Where Experience Matters: Asset Allocation and Asset Pricing with Opaque and Illiquid Assets*

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This version: December 4, 2015

Abstract

Alternative assets, such as private equity, hedge funds, and real assets, are illiquid and opaque, and thus pose a challenge to traditional models of asset allocation. In this paper, we study asset allocation and asset pricing in a general-equilibrium model with liquid assets and an alternative risky asset, which is opaque and incurs transaction costs, and investors who differ in their experience in assessing the alternative asset. We find that the optimal asset-allocation strategy of the relatively inexperienced investors is to initially tilt their portfolio away from the alternative asset and to hold more of it with experience. Counterintuitively, a decrease in the transaction cost for the alternative asset *increases* the portfolio tilt at the initial date, and hence, the liquidity discount. Transaction costs may induce inexperienced investors to hold a majority of the illiquid asset at later dates, even if they are pessimistic about future payoffs, and produce a sizable liquidity discount. During periods when the alternative asset is illiquid, investors trade the liquid equity index instead, leading to strong spillover effects.

Keywords: portfolio choice, alternative assets, private equity, transaction costs, heterogeneous beliefs, incomplete markets.
JEL: G11, G12

*We thank Bernard Dumas for extensive discussions. We are grateful for comments from Nöel Amenc, Harjoat Bhamra, Oliver Boguth (discussant), Julien Cujean (discussant), Victor DeMiguel, Vihang Errunza, Denis Gorea, Rob Heinkel, Christian Heyerdahl-Larsen, Philipp Illeditsch (discussant), Dietmar Leisen (discussant), Francis Longstaff, Lionel Martellini, Ludovic Phalippou, Sergei Sarkissian, Eli Talmor, Nikolas Tessaromatis, Josef Zechner, seminar participants at Goethe University Frankfurt, Frankfurt School of Finance & Management, University of Lausanne and the Ecole Polytechnique Fédérale de Lausanne, the 8th Private Equity Findings Symposium (Coller Institute), the 7th Biennial McGill Global Asset Management Conference, the Brunel Studies in Finance and Economics Conference, the Edhec-Princeton Academia-Meets-Practice Conference, and meetings of the European Finance Association and Northern Finance Association. Adrian Buss is affiliated with INSEAD, Boulevard de Constance, 77305 Fontainebleau, France; Email: `adrian.buss@insead.edu`. Raman Uppal is affiliated with CEPR and Edhec Business School, 10 Fleet Place, Ludgate, London, United Kingdom EC4M 7RB; Email: `raman.uppal@edhec.edu`. Grigory Vilkov is affiliated with Frankfurt School of Finance & Management, Sonnemannstrasse 9-11, 60314 Frankfurt, Germany; Email: `vilkov@vilkov.net`.

1 Introduction

Traditional models of portfolio choice and asset pricing assume that assets are liquid and can be traded without cost. While this is a reasonable assumption for government securities and publicly traded stocks of large companies, there are substantial costs for trading alternative asset classes, such as private equity, stocks of smaller companies, hedge funds, and real-estate funds,[1] and, consequently, the interval between trades in these assets can span long periods.[2] Moreover, because these alternative assets do not have long histories and regularly observed market values, the returns from investing in these assets are less transparent than the returns from public equity.[3] As Malmendier and Nagel (2011, 2014)) show, investors' expectations are influenced by their personal experience even in transparent markets. Accordingly, we allow investors' experience to play a crucial role in forming expectations for alternative asset classes, given the limited length and quality of their return data.[4]

Given the large and increasing role of alternative assets in portfolios,[5] our objective is to develop a model of asset allocation and asset pricing that takes into account both key characteristics of alternative assets: (1) trading is costly, and (2) returns are opaque, and thus, investors' assessments of these assets depend on their experience. Specifically, when assets can be traded only at a substantial cost, asset-allocation decisions depend not just on current market conditions and investors' experience, but also on past experience and past asset-allocation decisions, in addition to expected future experience and trading costs.

Thus, the *interaction* between opaqueness and illiquidity raises fundamental questions about asset allocation, asset pricing and their dynamics. How does the interaction between opaqueness and illiquidity affect an investor's portfolio? Should inexperienced investors hold

[1]Collett, Lizieri, and Ward (2003) estimate transaction costs of over 3% for institutional real estate, Beber and Pagano (2013) find that bid-ask spreads for smaller firms can be 10%, and costs of private equity transactions can exceed even 10% (Preqin Special Report (2013a)).

[2]Ang (2013) finds that even within liquid asset classes, subclasses can be highly illiquid, e.g., stocks in pink-sheet OTC markets may not trade for a week, and municipal bonds typically trade twice per year.

[3]The difficulties encountered in estimating returns of opaque assets are discussed by Phalippou (2009), Phalippou and Gottschalg (2009), and Ang and Sorensen (2013) for private equity, by Dhar and Goetzman (2005) for real estate, and by Ang, Ayala, and Goetzman (2014) for hedge funds and private equity.

[4]The empirical findings regarding the importance of personal experience are mirrored by the following quote from Albert Einstein: "The only source of knowledge is experience."

[5]Goetzman and Oster (2012) and Cejnek, Franz, Randl, and Stoughton (2014) report dramatic shifts toward alternative assets for endowment funds. While in 1985 Yale's asset allocation for alternative assets was 20%, in 2013 it invested 79.8% in alternatives, mostly absolute return, private equity, and real assets.

alternative assets at all, and how should they revise their portfolios over time as they gain experience but face substantial transaction costs? What risks arise because of illiquidity and investors' inexperience? What are the consequences for asset prices and risk premia and their evolution over time? Finally, what are the dynamics of the optimal risk-sharing arrangement between experienced and inexperienced investors?

To answer these questions, we develop a discrete-time model of a general-equilibrium exchange economy with heterogeneous investors who have Epstein and Zin (1989) and Weil (1990) utility functions. Our framework has a risk-free bond and two risky assets that differ in their level of liquidity and opaqueness. We interpret the first risky asset to be a liquid asset, such as the S&P 500 equity index that can be traded without cost, and the second asset to be a relatively illiquid and opaque alternative asset, which incurs a proportional transaction cost when traded. A transaction cost of less than 100% gives rise to *endogenous* illiquidity; that is, periods where investors choose not to trade the alternative asset. The extreme case of 100% transaction cost corresponds to the "blackout" studied in Longstaff (2009), which captures exogenous liquidity, such as lockout periods imposed by hedge funds.

In addition to transaction costs, a second feature of our model is that it has two classes of investors who differ in their experience, captured through differences in the precision of their prior beliefs about the expected growth rate of the alternative asset's dividends. In order to highlight the effect of experience, one investor is assumed to have an infinitely precise estimate of the expected growth rate of the alternative asset's dividends, while the other investor's estimate is much less precise.[6] Consequently, compared to the experienced investor, while using Bayes law to update her beliefs, the inexperienced investor assigns more weight to recent data and less weight to her prior. So, even if investors start out with beliefs that have the same prior mean, after observing the dividends of the alternative asset, their posterior beliefs differ with respect to both the posterior mean and the confidence (posterior variance) they have about these beliefs. This heterogeneity in posterior beliefs drives the dynamics of trade and asset prices in our model.

[6]We have also analyzed the setup where both investors have imperfect precision, with one investor's precision being higher than that of the other. However, the results are very similar, so we focus on the simpler case in which only one investor has imperfect precision.

There is strong empirical evidence that experience has a substantial effect on financial decision making. Vissing-Jorgenson (2003) shows that the beliefs of retail investors depend on their return experience, with inexperienced investors having the highest stock market expectation at the peak. Greenwood and Nagel (2009) show that this result holds even for well-trained, professional fund managers. Malmendier and Nagel (2011, 2014) show that investors' experience of macroeconomic shocks affects their beliefs and risk-taking.[7]

Our key results are summarized below. When one asset is opaque, the inexperienced investor recognizes the estimation risk inherent in this asset, and so she reduces her initial holdings of this asset. This is accompanied by an increase in the holding of the liquid equity index, and, driven by the desire for precautionary savings, a substantial increase in holdings of the bond. On the other hand, because of market clearing, the experienced investor's initial holdings of the alternative asset increase with its opacity. Consequently, neither investor holds the fully diversified market portfolio. As investors gain experience, they increase their investment in alternative assets. These asset-allocation characteristics are consistent with the empirically observed behavior of institutional investors: Lerner, Schoar, and Wang (2008) and Goetzman and Oster (2012) find that private (Ivy League) endowment funds have increased their target allocation for alternative assets over the last decades as they acquired (positive) experience and, nowadays, strongly overweight alternative assets in their portfolios.[8] In contrast, they find that relatively inexperienced endowment funds invest only marginally in alternative assets. A consequence of this asset allocation in our model is that the volatility of the inexperienced investor's consumption growth is lower than that of the experienced investor. However, in exchange for "insuring" the inexperienced investor, the experienced investor earns a higher expected return on her wealth. In line with this, Lerner, Schoar, and Wang (2008) find that endowment funds that entered earlier into alternative assets achieved higher returns and attribute this success to the importance of experience in interpreting ambiguous data.

[7]Additional evidence of "learning by trading" can be found in Kaustia and Knüpfer (2008), Seru, Shumway, and Stoffman (2010), and Barberis, Greenwood, Jin, and Shleifer (2014). There exist also a number of papers using experiments whose results support our modeling of experience, e.g., Smith, Suchanek, and Williams (1988), Dufwenberg, Lindqvist, and Moore (2005), Haruvy, Lahav, and Noussair (2007), and Hussam, Porter, and Smith (2008).

[8]See footnote 5 for evidence on the overweighting of alternative assets.

Surprisingly, because of the interaction between inexperience and illiquidity, the inexperienced investor's optimal initial holdings of the alternative asset can be *larger* with illiquidity. The intuition for this is that ideally the investor would like to start with a low holding of the alternative asset and increase this position over time as she gains experience. However, this would generate substantial trading costs. Accordingly, inexperienced investors that are new to an alternative asset class trade off the higher risk of initial over-investment versus the cost of future rebalancing. If the transaction cost dominates, it is optimal for such investors to reduce the portfolio tilt away from the alternative asset, and thus, hold more initially. Transaction costs also induce portfolio inertia. Consequently, in the presence of transaction costs, the inexperienced investor could end up holding a *majority* of the alternative asset over time even if she is more *pessimistic* about its growth rate than the experienced investor. This effect is driven by prior beliefs and the reluctance to change portfolio holdings in the presence of transaction costs.

As expected, transaction costs for trading the alternative asset reduce its turnover. This also triggers a decrease in bond turnover because the bond is used to finance trade in the alternative asset. However, the turnover of the liquid risky asset increases because it is used as a substitute for trading the alternative asset. This "spillover" effect can be so significant that the inexperienced investor's holdings in the liquid risky asset track perfectly the dividend dynamics of the *alternative asset*, introducing excess correlation and leading to more volatile holdings in the liquid equity index.

With transaction costs for the alternative asset, its price drops and this is mirrored by an increase in its expected return and also its volatility. For a transaction cost of 5%, we find an average price discount of more than 4% and a maximum discount of more than 12%, consistent with empirical data. For example, Franzoni, Nowak, and Phalippou (2012) find an illiquidity discount of 10% for private equity and Hege and Nuti (2011) report valuation gaps of 5%–10% in private-equity secondary markets. The price discount also implies a sizable increase in expected returns, in line with empirical evidence. Contrary to intuition, the illiquidity discount is stronger for moderate transaction costs than for full illiquidity. That is, the price discount is *larger* in a setting where investors incur a cost for trading the asset, compared to a setting where trade is precluded exogenously. Because the bond and the equity index serve as substitutes to the alternative asset, their prices increase in the transaction costs for the alternative asset.

In addition to the economic insights described above, our paper also makes a technical contribution: we demonstrate how to identify the equilibrium in a recursive fashion in a model where markets are incomplete, the decision to trade or not is endogenous, the risk-free interest rate is endogenous, investors have heterogeneous beliefs that depend on their experience, and preferences are given by Epstein-Zin-Weil utility functions.

The rest of the paper is organized as follows. In Section 2, we discuss the existing literature that is related to our work. In Section 3, we describe the general model of the economy we study. In Section 4, we characterize the equilibrium in this economy and explain how it can be described by a system of path-independent backward-only equations, instead of a system of backward-forward equations. In Section 5, we analyze the effect of transaction costs and differences in experience on asset allocation, asset prices, and consumption. We conclude in Section 6. Technical results are relegated to the appendix.

2 Related Literature

The alternative asset that we study has two key features: one, its return is opaque, so experienced and inexperienced investors typically disagree about the expected growth rate of its dividends; and two, trading the alternative asset is costly. Thus, our work is related to two streams of the literature, each of which considers one of the features in isolation.

Two important recent papers provide a comprehensive analysis of the implications of *experiential learning* in overlapping generation models assuming that young investors disregard history and learn only from their own experiences. While investors in Collin-Dufresne, Johannes, and Lochstoer (2014) have recursive preferences, Ehling, Graniero, and Heyerdahl-Larsen (2014) obtain an elegant analytical solution for log investors. Earlier papers studying the effects on asset prices of differences in beliefs, but without experiential learning, include Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero

(1998), Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2009), and Bhamra and Uppal (2014). See Basak (2005) for an extensive survey of asset-pricing models with disagreement between investors and Morris (1995) for a philosophical discussion of models where agents have difference in beliefs but do not learn from each others' behavior; that is, they "agree to disagree."

Second, there are several papers that study the effect of *transaction costs and illiquidity* on portfolio selection and equilibrium asset prices. In an influential paper with many insights, Longstaff (2009) shows that the effect of illiquidity on portfolio choice and asset prices can be substantial. He considers a setup with two log-investors who differ only in their rates of subjective time preference and face an exogenously specified blackout period, in which the asset cannot be traded at all. In an important extension to the work by Longstaff (2009), Ang, Papanikolaou, and Westerfield (2014) study asset allocation in a partial-equilibrium model where the duration and frequency of the blackout period are stochastic. Vayanos (1998) and Acharya and Pedersen (2005) consider illiquidity in the form of transaction costs in models where the investor's trading decisions are exogenously specified, e.g., for life-cycle reasons. Heaton and Lucas (1996) consider a general equilibrium model, but with quadratic transaction costs. Buss and Dumas (2014) study an equilibrium model with transaction costs, assuming that investors have power utility and full knowledge about all the parameters in the model. Garleânu and Pedersen (2014) provide portfolio choice and asset-pricing implications in a very tractable framework based on market-impact costs. For excellent surveys of the literature on portfolio choice and asset pricing with illiquidity, see the review papers by Amihud, Mendelson, and Pedersen (2005) and Vayanos and Wang (2011, 2012, 2013).

The main distinguishing features of our model from existing work are the following: we consider *both* experiential learning and illiquidity, which is modeled via the presence of transaction costs, so investors choose endogenously whether or not to trade; investors have recursive preferences that capture wealth effects (in contrast to exponential utility), intertemporal hedging demands (in contrast to log utility), and a preference for early or late resolution of uncertainty (in contrast to power utility); and, an endogenous risk-free rate.

Moreover, in contrast to the early literature studying the effect on asset prices of differences in beliefs, in our model investors differ with respect to both the posterior mean belief and confidence (posterior variance) they have in this belief.

3 Model

In this section, we describe the general-equilibrium model we use for our analysis. The model is set in discrete time with a finite horizon: $t \in \{1, ..., T\}$. We assume that there exists a single consumption good. The economy is populated by two types of investors, indexed by $k \in \{1, 2\}$, who derive utility from consumption and differ in their experience. Investors can trade three assets, indexed by $n \in \{0, 1, 2\}$, differing in their degree of riskiness, liquidity, and opacity. Asset $n = 0$ is liquid and risk-free; asset $n = 1$ is liquid but risky; and asset $n = 2$ is risky but illiquid with a dividend process that is opaque. In the rest of this section, we give the details of these financial assets and investors.

3.1 Financial Assets and Uncertainty

We consider a Lucas (1978) economy with two trees, $n \in \{1, 2\}$, each generating dividends $d(n, t)$. We assume that both dividends follow a binomial process, mimicking a Brownian motion with drift μ_n^D and volatility σ_n^D, implemented as in Jarrow and Rudd (1983). That is, an up-move (down-move) in the tree implies a relative change in dividend of $\exp\left(\left(\mu_n^D - \frac{1}{2}\left(\sigma_n^D\right)^2\right) \pm \sigma_n^D\right)$ with unconditional probabilities of 1/2 each. To capture a correlation ρ between the dividend processes, we set the *conditional* probability of an up-move (down-move) in the first dividend tree, conditional on an up-move (down-move) in the second dividend tree, to be $\nu = (1 + \rho)/2$.

There exist two risky assets, each modeled as a claim to the dividends of one of the trees. We normalize the number of shares outstanding for each asset to one. In addition to these two risky assets, there exists a one-period discount bond in zero net supply, indexed by $n = 0$. The number of shares of a particular asset n held by investor k at date t is denoted by $\theta(n,k,t)$; and the price of each risky asset n as perceived by investor k at date t, $S(n,k,t)$, is determined in equilibrium with the price on the terminal date being zero.

3.1.1 Transaction Costs

We assume that the discount bond and the first risky asset are perfectly liquid; that is, can be traded at any time without incurring costs. Thus, we interpret the bond as a risk-free government bond and the first risky asset as an equity index.

