(Crowd)funding Innovation *

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Abstract

Financing through crowdfunding, where future consumers fund production, is growing rapidly. We derive the optimal crowdfunding contract and show that it serves as a price-discrimination mechanism by forcing pivotal consumers to pay a premium above the expected future spot price, thus increasing the entrepreneur's profits even in the absence of financing constraints. This type of contract always increases production and may increase welfare. Interestingly, entrepreneurs respond to an increase in financing constraints by further increasing production. In such a setting reducing the cost of capital, a common policy instrument used for spurring innovation, may unintentionally reduce welfare.

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1 Introduction

Firms have historically responded to financing constraints with financial innovations in the form of new security design and financing arrangements (Stein, 2003; Tufano, 2003; Duffie and Rahi, 1995). Spurred by developments in information technology, a recent financing innovation that is increasingly deployed by start-ups is *crowdfunding,* which differs from typical investor participation through financial securities in striking ways. Investors in successful crowdfunding ventures often do not hold a financial security that gives ownership rights on the cash-flows of the firms. Rather, the investors have a claim on the yet to be developed product. The return to investors, therefore, depends on their subjective expected utility from the product. Thus, unlike the standard paradigm that separates optimal financing from the product market environment, investment financing via certain forms of crowdfunding is directly integrated with the real side of the firm, namely, the demand structure for its products. This raises fundamentally new issues regarding the optimal crowdfunding strategy and its relation to the real and financial environment of the firm.

However, while financing through crowdfunding is growing rapidly, there is little available analysis of crowdfunding and its implications for investment and production efficiency. In this paper, we examine these issues. We develop a framework for analyzing optimal crowdfunding strategy while incorporating salient aspects of crowdfunding platforms. Specifically, we consider the problem of a monopolist who wishes to introduce a new product that requires an initial investment that can be either financed externally by selling securities on future cash flows, or by approaching consumers through a crowdfunding platform. A unique feature of the typical crowdfunding platform is the ability of the firm to commit to a *pre-sale price* and *funding target.* In particular, consumers purchase the good in advance ahead of production at the pre-sale price and the monopolist commits to execute the project only if a funding target is met.[1] Thus, the pre-sale price and the funding target effectively determine a contract by the entrepreneur.

Consumers in our model realistically derive heterogenous utility from the project and choose

[1] The two largest crowdfunding platforms which operate in the rewards based category, supporting project financing via pre-sale contracts, endorse an all-or-nothing mechanism which is similar to the one described in this paper. Moreover, on Kickstarter, the largest platform of this type, this is the only contract available: the project owner must select a time frame (typically 30-60 days) and a financing goal. If the financing goal is not reached within the allocated time frame the financing campaign is cancelled.

between purchasing the good at the pre-sale price (through the crowdfunding platform) and purchasing the good in the spot market once, and if, production takes place. In addition to considering the price of the product, consumers therefore take into account their impact on the probability that the good will be produced. In sum, consumers must consider whether they are *pivotal* in the successful financing and initiation of production. Following the voting literature (Palfrey and Rosenthal, 1983; Ledyard, 1984), a consumer is pivotal in a particular contract when her utility from the product exceeds the pre-sale price and when the funding target is not met (in equilibrium) in the absence of her participation. Thus, the trade-off for the pivotal consumer is not whether to purchase the good early at the pre-sale price relative to expected future spot price, but rather whether to purchase the good at the pre-sale price or forego this opportunity. Recent empirical literature on crowdfunding has shown that consumers react to the targeted goal set by the crowdfunding contract. Mollick (2014) identifies that the typical project to succeed on Kickstarter does so by achieving the target goal by a thin margin. Kuppuswamy and Bayus (2014) show that as project campaigns get closer to their designated term consumers increase their propensity to participate.

Therefore, a crowdfunding contract determines a basic trade-offs for the consumers with respect to being pivotal or not. Our analysis addresses some interesting questions that emerge from this. Specifically: What is the optimal crowdfunding contract? Under what conditions will firms choose crowdfunding over conventional funding? How does such a mechanism impact product innovation and welfare ? Can the platform also benefit firms with low external funding costs? Can a government intervention policy that relaxes external financing costs to start-up firms have the unintended consequences of reducing production and welfare?

We first analyze the optimal crowdfunding contract when the monopolist has sufficient funds to finance the required investment balance not provided for by the crowdfunding contract. We then examine the role of financing constraints. Our main insight here is that the crowdfunding contract, and in particular the funding target, serve as a price discrimination mechanism: pivotal consumers are willing to pay a premium above the expected future spot price in order to ensure execution of the project. Therefore, the monopolist faces a trade-off: On the one hand, increasing the pre-sale price reduces the set of consumers that will participate in the crowdfunding stage; on the other hand, higher pre-sale price may increase the revenues from all consumers that participate. The optimal pre-sale contract balances these forces such that the monopolist always benefits from using the platform. That is, even in the absence of financing constraints, the monopolist benefits from funding her investment via the crowdfunding platform.

Besides increasing the profits of the monopolist, crowdfunding increases expected production and may increase total welfare or surplus. Moreover, an optimal crowdfunding contract leads to production in cases where there would be no production otherwise. Crowdfunding via product pre-sale always increases welfare when the marginal per unit cost of production is sufficiently low — a common feature of firms that utilize the crowdfunding platform in practice, such as hi-technology firms.

The optimal crowdfunding contract used by a deep-pocket monopolist sometimes sets the funding level below that required for investment. This can impose a cost on a financially-constrained monopolist. Such financing constraints may stem from transaction costs, lack of bargaining power, information asymmetries, or lack of reputation — all of which are challenges faced by entrepreneurs. Therefore, we also explore the optimal crowdfunding contract when it is costly to raise funds from financial markets.

It turns out that the financially-constrained monopolist optimally produces *higher* quantities and relies more on the crowdfunding platform as the cost of external financing increases or as her endowment of internal funds falls. In particular, the monopolist will lower the pre-sale price and increase the quantity of goods sold at the pre-sale stage in response to her financing constraint. While higher financing constraints reduce the profit of the monopolist they lower prices in both the pre-sale and spot markets. Overall, higher external financing costs increase total surplus as long as the monopolist finds it profitable to invest and production takes place. From a policy perspective, this suggests that government policies aimed at relaxing financing constraints for firms to spur innovation might in fact lead to lower welfare as they increase prices in the product market and potentially lower production. That is, there may be unintentional consequences for such government policies: Under certain conditions, when it is feasible to finance projects through pre-sale crowdfunding, welfare is increasing in the cost of capital and there is no social incentive to reduce the cost of capital for such firms.

To our knowledge, this study is among the first to analyze the design of optimal crowdfunding contracts, their implications for investment and production efficiency, and their interaction with financing constraints.[2] Our study is related to the literature on intertemporal price discrimination by a durable goods monopolist. The Coase conjecture (Coase (1972)) famously asserts that the

[2] Belleflamme et. al. (2014) and Sahn et al. (2014) incorporate explicit crowdfunding community benefits that drive consumer participation in the crowd funding stage, they compare the incentives of a monopolist to select equity over pre-sale crowdfunding and argue that crowdfunding would be preferred by a monopolist when crowdfunders experience additional community benefits from participation in the crowdfunding campaign per-se.

monopolist's time-consistent policy is to saturate the residual market at every point in time so that there is no effective intertemporal price discrimination. In contrast, the "Pacman conjecture" (Bagnoli et al. (1989)) holds that the infra-marginal consumers will realize that prices will only drop after they have exited the market, thereby allowing the monopolist to perfectly price discriminate through intertemporal market segmentation. Fehr and Kuhn (1995) show that the Coase conjecture will generally hold for a continuum of consumers — when buyers have a negligible effect on the seller's payoffs — and a finite set of prices. However, the Pacman outcome will hold when buyers are finite (and have a nonnegligible effect on the seller's payoffs) and there is a continuum of prices because the seller can induce infra-marginal consumers to pay a premium for early consumption. In our model, there is a continuum of consumers and prices; nevertheless, the monopolist is able to price discriminate because the pivotal consumers are willing to pay a premium to ensure the successful future production of the good, a consideration that is absent in the durable goods monopoly literature. In effect, we show the significant consequences of investment (or production) uncertainty on the monopolist's ability to intertemporally price discriminate.

Our analysis is also related to the literature on mechanisms that induce truthful revelation of private preferences for public goods and allow the socially efficient provision of public goods (Clarke (1971), Groves (1973), Green and Laffont (1977)). In such a mechanism, the social planner commits to a transfer (or tax) rule that is a function of self reported valuations where the tax of each consumer is also determined by the reports of others. In equilibrium such a mechanism can be designed to induce truthful reporting and optimal allocation. In our context, pivotal consumers pay a price premium in the pre-sale stage to ensure the provision of the (private) good ex post. Our analysis is also related to the literature on provision point mechanisms designed to elicit efficient private provisioning of a public good. Such mechanisms also make use of a target funding threshold which may determine the existence of the public good (Bagnoli and Lipman (1989)). A significant body of experimental literature studies the outcome and validity of provision point mechanisms and has shown that subjects react to the provision point challenge (Falkingern et al. (2000), Rondeau et. al. (1999)). However, there are major differences between the crowdfunding pre-sale contract and the pivotal mechanisms for the provision of public goods. In the latter, issues of intertemporal price patterns and market segmentation do not arise by definition. In contrast, the pre-sale contract, designed to maximize profit, is an intertemporal price-discrimination mechanism for a producer of private goods with market power, leading to a novel depiction of the role of pivotal consumers in financing product innovations. In addition, there are substantive issues in the design of pivotal pre-sale contracts when the producer is financially-constrained; such issues are, of course, not relevant in the public goods literature.

In addition, our study is related to the literature that explores the interaction between firms' financing and real investment and production decisions. It has been argued that information asymmetry and agency problems impose additional costs on capital providers and lead to under investment and capital rationing (Stiglitz and Weiss (1981), Myers and Majluf (1984), Stein (2003)). We add to this literature by examining the role of external financing constraints when firms have the alternative of funding through their consumers in the form of product pre-sale. While financing constraints can also lead to under-investment in our setting (when very extreme) we identify a channel through which higher financing costs actually leads to more production and increased total surplus.

Finally, our analysis is also related to the literature that explores the implications of capital structure and limited liability for the aggressiveness of firms in the product market (Brander and Lewis (1986), Maksimovic (1990)) and predation (Chevalier and Scharfstein (1996), Bolton and Schafstein (1990)).[3] For example, Brander and Lewis (1986) show that when financial and output decisions follow in a sequence, limited liability may commit a leveraged firm to be more aggressive. Bolton and Scharfstein (1990) show that by pursuing an aggressive strategy in the product market the rival firm can increase the probability of low profits and lead to higher probability of liquidation. It has also been shown that leverage might lead managers to alter their production levels and pricing behavior. Shleifer and Vishny (1992) have considered how capital structure affects the market for assets leading to an asymmetric market equilibrium outcome in which some firms remain unlevered in order to purchase assets of distress firms in the future. The implications of crowdfunding we emphasize here are quite different as they do not require competition and are relevant for firms that are producing innovative products that are likely to be (by definition) new to the market place.

