Risk Sharing Between Households and Financial Institutions in Credit Downturns

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Abstract

This paper develops a chained mortgage contracts model where both homeowner consumers and the financial institutions that securitize their mortgage loan are credit-constrained. The presence of financial frictions in both the consumer and the financial sectors brings the model closer to real financial markets and demonstrates the role of risk sharing between leveraged borrowers. Highly leveraged banks act as financial intermediaries in the mortgage contract so when their indebtedness worsens, they transfer some of the downturn to indebted consumers who have no financial recourse outside of the chained loan contracts. This consequence is observed both after easing borrowing conditions both for consumers and for financial institutions, but is especially evident following relaxed credit access for banks. After initially raising their leverage in response to the lax borrowing conditions, financial institutions repair their debt position relatively fast at the expense of consumers whose borrowing ability is squeezed although they are not the source of the disturbance. The result mirrors the recent subprime mortgage crisis characterized by a sharp but brief decline for banks and a protracted recovery for mortgaged households.

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1 Introduction

In 2004, Edmund Andrews, an economic reporter for The New York Times, joined millions of American home-buyers in purchasing a house at the peak of the real estate price bubble (Andrews 2009). The fact that he had regularly reported from the Federal Reserve, covered the Asian financial crisis of 1997, the Russian meltdown of 1998 and the dot-com collapse of 2000, did not prepare him for what was in store at the subprime mortgage party. “I had just come up with almost a half-million dollars, and I had barely lifted a finger. It had been so easy and fast,” Andrews said of obtaining his mortgage despite having modest disposable income and putting down very little downpayment. His mortgage was a classic subprime loan. The monthly payment first jumped from \$2,500 to \$3,700. If he kept the mortgage for two years, the interest rate could jump as high as 11.5 percent, and the monthly payments could ratchet up to \$4,500. After his wife lost her job, he fell behind on all payments from the mortgage to the electricity bill. When he finally defaulted, he was far from being the only one. In fact, he had outlived two of his three mortgage lenders. The first one collapsed overnight when the financial markets first froze up in August 2007 and the second one was forced out of the mortgage business by federal regulators.

Andrews' personal mortgage crisis would not have unfolded if highly leveraged financial institutions did not extend subprime loans under easy financing conditions in pursuit of higher returns. At the first tightening of the financial market, banks attempted to salvage their own balance sheets by increasing the repayment rate on loans like his (Andrews, 2009). With the downturn threatening their solvency, many highly leveraged financial institutions resorted to risk sharing in which they shifted some of the burden of their deteriorating indebtedness onto borrowing consumers. Raising the interest rates for mortgage-buyers and restricting their credit access were among the actions that banks undertook affording themselves a swift recovery at the expense of consumers who were faced with a protracted recession. Understanding these complex interactions and the risk sharing that happens requires a deeper knowledge of linked loan contracts.

The asset bubble that preceded the subprime mortgage crisis increased housing inventories to record high numbers (Coleman, LaCour-Little and Vandell, 2008). After the bubble burst, the oversupply of real estate reduced both the sale price of real estate and the value of the houses held by homeowners (Duca, Muellbauer and Murphy, 2010; Ellis, 2010). As a result, many mortgage-holders owed more on their housing loan than their residence was worth. Furthermore, at the peak of the asset bubble, many investors eager to tap into mortgage profits underwrote housing loans secured with very little downpayment and with a variable repayment rate, the so-called “subprime” loans (Duca, Muellbauer, and Murphy, 2009a and 2009b). These lax borrowing conditions allowed consumers to hold significant amounts of debt and to be at the mercy of the adjustable interest rate. Once the first signs of trouble raised the repayment rate, their risk premiums rose worsening their indebtedness (Demirgüç-Kunt, Evanoff and Kaufman, 2011; Laeven, Igan and Dell'Ariccia, 2008; Mian and Sufi, 2009). This caused a flurry of short sales and defaults as mortgage-buyers attempted to deleverage or declared bankruptcy. The result was a depressed housing market with even lower asset prices and sharply tightened credit access (Dennis, 2010; Duke, 2012; Madigan, 2012).

Before the financial crisis, the financial sector did not receive much attention in the literature of credit frictions. Papers like Kiyotaki and Moore (1997) and Iacoviello (2005) focus primarily on the demand side of credit and abstract from an active role of financial intermediation. The first models to consider the role of banks are Goodfriend and McCallum (2007) and Christiano, Trabandt and Walentin (2007). Both of these estimate the quantitative importance of the banking sector for central bank policy and for business cycles. Christiano, Motto and Rostagno (2010), Gerali, Neri, Sessa and Signoretti (2009), Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2011) and Luk and Vines (2011) develop a financial sector to explain specific precedents of the financial crisis such as excessive volatility, the proliferation of risk accumulation and the popularity of unconventional monetary policy. The contributions to the literature of the financial sector are discussed in detail in the next section. While these recent additions to the theoretical body of financial frictions augment the general knowledge of the financial crisis, they overwhelmingly focus on credit frictions in only one sector, namely the banking sector, and abstract from constraints in other sectors of the economy, including households. This makes them unsuitable to studying the extent of risk sharing that can occur with more than one type of financially-constrained agents. Only Hirakata, Sudo and Ueda (2009) introduce more than one leveraged sector modeling constraints in the production sector along with the financial sector but they also overlook the role of indebted consumers. This chapter models the chained contracts along the lines of Hirakata, Sudo and Ueda (2009) but the final borrowers are not entrepreneurs but credit-constrained households. It highlights the opportunities for risk sharing between both types of indebted agents and contrasts them with an alternative version similar to Luk and Vines (2011) where consumers have a fixed share arrangement with banks.

This paper develops a chained cosntracts model with credit frictions in both the consumer and the financial sectors demonstrate how opprtunities for risk sharing emerge. It does not attempt to explain the the housing bubble and all aspects of the subsequent collapse. Rather, it focuses on the lax financial conditions that permited increased borrowing. For this purpose, the paper extends the one-sector model of credit constraints of Bernanke, Gilchrist, Gertler (1999) to two sectors and shifts financial frictions from producers to homeowner consumers. A two-sector model can generate a pro-cyclical risk premium that widens following improvements in the housing market. This risk premium is function of the default risk that in turn depends on the size of the collateral and the value of the mortgage. So a shock that reduces housing prices makes real estate more affordable but also creates a Fisher-type effect where the price decrease worsens the value of the housing collateral of indebted households and increases their leverage. The higher leverage triggers an increase in the adjustable mortgage interest rate and the risk premium widens. A credit relaxation on the other hand impacts the leverage of mortgage-buyers directly but creating the incentive to borrow excessively. Hence, with credit constraints in the consumer sector, both a technological expansion in the housing sector and a financial easing imply a downturn for mortgaged consumers and the banks the securitize their mortgages thus bringing the model closer to the subprime crisis.

At the heart of the financial frictions setup is the inability of credit-constrained consumers to fully finance their housing purchase so they need to borrow external funds from risk-neutral investors. This borrowing is complicated by the presence of an idiosyncratic risk of default on the part of mortgage-buyers that is known to them but is unknown to lenders (Townsend, 1979). Unlike Cúrdia and Woodford (2009) where borrowers can choose whether to default, here bankruptcy is involuntary but depends on factors both endogenous (the size of the mortgage) and exogenous (macroeconomic shocks) to consumer decisions. If credit-constrained households default, investors must pay an auditing fee to assume possession of any remaining assets. Since investors cannot fully diversify away this risk, they charge borrowers a risk premium that would offset the expenses associated with eventual bankruptcy. Investors obtain their funds from the deposits of Ricardian consumers who have significant non-wage income from firm profits and share ownership. In contrast, credit-constrained consumers earn only labor income and have no savings. The economy is completed by two production sectors: one that produces conventional consumption and one that manufactures housing. Housing is a multi-period durable good whose manufacture follows the setup of Iacoviello and Neri (2010). Both production sectors are perfectly competitive and there is no idiosyncratic uncertainty in their returns.

When there are two types of credit-constrained agents, the chained loan contracts create opportunities for risk sharing that exacerbate the ensuing recession. As intermediaries of the two mortgage agreements, financial institutions have the ability to transfer some of the burden of a downturn to final borrower consumers. Furthermore, banks are often more leveraged so they experience a proportionally larger deterioration of their leverage ratio which is an added impetus for them to unload the leverage fast. The higher initial leverage and the sharper subsequent deterioration necessitate quick deleveraging in order to maintain solvency. That is the primary reason why banks resort to shifting part of the burden of financial tightening onto households. Two characteristics of the financial system allow them to do so. First, credit-constrained consumers have no recourse to alternative funding so they will participate in the mortgage contract even if the terms deteriorate. Second, the arrangement between banks and mortgage-buying households is such that the return from borrowing to consumers varies with the realization of the idiosyncratic uncertainty. When the uncertainty increases in times of a downturn, lenders can extract a larger share from indebted households. Banks can channel the increased proceeds toward repairing their own balance sheets during a deterioration of financial conditions. Consequentially, they can recover relatively fast at the cost of inducing a lasting recession for mortgaged households.

Banks resort to risk sharing following a relaxation of credit access for consumers and for themselves. Owing to their high initial indebtedness, any of the disturbances causes a more significant worsening in the indebtedness of banks than in the leverage of consumers. Banks recover their solvency by transferring some of the balance sheet deterioration onto consumers who have no financial resort outside of the chained mortgage contracts. Although the terms of the loan contract prevent excessive risk transfer and the consumer downturn is milder than that of banks, the recession of credit-constrained households is prolonged. As optimizers of the financial intermediation, banks can extract higher loan repayments from households to repair their balance sheets relatively fast while consumers suffer a protracted recession. The risk sharing is especially pronounced in the case with relaxed borrowing constraints for financial institutions. Although consumers are not the originators of the disturbance, they take a long time to recover from the bank-induced recession while financial institutions experience a sharp but relatively short downturn. The results of this case closely approximate the consequences of the subprime mortgage crisis.

