# Greater mutual aggravation*

Sebastian Ebert† Diego C. Nocetti‡ Harris Schlesinger§

December 18, 2015; AEA 2016 conference paper

**Abstract**

A large strand of research has identified conditions on preferences under which (i) a single risk is undesirable and (ii) two independent risks aggravate each other. We extend this line of inquiry by establishing conditions such that (iii) the degree of mutual aggravation is greater for more severe risks. Here, the severity of risks is characterized by means of general stochastic dominance shifts and also via moment-preserving stochastic transformations. Greater mutual aggravation is implied by all commonly used utility functions and may thus be regarded a typical property of expected utility preferences. We show that greater mutual aggravation determines the comparative statics of risk changes in several risk management problems, including precautionary saving, intertemporal risk-taking, and self-protection. Greater mutual aggravation further explains recent experimental findings on higher-order risk preferences. Finally, it offers a new, simple, and efficient method to elicit risk preferences up to "very" high orders.

**Keywords:** mixed risk aversion, mutual aggravation, risk apportionment, stochastic dominance, utility premium, prudence

**JEL codes:** D81, D91

*Due to his untimely death, Harris Schlesinger did not have the opportunity to (dis)approve the latest version of this paper. He presented its first version, which circulated under the title "Comparative Risk Apportionment", at the Annual Meeting of the European Group of Risk and Insurance Economists (EGRIE) in September 2013. We thank the conference participants and the discussant Johannes Maier for helpful comments and suggestions. The authors also thank Louis Eeckhoudt, Rachel J. Huang, and Larry Y. Tzeng for their helpful comments.

†Tilburg University, Department of Finance, Room K 913, PO Box 90153, 5000 LE Tilburg, The Netherlands. Phone: +31 13 466 3126, Email: S.Ebert@uvt.nl.

‡Clarkson University, School of Business, P.O. Box 5790, Potsdam, NY, USA. Email: dnocetti@clarkson.edu.

§Department of Finance, University of Alabama, Tuscaloosa AL 35487-0224, USA.

# 1 Introduction

Most economic decisions involve risk and oftentimes multiple sources of risk. Individuals, for example, cope with uncertainty over future health expenditures, the rate of taxation, the returns on human and financial capital, and the return on entrepreneurial activities to name a few. It is thus important to understand the welfare consequences of multiple risks as well as the strategies that decision makers adopt to manage those risks. Kimball (1993) used the terminology ”mutual aggravation of risks” in reference to a situation in which the presence of one risk increases the sensitivity to the presence of another, independent risk. Or, to put it differently, two risks are mutually aggravating if their joint occurrence is perceived to be more harmful than their occurrence in different states of nature. For example, a decision maker may perceive the risk of becoming unemployed and the risk of unexpected medical expenses to be more harmful if unemployment and sickness risk manifest at the same rather than at different dates.

More generally, we can say that two *changes* in risk – including the change from certainty to risk – aggravate each other when a deteriorating shift in the distribution of a random variable increases the sensitivity to a deteriorating shift in the distribution of another random variable. Eeckhoudt et al. (2009) refer to this sort of mutual aggravation as “preferences for combining good-with-bad”, in reference to the fact that the mentioned greater sensitivity can be depicted as a preference for an equal-chance lottery in which a good prospect and a bad prospect (e.g. defined via a stochastic dominance relation) are bundled in each state of nature over an equal-chance lottery in which two good prospects occur in one state of nature and two bad prospects occur in another state of nature.

Broadly stated, Eeckhoudt et al. (2009) showed that mutual aggravation of two stochastically dominated shifts is guaranteed by *mixed risk aversion* – the same trait of risk preferences that guarantees that each stochastic dominance trait by itself is undesirable. The term mixed risk aversion was coined by Caballé and Pomansky (1996) to describe an increasing utility function with derivatives alternating in sign. Brockett and Golden (1987) already noted that all commonly used risk-averse utility functions are mixed risk-averse. Mixed risk aversion comprises the important properties of risk aversion (Bernoulli (1738/1954)), prudence (Kimball (1990)), and temperance Kimball (1993).[1]

We summarize that a mixed risk averse decision maker (i) perceives stochastically dominated shifts of any order as undesirable (Hadar and Russel (1969), Hanoch and Levy (1970) and Jean (1980)) and (ii) perceives pairs of risk changes as mutually aggravating (Kimball (1993) and Eeckhoudt et al.

[1]The usefulness of prudence for precautionary saving was actually seen much earlier by Leland (1968), Sandmo (1970) and Drèze and Modigliani (1972), but it was Kimball (1990) who formalized their results into the concept of prudence. The conceptual idea of prudence was also exposited earlier by Menezes et al. (1980), who used the terminology ”aversion to downside risk.” Crainich and Eeckhoudt (2008) study the intensity of downside risk aversion. Temperance, in a slightly weaker form, likely first appears in the paper by Pratt and Zeckhauser (1987).

(2009)).

Following this line of inquiry, in this paper we show that mixed risk aversion implies that mutual aggravation is "increasing," in the sense that it is more severe when the risk changes themselves are more severe. Greater risks lead to greater mutual aggravation. While this result is intuitive, just as mutual aggravation (or a preference for combining good with bad) is intuitive, it follows from our analysis that mixed risk aversion is the necessary and sufficient property that ensures such consistent behavior. The result holds in any decision environment with changing risks; for concrete examples, see below. Our paper thus provides a new, reasonable, and influential implication of the vastly made assumption of mixed risk aversion. Our results further cater to the importance of the higher-order risk preferences of prudence and temperance, as implied by mixed risk aversion, that complement risk aversion in important ways.

More formally, when does a pair of risk changes generate greater mutual aggravation than another pair of risk changes? Consider pairs of preference relations of the form $\tilde{x}_i \succcurlyeq_i \tilde{y}_i$ and $\tilde{x}_i \succcurlyeq_i \tilde{z}_i$, for $i = 1, 2$, and where $\succcurlyeq_i$ represents a stochastic-dominance order. We are interested in finding conditions under which the two pairs of risk changes can be ordered by their degree of mutual aggravation. In short, we show that mixed risk aversion guarantees that the degree of mutual aggravation increases with the riskiness of risks: If $\tilde{y}_i$ dominates $\tilde{z}_i$ under the partial order $\succcurlyeq_i'$, the shifts $\tilde{x}_i \succcurlyeq \tilde{z}_i$ are more mutually aggravating than the shifts $\tilde{x}_i \succcurlyeq \tilde{y}_i$.[2]

Building upon Friedman and Savage's (1948) notion of a utility premium, our characterization of greater mutual aggravation is based on the comparison of two utility premia, each of which captures the utility gain from avoiding the joint occurrence of a pair of risk changes. We first characterize greater mutual aggravation for general stochastic dominance shifts. Then, we generalize our results to establish conditions for greater mutual aggravation when the stochastic dominance shifts preserve one or more moments of the distribution constant. In the process of doing so, we extend Eeckhoudt et al's (2009) characterization of mutual aggravation of stochastic dominance shifts (preferences for combining good-with-bad) to mutual aggravation with moment-preserving transformations.

Besides the obvious welfare implications of our analysis, we show that the concept of greater mutual aggravation has several other implications. First, we examine the relative valuation of various risk apportionment decisions in the model of Eeckhoudt and Schlesinger (2006). We show that, if the risk(s) to be apportioned are riskier, in the sense of $N$th-order stochastic dominance, a mixed risk averter will place a higher value on apportioning the risk. Building upon this result, we show that greater mutual aggravation has important implications for the design and analysis of the growing number of experiments on higher-order risk preferences. For example, our results imply that some

[2] While for sake of concreteness we focus on mixed risk averters throughout, using the methodology of Crainich et al. (2013) our results may well extend to mixed risk lovers.

results in the experiment of Ebert and Wiesen (2011) are indicative of temperance, even though originally designed to test for prudence. The logic behind this observation can be generalized to obtain a new experimental method to test for higher-order risk preferences that is based on greater mutual aggravation. It can be combined with existing methods and allows for an efficient and simple test of “very” higher-order risk preferences such as edginess or bentness. Finally, we show that greater mutual aggravation can shed light on various comparative statics results. We present precautionary saving, intertemporal risk taking, and self protection as examples.

Section 2 introduces notation, reinterprets Eeckhoudt et al. (2009) good-with-bad result in terms of mutual aggravation, and characterizes greater mutual aggravation for stochastic dominance. Section 3 generalizes this result and Eeckhoudt et al’s (2009) results to the moment-preserving stochastic order, which includes Ekern (1980) risk increases. Section 4 studies implications of our result for the influential risk apportionment model of (Eeckhoudt and Schlesinger, 2006), the experimental elicitation of risk preferences, and comparative statics problems.

## 2 Mutual aggravation for stochastic dominance

### 2.1 Stochastic dominance and mixed risk aversion

We first analyze risk changes in line with the well-known stochastic dominance order. Let $F$ denote the cumulative distribution function of a random variable, whose support is contained within the interval $[a, b]$. Define $F^{(0)}(x) \equiv F(x)$ and define $F^{(i)}(x) \equiv \int_a^x F^{(i-1)}(t)dt$ for $i \geq 1$.

**Definition 1 (Stochastic Dominance)** *The distribution $F$* dominates *the distribution $G$ in the sense of $N$th-order stochastic dominance (written $F \succeq_{NSD} G$) if*

*(i) $F^{(N-1)}(x) \leq G^{(N-1)}(x)$ for all $a < x < b$ with strict inequality for some $x \in (a, b)$.*

*(ii) $F^{(i)}(b) \leq G^{(i)}(b)$ for $i = 1, \ldots, N-2$.*

Given random variables $\tilde{x}$ and $\tilde{y}$ with cumulative distribution functions $F$ and $G$ respectively, we will say that $\tilde{x} \succeq_{N\text{SD}} \tilde{y}$. The following well-known result shows the link between Definition 1 and expected utility.[3]

**Theorem 1 (Stochastic dominance and expected utility)** *The following statements are equivalent:*

[3]See Hadar and Russel (1969) and Hanoch and Levy (1970) introduced second-order stochastic dominance to the economics literature. See Whitmore (1970), Jean (1980) as well as Ingersoll (1987) for extensions to higher orders of stochastic dominance.

*(i)* $F \succeq_{NSD} G$

*(ii)* $\int_a^b u(t)dF \geq \int_a^b u(t)dG$, *for all functions* $u$ *such that* $(-1)^n u^{(n)}(x) \leq 0$ *for* $n = 1, ..., N$ .

Caballé and Pomansky (1996) coined the term *mixed risk aversion,* which implies stochastic dominance preference of all orders.

**Definition 2 (Mixed risk aversion)** *An individual with utility function* $u$ *that satisfies* $(-1)^n u^{(n)}(x) \leq 0$ *for* $k = n, ..., N$ *is called* mixed risk-averse up to order $N$. *An individual with* $(-1)^k u^{(k)}(x) \leq 0$ *for all* $k \in \mathbb{N}$ *is called* mixed risk averse.