In contrast, investors pay a proportional cost for trading the second risky asset, labeled the *alternative* asset. The transaction cost $\tau(\cdot)$ depends on the (dollar) value of the trade:[9]

$$\tau\big(\theta(2,k,t),\theta(2,k,t-1),S(2,k,t)\big) \equiv \big|\theta(2,k,t)-\theta(2,k,t-1)\big| \times S(2,k,t) \times \kappa, \qquad (1)$$

where $\theta(2,k,t)$ denotes the number of shares of the alternative asset held by investor k at date t, and κ is the proportional transaction cost.[10] If κ equals zero, the asset can be traded without costs. On the other hand, if $\kappa = 100\%$, the asset becomes fully illiquid, resulting in a "blackout period" over which the asset cannot be traded at all, as in Longstaff (2009). To minimize the effect of the initial holdings on the equilibrium outcome, we assume that the investors can trade the alternative asset at no cost at the initial date. Thus, the optimal initial holdings depend only on the initial wealth of the investors—not its composition. Moreover, in the case of a full blackout, it gives the investors a chance to adjust their holdings once before the market shuts down, which allows us to compare our results with those in Longstaff (2009).[11]

[9] We could also consider the case where the transaction costs depend on the number of shares being traded, as in Vayanos (1998). Transaction costs could differ also across investors and over time. One could also allow them to vary with the state of the economy, for instance, being higher when aggregate consumption is low. However, for ease of exposition, we focus on the case of constant and identical costs.

[10] Because we are in a general-equilibrium setting, rather than assuming that the transaction cost is a deadweight loss to society, we assume that the transaction cost is added back to the investor's consumption *after* the investor has made her consumption and portfolio decisions; this eliminates any wealth effects arising from transaction costs.

[11] Our model easily allows for transaction costs at the initial date. In the quantitative analysis of our model, we study this case as a robustness exercise.

3.2 Investors: Preferences and Experience

Given that we wish to study how differences in experience between investors and illiquidity impacts asset allocation and asset pricing, we assume that each investor has the same initial endowment, half a share of each of the risky assets and a zero position in the bond, and homogeneous preferences.[12] Specifically, investors are assumed to have preferences that are of the Kreps and Porteus (1978) type. We adopt the Epstein and Zin (1989) and Weil (1990) specification of this utility function, in which lifetime utility $V(k,t)$ is defined recursively:

$$V(k,t) = \left[(1-\beta)\, c(k,t)^{1-\frac{1}{\psi}} + \beta E_t^k \left[V(k,t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}},$$

where E_t^k denotes the conditional expectation at time t under the investor's subjective probability measure, $c(k,t) > 0$ is the consumption of investor k at date t in state $\omega(t,s)$,[13] β denotes the subjective rate of time preference, $\gamma > 0$ is the coefficient of relative risk aversion, $\psi > 0$ is the elasticity of intertemporal substitution, and $\phi = \frac{1-\gamma}{1-1/\psi}$. The Epstein and Zin (1989) and Weil (1990) utility specification nests the more standard time-separable utility functions, but has the well-known advantage that the risk aversion parameter, which drives the desire to smooth consumption across states of nature, is distinct from the elasticity of intertemporal substitution parameter, which drives the desire to smooth consumption over time. The importance of using these utility functions in a model with learning is highlighted in Collin-Dufresne, Johannes, and Lochstoer (2013).

We now explain how we model the role of experience. We assume that both investors have perfect knowledge with respect to the dividend dynamics of the equity index and also know the correlation between the two dividend trees. For the alternative asset, which is subject to transaction costs, we assume that the first investor (the "experienced investor") has perfect knowledge about its dividend dynamics; that is, the investor's beliefs coincide with the objective probability measure. But, the second investor (the "inexperienced investor") has imperfect information about the asset's dividend dynamics and updates her

[12] We could easily allow the investors to differ along all three dimensions of their utility functions: time preference, risk aversion, and elasticity of intertemporal substitution. However, our focus is on the effect of illiquidity and investors' experience rather than preference heterogeneity.

[13] To simplify notation, we do not write explicitly the dependence on the state $\omega(t,s)$.

beliefs after each realization of the alternative asset's dividends using Bayesian learning. Thus, illiquidity and opacity reside *jointly* in the alternative asset.

Specifically, given the prior information and her observations, the inexperienced investor has to infer the probability for an up-move, $p_{2,u}$, in the binomial tree of the alternative asset. A conjugate prior for $p_{2,u}$ is the Beta distribution, Beta$(\alpha_1; \alpha_2)$, where the hyperparameters α_1 and α_2 can be interpreted as virtual (pseudo) counts of up- and down-moves before the start of the economy ("prior knowledge"). While larger values for α_1 and α_2 are in general associated with more precise prior knowledge about the distribution, the relative share of α_1 versus α_2 captures the prior expected value for $p_{2,u}$, as outlined below. Starting with the Beta prior and having observed u up-moves and d down-moves in the alternative asset's binomial tree, the investor's posterior beliefs are Beta distributed, with $p_{2,u} \sim \text{Beta}(\alpha_1 + u, \alpha_2 + d)$.[14] This implies that the posterior mean probability under the investor's subjective probability measure for the next move in the alternative asset's tree being an up-move is

$$E_t[p_{2,u}] = \frac{\alpha_1 + u}{\alpha_1 + u + \alpha_2 + d}, \tag{2}$$

with the posterior variance being

$$\text{Var}(p_{2,u}) = \frac{(\alpha_1 + u)(\alpha_2 + d)}{(\alpha_1 + u + \alpha_2 + d)^2(\alpha_1 + u + \alpha_2 + d + 1)}. \tag{3}$$

That is, the inexperienced investor updates her beliefs by simply counting the number of observed up-moves u and down-moves d in the tree. The predictive expected probability of the next move being up (down) is then just the relative share of observed up-moves (down-moves), incorporating in addition, the prior information corresponding to the "pseudo-counts." To incorporate the assumption that the inexperienced investor knows the correlation between the dividends for the two assets, we set the subjective conditional probability of an up move (down move) equal to the objective conditional probability.

Our learning algorithm highlights the importance of "learning from experience." Specifically, the inexperienced investor acts as a "trend-chaser," revising her beliefs upwards after a positive shock and downwards after a negative shock. There is substantial evidence in the

[14]See, for instance, Minka (2003).

literature for this type of learning behavior. For example, Malmendier and Nagel (2011, 2014) show that investors' experience of macroeconomic shocks affects their beliefs and risk-taking. Using data from the tech-bubble in the late 1990s, Vissing-Jorgenson (2003) shows that the beliefs of retail investors depend on their return experience, with inexperienced investors having the highest stock market expectation at the peak, and Greenwood and Nagel (2009) show that this result holds even for well-trained, professional fund managers.

4 Investor Optimization and Equilibrium

In this section, we first describe the optimization problem of each investor. We then impose market clearing and explain how one can solve for the equilibrium.

4.1 The Investors' Optimization Problem

The objective of each investor is to maximize expected lifetime utility by choosing consumption, $c(k,t)$, and the portfolio positions in each of the financial assets, $\theta(n,k,t)$:

$$V(k,t) = \max_{\{c(k,t),\theta(n,k,t)\}} \left[(1-\beta)\, c(k,t)^{1-\frac{1}{\psi}} + \beta E_t^k \left[V(k,t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}}, \tag{4}$$

subject to the budget equation

$$\begin{aligned} c(k,t) + \sum_{n=0}^{2} \theta(n,k,t) S(n,k,t) + \tau\big(\theta(2,k,t), \theta(2,k,t-1), S(2,k,t)\big) \quad &\leq \\ \theta(0,k,t-1) + \sum_{n=1}^{2} \theta(n,k,t-1)\Big(S(n,k,t) + d(n,t)\Big). \end{aligned} \tag{5}$$

Finally, because it is practically impossible to short alternative assets, we impose a shortsale constraint $\theta(2,k,t) \geq 0$.

The left-hand side of budget equation (5) is the amount of wealth allocated to consumption and the purchase of assets at date t, including transaction costs for trading the alternative asset, as specified in equation (1). The right-hand side captures the unit payout of the short-term bond as well as the prices and dividends of the two long-term risky assets at date t, scaled by the number of shares of each risky asset purchased at date $t-1$. This sum can be interpreted as the wealth with which the investor arrives at date t.

After some simplifications and rearranging, one gets the following kernel conditions:[15]

$$S(0,k,t) = E_t^k\left[\frac{M(k,t+1)}{M(k,t)}\right], \tag{6}$$

$$S(1,k,t) = E_t^k\left[\frac{M(k,t+1)}{M(k,t)}\Big(S(1,k,t+1)+d(1,t+1)\Big)\right], \tag{7}$$

$$\begin{aligned} S(2,k,t) &= \frac{1}{1-\lambda_{SSC}(k,t)}E_t^k\Bigg[\frac{M(k,t+1)}{M(k,t)}\times \\ &\quad \left(S(2,k,t+1)+d(2,t+1)-\frac{\partial\tau\big(\theta(2,k,t+1),\theta(2,k,t),S(2,k,t+1)\big)}{\partial\theta(2,k,t)}\right)\Bigg] \\ &\quad - \frac{1}{1-\lambda_{SSC}(k,t)}\times\frac{\partial\tau\Big(\theta(2,k,t),\theta(2,k,t-1),S(2,k,t)\Big)}{\partial\theta(2,k,t)}, \end{aligned} \tag{8}$$

where λ_{SSC} is associated with the Lagrange multiplier on the shortsale constraint and $\frac{M(k,t+1)}{M(k,t)}$ denotes investor k's pricing kernel:

$$\frac{M(k,t+1)}{M(k,t)} = \beta\left(\frac{V(k,t+1)^{1-\gamma}}{E_t^k\left[V(k,t+1)^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\left(\frac{c(k,t+1)}{c(k,t)}\right)^{-\frac{1}{\psi}}.$$

Note, the kernel condition for the alternative asset (8), equating the benefits from holding the asset to the benefit from selling the asset, deviates in two dimensions from the traditional kernel condition for fully liquid assets, as given in equation (7). First, the benefits derived from holding as well as selling the asset are both net of transaction costs, i.e., they are adjusted for costs associated with an immediate sale, through $\partial\tau\big(\theta(2,k,t),\theta(2,k,t-1),S(2,k,t)\big)/\partial\theta(2,k,t)$, as well as for potential future costs in case of a sale in the next period, given by $\partial\tau\big(\theta(2,k,t+1),\theta(2,k,t),S(2,k,t+1)\big)/\partial\theta(2,k,t)$. Second, the pricing kernel contains the shadow cost stemming from the shortsale constraint, reflected by $\frac{1}{1-\lambda_{SSC}(k,t)}$.

[15] Appendix A contains the detailed derivations of the first-order conditions.

4.2 Market-Clearing Conditions

In the economy we are considering, markets exist for the one-period risk-free bond, the two risky securities, and the consumption good. The market-clearing conditions for the financial assets imply that aggregate supply must equal aggregate demand:

$$\theta^{ss}(n) = \sum_{k=1}^{2} \theta(n,k,t), \quad \forall\ n = \{0,1,2\}, \tag{9}$$

where $\theta^{ss}(n)$ denotes the aggregate supply of asset n. For the two risky assets, aggregate supply $\theta^{ss}(n), n \in \{1,2\}$ is equal to 1, and for the risk-free bond, we have that $\theta^{ss}(0) = 0$.

If the financial markets clear, then Walras' law ensures that aggregate dividends must equal aggregate consumption:

$$\sum_{n=1}^{2} d(n,t) = \sum_{k=1}^{2} c(k,t).$$

4.3 Definition of Equilibrium

Equilibrium in this economy is defined as a set of consumption policies, $c(k,t)$, and asset-allocation policies, $\theta(n,k,t)$, along with the resulting price processes for the financial assets, $S(n,k,t)$, such that the consumption policy of each investor maximizes her lifetime utility, that this consumption policy is financed by the optimal asset-allocation policy, and markets for financial assets and the consumption good clear.

The budget constraint (5), the kernel conditions (6), (7), and (8), together with the complementary slackness conditions associated with the shortsale constraint, and the market-clearing conditions in (9) form the system of equations that characterize the equilibrium.

4.4 Solving for Equilibrium When Financial Markets are Incomplete

If financial markets are *complete*, one can separate the task of identifying the equilibrium into two distinct steps by exploiting the condition that investors can achieve perfect risk sharing. This condition can be used to first identify the optimal allocation of aggregate consumption across investors ("central planner's problem"). Then, in the second step, using these consumption choices one can determine asset prices and also the portfolio policy of each investor that supports this allocation.

In contrast, when financial markets are *incomplete*, one cannot separate the task of identifying the equilibrium into two steps because the consumption allocation one chooses must lie in the span of traded assets. Instead, one must solve for the consumption and portfolio policies *simultaneously.* In principle, one can identify the equilibrium by solving simultaneously the set of nonlinear first-order conditions for the investors along with the market-clearing conditions for all the states across all dates, as proposed in Cuoco and He (2001). The problem in implementing this approach is that the number of equations grows exponentially with the number of periods, so that a recursive approach would be preferable.

However, there are several problems in solving this system of equations recursively in a general-equilibrium setting. The first problem is that the current consumption and portfolio choices depend on the prices of assets, which, as shown in Equations (6), (7) and (8), depend on *future* consumption. But, in a general-equilibrium setting, when the investor attempts to solve for the optimal consumption and portfolio policies at date t, asset prices need to adjust in order for markets to clear. However, if one were solving the system of equations backward, these prices could *not* adjust because they depend on future consumption, which has already been determined in the previous step. Thus, to solve these equations, one would need to iterate backward *and* forward until the equations for all the nodes on the tree are satisfied. Dumas and Lyasoff (2012) address this problem by proposing a "time-shift" whereby at date t one solves for the optimal portfolio for date t but the optimal consumption for date $t+1$, instead of the optimal consumption for date t. Using this insight allows one to write the system of equations so that it is recursive.

Additional difficulties arise because of transaction costs, because whether a particular security is traded or not at a given node is now determined endogenously. Specifically, if investors choose to trade each of the assets, then they will agree on the prices of these assets. However, if investors find it optimal not to trade some of the assets, then investors will disagree on the prices of the assets that are not traded at that node. Consequently, the system of equations characterizing the solution depends on whether or not investors choose to trade all assets or only some of the assets.[16]

To address these problems, we observe that the past portfolio holdings enter the system of equations only through condition (8), as a first partial derivative of the transaction cost function $\tau(\cdot)$ with respect to the current portfolio investment. Under the assumption that the transaction costs are a constant proportion, κ, of the value of the asset being traded, there are only three possibilities for the form of this derivative. It is equal to zero when an investor decides not to trade; it is equal to $\kappa \times S(2,k,t)$ when the investor decides to increase the position in the asset; or, it is equal to $-\kappa \times S(2,k,t)$ when the investor sells the asset. Consequently, all the $\theta(2,k,t-1)$ values for which the investor decides to buy (sell) an asset at time t result in the same solution $\theta(2,k,t)$ for a given value of current consumption $c(k,t)$. In other words, instead of solving the problem over the undetermined wide grid of portfolio holdings at $t-1$, we can solve it first for the two trading decisions—sell or buy—at time t. The solution to this provides us with the bounds of the no-trade region—the region for which the portfolio investment from $t-1$ to t does not change. Within these bounds of the no-trade region that we have determined we use the past portfolio holdings as an endogenous state variable. We then solve the system of equations explicitly by restricting current portfolio holdings to equal past portfolio holdings and, in turn, eliminate the condition that investors agree on asset prices.

Using the "time-shift" and the insights described above, one can solve for the equilibrium recursively. Finally, after solving the dynamic program recursively up to the initial date, we undertake a simple single "forward step" for each simulated path of the underlying dividend processes to determine the equilibrium quantities. Additional details of this recursive solution method are given in Appendix B.

[16]For a discussion of the challenges in solving dynamic optimization problems with "occasionally binding constraints," see Christiano and Fisher (2000).

5 Analysis of the Model

In this section, we wish to evaluate the impact of illiquidity and opaqueness on asset allocation and asset prices. To do this, we undertake a quantitative analysis of the model described in the previous section.

5.1 Modeling Choices and Parameter Values

For our numerical illustration, we focus on the U.S. market. Thus, we interpret the risk-free asset as a U.S. Treasury bill and the risky, liquid asset as a broad U.S. equity index, such as the S&P 500. Given the tight link between dividend and return volatility in an exchange economy, we set the dividend volatility for the equity index equal to 20%, comparable to the 21.6% volatility reported in Longstaff (2009). We use a 1.2% expected growth rate for the index dividends, based on the value reported in Beeler and Campbell (2012, their Table 2). All returns are measured and compounded on a per annum basis.

In general, the alternative asset could represent any asset class that faces non-trivial costs for trading and has only a short time-series of historical price data, e.g., private equity, hedge funds, or real-estate funds. However, to give a better feel for the quantitative results, we first match the parameters to the private-equity asset class, varying selected parameters afterwards to accommodate other asset classes. Specifically, we interpret investments in the illiquid asset as investments into several private equity funds with different vintage years, e.g., for diversification purposes.[17] We mainly study a horizon of $T = 15$ years, longer than the typical ten years for single partnership contracts (Phalippou and Gottschalg (2009) and Harris, Jenkinson, and Kaplan (2014)). Later we report results for also 10 and 20 years.

