Valuing Time-Varying Attributes using the Hedonic Model: When is a Dynamic Approach Necessary?*

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Abstract

Despite the fact that dynamic models of location choice are well-motivated in the literature (as the housing market features large moving costs and predictability of amenities), it remains that the estimation of these models is not without substantial costs to the econometrician in terms of computation and/or additional data requirements. In this paper, we build off of the intuitive (static) modeling framework of Rosen (1974) and specify a simple, forward-looking model of location choice. We use this model, along with a series of insightful graphs, to describe more fully the potential biases associated with the static approach and relate this bias to the time-series trend of the amenity of interest. This allows the researcher to determine, *a priori*, the value of extending the empirical analysis to a forward-looking approach. In addition, we illustrate an empirically-relevant example where, despite a time-varying amenity, the static model and a forward-looking model arrive at the same estimate of willingness to pay. Finally, we apply these models to estimate the willingness-to-pay to avoid violent crime. Recovering estimates separately by (geographic) area allows us to illustrate various degrees of bias induced by assuming myopic agents.

Key Words: Hedonic Demand, Dynamics, Valuation, Willingness to Pay
JEL Classification Numbers: Q50, Q51, R21, R23

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1 Introduction

Based on the first-order conditions for utility maximization, Rosen's 1974 model of hedonic demand is both simple and intuitive. In addition, given the simple modeling framework, the standard hedonic model is straightforward to estimate, with estimation usually consisting of a single least-squares regression of the housing price function to obtain a local measure of willingness-to-pay. For this reason (combined with the increasing availability of housing transactions data), this approach has been used to value a myriad of local public and private goods, including school quality (Black (1999), Downes and Zabel (2002), Gibbons and Machin (2003)), climate (Albouy, Graf, Kellogg, and Wolff (2013)), safety (Gayer, Hamilton, and Viscusi (2000), Davis (2004), Greenstone and Gallagher (2008)), and environmental quality (Harrison and Rubinfeld (1978), Palmquist (1982), Chay and Greenstone (2005)).

An implicit assumption underlying the traditional model is that households are not forward-looking with respect to potentially time-varying housing and neighborhood amenities. Given the significant costs associated with purchasing a house, it is unlikely that households do not consider future levels of local amenities. When households are forward-looking in this manner, the traditional model will yield biased estimates of willingness-to-pay in most, but not all, cases. Recent literature has empirically quantified this bias in specific applications, comparing results from the traditional model to those obtained using fully-dynamic models of location choice (Bishop (2012), Bayer, McMillan, Murphy, and Timmins (2015), and Bishop and Murphy (2015)).

However, the estimation of dynamic models is not without substantial computational cost, even with the recent advances in the literature. Furthermore, fully dynamic models often require richer data. Therefore, we seek to describe more fully the potential bias associated with the static approach and to relate this bias to the time-series trend of the amenity of interest. In addition, we illustrate an empirically-relevant example where, despite a time-varying amenity, the static model and a forward-looking model arrive at the same estimate of willingness to pay. Thus, in this paper, we present a comprehensive description of the benefit of extending the empirical analysis to a forward-looking approach, allowing the researcher to determine, *a priori*, the relative costs.

As previously stated, the estimation procedure of the traditional Rosen model is quite intuitive. In a first stage, the parameters of the housing price function (and, at the same time, the parameters of the hedonic price gradient) are recovered through a regression of observed housing prices on amenities. In a second stage, the econometrician is able to back-out the implicit price of the amenities that each household *actually paid* using the family's observed consumption of the amenity and the parameters of the hedonic price gradient. The information provided by the first-order condition for utility maximization (i.e., that marginal cost will equal marginal benefit) allows the econometrician to invert this implicit price and arrive at the household's marginal willingness to pay for the amenity.

Thus, the basic premise of the Rosen model is to recover estimates of marginal willingness to pay from the information on (i) the quantity of the amenity the household chose to consume and (ii) the price schedule faced by the household. However, if households are choosing where to live based on some average stream of future amenities versus solely based on a measure of current amenities, the traditional model will get both (i) and (ii) wrong, resulting in biased estimates of willingness to pay. It is straightforward to see that by using the incorrect measure of quantity consumed the static model will either under- or over-attribute the true 'quantity' purchased. We refer to this as the quantity effect. Less obvious is what we refer to as the price effect: if the econometrician is recovering the implicit price of the amenity through the calculation of the price differential under quantity mismeasurement, the implicit price of the amenity will also be under- or over-stated. Using these notions of quantity- and price- effects, we seek to describe more fully the potential bias associated with the static approach and relate this bias to the time-series properties of the amenity of interest.

Within this framework, we show that in cases where the location-specific amenity levels are converging over time within the choice set, the traditional static model will typically understate willingness-to-pay (i.e., coefficients will be biased toward zero). For example, consider a household that purchases a house with a relatively low level of amenity in the current period. As a result of the convergence, the average stream of the consumed amenity will be higher than the current level. In addition, the true implicit price of the amenity will be relatively higher compared with the static approach. In other words, the household is buying relatively more of the amenity at a relatively higher price and the true willingness to pay is higher than that implied by the static model.[1] Analogously, the traditional model will typically overestimate willingness-to-pay (i.e., coefficients will be biased away from zero) in cases where the levels of the amenity are diverging across locations.[2] Intuitively, the size of

[1]If the household were consuming a relatively high amount of the amenity, the quantity and price effects go in opposite directions. That is, relative to the static model, the household is actually consuming less of the amenity but at a higher price.

[2]Consider again the household that purchases a house with a low level of an amenity at present. Now,

the bias is determined by how quickly the amenity is diverging through time.

Following this quantity- and price-effect intuition, we present a result where the forward-looking model and the static model yield the same estimate of willingness-to-pay, even when the amenity is evolving rapidly through time. This arises when the amenity of interest is rising or falling without converging or diverging,[3] thereby causing the effect of misspecified quantity to exactly offset the effect of misspecified price in the static model. For example, if each neighborhood received a one-unit increase in the amenity, then amenity levels would be increasing over time and the static model would understate quantity and overstate price. We show that in the specific case of linear utility, these effects exactly offset one another and the static model yields unbiased estimates of willingness to pay.

Finally, we show that in the simple model of linear utility, an adjustment factor may be easily derived. This adjustment factor, which is based on the time-series properties of the amenity of interest, can be then be used to convert the biased static estimates into the estimates that one would have obtained using a forward-looking model. Using a rich data set on housing transactions, we apply our adjustment measure to illustrate examples where the static model yields large biases and examples where the static model yields small biases. In particular, we use data from the Bay Area to estimate the marginal willingness to pay to avoid crime. We calculate the bias separately by county and find the static model produces a small bias in Alameda county while understating the willingness to pay to avoid crime by a factor of two in both Marin and San Mateo counties.

The remainder of the paper is organized as follows: Section 2 describes the traditional static model as well as a simple forward-looking model of hedonic demand; Section 3 describes the bias induced by the misspecified model under various transitions of the amenity of interest and provides guidance to answer the normative question of "when is the static model sufficient?"; Section 4 applies the framework to estimate the willingness to pay to avoid crime; and, finally, Section 5 concludes.

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both the consumption and the true implicit price of the amenity will be overstated by the static model. In other words, the household is buying relatively less of the amenity and paying a lower price and as such, the true willingness to pay is lower.

[3] In other words, the amenity follows a variance-preserving change-in-mean over time.

2 Model

In this section, we provide an overview of the traditional, Rosen-style static model, as well as a simple, forward-looking model of willingness-to-pay.

2.1 The Traditional, Static Model of Willingness to Pay

We first consider a static model of willingness to pay for a house or neighborhood amenity. In this model, households maximize current utility with respect to their choice of amenity consumption. Implicitly assumed in this modelling framework is the assumption of free mobility (or zero transaction costs). This means that households can freely reoptimize at the beginning of each period, so the problem of maximizing lifetime utility may be described as a series of independent, sequential decisions.

We begin by choosing a simple specification of household utility where household i has an individual-specific preference parameter, α_i, describing their preference for consumption of the amenity of interest, x_i. The household also receives utility from the consumption of the numeraire good, C_i.

$$U(x_i) = \alpha_i x_i + C_i \tag{1}$$

For simplicity, we consider a model where utility is increasing at a linear rate in the amenity x. Broadly speaking, the intuition developed here applies to non-linear specifications, which we present in Appendix A.

Households pay for their consumption of x as part of the bundle of goods described by housing. Housing price expenditure, relating the amenity level, x, to the annual user-cost of housing, is given by the housing price function, $P(x_i)$.[4] Incorporating the household's budget constraint that their numeraire consumption, C_i, is equal to their income, I_i, minus their cost of housing, $P(x_i)$, yields:

$$U(x_i) = \alpha_i x_i + I_i - P(x_i) \tag{2}$$

In the static problem, households are assumed to consider only current levels of the

[4] One could think of $P(x_i)$ as either being rent or as capturing the mortgage payment associated with choice of x.

amenity x and, therefore, maximize current utility with respect to their current choice of x.[5] Thus, the first-order condition for the optimal choice of x is given by:

$$U'(x_i) = \alpha_i - P'(x_i) = 0 \tag{3}$$

The first-order condition described by Equation (3) may then be used to solve for household i's marginal willingness to pay for amenity x. In other words, at their chosen level of x consumption, household i's individual-specific preference parameter, α_i, will equal their expenditure on x:

$$\alpha_i = P'(x)\Big|_{x=x_i^*} \tag{4}$$

In addition to the analytical model above, the estimation framework may be described intuitively through a series of graphs. In a first stage, the parameters of the housing price function (and, at the same time, the parameters of the hedonic price gradient, $P'(x)$), are recovered through a regression of observed housing prices on amenities. These relationships are shown in Figure (1).[6]

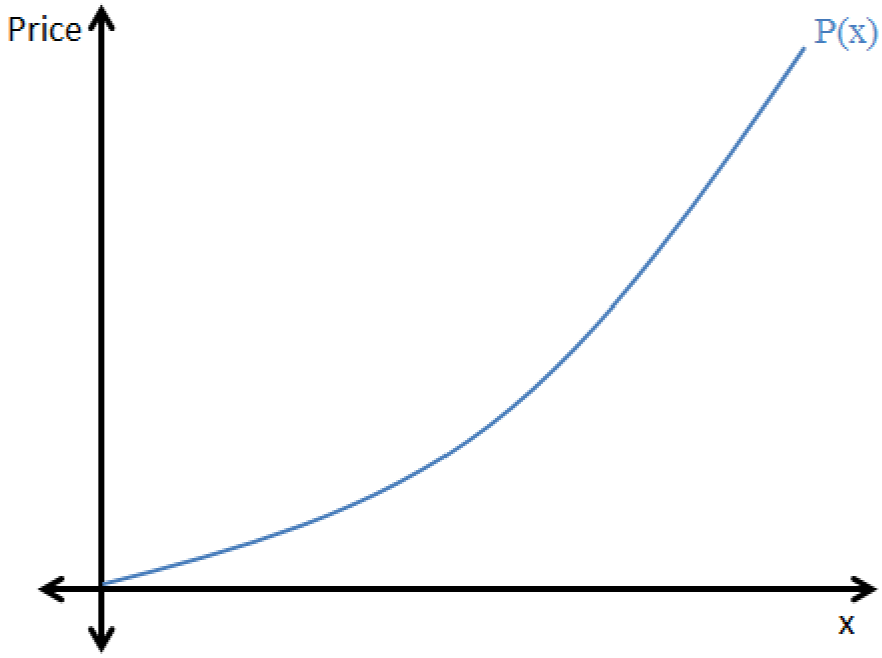
In a second stage, the econometrician is able to back-out the implicit price of x that each family *actually paid*, using the family's observed consumption of x and the hedonic gradient. The information provided by the first-order condition for utility maximization allows the econometrician to invert this implicit price to arrive at the structural preference parameters. This second stage is depicted in Figure (2).

