Does Variance Risk Have Two Prices?
Evidence from the Equity and Option Markets

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Abstract

We formally compare two versions of the market Variance Risk Premium (VRP) measured in the equity and option markets. Both VRPs follow common patterns and respond similarly to changes in volatility and economic conditions. However, we reject the null hypothesis that they are identical and find that their difference is strongly related to measures of the financial standing of intermediaries. These results shed new light on the information content of the VRP, suggest the presence of market frictions between the two markets, and are consistent with the key role played by intermediaries in setting option prices.

Key words: Variance Risk Premium, Option, Equity, Financial Intermediaries

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1. Introduction

The market Variance Risk Premium (VRP) is the compensation investors are willing to pay for assets that perform well when stock market volatility is high. Whereas this premium is embedded in the prices of various assets, notably equity portfolios exposed to market variance risk (the equity VRP), it can be easily computed using index options (the option VRP). For this reason, academics and policymakers alike commonly view the option VRP as the most readily available gauge of investors' risk aversion or, more colloquially, "fear".[3] However, recent studies provide evidence of potential mispricing between equity and option markets and stress the key role played by financial intermediaries (broker-dealers) in determining index option prices.[4] If option prices reflect local demand and supply forces in addition to broad economic fundamentals, the option VRP may behave quite differently from its equity-based counterpart.

In this paper, we formally test whether the two conditional market VRPs measured in the equity and option markets are equal. A key feature of our approach is that we do not compare the VRPs themselves, but their linear projections on a common set of predictive variables that capture volatility and economic conditions, as well as the financial standing of broker-dealers. This approach allows us to overcome the challenge of estimating the entire path of the premium, while guaranteeing that if the VRP projections are different, so are the VRPs. Therefore, a rejection of the null hypothesis of equal projections necessarily implies the same rejection for the VRPs.

Our conditional VRP measures are fully comparable, economically motivated, and simple to estimate. They are comparable across the two markets because they are conditioned on the same set of predictors. They allow for the measurement of the role played by several economically-motivated predictors in driving the prices of variance risk and their potential difference. Finally, they can be easily estimated using standard time-series and cross-sectional regressions. The only required inputs are price data on equity and index option portfolios that are sensitive to market variance shocks. For the equity market, we follow Ang et al. (2006) and extract the VRP projection

[3] See Bali and Zhou (2015); Bekaert and Hoerova (2014); Bollerslev et al. (2011); Drechsler and Yaron (2011), , as well as the recent report of the Bank for International Settlements (2014).

[4] The mispricing of SP500 index options is documented by Constantinides et al. (2011). The role of intermediaries in setting option prices is discussed by Adrian and Shin (2010); Bates (2003, 2008); Chen et al. (2015); Garleanu et al. (2009).

using a factor model that includes market variance risk. For the option market, we use the squared VIX index which measures the price of an index option portfolio that tracks market variance risk (see Carr and Wu (2009)).

Our results reveal strong commonalities between the two market VRP projections measured at a quarterly frequency. Comparing them between 1992 and 2014, we observe that they mostly take negative values, consistent with the notion that investors are willing to pay a premium to hedge against variance shocks. Their average values are close to -1.80% per year, which implies that a simple unconditional analysis would conclude that the two VRPs are identical. Finally, both premia increase in magnitude after volatility shocks and during recession periods. Their paths are therefore closely aligned and exhibit a correlation coefficient of 0.69.

However, the empirical evidence formally rejects the null hypothesis that the two premia are identical. The difference between the VRP projections exhibits several key features. First, it changes signs as the option VRP can be either below or above its equity-based counterpart. Second, it can be economically large—in 12 quarters out of 92, its magnitude is above 3.60% per year, which is two times the average premium itself. Third, it is not exclusively associated with crisis episodes such as the great recession in 2007-08. Finally, its variations are driven by two measures of the financial standing of intermediaries commonly used in the literature, namely the leverage ratio of broker-dealers and the quarterly return of the Prime Broker Index (PBI).[5] For instance, we observe that when these intermediaries take on leverage or make short-term gains, the magnitude of the option VRP decreases significantly, whereas the equity VRP remains unchanged. Equivalently, we find that during these periods a trading strategy that is long variance in the option market and short variance in the equity market delivers a positive alpha.

Before examining the implications of these results, we conduct an extensive analysis to confirm that the VRP difference is a robust feature of the data. First, we verify that it is not artificially caused by a misspecification of the factor model used to extract the equity VRP. We perform a large battery of tests and find it is not the case—in particular, the pricing errors are small, the model-implied mimicking portfolio closely tracks the market variance, and the inclusion of additional risk factors leaves the results unchanged. Second, we rely on theoretical and simulation analysis to

[5] See, for instance, Adrian and Shin (2010, 2014) who demonstrate empirically that the leverage ratio drops when intermediaries hit their risk constraints, and Boyson et al. (2010) who use the PBI return in the context of hedge fund contagion.

show that variance jumps are unlikely to drive our results. Finally, we document the same VRP difference when repeating the entire estimation using monthly data or individual stocks (instead of portfolios).

The VRP difference between the equity and option markets has several implications. First, it leads to a more nuanced view of the information content of the option VRP. The latter is frequently interpreted as a measure of investors' risk aversion and future economic activity. However, this interpretation could be misleading if the two broker-dealer variables that drive the option VRP mainly capture shocks that are specific to intermediaries. Consistent with this view, we observe that changes in both variables do not affect the risk attitude of equity investors towards stocks exposed to variance risk. In addition, we find that the equity VRP yields more accurate forecasts of the stock market return and economic activity than its option-based counterpart.

Second, the rejection of the null hypothesis that variance risk has the same price suggests the presence of market frictions between the equity and option markets. The simplest interpretation of this price difference is that investors face portfolio constraints that induce market segmentation.[6] In practice, such constraints may arise because equity investors face information costs or regulatory constraints that limit their positions in the option market or because broker-dealers do not have the mandate to trade stocks exposed to variance risk. An alternative explanation proposed by Garleanu and Pedersen (2011) is that investors with limited capital may value identical assets differently if they are traded in markets with different margin requirements—a situation observed in the equity and option markets. While the marginal contribution of each theory is difficult to determine without knowing all the constraints faced by investors, our empirical evidence suggests that the margin-based explanation, if used alone, cannot fully account for the path followed by the VRP difference. First, it cannot easily explain that the VRP difference takes both positive and negative values because margins are unlikely to be higher in the option market than in the equity market. Second, it predicts that the VRP difference should increase when investors' capital, or funding liquidity, is low (and vice-versa). However, direct measures of funding liquidity such as the default and TED spreads are weakly related to the VRP difference.

Finally, our results emphasize the key role played by financial intermediaries in the index option

[6] Market segmentation is also commonly used to explain mispricing across international markets. See e.g., Bekaert et al. (2011).

market. As shown empirically by Chen et al. (2015); Garleanu et al. (2009), broker-dealers supply index options to public investors in exchange for a premium for holding residual risk. Therefore, their ability to perform this task should depend on their ability to bear risk and take on leverage—if the latter declines, the option supply should drop and lead to higher option prices (and vice-versa). Consistent with this prediction, we find that a decrease in the leverage of broker-dealers has a positive impact on index option prices, which is not overturned when we treat leverage as endogenous and control for additional predictors. In addition, we show that deleveraging does not affect the prices of individual stock options, whose supply is not dominated by financial intermediaries. Taken together, these results point to supply variation as a plausible explanation for the strong relationship between leverage and the option VRP extracted from index option prices.

Our work is related to several strands of the literature. First, there is an extensive literature on the role played by market variance risk in the equity market. Ang et al. (2006) infer the unconditional VRP from the returns of portfolios exposed to volatility shocks, while Bansal et al. (2014); Campbell et al. (2015) derive an intertemporal CAPM with stochastic volatility to explain the cross-section of average stock returns.[7] Relative to these papers, we perform a conditional analysis of the equity VRP and study the drivers of its variation over time. Second, several studies examine the evolution of the market VRP using option prices (e.g., Bollerslev et al. (2011); Todorov (2010)). Our dynamic comparison with the equity market sheds new light on the informational content of the option VRP. Third, Constantinides et al. (2011) document violations of stochastic dominance bounds derived from stock market returns by call and put options written on the SP500 index. We provide a possible explanation for this mispricing, namely the difference in the pricing of market variance risk. Finally, Adrian and Shin (2010); Chen et al. (2015) show empirically that the behavior of financial intermediaries is an important driver of option prices. Relative to these papers, we find that these intermediaries affect the price of variance risk very differently in the equity and option markets.

The remainder of the paper is organized as follows. Section 2 presents the methodology to formally compare the conditional market VRPs in the equity and option markets. Section 3 describes the data and Section 4 the main empirical findings. Section 5 provides several interpretations for

[7] Note that Ang et al. (2006) present independent evidence on the pricing of both market (systematic) and idiosyncratic variance risks. Our work focuses exclusively on their analysis of market variance risk.

our main findings and Section 6 concludes. The online appendix available on the journal's website provides a detailed description of the methodology and reports additional results.

2. Empirical Framework

2.1. The Market Variance Risk Premium

We define the conditional market Variance Risk Premium (VRP) as

$$\lambda_{v,t} = E\left(rv_{t+1}|\, I_t\right) - E^Q\left(rv_{t+1}|\, I_t\right) = E\left(rv_{t+1}|\, I_t\right) - p_{rv,t}, \tag{1}$$

where rv_{t+1} is the realized variance of the market returns between time t and $t+1$, and $E\left(rv_{t+1}|\, I_t\right)$, $E^Q\left(rv_{t+1}|\, I_t\right)$ denote the physical and risk-neutral expectations of rv_{t+1} conditioned on all available information at time t. The term $E^Q\left(rv_{t+1}|\, I_t\right)$ is equal to the forward price of the variance payoff denoted by $p_{rv,t}$ (i.e., its price at time t multiplied by the gross risk-free rate).

Theory predicts that risk-averse investors wish to hedge against increases in aggregate variance because they represent a deterioration in investment opportunities. As a result, we expect the VRP defined in equation (1) to be negative. Stated differently, assets that perform well when realized market variance is high should earn lower average returns. Previous empirical studies confirm that the market VRP extracted from index options is negative on average (e.g., Bollerslev et al. (2011); Carr and Wu (2009)). The same result is also observed in the equity market where the market VRP is inferred from a cross-section of variance-risk sensitive equity portfolios (e.g., Ang et al. (2006)).

In this paper, we formally test whether the two conditional versions of the market VRP measured in the equity and option markets are equal. We develop a simple comparison approach based on the linear projection of the VRP on the space spanned by predictive variables that track the evolution of volatility and economic conditions, as well as the financial standing of intermediaries:

$$\lambda_{v,t}(z) = proj(rv_{t+1}|\, z_t) - proj(p_{rv,t}|\, z_t) = F_v' z_t - V_v' z_t, \tag{2}$$

where the J-vector z_t includes a constant and $J-1$ centered predictors, $F_v' z_t$ is the linear forecast of rv_{t+1}, and $V_v' z_t$ denotes the linear projection of $p_{rv,t}$ on z_t. By construction, if the conditional VRPs measured in both markets are the same, so are their linear projections—therefore, differences between projections signal periods when the prices of market variance risk differ.[8] Building on this insight, we compute the equity- and option-based estimates of $\lambda_{v,t}(z)$ as

$$
\begin{aligned}
\hat{\lambda}^{e}_{v,t}(z) &= (\hat{F}_v - \hat{V}^e_v)' z_t, \\
\hat{\lambda}^{o}_{v,t}(z) &= (\hat{F}_v - \hat{V}^o_v)' z_t,
\end{aligned} \tag{3}
$$

where $\hat{V}^{e\prime}_v z_t$ and $\hat{V}^{o\prime}_v z_t$ denote the projections of the forward variance prices formed in the equity and option markets, respectively. To compare the two markets, we simply take the difference between the two estimated VRP projections:

$$
\hat{D}_t(z) = \hat{\lambda}^{e}_{v,t}(z) - \hat{\lambda}^{o}_{v,t}(z) = (\hat{V}^o_v - \hat{V}^e_v)' z_t. \tag{4}
$$

The linear framework used here has several advantages. First, it guarantees that the two markets are fully comparable because both VRP projections are conditioned on the same information set. Second, it yields simple expressions for the VRP projections and their difference—in particular, $\hat{D}_t(z)$ only depends on $\hat{V}^e_v$ and $\hat{V}^o_v$ as the physical expectation term $\hat{F}_v z_t$ cancels out. Third, it allows us to measure the economic impact of each predictive variable on both VRPs. Finally, it is consistent with the extensive literature that uses linear regressions to forecast realized variance and measure risk premia.

We estimate the vector F_v from a simple time-series regression of rv_{t+1} on z_t (similar to Campbell et al. (2015); Paye (2012)). The two vectors of risk-neutral coefficients $\hat{V}^e_v$ and $\hat{V}^o_v$ are recovered from a set of equity and option portfolios that are exposed to the variance risk of the market. For sake of brevity, we describe the main steps of the procedure below and relegate in the online appendix additional details on the properties of the different estimators, which are all consistent and asymptotically normally distributed.

2.2. The Equity-Based Vector $\hat{V}^e_v$

The theoretical and empirical evidence presented above reveals that the market variance rv_{t+1} is a priced factor in the equity market. Building on this insight, we infer its premium from a set of

[8] Note that the opposite does not hold, i.e., the projections can be equal even if the VRPs differ. This situation occurs when the difference between the two VRPs is orthogonal to the predictors.

25 variance-risk sensitive equity portfolios. To mitigate data-mining concerns when forming these portfolios, we use the same approach as that of Ang et al. (2006) by sorting stocks monthly into quintiles based on their betas on the market and variance factors (see the online appendix for a detailed description). To estimate the equity-based vector V_v^e, we posit a parsimonious two-factor model for the excess return of each equity portfolio p ($p = 1, ..., 25$):

$$r_{p,t+1}^e = -p_{p,t} + b_{pv} \cdot rv_{t+1} + b_{pm} \cdot f_{m,t+1} + \epsilon_{p,t+1}, \tag{5}$$

where $f_{m,t+1}$ is the market excess return, b_{pv}, b_{pm} denote the portfolio betas, $\epsilon_{p,t+1}$ is the idiosyncratic component, and the equilibrium forward price $p_{p,t}$ is equal $b_{pv} \cdot p_{rv,t}^e + b_{pm} \cdot p_{fm,t}^e$, where $p_{rv,t}^e$ and $p_{fm,t}^e$ are the forward prices of the two risk factors formed in the equity market.[9,10] Specifying a two-factor model with constant betas is motivated by the fact that the 25 portfolios are (i) sorted along the market and variance dimensions, and (ii) rebalanced monthly to maintain stable exposures to both factors.

If we project $r_{p,t+1}^e$ on the space spanned by z_t, rv_{t+1}, and $f_{m,t+1}$ and use the equilibrium price condition, we can write the excess portfolio return as

$$r_{p,t+1}^e = -c_p' z_t + b_{pv} \cdot rv_{t+1} + b_{pm} \cdot f_{m,t+1} + e_{p,t+1}, \tag{6}$$

and the projected forward price as

$$proj(p_{p,t} | \, z_t) = c_p' z_t = (b_{pv} \cdot V_v^{e\prime} + b_{pm} \cdot V_m^{e\prime}) z_t, \tag{7}$$

where $V_v^{e\prime} z_t$ and $V_m^{e\prime} z_t$ denote the projections of $p_{rv,t}^e$ and $p_{fm,t}^e$ on z_t, respectively. Equations (6) and (7) serve as the two building blocks for our estimation procedure. The latter builds on recent work by Gagliardini et al. (2015) and is simply a conditional extension of the classic two-pass cross-

[9] This last equality is perfectly equivalent to the more familiar equality that applies to conditional returns. To see this, we can replace rv_{t+1} and $f_{m,t+1}$ with their demeaned versions, $\tilde{r}v_{t+1}$ and $\tilde{f}_{m,t+1}$, and use the fact that $\lambda_{v,t}^e = E\left(rv_{t+1} | \, I_t\right) - p_{rv,t}^e$ and $\lambda_{m,t}^e = E\left(f_{m,t+1} | \, I_t\right) - p_{fm,t}^e$ to rewrite equation (5) as $r_{p,t+1}^e = E(r_{p,t+1}^e \big| I_t) + b_{pv} \cdot \tilde{r}v_{t+1} + b_{pm} \cdot \tilde{f}_{m,t+1} + \epsilon_{p,t+1}$, where $E(r_{p,t+1}^e \big| I_t)$ must be equal to $b_{pv} \cdot \lambda_{v,t}^e + b_{pm} \cdot \lambda_{m,t}^e$ (see Cochrane (2005)).

[10] Since $f_{m,t+1}$ is an excess return, its forward price must be equal to zero ($p_{fm,t}^e = 0$). This condition provides us with a test of the validity of the model (see Section 4.4).

sectional regression.[11] In the first step, we run a time-series regression of $r^e_{p,t+1}$ on z_t, rv_{t+1}, and $f_{m,t+1}$ to estimate c_p, b_{pv}, and b_{pm} for each equity portfolio (equation (6)). In the second step, we exploit the condition that the vector c_p is equal to a linear combination of the two vectors V^e_m and V^e_v (equation (7))—by running a cross-sectional regression of each element of the estimated vector $\hat{c}_p$ on the estimated betas $\hat{b}_{pm}$ and $\hat{b}_{pv}$, we can therefore compute each element of $\hat{V}^e_v$.

This estimation procedure calls for two main comments. First, it requires the two-factor model to correctly price the 25 equity portfolios. If it is not the case, the estimated vector $\hat{V}^e_v$ could be biased and leads us to the wrong conclusion that the equity and option VRPs differ. The extensive tests performed later provide strong evidence that the two-factor model is correctly specified (see Section 4.4). Second, our approach should be distinguished from recent studies (e.g., Buraschi et al. (2014); Cao and Han (2013)) that use data on individual stock options to measure the premium attached to the variance of each stock (individual stock VRP). In contrast, we use data on individual stock returns to measure the premium attached to the variance of the aggregate market (market VRP).

2.3. The Option-Based Vector $\hat{V}^o_v$

In the option market, we build on previous work by Britten-Jones and Neuberger (2000); Carr and Wu (2009) who demonstrate that the realized market variance rv_{t+1} can be replicated by a portfolio of index options whose forward price is given by the squared VIX index vix^2_t.[12] As a result, the forward price of rv_{t+1} formed in the option market, denoted by $p^o_{rv,t}$, can be measured by vix^2_t.[13] Exploiting this result, we compute $\hat{V}^o_v$ from a simple time-series regression of vix^2_t on z_t since we have:

$$proj(p^o_{rv,t} \big| z_t) = proj(vix^2_t \big| z_t) = V^{o\prime}_v z_t. \tag{8}$$

The only challenge when estimating V^o_v stems from data limitations: whereas rv_{t+1} and z_t are

[11] Equations (6) and (7) are the conditional counterparts of those used in the traditional two-pass regression in which (i) the time-series regression becomes $r^e_{p,t+1} = -c_p + b_{pv} \cdot rv_{t+1} + b_{pm} \cdot f_{m,t+1} + e_{p,t+1}$; (ii) the cross-sectional regression becomes $c_p = b_{pv} \cdot V^e_v + b_{pm} \cdot V^e_m$, where V^e_v and V^e_m are the unconditional forward prices (i.e., $p^e_{rv} = V^e_v$, $p^e_{fm} = V^e_m$).

[12] The variance payoff can be replicated with a static portfolio of options that ensures a constant dollar gamma (unit beta to the variance factor) and a dynamic position in market futures to maintain delta-neutrality (zero beta to the market factor).

[13] As shown by Carr and Wu (2009); Jiang and Tian (2005), the equality between $p^o_{rv,t}$ and the squared VIX only holds approximately in case of large market movements. In the online appendix, we re-estimate the vector V^o_v using the SVIX index that is robust to jumps (see Martin (2013)) and document similar results.

observed over a long period beginning in 1970 (the long sample), vix_t^2 is only available in the early 1990's (the short sample). Therefore, we use the Generalized Method of Moments (GMM) for samples of unequal lengths developed by Lynch and Wachter (2013) to improve the precision of the estimated coefficients. The basic idea is to adjust the initial estimate of V_v^o obtained from vix_t^2 over the short sample using information about rv_{t+1} and z_t over the long sample. The intuition behind this adjustment can be easily illustrated with the following example. Suppose that we wish to estimate the averages of the realized variance and the squared VIX, denoted by rv and vix^2 (i.e., z_t equals 1). Now suppose that the estimated mean of rv_{t+1} over the short sample, denoted $\widehat{rv}_S$, is above the more precise estimate computed over the long sample. Because rv_{t+1} and vix_t^2 are positively correlated, $\widehat{vix}_S^2$ is also likely to be above average. Therefore, $\widehat{vix}_S^2$ is adjusted downward to produce the final estimate.

3. Data Description

3.1. Predictive Variables

We conduct our empirical analysis using quarterly data between April 1970 and December 2014. We employ a set of five macro-finance predictors to capture volatility and economic conditions: the lagged realized variance, the Price/Earnings (PE) ratio, the quarterly inflation rate, the quarterly growth in aggregate employment, and the default spread (all of which are expressed in log form). The theoretical motivation for using these variables as well as their ability to predict realized variance are discussed in the recent studies of Bollerslev et al. (2011); Campbell et al. (2015); Paye (2012). The online appendix provides more information on the definition of each predictor and displays some descriptive statistics.