When modeling the payout process for private equity, one has to answer two important questions: (1) Are its payouts similar to dividends? and (2) What are reasonable parameter values for the mean and volatility of the payouts? First, based on our interpretation of the illiquid investment being into several private-equity funds with different vintage years, one can easily imagine a 'dividend-like' payout structure. That is, while newly initiated

[17]For example, as of June 30, 2014, Calpers reported 295 active private equity investments with vintage years ranging from 1991 to 2014 and a median vintage year of 2006.

funds might still raise capital, the group of middle-aged and older funds would already realize investments and distribute the proceeds, so that the aggregate cash flow resembles a regular flow of dividends.[18] Second, a broad range of empirical evidence suggests that private-equity investments are more similar to traded securities than one might expect. For example, Moskowitz and Vissing-Jorgensen (2002) find that a portfolio of all private equity has a mean and standard deviation of returns close to that of the value-weighted index of traded stocks. Similarly, Kaplan and Schoar (2005), Cochrane (2005), and Phalippou and Gottschalg (2009) document comparable average returns for public and private equity, and the evidence in Metrick and Yasuda (2010) supports a volatility comparable to that of public equity.[19] In light of this evidence, for our base case we chose the same mean and standard deviation for private equity as for the equity index. This also allows us to isolate cleanly the effects of illiquidity and opaqueness from potential differences in the underlying payout process.

We set the correlation between dividends of public and private equity to be 50%, relying on Longstaff (2009), which is consistent also with the 40%–60% correlation between the assets' *returns*, documented in Ang, Papanikolaou, and Westerfield (2014) and Welch (2014).[20] Finally, we choose the share of initial income of the assets to match the ratio of the market capitalization of the assets. Specifically, we set the share of income provided by private equity to be 20% and that from the equity index to be 80%, so that the capitalized value of the alternative asset is consistent with the \$3.5 trillion value of assets under management for private equity relative to \$18 trillion U.S. stock-market capitalization.[21]

Transactions in alternative assets entail different types of costs. For example, transactions in private equity include participation in the initial offering and purchases or sales in the secondary market, so there would be costs for identifying the appropriate private eq-

[18] Calpers describes the aggregation as follows: "As the portfolio matures and distributions grow, the [Private Equity] Program is becoming self-sustaining."

[19] Metrick and Yasuda (2010) find a 60% volatility and 20% correlation for individual buyout firms and report that funds on average invest into 15 firms, implying an individual fund volatility of about 25%. Given that the investment into several funds yields additional diversification benefits, a volatility of around 15–20% for private equity as an asset class seems reasonable.

[20] The returns have a higher correlation than the underlying cash flows, due to the "pricing-kernel" channel.

[21] See Preqin Special Report (2014) for private-equity data; for stock-market capitalization, data are provided by the World bank at `http://data.worldbank.org/indicator/CM.MKT.LCAP.CD`

uity funds, closing the transaction, hiring specialized advisers or, in the secondary market, searching for a counterparty. In our model, we do not differentiate between these different types of costs; instead, we consider proportional transaction costs up to 10%, as reported by Preqin Special Report (2013a), as well as 100% to capture the case of full illiquidity.

In the second stage of our quantitative analysis, we vary selected parameters. To study the impact of the investment horizon, we consider setups with 10 and 20 years. Next, we report results when the alternative asset's dividend volatility is 12.5% or 27.5%, representing the lower volatility of hedge funds and real estate or the higher volatility of venture capital and small stocks.[22] We also consider correlations of 0.25 and 0.75 with the former representing, for example, the low correlation between buyout funds and equity indices (estimated to be 0.27 by Ang, Papanikolaou, and Westerfield (2014)) and the latter representing small stocks that have a correlation of about 0.80 with the market. Similarly, we report results for cases where the alternative asset's share of income is 10% and 30%. The former captures hedge funds and small stocks, and the latter represents commercial real estate.[23]

Our two groups of investors can be viewed as institutional investors, such as endowment or pension funds, that both have access to alternative asset classes, but differ in their experience, e.g., because of fund size or manager's ability. Specifically, we assume that the inexperienced investor has only little prior exposure to alternative assets, modeled as a $\text{Beta}(1, 1)$ prior distribution. Importantly, although having a lower precision, the prior mean of the inexperienced investor on the initial date coincides with the true probability. We also report results for different levels of prior precision.

Finally, we choose the preference parameters of the investors to match the risk-free rate, equity risk premium and equity-market volatility. Our choices for relative risk aversion, $\gamma = 1.75$, rate of time-preference, $\beta = 0.95$, and elasticity of intertemporal substitution, $\psi = 1.5$, imply a risk-free rate of 1.1%, an equity market risk premium of 6.41%, and a

[22]Buraschi, Kosowski, and Trojani (2014) report volatilities for hedge fund indices to range from 3.8% to 16.0% and Ghysels, Plazzi, Torous, and Valkanov (2012) document commercial real estate volatilities to range from 5.2% to 17.9%. In contrast, Ang, Papanikolaou, and Westerfield (2014) find a 27.8% volatility for venture capital and data from Kenneth French's web site implies a volatility of about 30% for small stocks.

[23]Preqin Special Report (2013b) reports assets of \$1.7 trillion for U.S. hedge funds while the S&P 600 market capitalization is below one trillion, implying a share of less than 10%. In contrast, Florance, Miller, Peng, and Spivey (2010) report a value of \$9.5 trillion for U.S. commercial real estate, which is about half of U.S. stock market capitalization.

market-return volatility of 21.2%, consistent with their empirical counterparts of 0.56%, 6.94%, and 20.17% estimated by Beeler and Campbell (2012). For comparative statics, we also consider variations in the preference parameters. The base-cases values for the parameters and the variations in them are summarized in Table 1.

5.2 Understanding the Effects of Inexperience

The effects of experience on the inexperienced investor's asset allocation exhibit themselves through two forces. First, with each new dividend realization the inexperienced investor updates her *posterior mean* for the probability of the next move in the alternative asset's dividend tree being an up-move. Second, with the passage of time, the inexperienced investor updates also her *posterior variance*. One can interpret the posterior mean as reflecting the investor's optimism and the posterior variance reflecting her confidence.

Figure 1 illustrates the dynamics of the economy for two specific simulations. Panels A and B show the evolution of the alternative asset's dividend (left-hand-side vertical scale) for each simulation, as well as the evolution of the posterior mean and posterior variance of the inexperienced investor's beliefs (right-hand-side vertical scale) about the probability of an up-move in the alternative asset's dividend tree.[24] The shaded gray areas highlight periods of negative dividend realizations; that is, periods in which the inexperienced investor adjusts her posterior mean downwards.[25] Note that the dividend dynamics for the equity index coincide for both paths, as shown in Panel C.

The revision in the posterior mean for the alternative asset can be upwards or downwards, depending on whether the most recent dividend shock was positive or negative. For example, in the first simulation (Panel A of Figure 1), the investor experiences a positive shock at date 2, revising her expected growth rate upwards. However, the negative shock at date 3 brings her back to her prior belief of 0.5. Finally, the inexperienced investor ends up being optimistic because of the "boom" over the last 10 periods. In contrast, in the second

[24]In the figure, we actually plot 2.5 times the posterior standard deviation in order to match the scale for the posterior mean so that we can plot both quantities in the same figure.

[25]For ease of exposition, we reproduce the shaded gray areas in all the figures that follow, facilitating their interpretation.

simulated path (Panel B), the alternative asset's dividends are first increasing, implying an increase in the inexperienced investor's optimism, before the dividends slowly decline over time, rendering the investor pessimistic relative to the earlier period.

In contrast to the posterior mean, the posterior variance is generally declining over time. But, the change is stochastic, depending not only on the passage of time, but also on whether the shocks experienced were positive or negative. One can see this by comparing across the two simulations the change in posterior variance between $t = 2$ and $t = 3$. In the second simulation, the investor experiences a second successive positive shock, which leads to a much stronger reduction in variance than for the first simulation, where the positive shock at $t = 2$ is followed by a negative shock at $t = 3$. Note that in certain cases the posterior variance can *increase* over time, as one can see in the second simulation, in which it increases over the period $t = 3$ to $t = 4$. This occurs because at dates 2 and 3 the alternative asset's dividend shocks were positive but the shock at $t = 4$ is negative, which makes the investor "less confident" about the beliefs she had formed on the basis of the earlier positive shocks.

Below, we study the quantities of interest for two specific paths in the simulation and also the average across 50,000 paths. Examining two particular paths allows us to illustrate the impact of changes in the posterior mean and precision. Studying the average allows us to understand whether the results for the two paths hold true in general, and also allows us to focus on the impact of changes in the posterior variance, because the changes in the posterior mean cancel out to a large extent when we average across paths.[26]

Note that for the experiential model of learning that we study, the posterior variance decreases over time on average. For our choice of parameter values, the posterior standard deviation decreases from 28.28% at the initial date to 12.25% at $t = 15$ (so, in the figure, the decrease shown is 70.71% ($= 2.5 \times 28.28\%$) to 30.61% ($2.5 \times 12.25\%$).[27] However, there is still considerable variation in the posterior mean across sample paths, as illustrated in

[26] Note that although the changes in the posterior mean cancel out when we average across paths, nonlinear responses to the these changes will not cancel out perfectly.

[27] Note that in a model with more than two states, the decline in the posterior variance with each observation would be even slower than it is in the model specified above.

Figure 2. For example, in more than 40% of the sample paths, the posterior mean differs from the true mean by at least 0.10. That is, even though the investor's confidence about her posterior mean is very high after fifteen periods, she may be confident about a mean that is substantially different from the true mean.

5.3 Asset Allocation

We start by investigating the implications of the model for the asset allocation choices of the two investors. The optimal holdings (number of shares) of the inexperienced investor for the three assets at the initial date are reported in the three panels of Table 2 and their evolution over time is shown in Figures 3–5.[28]

If both investors were equally experienced, then it would be optimal for each investor to hold 0.5 shares of each of the two risky assets (the market portfolio) and zero bonds. However, when one of the investors is relatively inexperienced, then neither investor holds the market portfolio; instead, the inexperienced investor *tilts* her portfolio away from the alternative asset, reducing her holding of this asset from 0.5 to 0.123—see Panel A of Table 2—and increasing her holdings of the bond and the equity index. The intuition for the portfolio tilt away from the alternative asset is that the presence of estimation risk induces a negative hedging demand for the alternative asset because of changes in perceived investment opportunities as the investor updates her beliefs. Because of the market-clearing condition, in response to the inexperienced investor's tilt away from the alternative asset, the experienced investor tilts her portfolio *toward* the alternative asset and holds 0.877 shares. These results for inexperienced investors are consistent with the results in partial-equilibrium models obtained by Brennan (1998), Barberis (2000), and Brandt, Goyal, Santa-Clara, and Stroud (2005).[29] In addition, the results for the experienced investor are consistent with the empir-

[28]Because of market-clearing, the optimal holding of the more experienced investor for each of the risky assets is one minus the holding for the inexperienced investor and for the bond it is the negative of the bond holding of the inexperienced investor.

[29]Williams (1977), Detemple (1986), Dothan and Feldman (1986), and Gennotte (1986) study the portfolio problem of an investor who does not know the true state of the economy but who knows the stochastic process governing the state variables. The key insight used in these papers to solve the investor's problem is that in continuous-time the investor's problem can be separated into a filtering problem, in which the investor estimates the current values of the state variables, and a standard investment problem in which the estimated values of the state variables replace the state variables themselves. Brennan (1998) analyzes in continuous time the simpler problem of an investor who is uncertain about the mean return on the risky asset. The investor learns about the mean as she observes returns over time, which one can view as a special case of the

ically observed asset holdings of endowment funds, e.g., Lerner, Schoar, and Wang (2008) and Goetzman and Oster (2012) find that large, experienced funds strongly overweight alternative assets relative to the market and their peers.

The *dynamics* of the asset-allocation decision of the inexperienced investor's holdings are intriguing. Panel A of Figure 3 shows that on average the inexperienced investor *increases* her holding of the alternative asset over time, with large increases in the first few periods. This is because the largest decreases in the posterior variance come in the first few periods (see Figure 1). We also find the surprising result that as the transaction costs for the alternative asset increase, the inexperienced investor may hold a *larger* share of this asset. For example, when the transaction cost increases from 2.5% to 10%, the initial holding of the alternative asset increases from 0.041 shares to 0.144 shares, and when the transaction cost is 100%, the holding of the alternative asset increases to 0.273 shares (see Panel A of Table 2). The intuition for this is that ideally the inexperienced investor would like to build up her position in the alternative asset slowly over time as she anticipates gaining experience (posterior variance declining). However, this strategy entails substantial transaction costs. Therefore, if future trading costs dominate, the inexperienced investor chooses to reduce the portfolio tilt away from the alternative asset, and thus, holds a larger share of this asset.[30] This intertemporal tradeoff is evident from Panel A of Figure 3, which shows that as the transaction costs increase, the initial holding increases while the investor cuts back on future volume trading, which is reflected in the flattening out of the plot.

Panels B and C of Figure 3 illustrate the effects of changes in the posterior mean as the inexperienced investor updates her beliefs. For example, in the case of the first simulated path and low transaction costs (black solid line in Panel B), the inexperienced investor substantially increases her holdings of the alternative asset due to the sizable change in her posterior mean triggered by positive dividend news at $t = 2$. For the next few periods, she

analyses of Detemple (1986), Dothan and Feldman (1986), or Gennotte (1986), in which the dynamics of the unobservable state variable, the mean return, are eliminated. Just like in the partial-equilibrium models of Barberis (2000) and Brandt, Goyal, Santa-Clara, and Stroud (2005), in our general-equilibrium model the investor learns about both the mean and the variance; in addition, investors also face transaction costs for trading the alternative asset.

[30] It is important to note that this result is not driven by the absence of transaction costs at the initial date. If we had a transaction cost also on the initial date, the inexperienced investor would deviate even less from her endowed holding of 0.5 shares, and so would be holding even more shares of the alternative asset.

does not change her holdings at all because the revisions in the posterior mean are too small to compensate for the transaction cost. Only during the boom phase (dates 6–10), when the posterior mean increases again, the investor strongly increases her holdings—in line with her optimistic beliefs. In contrast, for transaction costs of 10% the inexperienced investor never trades the alternative asset, because the changes in her beliefs are not substantial enough to overcome the cost of trading. In the case of full illiquidity, trading is precluded exogenously.

In the second simulated path, Panel C of Figure 3, the inexperienced investor increases her holdings at the start of the economy as a reaction to the two positive dividend shocks at dates 2 and 3. After this, the changes in dividends are small (alternating between negative and positive shocks), so the investor does not rebalance her position in order to save on transaction costs. However, in response to the series of negative shocks starting at $t = 8$, indicated by the gray background, the inexperienced investor reduces her holdings, but only for low levels of transaction costs—even though her posterior mean is declining dramatically over this period. This leads to the surprising result that the inexperienced investor ends up holding a majority of the alternative asset, despite being pessimistic about it. Even more surprising are the differences across the two simulations. For example, towards the end of the economy and for transaction costs of 10%, in Panel C the inexperienced investor holds more than twice as much of the alternative asset—despite being pessimistic—than in Panel B, where she is highly optimistic. This contrast nicely showcases the crucial interplay of experience and illiquidity and demonstrates the path dependence of asset holdings on the evolution of the economy. This path dependence implies that when evaluating a portfolio, it is not possible to say whether it is optimal or not unless one also knows the history of the economic conditions under which this portfolio was formed.

We now turn our attention to the equity index. Panels B and C of Figure 4 illustrate that the inexperienced investor uses the equity index, which is correlated with the alternative asset, as a substitute when she updates her beliefs but trading the alternative asset is too costly.[31] For example, for the first simulated path (Panel B) and for transaction costs of 10%

[31] Swensen (2009), the chief investment officer of Yale University, highlights this challenging interplay between illiquid assets and portfolio rebalancing, suggesting that "choosing the least bad alternative, investors

or full illiquidity—cases in which the investor abstains from trading the alternative asset—the holdings in the equity market *track perfectly* the dividend dynamics of the alternative asset. That is, after a positive shock to the alternative asset's dividends, the inexperienced investor increases her investment in the liquid equity index, and vice versa. This *spillover* or *contagion* effect materializes in an increase in the volatility of changes in the holdings of the equity index by the inexperienced investor—for the case of 10% transaction costs we find a relative increase in the volatility of the changes in the holdings of the equity index of more than 14% and also excess comovement in returns.

Similarly, Panels B and C of Figure 5 illustrate that the bond holdings of the inexperienced investor are closely linked to the dividend dynamics of the alternative asset. That is, after a positive dividend realization, the inexperienced investor is willing to borrow to trade on this positive news—either by buying the alternative asset directly or using the equity index as a substitute. The implications of an increase in precision can be clearly seen in Panel A, which gives the average bond holdings over time. As the inexperienced investor gains experience over time, she faces less estimation risk and, accordingly, reduces her bond holdings as her precautionary-savings motive diminishes.[32] The result that the change in holdings of the alternative asset are offset by changes in bond holdings is consistent with the empirical evidence in Brown and Tiu (2013).

We conclude this section by looking at the comparative statics for the asset-allocation decision. Table 2 shows that the asset allocation implications for the variations from the benchmark case are the same as for the base case. Strikingly, the portfolio tilt becomes stronger with the investor's investment horizon because of an increase in the size of the negative hedging demand; that is, the magnitude of the hedging demand is increasing in the investment horizon. For example, if the horizon increases from 15 to 20 years, the

should fund overallocations to private assets by reducing holdings of risky marketable securities" to control the portfolio risk.

