FEAR TRADING

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Abstract. We introduce a new class of swap trading strategies in incomplete markets, which disaggregate the tradeable compensation for time-varying nonlinear risks in aggregate market returns. While the price of Hellinger variance, a tradeable put-call symmetric measure of variance, has a leading contribution to the VIX volatility index, the higher-order contribution to the VIX is comprehensively captured by the price of tradeable skewness and kurtosis. Risk premia for trading Hellinger variance, skewness and kurtosis do not vanish after transaction costs and are all linked to non-tradeable indices of fear. Skew swaps appear as the most appropriate vehicles for trading fear and disaster risk, as they are best spanned by non-tradeable indices of fear and consistently price market skewness benchmarked to put-call symmetry.

Being frightened is an experience you can't buy.

Anthony Price, Sion Crossing (1984)

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1. Introduction

This paper introduces a systematic model-free approach for identifying the tradeable risk premium of time-varying market disaster risk. Our approach is based on a new class of general divergence trading strategies in incomplete option markets, which allow us to disaggregate the leading component of higher-order market risks from (second-order) market volatility risk.[1] We measure the risk premium and the price of disaster risk in a model-free way, from excess returns and forward prices of appropriate simple divergence swaps. We quantify the contribution of the price of disaster risk to the CBOE (2009) VIX volatility index, which is often interpreted by practitioners and academics as a proxy of investors' fear for market-wide disasters. Finally, we show that among simple divergence swaps, skew swaps are appropriate instruments for trading and pricing fear, which create a leading exposure to S&P 500 index (SPX) return skewness, are spanned by non-tradeable indicators of fear in the literature and consistently price return asymmetries benchmarked to put-call symmetry.

Economically, fear can be understood as an aversion to an increase in downside risk and it is naturally related to the economic concept of prudence (see Menezes et al., 1980, among others). Prudence can also be characterized as an aversion to lotteries with a payoff distribution exhibiting a more negative skewness, measured by lower odd moments of any order (Ebert, 2013). Consistently with this motivation, our preferred tradeable proxy of the price of fear directly reflects the price of realized market skewness captured by arbitrary odd higher-moments.

While implied variance indices such as VIX^2 are related to market skewness by construction, we find that the forward price of tradeable market skewness has an informational

[1]Divergence is a measure of discrepancy between random variables. Expected divergence is a measure of discrepancy between probability distributions, which generalizes standard measures of distance, such as the L_2-distance.

content different from other implied moments, which offers a convenient way to measure the price of fear as the price of an easily implementable skew trading strategy in incomplete option markets. Figure 1 highlights the weak time series relation between the implied variance and the implied skewness of S&P 500 (SPX) index returns, extracted from two time series of simple divergence swap rates. Indeed, we find that often implied variance is above average when implied skewness (kurtosis) is below (above) average, and vice versa. Moreover, while implied variance typically rises significantly for short periods of time, in connection with large downturns associated with adverse market news (as, e.g., after the Lehman default in September 2008), implied skewness can be persistently more negative than average in periods of market recovery and low average volatility.

Consistent with the economic motivation of fear as an aversion to a decrease in skewness, we identify the price of fear and fear risk premia through a new class of simple divergence swaps, which are tradeable in incomplete option markets and consistently disaggregate the compensation for variance and higher-order risks in SPX returns. Using such divergence swaps, we quantify, price and trade the components of VIX more directly related to investors' fear. We show that the leading contribution of variance risk to VIX and VIX risk premia is appropriately captured by Hellinger variance swaps, which measure the price of realized variance in a put-call symmetric way, unaffected by rotations of the implied volatility surface around the forward. Similarly, the leading contribution of skewness and kurtosis risks to VIX and VIX risk premia is comprehensively captured by Hellinger skew and kurtosis swaps.

Tradeability of our simple divergence swaps is ensured by the fact that they are implementable at moderate transaction costs in incomplete option markets, when only a discrete set of option strikes is available. This feature allows us to identify economically

relevant risk premia for time-varying second- and higher-order market risks net of transaction costs. We then specify tradeable corridor versions of simple divergence swaps, in order to localize the price and the risk premium for time-varying variance, skewness and kurtosis over the support of the distribution of SPX returns. In this way, we can directly observe how the prices and risk premia for variance, skewness and kurtosis depend on the time-varying risks generated by distinct regions of the payoff space. Finally, we link the forward price and the risk premium of simple variance, skewness and kurtosis swaps to non tradeable fear indices proposed in the literature, in order to support their interpretation as tradeable proxies for the price of fear and fear risk premia. Our empirical findings can be summarized as follows.

First, the risk premium (swap rate) for realized Hellinger variance is negative (positive) and dominated by a negative (positive) non-monotonic contribution of out-of-the money call and put (nearly at-the-money option) payoff risk premia (prices). Second, the risk premium (swap rate) for skewness risk is positive (negative) and the consequence of large positive risk premia (negative swap rates) for out-of-the-money put payoff risks, which are virtually monotonically decreasing (increasing) with option moneyness. Third, the risk premium (swap rate) for volatility feedback risk, which is tradeable using long-short semi-variance portfolios, is also positive (negative), but it depends in a more balanced, non-monotonic way on both out-of-the-money put and call option risks: At-the-money call and put option risks have a substantial influence on the risk premium (swap rate), which is positive (negative) for put (call) option payoffs, and hump-shaped for options that are progressively more out-of-the-money. Fourth, the kurtosis risk premium (swap rate) is negative (positive) and follows from a large negative (positive) contribution of risk premia (swap rates) induced by out-of-the-money put option payoffs, which is monotonically increasing (decreasing) with option moneyness. Finally, following this asymmetric

evidence on divergence risk premia generated by out-of-the-money put and call payoff risks, we investigate the interpretation of divergence swap rates and excess returns as tradeable measures of the price and the risk premium for fear. We find that while swap rates and excess returns of Hellinger variance, skewness, and kurtosis swaps all correlate with proxies of fear in the literature, Bollerslev et al. (2014)'s Tail Index (TI) alone explains already about 70% of the variation in implied Hellinger skews. Similarly, predictive regressions of skew swap returns on Bollerslev and Todorov (2011)'s Fear Index (FI) show that the latter largely correlates with the time-varying component of the premium for realized skewness, explaining about 30% of the variation in realized skew swap payoffs. We conclude that skew swaps are appropriate instruments for measuring the time-varying price of fear and for trading fear risk premia, in a way that helps to conveniently identify the leading contribution of the price of fear to the VIX. Hellinger-implied skewness is also a convenient tradeable measure of (forward neutral) skewness in incomplete option markets, having good theoretical properties with respect to empirical deviations from put-call symmetry and a large correlation with non tradeable indices of skewness used by practitioners, such as the CBOE (2010) Skew Index.[2]

Our work draws from a large literature studying the implications of time-varying uncertainty for the market price of variance and disaster risk and for the option-implied price of higher moments. Models emphasizing the important role of time-varying volatility or disaster risk for explaining key aggregate asset pricing features include Eraker and Shaliastovich (2008), Drechsler and Yaron (2011), Gabaix (2012) and Wachter (2013), among others. The tight relation between SPX option-implied moments, fear and disaster risk has been studied and documented by a large number of authors, including Bates (2000), Carr and Wu (2009), Backus et al. (2011), Kelly et al. (2011), Martin (2013), Jackwerth

[2]In the sample period from January 1990 to February 2014, the correlation between the implied Hellinger skewness and the CBOE skew is about 85%.

and Vilkov (2013) and Andersen et al. (2014a), among others. Evidence that aggregate higher moments risk is priced in the cross-section of asset returns has also been provided. Chang et al. (2013) show that stock exposure to shocks in option-implied skewness is priced with a negative premium. Cremers et al. (2014) estimate a lower excess return for stocks with positive exposure to the payoffs of market-neutral straddle portfolios mimicking time-varying volatility and jump risk, respectively. Jiang and Kelly (2012) document large and persistent exposures of hedge funds to downside tail risk, as well as a key role of tail risk for the cross-section of hedge fund returns. Our family of simple trading strategies can prove useful to characterize with a model-free approach the implications of this literature for the market price of tradeable market disaster risk, as the excess returns of simple skew swaps offer a natural measure of realized disasters for directly estimating time-varying market disaster risk premia and cross-sectional differences in exposure to disaster risk.

Our work is also related to the large literature on volatility trading. While a small subset of the trading strategies used in this paper has been considered in a previous version of this manuscript (Schneider, 2012) and in Bondarenko (2014), we borrow from Schneider and Trojani (2014)'s divergence framework to systematically isolate the tradeable properties of higher-order market risks from second-order volatility risk. This approach allows us to uniquely decompose VIX into the price contribution of even and odd implied moments of SPX returns, measured by the implied legs of a new family of simple divergence swaps that are tradeable at moderate transaction costs in incomplete option markets. The low transaction costs of these strategies are a consequence of the fact that dynamic trading during the lifetime of the swap is performed exclusively in the forward/futures market, in contrast to the dynamic option portfolio strategies necessary to replicate the skew swap contract proposed in Kozhan et al. (2013). Moreover, our simple divergence swaps are

all based on static option portfolios that are delta-hedged in a model-free way in the forward/futures market. Therefore, their payoffs have better statistical properties than those of naked buy-and-hold positions in single deep OTM options stressed in Broadie et al. (2009), allowing for a more accurate inference on divergence swap risk premia. Even though our family of simple divergence swaps offers a natural nesting framework also for different proxies of implied variance and corridor implied variance in the literature, such has VIX, SVIX (Martin, 2013) and the corridor variance indexes in Carr and Lewis (2004), Lee (2008), Andersen and Bondarenko (2009) and Andersen et al. (2011), among others, we focus on the isolation of the tradeable properties of higher-order risk premia attached to time-varying disasters. With this objective in mind, we specify a new class of simple variance, skewness and kurtosis swaps that allow us to identify natural tradeable proxies for the price and the risk premium of market fear in incomplete option markets.

The paper proceeds as follows. Section 2 introduces simple divergence trading strategies, together with the modifications needed to accommodate corridor divergence, in Section 2.5, and discrete option strikes, in Section 2.6. Section 3 studies the price and the risk premia of simple divergence strategies empirically, as well as their interpretation as prices and risk premia for fear. The distinct components of the price of VIX divergence are isolated in Section 3.2, using simple Hellinger contracts trading realized variance, skewness and kurtosis, while the relation to fear and tail indices is investigated in Section 3.7. Section 4 concludes. Section A of the Appendix contains tables and figures, while Section B of the Appendix contain proofs of the main results in the paper.

2. Simple Trading Strategies

2.1. **VIX and the Price of Fear.** The VIX implied volatility index is often referred to as an index of fear (CBOE, 2009), following the observation that states of particularly large VIX values are correlated with states in which option insurance against future market

catastrophe events is very expensive, as a consequence of investors' prudent assessment of market downside risk. This observation is a natural consequence of the definition of VIX as the price of a portfolio of out-of-the money options, in which out-of-the-money puts play a dominating role

$$VIX^2 \propto \sum_{i=1}^{n} \frac{Q(K_i)}{K_i^2}. \tag{1}$$

Here, $i = 1, \ldots, n$ indexes the set of available strikes K_i on a given date, for one month maturity out-of-the-money options with prices $Q(K_i)$. While VIX is mechanically related to the price of catastrophe insurance, it measures to first-order the forward neutral market variance, as can be seen from the cumulant expansion of VIX^2, under the assumption of a continuum of option prices (Martin, 2013)

$$VIX^2 = \kappa_2 + \frac{\kappa_3}{3} + \frac{\kappa_4}{12} + \frac{\kappa_5}{60} + \cdots \tag{2}$$

where κ_i is the $i-$forward-neutral cumulant of log forward SPX returns. This expression makes clear that VIX depends on the forward price of all realized even and odd moments of SPX returns, with a decreasing dependence on higher moments.

Motivated by Menezes et al. (1980), we can understand fear as an aversion to increases in downside risk, defined as an aversion to mean-variance preserving density transformations shifting variation from the right to the left of the return distribution. As shown in Menezes et al. (1980), aversion to downside risk is equivalent to the economic concept of prudence, which in the expected utility framework is equivalent to the requirement of a convex marginal utility. More generally, prudence attitudes are defined independently of the expected utility framework, using the apportionment approach of Eeckhoudt and Schlesiger (2006), and they are characterized by an aversion to lower odd moments of any order, which is independent of even moments of any order. This property is known

as the kurtosis robustness of prudence (Ebert, 2013). Therefore, fear can be understood statistically as an aversion to a decrease of odd moments of any order, which is independent of even moments. As a consequence, tradeable proxies for the price of fear should naturally reflect the price of an increase or a decrease of an arbitrary odd moment. At the same time, they should be as unrelated as possible to the price of even moments and, in particular, the price of variance. This economic intuition motivates our approach.

In order to isolate more properly the price of fear and to measure its contribution to the VIX, we specify a class of simple trading strategies that are suited to trade the leading contributions of even and odd realized moments of arbitrary order in incomplete option markets. These strategies are derived from the information-theoretic divergence swaps introduced by Schneider and Trojani (2014) for a complete option market setting. They are simple, because they are implementable using a discrete set of out-of-the money options requiring only a single static option position at inception. This last feature is important, in order to reduce the transaction costs implied by dynamic option-replication strategies for realized higher moments, such as for example the dynamic skew swaps considered in Kozhan et al. (2013). To specify our simple trading strategies in incomplete option markets, we proceed in two steps. We initially introduce appropriate portfolios of divergence swaps and their properties in a complete option market. We then extend the approach used in the definition of VIX in CBOE (2009), in order to propose suitable approximations of the prices and the payoffs of divergence swap portfolios in incomplete option markets.

2.2. **Simple Divergence Swaps.** We start from a frictionless arbitrage-free market, in which a continuum of European call and put options on the underlying SPX index, with prices $C_{t,T}(K)$ and $P_{t,T}(K)$ at time $t \in [0,T]$, respectively, are traded at strikes $K > 0$. We denote by $F_{t,T}$ ($p_{t,T}$) the SPX forward price (the risk-less zero coupon price) at time

$t \in [0,T]$ for delivery in T.[3] Given a discrete grid of trading dates $0 = t_0 < t_1 < t_2 < \ldots < t_n = T$, simple trading strategies are defined through a generating function $\Phi : \mathbb{R} \to \mathbb{R}$ which we assume to be twice-differentiable almost everywhere, and they pay as a floating leg the realized (generalized) divergence

$$D_n^\Phi := \sum_{i=1}^{n} D_\Phi(F_{i,T}; F_{i-1,T}) \ , \tag{3}$$

where for brevity $F_{i,T} := F_{t_i,T}$ and $D_\Phi(F_{i,T}; F_{i-1,T}) := \Phi(F_{i,T}) - \Phi(F_{i-1,T}) - \Phi'(F_{i-1,T})(F_{i,T} - F_{i-1,T})$ is a (generalized) $\Phi-$Bregman (1967) divergence between $F_{i,T}$ and $F_{i-1,T}$.[4] The forward price of realized divergence follows from the martingale property of forward prices and a telescoping sum property, as

$$S(D_n^\Phi) := E_0^{\mathbb{Q}_T}[D_n^\Phi] = E_0^{\mathbb{Q}_T}[D_1^\Phi] \ . \tag{4}$$

Therefore, the terminal cash-flow of a simple divergence swap is

$$Z_T^\Phi := D_n^\Phi - S(D_n^\Phi) = \Phi_{T,T} - \Phi_{0,T} - \sum_{i=1}^{N} \Phi'(F_{i-1,T})(F_{i,T} - F_{i-1,T}) \ , \tag{5}$$

where $\Phi_{t,T} := \mathbb{E}_t^{\mathbb{Q}_T}[\Phi(F_{T,T})]$ is the $t-$forward price of terminal payoff $\Phi(F_{T,T})$. By definition, the expected payoff of a divergence swap is a measure of the risk premium for trading realized divergence, i.e., the divergence premium. Similarly, $S(D_n^\Phi)$ is the (forward) price of future realized divergence. Therefore, divergence swaps that create a positive exposure to realized odd moments of SPX returns, in a way that is as independent as possible of the even moments of forward returns, are natural instruments to measure the price of fear and to trade fear risk premia.

[3] Under the assumption of a constant interest rate, $F_{t,T}$ can be interpreted as the SPX futures price.

[4] Strictly speaking, realized divergence and Bregman divergence are defined for a convex generating functions Φ, which ensures divergence to be nonnegative. We consider here a general generating function, because portfolios of proper divergence swaps give rise to simple trading strategies with generating functions that are not globally convex.

While VIX^2 equals the price of a divergence swap with generating function $\Phi(x) = -2\ln(x)$, VIX is not the most appropriate simple swap rate for measuring the price of fear, since, according to expansion (2), it depends on a positive contribution of all even and odd moments, with a dominating contribution of second moments.[5] More generally, we can introduce appropriate choices of generating functions Φ, in order to isolate more systematically the price and the risk premium of fear from the prices and risk premia of standard second-order risk aversion or other even higher-order attitudes to risk. Using simple divergence swaps, this task is achievable with a model-free approach, because the price (excess return) of a divergence swap can be computed from the price (payoff) of a static option portfolio that is dynamically delta-hedged in the forward market.

Result 2.1 (Simple Divergence Swaps)**.** *In a complete arbitrage-free option market, the floating leg (3) of a simple divergence swap can be generated by the payoff of a static delta-hedged option portfolio, as follows*

$$D_n^\Phi = \left(\int_0^{F_{0,T}} \Phi''(K) P_{T,T}(K) dK + \int_{F_{0,T}}^{\infty} \Phi''(K) C_{T,T}(K) dK \right) - \sum_{i=1}^{N} \left(\Phi'(F_{i-1,T} - \Phi'(F_{0,T}) \right) \left(F_{i,T} - F_{i-1,T} \right) . \tag{6}$$

The fixed leg (4) of a simple divergence swap has the following model-free representation, in terms of the forward price of an option portfolio

$$S(D_n^\Phi) = E_0^{\mathbb{Q}_T}[D_1^\Phi] = \frac{1}{p_{0,T}} \left(\int_0^{F_{0,T}} \Phi''(K) P_{0,T}(K) dK + \int_{F_{0,T}}^{\infty} \Phi''(K) C_{0,T}(K) dK \right) .$$

The divergence swap in Result 2.1 is called simple, because the replicating strategy for D_n^Φ in Eq. (6) is based on a static option portolio, with weights $\Phi''(K)$, and a dynamic

[5]Therefore, VIX actually reflects a combination of the price of several second- and higher-order attitudes to risk, such as, e.g., standard risk aversion, prudence and temperance.

delta-hedging that is performed exclusively in the forward market. Moreover, the fixed leg of a simple divergence swap is independent of the hedging frequency. Therefore, the forward price for realized divergence is independent of the observation frequency.[6] Table 1 illustrates the structure of the replicating strategy of simple divergence swap payoffs.

2.3. **Divergence Indices and Simple Power Divergence Swaps.** To isolate systematically the price and the risk premium of fear, we borrow from Schneider and Trojani (2014) and introduce a flexible class of divergence swaps, which is generated by power functions. Using portfolios of power divergence swaps, we can then more directly and more systematically identify the prices and the risk premia of odd higher-order risk attitudes.

Definition 2.2. (i) For any $q \in \mathbb{R}$, a power divergence swap of order q is the divergence swap generated by power function

$$\Phi_q(x) := \frac{x^q - 1}{q(q-1)} \; ; \; x \in \mathcal{D}_q \subset \mathbb{R} \tag{7}$$

where $\Phi_0(x) := -\ln(x)$ and $\Phi_1(x) := x\ln(x)$ are defined by continuity and $\mathcal{D}_q = \mathbb{R}$ for $q \in \mathbb{N} \setminus \{0,1\}$ and $\mathcal{D}_q = \mathbb{R}_+$ otherwise. We denote by $D_n^q := D_n^{\Phi_q}$ $(S(D_1^q) := S(D_1^{\Phi_q}))$ the floating (fixed) leg of a power divergence swap of order q. (ii) For any $q \in \mathbb{R}$, the (power) divergence index of order q is defined by[7]

$$DIX(q) := 2S(D_n^q / F_{0,T}^q) \; . \tag{8}$$

[6] In contrast to the skew swap in Kozhan et al. (2013), after the inception date the replicating strategy of simple divergence swaps does not take positions in implied moments. Another difference is the hedge ratio $\Phi'(F_{i-1,T})$ for the delta-hedging in the forward market, which in their case depends on the forward price of a nonlinear claim at time t_{i-1}, which can be computed in a model free-way using the cross section of option prices at time t_{i-1}.

[7] The scaling by F_0^q ensures the scale-invariance of $DIX(q)$ with respect to the initial forward price $F_{0,T}$; see Schneider and Trojani (2014).

(iii) For any $q \in \mathbb{R}$, the divergence index of order q, relative to Hellinger divergence, is defined by[8]

$$\overline{DIX}(q) := DIX(q)/DIX(1/2) \ . \tag{9}$$

Since, from Result 2.1, $VIX^2 = DIX(0)$ is a particular example of a divergence index, the floating leg of the simple divergence swap that has VIX as a fixed leg equals twice the Itakura and Saito (1968) divergence

$$2D_n^0 = -2\sum_{i=1}^{n} \left(\ln(F_{i,T}/F_{i-1,T}) - (F_{i,T}/F_{i-1,T} - 1)\right) \ . \tag{10}$$

In a continuous-time world with pure-diffusion price dynamics, this divergence is designed to trade exactly the quadratic variation of log returns.[9]

In general, the leading contribution to all power divergence indices derives from the second forward neutral moment of log returns. Explicitly,

$$DIX(q) = \mu_2 + 2\sum_{k=3}^{\infty} A_k(q) \cdot \frac{\mu_k}{k!} \ , \tag{12}$$

where $A_k(q) := (q^{k-1}-1)/(q-1)$ and μ_k is the $k-$th forward-neutral moment of log returns $y_T = \ln(F_{T,T}/F_{0,T})$. In other words, to first-order all divergence indices capture the second forward-neutral moment of log returns and differences between indices arise from different

[8]The terminology Hellinger divergence index for $DIX(1/2)$ derives from the fact that the $1/2-$power divergence equals half the squared Hellinger distance.