This static model will only return unbiased estimates of α_i when either moving is costless or when amenity levels are fixed through time. However, in any realistic application, the econometrician would face positive moving costs and time-varying amenities. Thus, we describe a forward-looking model in the next section.

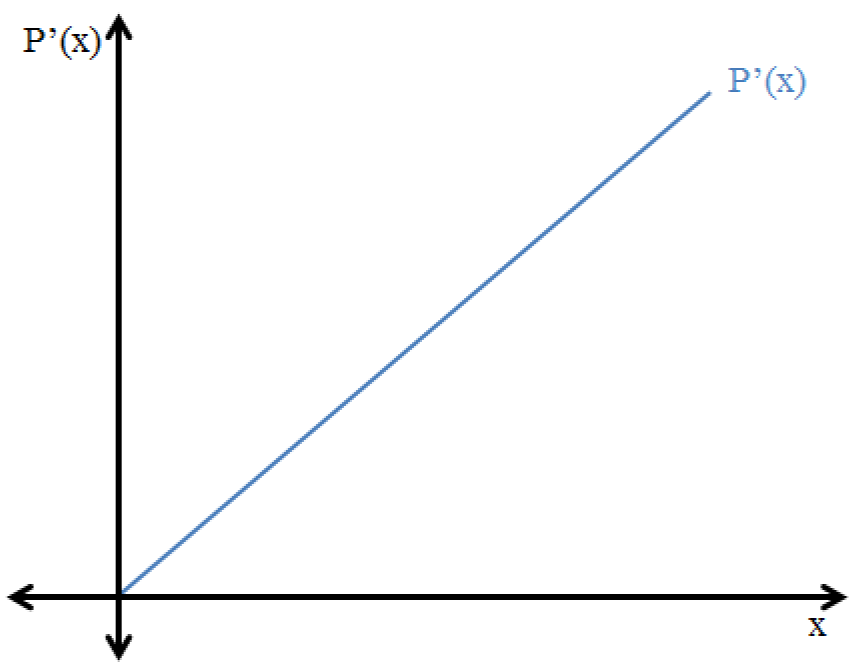
[5] If x is time-varying, this assumption is analogous to an assumption regarding households' moving costs; in a world with zero moving costs, households may costlessly reoptimize in every period (so looking to the future yields no benefit). However, many papers, including Kennan and Walker (2011), find evidence of substantial moving costs.

[6] In the figures, we illustrate a quadratic price function (linear price gradient) and an amenity that may be considered a "good", however, the intuition and results hold for any form of the price function.

Figure 1: Graphic Representation of the First-Stage Estimation

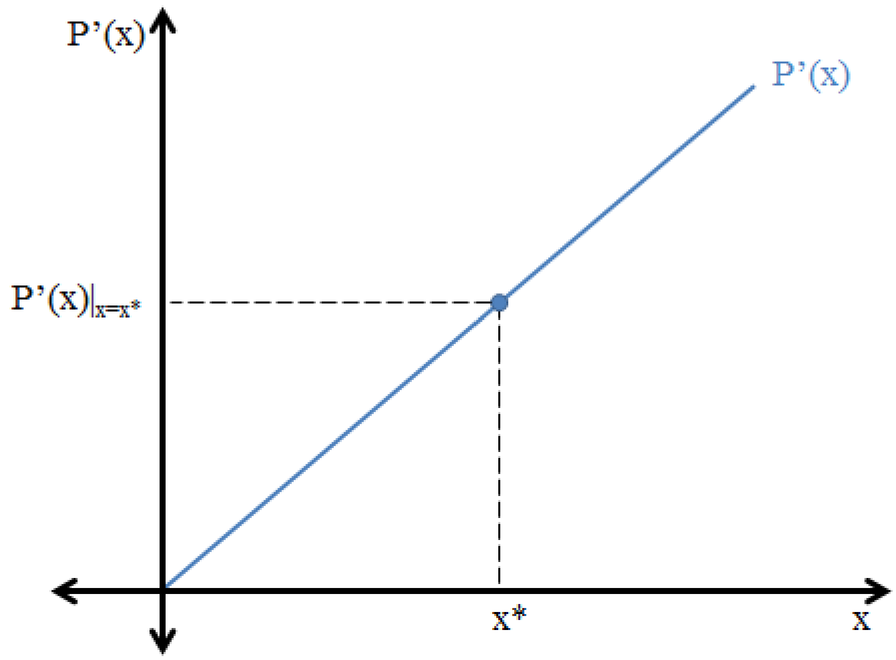


(a) The Hedonic Price Function, $P(x)$

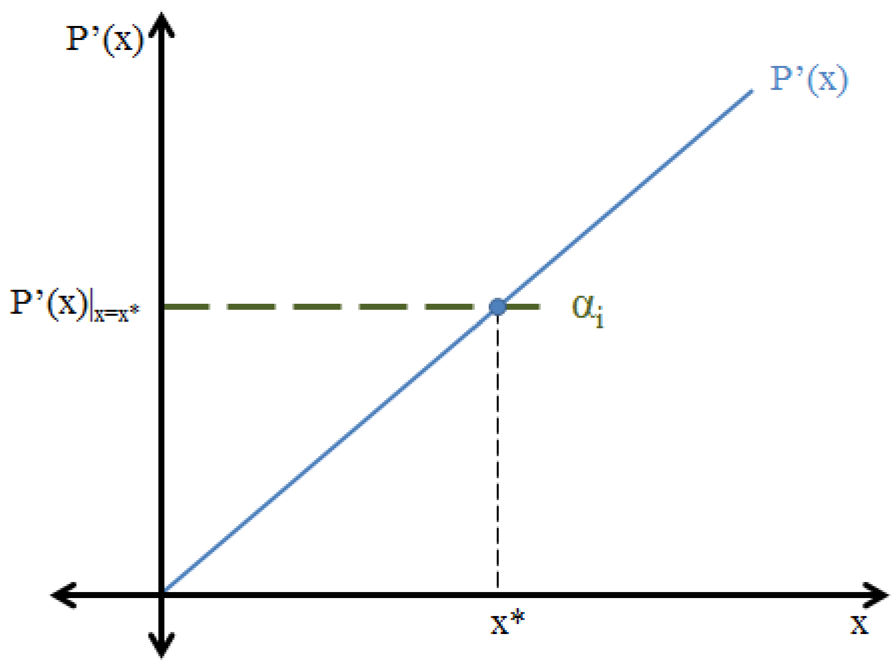


(b) The Hedonic Price Gradient, $P'(x) = \frac{\partial P(x)}{\partial x}$

Figure 2: Graphic Representation of the Second-Stage Estimation



(a) Household i's Observed Consumption of x and its Implicit Price



(b) Household i's Marginal Willingness to Pay for x

2.2 A Simple, Forward-Looking Model of Willingness to Pay

We now move to a forward-looking framework where households maximize a discounted sum of annual utility flows with respect to their current choice of x. In particular, we assume that households choose a residence based on the stream of associated utility flows for the next T years.[7] This is akin to assuming prohibitively high moving costs for the next T years.

In a fully-dynamic model, households would also maximize the discounted sum of annual utility flows (*i.e.*, lifetime utility), but would face positive, yet feasible, moving costs in each period. Households would then account not only for future utility flows, but for future reoptimization in their current choice. In the specification laid out here, we abstract from this future reoptimization to simplify the problem, while retaining the primary insights and intuition of a fully-dynamic model.

The housing price function still captures an annual user-cost of housing. As this price is determined in the current period, t, it is a function of current amenity levels and denoted $P(x_{i,t})$. For homeowners, which is our group of interest, it is natural to think of this annualized price as a mortgage payment: determined at the time of sale, it is a function of amenity levels in the period in which a household buys.[8]

The amenity of interest, x, is evolving through time. Households form expectations about future values of x, based on its known, Markovian transition process.[9]

Denoting the current period as t (and current choice of amenity level as $x_{i,t}$) and ignoring income, I_i, we can write the discounted sum of annual utility flows, i.e., the value function, as:

$$v(x_{i,t}) = E[\sum_{s=1}^{T} \beta^{s-1}(\alpha_i x_{i,s} - P(x_{i,t}))|x_{i,t}] \tag{5}$$

or as:

$$v(x_{i,t}) = \alpha_i \sum_{s=1}^{T} \beta^{s-1} E[x_{i,s}|x_{i,t}] - \sum_{s=1}^{T} \beta^{s-1} P(x_{i,t}) \tag{6}$$

For exposition purposes, we define a measure of expected "average" x consumption over

[7] In the application, we set T to seven years, which is the median household tenure in the United States over this period. See Section 4 for a discussion.

[8] For simplicity, we abstract away from any considerations about the post-T utility.

