$:

$$\Pr(\theta_2 < \theta_2^*(1, \rho, \theta_1) | \rho = \rho_H, \theta_1) < \Pr(\theta_2 < \theta_2^*(0, \rho, \theta_1) | \rho = \rho_H, \theta_1) \quad \forall \theta_1 \in (\underline{\theta}_1, \mu)$$

$$\Pr(\theta_2 < \theta_2^*(1, \rho, \theta_1) | \rho = 0, \theta_1) < \Pr(\theta_2 < \theta_2^*(0, \rho, \theta_1) | \rho = 0, \theta_1) \quad \forall \theta_1 < \underline{\theta}_1.$$

6 Testable implications

We offer testable implications of the wake-up call theory for the empirical contagion literature (section 6.1) and the experimental literature (section 6.2). An empirical literature studies the channels of contagion and the characteristics that make regions susceptible to contagion. We contribute to this literature by highlighting the role of the fundamental in the initially affected region and its non-linear effects on contagion. An advantage of laboratory experiments is that the information choice after a wake-up call is observed. We formulate three implications testable in experiments.

6.1 Implications for empirical work

There is a large literature on interdependence and contagion in international finance and financial economics with different approaches (see Forbes (2012) for a recent survey).³⁵ For an empirical literature that investigates (i) the channels of contagion during financial crises; and (ii) the dependence on the fundamental characteristics of the affected countries see Glick and Rose (1999), Van Rijckeghem and Weder (2001, 2003), and Dasgupta et al. (2011). This literature suggests that stronger trade or financial links and higher institutional similarity increase contagion.

In our model, the correlation of regional fundamentals captures such factors: $\rho \geq 0$ measures the intensity of trade or financial links and institutional similarities with the initially affected region, which has fundamentals θ_1 . Let $\Pr(\theta_2, \rho)$ be the probability of a crisis in another region with the characteristics θ_2 and ρ . This probability is conditional on a crisis in the initially affected region. Our model has two empirical implications.

Empirical implication 1.

$$\frac{d \frac{d \Pr(\theta_2, \rho)}{d \rho}}{d \theta_1} < 0. \quad (28)$$

³⁵The approaches include probability models (Eichengreen et al. (1996)), correlation analysis (Forbes and Rigobon (2002)), VAR models (Favero and Giavazzi (2002)), latent factor/GARCH models (Bekaert et al. (2014)), and extreme value analysis (Bae et al. (2003)).

Implication 1 states that a crisis in the first region due to a worse realized fundamental θ_1 is more likely to spread, the stronger the correlation ρ observed by the empiricist. This implication is based on updating and the mean effect (see Lemma 2). Implication 1 can also be implied by other models that build on the mean effect (e.g., Acharya and Yorulmazer (2008)).

Empirical implication 2. Under the conditions of Proposition 3:

$$\begin{aligned} \frac{d\Pr(\theta_2, \rho)}{d\rho} &< 0 \quad \text{if } \theta_1 > \underline{\theta}_1 \\ \frac{d\Pr(\theta_2, \rho)}{d\rho} &> 0 \quad \text{if } \theta_1 < \underline{\theta}_1. \end{aligned} \quad (29)$$

Implication 2 highlights the enhanced perception of risk after a wake-up call (see Lemma 5 and Proposition 3). It is based on the variance effect. In particular, after controlling for the contemporaneous fundamentals of the second region, θ_2 , there is a non-linear effect of the realized fundamental in the first region, θ_1 . A crisis in the first region due to *moderately low* fundamentals is *more* likely to spread if the empiricist observes no linkages, $\rho = 0$. By contrast, a crisis due to *extremely low* fundamentals is *less* likely to spread if the empiricist observes no linkages.

In sum, our wake-up call theory of contagion suggests a role for the fundamentals of the initially affected region. Further, the impact of these fundamentals is non-linear. For sufficiently low fundamentals in the initially affected region, $\theta_1 < \underline{\theta}_1$, contagion is more likely to occur when linkages are present, which is consistent with existing empirical findings. The non-linearity suggested by our theory can improve the measurement of contagion, especially for currency attacks and bank runs, where this non-linearity has been neglected so far.

6.2 Implications for experimental work

The literature on laboratory experiments contains several studies on financial contagion (for example, Cipriani and Guarino (2008) and Cipriani et al. (2013)) and

bank runs (Schotter and Yorulmazer (2009) and Garratt and Keister (2009)). There has been substantial interest in studying global games models in the laboratory, following Heinemann et al. (2004, 2009). However, contagion in a global games framework has received attention only recently. To our knowledge, Trevino (2013) offers the only such experimental study.

Our theory of contagion generates three implications for experiments. The first implication is the enhanced perception of risk after a wake-up call (Proposition 3). An experiment could analyze the impact of the number of informed participants when fundamentals are uncorrelated. Proposition 3 suggests that contagion can increase in the proportion of informed investors even if these observe a zero correlation. Hence, the probability of a crisis increases in the number of informed participants because of the elevated disagreement among informed participants.

A second set of implications relate to the acquisition of publicly available information about the correlation. While there is some experimental work on the acquisition of private signals, for instance by Szkup and Trevino (2012a), the acquisition of publicly available signals is still unexplored. We suggest to study three questions. How do the incentives to acquire information change with the known number of informed participants (Lemma 3)? This question can be analyzed for a given $\theta_1 < \theta_1^*$ by exogenously varying n and eliciting the willingness to pay for information. Furthermore, which thresholds of the information cost \bar{c} induce all (no) participant(s) to acquire information (Proposition 4 and Corollary 1)? This question can be analyzed by setting $n = 0$ ($n = 1$) in the previous exercise. Similarly, the information choices of participants are recorded for various information costs. Finally, under what conditions do the contagion results arise in laboratory experiments with endogenous information? This question requires to record, for various information costs, the information choices of participants after a wake-up call.

A third implication concerns the fundamental of the first region, θ_1 , in the previous two experiments. Proposition 3 and Lemma 5 imply an important role for the threshold θ_1 , highlighting the non-linear effect of fundamentals in the first region (see also section 6.1).

7 Conclusion

We propose a novel theory of contagion that explains how wake-up calls transmit financial crises. We study global coordination games of regime change with initial uncertainty about the correlation of fundamentals. A crisis in region 1 is a wake-up call for investors in region 2, inducing a re-assessment of fundamentals that increases the probability of a crisis in region 2. Contagion occurs even in the absence of ex-post correlated fundamentals, common lenders and balance sheet links.

Learning that fundamentals are uncorrelated leads to contagion for two reasons. First, the mean of the fundamental in region 2 is lower after the wake-up call, because not observing a crisis in region 1 would have been good news for investors since fundamentals may be positively correlated. This mean effect increases the probability of a crisis in region 2. Second, the variance of the fundamental in region 2 is higher after the wake-up call. When fundamentals are uncorrelated, observing a crisis in region 1 is uninformative for investors in region 2. Hence, there is greater disagreement among investors. This variance effect can increase the probability of a crisis in region 2. Both effects are aligned and induce investors to attack the regime more aggressively, resulting in a financial crisis in region 2. These results also prevail when investors optimally choose their private information.

The wake-up call theory of contagion has several applications. Currency speculators observe an exchange rate crisis elsewhere and are uncertain about the magnitude of trade and financial links. Uninsured bank creditors observe a run elsewhere and are uncertain about interbank linkages. Sovereign debt holders observe a default elsewhere and are uncertain about the resources and commitment of multilateral bail-out funds or the international lender of last resort.

Our theory of contagion is supported by existing evidence and creates new testable implications. We derive the empirical prediction that contagion depends non-linearly on the fundamental in the region of the initial crisis. Our implications are also attractive for experimental work, where the information choice is observed.

We wish to study implications for welfare and policy in subsequent work.

References

- Acharya, V. V. and T. Yorulmazer (2008). Information contagion and bank herding. *Journal of Money, Credit and Banking* 40(1), 215–231.
- Ahnert, T. and C. Bertsch (2013). A wake-up call: information contagion and strategic uncertainty. *Sveriges Riksbank Working Paper Series No. 282*.
- Ahnert, T. and A. Kakhbod (2014). Information, amplification and financial crises. *Bank of Canada Working Paper 2014-30*.
- Allen, F., A. Babus, and E. Carletti (2012). Asset commonality, debt maturity and systemic risk. *Journal of Financial Economics* 104(3), 519–534.
- Allen, F. and D. Gale (2000). Financial contagion. *Journal of Political Economy* 108(1), 1–33.
- Ammer, J. and J. Mei (1996). Measuring international economic linkages with stock market data. *Journal of Finance* 51, 1743–1764.
- Angeletos, G.-M., C. Hellwig, and A. Pavan (2006). Signaling in a global game: Coordination and policy traps. *Journal of Political Economy* 114(3), 452–484.
- Bae, K.-H., A. G. Karolyi, and R. M. Stulz (2003). A New Approach to Measuring Financial Contagion. *Review of Financial Studies* 16(3), 717–763.
- Bannier, C. E. and F. Heinemann (2005). Optimal Transparency and Risk-Taking to Avoid Currency Crises. *Journal of Institutional and Theoretical Economics* 161, 374–391.
- Basu, R. (1998). Contagion crises: the investors' logic. *mimeo*.
- Basu, R. (2002). Financial contagion and investor 'learning': an empirical investigation. *IMF Working Paper Series, No. 02/218*.
- Bekaert, G., M. Ehrmann, M. Fratzscher, and A. J. Mehl (2014, forthcoming). Global Crisis and Equity Market Contagion. *Journal of Finance*.
- Bordo, M. D. and A. P. Murshid (2000). Are financial crises becoming increasingly more contagious? What is the historical evidence on contagion? *NBER Working Paper 7900*.
- Calvo, G. A. and E. G. Mendoza (2000). Rational contagion and the globalization of securities markets. *Journal of International Economics* 51, 79–113.
- Carlsson, H. and E. van Damme (1993). Global games and equilibrium selection. *Econometrica* 61(5), 989–1018.

- Chen, H., Y. K. Lu, and W. Suen (2012). The Power of Whispers: A Theory of Rumor, Communication and Revolution. *mimeo*.
- Chen, H. and W. Suen (2013). Falling Dominoes: A Theory of Rare Events and Crisis Contagion. *mimeo*.
- Chen, Y. (1999). Banking panics: The role of the first-come, first-served rule and information externalities. *Journal of Political Economy* 107(5).
- Cipriani, M., G. Gardenal, and A. Guarino (2013). Financial contagion in the laboratory: The cross-market rebalancing channel. *Journal of Banking & Finance* 37(11), 4310–4326.
- Cipriani, M. and A. Guarino (2008). Herd Behavior and Contagion in Financial Markets. *The B.E. Journal of Theoretical Economics* 8(Contributions), Article 14.
- Corsetti, G., A. Dasgupta, S. Morris, and H. S. Shin (2004). Does One Soros Make a Difference? A Theory of Currency Crisis with Large and Small Traders. *Review of Economic Studies* 71(1), 87–113.
- Corsetti, G., B. Guimaraes, and N. Roubini (2006). International lending of last resort and moral hazard: A model of imf's catalytic finance. *Journal of Monetary Economics* 53(3), 441–471.
- Corsetti, G., P. Pesenti, and N. Roubini (1999). What caused the Asian currency and financial crisis? *Japan and the world economy* 11(3), 305–373.
- Dang, T. V., G. Gorton, and B. Holmström (2012). Ignorance, Debt and Financial Crises. *mimeo*.
- Dasgupta, A. (2004). Financial Congtagion Through Capital Connections: A Model of the Origin and Spread of Bank Panics. *Journal of the European Economic Association* 2, 1049–1084.
- Dasgupta, A. (2007). Coordination and delay in global games. *Journal of Economic Theory* 134(1), 195–225.
- Dasgupta, A., R. Leon-Gonzales, and A. Shortland (2011). Regionality revisited: An example of the direction of spread of currency crises. *Journal of International Money and Finance* 30, 831–848.
- Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 91, 401–419.
- Drazen, A. (1999). Political Contagion in Currency Crises. *NBER Working Paper* 7211.

- Edmond, C. (2013). Information Manipulation, Coordination, and Regime Change. *Review of Economic Studies* 80(4), 1422–1458.
- Eichengreen, B., A. K. Rose, and C. Wyplosz (1996). Contagious Currency Crisis. *NBER Working Paper* 5681.
- Favero, C. and F. Giavazzi (2002). Is the international propagation of financial shocks non-linear? Evidence from the ERM. *Journal of International Economics* 57, 231–246.
- Flood, R. and P. Garber (1984). Collapsing exchange-rate regimes: Some linear examples. *Journal of International Economics* 17(1-2), 1–13.
- Forbes, K. J. (2012). The 'Big C': Identifying and Mitigating contagion. *The Changing Policy Landscape. 2012 Jackson Hole Symposium hosted by the Federal Reserve Bank of Kansas City*, 23–87.
- Forbes, K. J. and R. Rigobon (2002). No contagion, only interdependence: measuring stock market comovements. *The Journal of Finance* 57(5), 2223–2261.
- Frankel, D., S. Morris, and A. Pauzner (2003). Equilibrium selection in global games with strategic complementarities. *Journal of Economic Theory* 108(1), 1–44.
- Garratt, R. and T. Keister (2009). Bank Runs as Coordination Failures : An Experimental Study. *Journal of Economic Behavior & Organization* 71(2), 300–317.
- Giordano, R., M. Pericoli, and P. Tommasino (2013). Pure or Wake-up-Call Contagion? Another Look at the EMU Sovereign Debt Crisis. *International Finance* 16(2), 131–160.
- Glick, R. and A. K. Rose (1999). Contagion and trade: Why are currency crises regional? *Journal of International Money and Finance* 18, 603–617.
- Goldstein, I. and A. Pauzner (2004). Contagion of Self-Fulfilling Financial Crises Due to Diversification of Investment Portfolios. *Journal of Economic Theory* 119, 151–183.
- Goldstein, I. and A. Pauzner (2005). Demand Deposit Contracts and The Probability of Bank Runs. *Journal of Finance* 60(3), 1293–1327.
- Goldstein, M. (1998). The Asian financial crisis causes, cures, and systematic implications. *Washington D.C.: Institute for International Economics*.
- Heinemann, F., R. Nagel, and P. Ockenfels (2004). The theory of global games on test: experimental analysis of coordination games with public and private information. *Econometrica* 72(5), 1583–1599.
- Heinemann, F., R. Nagel, and P. Ockenfels (2009). Measuring Strategic Uncertainty in Coordination Games. *The Review of Economic Studies* 76, 181–221.