2 The Chained Loan Contracts

2.1 Description of the Chained Loan Contracts

This section develops the consumer mortgage contract in a partial equilibrium framework, taking as given the price of the collateral and the risk-free rate of interest. The subsequent section endogenizes these variables as part of a general equilibrium solution. The model developed here extends the one-sector model of financial frictions of Bernanke, Gertler and Gilchrist (1999) by credit constraints in both the consumer and fiancial intermediation sectors. The model assumes that financial institutions act as intermediaries that borrow funds from investors and in turn lend to credit-constrained households. Both banks and borrowing consumers are subject to idiosyncratic uncertainty and participate in loan contracts. Taken together, the two transactions constitute two joined mortgage contracts similar to the ones in Hirakata, Sudo and Ueda (2009).

The economy consists of two sectors: one that produces generic consumption and one that manufactures housing (Figure 1). The consumption sector produces its output using sector-specific capital, labor and own technology as inputs. Following Iacoviello and Neri (2010), housing is a durable multi-period good. Housing manufacturers use housing capital, labor and sector-specific technology along with land, a finite resource. Each period only a fraction of housing deteriorates. The rest survives for the subsequent period. In each period, new housing production only replaces the depreciated housing. This approach to housing as a multi-period good brings a more staggered response to disturbances since newly produced output is only a fraction of total housing on the market. Both final good firms face no risk and they can borrow funds for capital financing at the risk-free rate as part of a fixed share arrangement. In addition to the two final goods, there are capital producing firms that supply final good manufacturers with sector-specific capital. Investment in both types of capital is the output of the consumption sector. The relationship between capital firms and final goods producers is a two-way one in the sense that capital firms buy used capital from goods producers and along with investment transform it into new capital.

In order to motivate credit frictions in the household sector, this model formally separates consumers into two types following Iacoviello and Neri (2010). Ricardian consumers possess no intrinsic risk of default and can borrow at the risk-free rate. They have significant non-wage income in the form of revenue from owning shares in final good firms and from absorbing the profits of capital producers. Their wealth allows them to finance their own housing purchase entirely. Ricardians save any unused income as deposits that are lent to investors and to final good firms. Credit-constrained consumers, on the other hand, receive only wage income so their net worth is not enough to finance their housing acquisition. Therefore, they must obtain external financing from investors.

The housing purchase of credit-constrained consumers is financed by a mortgage contract. The mortgage contract is necessitated by the existence of an agency problem that makes external borrowing for house purchase more expensive than own funds. The reason for this discrepancy is that credit-constrained consumers possess an inherent risk of default that is known to them but unobservable to lenders. In this case, lenders cannot perfectly observe the borrower's ability to repay and must pay an auditing cost in order to learn the bankruptcy prospect of the mortgage-buyer. When indebted consumers go bankrupt, lenders pay the monitoring fee and take possession of all of the borrower's remaining assets. In the household context, these auditing costs can be interpreted as the costs of legal proceedings to the value the borrower's assets and the administrative costs of selling the house to realize its collateral value. The presence of these financial frictions motivates the need for a loan (mortgage) contract as opposed to a share contract that usually takes place in the absence of idiosyncratic uncertainty. Additionally, the idiosyncratic uncertainty implies that credit-constrained consumers cannot borrow directly from their Ricardian counterparts. The intermediary services of lenders are necessary not only to disburse the loan but also to conduct monitoring.

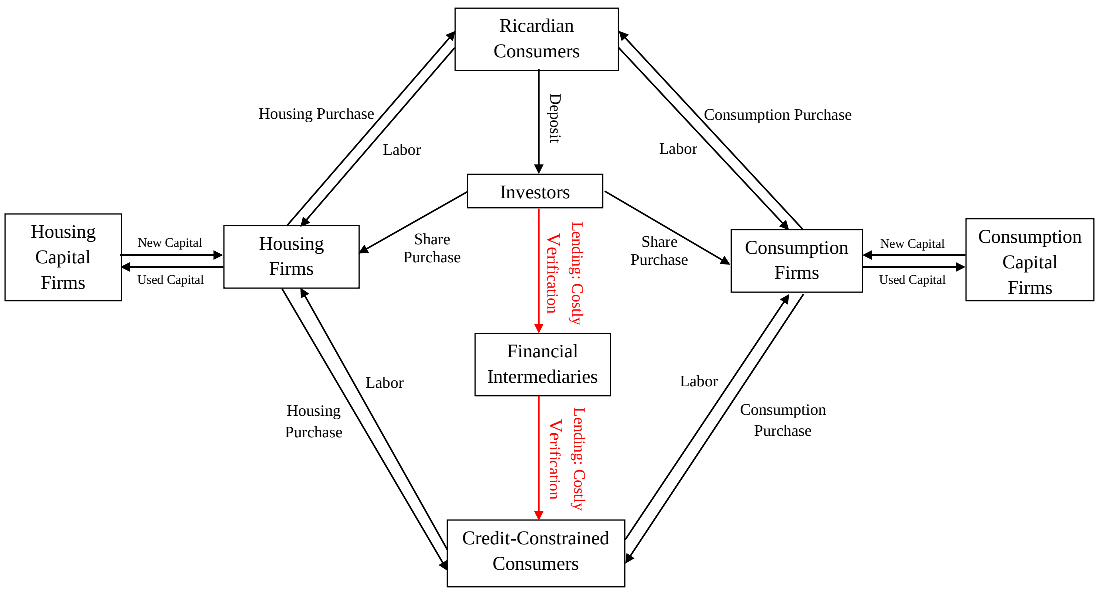


Figure 1: Model with Financial Frictions in the Consumer and Financial Sectors

Credit-constrained households obtain a loan from financial institutions who in turn borrow from investors. Financial institutions (i.e. banks) have an inherent probability of default just as credit-constrained consumers do but their risk is separate from that of borrowers Investors pool the deposits of Ricardian consumers, paying them the risk-free rate, and lend to financial institutions at a markup interest rate. Banks in turn lend to consumers at a different markup rate according to their risk profile. The difference between the borrowing and the lending rates, known as the external finance (risk) premium, exists in order to offset the expenses associated with a potential bankruptcy of mortgage-buyers. The greater the default prospect, the more likely the lender will have to pay the auditing cost so the risk premium will be higher. Hence the external finance premium is a function of the default rate, the collateral (net worth) of credit-constrained consumers and the value of the mortgage. Investors do not make any profit on their loans. The risk premium is exactly sufficient to offset the costs associated with a potential borrower default.

The distinction between investors and banks is not purely a model complication but mirrors real life financial markets. Investors can be characterized as representing "safe" mutual funds institutions that possess no idiosyncratic risk, while financial institutions correspond to investment banks that are highly leveraged. The emergence of such highly leveraged banks may be traced back to the desire for larger profits from riskier investments which safe financial agents such as the investors in this model would be unwilling to finance directly.

The presence of idiosyncratic uncertainty in the returns of banks and households necessitates two loan contracts that provide state-contingent returns to investors. At the beginning of the financial intermediation, investors lend to financial institutions in a loan arrangement that ensures them a fair return regardless of the realization of the idiosyncratic shock of banks. At the tail end of the financial intermediation, credit-constrained consumers borrow from financial institutions as part of another loan contract that ensures their participation. Banks are the intermediaries that participate in both loan contracts so they optimize both arrangements at the same time. This is also consistent with empirical observations of real life banks. Hence the two loan contracts must be chained in the sense that they are linked in the same optimization problem. The remainder of the paper explores the role of leveraged financial institutions and the psotiblity of risk sharing in the doual mrotage ocntract.

2.2 The Chained Loan Contracts

Financial institutions (i.e banks) borrow from investors one period in advance and lend to mortgage-buying consumers to finance their housing purchase. Both banks and credit-constrained consumers are borrowing-constrained in the sense that they each have a distinct probability of default that is known to them but unknown to other participants in the financial market who know only the distribution of the bankruptcy likelihood. Hence there are financial frictions both in the contract between investors and financial institutions and in the contract between banks and credit-constrained households. The presence of credit constraints necessitates that both arrangements are loan contracts where the returns are contingent on the realization of the two idiosyncratic shocks. Since financial institutions are intermediaries and participate in both contacts, the contracts are chained so that the returns from the first contract can provide sufficient funds to cover the lending in the second one.

There is an idiosyncratic shock ω^B associated with lending to financial institutions and a distinct idiosyncratic shock ω^C associated with lending to credit-constrained consumers. Similar to the financial institutions' loan contract, investors have to pay an auditing fee μ^B to learn the realization of ω^B. This makes lending to banks risky so financial institutions have to pay a premium on external funds. Financial institutions on their part, must pay a monitoring cost μ^C to learn the realization of the consumer default probability ω^C.