[32]Note, the average bond holdings of the inexperienced investor turn negative after some time. The reason for this is that, because of the negative hedging demand for the alternative asset induced by estimation risk, it is more likely that the inexperienced investor hits her shortsale constraint than that she owns all shares of the alternative asset. Thus, she is less constrained in buying shares of the alternative assets, financed by borrowing, than selling shares and investing the proceeds into the bond. This has a negative impact on the inexperienced investor's bond holdings which, when it dominates the precautionary savings motive, leads to a short position in the bond. As shown in the figure, transaction costs dampen this effect because they reduce the likelihood of the investor hitting her shortsale constraint.

inexperienced investor reduces her holding of the alternative asset from 0.123 shares to 0.022 shares in the absence of transaction costs. The magnitude of the portfolio tilt also depends on the risk-return benefits delivered by the alternative asset. For example, for a higher volatility of the alternative asset (e.g., small stocks or venture capital), and for a lower dividend correlation (e.g., hedge funds or real estate), the inexperienced investor reduces her tilt away from the market portfolio and, accordingly, holds more of the alternative assets as the diversification benefits dominate the negative hedging demand stemming from estimation risk. Similarly, when the relative size of the alternative asset in the economy increases, the inexperienced investor reduces the tilt away from the market. This is driven by the fact that a higher share of dividends increases the alternative asset's expected return, as shown by Cochrane, Longstaff, and Santa Clara (2008), so that the asset is more attractive to hold and the effect of estimation risk becomes relatively less important. We also study the dynamics of asset allocation for different levels of correlation, volatility, and precision in Figure 6. While most dynamic effects are similar to the base case, a higher initial precision implies smaller revision to her position over time and a lower correlation results in considerably less spillover effect as the equity index is not as good a substitute.

We also study the comparative statics with respect to the preference parameters of the investors; because these results are what one would expect, they are not reported in the tables. For instance, as the investors' risk aversion increases or her elasticity of intertemporal substitution decreases, the inexperienced investor is less willing to tolerate the estimation risk and associated variability in consumption over time, so that she reduces her holdings of the alternative asset. In some cases, the investor even completely abandons diversification, investing only in the bond and equity index.

5.4 Turnover

Table 3 shows that the effect of transaction costs on turnover is substantial.[33] From Panel A of Table 3, we see that when we introduce a 5% transaction cost for trading the alternative

[33] We measure turnover as the (absolute) dollar volume traded, first averaged across all states on a given date, and then across all dates.

asset its turnover drops to about half, and for a 10% transaction cost, the turnover drops by two-thirds. The majority of the trading in the alternative asset is driven by the stochastic nature of the inexperienced investor's updating of her prior mean, as can be seen from Panels B and C of Figure 3.[34]

From Panel B of Table 3, we see that the decrease in turnover of the alternative asset is offset by an increase in turnover of the liquid equity index of up to 45% for our base case in the presence of transaction costs of 10%. That is, trading the equity index becomes a substitute for trading in the alternative asset, as illustrated in Panels B and C of Figure 4.[35] In contrast, we observe from Panel C of Table 3 that the decrease in turnover of the alternative asset is accompanied by a decline in the average turnover of the risk-free bond.

Finally, examining the results of the variation in the benchmark parameters, we can see that the results are again qualitatively the same. Noticeably, the reduction in turnover is stronger for the cases of a low dividend correlation, more precise priors of the inexperienced investor, and a lower risk aversion (not reported). In all these cases, the investor has already invested considerably more in the initial period, so that her need for future trading in response to gains in experience are less crucial, implying that the trading costs dominate.

5.5 Asset Prices

When we move from the case where both investors have infinitely precise priors to the case where one investor is inexperienced, there are two opposing forces that impact the price of the alternative asset even in the absence of transaction costs. First, as explained by Dumas, Kurshev, and Uppal (2009), sentiment risk arising from differences in beliefs reduces the price of the asset. However, as explained by Miller (1977), shortsale constraints limit the expression of negative sentiment, and so increase the price. In all our setups, the effect of sentiment is dominated by the effect of the shortsale constraint for the alternative asset, and so its price increases. In contrast, the price of the equity index decreases, because beliefs

[34]Our computations show that about two-thirds of the trading volume in the model are driven by changes in the posterior mean, with the remaining one-third being due to changes in the posterior variance.

[35]Note also that in the presence of transaction costs, the turnover in the equity index is three to four times the turnover in the alternative asset; the turnover in the equity index would be even higher if we allowed for some learning about the dividends of this asset.

about its dividends are homogeneous, investors do not wish to short it, and so the positive price effect of the shortsale constraint is absent.

When we introduce costs for trading the alternative asset, the impact on equilibrium asset prices is significant. For example, Panel A of Table 4 shows that a transaction cost of 2.5% leads to a price decline of 2.83% and a transaction cost of 10% leads to a price decline of 5.17% at the initial date. At later dates, the price discount can be as large as 12.8% for a transaction cost of 10%, in line with the empirical findings of Hege and Nuti (2011) and Franzoni, Nowak, and Phalippou (2012) for the private-equity market. Contrary to intuition, the discount is *larger* in a setting where investors incur modest trading costs, compared to a setting where there is a blackout and all trade is precluded exogenously. This is because of the nonmonotone effect of transaction costs on the investors' asset-allocation decisions. Specifically, in the case of full illiquidity, the inexperienced investor buys *more* of the asset initially because she cannot trade in the future, which reduces the discount.

In contrast to the price of the alternative asset, we see from Panel B of Figure 4 that the price of the equity index increases in the cost for trading the alternative asset. The intuition for this is that if there is a cost for trading the alternative asset, then investors demand the public-equity index as a substitute, which boosts the price of the equity index.

Qualitatively, the results are the same for the variations from the base case. Noticeably, the price discount is larger for a higher dividend volatility of the alternative asset and a lower dividend correlation, because in these cases the diversification loss from tilting the portfolio is much greater, which increases the price discount. Similarly, when the supply of the alternative asset is low, the change in demand due to illiquidity has a greater effect on the price of this asset: The price discount is more than 7% for a transaction cost of 10%, which would further increase the price discount for asset classes such as hedge funds or small stocks. Moreover, the longer the horizon of the economy, the stronger is the effect because the discount applies to more periods. A more precise prior implies smaller tilts in the portfolio, and hence, a smaller price discount. Finally, the effects of preference parameters on the price discount of the alternative asset are small (not reported in table).

5.6 The Risk-Free Rate and Moments of Asset Returns

In a market where both investors have the same level of experience, there is no borrowing or lending between the two classes of investors. When one of the investors is inexperienced, she reduces her holding of the alternative asset, and uses the proceeds from the sale of the alternative asset to buy a little more of the market, with the rest of the funds invested in the bond because of precautionary savings. This increases the price of the bond and so the interest rate drops (see Panel C of Table 5). Costs for trading the alternative asset lead to an increase in bond holdings on average, driven by the desire for precautionary savings (see Panel A of Figure 5), which leads to a further decrease in the interest rate.

Next, we study the expected returns on the two risky assets. Because the equity index is traded at each date, its return can be computed in the standard fashion. However, in the presence of transaction costs, the alternative asset is not traded at each date, and its return cannot be computed in the standard way. The problem of measuring the returns on alternative assets such as private equity has been highlighted in several empirical papers; see, for example, Franzoni, Nowak, and Phalippou (2012), Ang and Sorensen (2012), and Ang, Papanikolaou, and Westerfield (2014). Franzoni, Nowak, and Phalippou advocate computing a "modified internal rate of return," which measures the average return for an investor who deposits dividends into an account that earns the interest rate. We, too, compute the return on the alternative asset using the modified internal rate of return.[36]

Panel A of Table 5 reports an increase in expected return for the alternative asset as we increase transaction costs. The observed effects mirror those for the price decrease of the alternative asset. The magnitudes are again quite large, e.g., for the base case we find an increase of more than 0.50% compared to the liquid counterpart, which is consistent with the empirical results in Metrick and Yasuda (2010), who estimate a 1% annual premium for venture capital. An increase in the transaction cost for the alternative asset increases demand for the equity index, thereby increasing its price. This price increase of the equity

[36] For robustness, we also use a second measure of return, which is computed the same way as the first measure but assumes reinvestment at a zero interest rate, which removes any equilibrium effects caused by changes in the endogenous interest rate. Expected returns using these two measure are very similar, and so we report only the results based on the first measure.

index is reflected in a decrease in its expected return; however, as we see from Panel B of Table 5, the magnitude of this effect is very small.[37] The results for the variations in the base-case parameters mirror those for asset prices, so we do not repeat this discussion.

Next, we study the implications for the equity risk premium. The equity risk premium increases with opaqueness because the presence of inexperienced investors introduces an additional risk in the market, for which investors need to be compensated. Transaction costs reduce both the expected return on the equity index and the interest rate. However, the effect on the interest rate is always greater, so the net effect on the equity risk premium is positive although the magnitude is extremely small.

Table 6 presents the impact of opaqueness and illiquidity on return volatilities.[38] Intuitively, the trading of inexperienced investors gives rise to an additional source of risk—sentiment risk—which is reflected in higher volatility for the returns of both risky assets and also higher correlation between their returns, relative to the perfect-market case. When the alternative asset can be traded only at a cost, this introduces an additional source of risk, driving up volatilities further. However, with transaction costs, there is considerably less information-based trading by the inexperienced investor, which reduces volatility. While for transaction costs of less than 100% the first effect always dominates, leading to higher volatility, the second effect typically dominates for the full-blackout cases, thus reducing the alternative asset's return volatility. Finally, transaction costs for trading the alternative asset have only a negligible effect on the volatility of the equity index.

5.7 Consumption Share and Consumption Growth Volatility

Above, our focus has been on the financial policies of the two investors. We now study the consumption generated by the asset-allocation decisions of the two investors. In particular, we look at the consumption share and the volatility of consumption growth. In the benchmark perfect-market case, the two investors are identical and have exactly the

[37]Note, because of the different procedures for computing the returns of the equity index and the alternative asset, the two are not directly comparable.

[38]For the alternative asset, volatility is measured as the standard deviation across paths of the measure based on the modified internal rate of return.

same consumption share and consumption-growth volatility. However, when one of the investors is inexperienced, she reduces her holdings of the alternative asset and increases her holdings of the bond. Consequently, the volatility of her consumption growth decreases (see Panel A of Table 7). The experienced investor, on the other hand, holds a levered portfolio with a much larger share of wealth invested in the alternative asset. Therefore, her consumption-growth volatility is higher than that in the benchmark case.

Of course, this risk-sharing arrangement, where the experienced investor offers "insurance" to the inexperienced investor, needs to be paid for.[39] This payment is reflected in the transfer of wealth from the inexperienced investor to the experienced one, which over time leads the experienced investor to consume a greater share of the aggregate dividend. This effect is illustrated in Panel A of Figure 7. Noticeably, as the inexperienced investor gains experience over time and there is a reduction in estimation risk (posterior variance), she is willing to accept more consumption risk (Panels B and C of Figure 7 show the *dynamics* of the risk-sharing arrangement). Accordingly, she reduces the premium for the "insurance" and, thus, the slope of the change in her consumption share declines (cf. Panel A).[40]

Focusing now on the effect of transaction costs, Panel A of Figure 7 depicts that the average consumption share of the inexperienced investor monotonically increases with transaction costs on the alternative asset. This is because with transaction costs the inexperienced investor reduces her portfolio tilt away from the alternative asset. As Table 7 shows, transaction costs have a nonmonotone effect on consumption volatility. This is because of two opposing forces. First, because of transaction costs, the inexperienced investor reacts less aggressively to new information, reducing her consumption volatility. Second, transaction costs introduce a friction into the optimal risk-sharing arrangement, and thus, lead to an increase in consumption-growth volatility. For high transaction costs, especially the case of a full blackout, the first effect dominates, thereby actually reducing consumption volatility for the inexperienced investor. Table 7 also shows how variations in the base-case parameters

[39] Alternatively, one can attribute this wealth transfer to the inexperienced investor's beliefs, which differ from the objective probability measure.

[40] The risk-sharing arrangement between the experienced and inexperienced investors is captured well by the Brian Tracy's quote that: "When a man with money meets up with a man with experience, the man with the experience is going to end up with the money and the man with money is going to end up with the experience." We are grateful to Francis Longstaff for bringing this quote to our attention.

affect the magnitude of these two effects. We also study the dynamics of the consumption share and consumption-growth volatility for different levels of correlation, volatility, and precision. Figure 8 shows that a lower correlation and a higher precision imply that the inexperienced investor is willing to accept a higher initial consumption-growth volatility as she buys more of the alternative asset. This, in turn, implies a smaller reduction in her consumption share.

6 Conclusion

Standard models of asset pricing and asset allocation assume that investors know the distribution of asset returns and that assets can be traded without cost. We study the implications of relaxing these two assumptions simultaneously. Specifically, we consider a general-equilibrium model of an exchange economy featuring an alternative asset that is illiquid and opaque, representing private equity, hedge funds, or real assets. The model features two groups of investors who differ in their experience, with the more experienced investors having a more precise estimate of the expected growth rate of the alternative asset's dividends.

The interplay between differences in investors' experience and differences in the liquidity of assets has strong implications for asset allocation. When investors differ in experience only, the inexperienced investors invest less in the alternative asset because of its "estimation risk," tilting their portfolio toward liquid assets, while experienced investors hold a much greater share of the alternative asset. As inexperienced investors gain experience, they increase their holdings of the alternative asset. Surprisingly, an increase in the transaction cost for the alternative asset can lead an investor to hold a *larger* share of this asset at the initial date. The intuition for this is that although the investor would ideally like to start with a low holding of the alternative asset, increasing it over time as she gains experience, this strategy entails substantial trading costs. Thus, it can be optimal for the inexperienced investor to reduce her portfolio tilt away from the alternative asset at the initial date, and thus, hold a larger share of this asset in order to economize on future trading costs. Another striking result is that, because of the portfolio inertia induced by transaction costs, the inexperienced investor might actually hold a majority of the alternative asset even though she is currently pessimistic about its growth rate. We also find interesting results for the inexperienced investor's holdings of the liquid assets. If investors find it optimal not to trade the alternative asset because of its transaction costs, then investors trade the liquid equity index instead, leading to strong spillover effects.

Similarly, experience and illiquidity have sizable effects on asset prices. In the presence of transaction costs for the alternative asset, its price drops substantially: for a 5% transaction cost, the average price discount is about 4%, with a maximum of more than 12%. Contrary to intuition, the illiquidity discount is stronger for moderate transaction costs than for full illiquidity. In contrast, prices for the liquid assets increase with transaction costs for the alternative asset, though this increase is smaller in magnitude.

These results are consistent with various empirical observations about alternative assets. For example, several papers (Lerner, Schoar, and Wang (2008) and Goetzman and Oster (2012), among others) document that large endowment funds have increased their exposure to alternative assets over the last years after experiencing positive returns and today strongly overweight alternative assets—relative to the market portfolio and relative to their less-experienced peers, confirming our static and dynamic asset-allocation results. Similarly, our asset pricing implications with an illiquidity discount of 5%–10% are comparable to results reported by Hege and Nuti (2011) and Franzoni, Nowak, and Phalippou (2012) for private equity.

We find several novel results for asset allocation and asset pricing, originating from incorporating both illiquidity and experiential learning into our model, characteristics that are absent from traditional models. Specifically, our work demonstrates how inexperienced investors should allocate wealth to new assets that are illiquid and opaque, how they should revise their investments as they gain experience, and what the pricing implications are for both new and existing assets.

A First-Order Conditions

The Lagrangian associated with equations (4), (5), and the shortsale constraint is:

$$
\begin{aligned}
\mathcal{L}(k,t) = & \sup_{c(k,t),\theta(n,k,t)} \inf_{\lambda_{BC}(k,t),\lambda_{SSC}(k,t)} \left[(1-\beta)\, c(k,t)^{1-\frac{1}{\psi}} + \beta E_t^k \left[V(k,t+1)^{1-\gamma}\right]^{\frac{1}{\phi}}\right]^{\frac{\phi}{1-\gamma}} \\
& + \lambda_{BC}(k,t) \Bigg[\theta(0,k,t-1) + \sum_{n=1}^{2} \theta(n,k,t-1)\Big(S(n,k,t)+d(n,t)\Big) \\
& \qquad -c(k,t) - \sum_{n=0}^{2} \theta(n,k,t) S(n,k,t) - \tau\left(\theta(2,k,t),\theta(2,k,t-1),S(2,k,t)\right)\Bigg] \\
& + \Bigg(\lambda_{BC}(k,t)\lambda_{SSC}(k,t) S(2,k,t)\Bigg) \times \theta(2,k,t), \qquad \text{(A1)}
\end{aligned}
$$

where $\lambda_{BC}(k,t)$ and $\lambda_{BC}(k,t)\lambda_{SSC}(k,t)S(2,k,t)$ denote the Lagrange multipliers associated with the budget constraint and the shortsale constraint, respectively.