- Hellwig, C. and L. Veldkamp (2009). Knowing what others know: Coordination motives in information acquisition. *Review of Economic Studies* 76, 223–251.
- Hirshleifer, J. (1971). The Private and Social Value of Information and the Reward to Inventive Activity. *American Economic Review* 61(4), 561–574.
- Iachan, F. S. and P. T. Nenov (2014). Information quality and crises in regime-change games. *Mimeo*.
- Kiyotaki, N. and J. Moore (2002). Balance-sheet contagion. *The American Economic Review* 92(2), 46–50.
- Kodres, L. E. and M. Pritsker (2002). A Rational Expectations Model of Financial Contagion. *Journal of Finance* 57(2), 769–799.
- Krugman, P. (1979). A model of balance-of-payments crises. *Journal of Money, Credit and Banking* 11(3), 311–25.
- Manz, M. (2010). Information-based contagion and the implications for financial fragility. *European Economic Review* 54, 900–910.
- Metz, C. E. (2002). Private and Public Information in Self-fulfilling Currency Crises. *Journal of Economics* 76(1), 65–85.
- Milgrom, P. R. (1981). Good News and Bad News: Representation Theorems and Applications. *The Bell Journal of Economics* 12(2), 380–391.
- Morris, S. and H. S. Shin (1998). Unique equilibrium in a model of self-fulfilling currency attacks. *American Economic Review* 88(3), 587–597.
- Morris, S. and H. S. Shin (2003). Global games: theory and applications. In M. Dewatripont, L. P. Hansen, and S. Turnovsky (Eds.), *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society)*. Cambridge University Press.
- Morris, S. and H. S. Shin (2007). Optimal Communication. *Journal of the European Economic Association* 5(2-3), 594–602.
- Obstfeld, M. (1986). Rational and self-fulfilling balance-of-payments crises. *American Economic Review* 76(1), 72–81.
- Oh, F. D. (2013). Contagion of a liquidity crisis between two firms. *Journal of Financial Economics* 107, 386–400.
- Pavlova, A. and R. Rigobon (2008). The Role of Portfolio Constraints in the International Propagation of Shocks. *Review of Economic Studies* 75(4), 1215–56.

- Prat, A. (2005). The Wrong Kind of Transparency. *American Economic Review* 95(3), 862–877.
- Radelet, S. and J. Sachs (1998). The Onset of the East Asian Financial Crisis. *NBER Working Paper* 6680.
- Ramirez, C. D. and W. Zandbergen (2013). Anatomy of Bank Contagion: Evidence from Helena, Montana During the Panic of 1893 . *GMU Working Paper in Economics No. 13-23*.
- Rochet, J.-C. and X. Vives (2004). Coordination failures and the lender of last resort: was bagehot right after all? *Journal of the European Economic Association* 2(6), 1116–1147.
- Schotter, A. and T. Yorulmazer (2009). On the dynamics and severity of bank runs: An experimental study. *Journal of Financial Intermediation* 18(2), 217–241.
- Szkup, M. and I. Trevino (2012a). Costly information acquisition in a speculative attack : Theory and experiments. *mimeo*.
- Szkup, M. and I. Trevino (2012b). Information Acquisition and Transparency in Global Games. *mimeo*.
- Taketa, K. (2004). Contagion of Currency Crises across Unrelated Countries. *IMES Discussion Paper No. 2004-E-22*.
- Trevino, I. (2013). Channels of Financial Contagion: Theory and Experiments. *mimeo*.
- Van Rijckeghem, C. and B. Weder (2001). Sources of contagion: is it finance or trade? *Journal of International Economics* 54(2), 293–308.
- Van Rijckeghem, C. and B. Weder (2003). Spillovers through banking centers: a panel data analysis of bank flows. *Journal of International Money and Finance* 22(4), 483–509.
- Vives, X. (2005). Complementarities and Games: New Developments. *Journal of Economic Literature* 43, 437–479.

A For Online Publication: Derivations and proofs

A.1 Bayesian equilibrium in region 1

The critical mass condition states that the equilibrium proportion of attacking investors equals the fundamental threshold below which a crisis occurs:

$$\begin{aligned} A(\theta_1^*) &= \Pr\{x_{i1} < x_1^* | \theta_1^*\} = \Phi(\sqrt{\gamma}(x_1^* - \theta_1^*)) = \theta_1^* \\ \Rightarrow x_1^* &= \theta_1^* + \frac{1}{\sqrt{\gamma}}\Phi^{-1}(\theta_1^*). \end{aligned} \quad (30)$$

The indifference condition states an investor who receives the signal threshold $x_{i1} = x_1^*$ is indifferent between attacking and not attacking:

$$b_1 \Pr\{\theta_1 < \theta_1^* | x_{i1} = x_1^*\} - l_1 \Pr\{\theta_1 \geq \theta_1^* | x_{i1} = x_1^*\} = 0 \quad (31)$$

where:

$$\Pr\{\theta_1 < \theta_1^* | x_{i1}\} = \Phi\left(\frac{\theta_1^* - E[\theta_1 | x_{i1}]}{\sqrt{\text{Var}[\theta_1 | x_{i1}]}}\right) = \Phi\left(\sqrt{\alpha + \gamma}\left[\theta_1^* - \frac{\alpha\mu + \gamma x_{i1}}{\alpha + \gamma}\right]\right),$$

which decreases in x_{i1} . Combining both equilibrium conditions leads to equation (6). Its right-hand side is constant and its left-hand side changes according to:

$$\frac{dF_1(\theta_1)}{d\theta_1} = \frac{\phi(\cdot)}{\sqrt{\alpha + \gamma}} \left[\alpha - \frac{\sqrt{\gamma}}{\phi(\Phi^{-1}(\theta_1))} \right]. \quad (32)$$

$F(\theta_1) \rightarrow 1$ as $\theta_1 \rightarrow 0$ and $F_1(\theta_1) \rightarrow 0$ as $\theta_1 \rightarrow 1$. Given $\frac{1}{1+b_1/l_1} \in (0, 1)$, precise private information, $\gamma > \underline{\gamma}_0 \equiv \frac{\alpha^2}{2\pi}$, ensures $\frac{dF_1(\theta_1)}{d\theta_1} < 0$, so a unique $\theta_1^* \in (0, 1)$ exists.

Comparative statics

$$\frac{d\theta_1^*}{d\mu} = \frac{\alpha}{\alpha - \frac{\sqrt{\gamma}}{\phi(\Phi^{-1}(\theta_1^*))}} < 0, \quad (33)$$

$$\frac{d\theta_1^*}{d(b_1/l_1)} = -\frac{\sqrt{\alpha + \gamma}}{\phi(\cdot)(1 + b_1/l_1)^2} \left[\alpha - \frac{\sqrt{\gamma}}{\phi(\Phi^{-1}(\theta_1^*))} \right] > 0. \quad (34)$$

A.2 Deriving the equilibrium in region 2

We proceed in two steps. First, we consider the special case in which all investors are informed, $n = 1$. The existence of a unique Bayesian equilibrium is just a corollary of Lemma 1 in this case. Second, we derive the equilibrium conditions for the general case in which some investors are uninformed, $n \in (0, 1]$. In Appendix A.3, we prove the existence of a unique monotone equilibrium for this general case.

A.2.1 All investors are informed

When all investors are informed, $n = 1$, they learn that the realized correlation. In the case of zero correlation, the updated prior of informed investors in region 2 coincides with that of investors in region 1 and the previous analysis applies. In the case of positive correlation, by contrast, a small change is required to obtain a corollary of Lemma 1. The modified threshold for the precision of private information is $\underline{\gamma}' \equiv \frac{\alpha^2}{2\pi(1-\rho_H^2)^2} \in (\underline{\gamma}_0, \infty)$. Moreover, the unique threshold fundamental $\theta_2^* = \theta_2^*(n = 1, \rho, \theta_1)$ is implicitly defined by:

$$F_2(\theta_2^*, \rho) \equiv \Phi\left(\frac{\alpha_2(\rho)[\theta_2^* - \mu_2(\rho, \theta_1)]}{\sqrt{\alpha_2(\rho) + \gamma}} - \sqrt{\frac{\gamma}{\alpha_2(\rho) + \gamma}} \Phi^{-1}(\theta_2^*)\right) = \frac{1}{1 + b_2/l_2} \quad (35)$$

for any realized correlation $\rho \in \{0, \rho_H\}$ and any observed fundamental $\theta_1 < \theta_1^*$.

Corollary 2 *Suppose all investors are informed about the correlation, $n = 1$, after a crisis in region 1, $\theta_1 < \theta_1^*$. If private information is sufficiently precise, $\gamma > \underline{\gamma}'$, then there exists a unique Bayesian equilibrium in region 2. This equilibrium is in threshold strategies, whereby a crisis occurs if the realized fundamental is below a threshold $\theta_2^*(1, \rho, \theta_1)$ defined by equation (35).*

A.2.2 Some investors are uninformed

Consider now the general case of $n \in [0, 1)$. After a crisis in region 1, uninformed investors use the observed θ_1 and their private signal x_{i2} to re-assess the local fundamental θ_2 . Uninformed investors do not learn the correlation of fundamentals.

Bayesian updating We show that the relationship between the posterior probability of zero correlation, $\hat{\rho}$, and the private signal, x_{i2} , is non-monotone. First, $\frac{d\hat{\rho}}{dx_{i2}} > 0$ if the private signal is relatively high. Intuitively, an investor places more weight on the probability of zero correlation after receiving a relatively good private signal. Instead, after a low private signal, $\frac{d\hat{\rho}}{dx_{i2}} > 0$ is not guaranteed. For extremely low signals, an even worse signal makes an uninformed investor infer that $\rho = 0$ is more likely due to the fatter tails of the more dispersed prior. Uninformed investors use Bayes' rule to form a belief about the correlation of fundamentals:

$$\hat{\rho} \equiv \Pr\{\rho = 0 | \theta_1, x_{i2}\} = \frac{p \Pr\{x_{i2} | \theta_1, \rho = 0\}}{p \Pr\{x_{i2} | \theta_1, \rho = 0\} + (1-p) \Pr\{x_{i2} | \theta_1, \rho = \rho_H\}}. \quad (36)$$

Computing $\Pr\{x_{i2} | \theta_1, \rho\}$ for each ρ , recall that the variance is independent of θ_1 :

$$\begin{aligned} \Pr\{x_{i2} | \theta_1, \rho = 0\} &= \frac{1}{\sqrt{\text{Var}[x_{i2} | \rho = 0]}} \phi\left(\frac{x_{i2} - \mathbb{E}[x_{i2} | \theta_1, \rho = 0]}{\sqrt{\text{Var}[x_{i2} | \rho = 0]}}\right) \\ &= \left(\frac{1}{\alpha} + \frac{1}{\gamma}\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i2} - \mu}{\sqrt{\frac{1}{\alpha} + \frac{1}{\gamma}}}\right) \end{aligned} \quad (37)$$

$$\begin{aligned} \Pr\{x_{i2} | \theta_1, \rho = \rho_H\} &= \frac{1}{\sqrt{\text{Var}[x_{i2} | \rho = \rho_H]}} \phi\left(\frac{x_{i2} - \mathbb{E}[x_{i2} | \theta_1, \rho = \rho_H]}{\sqrt{\text{Var}[x_{i2} | \rho = \rho_H]}}\right) \\ &= \left(\frac{1 - \rho_H^2}{\alpha} + \frac{1}{\gamma}\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i2} - [\rho_H \theta_1 + (1 - \rho_H)\mu]}{\sqrt{\frac{1 - \rho_H^2}{\alpha} + \frac{1}{\gamma}}}\right). \end{aligned} \quad (38)$$

Since $\rho_H > 0$, the derivatives of the posterior belief $\hat{\rho}$ are:

$$\frac{d\hat{\rho}}{d\theta_1} \begin{cases} \geq 0 & \text{if } x_{i2} \leq \rho_H \theta_1 + (1 - \rho_H)\mu \\ < 0 & \text{otherwise.} \end{cases} \quad (39)$$

First, if the private signal x_{i2} is sufficiently low, an increase in θ_1 induces uninformed investors to put a larger probability on uncorrelated regional fundamentals. The signs of this derivative would be reversed if we had $\rho_H < 0$.

Second, how does \hat{p} vary with the private signal x_{i2} ? We find that:

$$\frac{d\hat{p}}{dx_{i2}} \begin{cases} > 0 \text{ if } \rho_H > 0 \text{ and } x_{i2} \geq \rho_H \theta_1 + (1 - \rho_H)\mu \\ < 0 \text{ if } \rho_H < 0 \text{ and } x_{i2} \leq \rho_H \theta_1 + (1 - \rho_H)\mu \\ \leq 0 \text{ otherwise.} \end{cases} \quad (40)$$

Therefore, after receiving a relatively good private signal, $x_{i2} \geq \rho_H \theta_1 + (1 - \rho_H)\mu$, an investor places more weight on the probability of zero cross-regional correlation. If the private signal takes an intermediate value, $\frac{d\hat{p}}{dx_{i2}} > 0$ still holds. However, after receiving a relatively low private signal, $x_{i2} < \rho_H \theta_1 + (1 - \rho_H)\mu$, we have that $\frac{d\hat{p}}{dx_{i2}} \leq 0$ due to the more dispersed prior distribution if $\rho = 0$. For the same reason, an extremely high or low private signal induces uninformed investors to believe that fundamentals are uncorrelated across regions, $\lim_{x_{i2} \rightarrow +\infty} \hat{p} = 1 = \lim_{x_{i2} \rightarrow -\infty} \hat{p}$.

Equilibrium conditions when some investors are uninformed Analyzing the general case of some uninformed investors, we derive the system of equations - the critical mass and indifference conditions - that describe the equilibrium in region 2.

The critical mass conditions state that the proportion of attacking investors $A_2^*(\rho)$ equals the fundamental threshold $\theta_2^*(\rho)$ for each realized $\rho \in \{0, \rho_H\}$:

$$\theta_2^*(\rho) = n\Phi(\sqrt{\gamma}[x_{2I}^*(\rho) - \theta_2^*(\rho)]) + (1 - n)\Phi(\sqrt{\gamma}[x_{2U}^* - \theta_2^*(\rho)]). \quad (41)$$

We use the short-hands $\theta_2^*(\rho) \equiv \theta_2^*(n, \rho, \theta_1)$, $x_{2I}^*(\rho) \equiv x_{2I}^*(n, \rho, \theta_1)$, and $x_{2U}^* \equiv x_{2U}^*(n, \theta_1)$ for the fundamental threshold and the signal thresholds of informed and uninformed investors, respectively.