In addition to the macro-finance variables mentioned above, we consider two measures of the financial standing of broker-dealers (both expressed in log form). The first is the leverage ratio of broker-dealers using data from the Federal Reserve Flow of Funds Accounts (Table L 128).[14] Adrian and Shin (2010, 2014) provide supporting evidence that broker-dealers actively manage their leverage levels based on their risk-bearing capacity—in good times, they slowly increase their leverage and expand their asset base, whereas they deleverage in bad times, possibly because of

[14] The Federal Reserve defines broker-dealers as financial institutions that buy and sell securities for a fee, hold an inventory of securities for resale, or both.

tighter Value-at-Risk constraints or higher risk aversion levels. Second, we borrow from Boyson et al. (2010) and compute the value-weighted index of publicly-traded prime broker firms, including Goldman Sachs, Morgan Stanley, Bear Stearns, UBS, and Citigroup. The quarterly return of this Prime Broker Index (PBI) allows us to capture short-term changes in the financial strength of the major players in the brokerage sector.

3.2. The Set of Equity Portfolios

We summarize the properties of the 25 variance-risk sensitive portfolios in Table 1 by taking an equally-weighted average of all portfolios in the same variance beta quintile (Low, 2, 3, 4, High). For each portfolio, we measure the (post-formation) variance beta from the two-factor model in equation (6), where the market variance rv_{t+1} is proxied by the quarterly sum of the daily squared SP500 returns, and the market $f_{m,t+1}$ by the quarterly excess return of the CRSP index.

Consistent with theory, Panel A documents a strong and negative beta-return relationship (the cross-correlation equals -0.93). Specifically, the low-variance portfolio tends to perform poorly when aggregate variance increases (beta of -0.68) and therefore yields the highest average return (7.78% per year). As we move toward the high-variance portfolios, the post-ranking beta increases by 0.78 and the average return drop by 2.47% per year. Two additional results corroborate this negative beta-return relationship. First, the online appendix documents similar findings over the short sample between 1992 and 2014 (the cross-correlation equals -0.92). Second, we find that during the three largest variance shocks (Q4 1987, Q4 2008, Q3 2011), the market-hedged return of the high- minus low-variance portfolios is always positive (with an average return of 6.23% per quarter), whereas the opposite pattern holds during the three lowest variance shocks (Q1 2012, Q2 2008, Q1 1998). All of these results provide supportive evidence that the returns of the equity portfolios are exposed to market variance risk and can be used to extract information regarding its premium.

Next, Panel B examines whether commonly-used asset pricing models explain the average return difference across portfolios. Whereas high variance shocks are associated with stock market declines (the correlation between factor innovations equals -0.50), the two factors capture different dimensions of risk because the CAPM alphas exhibit the same pattern as the average portfolio returns. For the Fama-French model, the alphas remain different from zero, which is not surprising given that the portfolios have similar size and Book-to-Market (BM) levels (Panel A). Finally, the models still fail to capture the cross-section of average returns when we include traded momentum and Pastor-Stambaugh liquidity factors.[15]

[TABLE 1 HERE]

3.3. Market Variance Predictability

Before moving to the main empirical results, we report in Table 2 the vector $\hat{F}_v$ obtained from the predictive regression of the market realized variance on the predictors—as shown in equation (3), the predicted realized variance $\hat{F}_v z_t$ is a required input for measuring the equity and option VRP projections. To facilitate comparisons across the estimated coefficients, we standardize all predictors.

Panel A contains the estimated coefficients associated with the macro-finance variables. The lagged realized variance produces a strongly positive coefficient that captures the persistent component of the variance process. We also find a positive and statistically significant relationship between the default spread and the future realized variance. Because a risky bond is short the option to default, a low price signals that the future variance is expected to be above average. Conditional on the other predictors, a high PE ratio also signals above-average future variance and helps to capture episodes during which both stock prices and volatility are high. All of these results are in line with those documented by Campbell et al. (2015); Paye (2012) over the same quarterly frequency.

From previous work by Brunnermeier and Pedersen (2009), financial intermediation could potentially amplify shocks to asset markets. Contrary to this view, Panel B reveals that the incremental explanatory power of the broker-dealer variables (leverage and PBI return) is weak in the presence of macro-finance predictors.

[TABLE 2 HERE]

4. Main Empirical Results

We present our main results in four steps. First, we determine how the linear projection of the VRP in each market is related to the macro-finance and broker-dealer variables. Second, we formally compare the two VRP projections. Third, we conduct a short-sample analysis to evaluate

[15] The online appendix reveals that the rejection of these models is stronger during the short sample.

the stability of the results and the performance of a strategy that trades variance risk. Finally, we summarize the large battery of tests that verify the robustness of our empirical findings.

4.1. The Determinants of the Variance Risk Premia

4.1.1. Explanatory Power of the Macro-Finance Variables

We begin our analysis by measuring how the equity VRP varies with the set of macro-finance variables. The estimated vector associated with these variables is computed as $\hat{F}_v - \hat{V}_v^e$, where $\hat{F}_v$ is taken from Table 2 (Panel A) and the risk-neutral vector V_v^e is estimated using the conditional two-pass regression described in Section 2.2. The results in Panel A of Table 3 (first row) reveals several insights. First and consistent with our previous discussion, the average level of the equity VRP is negative and equal to -1.68% per year $(-0.42 \cdot 4)$. Second, the lagged realized variance has a significant impact on the equity VRP, both statistically and economically, i.e., a one-standard deviation increase in realized variance increases the magnitude of the VRP projection by 1.68% per year (-0.42 $\cdot$ 4). In volatile periods, assets that pay off when future volatility increases further becomes extremely valuable and this effect dominates the increase in expected future variance documented in Table 2 (i.e., $\hat{V}_v^{e\prime} z_t > \hat{F}_v^{\prime} z_t$). Third, we observe that the physical and risk-neutral expectation effects offset each other for both the PE ratio and the default spread. Therefore, these variables have a limited impact on the equity VRP despite being strong predictors of the realized variance (as shown in Table 2 and in previous studies). Finally, the coefficient for the inflation rate is both positive and significant. As this variable tends to be high during expansions, it helps capture the countercyclical component of the equity VRP.

Repeating the analysis for the option market, we compute the vector $\hat{F}_v - \hat{V}_v^o$ where the risk-neutral vector $\hat{V}_v^o$ is obtained by regressing the squared VIX index on the macro-finance variables using the GMM procedure described in Section 2.3. The VIX index is constructed from three-month SP500 option prices available over the short sample (1992-2014).[16] Similar to the equity market, Panel A (second row) reveals that the average level of the option VRP is negative (-1.80% per year) and that the coefficients for realized variance and inflation are both statistically significant. The only notable difference comes from the PE ratio whose coefficient is only significant in the option market.

[16] The quarterly VIX index is also referred to as the VXV index and is computed using the same methodology as the 30-day VIX index.

[TABLE 3 HERE]

4.1.2. Adding the Broker-Dealer Variables

Unlike the macro-finance variables, we find that the broker-dealer variables have a different impact on the two markets. Specifically, Panel B of Table 3 (first row) measures the incremental explanatory power of the two broker-dealer variables in the presence of the macro-finance variables. For the equity market, we find that their explanatory power is weak. The coefficients associated with the leverage ratio and PBI return are both close to zero and their t-statistics far below the conventional significance thresholds.

The results are strikingly different for the option market. Panel B (second row) reveals strong and positive relationships between the two broker-dealer variables and the option VRP projection. Periods when intermediaries deleverage or suffer short-term losses are associated with a higher magnitude for the option VRP (and vice-versa). The estimated coefficient for the leverage ratio is not only highly significant, it is also economically large, i.e., a one-standard deviation decrease in leverage increases the magnitude of the premium by 1.48% per year ($0.37 \cdot 4$). Because the two orthogonalized broker-dealer variables are negatively correlated (-0.28), the predictive information contained in the PBI return is obscured when used alone in the regression. Adding the leverage ratio clarifies the relationship between the PBI return and the option VRP and produces a positive and statistically significant coefficient (0.17).

4.2. Comparing the Equity and Option Markets

Next, we formally compare the equity and option markets by focusing on the estimated vector $\hat{V}_v^o - \hat{V}_v^e$ that drives the VRP difference. The results reported in Panels A and B of Table 3 (third row) highlight three important points. First, the average difference between the two VRPs is essentially zero (0.03% per quarter). It implies that a simple analysis of the unconditional premia is insufficient to uncover the large, but temporary discrepancies between the two markets. Second, the macro-finance variables are not relevant for explaining the VRP difference, i.e., none of the estimated coefficients is statistically significant. Therefore, the equity and option VRPs respond similarly to volatility and business cycle conditions. Third, the two broker-dealer variables play a key role in driving the VRP difference. For the leverage ratio, the estimated coefficient is highly significant and implies that a one-standard deviation decline in leverage increases the gap between the equity and option VRPs by 2.08% per year ($-0.52 \cdot 4$)—a change larger than the average premium itself. A similar result holds for the PBI return which yields a negative and significant coefficient of -0.28.

To visualize these findings, we plot in Figure 1 the equity and option VRP projections measured as $(\hat{F}_v - \hat{V}_v^e)' z_t$ and $(\hat{F}_v - \hat{V}_v^o)' z_t$.[17] We see that the two premia are closely aligned, especially over the last decade. Both are characterized by transitory spikes that follow large volatility shocks (e.g., burst of the dotcom bubble, 2008 crisis), and drop during the two recessions recorded between 1992 and 2012. This strong similarity results in a correlation coefficient of 0.69 between the two projections. However, Figure 1 also reveals important discrepancies between the two VRPs. Specifically, the magnitude of the option VRP is substantially larger during the 2008 and European debt crises, whereas the opposite pattern is observed during the late 1990s and early 2000s. As illustrated in Figure 2, these variations are closely associated with leverage—when intermediaries deleverage, we see that the price of variance risk is relatively higher in the option market (and vice-versa).

[FIGURE 1 HERE]

[FIGURE 2 HERE]

4.3. Analysis over the Short Sample

4.3.1. Comparing the Equity and Option Markets

Our estimation procedure exploits information over the long sample to maximize the accuracy of the estimated coefficients for the equity and option VRPs. To verify that the difference between the two markets is not an artefact of our econometric treatment of samples of unequal lengths, we repeat the analysis over the short sample only (1992-2014).

In Panel A of Table 4, we still find that the macro-finance variables drive the VRPs in both markets but not their difference. For the broker-dealer variables, Panel B (third row) reveals that leverage remains strongly related to the VRP difference, while the explanatory power of the PBI return becomes even stronger (its coefficient changes from -0.28 to -0.44). The overall evidence is therefore similar to the one documented over the full sample.

[TABLE 4 HERE]

[17] The path of the equity VRP is computed without the broker-dealer variables. Including these variables yields similar results as shown in the online appendix.

4.3.2. Trading Market Variance Risk

If the VRPs are not always equal, we should observe similar patterns in the returns of strategies that trade market variance risk in the equity and option markets. For the equity market, we define the excess return of the variance-mimicking portfolio $r^e_{s,t+1}$ as a linear combination of the (market-hedged) excess returns of the 25 equity portfolios, such that the variance of the hedging error is minimized and the variance beta equals one (see the online appendix for a detailed description). For the option market, the variance-mimicking portfolio is constructed using the approach of Carr and Wu (2009) described in Section 2.3, and its excess return $r^o_{s,t+1}$ is equal to $rv_{t+1} - vix^2_t$.

Next, we examine the performance of a trading strategy that is: (i) long the variance-mimicking equity portfolio; (ii) short the variance-mimicking option portfolio. Following past work (e.g., Christopherson et al. (1998)), we estimate the time-varying alpha of this strategy as a linear function of the predictors:

$$r_{s,t+1} = r^e_{s,t+1} - r^o_{s,t+1} = a'_s z_t + b'_s f_{t+1} + e_{s,t+1}, \tag{9}$$

where f_{t+1} is the vector of traded risk factors. Table 5 reports the estimated alpha coefficient for each predictor based on four models (CAPM, Fama-French (FF), momentum- and liquidity-based extensions of FF). Overall, the results mirror those documented for the VRP difference in Tables 3 and 4 and confirm the key role played by the two broker-dealer variables. Specifically, selling insurance against variance risk in the option market and hedging this risk in the equity market is profitable when these variables are below average. For instance, a one-standard deviation decline in leverage improves performance by approximately 2.10% per year (0.70·4).

[TABLE 5 HERE]

4.4. Robustness Analysis

To verify that the VRP difference is not driven by a misspecification of the two-factor model, we measure the magnitude of its pricing errors. Equation (7) implies that under the null hypothesis of correct specification, the J-vector c_p is equal to $b_{pv} \cdot V^e_v + b_{pm} \cdot V^e_m$. Therefore, we can perform a joint test based on the sum of the squared pricing errors $Q = \sum_{p=1}^{25} \zeta'_p \zeta_p$, where $\zeta_p = c_p - (b_{pv} \cdot V^e_v + b_{pm} \cdot V^e_m)$.[18] Table 3 reveals that the test statistic (J-stat) is far below the conventional rejection thresholds with or without the broker-dealer variables (the p-values range between 0.31 and 0.40).

[18] The distribution of the test statistic is described in the online appendix.

In the online appendix, we also perform an extensive analysis to evaluate the robustness of our main results. First, we further verify that the two-factor model is correctly specified by examining (i) the properties of the market risk premium, (ii) the hedging errors of the variance-mimicking portfolio, (iii) the impact of additional risk factors, and (iv) the degree of time variation in portfolio betas. Second, we demonstrate that variance jumps can potentially affect the equity and option VRPs, but can hardly explain their difference. Third, we document the same VRP difference when the estimation is based on monthly or individual stock data. In summary, the two broker-dealer variables reliably signal periods when the prices of variance risk differ across the two markets.

5. Interpreting the Evidence

In this section, we provide further interpretations of our main empirical results. First, we discuss the information contained in the equity and option VRPs. Second, we provide potential explanations for the VRP difference. Finally, we provide an economic interpretation for the strong relationship between the broker-dealer variables and the option VRP.

5.1. Information Content of the Variance Risk Premia

The VRP inferred from option prices is commonly interpreted by academics and policymakers as a measure of investors' risk aversion. Our empirical results reveal that this interpretation can be misleading because the information content of the option and equity VRPs is not always identical. Whereas both premia respond similarly to changes in economic and volatility conditions, the option VRP is disproportionately influenced by the broker-dealer variables. When financial intermediaries deleverage, the price of variance risk in the option market is high—yet, this does not imply that equity investors change their attitude towards stocks exposed to variance risk.

The information contained in the VRP is also used to forecast broad economic indicators (e.g., Bekaert and Hoerova (2014); Bollerslev et al. (2009)). If the broker-dealer variables capture shocks that are specific to intermediaries, we expect the option VRP projection to have a lower predictive ability than its equity-based counterpart. Consistent with this interpretation, we find that the ability of the broker-dealer variables to predict economic fundamentals is weak. Specifically, Panel A reports the estimated coefficients of the predictive regressions of the future quarterly market return and industrial production growth on a set of predictors that include the macro-finance variables, the broker-dealer variables, and the non-projected option VRP commonly used in previous studies and defined as $\hat{F}_v' z_t - vix_t^2$. The results show that several macro-finance variables are strong predictors of the market return and economic activity. On the contrary, none of the estimated coefficients for the leverage and PBI return is statistically significant.

Next, Panel B formally evaluates the predictive ability of: (i) the equity VRP projection, (ii) the option VRP projection, and (iii) the non-projected option VRP. In line with our previous findings (Panel A), we observe that the coefficients for the equity VRP projection are all statistically significant, while those for the option VRP projection are not.[19] We also find that the non-projected option VRP helps forecast the market return, which resonates with the results documented by Bollerslev et al. (2009).

[TABLE 6 HERE]

5.2. Possible Explanations for the VRP Difference

The rejection of the null hypothesis of equal VRPs means that the same variance risk is traded at different prices. This result can be explained by market frictions such as informational and regulatory constraints that limit risk-sharing between marginal investors in the equity and option markets. Theoretically, Basak and Croitoru (2000) demonstrate that when investors face portfolio constraints, markets are segmented and deviations from the law of one price can exist in equilibrium.[20] In practice, these constraints can take several forms. Retail investors may lack the expertise required to monitor option positions and mutual funds face limits on the amount of options held in their portfolios. On their side, index option trading desks generally trade exclusively in index futures to manage the delta of their option positions, but not in stocks exposed to market variance risk. When the magnitude of the option VRP is high, equity investors are therefore unable to write options in sufficient number, whereas broker-dealers fail to trade in variance-risk sensitive stocks

[19] Whereas the stock market coefficient is greater for the option-based projection, it is less precisely estimated because of the shorter sample size. Therefore, we cannot reject the null that the true coefficient is equal to zero.

[20] While the effects of trading costs on asset prices depend on multiple factors (including their form and magnitude), they could also induce market segmentation. This could be the case if, for instance, fixed trading costs are sufficiently large to limit investors' participation in the equity and option markets.

(and vice-versa).[21,22]

Alternatively, the gap between the two markets could be driven by different margin requirements. The margin-based asset pricing theory of Garleanu and Pedersen (2011) predicts that investors are willing to pay a higher price to be long in assets that carry lower margins because they consume less capital. Because margins are lower in the option market, this argument implies a higher price of variance risk in the option market or, equivalently, a positive difference between the equity and option VRPs. In addition, this difference should increase when investors' funding constraints are tight (i.e., when capital is scarce).

Whereas both explanations based on segmentation and margin requirements are likely to play a role, the second cannot be fully reconciled with the path followed by the VRP difference for two reasons. First, it cannot easily account for the positive and negative VRP differences observed in Figure 1 because margins in the option market are unlikely to become greater than in the equity market. Second, under the margin-based story, the explanatory power of the broker-dealer variables stems from their ability to track changes in investors' funding constraints. However, we find that alternative and arguably more direct measures of funding constraints, such as the default and TED spreads, do not produce a higher VRP difference, i.e., their coefficients are either not significant (default) or have the wrong sign (TED).

5.3. Broker-Dealer Variables and Option Supply

The VRP difference comes from the strong explanatory power of the broker-dealer variables in the option market. This finding resonates with the key role played by intermediaries in the option market. Chen et al. (2015); Garleanu et al. (2009) empirically demonstrate that public investors have a long net position in SP500 index options, particularly in deep out-of-the-money put options. By market clearing, financial intermediaries write options to satisfy this demand and are structurally short variance risk. As a result, these authors argue that changes in intermediaries' risk-bearing capacity should move the option supply curve and affect option prices.

[21] Anecdotal evidence suggests that during the recent crisis very few equity investors wrote put options in spite of their high prices. One notable exception is Warren Buffett whose short positions in equity put options reached a notional size of $40 billion in 2008 (Triana (2013)). Buffett built this position because he secured a deal in which puts were not marked-to-market in case of adverse market movements. Therefore, he benefitted from a special treatment that is not available to most investors.

[22] In addition, transaction costs can prevent the VRP difference that arises in segmented markets to be eliminated by unconstrained arbitrageurs. For instance, Figlewski (1989) argues that trading costs prevent professional investors from perfectly hedging their option positions in the equity market.

To test the validity of this supply-based mechanism, we examine the relationships between the broker-dealer variables and option prices. Provided that high leverage and PBI return signal a high risk-bearing capacity (Adrian and Shin (2010, 2014)), both variables should have a negative impact on option prices. In Table 7, we report the estimated vector $\hat{V}_v^o$ from the regression of the squared VIX index on the predictors. Because the VIX index is a measure of option expensiveness, $\hat{V}_v^o$ can be interpreted as the option price reaction to changes in the predictor values. The results in Panel B provide evidence in favor of supply effects, i.e., the coefficients are all strongly negative (-0.07 and -0.14) and imply that options become cheaper (expensive) when the leverage ratio and PBI return are high (low).

There are two potential concerns with this supply-based interpretation. First, the leverage ratio could be an endogenous variable if it also measures the quantity of options traded in the market. In a endogenous regression of price on quantity, Hamilton (1994) demonstrates that the slope coefficient is negative when supply shocks are the main determinants of the traded price and quantity. Therefore, the empirical evidence remains consistent with a supply-based mechanism. Second, the results could be affected by the omission of a relevant variable. While this case cannot be definitively ruled out, our baseline specification includes several macro-finance variables that potentially drive option prices. We also examine several additional predictors which all leave the explanatory power of the broker-dealer variables unchanged (see the online appendix).[23]

To shed further light on the supply-based mechanism, we repeat our analysis on (i) the price of individual stock variance risk (changes in individual stock variances) and (ii) the price of correlation risk (changes in the correlation structure of stocks).[24] Whereas the VIX is inferred from index options, the price of individual stock variance risk is computed from individual stock options whose supply is not dominated by financial intermediaries (see Garleanu et al. (2009)). Therefore, changes in their risk-bearing capacity are less likely to drive the prices of these options. Consistent with this interpretation, the results reported in the online appendix reveal that the relationship between leverage and the price of stock variance risk is positive and not statistically significant. For the price of correlation risk, we document similar results to those documented in Table 7 for the VIX index.

[23] Cheng et al. (2015); Etula (2013) also provide empirical evidence that the risk-bearing capacity of intermediaries negatively affect prices in commodity futures and derivatives markets. An important difference with these studies is that we control for a large set of macro-finance variables.

[24] We thank the referee for suggesting this analysis. We also thank Fabio Trojani, Andrea Vedolin, and Gregory Vilkov for sharing their data described in the online appendix.

This similarity resonates with the study by Driessen et al. (2009) which finds that the market VRP is mostly attributed to the premium for bearing correlation risk.

[TABLE 7 HERE]

6. Conclusion

In this paper, we formally compare two conditional versions of the market VRP inferred from equity and option prices. We find that the premia in both markets are, on average, in line with each other and respond similarly to changes in volatility and business cycle conditions. However, we identify episodes when they diverge and find that such differences are explained to a large extent by two broker-dealer variables that measure the financial standing of intermediaries. Specifically, an increase in the leverage or past performance of intermediaries decreases the magnitude of the option VRP (or vice-versa), but leaves the equity VRP unchanged.

