

# Policy Cooperation, Incomplete Markets and Risk Sharing

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## Abstract

The paper derives “targeting rules” for optimal policy in a simple two-country model in which financial markets are incomplete and policy is non-cooperative. The optimal rules are compared to the cooperative case. While the model is simple, it is complex enough so that the distortion introduced by incomplete financial markets matters. The complete markets case serves as a benchmark. Under complete markets, it is shown that the policy response in one state of the world influences outcomes in all other states through the effect on asset prices. It is noted that monetary policy cannot replicate an optimal tariff, so that the absence of a tariff instrument is a distortion even in the complete-markets economy. We show that optimal policy, even under complete markets and cooperation, does not try to minimize spillovers.

*Keywords:* cooperative policy, non-cooperative policy, strategic policy, targeting rules

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In the past few years, there has been increasing concern about the effects of monetary policy followed by the U.S. and other advanced countries on emerging markets. There are increasing calls for some sort of policy coordination or cooperation. At the very least, policymakers in advanced countries have been asked to consider the effects of their policies on the rest of the world. Raghuram Rajan (2014), the governor of the Central Bank of India says, “In its strong form, I propose that large country central banks, both in advanced countries and emerging markets, internalize more of the spillovers from their policies in their mandates, and are forced by new conventions on the ‘rules of the game’ to avoid unconventional policies with large adverse spillovers and questionable domestic benefits.” From a more technical perspective, Ostry and Ghosh (2013) note that “Taken as a whole, the various structural models and econometric studies suggest substantial cross-border spillovers operating through both direct and indirect effects. These may be especially large during times of crisis, but even in more normal times, they are sufficient to justify greater coordination of macroeconomic policies.”

This paper compares policy under coordination versus self-oriented strategies. There is a large academic literature that has studied the nature of cooperative versus non-cooperative monetary policy in open economies, which is surveyed briefly below, but no brief survey can do justice to the great volume of work done over the years. This study aims at making a contribution to the nature of strategic versus coordinated policy in the context of the New Keynesian literature.

The New Keynesian approach to monetary policy has been successful and influential in a number of ways. Some of the key contributions are built on rich dynamic models that are solved numerically and provide insights into the goals and tradeoffs for policy decisions. The New Keynesian literature has also contributed to our understanding of monetary policy even with models that are very simple, by developing intuition for what variables monetary policy should target and what tradeoffs face monetary policymakers.

For example, one of the early insights in the closed-economy New Keynesian macroeconomic literature was that under staggered price setting, inflation is distortionary. When firms do not all adjust prices at the same time, a general inflation will lead to misalignment of relative goods prices. Prices that have not been adjusted for some time will be too low – leading to inefficiently high demand for the products with these low prices. In this early literature, the “divine coincidence” emerged – the policy that drove inflation to zero also achieved the goal of

full employment. That coincidence arose in the simple models because, in the absence of inflation, the macroeconomy was undistorted.<sup>1</sup> If nominal prices were held constant by monetary policy, then there would be no need for price adjustment, so sticky nominal prices would not lead to any misallocations. Subsequent developments included other important macroeconomic distortions in the model, in which case the optimal policy potentially involved tradeoffs between the goals of inflation and full employment, for example.

The New Keynesian monetary policy literature has developed tools that have been very useful in clarifying the objectives of monetary policy. In essence, this literature treats monetary policy like a traditional public finance problem. The approach examines equilibrium macroeconomic models where the market outcome deviates from the efficient outcome due to one or more distortions, such as sticky nominal prices. It identifies distortions in the economy, and assesses the trade-offs for policy in abating those distortions. One of the key achievements of this literature has been to show how the objectives of the policymaker can be expressed in terms of a “targeting rule” for policy. The policymaker’s goal is assumed to be maximizing the welfare of households. The targeting rule is derived from the first-order conditions for that optimization problem. In general, policymakers cannot achieve the first-best outcome that is possible in an undistorted economy. The targeting rule shows how policymakers should trade off deviations from the efficient outcome. For example, in general, simultaneously achieving the goals of an efficient level of output and a non-distortionary level of inflation is not possible. The targeting rule shows how the deviations from these efficient outcomes should be traded off – for example, how much inflation should the policymaker accept in order to drive output closer to the full-employment level? The targeting rule does not have a unique representation, but often it can be expressed in simple and intuitive ways. For example, simple closed-economy New Keynesian models express the targeting rule in terms of a log-linear function of the output gap (the difference between actual output and the efficient level of output) and the deviation of inflation from the targeted rate.

An analogous literature developed for the open economy. Some simple models imply that the tradeoffs facing the policymaker in open economies are very similar or even identical to those for the closed economy. For example, the simple model of Clarida et al. (2002) derives the

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<sup>1</sup> That is, undistorted assuming that there was an appropriate constant subsidy to output to alleviate the underproduction that arises in the model from the monopoly power of producers.

optimal tradeoff between the objectives of reducing inflation and the output gap. It is the same targeting rule that holds for the corresponding closed economy. That does not mean that open economy considerations are unimportant. For example, the exchange rate is a key variable that affects the adjustment of inflation and output.

However, subsequent literature has introduced other distortions into the economy which have implications for the optimal targeting rule. Engel (2011), building on a substantial earlier literature, shows that when there is local-currency pricing – producers set prices for goods in the currency of consumers – the optimal targeting rule involves tradeoffs among the goals of zero inflation, zero output gap, and zero exchange rate misalignments. The latter are defined as deviations from the law of one price will arise under local currency pricing when the nominal exchange rate fluctuates in response to shocks. Corsetti et al. (2011a) show that when asset markets are incomplete, a particular deviation from allocations under optimal risk sharing may also be part of the targeting rule.

These latter two studies examine policy assuming cooperation between policymakers in different countries. However, the literature has not characterized the targeting rule for policymakers when there is not cooperation, except in some special cases. It is important to understand what the objectives of policy are under non-cooperation in order to gauge what would change if the world moved toward a regime of more policy coordination. The literature has demonstrated that, under non-cooperation, policymakers have an incentive to improve their own country's welfare relative to the rest of the world, and, as in the optimal tariff literature, that policy can manifest itself through manipulation of the country's terms of trade.

Our goal is to construct a targeting rule that expresses policy tradeoffs in terms of deviations from the undistorted economy. Our approach is to first characterize optimal policy in the case of complete markets. "Complete" means that households in the model can trade in a complete set of state-contingent claims. Obviously, the real world does not have such markets, so what do we gain from calculating optimal policy in such a world? First, it is a benchmark – it helps us to understand what is lost to the economy when markets are incomplete. Second, in fact in simple models, sometimes the outcomes of the economy under complete markets can be replicated when only a small number of assets such as stocks and bonds are traded.

The approach we take here adds two insights to the open-economy monetary policy literature, in addition to the derivation of the targeting rule. First, we note that while it has been

recognized that strategic policymakers have an incentive to influence the terms of trade in setting monetary policy (see Benigno and Benigno (2003), de Paoli (2009), and Corsetti, et al. (2011a)), what has not been fully recognized is the importance of targeting the terms of trade as a way to increase wealth at the expense of other countries. In the model, this channel works through state-contingent prices, but we can infer in a world in which stocks and bonds are traded that the policymaker may try to influence asset prices to favor their own residents.

Second, our complete markets model is still distorted. As has been recognized in the international trade literature for decades, tariffs are the optimal instrument to influence the terms of trade. The aim of the optimal tariff is to exercise a country's monopoly power on world markets. In monetary, sticky-price models, a monetary policy instrument is used to affect the terms of trade. In our model, which is a real model with no nominal price stickiness, but which is analogous to a class of sticky-price models, the policy instrument is a subsidy to employment. These instruments are inferior to a tariff because they introduce an internal distortion. In the sticky-price model, because prices do not adjust instantaneously, price does not equal marginal cost. For example, in a model with flexible wages and sticky prices, a policy that lowers aggregate demand will reduce wages but not prices. That policy drives a "wedge" between the marginal product of labor and the marginal rate of substitution of leisure for consumption. The subsidy to labor in the framework of this paper has a similar effect.<sup>2</sup>

It follows that the optimal policy under complete markets is not fully efficient from the strategic policymaker's standpoint if it cannot use tariffs flexibly to respond to shocks. The optimal policies are "constrained efficient." That in turn is reflected in the targeting rule that we derive under incomplete markets. We show that the policymaker trades off deviations of output and terms of trade under market incompleteness from the levels that would be achieved under the constrained efficient complete-markets policies, but with a correction also for the labor market wedge.

Besides characterizing the targeting rule for optimal policy under non-cooperation and incomplete markets, we can solve for the equilibrium of the model under cooperation and non-cooperation when markets are complete or incomplete. While the model is extremely simple, such comparisons help us to understand how cooperative policy might reduce spillovers. In

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<sup>2</sup> This wedge corresponds to the labor market wedge in the macroeconomics literature. See, for example, Hall (1997), Gali, et al. (2007), or Chari, et al. (2007).

particular, we find that spillovers, per se, are not something to be avoided. Spillovers are not evidence of a distorted world economy so, even under cooperative policy, there are spillovers of idiosyncratic shocks from one country to another.

The literature on optimal macroeconomic policy when policymakers act strategically goes back at least to the pioneering work of Hamada (1974, 1976, 1979.) Cooper (1985) surveys some of the early work in this area. As dynamic macroeconomic models advanced, so did the study of non-cooperative policy. Canzoneri and Henderson (1991) and Persson and Tabellini (1995) summarize the advances of the literature in the 1980s and 1990s.

This study contributes most directly to the literature on optimal policy under cooperation or non-cooperation in micro-founded New Keynesian models. The key early contributions of this literature are Obstfeld and Rogoff (1995, 1996, 2000, 2002a), Clarida et al. (2002) and Gali and Monacelli (2005).

