Asymmetric Information and Liquidity Provision

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Abstract

The presence of information asymmetry increases the probability that a potential predator will provide liquidity rather than engaging in predatory trading during liquidation by a distressed trader. More information asymmetry is associated with lower expected losses from liquidation for the distressed trader in illiquid markets. There is a negative correlation between the degree of information asymmetry and the returns from predatory trading, which is consistent with empirical findings. These results imply that strategic traders are more likely to stabilize markets by providing liquidity when information is asymmetric. These findings highlight a cost associated with disclosure and can explain the documented rarity of illiquidity episodes in financial markets.

Keywords: Asymmetric Information; Predatory trading; Liquidity

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Why do institutional investors at times stabilize markets by providing liquidity and at other times destabilize markets by engaging in predatory trading? We examine the determinants of the choice between providing liquidity and engaging in predatory trading when another large trader is forced to sell or buy a risky asset.[1] This choice has important implications for financial markets. Predatory trading reduces liquidity, usually at times when it is needed the most, and increases transaction costs for large traders. Moreover, Brunnermeier & Pedersen (2005) argue that predatory trading increases the risk of a financial crisis, amplifies financial contagion, and affects institutional investors' risk management strategies.

We argue that a key variable affecting this choice is the presence of information asymmetry between the distressed trader and her potential predators. A natural source of information asymmetry is the amount of assets to be sold or bought, which we assume is only known by the distressed trader.[2] This argument is supported by anecdotal evidence. For example, Lowenstein (2000) notes that the head of Long Term Capital Management "... bitterly complained to the Fed's Peter Fisher that Goldman, among others, was front- running, meaning trading against it on the basis of inside knowledge." Also, Wermers (2001) argues that "more frequent portfolio disclosure would enable increased front running by professional investors and speculators." Along the same line, the International Association for Quantitative Finance (IAQF) recommends that large institutional investors "limit granularity of reporting sufficiently to protect Investors against predatory trading against the Managers positions."[3,4]

We model the interaction between large traders in an illiquid market as a two-player nonzero-sum stochastic differential game. Illiquidity means that the price of the risky asset is a function of both the large traders' aggregate holding of this asset (long-term impact) and their aggregate trading rate of this asset (short-term impact). A distressed trader needs to liquidate the single risky

[1] Trading is non-informed, that is independent of risky asset's fundamental value. Vayanos (2001) argues that non-informed trading must represent a large subset of the trading activity in financial markets. Examples of non-informed trading include trading resulting from either index reconstitution or flow of fund to/from institutional investors. The extant literature documents significant non-informed trading activity in financial markets (see Chen et al. (2004), Coval & Stafford (2007), Zhang (2010), Petajisto (2011), and Bessembinder et al. (2014)).

[2] We refer to the amount to be liquidated as the liquidation size.

[3] See IAQF Investor Risk Committee Consensus Document: Findings on Hedge Fund Transparancy and Disclosure, July 27, 2001.

[4] See Table I in Brunnermeier & Pedersen (2005) for additional anecdotal evidence relating information asymmetry to predatory trading.

asset. A potential predator can either provide liquidity or engage in predatory trading but does not know the liquidation size, only the distribution it is drawn from. Profitable predation requires racing to sell the risky asset while its price is high, ahead of the distressed trader's price impact, and then buying it at a lower price later on. We consider closed loop equilibria to allow learning about the liquidation size through changes in the price of the risky asset. These changes are a function of changes in the asset's fundamental value and aggregate trading by the large traders. We provide closed-form solutions of the game under some parameter restrictions and use numerical techniques to solve for the equilibria without restrictions.

Our main finding is that information asymmetry reduces the probability that predatory trading occurs in illiquid markets. The intuition is that the potential predator faces higher losses when engaging in predatory trading relative to providing liquidity, losses due to errors made while estimating the liquidation size.[5] Predatory trading is associated with higher losses because it requires more aggressive trading to race the distressed trader to the market, which leads to more estimation errors and higher trading costs. Moreover, the distressed trader can partially forecast the potential predator's error in estimating the liquidation size. This forecast can lead to further losses to the potential predator when predation occurs.

We also find that market illiquidity affects the probability of predatory trading occurring. Predatory trading is less likely when the long-term price impact takes low values and is negligible when the long-term price impact is zero. This result is intuitive. As Brunnermeier & Pedersen (2005) note, "the predator derives profit from the price impact of the prey". Higher long-term price impacts lead to higher profits from predation. In addition, the resolution of uncertainty about the liquidation size, which reduces the degree of information asymmetry, is faster when the long-term price impact is high. The reason is that trading by the distressed trader explains a higher percentage of the changes in the price of the risky asset when the long-term price impact is high.

Our work highlights the welfare benefits of information asymmetry during crises. We show that an increase in information asymmetry generally benefits distressed traders, and hurts predators, and increases the large traders' aggregate wealth. These findings imply that there may be a

[5] These losses are increasing in the degree of information asymmetry. The potential predator never incurs losses in equilibrium in models with complete information. See Brunnermeier & Pedersen (2005), Carlin et al. (2007), and Schöneborn & Schied (2007).

cost associated with implementing recent policies requiring more transparency for institutional investors.[6]

Several of our predictions are consistent with existing empirical evidence. Parida & Teo (2011) provide evidence that funds reporting semiannually outperform funds reporting quarterly. Moreover, this difference in performance disappear when all funds are required to report quarterly. These results are consistent with our model's prediction that greater information asymmetry is associated with higher returns for the distressed trader. The model's prediction that a higher degree of information asymmetry leads to lower returns for the potential predator is consistent with the findings of Shive & Yun (2013). They show that returns from predatory trading are higher when mutual funds are required to have more frequent disclosure. Bessembinder et al. (2014) find empirical evidence that strategic traders provide liquidity when markets are resilient. They define market resiliency as the degree to which "some or all of the immediate price impact of trades is subsequently reversed." Their finding is consistent with our prediction that the potential predator provides liquidity when the long-term price impact is low. We predict that the potential predator's value is higher when the permanent price impact is higher. Both Shive & Yun (2013) and Arif et al. (2014) present empirical evidence consistent with this prediction. These papers find that the potential predators' values are higher when they trade in less liquid assets.

Related Literature

Our research is related to several strands of literature including models of liquidity crises, competition among strategic traders, and distressed liquidation of risky assets. The nature of the information structure makes our model unique. In our model, one agent is better informed than the other and the private information is about asset allocation and not the asset's fundamental value.

Our model is an extension of the first stage game in Carlin et al. (2007), who explain the puzzling fact that illiquidity is rare and episodic in financial markets.[7] They model predation as a breakdown in cooperation between institutional investors in a repeated game. We complement their work by showing that asymmetric information provides an alternative explanation for the episodic

[6] Siritto (2014), Fuchs et al. (2014), and Banerjee et al. (2015) arrive at similar conclusions in other settings.

[7] Infrequent and episodic illiquidity was puzzling because Brunnermeier & Pedersen (2005) showed that predatory trading is the equilibrium strategy during forced liquidations and forced liquidations are frequent in financial markets.

illiquidity. Our model applies to important types of interaction between institutional investors in which cooperation as described by Carlin et al. (2007) does not apply. These instances include interactions between high frequency traders/hedge-funds and mutual funds. All traders in their model must be able to execute a punishment strategy for the equilibrium to hold in their repeated game. In financial markets, mutual funds are unlikely to engage in predatory trading against high frequency traders and hedge-funds in part because of regulatory requirements and inferior technological sophistication.

Other research related to our model analyzes predatory trading under complete information. Predatory trading always occurs in equilibrium in Brunnermeier & Pedersen (2005). Schöneborn & Schied (2007) study predatory trading in a two-stage-game extension of the first stage game in Carlin et al. (2007). Carmona & Yang (2011) consider a similar two-stage extension but allow both strategic players to follow closed-loop strategies. Liquidity provision occurs in the models of both Schöneborn & Schied (2007) and Carmona & Yang (2011) when the permanent price impact is low, consistent with our results. Bessembinder et al. (2014) extend Brunnermeier and Pedersen's model to include resiliency. In their model predatory trading only occurs when markets are not resilient. In reality, it is often impossible to know the exact liquidation need of a trader even in nonanonymous markets. We complement this literature by highlighting the role of asymmetric information in determining the equilibrium outcome of the interaction between strategic traders when one trader is in distress.

Competition among strategic traders has been explored in extensions of Kyle (1985). Foster & Viswanathan (1996) and Back et al. (2000) characterize the trading behavior of informed strategic traders. There are two main differences between their models and ours. First, strategic traders have symmetric information *ex-ante* in their models. Second, the trading motives in their models are related to the risky asset's fundamental value. Choi et al. (2015) and Vayanos (2001) study the effect of non-informational trading by large traders. Vayanos investigates competition among large traders when trades are the result of risk-sharing needs. Choi *et al.* examine the equilibrium outcome of competition among two traders when one has a trading target and the other has private information about the value of the asset in a multi-period Kyle model. Predatory trading does not occur in the model of Choi et al. (2015). We complement this strand of the literature by studying the effect of information asymmetry on liquidity provision during distress liquidation.

Our paper is related to the literature on the Scholes liquidation problem, which is concerned with the optimal way to liquidate an illiquid asset (see Bertsimas & Lo (1998) , Huberman & Stanzl (2005), Moallemi et al. (2012), Carmona & Yang (2011), Obizhaeva & Wang (2013) and references therein). Moallemi et al. (2012) study the liquidation problem with asymmetric information in a discrete time setting. Carmona & Yang (2011) study the role of noise traders on predatory trading when both strategic traders follow closed-loop strategies. We complement this literature by focusing on the interaction between the degree of information asymmetry and the likelihood of predatory trading occurring and thus the implications of portfolio disclosure.