The rejection of the null hypothesis that the two VRPs are equal implies that caution should be exercised when the option VRP is used as an aggregate measure of investors' risk aversion. It also indicates the presence of frictions between the two markets that prevent the law of one price to apply. Finally, the close relationships between the broker-dealer variables and the option VRP are consistent with the key role played by financial intermediaries in the option market.

These results can be exploited in future theoretical work that attempts to explain the aggregate pricing of variance risk and to model local demand and supply factors in the option market. They also provide novel empirical evidence regarding the connection between risk-taking by financial intermediaries and asset prices. Understanding the nature of this connection is a major concern for policymakers (e.g., Bernanke and Kuttner (2005); Rajan (2006)) and an interesting avenue of future research.

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Table 1: Summary Statistics for the Variance Portfolios

Panel A shows the annualized excess mean, standard deviation, size (in log form), Book-to-Market (BM) ratio, and the pre-, post-rank variance betas of the quarterly returns of quintile portfolios formed by equally weighting all portfolios in the same variance beta quintile (Low, 2, 3, 4, High). For each quintile portfolio, the pre-rank beta is defined as the mean of the variance betas across stocks on the portfolio formation dates. The post-rank variance beta is computed from the time-series regression of the portfolio return on the variance and market factors (including all predictors). Panel B reports the annualized estimated alpha of each quintile portfolio using the CAPM, the Fama-French (FF) model that includes the market, size, and BM factors, and two extensions that include momentum and liquidity factors, respectively. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Unconditional Moments, Characteristics, and Variance Betas

Quintile	Mean (% p.a.)	St. Dev. (% p.a.)	Size	BM	Pre-rank beta		Post-rank beta	
Low	7.78	16.99	8.14	0.73	-0.70^{**}	(-2.24)	-0.68^{***}	(-3.16)
2	7.55	17.27	8.24	0.72	-0.32	(-0.84)	-0.56^{***}	(-3.06)
3	6.57	16.54	8.30	0.71	-0.03	(-0.09)	-0.47^{**}	(-2.53)
4	5.42	17.05	8.29	0.71	0.26	(0.67)	-0.22	(-1.48)
High	5.31	17.55	8.31	0.71	0.66^{**}	(2.12)	0.10	(0.44)
High-Low	-2.47	7.02	0.17	-0.02	1.36^{***}	(4.37)	0.78^{***}	(3.53)

Panel B: Alphas

Quintile	CAPM (% p.a.)		Fama-French (FF) (% p.a.)		FF+Momentum (% p.a.)		FF+Liquidity (% p.a.)	
Low	1.75^{**}	(1.97)	0.29	(0.32)	0.23	(0.28)	-0.04	(-0.05)
2	1.44	(1.47)	0.20	(0.24)	-0.12	(-0.11)	0.21	(0.24)
3	0.58	(0.79)	-0.65	(-0.95)	-0.11	(-0.16)	-0.70	(-1.07)
4	-0.75	(-1.03)	-1.85^{***}	(-2.85)	-2.27^{***}	(-3.55)	-1.99^{***}	(-2.91)
High	-0.96	(-1.06)	-2.12^{**}	(-2.56)	-1.74^{*}	(-1.80)	-2.22^{**}	(-2.53)
High-Low	-2.71^{**}	(-2.40)	-2.41^{**}	(-2.00)	-2.00^{*}	(-1.67)	-2.18^{*}	(-1.85)

Table 2: Market Variance Predictability

Panel A reports the estimated coefficients and the adjusted R^2 of the predictive regression of the quarterly realized market variance on the set of macro-finance variables that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on the future realized variance. Panel B examines the incremental predictive power of the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	R^2
Realized Variance	0.73^{***}	0.39^{***}	0.26^{***}	0.25^{**}	0.12	0.03	0.17
	(9.13)	(3.77)	(2.67)	(2.31)	(1.27)	(0.38)	

Panel B: Contribution of Broker-Dealer Variables

					Combined		
	Leverage (LEV)	R^2	PB Index (PBI)	R^2	Leverage (LEV)	PB Index (PBI)	R^2
Realized Variance	0.23	0.21	−0.06	0.17	0.23	−0.00	0.20
	(0.97)		(−1.12)		(0.88)	(−0.00)	

Table 3: Equity and Option Variance Risk Premia

Panel A examines the relationships between the macro-finance variables and the equity Variance Risk Premium (VRP), the option VRP, and their difference. The set of variables includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on the VRPs and their difference. The equity- and option-based coefficients are obtained from the conditional two-pass regression and the GMM for samples of unequal lengths, respectively. Panel B examines the incremental predictive power of the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. The J-statistic of the joint test and associated p-values in brackets determine whether the two-factor equity model is correctly specified. Details on the estimation procedure can be found in the appendix. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	J-stat.
Equity VRP	-0.42^{**}	-0.42^{*}	0.24	0.17	0.48^{*}	-0.02	4.55
	(-1.96)	(-1.65)	(0.65)	(0.47)	(1.75)	(-0.07)	[0.40]
Option VRP	-0.45^{***}	-0.34^{***}	0.35^{***}	0.01	0.19^{**}	-0.07	
	(-8.01)	(-3.70)	(3.42)	(0.12)	(2.22)	(-0.72)	
Difference	0.03	-0.09	-0.12	0.16	0.29	0.05	
	(0.10)	(-0.39)	(-0.59)	(0.55)	(1.44)	(0.25)	

Panel B: Contribution of Broker-Dealer Variables

					Combined		
	Leverage (LEV)	J-stat.	PB Index (PBI)	J-stat.	Leverage (LEV)	PB Index (PBI)	J-stat.
Equity VRP	-0.13	5.60	-0.10	5.34	-0.15	-0.11	6.29
	(-0.52)	[0.31]	(-0.47)	[0.40]	(-0.59)	(-0.58)	[0.33]
Option VRP	0.31^{***}		0.07		0.37^{***}	0.17^{**}	
	(3.84)		(0.83)		(4.68)	(2.03)	
Difference	-0.43^{***}		-0.17		-0.52^{***}	-0.28^{**}	
	(-5.32)		(-1.40)		(-5.90)	(-2.31)	

Table 4: Equity and Option Variance Risk Premia: Short Sample

Panel A examines the relationships between the macro-finance variables and the equity Variance Risk Premium (VRP), the option VRP, and their difference. The set of variables includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on the VRPs and their difference. The equity- and option-based coefficients are obtained from the conditional two-pass regression and the GMM for samples of unequal lengths, respectively. Panel B examines the incremental predictive power of the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. The J-statistic of the joint test and associated p-values in brackets determine whether the two-factor equity model is correctly specified. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	J-stat.
Equity VRP	−0.64***	−0.16	0.23	0.42	0.68***	0.17	5.34
	(−3.01)	(−0.54)	(0.66)	(1.06)	(2.96)	(0.51)	[0.18]
Option VRP	−0.32***	−0.25	0.19*	0.22	0.30***	−0.14	
	(−4.06)	(−1.55)	(1.85)	(0.97)	(2.65)	(−0.96)	
Difference	−0.31	0.08	0.04	0.19	0.38	0.31	
	(−0.95)	(0.13)	(0.29)	(0.72)	(1.20)	(1.16)	

Panel B: Contribution of Broker-Dealer Variables

					Combined		
	Leverage (LEV)	J-stat.	PB Index (PBI)	J-stat.	Leverage (LEV)	PB Index (PBI)	J-stat.
Equity VRP	0.15	7.45	−0.22	6.46	0.12	−0.18	8.93
	(0.67)	[0.06]	(−1.06)	[0.15]	(0.53)	(−1.05)	[0.03]
Option VRP	0.57***		0.09		0.64***	0.26**	
	(4.23)		(0.78)		(4.50)	(2.13)	
Difference	−0.41***		−0.31**		−0.51***	−0.44***	
	(−3.33)		(−2.06)		(−4.04)	(−3.52)	

Table 5: Performance of the Market Variance Trading Strategy

This table reports the performance of a trading strategy that is long the variance-mimicking equity portfolio and short the variance-mimicking option portfolio during the short sample (1992-2014). It reports the estimated alpha coefficients for the full set of predictors that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), the quarterly employment rate (EMP), the broker-dealer leverage ratio (LEV), and the quarterly return of the prime broker index (PBI). The coefficients determine the impact of a one-standard deviation change in the predictors on the alpha of the strategy using the CAPM, the Fama-French (FF) model that includes market, size, and BM factors, and two extensions that include momentum and liquidity factors, respectively. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	Leverage (LEV)	PB Index (PBI)	R^2
CAPM	−0.11	0.07	0.16	−0.08	0.01	−0.06	-0.61^{***}	-0.37^{***}	0.30
	(−0.95)	(0.38)	(0.91)	(−0.35)	(0.14)	(−0.29)	(−4.72)	(−3.01)	
Fama-French (FF)	−0.00	−0.04	0.21	−0.18	0.04	-0.31^{*}	-0.70^{***}	-0.28^{**}	0.40
	(−0.01)	(−0.20)	(1.38)	(−0.81)	(0.41)	(−1.68)	(−5.70)	(−2.54)	
FF+Momentum	0.00	−0.04	0.21	−0.18	0.04	-0.30^{*}	-0.70^{***}	-0.28^{**}	0.40
	(0.04)	(−0.21)	(1.34)	(−0.82)	(0.41)	(−1.68)	(−5.66)	(−2.51)	
FF+Liquidity	0.00	−0.04	0.22	−0.17	0.03	−0.30	-0.69^{***}	-0.29^{**}	0.40
	(0.08)	(−0.20)	(1.40)	(−0.77)	(0.25)	(−1.60)	(−5.78)	(−2.53)	

Table 6: Information Content of the Equity and Option Variance Risk Premia

Panel A reports the estimated coefficients and the adjusted R^2 of predictive regressions of the quarterly market returns and industrial production growth on a set of predictors that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP), the broker-dealer leverage ratio (LEV), the quarterly return of the prime broker index (PBI), and the non-projected option Variance Risk Premium (VRP) defined as $F_v' z_t - vix_t^2$. Panel B reports the estimated coefficients and the adjusted R^2 of univariate predictive regressions of the quarterly market returns and industrial production growth on the equity VRP projection, the option VRP projection, and the non-projected option VRP. The coefficients determine the impact of a one-standard deviation change in the variables on the predicted variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Informational Content of the Predictors

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	Leverage (LEV)	PB Index (PBI)	Option VRP (Non-projected)	R^2
Market	1.97^{***}	2.49^{**}	-3.39^{***}	-3.29^{**}	-1.64^{**}	1.02	-0.74	0.55	-2.94^{***}	0.21
	(2.68)	(2.23)	(-3.15)	(-2.10)	(-2.02)	(0.80)	(-0.58)	(0.55)	(-3.44)	
IP Growth	0.59^{***}	-0.24	-0.09	-0.72^{***}	-0.10	0.03	-0.29	0.22	-0.10	0.49
	(6.30)	(-1.53)	(-0.63)	(-3.45)	(-0.95)	(0.14)	(-1.61)	(1.53)	(-0.82)	

Panel B: Predictive Power of the Variance Risk Premia

	Equity VRP (projection)	R^2	Option VRP (projection)	R^2	Option VRP (non-projected)	R^2
Market	-1.34^{*}	0.02	-3.22	0.09	-3.90^{***}	0.19
	(-1.67)		(-1.20)		(-2.88)	
IP Growth	0.39^{**}	0.05	0.07	0.00	-0.00	0.00
	(2.42)		(0.12)		(-0.00)	

Table 7: The Squared VIX Index

Panel A reports the estimated coefficients and the adjusted R^2 of the regression of the quarterly squared VIX index on the set of macro-finance variables that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on the squared VIX and are computed using the GMM for samples of unequal lengths. Panel B examines the incremental predictive power of the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	R^2
Squared VIX	1.18*** (27.37)	0.73*** (9.47)	−0.09 (−0.97)	0.24** (3.46)	−0.07 (−1.22)	0.09 (1.17)	0.75

Panel B: Contribution of Broker-Dealer Variables

					Combined		
	Leverage (LEV)	R^2	PB Index (PBI)	R^2	Leverage (LEV)	PB Index (PBI)	R^2
Squared VIX	−0.07** (−1.97)	0.75	−0.14** (−2.07)	0.77	−0.14*** (−3.71)	−0.17** (−2.53)	0.77

Figure 1: Equity and Option Variance Risk Premia

This figure reports the paths of the quarterly equity (solid line) and option (dashed line) Variance Risk Premium (VRP) projections obtained with the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, the quarterly employment rate, the broker-dealer leverage, and the quarterly return of the prime broker index. The path of the option VRP is only reported during the short sample (1992-2014) because the quarterly VIX index is only available in 1992. The y-axis is in percent per quarter. Shaded areas correspond to NBER recession periods. Markers indicate the VRP for the quarter that follows the 1973 oil price shock (Oil Shock), the 1987 stock market crash (87 Crash), the beginning of the 1991 US military operation in Kuwait and Iraq (Gulf War), the 1998 collapse of the Long Term Capital Management fund (LTCM), the September 2001 terrorist attacks (9/11), the 2008 collapse of Lehman Brothers (Lehman), and the 2011 announcement of the Greek referendum on the exit from the Eurozone that followed the second rescue program (Greece).

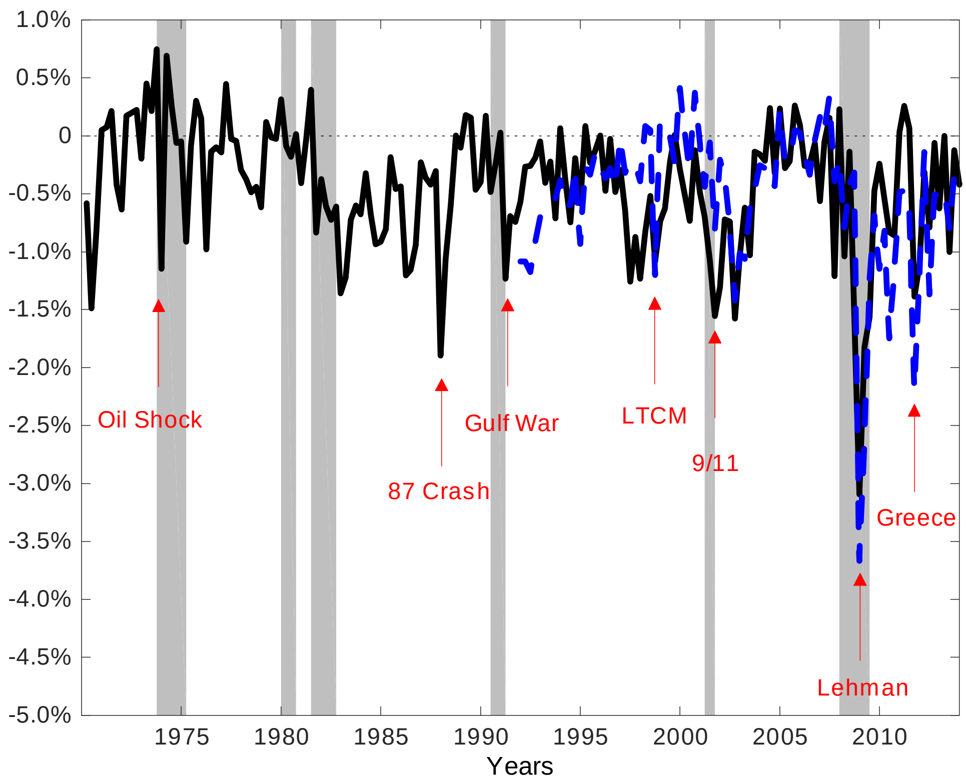
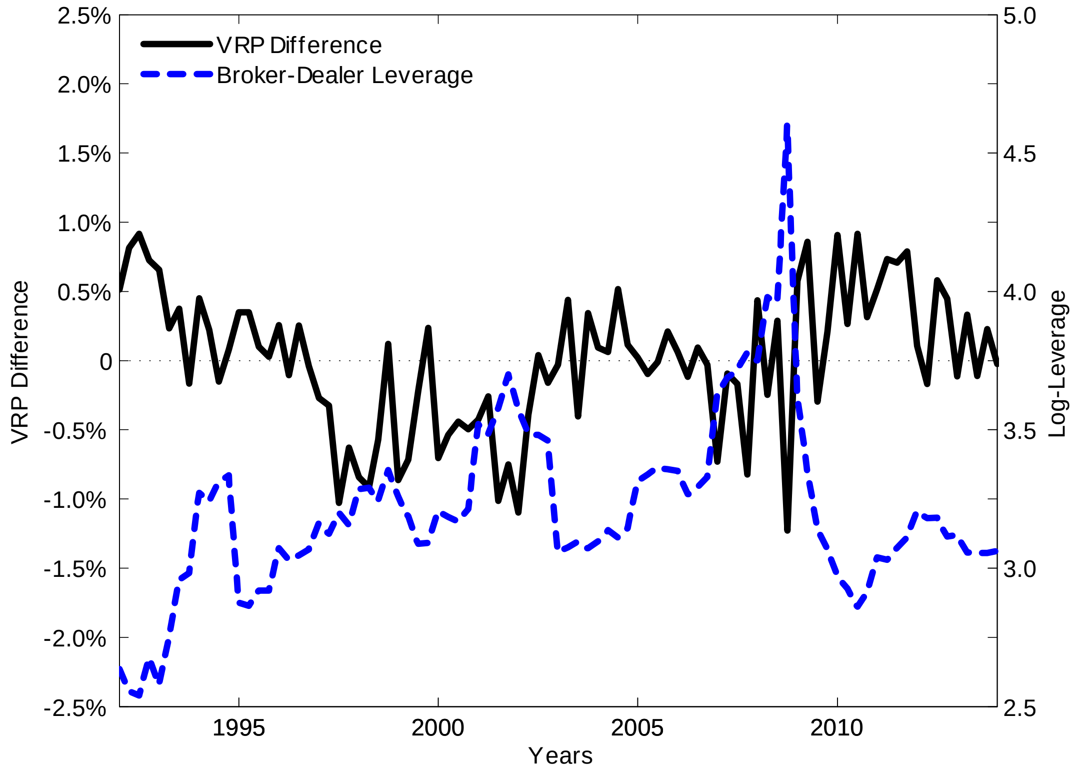


Figure 2: Variance Risk Premium Difference and Broker-Dealer Leverage

This figure plots the quarterly difference between the equity and the option Variance Risk Premia (solid line) obtained with the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, the quarterly employment rate, the broker-dealer leverage, and the quarterly return of the prime broker index. The dashed line shows the evolution of the quarterly leverage ratio of broker-dealers (in log form). The left y-axis is in percent per quarter.



Online Appendix for

Does Variance Risk Have Two Prices? Evidence from the Equity and Option Markets

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A Data Description

A.1 Predictive Variables

We provide more detail on the definition of the macro-finance variables used to capture the dynamics of the equity and option VRPs. The PE ratio is downloaded from Robert Shiller's webpage and is defined as the price of the SP500 divided by the 10-year trailing moving average of aggregate earnings. The quarterly inflation rate is computed from the Producer Price Index (PPI), aggregate employment is measured by the total number of employees in the nonfarm sector (seasonally-adjusted), and the default spread is defined as the yield differential between Moody's BAA- and AAA-rated bonds. These three series are downloaded from the Federal Reserve Bank of St. Louis.

Tables I and II report the unconditional moments and correlation matrix of the predictors over the long sample between 1970 and 2014 (179 quarterly observations) and over the short sample between 1992 and 2014 (92 quarterly observations). Overall, the summary statistics for the macro-finance studies are similar to those reported in previous studies. The two broker-dealer variables contain information at different frequencies—the leverage ratio is a slow-moving predictor, whereas the PBI return captures the short-term reaction of intermediaries to aggregate losses.

[TABLE I HERE]

[TABLE II HERE]

A.2 The Set of Equity Portfolios

A.2.1 Portfolio Formation

We use the approach developed by Ang, Hodrick, Xing, and Zhang (2006) to form the cross-section of 25 variance risk-sensitive portfolios. First, we estimate the market and variance betas of individual stocks each month using daily returns over the previous month. Using a daily frequency allows us to pin down the conditional risk loadings without specifying the conditioning information (see Lewellen and Nagel (2006) for a detailed discussion). For each stock with at least 17 daily observations, we regress its return on the CRSP market return $r_{m,d}$ and the innovation of the SP500 realized variance $u_{v,d}$. Computing a model-free variance innovation based on intraday return observations is not feasible because this data is only available in the latter part of the sample. To address this issue, we model the daily conditional variance of the SP500 return using a standard GARCH (1,1): $\sigma_d^2 = \gamma + \alpha\sigma_{d-1}^2 + \beta\varepsilon_d^2$, where ε_d^2 is the daily squared SP500 return. After estimating the parameters γ, α, and β using daily returns over a one-year rolling window, we compute $u_{v,d}$ as $\varepsilon_d^2 - \hat{\sigma}_{d-1}^2$, where $\hat{\sigma}_{d-1}^2$ is the conditional variance estimated on the previous day.[1]

Second, we sort stocks according to their exposures to the market and variance factors. Since short-window regressions can produce large estimation errors, we rank each stock i based on its beta t-statistics, $\hat{t}_{im,t} = \hat{b}_{im,t}/\hat{\sigma}_{b_{im,t}}$ and $\hat{t}_{iv,t} = \hat{b}_{iv,t}/\hat{\sigma}_{b_{iv,t}}$, where $\hat{\sigma}_{b_{im,t}}$, $\hat{\sigma}_{b_{iv,t}}$ denote the estimated volatilities of the estimated betas $\hat{b}_{im,t}$ and $\hat{b}_{iv,t}$.[2] Stocks are ranked first into quintiles based on their market t-statistic, and then into quintiles based on their variance t-statistic.

Third, we compute the average return of all stocks in each group. To mitigate liquidity concerns, we apply a value-weighting scheme and exclude NASDAQ stocks. Repeating these three steps each month over the entire sample period, we obtain the return time-series of the 25 variance portfolios.