[9] In Section 4, this transition process of x will be denoted $q(x_{i,t+1}|x_{i,t})$.

the horizon T:

$$\bar{x}_{i,t} = \bar{x}(x_{i,t}) = \frac{1}{B}\sum_{s=1}^{T}\beta^{s-1}E[x_{i,s}|x_{i,t}]$$

where $B = \sum_{s=1}^{T}\beta^{s-1}$. This allows us to rewrite Equation (6) in terms of this "average" x:

$$\tilde{v}(\bar{x}_{i,t}) = \alpha_i B\bar{x}_{i,t} - B\tilde{P}(\bar{x}_{i,t}) \tag{7}$$

where:

$$\tilde{P}(\bar{x}(x_{i,t})) \equiv P(x_{i,t}) \quad \text{and} \quad \tilde{v}(\bar{x}(x_{i,t})) \equiv v(x_{i,t}) \tag{8}$$

Assuming that $\bar{x}_{i,t}$ is a monotonic function of $x_{i,t}$, the household's problem is equivalent to choosing $\bar{x}_{i,t}$ to maximize $\tilde{v}(\bar{x}_{i,t})$, yielding the first-order condition:[10]

$$\tilde{v}'(\bar{x}_{i,t}) = \alpha_i B - B\tilde{P}'(\bar{x}_{i,t}) = 0 \tag{9}$$

The first-order condition described by Equation (9) may then be used to solve for an annualized value of household i's marginal willingness to pay for amenity x. In other words, at their chosen level of $\bar{x}$ consumption, household i's individual-specific preference parameter, α_i, can be recovered as:

$$\alpha_i = \tilde{P}'(\bar{x})\Big|_{\bar{x}=\bar{x}^*_{i,t}} \tag{10}$$

When compared with the analogous solution from the static model (Equation (4)), Equation (10) highlights the two effects that we previously referred to as the price effect and the quantity effect. The price effect is captured by the use of $\tilde{P}$ rather than P. The consumption effect is captured by the fact that we evaluate the functions at $\bar{x}_{i,t}$ rather than $x_{i,t}$. In the following section, we discuss the bias induced by each of these effects show an interesting result where these two effects cancel one another out.

Note that, graphically, the recovery of α_i for the forward-looking model appears similar to that of the static model depicted in Figure 2, but defined in $(\tilde{P}'(\bar{x}), \bar{x})$ space.

[10] If one were to work with $x_{i,t}$ instead of $\bar{x}_{i,t}$, the first-order condition would be given by: $\partial\tilde{v}(\bar{x}_{i,t})/\partial x_{i,t} = \alpha_i\partial\bar{x}_{i,t}/\partial x_{i,t} - \tilde{P}'(\bar{x}_{i,t})\partial\bar{x}_{i,t}/\partial x_{i,t} = 0$ which simplifies to (9).

3 Predicting and Understanding the Bias

When the amenity of interest, x, is time-varying and reopimization is not without cost, estimates of willingness to pay recovered using the static model will be biased. In this section, we first describe this bias analytically by discussing the mathematical difference between the estimate of marginal willingness to pay recovered from the static model (using Equation (4)) and from the forward-looking model (using Equation (10)). Secondly, we provide a detailed interpretation of the cause and of the sign of the bias by relating it to the time-series properties of the amenity x.

Using the definition that appears in Equation (8), $\tilde{P}(\bar{x}_{i,t}) \equiv P(x_{i,t})$, it follows that:

$$\begin{aligned} P'(x_{i,t}) &= \frac{\partial \tilde{P}(\bar{x}_{i,t}(x_{i,t}))}{\partial x_{i,t}} \\ &= \tilde{P}'(\bar{x}_{i,t}) \frac{\partial \bar{x}_{i,t}}{\partial x_{i,t}} \end{aligned} \tag{11}$$

and, therefore, that:

$$\alpha_i = \underbrace{\tilde{P}'(\bar{x})\Big|_{\bar{x}=\bar{x}^*_{i,t}}}_{\text{MWTP forward-looking model}} = \frac{1}{\frac{\partial \bar{x}_{i,t}}{\partial x_{i,t}}} \underbrace{P'(x)\Big|_{x=x^*_{i,t}}}_{\text{MWTP static model}} \tag{12}$$

Thus, the term describing the bias associated with using the static model when the underlying process is dynamic, can be easily seen in Equation (12). Said another way, the practical "correction" to the static model is straightforward – both the quantity and price effects are combined into a single adjustment term: $\partial \bar{x}_{i,t}/\partial x_{i,t}$. We further develop these notions in the examples that follow.

As can be seen in the bias term, $\partial \bar{x}_{i,t}/\partial x_{i,t}$, the sign and size of the bias will be driven by how a household's current amenity levels affects the average future stream of amenity levels. Thus, the transition properties of the amenity of interest will determine the sign and size of the bias and will, therefore, determine when the estimation of the dynamic model is most warranted. When we consider the time-series changes in housing and neighborhood amenities, we are interested both in the overall trend (are the amenity levels rising or falling?) and the change in variance across the houses (are the amenity levels converging or diverging?). In the rest of this section, we walk through the various potential paths of housing amenities and discuss their impacts on willingness-to-pay estimates.

3.1 The Amenity is Rising or Falling Through Time

Here we consider the case where the amenity of interest, x, is either rising or falling through time but in a manner that preserves the variance in x across locations. Consider the following, simple relationship:

$$\bar{x}(x_{i,t}) = x_{i,t} + c \tag{13}$$

In this example, c can be either positive or negative and we put no restrictions on the magnitude. When $c > 0$, the amenity x is rising through time. This would be the case if x were local expenditure on public schools and all schools received the same dollar increase in budget. Alternatively, when $c < 0$, the amenity is falling through time. In either case, we find that a change in current $x_{i,t}$ produces a one-for-one change in average future amenity consumption, $\bar{x}$:

$$\partial\bar{x}_{i,t}/\partial x_{i,t} = 1 \tag{14}$$

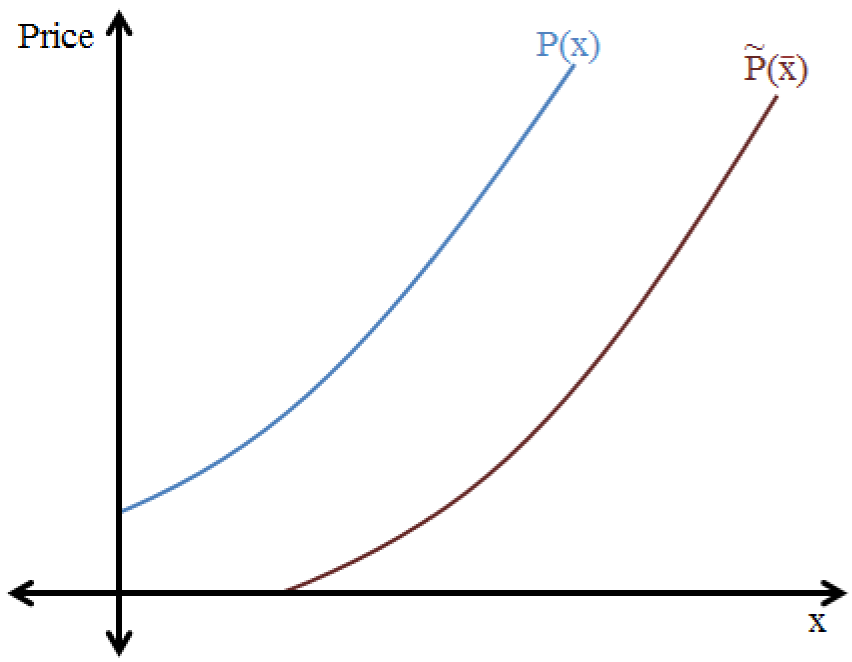
Returning to Equation (12), this implies that the term $\partial\bar{x}_{i,t}/\partial x_{i,t}$ drops out and the willingness-to-pay derived using the static model is identical to that derived using the dynamic one:

$$\tilde{P}'(\bar{x})\Big|_{\bar{x}=\bar{x}^*_{i,t}} = P'(x)\Big|_{x=x^*_{i,t}} \tag{15}$$

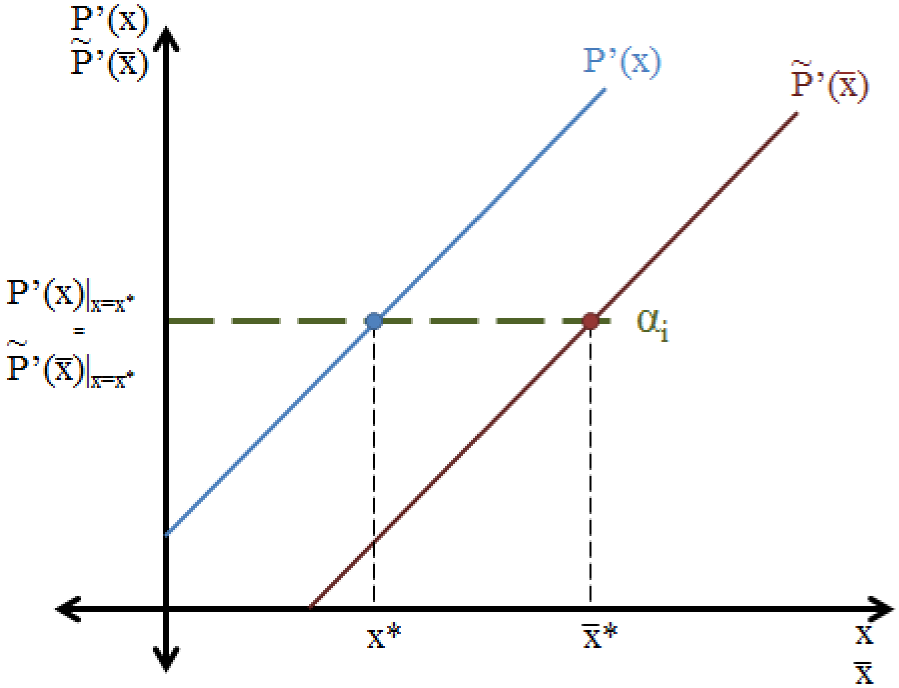
Thus, there is no bias associated with the static modeling framework, even when facing an amenity with potential strong time-trends (as long as the time-trend is variance-preserving across locations in the choice set); the econometrician will recover the true estimate of willingness-to-pay. Graphically, a variance-preserving increase in x is represented in Figure 3.

In Figure 3(a), we can see that the increase in x results in a dynamic price function that lies (in a parallel manner) to the right of the static one. In other words, for any given purchase price of a house, the average received level of associated amenities, $\bar{x}_{i,t}$, is higher than current level, $x_{i,t}$, would imply. The dynamic gradient, depicted in Figure 3(b), also lies (in a parallel manner) to the right of the static gradient, meaning that the implied implicit price of x is now lower.