The first indifference condition states that an uninformed investor with threshold signal $x_{i2} = x_{2U}^*$ is indifferent between attacking and not attacking:

$$\underbrace{\hat{p}^* \Psi(\theta_2^*(0), x_{2U}^*, 0) + (1 - \hat{p}^*) \Psi(\theta_2^*(\rho_H), x_{2U}^*, \rho_H)}_{\equiv J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_{2U}^*)} = \frac{1}{1 + b_2/l_2} \quad (42)$$

where $\hat{\rho}^* = \hat{\rho}(\theta_1, x_{2U}^*)$ and, for $d \in \{I, U\}$ and $\rho \in \{0, \rho_H\}$:

$$\Psi(\theta_{2d}^*, x_{2d}^*, \rho) \equiv \Phi \left(\theta_{2d}^* \sqrt{\alpha_2(\rho) + \gamma} - \frac{\alpha_2(\rho) \mu_2(\rho, \theta_1) + \gamma x_{2d}^*}{\sqrt{\alpha_2(\rho) + \gamma}} \right). \quad (43)$$

Two additional indifference conditions, one for each realized correlation, state that an informed investor is indifferent between attacking or not upon receiving the threshold signal $x_{i2} = x_{2I}^*(\rho)$:

$$\Psi(\theta_2^*(\rho), x_{2I}^*(\rho), \rho) = \frac{1}{1 + b_2/l_2} \quad \forall \rho \in \{0, \rho_H\}. \quad (44)$$

We have five equation in five unknowns. In the simplest case, in region 1, we had two thresholds x_1^* and θ_1^* . There, the objective was to establish aggregate behavior by inserting the critical mass condition, which states x_1^* in terms of θ_1^* , into the indifference condition. This yields one equation implicit in θ_1^* . We pursue a modified strategy here, solving this system of equations in order to express the equilibrium in terms of $\theta_2^*(0)$ and $\theta_2^*(\rho_H)$ only.

We also use the following insight. Since uninformed investors do not observe the realized cross-regional correlation, the signal threshold must be identical across these realizations, $x_{2U}^*(\rho = 0) = x_{2U}^*(\rho = \rho_H)$. In the following steps, we derive this threshold for either realization of the correlation ρ by using the fundamental threshold $\theta_2^*(\rho)$ and equalize both expressions. First, we use the critical mass condition in equation (41) for $\theta_2^*(0)$ to express x_{2U}^* as a function of $\theta_2^*(0)$ and $x_{2I}^*(0)$. Second, we use the indifference condition of informed investors in case of $\rho = 0$, equation (44), to obtain $x_{2I}^*(0)$ as a function of $\theta_2^*(0)$. Third, we use the critical mass condition in equation (41) for $\theta_2^*(\rho_H)$ to express x_{2U}^* as a function of $\theta_2^*(\rho_H)$ and $x_{2I}^*(\rho_H)$. Then, we use the indifference condition of informed investors in case of $\rho = \rho_H$, equation (44), to obtain $x_{2I}^*(\rho_H)$ as a function of $\theta_2^*(\rho_H)$. Thus, $\forall \rho$:

$$x_{2U}^*(\rho) = \theta_2^*(\rho) + \frac{\Phi^{-1} \left(\frac{\theta_2^*(\rho) - n \Phi \left(\frac{\alpha_2(\rho)(\theta_2^*(\rho) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha_2(\rho) + \gamma} \Phi^{-1} \left(\frac{1}{1 + b_2/l_2} \right)}{\sqrt{\gamma}} \right)}{1 - n} \right)}{\sqrt{\gamma}}. \quad (45)$$

Hence, for $\rho \in \{0, \rho_H\}$, a sufficient condition for the partial derivatives with respect to the fundamental thresholds to be strictly positive is $\gamma > \underline{\gamma}_1$:

$$\frac{dx_{2U}^*(\rho)}{d\theta_2^*(\rho)} > 0. \quad (46)$$

Since the signal threshold is the same for an uninformed investor, subtracting equation (45) evaluated at $\rho = 0$ from the same equation evaluated at $\rho = \rho_H$ must yield zero. This yields the first implicit relationships between $\theta_2^*(0)$ and $\theta_2^*(\rho_H)$:

$$K(n, \theta_2^*(0), \theta_2^*(\rho_H)) \equiv x_{2U}^*(0) - x_{2U}^*(\rho_H) = 0. \quad (47)$$

Now, we construct the second implicit relationship between the two aggregate thresholds $\theta_2^*(0)$ and $\theta_2^*(\rho_H)$ in two steps. First, insert equation (45) evaluated at $\rho = 0$ in $\Psi(\theta_2^*(0), x_{2U}^*(0), 0)$ and in \hat{p} as used in $J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_{2U}^*)$. Second, insert equation (45) evaluated at $\rho = \rho_H$ in $\Psi(\theta_2^*(\rho_H), x_{2U}^*(\rho_H), \rho_H)$. Combining both expressions yields:

$$L(n, \theta_2^*(0), \theta_2^*(\rho_H)) \equiv J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_{2U}^*(0), x_{2U}^*(\rho_H)) = \frac{1}{1 + b_2/l_2}. \quad (48)$$

A.2.3 All investors are uninformed

If all investors are uninformed, $n = 0$, the system of equations derived in Appendix section A.2.2 simplifies. Specifically, there is only one fundamental threshold and the system can be reduced to one equation in one unknown, where $\theta_2^*(0, 0, \theta_1) = \theta_2^*(0, \rho_H, \theta_1)$ in equation (42).

Using the results of Milgrom (1981) and Vives (2005), we show that the best-response function of an individual investor strictly increases in the threshold used by other investors. Therefore, there exists a unique equilibrium in threshold strategies if private information is sufficiently precise, as proven in the subsequent paragraph.

Monotonicity In contrast to the standard analysis of region 1, $J(0, \theta_2, \theta_1)$ is harder to characterize. The weights of the mixture distribution and the posterior beliefs about the correlation now depend on the threshold signal x_{2U}^* . Therefore, the question arises whether or not our focus on monotone equilibria is justified, in light of the global non-monotonicity of $\hat{p}(x_{2U}^*(\theta_2^*(0, 0, \theta_1)))$ in x_{2U}^* and, hence, in $\theta_2^*(0, 0, \theta_1)$, as established above. Fortunately, the best-response function of an individual investor i is proven to be strictly increasing in the threshold used by other investors:

$$r' = - \frac{\frac{d\Pr\{\theta_2 < \hat{\theta}_2(\hat{x}_2) | \theta_1, x_{i2}\}}{d\hat{x}_2}}{\frac{d\Pr\{\theta_2 < \hat{\theta}_2(\hat{x}_2) | \theta_1, \tilde{x}_{i2}\}}{d\tilde{x}_{i2}}} > 0, \quad (49)$$

where \tilde{x}_{i2} is the critical threshold of the private signal used by player i , \hat{x}_2 is the threshold used by all other investors, and $\hat{\theta}_2(\hat{x}_2)$ is the critical threshold of the fundamental in region 2 when $n = 0$. This is because $\Pr\{\theta_2 < \theta_2^* | \theta_1, x_{i2}\}$ is monotonically decreasing in x_{i2} , using a result of Milgrom (1981) (see below). Furthermore, given all other investors use a threshold strategy, $\Pr\{\theta_2 < \hat{\theta}_2(\hat{x}_2) | \theta_1, x_{i2}\}$ increases in \hat{x}_2 (again see below). Following Vives (2005), the best response of player i is to use a threshold strategy with attack threshold \tilde{x}_{i2} , where $\Pr\{\theta_2 < \hat{\theta}_2(\hat{x}_2) | \theta_1, \tilde{x}_{i2}\} = \frac{1}{1+b_2/l_2}$, implying $r' > 0$. Therefore, our focus on monotone equilibria is valid and we determine conditions sufficient for a unique monotone Bayesian equilibrium.

The conditional density function $f(x|\theta)$ is normal with mean θ and satisfies the monotone likelihood ratio property (MLRP): for all $x_i > x_j$ and $\theta' > \theta$, we have:

$$\frac{f(x_i|\theta')}{f(x_i|\theta)} \geq \frac{f(x_j|\theta')}{f(x_j|\theta)} \Leftrightarrow \frac{\phi(\sqrt{\gamma}(x_i - \theta'))}{\phi(\sqrt{\gamma}(x_i - \theta))} \geq \frac{\phi(\sqrt{\gamma}(x_j - \theta'))}{\phi(\sqrt{\gamma}(x_j - \theta))}. \quad (50)$$

Using Proposition 1 of Milgrom (1981), we conclude that $\Pr\{\theta_2 \leq \theta_2^* | \theta_1, x_{i2}\}$ monotonically decreases in x_{i2} . Hence, $\frac{d\Pr\{\theta_2 \leq \theta_2^* | \theta_1, \hat{x}_2\}}{d\theta_2^*} > 0$. Equation (42) then implies:

$$0 \leq \frac{d\hat{\theta}_2(\hat{x}_2)}{d\hat{x}_2} \leq \left(1 + \sqrt{\frac{2\pi}{\gamma}}\right)^{-1}. \quad (51)$$

Existence and uniqueness

Lemma 4 *Suppose there is a crisis in region 1, $\theta_1 < \theta_1^*$, and investors are uninformed about the correlation, $n = 0$. If private information is sufficiently precise, $\gamma > \underline{\gamma}'_1$, then there exists a unique monotone equilibrium in region 2. Each investor attacks if and only if the private signal is below the threshold x_{2U}^* . A crisis occurs if and only if the fundamental in region 2 is below the fundamental threshold $\theta_2^*(0, 0, \theta_1)$ defined by equation (42). This fundamental threshold is a weighted average of the thresholds that prevail if investors were informed:*

$$\min\{\theta_2^*(1, 0, \theta_1), \theta_2^*(1, \rho_H, \theta_1)\} < \theta_2^*(0, 0, \theta_1) < \max\{\theta_2^*(1, 0, \theta_1), \theta_2^*(1, \rho_H, \theta_1)\}.$$

Proof The proof is in three steps. First, we show that $J(0, \theta_2, \theta_1) \rightarrow 1 > \frac{1}{1+b_2/l_2}$ as $\theta_2 \rightarrow 0$, and $J(0, \theta_2, \theta_1) \rightarrow 0 < \frac{1}{1+b_2/l_2}$ as $\theta_2 \rightarrow 1$. Second, we show that $\frac{dJ(0, \theta_2, \theta_1)}{d\theta_2} < 0$ for some sufficiently high but finite values of γ , such that J strictly decreases in θ_2 . We denote this lower bound as $\underline{\gamma}'_1$. Therefore, if θ_2^* exists, it is unique. Third, by continuity, there exists a $\theta_2^*(0, 0, \theta_1)$ that solves $J(0, \theta_2, \theta_1) = \frac{1}{1+b_2/l_2}$.

Step 1: limiting behavior Observe that $J(0, \theta_2, \theta_1)$ is a weighted average of $F_2(\theta_2, 0)$ and $F_2(\theta_2, \rho_H)$. As $\theta_2 \rightarrow 0$, then $F_2(\theta_2, \rho) \rightarrow 1$ for any $\rho \in \{0, \rho_H\}$, so $J(0, \theta_2, \theta_1) \rightarrow 1 > \frac{1}{1+b_2/l_2}$. Likewise, as $\theta_2 \rightarrow 1$, then $F_2(\theta_2, \rho) \rightarrow 0$ for any $\rho \in \{0, \rho_H\}$, so $J(0, \theta_2, \theta_1) \rightarrow 0 < \frac{1}{1+b_2/l_2}$.

Step 2: strictly negative slope Using the indifference condition of uninformed investors to substitute x_{2U}^* in equation (42), the total derivative of J is:

$$\begin{aligned} \frac{dJ(0, \theta_2, \theta_1)}{d\theta_2} &= \hat{p}(\theta_2) \frac{dF_2(\theta_2, 0)}{d\theta_2} + (1 - \hat{p}(\theta_2)) \frac{dF_2(\theta_2, \rho_H)}{d\theta_2} \\ &+ \frac{d\hat{p}(\theta_1, x_{2U}(\theta_2))}{dx_{2U}} \frac{dx_{2U}(\theta_2)}{d\theta_2} [F_2(\theta_2, 0) - F_2(\theta_2, \rho_H)]. \quad (52) \end{aligned}$$

The proof proceeds by inspecting the individual terms of equation (52).

We know from our analysis of the case of informed investors that $\frac{dF_2(\theta_2, 0)}{d\theta_2} < 0$ if $\gamma > \underline{\gamma}_0$ and that $\frac{dF_2(\theta_2, \rho_H)}{d\theta_2} < 0$ if $\gamma > \underline{\gamma}'_0$. Moreover, these derivatives are also strictly negative in the limit when $\gamma \rightarrow \infty$. Thus, the first two components of the sum are negative and finite in the limit when $\gamma \rightarrow \infty$. By continuity, these terms are also negative for a sufficiently high but finite private noise.

The sign of the third summand in (52) is ambiguous: $F_2(\theta_2^*(0,0,\theta_1),0) \leq F_2(\theta_2^*(0,\rho_H,\theta_1),\rho_H)$ whenever $\theta_2^*(1,\theta_1,0) \leq \theta_2^*(1,\theta_1,\rho_H)$ and $F_2(\theta_2^*(0,0,\theta_1),0) > F_2(\theta_2^*(0,\rho_H,\theta_1),\rho_H)$ otherwise, where $\theta_2^*(0,0,\theta_1) = \theta_2^*(0,\rho_H,\theta_1)$. However, the difference vanishes in the limit when $\gamma \rightarrow \infty$.

The last term to consider is $\frac{d\hat{p}(\theta_1,x_{2U}(\theta_2))}{dx_{2U}(\theta_2)} \frac{dx_{2U}}{d\theta_2}$. Given the previous sufficient conditions on the relative precision of the private signal:

$$0 < \frac{dx_{2U}}{d\theta_2} = 1 + \frac{1}{\sqrt{\gamma}} \frac{1}{\phi(\Phi^{-1}(\theta_2))} < 1 + \frac{\sqrt{2\pi}}{\alpha}.$$

Finally, from section A.2.2, we know that the sign of $\frac{d\hat{p}}{dx_{2U}}$ is ambiguous. However, the derivative is finite for $\gamma \rightarrow \infty$. Taken together with the zero limit of the first factor of the third term, this term vanishes in the limit.

As a result, by continuity, there must exist a finite level of precision $\gamma > \underline{\gamma}'_1 \in (0,\infty)$ such that $\frac{dJ(0,\theta_2,\theta_1)}{d\theta_2} < 0$ for all $\gamma > \underline{\gamma}'_1$. This concludes the second step of the proof and therefore the overall proof of Lemma 4. (*q.e.d.*)

A.3 Proof of Proposition 1

The case of $n = 1$ is trivial, since it is merely a corollary of Lemma 1 (Morris and Shin (2003)). In what follows, we consider the case of a given $\theta_1 < \theta_1^*$ and $n < 1$, whereby some investors are uninformed. This proof establishes the conditions sufficient for the existence of a unique pair of fundamental thresholds by analyzing a system characterized by two equations, (47) and (48), in two unknowns, $\theta_2(0)$ and $\theta_2(\rho_H)$. The proof builds heavily on the description of the coordination stage in the case of potentially asymmetrically informed investors described in Appendix A.2. We show existence and uniqueness of the pair $(\theta_2^*(0), \theta_2^*(\rho_H))$. Then, the signal thresholds are uniquely backed out from $(\theta_2^*(0), \theta_2^*(\rho_H))$.