Brockett and Golden (1987) show that all commonly used utility functions imply mixed risk aversion. In this paper, we derive new and general implications of this important preference property.

## 2.2 Mutual aggravation of risk changes and the preference for combining "good" with "bad"

Given a set of independent random variables $\{\tilde{x}_1, \tilde{x}_2, \tilde{y}_1, \tilde{y}_2\}$, suppose that $\tilde{x}_1 \succcurlyeq_{N_1\text{SD}} \tilde{y}_1$ and $\tilde{x}_2 \succcurlyeq_{N_2\text{SD}} \tilde{y}_2$. From the results in the previous section it follows that an individual who is mixed risk averse up to order $\max(N_1, N_2)$ has the partial ranking $\tilde{x}_1 + \tilde{x}_2 \succcurlyeq \tilde{x}_i + \tilde{y}_j \succcurlyeq \tilde{y}_1 + \tilde{y}_2$ where $(i, j) \in \{(1, 2), (2, 1)\}$. Following Menezes and Wang (2005), we refer to the sums $\tilde{x}_i + \tilde{y}_j$ as the inner risks. We say that $\tilde{x}_1 + \tilde{x}_2$ is the outer least-risk and $\tilde{y}_1 + \tilde{y}_2$ is the outer greatest-risk. While the outer least-risk combines two "goods" and the outer greatest-risk combines two "bads", the inner risks combine "goods" with "bads". Eeckhoudt et al. (2009) establish that mixed risk averters prefer 50:50 lotteries that combine the inner risks over 50:50 lotteries that combine the outer risks. This means that mixed risk aversion implies a *preference for combining good with bad* within each state of nature over combining good with good in one state and bad with bad in the other state.

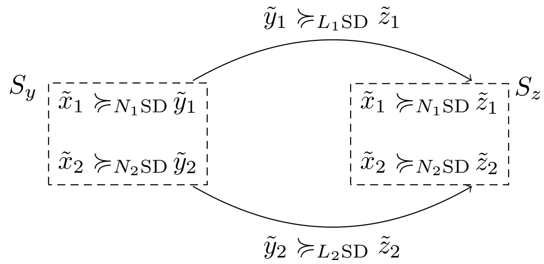
**Theorem 2 (Eeckhoudt, Schlesinger, and Tsetlin 2009)** *Suppose that* $\tilde{x}_1 \succcurlyeq_{N_1SD} \tilde{y}_1$ *and* $\tilde{x}_2 \succcurlyeq_{N_2SD} \tilde{y}_2$. *Then, lottery* $[\tilde{x}_1 + \tilde{x}_2; \tilde{y}_1 + \tilde{y}_2]$ *is dominated by lottery* $[\tilde{x}_1 + \tilde{y}_2; \tilde{y}_1 + \tilde{x}_2]$ *via* $(N_1 + N_2)$ *th degree stochastic dominance.*

Note that the lottery preference shown in Theorem 2 can be written as

$$\begin{aligned} & \mathbb{E}u(\tilde{x}_1 + \tilde{x}_2) + \mathbb{E}u(\tilde{y}_1 + \tilde{y}_2) \leq \mathbb{E}u(\tilde{x}_1 + \tilde{y}_2) + \mathbb{E}u(\tilde{y}_1 + \tilde{x}_2) \\ \iff \; & \mathbb{E}u(\tilde{x}_1 + \tilde{x}_2) - \mathbb{E}u(\tilde{y}_1 + \tilde{y}_2) \geq [\mathbb{E}u(\tilde{x}_1 + \tilde{x}_2) - \mathbb{E}u(\tilde{y}_1 + \tilde{x}_2)] + [\mathbb{E}u(\tilde{x}_1 + \tilde{x}_2) - \mathbb{E}u(\tilde{x}_1 + \tilde{y}_2)] \end{aligned}$$

Another interpretation of Eeckhoudt et al.'s (2009) result is thus the following. Mixed risk aversion up to order $N_1 + N_2$ implies that avoiding the joint occurrence of two risk changes – from $\tilde{x}_1$ to $\tilde{y}_1$

Figure 1: Stochastic dominance relationships




*Notes.* For $i = 1, 2$, $\tilde{x}_i$ dominates both $\tilde{y}_i$ and $\tilde{z}_i$ by $N_i$th-order stochastic dominance so that risk changes from $\tilde{x}_i$ to both $\tilde{y}_i$ and $\tilde{z}_i$ are undesirable for a mixed risk averse decision maker up to order $N_i$ (Theorem 1). Theorem 2 says that the risk changes $\tilde{x}_i$ to $\tilde{y}_i$ in set $S_y$ as well as the risk changes $\tilde{x}_i$ to $\tilde{z}_i$ in set $S_z$ are, respectively, mutually aggravating for a mixed risk averse decision maker up to order $N_1 + N_2$. Since $\tilde{y}_i$ dominates $\tilde{z}_i$ by $L_i$th-order stochastic dominance, Theorem 3 investigates under what condition the risk changes $\tilde{x}_i$ to $\tilde{z}_i$ are more mutually aggravating than the risk changes the risk changes $\tilde{x}_i$ to $\tilde{y}_i$.

and $\tilde{x}_2$ to $\tilde{y}_2$ as in the left-hand side of the above equation – is more beneficial than avoiding two risk changes that occur one at a time (the right-hand side of the above equation). In other words, mixed risk aversion implies that the risk changes *aggravate each other* as their joint occurrence is more harmful than the harm generated by their individual occurrences. Based on this intuition, the following definition extends Kimball's (1993) notion of mutually aggravating *risks* to risk changes.[4]

**Definition 3 (Mutually aggravating risk c)** *Risk changes $\tilde{x}_i \succcurlyeq_{N_iSD} \tilde{y}_i$, $i = 1, 2$, are mutually aggravating when the expected utility gain from avoiding both risk changes is greater than the sum of the expected utility gains from avoiding each risk change in isolation.*

## 2.3 Greater mutual aggravation and the comparative utility premium

Next, consider two sets of random variables $S_y = \{\tilde{x}_1, \tilde{x}_2, \tilde{y}_1, \tilde{y}_2\}$ and $S_z = \{\tilde{x}_1, \tilde{x}_2, \tilde{z}_1, \tilde{z}_2\}$ with $\tilde{x}_i \succcurlyeq_{N_i\mathrm{SD}} \tilde{y}_i$ and $\tilde{x}_i \succcurlyeq_{N_i\mathrm{SD}} \tilde{z}_i$ ($i = 1, 2$). Our objective is to compare the gain from combining good with bad in these sets when $\tilde{y}_1 \succcurlyeq_{L_1\mathrm{SD}} \tilde{z}_1$ and $\tilde{y}_2 \succcurlyeq_{L_2\mathrm{SD}} \tilde{z}_2$, i.e. the bads in $S_z$ are stochastically dominated by the bads in $S_y$. Figure 1 visualizes the relationships between the risks we study.

Intuition suggests that the utility gain from combining good with bad should be larger for the dominated risks. Likewise, the mutual aggravation of risk changes in $S_z$ should be more severe than

[4]Kimball considered the absence or presence of risks, which is a special case of 2nd degree stochastic dominance risk change when the means of the risks are non-positive.

the mutual aggravation of the risk changes in $S_y$. We thus look for a condition on preferences so that mutual aggravation is increasing in the riskiness of the risks.

Building upon Friedman and Savage (1948) and Eeckhoudt and Schlesinger (2006), our approach is based on the analysis of a utility premium and, in particular, on the comparison of two utility premia. For each set $S_y$ and $S_z$, respectively, the utility premia $\Psi_{\tilde{y}}$ and $\Psi_{\tilde{z}}$ capture the utility from combining good with bad or, equivalently, the degree of mutual aggravation:

$$\begin{aligned}\Psi_{\tilde{y}} &= \mathbb{E}u\left(\tilde{y}_1+\tilde{x}_2\right)+\mathbb{E}u\left(\tilde{x}_1+\tilde{y}_2\right)-\mathbb{E}u\left(\tilde{x}_1+\tilde{x}_2\right)-\mathbb{E}u\left(\tilde{y}_1+\tilde{y}_2\right) \text{ and} \\ \Psi_{\tilde{z}} &= \mathbb{E}u\left(\tilde{z}_1+\tilde{x}_2\right)+\mathbb{E}u\left(\tilde{x}_1+\tilde{z}_2\right)-\mathbb{E}u\left(\tilde{x}_1+\tilde{x}_2\right)-\mathbb{E}u\left(\tilde{z}_1+\tilde{z}_2\right).\end{aligned} \tag{1}$$

$\Psi_{\tilde{y}} \geq 0$ (resp. $\Psi_{\tilde{z}} \geq 0$) means that risks $\tilde{y}_1$ and $\tilde{y}_2$ (resp. $\tilde{z}_1$ and $\tilde{z}_2$) are mutually aggravating. By Theorem 2, $\Psi_{\tilde{y}} \geq 0$ and $\Psi_{\tilde{z}} \geq 0$ if the decision maker is mixed risk averse up to the $(N_1+N_2)\,th$ order. To determine when riskier risks are more mutually aggravating, we define the *increase in mutual aggravation* (equivalently, the increase in utility from combining good with bad) by a *comparative utility premium:*

$$\Delta\Psi_{(L_1,L_2)} = \Psi_{\tilde{z}} - \Psi_{\tilde{y}} \tag{2}$$

The condition that the mutual aggravation of two risks is more severe for riskier risks thus corresponds to $\Delta\Psi_{(L_1,L_2)} \geq 0$. We obtain the following result.

**Theorem 3 (Greater mutual aggravation for stochastic dominance)** *For $i=1,2$, suppose that $\tilde{x}_i \succcurlyeq_{N_iSD} \tilde{y}_i$, $\tilde{x}_i \succcurlyeq_{N_iSD} \tilde{z}_i$, and $\tilde{y}_i \succcurlyeq_{L_iSD} \tilde{z}_i$. If preferences display mixed risk aversion up to the order* $\max\left(N_1+L_2, N_2+L_1\right)$, *the degree of mutual aggravation of risk changes is larger in the set with dominated risks, i.e.* $\Delta\Psi_{(L_1,L_2)} \geq 0$.

Let us put this result in perspective to the earlier results on mixed risk aversion. First, the seminal result given in Theorem 1 shows that a utility function with derivatives of alternating sign implies that the decision maker dislikes stochastic dominance shifts of any order, thereby justifying the name "mixed risk aversion." Second, while it seems intuitive that two risk changes at a time are perceived as more harmful than two risk changes in isolation, it was not seen until Eeckhoudt and Schlesinger (2006) and Eeckhoudt et al. (2009) that this desirable consistency property follows from mixed risk aversion (Theorem 2). Third, Theorem 3 shows a new consistency property, namely, that mixed risk aversion implies that the mutual aggravation of risks increases when the risks are deteriorated via stochastic dominance. That is, Theorem 3 shows that mutual aggravation is monotone in the severity of the risks considered. Likewise, combining good with bad is more valuable when the risks to be combined are riskier.