Figure 3: The Amenity is Rising Through Time



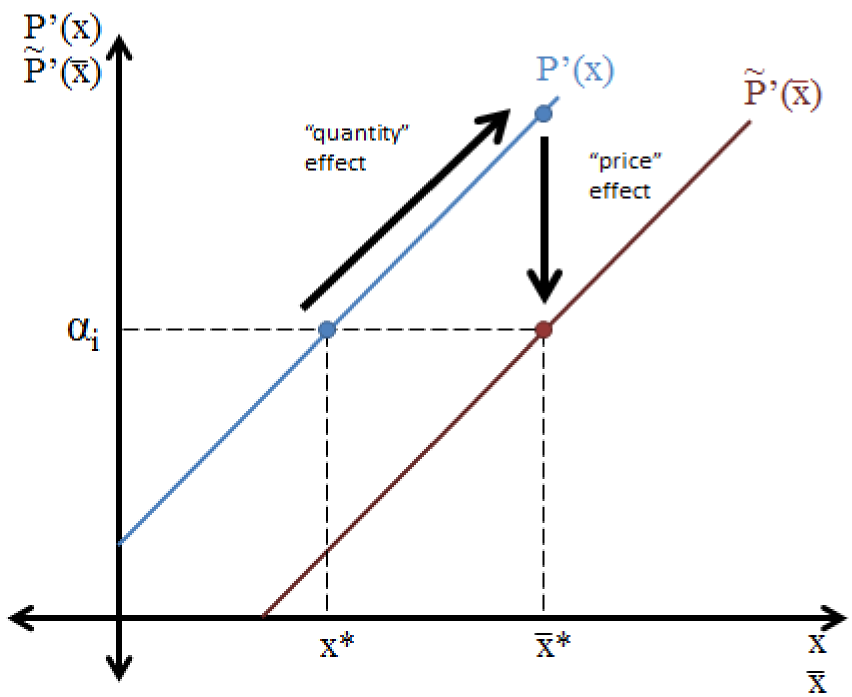
(a) The Hedonic Price Functions



(b) The Marginal Willingness to Pay for the Amenity

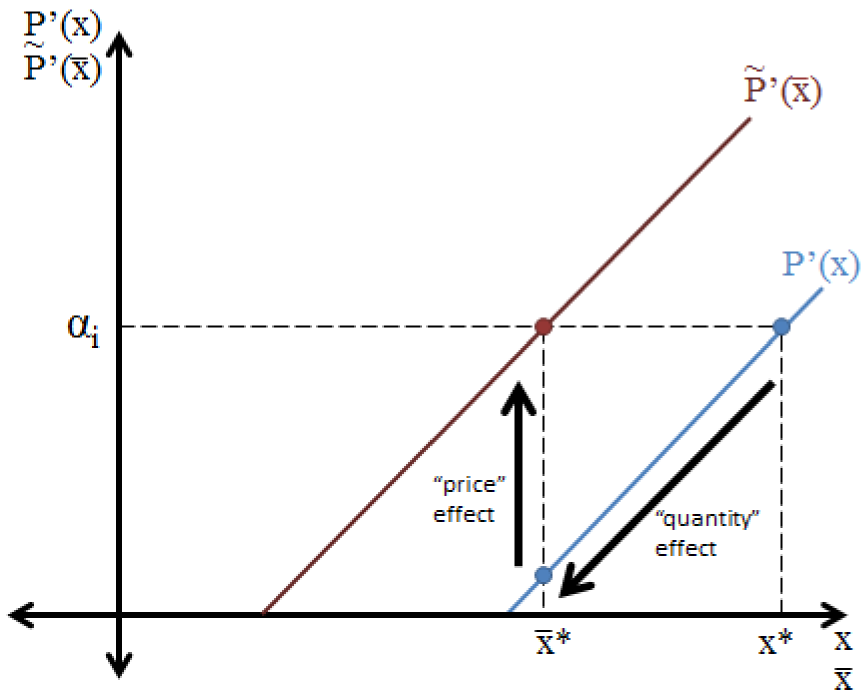
The lack of bias is actually the effect of the "price effect" and the "quantity effect" working in opposite directions and canceling one another out. This is because the household's average consumption is higher (compared with the naive static model) but the price of average consumption is lower (compared with the naive static model). Graphically, this may be seen in Figure 4.

Figure 4: The Competing Effects When x is Increasing



Analogously, Figure 5 shows the "price effect" and the "quantity effect" working in opposite directions and canceling one another out for a uniform-across-locations decrease in x. This is because the household's average consumption is lower (compared with the naive static model), but the price of average consumption is higher (compared with the naive static model).

Figure 5: The Competing Effects When x is Decreasing



3.2 The Amenity is Converging or Diverging Through Time

Here we consider the case when the time trend of the amenity of interest does not perfectly preserve the variance in x across locations in the choice set. In other words, we consider the cases where amenity levels are converging or diverging through time. The converging case might arise if shocks to amenity-levels arrive exogenously through time and these shocks decay. Alternatively, this case might arise endogenously. For example, in the case of school quality, local funds may be diverted to districts with the most underperformance in the prior period (at the expense of "over"-performers). The diverging case might arise with tipping – for example, if neighborhoods with a racial composition above a tipping point become more concentrated in that race and neighborhoods with a racial composition below a tipping point become less concentrated in that race. For our purposes, however, we do not distinguish between the causes of convergence or divergence.

3.2.1 Converging Amenity

Consider the simple extension of Equation (13) described by:

$$\bar{x}(x_{i,t}) = \gamma x_{i,t} + c \tag{16}$$

where $\gamma < 1$ and may be function of x. With $\gamma < 1$, the amenity will be converging (or mean-reverting). In particular, for any two levels of $x_{i,t}$ the difference will be larger than the difference in their implied average amenity, $\bar{x}$. Given this setup, and regardless of the overall trend, c, the necessary "adjustment" to the willingness-to-pay from the static model is given by:

$$\partial \bar{x}_{i,t} / \partial x_{i,t} = \gamma \tag{17}$$

implying that:

$$\tilde{P}'(\bar{x})\Big|_{\bar{x}=\bar{x}^*_{i,t}} = \frac{1}{\gamma} P'(x)\Big|_{x=x^*_{i,t}} \tag{18}$$

or that the willingness to pay derived by the static model is biased downwards by the factor γ.

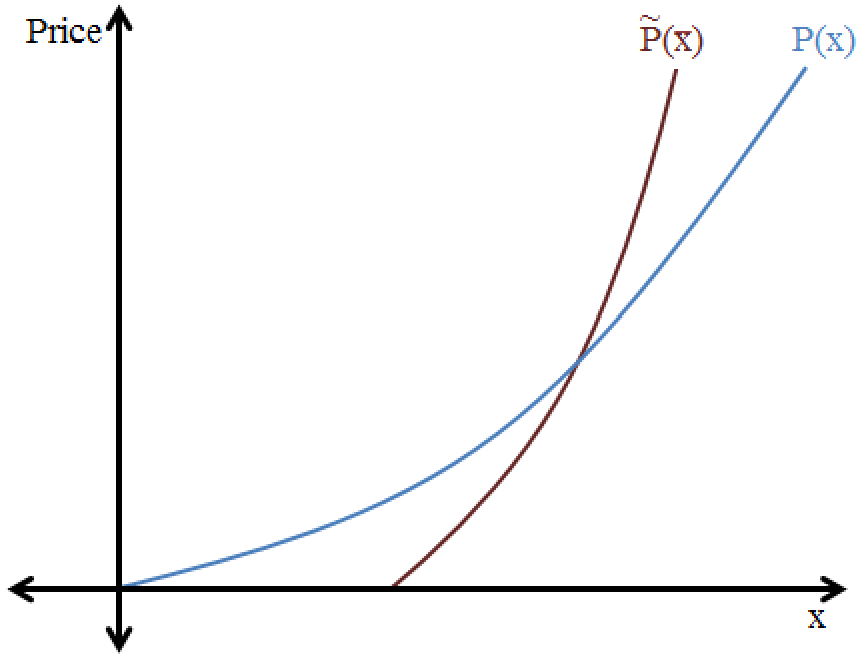
Graphically, this may be seen in Figures 6, 7, and 8, where we consider no overall trend in the amenity, x (i.e., c is defined so that the mean x is preserved through time). In Figure 6(a) we can see that price is increasing at a faster rate in $\bar{x}$ than x. Correspondingly, $\tilde{P}'(\bar{x})$ is always above $P'(x)$ as shown in Figure 6(b). In other words, the average amenity is more expensive than one would infer from the static gradient.

The bias is toward zero in the case of convergence regardless of whether or not the overall trend is of increasing or decreasing amenity levels. In addition, the bias is toward zero for all individuals buying in this market despite the fact that the price- and quantity-effects may work in the opposing direction.

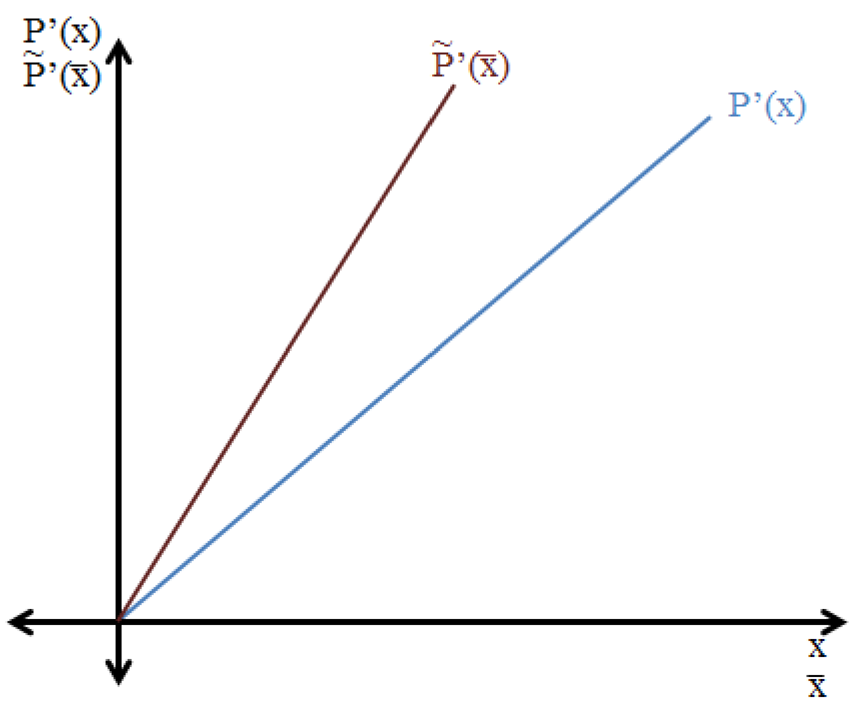
For households who purchase at a below-mean level of $x_{i,t}$, and therefore experience $\bar{x}_{i,t} > x_{i,t}$, both price and quantity effects are driving their true willingness-to-pay away from zero. This can be seen in Figure (7):

