# Optimal insurance for catastrophic risk: theory and application to nuclear corporate liability

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**Abstract**

This paper analyzes the optimal insurance for low-probability high-severity accidents, such as nuclear catastrophes, both from theoretical and applied standpoints. The risk premium of such catastrophic events may be a non-negligible proportion of individuals' wealth when the absolute risk aversion is very large in the accident state. The optimal indemnity schedule converges to a limit when the probability of the accident tends to zero. In the case of the limited liability of an industrial firm that may cause large scale damages, this limit schedule, and the associated corporate insurance contract, correspond to a straight deductible indemnification rule, in which victims are ranked according to the severity of their losses. The empirical part of the paper is an application of these general principles to the case of nuclear corporate liability. We calibrate a model using French data in order to estimate the optimal liability upper limit of a nuclear energy producer. We show that the upper limit adopted in 2004 through the revision of the Paris Convention is probably lower than the socially optimal level.

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# 1 Introduction

What qualifies a low-probability high-severity accident risk as a disaster risk? How should individuals and societies cover these risks? The present paper approaches these questions from theoretical and applied perspectives. Our motivation and ultimate objective is to analyze the case of nuclear accident risk.

We address the first question by characterizing individual preferences under which the normalized risk premium (i.e., the risk premium per unit of variance) may remain significant, even when the loss probability is very small. We then investigate the optimal insurance coverage of an individual who faces the risk of an accident with very low probability. We show that the normalized risk premium has a lower bound, which is a weighted average of absolute risk aversion values in the interval defined by the potential values of final wealth. In particular, under decreasing absolute risk aversion, a high absolute risk aversion (or, equivalently, a low risk tolerance) in the accident state may entail a large risk premium, even if the accident probability is very low. Concerning the optimal insurance coverage, we find that it converges to a limit when the accident probability goes to zero. This limit depends on the usual determinants of insurance demand: the insurance pricing rule and the individuals' wealth and degree of risk aversion.

In a second stage, we consider the risk of an industrial accident, such as a nuclear catastrophe, that may affect the entire population of a country. Should an accident occur, the firm has to indemnify the victims according to liability law, and it purchases insurance to prevent any insolvency. We characterize the indemnification rule that should be implemented by a utilitarian regulator. We show that it converges toward a straight deductible indemnity schedule, capped by an upper limit, when the accident probability goes to zero. In particular, the optimal coverage depends on the cost of capital that has to be levied to sustain the indemnification mechanism.

Finally, as an application of these theoretical principles, we consider the case of nuclear risk. Using studies conducted by experts in safety for a nuclear reactor in France, we calibrate a model of collective insurance choice and we characterize the optimal level of coverage for the victims of a large scale nuclear accident. In particular, we use data from the cat bond market to infer the premium that would be required by investors to set up an insurance deal for nuclear accidents. Our simulations suggest that the French nuclear liability law should be more ambitious that it currently is, even after the recent revision of the international Paris Convention in 2004.

The paper is organized as follows. Section 2 analyzes the risk premium and the insurance demand for a low-probability high-severity accident, from the perspective of a risk averse individual who is facing such a risk. Section 3 characterizes the optimal coverage and the corresponding corporate liability, when a large scale industrial accident may affect the whole population of a country. Section 4 illustrates these general results through a calibrated model of nuclear catastrophe coverage. Section 5 concludes, Section 6 is an appendix that contains proofs, tables and figures.

# 2 Risk premium and insurance demand for catastrophic risks

## 2.1 The risk premium of low-probability and high-severity risks

Consider an expected utility risk averse individual with a von Neumann-Morgenstern utility function $u(x)$ such that $u' > 0$ and $u'' < 0$, where $x$ is the individual's wealth. Let $A(x) = -u''(x)/u'(x)$ and $T(x) = 1/A(x)$ be her indices of absolute risk aversion and of risk tolerance, respectively. She holds an initial wealth $w$, and she is facing the risk of a loss $L < w$ with probability $p$. Thus $m(p, L) = pL$ and $\sigma^2(p, L) = p(1-p)L^2$ are the expected loss and the variance of the loss, respectively. The certainty equivalent $C(p, L)$ of this lottery is defined by

$$u(w - C) = (1-p)u(w) + pu(w - L).$$

We also denote

$$\theta(p, L) \equiv \frac{C(p, L) - m(p, L)}{\sigma^2(p, L)}$$

the normalized risk premium, that is the risk premium per unit of variance of the risk. Straightforward calculations give

$$\begin{aligned} C'_p(p, L) &= \frac{u(w) - u(w-L)}{u'(w-C)} > 0, \\ C''_{p^2}(p, L) &= -C'_p(p, L)^2 A(w - C) < 0. \end{aligned}$$

Thus, $C(p, L)$ is increasing and concave with respect to $p$, and of course we have $C(0, L) = 0$.

Put informally, the risk $(p, L)$ may be considered as catastrophic for the individual if $C(p, L)$ is non-negligible, for instance as a proportion of her initial wealth $w$, although $p$ is small or even very small. Obviously, this may occur if $C_p'(0,L)$ is large. We have

$$C_p'(0,L) = \frac{u(w) - u(w-L)}{u'(w)}. \tag{1}$$

Using l'Hôpital's Rule gives

$$\theta(0,L) \equiv \lim_{p \longrightarrow 0} \theta(p,L) = \frac{C_p'(0,L) - L}{L^2}. \tag{2}$$

Thus, for $L$ given, the larger $C_p'(0,L)$, the larger the normalized risk premium when $p$ goes to zero.

We know from the Arrow-Pratt approximation that the risk premium of low-severity risks per unit of variance is proportional to the index of absolute risk aversion. Indeed, we have

$$\lim_{L \longrightarrow 0} \theta(p,L) = \frac{A(w)}{2} \text{ for all } p \in (0,1),$$

which of course also holds when $p$ goes to 0, that is

$$\lim_{L \longrightarrow 0} \theta(0,L) = \frac{A(w)}{2}.$$

When $L$ is large, it is intuitive that the size of the risk premium depends on function $A(x)$ not only in the neighborhood of $x = w$, but over the whole interval $[w-L, L]$. Proposition 1 and its corollaries confirm this intuition. Proposition 1 provides an exact formula for $\theta(0,L)$ which is a weighted average of $A(x)\exp\{\int_x^w A(t)dt\}/2$ when $x$ is in $[w-L, w]$. Corollary 1 directly deduces a lower bound for $\theta(0,L)$, and Corollary 2 considers the case where $L = w$ and the index of relative risk aversion $R(x)$ is larger or equal to one.[1] In that case, the lower bound of $\theta(0,L)$ is the (non-weighted) average of $A(x)$ when $x \in [0, w]$.

**Proposition 1** *For all $L > 0$, we have*

$$\theta(0,L) = \frac{1}{2}\int_{w-L}^{w} [k(x)A(x)\exp\{\int_x^w A(t)dt\}]dx$$

*where $k(x) = 2[x - (w-L)]/L^2$ and*

$$\int_{w-L}^{w} k(x)dx = 1.$$

[1] Most empirical studies usually lead to values of $R(x)$ that are larger (and sometimes much larger) than one, and thus the assumption made in Corollary 2 does not seem to be very restrictive in practice.

From $A(t) > 0 \quad \forall t \in [x, w]$, we derive the following corollary.

**Corollary 1** *For all $L > 0$, we have*

$$\theta(0, L) > \frac{1}{2} \int_{w-L}^{w} k(x) A(x) dx.$$

**Corollary 2** *If $L = w$, $R(x) \equiv xA(x) \geq 1$ for all $x$ and $u(0) \in \mathbb{R}$ then*

$$\theta(0, L) > \frac{1}{2w} \int_{0}^{w} A(x) dx.$$

With the DARA case in mind, Proposition 1 and its corollaries suggest that $\theta(0, L)$ may be large if $A(x)$ is large when $x$ goes to $w - L$. A simple example, illustrated in Figure 1, is as follows. Assume $L = w$ and

$$u(x) = \begin{cases} 1 - \exp(-ax) & \text{if } x \leq \widehat{x} \\ b + cx & \text{if } x > \widehat{x} \end{cases}$$

where $b = 1 - w \exp(-a\widehat{x})/(w - \widehat{x})$ and $c = \exp(-a\widehat{x})/(w - \widehat{x})$, and $\widehat{x}$ is a fixed parameter such that $0 < \widehat{x} < w$. Thus $u(0) = 0, u(w) = 1$ and $A(x) = a$ if $x \leq \widehat{x}$ and $A(x) = 0$ if $x > \widehat{x}$.[2] When $a$ is increasing (with a given value of $\widehat{x}$), the individual becomes more risk averse in the neighborhood of the bad outcome $x = 0$, with unchanged normalization $u(0) = 0, u(w) = 1$. We then have $C_p'(0, L) = 1/c = (w - \widehat{x}) \exp(a\widehat{x})$ and thus $C_p'(0, L)$ is increasing with $a$ and goes to infinity when $a$ goes to infinity. Since $\widehat{x}$ is arbitrarily small, we learn from this example that $C_p'(0, L)$ may be large if the individual is highly risk averse in the neighborhood of the loss state $x = w - L$, or equivalently if her risk tolerance is very small around this state.

Symmetrically, Proposition 2 shows that, under non-increasing absolute risk aversion, the normalized risk premium $\theta(p, L)$ may be large when $p$ is close to zero only if $A(w - L)$ is very large, that is only when the individual's risk tolerance is very small in the accident state.

**Proposition 2** *Assume $R(x) \equiv xA(x) \leq \overline{\gamma}$ for all $x \in [w - L, w]$. Then, under non-increasing absolute risk aversion, we have*

$$\theta(0, L) < \frac{(\overline{\gamma} + 1) A(w - L)}{2},$$

*and*

$$C(p, L) < pL \left[ 1 + \frac{(\overline{\gamma} + 1) A(w - L)}{2} L \right].$$

[2] $u(x)$ is not strictly concave since $u''(x) = 0$ if $x > \widehat{x}$, but this is just for simplicity.

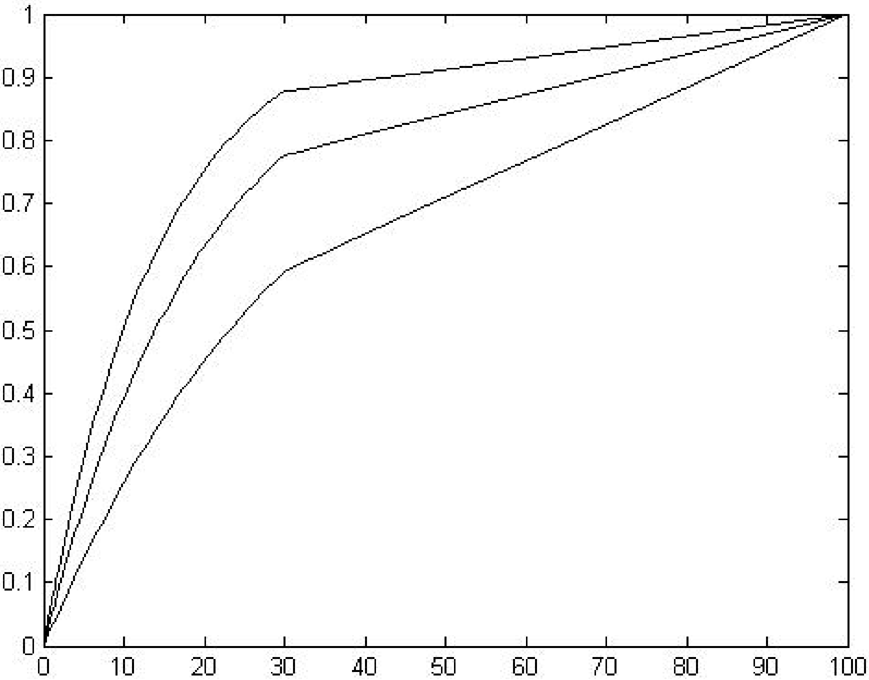



Figure 1: $\hat{x} = 30$, $w = 100$

Proposition 2 provides upper bounds for the normalized risk premium $\theta(0, L)$ and for the certainty equivalent $C(p, L)$ when the individual displays non-increasing risk aversion. $\overline{\gamma}$ is an upper bound for the index of relative risk aversion $R(x)$ when $x$ is in the interval $[w - L, w]$. The upper bound of $\theta(0, L)$ is proportional to $A(w - L)$, which is the index of absolute risk aversion in the loss state. Consequently, $C(p, L)$ may be non-negligible when $p$ is very small, say as a proportion of loss $L$, only if $A(w-L)$ is large. On the contrary, assume $A(w - L) = A(w)$, i.e., the index of absolute risk aversion remains constant in $[w - L, w]$. In that case, we would have $R(x) < R(w)$ for all $x < w$, and thus $\overline{\gamma} = R(w)$, which implies

$$C(p, L) < pL \left[1 + \frac{R(w)}{2} + \frac{R(w)^2}{2}\right].$$

Assuming $R(w) = 2$ or 3 would give $C(p, L) < 4pL$ or $C(p, L) < 7pL$, respectively. Thus, if $p$ is very small, then $C(p, L)/L$ is very small.[3]

[3] For the sake of numerical illustration, consider the case of a large scale nuclear disaster

Thus, under non-increasing absolute risk aversion, we may conclude that the risk premium of low-probability high-severity accidents may be non-negligible (and thus that the coverage of such a risk is a relevant issue) if and only if the risk tolerance is very low in such catastrophic cases.