For example, we mentioned in the introduction the possibility that a decision maker may perceive the risk of becoming unemployed and the risk of unexpected medical expenses to be more harmful if unemployment and sickness risk manifest at the same rather than at different dates. Theorem 3 shows, in addition, that if the risk of becoming unemployed and or the health risks become more severe, the degree of mutual aggravation of the risks will be more pronounced. Similarly, for a mixed risk averse decision maker the risky returns on a given financial investment will be aggravated by the risk on human capital returns, and the degree of aggravation will increase with the riskiness of the returns. In addition to these important welfare implications, we will illustrate several other implications of our characterization of greater mutual aggravation. Before we do that, we generalize the results obtained so far by looking at moment-preserving stochastic shifts.

# 3 Mutual aggravation of moment-preserving transformations

For *any* stochastic dominance shift, mixed risk aversion implies that (i) risks shifts are undesirable (Theorem 1), (ii) risk changes are mutually aggravating (Theorem 2), and (iii) mutual aggravation is increasing in the riskiness of risks (Theorem 3). Still, when more information on the type of stochastic dominance risk change is available, less than mixed risk aversion may be needed to obtain (i) to (iii). In particular, if the stochastic dominance shifts maintain one or more moments of the distribution constant, we can identify which orders of risk aversion are necessary for (i) to (iii). This will be insightful in the applications studied below. For example, when skewness but not mean and variance of the background risk in a precautionary saving model changes (cf. Eeckhoudt and Schlesinger (2008)), then prudence and temperance (mixed risk aversion from order 3 to order 4) are sufficient for precautionary saving to increase.

**Definition 4 ($M$-moments preserving $N$th-order stochastic dominance)** *Let $N \geq 1$ and $0 \leq M \leq N-1$. Risk $\tilde{x}$ dominates risk $\tilde{y}$ by $M$-*moments preserving *$N$*th-order stochastic dominance *($M/N$-order for short, written $\tilde{x} \succeq_{M/N} \tilde{y}$) if $\tilde{x} \succeq_{NSD} \tilde{y}$ and $\mathbb{E}[\tilde{x}^k] = \mathbb{E}[\tilde{y}^k]$ for $k = 0, \ldots, M \leq N-1$.*

Note that when $M = 0$ the latter condition reduces to $\mathbb{E}[\tilde{x}^0] = \mathbb{E}[\tilde{y}^0]$, which is trivially fulfilled. Therefore, when $M = 0$ we are back to stochastic dominance, i.e., the $0/N$-order is just the $N$SD-order. For $M = N-1$, the $M/N$-order corresponds to the concept of an $N$th-degree risk increase (Ekern (1980)). The most famous example of an $N$th-degree risk increase is the 2nd order risk increase, which is the mean-preserving spread studied by Rothschild and Stiglitz (1970). 3rd-degree risk increases corresponds to the downside risk increases defined by Menezes et al. (1980), which leave mean and variance unchanged, but decrease the skewness of a risk. Downside risk increases have received much attention in the finance literature subsequent to the recent financial crisis as well as in the behavioral literature which notes that individuals overweight the probability of unlikely, large losses. Liu and Meyer (2013) provide general preference conditions that characterize the tradeoffs between an $N$th-order and an $M$th-order risk increase. Denuit and Eeckhoudt (2013) recently studied mean-preserving stochastic dominance shifts, i.e., the $1/N$-order. In general, the $M/N$-order can describe many specific kinds of risk changes. As a final example, if $N = 4$ and $M = 2$, the set of changes under consideration corresponds to 4th-order stochastic dominance shifts that preserve mean and variance but may decrease skewness and increase kurtosis.

In the following, we generalize Theorems 1 to 3 by assuming that the shifts under consideration follow the $M/N$-order. In fact, the generalization of Theorem 1 has recently been made by Liu (2014). Using our notation, Liu (2014) showed

**Theorem 4 (Liu 2014)** *Suppose that $\tilde{x} \succcurlyeq_{M/N} \tilde{y}$. If the decision maker is mixed risk averse from the $(M+1)\,th$ order up to the $Nth$ order, then $\mathbb{E}u(\tilde{x}) \geq \mathbb{E}u(\tilde{y})$.*

Next, we generalize Theorem 2 by Eeckhoudt et al. (2009) from stochastic dominance to the moment-preserving stochastic dominance order.

**Theorem 5** *Risk changes $\tilde{x}_i \succcurlyeq_{M_i/N_i} \tilde{y}_i, (i = 1, 2)$ are mutually aggravating if the decision maker is mixed risk averse from the $(M_1 + M_2 + 2)\,th$ order up to the $(N_1 + N_2)\,th$ order.*

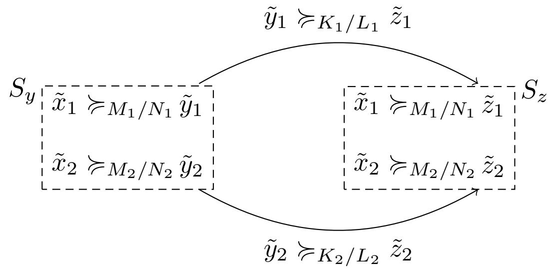
Theorem 5 pins down the set of decision makers that perceive risk changes as mutually aggravating. When $M_1 = M_2 = 0$ Theorem 5 recovers Theorem 2.[5] For $M_1 = N_1 - 1$ and $M_2 = N_2 - 1$, Theorem 5 shows that $N$th-degree risk increases à la Ekern (1980) are mutually aggravating for all decision makers that display $(N_1 + N_2)\,th$ degree risk aversion, a result stated separately by Eeckhoudt et al. (2009). An example of an implication not obtained by Eeckhoudt et al. (2009) is that two mean-preserving 3rd degree stochastic dominance shifts are mutually aggravating if the decision maker displays 4th, 5th, and 6th degree risk aversion. Finally, note that Theorem 5 not only presents the two important Eeckhoudt et al. (2009) results (the one for stochastic dominance and the one for risk increases) in a unified way, it also allows for "mixing" these risk shifts. For example, a mean-preserving spread ($M_1 = 1$ and $N_1 = 2$) and a third-order stochastic dominance deterioration ($M_2 = 0$ and $N_2 = 3$) are mutually aggravating in the case of mixed risk aversion from order 3 to 5.

Our final result in this section characterizes greater mutual aggravation for moment-preserving risk changes. Using the definitions of the utility premia $\Psi_{\tilde{z}}$ and $\Psi_{\tilde{y}}$ in equations (1), and now assuming that $\tilde{x}_i \succcurlyeq_{M_i/N_i} \tilde{y}_i$, $\tilde{x}_i \succcurlyeq_{M_i/N_i} \tilde{z}_i$, and $\tilde{y}_i \succcurlyeq_{K_i/L_i} \tilde{z}_i$, with $K_i \geq M_i$ (see Figure 2)[6], we define the

[5] In fact, another minor generalization is that our proof does not require $u' > 0$, but only mixed risk aversion starting at order 2.

[6] Notice that, since $\tilde{x}_i \succcurlyeq_{M_i/N_i} \tilde{y}_i$ and $\tilde{x}_i \succcurlyeq_{M_i/N_i} \tilde{z}_i$, the first $M_i$ moments of $\tilde{y}_i$ and $\tilde{z}_i$ must be the same, and this is the reason we impose the condition $K_i \geq M_i$.

Figure 2: Moment-preserving stochastic dominance relationships




*Notes.* This figure illustrates the risk changes for the general moments-preserving stochastic dominance order. For $M_1 = M_2 = K_1 = K_2 = 0$, we are back to the stochastic dominance case considered in Figure 1.

comparative utility premium as follows:

$$\Delta\Psi_{(K_1/L_1, K_2/L_2)} = \Psi_{\tilde{z}} - \Psi_{\tilde{y}}. \tag{3}$$

**Theorem 6** *For $i = 1, 2$, suppose that $\tilde{x}_i \succcurlyeq_{M_i/N_i} \tilde{y}_i$, $\tilde{x}_i \succcurlyeq_{M_i/N_i} \tilde{z}_i$, and $\tilde{y}_i \succcurlyeq_{K_i/L_i} \tilde{z}_i$, with $K_i \geq M_i$. If preferences display mixed risk aversion from the order $\min(M_2 + K_1 + 2, M_1 + K_2 + 2)$ up to the order $\max(N_1 + L_2, N_2 + L_1)$, the degree of mutual aggravation of risk changes is larger in the set with dominated risks, i.e. $\Delta\Psi_{(K_1/L_1, K_2/L_2)} \geq 0$.*

**Remark 1** *The proof of Theorem 6 shows that we have greater mutual aggravation also when only $\tilde{y}_2$ differs from $\tilde{z}_2$ while $\tilde{y}_1 = \tilde{z}_1$. The condition for greater mutual aggravation and a positive comparative utility premium, denoted by $\Delta\Psi_{(0, K_2/L_2)} \geq 0$, is mixed risk aversion from the order $(M_1 + K_2 + 2)$ up to the order $(N_1 + L_2)$. Similarly, when $\tilde{y}_1$ differs from $\tilde{z}_1$ and $\tilde{y}_2 = \tilde{z}_2$, Theorem 6 implies that $\Delta\Psi_{(K_1/L_1, 0)} \geq 0$ if the decision maker is mixed risk averse from the order $(M_2 + K_1 + 2)$ up to the order $(N_2 + L_1)$. Note that, as is the case for Theorem 5, Theorem 6 requires that the risk changes $\tilde{x}_i$ to $\tilde{y}_i$ and $\tilde{x}_i$ to $\tilde{z}_i$ must be non-trivial.*

# 4 Applications

## 4.1 Risk apportionment

To provide further intuition, we first apply Theorem 6 to study greater mutual aggravation in the influential risk apportionment model of Eeckhoudt and Schlesinger (2006).