[9]In a similar vein, $DIX(2) = SVIX^2$, the simple variance index introduced by Martin (2013). The floating leg of this simple divergence swap is

$$2D_n^2 = \sum_{i=1}^{n} \left(\frac{F_{i,T} - F_{i-1,T}}{F_{0,T}}\right)^2 \ , \tag{11}$$

and it is proportional to the realized variance of forward prices.

contributions of higher moments.[10] Therefore, portfolios of power divergence swaps can naturally isolate the contribution of higher moments to the price of divergence, in a way that is not mechanically related to second moments.

Expansion (12) raises the question of which divergence index is most suited to capture the price of divergence symmetrically, i.e., in a way that does not depend on a simple rotation of the option-implied volatility smile with respect to log moneyness. Schneider and Trojani (2014) show that $DIX(1/2)$, the Hellinger divergence index, is the single power divergence index that is invariant to rotations of implied volatilities.[11] Moreover, while empirically $DIX(1/2)$ and other power divergence indices look similar, expansion (12) shows that differences between such indices are informative about the price of higher moments risk. The relative divergence index $\overline{DIX}(q)$ is a direct natural measure of such differences, in terms of the relative price of asymmetric and symmetric divergence: While $DIX(1/2)$ is invariant to changes in the sign of the slope of the smile, $DIX(q)$ is not for $q \neq 1/2$. Finally, $\overline{DIX}(q)$ also has the obvious interpretation of the fixed leg of a divergence swap with floating leg

$$\overline{DIX}_n(q) := \frac{2D_n^q}{F_0^q DIX(1/2)}. \tag{14}$$

2.4. **Skewness, Quarticity and Higher-Order Hellinger Swaps.** To identify the prices and the risk premia of fear and, more generally, realized higher moments, we make use of higher-order Hellinger swaps. These swaps exploit the local structure of power

[10]For instance, the divergence index of order -1 depends exclusively on the even moments of forward returns

$$DIX(-1) = \mu_2 + 2\sum_{k=1}^{\infty} \frac{\mu_{2k}}{(2k)!}\ . \tag{13}$$

[11]The Hellinger divergence index also implies the smallest absolute contribution of higher moments with respect to symmetric divergence swap rates; see Schneider and Trojani (2014).

divergence swaps at $q = 1/2$, in order to uniquely decompose power divergence into put-call symmetric (put-call antisymmetric) contributions of even (odd) higher-order attitudes to uncertainty.

Definition 2.3. (i) A (scaled) simple Hellinger swap of order $j \in \mathbb{N}$ is a swap with floating leg

$$D_n^{1/2}(j) := \left.\frac{d^j(D_n^q/F_{0,T}^q)}{dq^j}\right|_{q=1/2} = \sum_{i=1}^{n} \left.\frac{d^j(D_q(F_{i,T};F_{i-1,T})/F_{0,T}^q)}{dq^j}\right|_{q=1/2}, \tag{15}$$

and fixed leg given by

$$S(D_n^{1/2}(j)) = E_0^{\mathbb{Q}_T}[D_1^{1/2}(j)] = \left.\frac{d^j E_0^{\mathbb{Q}_T}[D_1^q/F_{0,T}^q]}{dq^j}\right|_{q=1/2}. \tag{16}$$

(ii) We call the simple Hellinger swaps of order 0, 1, 2 and 3, Hellinger variance, skew, quarticity and quinticity swaps, respectively.

By construction, second- and higher-order simple Hellinger swaps in Definition 2.3 are simple divergence swaps generated by a corresponding generating function. Therefore, they can be replicated and priced in a model-free way using Result 2.1. Obviously, the zero-th order Hellinger swap is the (scaled) 1/2-power divergence swap, which measures the price of realized divergence symmetrically. It is defined through the generating function

$$\Phi_{1/2}^{(0)}(x/F_{0,T}) := \Phi_{1/2}(x/F_{0,T}) = -4((x/F_{0,T})^{1/2} - 1) . \tag{17}$$

The Hellinger skew swap is the simple divergence swap defined by the generating function

$$\Phi_{1/2}^{(1)}(x/F_{0,T}) := \left.\frac{d\Phi_q(x/F_{0,T})}{dq}\right|_{q=1/2} = -4(x/F_{0,T})^{1/2}\ln(x/F_{0,T}) . \tag{18}$$

Similarly, the simple Hellinger quarticity swap is generated by function

$$(19)\ \Phi^{(2)}_{1/2}(x/F_{0,T}) := \left.\frac{d^2\Phi_q(x/F_{0,T})}{dq^2}\right|_{q=1/2} = -4[(x/F_{0,T})^{1/2}(\ln^2(x/F_{0,T})+8)-8]\ .$$

From expansion (12) and Definition 2.3, the swap rate of a $k-$th order Hellinger swap has a leading contribution of moments of order k and a zero contribution of moments of order less than k. For instance, for Hellinger skew and quarticity swaps expansion (12) yields, respectively

$$(20)\quad S(D_1^{1/2}(1)) = \sum_{k=3}^{\infty}\left.\frac{dA_k(q)}{dq}\right|_{q=1/2}\frac{\mu_k}{k!} = -4\sum_{k=3}^{\infty}\frac{k(1/2)^{k-1}-1}{k!}\mu_k\ ,$$

$$(21)\quad S(D_1^{1/2}(2)) = \sum_{k=4}^{\infty}\left.\frac{d^2A_k(q)}{dq^2}\right|_{q=1/2}\frac{\mu_k}{k!} = -16\sum_{k=4}^{\infty}\frac{(1/2)^k(k(k-1)+2)-1}{k!}\mu_k\ .$$

It follows that $k-$th order Hellinger swap rates systematically isolate the leading contribution of the $k-$th order moments of log returns to the price of power divergence. Moreover, Schneider and Trojani (2014) show that even (odd) order Hellinger swap rates isolate such a contribution in a way that is consistent with put-call symmetry, i.e., put-call symmetrically (put-call antisymmetrically) with respect to rotations of the option-implied volatility smile. A useful implication of this property is that all odd Hellinger swap rates are zero under put-call symmetry. This feature is consistent with the intuition that in continuous-time pure diffusion models with no correlation between returns and volatility the price of fear should be zero; see Carr and Lee (2009) for a detailed characterization of put-call symmetry in continuous-time models.

Hellinger swaps decompose power divergence swaps into the leading contributions of higher-order risks, as a consequence of the fact that the generating functions of power

divergence and Hellinger swaps are related by

$$\Phi_q(x/F_{0,T}) = \sum_{j=0}^{\infty} \Phi_{1/2}^{(k)}(x/F_{0,T}) \frac{(q-1/2)^j}{j!} \ . \tag{22}$$

In this identity, even (odd) Hellinger swaps contribute identically (with opposite sign but identical absolute contribution) to all power divergence swaps of order q and $1-q$, respectively. Therefore, differences between power divergence swaps of order q and $1-q$ arise exclusively from the contribution of odd Hellinger swaps. Following these insights, we can use the analyticity of power divergences to obtain a convenient decomposition of power divergence swaps, which can be used to measure the contribution of the price of fear to the VIX.

Result 2.4. *For any $q \in \mathbb{R}$, the floating and the fixed leg of a simple power divergence swap of order q have the following unique decomposition*

$$D_n^q/F_{0,T}^q = \sum_{j=0}^{\infty} D_n^{1/2}(j) \frac{(q-1/2)^j}{j!} \ , \tag{23}$$

$$S(D_1^q/F_{0,T}^q) = \sum_{j=0}^{\infty} S(D_1^{1/2}(j)) \frac{(q-1/2)^j}{j!} \ . \tag{24}$$

In particular, VIX can be uniquely decomposed as follows, using Hellinger variance, Hellinger skewness and Hellinger quarticity

$$\begin{aligned} VIX^2 &= \sum_{j=0}^{\infty} 2S(D_1^{1/2}(j)) \frac{(-1)^j}{(2^j)j!} \\ &\approx 2 \cdot S(D_1^{1/2}(0)) - S(D_1^{1/2}(1)) + \frac{1}{4} S(D_1^{1/2}(2)) \ . \end{aligned} \tag{25}$$

In Result 2.4, the contribution of all even (odd) Hellinger swap rates to VIX^2 is strictly positive (negative). Increases (decreases) in uncertainty, which are reflected in a change of the level but not the slope of the smile, increase (decrease) the price of all even Hellinger

swaps and the VIX. In contrast, a positive (negative) deviation from put-call symmetry increases (decreases) the price of all odd Hellinger swaps and the VIX. Finally, the price of all odd Hellinger swaps is zero in absence of deviations from put-call symmetry. In this case, the VIX is completely determined by the price of the even Hellinger swaps.

Intuitively, it is difficult to motivate a non-zero price of fear and fear risk premium in economies where put-call symmetry holds, such as, e.g., a standard Black–Scholes economy or a continuous-time pure-diffusion economy with independent returns and volatilities, because of the symmetry of log returns under the physical and the forward-neutral probabilities in such settings. Consistent with this intuition, the swap rate of all odd Hellinger swaps is zero in such economies. Moreover, since even Hellinger swap rates are invariant to rotations of the implied volatility smile, they do not capture changes in the sign of the price of fear, when put-call symmetry deviations arise, e.g., from a flipping sign of the correlation between returns and volatility. In contrast, odd Hellinger swap rates are antisymmetric with respect to rotations of the smile and consistently reflect a change in the sign of the price of fear. We conclude that swap rates and excess returns of odd Hellinger swaps are more naturally related to the price of fear and to fear risk premia ex-ante. Note that the leading contribution of odd Hellinger swap rates to VIX^2 is produced by the Hellinger skew, the second-order contribution by the Hellinger quinticity, and so on. Therefore, Hellinger skew swaps are natural instruments for trading and pricing fear, using a model-free approach.

Remark 2.5. Result 2.4 implies in general that Hellinger variance, Hellinger skewness and Hellinger quarticity indices are sufficient to span quite accurately (i.e., up to terms

of order $O(\mu_5)$) power divergence indices, as follows

$$(26)\quad \begin{aligned} DIX(q) &\approx 2\cdot S(D_1^{1/2}(0)) + (q-1/2)\cdot 2S(D_1^{1/2}(1)) + (q-1/2)^2\cdot S(D_1^{1/2}(2)) \\ &=: DIX(1/2) + (q-1/2)SKEW(1/2) + (q-1/2)^2 QUART(1/2)\ . \end{aligned}$$

Empirically, decomposition (26) is indeed very accurate and virtually identical to the following exact decomposition of $DIX(q)$

$$(27)\quad DIX(q) = DIX(1/2) + (q-1/2)SKEW(q) + (q-1/2)^2 QUART(q)\ ,$$

where

$$(28)\quad SKEW(q) := \frac{DIX(q) - DIX(1-q)}{q-1/2}\ ,$$

$$(29)\quad QUART(q) := \frac{DIX(q) + DIX(1-q) - DIX(1/2)}{(q-1/2)^2}\ ,$$

are the (scaled) put-call antisymmetric skew and symmetric quarticity indices of order q in Schneider and Trojani (2014). Therefore, $SKEW(1/2)$ ($QUART(1/2)$) can be directly interpreted as the fixed leg of a long-short portfolio of power (symmetric) divergence swaps, giving rise to put-call antisymmetric (symmetric) skew (quarticity) swaps. Figure 2 of the Appendix illustrates the relation between decompositions (26) and (27) for power divergence swaps, showing that they are virtually identical for empirical purposes.

According to decomposition (26), the relative divergence index $\overline{DIX}(q)$, can also be naturally decomposed in terms of relative skew and quarticity indices

$$(30)\quad \overline{DIX}(q) \approx 1 + (q-1/2)\overline{SKEW}(1/2) + (q-1/2)^2\overline{QUART}(1/2)\ ,$$

where the relative Hellinger skew is $\overline{SKEW}(1/2) := SKEW(1/2)/DIX(1/2)$ and the relative Hellinger quarticity is $\overline{QUART}(1/2) := QUART(1/2)/DIX(1/2)$. The obvious floating legs associated with relative skew and quarticity are $\overline{SKEW}_n(1/2) := (2D_n^{1/2}(1))/DIX(1/2)$ and $\overline{QUART}_n(1/2) := D_n^{1/2}(2)/DIX(1/2)$, respectively

In terms of scale-invariant skew and quarticity indices, an equivalent useful decomposition of relative power divergence is

$$\overline{DIX}(q) \approx 1 + (q-1/2)DIX(1/2)^{1/2}\widetilde{SKEW}(1/2) + (q-1/2)^2 DIX(1/2)\widetilde{QUART}(1/2) \ ,$$

with the scale-invariant Hellinger skew and quarticity indices

$$\widetilde{SKEW}(1/2) := \frac{SKEW(1/2)}{DIX(1/2)^{3/2}} \ ; \ \widetilde{QUART}(1/2) := \frac{QUART(1/2)}{DIX(1/2)^2} \ . \tag{31}$$

Scale-invariant indices are useful to measure time-variations of the price of skew and quarticity that are not directly induced by a time-variation of the price of (symmetric) divergence. $\widetilde{SKEW}(1/2)$ and $\widetilde{QUART}(1/2)$ also have the obvious interpretation of the fixed legs of scale-invariant skew and quarticity swaps, having the floating legs

$$\widetilde{SKEW}_n(1/2) := \frac{2D_n^{1/2}(1)}{DIX(1/2)^{3/2}} \ ; \ \widetilde{QUART}_n(1/2) := \frac{D_n^{1/2}(2)}{DIX(1/2)^2} \ , \tag{32}$$

respectively. These swaps can be used to measure the risk premia traded using scale-invariant skew and quarticity.

2.5. **Corridor Contracts.** An important recent development in measuring implied variance is the corridor version of the VIX, the CVIX, proposed by Lee (2008), Andersen and Bondarenko (2009), Andersen et al. (2011), and Andersen et al. (2014b), among others. CVIX is constructed to measure to first-order the forward neutral variance of log returns,

while controlling for the illiquidity (or absence) of options far out-of-the-money in practice. CVIX can be interpreted as the fixed leg of a particular simple divergence swap with a corresponding generating function. Therefore, it is naturally embedded into the simple divergence swap framework established by Result 2.1. Given a generating function Φ, we can define for any $0 \leq a < b$ the corridor generating function $\Phi_{a,b}$, as follows:

$$\Phi_{a,b}(x) := \begin{cases} \Phi(a) + \Phi'(a)(x-a) & x < a \ , \\ \Phi(x) & a \leq x \leq b \ , \\ \Phi(b) + \Phi'(b)(x-b) & b < x \ . \end{cases} \tag{33}$$

The corridor function equals Φ inside the corridor and it is the linear extrapolation of Φ outside the corridor. When Φ is convex, the linear extrapolation of Φ outside the corridor implies the convexity of $\Phi_{a,b}$. Moreover, the realized divergence generated by function $\Phi_{a,b}$ outside the corridor is zero, because the corridor generating function is linear there. As a consequence, corridor divergence is concentrated on price changes inside the corridor, or price changes from regions inside (outside) the corridor to regions outside (inside) the corridor.

Given corridor generating function $\Phi_{a,b}$, corridor divergence swaps are easily replicated and priced, using a slightly modified version of Result 2.1. The floating leg of a corridor swap is simply $D_n^{\Phi_{a,b}}$, with the realized divergence in Eq. (3), where $\Phi'_{a,b}(x)$ is the left derivative of $\Phi_{a,b}$ in x. Denoting by $\Phi''_{a,b}(x)$ second left derivatives, we prove in Appendix B.2 the following modified version of Result 2.1, which gives the explicit weights of the option replicating portfolio for corridor divergence.

Corollary 2.6 (Simple Corridor Divergence Swaps)**.** *In a complete arbitrage-free option market, the floating leg* (3) *of a simple corridor divergence swap can be generated by the*

payoff of a static delta-hedged option portfolio

$$(34)\quad D_n^{\Phi_{a,b}} = \left(\int_a^{\min\{F_{0,T},b\}} \Phi''(K)P_{T,T}(K)dK + \int_{\max\{F_{0,T},a\}}^{\infty} \Phi''(K)C_{T,T}(K)dK\right) - \sum_{i=1}^{N} \left(\Phi'_{a,b}(F_{i-1,T}) - \Phi'_{a,b}(F_{0,T})\right)(F_{i,T} - F_{i-1,T}) \ .$$

The fixed leg (4) of a simple corridor divergence swap has the following model-free representation, in terms of the forward price of an option portfolio

$$S(D_n^{\Phi_{a,b}}) = \frac{1}{p_{0,T}}\left(\int_a^{\min\{F_{0,T},b\}} \Phi''(K)P_{0,T}(K)dK + \int_{\max\{F_{0,T},a\}}^{b} \Phi''(K)C_{0,T}(K)dK\right) \ .$$

The static option portfolio in the dynamic replication strategy for corridor divergence swaps in Corollary 2.6 has portfolio weights given by $\Phi''(K)$ inside the corridor and zero outside of the corridor, i.e., for appropriate choices of thresholds a and b, out-of-the-money call and/or put payoffs have no contribution to realized corridor divergence. For the case $a \leq F_{0,T} < b$ and a VIX generating function $\Phi(x) = -2\ln(x)$, the implied leg $S(D_n^{\Phi_{a,b}})$ of the corridor divergence swap gives the CVIX in Andersen and Bondarenko (2009) and Andersen et al. (2011). Equation (34) gives explicitly the replicating strategy for the realized corridor divergence, from which it follows that the floating leg of a corridor divergence swap having CVIX as implied leg is simply a corridor Itakura and Saito (1968) divergence. Panel (a) of Figure 3 illustrates the relation between realized power divergence and realized corridor power divergence for power $q = 0$.

Remark 2.7. Corridor divergence swaps can be introduced for general generating functions Φ, e.g., to define corridor Hellinger variance, Hellinger skew and Hellinger quarticity swaps, having a floating and a fixed leg that does not excessively depend on the payoffs and prices of out-of-the-money options. Allowing for a lower (upper) corridor threshold

$a = 0$ ($b = \infty$), one obtains upper and lower semi-divergence swaps. For instance, starting from the generating function $\Phi_{1/2}(x)$ of Hellinger divergence, we can obtain an upper Hellinger semi-variance index, based on an upper corridor threshold $b = \infty$

$$UDIX(1/2) := \frac{2}{p_{0,T}F_{0,T}^{1/2}}\left(\int_{F_{0,T}}^{\infty}\Phi''_{1/2}(K)C_{t,T}(K)dK\right). \tag{35}$$

Similarly, using a lower corridor threshold $a = 0$, we obtain a lower Hellinger semi-variance index

$$LDIX(1/2) := \frac{2}{p_{0,T}F_{0}^{1/2}}\left(\int_{0}^{F_{0,T}}\Phi''_{1/2}(K)P_{t,T}(K)dK\right). \tag{36}$$

$UDIX(1/2)$ and $LDIX(1/2)$ are natural measures of the price of symmetric divergence in upside and downside markets, respectively. Therefore, their difference captures to first order the price of volatility asymmetries in upside and downside markets, i.e., the price of volatility feedback effects, giving rise to the following Hellinger volatility feedback index:[12]

$$SKEW^{UL}(1/2) := UDIX(1/2) - LDIX(1/2)\ . \tag{37}$$

As volatility feedback is naturally related to returns asymmetries, we can interpret $SKEW^{UL}(1/2)$ as an additional tradeable proxy for the price of fear. Also $SKEW^{UL}(1/2)$, like the implied Hellinger skew, has a price of zero under put-call symmetry.

2.6. **Tradeable Divergence Swaps in Incomplete Option Markets.** In practice, option markets are incomplete. Therefore, a continuum of option prices to compute the

[12]See Turner et al. (1989), Campbell and Hentschel (1992), Bekaert and Wu (2000), Bollerslev et al. (2006) and Calvet and Fisher (2007), among others, for papers finding empirical support for volatility feedbacks. See also Lettau and Ludvigson (2010) for a review.

option portfolio in the replicating strategy for realized divergence is not in general available, even inside a compact corridor $[a, b]$. In practice realized divergence can be traded only approximately, using piecewise linear generating functions Φ and option portfolios that depend on a finite number of out-of-the-money options. These features are directly reflected also in definition (1) of the VIX by CBOE. The extent to which traded realized divergence accurately approximates the theoretical divergence generated by a function Φ depends on the granularity of the available option price information across strikes, which is a function of variables related to market liquidity and market structure. Whenever only a few strikes are traded, as it is the case for example in the foreign exchange option market, a replication of realized divergence based on a dynamic delta hedging using the hedge ratio implied by the theoretical generating function Φ, instead of its piecewise linear approximation, may induce non-negligible hedging errors. Therefore, in incomplete option markets we directly define realized divergence in terms of piecewise linear generating functions that allow a perfect replication with a finite number of option payoffs.