Outline of proof First, we analyze the relationship between $\theta_2(0)$ and $\theta_2(\rho_H)$ as governed by K . Using equations (47) and (46), $\frac{\partial K}{\partial \theta_2^*(0)} > 0$ and $\frac{\partial K}{\partial \theta_2(\rho_H)} < 0$. Hence,

$\frac{d\theta_2(0)}{d\theta_2(\rho_H)} > 0$ by the implicit function theorem.

Second, we analyze the relationship between $\theta_2(0)$ and $\theta_2(\rho_H)$ as governed by L . It can be shown that $\gamma > \underline{\gamma}'_0$ is sufficient for $\frac{\partial L}{\partial \theta_2(\rho_H)} < 0$. Thus, one can show that $\frac{dL}{d\theta_2(0)} < 0$ holds for a sufficiently high but finite value of γ . This is proven by generalizing the argument of the proof of Lemma 4, so $\lim_{\gamma \rightarrow \infty} [\Psi(\theta_2^*(0), x_{2U}^*, 0) - \Psi(\theta_2^*(\rho_H), x_{2U}^*, \rho_H)] = 0$. Hence, $\frac{d\theta_2(0)}{d\theta_2(\rho_H)} < 0$ in the limit. By continuity, there exists a finite precision, $\gamma > \underline{\gamma}'_1$, of private information that guarantees the inequality as well. Taken both of these points together, $(\theta_2^*(0), \theta_2^*(\rho_H))$ is unique if it exists. This arises from the established strict monotonicity and the opposite sign.

Third, we establish existence of $(\theta_2^*(0), \theta_2^*(\rho_H))$ by making two points: (i) for the highest permissible value of $\theta_2(0)$, the value of $\theta_2(\rho_H)$ prescribed by K is strictly larger than the value of $\theta_2(\rho_H)$ prescribed by L ; and (ii) for the lowest permissible value of $\theta_2(0)$, the value of $\theta_2(\rho_H)$ prescribed by K is strictly smaller than the value of $\theta_2(\rho_H)$ prescribed by L .

Formal argument To make these points, consider the following auxiliary step. For any $\theta_2(\rho) \geq \theta_2^*(1, \rho, \theta_1)$, it can be shown that:

$$\frac{\partial}{\partial n} \Phi^{-1} \left(\frac{\theta_2^*(\rho) - n \Phi \left(\frac{\alpha_2(\rho)(\theta_2^*(\rho) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha_2(\rho) + \gamma} \Phi^{-1} \left(\frac{1}{1+b_2/l_2} \right)}{\sqrt{\gamma}} \right)}{1-n} \right) \geq 0 \quad (53)$$

because $F_2(\theta_2(\rho), \rho) \leq \frac{1}{1+b_2/l_2}$ for any $\rho \in \{0, \rho_H\}$. Note that both the previous expression and the partial derivative hold with strict inequality if $\theta_2(\rho) > \theta_2^*(1, \rho, \theta_1)$.

Inspecting the inside of the inverse of the cdf, Φ^{-1} , we define the highest permissible value of $\theta_2(\rho)$ that is labelled $\bar{\theta}_2(\rho, n)$ for all ρ :

$$1 = \frac{\bar{\theta}_2(\rho, n) - n \Phi \left(\frac{\alpha_2(\rho)(\bar{\theta}_2(\rho, n) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha_2(\rho) + \gamma} \Phi^{-1} \left(\frac{1}{1+b_2/l_2} \right)}{\sqrt{\gamma}} \right)}{1-n}. \quad (54)$$

Hence, $1 \geq \bar{\theta}_2(\rho, 1) \geq \theta_2^*(1, \rho, \theta_1) \forall \rho$, where the first (second) inequality binds if and only if $n = 0$ ($n = 1$).

Next, evaluate K at the highest permissible value, $\theta_2(0) = \bar{\theta}_2(0, n)$, which yields $\theta_2(\rho_H) = \bar{\theta}_2(\rho_H, n)$. Likewise, evaluate L at the highest permissible value, $\theta_2(0) = \bar{\theta}_2(0, n)$, which yields $\theta_2(\rho_H) < \bar{\theta}_2(\rho_H, n)$. This proves point (i).

We now proceed with point (ii). We can similarly define the lowest permissible value of $\theta_2(\rho)$, which is labelled $\underline{\theta}_2(\rho, n)$ for all ρ . Now, $0 \leq \underline{\theta}_2(\rho, 1) \leq \theta_2^*(1, \rho, \theta_1) \forall \rho$, where the first (second) inequality binds if and only if $n = 0$ ($n = 1$).

Next, evaluate K at the lowest permissible value, $\theta_2(0) = \underline{\theta}_2(0, n)$, which yields $\theta_2(\rho_H) = \underline{\theta}_2(\rho_H, n)$. Likewise, evaluate L at $\theta_2(0) = \underline{\theta}_2(0, n)$, which yields $\theta_2(\rho_H) > \underline{\theta}_2(\rho_H, n)$. This proves point (ii) and completes the proof. (*q.e.d.*)

A.4 Comparative statics and fundamental threshold ranking

Section A.4.1 derives the conditions in Definition 1 and establishes comparative static results of the fundamental threshold when all investors are informed about the realized correlation, $n = 1$. We analyze the role of the precision of public and private information in section A.4.2, and the implications for the ranking of fundamental thresholds in section A.4.3. These results are useful for subsequent proofs. Finally, section A.4.4 analyzes the general case when some investors are uninformed, $n < 1$.

A.4.1 Constructing Definition 1

This definition allows us to distinguish between weak and strong priors about the fundamental. $X(\rho)$ and $Y(\rho)$ are derived by reformulating equation (35):

$$\begin{aligned} \Phi^{-1}(\theta_2^*(1, \rho, \theta_1)) - \frac{\alpha_2(\rho)}{\sqrt{\gamma}}(\theta_2^*(1, \rho, \theta_1) - \mu_2(\rho, \theta_1)) \\ = -\frac{\sqrt{\alpha_2(\rho) + \gamma}}{\sqrt{\gamma}}\Phi^{-1}\left(\frac{1}{1 + b_2/l_2}\right). \end{aligned} \quad (55)$$

First, $X(\rho)$ can be derived by setting $\theta_2^*(1, \rho, \theta_1) = \mu_2(\rho, \theta_1)$ and by isolating $\mu_2(\rho, \theta_1)$. A sufficient condition that assures that strong (weak) prior beliefs are associated with a low (high) incidence of attacks below (above) 50% is derived

from equation (55) by setting $\theta_2^* = \frac{1}{2}$. This leads to $Y(\rho)$.

A.4.2 Comparative statics: the precision of public and private information

The subsequent discussion draws in parts from Bannier and Heinemann (2005). We have the following partial derivatives of the fundamental thresholds:

$$\frac{d\theta_2^*(1, \rho, \theta_1)}{d\alpha} \begin{cases} < 0 \text{ if } \theta_2^*(1, \rho, \theta_1) < \mu_2(\rho, \theta_1) + \frac{1}{2\sqrt{\alpha_2(\rho)+\gamma}}\Phi^{-1}\left(\frac{1}{1+b_2/l_2}\right) \\ \geq 0 \text{ otherwise} \end{cases}$$

$$\frac{d\theta_2^*(1, \rho, \theta_1)}{d\gamma} \begin{cases} > 0 \text{ if } \theta_2^*(1, \rho, \theta_1) < \mu_2(\rho, \theta_1) + \frac{1}{\sqrt{\alpha_2(\rho)+\gamma}}\Phi^{-1}\left(\frac{1}{1+b_2/l_2}\right) \\ \leq 0 \text{ otherwise.} \end{cases}$$

If $b_2 \leq l_2$, then a strong prior about the fundamental, $\theta_2^*(1, \rho, \theta_1) < \mu_2(\rho, \theta_1) \forall \rho \in \{0, \rho_H\}$, implies that $\frac{d\theta_2^*}{d\alpha} < 0$ and $\frac{d\theta_2^*}{d\gamma} > 0$. If $b_2 > l_2$, then a weak prior, $\theta_2^*(1, \rho, \theta_1) > \mu_2(\rho, \theta_1) \forall \rho \in \{0, \rho_H\}$, implies that $\frac{d\theta_2^*}{d\alpha} > 0$ and $\frac{d\theta_2^*}{d\gamma} < 0$.

Instead, if $b_2 > l_2$, then $\theta_2^*(1, \rho, \theta_1) < \mu_2(\rho, \theta_1) \forall \rho \in \{0, \rho_H\}$ does not necessarily imply that $\frac{d\theta_2^*}{d\alpha} < 0$ and $\frac{d\theta_2^*}{d\gamma} > 0$. In other words, the inequalities involving $X(\rho)$ in Definition 1 are no longer sufficient if $b_2 > l_2$. However, Definition 1 provides a more restrictive definition of a strong (weak) prior about fundamentals by imposing additional conditions involving $Y(\rho)$, which assure that a strong (weak) prior belief is associated with a low (high) incidence of crises below (above) 50%. Hence, Definition 1 also ensures that a strong prior belief implies that $\frac{d\theta_2^*}{d\alpha} < 0$ and $\frac{d\theta_2^*}{d\gamma} > 0$ even if $b_2 > l_2$. Similarly, it ensures that a weak prior implies that $\frac{d\theta_2^*}{d\alpha} > 0$ and $\frac{d\theta_2^*}{d\gamma} < 0$ even if $b_2 \leq l_2$.

A.4.3 Comparative statics: the ranking of fundamental thresholds

This section analyzes the interaction between the *mean effect* and the *variance effect*. This interaction determines the ordering of fundamental thresholds $\theta_2^*(1, 0, \theta_1)$ and $\theta_2^*(1, \rho_H, \theta_1)$. However, note that our focus here is only on the ordering of fun-

damental thresholds, but not on the ordering of probability of a crisis. There is no one-to-one mapping between the ordering of fundamental thresholds and the ordering of the probability of a crisis, since the realized correlation also affects the conditional distribution of the fundamental, $\theta_2|\rho$.

Metz (2002) was one of the first to examine the dependence of the fundamental threshold on the precision of private and the public information (γ, α). An inspection of equation (35) for the special case $b_2 = l_2$ reveals that the fundamental threshold $\theta_2^*(1, 0, \theta_1)$ increases (decreases) in the precision of the private signal γ when the prior is strong (weak). This result is consistent with the findings of Rochet and Vives (2004). A related result is that the above relationship is opposite when considering a change in the precision of the public signal α .

Table 2 summarizes the effects of an increase in the correlation ρ if $\theta_1 < \mu$. This affects both the mean $\mu_2(\rho, \theta_1)$ and the precision $\alpha_2(\rho)$ of the updated prior about θ_2 . The effect of an increase in ρ on $\theta_2^*(1, \rho, \theta_1)$, and its impact on the ranking of fundamental thresholds, depends on the strength of the prior. The cases where the mean effect (ME) and the variance effect (VE) go in opposite directions are in bold. For potentially positive correlation, this requires a strong prior.

Prior belief	Effect of an increase in ρ on $\theta_2^*(1, \rho, \theta_1)$		Ordering of thresholds	
	Mean effect $\frac{d\theta_2^*(1, \rho, \theta_1)}{d\mu_2} \frac{d\mu_2(\rho, \theta_1)}{d\rho}$	Variance effect $\frac{d\theta_2^*(1, \rho, \theta_1)}{d\alpha_2} \frac{d\alpha_2(\rho)}{d \rho }$	$\rho_H > 0$	$\rho_H < 0$
strong	> 0 \forall $\rho \in (-1, 1)$	< 0	$\theta_2^*(1, \rho_H, \theta_1)$ $< \theta_2^*(1, 0, \theta_1)$ if VE > ME	$\theta_2^*(1, \rho_H, \theta_1)$ $< \theta_2^*(1, 0, \theta_1)$
weak		> 0	$\theta_2^*(1, \rho_H, \theta_1)$ $> \theta_2^*(1, 0, \theta_1)$	$\theta_2^*(1, \rho_H, \theta_1)$ $> \theta_2^*(1, 0, \theta_1)$ if VE > ME

Table 2: Effect of an increase in ρ on the ordering of the fundamental threshold in region 2 when all investors are informed after a crisis in region 1, $\theta_1 < \theta_1^* = \mu$.

To understand the mechanics behind the results in Table 2, recall $\frac{d\alpha_2(\rho)}{d|\rho|} > 0$.

As a result, the precision of the public signal is the lowest when fundamentals are uncorrelated, $\alpha < \alpha_2(\rho_H)$. Hence, the variance effect tends to decrease (increase) $\theta_2^*(1, \rho, \theta_1)$ if the prior belief is that fundamentals are strong (weak). Thus, for strong prior, there is a tension between the mean and the variance effect if $\rho_H > 0$. This tension is crucial for Lemma 5 derived below. By contrast, after no crisis in region 1, $\theta_1 \geq \mu$, there is no tension between the mean and variance effects since they go in the same direction. We use this last result in the proof of Lemma 2.

Threshold ranking Investors in region 2 re-assess the local fundamental θ_2 when learning about a positive correlation. Both the mean and the variance of the updated prior about θ_2 are lower after a crisis in region 1 (see Figure 1). Therefore, the relative size of these mean and variance effects determines the overall impact on the fundamental threshold relative to the case of zero correlation, $\theta_2^*(1, \rho_H, \theta_1) \leq \theta_2^*(1, 0, \theta_1)$.³⁶ We establish conditions for a ranking of thresholds after a crisis in region 1, specifically the sufficient conditions stated in Lemma 5.

Lemma 5 *Ranking of fundamental thresholds.* *Suppose private information is sufficiently precise and investors are informed, $n = 1$. The fundamental threshold ranking $\theta_2^*(1, 0, \theta_1) > \theta_2^*(1, \rho_H, \theta_1)$ is ensured by a strong prior about the fundamental in region 2 and an intermediate level of the realized fundamental in region 1, $\theta_1 \in (\underline{\theta}_1, \mu)$, where the lower bound $\underline{\theta}_1$ is defined in Proposition 3.*

Proof The threshold fundamental $\theta_2^* = \theta_2^*(n = 1, \rho, \theta_1)$ is implicitly defined by equation (35). For sufficiently precise private information, $\gamma > \underline{\gamma}'_0 \leq \underline{\gamma}_1$, $F_2(\theta_2^*, \rho)$ decreases in θ_2^* for a given ρ . Hence, the ranking is $\theta_2^*(1, 0, \theta_1) > \theta_2^*(1, \rho_H, \theta_1)$ if

³⁶The ranking of fundamental thresholds does not map one-to-one into a ranking of the probability of a crisis in region 2. The distribution of θ_2 conditional on θ_1 varies with the correlation of regional fundamentals. In particular, the distribution of $\theta_2 | \rho = \rho_H, \theta_1$ places greater weight on lower realizations than the distribution of $\theta_2 | \rho = 0, \theta_1$.