We organize the remaining paper as follows. Section 2 describes the basic model. In section 3 we analyze the optimal pre-sale crowdfunding contract by a financially unconstrained monopolist, while in section 4 we introduce financing constraints. Section 5 considers the pre-sale contract commitment and introduces probabilistic execution. Section 6 concludes.

[3] Zechner (1996) surveys this literature

2 The Model

Consider a monopolist firm contemplating the future production of a good with marginal production cost of $c \in [0, \bar{c}]$ and fixed setup or development cost of $I > 0$. That is, in order to undertake the project, the monopolist must allocate or raise I, which must be invested prior to production. We will first assume that the monopolist has sufficient funds of her own to internally finance the initial investment required for development of the product. Subsequently, we will consider a financially constrained monopolist.

There are three time periods in the model, $t = 0, 1, 2$. The good is produced and delivered to consumers at $t = 2$. The demand for the product is derived from a continuum of consumers with measure 1, with heterogeneous demand-intensity for product quality. Specifically, consumer-types are represented by $\theta_i \tilde{} U[0, 1]$ with utility $U_i = \theta_i \varphi - p$ from purchasing the good at cost p and quality $\varphi \in \{\varphi_\ell, \varphi_h\}$, $0 < \varphi_\ell < \varphi_h$. The product quality is realized at the beginning of $t = 2$, with common prior beliefs $P(\varphi = \varphi_h) = \lambda \in [0, 1]$. Meanwhile, pre-sale interactions occur at $t = 1$, with the expected product quality $E(\varphi) = \lambda\varphi_h + (1 - \lambda)\varphi_\ell$. Without loss of generality, for notational convenience, we normalize buyers' subjective discount rates on future consumption and the per-period return on their capital to zero. We focus our analysis on products where investment in innovation is significant while unit production costs are low compared to the expected product utility.

2.1 CrowdFunding Via Product Pre-Sale

Suppose a monopolist can finance all or part of the initial investment by pre-sale of products at $t = 1$. The crowdfunding or pre-sale contract, set at time $t = 0$, is defined by the pair $\langle p_1, F_c \rangle \in R^2_+$ and specifies the pre-sale price p_1 and the total *investment target* F_c, as follows. The pre-sale price represents the price at which consumers can purchase the good during the pre-sale stage. As we will discuss below, consumers alternatively may have the option to purchase the good in the spot market at $t = 2$ if investment takes place.

The monopolist commits to develop and deliver the good at the pre-sale price p_1 only if the investment target is met, i.e. total proceeds from pre-sale exceed the investment target F_c. That is, the crowdfunding mechanism enables firms to commit not to invest and develop the product if the investment target is not met. This ability of monopolist to commit to a pre-sale contract plays an important role in our model. Later, in section 5, we discuss the institutional background, describe conditions under which this commitment emerges endogenously, and also explicitly relax the full-commitment assumption itself.

If the investment target F_c is not raised during the pre-sale stage, then funding *fails*, i.e. no production takes place and the game ends. It is useful to define the set of consumers that have purchased in the pre-sale market at $t = 1$ as Θ^1. Thus, the remaining set of consumers that participate in the product market at $t = 2$ and can potentially purchase the good is $\Theta^2 = [0, 1] - \Theta^1$ (as discussed below). By definition, funding succeeds if and only if,

$$\text{Funding Succeeds} \Leftrightarrow p_1 \left(\int_{\theta \in \Theta^1} d\theta \right) \geq F_c \tag{1}$$

Since a consumer's utility from the product is increasing in her type θ, we assume that in any equilibrium the set of consumers that participate in the pre-sale stage belongs to the upper-tailed interval of types. That is, $\Theta^1 \equiv [\theta_1, 1]$ for some $\theta_1 \in [0, 1]$. This implies that the success of funding is equivalent to $p_1 (1 - \theta_1) \geq F_c$. While considering the unconstrained monopolist, we do not require any relation between F_c and I: thus, when $F_c > I$, the project can be fully financed via the product pre-sale contract; but, if $F_c < I$, it is implicitly assumed for now that the unconstrained monopolist can simply finance the balance via her own capital.

We define a consumer of type θ' as *pivotal* to the success of funding, with respect to the group of participants $\Theta^1 \equiv [\theta_1, 1]$, if non-participation of a neighborhood of measure ε of types around θ' will lead to failure of funding, for any $\varepsilon > 0$. Formally, let $\mathcal{A}$ =1 ($\mathcal{A} = 0$) represent whether a given type is pivotal or not to pre-sale contract (p_1, F_c) given θ_1:

$$\mathcal{A}(\theta'|\theta_1) = \begin{cases} 1 \text{ if } p_1 (1 - \theta_1) = F_c \text{ and } \theta' > \theta_1 \\ 0 \text{ otherwise.} \end{cases} \tag{2}$$

Intuitively, the above definition of a pivotal consumer captures the notion that the monopolist, by her choice of contract $\langle p_1, F_c \rangle$, can induce participation in the pre-sale market.

2.2 Post Investment Product Market

At $t = 2$, after observing the pre-sale demand Θ^1 and the realization of the product quality φ, the monopolist sets the market price p_2 .

2.3 Time Line

The time-line of the game is as follows:

- Time $t = 0$: The monopolist sets the crowdfunding contract $\langle p_1, F_c \rangle$
- Time $t = 1$: Consumers decide whether to participate in the pre-sale stage or not and demand $\Theta^1 = [\theta_1, 1]$ is determined. If funding succeeds (i.e., the investment target F_c is met) then investment of I takes place. If funding fails, then the game ends.
- Time $t = 2$: (If funding succeeds) The product quality φ is realized and is publicly observable. The monopolist sets price p_2 and sets production to clear the market.

2.4 Equilibrium

An equilibrium consists of a crowdfunding contract $\langle p_1, F_c \rangle$; the set of consumers Θ^1 that purchase the good in the pre-sale stage; and the market prices p_2 in the spot market that are a function of the realized product quality φ and the set of remaining consumers who have not participated in the pre-sale stage. We describe the equilibrium requirements for the consumers and the monopolist at each date, starting from the end of the game tree.

Consumers At $t = 2$: If funding succeeds at $t = 1$, then consumers that have not participated in the pre-sale market i.e., $\theta \notin \Theta^1$, rationally choose to participate in the product market if $\theta\varphi \geq p_2$. At $t = 1$, given contract $\langle p_1, F_c \rangle$, consumers participate in the pre-sale market to maximize their utility while anticipating the possibility that they are pivotal (i.e., if $\mathcal{A}(\theta'|\theta_1) = 1$) and the future expected product prices and quality $\langle p_2, \varphi \rangle$, given their beliefs regarding the pre-sale demand $\Theta^1 = [\theta_1, 1]$. We denote the best-response function of consumers to contract $\langle p_1, F_c \rangle$ by $\theta_1(p_1, F_c)$.

Monopolist At $t = 2$, if funding succeeds, the firm will set price p_2 to maximize profits given realized product quality φ and the residual potential consumer group $\Theta^2 \equiv [0, \theta_1]$. At $t = 0$, the monopolist will set the pre-sale contract $\langle p_1, F_c \rangle$ to maximize profits from both periods, given consumers' participation as determined by $\theta_1(p_1, F_c)$.

2.5 Benchmark

Consider the benchmark without pre-sale, which is the solution to the standard monopolist's profit maximization problem. That is, production follows investment of I and realization of the state of product quality. The monopoly produces quantity $q(\varphi)$ at price $p(\varphi)$ to solve:

$$\begin{aligned} \pi^{bm} &\equiv \max_{p} q(p-c) \\ \text{s.t. } q &= \left(1-\frac{p}{\varphi}\right)^{+} \end{aligned} \tag{3}$$

Note that the marginal consumer to purchase the good satisfies $\theta\varphi = p$. Thus, the benchmark outcome is:

$$p^{bm}(\varphi) = \frac{\varphi + c}{2}, q^{bm}(\varphi) = \frac{\varphi - c}{2\varphi} \Rightarrow \pi^{bm}(\varphi) = \frac{(\varphi - c)^2}{4\varphi}. \tag{4}$$

Prior to realization of φ the expected monopole equilibrium values are:

$$E[p^{bm}] = \frac{E(\varphi) + c}{2}, \tag{5}$$

$$E[q^{bm}] = \frac{1}{2} - \frac{c}{2}\left(\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_\ell}\right), \tag{6}$$

$$E[\pi^{bm}] = \frac{E(\varphi) - 2c}{4} + \frac{c^2}{4}\left(\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_\ell}\right). \tag{7}$$

The self-financing monopolist will undertake the project provided $I \leq E[\pi^{bm}]$. This result holds for any external financing which is executed under market conditions decoupled from the product market performance of the monopolist.

3 Equilibrium under a Crowdfunding Pre-Sale Market

To solve the equilibrium level of production when pre-sale is possible, we apply backwards induction and start by analyzing the product market equilibrium at $t = 2$, given a period $t = 1$ participation $\Theta^1 = [\theta_1, 1]$, and the realization of quality φ. The marginal consumer that purchases the good in the spot market given price p_2 and quality φ satisfies $\theta = \frac{p_2}{\varphi}$, provided that $\theta_1 > \frac{p_2}{\varphi}$. The monopolist chooses quantity and price as follows (where we denote the profit in the spot market when pre-sale is possible as π_2^{ps}):

$$\begin{aligned} \pi_2^{ps}(\theta_1, \varphi) &= \max_{p_2} q_2(p_2 - c) \\ \text{s.t., } q_2 &= \left(\theta_1 - \frac{p_2}{\varphi}\right)^{+} \end{aligned} \tag{8}$$

Thus, the spot product market outcome given the marginal consumer in the pre-sale market, θ_1, is:

$$p_2^{ps}(\theta_1) = \frac{\theta_1\varphi + c}{2},\ q_2^{ps}(\theta_1) = \frac{\theta_1\varphi - c}{2\varphi},\ \pi_2^{ps}(\theta_1) = \frac{(\theta_1\varphi - c)^2}{4\varphi}, \text{ provided that } \theta_1\varphi > c. \tag{9}$$

Alternatively, if $\theta_1\varphi \leq c$, then there is no production in the spot product market. We wish to focus on the case were the spot product market opens even for $\varphi = \varphi_\ell$. This assumption is applicable to digital products as well as technology products where the R&D and initial investments are significant but per unit production costs are low. To this end, we assume a sufficiently small marginal cost of production to insure opening of the spot product market, in particular we restrict attention to[4]

$$\text{Assumption } \mathbf{A1}\text{: } c < \frac{\varphi_\ell}{2} \tag{10}$$

Given the equilibrium in the sub-game of the spot-product market (and assuming that it opens in both states), the monopolist chooses the pre-sale contract $\langle p_1, F_c \rangle$ to maximize expected profits from both periods while taking into account the response of consumer $\theta_1(p_1, F_c)$. Thus, to establish the equilibrium pre-sale contract, we need to first derive the aforementioned response function of consumers.