The basic intuition to define a piecewise linear divergence generating function associated with a general generating function Φ borrows from identity (34) in Corollary 2.6, which implies

$$\begin{aligned}\widetilde{\Phi}_{a,b}(F_{T,T}) &:= \Phi_{a,b}(F_{T,T}) - \Phi_{a,b}(F_{0,T}) - \Phi'_{a,b}(F_{0,T})(F_{T,T} - F_{0,T}) \\ &= \int_a^{\min(F_{0,T},b)} \Phi''(K) P_{T,T}(K) dK + \int_{\max(F_{0,T},a)}^{b} \Phi''(K) C_{T,T}(K) dK \ .\end{aligned}$$

In this equation, note that functions $\widetilde{\Phi}_{a,b}$ and $\Phi_{a,b}$ generate the same corridor realized divergence, because Bregman divergence is invariant to affine transformations of the underlying generating function; see Banerjee et al. (2005), among others. As a consequence, the integral representation of $\widetilde{\Phi}_{a,b}(x)$ implied by the above identity gives a natural way to generate and trade realized divergence using a finite number of strikes.

Precisely, given an index set $\iota = \{1, 2, \ldots, M, M+1, \ldots, N-1, N\}$, enumerating the available option prices at time 0, such that $K_1 < \ldots < K_M \leq F_{0,T} < K_{M+1} < \ldots < K_N$, we substitute the two integrals in the definition of $\widetilde{\Phi}_{a,b}$ by their discrete approximation and introduce the following piecewise linear generating function

$$\Phi_{a,b}(x;\iota) := \sum_{i=1}^{M} \Phi''(K_i)(K_i - x)^+ \Delta K_i + \sum_{i=M+1}^{N} \Phi''(K_i)(x - K_i)^+ \Delta K_i \ , \tag{38}$$

where

$$\Delta K_i := \begin{cases} (K_{i+1} - K_{i-1})/2 & \text{if } 1 < i < N \ , \\ (K_2 - K_1) & \text{if } i = 1 \ , \\ (K_N - K_{N-1}) & \text{if } i = N \ . \end{cases} \tag{39}$$

The dynamic delta hedging in the definition of the realized divergence

$$D_n^{\Phi_{a,b}(\cdot;\iota)} := \Phi_{a,b}(F_{T,T};\iota) - \Phi_{a,b}(F_{0,T};\iota) - \sum_{i=1}^{n} \Phi'_{a,b}(F_{i-1,T};\iota)(F_{i,T} - F_{i-1,T}) \ , \tag{40}$$

generated by function $\Phi_{a,b}(\cdot;\iota)$, is computed in a straightforward way, using the left derivative

$$\Phi'(x;\iota) = -\sum_{i=1}^{M} \Phi''(K_i) \mathbb{1}_{\{K_i - x > 0\}} \Delta K_i + \sum_{i=M+1}^{N} \Phi''(K_i) \mathbb{1}_{\{x - K_i > 0\}} \Delta K_i \ . \tag{41}$$

From these findings, we define consistently with Result 2.1 a tradeable divergence swap, generated by function Φ in incomplete option markets.

Definition 2.8. In an incomplete option market, with strike prices K_i indexed by $i \in \iota$, a simple divergence swap generated by function Φ is the swap with floating leg $D_n^{\Phi_{a,b}(\cdot;\iota)}$

given in equation (40) and fixed leg given by

$$S\left(D_1^{\Phi_{a,b}(\cdot;\iota)}\right) = \frac{1}{p_{0,T}}\left(\sum_{i=1}^{M}\Phi''(K_i)P_{0,T}(K_i)\Delta K_i + \sum_{i=M+1}^{N}\Phi''(K_i)C_{0,T}(K_i)\Delta K_i\right) .$$

Besides being consistent with our general approach to trade realized divergence, the tradeable fixed leg in Definition 2.8 is also fully consistent with the definition of VIX by CBOE, which is obtained in our setting starting from a generating function $\Phi(x) = -2\ln(x)$ on a suitable corridor $[a, b]$. Panels (b) and (c) of Figure 3 illustrate the properties of realized corridor power divergence ($q = 0$) and Hellinger realized corridor skewness in complete and incomplete option markets, respectively.

3. Fear Trading

We make use of simple divergence swaps to disaggregate the VIX into the forward price of second-order Hellinger variance and the forward prices of higher-order Hellinger skewness and kurtosis, consistently with the VIX decomposition in Result 2.4. We focus on implied Hellinger variance, skewness and kurtosis, because they span the VIX very exhaustively. We also study Hellinger volatility feedback swaps, because they are additional natural trading strategies creating an exposure to market skewness. Using the corridor swap technology in Section 2.5, we further disaggregate the swap rates and the risk premia of Hellinger variance, skewness, kurtosis and volatility feedback over the support of SPX returns, in order to measure the local excess returns for realized variance, skewness and kurtosis risk over the support of SPX returns. From the excess returns (swap rates) of some of these swaps, we identify and characterize the tradeable components of fear risk premia (the price of fear).

Precisely, we first make use of corridor (VIX and Hellinger) variance swaps, in order to characterize the contribution of higher-order risks to variance excess returns. We then

compare these findings to those of corridor Hellinger skewness, kurtosis and volatility feedback swaps, in order to characterize the distinct asymmetric excess return properties of higher-order swaps. Finally, we link the swap rates and the excess returns of higher-order swaps to non-tradeable indices for the price and the risk premium of fear in the literature, in order to support and further motivate their interpretation as tradeable indicators of fear.

3.1. **Data Set and Corridor Choice.** We use monthly observations of the prices of SPX options, obtained from MarketDataExpress for the sample period from January 1990 until February 2014. Following Engle and Neri (2010), we exclude options with negative bid-ask spreads, with an implied volatility smaller than 0.001 or greater than 9, or with a Gamma greater than 0. Forward prices are computed from the option data through put-call parity, following CBOE (2009). We use this data to compute several monthly time series of the floating legs and the swap rates of corridor divergence swaps, generated by the piecewise linear approximation $\Phi_{a,b}(\cdot, \iota)$ in Eq. (38) for a generating function Φ. Such a linear approximation is defined from the set of observed option strikes without relying on any interpolation or extrapolation. As divergence functions Φ, we consider zero-th order power divergence (the VIX generating function) and the generating functions of Hellinger variance, skewness and quarticity swaps. Moreover, we compute the floating legs and swap rates of corridor versions of Hellinger volatility feedback swaps.

We fix a monthly horizon for all divergence swaps, starting and ending on the third Friday of each month, consistently with the maturity structure of option data. Options are struck at the special opening quotation (SOQ), which brings together the spot and the futures market.[13] The delta hedge in the floating leg of our swaps is performed on a daily frequency, in order to mimic as closely as possible OTC variance swaps, in which

[13]The CME reports significant deviations of the SOQ from the spot open price on around 30 occasions since 2004.

the floating leg is usually defined using daily squared log returns. As shown in Section 2, the implied swap leg of our divergence swaps is independent of the frequency of the hedging strategy. Given that the transaction costs implied by bid-ask spreads in option markets can be large, we compute effective excess payoffs and risk premia for short and long swap positions, using bid and ask prices, respectively, and we compare them to those implied by mid bid-ask prices.

We work with corridor swaps implied by the following corridor choices, $\{[a_i, b_i) : i = 1, \ldots, 11\}$, defined in percentage deviations from zero log moneyness: $(-\infty, -15)$, $[-15, -10)$, $[-10, -6)$, $[-6, -4)$, $[-4, -2)$, $[-2, 0)$, $[0, 2)$, $[2, 4)$, $[4, 6)$, $[6, 10)$ and $[10, \infty)$. This corridor choice implies a convenient trade-off between the degree of disaggregation of divergence risk premia over the support of SPX returns and the statistical properties of corridor divergence payoffs in the corridor. Such a tradeoff arises from the fact that finer corridors are typically associated with the delta-hedged payoffs of a more limited set of out-of-the-money options having strike price inside the given corridor.[14]

Broadie et al. (2009) show that the statistical properties of naked out-of-the-money option payoffs can imply a low accuracy of estimated option risk premia, especially for sufficiently deep out-of-the money options with an abnormally skewed empirical return distribution. Our corridor divergence approach overcomes this important issue in several ways. First, we make use of delta-hedged divergence payoffs that imply a market neutral payoff, as suggested by Broadie et al. (2009). Second, we select corridor widths implying a sufficient granularity of traded option strikes in the corridor. In this way, we can trade corridor divergence based on a sufficiently diversified portfolio of out-of-the-money

[14]As a robustness check, we also apply a slightly finer corridor stratification, which does however not change our results. The results are available in the Online Appendix. By construction, a too fine stratification implies a too low power for estimating accurately divergence risk premia, because of a too low number of traded out-of-the-money options with strike price inside the corridor, so that there is a natural limit to risk premium stratification.

options. Finally, we normalize realized corridor divergence payoffs by the implied leg of total realized divergence, instead of corridor divergence. This allows us to measure additively the contribution of corridor risk premia to total divergence risk premia and has the advantage of avoiding a scaling of corridor payoffs by the small implied legs of corridors that imply large deviations from zero log moneyness. In summary, this approach allows us to disaggregate in an easy way the excess returns of simple divergence swaps over the support of SPX returns, while preserving at the same time the efficiency of the estimation of corridor divergence risk premia.

Figure 4 illustrates some aspects of our corridor divergence methodology for disaggregating divergence risk premia. It also highlights the tradeoff between the choice of the corridor width and the statistical properties of corridor divergence swap payoffs. Panel 4a (Panel 4b) of Figure 4 plots the time series of returns of a short Hellinger skew swap in the corridor $[-10, -6)$ (in the corridor $(-\infty, 0)$). The empirical distribution of corridor skew returns in Panel 4a is generated by the payoffs of a long portfolio of out-of-the-money puts with a percentage log moneyness between -10% and -6%. In the vast majority of the cases, the payoff of the corridor skew swap equals zero and the swap yields a loss of 100%. At the same time, the empirical distribution of positive swap returns contains a number of quite large returns.[15] While this distribution is pronouncedly positively skewed, it is clearly less affected by extreme option returns than the empirical distribution of naked 6% out-of-the-money put returns, consistent with the findings in Broadie et al. (2009). When the corridor is wider or when it contains strikes of options that are less out-of-the-money, as in Panel 4b of Figure 4, the empirical distribution of corridor returns tends to be less dominated by extreme positive returns and by 100% losses. Aggregating further over all corridors, we obtain in Panel 4c the time series of short Hellinger skew returns, which

[15]Note that we plot a return of zero in all cases where no option with positive option price and with a strike price inside the relevant corridor is available for replicating the corridor divergence.

implies an empirical return distribution that is even more well-behaved.[16] Finally, scaling corridor divergence payoffs by implied divergence, instead of corridor implied divergence, allows us to quantify the contribution of corridor risk premia to total divergence risk premia and further regularizes the empirical distribution of corridor divergence payoffs, as shown in Figures IA.4 and IA.5 of the Online Appendix.

With the exception of corridors implied by extreme log moneyness deviations of about $\pm 15\%$ or more, for which the statistical uncertainty might make the estimation of expected corridor excess payoffs challenging for some types of divergence swaps, we conclude that our corridor methodology preserves a sufficient power for identifying corridor divergence risk premia.

3.2. **Power Divergence Risk Premia.** In this section, we first quantify the aggregate risk premia of power divergence swaps, while accounting for the bid-ask spreads in incomplete option markets. In a second step, we disaggregate these risk premia over the support of SPX returns using our corridor methodology. We focus on power divergence swaps of order 0 and 1/2, in order to decompose VIX in a second step into the return contribution of put-call symmetric Hellinger variance, Hellinger skewness and Hellinger kurtosis, respectively.

3.2.1. *Aggregate VIX and Hellinger Variance Risk Premia.* Table 2 summarizes the properties of the unconditional payoff of power divergence swaps, normalized by the price of Hellinger variance, estimated as the sample average of payoff

$$\overline{DIX}_n(q) - \overline{DIX}(q) = (2D_n^q/F_0^q - DIX(q))/DIX(1/2) \ , \tag{42}$$

[16]For comparison, we provide in the Online Appendix the box plots of the empirical distribution of corridor skew returns in Figure 4. We also provide a plot of the time series of corridor skew returns for a more extreme corridor $(-\infty, -10]$.

for $q = 0, 1/2$. By construction, the fixed leg of this payoff is the relative power divergence index $\overline{DIX}(q)$. Moreover, for $q = 1/2$, quantity (42) is the risk premium on Hellinger variance. We compute average normalized excess payoffs from the perspective of both a swap seller and a swap buyer, using mid, bid and ask prices, respectively.

In Table 2, we find that a short position on power divergence swaps executed at mid prices yields an annualized relative excess payoff of 28.34% (29.36%) for $q = 1/2$ ($q = 0$) over the full sample. The average excess payoff in the first (second) half of the sample is only larger (lower). In all cases, the average payoff of VIX-type swaps, evaluated at mid prices, is slightly larger than the one of Hellinger variance swaps.

These findings are the result of two distinct effects of demand and supply for divergence swaps, which can be highlighted using bid and ask divergence swap returns. While traded swap prices are likely strictly inside the bid-ask spread in phases of balanced option demand and supply, a divergence swap return evaluated at bid (ask) prices provides useful information about an upper (lower) bound for the return of a short (long) swap position, which could be more relevant in phases of excess net demand for some particular out-of-the-money put or call options. The effect of bid-ask spreads is large and economically relevant, with average excess payoffs around 6% lower (higher) for short (long) swap positions for VIX-type swaps. This asymmetric effect of bid ask spreads on divergence swap returns is a natural consequence of the larger loading on out-of-the-money puts in the option replicating portfolio of VIX relative to Hellinger variance. Consequently, the larger (smaller) returns for shorting (going long) variance swaps arise systematically for Hellinger swaps, i.e., shorting (going long) VIX-type swaps is on average more (less) expensive in phases of net buying pressure on out-of-the-money puts (calls).

Sharpe ratios for shorting power divergence swaps at mid prices are large and similar across contracts. They are about 49% for Hellinger as well as VIX variance over the

full sample. They have been about 12% higher (5% lower) in the first (second) part of the sample, making short divergence swaps particularly profitable from January 1990 to December 2002. The Sharpe ratios of long (short) swap positions are higher (lower) by about 15% for both Hellinger and VIX-type swaps, showing that bid-ask spreads also amplify the standard deviation of divergence returns. In summary, these findings show that the risk premia for VIX and Hellinger divergence are large, even after accounting for transaction costs, and linked to a similarly asymmetric risk-return trade-off.

3.2.2. *Corridor VIX and Hellinger Variance Risk Premia.* Unconditional differences between VIX and Hellinger variance swap payoffs and Sharpe ratios are rather small, as the leading contribution to the payoff of these swaps derives from realized second moments. However, differences between VIX and Hellinger variance swap rates (payoffs) are linked to the price (risk premium) of realized higher moments. To better isolate these small, but informative discrepancies, we disaggregate the risk premium of VIX and Hellinger variance swap payoffs in Eq. (42), into disjoint regions of the support of SPX returns, using the expected payoffs of VIX and Hellinger corridor variance swaps, based on the corridor technique introduced in Section 2.5.

We split the support of SPX returns into the following disjoint corridors $\{[a_i, b_i) : i = 1, \ldots, 11\}$, defined in percentage deviations from zero log moneyness: $(-\infty, -15)$, $[-15, -10)$, $[-10, -6)$, $[-6, -4)$, $[-4, -2)$, $[-2, 0)$, $[0, 2)$, $[2, 4)$, $[4, 6)$, $[6, 10)$ and $[10, \infty)$. Note that the sum of the swap rates and the floating legs of all these corridor swaps equals the price and the floating leg of divergence over the entire support of SPX returns. Owing to this additivity property, we thus identify more precisely the profitable regions of divergence swap payoffs from the unprofitable ones.

Figure 5 plots average payoffs of VIX and Hellinger corridor variance swaps, in percentage of $DIX(1/2)$ and as a function of the relevant corridor, together with their 95%

confidence interval. As before, we study short swap positions at mid (Panel 5a and 5b) and bid (Panel 5c and 5d) prices, as well as long swap positions at ask prices (Panel 5e and 5f). We find that the (negative) risk premium for VIX and Hellinger variance risk is concentrated in the out-of-the-money put and call payoff region. The premium is significant for log-moneyness deviations of at least about $\pm 2\%$ and less than about -10% from zero, despite the wide confidence intervals on average divergence swap payoffs, and on average it is slightly more negative for VIX variance.

Corridor divergence swap risk premia are asymmetric with respect to log moneyness, and tend to decrease in absolute value, even though non-monotonically, from lower to higher corridors. This suggests a more negative risk premium for VIX and Hellinger variance, conditional on states of low market valuations, which is reflected by more expensive out-of-the-money put portfolios relative to out-of-the-money call portfolios. At the same time, this evidence suggests the presence of a priced fear component in VIX and Hellinger divergence swap rates, which can be isolated more precisely using higher-order divergence swaps.

3.3. **Hellinger Corridor Skew and Kurtosis Risk Premia.** To isolate the price and the risk premium attached to higher-order risks, we make use of corridor Hellinger skew and quarticity (or kurtosis) swaps, which have swap rates and payoffs that uniquely decompose VIX swap rates and risk premia, into the leading contributions of put-call antisymmetric skewness and put-call symmetric quarticity, consistently with the decomposition in Result 2.4. In this way, we disaggregate the risk premium of Hellinger skewness

and Hellinger quarticity over the support of SPX returns, using the same corridors of Section 3.2.2. Precisely, we localize on each corridor the premium of the payoffs

$$\begin{aligned}\overline{SKEW}_n(1/2) - \overline{SKEW}(1/2) &= (SKEW_n(1/2) - SKEW(1/2))/DIX(1/2)\ ,\\ \overline{QUART}_n(1/2) - \overline{QUART}(1/2) &= (QUART_n(1/2) - QUART(1/2))/DIX(1/2)\ ,\end{aligned}$$

by localizing skew and quarticity payoffs across corridors and by normalizing them with Hellinger divergence index.

Figure 9 plots average payoffs of Hellinger corridor skew and quarticity swaps, in percentage of $DIX(1/2)$ and as a function of the relevant corridor, together with their 95% confidence intervals. As above, we focus on short swap positions at mid (Panel 6a and 6b) and bid (Panel 6c and 6d) prices, as well as long swap positions at ask prices (Panel 6e and 6f). We find a clearly positive (a small, but slightly negative) risk premium for Hellinger realized skewness in the out-of-the-money put (call) payoff region. The premium is significant for negative log-moneyness deviations between about -2% and -10% and positive deviations of at least $+2\%$, with confidence intervals on average skew swap excess returns that become substantially wider in states of low market valuations. The (long) skew risk premium is almost monotonically decreasing with log moneyness in the out-of-the-money put payoff region, but it is small in absolute value and almost flat in the out-of-the-money call payoff region, consistently with a large price for buying downside risk insurance and a moderate cost for buying positive skewness.

The risk premium for Hellinger realized quarticity is negative (not different from zero) in the out-of-the-money put (call) payoff region. It is significant for negative log-moneyness deviations of at least -4%, with confidence intervals on average excess returns that as before become clearly wider in states of low market valuations. Compared to Hellinger

skewness, the negative risk premium becomes significant for negative corridors more distant from the at-the-money region. At the same time, no significant corridor risk premium is observed for Hellinger quarticity linked to out-of-the-money call payoffs.

On an aggregate level, this evidence implies an economically important positive (negative) risk premium for long (short) Hellinger skewness and short (long) Hellinger quarticity, which is naturally related to the large price of downside risk insurance. On average, Hellinger variance risk premia evaluated at mid prices are between -3% and -4% in the out-of-the money put region. Hellinger skew (quarticity) risk premia, as a fraction of Hellinger variance, are between -0.01% (-0%) and -0.05% (-0.004%). The fact that Hellinger skew and quarticity payoffs make up such a small fraction of VIX and Hellinger variance payoffs makes it hard to identify the price and the risk premium for fear from tradeable payoffs that have a leading contribution of second moments.

3.4. **Scale-Invariant Hellinger Corridor Skew and Quarticity Risk Premia.** The different scaling of Hellinger variance, skewness and quarticity can be handled more conveniently using scale-invariant Hellinger skewness and quarticity, together with the corresponding corridor swaps. This approach produces a more natural measure of the excess return and the price of realized skewness and kurtosis, respectively.

3.4.1. *The Risk Premium on (scale-invariant) Hellinger Skewness and Kurtosis.* We disaggregate the risk premium of scale-invariant Hellinger skewness and quarticity over the support of SPX returns, using the same corridors of Section 3.2.2. Precisely, we localize on each corridor the premium of the payoffs

$$\begin{aligned} \widetilde{SKEW}_n(1/2) - \widetilde{SKEW}(1/2) &= (SKEW_n(1/2) - SKEW(1/2))/DIX(1/2)^{3/2} \,, \\ \widetilde{QUART}_n(1/2) - \widetilde{QUART}(1/2) &= (QUART_n(1/2) - QUART(1/2))/DIX(1/2)^2 \,. \end{aligned}$$

Figure 7 plots the average payoffs of scale-invariant Hellinger corridor skew and corridor quarticity. Similarly to Section 3.3, we find a clearly positive and large (a negative, but small) risk premium for Hellinger realized skewness in the out-of-the-money put (call) payoff region. The premium is significant for almost all negative log-moneyness deviations and for positive deviations of at least +6%. The risk premium for Hellinger quarticity is negative (negative, but not significant) in the out-of-the-money put (call) payoff region. It is significant for log-moneyness deviations of at least −2%.