$F_2(\theta_2^*(1, 0, \theta_1), 0) > F_2(\theta_2^*(1, 0, \theta_1), \rho_H)$, where $\alpha_2(0) = \alpha$ and $\mu_2(0, \theta_1) = \mu$:

$$\frac{\alpha}{\sqrt{\alpha + \gamma}} [\theta_2^*(1, 0, \theta_1) - \mu] - \sqrt{\frac{\gamma}{\alpha + \gamma}} \Phi^{-1}(\theta_2^*(1, 0, \theta_1)) > (56)$$

$$\frac{\alpha_2(\rho_H)}{\sqrt{\alpha_2(\rho_H) + \gamma}} [\theta_2^*(1, 0, \theta_1) - \mu_2(\rho_H, \theta_1)] - \sqrt{\frac{\gamma}{\alpha_2(\rho_H) + \gamma}} \Phi^{-1}(\theta_2^*(1, 0, \theta_1)).$$

Solving for θ_1 , which is implicit in $\mu_2(\rho_H, \theta_1)$, results in the lower bound on θ_1 , which is defined in equation (19).

Next, $\underline{\theta}_1 < \mu$ arises because, first, $\theta_2^* < \mu$, second, $[1 - \frac{\alpha_2}{\alpha_2(\rho_H)} \sqrt{\frac{\alpha_2(\rho_H) + \gamma}{\alpha_2 + \gamma}}] > 0$ and, third, $[\sqrt{\frac{\alpha_2(\rho_H) + \gamma}{\alpha_2 + \gamma}} - 1] > 0$. Finally, $\Phi^{-1}(\theta_2^*(1, 0, \theta_1)) < 0$ if $\mu_2(\rho, \theta_1) < Y(\rho) \forall \rho \in \{0, \rho_H\}$. Hence, $\theta_1 \in [\underline{\theta}_1, \mu]$ is non-empty and the inequality in Lemma 5 follows. (As an aside, if the definition of strong and weak priors only used X , and not also Y , then $[\underline{\theta}_1, \mu]$ may be empty under some parameter values.) (*q.e.d.*)

The existence of a unique threshold is again ensured by sufficiently precise private information. Under the sufficient conditions of Lemma 5, there is a positive mass of fundamentals, $\theta_1 \in (\underline{\theta}_1, \mu]$, that is conducive to both a crisis in region 1 and the threshold ranking $\theta_2^*(1, 0, \theta_1) > \theta_2^*(1, \rho_H, \theta_1)$ in region 2. This ranking of fundamental thresholds is important for the subsequent analysis, for example for Proposition 3. Note that the ranking reverses for low realized θ_1 , $\theta_2^*(1, 0, \theta_1) < \theta_2^*(1, \rho_H, \theta_1) \forall \theta_1 < \underline{\theta}_1$. (See also the proof of Lemma 6.)

At the core of Lemma 5 is the variance of the updated prior and its dependence on the realized correlation. As just derived in Table 2, the variance effect opposes the mean effect for a strong prior. To limit the size of the mean effect, we require a lower bound $\underline{\theta}_1$ to ensure that the variance effect dominates the mean effect, thereby generating the ranking $\theta_2^*(1, 0, \theta_1) > \theta_2^*(1, \rho_H, \theta_1)$. A decrease in the relative precision of public signals due to a lower realized ρ increases the disagreement between informed investors, which induces them to attack more aggressively.

A.4.4 General case

We now allow for some uninformed investors, $n < 1$.

Lemma 6 *Proportion of informed investors and fundamental thresholds.* Suppose there is a crisis in region 1, $\theta_1 < \theta_1^*$, and strong fundamentals in region 2. If private information is sufficiently precise, $\underline{\gamma} < \gamma < \infty$, and public information is sufficiently imprecise, $0 < \alpha < \bar{\alpha}$, then:

(A) Boundedness. *The fundamental thresholds in the polar case of informed investors bound the fundamental thresholds in the general case of asymmetrically informed investors:*

$$\begin{aligned} \text{if } \theta_1 \geq \underline{\theta}_1: & \quad \theta_2^*(1, \rho_H, \theta_1) \leq \theta_2^*(n, \rho, \theta_1) \leq \theta_2^*(1, 0, \theta_1) \quad \forall \rho \in \{0, \rho_H\} \quad \forall n \in [0, 1] \\ \text{if } \theta_1 < \underline{\theta}_1: & \quad \theta_2^*(1, 0, \theta_1) \leq \theta_2^*(n, \rho, \theta_1) \leq \theta_2^*(1, \rho_H, \theta_1) \quad \forall \rho \in \{0, \rho_H\} \quad \forall n \in [0, 1]. \end{aligned}$$

(B) Monotonicity. *The fundamental threshold in the case of zero (positive) cross-regional correlation increases (decreases) in the proportion of informed investors. Strict monotonicity is attained if and only if the fundamental thresholds are strictly bounded, that is $\forall \rho, n \in [0, 1)$:*

$$\frac{d\theta_2^*(n, 0, \theta_1)}{dn} \begin{cases} > 0 & \text{if } \theta_2^*(1, \rho_H, \theta_1) < \theta_2^*(n, \rho, \theta_1) < \theta_2^*(1, 0, \theta_1) \\ < 0 & \text{if } \theta_2^*(1, 0, \theta_1) < \theta_2^*(n, \rho, \theta_1) < \theta_2^*(1, \rho_H, \theta_1) \\ = 0 & \text{if } \theta_{2I}^*(\rho, \theta_1) = \theta_2^*(n, \rho, \theta_1) \end{cases} \quad (57)$$

$$\frac{d\theta_2^*(n, \rho_H, \theta_1)}{dn} \begin{cases} < 0 & \text{if } \theta_2^*(1, \rho_H, \theta_1) < \theta_2^*(n, \rho, \theta_1) < \theta_2^*(1, 0, \theta_1) \\ > 0 & \text{if } \theta_2^*(1, 0, \theta_1) < \theta_2^*(n, \rho, \theta_1) < \theta_2^*(1, \rho_H, \theta_1) \\ = 0 & \text{if } \theta_2^*(1, \rho, \theta_1) = \theta_2^*(n, \rho, \theta_1). \end{cases} \quad (58)$$

(C) Monotonicity in signal thresholds. *As a consequence of the monotonicity in fundamentals thresholds:*

$$\frac{d|x_{2I}^*(n, 0, \theta_1) - x_{2I}^*(n, \rho_H, \theta_1)|}{dn} \geq 0 \quad \forall n \in [0, 1). \quad (59)$$

Proof We prove the results of Lemma 6 in turn. A general observation is that the updated belief on the probability of positive cross-regional correlation becomes

degenerate: $\hat{p} \rightarrow p$ for $\alpha \rightarrow 0$. Results (A) and (B) are closely linked, so we start by proving them below.

Results (A) and (B). This prove has three steps.

Step 1: We show in the first step that both fundamental thresholds in the case of asymmetrically informed investors lie either within these bounds or outside of them. As a consequence of $\hat{p} \rightarrow p$, condition $L(n, \theta_2^*(0), \theta_2^*(\rho_H)) = 0$ prescribes that, for any n , the thresholds $\theta_2^*(0)$ and $\theta_2^*(\rho_H)$ are either simultaneously within or outside of the two bounds given by the fundamental thresholds if all investors are informed, $\theta_2^*(1, 0, \theta_1)$ and $\theta_2^*(1, \rho_H, \theta_1)$. This is proven by contradiction. First, suppose that $\theta_2^*(\rho_H) < \theta_2^*(1, \rho_H, \theta_1)$ and $\theta_2^*(0) < \theta_2^*(1, 0, \theta_1)$. This leads to a violation of $L(\cdot) = 0$ because $J(n, \theta_2^*(0), \theta_2^*(\rho_H)) > \frac{1}{1+b_2/l_2} \forall n$ if $\alpha \rightarrow 0$. Second, suppose that $\theta_2^*(\rho_H) > \theta_2^*(1, \rho_H, \theta_1)$ and $\theta_2^*(0) > \theta_2^*(1, 0, \theta_1)$. Again, leading to a violation because $J(n, \theta_2^*(0), \theta_2^*(\rho_H)) < \frac{1}{1+b_2/l_2} \forall n$ if $\alpha \rightarrow 0$.

Step 2: We now derive the derivatives of the fundamental thresholds with respect to the proportion of informed investors, $\frac{d\theta_2^*(\rho)}{dn}$ and $\frac{dx_{2I}^*(\rho)}{dn}$. Applying the implicit function theorem for simultaneous equations, we obtain these derivatives:

$$\frac{d\theta_2^*(n, 0, \theta_1)}{dn} = \frac{\begin{vmatrix} -\frac{\partial K}{\partial n} & \frac{\partial K}{\partial \theta_2(n, \rho_H, \theta_1)} \\ -\frac{\partial L}{\partial n} & \frac{\partial L}{\partial \theta_2(n, \rho_H, \theta_1)} \end{vmatrix}}{\begin{vmatrix} \frac{\partial K}{\partial \theta_2(n, 0, \theta_1)} & \frac{\partial K}{\partial \theta_2(n, \rho_H, \theta_1)} \\ \frac{\partial L}{\partial \theta_2(n, 0, \theta_1)} & \frac{\partial L}{\partial \theta_2(n, \rho_H, \theta_1)} \end{vmatrix}} \equiv \frac{|M_1|}{|M|} \quad (60)$$

where $|M| \equiv \det(M)$. We also find that:

$$\frac{d\theta_2^*(n, \rho_H, \theta_1)}{dn} = \frac{\begin{vmatrix} \frac{\partial K}{\partial \theta_2(n, 0, \theta_1)} & -\frac{\partial K}{\partial n} \\ \frac{\partial L}{\partial \theta_2(n, 0, \theta_1)} & -\frac{\partial L}{\partial n} \end{vmatrix}}{\begin{vmatrix} \frac{\partial K}{\partial \theta_2(n, 0, \theta_1)} & \frac{\partial K}{\partial \theta_2(n, \rho_H, \theta_1)} \\ \frac{\partial L}{\partial \theta_2(n, 0, \theta_1)} & \frac{\partial L}{\partial \theta_2(n, \rho_H, \theta_1)} \end{vmatrix}} \equiv \frac{|M_2|}{|M|}. \quad (61)$$

To find $|M|$, recall from the proof of Proposition 1 that $\frac{\partial K}{\partial \theta_2(0)} > 0$ and $\frac{\partial K}{\partial \theta_2(\rho_H)} < 0$.

Furthermore, $\frac{\partial L}{\partial \theta_2(\rho_H)} < 0$ and $\frac{\partial L}{\partial \theta_2(0)} < 0$ for a sufficiently high but finite value of γ . As a result, $|M| < 0$ for a sufficiently high but finite value of γ .

The proof proceeds by analyzing $|M_1|$ and $|M_2|$. To do this, we first examine the derivatives $\frac{\partial K}{\partial n}$ and $\frac{\partial L}{\partial n}$. Thereafter, we combine the results to obtain the signs of $|M_1|$ and $|M_2|$. We obtain $\forall n \in [0, 1)$:

$$\frac{\partial K}{\partial n} = \begin{cases} < 0 & \text{if } \theta_2^*(n, 0, \theta_1) < \theta_2^*(1, 0, \theta_1) \wedge \theta_2^*(n, \rho_H, \theta_1) > \theta_2^*(1, \rho_H, \theta_1) \\ > 0 & \text{if } \theta_2^*(n, 0, \theta_1) > \theta_2^*(1, 0, \theta_1) \wedge \theta_2^*(n, \rho_H, \theta_1) < \theta_2^*(1, \rho_H, \theta_1) \\ = 0 & \text{if } \theta_2^*(n, 0, \theta_1) = \theta_2^*(1, 0, \theta_1) \wedge \theta_2^*(n, \rho_H, \theta_1) = \theta_2^*(1, \rho_H, \theta_1). \end{cases}$$

After having found the partial derivative for one equilibrium condition (K), we turn to the other equilibrium condition (L). Here, we can invoke the envelope theorem in order to obtain $\frac{\partial L}{\partial n} = 0$. The idea is the following. Since L represents the indifference condition of an uninformed investor, the proportion of informed investors enters only indirectly via x_{2U}^* and we can write:

$$\frac{\partial L}{\partial n} = \frac{\partial J}{\partial x_{2U}^*} \frac{\partial x_{2U}^*}{\partial n} + \overbrace{\frac{\partial J}{\partial n}}^{=0}. \quad (62)$$

Since x_{2U}^* is the optimal signal threshold of an uninformed investor, it satisfies $J(\cdot, x_{2U}^*) = \frac{1}{1+b_2/l_2}$. Thus, we must have $\frac{\partial J}{\partial x_{2U}^*} = 0$, which corresponds to a first-order optimality condition. (This implicitly uses the result that the equilibrium is unique.)

Third, we obtain the derivatives of the fundamental thresholds for sufficiently small but positive values of α . We find that $\forall n \in [0, 1)$:

$$\frac{d\theta_2^*(n, 0, \theta_1)}{dn} = \begin{cases} > 0 & \text{if } \theta_2^*(n, 0, \theta_1) < \theta_2^*(1, 0, \theta_1) \wedge \theta_2^*(n, \rho_H, \theta_1) > \theta_2^*(1, \rho_H, \theta_1) \\ < 0 & \text{if } \theta_2^*(n, 0, \theta_1) > \theta_2^*(1, 0, \theta_1) \wedge \theta_2^*(n, \rho_H, \theta_1) < \theta_2^*(1, \rho_H, \theta_1) \\ = 0 & \text{if } \theta_2^*(n, 0, \theta_1) = \theta_2^*(1, 0, \theta_1) \wedge \theta_2^*(n, \rho_H, \theta_1) = \theta_2^*(1, \rho_H, \theta_1) \end{cases}$$

and $\forall n \in [0, 1)$:

$$\frac{d\theta_2^*(n, \rho_H, \theta_1)}{dn} = \begin{cases} < 0 & \text{if } \theta_2^*(n, 0, \theta_1) < \theta_2^*(1, 0, \theta_1) \wedge \theta_2^*(n, \rho_H, \theta_1) > \theta_2^*(1, \rho_H, \theta_1) \\ > 0 & \text{if } \theta_2^*(n, 0, \theta_1) > \theta_2^*(1, 0, \theta_1) \wedge \theta_2^*(n, \rho_H, \theta_1) < \theta_2^*(1, \rho_H, \theta_1) \\ = 0 & \text{if } \theta_2^*(n, 0, \theta_1) = \theta_2^*(1, 0, \theta_1) \wedge \theta_2^*(n, \rho_H, \theta_1) = \theta_2^*(1, \rho_H, \theta_1). \end{cases}$$

Step 3: In this final step, we combine the results from the previous two steps to show both boundedness and monotonicity. In particular, we use the result that the derivative of the fundamental threshold w.r.t. the proportion of informed investors is zero once the boundary is hit. Therefore, the thresholds in the general case of asymmetrically informed investors are always bounded, which proves Result (A). The distinction between the two cases arises because:

$$\theta_2^*(1, 0, \theta_1) = \begin{cases} > \theta_2^*(1, \rho_H, \theta_1) & \text{if } \theta_1 > \underline{\theta}_1 \\ < \theta_2^*(1, \rho_H, \theta_1) & \text{if } \theta_1 < \underline{\theta}_1 \\ = 0 & \text{if } \theta_1 = \underline{\theta}_1. \end{cases} \quad (63)$$

Given boundedness, in turn, the derivatives of the fundamental threshold can be clearly signed, yielding Result (B).