For households who purchase at an above-mean level of $x_{i,t}$, and therefore experience $\bar{x}_{i,t} < x_{i,t}$, there is a positive price effect and a negative quantity effect. However, the overall effect is unambiguous; their true willingness-to-pay is higher. This can be seen in Figure (8):

Figure 6: The Amenity is Mean Reverting Through Time



(a) The Hedonic Price Functions



(b) The Marginal Willingness to Pay for the Amenity

Figure 7: The Price and Quantity Effects When x is Mean Reverting

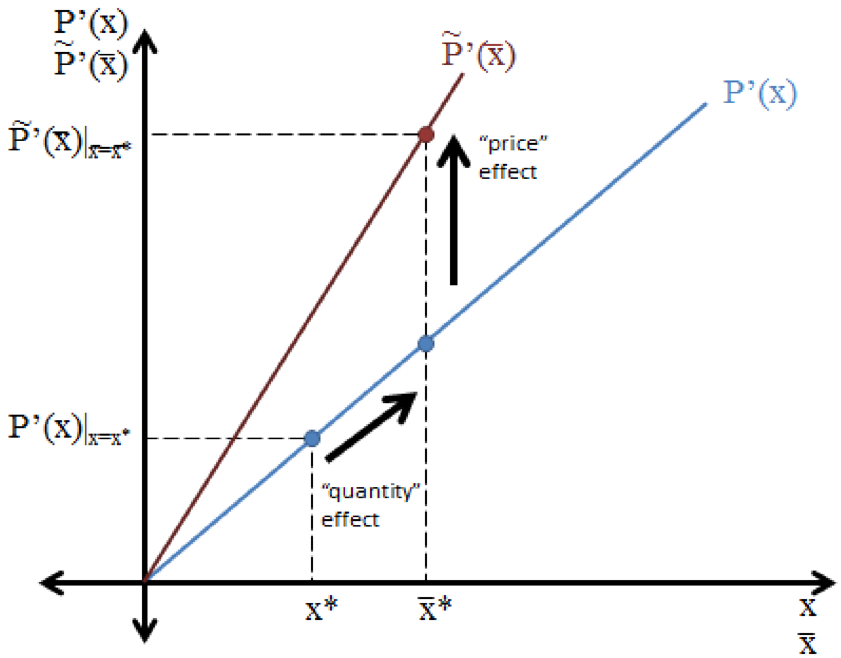
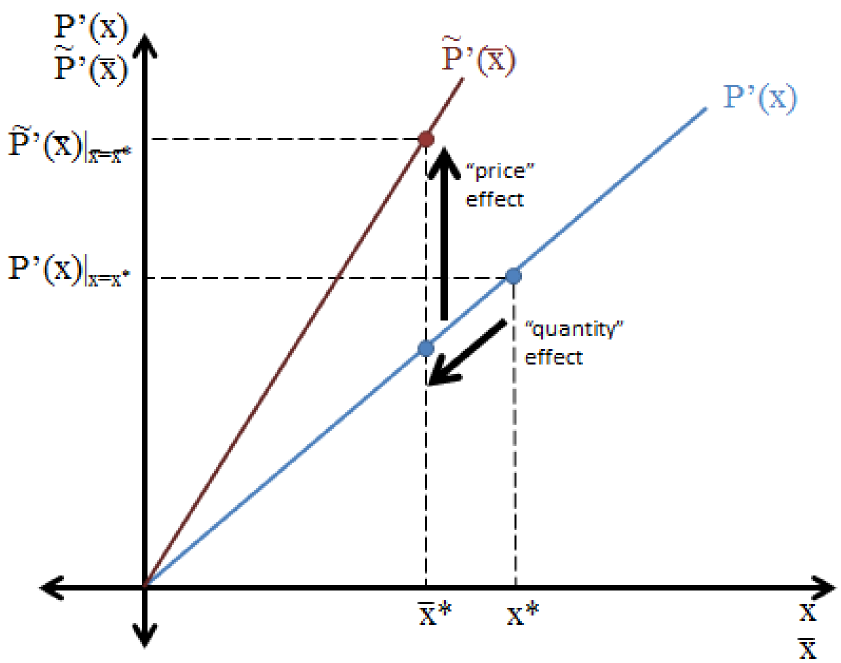


Figure 8: The Price and Quantity Effects When x is Mean Reverting



3.2.2 Diverging Amenity

Like the case of a converging amenity, we consider a case when the variance of the amenity of interest is not preserved across locations in the choice set through time. Specifically, we consider the case where the variance of the amenity, x, is increasing through time, i.e., amenity levels are diverging through time. The divergence of x across houses can take place whether the overall mean of x is rising, falling, or remaining constant.

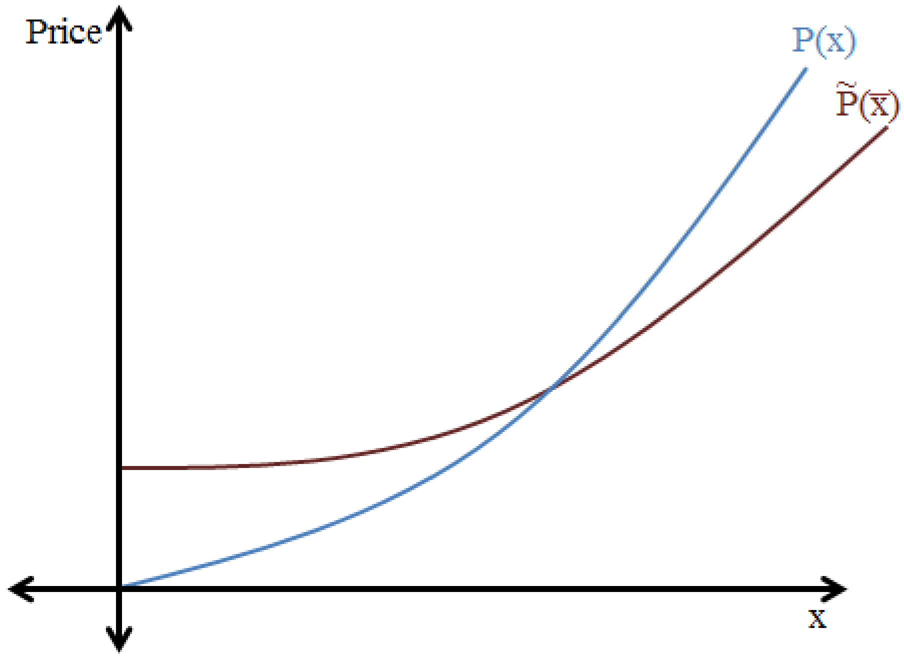
Consider again the process described by $\bar{x}(x_{i,t}) = \gamma x_{i,t} + c$. Now however, we assume that $\gamma > 1$. With $\gamma > 1$, the amenity will be diverging. Households who purchase a low $x_{i,t}$ will experience an even lower $\bar{x}_{i,t}$, while households who purchase a high $x_{i,t}$ will experience an even higher $\bar{x}_{i,t}$. Accordingly, for any two levels of $x_{i,t}$ the difference will be smaller than the difference in their implied average amenity, $\bar{x}_{i,t}$. Analogous to the convergence case, the necessary "adjustment" to the willingness-to-pay from the static model no longer falls out as $\partial\bar{x}_{i,t}/\partial x_{i,t} = \gamma$, implying that:

$$\tilde{P}'(\bar{x})\Big|_{\bar{x}=\bar{x}^*_{i,t}} = \frac{1}{\gamma}P'(x)\Big|_{x=x^*_{i,t}} \tag{19}$$

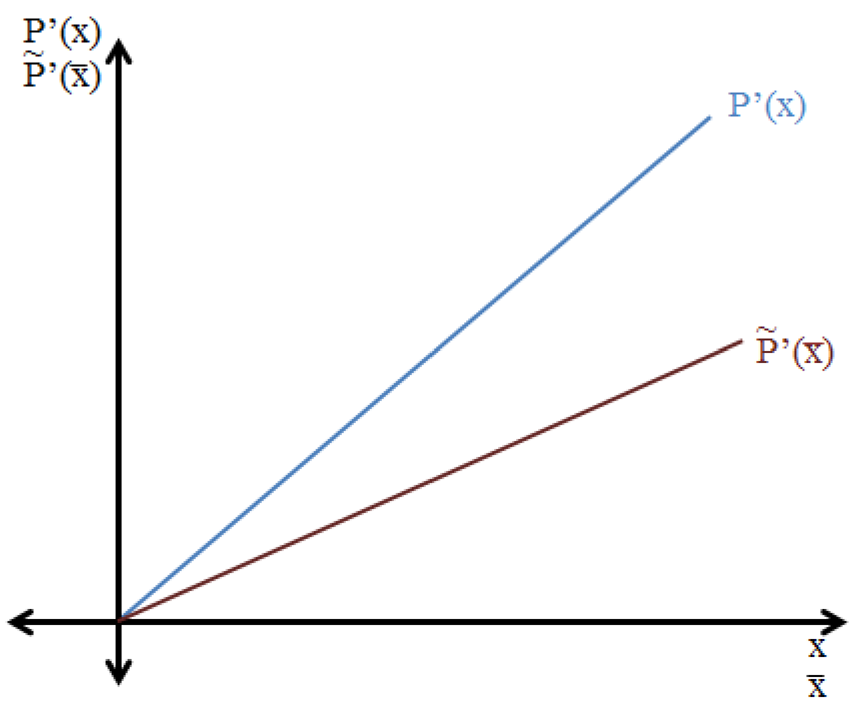
or that the willingness to pay derived by the static model is biased upward by the factor γ (as $\gamma > 1$). Graphically, this may be seen in Figures 9, 10, and 11, where we consider a diverging amenity (where the mean is held constant). In Figure 9(a) we can see that price is increasing at a slower rate in $\bar{x}$ than x. Correspondingly, $\tilde{P}'(\bar{x})$ is always below $P'(x)$ as shown in Figure 9(b). In other words, the average amenity is cheaper than one would infer from the static gradient.