### 4.1.1 Risk apportionment of order 3

Suppose that the sets of risk changes under consideration are $S_{\tilde{\epsilon}} = \{0, x^h, \tilde{\epsilon}, x^l\}$ and $S_{\tilde{\theta}} = \{0, x^h, \tilde{\theta}, x^l\}$ where $\tilde{\epsilon}$ and $\tilde{\theta}$ are mean zero risks while $x^l$ and $x^h$ are scalars such that $x^h > x^l$. Observe that $0 \succcurlyeq_{1/2} \tilde{\epsilon}$ (resp. $0 \succcurlyeq_{1/2} \tilde{\theta}$) and $x^h \succcurlyeq_{0/1} x^l$. The utility premium characterizing the degree of mutual aggravation for each set of risk changes is

$$\Psi_{\tilde{\epsilon}} = \left[u\left(x^l\right) + \mathbb{E}u\left(x^h + \tilde{\epsilon}\right)\right] - \left[u\left(x^h\right) + \mathbb{E}u\left(x^l + \tilde{\epsilon}\right)\right] \tag{4}$$

$$\Psi_{\tilde{\theta}} = \left[u\left(x^l\right) + \mathbb{E}u\left(x^h + \tilde{\theta}\right)\right] - \left[u\left(x^h\right) + \mathbb{E}u\left(x^l + \tilde{\theta}\right)\right] \tag{5}$$

Eeckhoudt and Schlesinger (2006) define prudence as a preference for "risk apportionment of order 3." Letting $[a; b]$ denote the lottery that yields $a$ and $b$ with equal probability, an individual is prudent if she prefers lottery $\tilde{B}_{3,\tilde{\epsilon}} = [x^l; x^h + \tilde{\epsilon}]$ over $\tilde{A}_{3,\tilde{\epsilon}} = [x^h; x^l + \tilde{\epsilon}]$ for all $x^l, x^h$ and $\tilde{\epsilon}$ as defined above. Therefore, note that $\Psi_{\tilde{\epsilon}} = \mathbb{E}[u(\tilde{B}_{3,\tilde{\epsilon}})] - \mathbb{E}[u(\tilde{A}_{3,\tilde{\epsilon}})]$ and $\Psi_{\tilde{\theta}} = \mathbb{E}[u(\tilde{B}_{3,\tilde{\theta}})] - \mathbb{E}[u(\tilde{A}_{3,\tilde{\theta}})]$. Since $N_1 = 2$, $N_2 = 1$, $M_1 = 1$, and $M_2 = 0$, our new Theorem 5 yields the well-known result that prudence (mixed risk aversion from order $1+0+2$ to order $1+2$) implies that $\tilde{B}_{3,\tilde{\epsilon}}$ is preferred over $\tilde{A}_{3,\tilde{\epsilon}}$ and, likewise, that $\tilde{B}_{3,\tilde{\theta}}$ is preferred over $\tilde{A}_{3,\tilde{\theta}}$.

Now suppose that $\tilde{\epsilon} \succcurlyeq_{K/L} \tilde{\theta}$. As before, the relative degree of mutual aggravation is captured by the comparative utility premium (for the notation see Remark 1)

$$\begin{aligned}
\Delta\Psi_{K/L,0} &= \Psi_{\tilde{\theta}} - \Psi_{\tilde{\epsilon}} \qquad (6)\\
&= \left\{\left[u\left(x^l\right) + \mathbb{E}u\left(x^h + \tilde{\theta}\right)\right] - \left[u\left(x^h\right) + \mathbb{E}u\left(x^l + \tilde{\theta}\right)\right]\right\}\\
&\quad - \left\{\left[u\left(x^l\right) + \mathbb{E}u\left(x^h + \tilde{\epsilon}\right)\right] - \left[u\left(x^h\right) + \mathbb{E}u\left(x^l + \tilde{\epsilon}\right)\right]\right\}.
\end{aligned}$$

$\Delta\Psi_{K/L,0} \geq 0$ means that the utility from risk apportionment is larger for the set with the dominated risk $\tilde{\theta}$. Intuitively, when there is an increase in the degree of mutual aggravation between two risk changes, the decision maker has a stronger incentive to apportion the riskier risks in such a way that they occur in different rather than in the same state of nature.

Noting that $L_2 = K_2 = 0$, $K_1 = K$, $L_1 = L$, Theorem 6 and Remark 1 readily imply the following result.

**Corollary 1** *The utility from third-order risk apportionment is greater when apportioning a riskier risk (i.e.* $\tilde{\epsilon} \succcurlyeq_{K/L} \tilde{\theta}$ *implies* $\Delta\Psi_{K/L,0} = \Psi_{\tilde{\theta}} - \Psi_{\tilde{\epsilon}} \geq 0$*) if the decision maker displays mixed risk aversion from the* $(K+2)$*th order up to the* $(L+1)$*th order.*

An important implication of this result is that, while prudence is all that is needed for mutual aggravation of a sure loss and the presence of a risk, the relative degree of mutual aggravation also depends on higher-order risk preferences. For example, if $K = 2$ and $L = 3$ (such that $\tilde{\theta}$ is a downside risk increase of $\tilde{\epsilon}$), the comparative utility premium $\Delta\Psi_{2/3}$ will be positive if the decision maker displays $4th$- degree risk aversion (temperance).

### 4.1.2 Risk apportionment of order 4

Next, let $S_{\{\tilde{\epsilon}_1,\tilde{\epsilon}_2\}} = \{0, 0, \tilde{\epsilon}_1, \tilde{\epsilon}_2\}$ and $S_{\left\{\tilde{\theta}_1,\tilde{\theta}_2\right\}} = \left\{0, 0, \tilde{\theta}_1, \tilde{\theta}_2\right\}$, where all risks have zero means and we assume that $\tilde{\epsilon}_i \succcurlyeq_{K_i/L_i} \tilde{\theta}_i$ (note that $K_i \geq 1$, so we restrict $L_i \geq 2$). In this case, mutual aggravation of the risk changes in each set coincides with risk apportionment of order 4 as defined by Eeckhoudt and Schlesinger (2006): The risk changes are mutually aggravating if the decision maker displays temperance. We wish to evaluate the sign of the comparative utility premium

$$\begin{aligned}\Delta\Psi_{(K_1/L_1,K_2/L_2)} &= \Psi_{\left\{\tilde{\theta}_1,\tilde{\theta}_2\right\}} - \Psi_{\{\tilde{\epsilon}_1,\tilde{\epsilon}_2\}} \\ &= \left\{\mathbb{E}\left[u\left(\tilde{\theta}_1\right) + u\left(\tilde{\theta}_2\right)\right] - \left[u\left(0\right) + \mathbb{E}u\left(\tilde{\theta}_1 + \tilde{\theta}_2\right)\right]\right\} \\ &\quad - \left\{\mathbb{E}\left[u\left(\tilde{\epsilon}_1\right) + u\left(\tilde{\epsilon}_2\right)\right] - \left[u\left(0\right) + \mathbb{E}u\left(\tilde{\epsilon}_1 + \tilde{\epsilon}_2\right)\right]\right\}\end{aligned} \tag{7}$$

Again using Theorem 6, we obtain,

**Corollary 2** *The utility from fourth-order risk apportionment is greater when apportioning riskier risks (i.e.* $\tilde{\epsilon} \succcurlyeq_{K_i/L_i} \tilde{\theta}, i = 1, 2$ *implies* $\Delta\Psi_{(K_1/L_1,K_2/L_2)} = \Psi_{\left\{\tilde{\theta}_1,\tilde{\theta}_2\right\}} - \Psi_{\{\tilde{\epsilon}_1,\tilde{\epsilon}_2\}} \geq 0$*) if the decision maker displays mixed risk aversion from order* $\min\left(K_1 + 3, K_2 + 3\right)$ *up to the order* $\max\left(L_1 + 2, L_2 + 2\right)$.

Thus, a temperate decision maker perceives independent risks as mutually aggravating, but the degree of mutual aggravation depends on the risks to be apportioned as well as the (higher-order) preferences of the decision maker as characterized precisely in Corollary 2. We will make use of this result in the next section.

## 4.2 A new approach to elicit risk preferences of (very) higher orders

An ever increasing number of experiments aims to elicit individual risk preferences. Not until the development of risk apportionment (Eeckhoudt and Schlesinger, 2006) and the "good"-with-"bad" theory of Eeckhoudt et al. (2009), however, knew researchers how to elicit higher-order risk preferences such as prudence and temperance. In these experiments (see Deck and Schlesinger (2010, 2014); Ebert and Wiesen (2011); Maier and Rüger (2011); Noussair et al. (forthcoming)), subjects repeatedly choose between two lotteries. One lottery indicates a preference for combining "good" with "bad" while the other lottery indicates a preference for combining "good" with "good" and "bad" with "bad". By designing lotteries with "goods" and "bads" of different orders, the experimenter can identify risk aversion, prudence, and temperance.

In this subsection, we show that the concept of greater mutual aggravation has important consequences for the interpretation of some of these experimental results. At the end of the subsection, we show how our result on greater mutual aggravation can improve existing designs used to elicit higher-order risk preferences. Moreover, we outline a method to test for risk preferences of the "very" high orders 5 ($u^{(5)} > 0$, edginess) and 6 ($u^{(6)} < 0$, bentness). The major advantage of the lotteries we propose is that they are no more complicated than those required to elicit prudence or temperance. Moreover, through proper design one can significantly reduce the number of choices subjects have to make.

### 4.2.1 Greater mutual aggravation and stochastic choice

Inference in the above mentioned experiments is often based on the assumption that subjects maximize utility, but also make "errors" when choosing between two lotteries, $\tilde{B}$ and $\tilde{A}$. These errors may arise, for example, because subjects devote limited attention to the task at hand. Choices between $\tilde{B}$ and $\tilde{A}$ are entirely random though only if the utility of $\tilde{B}$ and $\tilde{A}$ is the same. In general, the probability of choosing $\tilde{B}$ over $\tilde{A}$ is increasing in the difference of their (expected) utilities:

$$P \equiv \mathbb{P}(\text{Choose } \tilde{B} \text{ over } \tilde{A}) > 0.5 \Longleftrightarrow \mathbb{E}[u(\tilde{B})] - \mathbb{E}[u(\tilde{A})] > 0.$$

In the canonical probit model, for example, we have

$$P = \Phi\left(\frac{\mathbb{E}[u(\tilde{B})] - \mathbb{E}[u(\tilde{A})]}{\eta}\right)$$

where $\Phi(\cdot)$ denotes the cumulative distribution function for the standard normal distribution and where $\eta > 0$. The larger $\eta$, the larger is the probability that the subject does not decide according to her expected utility preferences, i.e., chooses $\tilde{B}$ over $\tilde{A}$ even when $\mathbb{E}[u(\tilde{B})] < \mathbb{E}[u(\tilde{A})]$.

Consider a subject's choice over risk apportionment lotteries of order 3 as defined above, $\tilde{A}_{3,\tilde{\epsilon}}$ and $\tilde{B}_{3,\tilde{\epsilon}}$, which employ the zero-mean risk $\tilde{\epsilon}$. According to the probit model, a subject makes a prudent choice with probability

$$P_{3,\tilde{\epsilon}} \equiv \Phi\left(\frac{\mathbb{E}[u(\tilde{B}_{3,\tilde{\epsilon},})] - \mathbb{E}[u(\tilde{A}_{3,\tilde{\epsilon}})]}{\eta}\right) = \Phi\left(\frac{\Psi_{\tilde{\epsilon}}}{\eta}\right) \tag{8}$$

where $\Psi_{\tilde{\epsilon}}$ is the utility premium from third-order risk apportionment defined in subsection 4.1.2. Therefore, since $\Phi(\cdot)$ is increasing, the probability $P_{n,\tilde{\epsilon}}$ is also increasing in the utility premium $\Psi_{n,\tilde{\epsilon}}$. A positive comparative utility premium thus tells us under which conditions a riskier zero-mean risk leads to a higher probability of choosing prudently. This has impact on choices as detailed in the following.