$CRRA$ preferences are an instance of such a case with $T(x) = \gamma x$, where $\gamma$ is the index of relative risk aversion. We then have $T(x) \longrightarrow 0$ and $A(x) \longrightarrow \infty$ when $x \longrightarrow 0$. However, $CRRA$ preferences are not very satisfactory from a theoretical standpoint, since the utility is not defined when wealth is nil. This corresponds to discontinuous preferences in which any lottery with zero probability for the zero wealth state is preferred to any lottery with a positive probability for this state. If preferences are of the $HARA$ type, then risk tolerance is a linear function of wealth, and we may write $T(x) = a + bx$, with $a > 0$ and $0 < b < 1$. In such a case, we have $A'(x) < 0, A(0) = 1/a$ and $R(x) > 1$. In particular, the individual's absolute risk aversion index is decreasing but upper bounded. A straightforward calculation then gives

$$\frac{1}{2w}\int_0^w A(x)dx = \frac{1}{2bw}\ln\left(1 + \frac{bw}{a}\right),$$

and thus, Corollary 2 shows that for all $M > 0$, we have $\theta(0, L) > M$ if

$$a < \frac{bw}{\exp(2bwM) - 1}.$$

The right-hand side of the previous inequality is positive and decreasing in $b$ and $M$. Thus, $\theta(0, L)$ is arbitrarily large if $a = T(0)$ is small enough and/or if $b = T'(x)$ is small enough. In words, the risk tolerance should be low in the neighborhood of the catastrophic state $x = 0$ for the normalized risk premium $\theta(0, L)$ to be large.

Proposition 3 establishes a sufficient condition under which $\theta(0, L)$ is (arbitrarily) large when the individual is sufficiently risk averse (or, equivalently, when her risk tolerance is sufficiently low) in the catastrophic loss state.

**Proposition 3** *Assume* $T(x) \equiv t(x, \varepsilon)$, *with* $\varepsilon > 0, t(w - L, 0) = t'_x(w - L, 0) = t''_{xx}(w - L, 0) = 0$ *and* $t'_x(x) > 0$ *for* $x > w - L$. *Then for all* $M > 0$, $\theta(0, L) > M$ *if* $\varepsilon$ *is small enough.*

that may occur with probability $p = 10^{-5}$, with total losses of \$100b evenly spread among 1 million inhabitants (think of people living in the neighborhood of the nuclear plant). In the case of an accident, each inhabitant would suffer a loss $L = \$100,000$, with expected loss $pL$ equal to \$1, and risk premium equal to \$4 or \$7, which would be negligible, say as a proportion of their annual electricity expenses. Postulating larger but still realistic values of the index of relative risk aversion would not substantially affect this conclusion.

In Proposition 3, it is assumed that the risk tolerance increases slowly (less than degree-two polynomials) when wealth increases in the neighbourhood of $w - L$. In such a setting, the normalized risk premium may be arbitrarily large if the risk tolerance in the loss state is small enough.

## 2.2 Insurance demand for catastrophic risks

We now assume that the individual can purchase insurance for a low-probability high-severity risk $(p, L)$. Insurance contracts specify the indemnity $I$ in the case of an accident, i.e., when the individual suffers the loss $L$, and the premium $P$ to be paid to the insurer, with $P = (1 + \sigma)pI$, where $\sigma > 0$ is the loading factor such that $p(1+\sigma) < 1$. The policyholder then faces the lottery $(w_1, w_2)$, with corresponding probabilities $1 - p$ and $p$, where $w_1$ and $w_2$ denote respectively the wealth in the no-loss and loss states, with $w_1 = w - P$ and $w_2 = w - P - L + I$. A straightforward calculation shows that feasible lotteries are defined by

$$[1 - p(1 + \sigma)]w_1 + (1 + \sigma)pw_2 = w - (1 + \sigma)pL, \tag{3}$$

with

$$w_2 - w_1 + L \geq 0, \tag{4}$$

for the sign condition $I \geq 0$ to be satisfied. The optimal lottery maximizes the individual's expected utility

$$(1 - p)u(w_1) + pu(w_2),$$

in the set of feasible lotteries. It is such that the marginal rate of substitution $-dw_2/dw_{1|Eu=ct.} = (1 - p)u'(w_1)/pu'(w_2)$ is equal to the slope (in absolute value) of the feasible lotteries lines, that is

$$(1 - p)(1 + \sigma)u'(w_1) = [1 - (1 + \sigma)p]u'(w_2). \tag{5}$$

Figure 2 shows the locus of optimal lotteries in the $(w_1, w_2)$ plane when $p$ changes. Point A represents the situation with no insurance, and point B represents the optimal lottery when $p$ goes to zero.

Let $w_1(p, L)$, $w_2(p, L)$ denote the optimal state-contingent wealth levels when $I > 0$, that is when $\sigma$ is not too large. Let us also denote

$$\begin{aligned} w_1^*(L) &\equiv \lim_{p \longrightarrow 0} w_1(p, L) = w, \\ w_2^*(L) &\equiv \lim_{p \longrightarrow 0} w_2(p, L), \end{aligned}$$

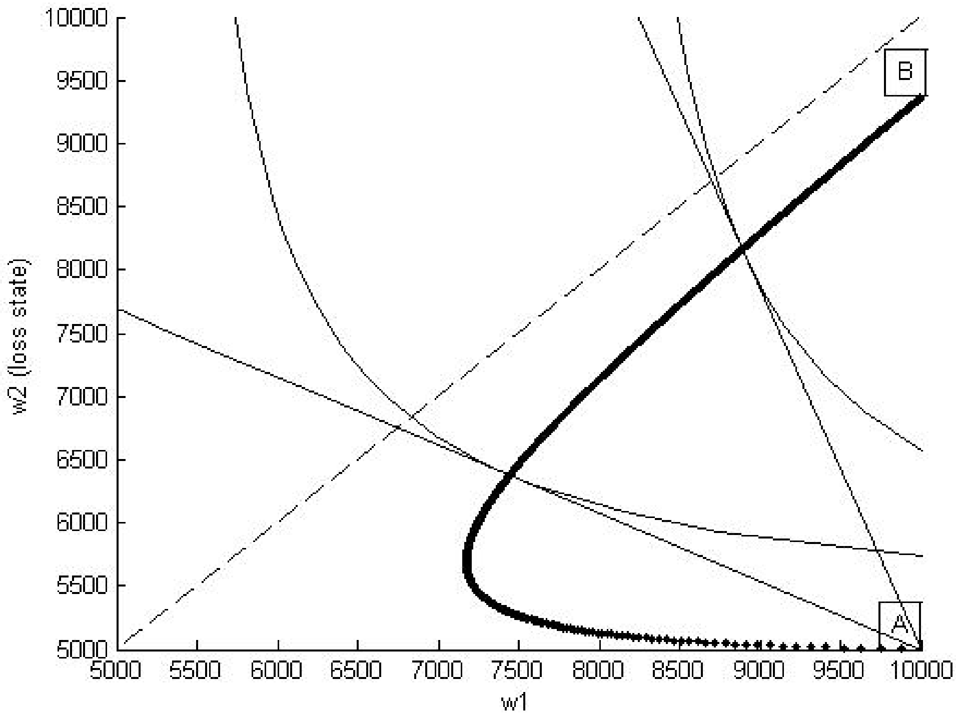



Figure 2: $w = 10000$, $L = 5000$, $u(x) = -\frac{x^{-3}}{3}$

with

$$u'(w_2^*(L)) = (1+\sigma)u'(w), \tag{6}$$

which implies $w_2^*(L) < w = w_1^*(L)$. Thus, when $p$ goes to 0, the optimal insurance contract $(P, I)$ goes to a limit $(P^*, I^*)$, with $P^* = 0$ and $I^* = w_2^*(L) + L - w_1^*(L) < L$. When $p$ is positive but close to 0, we still have $I < L$ and $P = (1+\sigma)pI \simeq (1+\sigma)pI^*$. Since $w_2^*(L) = w - L + I^*$, (6) gives

$$u'(w - L + I^*) = (1+\sigma)u'(w),$$

or

$$I^* = u'^{-1}((1+\sigma)u'(w)) - w + L,$$

and thus $I^*$ is decreasing with $\sigma$. The previous reasoning is valid only if $I^* > 0$, which holds if

$$u'(w - L) > (1+\sigma)u'(w),$$

that is, if the loading factor $\sigma$ is not too large.

**Lemma 1** *$\sigma \leq \theta(0, L)L$ is a sufficient condition for $I^* > 0$.*

Hence, the agent will be willing to buy a positive (and potentially large) amount of coverage if the normalized risk premium $\theta(0, L)$ is larger than the ratio of the loading factor $\sigma$ divided by the size of the loss $L$.

Finally, we may characterize the effect of a change in $L$ and/or $w$ on optimal insurance coverage. An increase $dL > 0$ for $w$ given induces an equivalent increase $dI^* = dL$. A simultaneous increase $dw = dL > 0$ induces an increase $dI^* > 0$ in coverage, while an increase in wealth with unchanged loss $dw > 0, dL = 0$ entails a decrease in optimal coverage $dI^* < 0$ under $DARA$ references, i.e., when $A' < 0$. Of course, there is nothing astonishing here. These are standard comparative statics results, which are extended here to the asymptotic characterization of catastrophic risk optimal insurance. They are summarized in Proposition 4.

**Proposition 4** *When $p$ goes to $0$, the optimal insurance coverage $I$ goes to a limit $I^*$, and when $p$ is close to $0$, coverage $I$ and premium $P$ are close to $I^*$ and $(1+\sigma)pI^*$, respectively. $I^*$ is lower than $L$, and it is decreasing with $\sigma$. A simultaneous uniform increase in $L$ and $w$ induces an increase in $I$ and $P$. Under DARA, an increase in $w$ with $L$ unchanged induces a decrease in $I$ and $P$.*

# 3 Optimal catastrophic risk coverage for a population

With the case of nuclear accident risk in mind, we now consider a population of individuals who face the risk of a catastrophic event (called "the accident") caused by a firm. Such an accident may affect the individuals differently, according to their risk exposure and also to their good or bad luck. The population has unit mass, and it is composed of $n$ groups or types indexed by $i = 1, ..., n$, and a proportion $\alpha_i$ of the population belongs to group $i$, with $\alpha_1 + \alpha_2 + ... + \alpha_n = 1$. In the case of a nuclear accident caused by a given reactor, the groups correspond to various locations that may be more or less distant from the nuclear power plant. The accident occurs with probability $\pi$. In the case of an accident, a proportion $q_i \in [0, 1]$ of type $i$ individuals suffer damages, with financial damages $\widetilde{x}_i$ for each individual in this subgroup of victims. $\widetilde{x}_i$ is a random variable, whose realization is denoted $x_i$, and which is distributed over the interval $[0, \overline{x}_i]$ with c.d.f. $F_i(x_i)$ and density $f_i(x_i) = F_i'(x_i)$. The random variables $\widetilde{x}_i$ are independently distributed among type $i$ individuals. Thus, we assume that in group $i$, the victims are randomly drawn with probability $q_i$, and the Law of Large Numbers guarantees that the proportion of affected individuals is equal to $q_i$, while their damages are independently distributed. The total cost of an accident is equal to

$$\sum_{i=1}^{n} \alpha_i q_i \left[ \int_0^{\overline{x}_i} x_i f(x_i) dx_i \right] = \sum_{i=1}^{n} \alpha_i q_i E\widetilde{x}_i.$$

Under our assumptions, this total cost is given, but the distribution of loss between members of each group is random.

Each type $i$ individual is covered by an insurance contract that specifies an indemnity $I_i(x_i) \geq 0$ for all $x_i$ in $[0, \overline{x}_i]$. This insurance coverage is taken out by the firm at price $P$. Once again, with the nuclear liability law in mind, we assume that the firm has to indemnify the victims according to the legal rule $I_i(x_i)$ and also - in order to prevent any bankruptcy risk - that it has to purchase insurance to cover its liability. Thus, $I_i(x_i)$ is at the same time the payment by the firm to type $i$ individuals and the transfer from the insurer to the firm. The firm pays a premium $P$ per individual, and this premium is passed on to the prices of the firm's product (say, on to the consumers' electricity bills). We assume that all consumers purchase the same quantity of the firm's products, and thus it is as if the insurance premium were paid by the individuals themselves.