In particular, given beliefs on θ_1, it follows from (9) that any consumer of type θ' who is not pivotal, i.e., for which $\mathcal{A}(\theta'|\theta_1) = 0$, will participate in the pre-sale only if the necessary conditions (11) and (12) apply[5]:

$$p_1 < \theta' E(\varphi) \tag{11}$$

$$p_1 < E[p_2] = \frac{\theta_1 E(\varphi) + c}{2} \tag{12}$$

In contrast to the above, any consumer of type θ' who is pivotal with respect to θ_1, i.e., for which $\mathcal{A}(\theta'|\theta_1) = 1$, expects that his participation is crucial to the existence of the future product

[4] Subsequently, we establish that the spot market opens in equilibrium in both states of the world as long as assumption A1 holds.

[5] Note that (11) and (12) are sufficient period 1 participation conditions for consumers who are of type $\theta' > \frac{\theta_1\varphi_l + c}{2\varphi_l}$ as such consumers will participate in the second period spot market regardless of the realization of φ. For consumers of type $\frac{\theta_1\varphi_l + c}{2\varphi_l} > \theta' > \frac{\theta_1\varphi_h + c}{2\varphi_h}$, who will only participate in the spot market following a high realization, replacing (12) with the condition $p_1 < \lambda\left(\frac{\theta_1\varphi_h + c}{2}\right) + \theta'\left((1-\lambda)\varphi_\ell\right)$ will constitute together with condition (11) a set of sufficient conditions for period 1 participation. Also note that $\lambda\left(\frac{\theta_1\varphi_h + c}{2}\right) + \theta'\left((1-\lambda)\varphi_\ell\right) < E[p_2]$.

market and will participate whenever condition (11) is satisfied regardless of his second period price expectations. This property of pivotal consumer participation shall allow the monopolist to design a richer set of contracts where $p_1 > E[p_2]$ and the participating consumers are pivotal.

First, we look for an equilibrium in which participating consumers in the pre-sale market are pivotal and the marginal participating consumer earns zero rents (we will later show that this is also the equilibrium outcome which provides maximum benefit to the monopolist). We call this type of contract a *pivotal contract.* Thus, the set of feasible pre-sale pivotal contracts reduces to the form:

$$\langle p_1, F_c \rangle \equiv \left\langle p_1, p_1 \left(1 - \frac{p_1}{E(\varphi)}\right)\right\rangle, \text{ for } p_1 \in (0, E(\varphi))\,. \tag{13}$$

In such a contract, all of the consumers who derive positive expected utility given p_1 are pivotal, i.e. $\mathcal{A}(\theta'|\theta_1) = 1$ for all $\theta' \geq \theta_1$, where the marginal consumer earns zero rents as in:

$$\theta_1(p_1, F_c) \equiv \theta_1(p_1) = \frac{p_1}{E(\varphi)}. \tag{14}$$

Of course, the sum raised in the pre-sale stage is $p_1(1 - \theta_1(p_1)) = F_c$. The residual demand in the spot market is therefore generated by consumers of type $\theta \in (0, \theta_1(p_1))$ and the optimal pivotal pre-sale contract set by the monopolist maximizes the sum of first period profit, $(p_1 - c)\left(1 - \frac{p_1}{E(\varphi)}\right)$, and the second period profits, $E\left(\pi_2^{ps}(\theta_1(p_1), \varphi)\right)$, where π_2^{ps} is given by (8) and $\theta_1(p_1)$ in the pivotal pre-sale contract is given by (14). Formally, the optimal pivotal pre-sale contract is given by the solution to the following problem:

$$\begin{aligned} \pi^{ps} \quad \equiv \quad & \max_{p_1 \in (0, E(\varphi))} (p_1 - c)\left(1 - \theta_1(p_1)\right) + E\left(\pi_2^{ps}(\theta_1(p_1), \varphi)\right) \\ & \text{s.t. } \theta_1(p_1) = \frac{p_1}{E(\varphi)}, \text{ and } \pi_2^{ps}(\theta_1(p_1), \varphi) \text{ is given by (8)} \end{aligned} \tag{15}$$

Note that the unconstrained monopolist will undertake the project provided that $I \leq \pi^{ps}$.

Proposition 1 ***:[Optimal Pre-Sale Contract]*** *Under assumption A1, the optimal pre-sale crowd-funding contract is given by (16), where the marginal participating consumer is* $\theta_1 = \frac{2E(\varphi)+c}{3E(\varphi)}$*, the spot market always opens and prices are set according to (9), and the expected future spot price is* *lower than the pre-sale price* $E[p_2^{ps}] = \frac{p_1^{ps}+c}{2} < p_1^{ps}$.

$$\langle p_1^{ps}, F_c^{ps} \rangle = \left\langle \frac{2E(\varphi)+c}{3}, \left(\frac{2E(\varphi)+c}{3}\right)\left(\frac{E(\varphi)-c}{3E(\varphi)}\right) \right\rangle \tag{16}$$

Effectively, the introduction of a pre-sale market allows the monopolist to price discriminate (see, e.g., Varian (1989)). Intuitively, the monopolist is able to extract higher surplus from the consumers with high intrinsic valuations of product quality (or demand-intensity) — the pivotal consumers — by making them pay a premium ex ante (or prior to production) to ensure the execution of the project. In this way, the monopolist raises its profits relative to the benchmark case. An unconstrained monopolist can always choose to finance the required investment utilizing other resources thus, one can conclude that the expected profits of an unconstrained monopolist are always higher under a pre-sale mechanism:

Proposition 2 ***[Profits]*** *Under assumption A1, the expected level of profits of the monopolist with a pre-sale crowdfunding market exceeds that of the monopolist when a pre-sale crowdfunding market does not exist. Formally, the gap in profitability is given by:*

$$\pi^{ps} - E[\pi^{bm}] = \frac{(E(\varphi)-c)^2}{12E(\varphi)}. \tag{17}$$

Denote by W^{bm} and W^{ps} as the total welfare levels in equilibrium, that are generated under the benchmark case (without a pre-sale market) and the crowdfunded pre-sale market, respectively. We explore next the production and welfare implications of the optimal pre-sale contract:

Proposition 3 ***[Welfare]*** *Under assumption A1, the levels of total expected production by the monopolist (or total expected welfare) is higher with pre-sale crowdfunding relative to the benchmark case; formally,* $E[W^{ps}] - E[W^{bm}] = \frac{5(E(\varphi)-c)^2}{72E(\varphi)} > 0$. *Moreover, some projects that are not feasible under the benchmark are executed with a pre-sale crowdfunding contract, i.e., projects where the required investment satisfies the condition:* $E[\pi^{bm}] < I < E[\pi^{ps}]$.

To see this, consider first a potential project such that $I < E[\pi^{bm}] < E[\pi^{ps}]$. This project is feasible under the benchmark, and consequently must also be feasible with a pre-sale market. Note that the expected second period price under pre-sale is lower than the benchmark price: $E[p_2^{bm}] - E[p_2^{ps}] = \frac{E(\varphi)-c}{6}$ thus the total quantity produced under a pre-sale contract is higher. Under the benchmark, the expected level of production (after investment) is $E[q^{bm}] = E\left[\frac{\varphi-c}{2\varphi}\right]$, as described in equation (6). Meanwhile, under the optimal pre-sale contract, total production is given by $E[q^{ps}] = q_1^{ps} + E[q_2^{ps}]$, as defined in equations (30) and (32), respectively. By comparing the two, we obtain explicitly the increase in production with the introduction of the pre-sale market:

$$E[q^{ps}] - E[q^{bm}] = \frac{E(\varphi) - c}{6E(\varphi)}. \tag{18}$$

Under assumption *A1,* this higher level of expected production implies an increase in expected welfare because $c < \varphi_\ell$.

Now, consider the case where $E[\pi^{bm}] < I < E[\pi^{ps}]$; in this case, the project is only profitable for the monopolist under the existence of a pre-sale market. In particular, the pre-sale mechanism allows the monopolist to price discriminate between the high-demand intensity customers and the remaining market segment that has relatively low utility value from the product. This mechanism enables the monopolist to extract additional surplus from consumers, increasing expected profit and enabling the execution of the project. This price discrimination ability also allows the monopolist to sell the product for a period-2 price that is lower than the monopolist's market price under regular financing, as detailed in Proposition 1.

Corollary 1 ***[Welfare, Expected Product Quality, and Cost of Production]*** *Under assumption A1, the improvement in welfare is increasing in the expected quality of the product and is decreasing in the marginal cost of production.*

In sum, the optimal pre-sale contract allows the monopolist to induce the pivotal (high valuation) consumers to reveal themselves through their willingness-to-pay a premium to ensure the execution of the project. In this fashion, the pre-sale contract is similar to the Groves-Clarke mechanisms that induce dominant strategy incentive compatible revelation of private-value preferences for public goods and the Bagnoli and Lipman (1989) provision point mechanism used to elicit the private provisioning of a public good utilizing a target threshold. In our context, pivotal consumers react to the pivotal mechanism and pay a price premium in the pre-sale stage to ensure the provision of the (private) good ex post. However, there are major differences between the crowdfunding pre-sale contract and the pivotal mechanisms for the provision of public goods, where issues of intertemporal price patterns and market segmentation do not arise by definition. The pre-sale contract is an intertemporal profit maximizing price-discrimination mechanism for a producer of private goods with market power, leading to a novel depiction of the role of pivotal consumers in financing innovation. In the public goods case, the pivotal consumers pay a tax or make a contribution such that, in equilibrium, the socially efficient level of the public good is provided. In our case, the pre-sale contract increases welfare relative to the one-shot monopoly benchmark by allowing consumers to be served who would otherwise be priced out in the monopoly situation. However, production of the good is not at the socially efficient level.

In the context of entrepreneurial innovation, financial constraints play an important role in determining not only the profits but also the feasibility set of potentially innovative projects. In the context of crowdfunding via a pre-sale contract such constraints may also impact the pivotal considerations as well as production and product market welfare. In what follows we evaluate these substantive issues in the design of pre-sale contracts when the producer is financially-constrained.

4 Financially Constrained Monopolist

We now consider the case where the monopolist is financially constrained. Namely, let A be the amount of funds that the monopolist has available for investment in this project, and thus $I - A$ is the amount of funds that need to be raised so as to implement the project. We assume that the cost of capital for the monopolist when raising funds in financial markets (outside of the pre-sale market) is given by $1 + R \geqslant 1$. When $R = 0$, there are effectively no financial constraints for the monopolist and the optimal crowdfunding contract is given by Proposition 1. However, because of financial regulations and transaction costs of financial intermediation, in practice there is no free entry in lending sectors (see, e.g., Demirguc-Kunt et al. (2004)). In particular, banking sectors are generally concentrated, allowing banks' bargaining power to extract some rents from entrepreneurs (Sharpe (1990), Rajan (1992), Rajan and Petersen (1997)).

The payoff to the financially constrained monopolist is determined, as before, by her profits from the pre-sale stage, $(p_1 - c)(1 - \theta_1(p_1))$, and the expected profits from the spot market, $E[\pi_2^{ps}(\theta_1(p_1), \varphi)]$. Now, however, the monopolist also faces the cost of external financing $Max[R(I - A - F_c), 0]$. This expected payoff to the monopolist shall be compared to the investment cost of I when deciding whether to invest or not (when $R = 0$ this payoff coincides with the expected payoff to the unconstrained monopolist). If the model parameters are such that the optimal pre-sale contract $\langle p_1^{ps}, F_c^{ps} \rangle$ derived earlier, in equation (16), results in the monopolist raising sufficient funds in the pre-sale stage, or $F_c^{ps} \geq (I - A)$, then it is still optimal for the financially constrained monopolist to use this contract and exclusively rely on crowdfunding without raising any funds externally; of course, this is true for any R.