#### 4.2.2 Prudent (and temperate) behavior in the experiment of Ebert and Wiesen (2011)

For sake of concreteness, consider again the example subsequent to Corollary 1: If $K = 2$ and $L = 3$ ($\tilde{\theta}$ is a downside risk increase of $\tilde{\epsilon}$), the comparative utility premium $\Delta\Psi_{2/3}$ will be positive if the decision maker displays $4th$- degree risk aversion (temperance). This is precisely the setting in the experiment of Ebert and Wiesen (2011). The authors test for prudence using two different types of zero-mean risks, which for convenience we also refer to as $\tilde{\epsilon}$ and $\tilde{\theta}$. While $\tilde{\epsilon}$ is right-skewed, $\tilde{\theta}$ is left-skewed. Since both risks only have two outcomes, it follows from Ebert (2015) that indeed $\tilde{\epsilon} \succcurlyeq_{2/3} \tilde{\theta}$. Ebert and Wiesen (2011, p. 1344, Result 5) find that "Significantly more subjects decide prudently when [the zero-mean risk] is left-skewed." Corollary 1 proves that this result of "more prudent choices" is, in fact, evidence for *temperance* since

$$\text{Temperance} \Longrightarrow \Delta\Psi_{L/K} \geq 0 \Longrightarrow \Psi_{n,\tilde{\epsilon}} \leq \Psi_{3,\tilde{\theta}} \Longrightarrow P_{3,\tilde{\epsilon}} \leq P_{3,\tilde{\theta}}.$$

A positive comparative utility premium thus accords well with the following intuition: Subjects devote greater attention to decisions in which the stakes are higher in the sense that the risks faced are riskier. Through the analysis of the comparative utility premium, Theorem 3 and its corollaries tell us which higher-order risk preferences ensure that riskier risks take the role of higher stakes.

#### 4.2.3 Elicitation of "very" high-order risk preferences

Greater mutual aggravation offers a new method to test for higher-order risk preferences in experiments – as was "accidentally" done in Ebert and Wiesen (2011). The authors only claimed evidence for prudence, but, in fact, also provided evidence for temperance. Here we show that, through greater mutual aggravation, one can test for "very" high-order risk preferences through comparatively simple tasks. Deck and Schlesinger (2014) test for risk apportionment of orders 5 and 6 using the lottery nesting procedure of Eeckhoudt and Schlesinger (2006) as well as "good" with "bad" decision tasks. However, each choice is between two doubly-compounded lotteries. The authors observe a lot of noise in subjects' answers and "attribute this phenomenon to the ever-increasing complexity involved with deciphering increasingly higher degrees of risk." (Deck and Schlesinger, 2014, p. 1916).

Our new approach to test for edginess or bentness does not involve doubly-compound lotteries. Moreover, a given set of lotteries can be used to identify *two* higher-order risk preferences. We now illustrate this idea by designing temperance lotteries such that choices over them can be used to test for both temperance and edginess. The logic will be the same as in the previous section where we showed that choices over properly designed pairs of prudence lotteries can identify both prudence and temperance.

Recall that temperance means that independent zero-mean risks are mutually aggravating, which is also referred to as risk apportionment of order 4; cf. subsection 4.1.2. Therefore, the experimenter elicits a couple of "good"-with-"bad" choices for sets of type $S_{\{\tilde{\epsilon}_1,\tilde{\epsilon}_2\}} = \{0, 0, \tilde{\epsilon}_1, \tilde{\epsilon}_2\}$. Next, the experimenter also elicits a couple of choices for sets of type $S_{\{\tilde{\theta}_1,\tilde{\theta}_2\}} = \{0, 0, \tilde{\theta}_1, \tilde{\theta}_2\}$ where $\tilde{\theta}_1 \succcurlyeq_{2/3} \tilde{\theta}_1$ and also $\tilde{\epsilon}_2 \succcurlyeq_{2/3} \tilde{\theta}_1$. For both sets $S_{\{\tilde{\epsilon}_1,\tilde{\epsilon}_2\}}$ and $S_{\{\tilde{\theta}_1,\tilde{\theta}_2\}}$, a choice indicating mutual aggravation of the two risks is evidence for temperance. In addition, by Corollary 2 with $K_1 = K_2 = 2$ and $L_1 = L_2 = 3$, the utility premium for choices in set $S_{\{\tilde{\theta}_1,\tilde{\theta}_2\}}$ is larger if the decision maker displays risk aversion of order 5, i.e., edginess. If a larger utility premium results in fewer choice errors as is commonly assumed, more temperate choices in set $S_{\{\tilde{\theta}_1,\tilde{\theta}_2\}}$ than in set $S_{\{\tilde{\epsilon}_1,\tilde{\epsilon}_2\}}$ are evidence for temperance. Standard tests of this hypothesis of more temperate choices with riskier zero-mean risks can be conducted within subject as well as between subjects, i.e., for aggregate choices. All the experimenter must do to obtain this test for edginess, given that he tests for temperance, is to deliberately choose the zero-mean risks. A test for the sixth-order preference bentness is easily employed by using, for example, temperance sets $S_{\{\tilde{\epsilon}_1,\tilde{\epsilon}_2\}}$ and $S_{\{\tilde{\theta}_1,\tilde{\theta}_2\}}$ as before, but with zero-mean risks that differ in kurtosis: $\tilde{\epsilon}_1 \succcurlyeq_{3/4} \tilde{\theta}_1$ and also $\tilde{\epsilon}_2 \succcurlyeq_{3/4} \tilde{\theta}_1$.

## 4.3 Greater mutual aggravation and the comparative statics of risk changes

In this section we show that the concept of greater mutual aggravation provides an intuitive way to obtain the comparative statics of risk changes in several risk-management decisions.

### 4.3.1 Precautionary saving

Consider first the familiar 2-date model of precautionary saving. In this model, the consumer has an income $w$ at both dates, faces a mean zero risk $\tilde{\epsilon}$ at date-1, and selects the level of savings $s$ to maximize intertemporal expected utility $v(w-s) + \mathbb{E}u(w+s+\tilde{\epsilon})$. Define

$$s_\epsilon \in \arg\max_{s\in B\subseteq\mathbb{R}} v(w-s) + \mathbb{E}u(w+s+\tilde{\epsilon}). \tag{9}$$

Alternatively, suppose that the consumer faces another zero-mean risk $\tilde{\theta}$, with $\tilde{\epsilon} \succcurlyeq_{K/L} \tilde{\theta}$. His problem can then be depicted as follows

$$s_\theta \in \arg\max_{s\in B\subseteq\mathbb{R}} v(w-s) + \mathbb{E}u\left(w+s+\tilde{\theta}\right). \tag{10}$$

The question that we want to address is how changes in risk affect the level of savings. In these problems, and the problems that follow, the constraint set $B$ needs not be convex and we do not impose any restriction on the function which does not include the risk, $v(x)$. Moreover, we will not restrict $u(x)$ to be concave – risk averse –, although we will assume that the solution to at least one of the problems is unique (i.e. $s_\epsilon$ and or $s_\theta$ are uniquely defined) and that $u(x)$ is smooth. These latter assumptions can be relaxed using Topkis (1978) monotonicity results. Nocetti (forthcoming) performs such an analysis. Eeckhoudt and Schlesinger (2008) also studied the problem of precautionary saving with higher order risks under the assumption that the solutions are unique and interior. We will demonstrate that establishing the sign of the comparative utility premium provides a simple and intuitive way to recover well-known results on precautionary saving.

Recall that a prudent decision maker is one that perceives a sure loss (i.e. lower wealth) and the presence of a mean-zero risk as mutually aggravating. Equivalently, this decision maker prefers to allocate a higher level of wealth to the state in which a risk is present rather than the state in which the risk is absent. Intuition then suggests that when a risk is introduced to date-1 income the prudent consumer will save more to ameliorate the presence of the risk, which is well known to be the case. Similarly, it is natural to expect that *greater* mutual aggravation arises when a sure loss is bundled with a dominated risk (Corollary 1) so that the decision maker increases precautionary savings even further. The following result corroborates this intuition.

**Proposition 1** *Consider problems (9) and (10), with* $\tilde{\epsilon} \succcurlyeq_{K/L} \tilde{\theta}$. *If the (date-1) comparative utility premium* $\Delta\Psi_{K/L} = \Psi_{\tilde{\theta}} - \Psi_{\tilde{\epsilon}}$ *(as defined in (6)) is positive, then* $s_\theta \geq s_\epsilon$.

We note that greater mutual aggravation, $\Delta\Psi_{K/L} \geq 0$, is all that is needed for $s_\theta \geq s_\epsilon$, irrespective of other properties of $u(x)$. Of course, Corollary 1 says that $\Delta\Psi_{K/L} \geq 0$ is ensured for any risk change in the form $\tilde{\epsilon} \succcurlyeq_{K/L} \tilde{\theta}$ if the decision maker displays mixed risk aversion from the $(K+2)\,th$ order up to the $(L+1)\,th$ order. For example, when $K = L-1$ extra precautionary saving will arise whenever $u(x)$ exhibits $(K+1)\,th$ degree risk aversion, as in Eeckhoudt and Schlesinger (2008). A new result that can be obtained with our more general method is the following. Let $K = 2$ and $L = 4$ to consider a shift in risk that leaves mean and variance unaffected but decreases skewness and increases kurtosis. Temperance and edginess guarantee that the decision maker increases saving.