Since the individual optimization problem of each bank can be aggregated to hold for the whole economy in the same way as before, the chained contracts setup proceeds directly on the aggregate level. Banks borrow funds from investors and in turn lend to credit-constrained households. Every period, they choose the optimal pair of cutoff risk $\bar{\omega}^B$ and housing H^C to maximize their next period expected share $1-\Gamma^B(\bar{\omega}^B_{t+1})$ of the total value of the contact that consists of the housing stock $p_t H^C_t$ minus the net worth of credit-constrained consumers N^C_t:

$$max\ E_t\left(\left(1-\Gamma^B(\bar{\omega}^B_{t+1})\right)R^B_{t+1}\right)\left(p_t H^C_t - N^C_t\right) \tag{1}$$

Banks lend to households at a state contingent markup rate $E_t R^C_{t+1}$ that is different from the rate $E_t R^B_{t+1}$ at which they repay their loans. The difference accounts for the distinct probabilities of default of both types of indebted agents. The expected earnings of financial institutions from lending to credit-constrained consumers equal the share $E_t\Gamma^C(\bar{\omega}^C_{t+1})$ they will receive tomorrow from the loan made today to households minus the auditing fee on insolvent consumer loans $\mu^C E_t G^C(\bar{\omega}^C_{t+1})$:

$$E_t\left(\left(\Gamma^C(\bar{\omega}^C_{t+1})-\mu^C G^C(\bar{\omega}^C_{t+1})\right)R^C_{t+1}\right)p_t H^C_t$$

The earnings of banks must equal the opportunity cost of lending to credit-constrained consumers, which is the value of the loan to banks at their markup interest rate $E_t R^B_{t+1}$. Hence the gross return on the banks' loan to credit-constrained consumers is:

$$E_t\left(\left(\Gamma^C(\bar{\omega}^C_{t+1})-\mu^C G^C(\bar{\omega}^C_{t+1})\right)R^C_{t+1}\right)p_t H^C_t = E_t R^B_{t+1}\left(p_t H^C_t - N^C_t\right) \tag{2}$$

The left hand side is the banks' share of the loan to borrowing households after monitoring and the right hand side is the gross return (the value of the housing purchase $p_t H^C_t$ minus the consumers' net worth N^C_t valued at the bank interest rate $E_t R^B_{t+1}$) from the housing purchase to financial institutions.

Credit-constrained households will participate in the chained loan contracts only if their participation constraint is met. Instead of taking part in the chained loan contracts, credit-constrained consumers can purchase housing using their own net worth N^C_t. In this alternative case, the ex-post return to their investments equals $E_t R^C_{t+1} N^C_t$. Hence credit-constrained consumers will participate in the chained contracts only if their share of the loan is at least equal to the value of their net worth:

$$E_t\left(\left(1-\Gamma^C(\bar{\omega}^C_{t+1})\right)R^C_{t+1}\right)p_t H^C_t \geq E_t R^C_{t+1} N^C_t \tag{3}$$

The first part of the chained contracts consists of investors lending to banks. Financial institutions split the gross profit from their loan to credit-constrained consumers with investors. This contract has the same costly state verification structure as the single loan contract. Banks own the net worth N^B_t and invest the amount $p_t H^C_t - N^C_t$ in the loan to credit-constrained consumers. They borrow the rest $p_t H^C_t - N^C_t - N^B_t$ from investors and repay the loan using their profits from lending to credit-constrained households. Investors receive a net share of the loan to banks $E_t\left(\Gamma^B(\bar{\omega}^B_{t+1}) - \mu^B G^B(\bar{\omega}^B_{t+1})\right)$ that includes the monitoring fees paid on failed banks. Financial institutions are subject to an idiosyncratic bankruptcy shock ω^B and their ex-post gross return on the loans to credit-constrained consumers is $\omega^B R^B$. Investors would participate in the chained contracts only if they get a fair return on their lending to financial institutions. Like in the single contract model, the lender's share of the profit in the contract with financial institutions $E_t\left(\Gamma^B(\bar{\omega}^B_{t+1})R^B_{t+1}\right)\left(p_t H^C_t - N^C_t\right)$ after paying the monitoring fee $\mu^B E_t G^B(\bar{\omega}^B_{t+1})$ must at least equal the opportunity cost of the investors' funds

$R_t\left(p_tH_t^C - N_t^C - N_t^B\right)$ valued at the risk-free rate R_t. Hence the zero profit participation constraint for investors must specify the amount of funds that banks borrow from investors $pH^C - N^C - N^B$, the cut-off value of the idiosyncratic shock $\bar{\omega}^B$ and the return rate of the loan to non-defaulting banks R^B. Since banks borrow at time t and repay the funds at $t+1$, the participation constraint of investors is:

$$E_t\left(\left(\Gamma^B(\bar{\omega}_{t+1}^B) - \mu^B G^B(\bar{\omega}_{t+1}^B)\right) R_{t+1}^B\right)\left(p_tH_t^C - N_t^C\right) \geq R_t\left(p_tH_t^C - N_t^C - N_t^B\right) \tag{4}$$

Lenders sign contracts with a lot of banks, to diversify away the idiosyncratic risk of financial institutions.

Substituting equation (2) into (4) and noting that both equations (3) and (4) bind at the optimum, eliminates the bank interest rate $E_tR_{t+1}^B$ and reduces the conditions that financial institutions must satisfy to two:

$$\begin{aligned} &E_t\left(\left(\Gamma^B(\bar{\omega}_{t+1}^B) - \mu^B G^B(\bar{\omega}_{t+1}^B)\right)\left(\Gamma^C(\bar{\omega}_{t+1}^C) - \mu^C G^C(\bar{\omega}_{t+1}^C)\right) R_{t+1}^C\right) p_tH_t^C = \\ &= R_t\left(p_tH_t^C - N_t^C - N_t^B\right) \end{aligned} \tag{5}$$

$$E_t\left(1 - \Gamma^C(\bar{\omega}_{t+1}^C)\right) p_tH_t^C = N_t^C \tag{6}$$

In the previous loan contract, credit-constrained consumers maximized the arrangement. The superior knowledge of borrowers about their own possibility of default allowed them to push lenders to their participation constraint and extract maximum returns from the loan. The chained loan contracts, however, are linked by the presence of financial institutions. Hence it is the intermediaries (financial institutions) which maximize their profits subject to satisfying the participation constraints of both credit-constrained consumers and of investors. In this case banks, which are borrowers in one part of the contract and lenders in the other part, optimize the two sides of the contract. The reason for banks maximizing their profits is twofold. First, it is empirical fact that financial institutions are often the ones which dictate both the lending and the borrowing terms. The reason for that may be informational asymmetry. Banks use monitoring technology to collect information on borrowers and lenders that would reduce the agency cost associated with lending. This process is costly and location-specific (a bank would not be willing to lend outside of its geographic and sectoral area of expertise). On the other hand, investors, who are geographically dispersed and consumers who lack the means find the cost of this monitoring technology prohibitive. Second, financial institutions, as the intermediaries that participate in both parts of the loan contract, have a transactional advantage. It is easier for them to optimize the two loan contracts together rather than optimizing only their borrowing arrangements with investors and leaving the second loan contract to consumers. The subsequent analysis demonstrates that the results from the chained contracts model are not analytically different from those of two separate loan arrangements.

In the chained contracts model, the presence of two sectors with idiosyncratic uncertainty has important implications for risk sharing between the indebted banks and households. As intermediaries in the two mortgage contracts, financial institutions are the first to experience a deterioration in their balance sheet. However, since they also maximize the chained contracts, they can shift some of the burden of the increased risk of default onto credit-constrained consumers. As end participants in the mortgage arrangement, households have no control over the leverage distribution between indebted sectors. In times of a downturn characterized by worsened leverage for financial institutions, they can improve their own position by forcing consumers to bear a share of the deteriorating leverage. Banks achieve this by extracting a larger return from the loan contract with households in times when the default probabilities of both borrowers are higher. The participation constraint of households, unlike the share agreement in the single loan contract of financial institutions, depends on changes in bankruptcy prospects. The increased bankruptcy prospects allow financial institutions to demand a higher repayment thus shifting the participation constraint (6) of consumers to the left. This can work because, in contrast to the share contract, consumers in the chained mortgage contracts have no recourse to funds outside of the borrowing arrangement due to their idiosyncratic uncertainty. Hence credit-constrained consumers have no choice but to bear some of the deterioration in indebtedness. Oftentimes, this affords banks a speedier recovery at the expense of indebted households who are faced with a protracted recession.

The chained loan optimization problem involves banks simultaneously maximizing both loan arrangements. Let for simplicity of expression $\Gamma^B(\bar{\omega}_t^B) - \mu^B G^B(\bar{\omega}_t^B) = \Psi^B(\bar{\omega}_t^B)$ and $\Gamma^C(\bar{\omega}_t^C) - \mu^C G^C(\bar{\omega}_t^C) = \Psi^C(\bar{\omega}_t^C)$. Banks choose the optimal level of $\bar{\omega}^C$, $\bar{\omega}^B$ and H^C by solving the following Lagrangian:

$$\begin{aligned} L_t^B &= E_t\left(\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)\Psi^C(\bar{\omega}_{t+1}^C)R_{t+1}^C\right)p_tH_t^C + \\ &+\lambda_{1,t}\left[E_t\left(\Psi^B(\bar{\omega}_{t+1}^B)\Psi^C(\bar{\omega}_{t+1}^C)R_{t+1}^C\right)p_tH_t^C - R_t\left(p_tH_t^C - N_t^C - N_t^B\right)\right] \\ &+\lambda_{2,t}\left[E_t\left(1-\Gamma^C(\bar{\omega}_{t+1}^C)\right)p_tH_t^C - N_t^C\right] \end{aligned}$$

The first order conditions yield two equations for the Lagrange multipliers:

$$\lambda_{1,t} = E_t\left(\frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\Psi_\omega^B(\bar{\omega}_{t+1}^B)}\right) \tag{7}$$

$$\lambda_{2,t} = E_t\left[\left[\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right) + \frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)\Psi^B(\bar{\omega}_{t+1}^B)}{\Psi_\omega^B(\bar{\omega}_{t+1}^B)}\right]R_{t+1}^C\frac{\Psi_\omega^C(\bar{\omega}_{t+1}^C)}{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}\right] \tag{8}$$

Here $\lambda_{1,t}$ is the marginal value of the internal funds to financial institutions, and $\lambda_{2,t}$ is the marginal increase in the profits of financial institutions per unit increase in the net worth of credit-constrained consumers.