But, this need not be the case when $F_c^{ps} < (I - A)$, for example when the level of internal funds A is relatively low. The monopolist can obtain additional funds externally at cost of R and/or by increasing the sums raised using crowdfunding by setting a contract of the form $\left\langle p_1^{fc}, F_c^{fc} \right\rangle$ where $p_1^{fc} < p_1^{ps}$ and $F_c^{fc} > F_c^{ps}$.

When external funds are available but costly, the monopolist may wish to continue and rely on crowdfunding for financing the full sum required, but this may not always be possible: the amount of funds that the monopolist can raise in the pre-sale market is bounded. In particular, the amount raised via crowdfunding, $p_1(1 - \theta_1(p_1))$, is maximized at level $\max_{p_1 \le E(\varphi)} p_1 \left(1 - \frac{p_1}{E(\varphi)}\right) = \frac{E(\varphi)}{4}$ for price $p_1 = \frac{E(\varphi)}{2}$. Thus, when $(I - A) > \frac{E(\varphi)}{4}$ the firm is compelled to raise funds externally.

When $(I - A) \in (F_c^{ps}, E(\varphi)/4)$ and the monopolist determines that it is optimal to raise all of the required financing using the crowdfunding presale contract she can do so by setting the pre-sale price at the level $p_1^{(I-A)}$:

$$p_1^{(I-A)} \equiv \frac{1}{2}\left[E(\varphi) + \sqrt{E^2(\varphi) - 4E(\varphi)(I - A)}\right]. \tag{19}$$

When using both sources of financing the monopolist balances the cost associated with distorting the crowdfunding contract - by lowering the pre-sale price and increasing the investment target away from the unconstrained optimum $\langle p_1^{ps}, F_c^{ps}\rangle$ - against the cost of capital R. Given that investment takes place by the financially constrained monopolist, the optimal crowdfunding contract is the solution to the following program:

$$\begin{aligned} \pi^{fc} \quad \equiv \quad & \max_{p_1 \le E(\varphi)} (p_1 - c)\left(1 - \theta_1(p_1)\right) + E\left[\pi_2^{ps}(\theta_1(p_1), \varphi)\right] - Max\left[R\left(I - A - F_c\right), 0\right] \\ & \text{s.t., } \theta_1(p_1) = \frac{p_1}{E(\varphi)},\ F_c = p_1(1 - \theta_1(p_1)) \text{ and } \pi_2^{ps}(\theta_1(p_1), \varphi) \text{ is given by (8).} \end{aligned} \tag{20}$$

We now present the solution to the optimal crowdfunding contract:

Proposition 4 ***: [Optimal Contract with Financing Constraints]*** *Under assumption A1, there exists a cutoff $\bar{R} > 0$ such that the monopolist will rely solely on crowdfunding when the cost of capital is sufficiently high $\left(\text{i.e. } R > \bar{R}\right)$ and it is feasible to do so (i.e. $(I - A) \in [F_c^{ps}, E(\varphi)/4]$) or when external funds are not required (i.e. $F_c^{ps} \ge (I - A)$); but, will use both methods of financing* *otherwise. The optimal crowdfunding contract is:*

$$\left\langle p_1^{fc}, F_c^{fc} \right\rangle = \begin{cases} \langle p_1^{ps}, F_c^{ps} \rangle & \text{if } (I-A) < F_c^{ps} \\ \left\langle p_1^{(I-A)}, I-A \right\rangle & \text{if } (I-A) \in (F_c^{ps}, E(\varphi)/4) \text{ and } R > \bar{R} \\ \langle p_1(R), F_c(R) \rangle & \text{otherwise} \end{cases}, \tag{21}$$

where, $p_1(R) \equiv \frac{2(1+R)E(\varphi)+c}{3+4R}$, $F_c(R) \equiv \left(\frac{2(1+R)E(\varphi)+c}{(3+4R)^2}\right)\left(\frac{E(\varphi)(1+2R)-c}{E(\varphi)}\right)$, *and* p_1^{ps} *and* $p_1^{(I-A)}$ *are defined by (16) and (19) respectively .*

Of course, the monopolist will execute the project only if $E[\pi^{fc}] \geq I$, under the optimal contract $\left\langle p_1^{fc}, F_c^{fc} \right\rangle$. The marginal participating consumer is now $\theta_1 = \frac{p_1^{cf}}{E(\varphi)}$, $F_c^{fc} = p_1^{fc}(1-\theta_1)$, and the spot market price is $p_2(\varphi) = \frac{\theta_1\varphi + c}{2}$. In order to understand how the optimal contract varies with the cost of external financing and the level of internal funds it is useful to distinguish between some distinct mutually exclusive cases:

Case 1 *The required funds are covered via the unconstrained optimal pre-sale contract, i.e.* $(I - A) \leq F_c^{ps}$.

Case 2 *The required funds can be fully covered via crowdfunding, i.e.* $(I - A) \in (F_c^{ps}, E(\varphi)/4]$, *and external financing costs are high, i.e.* $R > \bar{R}$.

Case 3 *The required funds can be fully covered via crowdfunding, i.e.* $(I - A) \in (F_c^{ps}, E(\varphi)/4]$, *and external financing costs are sufficiently low, i.e.* $R \leq \bar{R}$.

Case 4 *The required funds cannot be fully covered via crowdfunding, i.e.* $(I - A) > E(\varphi)/4$.

In cases 1 & 2 above the monopolist relies on a crowdfunding pre-sale mechanism as a sole source of financing. In case 3 & 4 the monopolist finds it optimal to secure funds using a combination of sums received via the crowdfunding pre-sale contract and external financing. The cost of capital cutoff level $\bar{R}$ used to derive the optimal contract in (21) solves the equality $F_c(\bar{R}) = (I - A)$ and is explicitly given by:

$$\bar{R} = \frac{1}{8}\sqrt{\frac{(E(\varphi)+2c)^2}{E(\varphi)\left(\frac{E(\varphi)}{4} - (I-A)\right)}} - \frac{3}{4}. \tag{22}$$

Note that the value of $\bar{R}$ is increasing with the amount of capital $(I - A)$ that the monopolist is required to raise in order to execute the project. Intuitively, for higher levels of internal funds or smaller investments, relying only on the crowdfunding is more likely.

[Insert Figure 1 here]

Figure 1 illustrates the relation between the cost of external funding R, the required financing $(I - A)$ and the pre-sale price as given by the optimal crowdfunding contract $\left\langle p_1^{fc}, F_c^{fc} \right\rangle$. In particular, one can see that the pre-sale price is weakly decreasing in both the cost of capital and the sums required. Intuitively, when more constrained, the monopolist relies more on the pre-sale market by further reducing the pre-sale price and increasing revenues to finance the investment. Note, a lower price in the pre-sale market is followed by a lower expected price in the spot market.

As suggested by Proposition 4, the pre-sale price also varies with the level of internal funds A (holding the investment level I fixed). When the level of internal funds is high enough, $A > I - F_c^{ps}$, the monopolist sets a high pre-sale price, $p_1^{fc} = p_1^{ps}$, and finances all funds via crowdfunding. For slightly lower levels of internal funds, fully relying on crowdfunding is still optimal but, it requires a lower price, $p_1^{fc} = p_1^{(I-A)}$ (and higher investment target). For further lower levels of internal funds, $A < \bar{A} \equiv I - F_c(R)$, the distortion in the crowdfunding contract is sufficiently costly such that external financing at cost of capital R becomes optimal and the presale price reaches its lowest level, $p_1^{fc} = p_1(R)$.

It follows from the above that the pre-sale price p_1^{fc} is weakly decreasing in the cost of capital and weakly increasing in the level of internal funds. Namely, when utilizing both sources of funds is optimal, a marginal increase in R decreases prices ($p_1(R)$ is decreasing in R); and when relying on crowdfunding to obtain the investment funds, a marginal increase in A increases prices ($p_1^{(I-A)}$ is increasing in A). Thus, it follows from Proposition 4 that the less financially constrained monopolist acts more aggressively in the product market, leading to higher expected prices and lower expected production.

Proposition 5 ***:[Production with Financing Constraints]****Under assumption A1, the expected level of production implied by the optimal crowdfunding contract* $\left\langle p_1^{fc}, F_c^{fc} \right\rangle$ *is weakly increasing [decreasing] in the cost of capital R [level of internal funds A]. However, the monopolist might (but need not) forego investment when the cost of capital is sufficiently high or the level of internal funds is sufficiently low.*

Increased production, induced by lack of internal funds or a high cost of capital, increases welfare as long as production takes place (recall that $c < \varphi_l$, as follows from assumption A1). Figure 2 illustrates this possible non-monotonic relation between the expected level of production, the cost of external funding R and the required financing $(I - A)$.

[Insert Figure 2 here]

To better understand the possible scenarios, it is useful to consider the cross-section plots presented in Figures 3 & 4 while considering the level of internal funds A and the cost of capital R separately. When the unconstrained monopolist raises sufficient funds to cover the financing needs, $F_c^{ps} > (I - A)$, production is not affected by the cost of capital, as demonstrated by Figure 3(a), or by marginal changes in the level of internal funds, as shown by Figure 4 in the aforementioned region. When financing requirements increase such that $F_c^{ps} \leq (I - A)$ the cost of capital has an initial inverse relationship with production, i.e. production increases together with the external cost of capital, as demonstrated in Figures 3(b) and 3(c). The reaction of a further increase in the cost of capital depends on the relation between the financing requirements, the maximum revenue attainable via a crowdfunding pre-sale mechanism and the monopolist profits. When $(I - A) \in (F_c^{ps}, E(\varphi)/4)$ and it is optimal to rely on both sources of funding (as $R < \bar{R}$) the pre-sale price is $p_1^{fc} = p_1(R)$ and any marginal increase in the cost of capital will increase production; however any further increase such that $R \in (\bar{R}, \widehat{R})$ (we define $\widehat{R}$ below) will have no marginal impact on production as in this region $p_1^{fc} = p_1^{(I-A)}$. An example of such a reaction can be viewed in Figure 3(b). When R increases beyond $\widehat{R}$ the project cannot justify the required investment and production is 0, as demonstrated in Figure 3(c).

[Insert Figure 3 here]

Note that while changes in the cost of capital and internal funds impact prices and production, their pronounced impact takes effect under different financing scenarios. The optimal pre-sale price set by a monopolist that utilizes both sources of financing $\left(p_1^{fc} = p_1(R)\right)$ is sensitive to the cost of capital but is not sensitive to the level of internal funds A; however, the price set when financing exclusively through crowdfunding $\left(p_1^{fc} = p_1^{(I-A)}\right)$ is not sensitive to the cost of capital R , but is increasing in the level of internal funds A. These different effects can be seen in Figure 4 where production is unchanged for $A \in (\widehat{A}, \overline{A})$ (we define $\widehat{A}$ below) , while production is decreasing in the level of internal funds for $A \in (\overline{A}, I - F_c^{ps})$. The result that financing constraints may lead to higher production suggests that a higher cost of capital need not slow down economic activity.