Now, for the case of $\theta_1 \geq \underline{\theta}_1$, we prove that $\theta_2^*(1, \rho_H, \theta_1) \leq \theta_2^*(\rho_H)$, $\theta_2^*(0) \leq \theta_2^*(1, 0, \theta_1)$ for all n if α sufficiently small. First, $\theta_2^*(1, \rho_H, \theta_1) < \theta_2^*(0) = \theta_2^*(\rho_H) = \theta_2^*(0, \rho, \theta_1) < \theta_2^*(1, 0, \theta_1)$ if $n = 0$, while $\theta_2^*(0) = \theta_2^*(1, 0, \theta_1)$ and $\theta_2^*(\rho_H) = \theta_2^*(1, \rho_H, \theta_1)$ if $n = 1$. Second, $\frac{d\theta_2^*(0)}{dn}|_{n=0} > 0$ and $\frac{d\theta_2^*(0)}{dn}|_{n=1} = 0$. Third, by continuity $\theta_2^*(0, \rho, \theta_1) < \theta_2^*(0) < \theta_2^*(1, 0, \theta_1)$ and $\frac{d\theta_2^*(n, \theta_1, 0)}{dn} > 0$ for small values of n . Fourth, if for any $\hat{n} \in (0, 1]$ $\theta_2^*(0) \nearrow \theta_2^*(1, 0, \theta_1)$ when $n \rightarrow \hat{n}$, then – for sufficiently small but positive values of α – it has to be true that $\theta_2^*(\rho_H) \searrow \theta_2^*(1, \rho_H, \theta_1)$ when $n \rightarrow \hat{n}$. This is because of the result in *step 1*. Fifth, given $\frac{d\theta_2^*(n, 0, \theta_1)}{dn} < 0$ if $\theta_2^*(0) > \theta_2^*(1, 0, \theta_1)$ and $\theta_2^*(\rho_H) < \theta_2^*(1, \rho_H, \theta_1)$, it follows by continuity that $\theta_2^*(0) = \theta_2^*(1, 0, \theta_1)$ and $\theta_2^*(\rho_H) = \theta_2^*(1, \rho_H, \theta_1)$ for all $n \geq \hat{n}$. In conclusion, $\theta_2^*(1, \rho_H, \theta_1) \leq \theta_2^*(\rho_H)$, $\theta_2^*(0) \leq \theta_2^*(1, 0, \theta_1)$ for all $n \in [0, 1]$ if α sufficiently small.

For the case $\theta_1 < \underline{\theta}_1$ it can be proven that $\theta_2^*(1, \rho_H, \theta_1) \geq \theta_2^*(\rho_H)$, $\theta_2^*(0) \geq \theta_2^*(1, 0, \theta_1) \forall n$ if α is sufficiently small using a similar argument (all signs in relation to fundamental thresholds flip).

Result (C). From equation (44),

$$x_{2I}^*(\rho) = \theta_2^*(\rho) + \frac{\theta_2^*(\rho) - \mu_2(\rho, \theta_1)}{\alpha_2(\rho, \theta_1)^{-1} \gamma} - \frac{\sqrt{\alpha_2(\rho, \theta_1) + \gamma}}{\gamma} \Phi^{-1}\left(\frac{1}{1 + b/l}\right) \quad (64)$$

we see that:

$$\frac{\gamma}{\alpha_2(\rho, \theta_1) + \gamma} \frac{dx_{2I}^*(\rho)}{dn} = \frac{d\theta_2^*(\rho)}{dn}. \quad (65)$$

Therefore, by continuity, there exists a sufficiently small but positive value of α that implies the required inequality, taking into account the monotonicity of the fundamental thresholds. Therefore, the distance between the fundamental thresholds is monotone for any $n > 0$, which implies $\frac{d|(x_{2I}^*(0) - x_{2I}^*(\rho_H))|}{dn} > 0$. (*q.e.d.*)

A.5 Proof of Lemma 2

Investors are uninformed about the realized correlation ρ , thereby considering the possibilities of both positively correlated and uncorrelated fundamentals. The proof considers two cases about when the realized fundamental θ_1 is observed. In the counter-factual case 1, investors always observe the realized θ_1 . In case 2, as assumed in the model, investors only observe θ_1 after a crisis in region 1, $\theta_1 < \mu$. Introducing this counter-factual is helpful for constructing the proof.

Case 1: First, it can be shown, by a direct extension of the proof of Proposition 1, that there exists a unique fundamental threshold $\theta_2^*(n = 0, \rho, \theta_1)$ if θ_1 is observed after no crisis in region 1, $\theta_1 \geq \mu$ if $\gamma > \underline{\gamma}_1 \in (0, \infty)$. This fundamental threshold is computed as a weighted average of $\theta_2^*(1, \rho_H, \theta_1)$ and $\theta_2^*(1, 0, \theta_1)$, following the logic of Proposition 1 and its proof.

Second, $\Pr\{\theta_2 \leq \theta_2^*(0, \rho, \theta_1) | \theta_1\}$ is continuous and monotonically decreasing in θ_1 for all $\gamma > \underline{\gamma}'_3$. To see this, consider equation (52) in the Proof of Proposition 1 and inspect its analog $\frac{dJ(0, \theta_2, \theta_1)}{d\theta_1}$. Observe that $\frac{dF_2(\theta_2^*, 0)}{d\theta_1} = 0$, $\frac{dF_2(\theta_2^*, \rho_H)}{d\theta_1} < 0$ and

$\frac{dx_{2U}(\theta_2)}{d\theta_1} = \frac{dx_{2U}(\theta_2^*)}{d\theta_2} \frac{d\theta_2^*}{d\theta_1}$. Using the same argument as in the proof of Proposition 1, there exists a finite level of precision $\gamma > \underline{\gamma}'_3 \in (0, \infty)$ such that $\frac{dJ(0, \theta_2, \theta_1)}{d\theta_1} < 0$ and:

$$\frac{d\theta_2^*(0, \rho, \theta_1)}{d\theta_1} = -\frac{\frac{dJ(0, \theta_2, \theta_1)}{d\theta_1}}{\frac{dJ(0, \theta_2, \theta_1)}{d\theta_2}} < 0. \quad (66)$$

This direct effect is exacerbated by an indirect effect via the conditional distribution of $\theta_2|\theta_1$. That is, the left-hand side of (16) is a weighted average over less favorable set of values of θ_1 than the right-hand side, with strictly positive weights on each θ_1 . Hence, inequality (16) holds for case 1.

Case 2: From case 1, the ranking of fundamental thresholds when θ_1 is observed is: $\Pr\{\theta_2 \leq \theta_2^*(0, \rho, \theta_1)|\theta_1 < \theta_1^*\} > \Pr\{\theta_2 \leq \theta_2^*(0, \rho, \theta_1)|\theta_1 = \theta_1^*\} \geq \Pr\{\theta_2 \leq \theta_2^*(0, \rho, \theta_1)|\theta_1 \geq \theta_1^*\}$. This ranking prevails if θ_1 is unobserved in the absence of a crisis in region 1, since the right-hand side of condition (16) is a weighted average over more favorable values of θ_1 . As a result, inequality (16) holds for sufficiently precise private information, where $\underline{\gamma}_3 < \infty$ denotes the maximum of the stated lower bounds on the precision of private information. (*q.e.d.*)

A.6 Proof of Proposition 2

After a crisis in region 1, $\theta_1 < \mu$, all investors observe the realized θ_1 and a proportion n of investors observe the realized correlation ρ . Consistent with our previous notation, $\theta_2^*(n=0, \rho, \theta_1) \equiv \theta_2^*|\theta_1 \geq \theta_1^*$ denotes the fundamental threshold of region 2 after no crisis in region 1 and $\theta_2^*(n, \rho, \theta_1) \equiv \theta_2^*|\theta_1 < \theta_1^*, n$ after a crisis.

The proof is constructed in four steps. First, we decompose the right-hand side of equation (17) for $E_3 \equiv \theta_2 < \theta_2^*(0, \rho, \theta_1)$ by the law of total probability:

$$\Pr\{E_3|\theta_1 \geq \theta_1^*\} = p\Pr\{E_3|\rho = 0, \theta_1 \geq \theta_1^*\} + (1-p)\Pr\{E_3|\rho = \rho_H, \theta_1 \geq \theta_1^*\}. \quad (67)$$

Since $p \in (0, 1)$, it then suffices to show both of the following inequalities:

$$\Pr\{\theta_2 < \theta_2^*(n, 0, \theta_1) | \rho = 0, \theta_1 < \theta_1^*\} > \Pr\{\theta_2 < \theta_2^*(0, \rho, \theta_1) | \rho = 0, \theta_1 \geq \theta_1^*\} \quad (68)$$

$$\Pr\{\theta_2 < \theta_2^*(n, 0, \theta_1) | \rho = 0, \theta_1 < \theta_1^*\} > \Pr\{\theta_2 < \theta_2^*(0, \rho, \theta_1) | \rho = \rho_H, \theta_1 \geq \theta_1^*\} \quad (69)$$

for all $n \in [0, 1]$, which we do below. In other words, we construct sufficient conditions without resorting to the ex-ante probability of positive correlation.

Second, we consider the case of $n = 0$. It can be shown, by a direct extension of Proposition 1, that there exists a unique $\theta_2^*(n = 0, \rho, \theta_1)$ after no crisis in region 1 (see the proof of Lemma 2). Given that the true distribution of θ_2 is the same on both sides of inequality (68), the result follows directly. We have that $\theta_2^* | \theta_1 \geq \theta_1^*$ must be strictly smaller than $\theta_2^*(n = 0, 0, \theta_1 < \theta_1^*)$, as the former consists of a weighted average of the fundamental thresholds $\theta_2^*(n = 0, 0, \theta_1)$ for each $\theta_1 \geq \theta_1^*$ with strictly positive weight on each $\theta_1 \geq \theta_1^*$. For inequality (69), observe that θ_2 is drawn from a more favorable distribution if $\rho = \rho_H$ because $\theta_1 \geq \theta_1^* = \mu$, which works for our result. Hence, inequality (69) is guaranteed to hold for $n = 0$.

Third, consider the case of $n = 1$. Recall that $[\theta_2^*(n = 0, \rho, \theta_1) | \theta_1 \geq \theta_1^*]$ is a weighted average of $\theta_2^*(n = 1, \rho_H, \theta_1)$ and $\theta_2^*(n = 1, 0, \theta_1)$ with strictly positive weights. Since $\theta_2^*(n = 1, \rho_H, \theta_1) < \theta_2^*(n = 1, 0, \theta_1)$ for all $\theta_1 > \underline{\theta}_1$ (Lemma 5) and, hence, for all $\theta_1 \geq \theta_1^*$, we have that $\theta_2^*(n = 1, 0, \theta_1) > [\theta_2^*(n = 0, \rho, \theta_1) | \theta_1 \geq \theta_1^*]$. Hence, inequality (68) holds. Given that θ_2 is drawn from a more favorable distribution if $\rho = \rho_H$, inequality (69) is guaranteed to hold.

Fourth, consider the case of $n \in (0, 1)$. Recall from Lemma 6 that $\theta_2^*(n, 0, \theta_1)$ is continuous and strictly monotone in n for $n \in (0, 1)$. Hence, (68) and (69) hold for all $n \in [0, 1]$. As a result, inequality (17) holds for sufficiently precise private information, where $\underline{\gamma}_4 < \infty$ denotes the maximum of the lower bounds on the precision of private information. (*q.e.d.*)

A.7 Proof of Proposition 3

The proof has five steps. First, consider the symmetric information cases of $n = 0$ and $n = 1$. Then, $\gamma > \max\{\underline{\gamma}'_0, \underline{\gamma}'_1\} < \infty$ meets the sufficient conditions of Proposition 1, so $\theta_2^*(1, \rho, \theta_1)$ and $\theta_2^*(0, \rho, \theta_1)$ are unique. Second, we have the threshold ranking $\theta_2^*(1, 0, \theta_1) > \theta_2^*(1, \rho_H, \theta_1)$ under the sufficient conditions of Lemma 5, that is an intermediate realized fundamental in region 1, $\theta_1 \in (\underline{\theta}_1, \mu]$, and a strong prior about the fundamental in region 2 (Definition 1).

Third, Proposition 1 implies that the fundamental threshold when all investors are uninformed, $\theta_2^*(0, \rho, \theta_1)$, is a weighted average of the fundamental thresholds used by informed investors. Since the weight satisfies $\hat{p} \in (0, 1)$, we have the following ranking:

$$\min\{\theta_2^*(1, 0, \theta_1), \theta_2^*(1, \rho_H, \theta_1)\} < \theta_2^*(0, \rho, \theta_1) < \max\{\theta_2^*(1, 0, \theta_1), \theta_2^*(1, \rho_H, \theta_1)\}. \quad (70)$$

Combined with the second point, we have: $\theta_2^*(1, 0, \theta_1) > \theta_2^*(0, \rho, \theta_1) \forall \theta_1 \in (\underline{\theta}_1, \mu]$.