Assume that the insurer allocates an amount of capital per individual $K$ in order to pay indemnities, should an accident occur. This is an amount of resource brought by investors, held in a liquid form, and that will be attributed to the insurer with probability $\pi$. A simple example (at least from a conceptual standpoint) is when the insurer issues a cat bond with par value $K$. The cat bond will pay some return (a spread above the risk-free rate of return), and it will be reimbursed to investors only if no accident occurs. Otherwise, the cat bond will default, and its proceeds will be used by the insurer to indemnify the victims.[4]

We know from the Law of large Numbers that the average indemnity paid to type $i$ victims in the case of an accident is

$$\int_0^{\overline{x}_i} I_i(x_i) f_i(x_i) dx_i,$$

[4]In practice, a Special Purpose Vehicle (SPV) is created by the sponsor as a legal entity able to host the cat bond. This SPV acts as an insurer or reinsurer with respect to the sponsor. It issues the bond, delivered to the investor in exchange for the principal payment, which entitles to a regular coupon. Upon the occurrence of a contractually defined event, called the trigger, the bond defaults and the sponsor gets to keep the principal. Cat bonds are used by insurers and reinsurers to hedge against large losses among their portfolios of insureds, and by large corporations to cover some catastrophic events.

and thus the total indemnity payment can be financed if

$$K = (1+\sigma)\sum_{i=1}^{n}\alpha_i q_i \int_0^{\overline{x}_i} I_i(x_i) f_i(x_i) dx_i,$$

where $\sigma$ is a loading factor that represents the claim handling costs that the insurer faces beyond the indemnification costs. This cost of capital is covered by the premiums raised by the insurer, so we have

$$P = c(\pi, K)$$

with capital cost $c(\pi, K)$, with $c'_K > 0, c'_{K^2} \geq 0, c'_\pi > 0$.[5]

Let $w_1$ and $w_{2i}(x_i)$ be the wealth of a type $i$ individual if she is not affected by an accident (which occurs with probability $1-\pi q_i$), and if she is affected with loss $x_i$ (which occurs with probability $\pi q_i$ and conditional loss density $f_i(x_i)$), respectively. We have

$$\begin{aligned} w_1 &= w - P, \\ w_2(x_i) &= w - P - x_i + I_i(x_i). \end{aligned}$$

All individuals have the same initial wealth $w$ and the same risk preferences represented by utility function $u$, with $u' > 0, u'' < 0$.

The set of feasible allocations $\{w_1, w_{21}(x_1), ..., w_{2n}(x_n), C_1, ..., C_n, K\}$ is defined by

$$u(w - C_i) = (1-\pi q_i)u(w_1) + \pi q_i \int_0^{\bar{x}_i} u(w_{2i}(x_i)) f_i(x_i) dx_i, \tag{7}$$

$$w_{2i}(x_i) - w_1 + x_i \geq 0 \text{ for all } i = 1, ..., n, \tag{8}$$

$$K = (1+\sigma)\sum_{i=1}^{n}\alpha_i q_i \int_0^{\overline{x}_i} I_i(x_i) f(x_i) dx_i, \tag{9}$$

$$w_1 = w - c(\pi, K). \tag{10}$$

Equation (7) defines the certainty equivalent loss $C_i$ incurred by type $i$ individuals, and equation (8) is a positivity constraint on the indemnity paid by the insurer. (9) defines the capital that is required to pay indemnities, and (10) follows from $w_1 = w - P$ and $P = c(\pi, K)$.

[5]If capital were levied through a cat bond, then $c(K, \pi)/K$ would be the spread over LIBOR, i.e., the compensation per euro required by investors for running the risk of losing their capital with probability $\pi$. A risk neutral investor would require $c(\pi, K) = \pi K$ to accept this risk. See Section 4 for further developments.

We consider a utilitarian regulator that designs the risk coverage mechanism in order to minimize the social cost of an accident, which is the weighted sum of certainty equivalent to individuals' losses. The corresponding optimization program is also a way of characterizing the Pareto optimal allocations when ex-ante transfers between groups are possible.[6] This may be written as minimizing

$$\sum_{i=1}^{n} \alpha_i C_i,$$

with respect to $\{w_1, w_{21}(x_1), ..., w_{2n}(x_n); C_1, C_2, ..., C_n, K\}$, subject to conditions (7),(8), (9) and (10). Proposition 5 characterizes the optimal solution of this problem when $\pi$ goes to 0.

**Proposition 5** *Assume $c'_K(\pi, K) \geq \pi$ and $\lim_{\pi \to 0} c'_K(\pi, K) = 0$. Then, when $\pi$ goes to zero, the optimal indemnity schedules $I_i(x_i)$ converge toward a unique straight deductible $I^*(x_i) = \max(x_i - d^*, 0)$. The deductible $d^*$ and the capital $K^*$ are defined by*

$$u'(w - d^*) = (1 + \sigma)u'(w) \lim_{\pi \to 0} \frac{c'_K(\pi, K^*)}{\pi},$$

$$K^* = (1 + \sigma) \sum_{i=1}^{n} \alpha_i q_i \left[ \int_{d^*}^{\overline{x}_i} (x_i - d^*) f_i(x_i) dx_i \right].$$

Proposition 5 shows that the optimal indemnity schedule for small $\pi$ involves full coverage of the victims above a straight deductible $d^*$ (the same for all individuals whatever their type).[7] This amounts to saying that the victims should be ranked in order of priority on the basis of their losses: the victims with loss $x_i$ should receive an indemnity only if the victims with loss $x'_i$ larger than $x_i$ receive at least $x'_i - x_i$. This simple characterization of optimal indemnification will be used in the simulation conducted in Section 4. As in the simple model of Section 2.1, we may derive comparative statics properties about the asymptotic deductible $d^*$. In particular, it is increasing in $\sigma$ and, under *DARA* preferences, it is increasing in wealth.

$d^*$ and $K^*$ are also affected by the marginal cost of capital. If the investors were risk neutral and perfectly aware of the probability of an accident, we

[6] See Proposition 6 in the appendix.

[7] The fact that the deductible does not depend on type $i$ is true only asymptotically when $\pi \longrightarrow 0$. Otherwise, the optimal indemnity schedule involves type-dependent deductibles $d_i$, with $I_i(x_i) = \max\{x_i - d_i, 0\}$. This is because lower deductibles would allow the regulator to transfer wealth from more risky types to less risky types (say from the groups with $q_i$ high to the groups with $q_i$ low if the conditional distribution of losses $F_i(x_i)$ is the same for all groups). This compensatory effect vanishes when $\pi$ goes to 0.

would have $c(\pi, K) = \pi K$, i.e., the cost of capital would just be equal to the risk premium that compensates the expected loss due to the default. We would have $c'_K(\pi, K) \equiv \pi$. In such a case, the cost of capital would not affect the optimal indemnity schedule.

However, as we will see in more detail in Section 4 with the example of the cat bond market for low-probability triggers, because of the aversion of investors towards risk or towards ambiguity, or for other reasons, it is much more realistic to keep the cost of capital in a more general form $c(\pi, K)$ In that case, as will be illustrated in Section 4, the cost of capital does affect the optimal indemnity schedule.

# 4 Nuclear catastrophe coverage

The liability of nuclear energy producers is regulated by the Paris (1960) and Brussels (1963) conventions in Europe and by the Price-Anderson act (1957) in the US. The no-fault liability is entirely channeled to the operator of the power plants, but it is limited to a given amount. The Price-Anderson act forces the nuclear industry to secure a coverage of approximately 10 billion US dollars, while the Paris and Brussels conventions have recently been revised to bring the coverage through corporate liability to a minimum of 700 million euros.

Our objective in this section is to characterize the optimal level of coverage $K^*$ for large-scale nuclear accidents. The probability of a nuclear disaster is difficult to assess because of the lack of data. This scarcity is of course a blessing for societies, but it prevents us from using the usual data analysis techniques. Neither the probabilities, nor the extent of economic damages can be inferred with a reasonable degree of accuracy from past events. Instead, we have to rely on the analysis developed by nuclear safety specialists. In particular, the Probabilistic Safety Assessment (PSA) studies seek to understand the odds and the stakes of a major accident along several dimensions: sanitary, environmental, economic, etc. Designed to improve prevention and the ex-post management of a crisis situation, they deliver, as a by-product, useful information about the probabilities of different scenarios, analyzed in detail in Dreicer et al. (1995) and Markandya (1995). Additional studies from international agencies, such as the French Institute for Radioprotection and Nuclear Safety (IRSN, 2013) and the Nuclear Energy Agency (NEA, 2000), also develop the methodology for estimating the costs associated with the various accident scenarios predicted by PSA studies.

As in Eeckhoudt et al. (2000), we make use of the aggregate information on costs and probabilities drawn from PSA studies to construct individual lotteries. These lotteries are subsequently used to estimate the social cost and optimal coverage of a nuclear accident.

## 4.1 The nuclear risk model

We consider the risk associated with one major[8] accident on the French territory. The 58 French nuclear reactors are gathered into 19 power plants. Based on Eeckhoudt et al. (2000), we assume that 2 million people live around each power plant. There are therefore 38 million people who live nearby a power plant (less than 100km) and 28 million people who live further away. We index these two groups by $i = 1, 2$. $\pi$ denotes the probability that a major nuclear accident affects one reactor. Since this probability is quite uncertain, we let it vary between $58 \times 10^{-6}$ and $58 \times 10^{-4}$.[9] In the case of an accident, an agent of group $i$ can face $S_i$ different situations, each state $s = 1, ..., S_i$ being characterized by a probability $f_{is}$, with $f_{i1} + f_{i2} + ... + f_{iS_i} = 1$. Losses can be either economic, environmental or sanitary. We monetize all of them by assuming that individuals have an initial global wealth $w$. This wealth is multiplicatively affected by a financial factor and a health factor. A health shock has two effects. On the one hand, it lowers current and future financial wealth due to the cost of treatment. On the other hand, it impedes the agent's ability to earn income in the future.[10]

The social planner decides the levels of coverage $K_1 = \sum_{s=1}^{S_1} f_{1s} I_{1s}$ and $K_2 = \sum_{s=1}^{S_2} f_{2s} I_{2s}$ dedicated to indemnify victims of groups 1 and 2, respectively, should an accident occur. $K_i$ entitles people from group $i$ to an indemnity $I_{is}(K) = \max(L_{is} - d_i)$ in state $s$, where $d_i$ is the corresponding deductible,[11] and $L_{is}$ is the money equivalent of individual $i$'s loss in state $s$, described in section 4.2. The insurance premium per individual is $P(K) = c(\pi, K)$, with

$$K = (1 + \sigma)(\frac{38}{66} K_1 + \frac{28}{66} K_2),$$

where $\sigma$ is a loading factor associated with claim handling costs. Our numerical simulations use $\sigma = 0.3$, which is seen as a good estimate for property

[8] We use ST21 as a benchmark for the number of direct victims in our baseline scenario. The PSA studies referenced above provide the technical background on which ST21 relies

[9] We ignore the probability that more than one reactor could be affected a given year.

[10] This is a crude but simple way to express the complementarity between health and wealth: the welfare gain of an increase in wealth is negatively affected if health worsens, and vice versa. See Finkelstein et al. (2013) on this complementarity.

[11] Proposition 5 established that optimal contracts feature a straight deductible for all groups.

and liability insurance.[12] Final wealth is then defined as

$$w_{is}^f(K_i) = w - L_{is} + I_{is}(K_i) - P(K),$$

and the certainty equivalent $C_i$ as

$$u(w - C_i) = (1-\pi)u(w - P(K)) + \pi \sum_{s=1}^{S_i} f_{is} u(w_{is}^f(K)). \tag{11}$$

For the sake of numerical tractability, we specify a Harmonic Absolute Risk Aversion (HARA) utility function

$$u(x) = \zeta\Big(\eta + \frac{x}{\gamma}\Big)^{1-\gamma},$$

whose domain is such that $\eta + (x/\gamma) > 0$, and with the condition $\zeta(1-\gamma)/\gamma > 0$, that guarantees that the function is indeed increasing and concave. The coefficient of absolute risk aversion is

$$A(x) = \Big(\eta + \frac{x}{\gamma}\Big)^{-1}. \tag{12}$$

The HARA class nests the Constant Relative Risk Aversion (CRRA) case when $\eta = 0$, and the Constant Absolute Risk Aversion (CARA) when $\gamma \rightarrow +\infty$. Except for the CARA and CRRA limit cases, HARA functions satisfy decreasing absolute risk aversion and increasing relative risk aversion. Studies on individual data, such as Levy (1994) and Szpiro (1986), have isolated a plausible range between 1 and 5 for the index of relative risk aversion.