#### 4.3.2 Intertemporal risk taking

An individual has income $x$ in each of two dates and an additional lifetime net asset value of $\tilde{\epsilon}_1$, with $\mathbb{E}\tilde{\epsilon}_1 = 0$, where $\epsilon_1 > 0$ indicates a net asset and $\epsilon_1 < 0$ indicates a net liability. In addition, there is another zero-mean income risk $\tilde{\epsilon}_2$ in the second period, which we interpret as labor-income risk. This individual must decide at date $t = 0$, before $\tilde{\epsilon}_1$ is realized, how to distribute the $\tilde{\epsilon}_1$-risk over two periods. His objective is to choose the level of the asset $\alpha \in [0,1]$ that solves the following program:

$$\alpha_\epsilon = \arg\max_{\alpha\in[0,1]} \mathbb{E}[v(x + \alpha\tilde{\epsilon}_1)] + \mathbb{E}[u(x + (1-\alpha)\tilde{\epsilon}_1 + \tilde{\epsilon}_2)]. \tag{11}$$

Now suppose that labor market conditions deteriorate in such a way that income risk $\tilde{\epsilon}_2$ becomes $\tilde{\theta}_2$, where $\tilde{\epsilon}_2 \succcurlyeq_{K/L} \tilde{\theta}_2$. Define

$$\alpha_\theta = \arg\max_{\alpha\in[0,1]} \mathbb{E}[v(x+\alpha\tilde{\epsilon}_1)] + \mathbb{E}[u(x+(1-\alpha)\tilde{\epsilon}_1+\tilde{\theta}_2)]. \tag{12}$$

Thinking in terms of (greater) mutual aggravation provides us immediately with the intuition of what the decision maker will do. If the risks in the second period are mutually aggravating and if mutual aggravation increases as the risk $\tilde{\epsilon}_2$ increases to $\tilde{\theta}_2$, then $\alpha$ should increase in order to compensate for the otherwise increased mutual aggravation in the second period. This is exactly what happens. In more detail, let us define $\tilde{\epsilon}_1^h \equiv (1-\alpha^h)\tilde{\epsilon}_1$ and $\tilde{\epsilon}_1^l \equiv (1-\alpha^l)\tilde{\epsilon}_1$, where $\alpha^h > \alpha^l$. Notice that, since $\mathbb{E}\tilde{\epsilon}_1 = \mathbb{E}\tilde{\epsilon}_2 = 0$, we have $\tilde{\epsilon}_1^h \succcurlyeq_{1/2} \tilde{\epsilon}_1^l$ and $0 \succcurlyeq_{1/2} \tilde{\epsilon}_2$. Define the utility premium for this set of risk changes as $\Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)} = \mathbb{E}u\left(x+\tilde{\epsilon}_1^h+\tilde{\epsilon}_2\right) + \mathbb{E}u\left(x+\tilde{\epsilon}_1^l\right) - \mathbb{E}u\left(x+\tilde{\epsilon}_1^l+\tilde{\epsilon}_2\right) - \mathbb{E}u\left(x+\tilde{\epsilon}_1^h\right)$. Similarly, given risk changes $\tilde{\epsilon}_1^h \succcurlyeq_{1/2} \tilde{\epsilon}_1^l$ and $0 \succcurlyeq_{1/2} \tilde{\theta}_2$ define the utility premium $\Psi_{\left(\tilde{\epsilon}_1,\tilde{\theta}_2\right)} = \mathbb{E}u\left(x+\tilde{\epsilon}_1^h+\tilde{\theta}_2\right) + \mathbb{E}u\left(x+\tilde{\epsilon}_1^l\right) - \mathbb{E}u\left(x+\tilde{\epsilon}_1^l+\tilde{\theta}_2\right) - \mathbb{E}u\left(x+\tilde{\epsilon}_1^h\right)$.

Theorem (5) implies that the two pairs of risk changes are mutually aggravating, so $\Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)} \geq 0$ and $\Psi_{\left(\tilde{\epsilon}_1,\tilde{\theta}_2\right)} \geq 0$, whenever the decision maker displays $4th$ degree risk aversion, i.e. temperance. A temperate individual would select to allocate a greater proportion of the asset to date-0 compared with that case in which the background risks are absent. If he also perceives the degree of mutual aggravation to be higher when there is a deterioration in the background risk, we should similarly expect him to favor a greater allocation of the asset towards date-0.

**Proposition 2** *Consider problems (11) and (12), with $\tilde{\epsilon}_2 \succcurlyeq_{K/L} \tilde{\theta}_2$. If the comparative utility premium $\Psi_{\left(\tilde{\epsilon}_1,\tilde{\theta}_2\right)} - \Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)}$ is positive (given mixed risk aversion from order $K+3$ up to order $L+2$), then $\alpha_\theta \geq \alpha_\epsilon$.*

If, for example, $L=4$ and $K=1$ the degree of mutual aggravation of the risks $\tilde{\epsilon}_1$ and $\tilde{\theta}_2$ will be greater than that for the risks $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ if the decision maker displays risk aversion of orders 4, 5, and 6, in which case this decision maker will select $\alpha_\theta \geq \alpha_\epsilon$.

### 4.3.3 Self protection

As a final example, we consider a self-protection model as an application of Theorem 3. In this problem there are two dates and the decision maker exerts effort $e$ at date-0 in order to reduce the probability of facing an adverse outcome $\tilde{\epsilon}_1$ at date-1. In addition, suppose that the decision maker faces an unavoidable background risk $\tilde{\epsilon}_2$ at date-1 and define

$$e_\epsilon = \arg\max_{e\in B\subseteq\mathbb{R}} v\left(y_0-e\right) + P\left(e\right)\mathbb{E}u\left(y_1+\tilde{\epsilon}_2\right) + \left[1-P\left(e\right)\right]\mathbb{E}u\left(y_1+\tilde{\epsilon}_2+\tilde{\epsilon}_1\right) \tag{13}$$

where $y_0$ and $y_1$ represent the non-stochastic endowments at date-0 and date-1, respectively.

Alternatively, consider the same problem but with the adverse outcome $\tilde{\theta}_1$ and the background risk $\tilde{\theta}_2$,

$$e_\theta = \arg\max_{e \in B \subseteq \mathbb{R}} v\left(y_0 - e\right) + P\left(e\right) \mathbb{E}u\left(y_1 + \tilde{\theta}_2\right) \ + \left[1 - P\left(e\right)\right] \mathbb{E}u\left(y_1 + \tilde{\theta}_2 + \tilde{\theta}_1\right) \tag{14}$$

where $\tilde{\epsilon}_1 \succcurlyeq_{L_1\mathrm{SD}} \tilde{\theta}_1$ and $\tilde{\epsilon}_2 \succcurlyeq_{L_2\mathrm{SD}} \tilde{\theta}_2$ and all random variables are assumed to have non-positive means. This last assumption implies that $0 \succcurlyeq_{2SD} \tilde{\epsilon}_i$ and $0 \succcurlyeq_{2SD} \tilde{\theta}_i$. The degree of mutual aggravation for these pairs of risk changes can be characterized as follows: $\Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)} = \left[\mathbb{E}u\left(y + \tilde{\epsilon}_1\right) + \mathbb{E}u\left(y + \tilde{\epsilon}_2\right)\right] - \left[u\left(y\right) + \mathbb{E}u\left(y + \tilde{\epsilon}_1 + \tilde{\epsilon}_2\right)\right]$ and $\Psi_{\left(\tilde{\theta}_1,\tilde{\theta}_2\right)} = \left[\mathbb{E}u\left(y + \tilde{\theta}_1\right) + \mathbb{E}u\left(y + \tilde{\theta}_2\right)\right] - \left[u\left(y\right) + \mathbb{E}u\left(y + \tilde{\theta}_1 + \tilde{\theta}_2\right)\right]$. Theorem 2 implies that $\Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)} \geq 0$ and $\Psi_{\left(\tilde{\theta}_1,\tilde{\theta}_2\right)} \geq 0$ if the decision maker is risk averse, prudent, and temperate. This decision maker will exert more effort in self-protection in the presence of background risks $\tilde{\epsilon}_2$ or $\tilde{\theta}_2$ than in the absence of these risks. But how would a decision maker react in the face of a deterioration of the background risk in the sense of $L_2th$ degree stochastic dominance together with a deterioration of the avoidable risk in the sense of $L_1th$ degree stochastic dominance? Our previous analysis suggests that this decision maker will exert additional effort if these changes exacerbate the perceived degree of mutual aggravation (i.e. if $\Psi_{4,\left(\tilde{\theta}_1,\tilde{\theta}_2\right)} > \Psi_{4,(\tilde{\epsilon}_1,\tilde{\epsilon}_2)}$), which is precisely what is shown in Proposition 3.

**Proposition 3** *Consider problems (13) and (14) with $\tilde{\epsilon}_1 \succcurlyeq_{L_1SD} \tilde{\theta}_1$ and $\tilde{\epsilon}_2 \succcurlyeq_{L_2SD} \tilde{\theta}_2$. If $P\left(x\right)$ is increasing and $\Delta\Psi_{(L_1,L_2)} = \Psi_{\left(\tilde{\theta}_1,\tilde{\theta}_2\right)} - \Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)} \geq 0$ (i.e. given mixed risk aversion up to the order* $\max\left(L_2 + 2, L_1 + 2\right)$, *then $e_\theta \geq e_\epsilon$.*

# 5 Conclusion

This paper characterizes a new fundamental property of risk preferences as implied by the expected utility model. In particular, we unify and extend two previous results on mixed risk aversion, a property that is satisfied by all commonly used utility functions. Most importantly, we provide a new result on the risk preferences implied by mixed risk aversion and study some of its implications.

The first of the two existing results says that a mixed risk averter dislikes any stochastic dominance deterioration. The second result says that a mixed risk averter perceives two risk changes at a time as more harmful than two risk changes in isolation, i.e., risk changes are mutually aggravating. Our new result says that mixed risk aversion implies greater mutual aggravation for greater risks.

Our notion of (greater) mutual aggravation of risk changes extends Kimball's (1993) notion of mutually aggravating risks. The second result mentioned above follows from the observation that the preference for combining "good" with "bad" put forward by Eeckhoudt et al. (2009) is equivalent to perceiving risk changes as mutually aggravating.

In general, our notion of “more risky” follows the moments-preserving stochastic dominance order. This order includes the well-known stochastic dominance as well as (higher-order) risk increases as special cases. We also generalize the results in Eeckhoudt et al. (2009) to this more general order. Thereby, we identify the exact orders of risk preference that ensure that risk changes are mutually aggravating and that mutual aggravation is increasing in the riskiness of risks.

We illustrate the importance of the concept of greater mutual aggravation through several applications. Greater mutual aggravation implies that the utility premium from disaggregating the harms in the risk apportionment model of Eeckhoudt and Schlesinger (2006) is increasing in the riskiness of the risks to be apportioned. This result clarifies earlier experimental evidence on higher-order risk preferences. It is also valuable for the design of future experiments and offers an efficient way to test for risk preferences of very high orders. We also show that greater mutual aggravation provides the intuition behind various comparative statics results. We present precautionary saving, intertemporal risk taking, and self protection as examples.