The first-order condition of (A1) with respect to consumption $c(k,t)$ is given by:

$$
\begin{aligned}
\frac{\partial \mathcal{L}(k,t)}{\partial c(k,t)} = & \frac{1}{1-\frac{1}{\psi}} \left\{(1-\beta)\, c(k,t)^{1-\frac{1}{\psi}} + \beta E_t \left[V(k,t+1)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}-1} \\
& \times (1-\beta)\left(1-\frac{1}{\psi}\right) c(k,t)^{1-\frac{1}{\psi}-1} - \lambda_{BC}(k,t) \\
= & (1-\beta)\, c(k,t)^{-\frac{1}{\psi}} \left[\left\{(1-\beta)\, c(k,t)^{1-\frac{1}{\psi}} + \beta E_t \left[V(k,t+1)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}\right]^{\frac{1}{\psi}} - \lambda_{BC}(k,t) \\
= & (1-\beta)\, c(k,t)^{-\frac{1}{\psi}} V(k,t)^{\frac{1}{\psi}} - \lambda_{BC}(k,t) \equiv 0. \qquad \text{(A2)}
\end{aligned}
$$

Second, the first-order condition of (A1) with respect to the Lagrange multiplier $\lambda_{BC}(k,t)$:

$$
\begin{aligned}
\frac{\partial \mathcal{L}(k,t)}{\partial \lambda_{BC}(k,t)} = & \Bigg[\theta(0,k,t-1) + \sum_{n=1}^{2} \theta(n,k,t-1)\Big(S(n,k,t)+d(n,t)\Big) - \sum_{n=0}^{2} \theta(n,k,t) S(n,k,t) \\
& -c(k,t) - \tau\left(\theta(2,k,t),\theta(2,k,t-1),S(2,k,t)\right)\Bigg] + \lambda_{SSC}(k,t) S(2,k,t)\theta(2,k,t) \equiv 0.
\end{aligned}
$$

Third, the derivative of (A1) with respect to the holdings of the first two assets ($n \in \{0, 1\}$):

$$
\begin{aligned}
\frac{\partial \mathcal{L}(k,t)}{\partial \theta(n,k,t)} &= \frac{1}{1-\frac{1}{\psi}} \left\{ (1-\beta)\, c(k,t)^{1-\frac{1}{\psi}} + \beta E_t^k \left[V(k,t+1)^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}-1} \times \\
&\quad \beta \frac{1-\frac{1}{\psi}}{1-\gamma} E_t^k \left[V(k,t+1)^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1} (1-\gamma)\, E_t^k \left[V(k,t+1)^{-\gamma} \frac{\partial V(k,t+1)}{\partial \theta(n,k,t)} \right] \\
&\quad -\lambda_{BC}(k,t) S(n,k,t) \\
&= \beta V(k,t)^{\frac{1}{\psi}} E_t^k \left[V(k,t+1)^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1} E_t^k \left[V(k,t+1)^{-\gamma} \frac{\partial V(k,t+1)}{\partial \theta(n,k,t)} \right] \\
&\quad -\lambda_{BC}(k,t) S(n,k,t) \equiv 0.
\end{aligned}
$$

Finally, taking the derivative of (A1) with respect to the holdings of the illiquid asset:

$$
\begin{aligned}
\frac{\partial \mathcal{L}(k,t)}{\partial \theta(2,k,t)} &= \frac{1}{1-\frac{1}{\psi}} \left\{ (1-\beta)\, c(k,t)^{1-\frac{1}{\psi}} + \beta E_t^k \left[V(k,t+1)^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}-1} \times \\
&\quad \beta \frac{1-\frac{1}{\psi}}{1-\gamma} E_t^k \left[V(k,t+1)^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1} (1-\gamma)\, E_t^k \left[V(k,t+1)^{-\gamma} \frac{\partial V(k,t+1)}{\partial \theta(2,k,t)} \right] \\
&\quad -\lambda_{BC}(k,t) \left(S(2,k,t) + \frac{\partial \tau\left(\theta(2,k,t), \theta(2,k,t-1), S(2,k,t)\right)}{\partial \theta(2,k,t)} \right) \\
&\quad + \lambda_{BC}(k,t) \lambda_{SSC}(k,t) S(2,k,t) \\
&= \beta V(k,t)^{\frac{1}{\psi}} E_t^k \left[V(k,t+1)^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1} E_t^k \left[V(k,t+1)^{-\gamma} \frac{\partial V(k,t+1)}{\partial \theta(2,k,t)} \right] \\
&\quad -\lambda_{BC}(k,t) \left(S(2,k,t) + \frac{\partial \tau\left(\theta(2,k,t), \theta(2,k,t-1), S(2,k,t)\right)}{\partial \theta(2,k,t)} \right) \\
&\quad + \lambda_{BC}(k,t) \lambda_{SSC}(k,t) S(2,k,t) \equiv 0.
\end{aligned}
$$

To summarize, the Karush-Kuhn-Tucker first-order conditions are:

$$
\lambda_{BC}(k,t) = (1-\beta) \cdot c(k,t)^{-\frac{1}{\psi}} \cdot V(k,t)^{\frac{1}{\psi}}, \tag{A3}
$$

$$
\begin{aligned}
0 = \theta(0,k,t-1) &+ \sum_{n=1}^{2} \theta(n,k,t-1) \Big(S(n,k,t) + d(n,t) \Big) \\
&- c(k,t) - \sum_{n=0}^{2} \theta(n,k,t) S(n,k,t) - \tau\Big(\theta(2,k,t), \theta(2,k,t-1), S(2,k,t) \Big),
\end{aligned} \tag{A4}
$$

$$S(n,k,t) = \frac{\beta V(k,t)^{\frac{1}{\psi}}}{\lambda_{BC}(k,t)} E_t^k \left[V(k,t+1)^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} \times$$

$$E_t^k \left[V(k,t+1)^{-\gamma} \frac{\partial V(k,t+1)}{\partial \theta(n,k,t)}\right], n \in \{0,1\}, \tag{A5}$$

$$S(2,k,t) = \frac{1}{1-\lambda_{SSC}(k,t)} \frac{\beta V(k,t)^{\frac{1}{\psi}}}{\lambda_{BC}(k,t)} \times$$

$$E_t^k \left[V(k,t+1)^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} E_t^k \left[V(k,t+1)^{-\gamma} \frac{\partial V(k,t+1)}{\partial \theta(2,k,t)}\right] \tag{A6}$$

$$- \frac{1}{1-\lambda_{SSC}(k,t)} \frac{\partial \tau\Big(\theta(2,k,t), \theta(2,k,t-1), S(2,k,t)\Big)}{\partial \theta(2,k,t)},$$

$$\theta(2,k,t) \geq 0; \qquad \lambda_{SSC}(k,t) \geq 0; \qquad \lambda_{BC}(k,t)\lambda_{SSC}(k,t)\theta(2,k,t) = 0. \tag{A7}$$

Equation (A3) equates the marginal utility of consumption to $\lambda_{BC}(k,t)$, the shadow price for relaxing the budget constraint. Equation (A4) is the budget constraint that the consumption and portfolio policies must satisfy. Equations (A5) and (A6) equate the benefit from holding an asset to the benefit from selling it—for the alternative asset, net of transaction costs and factoring in the shortsale constraint. Finally, equations (A7) are the complementary slackness and inequality conditions associated with the shortsale constraint.

From the envelope theorem we know that we can replace the derivative of the value function with the partial derivative of the Lagrangian, assuming that the decision variables are at their optimum and hence the first-order conditions with respect to decision variables are satisfied. Differentiating the original Lagrangian (A1) with respect to past holdings we get

$$\frac{\partial V(k,t)}{\partial \theta(0,k,t-1)} = \frac{\partial \mathcal{L}(k,t)}{\partial \theta(0,k,t-1)} = \lambda_{BC}(k,t) \times 1, \tag{A8}$$

$$\frac{\partial V(k,t)}{\partial \theta(1,k,t-1)} = \frac{\partial \mathcal{L}(k,t)}{\partial \theta(1,k,t-1)} = \lambda_{BC}(k,t) \times \Big(S(1,k,t) + d(1,t)\Big), \tag{A9}$$

$$\frac{\partial V(k,t)}{\partial \theta(2,k,t-1)} = \frac{\partial \mathcal{L}(k,t)}{\partial \theta(2,k,t-1)} = \lambda_{BC}(k,t) \times \Bigg(S(2,k,t) + d(2,t)$$

$$- \frac{\partial \tau\big(\theta(2,k,t), \theta(2,k,t-1), S(2,k,t)\big)}{\partial \theta(2,k,t-1)}\Bigg), \tag{A10}$$

such that one can rewrite the "kernel conditions" (A5) and (A6) by replacing the marginal value function with respect to asset holdings from (A8)–(A10) (shifted one period forward), and recognizing from the first-order conditions with respect to consumption (A2) that $\lambda_{BC}(k, t+1) = (1-\beta)\, c(k, t+1)^{-\frac{1}{\psi}} V(k, t+1)^{\frac{1}{\psi}}$, as follows:

$$S(0, k, t) = E_t^k \left[\frac{M(k, t+1)}{M(k, t)} \right], \tag{A11}$$

$$S(1, k, t) = E_t^k \left[\frac{M(k, t+1)}{M(k, t)} \Big(S(1, k, t+1) + d(1, t+1) \Big) \right], \tag{A12}$$

$$S(2, k, t) = \left(1 - \lambda_{SSC}(k, t) + \kappa \frac{\partial\, |\theta(2, k, t) - \theta(2, k, t-1)|}{\partial \theta(2, k, t)} \right)^{-1} \times \tag{A13}$$

$$E_t^k \left[\frac{M(k, t+1)}{M(k, t)} \left(S(2, k, t+1) \times \Big(1 - \kappa \frac{\partial \tau |\theta(2, k, t+1) - \theta(2, k, t)|}{\partial \theta(2, k, t)} \Big) + d(2, t+1) \right) \right],$$

where we used the transaction cost function in equation (1) to simplify its derivative, and where $\frac{M(k,t+1)}{M(k,t)}$ denotes investor k's pricing kernel:

$$\frac{M(k, t+1)}{M(k, t)} = \beta \left(\frac{V(k, t+1)^{1-\gamma}}{E_t^k \left[V(k, t+1)^{1-\gamma} \right]} \right)^{\frac{\frac{1}{\psi} - \gamma}{1-\gamma}} \left(\frac{c(k, t+1)}{c(k, t)} \right)^{-\frac{1}{\psi}}.$$

B Shifted System of Equations for Recursive Formulation

As explained in the main text, in a general-equilibrium setting when the investor attempts to solve for the optimal consumption and portfolio policies at date t, asset prices need to adjust in order for markets to clear. However, if one were solving the system of equations backward, these prices could *not* adjust because they depend on future consumption, which has already been determined in the previous step. Dumas and Lyasoff (2012) address this problem by proposing a "time-shift" whereby at date t one solves for the optimal portfolio for date t but the optimal consumption for date $t+1$, instead of the optimal consumption for date t. Using this insight allows one to write the system of equations so that it is recursive.

Assuming that there are K future states (nodes), the final system of equations to solve is given by both investors' ($k \in \{1, 2\}$) flow budget equations for the *next period* for all K

future states $\omega(t+1,s), s \in \{1, ..., K\}$:[41]

$$c(k,t+1) + \sum_{n=0}^{2} \theta(n,k,t+1) S(n,k,t+1) + \tau\Big(\theta(2,k,t+1), \theta(2,k,t), S(2,k,t+1)\Big)$$

$$= \theta(0,k,t) + \sum_{n=1}^{2} \theta(n,k,t)\Big(S(n,k,t+1) + d(n,t+1)\Big), \tag{B1}$$

together with the complementary slackness conditions and inequality conditions given in (A7), the kernel conditions given in (A11) to (A13), and the market-clearing conditions for all three assets:

$$\theta^{ss}(n) = \sum_{k=1}^{2} \theta(n,k,t), \quad \forall\ n \in \{0,1,2\}. \tag{B2}$$

Not counting the inequality conditions, these are $2K + 2 + 3 + 3$ equations ($2K$ flow budget equations, 2 complementarity-slackness conditions for the two investors, 3 kernel conditions for the prices of the three assets, and 3 financial-market-clearing conditions). The unknowns are next period's consumption $c(k,t+1)$ for both investors for all K states, both investor's current period's optimal asset holdings $\theta(n,k,t)$ for the three assets, together with each investor's Lagrange multiplier $\lambda_{SSC}(k,t)$ associated with the shortsale constraint; in total there are $2K + 6 + 2$ unknowns—the same number as the number of equations.

This system of equations is solved recursively based on interpolating functions for the future prices, $S(n,k,t{+}1)$, next period's utility functions, $V(k,t{+}1)$, as well as next period's asset holdings, $\theta(n,k,t+1)$, using the terminal conditions $S(n,k,T) = \theta(n,k,T) = 0$ for all states, all assets n, and all investors k.

After solving the system for all dates $t \in \{1, ..., T-1\}$, one has solved all equations from the global system—except the budget equations for the initial date, which have not been used so far because of the time shift. So, at the initial date, we need to solve the following system of equations for the two unknowns $c(k,1)$, $k \in \{1,2\}$:

$$c(k,1) + \sum_{n=0}^{2} \theta(n,k,1) S(n,k,1) = \theta(0,k,0) + \sum_{n=1}^{2} \theta(n,k,0)\Big(S(n,k,1) + d(n,1)\Big),$$

[41]Note, for ease of notation, we have suppressed the dependence of time $t+1$ variables on the state $\omega(t+1,s)$ in the equations that follow.

based on interpolating functions for the initial-date prices, $S(n, k, 1)$, and holdings, $\theta(n, k, 1)$. The endowed holdings $\theta(n, k, 0)$ for the three assets are exogenous to the system and reflect the initial period's incoming (endowed) wealth of the two investors.

Table 1: Model Parameters

Variable	Description	Base Case	Variation
T	Horizon	15	10; 20
β	Rate of time-preference	0.95	–
γ	Relative risk-aversion	1.75	1.5; 2.0
ϕ	Elasticity of intertemporal substitution	1.5	0.57; 1.25; 1.75
$\theta(0,k,0)$	Investor k's endowed holdings of bond	0	–
$\theta(2,k,0)$	Investor k's endowed holdings of risky assets	0.5	–
α_1, α_2	Hyperparameters of the prior beta distribution	1/1	2/2
μ_1^D	Mean of dividend growth rate for equity index	1.2%	–
μ_2^D	Mean of dividend growth rate for alternative asset	1.2%	–
σ_1^D	Volatility of dividend growth rate for equity index	20%	–
σ_2^D	Volatility of dividend growth rate for alternative asset	20%	12.5%; 27.5%
ρ	Correlation between dividend growth rates	0.5	0.25; 0.75
$\frac{d(1,1)}{d(1,1)+d(2,1)}$	Alternative asset's share of total initial dividends	0.2	0.1; 0.3

The table reports the model parameters used for our base case as well as the variations in the model parameters that we considered.

Table 2: Optimal Initial Holdings of the Inexperienced Investor

Parameter Values					Initial Holdings		Change in Initial Holdings Relative to Opaque Setup			
T	Volatility Altern.	Corr.	Size Altern.	Prior	Perfect Market	Opaque Constr.	2.5% TC	5% TC	10% TC	100% TC
Panel A: Opaque Asset										
15	20%	0.5	0.2	1/1	0.500	0.123	− 0.082	− 0.115	+ 0.021	+ 0.150
10	—	—	—	—	0.500	0.240	− 0.064	− 0.035	+ 0.009	+ 0.083
20	—	—	—	—	0.500	0.022	− 0.022	− 0.022	− 0.019	+ 0.215
—	12.5%	—	—	—	0.500	0.006	− 0.006	− 0.006	− 0.006	+ 0.118
—	27.5%	—	—	—	0.500	0.152	− 0.027	− 0.024	+ 0.075	+ 0.191
—	—	0.25	—	—	0.500	0.269	− 0.027	− 0.040	+ 0.045	+ 0.133
—	—	0.75	—	—	0.500	0.000	− 0.000	− 0.000	− 0.000	+ 0.082
—	—	—	0.1	—	0.500	0.000	− 0.000	− 0.000	− 0.000	+ 0.281
—	—	—	0.3	—	0.500	0.206	− 0.082	− 0.056	− 0.022	+ 0.066
—	—	—	—	2/2	0.500	0.243	− 0.035	+ 0.004	+ 0.030	+ 0.069
Panel B: Equity Index										
15	20%	0.5	0.2	1/1	0.500	0.508	+ 0.007	+ 0.007	− 0.032	− 0.066
10	—	—	—	—	0.500	0.504	+ 0.003	− 0.009	− 0.025	− 0.040
20	—	—	—	—	0.500	0.514	− 0.007	− 0.015	− 0.025	− 0.089
—	12.5%	—	—	—	0.500	0.510	− 0.011	− 0.017	− 0.022	− 0.041
—	27.5%	—	—	—	0.500	0.509	− 0.003	− 0.011	− 0.047	− 0.088
—	—	0.25	—	—	0.500	0.512	− 0.003	− 0.006	− 0.027	− 0.046
—	—	0.75	—	—	0.500	0.484	− 0.010	− 0.014	− 0.017	− 0.037
—	—	—	0.1	—	0.500	0.494	− 0.007	− 0.011	− 0.018	− 0.053
—	—	—	0.3	—	0.500	0.509	+ 0.013	− 0.006	− 0.029	− 0.068
—	—	—	—	2/2	0.500	0.504	− 0.001	− 0.013	− 0.022	− 0.032
Panel C: Risk-free Bond										
15	20%	0.5	0.2	1/1	0.000	0.736	+ 0.092	+ 0.143	+ 0.145	+ 0.142
10	—	—	—	—	0.000	0.370	+ 0.069	+ 0.087	+ 0.102	+ 0.082
20	—	—	—	—	0.000	1.106	+ 0.083	+ 0.128	+ 0.183	+ 0.192
—	12.5%	—	—	—	0.000	1.089	+ 0.063	+ 0.093	+ 0.122	− 0.017
—	27.5%	—	—	—	0.000	0.575	+ 0.052	+ 0.095	+ 0.181	+ 0.283
—	—	0.25	—	—	0.000	0.442	+ 0.062	+ 0.108	+ 0.066	+ 0.019
—	—	0.75	—	—	0.000	1.075	+ 0.054	+ 0.077	+ 0.095	+ 0.082
—	—	—	0.1	—	0.000	0.621	+ 0.038	+ 0.063	+ 0.100	+ 0.087
—	—	—	0.3	—	0.000	0.818	+ 0.134	+ 0.168	+ 0.211	+ 0.215
—	—	—	—	2/2	0.000	0.505	+ 0.069	+ 0.072	+ 0.088	+ 0.079

The table presents the change in optimal initial holdings of the inexperienced investor at the initial date $t = 1$ for different levels of transaction costs. Panel A reports the optimal holdings for the alternative asset, Panel B the optimal holdings for the equity index, and Panel C the optimal holdings for the risk-free bond. The first line of each panel refers to the base case; the remaining lines capture variations in parameters. T denotes the economy's horizon; Volatility Altern. denotes the alternative asset's dividend volatility; Corr. denotes the correlation between the dividends of the equity index and the alternative asset; Size Altern. denotes the initial share of the alternative asset's dividends relative to total dividends; Prior denotes the prior of the inexperienced investor, given by the two hyperparameters of the Beta distribution. The column titled *Perfect Market* reports the optimal initial holdings when both risky assets can be traded without costs and both investors have perfect knowledge. The column titled *Opaque Constr.* reports the optimal initial holdings when both risky assets can be traded without costs, but the expected growth rate of dividends is opaque for the inexperienced investor, and both investors face shortsale constraints.