Overall, this evidence provides an economically intuitive risk premium behaviour for scale-invariant Hellinger skewness and quarticity, over a wide moneyness range. Compared to scale-invariant Hellinger skewness, the absolute risk premium for scale-invariant Hellinger quarticity in low corridors is about three times the skewness risk premium. For comparison, while the mid-price risk premium for Hellinger variance in low corridors is between −3% and −4%, the risk premium for scale-invariant Hellinger skewness (quarticity) ranges between 4% and 15% (-3% and -40%).

3.4.2. *Implied Hellinger Skewness and Kurtosis.* The fixed leg $\widetilde{SKEW}(1/2)$ ($\widetilde{QUART}(1/2)$) of scale-invariant Hellinger skewness (quarticity) has the natural interpretation of a tradeable measure of statistical forward-neutral, or implied, skewness (kurtosis). Different non-tradeable measures of implied skewness have been proposed in the literature and in practice, such as the CBOE SKEW; see, e.g., the CBOE (2010) SKEW White Paper. Our measures of implied Hellinger skewness and quarticity provide a different, coherent way for quantifying forward-neutral skewness and kurtosis, as the price of tradeable skewness and quarticity swaps.

Figure 8 provides a summary of the time series properties of Hellinger implied skew and kurtosis, together with the CBOE SKEW. We find that while implied Hellinger skew has a correlation of about 85% with CBOE's skew, the two time series have clearly

distinct properties and can attain substantially different values in a number of cases. Moreover, despite the large correlation, $\widetilde{SKEW}(1/2)$ is conceptually different from CBOE SKEW, owing to its direct tradeable interpretation and its theoretical consistency with the implications of deviations from put-call symmetry. Similar to Hellinger implied skewness, Hellinger implied kurtosis $\widetilde{QUART}(1/2)$ is a natural tradeable measure of forward-neutral kurtosis, which is theoretically consistent with the implications of deviations from put-call symmetry. We find that while Hellinger implied kurtosis correlates quite extensively with Hellinger implied skewness, with a correlation of about -90%, the two series are clearly different and nonlinearly related, as highlighted in more detail by their scatter plot in Figure 8.

3.5. **Hellinger Corridor Volatility Feedback Risk Premia.** Asymmetric volatility is a well-known source of unconditional return skewness in many asset pricing models, which has a different origin than the conditional skewness generated, e.g., by asymmetric shocks in SPX returns. Since we can measure and trade realized asymmetric volatility using realized Hellinger volatility feedback, we can also quantify the risk premium and the price of asymmetric volatility.

3.5.1. *The Risk Premium on Hellinger Volatility Feedback.* We disaggregate the risk premium of Hellinger volatility feedback, over the support of SPX returns, based on the same corridors of Section 3.2.2. Precisely, we localize on each corridor the premium of the following payoff, introduced in Remark 2.7

$$\overline{SKEW}_n^{UL}(1/2) := (SKEW_n^{UL}(1/2) - SKEW^{UL}(1/2))/DIX(1/2) \ .$$

Figure 9 collects the average payoffs of Hellinger corridor volatility feedbacks. Interestingly, we find a clearly positive (negative) premium for a long volatility feedback position in the out-of-the-money put (call) payoff region. The premium is significant for virtually

all moneyness deviations of at least ±2%. In contrast to the risk premia for scale-invariant Hellinger skewness, which are almost monotonically decreasing in log moneyness and flat in the out-of-the-money call payoff region, the premia for volatility feedback exhibit a richer $S-$shaped structure. They are large in absolute value already for log-moneyness deviations of about ±2%. Moreover, they tend to flatten out (increase) rapidly for larger negative (positive) moneyness deviations, consistently with the intuition that risk premia for return asymmetries generated by volatility feedbacks are not primarily risk premia for extreme return skewness. In this sense, the risk premia for volatility feedback and for return skewness capture different properties of the market compensation for fear.

3.5.2. *Implied Hellinger Volatility Feedback.* The scaled implied leg $\overline{SKEW}^{UL}(1/2) := SKEW^{UL}(1/2)/DIX(1/2)$ of Hellinger volatility feedback has the natural interpretation of a tradeable scale-invariant measure of the price of asymmetric volatility, which can be naturally compared to measures of implied skewness or kurtosis, such as $\widetilde{SKEW}(1/2)$, CBOE SKEW or $\widetilde{QUART}(1/2)$. Figure 10 provides a summary of the time series properties of Hellinger implied volatility feedback, in comparison to $\widetilde{SKEW}(1/2)$ and CBOE SKEW. The correlation of $\overline{SKEW}^{UL}(1/2)$ with $\widetilde{SKEW}(1/2)$ (CBOE SKEW) is only 49% (35%), highlighting the distinct return higher-moment features captured by volatility feedbacks, relative to other measures of implied skewness. Implied volatility feedback also has a moderate correlation of only -29% with implied Hellinger kurtosis.

3.6. **Implied Corridor Divergence.** The asymmetric risk compensation for corridor divergence, highlighted in the previous sections, illustrates the particular characteristics of different divergence swaps, for creating distinct exposures to second- and higher-order SPX uncertainty. Such distinct properties are naturally related to the implied price of corridor divergence. Similar to the Black-Scholes option-implied volatility, which localizes as a function of moneyness the volatility price of an option, corridor implied divergence

localizes the option price of divergence with respect to moneyness. Figure 11 summarizes the unconditional properties of implied corridor divergence, for power divergence swaps, skew and kurtosis swaps, as well as for volatility feedback swaps.

Implied corridor VIX and Hellinger variance (panels (a) and (b)) attain a maximum for corridors that include the at-the-money strike region, and it decreases almost monotonically away for it, in a slightly asymmetric way, because variance payoffs generated by out-of-the-money puts are on average more expensive than those of out-of-the-money calls. This pattern is naturally linked to the asymmetric profile of corridor variance risk premia, which is tilted to more negative risk premia for out-of-the-money put payoffs, relative to out-of-the money call payoffs.

The implied leg of corridor Hellinger skewness (volatility feedback) in Panel (c) (Panel (d)) is negative (positive) for sufficiently out-of-the-money put (call) corridors. It is almost monotonically increasing with moneyness for skew corridors, while it has a more complex $S-$shape for volatility feedback corridors. This pattern is consistent with the positive (negative) corridor risk premia for Hellinger skews and Hellinger volatility feedbacks in corridors of out-of-the-money put (call) payoffs. The almost monotonically increasing ($S-$shaped) pattern of corridor implied skews (volatility feedbacks) is clearly reflected in the almost monotonically decreasing (reverse $S-$shaped) pattern of corridor skew (volatility feedback) risk premia.

Finally, despite the symmetry of realized Hellinger kurtosis, its implied leg in Panel (e) is monotonically decreasing with moneyness (flat and very small) in the out-of-the-money put (call) payoff region. Naturally, these features are associated with large (small) negative corridor risk premia that increase monotonically (are very flat) with respect to moneyness for out-of-the-money put (call) corridors.

3.7. **Trading Fear Risk Premia and the Price of Fear.** The asymmetric compensation for higher-order risks produced by Hellinger swaps suggests that Hellinger variance, Hellinger skewness, Hellinger quarticity, and Hellinger volatility feedback all correlate in different ways with information related to the price and the risk premium for investor fear. In this section, we produce a more direct interpretation of the prices and the risk premia of investor fear, in terms of traded Hellinger swaps, by linking Hellinger divergence swap rates and excess returns to non-tradeable proxies for the price and the risk premium of investor fear in the literature.

3.7.1. *Non-Tradeable Indices of Investor Fear and Tails.* The fear index (FI) from Bollerslev and Todorov (2011) is an estimate of a weekly risk premium for the ex-ante market compensation of upper vs. lower tail risk. It is estimated based on physical and risk-neutral information about SPX returns, using extreme-value theory methods, as the difference of the risk premia for payoffs in the upper and the lower tail of SPX returns, which are associated with the payoffs of 10% out-of-the-money call and put options.

FI is not explicitly defined as the ex-ante risk premium of a corresponding trading strategy. According to FI's definition, the obvious trading strategy would be a long-short portfolio of 10% out-of-the-money call and put options of one-week maturity. However, the excess returns of such a trading strategy would produce a quite noisy proxy of fear risk premia, because the underlying options are typically difficult to hedge and expire worthless in most cases. Figures IA.1 and IA.1b of the online Appendix illustrate this aspect more concretely, based on a corridor skew swap payoff generated by out-of-the-money put options having a log moneyness of less that -10% and a maturity of one month. Such options have produced only four positive returns over our full sample period, of which the largest (second largest) return was above +2000% (+900%). As a consequence,

FI is not easily tradeable and measurable, using the returns of such portfolios of 10% out-of-the-money options.

Similarly to FI, the tail index (TI) from Bollerslev et al. (2014) is an option-implied measure of the price of downside risk, which is derived from extreme-value theory estimates of the risk-neutral tails of out-of-the-money put options of maturity between 8 and 45 days. As a consequence, TI is also not easily tradeable. In order to obtain tradeable proxies for the price and the risk premium of fear, we follow a different approach, which considers the excess returns and implied legs of divergence swaps that are potentially consistent with the time series properties of FI and TI.

3.7.2. *Fear Risk Premia as (Tradeable) Divergence Risk Premia.* We test in this section whether the payoffs of appropriate simple divergence swaps can be interpreted as measures of realized fear risk. Intuitively, whenever the conditional expectation of simple divergence payoffs correlates with a time-varying fear risk premium proxied by Bollerslev and Todorov (2011)'s FI, we could interpret divergence payoffs themselves as noisy signals of (unobservable) fear risk premia. For this purpose, we first perform monthly predictive regressions of the form

$$Y_{t+1} = \alpha + \beta X_t + \varepsilon_{t+1} \ , \tag{43}$$

where X_t is Bollerslev and Todorov (2011)'s fear index at time t and Y_{t+1} the payoff in $t+1$ of a corridor divergence swap with one-month maturity. Recalling that FI is an estimate of the time-varying premium of 10% out-of-the-money call vs. put payoffs, this corridor approach can highlight in a flexible way the information provided by FI for future realized divergence, generated by out-of-the-money call and put payoff states, respectively. We make use of the family of disjoint corridors in Section 3.2.2 and focus for brevity on VIX

and Hellinger variance, as well as Hellinger skewness and quarticity, but the results for Hellinger volatility feedback are similar.

Figure 12 summarizes the results, by plotting the resulting predictive regression R^2's for the different simple divergence swaps, as a function of the relevant corridor. We find that FI forecasts corridor divergence payoffs in a similar way for all divergence swaps considered, with a $U-$shaped pattern of predictive R^2's around zero log moneyness. Consistently with the construction of FI, the largest significant contribution to predictive R^2's is generated for all divergence swaps by the $\pm 10\%$ out-of-the-money option corridors, even though with significantly different predictive regression coefficients across corridors. This evidence confirms that simple divergence swap payoffs contain useful information about fear risk premia.

The aggregation of divergence payoffs across corridors provides sharper information about which divergence swap better captures tradeable time-varying fear risk premia. Using aggregate payoffs, we can also easily test more formally their role as proxies of divergence risk premia, and vice versa, by testing the null hypothesis $\beta = 1$ in Eq. (44). Table 3 collects the resulting predictive regression results.

The results for Hellinger quarticity and quinticity consistently reject the null hypothesis $\beta = 1$, rejecting the interpretation of FI as a direct proxy for the risk premium of realized quarticity or quinticity. Despite the low power implied by very large confidence intervals, the results based on bid prices for VIX divergence, Hellinger divergence and Hellinger lower semi-variance also reject the null hypothesis $\beta = 1$. In contrast, this null hypothesis cannot be rejected for Hellinger skew and Hellinger volatility feedback swaps, suggesting FI as an observable proxy of time-varying skew or volatility feedback risk premia.

Given these findings, one would also be tempted to interpret skew and volatility feedback risk premia as fear risk premia. When considering in more detail the merit of these

premium interpretations, it is useful to note that fear risk premia are likely only imprecisely identified from realized signals of Hellinger volatility feedback, because of the large and very imprecise point estimate for β obtained. This insight is confirmed by the weak degree of predictability of FI for realized volatility feedback, with a predictive R^2 of 11.68% using mid prices that is less than half the R^2 of 28.13% implied for realized skewness. Based on this evidence, we conclude that simple Hellinger skew swaps are the most appropriate divergence strategies to trade fear risk premia.

3.7.3. *The Price of Fear as (Tradeable) Implied Divergence.* We finally test whether divergence swaps rates can be interpreted as measures of the price of fear, simply by performing monthly linear regressions of the form

$$Y_t = \alpha + \beta X_t + \varepsilon_t \ , \tag{44}$$

where X_t is Bollerslev et al. (2014)'s tail index in at time t and Y_t the implied leg of a divergence swap with one-month maturity. With these regressions, we quantify the fraction of divergence swap rate variation that can be explained by a variation of Bollerslev et al. (2014)'s proxy of the price of fear.

We find very large regression R^2's overall, typically larger than 55%, indicating, as expected, that all divergence swaps rate are sensitive to shocks in the price of fear. Perhaps surprisingly, implied lower Hellinger semi-variance is not the most sharp implied divergence proxy for the price of fear. In contrast, implied Hellinger skew and implied volatility feedback have the largest R^2's, with a largest explanatory power of about 68.5% for implied Hellinger skews, using mid prices. Based on this evidence, we conclude that implied Hellinger skewness can be interpreted as a tradeable proxy of the price of tail risk.

4. Conclusion

Time-varying market uncertainty depends on several distinct features, related to time-varying second- and higher-order risks of market returns, which are challenging to measure and to trade in isolation. In this context, excess returns of variance swaps are often interpreted as a compensation for crash risk, in addition to variance risk. Similarly, measures of implied volatility like the CBOE (2009) VIX are often interpreted as an index of investors' fear. In this paper, we adopt a systematic model-free approach for pricing and trading time-varying second- and higher-order risks. With this approach, we identify convenient tradeable proxies of the price of fear and fear risk premia, which are distinct from the leading contribution of the price of variance to the VIX.

Precisely, we measure market uncertainty with realized divergence and its price with implied divergence. In this way, we obtain a unique decomposition of VIX into the leading variance contribution and additional contributions from higher moments, offering a way to directly assess the price of investors' fear. We introduce simple trading strategies that are applicable at moderate transaction costs, in incomplete option markets with a discrete set of option strikes, which allow us to localize the price and risk premium contributions of realized variance, realized skewness, and realized kurtosis over the support of the distribution of SPX index returns.

These strategies originate from so-called simple Hellinger swaps and allow us to trade variance, skewness and kurtosis in a way that is benchmarked to put-call symmetry and deviations thereof. They are tradeable in the sense that they are implementable with available market instruments, with no inter- or extrapolations that depend on assumptions about the granularity of option strikes, continuous trading, or the absence of transaction costs.

From the excess returns of Hellinger realized variance, Hellinger realized skewness, Hellinger realized kurtosis and realized long-short Hellinger semi-variance (an instrument designed to create exposure to volatility-return feedback), we estimate large unconditional risk premia that are economically and statistically significant, even after transaction costs. However, while the risk premium for realized variance derives from a non-monotonic contribution of both out-of-the money call and put payoffs with respect to log moneyness, the risk premium contribution of realized skewness and realized kurtosis is concentrated in the out-of-the money put payoff region and it is upward and downward sloping, respectively. On the other hand, the risk premium contribution of realized volatility feedback is non-monotonic, dominated by the payoffs of options that are slightly out-of-the-money and it has a different sign for call and put option payoffs.

This asymmetric evidence on the risk premia generated by divergence swaps, suggests the price (risk premium) of some of these swaps as a directly tradeable measure of the leading component of the price (risk premium) for fear. Following this intuition, we study the relation between the price (risk premium) of Hellinger divergence swaps and well-known non-tradeable proxies for the price (risk premium) of fear in the literature, which are constructed from extreme-value theory estimates of the physical and risk-neutral tails of the distribution of SPX returns. Using predictive regressions of the payoffs of Hellinger skew swaps on Bollerslev and Todorov (2011)'s Fear Index, we find that the latter strongly correlates with the time-varying component of skew risk premia, explaining about 30% of the variation of Hellinger skew swap payoffs. Similarly, about 70% of the variation of implied Hellinger skew is explained by variations of Bollerslev et al. (2014)'s Tail Index alone. Based on these findings, we conclude that Hellinger skew swaps are appropriate instruments for trading and pricing fear, which also help to conveniently isolate the leading contribution of the price of fear to the VIX. Finally, using implied Hellinger skew and

implied Hellinger kurtosis, we propose new tradeable indices of skewness and kurtosis, which measure variations in the price of skewness and kurtosis not mechanically related to the price of variance.

Appendix A. Tables and Figures

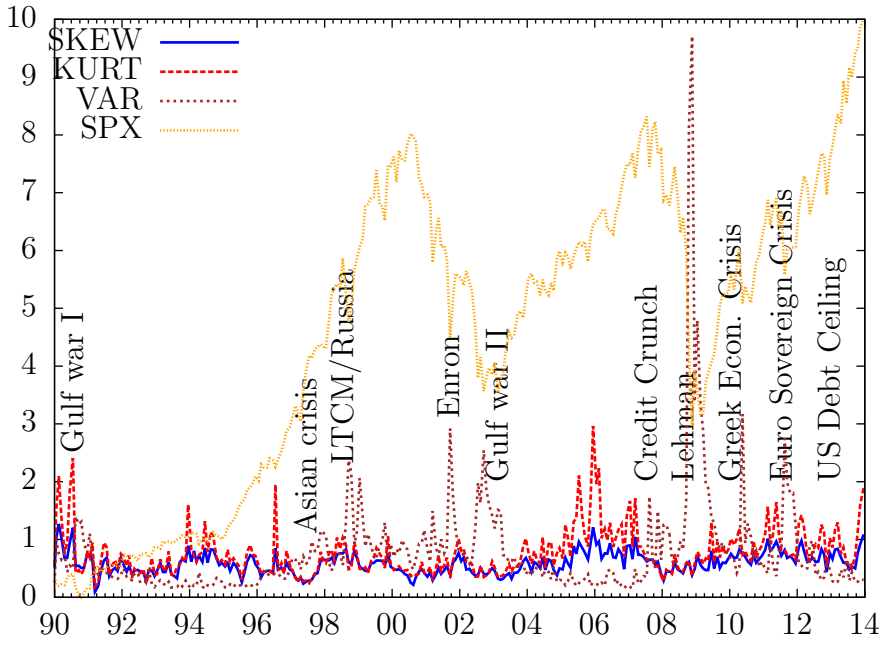
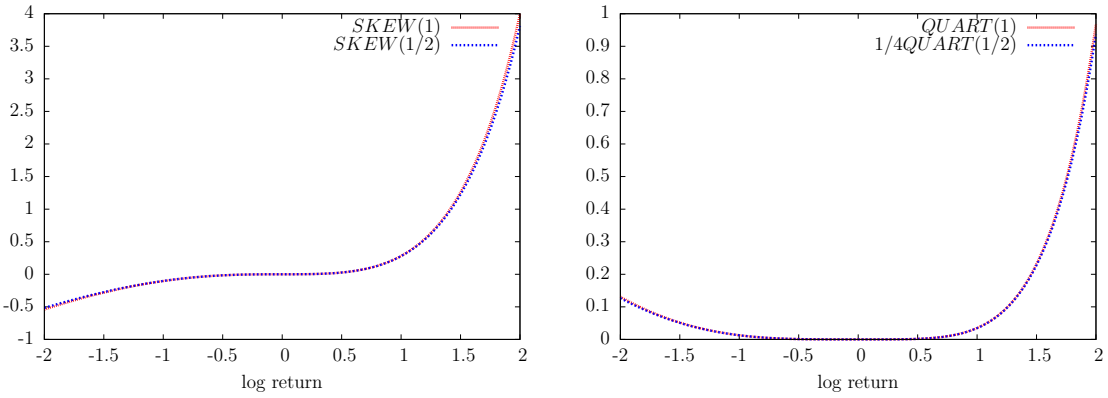
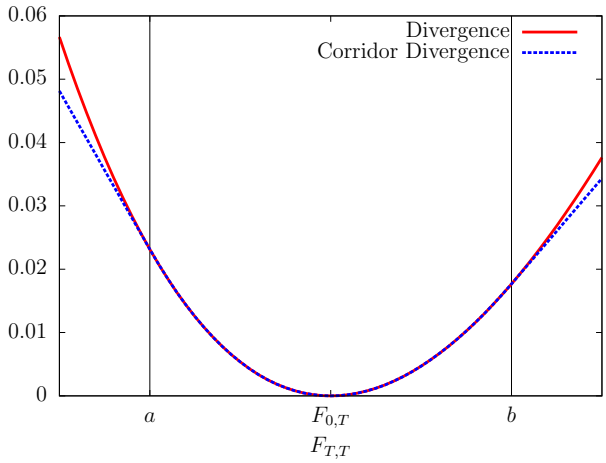


Figure 1. **Different Dimensions of Time-Varying Uncertainty Captured by Implied SPX Variance, Skewness and Kurtosis:** We plot the time series of the SPX index together with the time series of forward prices implied by divergence swaps trading realized variance, skewness and kurtosis, respectively. To compute these forward rates, we make use of SPX forward and option data from January 1990 to February 2014, provided by MarketDataExpress. Details on the definition and computation of tradeable divergence swap rates are given in the main text. For simplicity, all time series are scaled, in order to be displayed in a common plot. Additionally, we highlight in the graph the timing of some important events in our sample period.