Substituting both into the first order condition for H_t^C and rearranging gives the consumer risk premium $\frac{E_tR_{t+1}^C}{R_t}$:

$$\begin{aligned} \frac{E_tR_{t+1}^C}{R_t} &= E_t\left(\frac{\Gamma_\omega^C(\bar{\omega}_{t+1}^C)}{\left(1-\Gamma^C(\bar{\omega}_{t+1}^C)\right)\Psi_\omega^C(\bar{\omega}_{t+1}^C) + \Gamma_\omega^C(\bar{\omega}_{t+1}^C)\Psi^C(\bar{\omega}_{t+1}^C)}\right)\times \\ &\quad E_t\left(\frac{\Gamma_\omega^B(\bar{\omega}_{t+1}^B)}{\left(1-\Gamma^B(\bar{\omega}_{t+1}^B)\right)\Psi_\omega^B(\bar{\omega}_{t+1}^B) + \Gamma_\omega^B(\bar{\omega}_{t+1}^B)\Psi^B(\bar{\omega}_{t+1}^B)}\right) \end{aligned} \tag{9}$$

Equation (9) along with the two participation constraints determines the realized default probabilities $\bar{\omega}^B$ and $\bar{\omega}^C$. Even after a shock occurs, investors continue to receive the risk-free rate R_t so the model implements the realized participation constraints:

$$\begin{aligned}&\left(\Gamma^B(\bar{\omega}_t^B) - \mu^B G^B(\bar{\omega}_t^B)\right)\left(\Gamma^C(\bar{\omega}_t^C) - \mu^C G^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C = \\ &= R_{t-1}\left(p_{t-1} H_{t-1}^C - N_{t-1}^C - N_{t-1}^B\right)\end{aligned} \tag{10}$$

$$\left(1 - \Gamma^C(\bar{\omega}_t^C)\right) p_{t-1} H_{t-1}^C = N_{t-1}^C \tag{11}$$

along with the first order condition that contains the expectation of what will occur at $t+1$.

In order to complete the partial equilibrium setting, it is necessary to determine the evolution of the credit-constrained households' net worth. In any given period, the equity of the credit-constrained households, V_t^C, is the remaining share of the mortgage after replaying back lenders:

$$V_t^C = \left(1 - \Gamma^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C \tag{12}$$

Consumers can spend this dividend income on new housing. When house prices fall - and therefore the equity of the households V_t^C - households face the following decision problem. If they decrease housing demand today, current household utility would fall. But, if demand were kept constant, net worth would decrease, increasing the future external finance premium. Thus households face a trade-off between current housing purchase and future borrowing.

It is also necessary to make sure that credit-constrained consumers do not eventually grow out of their financial constraints. This paper assumes that every period a constant fraction $1-\nu^C$ of households retire. When they retire, they spend their remaining equity on consumption. The retirement consumption $C_{E,t}^C$ of credit-constrained consumers is:

$$C_{E,t}^C = \left(1 - \nu^C\right)\left(1 - \Gamma^C(\bar{\omega}_t^C)\right) R_t^C p_{t-1} H_{t-1}^C \tag{13}$$

Over time, the number of credit-constrained consumers decreases but their individual net worth increases as they cycle through many periods of mortgage contracts. Hence macroeconomically, the value of the net worth remains the same.

Credit-constrained consumers need to get started on their net worth with some income not devoted to purchasing consumption goods and housing. This is equivalent to establishing a savings account dedicated to the initial mortgage downpayment. The model assumes they provide one unit of labor inelastically to the production of housing that generates a wage w_t^C. This labor supply is solely for the purposes of starting their net worth accumulation and is weighted heavily so that it does not distort the overall labor supply. Having determined the period equity and the start-up net worth of credit-constrained consumers, is it easy to describe the evolution of their net worth. The evolution of the credit-constrained households' net worth is the sum of equity of non-retiring households plus their income from work:

$$N_t^C = \nu^C V_t^C + w_t^C \tag{14}$$

Similarly to consumers, financial institutions also have period equity that includes the return to banks $\Psi^C(\bar{\omega}_t^C)$ from lending to households:

$$V_t^B = \left(1-\Gamma^B(\bar{\omega}_t^B)\right) R_t^B \left(p_{t-1}H_{t-1}^C - N_{t-1}^C\right) = \\ = \left(1-\Gamma^B(\bar{\omega}_t^B)\right) \Psi^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \tag{15}$$

The consumption of banks $C_{E,t}^B$ on exit is:

$$C_{E,t}^B = \left(1-\nu^B\right)\left(1-\Gamma^B(\bar{\omega}_t^B)\right) \Psi^C(\bar{\omega}_t^C) R_t^C p_{t-1} H_{t-1}^C \tag{16}$$

3 The Complete Model

This section embeds the partial equilibrium of the loan contract derived in the previous section into a general equilibrium framework that endogenizes the risk-free rate R_t and the price of housing p_t. The economy consists of two production sectors: consumption and housing. Capital producers supply sector-specific capital to both types of final good firms. Households consume both consumption and housing.

3.1 Consumption Capital Sector

Firms that produce capital K_t^F for the consumption sector own technology that converts final goods into capital. They purchase depreciated capital from final good firms and make investments to produce new capital. The investment I_t^F is consumption. The newly produced capital is sold back to consumption producing firms.

There are standard quadratic adjustment costs to producing capital. The capital adjustment costs for the consumption capital are:

$$K_t^F = (1-\delta)K_{t-1}^F + J\left(\frac{I_t^F}{K_{t-1}^F}\right) K_{t-1}^F \tag{17}$$

The function J is such that $J' > 0$ and $J" < 0$. New capital is produced within the period and sold to final good producing firms at the price x_t^F. The optimal condition for investment is:

$$x_t^F J'\left(\frac{I_t^F}{K_{t-1}^F}\right) = 1 \tag{18}$$

3.2 Consumption Producers

Firms in the consumption sector use capital K_t^F, labor L_t^F and sector-specific technology A_t^F to produce their output. Consumption producing firms buy capital one period in advance. They borrow funds for the purchase of capital at the risk-free rate R_t which is equal to the expected return on capital. In order to do so, these firms issue claims to Ricardian consumers at the prevailing price of consumption capital x_t^F. At the end of each production period, they sell the remaining capital back to capital producing firms. The production function of consumption firms is:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_t^F)^{(1-\alpha_F)} \tag{19}$$

According to the share purchase setup, consumption firms supply inelastically one unit of labor to their own production in order to start the accumulation of their net worth. Factoring in this labor supply in the production function, the total labor supply in the consumption sector is:

$$L_t^F = (L_{F,t}^F)^{\Omega_F} (L_{H,t}^F)^{(1-\Omega_F)} = (L_{H,t}^F)^{(1-\Omega_F)} \tag{20}$$

where $L_{H,t}^F$ is the regular labor supply by both types of consumers and $L_{F,t}^F$ is the labor supply by consumption producers.

Recasting the production function only in terms of household labor yields:

$$Y_t^F = A_t^F (K_{t-1}^F)^{\alpha_F} (L_{H,t}^F)^{(1-\alpha_F)(1-\Omega_F)} \tag{21}$$

The firms in the sector are perfectly competitive so they maximize profits subject to input costs. The first-order conditions for optimal capital and labor are:

$$w_t = (1-\alpha_F)(1-\Omega_F)\left(\frac{Y_t^F}{L_{H,t}^F}\right) \tag{22}$$

$$R_t = \frac{\alpha_F \frac{Y_{t+1}^F}{K_t^F} + (1-\delta) x_{t+1}^F}{x_t^F} \tag{23}$$

The wage consumption firms receive for their labor supply is:

$$w_t^F = (1-\alpha_F)\,\Omega_F Y_t^F \tag{24}$$

3.3 Housing Capital Sector

Firms that produce capital K_t^H for the housing sector own technology that converts goods into capital. They purchase depreciated capital from final goods firms in the same sector and obtain investments to produce new capital. The investment I_t^H is consumption. The newly produced capital is sold back to housing producers.

Housing capital is subject to the same adjustment costs as consumption capital. The housing capital production equation is:

$$K_t^H = (1-\delta)K_{t-1}^H + J\left(\frac{I_t^H}{K_{t-1}^H}\right)K_{t-1}^H \tag{25}$$

New capital is produced within the period and sold to final good producing firms at the price x_t^H. The optimal level of investment in housing capital:

$$x_t^H J'\left(\frac{I_t^H}{K_{t-1}^H}\right) = 1 \tag{26}$$

3.4 Housing Producers

Housing is a multi-period good that survives for more than one period, unlike consumption which is not durable beyond the period in which it is produced. The production of housing follows closely Iacoviello and Neri (2010). Housing producers use capital K_t^H, labor L_t^H, land X_t and sector-specific technology A_t^H to produce new houses. Housing firms also buy capital one period earlier. In this version, housing firms have no aggregate uncertainty so they can borrow funds for the purchase of capital at the risk-free rate R_t which is equal to the expected return on capital. In order to do so, these firms issue claims to Ricardian consumers at the prevailing price of housing capital x_t^H. At the end of each production period, they sell the remaining capital back to housing firms. The production function of housing firms is:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (X_{t-1})^{\varepsilon} (L_t^H)^{(1-\alpha_H-\varepsilon)} \tag{27}$$

The amount of land is fixed and normalized to one. Furthermore, both the share purchase setup and the loan contracts assumed that housing producers, indebted households and financial institutions supply inelastically one unit of labor in order to start the accumulation of their respective net worth. Factoring in that labor supply, the total labor supply in the production of housing by origin is:

$$L_t^H = (L_{F,t}^H)^{\Omega_H} (L_{CC,t}^H)^{\Omega_C} (L_{B,t}^H)^{\Omega_B} (L_{H,t}^H)^{(1-\Omega_H-\Omega_C-\Omega_B)} = (L_{H,t}^H)^{(1-\Omega_H-\Omega_C-\Omega_B)} \tag{28}$$

where $L_{F,t}^H$ is the labor supply by housing firms, $L_{CC,t}^H$ the labor supply by credit-constrained consumers dedicated to starting their net worth, $L_{B,t}^H$. The labor supply by financial institutions and $L_{H,t}^H$ is regular household labor supply.