1. Basic Model

Our model is an extension of the first-stage game in Carlin et al. (2007). We consider a continuous time economy with two assets: a riskfree asset with zero return and a risky asset. There are two types of traders interacting in the market: long-term investors and strategic traders. Long-term investors have three key characteristics: (*i*) They are price takers, (*ii*) they have downward sloping demand curves, and (*iii*) their demand is a function of the margin of safety, that is the difference between the asset's fundamental value and its price .

Strategic traders are large, risk-neutral agents. Their trades affect the risky asset's price. Examples of strategic traders include hedge funds and proprietary trading firms. Both Brunnermeier & Pedersen (2005) and Carlin et al. (2007) assume that trades by each strategic trader are observable. An important departure of our model from this work is that we adopt the realistic assumption that trades by a strategic trader are her private information. A strategic trader can use changes in the price of the risky asset to estimate the trades/ asset holding of other strategic traders. Changes in price are a function of the strategic traders' trading rate and asset holding (because of illiquidity), and changes in the risky asset's fundamental value, none of which is observable. Thus, prices are not fully revealing in our model.

We assume that there are two strategic traders. The first strategic trader is the *distressed trader* who is required to sell (or buy) a certain amount of the risky asset between time 0 and time $T > 0$. We take as exogenous both the amount of assets to be sold and the time T by which she has to sell them. Distressed liquidation does not affect the risky asset's fundamental value and can arise as a result of risk management, regulatory requirements, or margin calls.

The second strategic trader is the *potential predator* who optimally buys (or sells) the risky asset in response to the distress event. The potential predator's optimal behavior has significant implications for the economy. She can either reduce liquidity by by engaging in predatory trading or supply additional liquidity to the market at a time when it is needed.

We model the interaction among strategic traders as a differential game; that is, a continuous time game. The game takes place in the time interval $[0, T]$. Let $\widetilde{\Delta x}$ denote the amount of the risky asset that the distressed trader must sell. We assume that $\widetilde{\Delta x}$ is a normal random variable with mean μ and variance σ^2. Our first key extension of the first-stage game in Carlin et al. (2007) is that we assume that Nature picks a realization Δx of $\widetilde{\Delta x}$ at $t = 0$ and announces it to the distressed trader, but not to the potential predator. We also assume that the potential predator receives a private signal $\tilde{S}$ that can contain information about the realization Δx. Formally, we assume that

$$\tilde{S} \;=\; \widetilde{\Delta x} + \tilde{\epsilon} \tag{1}$$

where $\tilde{\epsilon}$ is a normal random variable with mean zero and variance σ_0^2 independent of $\widetilde{\Delta x}$.

Let

$$\kappa \;\equiv\; \frac{\sigma_0^2}{\sigma^2 + \sigma_0^2} \qquad \text{and} \qquad \mathrm{R}^2 \;\equiv\; 1 - \kappa.$$

R^2 is the R-squared of the regression of $\widetilde{\Delta x}$ on $\tilde{S}$. It measures the quality of the signal $\tilde{S}$ in predicting the realization of the random variable $\widetilde{\Delta x}$. The information asymmetry between the distressed trader and the potential predator in our model is captured by κ. We refer to κ as the degree (or percentage) of information asymmetry between the distressed trader and the potential predator. There is no hierarchical information structure as in Townsend (1983) because the distressed trader does not observe $\tilde{S}$ and thus no trader has strictly superior information to the other.

We denote the amounts of the risky asset held by the distressed trader and the potential predator at time t by X_t^d and X_t^ℓ respectively. Similarly, Y_t^d and Y_t^ℓ denote the rates at which strategic traders are buying or selling the risky asset at time t; that is,

$$dX_t^d \;=\; Y_t^d dt \qquad \text{and} \qquad dX_t^\ell \;=\; Y_t^\ell dt.$$

The main state variable in the economy is the price P of the risky asset. P is the only variable (other than time t) that is observed by both strategic traders. Following Carlin et al. (2007) we assume that the price evolves as

$$dP_t \;=\; dF_t \;+\; \gamma dX_t \;+\; \lambda dY_t, \tag{2}$$

where F is the fundamental value of the risky asset, and we assume that $P_0 = F_0 + \lambda Y_0$. X_t is the sum of X_t^d and X_t^ℓ, and Y_t is the sum of Y_t^d and Y_t^ℓ. Following Carlin et al. (2007) we model F as a driftless Brownian motion with constant volatility $\sigma_F^2 = 1$. It is natural to assume that the fundamental value of the risky asset cannot be (perfectly) observed by all market participants independently of its price. The theoretical literature on informed trading relies on this assumption.

The strategic trader $i \in \{d, \ell\}$ knows both X_t^i and Y_t^i. Therefore, this strategic trader can estimate the following quantity when observing price's changes:

$$dZ_t \;=\; dF_t \;+\; \gamma dX_t^{-i} \;+\; \lambda dY_t^{-i},$$

where $\{-i\} = \{d, \ell\}\backslash\{i\}$. The price reveals neither X_t^{-i} nor Y_t^{-i} to the strategic trader i because the fundamental value is not observable.

Following Carlin et al. (2007), the constants γ and λ are called the permanent price impact and the temporary price impact respectively. To understand these definitions note that P satisfies

$$P_t \;=\; F_t \;+\; \gamma(X_t - X_0) \;+\; \lambda Y_t$$

in the partial equilibrium we consider; that is, when both γ and λ are constant. The price impact λY_t is called temporary because it vanishes if the traders' aggregate trading rate is zero (that is, if $dX = Y dt = 0$). The price impact γX_t is called permanent because it persists as long as the traders aggregate holding of the risky asset is non-zero.

The distressed trader's optimization problem is a trade-off between her desire to sell slowly to reduce trading costs and her need to sell faster to reduce the adverse effects of trades by the potential predator. She solves the following problem:

$$
\begin{aligned}
\max_{Y^d} \quad & \mathrm{E}^d\left[\int_0^T -P_t Y_t^d dt\right] \\
\text{subject to} \quad & \begin{cases} dP_t = \gamma dX_t + \lambda dY_t + dF_t \\ X_0^d = 0 \\ X_T^d = \Delta x \\ dX_t^d = Y_t dt. \end{cases}
\end{aligned} \tag{3}
$$

The potential predator faces a slightly different optimization problem. Following Brunnermeier & Pedersen (2005), we do not impose the restriction that the potential predator has zero excess holding of the risky asset at the end of the game, a restriction present in Carlin et al. (2007)'s first-stage game. However, we require that the potential predator liquidate her excess holding of the risky asset within a certain period after the game. Our modeling choice is more realistic than that of Carlin et al. (2007) since the potential predator is not in distress.[8]

We assume that the potential predator starts with zero shares of the risky asset. This assumption is without loss of generality. Her excess holding at the end of the game is X_T^ℓ. We model her return from liquidating her excess holding of the risky asset at the end of game by assuming that she has the following payoff at the end of the game

$$
X_T^\ell\left(F_T + \gamma X_T^d\right) - \frac{C}{2}\gamma\left(X_T^\ell\right)^2, \tag{4}
$$

where $C > 0$. This terminal payoff is the gain/loss from optimally liquidating X_T^ℓ, the excess assets bought/sold by the potential predator during the game, over a fixed period of time following the end of the game.[9]

[8] We show that the first-stage game with restriction in Carlin et al. (2007) is a limit of the model that we consider.

[9] Suppose that the potential predator wants/needs to liquidate X_T^ℓ within a time period ΔT. Her optimal strategy is to liquidate at constant rate $X_T^\ell/\Delta T$ (see Carlin et al. (2007)). The resulting return is

$$
\frac{X_T^\ell}{\Delta T}\int_T^{T+\Delta T} P_t dt = X_T^\ell\left(F_T + \gamma X_T^d\right) - \frac{1}{2}\gamma\underbrace{\left[-1 + \frac{T}{\Delta T} + \frac{2\lambda}{\Delta T}\right]}_{C}\left(X_T^\ell\right)^2,
$$

given that $P_T = F_T + \gamma(X_T^\ell + X_T^d) + \lambda X_T^\ell$. Note that the cost function evaluated at 0 is zero. Thus the potential predator's value reduces to that considered in Carlin et al. (2007) when $X_T^\ell = 0$.

The potential predator solves the following problem:

$$\max_{Y^\ell} \quad \mathrm{E}^\ell \left[\int_0^T -P_t Y_t^\ell dt \; + \; X_T^\ell \left(F_T + \gamma X_T^d \right) - \frac{C}{2} \gamma (X_T^\ell)^2 \right]$$

$$\text{subject to} \quad \begin{cases} dP_t \; = \; \gamma dX_t \; + \; \lambda dY_t \; + \; dF_t \\ X_0^\ell \; = \; 0 \\ dX_t^\ell \; = \; Y_t^\ell dt. \end{cases} \tag{5}$$

Next we define the set of feasible strategies. Learning about the distressed trader's liquidation is important for the potential predator. Therefore, we assume that the potential predator follows a closed-loop strategy, which is a departure from the extant predatory trading literature.[10] The potential predator updates her beliefs by observing the price dynamics and using Bayes' rule. The price dynamics generate a filtration $\{\mathfrak{F}(t), 0 \leq t < T\}$. The potential predator learns about $\widetilde{\Delta x}$ through this filtration. The potential predator's time t estimate of $\widetilde{\Delta x}$ is

$$\hat{X}_t \; \equiv \; \mathrm{E}\left[\widetilde{\Delta x} | \; \mathfrak{F}(t); \; \tilde{S} \right].$$

We consider strategies of the form[11]

$$Y_t^\ell \equiv \phi^\ell(t, \, X_t^\ell, \, \hat{X}_t).$$

For simplicity and following most of the predatory trading literature, we assume that the distressed trader follows a time-dependent (open-loop) strategy:

$$Y_t^d \equiv \phi^d(t).$$

We require that both ϕ^ℓ and ϕ^d be differentiable for feasible strategies.