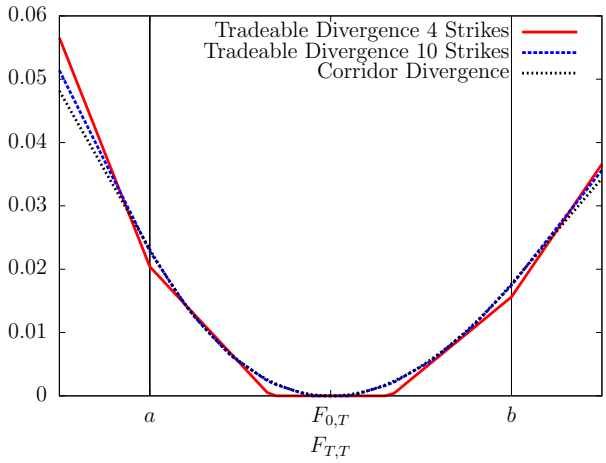


(A) Skewness Payoffs (B) Quarticity Payoffs

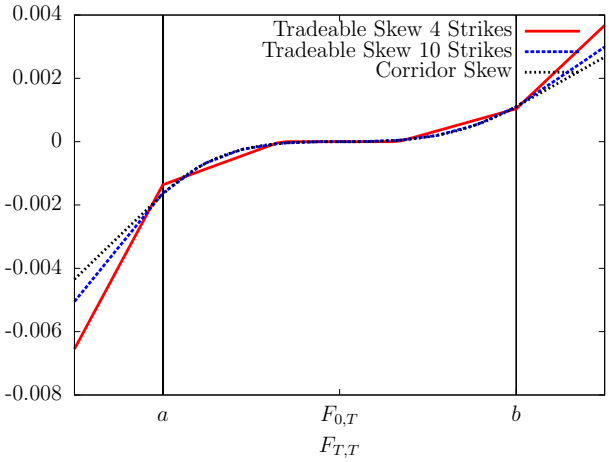
FIGURE 2. **Realized Skew and Quarticity:** We plot the floating leg of skew and quarticity swaps, as a function of log returns. $SKEW(1)$ ($SKEW(1/2)$) corresponds to the floating leg of the $q = 1$ (Hellinger) skew swap fixed leg in equation (26) (equation (27)). $QUART(1)$ ($QUART(1/2)$) corresponds to the floating leg of the $q = 1$ (Hellinger) quarticity swap fixed leg in equation (26) (equation (27)).



(A) Corridor Divergence

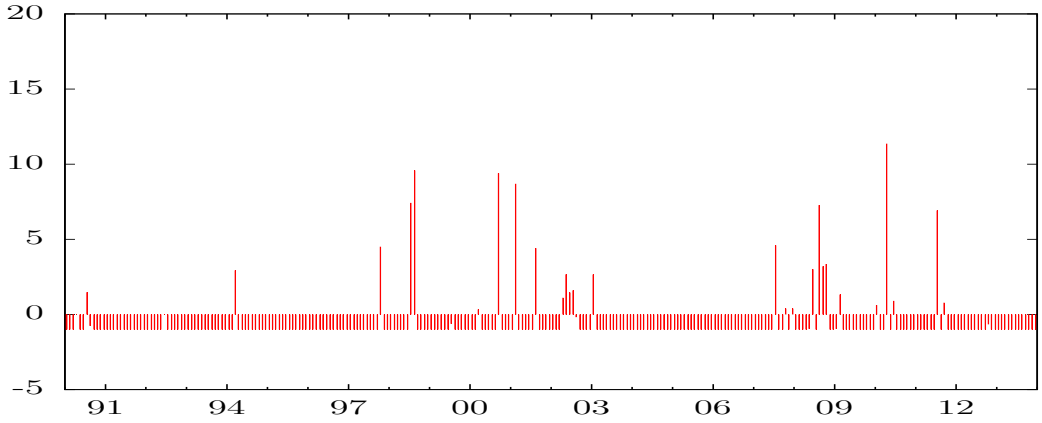


(B) Tradeable Corridor Divergence

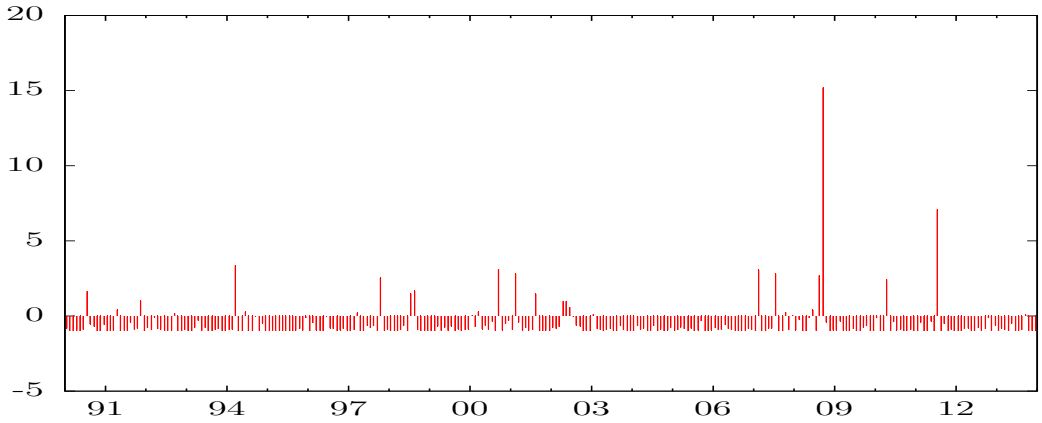


(C) Tradeable Corridor Skew

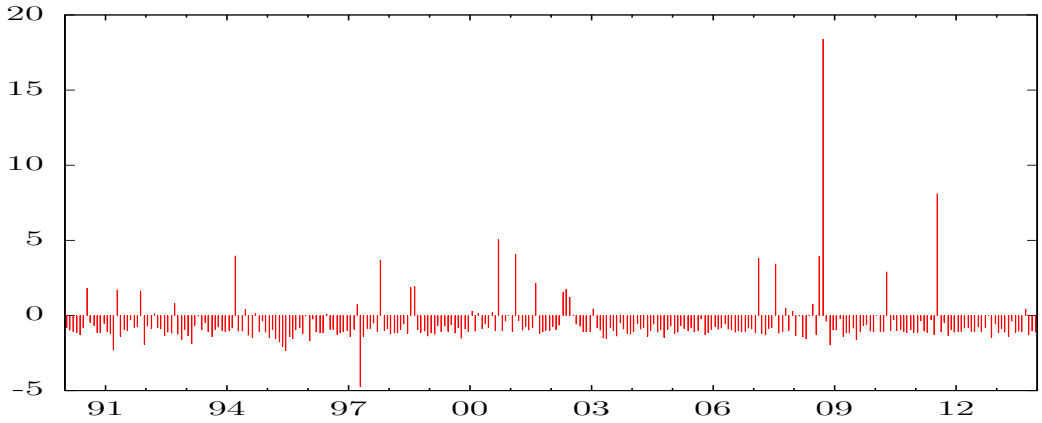
FIGURE 3. **Tradeable Corridor Divergence:** The figure shows different examples of realized corridor divergences, for $F_{0,T} = 1$, $F_{T,T} \in [0.7, 1.3]$ and a corridor $[a, b]$ such that $a = 0.8$ and $b = 1.2$. Panel (a) plots power and power corridor realized divergence for $p = 0$, both generated by the payoff of an option portfolio in complete option markets. Panel (b) plots power and power corridor realized divergence for $p = 0$, both generated by the payoff of an option portfolio in incomplete option markets, with 4 and 10 options, respectively, having equally spaced strikes in interval $[a, b]$. Panel (c) plots Hellinger and Hellinger corridor realized skewness, both generated by the payoff of an option portfolio in incomplete option markets, with 4 and 10 options, respectively, having equally spaced strikes in interval $[a, b]$. Both in Panel (b) and Panel (c), we also plot for comparison the realized corridor payoffs generated by an option portfolio in complete option markets.



(A) Log-Moneyness $[-10, -6)$

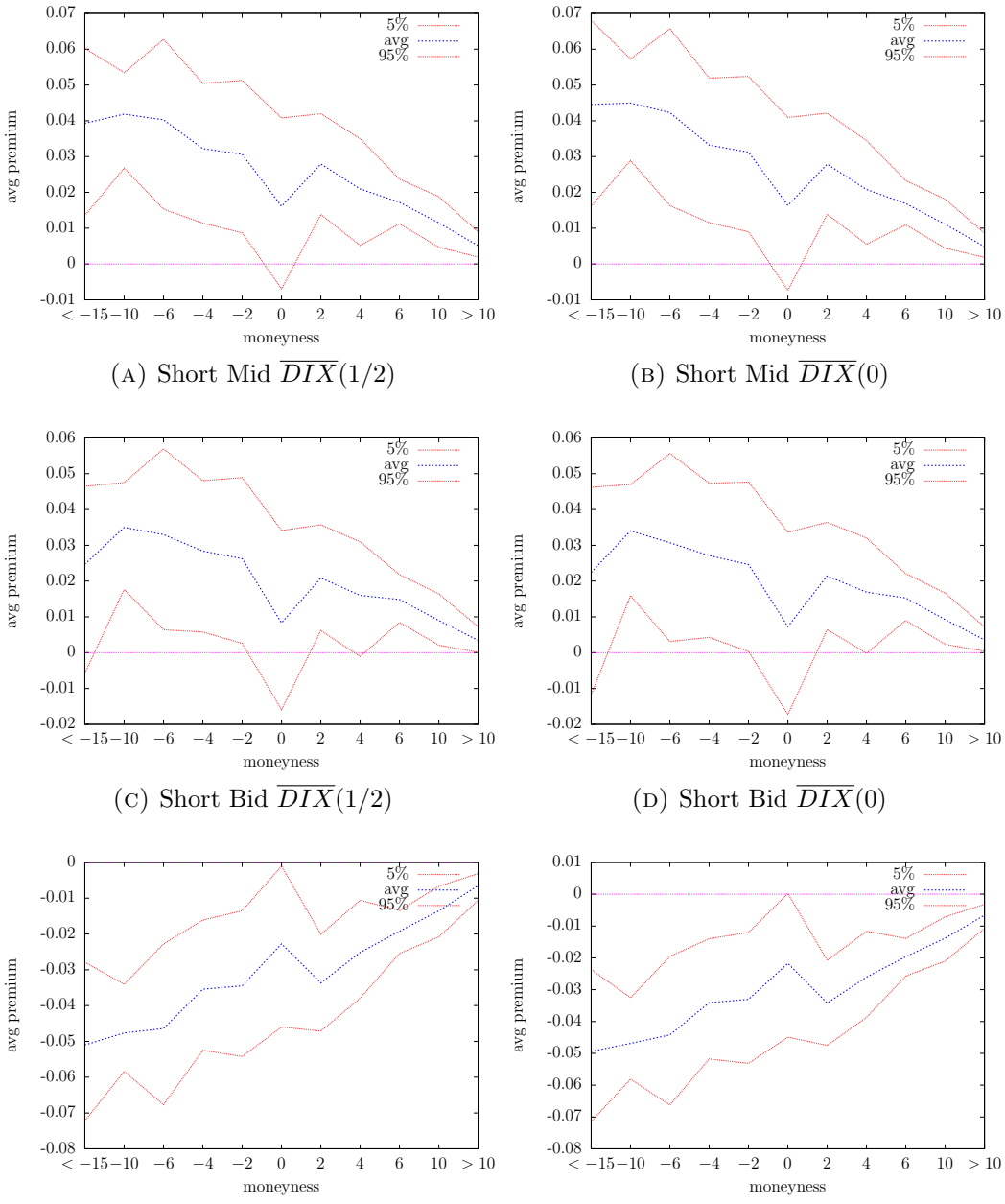


(B) Log-Moneyness $[-\infty, 0)$



(C) All

FIGURE 4. **Returns of Hellinger Corridor Skewness:** This figure plots in Panel (a) (Panel (b)) the time series of returns of a short corridor skew swap localized to all available options in the corridor $[-10, -6)$ $((-\infty, 0))$. Panel (c) plots the time series of returns of a short skew swap based on all available options on each day. The skew swap payoffs are constructed using SPX forward data from January 1990 to February 2014, provided by MarketDataExpress.

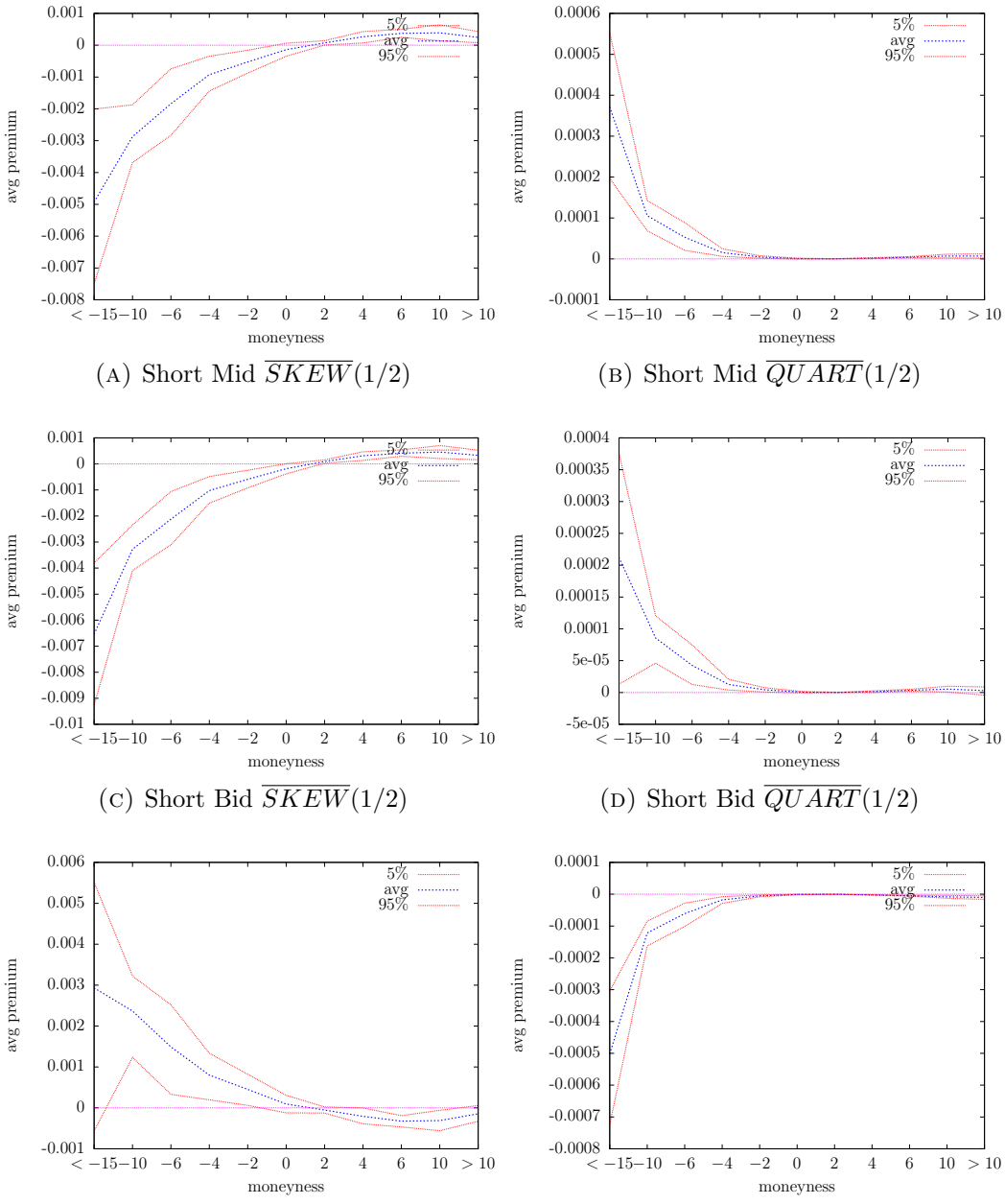


(A) Short Mid $\overline{DIX}(1/2)$ (B) Short Mid $\overline{DIX}(0)$

(C) Short Bid $\overline{DIX}(1/2)$ (D) Short Bid $\overline{DIX}(0)$

(E) Long Ask $\overline{DIX}(1/2)$ (F) Long Ask $\overline{DIX}(0)$

FIGURE 5. **VIX and Hellinger Corridor Divergence:** The charts present average monthly payoffs of corridor VIX and Hellinger variance swaps, normalized by $DIX(1/2)$. On the x-axis, the support of SPX log returns is stratified into 11 corridors of log moneyness. For each corridor, the excess return is computed from the payoff of corridor VIX and Hellinger variance swaps, based on the tradeable corridor swaps specified in Section 2.6. For instance, the average premium for moneyness -10% corresponds to the swap with corridor $[-15\%, -10\%)$. Similarly, the average premium for moneyness less than -15% (more than 10%) corresponds to the corridor $(-\infty, -15\%]$ ($[10\%, \infty)$). In the different subpanels, "Mid", "Bid", and "Ask" indicates short swap positions evaluated at mid prices, short swap positions evaluated at bid prices and long swap positions evaluated at ask prices, respectively. The data are SPX forward and option data from January 1990 to February 2014, provided by MarketDataExpress. Realized divergence is computed at a daily frequency. 95% confidence interval bands are obtained from a block bootstrap with 25,000 replications.

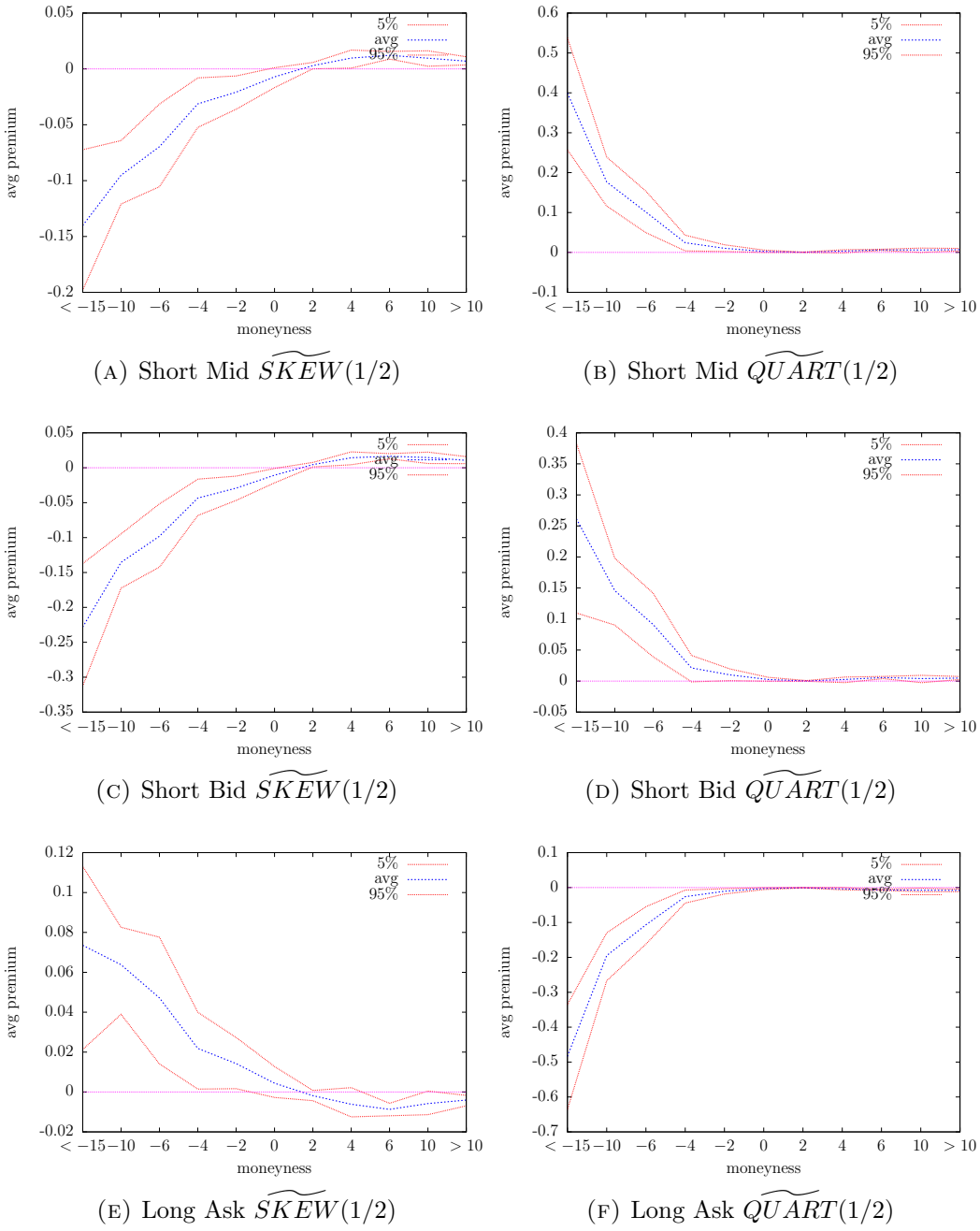


(A) Short Mid $\overline{SKEW}(1/2)$ (B) Short Mid $\overline{QUART}(1/2)$

(C) Short Bid $\overline{SKEW}(1/2)$ (D) Short Bid $\overline{QUART}(1/2)$

(E) Long Ask $\overline{SKEW}(1/2)$ (F) Long Ask $\overline{QUART}(1/2)$

FIGURE 6. **Hellinger Corridor Skew and Quarticity:** The charts present average monthly payoffs of Hellinger skewness and quarticity swaps, normalized by $DIX(1/2)$. On the x-axis, the support of SPX log returns is stratified into 11 corridors of log moneyness. For each corridor, the excess payoff is computed from the payoff of corridor Hellinger skewness and quarticity swaps, based on the tradeable corridor swaps specified in Section 2.6. For instance, the average premium for moneyness -10% corresponds to the swap with corridor $[-15\%, -10\%)$. Similarly, the average premium for moneyness less than -15% (more than 10%) corresponds to the corridor $(-\infty, -15\%]$ ($[10\%, \infty)$). In the different subpanels, "Mid", "Bid", and "Ask" indicates short swap positions evaluated at mid prices, short swap positions evaluated at bid prices and long swap positions evaluated at ask prices, respectively. The data are SPX forward and option data from January 1990 to February 2014, provided by MarketDataExpress. Realized divergence is computed at a daily frequency. 95% confidence interval bands are obtained from a block bootstrap with 25,000 replications.