We turn now to the investment decision of the monopolist and the definition of the cutoffs $\widehat{A}$, and $\widehat{R}$. In order for the monopolist to execute the project it must be that profits from production exceed the required capital investment (including the cost of external capital) i.e. $\pi^{fc}(I, A, R) \geq I$. For a given level of internal funds A, we define the cutoff cost of capital $\widehat{R}$ above which there is no investment as the solution to $\pi^{fc}(I, A, \widehat{R}) = I$, if such a solution exists, and $\widehat{R} = \infty$, otherwise. Note, $\widehat{R} < \infty$ whenever $I - A > \frac{E(\varphi)}{4}$ (it is not feasible to rely only on crowdfunding). Similarly, for a given cost of capital R, we define the cutoff level of internal funds $\widehat{A}$ below which there is no investment as the solution to $\pi^{fc}(I, \widehat{A}, R) = I$, if such a solution exists, and $\widehat{A} = 0$, otherwise. Figure 3(c) is an illustration of the case $\widehat{R} < \infty$ and Figure 4 is an illustration of the case $\widehat{A} > 0$.

[Insert Figure 4 here]

Government programs designed to support innovation, entrepreneurship and welfare often focus on increasing the availability of funds to productive yet constrained entrepreneurs. This can be done for example by offering business loans at favorable terms relative to the market or offering tax advantages to businesses that meet certain criteria. We show that under a pre-sale crowdfunding mechanism such policies may have an unexpected effect; i.e. a reduction in the cost of capital R or an increase in internal funds A may lower production and welfare.

Corollary 2 *: [Government Policy* *]Under assumption A1, when the monopolist invests and earns strictly positive expected profits under a particular pair $\langle R, A\rangle$; a marginal reduction in the cost of capital R or a marginal increase in the level of internal funds A, weakly increases the expected profitability of the monopolist, weakly increases expected prices, weakly reduces expected production, and therefore weakly decreases expected welfare.*

Recall that in propositions (3) and (2) we show how the existence of a crowdfunding pre-sale mechanism increases monopoly profits, production and welfare for the unconstrained monopolist. It is a direct result of our setup that these propositions also hold for the constrained case. Moreover it is a direct result of Corollary (2) that there exists a robust support such that the positive welfare increase generated by the existence of the pre-sale crowdfunding market is increasing in the cost of capital and is negatively related to the level of internal funds.

5 Evaluating the Pre-Sale Contract Commitment

In our analysis thus far, we have assumed that the monopolist can commit to the pre-sale contract in the sense that if the investment target is not met, production does not take place. In this section, we evaluate the implications of this commitment from a variety of perspectives. We first provide an institutional motivation for the pre-sale contract commitment assumption.

We then describe conditions under which this commitment and the resulting pivotal behavior are endogenous rather than an artifact of an exogenous assumption. We further examine the robustness of our results by extending our analysis and relaxing the full-commitment assumption; in particular, we assume that with some probability the firm can still invest following failure of funding on the crowdfunding platform. Finally, we relate the role of commitment in our model to the literature on intertemporal price discrimination.

5.1 Crowdfunding Institutions

From a practical point of view, most of the crowdfunding platforms which operate in the rewards-based category and support project financing via a pre-sale contract endorse an all or nothing mechanism that is similar to the one described in this paper. Moreover, on Kickstarter, the largest platform of this type, this is the only contract available. A project owner on Kickstarter must select a time-frame (typically 30-60 days) and a financing goal. If the financing goal is not reached within the allocated time-frame, the financing campaign is cancelled. There is empirical evidence that suggests an important role for the investment target. Namely, it is shown by Mollick (2014), while analyzing data from Kickstarter, that the goal attainment histogram describing the percentage of the goal achieved by all campaigns has two modals, the first is at the $0-10\%$ level and the second is at the $100-110\%$ level. That is, the most common campaign that succeeds in raising capital on Kickstarter typically does so by securing financing just above the 100% financing-goal mark. Indiegogo, one of the largest rewards based platforms which facilitates pre-selling of products, allows for entrepreneurs to select into an "All-or-Nothing" mechanism (where the entrepreneur receives the funds only if the financing goal is achieved) or a "Keep it All" mechanism in which they receive all sums pledged regardless of the total sum achieved. Cumming et al. (2015) analyze campaigns on Indiegogo and find that campaigns which have selected the All-or-Nothing mechanism attract more backers and have a significantly higher probability to secure the financing goal. These empirical findings suggest that the existence of a campaign target combined with an all or nothing mechanism has a significant impact on campaign outcome.

5.2 Probabilistic Project Execution and Robustness

Our analysis has so far assumed that the successful completion of the financing campaign is synonymous with the execution of the underlying project. This implies that consumers believe that the entrepreneurs cannot or will not attempt to find alternative funding following a failed financing campaign. Of course, this assumption may not generally hold. Following a failed financing campaign, project owners may obviously attempt alternative sources of capital, including other crowdfunding platforms. However, potential backers who wish to own the product do not know if this is the case, nor do they have a guarantee that they will have access to the product once it is financed on a different platform, possibly in a different geographic location. Alternatively as discussed in proposition (3) it may be that certain projects are only feasible when entrepreneurs are able to price discriminate via a crowdfunding presale contract as profits under external financing decoupled from the product market do not support the required investment. This may be true even when the monopolist is unconstrained.

We can incorporate such situations into our framework by utilizing the concept of probabilistic project execution. Let us assume that consumers have the beliefs that with probability $(1-\alpha)$, $\alpha \in [0,1]$, the project is executed regardless of the crowdfunding campaign. This also implies that consumers have the beliefs that with probability α the execution of the project is conditioned on the success of the current crowdfunding contract. Under probabilistic execution, and given beliefs on θ_1, it follows from (9) that any consumer of type θ' who is not pivotal (i.e., for whom $\mathcal{A}(\theta'|\theta_1) = 0$) will participate in the pre-sale iff conditions (11) and (12) (described above) apply. However, any consumer of type θ' who is pivotal with respect to θ_1 expects that with probability α his participation is crucial to the existence of the future product market, and therefore will participate whenever condition (23) is satisfied. Such a pivotal consumer is willing to pay a period-1 price (p_1) that reflects a premium over the expected period-2 price and is equal to the non-execution probability (α) multiplied by his expected utility gain from the existence of the project. That is, this consumer-type will participate in the pre-sale so long as,

$$p_1 - E[p_2] < \alpha \left(\theta' E(\varphi) - E[p_2]\right) \tag{23}$$

As before, the pivotal consumer or θ_1 depends on the crowdfunding pre-sale price p_1, the cost of production c, and the expected utility from the new product captured by $E(\varphi)$. Additionally, the pivotal consumer is determined by the probability α that the project is not executed if the funding target is not met:

$$\theta_1 = \frac{2p_1 - (1-\alpha)\,c}{(1+\alpha)\,E(\varphi)}. \tag{24}$$

We now consider the equilibrium in which all participating consumers in the pre-sale market are pivotal under probabilistic execution. Thus, the set of pre-sale contracts reduces to the form:

$$\langle p_1, F_c \rangle \equiv \left\langle p_1, p_1 \left(1 - \frac{2p_1 - (1-\alpha)\,c}{(1+\alpha)\,E(\varphi)} \right) \right\rangle . \tag{25}$$

In such a contract, all of the consumers who derive positive expected utility given p_1 are pivotal, i.e., $\mathcal{A}(\theta'|\theta_1) = 1$ for all $\theta' \geq \theta_1$, where the marginal consumer earns zero rents:

$$\theta_1(p_1, F_c, \alpha) \equiv \theta_1(p_1, \alpha) = \frac{2p_1 - (1-\alpha)\,c}{(1+\alpha)\,E(\varphi)} . \tag{26}$$

This contract has the same qualitative characteristics as the contract described in (13), it allows for price discrimination against the high demand customers while at the same time increasing total production and welfare (subject to assumption *A1*). The ability to price discriminate utilizing the pivotal mechanism and probabilistic project execution exists for any α. Note that consumer uncertainty regarding other model parameters may be incorporated into our setup in a similar fashion (for example when consumers only have probabilistic knowledge of θ_1 or the demand function).

5.3 Commitment and Intertemporal Price Discrimination

In more general terms, our market set-up can be viewed as an intertemporal price discrimination model for a durable goods monopolist, because consumers purchase at most a unit quantity of the product. One of our main insights is that the monopolist's ability to commit to the pre-sale contract, in conjunction with the strategic choice of the funding target, allow intertemporal price discrimination. Namely, the monopolist exploits the willingness of pivotal consumers to pay a premium (over the expected future spot prices) to ensure the execution of the project. The monopolist is thus able to segment the market in an intertemporal fashion and improve profits over the benchmark single-price monopoly profits.

Our analysis is therefore related to the large literature on intertemporal price discrimination by a durable goods monopolist. In particular, the ability of the durable goods monopolist to intertemporally price discriminate has received much attention. The literature forwards two polar outcomes with respect to the extraction of consumer surplus when the monopolist cannot credibly pre-commit to future price behavior. The Coase conjecture (Coase (1972)) famously asserts that in the absence of capacity constraints, the monopolist's time-consistent policy is to saturate the residual market at every point in time so that the monopolist would not be able to price above the marginal cost. That is, rational infra-marginal consumers will anticipate the monopolist's time-consistent policy to quickly serve the remaining market by dropping prices and not purchase at any price above the marginal cost (or the competitive price). In contrast, the "Pacman conjecture" (Bagnoli et al. (1989)) holds that the infra-marginal consumers will realize that prices will only drop after they have exited the market, thereby allowing the monopolist to perfectly price discriminate through intertemporal market segmentation.

Both the Coase conjecture and the Pacman conjectures are shown to hold, albeit in models with fundamentally different demand structures. Specifically, Gul et al. (1986) establish the Coase conjecture with a continuum of consumers and when the monopolist uses weak-Markov strategies. However, Bagnoli et al. (1989) establish the Pacman conjecture with a finite number of buyers. Fehr and Kuhn (1995) present a resolution of the Coase versus the Pacman conjectures based on the dimensionality of the set of consumers and prices. In particular, when there are a finite number of buyers and a continuum of prices, then buyers have nonnegligible effects on the seller's total payoffs. Hence, the threat of infra-marginal consumers to hold out for lower future prices is not credible because the seller can raise prices slightly, induce these consumers to pay a premium for early consumption, and make a non-trivial improvement in profits. If the seller is sufficiently patient, then there will be no reduction in prices as long as the high-value consumers stay in the market. In contrast, with a continuum of consumers and a finite set of prices, buyers have a negligible effect on the seller's payoffs, but price reductions have a nonnegligible effect. Hence, the threat of infra-marginal consumers to stay in the market till the price drops is credible — alternatively, the seller's threat of maintaining premium prices for a small set of consumers is not credible.