## 4.2 Recovering the individual lotteries

We use figures similar to Eeckhoudt et al. (2000) to calibrate our baseline scenario. The number of direct victims in the baseline scenario (scenario 1) is summarized in Table 1.

| Distance | Population | Financial loss | Death | Severe health effect |
|---|---|---|---|---|
| $<$ 100 km | 2 million | 10,000 | 500 | 1,000 |
| $\geq$ 100 km | 64 million | 0 | 3,000 | 6,000 |

Table 1: scenario 1

When an accident occurs, an individual of group 1, who lives nearby a power plant, has a probability 1/19 of living nearby the damaged power

[12]The numerical results are robust to realistic changes in the parameter $\sigma$.

plant ($< 100$ km), in which case she can die, suffer a severe health effect or a severe financial loss if she lives in the plume of radioactivity. With probability 18/19, she lives away from the damaged power plant ($\geq 100$ km) and can die or suffer a severe health effect, but with a lower probability.

Each person in the most exposed group 1 can potentially be in 6 distinct states (3 health states $\times$ 2 financial states). Individuals from group 2 never incur the direct financial loss, so they can only be in three different states. The lotteries associated with the baseline scenario are summarized in Tables 2 and 3. The initial wealth $w$ is calibrated in euros, as the sum of the asset value currently held, plus the expected discounted future wealth of the average French citizen, which yields $w = 870,000$ euros.[13]

In states $s = 1$ and $s = 2$, agents of group $i = 1$ die, which is why they suffer the same money-equivalent loss of $739,500$ euros. They may or may not suffer the financial loss, but in the case of death, this additional financial loss does not affect their welfare. In case of death, the agent is able to retain a fraction of her initial wealth, that can be understood as a bequest or subsistence parameter. Appendix 6.6 shows the robustness of our analysis to changes in this bequest parameter. Notice that our calibration implies Values of a Statistical Life (VSL) of the order of magnitude of several million euros, which is consistent with the estimates provided in Viscusi and Aldi (2003)'s meta-analysis.[14] In state $s = 3$, they face the combined consequences of a severe health and financial loss. In states $s = 4$ and $s = 5$, they suffer either the severe health effect or the financial shock, respectively.

| State | Direct loss | Total loss $L_{1s}$ | $f_{1s}$ (conditional) |
|---|---|---|---|
| $s = 1$ | 739,500 | 739,500 | 6.5789e-08 |
| $s = 2$ | 739,500 | 739,500 | 6.3844e-05 |
| $s = 3$ | 400,000 | 401,624 | 1.3158e-07 |
| $s = 4$ | 260,000 | 261,624 | 1.2769e-04 |
| $s = 5$ | 100,000 | 101,624 | 2.6296e-04 |
| $s = 6$ | 0 | 1,624.2 | 9.9978e-01 |

Table 2: lotteries for type $i = 1$

To these direct consequences, one must add more diffuse economic costs that are qualified as indirect costs in Schneider (1998) and subsequent works. They are difficult to quantify and to attribute to a given individual. Examples

[13] The details of this calibration are presented in Appendix 6.3.

[14] Weitzman (2009) discusses at length the link between the subsistence parameter and the Value of a Statistical Life, and the implications of VSL calibration for Cost-Benefit analysis of climate risks.

of such costs are : loss of attractiveness of an impacted territory, loss in terms of image for the industrial sector, etc.[15] For simplicity, we assume that these costs are evenly shared by all individuals in the economy[16] and we keep the total cost of the accident fixed at 100 billion euros. In group $i = 1$, agents in state $s = 6$ only face the indirect loss from the accident.

For group $i = 2$, individuals die in state $s = 1$, suffer the severe health effect in state $s = 2$ but only the indirect loss in state $s = 3$. Alternative scenarios (scenario 2,3,4 and 5) are generated by multiplying the number of direct victims considered in Table 1 by 2,3,4 and 5, respectively, while keeping the total cost fixed at 100 billion euros. The total cost of direct losses ranges from 5.4 billion euros, in scenario 1, to approximately 27 billion euros, in scenario 5. The total cost of indirect losses therefore varies between 73 and 94.6 billion euros. Because we assume that indirect losses are mutualized, they only affect marginally the optimal coverage level. The assumption that total cost is 100 billion euros is therefore innocuous.[17]

| State | Direct loss | Total loss $L_{2s}$ | $f_{2s}$ (conditional) |
|---|---|---|---|
| $s = 1$ | 739,500 | 739,500 | 5.3571e-05 |
| $s = 2$ | 260,000 | 261,624 | 1.0714e-04 |
| $s = 3$ | 0 | 1,624.2 | 9.9984e-01 |

Table 3: lotteries for type $i = 2$

## 4.3 The cost of capital

Bantwal and Kunreuther (2000) suggest that ambiguity aversion, loss aversion, uncertainty avoidance, as well as transaction costs due to legal and technical complexities, may account for the reluctance of investment managers to invest in the cat bond market. This argument suggests cat bonds prices may, in practice, be well above the actuarially fair cost.

In order to asses empirically the link between the probability $\pi$ that the trigger is activated, and the cost of a cat bond, we use the Artemis database[18],

[15]Here we do not discuss the effect of the catastrophe on growth as the literature has not reached a consensus on the growth effect of disasters. For example, Gignoux and Menéndez (2016) find a positive effect for the case of earthquake in India while Strobl (2012) finds a negative effect for the case of hurricanes in the Caribbeans.

[16]We could also treat these indirect costs as uninsurable background risks. Under the risk vulnerability assumption, these background risks would increase the degree of risk aversion toward insurable risks.

[17]In particular, assuming a total cost of 50 billions would not modify our results.

[18]http://www.artemis.bm/

which provides information on the main cat bonds transactions, including the nature of perils, types of trigger, probability of a capital loss, expected loss,[19] spreads, and identity of sponsors. The database contains more than two-hundred issues, some of which are divided in several tranches, characterized by different levels of risk, and therefore by different spreads. We only have complete information for 107 of the most recent tranches, spanning an interval of five years (2011-2015).

We estimate a model[20] of the form

$$s_i = \beta_0 + \beta_1 \mathbb{E}\ell_i + \beta_2 \Big(\mathbb{E}\ell_i\Big)^2 + \gamma \mathbf{X}_i + \varepsilon_i,$$

where cat bonds (or tranches of cat bonds in the case of multi-tranche cat bonds) are indexed by $i$. $s_i = c(\pi_i, K_i)/K_i$ is the spread over LIBOR, $\mathbb{E}\ell_i$ is the expected investor's loss expressed as a fraction of the cat bond's par value $K$, and $\mathbf{X}_i$ is a set of control variables. Table 4 delivers the OLS estimates of $\hat{\beta}_1$ and $\hat{\beta}_1$.[21] Both parameters are significant at the usual levels.[22]

| | Estimates |
|---|---|
| $\mathbb{E}\ell_i$ | $2.3329^{***}$ |
| | $(8.8959)$ |
| $\mathbb{E}\ell_i^2$ | $-7.787^{***}$ |
| | $(-3.3595)$ |
| $R^2$ | $0.8209$ |
| $\hat{s}(58 * 10^{-5}) - \hat{s}(0)$ | $1.3505 * 10^{-3}$ |

Table 4: OLS estimates

In order to insure the nuclear risk, we consider a simple cat bond, for which the capital is entirely transferred to the sponsor when the trigger is activated. Thus, the expected loss is simply equal to the probability of default: $\mathbb{E}\ell = \pi$. Using the estimated values of $\hat{\beta}_1$, $\hat{\beta}_1$ and $\hat{\gamma}$ allows us to write the expected spread of such a cat bond as

$$\hat{s}(\pi) = \hat{\beta}_0 + \hat{\beta}_1 \pi + \hat{\beta}_2 \pi^2 + \hat{\gamma}\mathbf{X}.$$

We compute the predicted cost of capital for a low-probability event as $\hat{c}(\pi, K) = [\hat{s}(\pi) - \hat{s}(0)]K$. We here consider $\hat{s}(0)$ as the cost of capital (over

[19] The probability of a capital loss and the distribution of losses are evaluated by modeling companies independent from the sponsor and the investor.

[20] Lane and Mahul (2008) use a similar data set to estimate a linear relation.

[21] The full table, along with alternative specifications is reported in Appendix 6.4.

[22] The $t$-statistics are reported in parenthesis below the estimates. *** : significant at 1% level.

LIBOR) that the firm would incur if $K$ was levied under the same liquidity and maturity conditions as with a cat bond, but without the reference to nuclear liability.[23]

This predicted cost is of the order of magnitude of the probability $\pi$ of a catastrophe, but it is above actuarially fair price. In addition, the concavity of the relationship between $s$ and $\pi$ suggests that low-probability events display a higher loading factor, making them more difficult to insure.

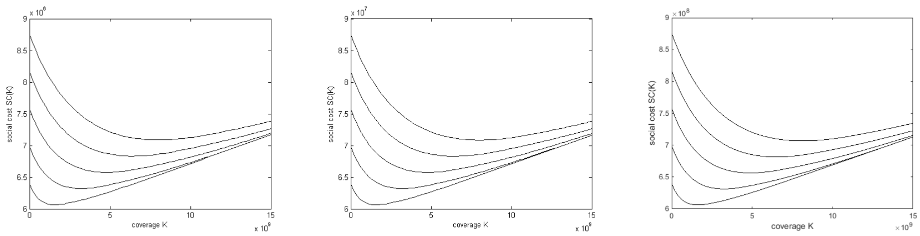
## 4.4 Optimal coverage

Using the definition (11) with $P(K) = \hat{c}(\pi, K)$, we are now in the position to simulate the certainty equivalents for individuals of both types $i = 1$ and $i = 2$. The optimal coverage levels $K_1^*$ and $K_2^*$ minimize the social cost $SC(K_1, K_2)$, where

$$SC(K_1, K_2) = [38\ C_1(K_1, K_2) + 28\ C_2(K_1, K_2)] \times 10^6.$$

We first discretize the space of possible values and minimize $SC(K_1, K_2)$ for all levels of $K$. This yields the optimal coverage levels $K_1(K)$ and $K_2(K)$ conditionally on total coverage. We then minimize $SC(K) \equiv SC(K_1(K), K_2(K))$, which yields $K^*$, $K_1^*$ and $K_2^*$.

Figures 3a, 3b and 3c display function $SC(K)$ for scenarios 1 to 5, with an upward shift when we go from scenario 1 to scenario 5. For $\pi = 58 \times 10^{-6}$, $\pi = 58 \times 10^{-5}$ and $\pi = 58 \times 10^{-4}$, the simulations reported here correspond to levels of relative risk aversion $\underline{R} = 1$ and $\overline{R} = 2$, where $\underline{R}$ and $\overline{R}$ respectively denote the relative risk aversion when the individual suffers the largest loss (state $s = 1$) or no loss at all ($s = 6$ or 3, according to the individual's type).




(a) $\pi = 58 * 10^{-6}$ (b) $\pi = 58 * 10^{-5}$ (c) $\pi = 58 * 10^{-4}$

**Result 1** *The optimal choice of coverage $K^*$ does not change with the accident probability $\pi$.*

[23]In other words, if nuclear accidents are the only potential source of insolvency, then $\hat{s}(0)+$ LIBOR is taken as the risk-less cost of capital. This cost was implicitly equal to 0 in the theoretical model of section 3.

| $\underline{R}$ | 1 | 2 |
|---|---|---|
| VSL | 4.967e+06 | 7.405e+06 |
| Scenario 1 | 1.755e+09 | 1.950e+09 |
| Scenario 2 | 3.705e+09 | 3.705e+09 |
| Scenario 3 | 5.460e+09 | 5.655e+09 |
| Scenario 4 | 7.410e+09 | 7.605e+09 |
| Scenario 5 | 9.165e+09 | 9.360e+09 |

Table 5: Coverage $K^*$, $\overline{R} = 2$

| $\underline{R}$ | 1 | 2 |
|---|---|---|
| VSL | 4.967e+06 | 7.405e+06 |
| Scenario 1 | 9.605e-02 | 1.652e-01 |
| Scenario 2 | 1.691e-01 | 2.746e-01 |
| Scenario 3 | 2.263e-01 | 3.524e-01 |
| Scenario 4 | 2.725e-01 | 4.105e-01 |
| Scenario 5 | 3.105e-01 | 4.556e-01 |

Table 6: Welfare gain, $\overline{R} = 2$

Notice that, for a given scenario, all the social cost curves have the same shape and therefore the same optimum point $K^*$. This is a direct application of Proposition 5.[24] Our numerical simulations confirm that the optimal coverage has converged, at least for values of $\pi$ lower than $58 \times 10^{-4}$.

From a policy perspective, this result implies that the exact value of the probability $\pi$ of a nuclear accident should not play any role in the optimal coverage choice. Whether the true probability is $58 \times 10^{-4}$ or $58 \times 10^{-6}$, our model suggests that the same level of coverage should be purchased, even though the price of coverage would not be the same.

**Result 2** *A collective risk should have a higher coverage when its consequences are more concentrated on a subset of individuals.*

We have assumed that the most serious accidents are characterized by more severe losses incurred by the victims (deaths, diseases and relocation), keeping constant the aggregate cost of the accident. To know what scenario best reflects the risk of a nuclear accident is a technical question, in which we have no say, but the economic insight is that the most severe losses, and the first that should be indemnified, are the ones that bring the agent's wealth close to her subsistence level. The indirect losses, that we have assumed to be already mutualized, play a marginal role on the optimal level of coverage $K^*$.