Recasting the production function only in terms of household labor and factoring in the fixed supply of land yields:

$$Y_t^H = A_t^H (K_{t-1}^H)^{\alpha_H} (L_{H,t}^H)^{(1-\alpha_H-\varepsilon)(1-\Omega_H-\Omega_C-\Omega_B)} \tag{29}$$

The price of housing is p_t. Housing firms in the sector are perfectly competitive so they maximize profits subject to input costs obtaining the following optimal conditions for housing capital and labor:

$$w_t = (1 - \Omega_H - \Omega_C - \Omega_B)(1 - \alpha_H - \varepsilon)\left(\frac{p_t Y_t^H}{L_{H,t}^H}\right) \tag{30}$$

$$R_t = \frac{\alpha_H \frac{p_{t+1} Y_{t+1}^H}{K_t^H} + (1-\delta)x_{t+1}^H}{x_t^H} \tag{31}$$

The wage housing firms receive for their labor supply is:

$$w_t^H = (1 - \alpha_H - \varepsilon)\,\Omega_H p_t Y_t^H \tag{32}$$

The wage credit-constrained consumers receive for the purpose of starting their net worth is:

$$w_t^C = (1 - \alpha_H - \varepsilon)\,\Omega_C p_t Y_t^H \tag{33}$$

3.5 Consumers

Both Ricardian and credit-constrained consumers have the same preferences. Households choose consumption C_t^i, housing H_t^i and labor L_t^i subject to their respective budget constraints. Here the superscript i denotes the type of consumers: R for Ricardian and C for credit-constrained. Housing is purchased one period in advance. This approach matches empirical reality better where acquiring a house involves transactional delays that involve search time, time spent with real estate agents and time to process escrow, payment and home insurance. Furthermore, there are financial motivations for the advance purchase. Since credit-constrained consumers purchase housing in advance of using it and they do not have non-wage income like Ricardians, they need the mortgage arrangement to facilitate the housing acquisition. Furthermore, the mortgage is intertemporal where consumers optimize their expected share of the loan at the time of the housing purchase but repay it only in the subsequent period after using the housing. As a result, they are exposed to unexpected shocks at the time of repayment so the mortgage is an inherently risky undertaking.

Each household seeks to maximize its lifetime expected utility:

$$U = E_t \sum_{t=0}^{\infty} \beta^t U(C_t^i, H_{t-1}^i, L_t^i) \tag{34}$$

The period utility of each household is given by:

$$U(C_t^i,\, H_{t-1}^i,\, L_t^i) = log(C_t^i) + \kappa log(H_{t-1}^i) - \gamma \frac{L_t^{i(1+\varphi)}}{1+\varphi} \tag{35}$$

The period utility function is separable in consumption C_t^i, housing H_{t-1}^i and labor L_t^i. Housing is purchased one period in advance and consumed the following period. At the end of the period, the remaining housing minus depreciation is sold back on the market. Following Iacoviello and Neri (2010), housing enters the utility function additively, rather than as part of a consumption aggregator in order to demonstrate its direct effect on consumer decisions. The additive nature of the utility function also facilitates housing to be be purchased both directly (by Ricardians) and via a mortgage (by credit-constrained consumers). There is a taste parameter κ that reflects the relative preference for consumption and housing.

3.5.1 Ricardian Consumers

Ricardian consumers purchase consumption and housing. Each period, they lend an amount B_t at the risk-free rate. Their lending covers the mortgage loan to credit-constrained households as well as finances the capital purchase of consumption firms and housing producers through their respective share arrangements. Ricardian consumers also absorb the profits Π_t of both capital sectors. The budget constraint of Ricardian households is:

$$C_t^R + p_t H_t^R + B_t = w_t L_t^R + (1-\delta) p_t H_{t-1}^R + R_{t-1} B_{t-1} + \Pi_t \tag{36}$$

Ricardian consumers maximize their utility function subject to this budget constraint. The left hand side reflects their consumption and housing purchase as well as their lending, while the right hand side represents their income from wages and from reselling the non-depreciated housing from the previous period as well as their returns from lending and from capital firms profits. The Lagrangian for Ricardian consumers yields three first-order conditions for consumption, housing and leisure. The first-order condition for the consumption-labor tradeoff is fairly standard:

$$\gamma (L_t^R)^{\varphi} C_t^R = w_t \tag{37}$$

The relationship between consumption and housing reflects the fact that housing is purchased one period in advance so the tradeoff between housing and consumption depends on both the current and on the future price of housing as well as on the intertemporal consumption substitution and the depreciation rate of housing:

$$\left(\frac{p_t}{\beta} E_t \left(\frac{C_{t+1}^R}{C_t^R} \right) - (1-\delta) p_{t+1} \right) H_t^R = \kappa E_t C_{t+1}^R \tag{38}$$

Ricardian consumers also have a standard Euler equation:

$$E_t \left(\frac{C_{t+1}^R}{C_t^R} \right) = \beta R_t \tag{39}$$

Combing the last two equations yields a simpler expression for the consumption-housing substitution:

$$\left(R_t p_t - (1-\delta)p_{t+1}\right) H_t^R = \kappa E_t C_{t+1}^R \tag{40}$$

3.5.2 Credit-Constrained Consumers

Just like Ricardian households, credit-constrained households also consume both consumption and housing. They earn income only from labor and do not own any shares. Since their income is not sufficient to allow them to purchase housing in full, they must obtain a mortgage from investors. Their mortgage is subject to an idiosyncratic risk of default so their borrowing is not riskless. Due to this probability of bankruptcy, they cannot borrow at the risk-free rate R_t and can do so only at the consumer interest rate R_t^C. As a result, buying a house is costly for them. Credit-constrained households also cannot optimize intertemporally their purchase of consumption since they have no access to risk-free financing. Hence their consumption needs must be met solely with their wages after their mortgage is repaid. The budget equation for credit-constrained households is:

$$C_t^C + R_t^C p_{t-1} H_{t-1}^C = w_t L_t^C + (1-\delta) p_t H_{t-1}^C \tag{41}$$

The left hand side reflects their consumption purchase as well as their housing mortgage, while the right hand side represents their income from wages and returns from reselling the non-depreciated housing from the previous period. Credit-constrained consumers maximize their utility function subject to this budget constraint. The optimization problem yields two first-order conditions for housing and leisure. The first-order condition for consumption-labor tradeoff is identical to that of Ricardian households:

$$\gamma (L_t^C)^{\varphi} C_t^C = w_t \tag{42}$$

The relationship between consumption and housing however depends on the consumer interest rate R_t^C instead of the risk-free rate R_t:

$$\left(R_t^C p_{t-1} - (1-\delta)p_t\right) H_{t-1}^C = \kappa C_t^C \tag{43}$$

The first-order condition for housing-consumption tradeoff for credit-constrained consumers is lagged, unlike that for Ricardian households which is forward-looking. This is due to the fact that the return rate at which credit-constrained consumers repay their mortgage is state-contingent and depends on the realization of the ex-post return on housing R_t^C that incorporates all shocks at the time of the repayment. The realized return on housing R_t^C depends on the past purchasing price p_{t-1} and the current selling price p_t of housing. It does not depend on the expected future price of housing which is unknown at the time of the loan contract. Hence equation (43) is a function of the ex-post return on housing R_t^C, while the corresponding equation for Ricardian households (40) is not lagged since Ricardians borrow at the risk-free rate which is not state-contingent.

Finally, credit-constrained consumers cannot optimize intertemporally their consumption purchase since they cannot borrow at the risk-free rate R_t. Their demand for consumption must be met by their income once all housing loans are repaid:

$$C_t^C = w_t L_t^C + ((1-\delta)p_t - R_t^C p_{t-1})H_{t-1}^C \tag{44}$$

Any disturbances that can increase the mortgage repayment, would crowd out regular consumption and the resulting diminished demand might impact negatively the consumption sector.

3.6 Market Clearing

Market clearing requires that the output of consumption must cover household consumption, consumption by conventional good producers on exit and consumption by credit-constrained consumers on retirement and the consumption of financial institutions on exit as well as investment in the two sectors:

$$Y_t^F = C_t^R + C_t^C + C_{E,t}^F + C_{E,t}^C + C_{E,t}^B + I_t^F + I_t^H \tag{45}$$

Housing is a multi-period good and each period a fraction δ of the housing available on the market depreciates. The remaining non-depreciated housing along with new production constitutes the available housing in the subsequent period. In each period, the sum of new production and leftover housing must meet the housing needs of consumers as well as the consumption of housing firms on exit and the monitoring costs of the mortgage contract:

$$\begin{aligned} p_t Y_t^H + (1-\delta)p_t(H_{t-1}^R + H_{t-1}^C) \quad &= \quad p_t(H_t^R + H_t^C) + C_{E,t}^H + \\ &\quad + \left[\mu^C G^C(\bar{\omega}_t^C) + \mu^B G^B(\bar{\omega}_t^B)\Psi^C(\bar{\omega}_t^C)\right] R_t^C p_{t-1} H_{t-1}^C \end{aligned} \tag{46}$$

The labor that both types of households supply equals the demand by housing and consumption firms:

$$L_{H,t}^F + L_{H,t}^H = L_t^R + L_t^C \tag{47}$$

Finally, Ricardian consumer lending must equal the loan to credit-constrained households as well as the share purchase of consumption firms and of housing firms:

$$B_t = p_t H_t^C - N_t^C + x_t^F K_t^F - N_t^F + x_t^H K_t^H - N_t^H \tag{48}$$

3.7 Model Calibration

The parameters that govern the general equilibrium for the chained contracts model are the same as those in the consumer mortgage contract. The chained contracts model satisfies the same steady state requirements as the respective single loan contracts for consumers and for financial institutions:

1. The steady state rate of the external consumer risk premium is 0.5% (Bernanke, Gertler and Gilchrist, 1999).

2. The steady state consumer leverage, i.e. value of housing stock to net worth ratio is $\frac{pH^C}{N^C} = 2$ (Bernanke, Gertler and Gilchrist, 1999).