A.2.2 Summary Statistics over the Short Sample

Table III summarizes the properties of the variance portfolios (Low, 2, 3, 4, High) over the short sample (1992-2014). Consistent with the results in Table 1 of the paper, Panel A

[1] Whereas Ang, Hodrick, Xing, and Zhang (2006) use the daily change in the VIX index to measure $u_{v,d}$, we follow a different approach for two reasons. First, as noted by these authors, the VIX is a noisy proxy of the variance innovation because it also captures changes in the risk premium itself. Second, data on the VIX is only available in the early 1990s whereas our sample begins in 1970.

[2] Several papers (e.g., Kosowski, Timmerman, Wermers, and White (2006)) show that the t-statistic allows for an improved ranking because it controls for the precision of the estimated coefficient. Consistent with these studies, we find that ranking based on t-statistics largely improves the post-ranking characteristics of the variance portfolios (see Section F.1).

reveals that both average returns and post-formation variance betas vary monotonically across portfolios. The spreads in average returns and variance betas between the high- and low-variance portfolios are both higher than the ones observed during the full sample (-5.09% vs -3.11% per year for returns, 1.49 vs 0.73 for betas). Panel B shows that the rejections of the commonly-used asset pricing models are stronger during the short sample. Across the four tested models, the estimated annual alphas of the high minus low variance portfolio range between -5.97% (CAPM) and -4.67% (Fama-French and liquidity) and are all highly significant.

[TABLE III HERE]

B Estimation Procedure

B.1 The Equity Market

B.1.1 Two-Pass Regression in a Conditional Setting

This section provides additional information on the procedure for estimating the Variance Risk Premium (VRP) projection in the equity market using the conditional two-pass regression approach developed by Gagliardini, Ossola, and Scaillet (2015). In the first step, we estimate, for each equity portfolio p ($p = 1, ..., n$), the coefficients of the following time-series regression:

$$r^e_{p,t+1} = -c'_p z_t + b_{pv} \cdot rv_{t+1} + b_{pm} \cdot f_{m,t+1} + e_{p,t+1}, \tag{B1}$$

where $r^e_{p,t+1}$ is the excess return of portfolio p, z_t is the J-vector of lagged predictors (including a constant), rv_{t+1} is the realized market variance, and $f_{m,t+1}$ is the market factor. The $(J+2)$-vector of coefficients $\beta_p = (-c'_p, b_{pv}, b_{pm})'$ is equal to $E[x_{t+1}x'_{t+1}]^{-1}E[x_{t+1}r^e_{p,t+1}]$, where $x_{t+1} = (z_t', rv_{t+1}, f_{m,t+1})'$. The OLS estimator of β_p is given by

$$\hat{\beta}_p = \left(\sum_{t=1}^{T} x_{t+1} x_{t+1}' \right)^{-1} \sum_{t=1}^{T} x_{t+1} r_{p,t+1}^e, \tag{B2}$$

where T is the total number of return observations.

In the second step, we compute the estimator of the $2J$-vector $V^e = (V_v^{e\prime}, V_m^{e\prime})'$ that drives the risk-neutral expectations of the two risk factors (variance and market). Specifically, we use a WLS approach to estimate the following cross-sectional regression:

$$c_p = B_p V^e, \tag{B3}$$

where B_p a $J \times 2J$ matrix equal to $[b_{pv} \cdot I_J, b_{pm} \cdot I_J]$, and I_J is the $J \times J$ identity matrix. For each portfolio p, we compute a $J \times J$ matrix of weights $w_p = diag(v_p)^{-1}$, where v_p is the covariance matrix of the J-vector of standardized errors $\sqrt{T}(\hat{c}_p - \hat{B}_p V^e)$. This matrix is equal to $v_p = C_{V^e}' Q_x^{-1} S_{pp} Q_x^{-1} C_{V^e}$, where $Q_x = E[x_{t+1} x_{t+1}']$, S_{pp} is a $(J+2) \times (J+2)$ matrix equal to $E[e_{p,t+1}^2 x_{t+1} x_{t+1}']$, C_{V^e} is a $(J+2) \times J$ matrix defined as $[E_1' - (I_J \otimes V^{e\prime}) J_A E_2']'$ with $E_1 = [I_J, \mathbf{0}_{J\times 2}]'$, $E_2 = [\mathbf{0}_{2\times J}, I_2]'$, $J_A = W_{J,J\cdot 2}(I_2 \otimes vec(I_J))$, $\mathbf{0}_{J\times 2}$ is a $J \times 2$ matrix of zeros, and $W_{J,J\cdot 2}$ is a $(J, J \cdot 2)$-commutation matrix.[3] The empirical counterpart of v_p is given by

$$\widehat{v}_p = C_{\hat{V}_1^e}' \hat{Q}_x^{-1} \hat{S}_p^{-1} \hat{Q}_x^{-1} C_{\hat{V}_1^e}, \tag{B4}$$

where $\hat{Q}_x = \frac{1}{T} \sum_{t=1}^{T} x_{t+1} x_{t+1}'$, $\widehat{S}_{pp} = \frac{1}{T} \sum_{t=1}^{T} \hat{e}_{p,t+1}^2 x_{t+1} x_{t+1}'$, $\hat{e}_{p,t+1} = r_{p,t+1}^e - \hat{\beta}_p' x_{t+1}$, $C_{\widehat{V}_1^e} = [E_1' - (I_J \otimes \hat{V}_1^{e\prime}) J_A E_2']'$, and $\hat{V}_1^e$ is the first-step OLS estimator of V^e obtained using unit weights, i.e., $\hat{V}_1^e = \left(\sum_{p=1}^{n} \hat{B}_p' \hat{B}_p \right)^{-1} \sum_{p=1}^{n} \hat{B}_p' \hat{c}_p$. Using the estimated matrix of weights

[3] The commutation matrix $W_{n,m}$ of order $n \cdot m \times n \cdot m$ is defined such that $W_{n,m} vec(A) = vec(A')$ for any matrix $A \in R^{m\times n}$.

$\hat{w}_p$, we obtain the following estimator of V^e :

$$\widehat{V}^e = \left(\sum_{p=1}^{n} \hat{B}_p' \hat{w}_p \hat{B}_p\right)^{-1} \sum_{p=1}^{n} \hat{B}_p' \hat{w}_p \hat{c}_p. \tag{B5}$$

Third, we turn to the estimation of the vector of the linear forecasts of the risk factors defined as $proj(f_{t+1}|\, z_t) = F'z_t$, where $f_{t+1} = (rv_{t+1}, f_{m,t+1})'$, and $F = [F_v, F_m]$ is a $J \times 2$ matrix of coefficients equal to $E[z_t z_t']^{-1} E[z_t f_{t+1}']$. These coefficients are obtained from an OLS regression of the factors on the lagged predictors, i.e.,

$$\hat{F} = \left(\sum_{t=1}^{T} z_t z_t'\right)^{-1} \sum_{t=1}^{T} z_t f_{t+1}'. \tag{B6}$$

Combining equations (B5) and (B6), we compute the projections of the variance and market risk premia as

$$\begin{aligned}
\hat{\lambda}_{v,t}^{e}(z) &= \left(\hat{F}_v - \hat{V}_v^e\right)' z_t, \\
\hat{\lambda}_{m,t}^{e}(z) &= \left(\hat{F}_m - \hat{V}_m^e\right)' z_t, .
\end{aligned} \tag{B7}$$

B.1.2 Distribution of the Estimated Coefficients

In an unconditional setting, Jagannathan and Wang (1998) show that the estimated vector of unconditional risk premia is consistent and asymptotically normally distributed. Its covariance matrix Σ_λ is equal to $\Sigma_f + \frac{1}{n}\Sigma_V$, where Σ_f is the covariance matrix of the risk factors, and Σ_{V^e} is the covariance matrix of the estimated risk factor expectations under the risk-neutral measure. Gagliardini, Ossola, and Scaillet (2015) demonstrate that a similar result holds in a conditional setting. Specifically, the $2J$-vector that drives the vector of conditional risk premia, $\hat{\Lambda}^e = vec(\hat{F}) - \hat{V}^e$, is consistent and asymptotically normally distributed, i.e.,

$$\sqrt{T}\left(\hat{\Lambda}^e - \Lambda^e\right) \Rightarrow N(\mathbf{0}_{2J\times 1}, \Sigma_{\Lambda^e}). \tag{B8}$$

The $2J \times 2J$ covariance matrix Σ_{Λ^e} is the sum of two terms, Σ_F and $\frac{1}{n}\Sigma_{V^e}$, defined as

$$\Sigma_F = (I_J \otimes Q_z^{-1})\Sigma_u(I_J \otimes Q_z^{-1}), \tag{B9}$$

$$\Sigma_{V^e} = \left(\frac{1}{n}B'WB\right)^{-1}\frac{1}{n}B'WVWB\left(\frac{1}{n}B'WB\right)^{-1}, \tag{B10}$$

where Σ_u is a $2J \times 2J$ matrix equal to $E(u_{t+1}u'_{t+1} \otimes z_t z'_t)$ with $u_{t+1} = f_{t+1} - F'z_t$, B is a $Jn \times 2J$ matrix defined as $[B'_1, ..., B'_n]'$, W is a $Jn \times Jn$ block diagonal matrix with elements $[w_p]_{p=1,...,n}$ on its diagonal, and V is a $Jn \times Jn$ matrix composed of $J \times J$ submatrices $[V_{pk}]_{p,k=1,...,n}$ with $V_{pk} = C'_{V^e}Q_x^{-1}S_{pk}Q_x^{-1}C_{V^e}$, and $S_{pk} = E[e_{p,t+1}e_{k,t+1}x_{t+1}x'_{t+1}]$. A consistent estimator of Σ_{Λ^e} can be obtained by replacing Σ_u, Q_z^{-1}, B, W, and V with their empirical counterparts.

B.1.3 Joint Test of Correct Specification

To determine whether the two-factor model is correctly specified, we use the test statistic proposed by Kan, Robotti, and Shanken (2013). Under the null hypothesis of correct specification, the sum of the squared pricing errors, $Q = \sum_{p=1}^{n}(c_p - B_pV^e)'(c_p - B_pV^e)$, is equal to zero and its estimated counterpart multiplied by T is asymptotically distributed as

$$T\hat{Q} = \frac{T}{n}\sum_{p=1}^{n}\hat{\zeta}'_p\hat{w}_p\hat{\zeta}_p \xrightarrow{d} \frac{1}{n}\sum_{p=1}^{nJ-2J} eig_p \cdot \chi_p^2, \tag{B11}$$

where $\hat{\zeta}_p$ is the J-vector of estimated errors computed as $\hat{c}_p - \hat{B}_p\hat{V}^e$, eig_p $(p = 1, ..., nJ-2J)$ are the non-zero eigenvalues of the matrix $D = V^{1/2}(W - WB(B'WB)^{-1}B'W)V^{1/2}$, and χ_p^2 $(p = 1, ..., nJ - 2J)$ are i.i.d. chi-square variables with one degree of freedom.

B.2 The Option Market

B.2.1 Treatment of Samples with Unequal Lengths

This section explains how to estimate the VRP projection in the option market. To compute the J-vector of risk-neutral coefficients V_v^o, we run a time-series regression of the squared VIX index vix_t^2 on the predictors. The main issue is that the periods over which the variables are observed have unequal lengths: the realized variance and predictors are available since 1970 (long sample), whereas the VIX index is only observed since the early 1990's (short sample). To exploit the information contained in the long sample, we use an extension of the Generalized Method of Moments (GMM) developed by Lynch and Wachter (2013). To begin, we denote by $g(V_{v,S}^o)$ and $g(F_{v,S})$ the J-vectors of moments associated with V_v^o and F_v over the short sample, and by $g(F_v)$ the J-vector of moments associated with F_v over the long sample:

$$\begin{aligned} g(V_{v,S}^o) &= \frac{1}{\lambda T}\sum^{\lambda T} f_t(V_{v,S}^o) = \frac{1}{\lambda T}\sum^{\lambda T} z_t u_{vix,t+1} = \frac{1}{\lambda T}\sum^{\lambda T} z_t(vix_t^2 - V_v^{o\prime} z_t), \\ g(F_{v,S}) &= \frac{1}{\lambda T}\sum^{\lambda T} f_t(F_{v,S}) = \frac{1}{\lambda T}\sum^{\lambda T} z_t u_{v,t+1} = \frac{1}{\lambda T}\sum^{\lambda T} z_t(rv_{t+1} - F_v' z_t), \\ g(F_v) &= \frac{1}{T}\sum^{T} f_t(F_v) = \frac{1}{T}\sum^{T} z_t u_{v,t+1} = \frac{1}{T}\sum^{T} z_t(rv_{t+1} - F_v' z_t), \end{aligned} \tag{B12}$$

where λT is the number of observations over the short sample. The procedure proposed by Lynch and Wachter (2013) consists of using a new set of moments to estimate V_v^o:

$$g(V_v^o) = g(V_{v,S}^o) - B_{V_v^o,F_v}(g(F_v) - g(F_{v,S})), \tag{B13}$$

where each row of the $J \times J$ matrix $B_{V_v^o,F_v}$ contains the coefficients of a regression of each element in $g(V_{v,S}^o)$ on the J-vector $g(F_{v,S})$. To compute these coefficients, we estimate the vectors V_v^o and F_v over the short sample. Then, we use equations (B12) to compute the J-vectors $f_t(\hat{V}_{v,S}^o)$ and $f_t(\hat{F}_{v,S})$ at each time t, and run a time-series regression of each element in $f_t(\hat{V}_{v,S}^o)$ on $f_t(\hat{F}_{v,S})$.

By construction, the estimated vector of long sample moments $g(\hat{F}_v)$ equals zero because we use it to compute $\hat{F}_v$, while the estimated short sample moment $g(\hat{F}_{v,S})$ is given by $\frac{1}{\lambda T} Z'(Y_{rv} - Z\hat{F}_v)$, where $Z = [z_{(1-\lambda)T+1}', ..., z_T']'$, and $Y_{rv} = [rv_{(1-\lambda)T+2}, ..., rv_{T+1}]'$.

Plugging these estimates into equation (B13), we have:

$$\begin{aligned} g(V_v^o) &= \frac{1}{\lambda T} Z'(Y_{vix} - ZV_v^o) - \hat{B}_{V_v^o,F_v} \left(\frac{1}{\lambda T} Z'(Y_{rv} - Z\hat{F}_v) \right) \\ &= \frac{1}{\lambda T} \left(Z'(Y_{vix} - ZV_v^o) - \hat{B}_{V_v^o,F_v} Z'(Y_{rv} - Z\hat{F}_v) \right), \end{aligned} \tag{B14}$$

where $Y_{vix} = [vix^2_{(1-\lambda)T+1}, ..., vix^2_T]'$. Therefore, the adjusted estimated vector given by

$$\hat{V}_v^o = (Z'Z)^{-1} \left(Z'Y_{vix} - \hat{B}_{V_v^o,F_v} Z'(Y_{rv} - Z\hat{F}_v) \right) = \hat{V}_{v,S}^o - \hat{A}, \tag{B15}$$

where $\hat{A}$ is the adjustment factor given by $(Z'Z)^{-1} \hat{B}_{V_v^o,F_v} Z'(Y_{rv} - Z\hat{F}_v)$.

B.2.2 Distribution of the Estimated Coefficients

Exploiting the results derived by Lynch and Wachter (2013), we can determine the properties of the $2J$-vector of estimated coefficients $\hat{C}^o = [\hat{F}_v', \hat{V}_v^{o\prime}]'$. Specifically, it is consistent and asymptotically normally distributed, i.e.,

$$\sqrt{\lambda T} \left(\hat{C}^o - C^o \right) \Rightarrow N(\mathbf{0}_{2J\times 1}, \Sigma_{C^o}). \tag{B16}$$

The $2J \times 2J$ covariance matrix Σ_{C^o} is equal to

$$\Sigma_{C^o} = (I_2 \otimes E[z_t z_t']^{-1}) S^A (I_2 \otimes E[z_t z_t']^{-1}), \tag{B17}$$

where S^A is defined as

$$S^A = \begin{bmatrix} \lambda S_{F_v} & \lambda S_{F_v,V_v^o} \\ \lambda S_{V_v^o,F_v} & S_{V_v^o} - (1-\lambda) S_{V_v^o,F_v} S_{F_v}^{-1} S_{F_v,V_v^o} \end{bmatrix}, \tag{B18}$$

with $S_{F_v} = \sum_{\tau=-\infty}^{\infty} E[f_t(F_v) f_{t-\tau}(F_v)']$, $S_{F_v,V_v^o} = \sum_{\tau=-\infty}^{\infty} E[f_t(F_v) f_{t-\tau}(V_v^o)']$, and $S_{V_v^o} = \sum_{\tau=-\infty}^{\infty} E[f_t(V_v^o) f_{t-\tau}(V_v^o)']$. To estimate these elements, we build on the procedure described by Stambaugh (1997) and Lynch and Wachter (2013). First, we use the White estimator to compute $\hat{S}_{F_v}$ over the full sample. Then, we use the estimated coefficient matrix $\widehat{B}_{V_v^o,F_v}$ and the estimated residual covariance matrix $\hat{\Sigma}$ from the regression of $f_t(\hat{V}_{v,S}^o)$ on $f_t(\hat{F}_{v,S})$ to compute the remaining terms:

$$\hat{S}_{F_v,V_v^o} = \hat{S}_{F_v}' \hat{B}_{V_v^o,F_v}', \tag{B19}$$

$$\hat{S}_{V_v^o} = \hat{\Sigma} + \hat{B}_{V_v^o,F_v} \hat{S}_{F_v} \hat{B}_{V_v^o,F_v}'. \tag{B20}$$

This approach guarantees that the estimator of $\hat{S}^A$ is positive-definite. Plugging this estimator in equation (B18) and replacing $E[z_t z_t']$ with its estimated value over the long sample, $\hat{Q}_z = \frac{1}{T}\sum_{t=1}^{T} z_t z_t'$, we obtain a consistent estimator of Σ_{C^o}.

Based on these results, we can determine the properties of the coefficients that affect the dynamics of the option VRP. Specifically, the J-vector of estimated coefficients $\hat{\Lambda}_v^o = \hat{F}_v - \hat{V}_v^o$ is asymptotically distributed as

$$\sqrt{\lambda T}(\hat{\Lambda}_v^o - \Lambda_v^o) \Rightarrow N(\mathbf{0}_{J\times 1}, \Sigma_{\Lambda_v^o}). \tag{B21}$$

The $J \times J$ covariance matrix $\Sigma_{\Lambda_v^o}$ is given by

$$\Sigma_{\Lambda_v^o} = \Sigma_{C^o}^1 + \Sigma_{C^o}^2 - 2\Sigma_{C^o}^{1,2}, \tag{B22}$$

where $\Sigma_{C^o}^1$ is the $J \times J$ upper block of Σ_{C^o}, $\Sigma_{C^o}^2$ is the $J \times J$ lower block of Σ_{C^o}, and $\Sigma_{C^o}^{1,2}$ is the off-diagonal block of Σ_{C^o}.

B.3 t-Statistics for the Difference in Estimated Coefficients

The difference between the equity and option VRPs is defined as $d_v' z_t$, where d_v is a J-vector equal to $V_v^o - V_v^e$. To determine whether each element of the estimated vector $\widehat{d_v}$ is significantly different from zero, we implement a bootstrap procedure. Consistent with the specification chosen to estimate both VRPs, we model the dynamics of the excess return of each equity portfolio p ($p = 1, ..., n$), the predictors, the market return, the realized variance, and the squared VIX index as

$$
\begin{aligned}
r^e_{p,t+1} &= -(b_{pv} \cdot V^{e\prime}_v + b_{pm} \cdot V^{e\prime}_m) z_t + b_{pv} \cdot rv_{t+1} + b_{pm} \cdot f_{m,t+1} + e_{p,t+1}, \\
z_{t+1} &= \Phi z_t + u_{z,t+1}, \\
rv_{t+1} &= F'_v z_t + u_{v,t+1}, \\
f_{m,t+1} &= F'_m z_t + u_{m,t+1}, \\
vix^2_t &= V^{o\prime}_v z_t + u_{vix,t}.
\end{aligned}
\tag{B23}
$$

After estimating the different coefficients in the system of equations (B23), we build a $\lambda T \times N$ matrix of estimated residuals, $\widehat{R} = [\hat{e}, \hat{u}_z, \hat{u}_v, \hat{u}_m, \hat{u}_{vix}]$, where λT is the number of observations over the short sample, and N is equal to $n + (J-1) + 3$. We have $\hat{e} = [\hat{e}'_{(1-\lambda)T+2}, ..., \hat{e}'_{T+1}]'$ with $\hat{e}_t = [\hat{e}_{1,t}, ..., \hat{e}_{n,t}]$, $\hat{u}_z = [\hat{u}'_{z,(1-\lambda)T+2}, ..., \hat{u}'_{z,T+1}]'$, $\hat{u}_v = [\hat{u}_{v,(1-\lambda)T+2}, ..., \hat{u}_{v,T+1}]'$, $\hat{u}_m = [\hat{u}_{m,(1-\lambda)T+2}, ..., \hat{u}_{m,T+1}]'$, and $\hat{u}_{vix} = [\hat{u}_{vix,(1-\lambda)T+1}, ..., \hat{u}_{vix,T}]'$.