Finally and despite the acknowledged difficulty of the calibration exercise, the two following results attempt to quantify the optimal levels of coverage and deductible.

**Result 3** *In our baseline assumption ($\overline{R} = 2$ or $\overline{R} = 3$ ), the optimal levels of coverage are higher than* 1.755 *billion euros, that is to say more than twice the* 700 *million euros provided for by the 2004 revision of the Paris Convention.*

[24]Note that the scale of the vertical axis varies with $\pi$ in Figures 3a, 3b and 3c.

| $\underline{R}$ | 1 | 2 |
|---|---|---|
| VSL | 4.967e+06 | 7.405e+06 |
| Scenario 1 | 3.977e+05 | 3.553e+05 |
| Scenario 2 | 3.760e+05 | 3.760e+05 |
| Scenario 3 | 3.831e+05 | 3.693e+05 |
| Scenario 4 | 3.764e+05 | 3.661e+05 |
| Scenario 5 | 3.809e+05 | 3.727e+05 |

Table 7: Deductible group 1, $\overline{R} = 2$

| $\underline{R}$ | 1 | 2 |
|---|---|---|
| VSL | 4.967e+06 | 7.405e+06 |
| Scenario 1 | 3.974e+05 | 3.545e+05 |
| Scenario 2 | 3.759e+05 | 3.759e+05 |
| Scenario 3 | 3.831e+05 | 3.688e+05 |
| Scenario 4 | 3.759e+05 | 3.652e+05 |
| Scenario 5 | 3.802e+05 | 3.716e+05 |

Table 8: Deductible group 2, $\overline{R} = 2$

For example, Table 5 and 6 show that, if we consider scenario 1, $\underline{R} = 2$ and $\overline{R} = 2$, then $K^*$ should be equal to 1.950 billion euros. The associated relative welfare gain, calculated as $[SC(K^*) - SC(0)]/SC(0)$ would be above 16.52%. Of course, welfare gains for group 2, taken separately, would be higher. Higher values for the coefficients of relative risk aversion, or a more pessimistic loss scenario would lead to much higher values of $K^*$ and substantially higher welfare gains. In addition, considering a higher probability of occurrence of the accident would yield a higher welfare gain.

**Result 4** *The deductibles implied by the choices of $K_1^*$ and $K_2^*$ are identical for the two groups and do not depend on the scenario under consideration.*

This result is also a direct application of Proposition 5. Tables 7 and 8 show that in the CRRA case $\overline{R} = \underline{R} = 2$, the deductible is between $350,000$ and $380,000$ euros for both groups and in any scenario.[25] These levels of deductible represent slightly less than half of the individual's maximum potential loss. It implies that only people in the worst states ($s = 1, 2, 3$ for group 1 and $s = 1$ for group 2) are indemnified. Finally, Tables 7 and 8 confirm the intuition that deductibles should decrease with risk aversion.

# 5 Conclusion

This paper has developed a theory of optimal insurance for low-probability high-severity events. Starting with a characterization of what is a catastrophic risk for an individual, we have then analyzed the optimal insurance scheme for a catastrophic accident, caused by an industrial activity. This has lead us to apply this approach to the case of nuclear accident risk.

[25] The small variation in deductible that we observe between the five scenarios and the two groups is only due to the coarseness of the discretization. Increasing tightness would yield values always closer from the theoretical predictions of Proposition 5.

We have shown that the risk premium of a low-probability high-severity event can remain large when the accident probability goes to zero, if the coefficient of absolute risk aversion is sufficiently large (or equivalently risk tolerance is sufficiently low) in the accident state. In addition, the optimal indemnity converges to a positive limit. In the case of an industrial catastrophe that may affect the whole population of a country, the asymptotic indemnity schedule is characterized by a straight deductible, common to all individuals.

Our results also suggest that the nuclear liability law could be much more ambitious than what it currently is in France, where the nuclear corporate liability just meets the requirements of the revised Paris convention, contrary to other countries, such as Germany, where this liability has been extended far beyond these requirements.

# 6 Appendix

## 6.1 Proposition 6

Let us assume that the government can redistribute wealth between groups through ex ante lump sum transfers. We denote $t_i$ the net transfer paid to each individual of group $i$, the government budget constraint being written as

$$\sum_{i=1}^{n} \alpha_i t_i = 0.$$

Now we have

$$\begin{aligned} w_1 &= w - P + t_i, \\ w_{2i}(x_i) &= w - P - x_i + I_i(x_i) + t_i. \end{aligned}$$

and the certainty equivalent loss incurred by type $i$ individuals is still denoted by $C_i$, with

$$\begin{aligned} u(w - C_i + t_i) &= (1 - \pi q_i) u(w_1 + t_i) \\ &\quad + \pi q_i \int_0^{\overline{x}_i} u(w_{2i}(x_i) + t_i) f(x_i) dx_i. \end{aligned} \tag{13}$$

An allocation is written as $\mathcal{A} = \{w_1, w_{21}(x_1), ..., w_{2n}(x_n), C_1, ..., C_n, t_1, ..., t_n, K\}$, and $\mathcal{A}$ is feasible if (8), (9),(10) and (13) are satisfied.

**Definition 2** *$\mathcal{A}$ is Pareto-optimal if it is feasible and if there does not exist another feasible allocation $\widehat{\mathcal{A}} = \{\widehat{w}_1, \widehat{w}_{21}(x_1), ..., \widehat{w}_{2n}(x_n), \widehat{C}_1, ..., \widehat{C}_n, \widehat{t}_1, ..., \widehat{t}_n, \widehat{K}\}$ such that $\widehat{C}_i - \widehat{t}_i \leq C_i - t_i$ for all $i = 1, ..., n$, with $\widehat{C}_{i_0} - \widehat{t}_{i_0} < C_{i_0} - t_{i_0}$ for at least one group $i_0$.*

**Proposition 6** *$\mathcal{A} = \{w_1, w_{21}(x_1), ..., w_{2n}(x_n), C_1, ..., C_n, t_1, ..., t_n, K\}$ is a Pareto-optimal allocation if and only if it minimizes $\sum_{i=1}^{n} \alpha_i C_i$ in the set of feasible allocations.*

## 6.2 Proofs

**Proof of Proposition 1**

From equation (1), we have

$$C_p'(0, L) = \frac{u(w) - u(w - L)}{u'(w)} = \int_{w-L}^{w} \frac{u'(x)}{u'(w)} dx.$$

Since

$$u'(x) = u'(w) - \int_x^w u''(t)dt,$$

for all $x \in [w-L, w]$, we may write

$$\begin{aligned} C_p'(0,L) &= L - \int_{w-L}^{w} \frac{u'(x)}{u'(w)}dx \\ &= L - \int_{w-L}^{w} \left[ \int_x^w \frac{u''(t)}{u'(w)}dt \right] dx \\ &= L + \int_{w-L}^{w} \left[ \int_x^w A(t)\frac{u'(t)}{u'(w)}dt \right] dx, \end{aligned}$$

and thus

$$\theta(0,L) = \frac{1}{L^2}\int_{w-L}^{w} \left[ \int_x^w A(t)\frac{u'(t)}{u'(w)}dt \right] dx.$$

Integrating by parts gives

$$\theta(0,L) = \frac{1}{2}\int_{w-L}^{w} k(x)A(x)\frac{u'(x)}{u'(w)}dx, \tag{14}$$

where $k(x) = 2\frac{x-(w-L)}{L^2}$, with

$$\int_{w-L}^{w} k(x)dx = 1.$$

In addition, we have

$$u'(x) = u'(w)\exp\{\int_x^w A(x)dx\},$$

, which yields the result.

**Proof of Corollary 2**

When $L = w$, we have

$$\theta(0,L) > \frac{1}{w}\int_0^w \frac{xu'(x)}{wu'(w)}A(x)dx,$$

from Proposition 1. Furthermore, we have

$$\begin{aligned} \frac{d[xu'(x)]}{dx} &= xu''(x) + u'(x) \\ &= -u'(x)[R(x) - 1], \end{aligned}$$

and thus

$$\frac{d[xu'(x)]}{dx} \leq 0 \text{ if } R(x) \geq 1.$$

We deduce

$$\theta(0,L) > \frac{1}{w}\int_0^w A(x)dx \text{ if } R(x) \geq 1.$$

**Proof of Proposition 2**

Using $A' \leq 0$ in equation (14) allows us to write

$$\theta(0,L) \leq \frac{A(w-L)}{L^2u'(w)}\int_{w-L}^{w}[x-(w-L)]u'(x)dx$$

Using $R(x) \leq \overline{\gamma}$ and $u''(x) < 0$ yields

$$\begin{aligned}\frac{d}{dx}[\Big(x-(w-L)\Big)u'(x)] &= u'(x)[1-R(x)-\frac{u''(x)}{u'(x)}(w-L)] \\ &\geq u'(x)[1-R(x)] \\ &\geq u'(x)(1-\overline{\gamma}) \\ &\geq u'(w)(1-\overline{\gamma}),\end{aligned}$$

for all $x \in [w-L, w]$. Hence, we have

$$\begin{aligned}\Big(x-(w-L)\Big)u'(x) + (w-x)u'(w)(1-\overline{\gamma}) &\leq [w-(w-L)]u'(w) \\ \Big(x-(w-L)\Big)u'(x) &\leq Lu'(w) + (w-x)u'(w)(\overline{\gamma}-1) \\ &= u'(w)[L+(w-x)(\overline{\gamma}-1)],\end{aligned}$$

for all $x \in [w-L, w]$. Consequently,

$$\begin{aligned}\theta(0,L) &\leq \frac{A(w-L)}{L^2u'(w)}\int_{w-L}^{w}\{u'(w)[L+(w-x)(\overline{\gamma}-1)]\}\,dx \\ &= \frac{A(w-L)}{L^2}\left[\frac{L^2(\overline{\gamma}+1)}{2}\right] \\ &= \frac{A(w-L)(\overline{\gamma}+1)}{2}.\end{aligned}$$

Using $C_p'' < 0$ and $C(0,L) = 0$ allows us to write

$$\begin{aligned}C(p,L) &< C'(0,L)p \\ &= pL + \theta(0,L)pL^2 \\ &\leq pL\left[1+\frac{A(w-L)(\overline{\gamma}+1)L}{2}\right].\end{aligned}$$

**Proof of Proposition 3**

$T_\varepsilon(x) \equiv t(x,\varepsilon)$, with $\varepsilon > 0, t(w-L,0) = t'_x(w-L,0) = t''_{xx}(w-L,0) = 0$ and $t'_x(x,0) > 0$ for $x > w-L$. Let $M > 0$. Then, for $\varepsilon$ small enough, there exist $x_0(M,\varepsilon)$ and $x_1(M,\varepsilon)$ such that

$$\begin{aligned}
w-L &< x_0(M,\varepsilon) < x_1(M,\varepsilon),\\
T_\varepsilon(x_0(M,\varepsilon)) &= [x_0(M,\varepsilon)-(w-L)]^2/L^2M\ ,\\
T_\varepsilon(x_1(M,\varepsilon)) &\leq [x_1(M,\varepsilon)-(w-L)]^2/L^2M,\\
T_\varepsilon(x) &< [x-(w-L)]^2/L^2M \text{ if } x_0(M,\varepsilon) < x < x_1(M,\varepsilon),\\
x_0(M,\varepsilon) &\longrightarrow w-L \text{ when } \varepsilon \longrightarrow 0,\\
x_1(M,\varepsilon) &\longrightarrow x_1^*(M) > 0 \text{ when } \varepsilon \longrightarrow 0.
\end{aligned}$$

Thus, we have

$$T_\varepsilon(x) \leq \frac{[x_1(M,\varepsilon)-(w-L)][x-(w-L)]}{L^2M}\ ,$$

or equivalently

$$A_\varepsilon(x) > \frac{L^2M}{[x_1(M,\varepsilon)-(w-L)][x-(w-L)]},$$

if $x_0(M,\varepsilon) < x < x_1(M,\varepsilon)$. Hence, we may write

$$\begin{aligned}
\theta(0,L) &> \frac{1}{2}\int_{w-L}^{w} k(x)A(x)dx\\
&> \frac{1}{2}\int_{x_0(M,\varepsilon)}^{x_1(M,\varepsilon)} \left(\frac{2[x-(w-L)]}{L^2} \times \frac{L^2M}{[x_1(M,\varepsilon)-(w-L)][x-(w-L)]}\right)dx\\
&> \int_{x_0(M,\varepsilon)}^{x_1(M,\varepsilon)} \frac{M}{x_1(M,\varepsilon)-(w-L)}dx\\
&= M \times \frac{x_1(M,\varepsilon)-x_0(M,\varepsilon)}{x_1(M,\varepsilon)-(w-L)}.
\end{aligned}$$

Since $x_0(M,\varepsilon) \longrightarrow w-L$ and $x_1(M,\varepsilon) \longrightarrow x_1^*(M)$ when $\varepsilon \longrightarrow 0$, the right-hand side of the previous inequality goes to $M$ when $\varepsilon \longrightarrow 0$, and we deduce that $\theta(0,L)$ is larger than $M$ for $\varepsilon$ small enough.