Table 3: Average Turnover

Parameter Values					Avg. Turnover		Percentage Change in Average Turnover Relative to Opaque Setup			
T	Volatility Altern.	Corr.	Size Altern.	Prior	Perfect Market	Opaque Constr.	2.5% TC	5% TC	10% TC	100% TC
Panel A: Opaque Asset										
15	20%	0.5	0.2	1/1	0.000	0.627	− 42.17	− 49.80	− 69.11	− 89.47
10	—	—	—	—	0.000	0.498	− 38.92	− 56.72	− 72.99	− 88.52
20	—	—	—	—	0.000	0.697	− 47.76	− 56.10	− 64.51	− 90.15
—	12.5%	—	—	—	0.000	0.738	− 51.79	− 61.20	− 70.93	− 83.58
—	27.5%	—	—	—	0.000	0.535	− 41.00	− 51.42	− 68.54	− 92.47
—	—	0.25	—	—	0.000	0.600	− 45.10	− 57.20	− 74.56	− 95.03
—	—	0.75	—	—	0.000	0.632	− 52.84	− 60.70	− 70.62	− 81.80
—	—	—	0.1	—	0.000	0.375	− 41.73	− 51.14	− 60.86	− 91.03
—	—	—	0.3	—	0.000	0.809	− 45.53	− 58.65	− 71.11	− 88.25
—	—	—	—	2/2	0.000	0.535	− 56.23	− 68.22	− 80.30	− 89.80
Panel B: Equity Index										
15	20%	0.5	0.2	1/1	0.000	0.463	+ 28.01	+ 35.95	+ 44.67	+ 58.48
10	—	—	—	—	0.000	0.350	+ 33.19	+ 46.12	+ 59.43	+ 70.72
20	—	—	—	—	0.000	0.513	+ 28.06	+ 33.76	+ 36.67	+ 55.03
—	12.5%	—	—	—	0.000	0.507	+ 24.47	+ 27.23	+ 27.29	+ 35.26
—	27.5%	—	—	—	0.000	0.407	+ 35.43	+ 47.17	+ 69.13	+ 89.82
—	—	0.25	—	—	0.000	0.256	+ 14.80	+ 25.94	+ 35.88	+ 44.40
—	—	0.75	—	—	0.000	0.662	+ 40.33	+ 45.76	+ 50.84	+ 56.54
—	—	—	0.1	—	0.000	0.535	+ 19.21	+ 25.30	+ 28.47	+ 41.51
—	—	—	0.3	—	0.000	0.376	+ 52.41	+ 61.60	+ 74.08	+ 90.16
—	—	—	—	2/2	0.000	0.278	+ 51.70	+ 61.93	+ 72.29	+ 79.25
Panel C: Risk-free Bond										
15	20%	0.5	0.2	1/1	0.000	1.063	− 8.68	− 10.40	− 14.57	− 21.05
10	—	—	—	—	0.000	0.890	− 8.42	− 12.49	− 16.83	− 21.40
20	—	—	—	—	0.000	1.146	− 9.31	− 10.74	− 13.66	− 21.46
—	12.5%	—	—	—	0.000	1.274	− 18.04	− 22.38	− 28.37	− 33.34
—	27.5%	—	—	—	0.000	0.898	− 1.16	− 0.59	− 0.15	− 4.67
—	—	0.25	—	—	0.000	0.805	− 20.59	− 26.31	− 34.71	− 47.21
—	—	0.75	—	—	0.000	1.319	− 3.04	− 4.08	− 6.53	− 9.15
—	—	—	0.1	—	0.000	0.950	− 5.21	− 6.08	− 7.99	− 12.80
—	—	—	0.3	—	0.000	1.151	− 11.58	− 14.68	− 18.80	− 25.91
—	—	—	—	2/2	0.000	0.803	− 14.58	− 18.42	− 23.53	− 27.61

The table presents the percentage change in average turnover (average number of shares traded between $t = 1$ and T) for different levels of transaction costs. Panel A reports the average turnover for the alternative asset, Panel B the average turnover for the equity index, and Panel C the average turnover for the risk-free bond. The first line of each panel refers to the base case; the remaining lines capture variations in parameters. T denotes the economy's horizon; Volatility Altern. denotes the alternative asset's dividend volatility; Corr. denotes the correlation between the dividends of the equity index and the alternative asset; Size Altern. denotes the initial share of the alternative asset's dividends relative to total dividends; Prior denotes the prior of the inexperienced investor, given by the two hyperparameters of the Beta distribution. The column titled *Perfect Market* reports the average turnover when both risky assets can be traded without costs and both investors have perfect knowledge. The column titled *Opaque Constr.* reports the average turnover when both risky assets can be traded without costs, but the expected growth rate of dividends is opaque for the inexperienced investor, and both investors face shortsale constraints.

Table 4: Initial Asset Prices

Parameter Values					Asset Price		Percentage Asset Price Change Relative to Opaque Setup			
T	Volatility Altern.	Corr.	Size Altern.	Prior	Perfect Market	Opaque Constr.	2.5% TC	5% TC	10% TC	100% TC
Panel A: Opaque Asset										
15	20%	0.5	0.2	1/1	2.059	2.125	− 2.83	− 4.37	− 5.17	− 4.41
10	—	—	—	—	1.466	1.509	− 2.53	− 3.46	− 4.12	− 3.54
20	—	—	—	—	2.528	2.599	− 2.78	− 4.31	− 6.11	− 4.73
—	12.5%	—	—	—	2.259	2.351	− 2.43	− 3.50	− 4.33	− 3.86
—	27.5%	—	—	—	1.837	1.878	− 2.85	− 4.16	− 5.33	− 4.41
—	—	0.25	—	—	2.155	2.269	− 3.26	− 4.74	− 6.19	− 5.51
—	—	0.75	—	—	1.958	1.984	− 1.85	− 2.60	− 3.25	− 2.91
—	—	—	0.1	—	1.085	1.162	− 3.21	− 5.20	− 7.54	− 7.28
—	—	—	0.3	—	2.974	3.016	− 2.45	− 3.35	− 4.02	− 3.10
—	—	—	—	2/2	2.059	2.083	− 1.99	− 2.65	− 2.91	− 2.46
Panel B: Equity Index										
15	20%	0.5	0.2	1/1	7.310	7.286	+ 0.07	+ 0.17	+ 0.17	+ 0.41
10	—	—	—	—	5.373	5.360	+ 0.04	+ 0.09	+ 0.16	+ 0.38
20	—	—	—	—	8.743	8.708	+ 0.06	+ 0.17	+ 0.34	+ 0.36
—	12.5%	—	—	—	7.187	7.131	+ 0.10	+ 0.24	+ 0.40	+ 0.61
—	27.5%	—	—	—	7.453	7.465	+ 0.04	+ 0.05	+ 0.06	+ 0.19
—	—	0.25	—	—	7.274	7.262	+ 0.04	+ 0.06	+ 0.11	+ 0.45
—	—	0.75	—	—	7.353	7.316	+ 0.09	+ 0.16	+ 0.30	+ 0.40
—	—	—	0.1	—	8.237	8.186	+ 0.03	+ 0.10	+ 0.22	+ 0.49
—	—	—	0.3	—	6.427	6.433	+ 0.09	+ 0.11	+ 0.11	+ 0.26
—	—	—	—	2/2	7.310	7.298	+ 0.02	+ 0.02	+ 0.06	+ 0.16

The table presents the percentage change in initial stock prices, as we vary transaction costs. Panel A reports the initial stock price for the alternative asset and Panel B the initial stock price for the equity index. The first line of each panel refers to the base case; the remaining lines capture variations in parameters. T denotes the economy's horizon; Volatility Altern. denotes the alternative asset's dividend volatility; Corr. denotes the correlation between the dividends of the equity index and the alternative asset; Size Altern. denotes the initial share of the alternative asset's dividends relative to total dividends; Prior denotes the prior of the inexperienced investor, given by the two hyperparameters of the Beta distribution. The column titled *Perfect Market* reports the initial stock price when both risky assets can be traded without costs and both investors have perfect knowledge. The column titled *Opaque Constr.* reports the initial stock price when both risky assets can be traded without costs, but the expected growth rate of dividends is opaque for the inexperienced investor, and both investors face shortsale constraints.

Table 5: Risk-Free Rate and Expected Returns

Parameter Values					Expected Return		Change in Percentage Expected Return Relative to Opaque Setup			
T	Volatility Altern.	Corr.	Size Altern.	Prior	Perfect Market	Opaque Constr.	2.5% TC	5% TC	10% TC	100% TC
Panel A: Opaque Asset										
15	20%	0.5	0.2	1/1	4.295	4.265	+ 0.301	+ 0.459	+ 0.528	+ 0.344
10	—	—	—	—	4.051	3.901	+ 0.364	+ 0.492	+ 0.569	+ 0.411
20	—	—	—	—	4.618	4.727	+ 0.215	+ 0.344	+ 0.481	+ 0.258
—	12.5%	—	—	—	3.734	3.461	+ 0.236	+ 0.324	+ 0.363	+ 0.274
—	27.5%	—	—	—	5.055	5.367	+ 0.333	+ 0.482	+ 0.611	+ 0.353
—	—	0.25	—	—	4.139	3.914	+ 0.346	+ 0.504	+ 0.665	+ 0.535
—	—	0.75	—	—	4.507	4.649	+ 0.188	+ 0.251	+ 0.260	+ 0.150
—	—	—	0.1	—	3.474	3.034	+ 0.326	+ 0.530	+ 0.768	+ 0.676
—	—	—	0.3	—	4.869	5.038	+ 0.262	+ 0.344	+ 0.395	+ 0.178
—	—	—	—	2/2	4.263	4.385	+ 0.243	+ 0.309	+ 0.313	+ 0.170
Panel B: Equity Index										
15	20%	0.5	0.2	1/1	7.512	7.639	− 0.016	− 0.031	− 0.035	− 0.071
10	—	—	—	—	7.553	7.672	− 0.017	− 0.029	− 0.043	− 0.097
20	—	—	—	—	7.498	7.631	− 0.026	− 0.038	− 0.059	− 0.052
—	12.5%	—	—	—	7.829	8.015	− 0.019	− 0.039	− 0.064	− 0.092
—	27.5%	—	—	—	7.165	7.256	− 0.012	− 0.018	− 0.023	− 0.049
—	—	0.25	—	—	7.567	7.655	− 0.018	− 0.028	− 0.037	− 0.075
—	—	0.75	—	—	7.431	7.652	− 0.012	− 0.021	− 0.040	− 0.052
—	—	—	0.1	—	7.506	7.667	− 0.009	− 0.019	− 0.037	− 0.076
—	—	—	0.3	—	7.408	7.481	− 0.018	− 0.027	− 0.027	− 0.053
—	—	—	—	2/2	7.504	7.584	− 0.000	− 0.002	− 0.006	− 0.035
Panel C: Risk-free Bond										
15	20%	0.5	0.2	1/1	1.100	0.826	− 0.027	− 0.057	− 0.066	− 0.064
10	—	—	—	—	1.099	0.938	− 0.027	− 0.044	− 0.062	− 0.090
20	—	—	—	—	1.100	0.750	− 0.027	− 0.053	− 0.094	− 0.046
—	12.5%	—	—	—	1.750	1.545	− 0.032	− 0.070	− 0.110	− 0.112
—	27.5%	—	—	—	0.432	0.096	− 0.018	− 0.029	− 0.036	− 0.009
—	—	0.25	—	—	1.562	1.539	− 0.020	− 0.031	− 0.042	− 0.041
—	—	0.75	—	—	0.645	0.061	− 0.033	− 0.056	− 0.092	− 0.089
—	—	—	0.1	—	0.745	0.532	− 0.010	− 0.030	− 0.064	− 0.069
—	—	—	0.3	—	1.328	0.972	− 0.033	− 0.052	− 0.061	− 0.045
—	—	—	—	2/2	1.099	0.892	− 0.011	− 0.018	− 0.025	− 0.040

The table presents the change in percentage expected returns for different levels of transaction costs. Panel A reports the modified internal rate of return for the alternative asset, Panel B the expected return for the equity index, and Panel C the risk-free rate for the bond. The first line of each panel refers to the base case; the remaining lines capture variations in parameters. T denotes the economy's horizon; Volatility Altern. denotes the alternative asset's dividend volatility; Corr. denotes the correlation between the dividends of the equity index and the alternative asset; Size Altern. denotes the initial share of the alternative asset's dividends relative to total dividends; Prior denotes the prior of the inexperienced investor, given by the two hyperparameters of the Beta distribution. The column titled *Perfect Market* reports the percentage expected returns when both risky assets can be traded without costs and both investors have perfect knowledge. The column titled *Opaque Constr.* reports the percentage expected returns when both risky assets can be traded without costs, but the expected growth rate of dividends is opaque for the inexperienced investor, and both investors face shortsale constraints.

Table 6: Return Volatility

Parameter Values					Return Volatility		Change in Percentage Return Volatility Relative to Opaque Setup			
T	Volatility Altern.	Corr.	Size Altern.	Prior	Perfect Market	Opaque Constr.	2.5% TC	5% TC	10% TC	100% TC
Panel A: Opaque Asset										
15	20%	0.5	0.2	1/1	5.566	6.647	+ 0.156	+ 0.231	+ 0.233	− 0.191
10	—	—	—	—	5.889	6.702	+ 0.154	+ 0.200	+ 0.181	− 0.053
20	—	—	—	—	5.721	7.164	+ 0.145	+ 0.214	+ 0.266	− 0.410
—	12.5%	—	—	—	3.158	3.769	+ 0.077	+ 0.096	+ 0.042	− 0.114
—	27.5%	—	—	—	8.487	10.548	+ 0.241	+ 0.335	+ 0.391	− 0.462
—	—	0.25	—	—	5.515	6.308	+ 0.185	+ 0.265	+ 0.333	+ 0.017
—	—	0.75	—	—	5.657	7.063	+ 0.092	+ 0.113	+ 0.033	− 0.292
—	—	—	0.1	—	5.196	5.805	+ 0.184	+ 0.297	+ 0.414	+ 0.158
—	—	—	0.3	—	5.741	7.097	+ 0.112	+ 0.134	+ 0.099	− 0.393
—	—	—	—	2/2	5.576	6.535	+ 0.073	+ 0.095	+ 0.037	− 0.181
Panel B: Equity Index										
15	20%	0.5	0.2	1/1	21.194	21.590	− 0.015	− 0.015	+ 0.009	+ 0.077
10	—	—	—	—	21.127	21.445	− 0.014	− 0.007	+ 0.022	+ 0.053
20	—	—	—	—	21.188	21.646	− 0.015	− 0.016	− 0.004	+ 0.088
—	12.5%	—	—	—	21.358	21.921	− 0.007	− 0.001	+ 0.045	+ 0.091
—	27.5%	—	—	—	21.075	21.395	− 0.016	− 0.020	− 0.010	+ 0.035
—	—	0.25	—	—	21.154	21.155	− 0.021	− 0.027	− 0.020	+ 0.009
—	—	0.75	—	—	21.188	22.317	+ 0.003	+ 0.021	+ 0.085	+ 0.165
—	—	—	0.1	—	21.242	21.720	− 0.008	− 0.008	+ 0.006	+ 0.051
—	—	—	0.3	—	21.088	21.412	− 0.016	− 0.012	+ 0.018	+ 0.095
—	—	—	—	2/2	21.185	21.454	+ 0.002	+ 0.008	+ 0.031	+ 0.044