Definition 1. *An equilibrium is a set of feasible strategies $\{Y^d, Y^\ell\}$ such that Y^d is a solution of the optimization problem (3) given Y_t^ℓ while Y^ℓ solves the optimization problem (5) given Y_t^d.*

[10]Carlin et al. (2007) discuss the closed-loop equilibrium of their first-stage model.

[11]We also solve the game under the assumption that the potential predator follows open-loop strategies. The results are qualitatively similar.

The presence of the potential predator has an effect on the losses the distressed trader faces due to distress liquidation in an illiquid market. We study this effect by comparing the distressed trader's value from distress liquidation in the presence of the potential predator to that in its absence. We say that the potential predator engages in *predatory trading* in a given state of the world if the distressed trader loses more than she would have lost in the absence of the potential predator. Otherwise we say that the potential predator *provides liquidity.*[12]

Discussion of assumptions

Our model adopts two features shared by models in the theoretical literature concerned with the interaction between strategic traders in illiquid markets (see Kyle & Xiong (2001), Pritsker (2009), Morris & Shin (2004), Attari et al. (2005), Huberman & Stanzl (2004), Oehmke (2014), and references therein). The first feature is the presence of long-term (non-strategic) traders who have a downward sloping demand curve. The demand curve that is a function of Graham (1973)'s safety margin, that is, the difference between the asset's fundamental value and its price. The assumption that long-term investors have downward sloping demand curves is motivated by the fact that long-term investors need to be rewarded with higher returns to change their long-run equilibrium holding of the risky asset, possibly because of risk-aversion. Higher returns are achieved through lower prices. Long-term investors do not take advantage of short-term opportunities unrelated to the risky asset's fundamental value, such as asset fire sales. They provide liquidity to the market by buying (selling) the risky asset when its price is below (above) its fundamental value. Examples of long-term investors include retail investors. Kaniel et al. (2008) find empirical evidence that individual investors provide liquidity to strategic traders which enables the latter to trade more frequently. Shleifer (1986), Wurgler & Zhuravskaya (2002), and Krishnamurthy & Vissing-Jorgensen (2012) find empirical support for the assumption of downward sloping demand curves.

The second feature is that the changes in the price of risky asset are a linear function of

[12]This definition is consistent with that of Brunnermeier & Pedersen (2005), who define predatory trading as "trading that induces and/or exploits the need of other investors to reduce their positions". We considered the following alternative definitions of predatory trading: (1) A potential predator *predates* if the excess holding of the risky asset by the potential predator at time T is negative, that is, if $X_T^{\ell} < 0$. (2) A potential predator *predates* if the aggregate amount of time she trades in the same direction as the distressed trader during the game is greater than $T/2$. Our results did not change qualitatively under these alternative definitions of predatory trading/providing liquidity.

the changes in the asset's fundamental value, the strategic traders' aggregate order flow, and the aggregate changes in their order flow. These price dynamics reflect the notion that changes in price are due to both changes in the asset's fundamental value and market frictions. That is, the risky asset is *illiquid* and trading by strategic traders is associated with a price impact. The price impact has two components: a long-term component related to the aggregate holding of the risky asset by the strategic traders and a short-term component related to the strategic traders' aggregate order flow. Both Kyle (1985) and Pritsker (2009) present models that endogenously generate permanent price impacts. Madhavan & Cheng (1997), Glosten & Harris (1988), and Sadka (2006) find empirical evidence supporting the assumption that large trades have distinct permanent and temporary price impacts.

We relax Carlin et al. (2007)'s assumption that the potential predator finishes the game with zero excess liquidity. The potential predator is not in distress, thus it is more realistic to allow her to choose her excess holding of the risky asset at the end of the game. This modeling choice is made by both Brunnermeier & Pedersen (2005) and Schöneborn & Schied (2007).[13] We assume that the potential predator liquidate her excess holding of the risky asset within a certain period after the game. The potential predator deviates from her long-run holding of the risky asset to take advantage of distress liquidation so it natural to assume that she eventually returns to her initial holding of the risky asset. The time by which the potential predator returns to her initial holding of the risky asset, which is $T + \Delta T$ in our model, determines the constant C. In practice, this time depends on several factors (e.g.: Regulation, disclosure of information about the firm, etc).[14] We obtain our theoretical results for arbitrary but finite C. Our main numerical results are derived under the assumption that

$$C \equiv 1.$$

We also consider the equilibrium when

$$C \rightarrow \infty \qquad \Longleftrightarrow \qquad \Delta T \rightarrow 0$$

[13]The time at which the distressed trader completes liquidation in endogenously determined in Brunnermeier & Pedersen (2005). However, this time is known at the start of the game because they assume perfect information.

[14]We implicitly assume that $\Delta T > 0$.

in the Appendix. The potential predator does not have time to liquidate her excess holding of the risky asset at the end of the game in the later limit. We show that she has zero excess holding of the risky asset almost surely in this case, which coincides with the first-stage game of Carlin et al. (2007).[15] The results are qualitaively similar in both cases.

2. Equilibrium

Equilibria in the game are determined by trade-offs faced by the players. The distressed trader faces a trade-off between two forces. Trading costs lead her to try to liquidate the risky asset at a slow rate. On the other hand, trades by the potential predator have a price impact. This impact reduces the distressed trader's value when the potential predator is trading in the same direction as the distressed trader, ceteris paribus. Thus, the distressed trader to sell at a higher rate in response to the potential trader racing to the market.

The potential predator generates profits from the distressed trader's price impact by selling high and buying low. She can first race the distressed trader to the market and sell the risky asset when the price is high. We call this strategy *sell-first*. For this strategy to be profitable, she needs trades by the distressed trader to have permanent price impact so she can buy the risky asset at a lower price later on. Alternatively, the potential predator can first buy the risky asset. We call this strategy *buy-first*. This strategy is profitable if the potential predator can sell the risky asset at a higher price later on, which can occur when prices recover following the distressed trader's exit from the market. The first strategy is associated with faster trading by the potential predator because of the need to race to the market. This difference in trading speed affects the choice between the two strategies when trading costs, captured by the temporary price impact, are non-zeros.

In the presence of asymmetric information, the potential predator can incur losses in some states of the world because she can sell too much or too little of the risky asset due to the fact that she does not know the liquidation size. These losses depend on the rate at which the potential predator is trading. The potential predator's value is non-linear in her estimate of the liquidation size and thus the potential predator is not risk-neutral with respect to uncertainty about the liquidation

[15]Either one of the following two conditions can lead the potential predator to find it optimal to have non-zero excess holding risky asset even if she cannot liquidate at the end of the game under two conditions: (1) Information about the fundamental value of the risky asset is released during the game; (2) The potential predator's initial holding of the risky asset was sub-optimal. We rule out both of these possibilities.

size. That is, the degree of information asymmetry matters. As a result, information asymmetry and learning are important in determining the equilibrium in our model and affect the probability that the potential predator chooses to provide liquidity.

We start by solving for the equilibrium in the special case of no permanent price impact, a case where we can solve the game in closed-form.

2.1. No permanent price impact case

The following proposition characterizes the equilibrium in the absence of a permanent price impact.[16] The closed-form solutions will help build the intuition for the general case we present in the next subsection.

Proposition 1. *Suppose that there is no permanent price impact, that is* $\gamma = 0$. *Then there exists a unique equilibrium* (Y^d, Y^ℓ) *with*

$$\begin{aligned} Y_t^\ell &= -\frac{1}{2T}\hat{X}_t = -\frac{1}{2T}\left[\mu + (1-\kappa)(\tilde{S}-\mu)\right] \\ Y_t^d &= \frac{1}{T}\Delta x. \end{aligned}$$

The distressed trader's equilibrium expected value is

$$V^d = V^{0,d} + \frac{\lambda}{2T}\left[(1-\kappa)\sigma^2 + \mu^2\right], \tag{6}$$

where $V^{0,d}$ *is the value obtained by the distressed trader in the absence of the potential predator. The probability that the potential predator provides liquidity in equilibrium is*

$$\begin{aligned} \Pr = &\int_0^\infty \Phi\left(\frac{1}{\sigma}\left[\sqrt{\frac{\kappa}{1-\kappa}}\mu + \sqrt{\frac{1-\kappa}{\kappa}}x\right]\right)\phi\left(\frac{x-\mu}{\sigma}\right)dx \\ &+ \int_{-\infty}^0 \Phi\left(-\frac{1}{\sigma}\left[\sqrt{\frac{\kappa}{1-\kappa}}\mu + \sqrt{\frac{1-\kappa}{\kappa}}x\right]\right)\phi\left(\frac{x-\mu}{\sigma}\right)dx \end{aligned}$$

[16] We consider the problem where $\gamma = 0$ but the cost function is of the form

$$X_T^\ell F_T - \frac{C}{2}\lambda\left(X_T^\ell\right)^2,$$

Numerical solutions are qualitatively similar to the proposition.

where Φ (ϕ) is the cumulative distribution (probability density) function of the standard normal distribution.

Proof. See Appendix Appendix A. $\square$

The results in Proposition 1 are consistent with the empirical evidence in Bessembinder et al. (2014). They find that potential predators provide liquidity when markets are resilient. They define resilient markets as markets where "some or all of the immediate price impact of trades is subsequently reversed". Resilient markets can be viewed as markets with negligible permanent price impacts.

Proposition 1 also provides a new rationale for "sunshine trading", the practice of pre-announcing order size. It shows that it is optimal for the distressed trader to reduce information asymmetry about the liquidation size when the permanent price impact is negligible (see Equation 6). The mechanism is in our paper is complementary to that in Admati & Pfleiderer (1991).[17]

Liquidation by the distressed trader has no permanent price impact when $\gamma = 0$, which implies that prices recover quickly after trades by the distressed trader. Thus, a sell-first strategy is not profitable. In equilibrium, the potential predator follows a buy-first strategy based on her estimate of the liquidation size. The buying reduces the magnitude of the aggregate price impact of strategic traders ($\lambda|\Delta x - \tilde{X}/2|/T$) relative to the case without the potential predator ($\lambda|\Delta x|/T$). Therefore, the price at which the distressed trader liquidates the risky asset ($P_t = F_t + \lambda(\Delta x - \tilde{X}_t/2)/T$) is higher on average relative to the case without the potential predator ($P_t = F_t + \lambda \Delta x/T$). The higher price means lower losses from distress liquidation for the distressed trader. Hence, in expectation, the potential predator provides liquidity when $\gamma = 0$.