3. The steady state leverage of financial institutions is $\frac{pH^C - N^C}{N^B} = 5$ (Hirakata, Sudo and Ueda, 2009).

4. The failure rate of both credit-constrained consumers $F^C(\bar{\omega}^C)$ and financial institutions $F^B(\bar{\omega}^B)$ is 2% (Bernanke, Gertler and Gilchrist, 1999).

The loan parameters that satisfy the chained contracts model are in Table 1.

Table 1: Loan Contract Parameters in the Chained Contracts Economy

Parameter	ν^F	ν^H	ν^C	ν^B	μ^C	μ^B
Value	0.99	0.99	0.98	0.97	0.02	0.03

The complete model is solved for the deterministic steady state and then log-linearized around that steady state. The steady state values of the model variables are in Table 2.

Table 2: Steady State Variables in the Chained Contracts Economy

Variable	Y^F	Y^H	C^R	C^C	H^R	H^C	L^R	L^C	I^F	I^H
Value	100	10	32.5	29.9	210	169	15.6	19.9	24.9	3.35
Variable	K^F	K^H	L^F	L^H	R^C	R^B	N^F	C_E^F	N^H	C_E^H
Value	997	134	30.0	5.53	1.015	1.0147	498	5.68	67.0	0.80
Variable	N^C	C_E^C	N^B	C_E^B	$\bar{\omega}^C$	σ^C	Γ^C	G^C	Γ_ω^C	G_ω^C
Value	159	2.53	31.9	1.11	0.5	0.31	0.50	0.01	0.98	0.15
Variable	Γ_σ^C	G_σ^C	$\Gamma_{\omega\omega}^C$	$G_{\omega\omega}^C$	$\Gamma_{\omega\sigma}^C$	$G_{\omega\sigma}^C$	$\bar{\omega}^B$	σ^B	Γ^B	G^B
Value	-0.02	0.16	-0.31	2.02	-0.36	1.89	0.79	0.11	0.79	0.02
Variable	Γ_ω^B	G_ω^B	Γ_σ^B	G_σ^B	$\Gamma_{\omega\omega}^B$	$G_{\omega\omega}^B$	$\Gamma_{\omega\sigma}^B$	$G_{\omega\sigma}^B$	p	w
Value	0.98	0.45	-0.04	0.74	-0.57	10.8	-0.97	14.4	1.88	2.15
Variable	Ψ^B	Ψ_ω^B	Ψ_σ^B	$\Psi_{\omega\omega}^B$	$\Psi_{\omega\sigma}^B$	Ψ^C	Ψ_ω^C	Ψ_σ^C	$\Psi_{\omega\omega}^C$	$\Psi_{\omega\sigma}^C$
Value	0.79	0.97	-0.05	-0.83	-1.32	0.50	0.98	-0.02	-0.36	-0.41

4 Risk Sharing in Chained Loan Contracts

In the chained contracts setting, both consumers and financial institutions can experience a credit easing on their own and together. Relaxing borrowing access for households spreads the resulting leverage worsening across both types of credit-constrained agents but the combined downturn is deeper than that of households in their sole borrowing arrangement due to the larger combined leverage (Figures 2 and 3). However, a financial relaxation for banks implies stronger risk sharing that is more tilted toward consumers. Banks struggle to repair their debt positions after the credit easing in their own sector so they shift a large share of the downturn onto households. While the downturn is sharp but brief for banks, consumers are drawn into a prolonged recession.

The decreased consumer volatility within the chained contracts reduces not only the household risk premium but also the risk premium of banks since the two loan contracts are linked in the same maximization problem (Figures 2.27 and 2.28). The reduction in risk premium is passed from consumers to financial institutions. The improvements in consumer credit access also affect financial institutions which, as intermediaries in the loan contract, need less downpayment to secure funding from investors on behalf of consumers (Figure 2.20). Less net worth implies higher indebtedness both for financial institutions and for households.

However, as optimizing agents in the chained loan contract, financial institutions experience most of the consequences of the eased borrowing on their own debt position (Figure 2.26). Their indebtedness rises more than that of consumer since they begin more leveraged. However, the presence of two chained loan contracts allows them to share the downturn with consumers. The leverage of banks rises but by less than when they are the only borrowers and at the expense of that of credit-constrained consumers (Figure 2.25). Overall, the risk sharing in the chained loan contract dampens the effect of the shock, spreading it over both types of indebted agents while at the same time concentrating a larger share in the intermediary sector that optimizes the dual contracts. The default probability of both agents also increases but by less than before (Figures 2.23 and 2.24).

Taken together, the worsening in the leverage of both credit-constrained consumers and banks is larger than that of only consumers in the single loan contract. This is due to the higher initial indebtedness of banks which reflects on both agents. Despite the large combined downturn in the chained contracts model, the debt position of financial institutions recovers relatively fast and about 70 quarters (14.5 years) after the shock, their leverage ratio and bankruptcy prospects return to pre-shock levels (Figure 2.26). Bank recovery, however, is at the expense of the revival of the borrowing ability of households. As maximizers of the chained contracts, financial institutions can push consumers to their participation constraint so most of the subsequent improvements in housing equity benefit them at the cost of delaying the recovery of mortgaged households. The indebtedness of credit-constrained consumers unwinds slowly and remains above its pre-shock level for close to 100 quarters (25 years) (Figures 2.25 and 2.26). Risk sharing implies that the net worth of banks and their leverage position improve relatively fast, while consumers are faced with a protracted recovery.

A model with relaxed credit access for financial institutions produces a deeper recession than that with relaxed consumer credit (Figure 3). Following a reduction in their own volatility, financial institutions are faced with a bigger leverage rise than in the previous scenario due to their higher initial leverage. Since the default probability of financial institutions is initially much higher than that of households, a reduction in their own volatility leads to a more significant tightening of their financial conditions. In order to mitigate this threat to their own solvency, financial institutions shift a considerable share of the burden onto households. They again see a smaller increase in their leverage than when they are the only borrowers since some of the increase is and transferred onto consumers (Figures 3.25 and 3.26).

Consumers again have no ability to borrow outside of the loan contract due to their idiosyncratic uncertainty so banks can afford to transfer some of the downturn onto them without turning them away from the mortgage purchase. Hence households are forced to absorb some of the deterioration in the balance sheets of banks although they are not the source of the disturbance. Following a shock to the volatility of banks, the leverage of mortgaged households rises by as much as when their own volatility is reduced in the same chained contracts setting (Figure 3.25). The subsequent recovery again benefits financial institutions more than credit-constrained consumers. As optimizers of the chained loan contract, financial institutions can repair their debt position faster at the expense of consumers who experience a smaller leverage worsening and despite not being the source of the disturbance, undergo a slower recovery.

The two chained contract simulations demonstrate that consumers share some of the burden of higher indebtedness with financial institutions so their leverage does not deteriorate as much as before. Nevertheless, their subsequent recovery could be more protracted since banks can use their optimizing position to extract more equity from consumers in order to improve their own debt position. This is especially evident in the case with a decrease in financial volatility. Since consumers are not the source of the disturbance, they should be relatively protected by their participation constraint. However, the deterioration in the balance sheets of banks is so considerable that financial institutions resort to extracting as much equity from households as possible in order to repair their own debt position. Credit-constrained consumers suffer the negative consequences of a reduction in bank volatility with the same magnitude as following a decrease in their own volatility. The financial troubles of banks draw them into a prolonged downturn and their leverage remains above its steady state value for more than 100 quarters (25 years). The last scenario describing a reduction in bank volatility in a chained contracts setup offers the closest cautionary tale to the subprime recession. Leading up to the subprime crisis, the balance sheets of banks were deemed sufficiently low risk and fairly stable allowing them to accumulate excessive debt. The resulting downturn was passed on to consumers who experienced significant worsening in their debt positions that led to a prolonged tightening of credit access.