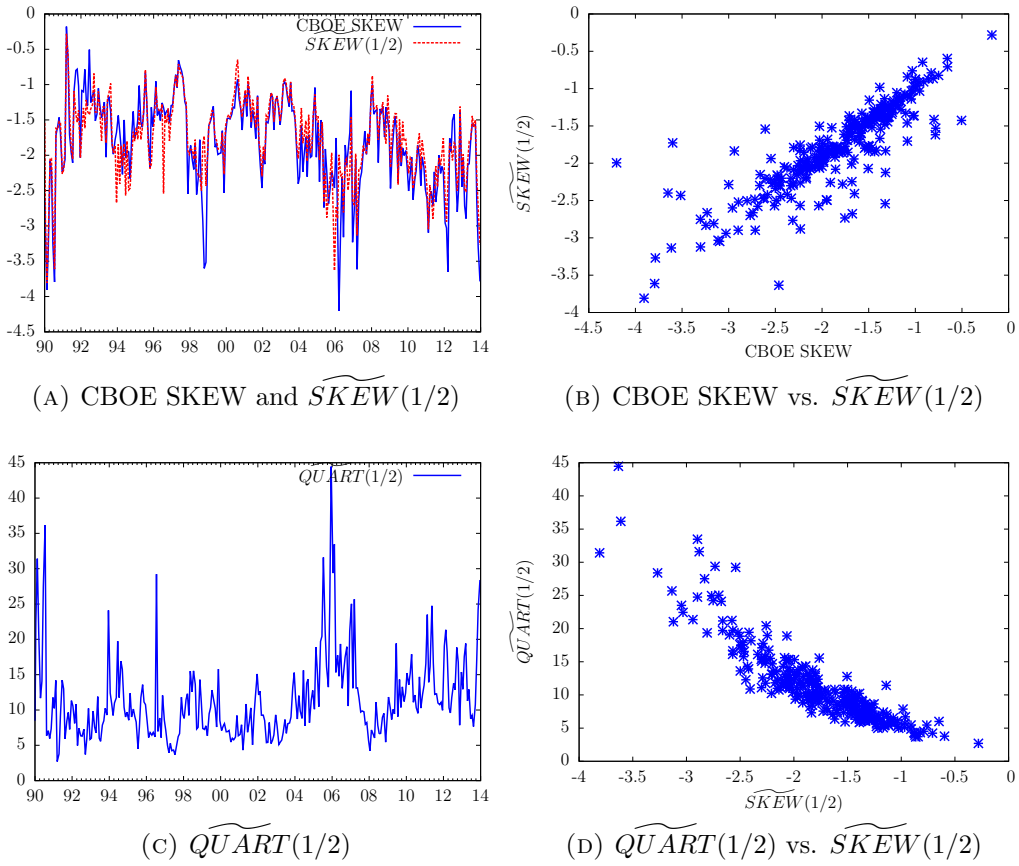


(A) Short Mid $\widetilde{SKEW}(1/2)$ (B) Short Mid $\widetilde{QUART}(1/2)$

(C) Short Bid $\widetilde{SKEW}(1/2)$ (D) Short Bid $\widetilde{QUART}(1/2)$

(E) Long Ask $\widetilde{SKEW}(1/2)$ (F) Long Ask $\widetilde{QUART}(1/2)$

FIGURE 7. **Scale-Invariant Hellinger Corridor Skew and Quarticity:** The charts present average monthly payoffs of Hellinger skewness (quarticity) swaps, normalized by $DIX(1/2)^{3/2}$ ($DIX(1/2)^2$). On the x-axis, the support of SPX log returns is stratified into 11 corridors of log moneyness. For each corridor, the excess payoff is computed from the payoff of corridor VIX and Hellinger variance, Hellinger skewness and Hellinger quarticity swaps, based on the tradeable corridor swaps specified in Section 2.6. For instance, the average premium for moneyness -10% corresponds to the swap with corridor $[-15\%, -10\%)$. Similarly, the average premium for moneyness less than -15% (more than 10%) corresponds to the corridor $(-\infty, -15\%]$ ($[10\%, \infty)$). In the different subpanels, "Mid", "Bid", and "Ask" indicates short swap positions evaluated at mid prices, short swap positions evaluated at bid prices and long swap positions evaluated at ask prices, respectively. The data are SPX forward and option data from January 1990 to February 2014, provided by MarketDataExpress. Realized divergence is computed at a daily frequency. 95% confidence interval bands are obtained from a block bootstrap with 25,000 replications.



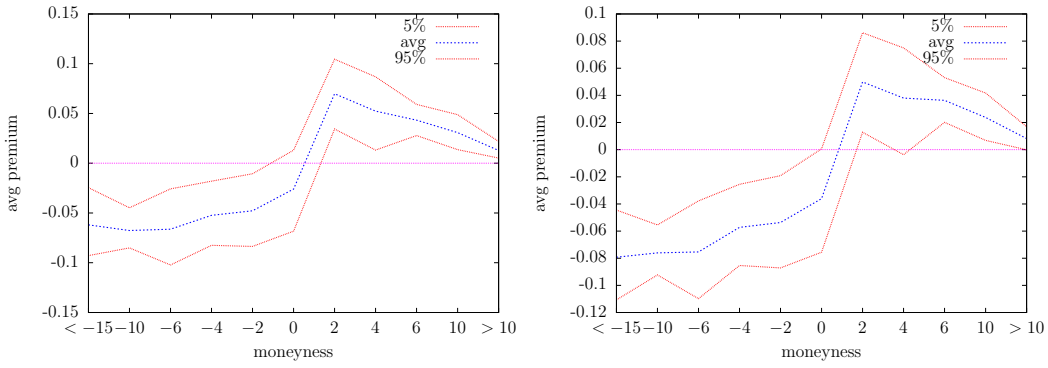
(A) CBOE SKEW and $\widetilde{SKEW}(1/2)$ (B) CBOE SKEW vs. $\widetilde{SKEW}(1/2)$

(C) $\widetilde{QUART}(1/2)$ (D) $\widetilde{QUART}(1/2)$ vs. $\widetilde{SKEW}(1/2)$

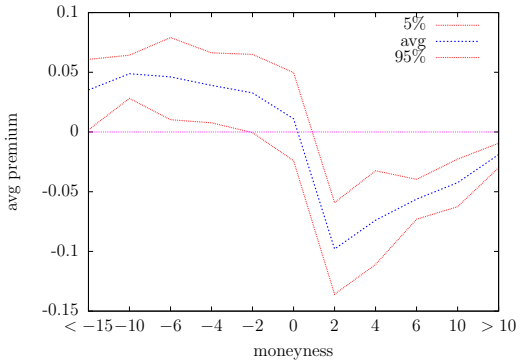
FIGURE 8. **Implied Hellinger Skewness and Kurtosis:** The figure illustrates the properties of monthly implied Hellinger Skew and Kurtosis. Panel (a) compares the time series of the Implied Hellinger skew $\widetilde{SKEW}(1/2)$ in equation (31) with (for better comparability) the time series of an affine transformation of the CBOE SKEW, defined by

$$CBOE\ SKEW := (100 - Z)/10 = \frac{\mathbb{E}_0^{\mathbb{Q}_T}\left[\left(R_{0,T} - \mathbb{E}_0^{\mathbb{Q}_T}\left[R_{0,T}\right]\right)^3\right]}{\left(\mathbb{E}_0^{\mathbb{Q}_T}\left[R_{0,T}\right] - \mathbb{E}_0^{\mathbb{Q}_T}\left[R_{0,T}\right]^2\right)^{3/2}},$$

where Z is the skew measure published by the CBOE and $R_{t,T} := \log(F_{T,T}/F_{0,T})$. Panel (c) plots the time series of implied Hellinger kurtosis $\widetilde{QUART}(1/2)$ in equation (31). Panel (b) (Panel (d)) exhibits a scatter plot of $\widetilde{SKEW}(1/2)$ and the CBOE SKEW ($\widetilde{QUART}(1/2)$). The data are SPX forward and option data from January 1990 to February 2014, provided by MarketDataExpress.

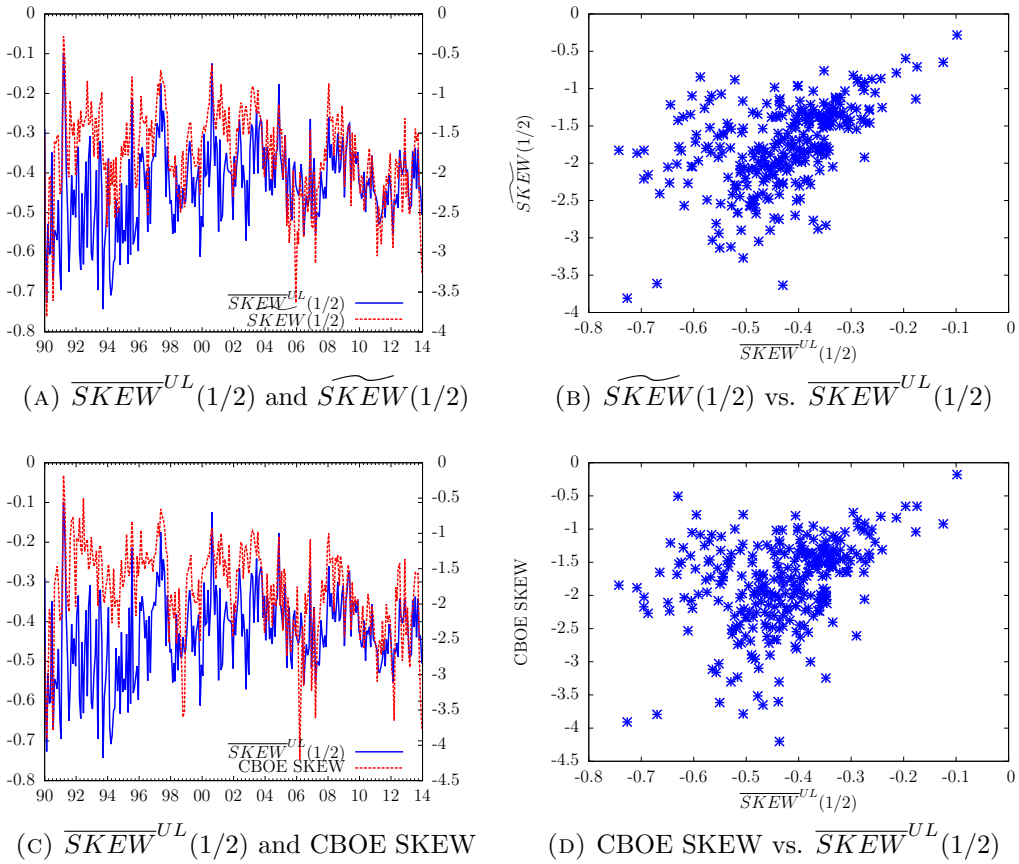


(A) Short Mid $\overline{SKEW}^{UL}(1/2)$ (B) Short Bid $\overline{SKEW}^{UL}(1/2)$



(C) Long Ask $\overline{SKEW}^{UL}(1/2)$

FIGURE 9. **Hellinger Volatility Feedback:** The figure plots average monthly payoffs of Hellinger volatility feedback swaps, normalized by $DIX(1/2)$. On the x-axis, the support of SPX log returns is stratified into 11 corridors of log moneyness. For each corridor, the excess payoff is computed from the payoff of corridor Hellinger volatility feedbacks, based on the tradeable corridor swaps specified in Section 2.6. For instance, the average premium for moneyness -10% corresponds to the swap with corridor $[-15\%, -10\%)$. Similarly, the average premium for moneyness less than -15% (more than 10%) corresponds to the corridor $(-\infty, -15\%]$ ($[10\%, \infty)$). In the different subpanels, "Mid", "Bid", and "Ask" indicates short swap positions evaluated at mid prices, short swap positions evaluated at bid prices and long swap positions evaluated at ask prices, respectively. The data are SPX forward and option data from January 1990 to February 2014, provided by MarketDataExpress. Realized divergence is computed at a daily frequency. 95% confidence interval bands are obtained from a block bootstrap with 25,000 replications.



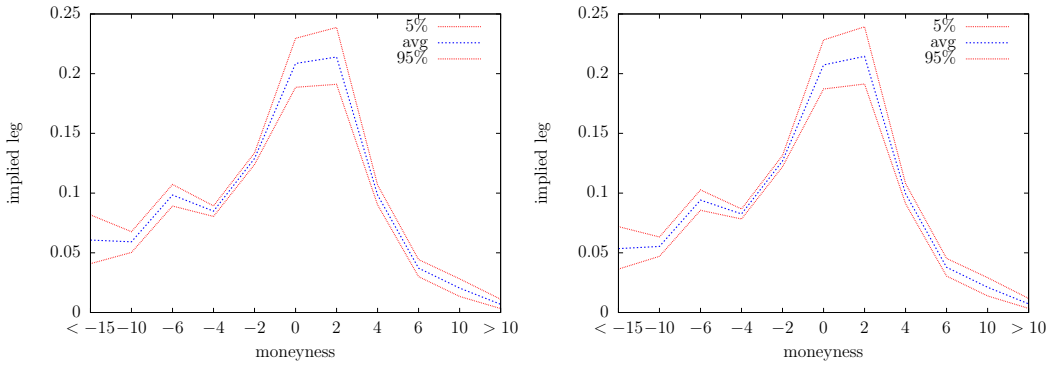
(A) $\overline{SKEW}^{UL}(1/2)$ and $\widetilde{SKEW}(1/2)$

(B) $\widetilde{SKEW}(1/2)$ vs. $\overline{SKEW}^{UL}(1/2)$

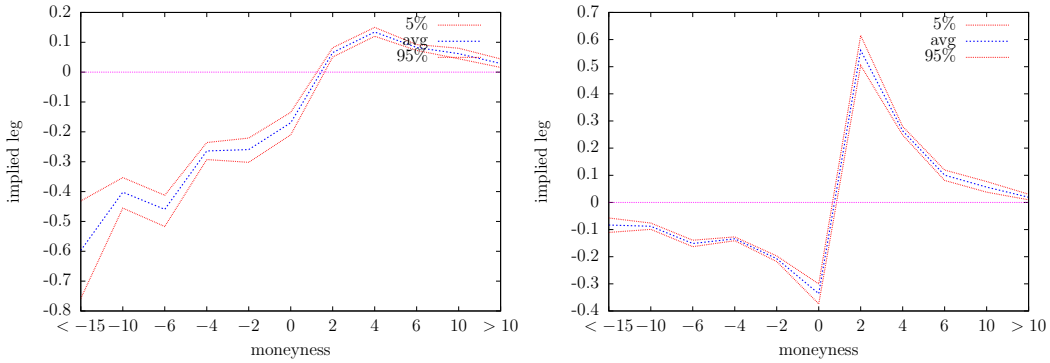
(C) $\overline{SKEW}^{UL}(1/2)$ and CBOE SKEW

(D) CBOE SKEW vs. $\overline{SKEW}^{UL}(1/2)$

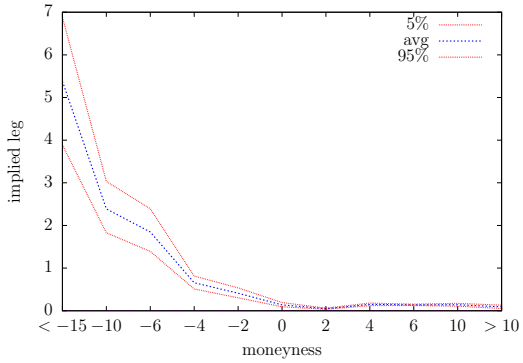
FIGURE 10. **Implied Hellinger Volatility Feedback:** The figure illustrates the properties of monthly implied Hellinger volatility feedbacks. Panel (a) plots the time series of implied Hellinger volatility feedbacks $\overline{SKEW}^{UL}(1/2)$ (left y-axis), defined in equation (37) and $\widetilde{SKEW}(1/2)$ (right y-axis). Panel (b) exhibits a scatter plot of the same data. Panel (c) plots the time series of implied Hellinger volatility feedback skew $\overline{SKEW}^{UL}(1/2)$ (left y-axis) and CBOE SKEW (right y-axis), whereas panel (d) exhibits a scatter plot of the same data. The data are SPX forward and option data from January 1990 to February 2014, provided by MarketDataExpress.



(A) $\overline{DIX}(0)$ (B) $\overline{DIX}(1/2)$

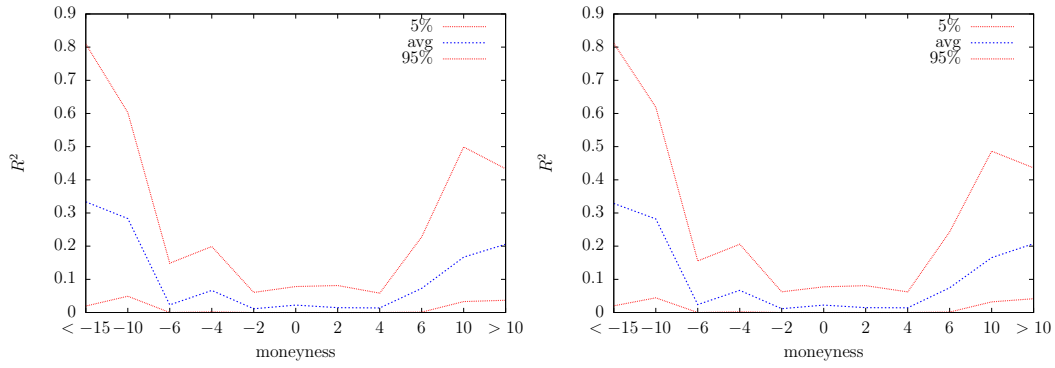


(C) $\widetilde{SKEW}(1/2)$ (D) $\overline{SKEW}^{UL}(1/2)$

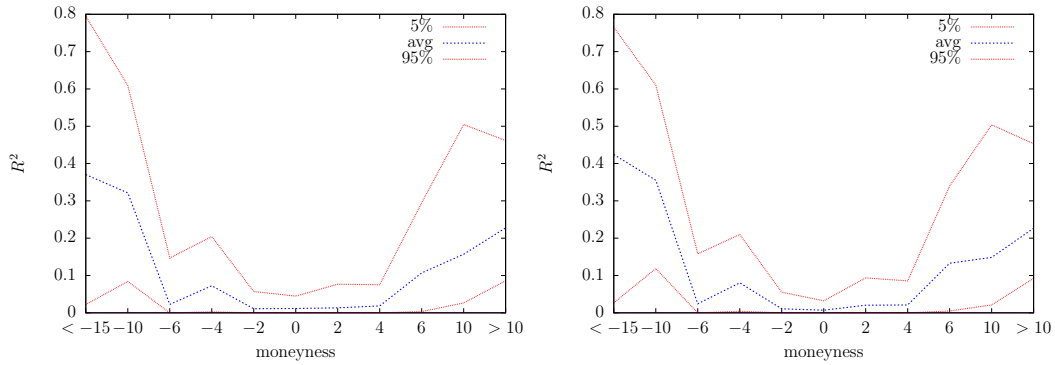


(E) $\widetilde{QUART}(1/2)$

FIGURE 11. **Implied Divergence:** The figure plots average normalized swap rates of different corridor divergence swaps: power divergence (panels (a) and (b)), Hellinger skew (panel (c)), Hellinger volatility feedback (panel (d)), and Hellinger quarticity (panel (e)). On the x-axis, the support of SPX log returns is stratified into 11 corridors of log moneyness. For each corridor, the average swap rate is computed from the implied leg of corridor divergence swaps, based on the tradeable corridor swaps specified in Section 2.6. For instance, the average swap rate for moneyness -10% corresponds to the corridor swap with corridor $[-15\%, -10\%)$. Similarly, the average swap rate for moneyness less than -15% (more than 10%) corresponds to the corridor $(-\infty, -15\%]$ ($[10\%, \infty)$). For brevity, all swap rates evaluated at mid prices. The data are SPX forward and option data from January 1990 to February 2014, provided by MarketDataExpress. 95% confidence interval bands are obtained from a block bootstrap with 25,000 replications.