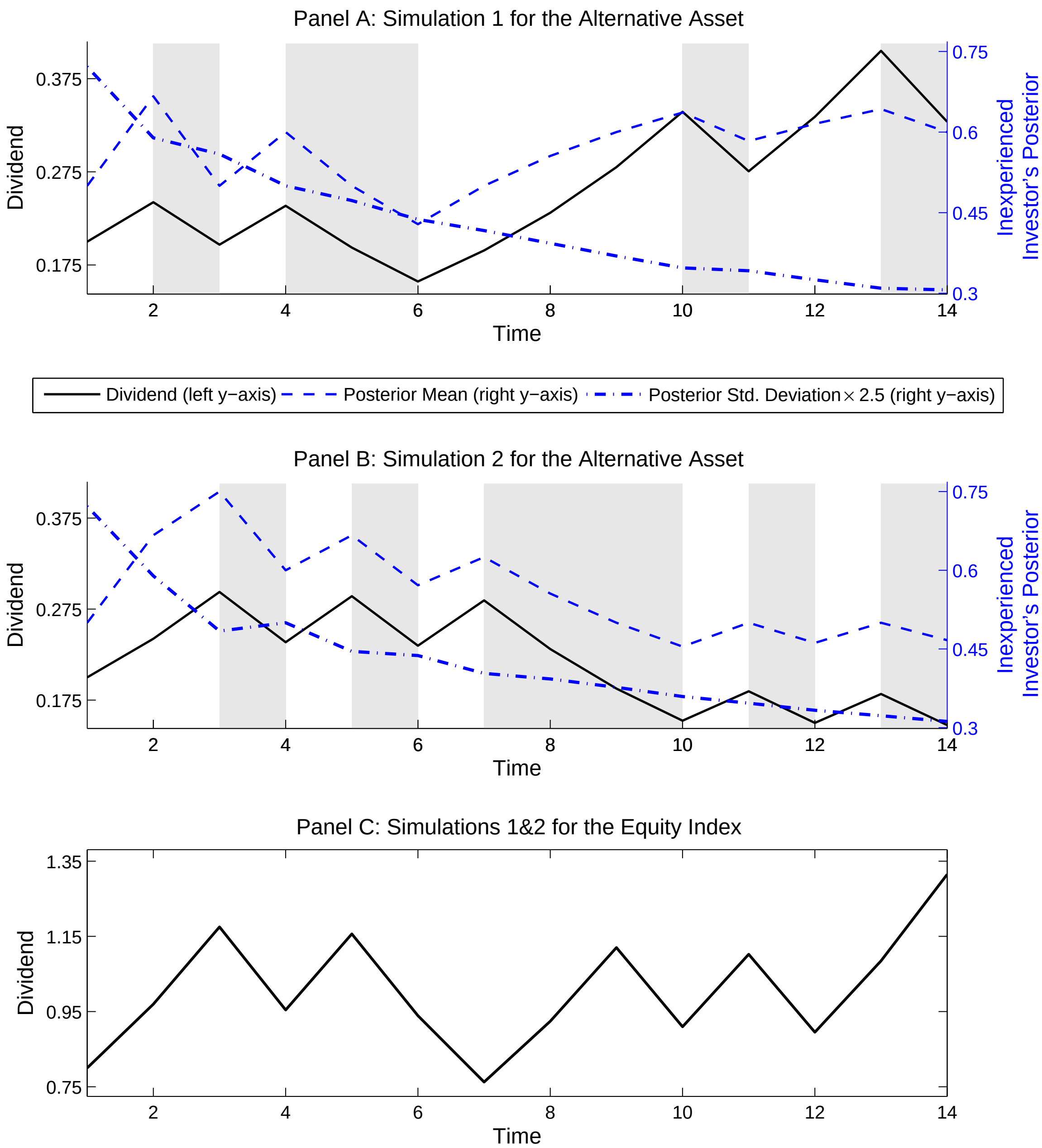
The table presents the change in percentage volatility for different levels of transaction costs. Panel A reports the dispersion across paths in the modified internal rate of return for the alternative asset and Panel B the volatility for the equity index. The first line of each panel refers to the base case; the remaining lines capture variations in parameters. T denotes the economy's horizon; Volatility Altern. denotes the alternative asset's dividend volatility; Corr. denotes the correlation between the dividends of the equity index and the alternative asset; Size Altern. denotes the initial share of the alternative asset's dividends relative to total dividends; Prior denotes the prior of the inexperienced investor, given by the two hyperparameters of the Beta distribution. The column titled *Perfect Market* reports the percentage volatility when both risky assets can be traded without costs and both investors have perfect knowledge. The column titled *Opaque Constr.* reports the percentage volatility when both risky assets can be traded without costs, but the expected growth rate of dividends is opaque for the inexperienced investor, and both investors face shortsale constraints.

Table 7: Consumption Volatility

Parameter Values					Cons. Volatility		Change in Percentage Cons. Volatility Relative to Opaque Setup			
T	Volatility Altern.	Corr.	Size Altern.	Prior	Perfect Market	Opaque Constr.	2.5% TC	5% TC	10% TC	100% TC
Panel A: Inexperienced Investor										
15	20%	0.5	0.2	1/1	5.566	6.647	+ 0.156	+ 0.231	+ 0.233	− 0.191
10	—	—	—	—	5.889	6.702	+ 0.154	+ 0.200	+ 0.181	− 0.053
20	—	—	—	—	5.721	7.164	+ 0.145	+ 0.214	+ 0.266	− 0.410
—	12.5%	—	—	—	3.158	3.769	+ 0.077	+ 0.096	+ 0.042	− 0.114
—	27.5%	—	—	—	8.487	10.548	+ 0.241	+ 0.335	+ 0.391	− 0.462
—	—	0.25	—	—	5.515	6.308	+ 0.185	+ 0.265	+ 0.333	+ 0.017
—	—	0.75	—	—	5.657	7.063	+ 0.092	+ 0.113	+ 0.033	− 0.292
—	—	—	0.1	—	5.196	5.805	+ 0.184	+ 0.297	+ 0.414	+ 0.158
—	—	—	0.3	—	5.741	7.097	+ 0.112	+ 0.134	+ 0.099	− 0.393
—	—	—	—	2/2	5.576	6.535	+ 0.073	+ 0.095	+ 0.037	− 0.181
Panel B: Experienced Investor										
15	20%	0.5	0.2	1/1	21.194	21.590	− 0.015	− 0.015	+ 0.009	+ 0.077
10	—	—	—	—	21.127	21.445	− 0.014	− 0.007	+ 0.022	+ 0.053
20	—	—	—	—	21.188	21.646	− 0.015	− 0.016	− 0.004	+ 0.088
—	12.5%	—	—	—	21.358	21.921	− 0.007	− 0.001	+ 0.045	+ 0.091
—	27.5%	—	—	—	21.075	21.395	− 0.016	− 0.020	− 0.010	+ 0.035
—	—	0.25	—	—	21.154	21.155	− 0.021	− 0.027	− 0.020	+ 0.009
—	—	0.75	—	—	21.188	22.317	+ 0.003	+ 0.021	+ 0.085	+ 0.165
—	—	—	0.1	—	21.242	21.720	− 0.008	− 0.008	+ 0.006	+ 0.051
—	—	—	0.3	—	21.088	21.412	− 0.016	− 0.012	+ 0.018	+ 0.095
—	—	—	—	2/2	21.185	21.454	+ 0.002	+ 0.008	+ 0.031	+ 0.044

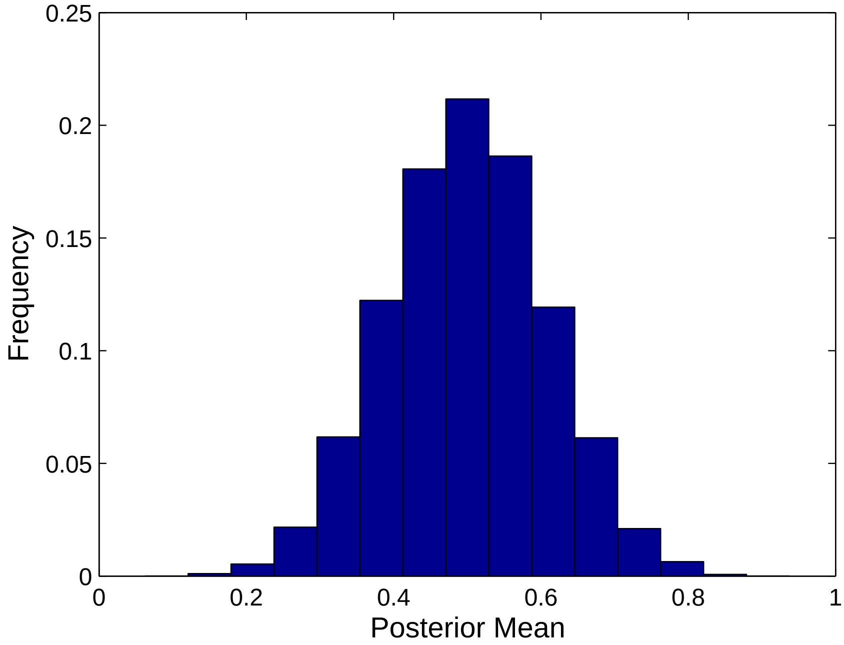
The table presents the change in percentage consumption volatility for the two investors for different levels of transaction costs. Panel A reports the consumption volatility for the inexperienced investor and Panel B for the experienced investor. The first line of each panel refers to the base case; the remaining lines capture variations in parameters. T denotes the economy's horizon; Volatility Altern. denotes the alternative asset's dividend volatility; Corr. denotes the correlation between the dividends of the equity index and the alternative asset; Size Altern. denotes the initial share of the alternative asset's dividends relative to total dividends; Prior denotes the prior of the inexperienced investor, given by the two hyperparameters of the Beta distribution. The column titled *Perfect Market* reports the percentage consumption volatility when both risky assets can be traded without costs and both investors have perfect knowledge. The column titled *Opaque Constr.* reports the percentage consumption volatility when both risky assets can be traded without costs, but the expected growth rate of dividends is opaque for the inexperienced investor, and both investors face shortsale constraints.

Figure 1: Simulated Dividends for Alternative Asset and Equity Index



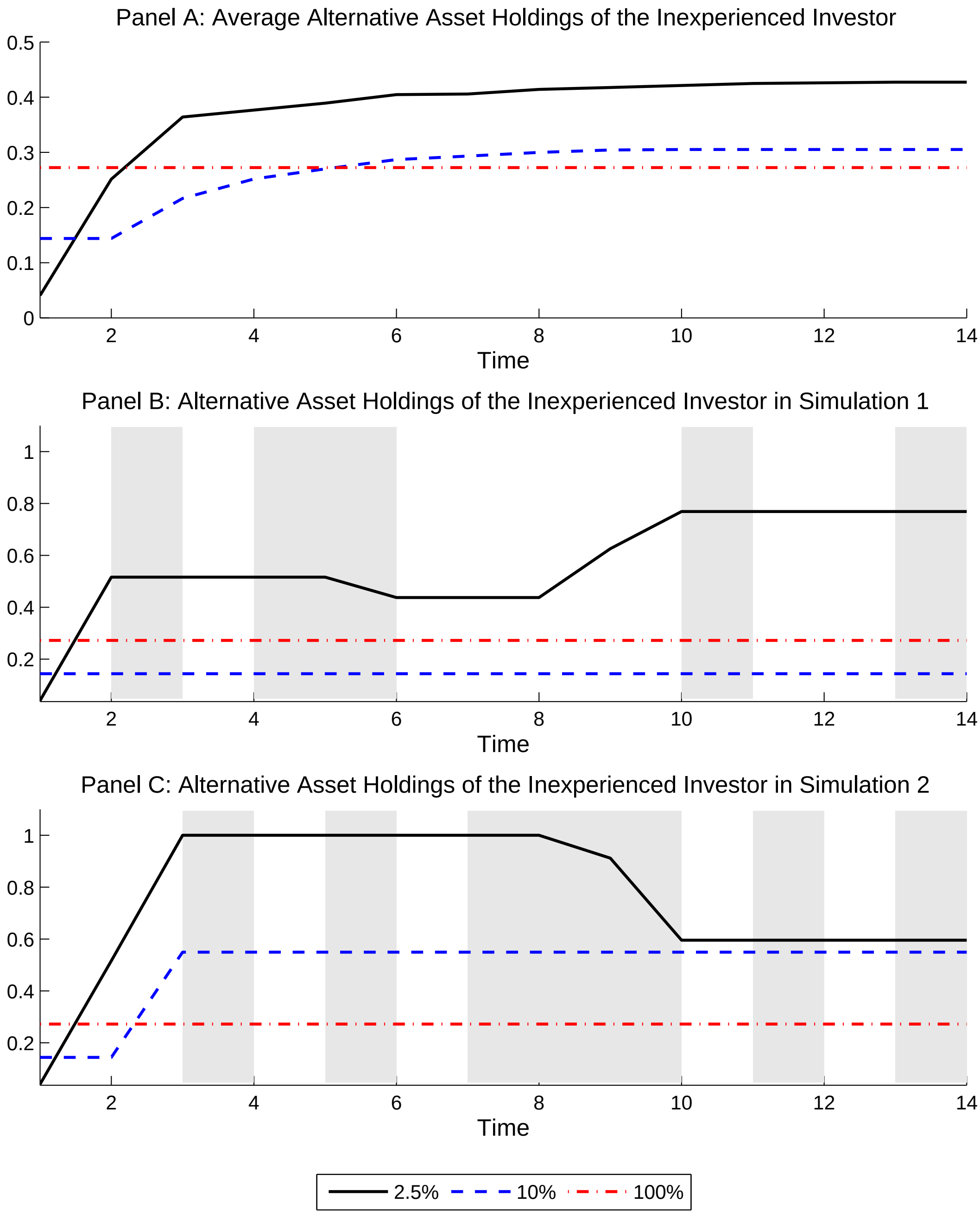
The figure shows the evolution of the dividends for the alternative asset and the equity index over time for two simulated paths of the economy. Panels A and B plot the dividend dynamics of the alternative asset (left-hand-side vertical scale), together with the resulting posterior mean and the scaled posterior standard deviation for the probability of an up-move in the alternative asset's dividend tree (both on right-hand-side vertical scale), as perceived by the inexperienced investor. The shaded gray areas indicate periods of negative dividend realizations. Panel C depicts the evolution of the dividends of the equity index, coinciding for both paths. Simulations are based on the parameter values for the base case.

Figure 2: Frequency Diagram of the Posterior Mean at $t = 15$



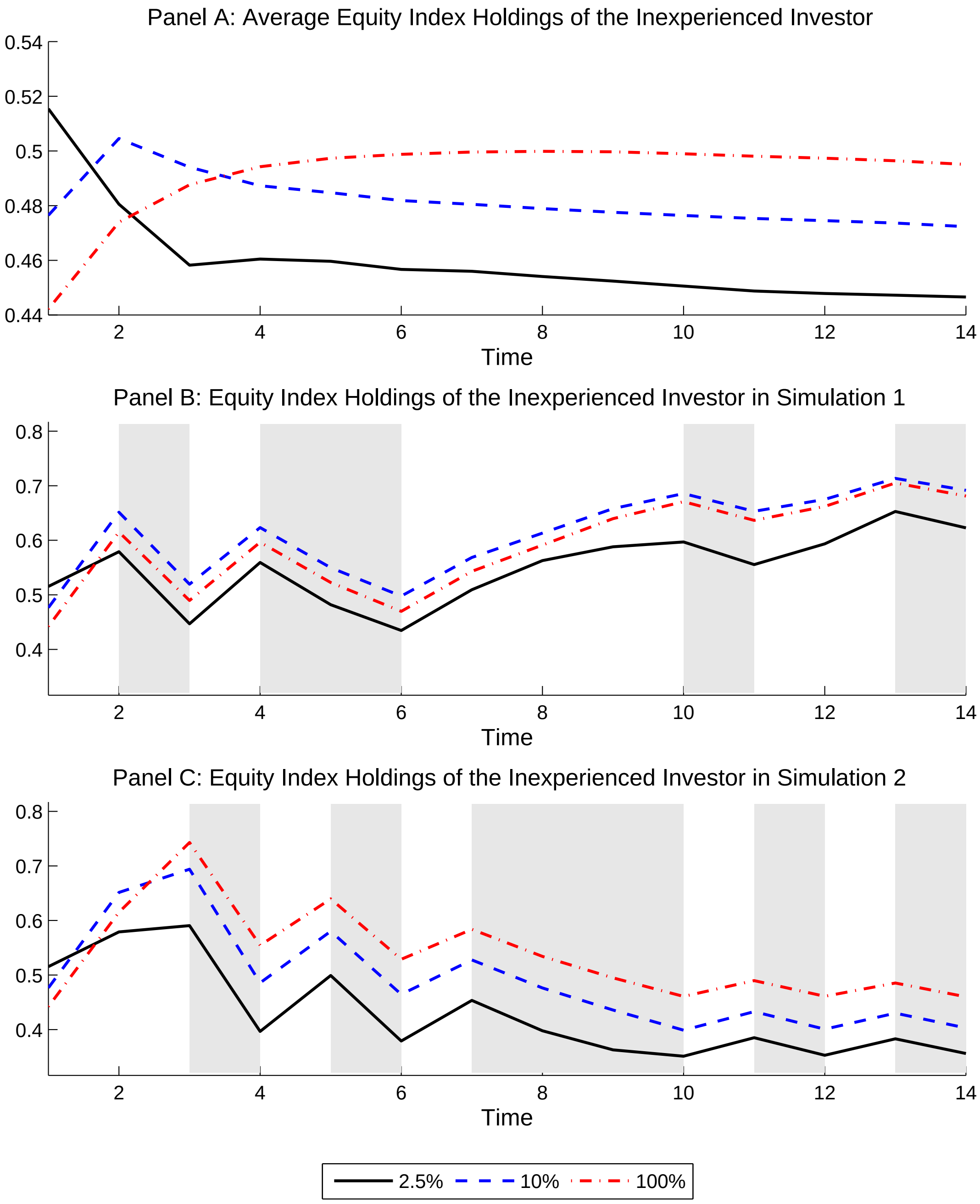
The figure shows the frequency diagram for the inexperienced investor's posterior mean of the probability of an up-move at date $T = 15$, as given in equation (2). The computations are based on a Beta(1,1) prior and 50,000 simulated paths of the underlying dividend tree.