Figure 2: Reduced Consumer Volatility in a Chained Contracts Model

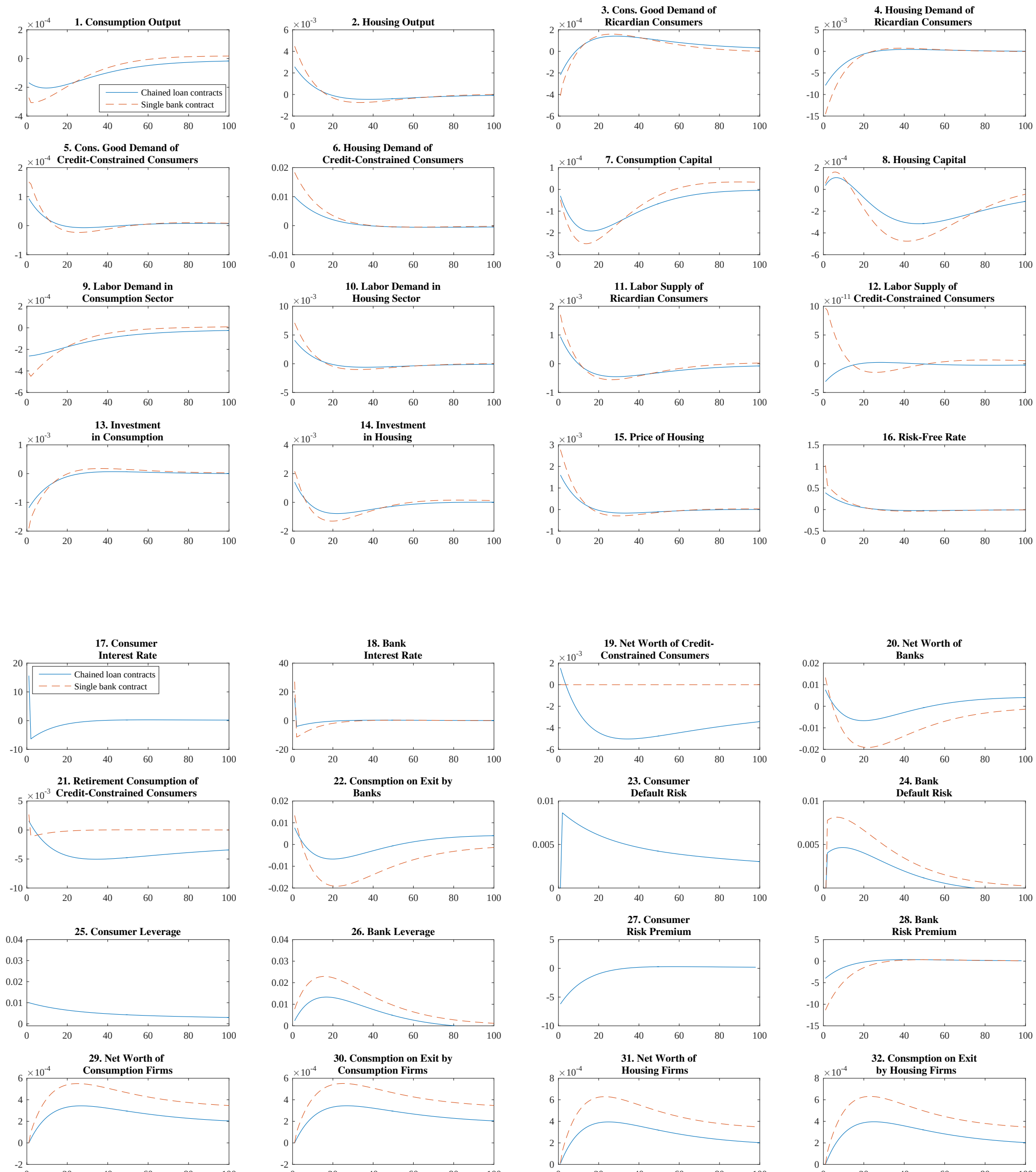
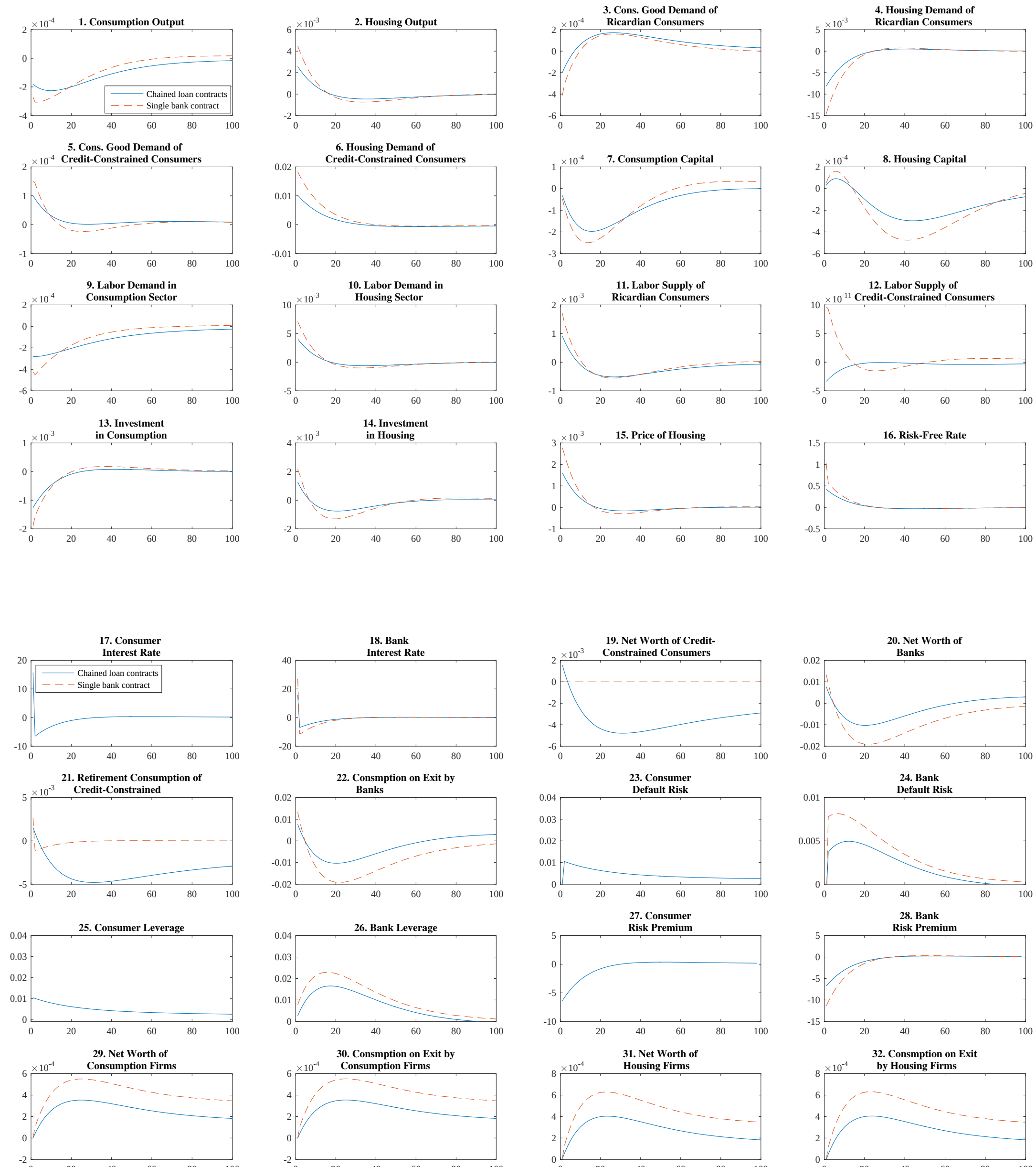


Figure 3: Reduced Bank Volatility in a Chained Contracts Model



5 Conclusion

The subprime mortgage crisis dashed the hopes of many for home ownership and set off a deep recession. Loan applicants saw their financing prospects reduced for years ahead as suddenly prudent banks struggled to improve their balance sheet positions. Many financial institutions were brought to the brink of collapse or saved by the too big to fail policy only to push through their recovery at the expense of credit-squeezed consumers. The consequences of excessive leverage and excessive lending as subprime loans had many calling for more stringent supervision of borrowing transactions. An improved regulation framework cannot emerge without understanding how the subprime mortgage crisis happened to be.

The straightforward setup of financial frictions in the household and financial sectors in this paper updates the financial accelerator approach of Bernanke, Gertler and Gilchrist (1999) to include financial institutions acting as intermediaries between investors and credit-constrained consumers. The result is two chained mortgage contracts in which both banks and credit-constrained households are subject to idiosyncratic uncertainty. The model not only has the benefit of depicting financial markets more accurately but also demonstrates that there are opportunities for risk sharing between financial institutions and credit-constrained consumers. Since banks participate in both loan arrangements - they borrow from investors and in turn lend to mortgage-buying consumers - they maximize the chained loan contracts. Their position as optimizers allows them to shift some of the negative consequences of a disturbance onto households. Hence in the chained contracts model banks can engage in risk sharing where they extract a larger return from consumers in order to mitigate the negative effects on their leverage and thereby prolonging the downturn for consumers.

Banks resort to risk sharing following a relaxation of credit access for consumers and for themselves. Owing to their high initial indebtedness, any of the disturbances causes a more significant worsening in the indebtedness of banks than in the leverage of consumers. Banks recover their solvency by transferring some of the balance sheet deterioration onto consumers who have no financial resort outside of the chained mortgage contracts. Although the terms of the loan contract prevent excessive risk transfer and the consumer downturn is milder than that of banks, the recession of credit-constrained households is prolonged. As optimizers of the financial intermediation, banks can extract higher loan repayments from households to repair their balance sheets relatively fast while consumers suffer a protracted recession. The risk sharing is especially pronounced in the case with relaxed borrowing constraints for financial institutions. Although consumers are not the originators of the disturbance, they take a long time to recover from the bank-induced recession while financial institutions experience a sharp but relatively short downturn. The results of this case closely approximate the consequences of the subprime mortgage crisis.

The model, while aptly demonstrating the role of leverage in triggering a financial downturn, could be enriched further to offer a deeper understanding of credit mechanisms. When the crisis started, many banks attempted to salvage their debt position by recalling loans to other financial institutions, rather than merely bearing out the increasing bankruptcy risk as described in this paper. The model can be augmented by adding inter-bank relationships and loan networks that may trigger a domino-like effect of rising default risk. Furthermore, the potential role of a bailout policy could be explored by adding government to the existing setup. It may also be a cautionary tale to consider the possibility of endogenous steady state leverage ratio for both banks and consumers. As history leading to the crisis demonstrates, leverage limits were poorly regulated and enforced prior to the subprime mortgage crisis allowing instead financial institutions to reach dangerously high debt to equity ratios. Finally, an important contribution could be to model the role of heterogeneous consumer expectations. It may be especially interesting to consider to what extent departures from the representative agent theory could explain the collective failure of agents to foresee the subprime mortgage crisis.

References

Andrews, E. (2009). "My Personal Credit Crisis," *The New York Times,* May 14, 2009.

Aoki, K., J. Proudman and G. Vlieghe (2004). "House Prices, Consumption, and Monetary Policy: A Financial Accelerator Approach," *Journal of Financial Intermediatio*n, vol. 13, pp. 414-435.

Aron, J., J. Duca, J. Muellbauer, K. Murata and A. Murphy (2009). "Credit, Housing Collateral and Consumption: the UK, Japan and the US," Centre for Economic Policy Research Discussion Paper No. 7876.

Benhabib, J., R. Rogerson, and R. Wright (1991). "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, vol. 99, iss.6, pp. 1166-1187.

Bernanke, B., M. Gertler and S. Gilchrist (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework," In Vol. 1 of *Handbook of Macroeconomics*, edited by J. B. Taylor and M. Woodford, Chapter 21, pp. 1341-1393. Elsevier.