For each bootstrap iteration b ($b = 1, ..., 1,000$), we first draw with replacement a set of T rows from the matrix $\widehat{R}$. This procedure allows us to preserve the cross-sectional correlation between the residuals. Second, we plug the estimated coefficients and the bootstrapped residuals into equations (B23) to build, for each time t ($t = 1, ..., T$), the J-vector of predictors $z_t(b)$, the excess return of the variance portfolios $r^e_{p,t+1}(b)$, the realized variance $f_{v,t+1}(b)$, and the market return $f_{m,t+1}(b)$. Third, we construct the squared VIX $vix^2_t(b)$ using the bootstrapped residuals over the short sample. Fourth, we take all of these bootstrapped time-series and re-estimate the J-vectors of coefficients $\hat{V}^e_v(b)$ and $\hat{V}^o_v(b)$ using the approaches proposed by Gagliardini, Ossola, and Scaillet (2015) and Lynch and Wachter (2013), and compute $\hat{d}_v(b)$ as $\hat{V}^o_v(b) - \hat{V}^e_v(b)$. After repeating these four steps 1,000 times, we compute the t-statistic of each element, $\hat{d}_{v,j}$ ($j = 1, ..., J$), as $\frac{\hat{d}_{v,j}}{\hat{\sigma}_{d_{v,j}}}$, where $\hat{\sigma}_{d_{v,j}}$ is the standard deviation of the 1,000 bootstrapped values.

C Construction of the Variance Mimicking Portfolio

This section explains how we form a mimicking portfolio that tracks the market variance payoff based on the return information contained in the 25 equity portfolios. Specifically, we can use equation (B1) to write the (market-hedged) excess return of each portfolio as $r^e_{p,t+1} = -b_{pv} \cdot p^e_{rv,t} + b_{pv} \cdot rv_{t+1} + \epsilon_{p,t+1}$, where $p^e_{rv,t}$ is the forward price of the realized variance formed in the equity market, and $\epsilon_{p,t+1}$ is the idiosyncratic component. Whereas we do not observe $p^e_{rv,t}$, we can replace it by its projection implied by the two-factor model $proj(p^e_{rv,t} \big| z_t) = V^{e\prime}_v z_t$, and form a strategy that invests: (i) one dollar in portfolio p financed at the risk-free rate; (ii) $\frac{b_{pv} \cdot proj(p^e_{rv,t} | z_t)}{(1+r_{ft})}$ dollars at the risk-free rate. The resulting payoff is equal to

$$x^e_{p,t+1} = r^e_{p,t+1} + b_{pv} \cdot proj(p^e_{rv,t} \big| z_t) = b_{pv} \cdot rv_{t+1} + e_{p,t+1}, \tag{C1}$$

where $e_{p,t+1}$ is equal to $\epsilon_{p,t+1} + b_{pv}(proj(p^e_{rv,t} \big| z_t) - p^e_{rv,t})$. After stacking the portfolio payoffs and variance betas to form the vectors $x^e_{t+1} = [x^e_{1,t+1}, ..., x^e_{n,t+1}]'$ and $b_v = [b_{1v}, ..., b_{nv}]'$, we can construct the variance-mimicking equity portfolio by solving the following minimization problem:

$$\min_b \; var(rv_{t+1} - b' x^e_{t+1}) \quad \text{s.t. } b' b_v = 1. \tag{C2}$$

The optimal coefficient b^* is given by

$$b^* = E(x^e_{t+1} x^{e\prime}_{t+1})^{-1} \left[E(x^e_{t+1} rv_{t+1}) - q b_v \right], \tag{C3}$$

where the constant q is equal to $\frac{b'_v \left[E(x^e_{t+1} x^{e\prime}_{t+1})^{-1} E(x^e_{t+1} rv_{t+1}) \right] - 1}{b'_v E(x^e_{t+1} x^{e\prime}_{t+1})^{-1} b_v}$. The final payoff of this portfolio is equal to the sum of the realized variance and a residual term, i.e.,

$$x^e_{s,t+1} = b^{*\prime} x^e_{t+1} = rv_{t+1} + b^{*\prime} e_{t+1}, \tag{C4}$$

and its excess return is given by

$$r^e_{s,t+1} = b^{*\prime} r^e_{t+1} = rv_{t+1} + b^{*\prime} e_{t+1} - proj(p^e_{rv,t} \big| \, z_t), \tag{C5}$$

where $e_{t+1} = [e_{1,t+1}, ..., e_{n,t+1}]'$ and $r^e_{t+1} = [r^e_{1,t+1}, ..., r^e_{n,t+1}]'$. To compute the optimal vector b^* from the data, we first use equation (C1) and measure the payoff vector x^e_{t+1} as $r^e_{t+1} + \hat{b}_v \cdot \hat{V}^{e\prime}_v z_t$, where the estimated vector $\hat{b}_v$ is obtained from the time-series regression of the two-factor model with all predictors, and $\hat{V}^e_v$ is the estimated equity vector estimated using the conditional two-pass regression. Second, we replace $E(x^e_{t+1} x^{e\prime}_{t+1})$ with $\frac{1}{T}\sum_{t=1}^{T} x^e_{t+1} x^{e\prime}_{t+1}$, $E(x^e_{t+1} rv_{t+1})$ with $\frac{1}{T}\sum_{t=1}^{T} x^e_{t+1} rv_{t+1}$, and b_v with $\hat{b}_v$ in equation (C3) to compute b^*. Third, we compute the payoff and excess return the mimicking portfolio as $\hat{b}^{*\prime} x^e_{t+1}$ and $\hat{b}^{*\prime} r^e_{t+1}$, respectively.

Our construction of the mimicking equity portfolio is closely related to the one examined by Ferson, Siegel, and Xu (2006) and Lamont (2001), in which they maximize the correlation between the risk factor and the mimicking portfolio return conditional on a set of predictors z_t. Applying their approach to the variance factor, we obtain a mimicking portfolio whose excess return is given by $b' r^e_{t+1}$, where b is the coefficient vector from the following time-series regression: $rv_{t+1} = c' z_t + b' r^e_{t+1} + e_{t+1}$. The optimization problem in equation (C2) is similar except that we impose two restrictions: (i) the variance beta of the mimicking portfolio is equal to one (to make it comparable to the variance-mimicking option portfolio); (ii) the intercept $c' z_t$ is consistent with the model restriction, i.e., $c' z_t = proj(p^e_{rv,t} \big| \, z_t) = V^{e\prime}_v z_t$.

D Specification Tests

D.1 The Market Risk Premium

As discussed in Section B, the estimation procedure yields an estimate of the Market Risk Premium (MRP) projection. Studying its properties provides an additional specification test of the two-factor model. This test uses the restriction that the market factor $f_{m,t+1}$ is an excess return which has, by construction, a zero forward price ($p^e_{fm,t} = 0$). Therefore, if the two-factor model is correctly specified, the linear projection of the MRP must be equal to the linear projection of the market factor, i.e.,

$$\lambda^e_{m,t}(z) = proj(f_{m,t+1} | z_t) = F'_m z_t. \tag{D1}$$

A testable implication of this restriction is that each element of the risk-neutral vector V^e_m equals zero. The empirical evidence in Table IV reveals that the equality $V^e_m = 0$ is not rejected by the data because none of the estimated coefficients is statistically significant.

We also examine the relationships between the predictors and the MRP projection by reporting the estimated vector $\hat{F}_m - \hat{V}^e_m$ in Table V. Panel A reveals that the PE ratio is the most important explanatory variable with a negative and significant coefficient of -1.96. The resulting premium is strongly countercyclical, as depicted in Figure I which plots its time variation over the long sample (1970-2014). Overall, the properties of the market risk premium are consistent with those documented in the previous literature (e.g., Fama and French (1989) and Keim and Stambaugh (1986)).

[TABLE IV HERE]

[TABLE V HERE]

[FIGURE I HERE]

D.2 Comparison of the Portfolio Return Projections

To further assess the specification of the two-factor model, we study the properties of the projection of the equity portfolio returns on the space spanned by the predictors, defined as

$$proj(r^e_{p,t+1} | z_t) = h'_p z_t, \tag{D2}$$

where the coefficient vector h_p is obtained from a time-series regression of the portfolio excess return on the predictors. This projection is unconstrained in the sense that it can be estimated without specifying the identity and/or the number of risk factors. As such, it can be compared with the constrained version implied by the two-factor model:

$$proj^M(r^e_{p,t+1} \,|\, z_t) = b_{pv}\lambda^e_{v,t}(z) + b_{pm}\lambda^e_{m,t}(z) = b_{pv}\left(F_v - V^e_v\right)' z_t + b_{pm}\left(F_m - V^e_m\right)' z_t. \quad \text{(D3)}$$

If the two-factor model omits relevant risk factors, we expect the two projections to follow different patterns. To examine this issue, we run a time-series regression of the estimated value of $proj(r^e_{p,t+1} \,|\, z_t)$ on that of $proj^M(r^e_{p,t+1} \,|\, z_t)$ for the five quintile portfolios described in Table 1 of the paper. Table VI reveals that the constrained version tracks its unconstrained counterpart almost perfectly, with adjusted R^2s ranging between 0.95 and 0.98. This analysis provides further evidence that the two-factor model performs well at capturing the dynamics of the equity portfolio returns.

[TABLE VI HERE]

D.3 Hedging Errors of the Variance-Mimicking Portfolio

If the two-factor model is correctly specified, two predictions can be made on the hedging error of the variance-mimicking equity portfolio. First, its volatility must be small because the idiosyncratic term is largely diversified away. We find that the volatility of the hedging error represents only 19% of the average residual volatility of the 25 equity portfolios. To visualize this result, we plot the payoff of the mimicking equity portfolio, alongside with that of its option-based counterpart. Whereas the former logically exhibits greater volatility because of the residual term, Figure II shows that it is able to closely track realized variance with a correlation coefficient of 0.80.

Second, the hedging error should be uncorrelated with the macro-finance and broker-dealer variables because the difference between the forward price $p^e_{rv,t}$ and its model-based projection $V_v^{e\prime} z_t$ is unpredictable. Consistent with this prediction, the regression analysis reveals that none of the coefficients is statistically significant.

[FIGURE II HERE]

D.4 Additional Risk Factors

We examine whether our main results change when we include additional risk factors that could potentially drive the cross-section of equity portfolio returns. To begin, we consider three extensions of the two-factor model that include commonly-used risk factors:

$$\begin{aligned}
r_{p,t+1}^{e} &= -c_p' z_t + b_p' f_{t+1} + s_p \cdot r_{smb,t+1} + h_p \cdot r_{hml,t+1} + e_{p,t+1}, \\
r_{p,t+1}^{e} &= -c_p' z_t + b_p' f_{t+1} + s_p \cdot r_{smb,t+1} + h_p \cdot r_{hml,t+1} + m_p \cdot r_{mom,t+1} + e_{p,t+1}, \\
r_{p,t+1}^{e} &= -c_p' z_t + b_p' f_{t+1} + s_p \cdot r_{smb,t+1} + h_p \cdot r_{hml,t+1} + l_p \cdot r_{liq,t+1} + e_{p,t+1}, \qquad \text{(D4)}
\end{aligned}$$

where $b_p = [b_{pv}, b_{pm}]'$, $f_{t+1} = [rv_{t+1}, f_{m,t+1}]'$, $r_{smb,t+1}$, $r_{hml,t+1,}$ $r_{mom,t+1}$, $r_{liq,t+1}$ are the returns on zero-investment factor-mimicking portfolios for size, book-to-market, momentum, and liquidity obtained from Kenneth French's and Lubos Pastor's websites, and s_p, h_p, m_p, l_p are the associated risk loadings.

Next, we allow the equity portfolios to load differently on the different components of the market realized variance. We build on previous work by Adrian and Rosenberg (2008) and consider a three-factor model that distinguishes between the low- and high-frequency components of the realized variance denoted by $rv_{1,t+1}$ and $rv_{2,t+1}$, respectively. Both components are extracted from the daily squared returns of the SP500 using the Hodrick–Prescott filter and their sum is equal to rv_{t+1}. The resulting three-factor model can be written as

$$r_{p,t+1}^{e} = -c_p' z_t + b_{pv} \cdot rv_{t+1} + b_{pv2} \cdot rv_{2,t+1} + b_{pm} \cdot f_{m,t+1} + e_{p,t+1}. \qquad \text{(D5)}$$

Finally, we allow for a non-linear relationship between the equity portfolio returns and the realized variance. If we model the variance beta as a linear function of rv_{t+1}, we obtain the following three-factor model:

$$r^e_{p,t+1} = -c'_p z_t + b_{pv} \cdot rv_{t+1} + b_{pv2} \cdot rv^2_{t+1} + b_{pm} \cdot f_{m,t+1} + e_{p,t+1}. \tag{D6}$$

For each model, we use the conditional two-pass regression to compute the VRP equity projection, and compare the coefficients for the broker-dealer variables in the equity and option markets. The results in Table VII confirm that the two variables continue to play a significantly different role in the equity and option markets. In addition, the alternative equity VRP projections are all similar to the baseline projection with pairwise correlations ranging between 0.75 and 0.97.

[TABLE VII HERE]

D.5 Time-Varying Portfolio Betas

We model the time variation of the variance and market betas as linear functions of each predictor $z_{j,t} : b_{pv,t} = b_{pv} + b_{pv,j} z_{j,t-1}$ and $b_{pm,t} = b_{pm} + b_{pm,j} z_{j,t-1}$ (for $j = 1, ..., J$). In this case, the factor model can be written as

$$\begin{aligned} r^e_{p,t+1} &= -c'_{pt} z_t + b_{pv,t} \cdot rv_{t+1} + b_{pm,t} \cdot f_{m,t+1} + e_{p,t+1}, \\ &= -c'_{pt} z_t + b_{pv} \cdot rv_{t+1} + b_{pm} \cdot f_{m,t+1} + b_{pv,j} \cdot f_{1,t+1} + b_{pm,j} \cdot f_{2,t+1} + e_{p,t+1}, \end{aligned} \tag{D7}$$

where $f_{1,t+1} = z_{j,t} rv_{t+1}$, $f_{2,t+1} = z_{j,t} f_{m,t+1}$, and the equilibrium value for c_{pt} is given by $b_{pv,t} \cdot V^{e\prime}_v + b_{pm,t} \cdot V^{e\prime}_m$.

To measure the degree of time variation of the portfolio betas, we estimate the coefficients $b_{pv,j}$ and $b_{pm,j}$ for each portfolio and each predictor (realized variance, PE ratio, default spread, inflation, employment, leverage, and PBI return). In Table VIII, we report the coefficient t-statistics for the variance factor and find little evidence of time variation in betas. Only 10% of the coefficients are significant at the 5% level, and these occurrences are evenly distributed across predictors. For the market factor, Table IX shows that the proportion of significant coefficients is slightly higher at 18% and most of them are attributed to the PE ratio. Despite this higher concentration, its impact on the coefficients associated with the broker-dealer variables, $\widehat{v}_v^e(lev)$ and $\widehat{v}_v^e(pbi)$, is expected to be small. The reason is that the magnitude of the bias of these coefficients for each portfolio depends on the three covariances between the omitted factor $f_{2,t+1}$ and lev_t, pbi_t, $f_{m,t+1}$, which are all statistically indistinguishable from zero.

[TABLE VIII HERE]

[TABLE IX HERE]

E Potential Impact of Jump Risk

E.1 The Peso Problem

As discussed by Ang, Hodrick, Xing, and Zhang (2006), the estimation of the VRP can be affected by the Peso problem, i.e., the occurrence of large but infrequent variance jumps. To illustrate, suppose that we want to estimate the average option VRP defined as the difference between the average realized variance $\widehat{rv}$ and the average squared VIX $\widehat{vix}^2$. If the number of variance spikes during the sample is smaller than the option market expected ex-ante (measured by the risk-neutral expectation), $\widehat{rv}$ is lower than $\widehat{vix}^2$ and the magnitude of the estimated VRP is inflated.

In our setting, $\widehat{rv}$ is replaced with the more general expression $\hat{F}_v' z_t$ but the analysis remains unchanged. Therefore, the equity and option VRPs should be interpreted with some caution. However, the VRP difference $\hat{D}_t(z)$ mitigates this problem because the term $\hat{F}_v' z_t$ cancels out (see equation (4) in the paper). Therefore, as long as the risk-neutral equity expectation $\hat{V}_v^{e\prime} z_t$ is not systematically biased—a point discussed below—, $\hat{D}_t(z)$ provides meaningful information about the price difference in both markets.

E.2 Jump Risk and the Omitted-Factor Bias

The equity vector $\hat{V}_v^e$ can potentially be biased if jump risk is required for explaining the cross-section of equity portfolio returns. Although our previous analysis strongly suggests that the two-factor is correctly specified, we carefully examine the theoretical properties of the bias from omitting the jump risk factor.[4] Without loss of generality, we focus on leverage and assume that its (true) risk-neutral coefficients are the same in the equity and option markets (i.e., $v_v^e(lev) = v_v^o(lev)$). Then, we then determine under which conditions $\hat{v}_v^e(lev)$ is positively biased and leads to the negative difference between $\hat{v}_v^o(lev)$ and $\hat{v}_v^e(lev)$ documented in Table 3 of the paper (Panel B).

To summarize our theoretical analysis presented in Section G, we demonstrate that the jump risk premium must be sensitive to leverage (similar to the VRP). In addition, the jump and variance betas must have opposite signs which implies that equity portfolios must combine two properties difficult to reconcile: their returns must be positive when variance is high, but negative when a jump occurs. Assuming that these conditions hold, we further examine whether the bias can quantitatively reproduce the results in Table 3 using a Monte-Carlo simulation analysis that matches the salient features of the data. We find that the sensitivity of the jump risk premium to leverage must be economically large and the portfolio betas on the jump and variance risks must be highly negatively correlated. To summarize, the bias of $\hat{v}_v^e(lev)$ can only explain the observed VRP difference under strong theoretical and empirical conditions that are unlikely to be met.

E.3 Extreme Variance Observations

Besides the Peso problem, it is well known that a single large data observation can have a disproportionate impact on estimated coefficients in linear regression models. To evaluate this impact, we repeat the analysis after winsorizing 2.5% and 5% of the most extreme

[4] Whereas we mainly focus on the jump risk factor, the theoretical analysis of the bias presented in Section G is general and applies to any omitted factor.

market variance observations (1% and 2.5% in each tail). Table X reveals that the estimated coefficients associated with the two broker-dealer variables remain statistically significant in the option market. Therefore, the ability of these variables to explain the VRP difference is not driven by a few extreme observations.

[TABLEX HERE]

E.4 The SVIX Index

When the market is subject to large price movements, the squared VIX computed from option prices is not necessarily equal to the forward price of the realized variance. To address this issue, we examine the properties of the SVIX index that is robust to market jumps (see Martin (2013)). In Table XI, we examine the relationships between this index and the different predictors between January 1996 and January 2012 (period during which the SVIX index is available). Similar to the VIX index, the results confirm the important role played by the two broker-dealer variables in the option market as their estimated coefficients remain both negative and significant.

[TABLE XI HERE]

F Further Evidence

F.1 Alternative Approaches for Forming Equity Portfolios

We consider alternative procedures that could reasonably be used to form the set of variance risk-sensitive portfolios. To begin, we construct these portfolios after modifying the population of stocks in two ways. First, we remove tiny stocks as an alternative to excluding NASDAQ stocks (All but tiny stocks). Similar to Fama and French (2008), we define a stock as tiny if its monthy market capitalization falls below the 20th percentile of the market capitalization for NYSE stocks. Second, we include all existing stocks (All stocks) to address the concern that relevant information on the market VRP is lost when specific stocks are excluded (All stocks).

Next, we consider three approaches for measuring the stock exposures to market and variance risk. First, it is well-known that OLS coefficients are sensitive to outliers (e.g., Martin and Simin (2003)). To address this issue, we compute robust beta t-statictics (Robust betas) using the Huber loss function (Huber (1981)). Second, we expand the time-horizon over which betas are estimated from monthly to quarterly (Quarterly betas). Third, we rank stocks according to their estimated betas to evaluate the importance of controling for estimation errors using t-statistics (No t-statistics).

Finally, nonsynchronous price movements can have a significant impact on stock betas measured at the daily frequency (see Lo and MacKinlay (1990)). In the spirit of Dimson (1979), we therefore add the lagged daily market return $r_{m,d-1}$ and lagged variance innovation $u_{v,d-1}$ in the time-series regresssions performed each month (Lagged factors): $r_{i,d} = \gamma + b^1_{im,t} r_{m,d} + b^2_{im,t} r_{m,d-1} + b^1_{irv,t} u_{v,d} + b^2_{irv,t} u_{v,d-1} + \epsilon_{i,d}$, where $r_{i,d}$ is the return of stock i on day d. The estimated market and variance betas are then computed as $\hat{b}_{im,t} = \hat{b}^1_{im,t} + \hat{b}^2_{im,t}$ and $\hat{b}_{irv,t} = \hat{b}^1_{irv,t} + \hat{b}^2_{irv,t}$, respectively. Alternatively, we also exclude the daily return observations equal to zero for each month and each stock (Zero returns). This approach is motivated by Bekaert, Harvey, and Lundblad (2007) who use the number of zero daily returns as their main illiquidity measure.

Table XIII reports the coefficients for the broker-dealer variables in the equity and option markets. In all of these cases, we still find that the leverage ratio is a key determinant of the difference between the two VRPs. For the PBI return, the difference between the estimated coefficients remains highly significant in all but two cases. When stocks are ranked based on the estimated betas (no t-statistics), the negative relationship between variance betas and average returns weakens and the correlation with the baseline equity VRP drops to 0.44. These results highlight the importance of using t-statistics to reduce estimation errors in short-window regressions. Similar results are obtained with quarterly betas, which is consistent with Ang, Hodrick, Xing, and Zhang (2006) who note that using longer windows reduces the information content of the beta estimates.

[TABLE XII HERE]

F.2 Broker-Dealer Variables in the Equity Market

In Table 3 of the paper, we show that the broker-dealer variables (leverage and PBI return) play no role in driving the time variation of the equity VRP projection. To visualize this result, we plot in Figure III the projections with and without the broker-dealer variables and confirm that they are nearly indistinguishable.