The proposition highlights that a key force determining the occurrence of predatory trading is whether or not trading by the potential predator amplifies the distressed trader's price impact. We define the notion of *gap* to study this key force. A quantity's *gap* is the difference between the quantity's value when the potential predator is present in the market from the value when the

[17] Admati & Pfleiderer (1991) argue that sunshine trading improves liquidity by reducing adverse selection, which lead to higher value for the distressed trader.

potential predator is absent from the market. In the case $\gamma = 0$, the strategic traders' aggregate trading rate gap is $\Delta x/T - \hat{X}_t/(2T) - \Delta x/T = -\hat{X}_t/(2T)$. This gap is positive on average. As a result, the price gap is positive on average because it is linear and increasing in the strategic traders' aggregate trading rate gap. That is, the distressed trader sells the risky asset at a higher price in the presence of the potential predator. Hence, the distressed trader's expected value gap is positive since her trading rate gap is zero.

In general, the price gap is a linear function of both the strategic traders' aggregate holding and trading rate gaps. We shall study how information asymmetry and market illiquidity affect these gaps to understand how they impact the choice between predatory trading and providing liquidity in the general case.

The potential predator's presence in the market can reduce the distressed trader's value through two channels: (1) A direct channel which occurs when the potential predator attempts to trade in the same direction as the distressed trader for the majority of the game and is successful in doing so. (2) An indirect channel which occurs when the potential predator attempts to trade in the opposite direction as the distressed trader for the majority of the game but fails to do so because her estimate of the liquidation size has the opposite sign as the true realization.[18] The direct channel is absent when $\gamma = 0$. We shall see that the direct channel is present and dominates in the general case for higher values of γ.

2.2. General case

We characterize the equilibrium strategies in terms of a system of differential equations (A.34)—(A.40) provided in Appendix Appendix A. The proof of the following is in Appendix Appendix A.

Theorem 1. *Given a set of time-dependent functions $(c_1, c_2, c_3, a_1, a_2)$ satisfying the system of first-order differential equations (A.34)—(A.40) in Appendix Appendix A, a linear equilibrium*

[18]The indirect channel is possible in equilibrium because we model the liquidation size as a normal random variable, allowing for both positive and negative realizations of $\widetilde{\Delta x}$ for any choice of μ and $\sigma \neq 0$. However, the probability of the indirect channel occurring in equilibrium in our model is negligible for realistic families of parameters.

(Y^d, Y^ℓ) is defined by

$$
\begin{aligned}
Y_t^\ell &= c_1(t) X_t^\ell + c_2(t) \hat{X}_t + c_3(t) \\
Y_t^d &= a_1(t) + a_2(t) \Delta x.
\end{aligned}
$$

In this equilibrium, the amount of the risky asset held by the potential predator at time t is

$$
X_t^\ell = \int_0^t \frac{c_1(s)}{c_1(t)} \Big(c_2(s) \hat{X}(s) + c_3(s) \Big)\, ds. \tag{7}
$$

Moreover, the uncertainty faced by the potential predator about the realization of the liquidation size $\widetilde{\Delta x}$ is a decreasing function of time.

We solve the system of first-order differential equations characterizing c_2, c_3, a_1 and a_2 numerically. See Appendix Appendix B for details. We obtain the closed-form solution for c_1:

$$
c_1(t) = -\frac{\rho(C+1)}{2 + \rho(C+1)(T-t)} \qquad \text{where} \qquad \rho = \frac{\gamma}{\lambda}.
$$

c_1 is negative and a decreasing function of t. Therefore, the potential predator reduces her excess holding of the risky asset toward the end of the game, *ceteris paribus* (assuming that $\gamma \neq 0$). The potential predator reduces her excess holding of the risky asset to zero when she has no time to liquidate at the end of the game:

Corollary 1. *The potential predator has zero excess holding of the risky asset almost surely when she has no time to liquidate the risky asset at the end of the game. That is,*

$$
X_T^\ell = 0 \text{ a.a.} \qquad \textit{in the limit} \qquad \Delta T \to 0.
$$

The corollary shows that Carlin et al. (2007)'s requirement that the potential predator holds zero excess return is a limit of the game we consider. The term c_1 satisfies

$$
\lim_{C \to \infty} c_1(t) = -\frac{1}{T-t}.
$$

Thus,

$$dX_t^\ell \;=\; -\frac{1}{T-t}X_t^\ell + c_2(t)\hat{X}_t + c_3(t)$$

which implies that X_t^ℓ is a Brownian bridge.

We fix the value of the constant C in the cost function (see Equation (4))

$$C \;\equiv\; 1$$

for the remainder of the paper.

2.3. Properties of the Equilibrium

We examine how information asymmetry and market illiquidity affect the linear equilibrium. The trade-offs faced by the traders determine the equilibrium and the occurrence of either liquidity provision or predatory trading.

The distressed trader liquidates faster in response to the potential predator racing to the market. Thus, both traders trade faster and in the same direction on average in equilibria where the potential predator chooses a sell-first strategy characterized by racing to the market for a long period. In such case, the strategic traders' aggregate trading rate gap is negative on average, which leads to a negative aggregate holding gap and thus a negative price gap on average. A lower price gap means that the distressed trader liquidates the asset at a lower price. Thus, the distressed trader's value is lower when the price gap is lower. Therefore, predatory trading occurs when the potential predator's presence results in a negative price gap on average.

We illustrate the linear equilibrium and the effects of information asymmetry in Figure 1. We simulate 100×100 equilibrium paths of the game for 100 realizations of the liquidation size $\widetilde{\Delta x}$ and 100 paths of the risky asset's fundamental value for each of two values of the degree of information asymmetry. We plot the *average* equilibrium strategies for both the distressed trader (Fig 1 (a)) and the potential predator (Fig 1 (b)). We also plot the strategic traders' aggregate holding gap of the risky asset (Fig 1 (c)) and the price gap (Fig 1 (d)). Finally, we plot the dynamics of both the distressed trader's expected value gap (Fig 1 (e)) and the potential predator's expected value gap (Fig 1 (f)).

Figure 1 (e) indicates that the distressed trader's expected value is higher with a higher degree of information asymmetry. It follows that liquidity provision is more likely to occur with a higher degree of information asymmetry. When information asymmetry is lower, the potential predator faces lower losses from estimation errors. Thus, she can follow a more aggressive sell-first strategy, which results in a lower price gap (see Fig 1 (d)) through faster trading on average by both traders (see Fig 1 (a) and (b)) and a lower aggregate holding gap (see Fig 1 (c)).

[Insert Figure 1 here]

Figure 2 (e) shows that predatory trading occurs in markets with high permanent price impacts and liquidity provision occurs in markets with low permanent price impacts. The intuition is as follows. A higher permanent price impact increases the profits to the potential predator of racing to the market. As a result, the potential predator chooses a more aggressive self-first strategy which leads to a negative price gap (see Fig 2 (d)) through faster trading on average by both traders (see Fig 2 (a) and (b)) and a negative aggregate holding gap (see Fig 2 (c)). The case with low permanent price impact follows from a similar argument.

[Insert Figure 2 here]

We now examine the learning dynamics in the equilibrium. The potential predator learns about the liquidation size by observing fluctuations in the price of the risky asset. Her estimate of the liquidation size is

$$\hat{X}_t \;\equiv\; \mathrm{E}\left[\widetilde{\Delta x}|\; \mathfrak{F}(t);\; \tilde{S}_1\right].$$

The degree of uncertainty about the liquidation size is characterized by the variance of the random variable $\hat{X}_t$, denoted $\Omega(t)$. A sufficient statistic for learning in our model is the percentage of uncertainty left at time t, which we denote $\delta(t)$:

$$\delta(t) \;\equiv\; \frac{\Omega(t)}{\Omega(0)} \;=\; \frac{\Omega(t)}{\kappa\sigma^2}.$$

Learning by the potential predator is a function of changes in the price of the risky asset resulting from the distressed trader's action. Therefore, it follows from Equation (2) that learning is driven by two forces in our model. The first is the rate (and acceleration) at which the distressed trader trades. The second is the set of price impacts. Learning in equilibrium depends on how these two forces interact.

Figure 3 (a) shows that learning is faster when the permanent price impact is higher. The reason is two-fold. First, a higher permanent price impact means that changes in price are more informative about changes in the distressed trader's holding of the risky asset. Second, a higher permanent price impact is associated with greater changes in the rate at which the distressed trader trades in equilibrium (see Figure 2 (a)). These two forces combine to improve learning when there is a higher permanent price impact.

Figure 3 (b) illustrates that the effect of the temporary price impact on learning is ambiguous. The two forces driving learning can work in opposite directions in equilibrium when varying the temporary price impact. Increasing the temporary price impact increases the learning rate, ceteris paribus. However, in equilibrium, both traders trade less aggressively in markets with higher temporary price impacts. A lower trading rate by the distressed trader decreases the rate at which the potential predator learns about the liquidation size. This effect can dominate the positive direct effect of a higher temporary price impact on learning. Therefore, in equilibrium, learning can be slower at some point in time in markets with higher temporary price impacts.

[Insert Figure 3 here]

2.4. Predatory trading versus liquidity provision

The previous subsection studied the distressed trader's value, which is an expectation. Here, we study the probability of predatory trading occurring in equilibrium.

We estimate the probability that the potential predator will predate for several sets of values of the permanent price impact, the temporary price impact, and the degree of information asymmetry and report the results in Table 1.