Brueckner, J., P. Calem and L. Nakamura (2011). "Subprime Mortgages and the Housing Bubble," *Journal of Urban Economics*, vol. 71, iss. 2, pp. 230-243.

Calomiris, C. (2009). "The Subprime Turmoil: What's Old, What's New, and What's Next," *The Journal of Structured Finance,* vol. 15, iss 1, pp. 6-52.

Calza, A., T. Monacelli and L. Stracca (2007). "Mortgage Markets, Collateral Constraints, and Monetary Policy: Do Institutional Factors Matter?" Centre for Economic Policy Research Discussion Paper No. 6231.

Chang, Y. (2000). "Comovement, Excess Volatility and Home Production," *Journal of Monetary Economics*, vol. 46, iss. 2, pp. 385-396.

Coleman, M., M. LaCour-Little and K. Vandell (2008). "Subprime Lending and the Housing Bubble: Tail Wags Dog?" *Journal of Housing Economics*, vol. 17, iss. 4, pp. 272–290.

Corradin, S. (2012). "Household Leverage," European Central Bank Working Paper No. 1452.

Cúrdia, V. and M. Woodford (2009). "Credit Frictions and Optimal Monetary Policy," Federal Reserve Bank of New York Working Paper.

Davis, M. and J. Heathcote (2005). "Housing and the Business Cycle," *International Economic Review*, vol. 46, iss. 3, pp. 751-784.

Degen, R. J. (2009). "Moral Hazard and the Financial Crisis of 2007-9: An Explanation for Why the Subprime Mortgage Defaults and the Housing Market Collapse

Produced a Financial Crisis That Was More Severe Than the Previous Crashes (With Exception of the Great Depression of 1929)," Polytechnic Institute of Leiria Working Paper No. 46.

Demirgüç-Kunt, A., D, Evanoff and G. Kaufman (2011). *The International Financial Crisis: Have the Rules of Finance Changed?* World Scientific Publishing Company.

Demyanyk Y. and O. Van Hemert (2008). "Understanding the Subprime Mortgage Crisis," Federal Reserve Bank of St. Louis Supervisory Policy Analysis Working Papers No. 2007-05.

Dennis, W. (2012). "Small Business, Credit Access, and a Lingering Recession," NFIB Research Foundation.

Díaz, A. and M. Luengo-Prado (2010). "The Wealth Distribution with Durable Goods," *International Economic Review*, vol. 51, iss. 1, pp. 143-170.

Disney, R., S. Bridges and J. Gathergood (2006). "Housing Wealth and Household Indebtedness: Is there a Household 'Financial Accelerator'?" Paper presented at Bank of Spain Conference on Household Finances and Housing Wealth, April 2007.

Duca, J., J. Muellbauer and A. Murphy (2010). "Housing Markets and the Financial Crisis of 2007–2009: Lessons for the Future," *Journal of Financial Stability*, vol. 6, iss. 4, pp. 203–217.

Duca, J., J. Muellbauer and A. Murphy (2009a). "Credit Constraints and House Prices: Making Sense of the U.S. Experience," Centre for Economic Policy Research Discussion Paper No. 8360.

Duca, J., J. Muellbauer and A. Murphy (2009b). "The Financial Crisis and Consumption," Oxford University Working Paper.

Duke, E. (2012). "Economic Developments, Risks to the Outlook, and Housing Market Policies," Federal Reserve Board Speech at the Virginia Bankers Association/Virginia Chamber of Commerce 2012 Financial Forecast, Richmond, Virginia.

Ellis, L. (2010). "The Housing Meltdown: Why Did It Happen in the United States?" *International Real Estate Review*, vol 13, iss. 3, pp. 351-394.

Favilukis, J., S. Ludvigson and S. Van Nieuwerburgh (2009). "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium," Unpublished paper, London School of Economics.

Fernández-Villaverde, J. (2010) "Fiscal Policy in a Model with Financial Frictions," *American Economic Review: Papers and Proceedings*, vol. 100, iss. 2, pp. 1–9.

Fisher, J. (2007). "Why Does Household Investment Lead Business Investment over the Business Cycle?" *Journal of Political Economy*, vol. 115, iss. 1, pp. 141-168.

Foote, C., K. Gerardi, L. Goette and P. Willen (2008). "Just the Facts: An Initial Analysis of Subprime's Role in the Housing Crisis," *Journal of Housing Economics*, vol. 17, iss. 4, pp. 291–305.

Geanakoplos, J. (2010). "The Leverage Cycle," Cowles Foundation Discussion Papers No. 1715R.

Gertler, M., and N. Kiyotaki (2010). "Financial Intermediation and Credit Policy in Business Cycle Analysis," New York University Working Paper.

Gervais, M. (2002). "Housing Taxation and Capital Accumulation," *Journal of Monetary Economics*, vol. 49, iss. 7, pp. 1461-1489.

Glick, R. and K. Lansing (2010). "Global Household Leverage, House Prices, and Consumption," *RBSF Economic Letter*, Federal Reserve Bank of San Francisco, iss. January.

Hirakata, N., N. Sudo and K. Ueda (2009). "Chained Credit Contracts and Financial Accelerators," IMES Discussion Paper 2009-E-30, Institute for Monetary and Economic Studies, Bank of Japan.

Iacoviello, M. (2005). "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle," *American Economic Review*, vol. 95, iss. 3, pp. 739-764.

Iacoviello, M. and S. Neri (2010). "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics*, vol. 2, iss. 2, pp.125-64.

Johnson, K. and G. Li (2011). "Are Adjustable-Rate Mortgage Borrowers Borrowing Constrained?" Federal Reserve Board Series on Finance and Economics Discussion Paper No. 2011-21.

Kim, Y. and A. Isaac (2010). "The Macrodynamics of Household Debt," Trinity College, Department of Economics Working Paper No. 1010.

Kiyotaki, N., A. Michaelides, and K. Nikolov (2008). "Winners and Losers in Housing Markets," Unpublished paper, London School of Economics.

Kiyotaki, N. and J. Moore (1997). "Credit Cycles," *Journal of Political Economy,* vol. 105, iss. 2, pp. 211-248.

Kuralbayeva, K. and D. Vines (2007). "The Role of the Housing Sector in the Transmission of External Shocks in a Small Open Economy: A Financial Accelerator Framework," Working Paper, Department of Economics, University of Oxford.

Laeven, L., D. Igan and G. Dell'Ariccia (2008). "Credit Booms and Lending Standards: Evidence from the Subprime Mortgage Market," International Monetary Fund Working Papers No. 08/106.

Luk. S, and D. Vines (2011). “Financial-Friction Macroeconomics with Highly Leveraged Financial Institutions,” Centre for Economic Policy Research Discussion Paper No. 8576.

Madigan, K. (2012) “Housing Is Still ‘Shadowed’ by Excess Supply,” *Wall Street Journal* March 21, 2012.

Mian, A. and A. Sufi (2009a). "The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis,” *Quarterly Journal of Economics*, vol. 124, iss. 4, pp. 1449-1496.

Mian, A. and A. Sufi (2009b). "Household Leverage and the Recession of 2007 to 2009,” University of Chicago, Booth School of Business Working Paper.

Mian A. and A. Sufi (2010). “Household Leverage and the Recession of 2007 to 2009,” National Bureau of Economic Research Working Paper No. 15896.

Midrigan, V. and T. Philippon (2011). “Household Leverage and the Recession,” Centre for Economic Policy Research Discussion Paper No. 8381.

Muellbauer, J. (2007). “Housing, Credit and Consumer Expenditure,” *Proceedings*, Federal Reserve Bank of Kansas City, pp. 267- 334.

Muellbauer, J. (2010). “Household Decisions, Credit Markets and the Macroeconomy: Implications for the Design of Central Bank Models,” Bank for International Settlements Working Paper No. 306.

Muellbauer, J. and A. Murphy (1997). “Booms and Busts in the UK Housing Market,” *Economic Journal*, vol. 107, iss. 445, pp. 1701–1727.

Muellbauer, J. and A. Murphy (1990). “Is the UK Balance of Payments Sustainable?” *Economic Policy,* vol. 11, pp. 345–383.

Nelson, A. (2011). “The New California Dream: How Demographic and Economic Trends May Shape the Housing Market a Land Use Scenario for 2020 and 2035,” Urban Land Institute Report.

Ortalo-Magné, F. and S. Rady (2006). “Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints,” *Review of Economic Studies*, vol. 73, iss. 2, pp. 459-485.

Pennington-Cross, A. and G. Ho (2010). “The Termination of Subprime Hybrid and Fixed-Rate Mortgages,” *Real Estate Economics*, vol. 38, iss. 3, pp. 399-426.

Peterson, B. (2006). “Idiosyncratic Uncertainty, Individual Uncertainty and Illiquidity: Housing and the Business Cycle,” Indiana University, mimeo.

Piazzesi, M. and M. Schneider (2009). “Momentum Traders in the Housing Market: Survey Evidence and a Search Model,” National Bureau of Economic Research Working Paper No. 14669.

Ríos-Rull, J. and V. Sánchez-Marcos (2009). “Aggregate Shocks and House Price Fluctuations,” Paper presented at the Conference on Housing, Financial Assets and the Economy, London School of Economics, May 2009.

Scheinkman, J. and W. Xiong (2003). “Overconfidence and Speculative Bubbles,” *Journal of Political Economy*, vol. 111, iss. 6, pp. 1183-1219.

Sengupta, R. and Y. Tam (2010). “Why HARM the Subprime Borrower?” *The Regional Economist*, Federal Reserve Bank of St. Louis, iss. April, pp. 21-22.

Townsend, R. M. (1979). “Optimal Contracts and Competitive Markets with Costly State Verification,” *Journal of Economic Theory*, vol. 21, pp. 265-93.

Vines, D. (2011). “The Macroprudential Quandary: A Review of Balancing the Banks: Global Lessons from the Financial Crisis, by Dewatripont, M., J-C. Rochet, and J. Tirole,” *Economic Journal*.