Figure 3: Holdings of the Inexperienced Investor in the Alternative Asset



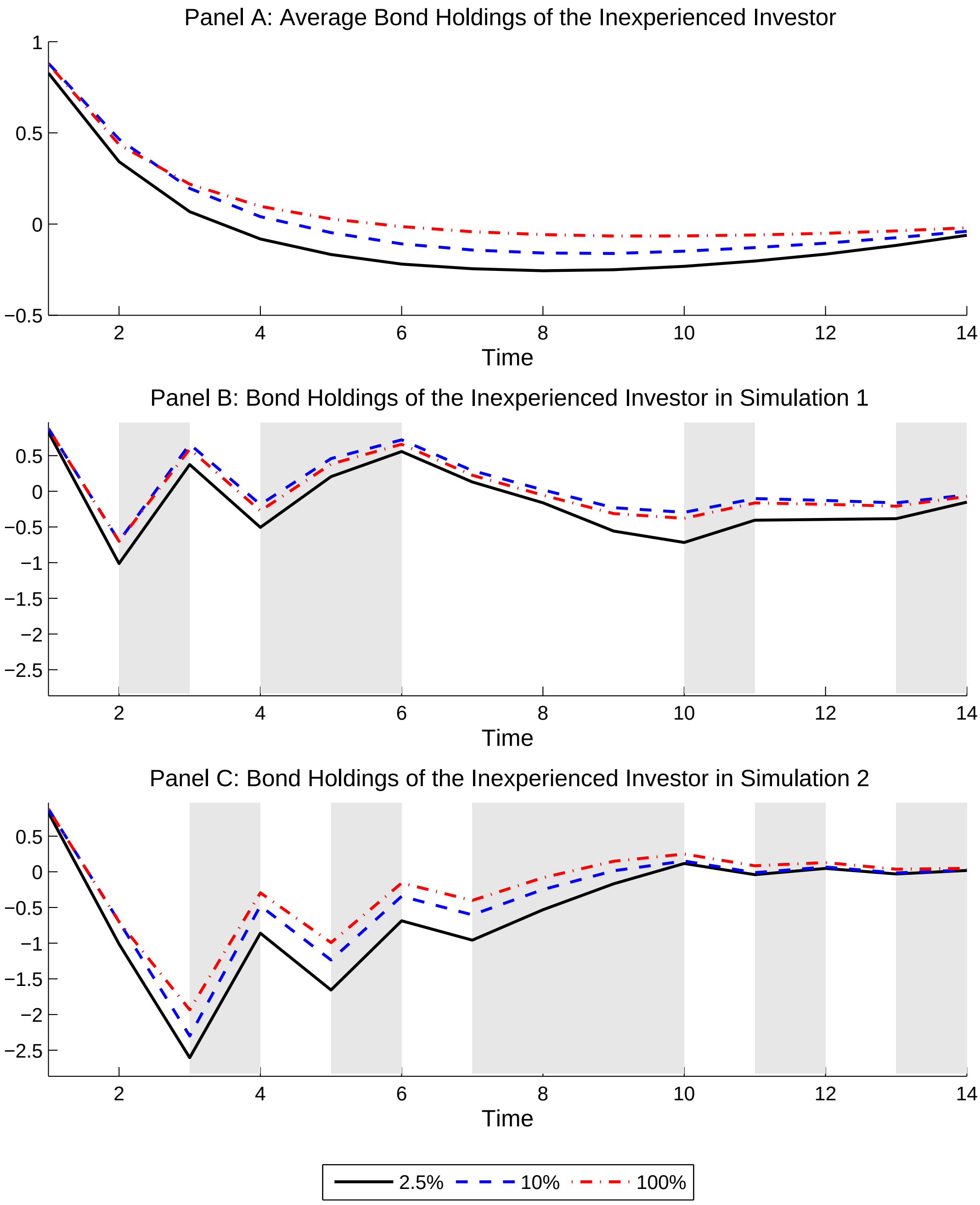
The figure plots the holdings of the inexperienced investor in the alternative asset over time. Panel A shows the average holdings (across paths), whereas Panels B and C depict the holdings for the two simulated paths of the economy, which are described in Figure 1. The shaded gray areas indicate periods of negative dividend realizations for the alternative asset. Results are based on the parameter values for the base case and under the assumption that the alternative asset is opaque. The dynamics are shown for transaction cost levels of 2.5%, 10%, and 100%.

Figure 4: Holdings of the Inexperienced Investor in the Equity Index



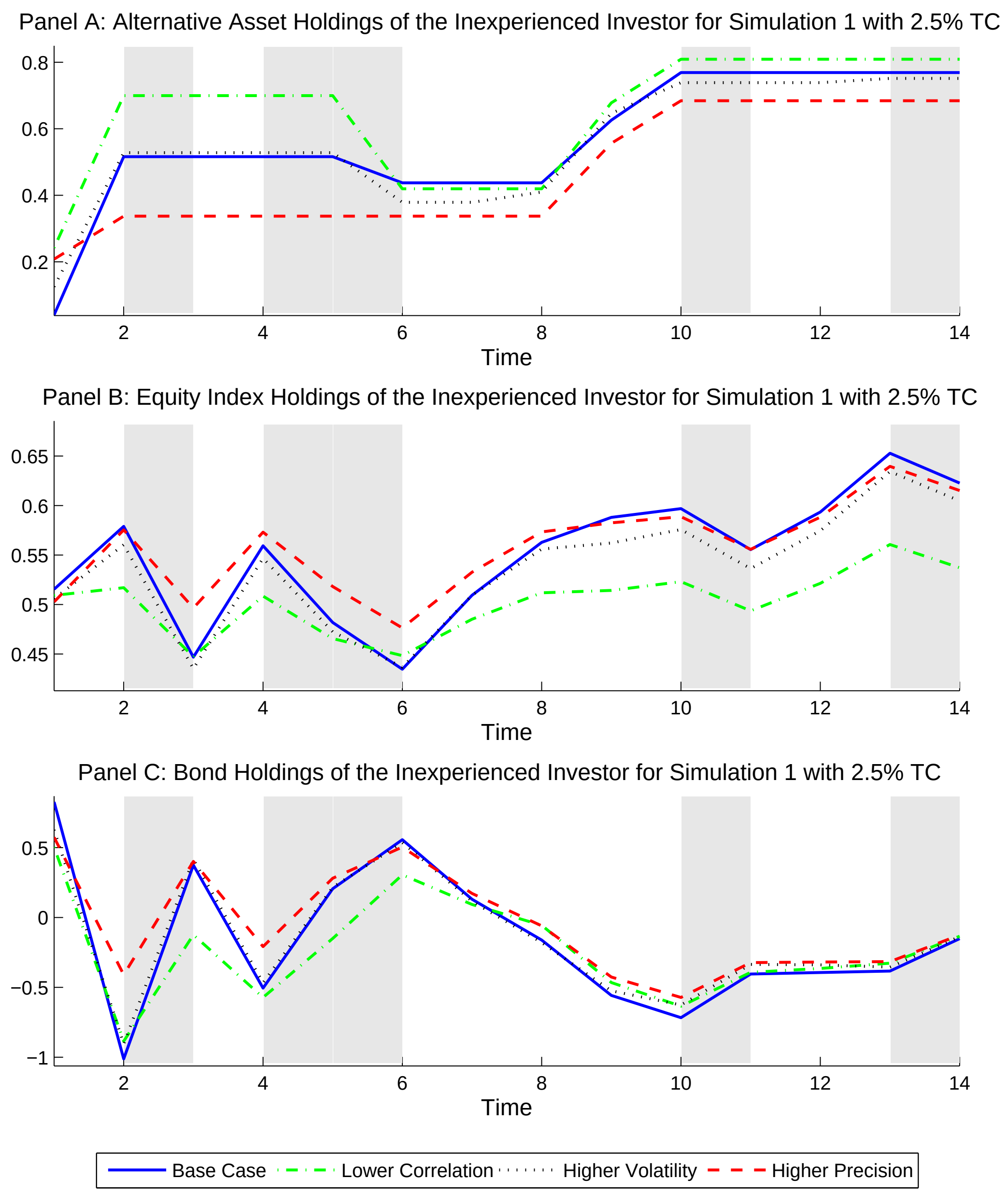
The figure plots the holdings of the inexperienced investor in the equity index over time. Panel A shows the average holdings (across paths), whereas Panels B and C depict the holdings for the two simulated paths of the economy, which are described in Figure 1. The shaded gray areas indicate periods of negative dividend realizations for the alternative asset. Results are based on the parameter values for the base case and under the assumption that the alternative asset is opaque. The dynamics are shown for transaction cost levels of 2.5%, 10%, and 100%.

Figure 5: Holdings of the Inexperienced Investor in the Bond



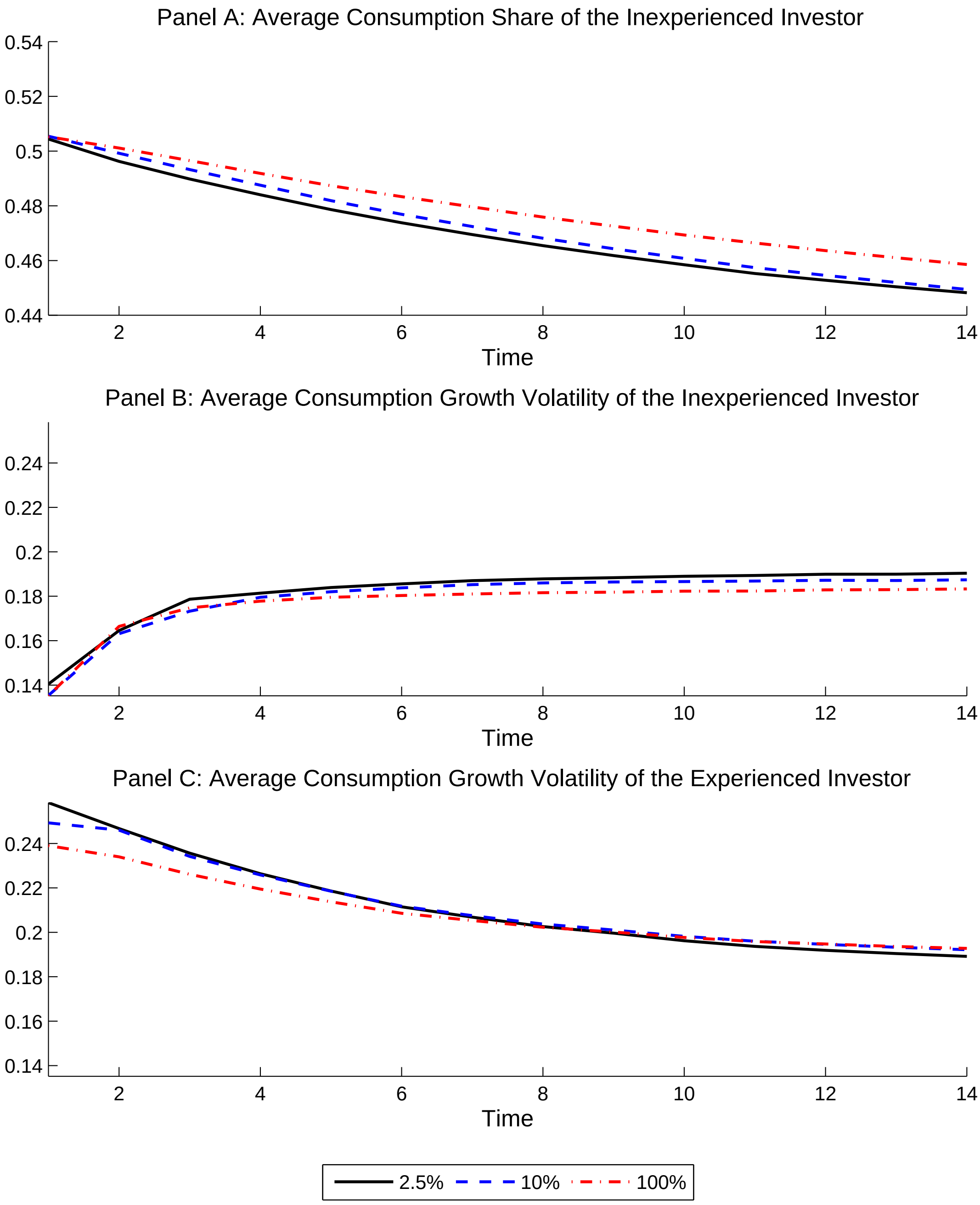
The figure plots the holdings of the inexperienced investor in the bond over time. Panel A shows the average holdings (across paths), whereas Panels B and C depict the holdings for the two simulated paths of the economy, which are described in Figure 1. The shaded gray areas indicate periods of negative dividend realizations for the alternative asset. Results are based on the parameter values for the base case and under the assumption that the alternative asset is opaque. The dynamics are shown for transaction cost levels of 2.5%, 10%, and 100%.

Figure 6: Asset Holdings For Alternative Specifications of Parameter Values



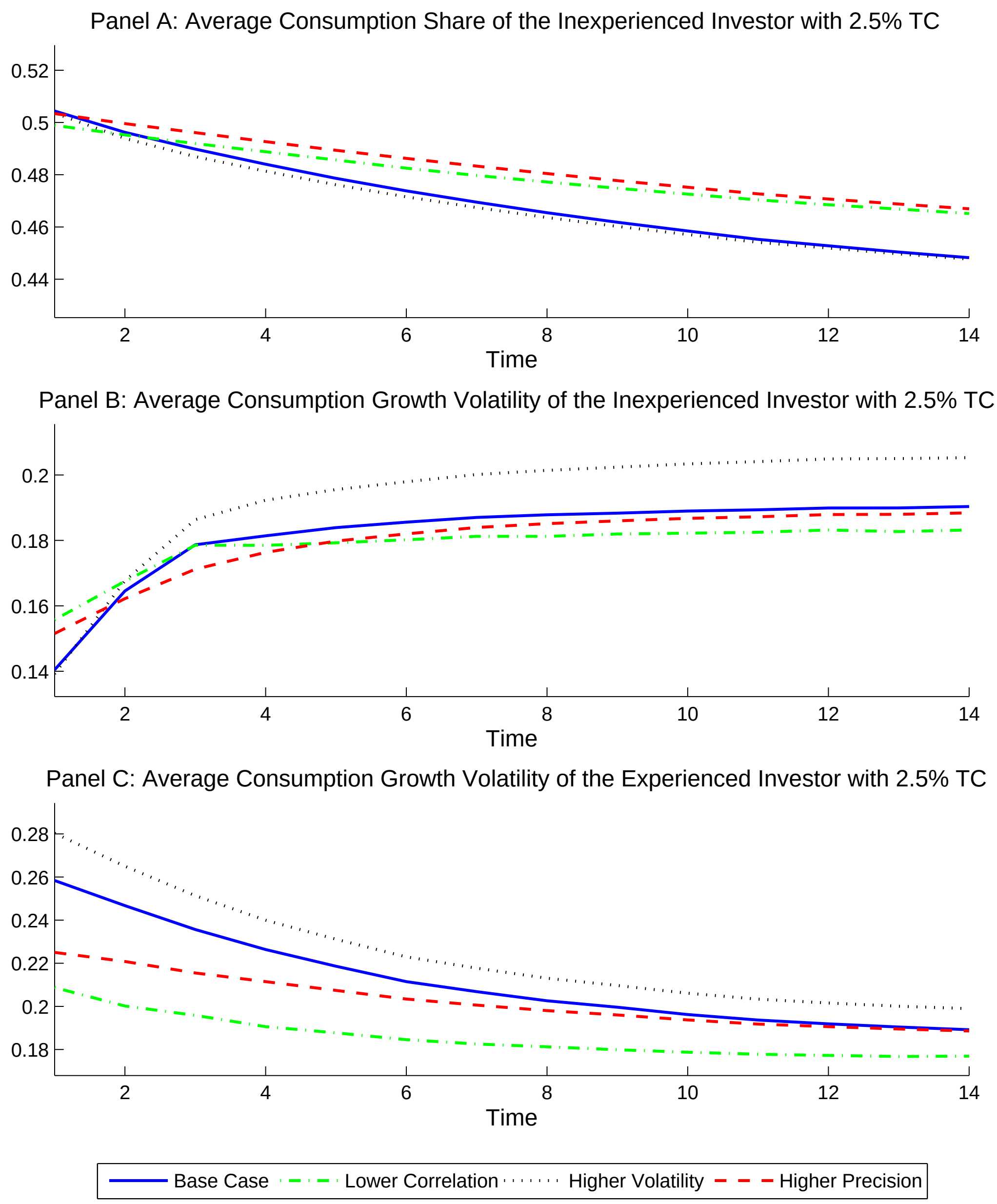
The figure shows the dynamics of the asset allocation of the inexperienced investor for a simulated path of the economy, which is described in Panel A of Figure 1, for various alternative specifications of the parameter values. Panel A shows the holdings in the alternative asset, Panel B the holdings in the equity index, and Pane C the holdings in the risk-free bond—in each case plotted over time. The shaded gray areas indicate periods of negative dividend realizations for the alternative asset. Results are shown for the base case, the case in which the alternative asset's dividend growth rate has a volatility of 27.5% ("Higher Volatility"), the case in which the correlation between the risky assets' dividends is 0.25 ("Lower Correlation"), and the case in which the inexperienced investor starts out with a more precise prior ("Higher Precision"). All results are for a transaction cost of 2.5%.

Figure 7: Consumption Dynamics



The figure shows the dynamics of the two investors' consumption growth over time. Panel A shows the evolution of the inexperienced investor's average consumption share. Panels B and C show the evolution of the volatility of consumption growth faced by the inexperienced and the experienced investors, respectively. Results are based on the parameters values for the base case and under the assumption that the alternative asset is opaque. The dynamics are shown for transaction cost levels of 2.5%, 10%, and 100%.

Figure 8: Consumption Dynamics For Alternative Specifications of Parameter Values



The figure shows the consumption growth dynamics for alternative specifications of the parameter values. Panel A shows the evolution of the inexperienced investor's average consumption share. Panels B and C show the evolution of the volatility of consumption growth faced by the inexperienced and the experienced investor. Results are shown for the base case, the case in which the alternative asset's dividend growth rate has a volatility of 27.5% ("Higher Volatility"), the case in which the correlation between the risky assets' dividends is 0.25 ("Lower Correlation"), as well as the case in which the inexperienced investor starts out with a more precise prior ("Higher Precision"). All results are for a transaction cost of 2.5%.

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