For households who purchase at a below-mean level of $x_{i,t}$, and therefore experience $\bar{x}_{i,t} < x_{i,t}$, both price and quantity effects driving their true willingness to pay towards zero. This can be seen in Figure (10); for households who purchase at an above-mean level of $x_{i,t}$, and therefore experience $\bar{x}_{i,t} > x_{i,t}$, there is a negative price effect and a positive quantity effect. However, the overall effect is unambiguous; their true willingness-to-pay is lower. This can be seen in Figure (11):

Figure 9: The Amenity is Mean Diverging Through Time



(a) The Hedonic Price Functions



(b) The Marginal Willingness to Pay for the Amenity

Figure 10: The Price and Quantity Effects When x is Diverging

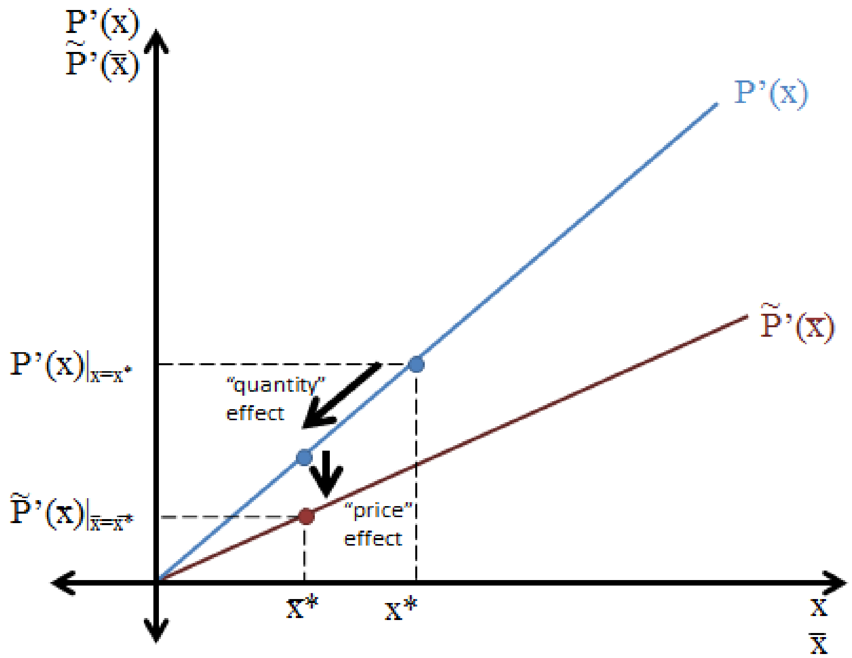
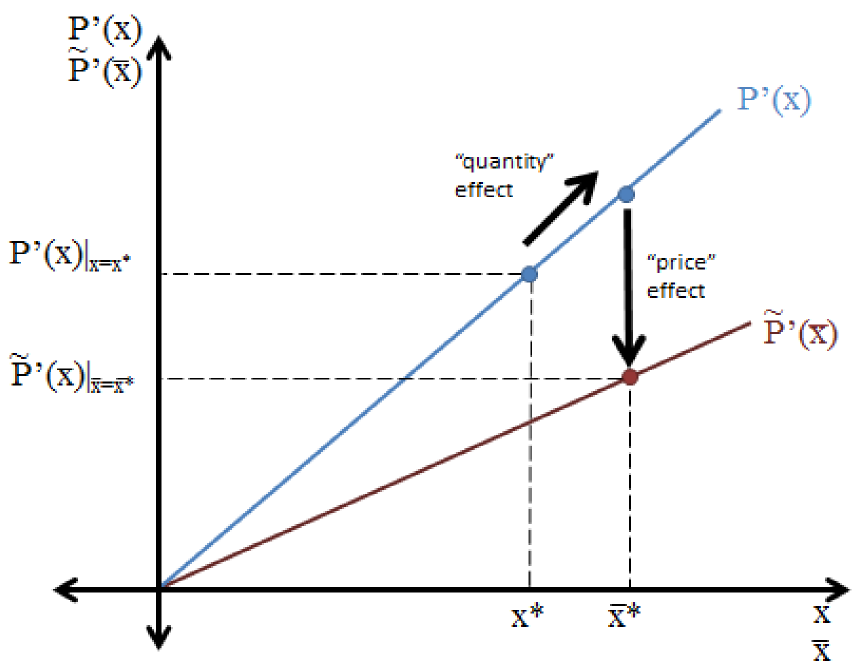


Figure 11: The Price and Quantity Effects When x is Diverging



3.3 Nonlinear Utility

In Appendix A, we derive the nonlinear case in greater detail. However, we summarize the key take-aways here. The effects discussed in Section 3.2 still hold. That is, if the amenity is converging through time the static model tends to underestimate the marginal willingness to pay and if the amenity is diverging through time the static model tends to overestimate the marginal willingness to pay. The effects discussed in Section 3.1 also partially hold. The price and quantity effects work still work in opposite directions. However, due to the nonlinearity of the utility function, they no longer exactly cancel each other out. A natural assumption about the utility function is that utility is increasing and concave in the amenity. In this case, an increasing trend in the amenity will mean that $\bar{x}_{i,t}$ lies on a flatter portion of the utility function than $x_{i,t}$ and that the static model will underestimate the marginal willingness to pay.[11] Analogously, an decreasing trend in the amenity will mean that $\bar{x}_{i,t}$ lies on a steeper portion of the utility function than $x_{i,t}$ and that the static model will overestimate the marginal willingness to pay.

4 Application

4.1 Data

In our application, we use a dataset describing housing transactions and violent crime rates for five counties in the Bay Area of California (Alameda, Contra Costa, Marin, San Mateo, and Santa Clara) over the period 1990 to 2008. This is the same data used in Bishop and Murphy (2011). As such, we can compare the results obtained from the simple adjustment discussed here with the fully dynamic approach used there. The data are richer than required for illustrating the concepts discussed in this paper, as they allow the econometrician to follow households through time. This richness is however needed for the fully dynamic approach.

The real estate transactions data were purchased from Dataquick and include dates, prices, loan amounts, and buyers', sellers', and lenders' names for all transactions. In addition, the data for the final observed transaction for each house include characteristics such

[11] If the amenity is not changing in expectation, but there is uncertainty, the static model will overestimate marginal willingness to pay. Therefore, for a small increasing trend, the static model may overestimate marginal willingness to pay.

as exact street address, square footage, year built, lot size, number of rooms, number of bathrooms, and number of units in the building. The process of cleaning the data involves a number of cuts which we discuss in more detail in Appendix B.

Crime statistics come from the RAND California database. These data are organized by city and are measured as incidents per 100,000 residents.[12] The data describe annual violent crime rates for each of the cities within the San Francisco Metropolitan area. In this dataset, violent crime is defined as "crimes against people, including homicide, forcible rape, robbery, and aggravated assault." Crime rates are imputed for each house in our data set using an inverse-distance weighted average of the crime rate in each city. Specifically, we weight the contribution of each city by the inverse of distance-squared, computing distance using the "great circle" calculation.

Figure A.1 in Appendix B illustrates the location of these city centroids. Figures 12 and 13 illustrate the cross-sectional distribution of violent crime rates and the time-series of violent crime rates, respectively. There is a noticeable downward trend in violent crime, consistent with the decrease experienced by most of the US over this period.

The final sample includes 541,415 transactions which are used to estimate the price functions separately by county. We then calculate estimates of α_i for our sample of 369,015 households.

4.2 Empirical Specification

We assume the familiar, log-linear specification for the hedonic price function:

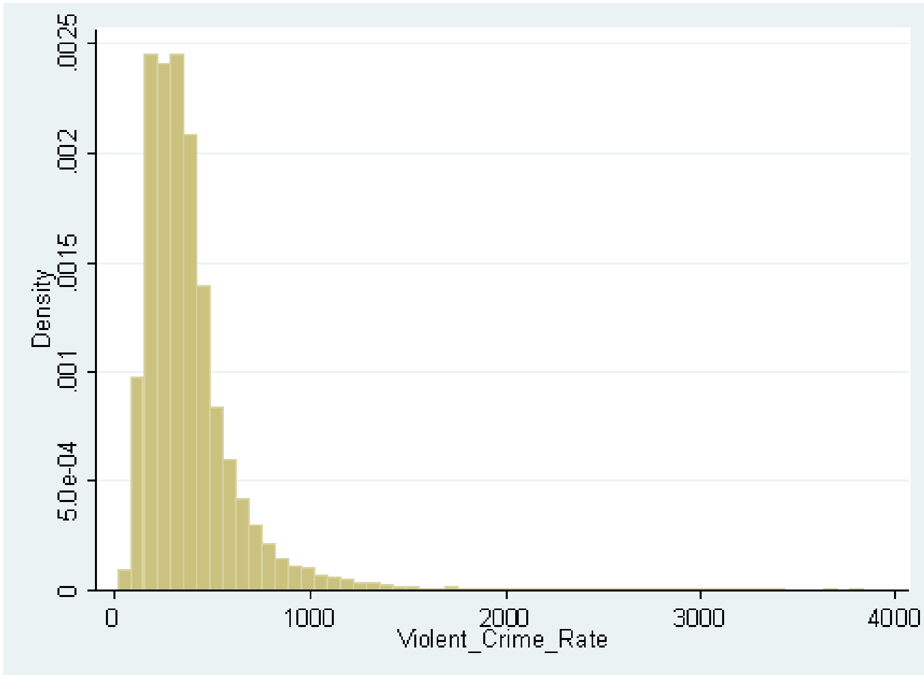
$$log(P_{j,k,t}) = \theta_{0,k} + \theta_{1,k}x_{j,k,t} + \theta_{2,k}x^2_{j,k,t} + H'_{j,k,t}\theta_{3,k} + \epsilon_{j,k,t} \tag{20}$$

where j denotes the property, k denotes the county, and t denotes the year of sale. The vector of housing attributes, $H_{j,k,t}$, includes property age, square footage, lot size, number of rooms, year of sale, and a set of fixed effects at the Census-tract level.

While the functional form of the price function is important for correctly estimating households' marginal willingness to pay, the ratio of static and forward-looking estimates is invariant to the choice of functional form and is solely determined by the transition-process

[12]There are 75 reporting cities in the five counties of analysis.