(A) FI - Corridor $DIX(0)$ (B) FI - Corridor $DIX(1/2)$



(C) FI - Corridor $SKEW(1/2)$ (D) FI - Corridor $QUART(1/2)$

FIGURE 12. **Predictive Regression of Corridor Realized Divergence on the Fear Index:** We plot the predictive R^2s implied by a predictive regression of monthly payoffs of corridor divergence on Bollerslev and Todorov (2011)'s Fear Index. On the x-axis, the support of SPX log returns is stratified into 11 corridors of log moneyness. For each corridor, the excess payoff is computed from the payoff of corridor VIX and Hellinger variance, Hellinger skewness and Hellinger quarticity swaps, based on the tradeable corridor swaps specified in Section 2.6 and using mid bid-ask prices fro brevity. For instance, the average premium for moneyness -10% corresponds to the swap with corridor $[-15\%, -10\%)$. Similarly, the average premium for moneyness less than -15% (more than 10%) corresponds to the corridor $(-\infty, -15\%]$ ($[10\%, \infty)$). The data are SPX forward and option data from January 1990 to February 2014, provided by MarketDataExpress. Realized divergence is computed at a daily frequency. 95% confidence interval bands are obtained from a block bootstrap with 25,000 replications.

Time	Position	Forw. Price in t_i	Payoff in $t_n = T$
t_0	long D_1^Φ	$S(D_1^\Phi)$	$D_1^\Phi - S(D_1^\Phi)$
t_1	lo. $\Phi'(F_{0,T})$ fw. on S_T	$\Phi'(F_{0,T})F_{1,T}$	$\Phi'(F_{0,T})(F_{T,T} - F_{1,T})$
	sh. $\Phi'(F_{1,T})$ fw. on S_T	$\Phi'(F_{1,T})F_{1,T}$	-$\Phi'(F_{1,T})(F_{T,T} - F_{1,T})$
t_2	lo. $\Phi'(F_{1,T})$ fw. on S_T	$\Phi'(F_{1,T})F_{2,T}$	$\Phi'(F_{1,T})(F_{T,T} - F_{2,T})$
	sh. $\Phi'(F_{2,T})$ fw. on S_T	$\Phi'(F_{2,T})F_{2,T}$	-$\Phi'(F_{2,T})(F_{T,T} - F_{2,T})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t_{n-1}	lo. $\Phi'(F_{n-2,T})$ fw. on S_T	$\Phi'(F_{n-2,T})F_{n-1,T}$	$\Phi'(F_{n-2,T})(F_{T,T} - F_{n-1,T})$
	sh. $\Phi'(F_{n-1,T})$ fw. on S_T	$\Phi'(F_{n-1,T})F_{n-1,T}$	-$\Phi'(F_{n-1,T})(F_{n,T} - F_{n-1,T})$
Total	–	–	$D_n^\Phi - S(D_n^\Phi)$

TABLE 1. **Structure of Simple Divergence Trading Strategy:** This table illustrates the structure of the simple trading strategy replicating the realized divergence payoff in Eq. (5), for an investment horizon $[0, T]$ and a sampling frequency defined by trading dates $0 = t_0 < t_1 < \cdots < t_n = T$. We denote by $P_{t,T}(K)$ ($C_{t,T}(K)$) the price at time $0 \leq t \leq T$ of a European put (call) option on underlying S, with strike K and maturity T. The price of a zero coupon bond at time 0 with maturity T is denoted by $p_{0,T}$. The payoff of the replicating strategy at time T is obtained by summing the payoffs in the last column. Precisely, the definition of realized divergence gives

$$\begin{aligned} D_n^\Phi - S(D_n^\Phi) &= \sum_{i=1}^n \{\Phi(F_{i,T}) - \Phi(F_{i-1,T}) - \Phi'(F_{i-1,T})(F_{i,T} - F_{i-1,T})\} - S(D_1^\Phi) \\ &= \Phi(F_{T,T}) - \Phi(F_{0,T}) - \sum_{i=1}^n \Phi'(F_{i-1,T})(F_{i,T} - F_{i-1,T}) - S(D_1^\Phi) \ . \end{aligned}$$

Recalling that $D_1^\Phi = \Phi(F_{T,T}) - \Phi(F_{0,T}) - \Phi'(F_{0,T})(F_{T,T} - F_{0,T})$, it follows

$$\begin{aligned} D_n^\Phi - S(D_n^\Phi) &= D_1^\Phi - S(D_1^\Phi) + \Phi'(F_{0,T})(F_{T,T} - F_{1,T}) - \sum_{i=2}^n \Phi'(F_{i-1,T})(F_{i,T} - F_{i-1,T}) \\ &= D_1^\Phi - S(D_1^\Phi) + \Phi'(F_{0,T})(F_{T,T} - F_{1,T}) - \sum_{i=2}^n \Phi'(F_{i-1,T})((F_{T,T} - F_{i-1,T}) - (F_{T,T} - F_{i,T})) \\ &= D_1^\Phi - S(D_1^\Phi) + \sum_{i=1}^{n-1} (\Phi'(F_{i-1,T}) - \Phi'(F_{i,T}))(F_{T,T} - F_{i,T}) \ . \end{aligned}$$

The first (second) term on the right hand side of the last equality is the payoff generated by the static (dynamic) option replicating portfolio (trading strategy in the forward market). Together, these two payoffs produce the payoff identity derived in the last column of the table above. Consistently with Result 2.1, we also have:

$$D_1^\Phi - S(D_1^\Phi) = \int_0^{F_{0,T}} \Phi''(K) \left(P_{T,T}(K) - \frac{P_{0,T}(K)}{p_{0,T}} \right) dK + \int_{F_{0,T}}^\infty \Phi''(K) \left(C_{T,T}(K) - \frac{C_{0,T}(K)}{p_{0,T}} \right) dK \ ,$$

which identifies the weights and the payoff of the static option replicating portfolio. Similarly,

$$-\sum_{i=1}^n (\Phi'(F_{i-1,T}) - \Phi'(F_{0,T}))(F_{i,T} - F_{i-1,T}) = \sum_{i=1}^{n-1} (\Phi'(F_{i-1,T}) - \Phi'(F_{i,T}))(F_{T,T} - F_{i,T}) \ ,$$

which identifies the self-financed dynamic replicating strategy in the forward market.

Returns	bid	mid	ask
		Entire Sample	
$\overline{DIX}(1/2)$	22.00 (12.50,29.85)	28.34 (19.99,35.47)	-33.64 (-40.15,-26.10)
$\overline{DIX}(0)$	21.21 (11.15,29.57)	29.35 (20.55,36.72)	-32.98 (-39.87,-24.92)
$\overline{SKEW}^{UL}(1/2)$	-20.64 (-28.54,-11.93)	-14.29 (-22.92,-4.73)	6.83 (-3.48,16.33)
$\widetilde{SKEW}(1/2)$	-39.86 (-48.72,-29.94)	-32.30 (-41.79,-21.22)	21.98 (8.94,32.36)
$\widetilde{QUART}(1/2)$	54.98 (35.33,71.13)	73.64 (55.88,91.50)	-85.21 (-104.15,-67.64)
		First Half	
$\overline{DIX}(1/2)$	24.03 (16.74,30.83)	28.96 (21.94,35.50)	-33.25 (-39.45,-26.68)
$\overline{DIX}(0)$	23.61 (16.18,30.62)	30.05 (22.94,36.62)	-32.82 (-39.16,-25.98)
$\overline{SKEW}^{UL}(1/2)$	-22.93 (-34.03,-11.05)	-18.16 (-30.03,-5.59)	12.40 (-1.25,25.30)
$\widetilde{SKEW}(1/2)$	-38.64 (-46.38,-30.44)	-33.46 (-41.62,-24.45)	26.92 (16.95,36.13)
$\widetilde{QUART}(1/2)$	54.52 (46.57,62.44)	64.96 (55.60,74.72)	-72.28 (-83.28,-62.27)
		Second Half	
$\overline{DIX}(1/2)$	20.07 (3.22,33.84)	27.76 (12.96,39.67)	-34.04 (-44.91,-20.84)
$\overline{DIX}(0)$	18.96 (1.09,33.29)	28.70 (13.43,41.26)	-33.05 (-44.46,-18.95)
$\overline{SKEW}^{UL}(1/2)$	-18.45 (-28.08,-7.03)	-10.91 (-21.38,1.69)	1.69 (-12.39,13.21)
$\widetilde{SKEW}(1/2)$	-41.02 (-54.91,-24.48)	-31.13 (-46.41,-12.03)	17.28 (-5.10,34.14)
$\widetilde{QUART}(1/2)$	55.60 (19.45,83.19)	81.87 (49.74,110.98)	-97.48 (-127.04,-67.68)
Sharpe Ratios x 100			
		Entire Sample	
$\overline{DIX}(1/2)$	35.10 (14.02,62.46)	49.67 (25.85,80.42)	-63.78 (-97.69,-37.98)
$\overline{DIX}(0)$	32.61 (12.03,59.88)	49.30 (25.60,80.50)	-60.27 (-94.57,-34.32)
$\overline{SKEW}^{UL}(1/2)$	-31.04 (-48.95,-15.40)	-20.17 (-36.81,-5.84)	9.11 (-3.71,24.05)
$\widetilde{SKEW}(1/2)$	-61.38 (-104.08,-31.85)	-44.14 (-82.87,-18.69)	26.60 (6.11,59.45)
$\widetilde{QUART}(1/2)$	56.22 (17.36,129.59)	80.68 (36.83,142.65)	-99.00 (-149.80,-56.06)
		First Half	
$\overline{DIX}(1/2)$	48.16 (28.11,74.31)	62.10 (39.61,92.25)	-75.73 (-109.29,-50.67)
$\overline{DIX}(0)$	46.20 (26.59,72.50)	62.59 (40.03,92.42)	-72.79 (-106.11,-48.30)
$\overline{SKEW}^{UL}(1/2)$	-38.21 (-61.75,-17.36)	-28.27 (-50.84,-7.97)	18.36 (-1.48,39.53)
$\widetilde{SKEW}(1/2)$	-80.42 (-116.42,-53.87)	-63.90 (-96.08,-40.21)	46.75 (24.67,76.82)
$\widetilde{QUART}(1/2)$	108.08 (80.79,147.46)	128.42 (101.30,166.62)	-138.98 (-175.00,-113.27)
		Second Half	
$\overline{DIX}(1/2)$	29.92 (2.88,75.36)	45.10 (13.75,97.81)	-59.86 (-118.42,-24.60)
$\overline{DIX}(0)$	27.26 (0.86,71.49)	44.51 (12.96,97.99)	-55.92 (-114.55,-21.01)
$\overline{SKEW}^{UL}(1/2)$	-26.81 (-52.16,-7.57)	-15.04 (-36.82,1.76)	3.11 (-10.83,20.66)
$\widetilde{SKEW}(1/2)$	-56.84 (-127.69,-19.73)	-38.84 (-102.52,-8.25)	20.23 (-2.81,70.89)
$\widetilde{QUART}(1/2)$	49.22 (6.91,157.19)	74.20 (23.73,164.26)	-94.41 (-167.35,-41.76)

TABLE 2. **Returns and Sharpe Ratios on Variance, Skew, Quarticity and Volatility Feedback Swaps:** The table presents summary statistics on average annualized payoffs and payoff Sharpe ratios of Hellinger and VIX power divergence swaps (with payoffs scaled by $DIX(1/2)$), scale-invariant Hellinger skew and quarticity swaps, as well as Hellinger volatility feedback swaps (with payoffs scaled by $DIX(1/2)$). For instance, for the case of power divergence swaps, we estimate the average payoff $\mathbb{E}^{\mathbb{P}}\left[(2D_n^q - 2S(D_n^q))/DIX(1/2)\right]$ and the payoff Sharpe ratio $\mathbb{E}^{\mathbb{P}}\left[(D_n^q - S(D_n^q))/\sqrt{\mathbb{V}^{\mathbb{P}}(D_n^q - S(D_n^q))}\right]$ for $q = 0, 1/2$. The columns headed by "Bid" and "Mid" denote returns from short positions at bid and mid option prices, respectively. The column headed by "Ask" contains returns from long positions. The data are SPX forward and option data from January 1990 to February 2014, provided by MarketData-Express. The first (second) half of the sample goes from January 1990 (January 2003) to December 2002 (February 2014). Realized divergence is computed at a daily frequency. 95% confidence interval bands in parentheses are obtained from a block bootstrap with 25,000 replications.

R^2	bid	mid	ask
$DIX(1/2)$	5.64 (0.024,24.58)	10.07 (0.62,33.19)	16.15 (2.43,41.08)
$DIX(0)$	5.43 (0.025,24.62)	11.61 (0.98,35.98)	14.94 (2.02,40.03)
$LDIX(1/2)$	7.72 (0.23,28.25)	11.43 (0.72,32.54)	15.65 (1.60,37.30)
$SKEW^{UL}(1/2)$	7.29 (0.19,24.22)	11.68 (0.32,29.64)	16.64 (1.06,34.66)
$SKEW(1/2)$	19.24 (0.96,42.52)	28.13 (3.01,48.46)	34.98 (4.53,53.07)
$QUART(1/2)$	31.99 (2.65,69.47)	40.80 (3.94,70.05)	43.41 (5.60,71.14)
$QUINT(1/2)$	43.16 (2.23,74.54)	42.98 (3.63,72.95)	40.33 (5.23,71.08)

β	bid	mid	ask
$DIX(1/2)$	0.48 (-0.11,0.96)	0.81 (0.27,1.19)	1.14 (0.56,1.57)
$DIX(0)$	0.50 (-0.14,0.99)	0.96 (0.39,1.37)	1.16 (0.56,1.63)
$LDIX(1/2)$	0.78 (0.056,1.31)	1.01 (0.24,1.60)	1.25 (0.43,1.93)
$SKEW^{UL}(1/2)$	-0.88 (-1.66,0.086)	-1.22 (-2.13,-0.13)	-1.55 (-2.64,-0.32)
$SKEW(1/2)$	-0.58 (-1.01,-0.17)	-0.83 (-1.47,-0.24)	-1.09 (-1.94,-0.29)
$QUART(1/2)$	0.27 (0.10,0.44)	0.51 (0.12,0.94)	0.76 (0.14,1.45)
$QUINT(1/2)$	-0.22 (-0.41,-0.037)	-0.48 (-0.98,-0.047)	-0.74 (-1.55,-0.057)

$\alpha \times 100$	bid	mid	ask
$DIX(1/2)$	0.28 (-0.51,1.2)	0.45 (-0.47,-1.7)	0.6 (-0.56,2.1)
$DIX(0)$	0.38 (-0.48,1.4)	0.65 (-0.45,2.1)	0.69 (-0.56,2.3)
$LDIX(1/2)$	0.98 (-0.23,2.5)	1.1 (-0.25,2.8)	1.2 (-0.33,3.1)
$SKEW^{UL}(1/2)$	-1.6 (-3.6,0.058)	-1.8 (-4,0.11)	-1.9 (-4.5,0.18)
$SKEW(1/2)$	-0.85 (-2,-0.058)	-1.1 (-2.5,-0.041)	-1.4 (-3.1,-0.016)
$QUART(1/2)$	0.36 (0.042,0.82)	0.66 (0.035,1.4)	0.95 (0.029,2.1)
$QUINT(1/2)$	-0.3 (-0.66,-0.018)	-0.64 (-1.4,-0.016)	-0.97 (-2.1,-0.014)

TABLE 3. **Predictive Power of Fear Index:** We run predictive regressions of the form $Y_{t+1} = \alpha + \beta X_t + \varepsilon_t$, where Y_{t+1} is the monthly payoff of one of our divergence swaps and X_t the fear index in Bollerslev and Todorov (2011). We consider a number of divergence swaps introduced in Section 2, with floating leg given by corridor VIX and Hellinger variance, lower Hellinger semivariance, Hellinger volatility feedback, corridor Hellinger skewness, quarticity and quinticity, respectively. We report in the top, middle and bottom panel regression R^2's and point estimates for α and β, together with 95% confidence intervals in parentheses, obtained from a block bootstrap with 25,000 replications. All divergence swaps are tradeable, in the sense that they make use of finitely many options for replication of their floating leg; see Section 2.6 for details. The time series of monthly payoffs of our divergence swaps is computed based on SPX options and forwards from January 1990 to February 2014, using a daily frequency for the computation of realized divergence.

TI	bid	mid	ask
$DIX(1/2)$	55.72 (26.62,69.68)	56.17 (27.83,69.44)	56.18 (28.95,69.07)
$DIX(0)$	55.99 (27.02,69.79)	56.88 (29.00,70.10)	56.20 (28.58,69.11)
$LDIX(1/2)$	58.51 (28.62,72.44)	59.03 (30.35,72.14)	59.19 (31.28,71.76)
$SKEW^{UL}(1/2)$	63.05 (30.17,82.68)	64.08 (32.46,78.35)	62.89 (34.19,75.09)
$SKEW(1/2)$	74.98 (52.22,84.34)	68.55 (50.89,80.58)	65.00 (49.47,79.52)
$QUART(1/2)$	65.25 (48.86,78.37)	60.24 (44.24,76.77)	57.11 (40.91,75.08)
$QUINT(1/2)$	65.63 (48.21,78.29)	54.54 (37.29,72.35)	50.15 (33.68,69.50)

TABLE 4. **Relation between Implied Divergence and Tail Index:** We run linear regressions of the form $Y_t = \alpha + \beta X_t + \varepsilon_t$, where Y_t is the implied leg of one of our divergence swaps and X_t the tail index in Bollerslev et al. (2014). We consider a number of divergence swaps introduced in Section 2, with implied leg given by implied corridor VIX and Hellinger variance, lower Hellinger semivariance, Hellinger volatility feedback, corridor Hellinger skewness, quarticity and quinticity, respectively. We report regression R^2's, together with 95% confidence intervals in parentheses, obtained from a block bootstrap with 25,000 replications. All divergence swaps are tradeable, in the sense that they make use of finitely many options for replication of their floating leg; see Section 2.6 for details. The time series of fixed legs of our divergence swaps is computed based on SPX options and forwards from January 1990 to February 2014.

Appendix B. Proofs

B.1. **Proof of Result 2.1.**

Proof. Given a twice continuously differentiable generating function Φ almost everywhere, Lagrange's remainder (see, e.g., Definition 3.3 in Hobson and Klimmek, 2012) gives

$$\begin{aligned}\Phi(F_{T,T}) - \Phi(F_{0,T}) &= \Phi'(F_{0,T})(F_{T,T} - F_{0,T}) \\ &\quad + \int_0^{F_{0,T}} \Phi''(K) P_{T,T}(K) dK + \int_{F_{0,T}}^{\infty} \Phi''(K) C_{T,T}(K) dK \ .\end{aligned} \tag{45}$$

From Eq. (3), we thus obtain

$$\begin{aligned}D_n^{\Phi} &= \sum_{i=1}^{n} [\Phi(F_{i,T}) - \Phi(F_{i-1,T}) - \Phi'(F_{i-1,T})(F_{i,T} - F_{i-1,T})] \\ &= \Phi(F_{T,T}) - \Phi(F_{0,T}) - \sum_{i=1}^{n} \Phi'(F_{i-1,T})(F_{i,T} - F_{i-1,T}) \\ &= \int_0^{F_{0,T}} \Phi''(K) P_{T,T}(K) dK + \int_{F_{0,T}}^{\infty} \Phi''(K) C_{T,T}(K) dK + \Phi'(F_{0,T})(F_{T,T} - F_{0,T}) \\ &\quad - \sum_{i=1}^{n} \Phi'(F_{i-1,T})(F_{i,T} - F_{i-1,T}) \\ &= \int_0^{F_{0,T}} \Phi''(K) P_{T,T}(K) dK + \int_{F_{0,T}}^{\infty} \Phi''(K) C_{T,T}(K) dK \\ &\quad - \sum_{i=1}^{n} (\Phi'(F_{i-1,T}) - \Phi'(F_{0,T}))(F_{i,T} - F_{i-1,T}) \ .\end{aligned}$$

Taking $\mathbb{Q}_T$ expectations and using the martingale property of forward prices finally yields the desired result. □

B.2. **Proof of Corollary 2.6.**

Proof. Given a generating function Φ that is twice continuously differentiable almost everywhere, so is the corridorized generating funciton $\Phi_{a,b}$. Lagrange's remainder (see,

e.g., Definition 3.3 in Hobson and Klimmek, 2012) gives

$$\begin{aligned}
\Phi_{a,b}(F_{T,T}) &= \Phi_{a,b}(F_{0,T}) + \Phi'_{a,b}(F_{0,T})(F_{T,T} - F_{0,T}) + \int_{F_{0,T}}^{F_{T,T}} \Phi''_{a,b}(K)(F_{T,T} - K)^+ dK \\
&= \Phi_{a,b}(F_{0,T}) + \Phi'_{a,b}(F_{0,T})(F_{T,T} - F_{0,T}) + \int_{0}^{F_{0,T}} \Phi''_{a,b}(K)(K - F_{T,T})^+ dK \\
&\quad + \int_{F_{0,T}}^{\infty} \Phi''_{a,b}(K)(F_{T,T} - K)^+ dK \\
&= \Phi_{a,b}(F_{0,T}) + \Phi'_{a,b}(F_{0,T})(F_{T,T} - F_{0,T}) + \int_{a}^{\min(F_{0,T},b)} \Phi''(K)(K - F_{T,T})^+ dK \\
&\quad + \int_{\max(F_{0,T},a)}^{b} \Phi''(K)(F_{T,T} - K)^+ dK \ .
\end{aligned}$$