Benigno (2002) is one of the first to couch the non-cooperative policy game in a New Keynesian setting. That paper shows how the policymaker tries to exploit its monopoly power in the world market for the good it exports to improve the welfare of its residents. Obstfeld and Rogoff (2002b) is a seminal work. That paper argues that in practice, the gains from monetary policy cooperation are small. Benigno and Benigno (2003), Devereux and Engel (2003) and Corsetti and Pesenti (2005) further examine whether there are gains from cooperation under some generalizations of the Obstfeld and Rogoff model. Other contributions in the New Keynesian literature to the debate on the gains from cooperation include Pappa (2004) and Canzoneri, et al. (2005). Corsetti, et al. (2011a) survey the literature on optimal monetary policy in open economies, and consider the differences between non-cooperative and cooperative policy.

This study does not attempt to address the question of whether there are gains from cooperation. Indeed, the model is far too simple to make a useful contribution to that debate. Instead, the aim is to give insights into the objectives of policy when there are strategic considerations, and to compare those to the case of cooperation, in a more general setting than the previous literature, where asset market considerations matter.

## 1. The Model

The model laid out here is very simple. It is possibly the simplest two-country model that is still general enough to demonstrate some of the principles behind optimal policy when markets are incomplete. In an international setting, when all households have identical Cobb-Douglas preferences over consumption of Home and Foreign goods, or have log utility, then markets are effectively complete even when there is no asset trade because terms of trade movements provide complete insurance. This result is well known in the literature from the work of Cole and Obstfeld (1991). This model deviates from that case by assuming preferences have constant relative risk aversion but not necessarily logarithmic, and with Cobb-Douglas utility but home bias in preferences.

There are two countries, Home and Foreign. Each produces a single good in competitive markets, using labor as an input. Households in each country consume the output of both countries. There is home bias in preferences of goods consumption. Households in each country get utility from consumption but lose utility from work. There is a representative household in each country. The model is static.

The model laid out here is one in which prices are flexible, and markets are competitive. The optimal policies to deal with market incompleteness can be supported with a flexible taxation policy. However, as we explain below, the crucial equations of the model can also be derived in a sticky-price setting with monopolistic producers, with only some slight modifications. Hence, the policy rules we derive may apply in this standard (albeit static) New Keynesian setting.

Here we present the equations describing the Home country. The Foreign country is symmetric. Then we present equilibrium conditions.

In the notation here, a subscript on a variable means the variable is a function of the state denoted by that subscript. For example, consumption in state  $j$ ,  $C_j$ , can be interpreted to mean  $C(\nabla_j)$ , where  $\nabla_j$  denotes state  $j$ . Before the realization of the random variables (which are productivity levels in the Home and Foreign countries) the probability of state  $j$  is  $\pi_j$ . It is necessary to carefully designate consumption, output and prices in each state, and the probability of each state, because under one specification of the model, state-contingent claims are traded

before the realization of the state. In the other specification, international asset markets are exogenously incomplete – we assume trade in goods is balanced, and there is no asset trade.

*Households:*

The objective of Home households is to maximize:

$$(1) \quad U = \frac{1}{1-\sigma} \sum \pi_j C_j^{1-\sigma} - \sum \pi_j N_j .$$

Here,  $C_j$  denotes the consumption aggregate, which is defined by:

$$C_j = C_{H,j}^{\nu/2} C_{F,j}^{(2-\nu)/2} ,$$

where  $C_{H,j}$  is the consumption of the good produced in Home, and  $C_{F,j}$  is consumption produced in Foreign. There is home-bias in consumption, so  $1 < \nu \leq 2$  . (The Foreign consumer puts a weight of  $\nu/2$  on the Foreign good.)  $N_j$  is labor input.

We can derive the standard demand curves.:

$$(2) \quad C_{H,j} = \frac{\nu}{2} P_{H,j}^{-1} P_j C_j \quad \text{and} \quad C_{F,j} = \frac{2-\nu}{2} P_{F,j}^{-1} P_j C_j ,$$

where  $P_j \equiv k P_{H,j}^{\nu/2} P_{F,j}^{(2-\nu)/2}$ , where  $k \equiv (\nu/2)^{-\nu/2} ((2-\nu)/2)^{(\nu-2)/2}$  .

The analogous conditions for the Foreign household are:

$$(3) \quad C_{F,j}^* = \frac{\nu}{2} P_{F,j}^{*-1} P_j^* C_j^* \quad \text{and} \quad C_{H,j}^* = \frac{2-\nu}{2} P_{H,j}^{*-1} P_j^* C_j^* ,$$

where  $P_j^* = k P_{F,j}^{*\nu/2} P_{H,j}^{*(2-\nu)/2}$  .

When markets are complete, prior to the realization of the state, the household can buy or sell a state-contingent claim that pays off one unit in nominal terms if the state is realized.<sup>3</sup>

In any given state  $j$ , when the state is realized, the household faces the budget constraint:

$$(4) \quad P_j C_j = W_j N_j - T_j + D_j .$$

Here,  $W_j$  is the wage,  $T_j$  is a lump-sum tax imposed on the household (or subsidy if  $T_j$  is negative), and  $D_j$  is the amount of state-contingent bonds purchased for state  $j$ , which could be negative.

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<sup>3</sup> The numeraire for state contingent claims is irrelevant. We express things in nominal terms here so that the model can be easily generalized to the sticky nominal price case.



Before the realization of the state, under complete markets, the household chooses its quantity of state-contingent bonds, and its consumption and labor supply for each state, to maximize (1) subject to:

$$\sum Z_j P_j C_j = \sum Z_j (W_j N_j - T_j) + \sum Z_j D_j$$

Here,  $Z_j$  is the price of a claim on state  $j$ . The last term in this expression must be zero – the value of consumption at state-contingent prices must equal the value of net income, so

$$\sum Z_j D_j = 0.$$

After the realization of the state, the household receives its state-contingent payoff (which may be negative) and consumes and supplies labor according to its state-contingent plan.

It is useful to write out the Lagrangian for the Home household's problem:

$$\mathcal{L} = \frac{1}{1-\sigma} \sum \pi_j C_j^{1-\sigma} - \sum \pi_j N_j + \lambda \left( \sum Z_j (W_j N_j - T_j) - \sum Z_j P_j C_j \right).$$

First-order conditions are:

$$\pi_j C_j^{-\sigma} = \lambda P_j Z_j$$

$$\pi_j = \lambda W_j Z_j,$$

which can be summarized as:

$$(5) \quad \frac{C_j^{-\sigma}}{\lambda P_j} = \frac{1}{\lambda W_j} = \frac{Z_j}{\pi_j}.$$

Similarly, for Foreign households, we can conclude:

$$(6) \quad \frac{C_j^{*- \sigma}}{\lambda^* E_j P_j^*} = \frac{1}{\lambda^* E_j W_j^*} = \frac{Z_j}{\pi_j}.$$

(We have expressed the Foreign nominal price level,  $P_j^*$ , and the Foreign nominal wage,  $W_j^*$ , in units of Foreign currency. Then  $E_j$  is the nominal exchange rate, expressed as Home currency price of Foreign currency. Note that in this paper,  $E_j$  denotes the exchange rate, while  $E$  will denote expectations.  $Z_j$ , the price of a contingent claim in units of Home currency, is the same for Foreign and Home households.)

### *Firms*

A representative competitive firm produces output according to the production function:

$$Y_j = A_j N_j$$

$Y_j$  is output of the Home good, and  $A_j$  is productivity for Home production which is an exogenously given random variable. (Foreign output is denoted  $Y_j^*$  and Foreign productivity is  $A_j^*$ .)

The firm is a price taker and sells output to Home and Foreign households. We assume the law of one price holds,  $P_{H,j} = E_j P_{H,j}^*$ . The firm chooses output to maximize:

$$(7) \quad P_{H,j} Y_j - (1 - \tau_j) W_j Y_j / A_j ,$$

where  $\tau_j$  is a government subsidy to employment.

The first-order condition is given by:

$$(8) \quad P_{H,j} = (1 - \tau_j) W_j / A_j .$$

The law of one price also holds for the Foreign firm, and the analogous first-order condition is:

$$(9) \quad P_{F,j}^* = (1 - \tau_j^*) W_j^* / A_j^* .$$

### *Equilibrium under complete markets*

First-order conditions (2)-(9) hold.

Define the Home country's terms of trade as the price of its imports relative to its exports:

$$S_j = P_{F,j} / P_{H,j} .$$

With this definition and the definition of the consumer price levels in each country, we note that equations (5) and (6) imply

$$(10) \quad C_j^\sigma \lambda = S_j^{\nu-1} C_j^{*\sigma} \lambda^* .$$

Solving out for wages from equations (5)-(9), we can write:

$$(11) \quad 1 = (1 - \tau_j) k A_j^{-1} S_j^{\frac{2-\nu}{2}} C_j^\sigma , \quad \text{and}$$

In addition, we have goods market clearing:

$$(12) \quad Y_j = C_{H,j} + C_{H,j}^* , \quad \text{and} \quad Y_j^* = C_{F,j} + C_{F,j}^* .$$

Then, using (2) and (3), these equations may be rewritten as:

$$(13) \quad Y_j = k \left( \frac{\nu}{2} S_j^{(2-\nu)/2} C_j + \frac{2-\nu}{2} S_j^{\nu/2} C_j^* \right)$$

and

$$(14) \quad Y_j^* = k \left( \frac{\nu}{2} S_j^{(\nu-2)/2} C_j^* + \frac{2-\nu}{2} S_j^{-\nu/2} C_j \right).$$

We do not treat the Lagrange multipliers,  $\lambda / \lambda^*$ , in equation (10) as constants, as does much of the literature. In fact, we will see that optimal strategic policies tries to influence this ratio. The Appendix shows

$$(15) \quad \frac{\lambda}{\lambda^*} = \frac{E(C^{1-\sigma})}{E(C^{*1-\sigma})}.$$

We can use (10) and (15) to solve for the terms of trade as:

$$(16) \quad S_j = C_j^{\sigma/(v-1)} C_j^{*-\sigma/(v-1)} \left[ E(C^{1-\sigma}) \right]^{1/(v-1)} \left[ E(C^{*1-\sigma}) \right]^{-1/(v-1)}.$$

### *Incomplete markets*

Our goal is to compare cooperative and non-cooperative policy under incomplete markets with the optimal cooperative and non-cooperative policies when markets are complete. The case of incomplete markets we consider here is one in which there is no asset trade at all. In this case, there are no state-contingent claims payoffs, so the state-by-state budget constraint for Home households becomes simply:

$$P_j C_j = W_j N_j - T_j.$$

Households choose consumption and labor input state-by-state subject to this constraint. The demand equations (2) and (3) still hold. The consumption-leisure tradeoffs also still hold:

$$(17) \quad \frac{C_j^{-\sigma}}{P_j} = \frac{1}{W_j} \quad \text{and} \quad \frac{C_j^{*-\sigma}}{P_j^*} = \frac{1}{W_j^*}.$$

Firms' decisions are the same as under complete markets.