In our model, there is a continuum of consumers and no restriction on prices, that is, our market structure is similar to situations where the Coase conjecture should hold. However, because of the endogenous uncertainty regarding the execution of the project — and the production of the good desired by consumers — pivotal consumers are willing to pay a premium (to ensure the successful future availability of the product). The monopolist, of course, exploits this willingness of infra-marginal consumers to pay this premium to price discriminate even when there are a continuum of consumers and no ex ante restrictions on feasible prices. The crucial point is that in the crowdfunding of innovations, infra-marginal consumers pay a premium for ensuring the successful execution of the project, and not for early consumption — as is the case in the standard durable goods monopoly model, where there is no uncertainty on the availability of the product. Alternatively stated, project (or production) uncertainty implies that pivotal consumers have nonnegligible effects on monopoly profits and induce such consumers to pay a premium even when there are a continuum of consumers. This, rather than the commitment assumption, is the basic distinction between our model and the durable goods monopoly literature.[6] In particular, the funding uncertainty for the project paves the way for the important roles of the investment target and external financing costs in the monopolist's price discrimination ability in our analysis.

6 Conclusion

Financing through crowdfunding is growing rapidly, especially for start-ups. Unlike the standard text book paradigm that separates optimal financing from the product market environment, investment financing via crowdfunding is directly integrated with the real side of the firm, namely, the demand structure for its products. In particular, future consumers may potentially provide all or part of the required resources for the creation of new products. Crowdfunding contracts that include a commitment to develop and produce a product subject to achieving a predetermined funding goal have become a common phenomenon with crowdfunding platforms enforcing this commitment by transferring the accumulated proceeds to the entrepreneur only if the goal is met. Thus, the pre-sale price and the funding target effectively determine a contract by the entrepreneur.

The presentation of such crowdfunding contracts generates interesting trade-offs for consumers. In addition to considering the price of the product, consumers need to take into account their impact on the probability that the good will be produced — that is, consumers must consider whether they are pivotal in the successful financing and initiation of production.

We find that an optimal crowdfunding contract, which specifies a pre-sale price and a funding target, may serve as a price discrimination mechanism by forcing pivotal consumers to pay a premium above the expected future spot price. Strikingly, the optimal pre-sale contract is such that the monopolist always benefits from using the platform. That is, even in the absence of financing constraints, the monopolist benefits from funding her investment via the crowdfunding platform.

Moreover, crowdfunding via product pre-sale always increases welfare when the marginal per unit cost of production is sufficiently low - a common feature of firms that utilize the crowdfunding platform in practice, such as hi-technology firms. Crowdfunding may also lead to production in cases where there would be no production otherwise.

[6] We note that there also exists product quality uncertainty (at the pre-sale stage) in our model. However, as shown by Kumar (2006), with a continuum of buyers and unrestricted price-quality combinations, such variable quality combinations of future products is not sufficient to allow the monopolist to price discriminate.

For financially constrained entrepreneurs, the use of the crowdfunding platform is positively related to the cost of external financing and negatively related to their endowment of internal funds. While higher financing costs reduce the profit of the monopolist they lower prices in both the pre-sale and spot markets. Overall, higher external financing costs increase total production as long as production takes place.

Interestingly, when production costs are sufficiently low and projects are fundable via pre-sale crowdfunding, higher financing costs increase welfare. From a policy perspective this implies that government policies aimed at relaxing financing constraints for firms so as to spur innovation might in fact lead to lower welfare as they increase prices in the product market and potentially lower production.

7 Appendix

Proof. **[Proposition 1]** We start by deriving the optimal pivotal contract and then subsequently we establish that this is without-loss-of-generality. Considering pivotal contract $\langle p_1, F_c = p_1(1-\theta_1)\rangle$, where $p_1 \in (0, E(\varphi))$, the marginal participating consumer is $\theta_1 = \frac{p_1}{E[\varphi]}$ and set of pivotal consumers is $\Theta^1 = \{\theta : \theta \in [\theta_1, 1]\}$. Assuming that the spot market opens in both states of product quality, the monopolist's problem reduces to finding the optimal pre-sale price by solving (15). We will show that the spot market indeed opens in both states of product quality under this solution as long as the marginal cost of production is sufficiently low, as required by assumption A1.

$$\pi^{ps} \equiv \max_{p_1 \in (0, E(\varphi))} [(p_1 - c)\,(1 - \frac{p_1}{E(\varphi)}) + E\left[\pi_2^{ps}\left(\frac{p_1}{E(\varphi)}\right)\right]. \tag{27}$$

The objective function can be simplified to,

$$\pi^{ps} = p_1\left(1 + \frac{c}{2E(\varphi)}\right) - \frac{3p_1^2}{4E(\varphi)} + E\left[\frac{c^2}{4\varphi}\right] - c,$$

the unique interior solution is $p_1^{ps} = \frac{2E(\varphi)+c}{3}$, and the implied optimal investment target is,

$$F_c^{ps} = p_1(1 - \frac{p_1}{E(\varphi)}) = \frac{2E(\varphi)+c}{3}\frac{E(\varphi)-c}{3E(\varphi)}. \tag{28}$$

To summarize, the optimal crowdfunding contract $\langle p_1^{ps}, F_c^{ps}\rangle$ is given by,

$$\left(p_1^{ps} = \frac{2E(\varphi)+c}{3}, F_c^{ps} = \frac{2E(\varphi)+c}{3}\frac{E(\varphi)-c}{3E(\varphi)}\right) \tag{29}$$

For a given realization of product quality φ the monopolist's profit from the spot market is $\pi_2^{ps}\left(\frac{p_1}{E(\varphi)}\right) = \frac{1}{4\varphi}\left(\frac{p_1}{E(\varphi)}\varphi - c\right)^2$. Thus, the monopolist will indeed choose positive production in both states of product quality if $\frac{p_1}{E(\varphi)}\varphi_\ell > c$ - which under the optimal contract requires that, $0 \leq c \leq \frac{2}{\left(3 - \frac{\varphi_l}{E(\varphi)}\right)}\varphi_\ell$ as follows from assumption A1.

At the optimal crowdfunding contract we obtain:

$$q_1^{ps} = \frac{E(\varphi)-c}{3E(\varphi)} = \frac{1}{3} - \frac{c}{3E(\varphi)} \tag{30}$$

$$E[p_2^{ps}] = E\left[\frac{\theta_1\varphi + c}{2}\right] = \frac{E(\varphi)+2c}{3} \tag{31}$$

$$E[q_2^{ps}] = E\left[\frac{\theta_1\varphi - c}{2\varphi}\right] = \frac{1}{3} + \frac{c}{6E(\varphi)} - \frac{c}{2}\left(\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_\ell}\right) \tag{32}$$

The implied expected profit is,

$$\begin{aligned}
\pi^{ps} &= q_1^{ps}(p_1^{ps} - c) + E[q_2^{ps}(p_2^{ps} - c)] \\
&= \left(\frac{E(\varphi)-c}{3E(\varphi)}\right)\left(\frac{2E(\varphi)+c}{3} - c\right) + E[q_2^{ps}(p_2^{ps} - c)] \\
&= \frac{2\left(E(\varphi)-c\right)^2}{9E(\varphi)} + \frac{E(\varphi)^2 - 2E(\varphi)c}{9E(\varphi)} - \frac{5c^2}{36E(\varphi)} + \frac{c^2}{4}\left(\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_\ell}\right)
\end{aligned}$$

Now, we verify the sub optimality (from the monopolist perspective) of any alternative contract to implement the investment. Suppose, by contradiction that the optimal contract is not a pivotal contract. Recall, according to our definition, a pivotal contract $\langle p_1, F_c = p_1(1-\theta_1)\rangle$ must induce a pivotal state on a non-zero set of consumers, $\theta \in [\theta_1, 1]$, and that under such a contract the marginal participating consumer earns zero rents on average, i.e., $\theta_1 = \frac{p_1}{E(\varphi)}$.

Consider first the case that under this alternative contract there exist pivotal consumers, i.e., $F_c = p_1(1-\theta_1)$ (see definition (2)), but this contract, as assumed, is not a pivotal contract as the marginal consumer to participate in the funding earns positive rents, i.e., $\theta_1 > \frac{p_1}{E(\varphi)}$. Thus, the marginal consumer is $\theta_1 = 1 - \frac{F_c}{p_1}$. But, an improvement for the monopolist would be a contract where $p_1' = p_1 + \varepsilon$ for some $\varepsilon > 0$, such that $p_1 + \varepsilon < \left(1 - \frac{F_c}{p_1}\right) E(\varphi)$, and $F_c' = \frac{F_c}{p_1}(p_1 + \varepsilon)$. This proposed contract will yield higher profits to the monopolist without changing the set of pivotal consumers (though, inducing them to pay a higher pre-sale price).

Second, consider the case in which there is no pivotal consumer, i.e., any consumer that participate in the pre-sale market is not pivotal. Now, consider the equilibrium induced by this optimal contract with prices $\langle p_1, p_2^h, p_2^\ell\rangle$, where $p_1 \leq E(\varphi)$, $p_2^h \leq \varphi_h$, $p_2^\ell \leq \varphi_\ell$. The payoff from participating in the pre-sale market is $\theta E(\varphi) - p_1$ and the payoff from purchasing the good in the spot market is $\theta\varphi_S - p_2^S$ for $S = \ell, h$. As participating consumers are not pivotal they will participate in the pre-sale market only if $\theta E(\varphi) - p_1 \geq E\left(\theta\varphi_S - p_2^S\right)^+$. Due to the monotonicity of preferences in θ, there exists a unique cutoff $\hat{\theta}$ above which the best response of consumers of type $\theta \geq \hat{\theta}$ is to purchase the good in the pre-sale market, where $\hat{\theta} \in [0,1]$. If $\hat{\theta} = 1$ (no consumer purchases in the pre-sale stage) then the monopolist at most obtains profits π^{bm} and, as follows from above, $\pi^{ps} \geq \pi^{bm}$. If $\hat{\theta} < 1$ one can offer an alternative contract with higher price in the pre-sale market under which the participating consumers become pivotal. In particular, an alternative contract $p_1' = p_1 + \varepsilon$ for some $\varepsilon > 0$, such that $p_1 + \varepsilon < \hat{\theta}E(\varphi)$, and $F_c' = p_1'(1-\hat{\theta})$ will yield higher profits to the monopolist by turning the set of participating consumers to pivotal consumers and by doing that, inducing them to pay a higher pre-sale price.