Fourth, given that the realized correlation of regional fundamentals is zero, $\rho = 0$, the ordering of thresholds implies an ordering of probabilities. That is, the probability of a crisis in region 2 is higher when all investors are informed than when all investors are uninformed:

$$Pr\{\theta_2 < \theta_2^*(n = 1, \rho = 0, \theta_1)\} > Pr\{\theta_2 < \theta_2^*(n = 0, \rho = 0, \theta_1)\}, \forall \theta_1 \in (\underline{\theta}_1, \mu]. \quad (71)$$

Fifth, we generalize the result to any proportion of informed investors, $n \in (0, 1)$, which yields the result stated in equation (18). From Lemma 6, we have $\frac{d\theta_2^*(n, \rho=0, \theta_1)}{dn} > 0 \forall \theta_1 \in (\underline{\theta}_1, \mu]$ if private information is sufficiently precise, $\underline{\gamma} < \gamma < \infty$, and public information is sufficiently imprecise, $0 < \alpha < \bar{\alpha}$. Finally, we denote $\underline{\gamma}_2 < \max\{\underline{\gamma}'_0, \underline{\gamma}'_1, \underline{\gamma}\} < \infty$ as the maximum of the stated lower bounds on the precision of private information. The result of Proposition 3 follows. (*q.e.d.*)

A.8 Intuition: costs and benefits from a tailored signal threshold

Consider the benefit from using a tailored signal threshold. An informed investor's marginal benefit of using a higher signal threshold $\widehat{x}_{2I}(n, \rho, \theta_1)$ is given by:

$$\begin{aligned} & b \int_{-\infty}^{\theta_2^*(n, \rho, \theta_1)} g(\widehat{x}_{2I}(n, \rho, \theta_1) | \theta_2) f(\theta_2, \rho, \theta_1) d\theta_2 \\ & - l \int_{\theta_2^*(n, \rho, \theta_1)}^{+\infty} g(\widehat{x}_{2I}(n, \rho, \theta_1) | \theta_2) f(\theta_2, \rho, \theta_1) d\theta_2, \end{aligned} \quad (72)$$

which is zero when evaluated at $\widehat{x}_{2I}(n, \rho, \theta_1) = x_{2I}^*(n, \rho, \theta_1) \forall \rho \in \{0, \rho_H\}$ by optimality. Furthermore, equation (72) decreases monotonically in $\widehat{x}_{2I}(n, \rho, \theta_1)$:

$$\frac{dg(\widehat{x}_{2I}(n, \rho, \theta_1) | \theta_2)}{d\widehat{x}_{2I}(n, \rho, \theta_1)} = \begin{cases} > 0 & \text{if } \widehat{x}_{2I}(n, \rho, \theta_1) < \theta_2 \\ \leq 0 & \text{if } \widehat{x}_{2I}(n, \rho, \theta_1) \geq \theta_2. \end{cases} \quad (73)$$

and $\lim_{\gamma \rightarrow \infty} x_{2I}^*(n, \rho, \theta_1) = \theta_2^*(n, \rho, \theta_1) \forall \rho \in \{0, \rho_H\}$.

When $\theta_2^*(1, 0, \theta_1) > \theta_2^*(1, \rho_H, \theta_1)$ we have that $x_{2I}^*(n, 0, \theta_1) > x_{2U}^*(n, \theta_1) > x_{2I}^*(n, \rho_H, \theta_1)$. Therefore, the marginal benefit from increasing $x_{2I}^*(n, 0, \theta_1)$ above $x_{2U}^*(n, \theta_1)$ is:

$$p \left(\begin{array}{l} b \int_{-\infty}^{\theta_2^*(n, 0, \theta_1)} g(x_{2U}^* | \theta_2) f(\theta_2) d\theta_2 \\ - l \int_{\theta_2^*(n, 0, \theta_1)}^{+\infty} g(x_{2U}^* | \theta_2) f(\theta_2) d\theta_2 \end{array} \right) > 0, \quad (74)$$

while the marginal benefit from increasing $x_{2I}^*(n, \rho_H, \theta_1)$ above $x_{2U}^*(n, \theta_1)$ is:

$$(1-p) \left(\begin{array}{l} b \int_{-\infty}^{\theta_2^*(n, \rho_H, \theta_1)} g(x_{2U}^* | \theta_2) f(\theta_2 | \rho_H, \theta_1) d\theta_2 \\ - l \int_{\theta_2^*(n, \rho_H, \theta_1)}^{+\infty} g(x_{2U}^* | \theta_2) f(\theta_2 | \rho_H, \theta_1) d\theta_2 \end{array} \right) < 0. \quad (75)$$

These expressions are best understood in terms of type-I and type-II errors. Let the null hypothesis be that there is a crisis in region 2, such that $\theta_2 < \theta_2^*$. Each of the expressions in equations (74) and (75) have two components. The first component in each equation represents the marginal benefit from attacking when a crisis occurs. (Equivalently, this is the marginal loss from not attacking when a crisis occurs (type-I error)). The second component in each equation is negative and represents the marginal cost of attacking when no crisis occurs (type-II error).

Lemma 6 together with Proposition 1 imply the following. After a crisis in region 1, we have for strong fundamentals in region 2, a sufficiently precise private information, and a sufficiently imprecise public information that $\theta_2^*(n, \rho_H, \theta_1) < \theta_2^*(n, 0, \theta_1) \forall n \in [0, 1]$ if $\theta_1 \in (\underline{\theta}_1, \theta_1^*)$. Hence, the marginal benefit from increasing $x_{2I}^*(n, 0, \theta_1)$ above $x_{2U}^*(n, \theta_1)$ is positive because the type-I error is relatively more costly than the type-II error. By contrast, the marginal benefit from decreasing $x_{2I}^*(n, \rho_H, \theta_1)$ below $x_{2U}^*(n, \theta_1)$ is positive because the type-II error is more costly. In sum, informed investors attack more aggressively upon learning that $\rho = 0$.

A.9 Proof of Lemma 3

The proof has three cases and builds on equation (22). Equation (22) is constructed from EU_I and EU_U . The expected utility of an informed investor writes:

$$\begin{aligned} & \mathbb{E}[u(d_i = I, n)] \equiv EU_I - c \tag{76} \\ = & -c + p \left(\begin{aligned} & b \int_{-\infty}^{\theta_2^*(n, 0, \theta_1)} \int_{x_{i2} \leq x_{2I}^*(n, 0, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | 0, \theta_1) d\theta_2 \\ & - l \int_{\theta_2^*(n, 0, \theta_1)}^{+\infty} \int_{x_{i2} \leq x_{2I}^*(n, 0, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | 0, \theta_1) d\theta_2 \end{aligned} \right) \\ & + (1-p) \left(\begin{aligned} & b \int_{-\infty}^{\theta_2^*(n, \rho_H, \theta_1)} \int_{x_{i2} \leq x_{2I}^*(n, \rho_H, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | \rho_H, \theta_1) d\theta_2 \\ & - l \int_{\theta_2^*(n, \rho_H, \theta_1)}^{+\infty} \int_{x_{i2} \leq x_{2I}^*(n, \rho_H, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | \rho_H, \theta_1) d\theta_2 \end{aligned} \right). \end{aligned}$$

By contrast, the expected utility of an uninformed investor writes:

$$\begin{aligned} & \mathbb{E}[u(d_i = U, n)] \equiv EU_U \tag{77} \\ = & p \left(\begin{aligned} & b \int_{-\infty}^{\theta_2^*(n, 0, \theta_1)} \int_{x_{i2} \leq x_{2U}^*(n, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | 0, \theta_1) d\theta_2 \\ & - l \int_{\theta_2^*(n, 0, \theta_1)}^{+\infty} \int_{x_{i2} \leq x_{2U}^*(n, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | 0, \theta_1) d\theta_2 \end{aligned} \right) \\ & + (1-p) \left(\begin{aligned} & b \int_{-\infty}^{\theta_2^*(n, \rho_H, \theta_1)} \int_{x_{i2} \leq x_{2U}^*(n, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | \rho_H, \theta_1) d\theta_2 \\ & - l \int_{\theta_2^*(n, \rho_H, \theta_1)}^{+\infty} \int_{x_{i2} \leq x_{2U}^*(n, \rho_H, \theta_1)} g(x_{i2} | \theta_2) dx_{i2} f(\theta_2 | \rho_H, \theta_1) d\theta_2 \end{aligned} \right). \end{aligned}$$

First, for $\theta_1 = \underline{\theta}_1$ there are no benefits from acquiring information because $x_{2I}^*(n, \rho, \underline{\theta}_1) = x_{2U}^*(n, \underline{\theta}_1) \forall \rho$. Hence, $\bar{c}(n, \underline{\theta}_1) = 0 \forall n \in [0, 1]$ from equation (22).

Second, if $\underline{\theta}_1 < \theta_1 < \theta_1^*$ then $\theta_2^*(n, 0, \theta_1) > \theta_2^*(n, \rho_H, \theta_1)$ and $x_{2I}^*(n, 0, \theta_1) >$

$x_{2U}^*(n, \theta_1) > x_{2I}^*(n, \rho_H, \theta_1)$ under the sufficient conditions of Lemma 5 and Lemma 6. We will prove that $\frac{d\bar{c}(n, \theta_1)}{dn} \geq 0 \forall \theta_1 \in (\underline{\theta}_1, \theta_1^*)$ and $\bar{c}(n, \theta_1) > 0 \forall \theta_1 \in (\underline{\theta}_1, \theta_1^*)$.

An increase in the proportion of informed investors is associated with a (weak) increase in both $\theta_2^*(0)$ and $x_2^*(0)$ as well as a (weak) decrease in both $\theta_2^*(\rho_H)$ and $x_2^*(\rho_H)$. Furthermore, $x_{2U}^*(n, \theta_1)$ is unaffected. An increase in n leads to a relative increase of the benefit component in the first summand of equation (22) and a relative increase of the loss component in the second summand. For this reason, the left-hand side of equation (22) increases in n . Thus, $\frac{d\bar{c}(n, \theta_1)}{dn} \geq 0 \forall \theta_1 \in (\underline{\theta}_1, \theta_1^*)$.

It remains to consider the case of $\theta_1 < \underline{\theta}_1$. Here, we have $\theta_2^*(1, 0, \theta_1) < \theta_2^*(1, \rho_H, \theta_1)$ and $\theta_2^*(1, 0, \theta_1) \leq \theta_2^*(n, \rho, \theta_1) \leq \theta_2^*(1, \rho_H, \theta_1) \forall \rho \in \{0, 1\}$. Hence, $x_{2I}^*(n, 0, \theta_1) < x_{2U}^*(n, \theta_1) < x_{2I}^*(n, \rho_H, \theta_1)$. We will prove that $\frac{d\bar{c}(n, \theta_1)}{dn} \geq 0 \forall \theta_1 < \underline{\theta}_1$ and $\bar{c}(n, \theta_1) > 0 \forall \theta_1 < \underline{\theta}_1$.

Again, it is optimal to purchase information if the differential expected payoff is positive. Given that $\theta_2^*(1, 0, \theta_1) < \theta_2^*(1, \rho_H, \theta_1)$, the first two summands in (22) are strictly positive and, thus, $\bar{c}(n, \theta_1) > 0 \forall \theta_1 < \underline{\theta}_1$. Furthermore, an increase in n is associated with a (weak) decrease in $\theta_2^*(0)$ and $x_2^*(0)$, and a (weak) increase in $\theta_2^*(\rho_H)$ and $x_2^*(\rho_H)$. For this reason, an increase in n leads to a relative increase of the loss component in the first summand of equation (22) and a relative increase in the benefit component in the second summand. As a result, we have that the left-hand side of equation (22) increases in n . Thus, $\frac{d\bar{c}(n, \theta_1)}{dn} \geq 0 \forall \theta_1 < \underline{\theta}_1$, which concludes the proof. (*q.e.d.*)

A.10 Proof of Proposition 4

The result follows from Lemma 3 in combination with Proposition 2. From Lemma 3 there exists a strictly positive cost level, $c < \bar{c}(0, \theta_1)$, such that information acquisition occurs for all $\theta_1 \neq \underline{\theta}_1$, i.e. $n^* = 1$. Hence, there exists a unique pure-strategy PBE where the wake-up call contagion effect arises if private signals are sufficiently precise, $\gamma > \max\{\underline{\gamma}_2, \underline{\gamma}_4\}$, and the public signal sufficiently imprecise, $\alpha < \bar{\alpha}$. (*q.e.d.*)

A.11 Proof of Corollary 1

The result follows from Lemma 3 in combination with Lemma 5. From Lemma 3 there exists a strictly positive cost level such that information acquisition occurs for all $c \leq c(0, \theta_1)$ and does not occur for all $c \geq c(1, \theta_1)$, provided that $\theta_1 \in (\underline{\theta}_1, \mu)$ in both cases. Hence, there does exist a unique pure-strategy PBE with $n^* = 1$ in the former case and with $n^* = 0$ in the latter case, provided the private signal is sufficiently precise, $\gamma > \underline{\gamma}_2$, and the public signal sufficiently imprecise, $\alpha < \bar{\alpha}$. More specifically, for both cases there exist unique optimal attacking rules at the coordination stage and a unique information acquisition rule at the information stage. Given Proposition 3, the probability of a crisis is higher if $c < \bar{c}(0, \theta_1)$ (informed) than if $\bar{c}(1, \theta_1) < c$ (uninformed). (*q.e.d.*)

A.12 Derivations for endogenous private information precision

As in Szkup and Trevino (2012b) we consider a cost function for private signal precision, $C(\gamma_{i2})$, that is strictly increasing and convex with $\lim_{\gamma_{i2} \rightarrow \infty} C'(\gamma_{i2}) = \infty$ and $C'(\underline{\gamma}_5) = 0$, where $\underline{\gamma}_5 < \infty$. The model is solved backwards. Let $\Gamma(\gamma_2)$ be the proportion of investors who chose precision $\gamma_{i2d} \leq \gamma_2$. Szkup and Trevino prove in a similar global games model with a single region, that for any Γ there exists a unique equilibrium in the coordination stage, when the smallest private signal precision in Γ is sufficiently high. Further, the authors show that there exists a unique equilibrium in the private information acquisition game for a sufficiently high $\underline{\gamma}_5$, characterized by symmetric private signal precision choices.