# A Proofs

## A.1 Proof of Theorem 3

By definition of $\Delta\Psi_{(L_1,L_2)}$

$$\begin{aligned}\Delta\Psi_{(L_1,L_2)} &= \Psi_{\tilde{z}} - \Psi_{\tilde{y}} \qquad (15)\\ &= \mathbb{E}\left\{\left[u\left(\tilde{x}_1+\tilde{z}_2\right)+u\left(\tilde{x}_2+\tilde{z}_1\right)\right]-\left[u\left(\tilde{x}_1+\tilde{x}_2\right)+u\left(\tilde{z}_2+\tilde{z}_1\right)\right]\right\} - \\ &\quad \mathbb{E}\left\{\left[u\left(\tilde{x}_1+\tilde{y}_2\right)+u\left(\tilde{x}_2+\tilde{y}_1\right)\right]-\left[u\left(\tilde{x}_1+\tilde{x}_2\right)+u\left(\tilde{y}_2+\tilde{y}_1\right)\right]\right\}\end{aligned}$$

Fix $\tilde{z}_1 = \tilde{y}_1$ and define the resulting comparative utility premium as $\Delta\Psi_{(0,L_2)}$. Then,

$$\Delta\Psi_{(0,L_2)} = \mathbb{E}\left[u\left(\tilde{x}_1+\tilde{z}_2\right)+u\left(\tilde{y}_2+\tilde{y}_1\right)\right]-\mathbb{E}\left[u\left(\tilde{z}_2+\tilde{y}_1\right)+u\left(\tilde{x}_1+\tilde{y}_2\right)\right]. \qquad (16)$$

Since $\tilde{x}_1 \succcurlyeq_{N_1\mathrm{SD}} \tilde{y}_1$ and $\tilde{y}_2 \succcurlyeq_{L_2\mathrm{SD}} \tilde{z}_2$, Theorem 2 implies that $\Delta\Psi_{(0,L_2)} \geq 0$ if the decision maker displays mixed risk aversion up to the $(N_1+L_2)\,th$ order. We now show that $\Delta\Psi_{(L_1,L_2)} - \Delta\Psi_{(0,L_2)} \geq 0$. We have

$$\Delta\Psi_{(L_1,L_2)} - \Delta\Psi_{(0,L_2)} = \mathbb{E}\left[u\left(\tilde{x}_2+\tilde{z}_1\right)+u\left(\tilde{z}_2+\tilde{y}_1\right)\right]-\mathbb{E}\left[u\left(\tilde{z}_2+\tilde{z}_1\right)+u\left(\tilde{x}_2+\tilde{y}_1\right)\right] \qquad (17)$$

Since $\tilde{x}_2 \succcurlyeq_{N_2\mathrm{SD}} \tilde{z}_2$ and $\tilde{y}_1 \succcurlyeq_{L_1\mathrm{SD}} \tilde{z}_1$, Theorem 2 implies that the lottery $[\tilde{x}_2+\tilde{z}_1;\tilde{z}_2+\tilde{y}_1]$ dominates the lottery $[\tilde{z}_2+\tilde{z}_1;\tilde{x}_2+\tilde{y}_1]$ via $(N_2+L_1)\,th$ degree stochastic dominance. In total we thus have

$$\Delta\Psi_{(L_1,L_2)} \geq \Delta\Psi_{(0,L_2)} \geq 0$$

if the decision maker is mixed risk averse up to the order $\max\left(N_1+L_2, N_2+L_1\right)$.■

## A.2 Proof of Theorem 5

Define $v(w) \equiv \mathbb{E}u(\tilde{x}_2 + w) - \mathbb{E}u(\tilde{y}_2 + w)$. Thus, $\frac{1}{2}\mathbb{E}u(\tilde{x}_1 + \tilde{y}_2) + \frac{1}{2}\mathbb{E}u(\tilde{x}_2 + \tilde{y}_1) \geq \frac{1}{2}\mathbb{E}u(\tilde{x}_1 + \tilde{x}_2) + \frac{1}{2}\mathbb{E}u(\tilde{y}_2 + \tilde{y}_1)$ holds if and only if $\mathbb{E}v(\tilde{y}_1) - \mathbb{E}v(\tilde{x}_1) \geq 0$. By Theorem 4, $\mathbb{E}v(\tilde{y}_1) \geq \mathbb{E}v(\tilde{x}_1)$ holds if and only if $(-1)^{1+k_1} v^{(k_1)}(w) \geq 0$ for $k_1 = M_1 + 1, ..., N_1$. Since $v^{(k_1)}(w) = \mathbb{E}u^{(k_1)}(\tilde{x}_2 + w) - \mathbb{E}u^{(k_1)}(\tilde{y}_2 + w)$, this condition is equivalent to $(-1)^{1+k_1}\left[\mathbb{E}u^{(k_1)}(\tilde{x}_2 + w) - \mathbb{E}u^{(k_1)}(\tilde{y}_2 + w)\right] \geq 0$ for $k_1 = M_1 + 1, ..., N_1$. Using Theorem 4 again, the term in brackets is positive if $(-1)^{k_2} u^{(k_1+k_2)}(x) \geq 0$ for $k_2 = M_2 + 1, ..., N_2$. We can then conclude that $\mathbb{E}v(\tilde{y}_1) - \mathbb{E}v(\tilde{x}_1) \geq 0$ if $(-1)^{k_1+k_2+1} u^{(k_1+k_2)}(x) \geq 0$ for $k_1 = M_1 + 1, ..., N_1$ and $k_2 = M_2 + 1, ..., N_2$, or equivalently, if $(-1)^{k+1} u^{(k)}(x) \geq 0$ for $k = M_1 + M_2 + 2, ..., N_1 + N_2$.■

## A.3 Proof of Theorem 6

Fix $\tilde{z}_1 = \tilde{y}_1$ and define the resulting comparative utility premium as $\Delta\Psi_{(0,K_2/L_2)}$. Just as in the proof of Theorem 3 we find that

$$\Delta\Psi_{(0,K_2/L_2)} = \mathbb{E}\left[u(\tilde{x}_1 + \tilde{z}_2) + u(\tilde{y}_2 + \tilde{y}_1)\right] - \mathbb{E}\left[u(\tilde{z}_2 + \tilde{y}_1) + u(\tilde{x}_1 + \tilde{y}_2)\right]. \tag{18}$$

By assumption, $\tilde{x}_1 \succcurlyeq_{M_1/N_1} \tilde{y}_1$ and $\tilde{y}_2 \succcurlyeq_{K_2/L_2} \tilde{z}_2$. Theorem 5 readily implies that $\Delta\Psi_{(0,K_2/L_2)} \geq 0$ if the decision maker is mixed risk averse from the $(M_1 + K_2 + 2)\,th$ order up to the $(N_1 + L_2)\,th$ order. We now show that $\Delta\Psi_{(K_1/L_1,K_2/L_2)} - \Delta\Psi_{(0,K_2/L_2)} \geq 0$. We have

$$\Delta\Psi_{(K_1/L_1,K_2/L_2)} - \Delta\Psi_{(0,K_2/L_2)} = \mathbb{E}\left[u(\tilde{x}_2 + \tilde{z}_1) + u(\tilde{z}_2 + \tilde{y}_1)\right] - \mathbb{E}\left[u(\tilde{z}_2 + \tilde{z}_1) + u(\tilde{x}_2 + \tilde{y}_1)\right] \tag{19}$$

By assumption, $\tilde{x}_2 \succcurlyeq_{M_2/N_2} \tilde{z}_2$ and $\tilde{y}_1 \succcurlyeq_{K_1/L_1} \tilde{z}_1$. Theorem 5 then implies that $\Delta\Psi_{(K_1/L_1,K_2/L_2)} - \Delta\Psi_{(0,K_2/L_2)} \geq 0$ if the decision maker is mixed risk averse from the $(M_2 + K_1 + 2)\,th$ order up to the $(N_2 + L_1)\,th$ order. As in the proof of Theorem 3 it follows that $\Delta\Psi_{(K_1/L_1,K_2/L_2)} \geq 0$ if the decision maker is mixed risk averse from the order $\min(M_2 + K_1 + 2, M_1 + K_2 + 2)$ up to the order $\max(N_1 + L_2, N_2 + L_1)$.■

## A.4 Proof of Proposition 1

From (6) we have $\Delta\Psi_{K/L} = \Psi_{\tilde{\theta}} - \Psi_{\tilde{\epsilon}} = \mathbb{E}u\left(w + x^h + \tilde{\theta}\right) + \mathbb{E}u\left(w + x^l + \tilde{\epsilon}\right) - \mathbb{E}u\left(w + x^l + \tilde{\theta}\right) - \mathbb{E}u\left(w + x^h + \tilde{\epsilon}\right)$. Suppose that $\Delta\Psi_{K/L} \geq 0$, that $s_\epsilon$ (respectively $s_\theta$) is a unique maximizer, and that $s_\epsilon > s_\theta$. We have

$$\begin{aligned} 0 &\geq \left[v(w - s_\epsilon) + \mathbb{E}u\left(w + s_\epsilon + \tilde{\theta}\right)\right] - \left[v(w - s_\theta) + \mathbb{E}u\left(w + s_\theta + \tilde{\theta}\right)\right] \\ &\geq \left[v(w - s_\epsilon) + \mathbb{E}u(w + s_\epsilon + \tilde{\epsilon})\right] - \left[v(w - s_\theta) + \mathbb{E}u(w + s_\theta + \tilde{\epsilon})\right] \\ &> 0 \end{aligned}$$

The first inequality follows from the definition of $s_\theta$, the second inequality follows from the assumption that $\Delta\Psi_{K/L} \geq 0$ and $s_\epsilon > s_\theta$, and the third strict inequality follows from the definition of $s_\epsilon$ and the assumption that $s_\epsilon$ is a unique maximizer in (9). Since the inequalities are enclosed by zeroes, we have a contradiction and we conclude that $s_\theta \geq s_\epsilon$.■

## A.5 Proof of Proposition 2

Notice that $\Psi_{\left(\tilde{\epsilon}_1,\tilde{\theta}_2\right)} - \Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)} = \mathbb{E}u\left(x+\tilde{\epsilon}_1^h+\tilde{\theta}_2\right) + \mathbb{E}u\left(x+\tilde{\epsilon}_1^l+\tilde{\epsilon}_2\right) - \mathbb{E}u\left(x+\tilde{\epsilon}_1^l+\tilde{\theta}_2\right) - \mathbb{E}u\left(x+\tilde{\epsilon}_1^h+\tilde{\epsilon}_2\right).$ Suppose that $\Psi_{\left(\tilde{\epsilon}_1,\tilde{\theta}_2\right)} - \Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)} \geq 0$, that $\alpha_\epsilon$ (alternatively $\alpha_\theta$) is a unique maximizer, and that $\alpha_\epsilon > \alpha_\theta$. We have

$$
\begin{aligned}
0 \quad &\geq \quad \left[\mathbb{E}[v(x+\alpha_\epsilon\tilde{\epsilon}_1)] + \mathbb{E}[u(x+(1-\alpha_\epsilon)\tilde{\epsilon}_1+\tilde{\theta}_2)\right] - \left[\mathbb{E}[v(x+\alpha_\theta\tilde{\epsilon}_1)] + \mathbb{E}[u(x+(1-\alpha_\theta)\tilde{\epsilon}_1+\tilde{\theta}_2)]\right] \\
&\geq \quad [\mathbb{E}[v(x+\alpha_\epsilon\tilde{\epsilon}_1)] + \mathbb{E}[u(x+(1-\alpha_\epsilon)\tilde{\epsilon}_1+\tilde{\epsilon}_2)]] - [\mathbb{E}[v(x+\alpha_\theta\tilde{\epsilon}_1)] + \mathbb{E}[u(x+(1-\alpha_\theta)\tilde{\epsilon}_1+\tilde{\epsilon}_2)]] \\
&> \quad 0
\end{aligned}
$$