Table 1 shows that predatory trading occurs with certainty for higher (lower) values of the permanent (temporary) price impact when the degree of information asymmetry is zero. This result is consistent with the findings of Brunnermeier & Pedersen (2005), and Schöneborn & Schied (2007). We define *predation markets* as markets where predatory trading occurs with certainty when there is no information asymmetry.[19] We shall focus our discussions on predation markets.[20]

Table 1 indicates that the probability of predatory trading occurring decreases as the degree of information asymmetry increases in predation markets. This result is intuitive. Increasing the degree of information asymmetry decreases the marginal value of racing to the market by increasing the expected losses to the potential predator due to estimations errors. Less aggressive racing to the market (or no racing in the case of the buy-first strategy) increases the likelihood of liquidity provision.

We also observe from Table 1 that there is a positive relation between the probability of predatory trading occurring and the permanent price impact in predation markets. Increasing the permanent price impact improves learning and increases the profits from racing to the market. Both effects combine to increase the marginal value of racing to the market in equilibrium.

[Insert Table 1 here]

2.5. Welfare

We examine the effects of uncertainty about the amount of the asset to be liquidated on the strategic traders' aggregate and individual values.

In the partial equilibrium that we consider, the permanent price impact is a transfer of wealth from the strategic traders to the long-term investors. This transfer of wealth is a function of the aggregate change in holding of the risky asset by the strategic traders. The transfer occurs because long-term investors have a downward sloping demand curve which is characterized by the permanent price impact in the price function. The downward sloping nature of the demand curve represents the compensation required by long-term investors when strategic traders change their aggregate positions, walking up or down the demand curve. Long-term investors require this compensation because they are risk-averse.

The temporary price impact is a deadweight cost for the strategic traders' collective trading. This deadweight cost is due to trading costs such as inventory costs, search costs, bid-ask spread,

[19] Not all markets are predation markets in our model (see Proposition 1 and the first rows of Table 1 (a) and (b)).

[20] In reality, information asymmetry is mainly relevant to predatory trading in the context of predation markets: The distressed trader will reduce information asymmetry in non-predation markets where predatory trading would otherwise occur. This point is consistent with Proposition 1 and the presence of sunshine trading in financial markets.

clearing fees, etc.

We will focus on the welfare gains/losses to the strategic traders due to the presence of the potential predator. We use simulated equilibrium paths to compute each trader's equilibrium expected value. Table 2 presents the strategic traders' aggregate value as a percentage change relative to the case without the potential predator. It also contains the potential predator expected value and the distressed trader expected value as a percentage change over her value in the absence of the potential predator.

Relating Table 2 to Table 1, we observe that the percentage change in the strategic traders aggregate expected value is negatively related to the probability of predatory trading occurring in predation markets. The strategic traders trade in the same direction and more aggressively on average when predatory trading occurs. As a result, there is a larger transfer of wealth from the strategic traders to the long-term investors and a larger deadweight loss in trading costs when predatory trading occurs. The potential predator's gains from predation are lower that the distressed trader losses. Hence, information asymmetry improves the strategic traders' aggregate welfare in predation markets.

Table 2 shows that the potential predator's value decreases as the degree of information asymmetry increases in predation markets. This result is consistent with our argument that the presence of information asymmetry can lead to losses due to estimation errors. We explore the mechanism yielding this relation. The value achieved by the potential predator is:

$$\mathrm{E}^{\ell}\left\{-\int_0^T \underbrace{\left[F_t+\gamma(X_t^d+X_t^{\ell})+\lambda(Y_t^d+Y_t^{\ell})\right]}_{P_t} Y_t^{\ell}dt \;+\; X_T^{\ell}\left(F_T+\gamma X_T^d\right) \;-\; \frac{C}{2}\gamma(X_T^{\ell})^2\right\}$$

Assume that both players play linear strategies on the form given in Theorem 1 (we do not make this assumption when deriving Theorem 1). We can use conditional expectation to write this value as

$$\int_0^T -\mathrm{E}^{\ell}\Big\{\Big[\Big(F_t+\gamma\left[\bar{a}_1(t)+\bar{a}_2(t)\hat{X}_t+X_t^{\ell}\right]+\lambda\left[a_1(t)+(a_2(t)+c_2(t))\hat{X}_t+c_1(t)X_t^{\ell}+c_3(t)\right]\Big)$$
$$\times\Big(c_1(t)X_t^{\ell}+c_2(t)\hat{X}_t+c_3(t)\Big)\Big]dt \;+\; X_T^{\ell}\Big(F_T+\gamma\left[\bar{a}_1(T)+\bar{a}_2(T)\right]\hat{X}_T\Big) \;-\; \frac{C}{2}\gamma(X_T^{\ell})^2\Big\}$$

where

$$\bar{a}_i(t) = \int_0^t a_i(s)ds, \quad i \in \{1, 2\}.$$

The source of uncertainty is $\hat{X}_t$, potential predator's estimate of the liquidation size. Focusing on powers of $\hat{X}_t$, the expression inside the expectation is quadratic in $\hat{X}_t$, and the coefficient associated with $\hat{X}_t^2$ is

$$-\left[\gamma \bar{a}_2(t) + \lambda(a_2(t) + c_2(t))\right] c_2(t).$$

Assuming that a_2 (and thus, $\bar{a}_2$) is positive (our simulations indicate that this holds in equilibrium), the expression above is negative if $c_2(t)$ is positive. That is, the coefficient associated with $\hat{X}_t^2$ is negative when the potential predator attempts to trade in the same direction as the distressed trader. An increase in the degree of information asymmetry increases the variance of $\hat{X}_t$ without changing its mean. Thus, by Jensen's inequality, an increase in uncertainty about the liquidation size decreases the potential predator's value when she races the distressed trader to the market.

This heuristic argument implies that, in terms of her payoff and equilibrium strategy, the potential predator is averse to uncertainty about the liquidation size when engaging in predatory trading.

We observe from Table 2 that there is a positive association between the potential predator's value and the permanent price impact. This relation is driven by the positive association between the permanent price impact and the probability of predatory trading occurring. The relation is consistent with the empirical evidence of both Shive & Yun (2013) and Arif et al. (2014). They find that potential predators earn higher profits when trading in less liquid assets.

[Insert Table 2 here]

3. Conclusion

We characterized the effect of asymmetric information on a strategic investor's decision to either provide liquidity or engage in predatory trading when another strategic trader is in distress. The potential predator estimates the liquidation size, that is the amount of assets to be liquidated by the distressed trader, by observing price dynamics. She can either provide liquidity or engage in predatory trading. We define providing liquidity as increasing the value achieved by the distressed trader (relative to the case without the potential predator).

There is a unique equilibrium in the absence of a permanent price impact, that this equilibrium is linear, and we provide the equilibrium strategies in closed-form. In equilibrium, the potential predator always trades in the direction opposite to her estimate of the liquidation size. That is, she attempts to provide liquidity. This result is consistent with empirical evidence.

We provide conditions under which a linear equilibrium exists without parameter restrictions. We find that predatory trading always occurs in the absence of information asymmetry in markets with large permanent price impact. We call these markets predations markets.

Our main finding is that introducing information asymmetry in predation markets increases the probability that the potential predator will provide liquidity. The mechanism driving this result is the fact that the potential predator is adverse to uncertainty about the liquidation size when engaging in predatory trading. We observe that information asymmetry reduces the potential predator's value. This result is consistent with the existing empirical evidence that more frequent disclosure by mutual funds is associated with returns from "front-running" mutual funds.

Overall, our results show that, in the presence of information asymmetry, potential predators are more likely to stabilize markets by providing liquidity. Therefore information asymmetry can explain the observed episodic illiquidity in financial markets.

The potential predator's choice is also a function of market liquidity, represented by both the permanent price impact and the temporary price impact. Our numerical simulations show that the potential predator is more likely to provide liquidity in markets with low (resp. high) permanent (resp. temporary) price impact. The potential predator also achieves higher value for higher permanent price impact, consistent with empirical findings.

This paper highlighted some benefits to having information asymmetry in financial markets. These benefits are relevant when evaluating (recent) policies/regulations requiring more transparency for institutional investors. Understanding the role of information asymmetry before a crisis such as distress liquidation occurs remains an open question.

Appendix A. Equilibrium

We prove Theorem 1 by characterizing the set of best-response strategies for each player and then the equilibrium strategies.

Appendix A.1. Potential predator best-response

We first assume that the distressed trader follows a linear strategy of the form:

$$Y^d(t) \;\; = \;\; a_1(t) \; + \; a_2(t)\Delta x \tag{A.1}$$

where a_1 and a_2 are continuously differentiable. Let

$$\bar{a}_1(t) \;=\; \gamma \int_0^t a_1(s)ds + \lambda a_1(t); \qquad \bar{a}_2(t) \;=\; \gamma \int_0^t a_2(s)ds + \lambda a_2(t).$$

The state variables relevant to the potential predator's optimization problem are the price P, her asset holding X^ℓ, and her estimate of $\widetilde{\Delta x}$ which we denote $\hat{X}$. The price component providing additional information to the potential predator is the variable Z defined as

$$Z_t \;\; \equiv \;\; \gamma X_t^d + \lambda Y_t^d + F_t. \tag{A.2}$$

The informative component of price (to the potential predator) generates a filtration $\{\mathfrak{F}(t), 0 \leq t < T\}$. The potential predator learns about $\widetilde{\Delta x}$ as follows:

Lemma 2. *Suppose that the distressed trader follows a strategy of the form given in Equation (A.1). Then the time t estimate of $\widetilde{\Delta x}$, denoted*

$$\hat{X}_t \;=\; \mathrm{E}\left[\widetilde{\Delta x} | \; \mathfrak{F}(t); \tilde{S}_1\right],$$

is

$$\hat{X}_t \;\; = \;\; \hat{X}_0 + \int_0^t \sigma_\ell(t) dW(u)$$

where

$$\hat{X}_0 = \mu + (1-\kappa)(\tilde{S}-\mu); \qquad \Omega(t) = \left[\int_0^t (\bar{a}_2'(u))^2 du + \frac{1}{\kappa\sigma^2}\right]^{-1}; \tag{A.3}$$

$$\sigma_\ell(t) = \bar{a}_2'(t)\Omega(t); \qquad dW = dB + \bar{a}_2'(\Delta x - \hat{X})dt. \tag{A.4}$$