Figure 12: Pooled Cross-Sectional Variation in Violent Crimes per 100,000 Residents



of the amenity of interest. In this application, we estimate the transition of violent crime by assuming that it follows an AR(1) process. Specifically, we estimate the following equation separately for each county, k:

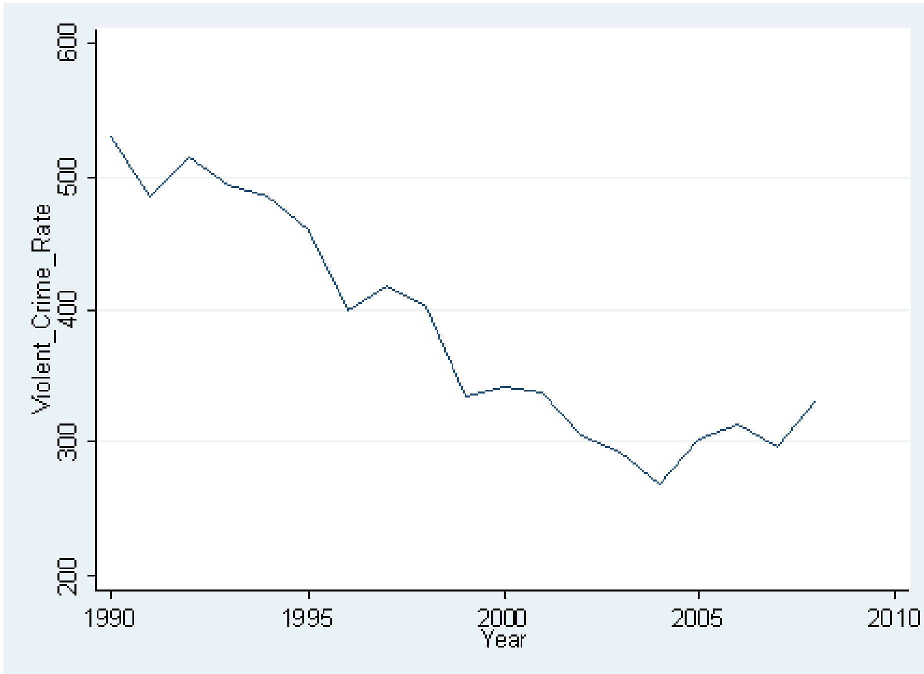
$$x_{j,k,t} = \rho_{0,k} + \rho_{1,k} x_{j,k,t-1} + \rho_{2,k} t + \varepsilon_{j,k,t} \tag{21}$$

With this simple transition process, $\bar{x}$ may be expressed as $\bar{x}_t = \gamma_k x_t + c_{k,t}$ where $\gamma_k = \frac{1}{B} \sum_{s=1}^{T} \beta^{s-1} \rho_{1,k}^{s-1}$.

It is important to note that, while the parameter $c_{k,t}$ may also be recovered, it describes only the variance-preserving change in violent crime. In other words, the bias is solely determined by the parameter γ_k, following the intuition laid out in Section 3.

Each household's marginal willingness to pay to avoid violent crime is recovered as the implicit price (i.e. the value of the gradient) at their chosen level of violent crime exposure. For the static estimates, marginal willingness to pay is recovered according to Equation 4. For forward-looking estimates, marginal willingness to pay is recovered according to Equation 12

Figure 13: Pooled Time-Series Variation in Violent Crimes per 100,000 Residents



with β set to 0.95 and T set to seven years.

4.3 Results

We first estimate the transition probability function for violent crime separately for each of the five counties. The transition probability parameters (the ρs) are reported in Table A.1 in Appendix C. This allows us to calculate the corresponding values of γ_k for each of the five counties. These γ_k parameters, which determine the size and sign of the bias are given in Table 1. It is useful to note that all of the values of γ_k are less than one, indicating that violent crime is converging through time in each of the counties.[13]

We estimate the price function, $P(x)$, separately for each of the five counties.[14] For exposition, in Figure 14, we show both the hedonic price function and the gradient function

[13]The corresponding values of $c_{k,t}$ (not shown) and indicate that crime is falling over time in the Bay Area. As discussed in Section 3, this overall trend will not affect how our forward-looking model differs from the static model.

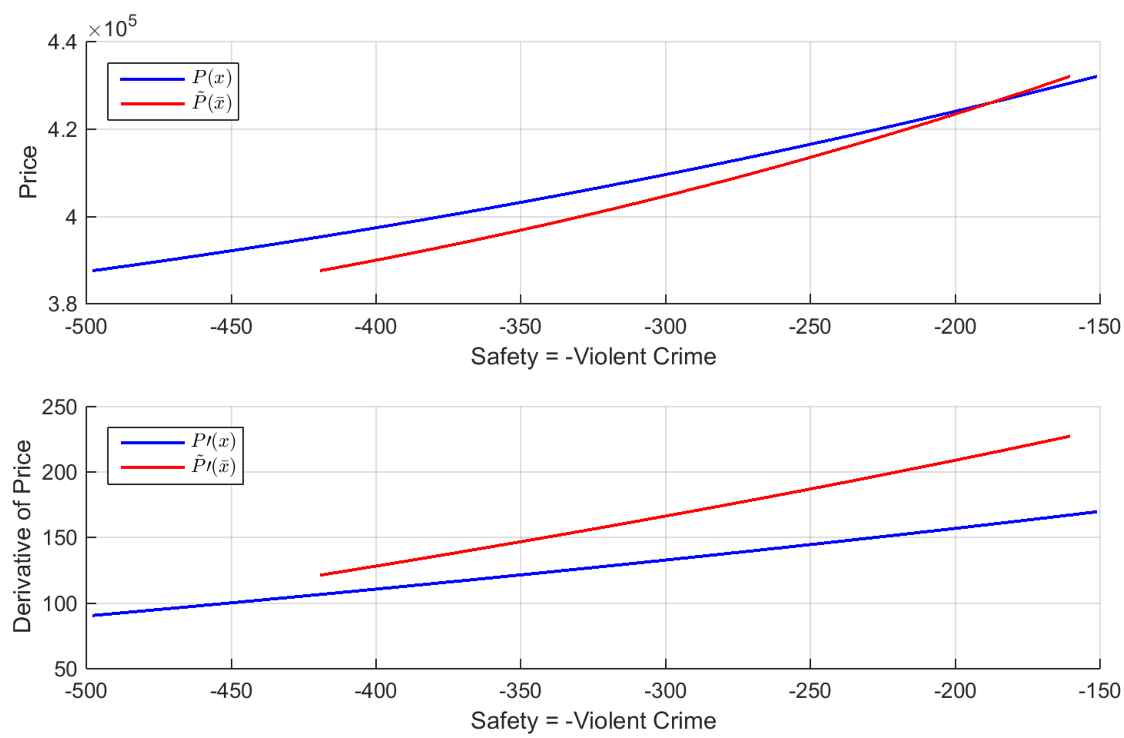
[14]To convert price measures into annualized user costs, we follow the literature and multiply prices by 0.075.

Table 1: County-Specific Estimates of γ

Alameda	0.88
Contra Costa	0.83
Marin	0.51
San Mateo	0.51
Santa Clara	0.75

the most populous county in our sample, Santa Clara. To keep the figures consistent with our earlier discussion (i.e., in the first quadrant describing a “good”), we use the negative of crime on the horizontal axis. As such, one can think of this as capturing how price varies with a measure of safety.

Figure 14: Price Function and Hedonic Gradient Results for Santa Clara County



It is clear from Figure 14 that price is increasing faster in expected average safety than it is in current safety. In other words, safety is actually more expensive than the naive model would suggest. Graphically, this can be seen by the fact that $\tilde{P}'(\bar{x})$ is always lying above $P'(x)$.

Figures for the other four counties are similar with the same implied biases and intuition. These figures are presented in Appendix C. In all cases, price is increasing in safety at an increasing rate (implying an upward-sloping gradient).

As we find convergence in violent crime over the period of our sample, households who are currently consuming a low amount of violent crime are actually consuming a relatively higher amount in expectation. Households that are currently consuming a high amount of violent crime are actually consuming a relatively lower amount in expectation. This translates to positive and negative quantity effects, respectively. However, the discount on housing price received is smaller than that implied by the static model and, as such, the price effect leads the static model to understate the absolute taste for crime. The overall effect is that the marginal willingness to pay recovered from the static model is biased toward zero for all initial choices of crime.

The static model's estimate of α_i is obtained by observing the implicit price (i.e., the value of the gradient) at each household's observed level of consumption. We do this for all households in the dataset (over all five counties) and the sample mean of this estimate is -\$10.66. In other words, the static model implies that the average household dislikes violent crime and is willing to pay \$10.66 per year to avoid one additional crime per 100,000 residents. This translates to a willingness to pay of \$374.08 per year to reduce total violent crime by ten percent at the average level of violent crime (350.92 incidents per 100,000 residents).

The forward-looking model's estimate of α_i for each household is obtained by "correcting" the static model's estimate with the term γ_k. In other words, following the discussion in Section 3, we divide each household's estimate of willingness to pay (from the static model) by their county-specific estimate of γ_k. As all estimated values of γ_k are less than one (i.e., violent crime rates are converging), our forward-looking willingness to pay measures will be larger in absolute value. The mean estimate of marginal willingness to pay from the forward-looking model is -\$14.28. This implies that the naive, static approach leads to an almost thirty percent bias.

As noted previously, the model laid out here is not fully dynamic in that households may not choose to reoptimize within the seven-year time horizon. However, we are able to compare the estimates here to those obtained in Bishop and Murphy (2011). In that paper, the fully dynamic model is estimated using the same data. Interestingly, the forward-looking estimate (of -$14.28) is reasonably close to the fully dynamic estimate (-$13.45).

In this paper, we estimate all key equations separately by county. This allows us to present the geographic heterogeneity in marginal willingness to pay. Table 2 shows the mean marginal willingness to pay estimates separately for each county.