**Proof of Lemma 1**

We have $I^* > 0$ iff

$$\begin{aligned}
\sigma &< \frac{u'(w-L)-u'(w)}{u'(w)}\\
&= -\frac{1}{u'(w)}\int_{w-L}^{L} u''(x)dx\\
&= \int_{w-L}^{L} A(x)\frac{u'(x)}{u'(w)}dx.
\end{aligned}$$

Using $Lk(x)/2 < 1$ for all $x \in (w-L, w]$ gives

$$\int_{w-L}^{L} A(x)\frac{u'(x)}{u'(w)}dx > \frac{L}{2}\int_{w-L}^{L} k(x)A(x)\frac{u'(x)}{u'(w)}dx = L\theta(0,L),$$

and thus we have $I^* > 0$ if $\theta(0,L)L \geq \sigma$.

**Proof of Proposition 5**

The planner's program is to minimize $\sum_i \alpha_i C_i$ under constraints (7), (8), (9) and (10). The Kuhn-Tucker multipliers associated with each set of constraints are respectively $\gamma_i$, $\phi_i(x_i)$, $\eta$ and $\rho$. The optimality conditions are

$$\alpha_i - \gamma_i u'(w - C_i) = 0 \tag{15}$$

$$\gamma_i \pi q_i u'(w_{2i}(x_i)) f_i(x_i) - \eta(1+\sigma)\alpha_i q_i f_i(x_i) + \phi_i(x_i) = 0, \tag{16}$$

$$u'(w_1)\sum_{i=1}^{n}(1-\pi q_i)\gamma_i - \sum_{i=1}^{n}\int_0^{\bar{x}_i}\phi_i(x_i)dx_i - \rho + \eta(1+\sigma)\sum_{i=1}^{n}\alpha_i q_i = 0, \tag{17}$$

$$-\eta + \rho c'_K(\pi, K) = 0, \tag{18}$$

$$\phi_i(x_i) \geq 0 \qquad \text{and} \qquad \phi_i(x_i) = 0 \quad \text{if} \quad w_{2i}(x_i) - w_1 + x_i > 0 \qquad \forall i. \tag{19}$$

Let $x_i$ be such that $w_{2i}(x_i) - w_1 + x_i > 0$. Thus, we have $\phi_i(x_i) = 0$ from (19) and (16) gives

$$\pi\gamma_i u'(w_{2i}(x_i)) = \eta(1+\sigma)\alpha_i. \tag{20}$$

(15) and (20) yield

$$u'(w_{2i}(x_i)) = \frac{\eta}{\pi}(1+\sigma)u'(w - C_i). \tag{21}$$

Hence, if there exist $x_i^0, x_i^1 \in [0, \bar{x}_i]$ such that $w_{2i}(x_i^0) - w_1 + x_i^0 > 0$ and $w_{2i}(x_i^1) - w_1 + x_i^1 > 0$, then we must have

$$u'(w_{2i}(x_i^0)) = u'(w_{2i}(x_i^1)),$$

which implies

$$w_{2i}(x_i^0) = w_{2i}(x_i^1).$$

Consequently, $w_{2i}(x_i)$ is constant over the set of $x_i$ for which $w_{2i}(x_i) - w_1 + x_i > 0$, and we can write

$$w_{2i}(x_i) = w_1 - d_i,$$

with $d_i < x_i$ for all $x_i$ in this set and

$$u'(w_1 - d_i) = \frac{\eta}{\pi}(1+\sigma)u'(w - C_i). \tag{22}$$

Now let $x_i$ be such that $w_{2i}(x_i) - w_1 + x_i = 0$. Using (15), (16) and (19) allows us to write

$$u(w_{2i}(x_i)) = u'(w_1 - x_i) \leq \frac{\eta}{\pi}(1+\sigma)u'(w - C_i).$$

Using (21), and $u'' < 0$ we deduce $x_i \leq d_i$. Thus, we have established that there exists a $d_i$ such that

$$w_{2i}(x_i) = w_1 - d_i \quad \text{if} \quad x_i > d_i, \tag{23}$$
$$w_{2i}(x_i) = w_1 - x_i \quad \text{if} \quad x_i \leq d_i. \tag{24}$$

When $\pi \to 0$, we have $w_1 \longrightarrow w$ and $C_i \longrightarrow 0$ from (10) and (7) respectively. (22) then gives $d_i \longrightarrow d^* \quad \forall i$ with

$$u'(w - d^*) = (1+\sigma)u'(w) \lim_{\pi \to 0} \frac{\eta}{\pi}. \tag{25}$$

(15), (17) and (18) imply

$$\lim_{\pi \to 0} 1 - \frac{\eta}{c'_K(\pi, K)} + \eta(1+\sigma)\sum_{i=1}^{n} \alpha_i q_i - \sum_{i=1}^{n} \int_0^{\bar{x_i}} \phi_i(x_i)dx_i = 0. \tag{26}$$

Suppose that $\eta$ does not go to zero when $\pi$ does. In such a case, we would have $\frac{\eta}{c'_K(\pi,K)} \longrightarrow +\infty$ when $\pi \longrightarrow 0$ since $c'_K(\pi, K) \longrightarrow 0$, and thus

$$\lim_{\pi \to 0} \eta[\frac{1}{c'_K(\pi, K)} + (1+\sigma)\sum_{i=1}^{n} \alpha_i q_i] = +\infty.$$

Since $\phi_i(x_i) \geq 0 \quad \forall i$, this is in contradiction with (26). Thus, we have

$$\lim_{\pi \longrightarrow 0} \left[1 - \frac{\eta}{c'_K(\pi, K)} - \sum_{i=1}^{n} \int_0^{\bar{x_i}} \phi_i(x_i)dx_i\right] = 0. \tag{27}$$

If $d_i \leq 0$, we have $w_{2i}(x_i) - w_1 + x_i > 0$ and $\phi_i(x_i) = 0 \quad \forall x_i$. Hence

$$\sum_{i=1}^{n} \int_0^{\bar{x_i}} \phi_i(x_i) = 0.$$

If $d_i > 0$, we have $\phi_i(x_i) = 0$ for $x_i > d_i$, and thus (15), (16) and (24) give

$$\int_0^{\bar{x_i}} \phi_i(x_i)dx_i = \int_0^{d_i} \phi_i(x_i)dx_i \tag{28}$$
$$= -\pi\alpha q_i \int_0^{d_i} [\frac{u'(w - x_i)}{u'(w - C_i)} - \frac{\eta}{\pi}(1+\sigma)]f_i(x_i)dx_i. \tag{29}$$

Using the fact that $\eta \longrightarrow 0$ when $\pi \longrightarrow 0$ gives

$$\lim_{\pi \to 0} \int_0^{\bar{x_i}} \phi_i(x_i) dx_i = 0,$$

and from (26) we derive

$$\lim_{\pi \to 0} \frac{\eta}{c'_K(\pi, K)} = 1.$$

Using (25), we finally deduce

$$\begin{aligned} u'(w - d^*) &= (1+\sigma) u'(w) \lim_{\pi \to 0} \frac{c'_K(\pi, K)}{\pi} \\ &> u'(w), \end{aligned}$$

where the last inequality derives from $\sigma > 0$ and $c'_K(\pi, K) \geq \pi$. We obtain $d^*$. Since $I_i(x_i) = w_{2i}(x_i) + x_i - w_1$, we deduce that $I_i(x_i) \longrightarrow I^*(x_i) = \max(x_i - d^*, 0)$ when $\pi \longrightarrow 0$.

**Proof of Proposition 6**

Assume that $\mathcal{A}$ minimizes $\sum_{i=1}^n \alpha_i C_i$ in the set of feasible allocations, and suppose that it is not Pareto-optimal. Then there exists a feasible allocation $\widehat{\mathcal{A}}$ and a group $i_0$ such that $\widehat{C}_i - \widehat{t}_i \leq C_i - t_i$ for all $i$ and $\widehat{C}_{i_0} - \widehat{t}_{i_0} < C_{i_0} - t_{i_0}$. Consequently,

$$\sum_{i=1}^n \alpha_i (\widehat{C}_i - \widehat{t}_i) < \sum_{i=1}^n \alpha_i (C_i - t_i). \tag{30}$$

Since $\mathcal{A}$ and $\widehat{\mathcal{A}}$ are feasible, we have

$$\sum_{i=1}^n \alpha_i t_i = \sum_{i=1}^n \alpha_i \widehat{t}_i = 0, \tag{31}$$

and thus (30) and (31) give

$$\sum_{i=1}^n \alpha_i \widehat{C}_i < \sum_{i=1}^n \alpha_i C_i,$$

which contradicts the fact that $\mathcal{A}$ minimizes $\sum_{i=1}^n \alpha_i C_i$ in the set of feasible allocations.

Conversely, assume that $\mathcal{A}$ is a Pareto-optimal allocation, and suppose that it does not minimize $\sum_{i=1}^n \alpha_i C_i$ in the set of feasible allocations. Thus there exists a feasible allocation $\widehat{\mathcal{A}}$ such that $\sum_{i=1}^n \alpha_i \widehat{C}_i < \sum_{i=1}^n \alpha_i C_i$, and thus

$$\sum_{i=1}^n \alpha_i (\widehat{C}_i - \widehat{t}_i) < \sum_{i=1}^n \alpha_i (C_i - t_i). \tag{32}$$

Let us choose $\widehat{t}_i$ such that

$$\widehat{t}_i = \widehat{C}_i + t_i - C_i$$

for all $i \neq i_{i_0}$, which does not contradict the feasibility of $\widehat{\mathcal{A}}$ if we choose

$$\widehat{t}_{i_0} = -\sum_{i \neq i_0} \widehat{t}_i. \tag{33}$$

We have

$$\widehat{C}_i - \widehat{t}_i = C_i - t_i \text{ for all } i \neq i_0. \tag{34}$$

Furthermore, (32),(33) and (34) give

$$\widehat{C}_{i_0} - \widehat{t}_{i_0} < C_{i_0} - t_{i_0}. \tag{35}$$

(34) and (35) contradict the fact that $\mathcal{A}$ is Pareto-optimal.

## 6.3 Calibration of initial wealth and losses

The French national statistical agency, INSEE provides an average estimated Gross National Product per capita of 32,227 euros[26] and an average age of 39.2 years old[27]. The French National Institute on Demographics (INED) provides an estimated life expectancy of 73.2 for the average 39.2[28] years old citizen. Lifetime wealth is obtained as the annual GDP per capita discounted at rate 2% on a 34 years horizon. This yields an expected discounted future wealth of 805 310 euros, rounded to 800 000 euros. The INSEE also provides an estimated average of 70 000 euros of current assets. We therefore consider that initial wealth is 870 000 euros.

The worst case scenario is a fatal outcome that occurs in states $s = 1$ and $s = 2$. As in Eeckhoudt et al. (2000), we assume that when this worst state materializes, the agent is only able to retain a fraction, equal to $\theta = 10\%$ of her initial wealth, that can be interpreted as a subsistence parameter. Whether or not the agent suffers a financial loss does not matter in this case. In state $s = 3$, the agent suffers a severe health loss, due to radioactivity exposure as well as a direct financial loss. The cost of health treatment and the health induced reduction in future income is estimated in Eeckhoudt et al.(2000) at 260,000 euros. In addition, we assume that the combination of the health and wealth shock further impede initial wealth by an additional 70,000 euros. Total loss in this state is therefore estimated at 400,000 euros. In state $s = 4$, the agent only faces the $260,000$ euros health loss. In state $s = 5$, she faces the 70,000 euros financial loss. In addition, the obligation to move away from her house generates an additional reduction of 30,000 on future wealth.

The Value of a Statistical Life (VSL) represents the agent's willingness to pay to marginally decrease her probability of dying. For an individual of group 1, let $f_d$ be the probability of dying, $u(w)$ the utility associated with the initial state and $u(w_d)$, the utility associated with the death state. The willingness to pay for a marginal decrease in the death probability is called VSL and is given by

$$\frac{df_d}{dw} = \frac{u(w) - u(w_d)}{(1 - f_d)u'(w) + f_d u'(w_d)}.$$

In our calibration and for an individual of group 1, we have $f_d = f_{11} + f_{12}$, $w = 870,000$, $w_d = 870,000 - 848,350 = 21,650$ and $u$ is a HARA function.