The goods market clearing equation (12) still holds. The asset-market equilibrium condition (10) is replaced by the balanced trade equation:

$$(18) \quad C_j = S_j^{\nu-1} C_j^*.$$

or, solving for the terms of trade:

$$(19) \quad S_j = C_j^{\frac{1}{\nu-1}} C_j^{*-1}.$$

*Discussion of the approach to the optimal policy problems*

Policymakers in the Home country commit to a policy before state contingent claims are traded in the complete-markets economy. The opposite timing – that asset markets open before policymakers commit to a policy – would constitute a case of discretionary policy. The portfolio of state-contingent claims would be determined before policy is implemented. While such a case is interesting, this study is concerned strictly with policymaking under commitment.<sup>4</sup>

We assume the Home policymaker chooses  $C_j$  for all  $j$ , taking  $C_j^*$  for all  $j$  as given. We take consumption in each state for the policymaker's strategy because it is the natural way to think of demand management in this static setting. However, in general, the outcome of the model may depend on the strategy – for example, if output were the strategy variable, we may derive a different targeting rule and find a different equilibrium under non-cooperation.

The Home policymaker in the non-cooperative case wants to influence the terms of trade in order to increase the value of the Home household's income. When markets are complete, the terms of trade for state  $j$  are influenced not just by the level of output in state  $j$ . The policy rule aims to increase the Home household's wealth when the market for state-contingent claims open, which can be seen as influencing  $\lambda / \lambda^*$  given by  $E(C^{1-\sigma}) / E(C^{*1-\sigma})$ , as shown in equations (15) and (16). In essence, the policymaker is trying to influence asset prices, so as to make his own residents wealthier.

The policy strategies in each country under non-cooperation involve committing to a consumption level in each state. However, that does not describe the implementation of the policy. The policy is implemented by choosing the employment subsidy in each country. In the Home country that is given by  $\tau_j$  in equation (7). Once optimal policies have been determined for each state in equilibrium, then the output level and terms of trade are determined by equations (13), (14), and (16) under complete markets or (18) under incomplete markets. Given

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<sup>4</sup> See Senay and Sutherland (2007, 2013) for a comparison of optimal policy in models with asset trade, in the cases of commitment versus discretionary policy (policy chosen after portfolios have been set.)

the equilibrium values of  $C_j$ ,  $Y_j$ , and  $S_j$ , the subsidy that supports the equilibrium can be derived directly from equation (11).<sup>5</sup> As is well known in the literature, if either there is no home bias in preferences ( $\nu = 1$ ) or preferences are logarithmic in aggregate consumption ( $\sigma = 1$ ), then even if there is no asset trade at all and trade is balanced, markets are effectively complete. There is no difference, in other words, between the allocations under complete markets world and balanced trade in those cases.

### *Keynesian version of the model*

We have set up the model with perfectly competitive firms and flexible prices, where the policy instrument is a time-varying subsidy. However, the model could be converted to a New Keynesian model with sticky nominal prices. The budget constraints and objectives of the firm have been written in nominal terms. Nominal prices are not determined in the flexible price model set out here. But if prices were sticky, these same equations could be used to characterize optimal monetary policy. Sticky prices here means that nominal prices are set in advance, before the realization of the state. They are set simultaneously with the purchase and sale of state-contingent claims – that is, after the policy commitment and before the realization of the state. To transform the model to a Keynesian sticky price model, we would need to assume that each good is produced by a monopolist. The price of home goods,  $P_{H,j}$ , would be a price index over an infinite variety of goods produced by monopolists, with a constant elasticity of substitution (greater than one.) This is the set-up, for example, in Devereux and Engel (2003) and Obstfeld and Rogoff (2004). Because the model is static, these prices are set and are independent of the realization of the state. Under producer currency pricing (which means that each firm sets a single price in units of its own currency),  $P_{H,j}$  and  $P_{F,j}^*$  are fixed and do not depend on the state  $j$ .

While introducing monopolistic producers also introduces another distortion into the model – inefficient levels of production due to the monopoly power of firms – this is not a distortion that monetary policy under commitment is able to address. If monetary policy tried to introduce an expansionary bias in order to offset the underproduction arising from the monopoly

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<sup>5</sup> The Appendix derives the solution to the model under balanced trade and complete markets, in terms of productivity shocks and the employment subsidy.

distortion, firms would simply set higher ex ante nominal prices. In the monetary model, the policymakers do not have access to an employment subsidy,  $\tau_j$ , that can vary across states, but we will assume that they set a state-independent subsidy,  $\tau$ , that would lead to an efficient level of output in the absence of productivity shocks. These assumptions follow common ones in the New Keynesian literature. On the one hand, it is usually assumed that fiscal policy is not as flexible as monetary policy, so while monetary policy can be changed state by state, fiscal policy cannot. On the other hand, state-independent fiscal policy is feasible. The assumption that it is efficient in the case of no shocks is one of convenience.

Given these modifications, it is easy to see how our model can be interpreted as one for optimal monetary policy. Written in nominal terms,  $P_{H,j}$  and  $P_{F,j}^*$  are each set equal to one, and the relative price,  $E_j P_{F,j}^* / P_{H,j}$  is just equal to the nominal exchange rate  $E_j$ . That is essentially equivalent to a model in which  $P_{H,j}$  and  $P_{F,j}^*$  are each set in advance. The average level of output in such a model would be different than that in the competitive model because of the monopoly distortion, but the state-dependent parts of the model would be identical.

We also do not derive an “instrument” rule for the Keynesian case. In fact, we have not even stated what the instrument of monetary policy would be in this model. In a dynamic model, much of the literature assumes that the nominal interest rate is the instrument. There is no interest rate in this model because it is static. Perhaps a static analog to the nominal interest rate would be the level of nominal consumption,  $P_j C_j$ . That is, we might posit that our policymakers directly control the level of nominal expenditure in the economy. As in much of the New Keynesian literature, we do not introduce explicitly the markets through which the policymaker controls its instrument. We assume that the policymaker can control nominal spending just as the standard literature assumes the policymaker directly controls the nominal interest rate.

There is an important subtle distinction between the monetary policy interpretation and the competitive, tax policy interpretation of the model. Below we approximate the model in a non-stochastic steady state. We assume optimal policy is followed in the steady state. In the tax model, that assumption is straightforward – the optimal tax rule is simply applied in the steady state case. However, monetary policy only is effective in targeting fluctuations, and will have no effect on the steady state. So, under the monetary policy interpretation, we still must assume that

policymakers can impose a non-state contingent tax that gives the efficient outcome in the non-stochastic steady state.

## 2. Policymaking

### a. *Non-cooperative case*

Policymakers in the Home country seek to choose  $C$  in each state to maximize the welfare of Home households (taking  $C^*$  in each state as given.) Using the equilibrium condition for output (13), we can rewrite the objective (1) as

$$(20) \quad \frac{1}{1-\sigma} E(C^{1-\sigma}) - E\left(kA^{-1} \left( \frac{\nu}{2} S^{(2-\nu)/2} C + \frac{2-\nu}{2} S^{\nu/2} C^* \right)\right),$$

whether markets are complete or incomplete. In choosing consumption, the Home policymaker trades off the additional utility from consumption with the additional work effort required to produce the consumption goods that arises because an increase in Home consumption increases demand for the Home good. If the policymaker took the terms of trade as given, the first-order condition for optimal policy would be:

$$C^{-\sigma} - kA^{-1} \frac{\nu}{2} S^{(2-\nu)/2} = 0,$$

and would be the same under incomplete or complete markets.<sup>6</sup> The difference in policy choices arises because altering the level of aggregate demand,  $C$ , affects the terms of trade differently under balanced trade and complete markets (see equations (16) and (19).)

It is insightful to examine the objectives and first-order conditions for optimal policy under balanced trade and compare it to the case of complete markets. In both cases, we can write Lagrangians that incorporate how the terms of trade are affected by the policy choice, from equations (16) and (19). In the case of balanced trade, we have:

$$L = E \left[ \frac{1}{1-\sigma} C^{1-\sigma} - kA^{-1} \left( \frac{\nu}{2} S^{(2-\nu)/2} C + \frac{2-\nu}{2} S^{\nu/2} C^* \right) + \omega \left( S - C^{\frac{1}{\nu-1}} C^{*\frac{-1}{\nu-1}} \right) \right],$$

where  $\omega$  is the Lagrange multiplier. Choosing  $C$  and  $S$ , holding  $C^*$  constant, we can derive:

$$(21) \quad C^{-\sigma} = \frac{\nu}{2(\nu-1)} kA^{-1} S^{\frac{2-\nu}{2}}.$$

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<sup>6</sup> Hereinafter, we omit the state subscript  $j$  when there is no possibility of confusion.