[FIGURE III HERE]

F.3 Alternative Predictive Variables

In this section, we measure whether the impact of the broker-dealer variables (leverage and PBI return) on the equity and option VRPs changes with the identity of the macro-finance variables. First, we replace the PE ratio with the dividend yield computed from the CRSP index. Second, we replace the quarterly growth rate in employment with two alternative indicators of real activity: the seasonally-adjusted quarterly growth rate in industrial production and the business cycle indicator constructed by Aruoba, Diebold, and Scotti (2009) which aggregates information regarding employment, industrial production, and interest rates. Third, we take the initial set of macro-finance variables and add two commonly-used interest rate variables: the 3-month T-bill rate and the term spread, defined as the difference between the 10- and 1-year T-bond yields. Fourth, we add the quarterly variance of the inflation rate following recent work by Paye (2012) who finds that this variable helps to predict the future quarterly volatility. For each specification, Table XIII reports the coefficients for the broker-dealer variables in the equity and option markets. In all of these cases, the two broker-dealer variables continue to play a significantly different role in the equity and option markets.

We also consider alternative specifications in which we add to the baseline set of predictors the squared values of the macro-finance variables. The motivation for these tests is to determine whether the explanatory power of the broker-dealer variables stems from their ability to capture any non-linear relationships between the macro-finance variables and the VRPs. The results documented in Table XIV show that it is not the case.

[TABLE XIII HERE]

[TABLE XIV HERE]

F.4 Implied Individual Stock Variance and Correlation

The market variance is equal to the sum of the individual stock variances and their covariances. Therefore, the VIX index contains information about the prices of both individual stock variance risk (changes in individual stock variances) and correlation risk (changes in the correlation structure of stocks). In this section, we examine how the broker-dealer variables affect each of these two prices. We extract the price of individual stock variance from individual option prices as the equally-weighted average of the implied variances of the SP500 stocks. For correlation risk, its price is measured by the implied correlation among SP500 stocks computed from index and individual option prices. Both series are computed monthly and are available between January 1996 and August 2013.[5]

The relationships between the broker-dealer variables and the implied stock variance is reported in Panel B of XV (first row). Contrary to the squared VIX, the coefficient associated with leverage is positive and is not statistically significant when considered jointly with the PBI return (with a t-statistic of 1.47). This finding is consistent with the empirical role played by intermediaries in the option market. Whereas the VIX is inferred from index options, the implied stock variance is computed from individual stock options whose supply is not dominated by financial intermediaries (see Garleanu, Pedersen, and Poteshman (2009)). Changes in their risk-bearing capacity are therefore less likely to

[5] Driessen, Maenhout, and Vilkov (2009) provide a detailed description of the construction of these variables.

drive the prices of these options.

Repeating this analysis for the implied correlation, we observe in Panel B (second row) that it shares strong similarities with the squared VIX as the leverage coefficient is both negative and highly significant. Therefore, periods when intermediaries deleverage are associated with an increase in the prices of both aggregate variance and correlation risks. This similarity resonates with the study by Driessen, Maenhout, and Vilkov (2009) which finds that the market VRP is mostly attributed to correlation risk.

[TABLE XV HERE]

F.5 Analysis based on Monthly Data

In this section, we re-estimate the equity and option VRPs over a monthly time-horizon using the same estimation procedure as the one described in Section B. All of the variables (risk factors, portfolio returns, predictors) are available at the monthly frequency, except for the leverage ratio of broker-dealers. To address this issue, we linearly interpolate the quarterly leverage values (similar to Kan, Robotti, and Shanken (2013), and Vissing-Jorgensen and Attanasio (2003)). We also replace the quarterly VIX with its monthly counterpart computed from the prices of one-month SP500 options.

Table XVI examines the explanatory power of the predictors on the monthly VRPs and reveals that the two broker-dealer variables remain the only significant drivers of the VRP difference (the t-statistics are equal -4.26 for leverage and -5.08 for the PBI return). We observe that the explanatory power of the PBI return becomes stronger at the monthly horizon, consistent with the fact that changes in this variable are short-lived. For the equity market, using monthly data seems to introduce some noise in the relationship between the macro-finance variables and the VRP as the t-statistics decrease and the model is marginally rejected (when the 10% threshold is used). However, the coefficients share strong commonalities with their quarterly-based counterparts. In most cases, their signs remain the same and their ratios are close to one third.

[TABLE XVI HERE]

F.6 Analysis based on Individual Stocks

F.6.1 Estimation Procedure

The analysis of the equity VRP presented in the paper is based on a set of variance risk-sensitive equity portfolios. An alternative procedure advocated, among others, by Ang, Liu, and Schwarz (2010), and Gagliardini, Ossola, and Scaillet (2015) is to directly use individual stock data as inputs for estimation. Whereas the extended two-pass regression is similar to the one presented in Section B, Gagliardini, Ossola, and Scaillet (2015) explain that two important differences must be properly accounted for. First, the panel of individual stock returns is unbalanced and contains stocks with short-return histories. These stocks may yield regression coefficients that are either highly volatile or impossible to compute (if the matrix inversion cannot be performed). To address this issue, it is necessary to introduce an appropriate trimming mechanism to reduce the cross-section of stocks. Second, the number of individual stocks is extremely large (more than 7,000 in our sample). The econometric theory must account for this feature by letting both the number of return observations and the number of stocks grow large (double asymptotics). It implies that the asymptotic distributions of the estimated coefficients and the test statistic differ from those obtained with portfolios.

The estimation procedure can be summarized as follows. In the first step, we compute, for each individual stock i ($i = 1, ..., m$), the OLS estimator of the $(J+2)$-vector of coefficients $\beta_i = (-c_i', b_{iv}, b_{im})'$ as

$$\hat{\beta}_i = \left(\sum_{t=1}^{T} I_{i,t} x_t x_t' \right)^{-1} \sum_{t=1}^{T} I_{i,t} x_t r_{i,t}^e, \tag{F1}$$

where $r_{i,t}^e$ is the stock excess return, x_{t+1} is a $(J+2)$-vector defined as $(z_t', rv_{t+1}, f_{m,t+1})'$, T is the total number of observations, and $I_{i,t}$ is a indicator function equal to 1 if $r_{i,t}^e$ is observed. Following Gagliardini, Ossola, and Scaillet (2015), we introduce a trimming device that keeps stock i in the cross-section only if $CN(\hat{Q}_{x,i}) \leq \chi_{1,T}$ and $\tau_{i,T} \leq \chi_{2,T}$, where $CN(\hat{Q}_{x,i}) = \left(eig_{\max}(\hat{Q}_{x,i})/eig_{\min}\left(\hat{Q}_{x,i}\right)\right)^{\frac{1}{2}}$ denotes the condition number of $\hat{Q}_{x,i} = \frac{1}{T}\sum_{t=1}^{T} I_{i,t}x_t x_t'$, $\tau_{i,T} = \frac{T}{T_i}$, and $T_i = \sum_{t=1}^{T} I_{i,t}$ is the total number of return observations for stock i. As advocated by Gagliardini, Ossola, and Scaillet (2015), we set $\chi_{1,T} = 15$ and $\chi_{2,T} = 2.275$ (which implies a minimum of 80 quarterly return observations).[6]

In the second step, we compute the estimator of the $2J$-vector $V^e = (V_v^{e\prime}, V_m^{e\prime})'$ that drives the risk-neutral expectations of the two risk factors (variance and market) using the non-trimmed stocks only. Similar to the portfolio approach, we use a WLS approach in which the $J \times J$ matrix of estimated weights for each stock i is computed as $\hat{w}_i = diag(1_i^{\chi}\hat{v}_i)^{-1}$, where $\hat{v}_i$ is given by equation (B4), and 1_i^{χ} is $J \times J$ matrix whose diagonal elements are equal to one if stock i is kept in the cross-section and zero otherwise. Using the estimated matrix of weights $\hat{w}_i$, we obtain the following estimator of V^e :

$$\widehat{V}^e_{stock} = \left(\sum_{i=1}^{m} \hat{B}_i'\hat{w}_i\hat{B}_i\right)^{-1}\sum_{i=1}^{m} \hat{B}_i'\hat{w}_i\hat{c}_i, \tag{F2}$$

where $\hat{B}_i$ a $J \times 2J$ matrix equal to $[\hat{b}_{iv} \cdot I_J, \hat{b}_{im} \cdot I_J]$. The J-vector of estimated coefficients $\widehat{V}^e_{v,stock}$ can be then plugged in equation (B7) to obtain the VRP projection in the equity market, $\hat{\lambda}^e_{v,t}(z) = \left(\hat{F}_v - \hat{V}^e_{v,stock}\right)' z_t$.

When T and m grows large and the two-factor model is correctly specified, Gagliardini, Ossola, and Scaillet (2015) demonstrate that the $2J$-vector $\hat{\Lambda}^e_{stock} = vec(\hat{F}) - \hat{V}^e_{stock}$ is consistent and normally distributed, i.e.,

$$\sqrt{T}\left(\hat{\Lambda}^e_{stock} - \Lambda^e\right) \Rightarrow N(\mathbf{0}_{2J\times 1}, \Sigma_F). \tag{F3}$$

where $\Sigma_F = (I_J \otimes Q_z^{-1})\Sigma_u(I_J \otimes Q_z^{-1})$. As discussed above, the asymptotic distribution

[6] Specifically, Gagliardini, Ossola, and Scaillet (2015) conduct a Monte-Carlo analysls based on $T = 546$ monthly observations, and note that $\chi_{2,T}$ must be equal to 2.275 $\left(\frac{546}{200}\right)$ in order to estimate the risk premium coefficients and the model test statistic (see p. 38). In our sample, T is equal to 179, which implies that each stock must have at least 80 return observations $\left(\text{i.e., } \frac{179}{80} = 2.23\right)$.

of $\hat{\Lambda}^e_{stock}$ differs from its portfolio-based counterpart because its covariance matrix only depends on the variance of the coefficients in the factor predictive regressions (i.e., the term $\frac{1}{n}\Sigma_{V^e}$ that appears in equation (B8) vanishes here).

The statistic for testing whether the two-factor model is correctly specified is also different from its portfolio-based counterpart in equation (B11). The test developed by Gagliardini, Ossola, and Scaillet (2015) is based on the statistic $\hat{\xi}_{m,T}$ defined as $T\sqrt{m}(\hat{Q} - \frac{1}{T}J)$, where $\hat{Q} = \frac{1}{m}\sum_{i=1}^{m}\hat{\zeta}_i'\hat{w}_i\hat{\zeta}_i$, and $\hat{\zeta}_i = \hat{c}_i - \hat{B}_i\hat{V}^e_{stock}$. As T and n grow large, it can be shown that

$$\hat{\xi}_{m,T} \Rightarrow N(0, \Sigma_\xi). \tag{F4}$$

The variance term is defined as

$$\Sigma_\xi = 2\lim_{m\to\infty} E\left[\frac{1}{n}\sum_{i,j}\frac{\tau^2_{i,T}\tau^2_{j,T}}{\tau^2_{i,j,T}} trace\left[\left(C'_{V^e}Q^{-1}_{x,i}S_{ij}Q^{-1}_{x,j}C_{V^e}\right)w_i\left(C'_{V^e}Q^{-1}_{x,j}S_{ji}Q^{-1}_{x,i}C_{V^e}\right)w_j\right]\right],$$

where $\tau_{i,j,T} = \frac{1}{T}\sum_{t=1}^{T} I_{i,t}I_{j,t}$. To implement this test, we simply need to replace C_{V^e}, $Q_{x,i}$, $Q_{x,j}$, S_{ij}, w_i, and w_j with their empirical counterparts to obtain a consistent estimator of Σ_ξ. Finally, we use the same bootstrap procedure as the one outlined in Section B.3 to examine the VRP difference between the equity and option markets. To make the bootstrap approach tractable, we assume that the return residuals from the two-factor model are uncorrelated across stocks.

F.6.2 Empirical Results

The results in Table XVII reveal that our main results hold again with individual stocks. The leverage ratio and PBI return remain the most important drivers of the VRP difference with t-statistics equal to -3.66 and -6.36, respectively. Similar to the results documented by Gagliardini, Ossola, and Scaillet (2015) for commonly-used asset pricing models (e.g., CAPM, Fama-French), our specification test largely rejects the null hypothesis that the two-factor model is able to price individual stocks. This finding resonates with the previous literature that highlights the challenges of explaining the cross-section of individual stock returns as (i) individual stocks are likely to be exposed to a wide range of risk factors (e.g., Lewellen, Nagel, and Shanken (2010), Daniel and Titman (2012)); (ii) their betas are likely to change over time (e.g., Andersen, Bollerslev, Diebold, and Wu (2006)).[7] The rejection of the two-factor model contrasts with its ability to explain the cross-section of portfolio returns used in our baseline specification. Therefore, the portfolio-based approach described in Section B allows for a more accurate estimation of the equity VRP.

[TABLE XVII HERE]

G Analysis of the Omitted-Factor Bias

G.1 Theoretical Analysis

In this section, we study the potential impact of omitting a relevant factor on the estimated risk-neutral equity vector $\hat{V}_v^e$. Whereas we focus on the coefficient associated with leverage, $\hat{v}_v^e(lev)$, the same analysis applies to the PBI return. Without loss of generality, we make several assumptions to make our presentation as simple as possible. First, we assume that the portfolio returns are driven by the standardized leverage lev_t and two demeaned factors, the realized variance rv_{t+1} and an additional factor $f_{1,t+1}$. To allow for a non-zero correlation ρ between the two factors, we write $f_{1,t+1}$ as $\rho rv_{t+1} + b_\varepsilon \varepsilon_{1,t+1}$, where b_ε is equal to $\sqrt{(1-\rho^2)}$ and the factor $\varepsilon_{1,t+1}$ is uncorrelated with rv_{t+1}. Second,

[7] In theory, we could incorporate several sources of risk and explicitly specify the beta dynamics. However, such models are difficult to estimate because of the large number of parameters. In addition, Ghysels (1998) shows that a wrong specification of time-varying betas may result in large pricing errors (possibly greater than those produced by a constant beta model).

we define the covariance matrix of the vector $[lev_t, rv_{t+1}, \varepsilon_{1,t+1}]'$ as

$$\Omega = \begin{bmatrix} 1 & f_v^e(lev) & f_\varepsilon^e(lev) \\ f_v^e(lev) & \sigma^2 & 0 \\ f_\varepsilon^e(lev) & 0 & \sigma^2 \end{bmatrix}, \tag{G1}$$

where the volatilities of rv_{t+1}, $\varepsilon_{1,t+1}$ (and thus $f_{1,t+1}$) are captured by the same parameter σ. Based on the coefficients in equation (G1), we can write the projections of the two factors on the space spanned by leverage as

$$proj(rv_{t+1}|\, z_t) = f_v^e(lev) \cdot lev_t, \qquad proj(f_{1,t+1}|\, z_t) = (\rho f_v^e(lev) + b_\varepsilon f_\varepsilon^e(lev)) \cdot lev_t. \tag{G2}$$

Similarly, we define the projections of the forward factor prices as

$$proj(p_{rv,t}^e|\, z_t) = v_v^e(lev) \cdot lev_t, \qquad proj(p_{f1,t}^e|\, z_t) = (\rho v_v^e(lev) + b_\varepsilon v_\varepsilon^e(lev)) \cdot lev_t. \tag{G3}$$

Third, we assume that the vector of portfolio betas on rv_{t+1} and $f_{1,t+1}$, denoted by $b_p = [b_{pv}, b_{pf}]'$, is drawn from a bivariate normal distribution with a zero mean and a covariance matrix Ω_b in which the variance terms are both equal to σ_b^2 and the covariance is given by $\rho_b\sigma_b^2$. The correlation coefficient ρ_b provides a convenient way to examine different scenarios regarding the signs of the betas: if ρ_b is positive, b_{pv} and b_{pf} tend to have the same signs, while the opposite holds if ρ_b is negative. The resulting factor representation of the excess return of each portfolio is defined as

$$r_{p,t+1}^e = -c_p \cdot lev_t + b_{pv} \cdot rv_{t+1} + b_{pf} \cdot f_{1,t+1} + e_{p,t+1}, \tag{G4}$$

where the restriction imposed by the model on the intercept is given by

$$c_p = b_{pv} \cdot v_v^e(lev) + b_{pf} \cdot (\rho v_v^e(lev) + b_\varepsilon v_\varepsilon^e(lev)). \tag{G5}$$

We study the properties of $\hat{v}_v^e(lev)$ when the model used for estimation includes lev_t and rv_{t+1}, but omits the relevant factor $f_{1,t+1}$. Standard results on regression analysis reveal that the estimated coefficient vector $\hat{\beta}_p = (-\hat{c}_p, \hat{b}_{pv})'$ is biased, i.e.,

$$bias(\hat{\beta}_p) = \Omega_{lev,rv}^{-1} \begin{bmatrix} cov(lev_t, \rho rv_{t+1}) \\ cov(rv_{t+1}, \rho rv_{t+1}) \end{bmatrix} b_{pf} + \Omega_{lev,rv}^{-1} \begin{bmatrix} cov(lev_t, b_\varepsilon \varepsilon) \\ cov(rv_{t+1}, b_\varepsilon \varepsilon) \end{bmatrix} b_{pf}, \quad \text{(G6)}$$

where $\Omega_{lev,rv}$ denotes the covariance matrix of lev_t and rv_{t+1}. From equation (G6), the average values of the two estimated coefficient can be written as

$$\begin{aligned} c_p^* &= E(\hat{c}_p) = c_p - \frac{\sigma^2 b_\varepsilon f_\varepsilon^e(lev)}{\sigma^2 - f_v^e(lev)^2} b_{pf}, \\ b_{pv}^* &= E(\hat{b}_{pv}) = b_{pv} + \rho b_{pf} - \frac{b_\varepsilon f_\varepsilon^e(lev) f_v^e(lev)}{\sigma^2 - f_v^e(lev)^2} b_{pf}. \end{aligned} \quad \text{(G7)}$$

Any bias in the coefficient vector $\hat{\beta}_p$ across portfolios can potentially affect the average value of $\hat{v}_v^e(lev)$ because the latter is obtained from the cross-sectional regression of the vector $c^* = [c_1^*, ..., c_n^*]'$ on the vector $b_v^* = [b_{1v}^*, ..., b_{nv}^*]'$, i.e.,

$$v_v^e(lev)^* = E(\hat{v}_v^e(lev)) = (b_v^{*\prime} b_v^*)^{-1} b_v^{*\prime} c^*. \quad \text{(G8)}$$

Intuitively, $v_v^e(lev)^*$ can be interpreted as a cross-sectional average of the portfolio pseudo-values, where each pseudo-value, $v_v^e(lev)_p^*$, is defined such that the equilibrium condition applied to the misspecified model holds (for $p = 1, ..., n$):

$$c_p^* = b_{pv}^* \cdot v_v^e(lev)_p^*. \quad \text{(G9)}$$

Therefore, we can determine the properties of the bias of $\hat{v}_v^e(lev)$ by studying the difference between the pseudo-value $v_v^e(lev)_p^*$ and the true value $v_v^e(lev)$. Taking the difference between the LHSs and RHSs of equations (G5) and (G9) and rearranging terms, we can express the difference between $v_v^e(lev)_p^*$ and $v_v^e(lev)$ as

$$\frac{b_{pf}}{b_{pv}^*}\left[b_\varepsilon\left(v_\varepsilon^e(lev)-\frac{\sigma^2+f_\varepsilon^e(lev)f_v^e(lev)}{\sigma^2-f_v^e(lev)^2}f_\varepsilon^e(lev)\right)\right]=v_v^e(lev)_p^*-v_v^e(lev). \tag{G10}$$

This expression can be simplified if we set $f_v^e(lev)$ equal to zero—an assumption consistent with the fact that the estimated coefficient $\hat{f}_v^e(lev)$ is not significant (see Table 2 of the paper). In this case, b_{pv}^* is equal to $b_{pv}+\rho b_{pf}$ and we have

$$\frac{b_{pf}}{b_{pf}(\rho_b+\rho)+\varepsilon_b}\left[b_\varepsilon(v_\varepsilon^e(lev)-f_\varepsilon^e(lev))\right]=v_v^e(lev)_p^*-v_v^e(lev), \tag{G11}$$

where b_{pv} is decomposed into its two orthogonal components, $b_{pf}\rho_b$ and ε_b.

Equation (G11) reveals three cases where the estimated leverage coefficient is unbiased. First, $bias(\hat{v}_v^e(lev))$ equals zero if the risk premium of the orthogonal factor $\varepsilon_{1,t+1}$, $\lambda_{\varepsilon,t}(z)$, is unrelated to leverage. This condition implies that $v_\varepsilon^e(lev)-f_\varepsilon^e(lev)$ is null and that $v_v^e(lev)_p^*$ equals $v_v^e(lev)$ for each portfolio. Second, we obtain the same result if the factor correlation ρ tends to ±1 because $b_\varepsilon=\sqrt{(1-\rho^2)}$ tends to zero. Third, $\hat{v}_v^e(lev)$ is unbiased when the sum of the correlations, $\rho_b+\rho$, is equal to zero. In this case, the first term on the LHS becomes $\frac{b_{pf}}{\varepsilon_b}$ and the two elements of this ratio are uncorrelated. Because $\frac{b_{pf}}{\varepsilon_b}$ randomly takes negative and positive values, the cross-portfolio differences between $v_v^e(lev)_p^*$ and $v_v^e(lev)$ offset one another and $\hat{v}_v^e(lev)$ is unbiased.

If none of these conditions is met, $\hat{v}_v^e(lev)$ is biased either positively or negatively depending on the signs of: (i) the ratio $\frac{b_{pf}}{b_{pv}^*}$ (as measured by the sign of $\rho_b+\rho$); (ii) the coefficient $v_\varepsilon^e(lev)-f_\varepsilon^e(lev)$ that relates leverage to $\lambda_{\varepsilon,t}(z)$. If both terms are positive or negative for most portfolios, the LHS of equation (G11) is positive, which implies that the bias is positive, i.e., $\hat{v}_v^e(lev)$ is, on average, higher than $v_v^e(lev)$. On the other hand, if the two sums have opposite signs for most portfolios, the LHS of equation (G11) is negative and the resulting bias is negative, i.e., $\hat{v}_v^e(lev)$ is, on average, lower than $v_v^e(lev)$.