[26]http://www.bdm.insee.fr
[27]http://www.insee.fr/
[28]http://www.ined.fr/

## 6.4 OLS Estimates

$D_{2011}$, $D_{2012}$ and $D_{2013}$ are three dummies taking value 1 if cat bond $i$ was issued in 2011,2012 and 2013 (respectively). $D_{2014}$ is left out of the regression and defines the reference group. We see that all prices were significantly above the 2014 prices, with a peak in 2011. $D_{Japan}$, $D_{US}$ and $D_{Other}$ take value 1 if cat bond $i$ was issued in Japan, US or in any other country outside Europe. The reference group is Europe. $D_{First}$ takes value 1 if the cat bond corresponds to a sponsor's first issue. $D_{Insur}$ takes value 1 if the cat bond was issued by an insurance (or reinsurance company). $D_{Private}$ or by a private company other than an insurance company. The dummy that take value 1 when the cat bond is issued by a state sponsored program such as the World Bank or the Caribbeans Countries Risk Insurance Facility (CCRIF) is left outside as the reference. $D_{Index}$ and $D_{Param}$ take value 1 respectively if the trigger of the cat bond is an industry loss index or a parametric trigger. The reference group is the indemnity trigger.

| | Quad 1 | Quad 2 | Linear 1 | Linear 2 |
|---|---|---|---|---|
| constant | $-0.0064$ | | $-0.0017$ | |
| | $(-0.5822)$ | | $(-0.1487)$ | |
| $\mathbb{E}(l)_i$ | $2.3329^{***}$ | $2.3156^{***}$ | $1.5129^{***}$ | $1.5131^{***}$ |
| | $(8.8959)$ | $(8.8725)$ | $(14.9794)$ | $(14.9812)$ |
| $(\mathbb{E}(l)_i)^2$ | $-7.7870^{***}$ | $-7.6155^{***}$ | | |
| | $(-3.3595)$ | $(-3.3070)$ | | |
| $D_{2011}$ | $0.0534^{***}$ | $0.0536^{***}$ | $0.0544^{***}$ | $0.0544^{***}$ |
| | $(3.6594)$ | $(3.6694)$ | $(3.5387)$ | $(3.5427)$ |
| $D_{2012}$ | $0.0376^{***}$ | $0.0372^{***}$ | $0.0424^{***}$ | $0.0423^{***}$ |
| | $(6.8104)$ | $(6.7814)$ | $(7.5474)$ | $(7.6334)$ |
| $D_{2013}$ | $0.0116^{**}$ | $0.0112^{***}$ | $0.0134^{***}$ | $0.0428^{***}$ |
| | $(2.5888)$ | $(2.5238)$ | $(2.8499)$ | $(2.8694)$ |
| $D_{Japan}$ | $0.0059$ | $0.0022$ | $0.0080$ | $0.0133$ |
| | $(0.5237)$ | $(0.2365)$ | $(0.6731)$ | $(0.7185)$ |
| $D_{US}$ | $0.0170^{*}$ | $0.0138^{**}$ | $0.0217^{**}$ | $0.0070^{***}$ |
| | $(1.9455)$ | $(2.0349)$ | $(2.3829)$ | $(3.0678)$ |
| $D_{Other}$ | $0.0081$ | $0.0054$ | $0.0121$ | $0.0208$ |
| | $(0.7553)$ | $(0.5610)$ | $(1.0778)$ | $(1.1285)$ |
| $D_{First}$ | $0.0015$ | $0.0010$ | $0.0030$ | $0.0113$ |
| | $(0.3256)$ | $(0.2061)$ | $(0.5949)$ | $(0.5762)$ |
| $D_{Private}$ | $0.0225$ | $0.0164$ | $0.0360^{**}$ | $0.0028^{**}$ |
| | $(1.3059)$ | $(1.1958)$ | $(2.0395)$ | $(2.5785)$ |
| $D_{Insur}$ | $0.0087$ | $0.0062$ | $0.0063$ | $0.0057$ |
| | $(1.0497)$ | $(0.8733)$ | $(0.7229)$ | $(0.7529)$ |
| $D_{Index}$ | $-0.0128$ | $-0.0135^{*}$ | $-0.0093$ | $-0.0095$ |
| | $(-1.6562)$ | $(-1.7622)$ | $(-1.1561)$ | $(-1.1993)$ |
| $D_{Param}$ | $-0.0131^{***}$ | $-0.0138^{***}$ | $-0.0108^{**}$ | $-0.0110^{***}$ |
| | $(-2.9995)$ | $(-3.2566)$ | $(-2.3844)$ | $(-2.5310)$ |
| $R^2$ | $0.8209$ | $0.8204$ | $0.8013$ | $0.8013$ |
| $\hat{s}(10^{-5})$ | $2.3329 * 10^{-5}$ | $2.3156 * 10^{-5}$ | $1.5129 * 10^{-5}$ | $1.5131 * 10^{-5}$ |

Table 9: OLS estimates

## 6.5 Figures

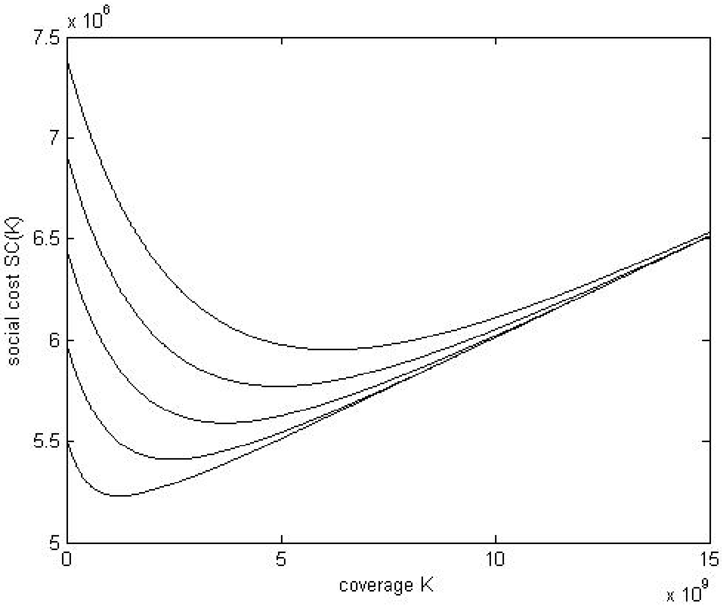



Figure 4: $\pi = 10^{-4}$

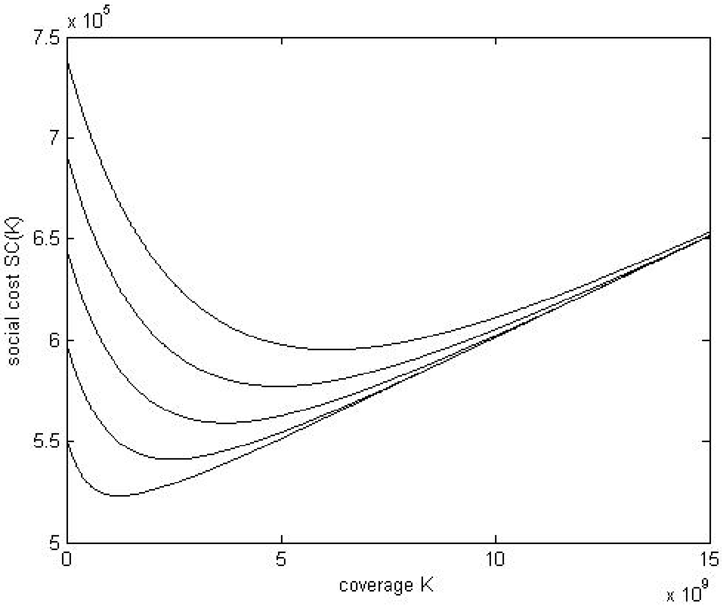



Figure 5: $\pi = 10^{-5}$

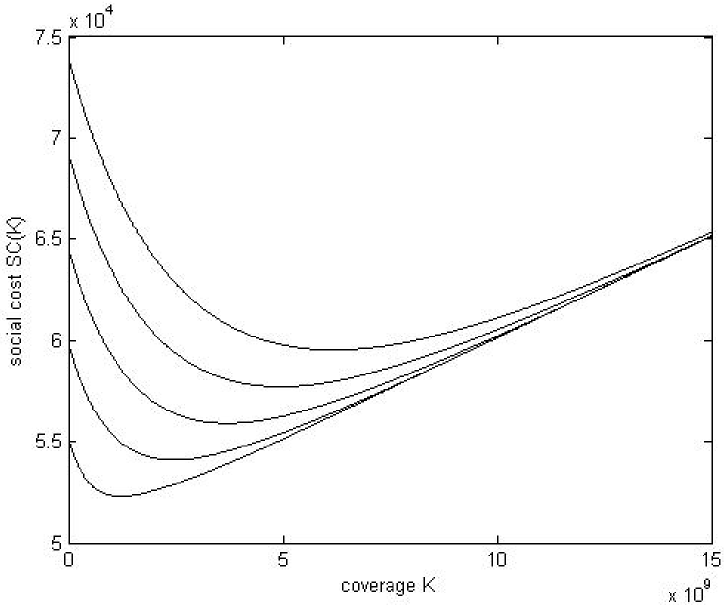



Figure 6: $\pi = 10^{-6}$

## 6.6 Tables

The following tables summarize the numerical results of section 4. Each table presents the either optimal coverage or the welfare gain for a given set of hypotheses. The cost of handling claims is set to $\sigma = 0.3$, which is viewed as a reasonable estimate in the literature. However changes in this parameter have a very limited impact on the simulation results. As we have shown in the previous section, the calibration of $\pi$ does not play any role in the determination of $^*$. Here we use $\pi = 10^{-5}$.

$\underline{R}$ and $\overline{R}$ are the indexes of relative risk aversion in the death state and in the non-loss state, respectively. Since HARA functions feature Increasing Relative Risk Aversion, it must be that $\underline{R} < \overline{R}$. The scenarios that are considered vary across lines. All results are expressed in euros.

Within each table, we fix $\overline{R}$ and we let $\underline{R}$ vary through the columns. From left to right, we therefore increase the agent's risk aversion. For each level of $\overline{R}$, we provide two tables. The first delivers our estimates for the optimal level of coverage and the second computes the welfare gain relative to the no-coverage situation.

The most sensitive parameter is usually the subsistence level $\theta$. Our results indicate that while the optimal coverage is robust to changes in $\theta$, the estimated welfare gains are quite sensitive. As expected, optimal coverage increases with the severity of the scenario under consideration and with the degree of risk aversion.

### 6.6.1 Optimal coverage and welfare gains with $\theta = 0.90$

| $\underline{\text{R}}$ | 1 |
|---|---|
| Scenario 1 | 9.750e+08 |
| Scenario 2 | 1.755e+09 |
| Scenario 3 | 2.730e+09 |
| Scenario 4 | 3.705e+09 |
| Scenario 5 | 4.485e+09 |

Table 10: Coverage, $\overline{R} = 1$

| $\underline{\text{R}}$ | 1 |
|---|---|
| Scenario 1 | 1.443e-02 |
| Scenario 2 | 2.768e-02 |
| Scenario 3 | 3.985e-02 |
| Scenario 4 | 5.104e-02 |
| Scenario 5 | 6.138e-02 |

Table 11: Welfare gain, $\overline{R} = 1$

| $\underline{R}$ | 1 | 2 |
|---|---|---|
| VSL | 4.9665e+06 | 7.4048e+06 |
| Scenario 1 | 1.755e+09 | 1.950e+09 |
| Scenario 2 | 3.705e+09 | 3.705e+09 |
| Scenario 3 | 5.460e+09 | 5.655e+09 |
| Scenario 4 | 7.410e+09 | 7.605e+09 |
| Scenario 5 | 9.165e+09 | 9.360e+09 |

Table 12: Coverage, $\overline{R} = 2$

| $\underline{R}$ | 1 | 2 |
|---|---|---|
| VSL | 4.9665e+06 | 7.4048e+06 |
| Scenario 1 | 9.605e-02 | 1.652e-01 |
| Scenario 2 | 1.691e-01 | 2.746e-01 |
| Scenario 3 | 2.263e-01 | 3.524e-01 |
| Scenario 4 | 2.725e-01 | 4.105e-01 |
| Scenario 5 | 3.105e-01 | 4.556e-01 |

Table 13: Welfare gain, $\overline{R} = 2$

| $\underline{R}$ | 1 | 2 | 3 |
|---|---|---|---|
| VSL | 6.2319e+06 | 1.1386e+07 | 1.6135e+07 |
| Scenario 1 | 2.145e+09 | 2.145e+09 | 2.145e+09 |
| Scenario 2 | 4.095e+09 | 4.290e+09 | 4.290e+09 |
| Scenario 3 | 6.240e+09 | 6.435e+09 | 6.435e+09 |
| Scenario 4 | 8.190e+09 | 8.580e+09 | 8.580e+09 |
| Scenario 5 | 1.034e+10 | 1.073e+10 | 1.073e+10 |