From (21) and (11), it is clear that the optimal policy holds the policy instrument,  $\tau$ , constant across states. The policy is to tax employment, so  $\tau = -\frac{2-\nu}{2(\nu-1)}$ . The Home policymaker, taking foreign consumption as given, restricts fluctuations in employment, acting like a monopolist.

Under complete markets, the Lagrangian is given by:

(22)

$$L = E \left[ \frac{1}{1-\sigma} C^{1-\sigma} - kA^{-1} \left( \frac{\nu}{2} S^{(2-\nu)/2} C + \frac{2-\nu}{2} S^{\nu/2} C^* \right) + \omega \left( S - C^{\frac{\sigma}{\nu-1}} C^{*\frac{-\sigma}{\nu-1}} (EC^{1-\sigma})^{\frac{1}{\nu-1}} (EC^{*1-\sigma})^{\frac{-1}{\nu-1}} \right) \right].$$

We can write the first-order conditions as:

$$(23) \quad C^{-\sigma} - k \frac{\nu}{2} A^{-1} S^{\frac{2-\nu}{2}} - \omega \frac{\sigma}{\nu-1} S C^{-1} + \frac{\sigma-1}{\nu-1} C^{-\sigma} (EC^{1-\sigma})^{-1} E(\omega S) = 0.$$

$$(24) \quad \omega = kA^{-1} \frac{\nu(2-\nu)}{4} \left( S^{\frac{-\nu}{2}} C + S^{\frac{-(2-\nu)}{2}} C^* \right).$$

In equation (23), the last of the four terms that appears in that equation arises because of the effect of changing consumption in state  $j$  on the ex ante wealth of Home relative to Foreign households. That is, this term reflects the influence of the effect of a change in  $C_j$  on  $\lambda / \lambda^*$  in equation (15). Most of the previous literature treats  $\lambda / \lambda^*$  as a constant and mistakenly omits this term from the first-order conditions of the policymaker in the non-cooperative case.

The optimization problem expressed in (22) is characteristic of a broader class of problems of policymaking under commitment when there are assets traded. In the simple model here, the assets are state-contingent claims, though we have noted that in many models the payoffs to state-contingent claims can be spanned by a smaller set of assets such as bonds and equities. The policymaker choosing consumption for state  $j$  must consider not only the effects of that policy on welfare in state  $j$ , but also in other states, because the choice of  $C_j$  affects

consumption in all states through the term  $(EC^{1-\sigma})^{\frac{1}{\nu-1}} (EC^{*1-\sigma})^{\frac{-1}{\nu-1}}$  that appears in (22). The objective function of the policymaker is not additively separable over the states of the world. In other words, when the policymaker commits to a given plan, the choice of policy for each state affects asset prices, and those asset prices determine household wealth, and therefore welfare in every state.



Under complete markets, the policy instrument is not constant across states. Home households benefit from a drop in the price of the Home good, because the state-contingent contracts pay off more when Home consumer price levels are low. Under home bias in preferences, the Home consumer price level falls when the price of the Home good falls. When  $\sigma > 1$ , the policymaker's incentive when Home productivity rises is to expand Home output more and let the price of Home goods drop more compared to the balanced trade case. Although this paper does not attempt to show how trade in equities or other assets could mimic the trade in state-contingent claims, intuitively these same effects may be present when there is home bias in equity holdings.<sup>7</sup> Allowing more expansion of production when productivity shocks occur may increase firms' profits in a given state.

*b. Cooperative policy*

The objective function can be written as:

$$(25) \quad \frac{1}{1-\sigma} E(C^{1-\sigma}) + \frac{1}{1-\sigma} E(C^{*1-\sigma}) - E\left( A^{-1} \left( \frac{\nu}{2} S^{(2-\nu)/2} C + \frac{2-\nu}{2} S^{\nu/2} C^* \right) \right) - E\left( A^{*-1} \left( \frac{\nu}{2} S_j^{(\nu-2)/2} C_j^* + \frac{2-\nu}{2} S_j^{-\nu/2} C_j \right) \right).$$

Under cooperation, there is no incentive to manipulate the terms of trade whether markets are complete or not.

The cooperative policy problem under complete markets can be greatly simplified by noticing that a policymaker that wishes to maximize the sum of the utility of Home and Foreign households has no incentive to influence  $\lambda / \lambda^*$ . The standard practice in the literature, which is completely correct, is to set  $\lambda / \lambda^* = 1$ , in which case equation (16) simplifies to

$$S_j = C_j^{\sigma/(v-1)} C_j^{*-\sigma/(v-1)}.$$

### 3. Targeting Rules

In this section, we characterize the targeting rule for policymakers. The targeting rule shows in a simple way how the policymaker trades off competing targets for policy – how much

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<sup>7</sup> See Engel and Matsumoto (2009) and Heathcote and Perri (2013) for examples of similar models where there is substantial home bias in equity holdings when non-state-contingent bonds are also traded.

it should weigh competing objectives. We compare the targeting rule under cooperation versus under strategic behavior by individual country policymakers. How are the objectives different? What does the global policymaker care about compared to the self-oriented national policymaker?

Targeting rules for this simple model are easily derived from the welfare functions of the previous section. The policymaker sets consumption to maximize utility, and the objective functions are expressed in terms of consumption, so the first-order condition from the unconstrained optimization problem gives us the targeting rules.

### *Cooperative case*

We can begin by deriving the targeting rule of the global policymaker.<sup>8</sup> Log-linear approximations to the first-order conditions for maximizing global welfare (that is, maximizing (25)) when there is no trade in state-contingent claims and trade is balanced are given by (in each state, and dropping the subscript for the state):

$$\sigma(\nu-1)^2 c^R + \nu(2-\nu)c^R - (\nu-1)a^R = 0$$

$$\sigma c^W - a^W = 0.$$

In the notation introduced here, for any variable  $x$ , we define an average “world” variable as

$x^W \equiv \frac{x+x^*}{2}$ , and an average “relative” variable as  $x^R \equiv \frac{x-x^*}{2}$ . Lower case letters denote the log

of the deviation of the corresponding upper case variable from the log of the variable under the non-stochastic “steady state” (which is the outcome if productivity were constant and equal to one in all states in both countries.)

To understand the objectives of the global policymaker, it is helpful to look at some special cases. If there is no home bias in consumption (if  $\nu = 1$ ), then the policymaker would set Home and Foreign consumption equal. We would have  $c^R = 0$ , and  $c = c^* = c^W = a^W / \sigma$ . The policymaker would insure that the two countries shared the global shock to productivity. At the other extreme, if there were complete home bias in consumption ( $\nu = 2$ ), each country’s consumption would respond only to its own changes in productivity:  $c = a / \sigma$ ,  $c^* = a^* / \sigma$ . The

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<sup>8</sup> These derivations and results correspond to those in Corsetti, et al. (2011a).

intermediate case involves some but not complete spillovers from the Foreign country to Home. This is explored in more depth in section 4.

Under complete markets, the corresponding first-order conditions are given by:

$$\sigma \bar{c}^R - (\nu - 1) a^R = 0$$

$$\sigma \bar{c}^W - a^W = 0 .$$

Here we use an overbar on the consumption variables to denote the value of consumption under complete markets. The condition for sharing of the world shock is the same as under incomplete markets, but the effect of relative shocks on relative consumption is different under complete markets.

One can express the objectives of the policymaker under incomplete markets in terms of distortions – the difference between the target under incomplete markets and under complete markets:

$$\sigma \tilde{c}^R - (\sigma - 1) \nu (2 - \nu) c^R = 0$$

$$\tilde{c}^W = 0 .$$

Here, for any variable  $x$ , we define  $\tilde{x}$  as the deviation of the variable under incomplete markets from the value it takes under complete markets:  $\tilde{x} = x - \bar{x}$ . In this case of a global policymaker,  $\tilde{c}$  is the difference between the outcomes for the log of consumption under incomplete markets and complete markets for given realizations of the Home and Foreign productivity shocks.<sup>9</sup>

The second of these “targeting rules” is simple – the policymaker will set world consumption equal to the value it takes when markets are complete. The first, however, reveals a tradeoff. The policymaker cannot set relative consumption at the same levels as are achievable under complete markets. The policymaker would like to be able to do that and achieve the globally efficient outcome that can be achieved under complete markets (and cooperative policy.) But markets are not complete and the policymaker is constrained by the balanced trade condition. He could set  $\tilde{c}^R$  equal to zero, but then would not be able to reach the target of  $\tilde{c}^W = 0$ . The optimality condition says that (assuming  $\sigma > 1$ ) the policymaker will set  $c^R > \bar{c}^R$  whenever  $c^R > 0$ .

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<sup>9</sup> In order to take these differences, it is important that the log linearizations in both cases of incomplete and complete markets are around the same steady state.

To understand this tradeoff, we introduce the variable  $M_j$ , the deviation from risk sharing that occurs under complete markets. Specifically, the Foreign marginal utility of a unit of expenditure is given by  $C_j^{*-\sigma} / P_j^*$  and the Home marginal utility of a unit of expenditure is given by  $C_j^{-\sigma} / P_j$ .  $M_j$  is defined as the ratio of  $C_j^{*-\sigma} / P_j^*$  to  $C_j^{-\sigma} / P_j$ :

$$M_j \equiv \left[ C_j^{*-\sigma} / P_j^* \right] / \left[ C_j^{-\sigma} / P_j \right] = C_j^\sigma C_j^{*-\sigma} S_j^{1-\nu}.$$

In the complete markets cooperative equilibrium, ex ante the global policymaker treats the two countries equally, so  $E(C^{1-\sigma}) = E(C^{*1-\sigma})$ . From (10) and using (15), we have  $M_j = 1$  in the complete markets equilibrium. However, under the assumption of balanced trade,

$M_j = C_j^{\sigma-1} C_j^{*-(\sigma-1)}$ . Hence,  $m_j$  is the log of the deviation from the outcome from complete markets risk sharing. It is the risk-sharing distortion, as defined in Corsetti et al. (2011a).