G.2 Simulation Analysis

To quantify the magnitude of this bias, we recourse to a Monte-Carlo simulation analysis. The parameter values are chosen according to our empirical findings. The factor volatility σ is equal to 0.012 (the standard deviation of the realized variance), and the beta volatility σ_b is equal to 0.51 (the cross-sectional standard deviation of the variance betas). Importantly, we set $v_v^e(lev)$ equal to -0.37 so that the equity VRP, defined here as $(f_v^e(lev) - v_v^e(lev))\, lev_t = -v_v^e(lev) lev_t$, exhibits the same relationship with leverage as the one estimated in the option market (see Table 3 of the paper). Therefore, the (true) impact of leverage on the equity and option VRPs is exactly the same. In both markets, a decrease in leverage increases the compensation for hedging against variance risk (i.e., both VRPs decrease).

The simulation analysis is conducted over 1,000 trials, where each trial s ($s = 1, ..., 1,000$) includes three steps. First, we randomly draw, for each portfolio p ($p = 1, ..., 25$), the beta vector b_p^s from the bivariate normal distribution. Second, we compute the values taken by the two coefficients $c_p^{*,s}$ and $b_{p,v}^{*,s}$ for each portfolio using equations (G7). Third, we create the two vectors $c^{*,s} = [c_1^{*,s}, ..., c_{25}^{*,s}]'$ and $b_v^{*,s} = [b_{1v}^{*,s}, ..., b_{25v}^{*,s}]'$, and use equation (G8) to compute $v_v^e(lev)^{*,s}$. After repeating these three steps over 1,000 draws of portfolio betas, we compute $bias(\hat{v}_v^e(lev))$ as $\frac{1}{1,000}\sum_{s=1}^{1,000} v_v^e(lev)^{*,s} - v_v^e(lev)$.

In our first scenario, we interpret the omitted factor $f_{1,t+1}$ as a jump risk factor that has the same premium properties as those of the realized variance by setting $f_f^e(lev) = 0$ and $v_f^e(lev) = -0.37$. The negative value for $v_f^e(lev)$ implies that the jump risk premium, $\left(f_f^e(lev) - v_f^e(lev)\right) lev_t$, is strongly negative when intermediaries' leverage is low (similar to the VRP itself). In addition, a jump factor is positively correlated with realized variance by construction (Todorov (2010)). To account for this positive relationship, we set ρ is equal to 0.55 (the correlation between the realized variance and the high-frequency variance component computed by Adrian and Rosenberg (2008)). In Panel A of Figure IV, we plot the relative bias, defined as $\frac{bias(\hat{v}_v^e(lev))}{abs(v_v^e(lev))}$, across different values for the beta correlation ρ_b ranging between -0.9 and 0.9. Because $v^e_\varepsilon(lev) - f^e_\varepsilon(lev)$ is negative, we have a positive bias when $\rho_b + \rho$ is negative (i.e., $\frac{b_{pf}}{b^*_{pv}}$ tends to be negative). At $\rho_b = -0.53$, the sum $\rho_b + \rho$ is null and the resulting bias is equal to zero. Finally, the bias becomes negative when $\rho_b > -0.53$ because $\frac{b_{pf}}{b^*_{pv}}$ tends to be positive. All of these results are consistent with the predictions of equation (G11).

This analysis is important for interpreting the main result of the paper that leverage drives the option VRP, but not the equity VRP. To attribute this result to the omission of a jump risk factor, $v^e_v(lev)^*$ must be equal to zero. Given that $v^e_v(lev)$ equals -0.37, the relative bias must therefore be positive and equal to 100%. However, Panel A reveals that the highest bias is close to 90%. A bias of 100% is achievable if we are willing to assume that the risk premium of $f_{1,t+1}$ is extremely sensitive to leverage. For instance, suppose that the term $v^e_\varepsilon(lev) - f^e_\varepsilon(lev)$ is doubled from -0.37 to -0.74 so that a one-standard deviation decline in leverage increases the magnitude of the jump premium by 3.0% per year. In this case, Panel A shows that the bias reaches 100% provided that portfolio betas have opposite signs 79% of the time (i.e., ρ_b must be equal to -0.80).[8] This condition implies that the equity portfolios must combine two properties that are difficult to reconcile: their returns must be high (low) when the realized variance is high and, at the same time, low (high) when the market return exhibits a jump.

In the second scenario, we assume that $f_{1,t+1}$ has the opposite premium properties to the ones of the realized variance, i.e., $f^e_f(lev) = 0$ and $v^e_f(lev) = 0.37$. The omitted factor can be interpreted as a recession risk factor whose premium, $\left(f^e_f(lev) - v^e_f(lev)\right) lev_t$, increases when intermediaries' leverage is below average (contrary to the VRP). The main difference with the previous scenario is that $v^e_\varepsilon(lev) - f^e_\varepsilon(lev)$ turns positive, which produces a positive relationship between ρ_b and $bias(\hat{v}^e_v(lev))$. To plot this relationship, we set $v^e_f(lev)$ equal to 0.37 and ρ equal to -0.41 (the average correlation between the realized variance and the market return). The results in Panel B reveal that the bias

[8] This proportion is obtained by simulating a large number of draws for the beta vector $b_p = [b_{pv}, b_{pf}]'$ from the bivariate normal distribution (with $\rho_b = -0.85$), and then counting the number of times b_{pv} and b_{pf} have different signs.

reaches 100% only if the portfolio betas have the same sign 85% of the time (i.e., ρ_b must be equal to 0.89). Similar to the first scenario, this last condition implies that the portfolios must combine two properties that are difficult to reconcile: they must perform well (poorly) both when the realized variance is high and when the recession hits. If we are willing to double $v_f^e(lev)$ from 0.37 to 0.74, the required beta correlation remains high at 0.73.

[FIGURE IV HERE]

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Table I: Unconditional Moments of the Predictive Variables

Panel A reports the first four moments as well as the first-, and second-order partial autocorrelation coefficients of the predictors used to capture the dynamics of the Variance Risk Premium (VRP) over the long sample from 1970 to 2014 (179 quarterly observations). The set of predictors (all expressed in log form) includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate of the producer price index (PPI), the quarterly growth rate of the seasonally-adjusted number of employees in the nonfarm sector (EMP), the leverage ratio of broker-dealers (LEV), and the quarterly return of the prime broker index (PBI). Panel B reports the same statistics over the short sample from 1992 to 2014 (92 quarterly observations).

Panel A: Long Sample (1970-2014)

	Mean	Std.	Skew.	Kurt.	AC1	AC2
Lagged Realized Variance (RV)	−5.34	0.79	0.96	4.57	0.66	0.14
Price/Earnings Ratio (PE)	2.86	0.45	-0.14	2.18	0.98	−0.14
Default Spread (DEF)	1.01%	0.41%	1.85	8.29	0.83	−0.11
Producer Price Index (PPI)	0.92%	1.28%	0.20	6.22	0.41	0.21
Employment Growth (EMP)	0.37%	0.56%	−0.83	4.84	0.75	0.04
Broker-Dealer Leverage (LEV)	2.71	0.60	0.06	2.20	0.96	0.12
Prime Broker Index (PBI)	1.98%	17.6%	−0.57	4.62	0.05	−0.14

Panel B: Short Sample (1992-2014)

	Mean	Std.	Skew.	Kurt.	AC1	AC2
Lagged Realized Variance (RV)	−5.26	0.89	0.76	3.41	0.72	0.13
Price/Earnings Ratio (PE)	3.22	0.24	0.49	3.22	0.94	−0.19
Default Spread (DEF)	0.89%	0.40%	3.21	17.00	0.81	−0.24
Producer Price Index (PPI)	0.55%	1.09%	−1.24	10.09	0.14	0.00
Employment Growth (EMP)	0.27%	0.47%	−1.71	6.79	0.82	0.17
Broker-Dealer Leverage (LEV)	3.20	0.31	1.08	6.38	0.84	0.03
Prime Broker Index (PBI)	2.07%	16.3%	−1.31	6.19	0.05	−0.10

Table II: Correlation Matrix of the Predictive Variables

Panel A reports the correlation matrix of the predictors over the long sample from 1970 to 2014 (179 quarterly observations). The set of predictors (all expressed in log form) includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate of the producer price index (PPI), the quarterly growth rate of the seasonally-adjusted number of employees in the nonfarm sector (EMP), the leverage ratio of broker-dealers (LEV), and the quarterly return of the prime broker index (PBI). Panel B reports the same statistics over the short sample from 1992 to 2014 (92 quarterly observations).

Panel A: Long Sample (1970-2012)

	PE	DEF	PPI	EMP	LEV	PBI
Lagged Realized Variance (RV)	0.06	0.41	−0.08	−0.44	0.11	−0.28
Price/Earnings Ratio (PE)		−0.53	−0.30	−0.02	0.78	0.04
Default Spread (DEF)			−0.09	−0.52	−0.22	−0.04
Producer Price Index (PPI)				0.14	−0.30	−0.02
Employment Growth (EMP)					−0.22	−0.04
Broker-Dealer Leverage (LEV)						−0.07

Panel B: Short Sample (1992-2014)

	PE	DEF	PPI	EMP	LEV	PBI
Lagged Realized Variance (RV)	0.05	0.62	−0.20	−0.53	0.40	−0.45
Price/Earnings Ratio (PE)		−0.47	0.10	0.39	0.19	0.21
Default Spread (DEF)			−0.35	−0.76	−0.32	−0.40
Producer Price Index (PPI)				0.23	0.04	0.15
Employment Growth (EMP)					−0.27	0.21
Broker-Dealer Leverage (LEV)						−0.30

Table III: Summary Statistics for the Equity Portfolios: Short Sample

Panel A shows the annualized excess mean, standard deviation, size (in log form), Book-to-Market (BM) ratio, and the pre-, post-rank variance betas of the quarterly returns of quintile portfolios formed by equally weighting all portfolios in the same variance beta quintile (Low, 2, 3, 4, High). For each quintile portfolio, the pre-rank beta is defined as the mean of the variance betas across stocks on the portfolio formation dates. The post-rank variance beta is computed from the time-series regression of the portfolio return on the variance and market factors (including all predictors). Panel B reports the annualized estimated alpha of each quintile portfolio using the CAPM, the Fama-French (FF) model that includes the market, size, and BM factors, and two extensions that include momentum and liquidity factors, respectively. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% level, respectively.

Panel A: Unconditional Moments, Characteristics, and Variance Betas

Quintile	Mean (% p.a.)	St. Dev. (% p.a.)	Size	BM	Pre-rank beta		Post-rank beta	
Low	10.29	14.74	9.33	0.54	−0.65	(−2.25)	-1.66^{***}	(−4.55)
2	9.45	15.52	9.44	0.55	−0.29	(−0.82)	-1.01^{***}	(−2.91)
3	7.47	14.93	9.49	0.52	−0.02	(−0.06)	-0.99^{***}	(−2.83)
4	5.54	14.86	9.44	0.54	0.25	(0.70)	-0.59^{**}	(−2.15)
High	5.19	16.35	9.46	0.53	0.61	(2.15)	−0.17	(−0.48)
High-Low	−5.09	8.31	0.13	−0.01	1.22	(4.40)	1.49^{***}	(3.32)

Panel B: Alphas

Quintile	CAPM (% p.a.)		Fama-French (FF) (% p.a.)		FF+Momentum (% p.a.)		FF+Liquidity (% p.a.)	
Low	4.07^{**}	(2.76)	2.40^{*}	(1.73)	2.64^{*}	(1.94)	1.54	(1.28)
2	2.95^{*}	(1.72)	1.62	(1.17)	0.96	(0.58)	1.30	(0.87)
3	0.94	(0.73)	−0.04	(−0.38)	0.04	(0.03)	−0.06	(−0.61)
4	−0.99	(−0.84)	-2.07^{**}	(−2.26)	-2.37^{***}	(−2.63)	-2.49^{***}	(−2.58)
High	-1.90^{**}	(−1.31)	-2.96^{**}	(−2.31)	-2.92^{**}	(−2.01)	-3.13^{**}	(−2.24)
High-Low	-5.97^{***}	(−3.13)	-5.36^{***}	(−2.72)	-5.56^{***}	(−2.83)	-4.67^{**}	(−2.42)

Table IV: Market Factor: Risk Neutral Expectation

Panel A reports the estimated coefficients that drive the risk-neutral expectation of the market factor for the set of macro-finance variables that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on the risk-neutral expectation of the market factor and are obtained from the conditional two-pass regression. Panel B examines the incremental predictive power of the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)
Risk Neutral	0.01	−0.18	−0.12	−0.02	−0.01	−0.22
Expectation	(0.08)	(−1.19)	(−0.48)	(−0.08)	(−0.10)	(−1.32)

Panel B: Contribution of Broker-Dealer Variables

			Combined	
	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)
Risk Neutral	−0.14	0.16	−0.11	0.14
Expectation	(−1.02)	(1.35)	(−0.77)	(1.14)

Table V: Market Factor: Risk Premium

Panel A examines the relationships between the macro-finance variables and the Market Risk Premium (MRP). The set of variables includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on the MRP and are obtained from the conditional two-pass regression. Panel B examines the incremental predictive power of the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)
Market Risk Premium	1.70^{**}	0.68	-1.96^{**}	-1.40	-1.17	-1.62^{**}
	(2.55)	(0.88)	(-1.96)	(-1.02)	(-1.29)	(-1.98)

Panel B: Contribution of Broker-Dealer Variables

			Combined	
	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)
Market Risk Premium	-0.56	0.91	-0.34	0.83
	(-0.62)	(1.26)	(-0.34)	(1.01)

Table VI: Constrained versus Unconstrained Portfolio Return Projections

This table reports the slope and adjusted R^2 of a time-series regression of the unconstrained return projection on the model-implied projection for each quintile portfolio formed by equally weighting all equity portfolios in the same variance beta quintile (Low, 2, 3, 4, High). The unconstrained expectation is expressed as a linear function of the set of predictors that includes the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, the quarterly employment rate, the broker-dealer leverage ratio, and the quarterly return of the prime broker index. The constrained version is computed from the estimated coefficients of the two-factor model. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% level, respectively.

	Equity Portfolios				
Quintile	Low	2	3	4	High
Slope	1.03***	0.88***	1.02***	1.03***	1.17***
	(52.92)	(94.91)	(89.21)	(69.41)	(53.58)
R^2	0.96	0.98	0.98	0.96	0.95

Table VII: Alternative Factor Models for the Equity Market

This table examines the robustness of the explanatory power of the broker-dealer variables to changes in the factor model used in the equity market. The first specification includes the initial factors (realized variance (RV) and market return), and the Fama-French size and Book-to-Market (BM) factors. The second and third specifications add the momentum and liquidity factors to the previous specification. The fourth specification includes the initial factors and the high-frequency RV component obtained from the Hodrick-Prescott filter. The fifth specification includes the initial factors and the squared RV. For each specification, the first column reports the correlation between the equity Variance Risk Premium (VRP) projection and its baseline counterpart presented in the paper. The remaining columns contain the estimated coefficients that drive the equity and option VRPs (as well as their difference) for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The coefficients determine the impact of a one-standard deviation change in the predictors on the equity and option VRPs, as well as their difference. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels, respectively.

		Equity VRP		Option VRP		VRP Difference	
	Corr. Baseline	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)
Size+BM	0.75	−0.18	−0.09	0.37***	0.17**	−0.55***	−0.26**
		(−0.90)	(−0.48)	(4.68)	(2.03)	(−6.43)	(−2.03)
Size+BM +Momentum	0.79	−0.14	−0.06	0.37***	0.17**	−0.51***	−0.23*
		(−0.67)	(−0.30)	(4.67)	(2.03)	(−5.84)	(−1.79)
Size+BM +Liquidity	0.75	−0.17	−0.05	0.37***	0.17**	−0.54***	−0.22*
		(−0.84)	(−0.23)	(4.67)	(2.03)	(−6.24)	(−1.70)
High-frequency RV	0.97	−0.26	−0.09	0.37***	0.17**	−0.63***	−0.26**
		(−1.17)	(−0.47)	(4.67)	(2.03)	(−7.27)	(−2.11)
Squared RV	0.97	−0.19	−0.14	0.37***	0.17**	−0.56***	−0.31**
		(−0.83)	(−0.71)	(4.67)	(2.03)	(−6.54)	(−2.53)

Table VIII: Time-Varying Betas: Variance Factor

This table reports, for each equity portfolio sorted along the Market (M) and Variance (V) dimensions, the t-statistic of the coefficients measuring the sensitivity of the variance beta to changes in the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), the quarterly employment rate (EMP), the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI). * designates statistical significance at the 5% level or lower.

	R. Var. (RV)	PE Ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	Leverage (LEV)	PB Index (PBI)
M1V1	−1.98*	−0.50	1.36	−1.15	0.47	−0.88	0.38
M1V2	−0.83	1.33	0.25	−1.41	−0.16	0.44	−0.22
M1V3	−1.13	−1.03	1.05	1.09	0.53	0.46	−0.89
M1V4	−3.43*	1.10	−0.65	−1.70	0.84	0.01	−0.84
M1V5	−2.64*	−0.50	−0.66	−2.44*	2.85*	−1.17	1.88
M2V1	−3.95*	0.24	−1.95	−1.85	2.72*	−1.26	0.78
M2V2	−0.21	1.64	0.46	−2.10	0.81	0.26	0.22
M2V3	−1.37	−0.19	1.02	−0.70	−0.89	0.15	1.21
M2V4	-1.74	1.02	0.96	−0.62	0.00	0.28	0.36
M2V5	0.43	−0.35	1.58	0.58	−0.35	2.06*	1.46
M3V1	−2.59*	−0.58	0.67	−1.26	0.66	−1.87	1.61
M3V2	0.89	−0.96	0.39	−0.50	0.42	0.83	−0.18
M3V3	−2.05*	−0.67	−2.00*	−1.02	3.00*	0.37	0.20
M3V4	-0.64	−0.15	1.24	−0.29	0.10	−0.32	1.35
M3V5	−0.42	0.31	−0.73	0.47	0.93	1.54	−1.69
M4V1	−0.99	−0.17	1.44	−0.42	−1.46	1.60	0.18
M4V2	−2.73*	−0.24	0.03	−0.91	0.00	−1.96*	0.00
M4V3	0.15	−1.80	4.17*	0.62	−2.35*	1.02	0.73
M4V4	0.02	−0.05	1.37	−0.31	0.67	1.91	1.34
M4V5	0.19	−1.66	1.43	−0.45	−0.60	4.07*	−0.96
M5V1	−1.19	−1.47	−0.31	−0.62	0.11	−1.29	−0.29
M5V2	-0.68	−0.19	0.57	−1.92	0.56	0.05	0.74
M5V3	−0.54	1.50	−0.53	−0.86	1.19	0.31	−0.22
M5V4	0.52	−0.46	1.80	−0.26	−0.64	1.08	0.88
M5V5	1.77	−1.23	1.69	1.15	−0.61	0.82	0.00

Table IX: Time-Varying Betas: Market Factor

This table reports, for each equity portfolio sorted along the Market (M) and Variance (V) dimensions, the t-statistic of the coefficients measuring the sensitivity of the market beta to changes in the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), the quarterly employment rate (EMP), the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI). * designates statistical significance at the 5% level or lower.