Proof. The proof follows from applying the Kalman Bucy filter and basic conditional expectation formulas for multivariate normal random variables. □

We now study the dynamics of $\hat{X}_t$. It follows from (A.3) and (A.4) that

$$\begin{aligned} \Omega(t)' &= -(\bar{a}_2'(t))^2\Omega(t)^2 \\ &= -\sigma_\ell(t)\bar{a}_2'(t)\Omega(t) \\ \Rightarrow \int_0^t \sigma_\ell(s)\bar{a}_2'(s)ds &= -\int_0^t \frac{\Omega(t)'}{\Omega(t)}ds \\ &= -\ln\frac{\Omega(t)}{\Omega(0)}. \end{aligned} \tag{A.5}$$

Let

$$\delta(t) = \frac{\Omega(t)}{\Omega(0)} \quad \Rightarrow \quad \left(\frac{1}{\delta(t)}\right)' = \Omega(0)\left(\bar{a}_2'(t)\right)^2. \tag{A.6}$$

$\delta(t)$ is the percentage of the initial variance remaining at time t. We shall refer to $\delta(t)$ as the percentage of uncertainty left at time t. Lemma 5 implies that the variable $\hat{X}_t$ satisfies

$$d\hat{X}_t = \sigma_\ell(t)\bar{a}_2'(t)(\Delta x - \hat{X}_t)dt + \sigma_\ell(t)dB.$$

This implies that

$$\begin{aligned} d\left(\exp\left[\int_0^t \sigma_\ell(s)\bar{a}_2'(s)ds\right]\hat{X}_t\right) &= \left[\sigma_\ell(t)\bar{a}_2'(t)\hat{X}_t dt + d\hat{X}_t\right]\exp\left[\int_0^t \sigma_\ell(s)\bar{a}_2'(s)ds\right] \\ &= \left[\sigma_\ell(t)\bar{a}_2'(t)\Delta x dt + \sigma_\ell(t)dB\right]\exp\left[\int_0^t \sigma_\ell(s)\bar{a}_2'(s)ds\right]. \end{aligned}$$

Therefore,

$$\begin{aligned}\frac{1}{\delta(t)}\hat{X}_t - \hat{X}_0 &= \int_0^t \left\{ \left[\sigma_\ell(u)\bar{a}_2'(u)\Delta x du + \sigma_\ell(u) dB_u\right] \frac{1}{\delta(u)} \right\} \\ &= \Delta x \int_0^t \Omega(0)(\bar{a}_2'(u))^2 du + \Omega(0) \int_0^t \bar{a}_2'(u) dB_u.\end{aligned}$$

Hence,

$$\hat{X}_t = \Delta x + \left[\hat{X}_0 - \Delta x\right] \delta(t) + \Omega(t) \int_0^t \bar{a}_2'(u) dB(u). \tag{A.7}$$

Next we turn our attention to the potential predator's optimization problem. Let J denote the potential predator's value. J is a function $(Z, X_t^\ell, \hat{X}_t, t)$. Given the state variables dynamics, the HJB equation associated with the potential predator's optimization problem is

$$\begin{aligned}&\max_Y \left\{ \left[J_X - Z - \gamma X\right] Y - \lambda Y^2 \right\} \\ &\qquad + J_t + \left(\bar{a}_1' + \bar{a}_2'\hat{X}\right) J_Z + \frac{1}{2} J_{ZZ} + \frac{1}{2}\sigma_{\hat{X}}^2 J_{\hat{X}\hat{X}} + \sigma_{\hat{X}} J_{Z\hat{X}} = 0.\end{aligned} \tag{A.8}$$

The optimal strategy is then

$$Y^* = \frac{1}{2\lambda}\left[J_X - Z - \gamma X\right]. \tag{A.9}$$

Equation (A.8) becomes

$$0 = J_t + \left(\bar{a}_1' + \bar{a}_2'\hat{X}\right) J_Z + \frac{1}{2} J_{ZZ} + \frac{1}{2}\sigma_{\hat{X}}^2 J_{\hat{X}\hat{X}} + \sigma_{\hat{X}} J_{Z\hat{X}} + \frac{1}{4\lambda}\left[J_X - Z - \gamma X\right]^2. \tag{A.10}$$

We conjecture a solution of the form:

$$\begin{aligned}J(t, Z, X, \hat{X}) &= b_1(t)Z^2 + b_2(t)X^2 + b_3(t)\hat{X}^2 + b_4(t)ZX + b_5(t)Z\hat{X} + b_6(t)X\hat{X} + b_7(t)Z \\ &\qquad + b_8(t)X + b_9(t)\hat{X} + b_{10}(t).\end{aligned} \tag{A.11}$$

The terminal value of the optimization problem implies the following terminal values for $b_i, i =$

$1, \cdots 10:$

$$b_1(T) = 0; \qquad b_2(T) = -\frac{C}{2}\gamma; \qquad b_3(T) = 0; \qquad b_4(T) = 1; \qquad b_5(T) = 0;$$
$$b_6(T) = -\lambda a_2(T); \qquad b_7(T) = 0; \qquad b_8(T) = -\lambda a_1(T); \qquad b_9(T) = 0; \qquad b_{10}(T) = 0.$$

Define the liquidity ratio as

$$\rho = \frac{\gamma}{\lambda}.$$

Plugging (A.11) into Equation (A.10) we obtain the following system of equations:

$$b_1' + \frac{1}{4\lambda}(b_4 - 1)^2 = 0. \tag{A.12}$$
$$b_2' + \frac{1}{4\lambda}(2b_2 - \gamma)^2 = 0. \tag{A.13}$$
$$b_3' + \bar{a}_2' b_5 + \frac{1}{4\lambda} b_6^2 = 0. \tag{A.14}$$
$$b_4' + \frac{1}{2\lambda}(2b_2 - \gamma)(b_4 - 1) = 0. \tag{A.15}$$
$$b_5' + 2\bar{a}_2' b_1 + \frac{1}{2\lambda} b_6(b_4 - 1) = 0. \tag{A.16}$$
$$b_6' + \bar{a}_2' b_4 + \frac{1}{2\lambda} b_6(2b_2 - \gamma) = 0. \tag{A.17}$$
$$b_7' + 2\bar{a}_1' b_1 + \frac{1}{2\lambda} b_8(b_4 - 1) = 0. \tag{A.18}$$
$$b_8' + \bar{a}_1' b_4 + \frac{1}{2\lambda} b_8(2b_2 - \gamma) = 0. \tag{A.19}$$
$$b_9' + \bar{a}_1' b_5 + \bar{a}_2' b_7 + \frac{1}{2\lambda} b_8 b_6 = 0. \tag{A.20}$$
$$b_{10}' + \bar{a}_1' b_7 + b_1 + \sigma_{\hat{X}}^2 b_3 + \sigma_{\hat{X}} b_5 + \frac{1}{4\lambda} b_8^2 = 0. \tag{A.21}$$

The general solutions to equations (A.12), (A.13), and (A.15) are

$$b_2(t) = \frac{1}{2}\gamma \left[1 - \frac{2(C+1)}{2 + \rho(C+1)(T-t)}\right] \tag{A.22}$$
$$b_4(t) = 1 \tag{A.23}$$
$$b_1(t) = 0. \tag{A.24}$$

Substituting these into the previous system we get that

$$
\begin{aligned}
b_1(t) &= 0 \\
b_3' + \frac{1}{4\lambda} b_6^2 &= 0. \\
b_5(t) &= 0. \\
b_7(t) &= 0 \\
b_9' + \frac{1}{2\lambda} b_8 b_6 &= 0. \\
b_{10}' + \sigma_{\hat{X}}^2 b_3 + \frac{1}{4\lambda} b_8^2 &= 0.
\end{aligned}
$$

Therefore we obtain the optimal strategy Y^* once we solve for b_6 and b_8. Equations (A.17) and (A.19) have the same homogeneous solution:

$$
-\frac{(C+1)}{1+\rho(C+1)(T-t)}.
$$

It is straightforward to obtain the homogeneous solutions to the remaining equations. The existence and uniqueness results for the equations follow from the assumption that a_1 and a_2 are continuously differentiable.