We can rewrite the first targeting rule as:

$$\sigma \tilde{c}^R + 2\nu(2-\nu)m = 0.$$

The policymaker under complete markets faces a tradeoff between the objective of achieving the relative consumption that is desired under complete markets and the objective of minimizing the consumption distortion introduced by market incompleteness. Under complete markets, policymakers set:

$$\bar{c}^R = \frac{\nu-1}{\sigma} a^R.$$

Under incomplete markets,

$$c^R = \frac{\nu-1}{\sigma(\nu-1)^2 + \nu(2-\nu)} a^R.$$

When  $\sigma > 1$ ,  $c^R$  responds more to relative productivity shocks under incomplete markets than under complete markets. The policymaker must allow for more responsiveness of relative consumption in order to help alleviate the risk sharing distortion.

We can recast these targeting rules in terms of output gaps instead of consumption gaps.

Taking approximations to the goods market clearing conditions, we find:

$$\tilde{c}^R = \frac{\nu-1}{\sigma\nu(2-\nu) + (\nu-1)^2} \tilde{y}^R + \frac{\nu(2-\nu)}{2(\sigma\nu(2-\nu) + (\nu-1)^2)} m, \text{ and } \tilde{c}^W = \tilde{y}^W.$$

Under cooperation, the targeting rules can then be written as:

$$(26) \quad \sigma \tilde{y}^R + \frac{(\sigma-1)(\nu-1)\nu(2-\nu)}{2} m = 0$$

$$(27) \quad \tilde{y}^W = 0$$

The second condition simply says the global policymaker sets the world output gap to zero – it sets global output at the same level as would be achieved under complete markets. But the first condition demonstrates a tradeoff between setting the relative output gap to zero and minimizing the risk-sharing distortion.

### *Non-cooperative equilibrium*

How does the targeting rule for the non-cooperative policymaker compare to that of the cooperative policymaker? Does the strategic policymaker care about deviations from risk sharing? How are strategic elements reflected in the targeting rule?

Targeting rules under non-cooperation are, in essence, reaction functions. The Home policymaker's targeting rule is derived from the first-order condition for choosing optimal consumption taking the productivity shocks in each country as given *and* taking Foreign consumption as given.

It may seem superfluous to characterize the targeting rule because the instrument rule is so simple under balanced trade – a constant employment tax, as we showed above. However, deriving the targeting rule in this simple model may be helpful for understanding optimal policy in a richer environment, and it helps facilitate comparison with the cooperative case.

It is important to note that the steady state of the two economies is the same. That may seem obvious, because when there are no shocks (as in steady state), then there is only one state, so trade must be balanced even if trade in assets were allowed. The reason that this is worth pointing out, however, is that if one ignored the effects of policy on  $\lambda / \lambda^*$  in deriving the optimality condition (23) (as much of the literature has done), the steady states would not be the same.

We will express the targeting rule in terms of output and the terms of trade, as is familiar from the literature. A log-linear approximation to the first-order condition (21) for the Home policymaker under incomplete markets, the goods market equilibrium condition (13) and the balanced trade condition (18) gives us:

$$(28) \quad \sigma y - \frac{(\sigma-1)(2-\nu)}{2} s - a = 0.$$

Under complete markets, log-linearizing the first-order conditions for the Home policymaker (given by (23) and (24)), and using the goods market equilibrium condition (13) and the asset-market equilibrium condition (16), we find:

$$(29) \quad \sigma \bar{y} - \frac{(\sigma-1)\nu(2-\nu)}{2} \bar{s} - a + (\sigma-1)(2-\nu) \left( -a + \frac{\bar{s}}{2} + \sigma \bar{c} \right) = 0.$$

where the overbar denotes quantities under complete markets (so as to distinguish these from the values they take on under balanced trade.) The targeting rule (29) has been written in a way to highlight the differences between the objectives under complete markets and those under incomplete markets, given in (28). (Recall that the lower case letters are deviations from the “steady state”, as defined above. To a first order, the expected logs of any variable are equal to their steady state value, so, for example,  $E\bar{c} = E\bar{c}^* = 0$ .)

Now we can use the first-order conditions for households and firms in the home country to write:

$$(30) \quad \sigma \bar{c} + \bar{p} = \bar{p}_H + a + \bar{\tau}$$

We can then rewrite (29) as:

$$(31) \quad \sigma \bar{y} - \frac{(\sigma-1)(2-\nu)}{2} \bar{s} - a + (\sigma-1)(2-\nu) \bar{\tau} = 0.$$

Under complete markets, the policymaker has an output objective, but also a terms of trade objective. When the home country experiences an increase in productivity, the policymaker would like output to rise. However, in a competitive economy, the terms of trade for this country would fall more than is socially optimal. The policymaker trades off its output objective with its desire to affect the terms of trade. Equation (31) also includes an expression involving the labor wedge,  $\bar{\tau}$ . The policymaker in this complete market economy does not have access to a tariff instrument, so its employment subsidy introduces a distortion in labor markets. It must trade off that distortion against its output and terms of trade objectives.

The targeting rule can be expressed by taking (28) less (31):

$$(32) \quad \sigma \tilde{y} - \frac{(\sigma-1)(2-\nu)}{2} \tilde{s} + (\sigma-1)(2-\nu) \tilde{\tau} = 0.$$

Here, we have used the fact that in the balanced trade model, the tax on labor is a constant, and therefore equal to the steady-state value, so  $\tilde{\tau} = -\bar{\tau}$ . Although we can express the optimal policy under complete markets simply as a constant subsidy as in section 2.a above, we write the “targeting rule” that expresses the tradeoffs the policymaker under incomplete markets faces when trying to replicate the more desirable complete markets outcome. This expression shows that when trade in contingent claims is shut down and there is no asset trade, the policymaker targets the output gap,  $\tilde{y}$  and the terms of trade gap,  $\tilde{s}$  and the labor wedge under complete markets.

The complete-markets economy we have examined still is distorted. The policy of taxing or subsidizing labor introduces a “labor market wedge” – that is, a gap between the marginal rate of substitution of leisure for consumption and the marginal product of labor. A different approach to this problem would be the following: We could examine non-cooperative policy in a fully undistorted economy. The undistorted economy would have complete asset markets, but also access to a tariff (rather than just a labor subsidy) as a policy instrument. As in the optimal tariff literature, from the standpoint of the Home economy, the undistorted equilibrium is one in which the policymaker restricts output of the Home economy in order to exploit its monopoly power in the production of the Home good. By using a tariff to achieve this goal, the Home policymaker leaves the Home economy undistorted, but imposes an international distortion by driving a wedge between relative prices that Home agents face and relative prices in the rest of the world.

From that perspective, there are two imperfections in our economy with balanced trade. There is the lack of asset trade, but also the policymaker lacks the type of instrument that can reproduce the optimal tariff policy. Without access to a tariff the policymaker (regardless of market completeness) cannot drive a wedge between the prices faced at Home and in the rest of the world. By analogy, monetary policy also lacks the ability to manipulate Home versus rest of the world relative prices.

The non-cooperative policymaker would like to minimize the output gap, and in addition has the goal of manipulating the terms of trade as a monopolist would, but also must be mindful of the internal distortion introduced by manipulating the terms of trade without access to a tariff.

The tax or wedge under complete markets can be expressed in terms of the terms of trade:

$$(33) \quad \bar{\tau} = \frac{-(\sigma-1)(2-\nu)(\nu-1)}{2(1+(\sigma-1)(2-\nu))} \frac{1}{s}.$$

From (33), we can see that under complete markets, a policy that is introduced to influence the terms of trade must necessarily lead to an internal distortion (as it would in a monetary sticky-price model as well.) The policymaker under complete markets that does not have access to a tariff must trade off its objectives for the terms of trade with its desire to drive the labor market wedge to zero.

Why did we not examine the optimal policy using the tariff instrument instead of the subsidy to employment? In general, policymakers have few levers that are flexible enough that they can respond in a timely way to business cycle fluctuations. Changing tariffs in most countries is a time-consuming process, especially for countries that must abide by WTO regulations. But is changing the employment subsidy a more viable option? The answer is probably not, but we examine that case because of the analogy between an employment subsidy in this model, and monetary policy in a Keynesian model. Monetary policy certainly is a flexible tool, at least when central banks have independence and so are not constrained by the political process. In a Keynesian model, households adjust consumption and labor input so that the marginal rate of substitution between leisure and consumption equals the real wage. However, firms in the short run that have set nominal prices determine employment by hiring the amount of workers required to produce the amount of output that is demanded. In other words, they do not hire workers up until the point where the marginal product of labor equals the real wage. Monetary policy can alter aggregate demand for goods, and therefore alter the gap between the marginal product of labor and the marginal rate of substitution of leisure for consumption. In the model here, employment subsidies play a similar role in driving this labor wedge.

We can conclude that compared to the cooperative case, the strategic policymaker is concerned not with the sharing of risk, but with using its market power to shift consumer's purchasing power favorably. In the cooperative case, we saw that the risk-sharing distortion, defined by  $m$  above, was one concern of the policymaker. Equations (26) and (27) show how the global policymaker trades off  $m$  with the Home and Foreign output gaps. In the non-cooperative case, each policymaker is not concerned with this risk-sharing distortion. Instead, they want to manipulate the terms of trade in their favor, and do not weigh the cost that imposes on the households in the other country. Equation (32) shows that the Home policymaker trades off the



output gap not with a risk-sharing distortion, but instead with the deviation of the terms of trade from what it would achieve optimally under complete markets.