Q.E.D ■

Proof. [Proposition 2] Expected profits under an optimal crowdfunding presale contract compared to benchmark is (see ((17)),

$$\begin{aligned} &\pi^{ps} - E[\pi^{bm}] = \\ = \;& \frac{2(E(\varphi)-c)^2}{9E(\varphi)} + \frac{2E(\varphi)+c}{12E(\varphi)}\left(\frac{2E(\varphi)-5c}{3}\right) + \frac{c^2}{4}\left(\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_\ell}\right) - \left[\frac{E(\varphi)-2c}{4} + \frac{c^2}{4}\left(\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_\ell}\right)\right] = \\ = \;& \frac{(E(\varphi)-c)^2}{12E(\varphi)} > 0 \text{ (follows from assumption A1).} \end{aligned}$$

Q.E.D ■

Proof. Proposition 3 Consider a potential project such that $I < E[\pi^{bm}] < E[\pi^{ps}]$ where $p_1^{ps}(1 - \frac{p_1^{ps}}{E(\varphi)}) > I$. This project is desirable from the perspective of the monopolist (positive net present value) both in the benchmark case and when crowdfunding is available. In the benchmark, $E[q^{bm}] = E\left[\frac{\varphi-c}{2\varphi}\right] = \frac{1}{2} - \frac{c}{2}\left(\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_\ell}\right)$ as described in equation (6). Under an optimal Crowdfunding contract total production is: $E[q^{ps}] = q_1^{ps} + E[q_2^{ps}]$ as defined in equations (30) and (32) respectively. Comparing total expected production under these two scenarios shows that it is higher under the optimal crowdfunding contract (deploying assumption A1):

$$E[q^{ps}] - E[q^{bm}] = \frac{E(\varphi)-c}{6E(\varphi)} > 0.$$

Recall that $E[p_2^{bm}] - E[p_2^{ps}] = \frac{\bar{\varphi}-c}{6}$, thus the average additional consumer will get a surplus of $\frac{\bar{\varphi}-c}{12}$ and the total additional welfare created under the first best crowdfunding contract is (after some algebraic manipulations):

$$\begin{aligned} E[W^{ps}] - E[W^{bm}] &= \frac{\bar{\varphi}-c}{6\bar{\varphi}}\frac{\bar{\varphi}-c}{12} + \frac{\bar{\varphi}-c}{6\bar{\varphi}}\frac{\bar{\varphi}-c}{3} = \\ &= \frac{(\bar{\varphi}-c)^2}{72\bar{\varphi}} + \frac{(\bar{\varphi}-c)^2}{18\bar{\varphi}} = \frac{5\,(\bar{\varphi}-c)^2}{72\bar{\varphi}}. \end{aligned}$$

Q.E.D ■

Proof. **[Proposition 4]** The net payoff to the monopolist after investing I from a particular crowdfunding contract (where $\max(I-A-F_c,0)$ are externally financed) is, $A+\pi_1+E(\pi_2)-I-R(\max(I-A-F_c,0))$, while her reservation utility from not investing is just A. Comparing the two alternatives yields that investment is optimal as long as, $\pi_1+E(\pi_2)-R(\max(I-A-F_c,0)) > I$.

When $F_c^{ps} \geq I-A$ The optimal crowdfunding contract $\langle p_1^{ps}, F_c^{ps}\rangle$, presented in Proposition **1,** maximizes $\pi_1+E(\pi_2)-R(\max(I-A-F_c,0))$; otherwise, the constrained monopolist solves the following problem,

$$\pi^{fc} \equiv \max_{p_1 \leq E(\varphi)} (p_1-c)(1-\frac{p_1}{E(\varphi)}) + E\left[\frac{(\theta_1\varphi-c)^2}{4\varphi}\right] - R\left((I-A) - \left[p_1(1-\frac{p_1}{E(\varphi)})\right]\right)$$

where

$$\begin{aligned} E\left[\frac{(\theta_1\varphi-c)^2}{4\varphi}\right] &= E\left[\frac{(\theta_1\varphi)^2+c^2-2c\theta_1\varphi}{4\varphi}\right] = \frac{1}{4}E\left[\theta_1^2\varphi + \frac{c^2}{\varphi} - 2c\theta_1\right] \\ &= \frac{1}{4}\left[\theta_1^2 E(\varphi) + \lambda\frac{c^2}{\varphi_h} + (1-\lambda)\frac{c^2}{\varphi_\ell} - 2c\theta_1\right] \end{aligned}$$

It is useful to note that

$$\begin{aligned} \frac{\partial}{\partial p_1}E\left[\frac{(\theta_1\varphi-c)^2}{4\varphi}\right] &= \frac{\partial}{\partial p_1}\frac{1}{4}\left[\left(\frac{p_1}{E(\varphi)}\right)^2 E(\varphi) + \lambda\frac{c^2}{\varphi_h} + (1-\lambda)\frac{c^2}{\varphi_\ell} - \frac{2cp_1}{E(\varphi)}\right] \\ &= \frac{\partial}{\partial p_1}\frac{1}{4}\left[\frac{p_1^2}{E(\varphi)} + \lambda\frac{c^2}{\varphi_h} + (1-\lambda)\frac{c^2}{\varphi_\ell} - \frac{2cp_1}{E(\varphi)}\right] \\ &= \frac{p_1-c}{2E(\varphi)} \end{aligned}$$

Thus, the interior solution, as defined by the First-Order-Condition is given by $\frac{\partial \pi^{fc}}{\partial p_1} = 0$ where,

$$\begin{aligned}
\frac{\partial \pi^{fc}}{\partial p_1} &\equiv 1 - \frac{p_1}{E(\varphi)} - \frac{(p_1 - c)}{E(\varphi)} + \frac{p_1 - c}{2E(\varphi)} + R\left[(1 - \frac{p_1}{E(\varphi)}) - \frac{p_1}{E(\varphi)}\right] \\
&= 1 + \frac{c}{2E(\varphi)} + R - \frac{p_1}{2E(\varphi)}(4R + 3).
\end{aligned}$$

The unique interior solution is,

$$p_1(R) \equiv \frac{2(1+R)E(\varphi) + c}{3 + 4R} \in \left[\frac{E(\varphi)}{2}, \frac{2E(\varphi) + c}{3}\right].$$

Notice that $p_1(0) = \frac{2E(\varphi)+c}{3}$ and $p_1(\infty) = \frac{E(\varphi)}{2}$. But more generally,

$$\frac{p_1(R)}{\partial R} \propto (3 + 4R)\, E(\varphi) - 2\,(2(1+R)E(\varphi) + c) = -E(\varphi) - 2c < 0$$

That is, as the external cost of capital increases, the firm raises more funds in the pre-sale market by decreasing the pre-sale price. One can easily verify that, as before, the expected spot price $E\left[p_2(\varphi)\right] = \frac{\theta_1 E\varphi + c}{2}$ is higher than the pre-sale price p_1 where $\theta_1 = \frac{p_1}{E\varphi}$; indeed, $p_1 > E\left[p_2(\varphi)\right] \Leftrightarrow p_1 > \frac{p_1+c}{2} \Leftrightarrow p_1 > c$ (provided from optimality of p_1). One can also verify that the second period or spot market opens in the low state of product quality, i.e., that $\theta_1\varphi_l > c \Leftrightarrow \frac{p_1}{E(\varphi)}\varphi_l > c$. Evaluating the latter condition under the lowest possible equilibrium value of $p_1 = \frac{E(\varphi)}{2}$ requires that $\frac{\frac{E(\varphi)}{2}}{E(\varphi)}\varphi_l > c \Leftrightarrow c < \frac{\varphi_l}{2}$ which follows from assumption A1.

Now, moving on to the level of funds raised via pre-sale: since $F_c = p_1\left(1 - \frac{p_1}{E(\varphi)}\right)$ we obtain that at the interior optimum

$$\begin{aligned}
F_c(R) &= \left(\frac{2(1+R)E(\varphi) + c}{3 + 4R}\right)\left(1 - \frac{2(1+R)E(\varphi) + c}{(3 + 4R)\, E(\varphi)}\right) \\
&= \left(\frac{2(1+R)E(\varphi) + c}{(3 + 4R)^2}\right)\left(\frac{E(\varphi)\,(1 + 2R) - c}{E(\varphi)}\right).
\end{aligned}$$

Note, for $R = 0$ we obtain the level of funds raised in the pre-sale market by an unconstrained monopolist: $F_c^{ps} = \left(\frac{2E(\varphi)+c}{3}\right)\left(\frac{E(\varphi)-c}{3E(\varphi)}\right)$. As suggested above, an increase in the cost of capital will increase the level of funds raised during the pre-sale market:

$$\begin{aligned}
\frac{\partial F_c(R)}{\partial R} &= \frac{\partial F_c}{\partial p_1}\frac{\partial p_1(R)}{\partial R} \\
&= \left(1-\frac{2p_1}{E(\varphi)}\right)\frac{2E(\varphi)\left(3+4R\right)-4\left(2(1+R)E(\varphi)+c\right)}{\left(3+4R\right)^2} \\
&= \left(1-\frac{2p_1}{E(\varphi)}\right)\left(\frac{2E(\varphi)}{3+4R}-\frac{4p_1}{3+4R}\right)=\frac{2\left(E(\varphi)-2p_1\right)^2}{E(\varphi)\left(3+4R\right)}>0.
\end{aligned}$$

This implies that the higher the cost of external financing, the monopolist will rely more on the pre-sale market and the level of external financing reduces. Since we have explored the interior solution, we shall verify that the amount of funds raised via the pre-sale market does not already suffice to cover the investment required. Namely,

$$F_c(R) < (I-A) \Leftrightarrow \left(\frac{2(1+R)E(\varphi)+c}{\left(3+4R\right)^2}\right)\left(\frac{E(\varphi)\left(1+2R\right)-c}{E(\varphi)}\right) < (I-A).$$

We now distinguish between two cases, (i) $(I-A) > \frac{E(\varphi)}{4}$ and (ii) $(I-A) \in \left(F_c^{ps}, \frac{E(\varphi)}{4}\right)$. For case (i) we have shown that external financing is required for production, but for case (ii) we define the critical and unique value $\bar{R}$ such that (notice the strictly positive value of $\frac{\partial F_c(R)}{\partial R}$),

$$F_c(\bar{R}) = (I-A). \tag{33}$$

Still considering case (ii): for $R \leq \bar{R}$, we have $F_c(R) < (I-A)$ and therefore, some external financing is optimal, i.e., the interior solution derived above holds. But, for $R > \bar{R}$, the optimal level of funds raised via the pre-sale market is not given by $F_c(R)$ which would have resulted it raising more funds in the pre-sale market than required, reducing profit, thus it is exactly equal to $(I-A)$. Accordingly, the price in the pre-sale market in this case, denoted by $p_1^{(I-A)}$, is given by

$$p_1^{(I-A)} = \frac{1}{2}\left[E(\varphi)+\sqrt{E^2(\varphi)-4E(\varphi)(I-A)}\right] \in \left(\frac{E(\varphi)}{2}, p_1^{ps}\right).$$

We can define the optimal pre-sale price, given that the firm wishes to produce as,

$$p_1^{fc}(R) = \begin{cases} p_1(R), & \text{if } (I-A) > E(\varphi)/4 \\ p_1^{(I-A)}, & \text{if } (I-A) \in (F_c^{ps}, E(\varphi)/4) \text{ and } R > \bar{R} \\ p_1(R), & \text{if } (I-A) \in (F_c^{ps}, E(\varphi)/4) \text{ and } R \leq \bar{R} \\ p_1^{ps}, & \text{if } (I-A) < F_c^{ps} \end{cases}$$