Table 2: Average Marginal Willingness to Pay by County

	Static MWTP in $ per year)	Forward-Looking MWTP in $ per year	Implied Bias in percentage points ($= \gamma_k - 1$)
Alameda	-5.60	-6.40	-12
Contra Costa	-16.54	-19.90	-17
Marin	-16.00	-31.42	-49
San Mateo	-4.24	-8.35	-49
Santa Clara	-11.61	-15.55	-25

5 Conclusion

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Appendix A: Nonlinear Utility

A.1 Static Problem

Households choose x_i to maximize $U(x_i)$, where $U(x_i)$ is given by:

$$U(x_i) = u(x_i) + I_i - P(x_i) \tag{A.1}$$

This yields the first order condition:

$$U'(x_i) = u'(x_i) - P'(x_i) = 0 \tag{A.2}$$

Econometrician can recover $u'(x_i)$ as

$$u'(x)\Big|_{x=x_i^*} = P'(x)\Big|_{x=x_i^*} \tag{A.3}$$

A.2 Dynamic Problem

Households choose $x_{i,t}$ to maximize $v(x_{i,t})$, where ignoring income, I_i, $v(x_{i,t})$ is given by:

$$v(x_{i,t}) = E[\sum_{s=t}^{T} \beta^{s-1}(u(x_{i,s}) - P(x_{i,t}))|x_{i,t}] \tag{A.4}$$

we can rewrite (A.4) as:

$$v(x_{i,t}) = \sum_{s=t}^{T} \beta^{t-1} E[u(x_{i,s})|x_{i,t}] - \sum_{s=t}^{T} \beta^{s-1} P(x_{i,t})) \tag{A.5}$$

As before, we let $B = \sum_{s=1}^{T} \beta^{s-1}$. Rewrite (A.5) as

$$v(x_{i,t}) = \sum_{s=t}^{T} \beta^{t-1} E[u(x_{i,s})|x_{i,t}] - BP(x_{i,t})) \tag{A.6}$$

Choosing $x_{i,t}$ to maximize the value function yields the first order condition:

$$\frac{1}{B}v'(x_{i,t}) = \frac{1}{B}\sum_{s=t}^{T}\beta^{s-1}\frac{\partial E[u(x_{i,s})|x_{i,t}]}{\partial x_{i,t}} - P'(x_{i,t}) = 0 \tag{A.7}$$

which can be rewritten as to:

$$\begin{aligned} P'(x_{i,t}) &= \frac{1}{B}\sum_{s=t}^{T}\beta^{t-1}\frac{\partial E[u(x_{i,s})|x_{i,t}]}{\partial x_{i,t}} \\ P'(x_{i,t}) &= \frac{1}{B}u'(x_{i,t}) + \frac{1}{B}\sum_{s=t+1}^{T}\beta^{t-1}\frac{\partial E[u(x_{i,s})|x_{i,t}]}{\partial x_{i,t}} \end{aligned} \tag{A.8}$$

The bias is determined by whether or not $\frac{1}{B}\sum_{s=t}^{T}\beta^{t-1}\frac{\partial E[u(x_{i,s})|x_{i,t}]}{\partial x_{i,t}} > \frac{\partial u(x_{i,t})}{\partial x_{i,t}}$.

Appendix B: Data Cleaning Details

The process of cleaning the data involves a number of cuts. Many of these are made in order to deal with the fact that we only see housing characteristics at the time of the last sale, but we need to use housing characteristics from all sales as controls in our hedonic price regressions. We therefore seek to eliminate any observations that reflect major housing improvement or degradation. First, to control for land sales or re-builds, we drop all transactions where "year built" is missing or with a transaction date that is prior to "year built". Second, in order to control for property improvements (*e.g.*, an updated kitchen) or degradations (e.g., water damage) that do not present as re-builds, we drop any house that ever appreciates or depreciates in excess of 50 percentage points of the county-year mean price change. We also drop any house that moves more than 40 percentile points between consecutive sales in the county-year distribution. Additionally, we drop transactions where the price is missing, negative, or zero. After using the consumer price index to convert all transaction prices into 2000 dollars, we drop one percent of observations from each tail to minimize the effect of outliers. Finally, as we merge-in data describing local crime rates using each property's geographic coordinates, we drop properties where latitude and longitude are missing.

A number of additional cuts were made to create the data for Bishop and Murphy (2011).

Using the common variables of date, Census tract, loan value, and lender, we merge-in data describing household race and income from the Home Mortgage Disclosure Act dataset (available for all households taking out a mortgage). We successfully match approximately 75% of individuals in the transactions sample to the HMDA sample. Based on the algorithm for tracking households through time, we keep only those households observed to purchase three or fewer times during the sample period. Finally, we drop households where race or income are missing and households with income less than $25,000 or more than $500,000 income (in 2000 dollars). Note that this accounts for less than two percent of the remaining sample.

Figure A.1: Cities within the San Francisco Metropolitan Area

Appendix C: Results

Table A.1: Transition Probability Estimates

	$\rho_{0,k}$	$\rho_{1,k}$	$\rho_{2,k}$
Alameda	30.48	0.95	-1.22
Contra Costa	17.18	0.93	0.05
Marin	110.66	0.74	-3.88
San Mateo	94.60	0.74	-0.59
Santa Clara	49.56	0.89	-2.31

Figure A.2: Alameda

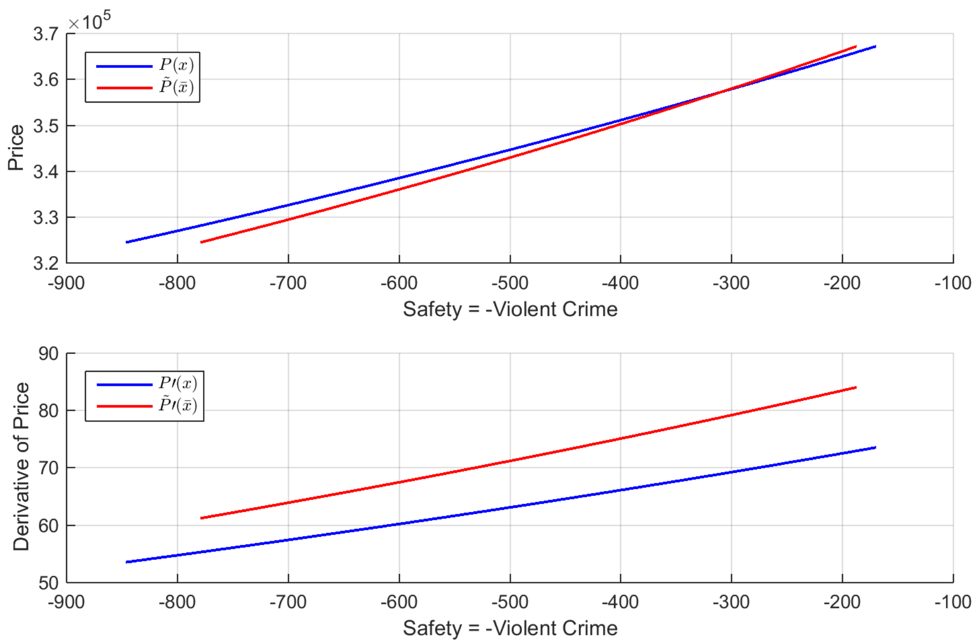

Figure A.3: Contra Costa

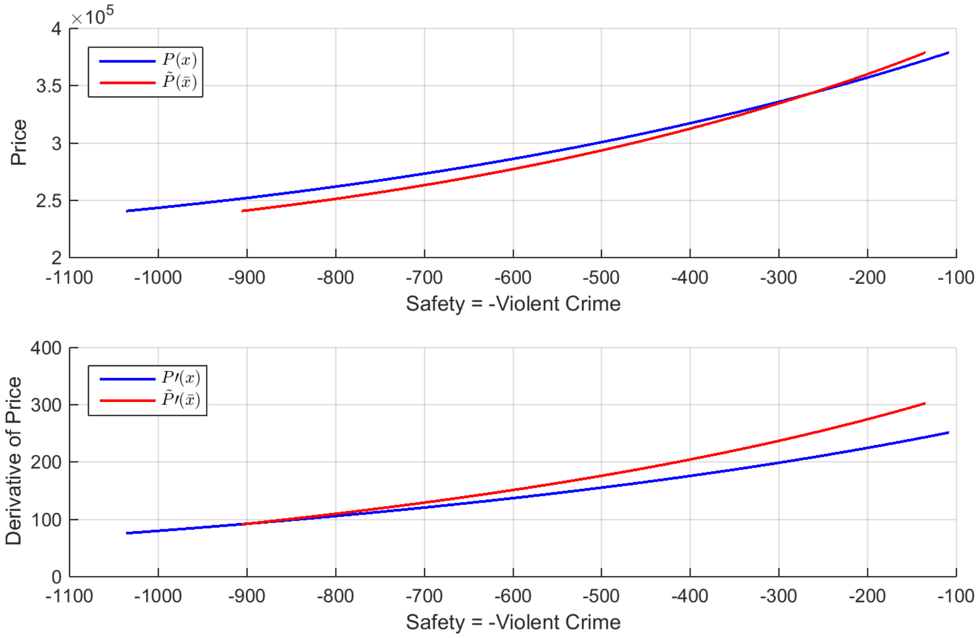

Figure A.4: Marin

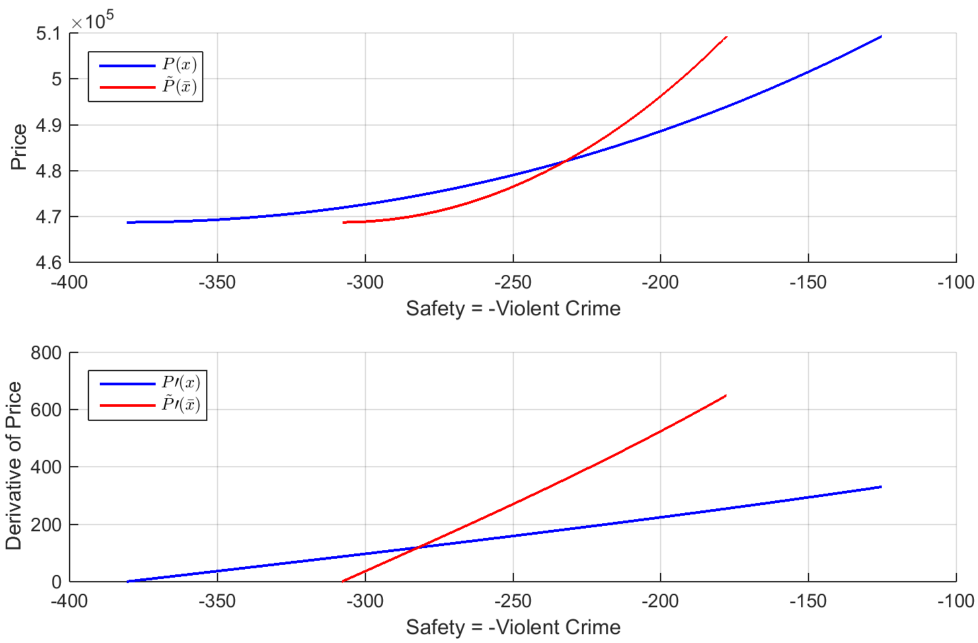

Figure A.5: San Mateo