Put another way, while the cooperative policymaker would like to maximize the sum of expected utility in the Home and Foreign countries, this is not equivalent to minimizing the sum of the Home and Foreign policymakers' loss functions. From the Home perspective, for example, in the non-cooperative case, it is a loss when the terms of trade are not equal to the optimal terms of trade taking the Foreign policy as given. The Home loss function does not incorporate the spillovers on the Foreign country, nor vice-versa. From the cooperative standpoint, however, the objective is not to achieve a terms of trade objective, but rather to ameliorate the risk-sharing distortion (trading that off with the output objectives.)

### *Discussion*

There has been a widespread concern about the spillovers of Federal Reserve policy, and to a lesser extent, European Central Bank policy, in recent years. The flashpoint of that discussion was the comment in September 2010 by Guido Mantega, the Brazilian finance minister, that “we are in the middle of a currency war”<sup>10</sup>, referring to the depreciation of the major currencies against those of Brazil and other emerging markets. In April 2014, Raghuram Rajan, the governor of the Reserve Bank of India, noted that the major countries were not sufficiently considering the effects of spillovers of their monetary policies, noting the “initiation of unconventional policy as well as an exit whose pace is driven solely by conditions in the source country,” specifically aiming his remarks at monetary policy in the U.S. and other industrial countries that “hold interest rates near zero for long, as well as balance sheet policies such as quantitative easing or exchange intervention, that involve altering central bank balance sheets in order to affect certain market prices.”<sup>11</sup> Olivier Blanchard, Jonathan Ostry and Atish Ghosh of the International Monetary Fund observe “if large players in the global economy are responsible for significant adverse spillovers across a swath of smaller countries, this needs to be acknowledged as well, and feasible remedies considered.”<sup>12</sup>

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<sup>10</sup> See Financial Times, 2010.

<sup>11</sup> See Rajan, 2014.

<sup>12</sup> Blanchard, Ostry and Ghosh, 2013.

The response by the Federal Reserve has been to note that their legal mandate is to keep prices stable and unemployment low, while recognizing the potential for spillovers to the global economy. Ben Bernanke (2012) notes at the outset of his address that “All of the Federal Reserve's monetary policy decisions are guided by our dual mandate to promote maximum employment and stable prices.” He goes on to argue that nonetheless “it is not at all clear that accommodative policies in advanced economies impose net costs on emerging market economies, for several reasons.” He argues that “by boosting U.S. spending and growth, it has the effect of helping support the global economy as well.”

Stanley Fischer (2014) states “the Federal Reserve's objectives are given by its dual mandate to pursue maximum sustainable employment and price stability, and our policy decisions are targeted to achieve these dual objectives.” He goes on to observe “The state of the U.S. economy is significantly affected by the state of the world economy. A wide range of foreign shocks affect U.S. domestic spending, production, prices, and financial conditions. To anticipate how these shocks affect the U.S. economy, the Federal Reserve devotes significant resources to monitoring developments in foreign economies, including emerging market economies (EMEs), which account for an increasingly important share of global growth. The most recent available data show 47 percent of total U.S. exports going to EME destinations. And of course, actions taken by the Federal Reserve influence economic conditions abroad. Because these international effects in turn spill back on the evolution of the U.S. economy, we cannot make sensible monetary policy choices without taking them into account.”

Neither Bernanke nor Fischer claim that U.S. policy is cooperative. They both note that the Federal Reserve tries to achieve its internal objectives. It does not ignore the rest of the world, but it does not alter its policies to take into account their effect on the rest of the world, except insofar as global repercussions can then feed back onto the U.S. economy. Non-cooperative policy does not mean that the policymaker is actively trying to take advantage of other economies. It means exactly that it is concerned with only its own objectives when setting policy rules. In that sense, U.S. monetary policy fits into the non-cooperative framework.

However, note that the Federal Reserve policy is aimed at “employment and price stability.” We have seen that the targeting rule in the global economy involves trading off the employment (or output) objective with a terms of trade objective. (Inflation is not present in our static model, but in a dynamic version of this model with staggered pricing, the targeting rule

would surely trade off inflation as well.) One interpretation of the remarks by Bernanke and Fischer is that while policy is not coordinated, the Fed also ignores the terms of trade objective. It does not attempt to exploit its monopoly power in export markets.

The exploitation of that monopoly power involves limiting the terms of trade fluctuations, at least in response to the productivity changes that are the exogenous shocks in this model. When output is short of potential, monetary policy needs to be expansionary but the strategic central bank will dampen the potential depreciation needed to reach full employment. In other words, Mantega is unlikely to be satisfied with a U.S. monetary policy that focuses on employment and inflation, because it is still the case that the expansionary effects of U.S. monetary policy during times of employment shortfalls may have negative spillovers on Brazil through the exchange rate channel.

In the next section, we compare the extent of spillovers under cooperative policy versus non-cooperative policy.

#### 4. Spillovers

In this section, we solve the model under complete markets and incomplete markets, with cooperation and without cooperation, in order to understand the nature of spillovers. Our focus will be on the spillovers to consumption, but similar analysis would apply to output.

It is important to note that our focus is on spillovers – the elasticity of consumption in one country with respect to a shock in the other country – but not on the overall level of consumption. As is well known, the level of consumption is lower under non-cooperation. Our analysis is couched in terms of the effects of shocks on the deviation of the log of consumption from its steady-state value, but the steady-state value of consumption is lower under non-cooperation compared to cooperation. In particular, under cooperation, the steady state value of Home or Foreign consumption is given by  $C = k^{-1/\sigma}$  (when productivity levels in both countries take on a value of one.) But under non-cooperation,  $C = \left( k^{-1} \frac{2(\nu-1)}{\nu} \right)^{1/\sigma}$  which is strictly less than the value under cooperation, as long as there is not complete home bias.

We begin our analysis by noting that, by definition,  $c = c^R + c^W$ . In all cases – complete markets, incomplete, cooperation or non-cooperation – we find

$$c^W = a^W / \sigma .$$

Whether there is cooperation or not, and independently of the completeness of financial markets, under optimal policy, the elasticity of consumption with respect to the “world” shock is given by  $1/\sigma$ . Any differences in the spillovers of Foreign productivity shocks to Home consumption comes through differences in the elasticity of  $c^R$  with respect to  $a^R$ .

It is useful to begin by considering the solution for Home consumption under complete markets with cooperation, because allocations are globally efficient in this case. There we find:

$$c^R = \frac{\nu - 1}{\sigma} a^R .$$

If Home and Foreign households had the same consumption preferences, so  $\nu = 1$ , there would be no difference in their consumption in equilibrium under the optimal policy:  $c^R = 0$ . Relative shocks have an effect on relative consumption in the efficient allocation because of home bias in preferences. Each country’s consumption is more susceptible to its own country’s productivity shocks because each country consumes more of its own output.

Under non-cooperation (and complete markets),

$$c^R = \frac{(\nu - 1)(1 + (\sigma - 1)(2 - \nu))}{\sigma(1 + (\sigma - 1)\nu(2 - \nu))} a^R .$$

In the case of home bias and in the empirically plausible case of  $\sigma > 1$ , the elasticity of  $c^R$  with respect to  $a^R$  is *smaller* under non-cooperation. In this sense, spillovers are smaller when policymakers follow strategic objectives rather than cooperate.

To understand this, note that in equilibrium, under complete markets, from (16),

$$s = \frac{2\sigma}{\nu - 1} c^R .$$

Here we have used the equilibrium result  $Ec = Ec^* = 0$ . As we have discussed above, the terms of trade fluctuations are smaller under non-cooperation compared to cooperation, because the strategic policymaker acts as a monopolist to limit the response of its price to supply shocks.

Notice that in the case of complete home bias in preferences ( $\nu = 2$ ) the spillovers are the same under cooperation and non-cooperation. That is not surprising, because in this instance, the two economies are closed, and so cooperation is irrelevant. At the other extreme, we can take the case of no home bias, so  $\nu = 1$ , to proxy for complete openness of markets. In that case,  $c^R = 0$

under both cooperation and non-cooperation. Under complete risk sharing, if the consumption baskets are the same, then as we noted above, aggregate consumption is equalized across households.

It is therefore only at intermediate levels of openness that strategic policymaking leads to smaller spillovers. As an example, set  $\sigma = 2$ , which is a standard calibration of the coefficient of relative risk aversion. The effect of productivity shocks on relative consumption ( $dc^R / da^R$ ) under non-cooperation, relative to under cooperation, is maximized when  $\nu = 1.59$ , so that about 80% of expenditures fall on goods produced within the country.