Consider the incentives to acquire private information after a crisis in region 1 where θ_1 is observed and suppose that $n = 1$. Imposing a symmetric equilibrium in the private information acquisition game at date 2, γ_{2I}^* solves:

$$\max_{\gamma_{2I}} E[u(\gamma_{2I}; \Gamma)] \equiv B(\gamma_{2I}; \Gamma) - C(\gamma_{2I}) \quad (78)$$

where $\theta_2^*(\rho; \Gamma) \equiv \theta_2^*(n = 1, \rho, \theta_1; \Gamma)$ solves $F_2(\theta_2^*(\rho; \Gamma), n = 1, \rho, \theta_1) = 0$

$$\Gamma = \Gamma(\gamma_{i2I})$$

$$B(\gamma_{i2I}; \Gamma) = \left(\begin{array}{l} b_2 \int_{-\infty}^{\theta_2^*(\rho; \Gamma)} \int_{x_{i2I} \leq x_2^*(\rho, \gamma_{i2I}; \Gamma)} g(x_{i2I} | \theta_2, \gamma_{i2I}) dx_{i2I} f(\theta_2 | \rho, \theta_1) d\theta_2 \\ -l_2 \int_{\theta_2^*(\rho; \Gamma)}^{\infty} \int_{x_{i2I} \leq x_2^*(\rho, \gamma_{i2I}; \Gamma)} g(x_{i2I} | \theta_2, \gamma_{i2I}) dx_{i2I} f(\theta_2 | \rho, \theta_1) d\theta_2 \end{array} \right).$$

$B(\gamma_{i2I}; \Gamma)$ is the net benefit of attacking, which can be re-written as:

$$B(\gamma_{i2I}; \Gamma) = \left(\begin{array}{l} b_2 \int_{-\infty}^{\theta_2^*(\rho; \Gamma)} \Phi \left(\frac{x_2^*(\rho, \gamma_{i2I}; \Gamma) - \theta_2}{1/\sqrt{\gamma_{i2I}}} \right) \sqrt{\alpha_2} \phi \left(\frac{\theta_2 - \mu_2}{1/\sqrt{\alpha_2}} \right) d\theta_2 \\ -l_2 \int_{\theta_2^*(\rho; \Gamma)}^{\infty} \Phi \left(\frac{x_2^*(\rho, \gamma_{i2I}; \Gamma) - \theta_2}{1/\sqrt{\gamma_{i2I}}} \right) \sqrt{\alpha_2} \phi \left(\frac{\theta_2 - \mu_2}{1/\sqrt{\alpha_2}} \right) d\theta_2 \end{array} \right). \quad (79)$$

Next, we show that the marginal private benefit from a higher private signal precision in equation (79) exactly coincides with the marginal private benefit from a higher private signal precision in Szkup and Trevino (2012a) for the special case when $b_2 = l_2 = 1/2$ in our model and $T = 1/2$ in their model. To see this, we switch to their notation of signal precisions ($\gamma_{i2I} = \tau_i$, $\alpha_2 = \tau_\theta$) and do the necessary adjustments due the difference in payoffs in the two models ($\theta_2 = 1 - \theta$, $\mu_2 = 1 - \mu_\theta$). Resulting from the difference in payoffs we also have that $x_2^*(\rho, \gamma_{i2I}; \Gamma) = 1 - x_i^*$ and $\theta_2^*(\rho; \Gamma) = 1 - \theta^*$. Hence, equation (79) can be re-written as:

$$\begin{aligned} B(\gamma_{i2I}; \Gamma) &= \int_{-\infty}^{+\infty} (\mathbf{1}_{\{\theta_2 < \theta_2^*(\rho; \Gamma)\}} - \frac{1}{2}) \Phi \left(\frac{x_2^*(\rho, \gamma_{i2I}; \Gamma) - \theta_2}{1/\sqrt{\gamma_{i2I}}} \right) \sqrt{\alpha_2} \phi \left(\frac{\theta_2 - \mu_2}{1/\sqrt{\alpha_2}} \right) d\theta_2 \\ &= \int_{+\infty}^{\theta^*} \Phi \left(\frac{\theta - x_i^*}{1/\sqrt{\tau_i}} \right) \sqrt{\tau_\theta} \phi \left(\frac{\mu_\theta - \theta}{1/\sqrt{\tau_\theta}} \right) (-1) d\theta \\ &\quad - \frac{1}{2} \int_{-\infty}^{+\infty} \Phi \left(\frac{\theta - x_i^*}{1/\sqrt{\tau_i}} \right) \sqrt{\tau_\theta} \phi \left(\frac{\mu_\theta - \theta}{1/\sqrt{\tau_\theta}} \right) (-1) d\theta \\ &= B^i \equiv \int_{-\infty}^{\infty} (\mathbf{1}_{\{\theta > \theta^*(\Gamma)\}} - \frac{1}{2}) \Phi \left(\frac{\theta - x_i^*}{1/\sqrt{\tau_i}} \right) \sqrt{\tau_\theta} \phi \left(\frac{\theta - \mu_\theta}{1/\sqrt{\tau_\theta}} \right) d\theta, \end{aligned} \quad (80)$$

where the last equation is an identical marginal benefit of increasing the private signal precision as in the Proof of Claim A.1 of Szkup and Trevino (2012a).

In the Proof of Claim A.1 Szkup and Trevino show that the derivative of equation (80) is (where $T = 1/2$ is required for the mapping to our model):

$$\frac{\partial B_i}{\partial \tau_i} = \frac{1}{2} \tau_i^{-1} \frac{\sqrt{\tau_i \tau_\theta}}{\tau_i + \tau_\theta} \phi \left(\frac{x_i^* - \mu_\theta}{\sqrt{\frac{\tau_i + \tau_\theta}{\tau_i \tau_\theta}}} \right) \phi(\Phi^{-1}(1 - T)) > 0. \quad (81)$$

Given the convexity of the cost function and the concavity of the net benefit of increasing the private signal precision, Szkup and Trevino show that there is a unique private signal precision solving:

$$\frac{\partial B_i}{d\tau^*} - \frac{dC(\tau^*)}{d\tau^*} = 0. \quad (82)$$

As show by Szkup and Trevino in equation (16) in the Proof of Proposition 4:

$$\begin{aligned} \frac{\partial^2 B_i}{\partial \tau_i \partial \tau_\theta} = & \frac{1}{2} \tau_i^{-1} \frac{\sqrt{\tau_i \tau_\theta}}{\tau_i + \tau_\theta} \phi \left(\frac{x_i^* - \mu_\theta}{\sqrt{\frac{\tau_i + \tau_\theta}{\tau_i \tau_\theta}}} \right) \phi(\Phi^{-1}(1 - T)) \cdot \\ & \cdot \left(\frac{\tau_i - \tau_\theta}{2\tau_\theta(\tau_i + \tau_\theta)} - \tau_i \tau_\theta \left(\frac{x_i^* - \mu_\theta}{\tau_i + \tau_\theta} \right) \left(\frac{\partial x_i^*}{\partial \tau_\theta} + \frac{\tau_i(x_i^* - \mu_\theta)}{2\tau_\theta(\tau_i + \tau_\theta)} \right) \right). \end{aligned} \quad (83)$$

Szkup and Trevino (2012b) demonstrate that the derivative in equation (83) is strictly negative if the prior is that fundamentals are either sufficiently weak $\mu_\theta < -\frac{1}{\tau_\theta} + \frac{1}{2}$ or sufficiently strong $\mu_\theta > \frac{1}{\tau_\theta} + \frac{1}{2}$.

Next, extending analysis of Szkup and Trevino, we consider:

$$\begin{aligned} \frac{\partial^2 B_i}{\partial \tau_i \partial \mu_\theta} = & \frac{1}{2} \tau_i^{-1} \frac{\sqrt{\tau_i \tau_\theta}}{\tau_i + \tau_\theta} \phi \left(\frac{x_i^* - \mu_\theta}{\sqrt{\frac{\tau_i + \tau_\theta}{\tau_i \tau_\theta}}} \right) \phi(\Phi^{-1}(1 - T)) \cdot \\ & \cdot \left(\frac{\mu_\theta - x_i^*}{\sqrt{\frac{\tau_i + \alpha_2}{\tau_i \tau_\theta}}} \right) \left(\frac{dx_i^*}{d\mu_\theta} - 1 \right) \sqrt{\frac{\tau_i \tau_\theta}{\tau_i + \tau_\theta}}. \end{aligned} \quad (84)$$

We find that equation (84) is negative for weak fundamentals, i.e. if μ_θ small.

We proceed by considering the case where $T = 1/2$ and continue the analysis for our model after substituting in for the respective variables. First, we find that:

$$\begin{aligned} \frac{\partial^2 B(\gamma_{2I}; \Gamma)}{\partial \gamma_{2I} \partial \mu_2} = & \frac{1}{2} \gamma_{2I}^{-1} \frac{\sqrt{\gamma_{2I} \alpha_2}}{\gamma_{2I} + \alpha_2} \phi \left(\frac{\mu_2 - x_2^*(\rho, \gamma_{2I}; \Gamma)}{\sqrt{\frac{\gamma_{2I} + \alpha_2}{\gamma_{2I} \alpha_2}}} \right) \phi(0) \cdot \\ & \cdot \left(\frac{x_2^*(\rho, \gamma_{2I}; \Gamma) - \mu_2}{\sqrt{\frac{\gamma_{2I} + \alpha_2}{\gamma_{2I} \alpha_2}}} \right) \left(1 - \frac{dx_2^*(\rho, \gamma_{2I}; \Gamma)}{d\mu_2} \right) \sqrt{\frac{\gamma_{2I} \alpha_2}{\gamma_{2I} + \alpha_2}} < 0, \end{aligned} \quad (85)$$

because $\frac{dx_2^*(\rho, \gamma_{2I}; \Gamma)}{d\mu_2} = \frac{\gamma_{2I} + \alpha_2}{\gamma_{2I}} \frac{d\theta_2^*(\rho; \Gamma)}{d\mu_2} < 0$. Further, $x_2^*(\rho, \gamma_{2I}; \Gamma) < \theta_2^*(\rho; \Gamma) < \mu_2$ if fundamentals are strong (see Definition 1).

Second, we have:

$$\begin{aligned} \frac{\partial^2 B(\gamma_{2I}; \Gamma)}{\partial \gamma_{2I} \partial \alpha_2} = & \frac{1}{2} \gamma_{2I}^{-1} \frac{\sqrt{\gamma_{2I} \alpha_2}}{\gamma_{2I} + \alpha_2} \phi \left(\frac{\mu_2 - x_2^*(\rho; \gamma_{2I}; \Gamma)}{\sqrt{\frac{\gamma_{2I} + \alpha_2}{\gamma_{2I} \alpha_2}}} \right) \phi(0) \cdot \\ & \cdot \left(\frac{\gamma_{2I} - \alpha_2}{2\alpha_2(\gamma_{2I} + \alpha_2)} - \frac{\mu_2 - x_2^*(\rho; \gamma_{2I}; \Gamma)}{\frac{\gamma_{2I} + \alpha_2}{\gamma_{2I} \alpha_2}} \left(-\frac{dx_2^*(\rho; \gamma_{2I}; \Gamma)}{d\alpha_2} + \frac{\gamma_{2I}(\mu_2 - x_2^*(\rho; \gamma_{2I}; \Gamma))}{2\alpha_2(\gamma_{2I} + \alpha_2)} \right) \right), \end{aligned} \quad (86)$$

where $\frac{dx_2^*(\rho; \gamma_{2I}; \Gamma)}{d\alpha_2} = \frac{\gamma_{2I} + \alpha_2}{\gamma_{2I}} \frac{d\theta_2^*(\rho; \Gamma)}{d\alpha_2} + (\theta_2^*(\rho; \Gamma) - \mu_2) < 0$ if the prior is that fundamentals are strong. The first line of equation (86) is strictly positive. It remains to show under what conditions the expression in the second line of equation (86) is negative so that the cross-derivative is negative.

The analysis is similar to Szkup and Trevino (2012a) and we find that:

$$\begin{aligned} \frac{\partial}{\partial \gamma_{2I}} \left(\frac{\gamma_{2I} - \alpha_2}{2\alpha_2(\gamma_{2I} + \alpha_2)} \right) &> 0 \quad (87) \\ \frac{\partial}{\partial \gamma_{2I}} \frac{\mu_2 - x_2^*(\rho; \gamma_{2I}; \Gamma)}{\frac{\gamma_{2I} + \alpha_2}{\gamma_{2I} \alpha_2}} \left(-\frac{dx_2^*(\rho; \gamma_{2I}; \Gamma)}{d\alpha_2} + \frac{\gamma_{2I}(\mu_2 - x_2^*(\rho; \gamma_{2I}; \Gamma))}{2\alpha_2(\gamma_{2I} + \alpha_2)} \right) &< 0 \end{aligned}$$

and:

$$\begin{aligned} \lim_{\gamma_{2I} \rightarrow \infty} \left(\frac{\gamma_{2I} - \alpha_2}{2\alpha_2(\gamma_{2I} + \alpha_2)} \right) &= \frac{1}{2\alpha_2} \quad (88) \\ \lim_{\gamma_{2I} \rightarrow \infty} \frac{\mu_2 - x_2^*(\rho; \gamma_{2I}; \Gamma)}{\frac{\gamma_{2I} + \alpha_2}{\gamma_{2I} \alpha_2}} \left(-\frac{dx_2^*(\rho; \gamma_{2I}; \Gamma)}{d\alpha_2} + \frac{\gamma_{2I}(\mu_2 - x_2^*(\rho; \gamma_{2I}; \Gamma))}{2\alpha_2(\gamma_{2I} + \alpha_2)} \right) \\ &= \alpha_2(\mu_2 - \theta_2^*(\rho; \Gamma)) \left(-\frac{d\theta_2^*(\rho; \Gamma)}{d\alpha_2} + \frac{\mu_2 - \theta_2^*(\rho; \Gamma)}{2\alpha_2} \right). \end{aligned}$$

Given that both summands in the second line of equation (86) are increasing in γ_{2I} , the expression is strictly negative for all $0 < \gamma_{2I} < \infty$ if:

$$\frac{1}{2\alpha_2^2} < (\mu_2 - \theta_2^*(\rho; \Gamma)) \left(-\frac{d\theta_2^*(\rho; \Gamma)}{d\alpha_2} + \frac{\mu_2 - \theta_2^*(\rho; \Gamma)}{2\alpha_2} \right). \quad (89)$$

With strong fundamentals $-\frac{d\theta_2^*(\rho; \Gamma)}{d\alpha_2} > 0$. Hence, inequality (89) holds if:

$$\frac{1}{\alpha_2} < (\mu_2 - \theta_2^*(\rho; \Gamma))^2 \Leftrightarrow \mu_2 > \alpha_2^{-1/2} + \theta_2^*(\rho; \Gamma), \quad (90)$$

which, given Definition 1, is guaranteed to hold if $\mu_2 > \alpha_2^{-1/2} + 1/2$.

As a result, $\frac{\partial^2 B(\gamma_{2I}; \Gamma)}{\partial \gamma_{2I} \partial \mu_2} < 0$ and $\frac{\partial^2 B(\gamma_{2I}; \Gamma)}{\partial \gamma_{2I} \partial \alpha_2} < 0$ for a prior that fundamentals are sufficiently strong, i.e. if $\mu_2(\rho, \theta_1) > \max\{\alpha_2^{-1/2} + 1/2, X(\rho), Y(\rho)\}$. In words, investors have a higher incentive to acquire more precise private signals if the mean of public information, μ_2 , is lower or if the public signal precision, α_2 , is lower. This derivations are the basis of our discussion in section 5.

If a crisis in region 1 takes place, both μ_2 and α_2 depend on what investors learn about θ_1 and ρ . Proposition 4 states conditions such that, for the game with acquisition of a publicly available signal only, there exists a unique monotone pure-strategy PBE where $n^* = 1$ after a crisis in region 1 if the information cost is sufficiently small. In our discussion we assume that these conditions are satisfied. Particularly, we assume that $\gamma > \underline{\gamma}_5 > \max\{\underline{\gamma}_2, \underline{\gamma}_4\} \in (0, \infty)$. Hence, the result of Proposition 4 prevails with endogenous private signal precision, as the endogenous γ_{2d} 's are guaranteed to be sufficiently high.