The existence of a solution to the HJB equation implies the existence of a unique best response strategy. It follows from Equation (A.9) that the unique best response strategy is the linear strategy

$$
\begin{aligned}
Y^* &= \frac{1}{2\lambda}\left[(2a_2(t)-\gamma)X + a_6(t)\hat{X} + a_8(t)\right] \\
&= -\frac{(C+1)\rho}{2+\rho(C+1)(T-t)} X + \frac{1}{2\lambda}\left[a_6(t)\hat{X} + a_8(t)\right]. \qquad \text{(A.25)}
\end{aligned}
$$

Appendix A.2. Distressed trader best-response

Assume that the potential predator follows a linear strategy

$$
Y^\ell(t, Z, X^\ell, \hat{X}) = c_1(t)X^\ell + c_2(t)\hat{X} + c_3(t)
$$

where c_1, c_2, and c_3 are continuously differentiable. Then X^ℓ evolves as

$$dX^\ell \;=\; Y^\ell dt \;=\; \left[\left(c_2\hat{X} + c_3\right) + c_1 X^\ell\right] dt.$$

Therefore,

$$\begin{aligned} X_t^\ell &= A(t)\int_0^t A(-s)\left[c_2(s)\hat{X}_s + c_3(s)\right] ds \\ \Rightarrow \mathrm{E}^d[X_t^\ell] &= A(t)\int_0^t A(-s)\left[c_2(s)B(s) + c_3(s)\right] ds, \end{aligned}$$

where

$$A(t) \;=\; \exp\left[\mathrm{Sign}(t)\int_0^{|t|} c_1(s)ds\right] \qquad \text{and} \qquad B(t) \;=\; \mathrm{E}^d[\hat{X}_t].$$

Equation (A.7) and standard Normal-Normal updating results imply that

$$B(t) \;=\; \left[1 - \kappa\delta(t)\right]\Delta x + \mu\kappa\delta(t). \tag{A.26}$$

We now consider the distressed trader's optimization problem. Recall that

$$\begin{aligned} P(t) &= U + \gamma(X_t^d + X_t^\ell) + \lambda(Y_t^d + Y_t^\ell) \\ &= U + \gamma X_t^d + \lambda Y_t^d + (\gamma + \lambda c_1(t))X_t^\ell + \lambda c_2(t)\hat{X}_t + \lambda c_3(t). \end{aligned}$$

We can rewrite the optimization problem as

$$\begin{aligned} &\max_{Y\in\mathcal{Y}} && \left[\int_0^T \mathcal{L}\left(t, X^d, Y^d\right) dt\right] \\ &\text{subject to} && \begin{cases} X_0^d \;=\; 0 \\ X_T^d \;=\; \Delta x \\ dX^d \;=\; Y^d dt \end{cases} \end{aligned} \tag{A.27}$$

where

$$
\begin{aligned}
\mathcal{L}\left(t, X^d, Y^d\right) &= -Y_t^d\left\{u+\gamma X_t^d+\lambda Y_t^d+\lambda c_2(t)B(t)+\lambda c_3(t)+h(t)\right\}. \\
h(t) &= (\gamma+\lambda c_1(t))A(t)\int_0^t A(-s)\left(c_2(s)B(s)+c_3(s)\right)ds. \qquad \text{(A.28)}
\end{aligned}
$$

Using standard techniques, that is the Pontryagin Maximization Principle (PMP), we obtain that the optimal Y, if it exists, satisfies the following Euler-Lagrange equation:

$$
\frac{d}{dt}Y(t) = -\frac{1}{2\lambda}\frac{d}{dt}\left[h(t)+\lambda\left(B(t)c_2(t)+c_3(t)\right)\right].
$$

We deduce that Y_t is of the form

$$
Y_t = \text{cst} - \frac{1}{2\lambda}\left[h(t)+\lambda(B(t)c_2(t)+c_3(t))\right]; \qquad \text{cst} = Y_0+\frac{1}{2}(B(0)c_2(0)+c_3(0)).
$$

The boundary conditions in Equation (A.27) imply that

$$
\Delta x = \int_0^T Y_t dt \Rightarrow \Delta x - \text{cst}\times T = -\int_0^T \frac{1}{2\lambda}\left[h(s)+\lambda(B(s)c_2(s)+c_3(s))\right]ds.
$$

Therefore,

$$
Y_0 = -\frac{1}{2}(B(0)c_2(0)+c_3(0)) + \frac{1}{T}\left[\Delta x + \int_0^T \frac{1}{2\lambda}\left[h(s)+\lambda(B(s)c_2(s)+c_3(s))\right]ds.\right]
$$

Hence, the distressed trader's best-response, if it exists, is

$$
Y_t^d = a_{11}(t)+a_{21}(t)\Delta x \qquad \text{(A.29)}
$$

where

$$
\begin{aligned}
a_{12}(t) &= \frac{1}{T}-\frac{1}{2\lambda}\left[h_0(t)+\lambda B_0(t)c_2(t)\right]+\frac{1}{2\lambda T}\int_0^T\left[h_0(s)+\lambda B_0(s)c_2(s)\right]ds \qquad \text{(A.30)} \\
a_{11}(t) &= -\frac{1}{2\lambda}\left[h_1(t)+\lambda(B_1(t)c_2(t)+c_3(t))\right]+\frac{1}{2T\lambda}\int_0^T\left[h_1(s)+\lambda(B_1(s)c_2(s)+c_3(s))\right]ds, \\
&\qquad \text{(A.31)}
\end{aligned}
$$

and

$$B_0(t) = 1 - \kappa\delta(t); \qquad h_0(t) = (\gamma + \lambda c_1(t))A(t)\int_0^t A(-s)c_2(s)B_0(s)ds;$$
$$B_1(t) = \mu\kappa\delta(t); \qquad h_1(t) = (\gamma + \lambda c_1(t))A(t)\int_0^t A(-s)\left[c_2(s)B_1(s) + c_3(s)\right]ds.$$

The differentiability of c_1, c_2 and c_3 implies that a_{11} and a_{12} are well-defined and differentiable.

Equation (A.29) gives the form the distressed trader's best-response *necessarily* takes if it exists. The following lemma proves the existence of the distressed trader's best-response:

Lemma 3. *Suppose the potential predator's strategy is linear with continuous coefficients. Then the distressed trader's best-response strategy is*

$$Y_t^d \;=\; a_{11}(t) + a_{21}(t)\Delta x$$

where a_{11} and a_{12} are given by Equations (A.30) and (A.31).

Proof. The integrand in Equation (A.27) is concave. Theoreom 3 in Rockafellar (1974) then implies that the integral functional we are optimizing is concave. Therefore the necessary conditions are also sufficient. □

Appendix A.3. Equilibrium

Solving for the equilibrium is done by combining the results from the previous two sections. Linear equilibrium strategies are of the form

$$\begin{aligned} Y^\ell &= c_1(t)X^\ell + c_2(t)\hat{X} + c_3(t). \\ Y^d &= a_1(t) + a_2(t)\Delta x. \end{aligned}$$

The distressed trader's strategy satisfies

$$\int_0^T Y^d(t)dt \;=\; \Delta x \quad \forall\, \Delta x.$$

For a linear strategy, this implies that

$$\int_0^T a_2(t)dt = 1. \tag{A.32}$$

$$\int_0^T a_1(t)dt = 0. \tag{A.33}$$

The coefficients c_1, c_2, c_3, a_1, and a_2 are related through Equations (A.25) and (A.29). Using the results in the previous two sections, we have the following relations between the coefficients:

$$\begin{aligned}
a_1(t) &= -\frac{1}{2}\left[\rho\Big(1 - C + \rho(C+1)(T-t)\Big)H_1(t) + B_1(t)c_2(t) + c_3(t)\right] + \mu_{a_1}. \\
a_2(t) &= -\frac{1}{2}\left[\rho\Big(1 - C + \rho(C+1)(T-t)\Big)H_0(t) + B_0(t)c_2(t)\right] + \mu_{a_2}. \\
\mu_{a_1} &= \frac{1}{2T}\int_0^T \left[\rho\Big(1 - C + \rho(C+1)(T-s)\Big)H_1(s) + B_1(s)c_2(s) + c_3(s)\right] ds. \\
\mu_{a_2} &= \frac{1}{T} + \frac{1}{2T}\int_0^T \left[\rho\Big(1 - C + \rho(C+1)(T-s)\Big)H_0(s) + B_0(s)c_2(s)\right] ds. \\
B_0(t) &= 1 - \kappa\delta(t). \\
B_1(t) &= \mu\kappa\delta(t). \\
H_0(t) &= \int_0^t \frac{c_2(s)B_0(s)}{2 + \rho(C+1)(T-s)} ds. \\
H_1(t) &= \int_0^t \frac{c_2(s)B_1(s) + c_3(s)}{2 + \rho(C+1)(T-s)} ds. \\
c_1(t) &= -\frac{\rho(C+1)}{2 + \rho(C+1)(T-t)}. \\
0 &= c_2' - \frac{\rho(C+1)}{2 + \rho(C+1)(T-t)} c_2 + \frac{1}{2\lambda}\bar{a}_2'. \\
0 &= c_3' - \frac{\rho(C+1)}{2 + \rho(C+1)(T-t)} c_3 + \frac{1}{2\lambda}\bar{a}_1'. \\
c_2(T) &= -\frac{1}{2}a_2(T). \\
c_3(T) &= -\frac{1}{2}a_1(T).
\end{aligned}$$

Therefore, solving for the equilibrium is equivalent to solving for a fixed-point problem in (a_1, a_2, c_2, c_3). This fixed-point problem can be broken into two fixed-point problems, the first involving only a_2 and c_2. We do not have existence and uniqueness results regarding this fixed-point problem, and standard techniques do not apply here. We shall transform this fixed-point problem into a system of differential equations that we will solve numerically. Using some algebra, we obtain from the relations above the following system of equations

$$0 = \delta'(t) + \lambda^2\kappa\sigma^2\big[\rho a_2(t) + a_2'(t)\big]^2\delta^2(t) \tag{A.34}$$

$$0 = H_0'(t) - \frac{[1-\kappa\delta(t)]\,c_2(t)}{2+\rho(C+1)(T-t)} \tag{A.35}$$

$$0 = H_1'(t) - \frac{\mu\kappa\delta(t)c_2(t) + c_3(t)}{2+\rho(C+1)(T-t)} \tag{A.36}$$

$$0 = c_2'(t) + c_1(t)c_2(t) + \frac{1}{2}\big[\rho a_2(t) + a_2'(t)\big] \tag{A.37}$$

$$0 = c_3'(t) + c_1(t)c_3(t) + \frac{1}{2}\big[\rho a_1(t) + a_1'(t)\big] \tag{A.38}$$

$$\begin{aligned} 0 = a_2'(t) + \frac{1}{2}\Big[&-\rho^2(C+1)H_0(t) + \rho[1-\kappa\delta(t)]c_2(t) + \lambda^2\kappa^2\sigma^2\big[\rho a_2(t) + a_2'(t)\big]^2\delta^2(t)c_2(t) \\ &-\frac{1}{2}[1-\kappa\delta(t)]\big[\rho a_2(t) + a_2'(t)\big]\Big] \end{aligned} \tag{A.39}$$

$$\begin{aligned} 0 = a_1'(t) - \frac{1}{3}\rho a_1(t) - \frac{2}{3}\big[&\rho^2(C+1)H_1(t) - \rho\left(\mu\kappa\delta(t)c_2(t) + c_3(t)\right) \\ &+ \lambda^2\mu\kappa^2\sigma^2\big[\rho a_2(t) + a_2'(t)\big]^2\delta^2(t)c_2(t) + \frac{\mu\kappa\delta(t)}{2}\big[\rho a_2(t) + a_2'(t)\big]\Big] \end{aligned} \tag{A.40}$$

with boundary conditions

$$\begin{aligned} &H_0(0) = 0; \qquad a_2(0) = \mu_{a_2} - \frac{1}{2}(1-\kappa)c_2(0); \qquad c_2(T) = -\frac{1}{2}a_2(T); \\ &H_1(0) = 0; \qquad a_1(0) = \mu_{a_1} - \frac{1}{2}\left[\mu\kappa c_2(0) + c_3(0)\right]; \qquad c_3(T) = -\frac{1}{2}a_1(T); \qquad \delta(0) = 1. \end{aligned}$$

The existence and uniqueness results from the HJB theory and both the PMP and Lemma 3 imply that a linear equilibrium exists if and only if the system of equations (A.34)—(A.40) has a solution. This result completes the proof of Theorem 1.