There is, however, another way to view spillovers. When we take the elasticity of  $c^R$  with respect to  $a^R$  as a measure of spillovers, we are holding constant the effect the Foreign shock has on Home consumption through its effect on  $c^W$ . In general, a shock to  $a^*$  will have effects on both  $a^R$  and  $a^W$ . The latter will be unaffected only when a shock to  $a$  exactly offsets the  $a^*$  disturbance. We can represent the solutions for Home consumption under cooperation and non-cooperation as:

$$c_j^{COOP} = b^C a_j^R + \frac{1}{\sigma} a_j^W$$

$$c_j^{NON} = b^N a_j^R + \frac{1}{\sigma} a_j^W ,$$

where  $b^C$  and  $b^N$  respectively, refer to the coefficients on  $a^R$  in the solutions for the cooperative and non-cooperative cases above, respectively. We have seen that  $b^C > b^N > 0$ , so that the relative shock has a larger effect on consumption in the cooperative case. But the key thing to recognize is that the equilibrium terms of trade movements serve to modify the effect of the Foreign shock on Home consumption. If there is a decline in  $a^*$ , then there will be a global drop in consumption, but the relative price of the Foreign good,  $s$ , increases. This works to increase Home consumption relative to Foreign consumption, thus modifying the impact of the shock on Home consumption. The overall transmission is positive – a decline in Foreign productivity leads to a decline in both Foreign and Home consumption. This is true under both cooperation, so  $\frac{1}{\sigma} - b^C > 0$ , and under non-cooperation,  $\frac{1}{\sigma} - b^N > 0$ . It follows from  $b^C > b^N > 0$  that the overall spillovers are smaller under cooperation than under non-cooperation:

$$\frac{1}{\sigma} - b^N > \frac{1}{\sigma} - b^C > 0.$$

The same analysis holds for incomplete markets. There we find:

$$b^N = \frac{\nu - 1}{\sigma - (\sigma - 1)(2 - \nu)}$$

$$b^C = \frac{\nu - 1}{\sigma - (\sigma - 1)\nu(2 - \nu)}.$$

The spillovers from relative shocks are greater under cooperation:  $b^C > b^N > 0$ . But the spillovers from idiosyncratic shocks are smaller under cooperation:  $\frac{1}{\sigma} - b^N > \frac{1}{\sigma} - b^C > 0$ .

Productivity shocks in one country have a significant effect on the world productivity level only when the country is large in the global economy. But our model of strategic interaction relies on some countries being large. If a country is too small, and does not form a cartel with other countries to set policy, then there is no strategic element to policymaking. However, as in de Paoli (2009), a country could be small relative to overall global output, but be a large player in the market for its exports.

So there is some subtlety required in thinking about whether cooperation reduces international spillovers. Under non-cooperation, the terms of trade movements in response to supply shocks are smaller than under cooperation because of the duopolistic behavior of strategic policymakers. However, since terms of trade fluctuations serve to modify the transmission of shocks that comes through their effects on global productivity, the net spillovers are smaller under cooperation. Moreover, these findings are specific to the simple set-up of this model, and hence are illustrative rather than definitive.

It is important to emphasize that this is not a welfare analysis. Here we are only addressing the question of whether we can reasonably expect cooperation to reduce spillovers, not whether the outcome is desirable.

## 5. Conclusions

The central objective of this study was to derive a targeting rule for monetary policy in a non-cooperative, strategic environment, and to compare it with the targeting rule under cooperation. In order to do so, we built a very simple static model. The targeting rule is based on

a comparison of optimal policy under incomplete markets and complete financial markets. The model is far too stylized to draw realistic conclusions about the conduct of monetary policy. It is, perhaps, a building block for richer models.

In particular, of course, the model is static. In a dynamic model of incomplete markets, the relative wealth positions of countries change over time. In addition, in a monetary version of the model, staggered nominal price setting will introduce distortions as in standard New Keynesian models. The targeting rules derived here will certainly need to be altered to take into account these dynamic features.<sup>13</sup>

Engel (2015) surveys some of the challenges for the academic literature on non-cooperative monetary policy. Here, briefly, we make note of three.

Policymakers appear to be very concerned about the effects of monetary policy on the stability of capital flows. Rajan (2014), for example, states “ideally, recipient countries would wish for stable capital inflows, and not flows pushed in by unconventional policy.” Benoit Couéré of the Executive Board of the European Central Bank asserts “The volatility in capital flows to and from emerging economies over the last year was also mainly related to expectations of US monetary policy.” Fischer (2014) observes “EME critics argued that U.S. policy accommodation contributed to a surge of capital inflows and excessive credit growth in their economies, creating risks of financial instability. But, as time wore on, most EMEs seemed glad to receive those flows.” Most modern open-economy macroeconomic models do not accord a very prominent role for capital flows. There may be rebalancing of portfolios of assets in response to shocks, but typically in monetary policy analysis those shifts are kept in the background. In any case, it is not clear from the perspective of these models why capital flows present a problem. The models apparently do not capture the concerns of policymakers.

Second, the literature, including this paper, treats monetary policy coordination as if a single monetary policymaker would choose policy to maximize the joint welfare of all cooperating countries. In practice, it is more likely that further coordination would be reached as a result of meetings among monetary policymakers, where each policymaker enters the room hoping to improve his own country’s welfare but recognizing that there are gains to be made from cooperation. Perhaps a more fruitful model of such a process would be a bargaining game.

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<sup>13</sup> See Corsetti et al. (2011b), Engel (2014) and Bhattarai et al. (2015) for examples of dynamic two-country New Keynesian models with incomplete markets, where policymaking under cooperation is considered.

Such a model could explore the roles of the smaller economies and the potential gains from forming coalitions.

Third is the possibility of political influences in setting monetary policy. Couéré points to this: “The first factor, which is often ignored in the academic literature, is the political economy of coordination. In the real world, the ability of central banks to deviate from a pre-determined path of action differs across countries. Central banks operate in different economic structures and institutional set-ups, notably in terms of mandates, time horizons and objectives. There are also differences in accountability arrangements and in economic and political cycles. All of this affects domestic policy incentives.” In the literature on international trade policies, the policymaker is modeled as facing a tradeoff between domestic welfare objectives and political constraints. That approach may be fruitful in understanding monetary policy as well.

Fischer (2014) observes “My teacher Charles Kindleberger argued that stability of the international financial system could best be supported by the leadership of a financial hegemon or a global central bank. But I should be clear that the U.S. Federal Reserve System is not that bank. Our mandate, like that of virtually all central banks, focuses on domestic objectives. As I have described, to meet those domestic objectives, we must recognize the effect of our actions abroad, and, by meeting those domestic objectives, we best minimize the negative spillovers we have to the global economy.” It is clear that non-cooperative policy entails focusing on domestic objectives. Does the Fed, in fact, pursue those objectives to the fullest, including exploiting monopoly power in the terms of trade? Or, conversely, do central banks implicitly cooperate in their policy, recognizing the dangers of pursuing purely strategic policy? These are surely questions for further theoretical and empirical research.



## References

- Benigno, Gianluca, and Pierpaolo Benigno. 2003. "Price Stability in Open Economies." *Review of Economic Studies* 70, 743-764.
- Benigno, Pierpaolo. 2002. "A Simple Approach to International Monetary Policy Coordination." *Journal of International Economics* 57, 177-196.
- Bernanke, Ben S. 2012. "U.S. Monetary Policy and International Implications." Board of Governors of the Federal Reserve System. Speech at the "Challenges of the Global Financial System: Risks and Governance under Evolving Globalization," sponsored by Bank of Japan-International Monetary Fund, Tokyo, Japan
- Bhattarai, Saroj; Jae Won Lee; and, Woong Yong Park. 2015. "Optimal Monetary Policy in a Currency Union with Interest Rate Spreads." *Journal of International Economics* 96, 375-397.
- Blanchard, O.; J.D. Ostry; and, A.R. Ghosh. 2013. "International Policy Coordination: The Loch Ness Monster." International Monetary Fund, IMFDirect.
- Canzoneri, Matthew; Robert Cumby; and, Behzad Diba. 2005. "The Need for International Policy Coordination: What's Old? What's New? and What's Yet to Come?" *Journal of International Economics* 66. 363-384.
- Canzoneri, Matthew, and Dale Henderson. 1991. *Monetary Policy in Interdependent Economies: A Game Theoretic Approach* (MIT Press).
- Clarida Richard; Jordi Gali; and, Mark Gertler 2002. "A Simple Framework for International Monetary Policy Analysis. *Journal of Monetary Economics* 49, 879-904.
- Cooper, Richard. 1985. "Economic Interdependence and Coordination of Economic Policies." In Ronald Jones and Peter Kenen, eds., *Handbook of International Economics, vol. 2* (North-Holland).
- Corsetti, Giancarlo, and Paolo Pesenti. 2005. "International Dimensions of Optimal Monetary Policy." *Journal of Monetary Economics* 52, 281-305.
- Corsetti, Giancarlo; Luca Dedola; and, Sylvain Leduc. 2011a. "Optimal Monetary Policies in Open Economies." In Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics, vol. 3* (Elsevier).
- Corsetti, Giancarlo; Luca Dedola; and, Sylvain Leduc. 2011b. "Demand Imbalances, Exchange-Rate Misalignments and Monetary Policy." Working paper, Cambridge University.
- de Paoli, Bianca. 2009. "Monetary Policy and Welfare in a Small Open Economy." *Journal of*

*International Economics* 77, 11-22.

- Devereux, Michael B., and Charles Engel. 2003. "Monetary Policy in the Open Economy Revisited: Exchange Rate Flexibility and Price Setting Behavior." *Review of Economic Studies* 70, 765-803.
- Engel, Charles. 2011. "Currency Misalignments and Optimal Monetary Policy: A Reexamination." *American Economic Review* 101, 2796-2822.
- Engel, Charles. 2014. "Exchange Rate Stabilization and Welfare." *Annual Review of Economics* 6, 155-177.
- Engel, Charles. 2016. "International Coordination of Central Bank Policy." *Journal of International Money and Finance*, forthcoming.
- Engel, Charles, and Akito Matsumoto. 2009. "The International Diversification Puzzle when Goods Prices are Sticky: It's Really about Exchange-Rate Hedging, not Equity Portfolios." *American Economic Journal: Macroeconomics* 1, 155-188.
- Financial Times. 2010. "Brazil in 'currency war' Alert." September 27, 2010.
- Fischer, Stanley. 2014. "The Federal Reserve and the Global Economy." Board of Governors of the Federal Reserve System. Speech at the Per Jacobsson Foundation Lecture, 2014 Annual Meetings of the International Monetary Fund and the World Bank Group, Washington, D.C.
- Gali, Jordi, and Tommaso Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *Review of Economic Studies* 72, 707-734.
- Hamada, Koichi. 1974. "Alternative Exchange Rate Systems and the Interdependence of Monetary Policies." In Robert Aliber, ed. *National Monetary Policies and the International Financial System* (Chicago).
- Hamada, Koichi. 1976. "A Strategic Analysis of Monetary Interdependence." *Journal of Political Economy* 84, 677-700.
- Hamada, Koichi. 1979. "Macroeconomic Strategy and Coordination under Alternative Exchange Rates." In Rudiger Dornbusch and Jacob Frenkel, eds. *International Economic Policy* (Johns Hopkins University Press).
- Heathcote, Jonathan, and Fabrizio Perri. 2013. "The International Diversification Puzzle is Not as Bad as You Think." *Journal of Political Economy* 121, 1108-1159.