The first inequality follows from the definition of $\alpha_\theta$, the second inequality follows from the assumption that $\Psi_{\left(\tilde{\epsilon}_1,\tilde{\theta}_2\right)} - \Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)} \geq 0$ and $\alpha_\epsilon > \alpha_\theta$, and the third strict inequality follows from the definition of $\alpha_\epsilon$ and the assumption that $\alpha_\epsilon$ is a unique maximizer. Since the inequalities are enclosed by zeroes, we have a contradiction and we conclude that $\alpha_\theta \geq \alpha_\epsilon$.■

## A.6 Proof of Proposition 3

The comparative utility premium is given by

$$
\begin{aligned}
\Delta\Psi_{(L_1,L_2)} \quad &= \quad \Psi_{\left(\tilde{\theta}_1,\tilde{\theta}_2\right)} - \Psi_{(\tilde{\epsilon}_1,\tilde{\epsilon}_2)} \\
&= \quad \left\{\mathbb{E}\left[u\left(\tilde{\theta}_1\right)+u\left(\tilde{\theta}_2\right)\right] - \mathbb{E}\left[u\left(0\right)+u\left(\tilde{\theta}_1+\tilde{\theta}_2\right)\right]\right\} - \left\{\left[\mathbb{E}u\left(\tilde{\epsilon}_1\right)+\mathbb{E}u\left(\tilde{\epsilon}_2\right)\right] - \left[u\left(0\right)+\mathbb{E}u\left(\tilde{\epsilon}_1+\tilde{\epsilon}_2\right)\right]\right\}
\end{aligned}
$$

Suppose that $\Delta\Psi_{(L_1,L_2)} \geq 0$, that $e_\epsilon$ (alternatively $e_\theta$) is a unique maximizer, and that $e_\epsilon > e_\theta$. We have

$$
\begin{aligned}
0 \quad &\geq \quad \left[v\left(y_0-e_\epsilon\right) + P\left(e_\epsilon\right)\mathbb{E}u\left(y_1+\tilde{\theta}_2\right) \; + \left[1-P\left(e_\epsilon\right)\right]\mathbb{E}u\left(y_1+\tilde{\theta}_2+\tilde{\theta}_1\right)\right] \\
&\quad\; - \left[v\left(y_0-e_\theta\right) + P\left(e_\theta\right)\mathbb{E}u\left(y_1+\tilde{\theta}_2\right) \; + \left[1-P\left(e_\theta\right)\right]\mathbb{E}u\left(y_1+\tilde{\theta}_2+\tilde{\theta}_1\right)\right] \\
&= \quad \left[v\left(y_0-e_\epsilon\right) - v\left(y_0-e_\theta\right)\right] + \left[P\left(e_\epsilon\right) - P\left(e_\theta\right)\right]\left[\mathbb{E}u\left(y_1+\tilde{\theta}_2\right) - \mathbb{E}u\left(y_1+\tilde{\theta}_2+\tilde{\theta}_1\right)\right] \\
&\geq \quad \left[v\left(y_0-e_\epsilon\right) - v\left(y_0-e_\theta\right)\right] + \left[P\left(e_\epsilon\right) - P\left(e_\theta\right)\right]\left[\mathbb{E}u\left(y_1+\tilde{\epsilon}_2\right) - \mathbb{E}u\left(y_1+\tilde{\epsilon}_2+\tilde{\epsilon}_1\right)\right] \\
&> \quad 0
\end{aligned}
$$

The first inequality follows from the definition of $e_\theta$. The second inequality follows from the assumption that $\Delta\Psi_{(L_1,L_2)} \geq 0$, $e_\epsilon > e_\theta$ and that $P(x)$ is increasing. Specifically, given that $P(x)$ is increasing, notice that the second inequality holds if $\left[\mathbb{E}u\left(y_1+\tilde{\theta}_2\right)-\mathbb{E}u\left(y_1+\tilde{\theta}_2+\tilde{\theta}_1\right)\right]-\left[\mathbb{E}u\left(y_1+\tilde{\epsilon}_2\right)-\mathbb{E}u\left(y_1+\tilde{\epsilon}_2+\tilde{\epsilon}_1\right)\right] \geq 0$. This inequality is equivalent to the condition $\Delta\Psi_{(L_1,L_2)} + \mathbb{E}\left[u\left(\tilde{\epsilon}_1\right)-u\left(\tilde{\theta}_1\right)\right] \geq 0$. Given mixed risk aversion up to the order $\max\left(L_1+2, L_2+2\right)$ we have $\Delta\Psi_{(L_1,L_2)} \geq 0$ and also $\mathbb{E}\left[u\left(\tilde{\epsilon}_1\right)-u\left(\tilde{\theta}_1\right)\right] \geq 0$. Thus, $\Delta\Psi_{(L_1,L_2)} + \mathbb{E}\left[u\left(\tilde{\epsilon}_1\right)-u\left(\tilde{\theta}_1\right)\right] \geq 0$. Finally, the third strict inequality follows from the definition of $e_\epsilon$ and the assumption that $e_\epsilon$ is a unique maximizer. Since the inequalities are enclosed by zeroes, we have a contradiction and we conclude that $e_\theta \geq e_\epsilon$.■

## References


Bernoulli, D. (1738/1954): "Specimen Theoriae Novae de Mensura Sortis. Commentarii Academiae Scientiarum Imperialis Petropolitanae. Translated, Bernoulli, Daniel, 1954. Exposition of a New Theory on the Measurement of Risk," *Econometrica*, 22, 23–36.

Brockett, P. L. and L. L. Golden (1987): "A Class of Utility Functions Containing All the Common Utility Functions," *Management Science*, 33, 955–964.

Caballé, J. and A. Pomansky (1996): "Mixed Risk Aversion," *Journal of Economic Theory*, 71, 485–513.

Crainich, D. and L. Eeckhoudt (2008): "On the Intensity of Downside Risk Aversion," *Journal of Risk and Uncertainty*, 36, 267–276.

Crainich, D., L. Eeckhoudt, and A. Trannoy (2013): "Even (Mixed) Risk Lovers are Prudent," *American Economic Review*, 104, 1529–1535.

Deck, C. and H. Schlesinger (2010): "Exploring Higher Order Risk Effects," *Review of Economic Studies*, 77, 1403–1420.

——— (2014): "Consistency of Higher-Order Risk Preferences," *Econometrica*, 82, 1913–1943.

Denuit, M. M. and L. Eeckhoudt (2013): "Risk Attitudes and the Value of Risk Transformations," *International Journal of Economic Theory*, 9, 245–254.

Drèze, J. and F. Modigliani (1972): "Consumption Decisions under Uncertainty," *Journal of Economic Theory*, 5, 308–335.

Ebert, S. (2015): "On Skewed Risks in Economic Models and Experiments," *Journal of Economic Behavior and Organization*, 4, 85–97.

Ebert, S. and D. Wiesen (2011): “Testing for Prudence and Skewness Seeking,” *Management Science*, 57, 1334–1349.

Eeckhoudt, L. and H. Schlesinger (2006): “Putting Risk in Its Proper Place,” *American Economic Review*, 96, 280–289.

——— (2008): “Changes in Risk and the Demand for Saving,” *Journal of Monetary Economics*, 55, 1329–1336.

Eeckhoudt, L., H. Schlesinger, and I. Tsetlin (2009): “Apportioning of Risks via Stochastic Dominance,” *Journal of Economic Theory*, 144, 994–1003.

Ekern, S. (1980): “Increasing $N$th Degree Risk,” *Economics Letters*, 6, 329–333.

Friedman, M. and L. Savage (1948): “The Utility Analysis of Choices Involving Risk,” *Journal of Political Economy*, 56, 279–304.

Hadar, J. and W. Russel (1969): “Rules for Ordering Uncertain Prospects,” *American Economic Review*, 59, 25–34.

Hanoch, G. and H. Levy (1970): “Efficiency Analysis of Choices Involving Risk,” *Review of Economic Studies*, 36, 335–346.

Ingersoll, J. (1987): *Theory of Financial Decision Making*, Rowman & Littlefield, Lanham, MD.

Jean, W. (1980): “The Geometric Mean and Stochasic Dominance,” *Journal of Finance*, 35, 151–158.

Kimball, M. S. (1990): “Precautionary Savings in the Small and in the Large,” *Econometrica*, 58, 53–73.

——— (1993): “Standard Risk Aversion,” *Econometrica*, 61, 589–611.

Leland, H. E. (1968): “Saving and Uncertainty: The Precautionary Demand for Saving,” *Quarterly Journal of Economics*, 82, 465–473.

Liu, L. (2014): “Precautionary Saving in the Large: $n$th Degree Deteriorations in Future Income,” *Journal of Mathematical Economics*, 52, 169–172.

Liu, L. and J. Meyer (2013): “Substituting One Risk Increase for Another: A Method for Measuring Risk Aversion,” *Journal of Economic Theory*, 148, 2706–2718.

Maier, J. and M. Rüger (2011): “Experimental Evidence on Higher-Order Risk Preferences with Real Monetary Losses,” Working Paper, University of Munich.

Menezes, C. F., C. Geiss, and J. Tressler (1980): "Increasing Downside Risk," *American Economic Review*, 70, 921–932.

Menezes, C. F. and X. H. Wang (2005): "Increasing Outer Risk," *Journal of Mathematical Economics*, 41, 875–886.

Nocetti, D. (forthcoming): "Robust Comparative Statics of Risk Changes," *Management Science*.

Noussair, C., S. Trautmann, and G. Van de Kuilen (forthcoming): "Higher Order Risk Attitudes, Demographics, and Financial Decisions," *Review of Economic Studies*.

Pratt, J. W. and R. J. Zeckhauser (1987): "Proper Risk Aversion," *Econometrica*, 55, 143–154.

Rothschild, M. D. and J. E. Stiglitz (1970): "Increasing Risk: I. A Definition," *Journal of Economic Theory*, 2, 225–243.

Sandmo, A. (1970): "The Effect of Uncertainty on Saving Decisions," *Review of Economic Studies*, 37, 353–360.

Topkis, D. (1978): "Minimizing a Submodular Function on a Lattice," *Operations Research*, 26, 305–321.

Tsetlin, I. and R. Winkler (2009): "Multiattribute Utility Satisfying a Preference for Combining Good with Bad," *Management Science*, 55, 1942–1952.

Whitmore, G. (1970): "Third-Degree Stochastic Dominance," *American Economic Review*, 60, 457–4595.