Table 14: Coverage, $\overline{R} = 3$

| $\underline{R}$ | 1 | 2 | 3 |
|---|---|---|---|
| VSL | 6.3723e+06 | 1.2257e+07 | 1.8843e+07 |
| Scenario 1 | 1.293e-01 | 2.555e-01 | 3.598e-01 |
| Scenario 2 | 2.204e-01 | 3.947e-01 | 5.164e-01 |
| Scenario 3 | 2.879e-01 | 4.823e-01 | 6.040e-01 |
| Scenario 4 | 3.399e-01 | 5.426e-01 | 6.599e-01 |
| Scenario 5 | 3.813e-01 | 5.865e-01 | 6.988e-01 |

Table 15: Welfare gain, $\overline{R} = 3$

| R | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| VSL | 2.2109e+07 | 4.3029e+07 | 4.6975e+07 | 4.2608e+07 |
| Scenario 1 | 2.925e+09 | 3.120e+09 | 3.120e+09 | 3.120e+09 |
| Scenario 2 | 5.850e+09 | 6.045e+09 | 6.045e+09 | 6.045e+09 |
| Scenario 3 | 8.775e+09 | 9.165e+09 | 9.165e+09 | 9.165e+09 |
| Scenario 4 | 1.170e+10 | 1.209e+10 | 1.229e+10 | 1.229e+10 |
| Scenario 5 | 1.463e+10 | 1.521e+10 | 1.521e+10 | 1.541e+10 |

Table 16: Coverage, $\overline{R} = 4$

| R | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| VSL | 2.2109e+07 | 4.3029e+07 | 4.6975e+07 | 4.2608e+07 |
| Scenario 1 | 4.310e-01 | 7.125e-01 | 8.415e-01 | 9.038e-01 |
| Scenario 2 | 5.878e-01 | 8.234e-01 | 9.090e-01 | 9.465e-01 |
| Scenario 3 | 6.689e-01 | 8.685e-01 | 9.339e-01 | 9.616e-01 |
| Scenario 4 | 7.185e-01 | 8.929e-01 | 9.469e-01 | 9.693e-01 |
| Scenario 5 | 7.519e-01 | 9.082e-01 | 9.549e-01 | 9.740e-01 |

Table 17: Welfare gain, $\overline{R} = 4$

| R | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| VSL | 3.8504e+07 | 6.3288e+07 | 5.5351e+07 | 4.4793e+07 | 3.6861e+07 |
| Scenario 1 | 3.510e+09 | 3.510e+09 | 3.510e+09 | 3.510e+09 | 3.510e+09 |
| Scenario 2 | 7.020e+09 | 7.020e+09 | 7.215e+09 | 7.215e+09 | 7.215e+09 |
| Scenario 3 | 1.034e+10 | 1.053e+10 | 1.073e+10 | 1.073e+10 | 1.073e+10 |
| Scenario 4 | 1.385e+10 | 1.424e+10 | 1.424e+10 | 1.424e+10 | 1.443e+10 |
| Scenario 5 | 1.736e+10 | 1.775e+10 | 1.775e+10 | 1.794e+10 | 1.794e+10 |

Table 18: Coverage, $\overline{R} = 5$

| R | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| VSL | 3.8504e+07 | 6.3288e+07 | 5.5351e+07 | 4.4793e+07 | 3.6861e+07 |
| Scenario 1 | 6.090e-01 | 8.730e-01 | 9.483e-01 | 9.747e-01 | 9.860e-01 |
| Scenario 2 | 7.447e-01 | 9.279e-01 | 9.717e-01 | 9.863e-01 | 9.925e-01 |
| Scenario 3 | 8.045e-01 | 9.478e-01 | 9.798e-01 | 9.903e-01 | 9.947e-01 |
| Scenario 4 | 8.382e-01 | 9.580e-01 | 9.838e-01 | 9.923e-01 | 9.957e-01 |
| Scenario 5 | 8.598e-01 | 9.643e-01 | 9.863e-01 | 9.934e-01 | 9.964e-01 |

Table 19: Welfare gain, $\overline{R} = 5$

### 6.6.2 Optimal coverage and welfare gains with $\theta = 0.975$

| R | 1 |
|---|---|
| Scenario 1 | 9.300e+08 |
| Scenario 2 | 1.875e+09 |
| Scenario 3 | 2.805e+09 |
| Scenario 4 | 3.750e+09 |
| Scenario 5 | 4.680e+09 |

Table 20: Coverage, $\overline{R} = 1$

| R | 1 |
|---|---|
| Scenario 1 | 4.653e-02 |
| Scenario 2 | 8.647e-02 |
| Scenario 3 | 1.211e-01 |
| Scenario 4 | 1.514e-01 |
| Scenario 5 | 1.782e-01 |

Table 21: Welfare gain, $\overline{R} = 1$

| R<br>VSL | 1<br>1.8926e+07 | 2<br>3.3396e+07 |
|---|---|---|
| Scenario 1 | 1.665e+09 | 1.680e+09 |
| Scenario 2 | 3.345e+09 | 3.360e+09 |
| Scenario 3 | 5.010e+09 | 5.040e+09 |
| Scenario 4 | 6.690e+09 | 6.720e+09 |
| Scenario 5 | 8.355e+09 | 8.400e+09 |

Table 22: Coverage, $\overline{R} = 2$

| $\underline{R}$<br>VSL | 1<br>1.8926e+07 | 2<br>3.3396e+07 |
|---|---|---|
| Scenario 1 | 3.599e-01 | 5.139e-01 |
| Scenario 2 | 5.191e-01 | 6.699e-01 |
| Scenario 3 | 6.088e-01 | 7.453e-01 |
| Scenario 4 | 6.664e-01 | 7.897e-01 |
| Scenario 5 | 7.065e-01 | 8.190e-01 |

Table 23: Welfare gain, $\overline{R} = 2$

| $\underline{R}$<br>VSL | 1<br>1.1036e+08 | 2<br>2.8702e+08 | 3<br>4.2413e+08 |
|---|---|---|---|
| Scenario 1 | 2.025e+09 | 2.025e+09 | 2.040e+09 |
| Scenario 2 | 4.050e+09 | 4.065e+09 | 4.065e+09 |
| Scenario 3 | 6.075e+09 | 6.090e+09 | 6.105e+09 |
| Scenario 4 | 8.100e+09 | 8.130e+09 | 8.130e+09 |
| Scenario 5 | 1.013e+10 | 1.016e+10 | 1.017e+10 |

Table 24: Coverage, $\overline{R} = 3$

| $\underline{R}$<br>VSL | 1<br>1.1036e+08 | 2<br>2.8702e+08 | 3<br>4.2413e+08 |
|---|---|---|---|
| Scenario 1 | 7.864e-01 | 9.192e-01 | 9.579e-01 |
| Scenario 2 | 8.755e-01 | 9.560e-01 | 9.775e-01 |
| Scenario 3 | 9.099e-01 | 9.690e-01 | 9.842e-01 |
| Scenario 4 | 9.281e-01 | 9.756e-01 | 9.876e-01 |
| Scenario 5 | 9.394e-01 | 9.796e-01 | 9.897e-01 |

Table 25: Welfare gain, $\overline{R} = 3$

| $\underline{R}$<br>VSL | 1<br>5.0096e+08 | 2<br>9.8738e+08 | 3<br>8.6853e+08 | 4<br>6.9773e+08 |
|---|---|---|---|---|
| Scenario 1 | 2.580e+09 | 2.595e+09 | 2.595e+09 | 2.595e+09 |
| Scenario 2 | 4.800e+09 | 4.830e+09 | 4.830e+09 | 4.830e+09 |
| Scenario 3 | 7.035e+09 | 7.065e+09 | 7.065e+09 | 7.080e+09 |
| Scenario 4 | 9.270e+09 | 9.300e+09 | 9.300e+09 | 9.315e+09 |
| Scenario 5 | 1.149e+10 | 1.154e+10 | 1.154e+10 | 1.155e+10 |

Table 26: Coverage, $\overline{R} = 4$

| R | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| VSL | 5.0096e+08 | 9.8738e+08 | 8.6853e+08 | 6.9773e+08 |
| Scenario 1 | 9.518e-01 | 9.903e-01 | 9.965e-01 | 9.983e-01 |
| Scenario 2 | 9.741e-01 | 9.949e-01 | 9.982e-01 | 9.991e-01 |
| Scenario 3 | 9.818e-01 | 9.964e-01 | 9.987e-01 | 9.994e-01 |
| Scenario 4 | 9.857e-01 | 9.972e-01 | 9.990e-01 | 9.995e-01 |
| Scenario 5 | 9.880e-01 | 9.977e-01 | 9.992e-01 | 9.996e-01 |

Table 27: Welfare gain, $\overline{R} = 4$

| R | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| VSL | 1.3733e+09 | 1.2765e+09 | 8.9388e+08 | 6.7680e+08 | 5.4322e+08 |
| Scenario 1 | 2.970e+09 | 2.985e+09 | 2.985e+09 | 3.000e+09 | 3.000e+09 |
| Scenario 2 | 5.340e+09 | 5.355e+09 | 5.355e+09 | 5.355e+09 | 5.355e+09 |
| Scenario 3 | 7.695e+09 | 7.710e+09 | 7.725e+09 | 7.725e+09 | 7.725e+09 |
| Scenario 4 | 1.005e+10 | 1.008e+10 | 1.010e+10 | 1.010e+10 | 1.010e+10 |
| Scenario 5 | 1.242e+10 | 1.245e+10 | 1.245e+10 | 1.247e+10 | 1.247e+10 |

Table 28: Coverage, $\overline{R} = 5$

| R | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| VSL | 1.3733e+09 | 1.2765e+09 | 8.9388e+08 | 6.7680e+08 | 5.4322e+08 |
| Scenario 1 | 9.893e-01 | 9.987e-01 | 9.997e-01 | 9.999e-01 | 9.999e-01 |
| Scenario 2 | 9.943e-01 | 9.993e-01 | 9.998e-01 | 9.999e-01 | 1.000e+00 |
| Scenario 3 | 9.960e-01 | 9.995e-01 | 9.999e-01 | 1.000e+00 | 1.000e+00 |
| Scenario 4 | 9.969e-01 | 9.996e-01 | 9.999e-01 | 1.000e+00 | 1.000e+00 |
| Scenario 5 | 9.974e-01 | 9.997e-01 | 9.999e-01 | 1.000e+00 | 1.000e+00 |

Table 29: Welfare gain, $\overline{R} = 5$

# 7 Bibliography

## References


[1] Arrow K.J. and Lind R.C. (1970). Uncertainty and the evaluation of public investment decisions, *The American Economic Review*, Vol. 60, No. 3, pp. 364-378.

[2] Bantwal, V.J. and Kunreuther, H.C. (2000). A catbond premium puzzle, *Journal of Behavioral Finance*, vol.1, No. 1, 76-91.

[3] Eeckhoudt L., Schieber C. and Schneider T. (2000). Risk Aversion and the External Cost of a Nuclear Accident, *Journal of Environmental Management*, Vol. 58, pp. 109-117.

[4] Dreicer M., Tort V., and Margerie H. (1995). The External Costs of the Fuel Cycle: Implementation in France, CEPN report R-238.

[5] Finkelstein, A., Luttmer, E.F.P., and Notowidigdo, M.J. (2013). What good is wealth without health? The effect of health on the marginal utility of consumption, *Journal of the European Economic Association*, 11(S1), 221-258.

[6] Gignoux, J., Menéndez, M. (2016). Benefit in the wake of disaster: Long-run effects of earthquakes on welfare in rural Indonesia. *Journal of Development Economics*, 118, 24-44.

[7] IRSN (2013), Methodology applied by the IRSN to estimate the costs of a nuclear accident in France. Report PRP-CRI/SESUC/2013-00261.

[8] Lane, M. and Mahul, O. (2008). Catastrophe Risk Pricing: An Empirical Analysis. *World Bank Policy Research Working Paper Series.*

[9] Levy, H. (1994). Absolute and relative risk aversion: an experimental study. *Journal of Risk and Uncertainty* 8, 289-307.

[10] Markandya, A. (1995). Externalities of Fuel Cycles ExternE Project. Report Economic Valuation: An Impact Pathway Approach. European Commission DG XII.

[11] NEA (2000). Methodologies for Assessing the Economic Consequences of Nuclear Reactor Accidents, OECD Nuclear Energy Agency, Paris.

[12] Schneider T. (1998). Integration of the Indirect Costs Evaluation, CEPN Report No. 260.

[13] Strobl, E. (2012). The economic growth impact of natural disasters in developing countries: Evidence from hurricane strikes in the Central American and Caribbean regions. *Journal of Development economics*, 97(1), 130-141.

[14] Szpiro, G. (1986). Measuring risk aversion: an alternative approach. *The Review of Economics and Statistics*, 68, 156-159.

[15] Viscusi, W. K., Aldy, J. E. (2003). The value of a statistical life: a critical review of market estimates throughout the world. *Journal of risk and uncertainty*, 27(1), 5-76.

[16] Weitzman, M. L. (2009). On modeling and interpreting the economics of catastrophic climate change. *The Review of Economics and Statistics*, 91(1), 1-19.