	R. Var. (RV)	PE Ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	Leverage (LEV)	PB Index (PBI)
M1V1	−2.81*	−2.61*	0.91	0.94	1.22	−0.98	2.79*
M1V2	−2.45*	−2.15*	1.26	2.40	−0.08	0.59	0.12
M1V3	−2.22*	−1.94	1.69	4.65	0.26	−0.43	−0.03
M1V4	−2.34*	−2.87*	1.48	1.55	−0.56	0.48	−0.15
M1V5	−1.88	−2.28*	0.96	−0.04	0.86	−0.16	1.08
M2V1	−2.18*	−2.63*	1.49	−0.25	0.44	−0.10	1.78
M2V2	−1.89	−1.63	0.70	0.25	1.41	−0.30	1.43
M2V3	−1.92	−2.23*	1.50	0.71	0.08	−0.31	0.94
M2V4	−3.44*	−4.48*	0.79	2.98*	0.86	0.15	0.39
M2V5	−1.49	−2.28*	1.19	2.14*	−0.70	1.21	1.42
M3V1	−1.79	−4.50*	2.13*	1.28	−0.90	−0.74	1.93
M3V2	−0.31	−2.24*	−0.06	2.13*	0.81	0.32	−0.36
M3V3	−1.41	−1.54	−0.11	0.63	0.64	0.82	0.09
M3V4	−1.41	−2.77*	2.29*	0.99	0.29	−1.01	1.59
M3V5	−1.26	−2.17*	0.78	0.91	0.80	−0.07	−0.82
M4V1	−1.18	−1.77	1.10	−0.34	−0.02	1.42	1.40
M4V2	−2.79*	−4.07*	1.68	1.39	1.00	0.13	1.16
M4V3	−3.94*	−2.54*	1.22	0.14	0.01	1.09	0.13
M4V4	−0.14	−1.35	1.21	−0.19	1.15	0.88	1.94
M4V5	−0.71	−1.93	1.69	0.17	−0.73	3.53*	1.05
M5V1	−0.39	−1.44	0.83	0.84	−1.53	−0.44	−0.03
M5V2	0.98	−2.10*	1.56	0.55	0.27	1.59	0.11
M5V3	0.49	−0.96	0.53	−1.64	0.51	2.59*	−2.55*
M5V4	−1.21	0.12	−0.03	−0.87	−0.24	1.31	0.35
M5V5	1.42	0.09	1.53	−0.92	−0.61	0.71	0.32

Table X: Winsorized Variance Observations

This table examines the robustness of the explanatory power of the broker-dealer variables to winsorizing extreme observations of the realized variance. For each threshold (1% and 2.5% of observations winsorized at each end), the columns report the estimated coefficients that drive the equity and option Variance Risk Premia (VRPs), as well as their difference for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The coefficients determine the impact of a one-standard deviation change in the predictors on the equity and option VRPs, as well as their difference. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Equity VRP		Option VRP		VRP Difference	
	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)
2% winsorized	-0.15	-0.12	0.32^{***}	0.15^{*}	-0.47^{***}	-0.27^{***}
	(-0.61)	(-0.63)	(4.03)	(1.75)	(-6.12)	(-2.53)
5% winsorized	-0.07	-0.09	0.20^{***}	0.11	-0.27^{***}	-0.20^{***}
	(-0.71)	(-0.99)	(3.66)	(1.37)	(-4.50)	(-2.91)

Table XI: The Squared SVIX Index: Period 1996-2012

Panel A reports the estimated coefficients and the adjusted R^2 of the regression of the quarterly squared SVIX index on the set of macro-finance variables that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on the squared SVIX and are computed using the GMM for samples of unequal lengths. Panel B examines the incremental predictive power of the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	R^2
Squared SVIX	1.18^{***}	0.69^{***}	-0.11	0.15^{**}	-0.05	0.11	0.76
	(31.50)	(10.31)	(-1.37)	(2.29)	(-0.87)	(1.58)	

Panel B: Contribution of Broker-Dealer Variables

	Leverage (LEV)	R^2	PB Index (PBI)	R^2	Combined Leverage (LEV)	Combined PB Index (PBI)	Combined R^2
Squared SVIX	-0.09^{**}	0.76	-0.12^{**}	0.77	-0.15^{***}	-0.17^{***}	0.78
	(-2.57)		(-2.17)		(-4.37)	(-3.08)	

Table XII: Alternative Approaches for Forming Portfolios

This table examines the robustness of the explanatory power of the broker-dealer variables to changes in the portfolio formation procedure. The first specification includes all but tiny stocks, whereas the second includes all stocks in the population. The third, fourth, and fifth specifications rank stocks each month based on their robust beta t-statistics, their quarterly beta t–statistics, and their estimated betas. The sixth and seventh specifications account for nonsynchronous trading by including the lagged factors in the stock return regressions, and by excluding return observations equal to zero. For each specification, the first column reports the correlation between the equity Variance Risk Premium (VRP) projection and its baseline counterpart presented in the paper. The remaining columns contain the estimated coefficients that drive the equity and option VRPs (as well as their difference) for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The coefficients determine the impact of a one-standard deviation change in the predictors on the equity and option VRPs, as well as their difference. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels, respectively.

		Equity VRP		Option VRP		VRP Difference	
	Corr. Baseline	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)
All but Tiny Stocks	0.82	-0.08 (-0.34)	-0.11 (-0.34)	0.37^{***} (4.67)	0.17^{**} (2.03)	-0.45^{***} (-3.32)	-0.28^{**} (-2.10)
All Stocks	0.81	0.00 (0.02)	-0.23 (-1.18)	0.37^{***} (4.67)	0.17^{**} (2.03)	-0.37^{***} (-3.55)	-0.40^{***} (-2.96)
Robust Betas	0.97	-0.08 (-0.33)	0.03 (0.16)	0.37^{***} (4.67)	0.17^{**} (2.03)	-0.45^{***} (-5.40)	-0.14 (-1.30)
Quarterly Betas	0.44	-0.04 (-0.17)	-0.28 (-1.19)	0.37^{***} (4.67)	0.17^{**} (2.03)	-0.42^{***} (-4.63)	-0.45^{***} (-3.69)
No t-statistics	0.62	-0.23 (-1.14)	0.08 (0.54)	0.37^{***} (4.67)	0.17^{**} (2.03)	-0.60^{***} (-8.50)	-0.09 (-1.40)
Lagged Factors	0.76	-0.21 (-1.25)	-0.23 (-1.34)	0.37^{***} (4.67)	0.17^{**} (2.03)	-0.52^{***} (-7.04)	-0.41^{***} (-3.58)
Zero Returns	0.86	-0.23 (-0.86)	-0.15 (-0.77)	0.37^{***} (4.67)	0.17^{**} (2.03)	-0.61^{***} (-7.28)	-0.32^{***} (-2.83)

Table XIII: Alternative Set of Macro-Finance Variables

This table examines the robustness of the explanatory power of the broker-dealer variables to changes in the set of macro-finance variables. The first specification replaces the price/earnings ratio with the dividend yield. The second and third specifications replace the quarterly growth rate in employment with the quarterly growth rate in industrial production and the business cycle indicator proposed by Aruoba, Diebold, and Scotti (2009), respectively. The fourth, fifth, and sixth specifications add two bond variables (the three-month T-Bill rate and the term spread), and the quarterly volatility of the inflation rate to the initial set of predictors. For each specification, the columns report the estimated coefficients that drive the equity and option Variance Risk Premia (VRPs) (as well as their difference) for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The coefficients determine the impact of a one-standard deviation change in the predictors on the equity and option VRPs, as well as their difference. The figures in parentheses report the heteroskedasticity-robust t-statistics ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Equity VRP		Option VRP		VRP Difference	
	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)
Dividend Yield	−0.05	−0.05	0.41***	0.18**	−0.41***	−0.24**
	(−0.02)	(0.27)	(4.59)	(2.16)	(−3.42)	(−1.95)
Industrial Production	−0.17	−0.12	0.36***	0.17**	−0.54***	−0.30**
	(−0.70)	(−0.64)	(4.57)	(2.04)	(−3.93)	(−2.28)
Business Cycle	−0.07	−0.12	0.31***	0.19**	−0.38***	−0.30**
	(−0.31)	(−0.60)	(4.20)	(2.31)	(−3.93)	(−2.32)
Short Rate	−0.11	−0.12	0.38***	0.17**	−0.48***	−0.30**
	(0.48)	(−0.65)	(4.62)	(2.07)	(−3.68)	(−2.27)
Term Spread	−0.17	−0.13	0.38***	0.18**	−0.55***	−0.31**
	(−0.69)	(−0.65)	(4.76)	(2.18)	(−4.05)	(−2.30)
Vol. Inflation	−0.16	−0.10	0.35***	0.16**	−0.52***	−0.26**
	(−0.68)	(−0.53)	(4.56)	(1.97)	(−3.91)	(−2.06)

Table XIV: Squared Macro-Finance Variables

This table examines the robustness of the explanatory power of the broker-dealer variables to changes in the set of macro-finance variables. The specifications include the initial set of predictors (lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, the quarterly employment rate) and their squared values taken one at a time. For each specification, the columns report the estimated coefficients that drive the equity and option Variance Risk Premia (VRPs) (as well as their difference) for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The coefficients determine the impact of a one-standard deviation change in the predictors on the equity and option VRPs, as well as their difference. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Equity VRP		Option VRP		VRP Difference	
	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)	Leverage (LEV)	PB Index (PBI)
Realized Variance^2	−0.09	−0.16	0.35***	0.18***	−0.43***	−0.34***
	(−0.36)	(0.80)	(4.33)	(2.32)	(−3.54)	(−2.74)
PE Ratio^2	−0.18	−0.07	0.26***	0.18**	−0.45***	−0.25**
	(−0.78)	(0.34)	(3.56)	(2.19)	(−3.46)	(−2.01)
Default Spread^2	−0.14	−0.08	0.37***	0.15*	−0.51***	−0.23*
	(−0.57)	(−0.41)	(4.80)	(1.86)	(−3.76)	(−1.73)
Inflation^2	−0.14	−0.09	0.35***	0.19**	−0.49***	−0.28**
	(−0.58)	(−0.44)	(4.45)	(2.38)	(−3.69)	(−2.14)
Employment^2	−0.18	−0.14	0.35***	0.22***	−0.54***	−0.37***
	(−0.74)	(−0.75)	(4.54)	(2.65)	(−4.06)	(−2.80)

Table XV: Implied Stock Variance and Implied Correlation

Panel A reports the estimated coefficients and adjusted R^2 of regressions of the equally-weighted average of the monthly implied variances of individual stocks (Implied Stock Variance) and the monthly implied correlation (Implied Correlation) on the set of macro-finance predictors that include the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on Implied Stock and Implied Correlation, and are computed using the GMM for samples of unequal lengths. Panel B examines the incremental predictive power of the orthogonalized broker-dealer variables, the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI), in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	R^2
Implied Stock Variance	1.74*** (36.49)	0.67*** (10.91)	0.50*** (5.41)	0.51*** (5.84)	−0.01 (−0.15)	0.25*** (−3.39)	0.69
Implied Correlation	40.63*** (67.51)	8.67*** (12.20)	−10.62*** (−8.65)	−5.15*** (−4.96)	−1.39** (−2.05)	3.26*** (4.04)	0.50

Panel B: Contribution of Broker-Dealer Variables

	Leverage (LEV)	R^2	PB Index (PBI)	R^2	Combined: Leverage (LEV)	Combined: PB Index (PBI)	Combined: R^2
Implied Stock Variance	0.18*** (2.68)	0.73	−0.21*** (−3.94)	0.71	0.10 (1.47)	−0.18*** (−3.38)	0.73
Implied Correlation	−1.16** (−2.34)	0.51	0.26 (0.43)	0.50	−1.45*** (−2.91)	−0.54 (−0.86)	0.51

Table XVI: Variance Risk Premia: Monthly Analysis

Panel A examines the relationships between the macro-finance variables and the equity Variance Risk Premium (VRP), the option VRP, and their difference at the monthly frequency. The set of variables includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on the VRPs and their difference. The equity- and option-based coefficients are obtained from the conditional two-pass regression approach and the GMM for samples of unequal lengths, respectively. Panel B examines the incremental predictive power of the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. The J-statistic of the joint test and associated p-values in brackets determine whether the two-factor equity model is correctly specified. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	J-stat.
Equity VRP	−0.09	−0.06	0.09	0.04	0.14*	0.07	5.76
	(−1.15)	(−0.56)	(0.78)	(0.34)	(1.75)	(0.77)	[0.04]
Option VRP	−0.13***	−0.09***	0.08***	0.06***	0.05***	−0.01	
	(−12.34)	(−5.87)	(4.36)	(2.87)	(3.08)	(−0.79)	
Difference	0.04	0.04	0.01	−0.02	0.08	0.08	
	(0.49)	(0.59)	(0.14)	(−0.16)	(1.45)	(1.09)	

Panel B: Contribution of Broker-Dealer Variables

					Combined		
	Leverage (LEV)	J-stat.	PB Index (PBI)	J-stat.	Leverage (LEV)	PB Index (PBI)	J-stat.
Equity VRP	0.02	6.90	−0.20***	6.76	−0.05	−0.20**	7.71
	(0.24)	[0.03]	(−2.58)	[0.04]	(−0.47)	(−2.49)	[0.04]
Option VRP	0.07***		0.00		0.08***	0.03**	
	(3.56)		(0.32)		(4.37)	(2.07)	
Difference	−0.05*		−0.20***		−0.13***	−0.23***	
	(−1.67)		(−4.53)		(−4.26)	(−5.08)	

Table XVII: Variance Risk Premia: Individual Stocks

Panel A examines the relationships between the macro-finance variables and the equity Variance Risk Premium (VRP) inferred from individual stocks, the option VRP, and their difference. The set of variables includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The coefficients determine the impact of a one-standard deviation change in the variables on the VRPs and their difference. The equity- and option-based coefficients are obtained from the conditional two-pass regression applied to individual stocks and the GMM for samples of unequal lengths, respectively. Panel B examines the incremental predictive power of the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The figures in parentheses report the heteroskedasticity-robust t-statistics. The J-statistic of the joint test and associated p-values in brackets determine whether the two-factor equity model is correctly specified. ***, **, and * designate statistical significance at the 1%, 5%, and 10% levels.

Panel A: Macro-Finance Variables

	Mean	R. Var. (RV)	PE ratio (PE)	Default (DEF)	Inflation (PPI)	Employ. (EMP)	J-stat.
Equity VRP	0.05	0.01	0.07	0.01	0.08	0.22***	4.29
	(0.68)	(0.08)	(0.67)	(0.10)	(0.82)	(3.26)	[0.00]
Option VRP	−0.45***	−0.34***	0.35***	0.01	0.19**	−0.07	
	(−8.01)	(−3.70)	(3.42)	(0.12)	(2.22)	(−0.72)	
Difference	0.50**	0.34***	−0.29***	0.00	−0.11*	0.28***	
	(2.52)	(3.41)	(−3.56)	(−0.01)	(−1.72)	(3.47)	

Panel B: Contribution of Broker-Dealer Variables

					Combined		
	Leverage (LEV)	J-stat.	PB Index (PBI)	J-stat.	Leverage (LEV)	PB Index (PBI)	J-stat.
Equity VRP	0.21	4.55	−0.25***	4.28	0.14	−0.22**	4.79
	(0.89)	[0.00]	(−4.41)	[0.00]	(−0.52)	(−2.11)	[0.00]
Option VRP	0.31***		0.07		0.37***	0.17**	
	(3.84)		(0.83)		(4.68)	(2.03)	
Difference	−0.09*		−0.32***		−0.23***	−0.39***	
	(−1.68)		(−5.05)		(−3.66)	(−6.36)	

Figure I: Market Risk Premium

This figure reports the path of the quarterly Market Risk Premium (MRP) projection obtained with the set of macro-finance predictors that includes the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, and the quarterly employment rate. Shaded areas correspond to NBER recession periods. The y-axis is in percent per quarter.

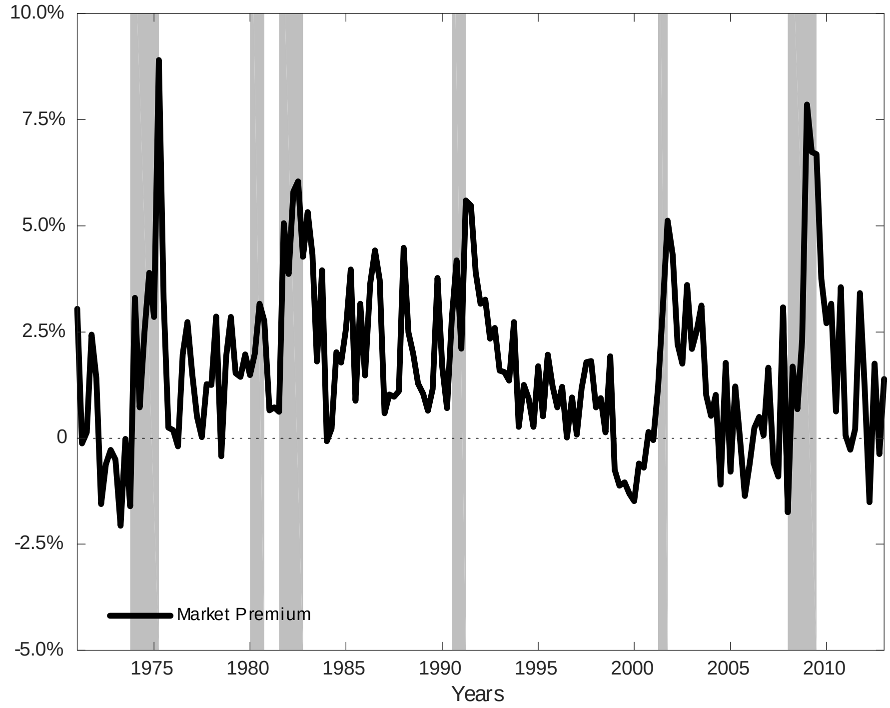

Figure II: Payoffs of the Variance Mimicking Portfolios

This figure plots the quarterly payoffs of the mimicking portfolios formed in the equity and option markets. The construction of the mimicking option portfolio (solid line) is based on the approach developed by Carr and Wu (2009). The mimicking equity portfolio (dashed line) is obtained from a linear combination of the equity portfolios inferred from the two-factor model. The quarterly realized variance is almost identical to the payoff of the option portfolio and is therefore not shown.

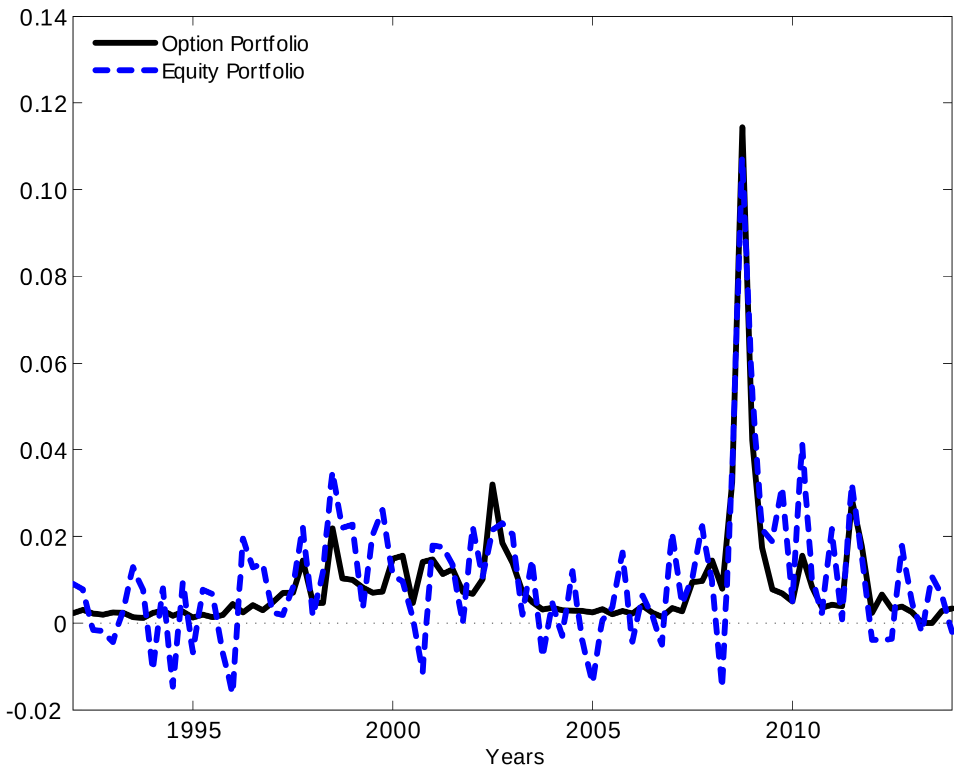

Figure III: Equity Variance Risk Premium: Impact of the Broker-Dealer Variables

This figure compares the paths followed by two versions of the quarterly equity Variance Risk Premium (VRP) projection. The first version is based on the set of macro-finance variables that includes the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, and the quarterly employment rate. The second version is obtained using the macro-finance predictors as well as the two broker-dealer variables, which are the leverage ratio of broker-dealers and the quarterly return of the prime broker index. The y-axis is in percent per quarter.

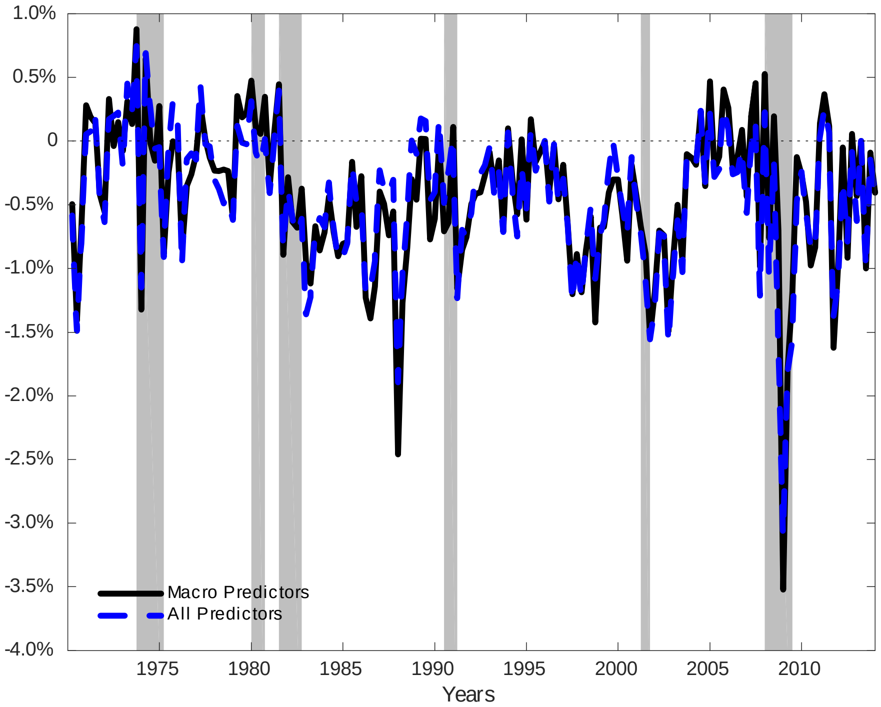

Figure IV: Simulated Omitted-Factor Bias

Panel A reports the bias of the estimated risk-neutral leverage coefficient for the equity Variance Risk Premium (VRP), $\hat{v}_v^e(lev)$, when the asset pricing model omits a factor whose risk premium is negatively related to leverage. The bias is defined in relative terms as a fraction of the true leverage coefficient $v_v^e(lev)$, which is set equal to -0.37 (similar to the option-based estimate in Table 3 of the paper). For each value taken by the correlation between the portfolio betas (on the variance and omitted factors), we compute the bias via a simulation analysis that replicates the salient feature of the data (1,000 trials are used for each scenario). We consider two different values for the coefficient that relates leverage to the quarterly risk premium of the omitted factor: -0.37% (solid line), and -0.74% (dashed line). Panel B repeats the analysis for an omitted factor whose risk premium is positively related to leverage.