- Obstfeld, Maurice, and Kenneth Rogoff. 1995. "Exchange Rate Dynamics Redux." *Journal of Political Economy* 103, 624-660.
- Obstfeld, Maurice, and Kenneth Rogoff. 1996. *Foundations of International Macroeconomics* (MIT Press).
- Obstfeld, Maurice, and Kenneth Rogoff. 2000. "New Directions for Stochastic Open Economy Models." *Journal of International Economics* 50, 117-153.
- Obstfeld, Maurice, and Kenneth Rogoff. 2002a. "Risk and Exchange Rates." In Elhanan Helpman and Efraim Sadka, eds., *Contemporary Economic Policy: Essays in Honor of Assaf Razin* (Cambridge University Press).
- Obstfeld, Maurice, and Kenneth Rogoff. 2002b. "Global Implications of Self-Oriented National Monetary Rules." *Quarterly Journal of Economics* 117, 503-535.
- Ostry, Jonathan D., and Atish R. Ghosh. 2013. "Obstacles to International Policy Coordination and How to Overcome Them." Working paper, Research Department, International Monetary Fund.
- Pappa, Evi. 2004. "Do the ECB and the Fed Really Need to Cooperate? Optimal Monetary Policy in a Two-Country World." *Journal of Monetary Economics* 51, 753-779.
- Persson, Torsten, and Guido Tabellini. 1995. "Double-Edged Incentives: Institutions and Policy Coordination." In Gene Grossman and Kenneth Rogoff, eds., *Handbook of International Economics, vol. 3* (North-Holland).
- Rajan, Raghuram. 2014. "Competitive Monetary Easing: Is It Yesterday Once More?" Reserve Bank of India.
- Senay, Ozge, and Alan Sutherland. 2007. "Optimal Monetary Policy and the Timing of Asset Trade in Open Economies." *Economic Letters* 95, 297-302.
- Senay, Ozge, and Alan Sutherland. 2013. "The Timing of Asset Trade and Optimal Policy in Dynamic Open Economies." *Macroeconomic Dynamics* 17, 1543-1573.

## Appendix

### *Derivation of equilibrium under complete markets*

Here, we solve for the ratio of Lagrange multipliers,  $\lambda / \lambda^*$ , in equation (10).<sup>14</sup> From the zero profit condition under free entry in competitive markets and the budget constraint (4), using the definition of profits (7), we have:

$$P_j C_j = P_{H,j} Y_j + D_j.$$

Then using the equilibrium condition (12) and the demand equations (2) and (3), we can write this equation as:

$$P_j C_j = \frac{\nu}{2} P_j C_j + \frac{2-\nu}{2} E_j P_j^* C_j^* + D_j$$

Multiply by the price of contingent claims, sum up over states, and recall  $\sum Z_j D_j = 0$ , to get:

$$\sum Z_j P_j C_j = \sum Z_j E_j P_j^* C_j^*.$$

Then use the first-order conditions (5) and (6) to write

$$Z_j = \frac{\pi_j C_j^{-\sigma}}{\lambda P_j} \quad \text{and} \quad Z_j = \frac{\pi_j C_j^{*-\sigma}}{\lambda^* E_j P_j^*}.$$

Substituting into the previous expression, we find:

$$\begin{aligned} \frac{1}{\lambda} \sum \pi_j C_j^{1-\sigma} &= \frac{1}{\lambda^*} \sum \pi_j C_j^{*1-\sigma}, \text{ or} \\ (34) \quad \frac{\lambda}{\lambda^*} &= \frac{\sum \pi_j C_j^{1-\sigma}}{\sum \pi_j C_j^{*1-\sigma}} = \frac{E(C^{1-\sigma})}{E(C^{*1-\sigma})}. \end{aligned}$$

### *Solution to model in terms of productivity shocks and labor subsidies*

The equivalent equation to (11) for the Foreign country is given by:

$$(35) \quad 1 = (1 - \tau_j^*) k A^{*-1} S^{\frac{\nu-2}{2}} C^{*\sigma}$$

Using this equation and (11), we find:

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<sup>14</sup> These steps draw on the Appendix of Devereux and Engel (2003).

$$(36) \quad \frac{A}{A^*} S^{\nu-2} = \frac{(1-\tau) C^\sigma}{(1-\tau^*) C^{*\sigma}}.$$

Then using equation (18), the trade-balance equation, we can derive:

$$(37) \quad S = \left( \frac{A/(1-\tau)}{A^*/(1-\tau^*)} \right)^{\frac{1}{\sigma(\nu-1)+2-\nu}}.$$

Substitute this equation back into (11) and we find:

$$(38) \quad C = k^{\frac{-1}{\sigma}} \left( \frac{A}{1-\tau} \right)^{\frac{2\sigma(\nu-1)+2-\nu}{2\sigma(\sigma(\nu-1)+2-\nu)}} \left( \frac{A^*}{1-\tau^*} \right)^{\frac{2-\nu}{2\sigma(\sigma(\nu-1)+2-\nu)}}$$

Analogously, for the Foreign country, we have:

$$(39) \quad C^* = k^{\frac{-1}{\sigma}} \left( \frac{A^*}{1-\tau^*} \right)^{\frac{2\sigma(\nu-1)+2-\nu}{2\sigma(\sigma(\nu-1)+2-\nu)}} \left( \frac{A}{1-\tau} \right)^{\frac{2-\nu}{2\sigma(\sigma(\nu-1)+2-\nu)}}.$$

Under complete markets, we can combine equation (36) with the condition for equilibrium (16) to find:

$$(40) \quad \frac{C}{C^*} = \left( \frac{A/(1-\tau)}{A^*/(1-\tau^*)} \right)^{\frac{\nu-1}{\sigma}} \left( \frac{E(C^{1-\sigma})}{E(C^{*1-\sigma})} \right)^{\frac{\nu-2}{\sigma}}$$

We can use this to derive:

$$(41) \quad S = \frac{A/(1-\tau) E(C^{1-\sigma})}{A^*/(1-\tau^*) E(C^{*1-\sigma})}$$

Also, using (11), (35), and (40) and (41), we find:

$$(42) \quad C = k^{\frac{-1}{\sigma}} \left( \frac{A}{1-\tau} \right)^{\frac{\nu}{2\sigma}} \left( \frac{A^*}{1-\tau^*} \right)^{\frac{2-\nu}{2\sigma}} \left( \frac{E(C^{1-\sigma})}{E(C^{*1-\sigma})} \right)^{\frac{\nu-2}{2\sigma}}$$

$$(43) \quad C^* = k^{\frac{-1}{\sigma}} \left( \frac{A}{1-\tau} \right)^{\frac{2-\nu}{2\sigma}} \left( \frac{A^*}{1-\tau^*} \right)^{\frac{\nu}{2\sigma}} \left( \frac{E(C^{1-\sigma})}{E(C^{*1-\sigma})} \right)^{\frac{2-\nu}{2\sigma}}$$

From these expressions, we can solve:

$$\frac{E(C^{1-\sigma})}{E(C^{*1-\sigma})} = \left\{ \frac{E \left[ \left( \frac{A}{1-\tau} \right)^{\frac{\nu(1-\sigma)}{2\sigma}} \left( \frac{A^*}{1-\tau^*} \right)^{\frac{(2-\nu)(1-\sigma)}{2\sigma}} \right]}{E \left[ \left( \frac{A^*}{1-\tau^*} \right)^{\frac{\nu(1-\sigma)}{2\sigma}} \left( \frac{A}{1-\tau} \right)^{\frac{(2-\nu)(1-\sigma)}{2\sigma}} \right]} \right\}^{\frac{\sigma}{2-\nu+\sigma(\nu-1)}} .$$

This expression can be substituted into (41), (42) and (43) to complete the characterization of the solution.

As noted in the text, there are two well-known cases in which the balanced-trade and complete-markets solution are the same. The first is when  $\sigma = 1$ , in which case we find

$$S = \frac{A/(1-\tau)}{A^*/(1-\tau^*)},$$

$$C = k^{-1} \left( \frac{A}{1-\tau} \right)^{\frac{\nu}{2}} \left( \frac{A^*}{1-\tau^*} \right)^{\frac{2-\nu}{2}},$$

$$C^* = k^{-1} \left( \frac{A^*}{1-\tau^*} \right)^{\frac{\nu}{2}} \left( \frac{A}{1-\tau} \right)^{\frac{2-\nu}{2}}.$$

The second case is when  $\nu = 1$ , in which case we find

$$S = \frac{A/(1-\tau)}{A^*/(1-\tau^*)},$$

$$C = C^* = k^{\frac{-1}{\sigma}} \left( \frac{A}{1-\tau} \right)^{\frac{1}{2\sigma}} \left( \frac{A^*}{1-\tau^*} \right)^{\frac{1}{2\sigma}}.$$