Q.E.D ■

Proof. [Proposition 5] The expected level of production, given that the optimal crowdfunding contract is implemented by the monopolist and investment takes place, is:

$$q_1^{fc}(R,A) = \begin{cases} \frac{(1+2R)E(\varphi)-c}{(3+4R)E(\varphi)} & \text{if } (I-A) > E(\varphi)/4 \\ \frac{1}{2} - \frac{\sqrt{1-4\frac{(I-A)}{E(\varphi)}}}{2} & \text{if } (I-A) \in (F_c^{ps}, E(\varphi)/4) \text{ and } R > \bar{R} \\ \frac{(1+2R)E(\varphi)-c}{(3+4R)E(\varphi)} & \text{if } (I-A) \in (F_c^{ps}, E(\varphi)/4) \text{ and } R \leq \bar{R} \\ \frac{1}{3} - \frac{c}{3E(\varphi)} & \text{if } (I-A) < F_c^{ps} \end{cases} \tag{34}$$

$$E[q_2^{fc}(R,A)] = \begin{cases} \frac{2(1+R)E(\varphi)+c}{2(3+4R)E(\varphi)} - \frac{c}{2}\left[\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_l}\right] & \text{if } (I-A) > E(\varphi)/4 \\ \frac{1}{4}\left[1+\sqrt{1-4\frac{(I-A)}{E(\varphi)}}\right] - \frac{c}{2}\left[\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_l}\right] & \text{if } (I-A) \in (F_c^{ps}, E(\varphi)/4) \text{ and } R > \bar{R} \\ \frac{2(1+R)E(\varphi)+c}{2(3+4R)E(\varphi)} - \frac{c}{2}\left[\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_l}\right] & \text{if } (I-A) \in (F_c^{ps}, E(\varphi)/4) \text{ and } R \leq \bar{R} \\ \frac{1}{3} + \frac{c}{6E(\varphi)} - \frac{c}{2}\left[\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_l}\right] & \text{if } (I-A) < F_c^{ps} \end{cases} \tag{35}$$

Note that for an internal solution when $R < \bar{R}$ and production takes place,
$E[q_1^{fc}(R,A) + q_2^{fc}(R,A)] = \frac{(1+2R)E(\varphi)-c}{(3+4R)E(\varphi)} + \frac{2(1+R)E(\varphi)+c}{2(3+4R)E(\varphi)} - \frac{c}{2}\left[\frac{\lambda}{\varphi_h} + \frac{(1-\lambda)}{\varphi_l}\right]$
and $\frac{\partial E[q_1^{fc}(R,A)+q_2^{fc}(R,A)]}{\partial R} = \frac{1}{(3+4R)^2} + \frac{2c}{(3+4R)^2 E(\varphi)} > 0$

It follows directly from the above that expected production as implied by $\left\langle p_1^{fc}, F_c^{fc}\right\rangle$ is weakly increasing in the cost of capital R.

Next, we derive the relation with respect to the level of internal funds A. It is useful to restate the optimal contracts using notation (for $I > E\varphi/4$),

$$\bar{A} = I - F_c(R) \in [I - E(\varphi)/4, I - F_c^{ps}]. \tag{36}$$

As noted before (but now in terms of A), for $A < \bar{A}$, we have $F_c(R) < I - A$ and therefore, some external financing is optimal, i.e., the interior solution derived above holds. But, for the case $A > \bar{A}$, we have $F_c(R) > I - A$ and therefore, the optimal level of funds raised via the pre-sale market is not given by $F_c(R)$ but rather by $I - A$. Accordingly, the price in the latter case is $p_1^{(I-A)}$ defined above. Note that, $p_1^{(I-A)}$ is increasing in A. To summarize, we can restate the optimal contract as follows (for $I > E(\varphi)/4$),

$$p_1^{fc} = \begin{cases} p_1(R), & \text{if } A \in [0, \bar{A}] \\ p_1^{(I-A)}, & \text{if } A \in \left[\bar{A}, I - F_c^{ps}\right] \\ p_1^{ps}, & \text{if } A \in [I - F_c^{ps}, I] \end{cases} .$$

It follows that the optimal price is weakly increasing in the level of internal funds A. When addressing the monopolist's investment decision one must consider whether $\pi^{fc}(I, R, A) \geq I$. Since the expected profits of the monopolist are weakly decreasing in the cost of capital and weakly increasing in the level of internal funds (as follows from above analysis) it is possible that for sufficiently high cost of capital or sufficiently low level of internal funds the monopolist will optimally choose not to invest. But, for sufficiently profitable product market it is the case that relaxing financing constraints weakly leads to more production, while it is optimal to invest. **Q.E.D** ■

Proof. **[Corollary 2]** This follows directly from Proposition (4) and the analysis in the proof of Proposition (**5**). **Q.E.D** ■

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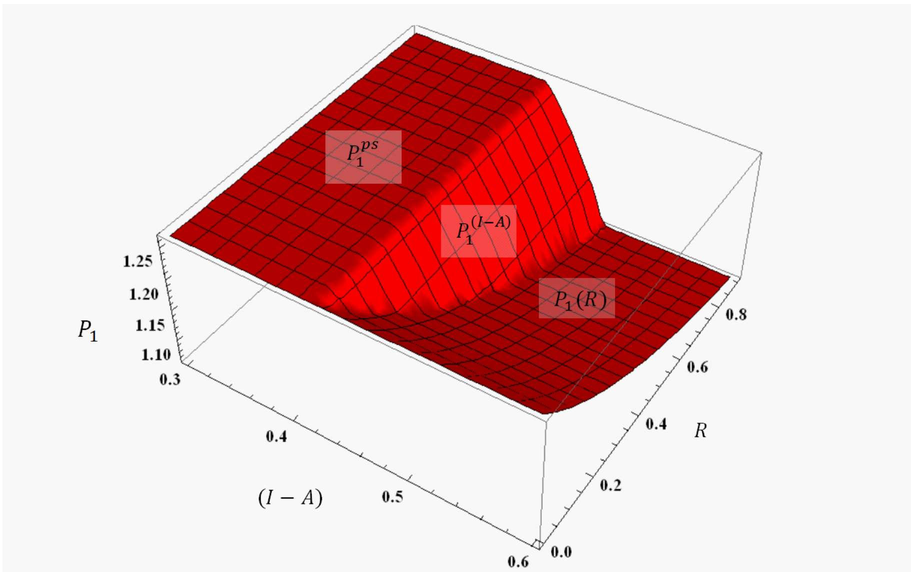

Figure 1: Pre-sale price as a function of capital required $(I - A)$ and cost of capital (R)

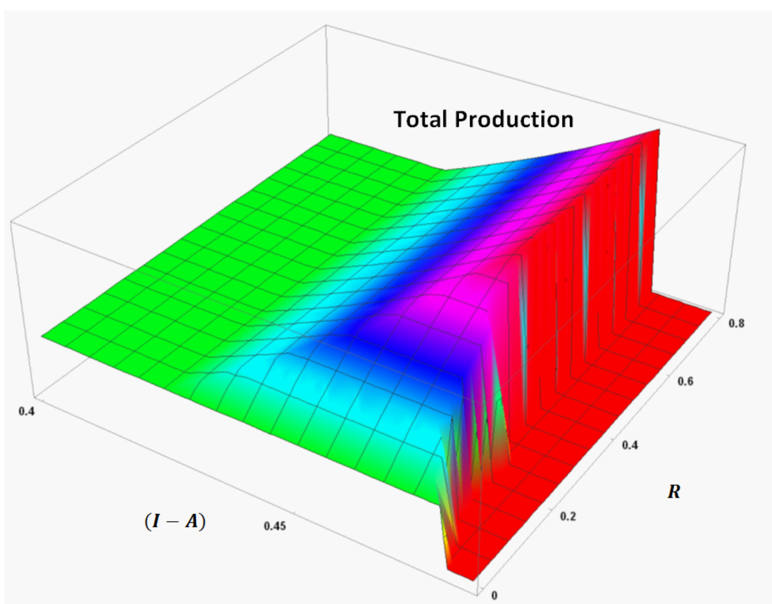

Figure 2: Total Production as a function of capital required $(I - A)$ and cost of capital (R)

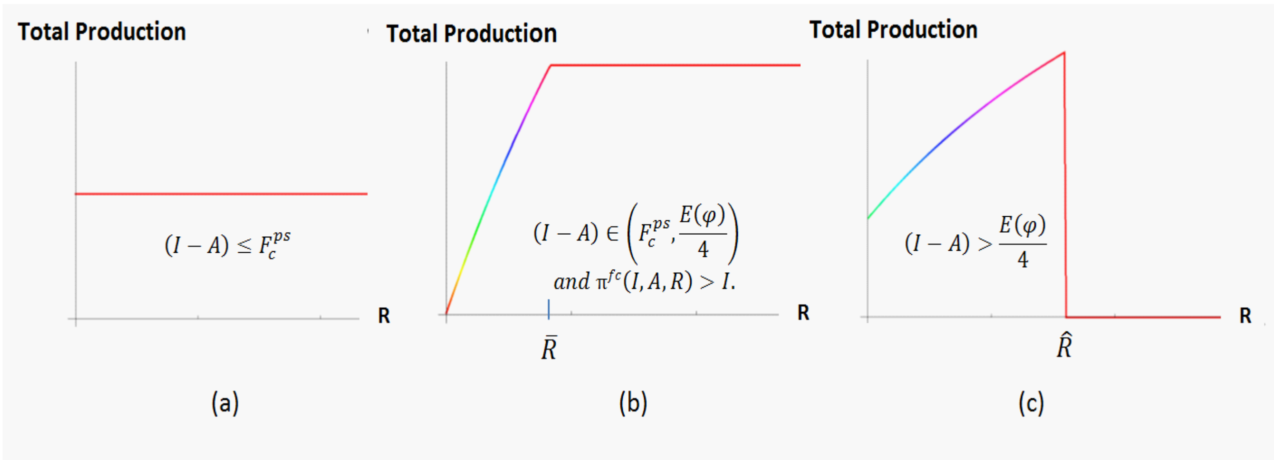

Figure 3: The impact of Cost of Capital (R) on Total Production under different levels of financing requierments $(I-A)$

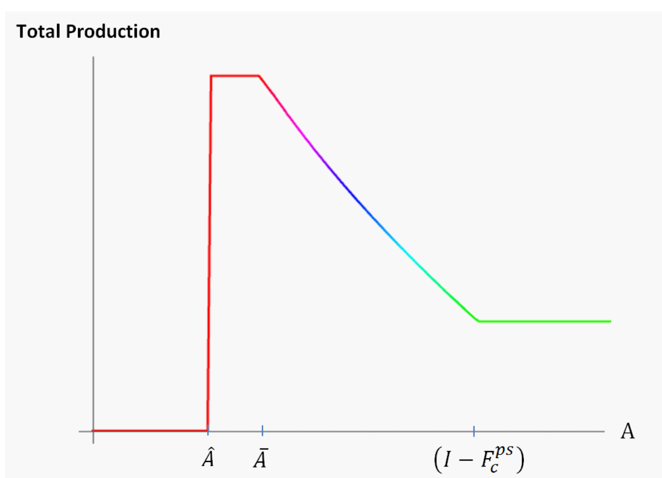

Figure 4: The impact of Internal Funds (A) on Total Production under an optimal crowdfundiung presale contract