Expression (34) is obtained analogously to the previous proof in Section B.1 of this Appendix. Taking $\mathbb{Q}_T$ expectations and using the martingale property of forward prices finally yields the desired result. □

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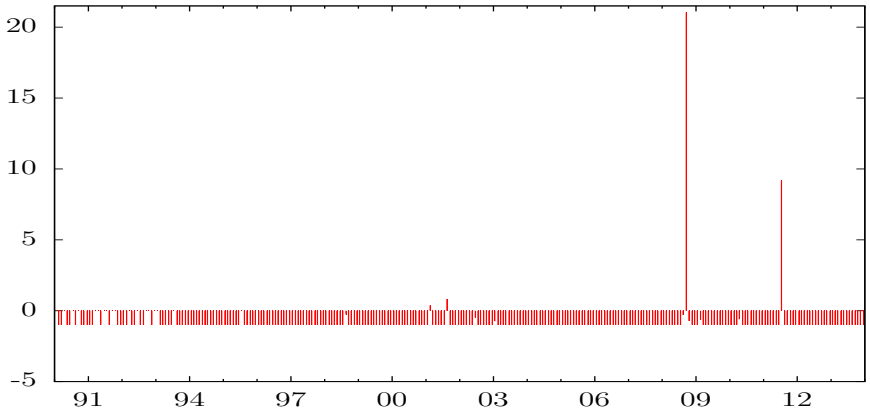
Internet Appendix for

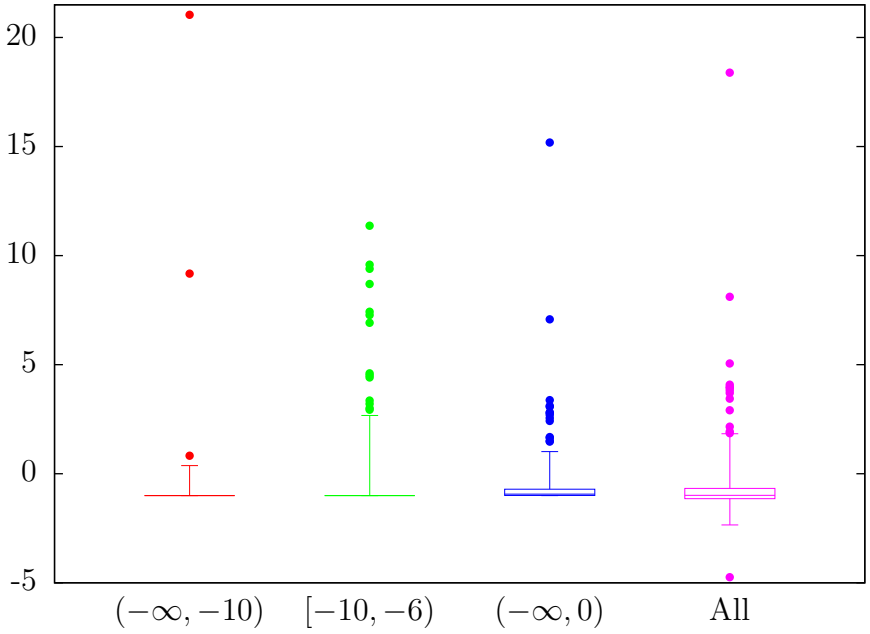
Fear Trading

(not for publication)

January 16, 2015

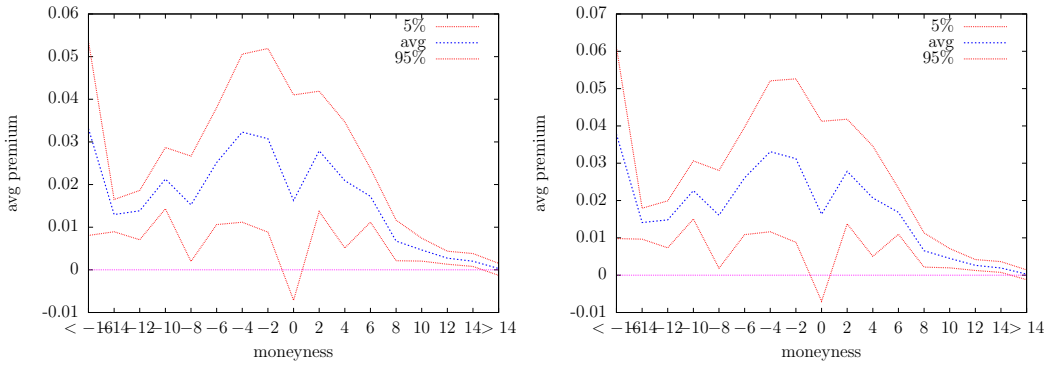


(A) Log-Moneyness $(-\infty, -10)$

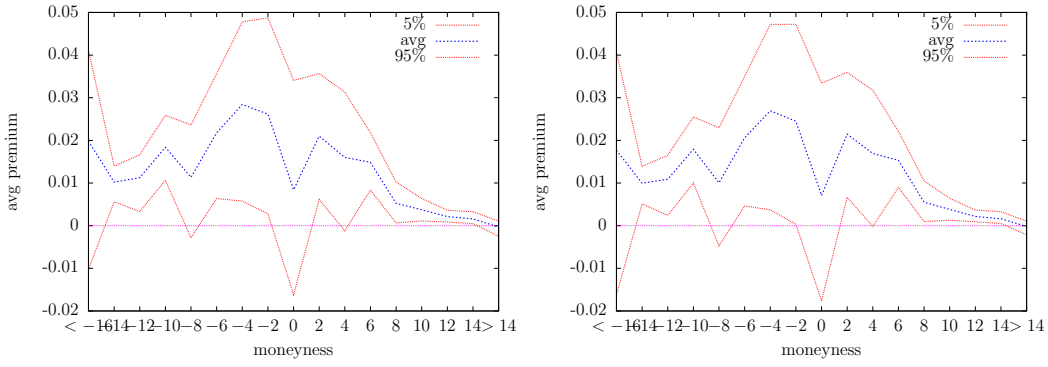


(B) Boxplot

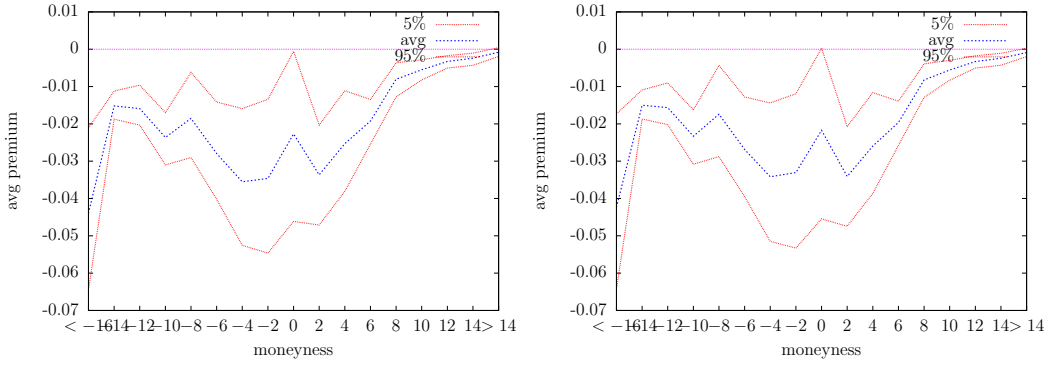
FIGURE IA.1. **Returns of Hellinger Corridor Skewness:** Panel (a) plots the time series of returns of a short corridor skew swap, localized to all available options in the corridor $(-\infty, -10)$. Panel (b) shows box plots of returns of short corridor skew swaps, localized to all available options in the corridors $(-\infty, -10)$, $[-10, -6)$, $(-\infty, 0)$ and $(-\infty, \infty)$, respectively. In all box plots, extreme observations outside the 95% interquartile range of the data are represented as dots. The skew swap payoffs and fixed legs are constructed using SPX forward data from January 1990 to February 2014, provided by MarketDataExpress.



(A) Short Mid $\overline{DIX}(1/2)$ (B) Short Mid $\overline{DIX}(0)$

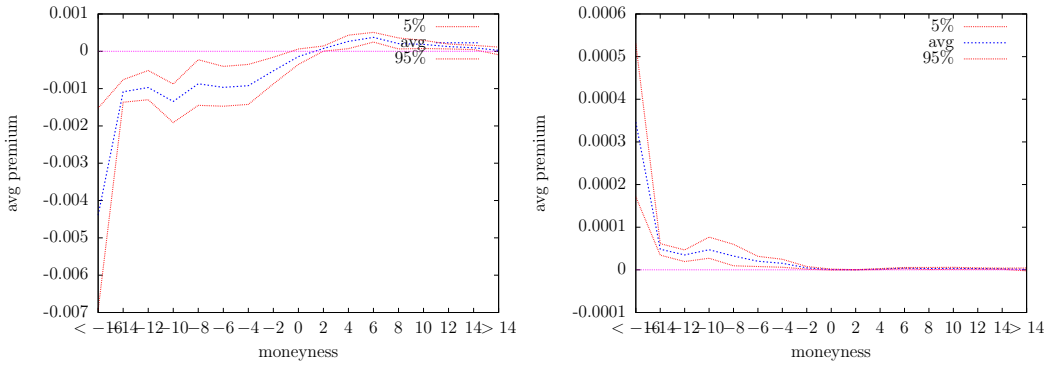


(C) Short Bid $\overline{DIX}(1/2)$ (D) Short Bid $\overline{DIX}(0)$

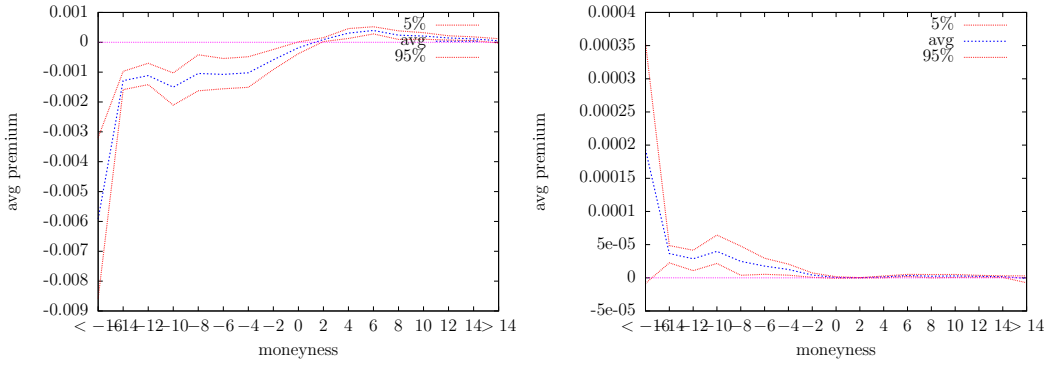


(E) Long Ask $\overline{DIX}(1/2)$ (F) Long Ask $\overline{DIX}(0)$

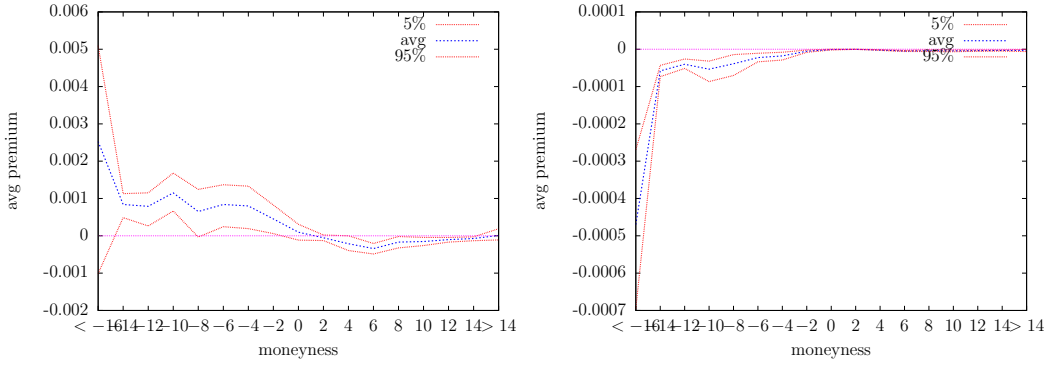
FIGURE IA.2. **Corridor Divergence (Fine Corridors):** The charts show average monthly returns from divergence swaps. The x-axis stratifies the region around the forward price into 17 intervals between log moneyness levels from -16% and below to +14% and above. The average excess returns are computed from tradeable corridor contracts outlined in Section 2.6. As an example, the average premium at moneyness -10% refers to the moneyness corridor [-12%, -10%). The entry at < -16 refers to the corridor [left-most available strike, -16%), and similarly the entry at > 14 refers to the corridor (14%, right-most available strike]. The name code reflects whether the options were purchased at "mid", "bid", or "ask" price. The data are S&P 500 forward and options data from January 1990 until February 2014 from MarketDataExpress, and the floating legs are computed from daily observations. The average premium and the 5% and 95% confidence intervals are obtained from a block bootstrap with 25,000 replications.



(A) Short Mid $\overline{SKEW}(1/2)$ (B) Short Mid $\overline{QUART}(1/2)$

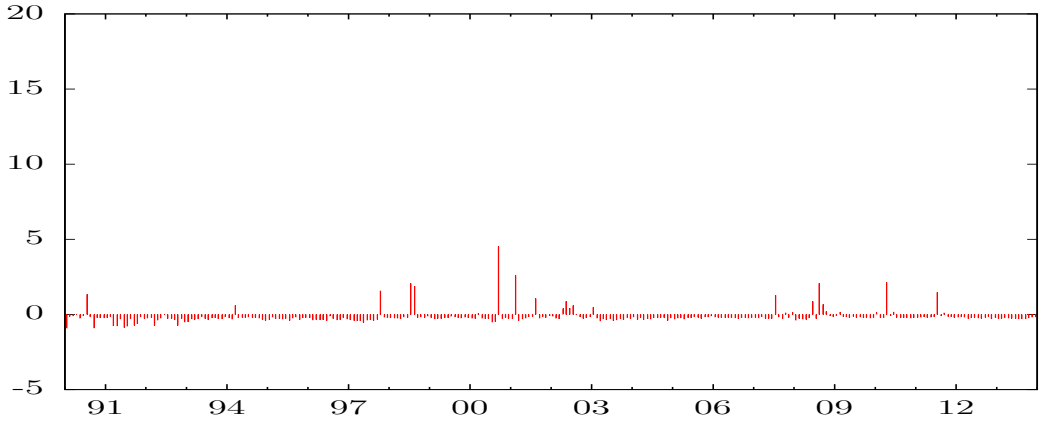


(C) Short Bid $\overline{SKEW}(1/2)$ (D) Short Bid $\overline{QUART}(1/2)$

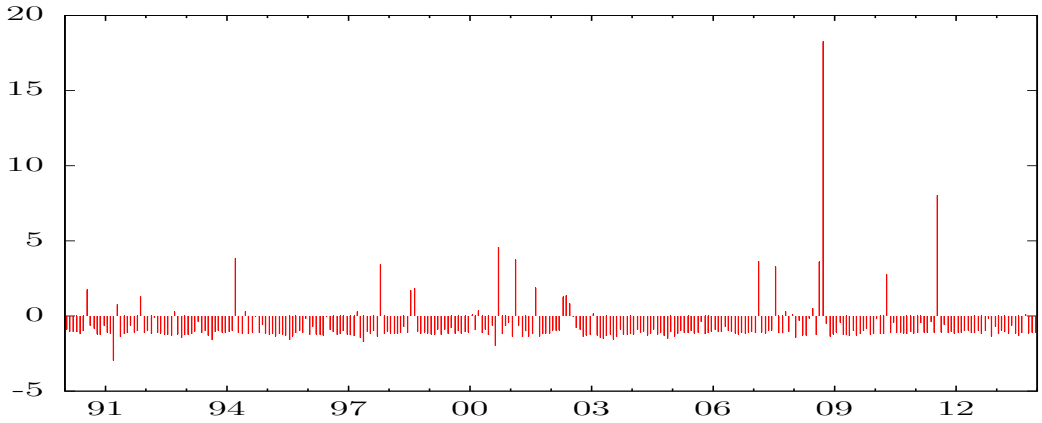


(E) Long Ask $\overline{SKEW}(1/2)$ (F) Long Ask $\overline{QUART}(1/2)$

FIGURE IA.3. **Corridor Skew and Quarticity (Fine Corridors):** The charts show average monthly returns from skewness and quarticity swaps. The x-axis stratifies the region around the forward price into 17 intervals between log moneyness levels from -16% and below to +14% and above. The average excess returns are computed from tradeable corridor contracts outlined in Section 2.6. As an example, the average premium at moneyness -10% refers to the moneyness corridor [-12%, -10%). The entry at < -16 refers to the corridor [left-most available strike, -16%), and similarly the entry at > 14 refers to the corridor (14%, right-most available strike]. The name code reflects whether the options were purchased at "mid", "bid", or "ask" price. The data are S&P 500 forward and options data from January 1990 until February 2014 from MarketDataExpress, and the floating legs are computed from daily observations. The average premium and the 5% and 95% confidence intervals are obtained from a block bootstrap with 25,000 replications.

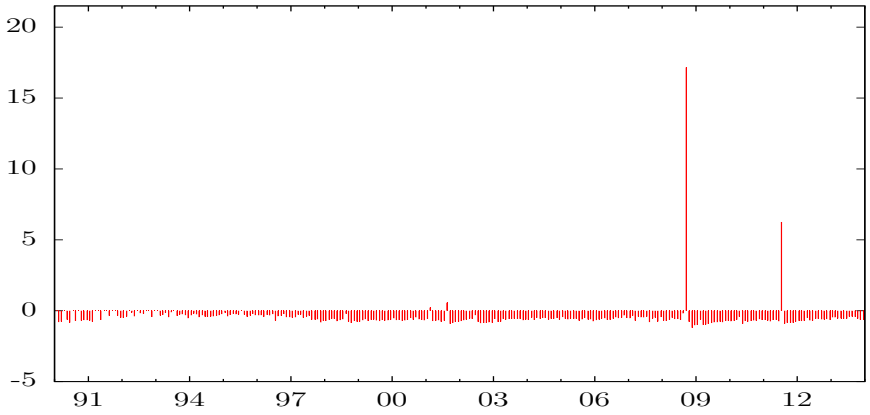


(A) Log-Moneyness $[-10, -6)$

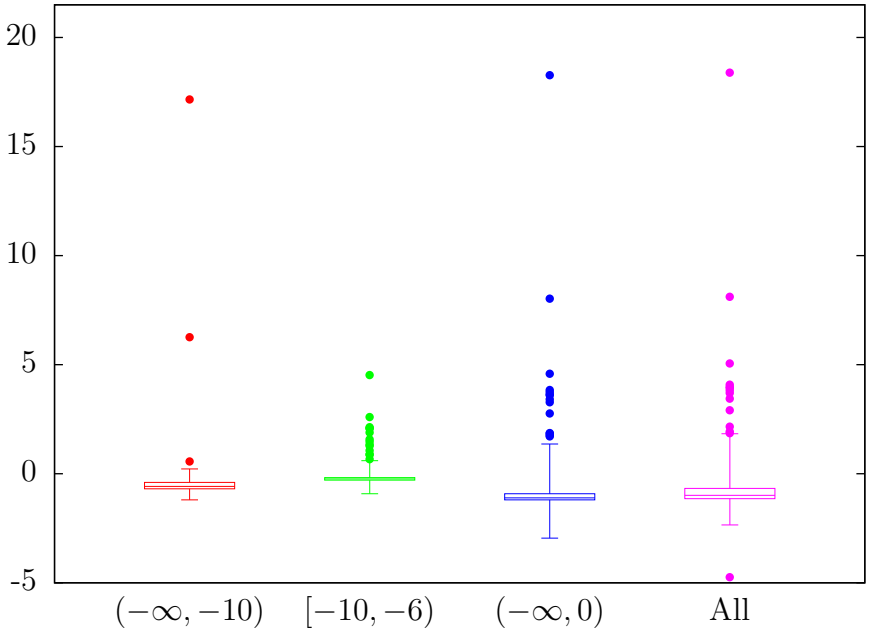


(B) Log-Moneyness $(-\infty, 0)$

FIGURE IA.4. **Payoffs of Hellinger Corridor Skewness Normalized by Implied Hellinger Skewness:** This figure plots in Panel (a) (Panel (b)) the time series of payoffs of a short corridor skew swap, localized to all available options in the corridor $[-10, -6)$ $((-\infty, 0))$ and normalized by implied Hellinger skewness. The corridor skew swap payoffs and the implied skewness legs are constructed using SPX option data from January 1990 to February 2014, provided by MarketDataExpress.



(A) Log-Moneyness $(-\infty, -10)$



(B) Boxplot

FIGURE IA.5. **Payoffs of Hellinger Corridor Skewness Normalized by Implied Hellinger Skewness:** Panel (a) plots the time series of payoffs of a short corridor skew swap, localized to all available options in the corridor $(-\infty, -10)$ and normalized by implied Hellinger skewness. Panel (b) shows box plots of payoffs of short corridor skew swaps, localized to all available options in the corridors $(-\infty, -10)$, $[-10, -6)$, $(-\infty, 0)$ and $(-\infty, \infty)$, respectively, and normalized by implied Hellinger skewness. In all box plots, extreme observations outside the 95% interquartile range of the data are represented as dots. The skew swap payoffs and fixed legs are constructed using SPX option data from January 1990 to February 2014, provided by MarketDataExpress.