The system of equations (A.34)—(A.40) has a unique solution on a subset of $(0,\,T)$ for any given set of initial values since $[2+\rho(C+1)(T-t)]^{-1}$ is smooth on $(0,\,T)$. The existence and uniqueness problem we face is more complicated because we our problem is a boundary value problem.

Appendix A.4. Proof of the Corollary

Suppose that

$$\gamma \;=\; 0.$$

This implies that

$$c_1 \equiv 0.$$

The system of equations (A.34)—(A.40) then reduces to

$$0 = \delta'(t) + \lambda^2\kappa\sigma^2\left[a_2'(t)\right]^2\delta^2(t) \tag{A.41}$$

$$0 = H_0^{'}(t) - \frac{\left[1-\kappa\delta(t)\right]c_2(t)}{2} \tag{A.42}$$

$$0 = H_1^{'}(t) - \frac{\mu\kappa\delta(t)c_2(t) + c_3(t)}{2} \tag{A.43}$$

$$0 = c_2^{'}(t) + \frac{1}{2}a_2'(t) \tag{A.44}$$

$$0 = c_3^{'}(t) + \frac{1}{2}a_1'(t) \tag{A.45}$$

$$0 = a_2'(t) + \frac{1}{2}\left[\lambda^2\kappa^2\sigma^2\left[a_2'(t)\right]^2\delta^2(t)c_2(t) - \frac{1}{2}[1-\kappa\delta(t)]a_2'(t)\right] \tag{A.46}$$

$$0 = a_1'(t) - \frac{2}{3}\left[\lambda^2\mu\kappa^2\sigma^2\left[a_2'(t)\right]^2\delta^2(t)c_2(t) + \frac{\mu\kappa\delta(t)}{2}a_2'(t)\right] \tag{A.47}$$

with boundary conditions

$$\begin{array}{llll} H_0(0) = 0; & a_2(0) = \mu_{a_2} - \frac{1}{2}(1-\kappa)c_2(0); & c_2(T) = -\frac{1}{2}a_2(T); & \\ H_1(0) = 0; & a_1(0) = \mu_{a_1} - \frac{1}{2}\left[\mu\kappa c_2(0) + c_3(0)\right]; & c_3(T) = -\frac{1}{2}a_1(T); & \delta(0) = 1. \end{array}$$

Equations (A.44) and (A.45), together with the terminal boundary conditions for c_2 and c_4, imply that

$$c_2(t) = -\frac{1}{2}a_2(t) \qquad \text{and} \qquad c_3(t) = -\frac{1}{2}a_1(t).$$

Plugging the first equality above into Equation (A.46) leads to

$$\begin{aligned} 0 &= (3+\kappa\delta(t))a_2'(t) - \lambda^2\kappa^2\sigma^2\left[a_2'(t)\right]^2\delta^2(t)a_2(t) \\ &= (3+\kappa\delta(t))a_2'(t) + \kappa\delta'(t)a_2(t) \\ \Rightarrow a_2(t) &= a_2(0)\frac{3+\kappa}{3+\kappa\delta(t)}. \end{aligned}$$

We used Equation (A.41) to obtain the second equality. Taking the derivative of a_2 with respect to t and plugging the result in Equation (A.41) yields

$$0 \;=\; \delta'(t)\left([3+\kappa\delta(t)]^4 + D\delta^2(t)\delta'(t)\right) \quad \text{where} \quad D \;=\; \lambda^2\kappa^3\sigma^2 a_2^2(0)[3+\kappa]^2.$$

The solution δ thus satisfies either

$$0 \;=\; \delta'(t) \;\; \forall\, t \in \; [0,T] \qquad \text{or} \qquad 0 \;=\; [3+\kappa\delta(t)]^4 + D\delta^2(t)\delta'(t) \;\; \forall\, t \in \; [0,T]$$

because we require smooth solutions. We shall show that the unique solution is

$$\delta'(t) \;=\; 0 \;\; \forall\, t \in \; [0,T].$$

To do so, we show that the solution to the ODE

$$\delta'(t) \;=\; -\frac{1}{D}\frac{[3+\kappa\delta(t)]^4}{\delta^2(t)}$$

cannot be smooth and satisfy the requirement that

$$\delta(t) \geq 0 \quad \forall\, t,$$

that is, the requirement that the percentage of uncertainty remaining in the game is non-negative. Suppose that δ is smooth,

$$\delta'(t) \;=\; -\frac{1}{D}\frac{[3+\kappa\delta(t)]^4}{\delta^2(t)}, \qquad \text{and} \qquad \delta(t) \geq 0 \quad \forall\, t.$$

The expression for $a_2(t)$ yields that

$$a_2(0) \;\leq\; a_2(t) \;\leq\; a_2(0)\frac{3+\kappa}{3}$$

since $0 \leq \delta(t) \leq 1$ for all $t \geq 0$. It thus follows from Equation (A.32) that

$$a_2(0)T \;\leq\; \int_0^T a_2(t)dt = 1 \;\leq\; a_2(0)\frac{3+\kappa}{3}T \qquad \Rightarrow \qquad \frac{3}{3+\kappa}\frac{1}{T} \;\leq\; a_2(0) \;\leq\; \frac{1}{T}$$

Moreover, for $t > 0$,

$$\delta'(t) \;<\; -\frac{[3+\kappa]^2}{\lambda^2\kappa^3\sigma^2 a_2^2(0)} \quad \Rightarrow \quad \delta(t) \;<\; 1 - \frac{[3+\kappa]^2}{\lambda^2\kappa^3\sigma^2 a_2^2(0)}t$$

since the function $-([3+\kappa x]^4)/x^2$ is an increasing function for $x \in (0,1]$ and $\delta(t)$ is bounded above by 1. It thus follows that $\delta(t) < 0$ for

$$t > \frac{\lambda^2\kappa^3\sigma^2 a_2^2(0)}{[3+\kappa]^2} > \frac{9\lambda^2\kappa^3\sigma^2}{[3+\kappa]^4}\frac{1}{T^2}.$$

This result contradicts both the assumption that $\delta(t) \geq 0$ and that $\delta(t)$ is smooth since $\delta(0) = 1$ and $\delta'(t)$ is not defined for $\delta(t) = 0$ (the contradiction holds for T sufficiently large). The contradiction implies that the only possible solution is

$$0 \;=\; \delta'(t) \quad \forall\, t \in\; [0,T] \quad \Rightarrow \quad 1 \;=\; \delta(t) \quad \forall\, t \in\; [0,T].$$

For this solution, we have

$$a_2(t) \;=\; a_2(0) \quad \forall\, t \in\; [0,T] \qquad \Rightarrow \qquad -2c_2(t) \;=\; a_2(t) \;=\; \frac{1}{T} \quad \forall\, t \in\; [0,T].$$

It thus follows from Equations (A.47) and (A.33) that

$$a_1(t) \;=\; a_1(0) \quad \forall\, t \in\; [0,T] \qquad \Rightarrow \qquad a_1(t) \;=\; c_3(t) \;=\; 0 \quad \forall\, t \in\; [0,T].$$

The assumption $\gamma = 0$ and the fact that a_2 is constant imply that

$$\bar{a}_2'(t) \;=\; 0.$$

Thus, Equations (A.3) and (A.7) imply that

$$\hat{X}_t \;=\; \hat{X}_0 \;=\; \mu + (1-\kappa)(\tilde{S} - \mu).$$

This completes the derivation of the equilibrium strategies.

We now derive the distressed trader's equilibrium expected value and the probability of the potential predator providing liquidity in equilibrium.

$$\begin{aligned}
V^d &= \mathrm{E}^d \left\{ \int_0^T - \left[F_t + \lambda(Y^d + Y^\ell) \right] Y^d dt \right\} \\
&= V^{d,0} - \lambda T \, \mathrm{E}^d \left[Y^\ell Y^d \right] \\
&= V^{d,0} + \frac{\lambda}{2T} \, \mathrm{E}^d \left[\widetilde{\Delta x} \, \mathrm{E}^d \left[\hat{X}_t | \widetilde{\Delta x} \right] \right] \\
&= V^{d,0} + \frac{\lambda}{2T} \, \mathrm{E}^d \left[\widetilde{\Delta x} \, \Big(\mu + (1-\kappa)(\widetilde{\Delta x} - \mu) \Big) \right] \\
&= V^{d,0} + \frac{\lambda}{2T} \left[\mu^2 + (1-\kappa)\sigma^2 \right].
\end{aligned}$$

Here, $V^{d,0}$ is the distressed trader's equilibrium expected value in the absence of the potential predator. We rewrite the signal $\tilde{S}$ as

$$\tilde{S} \;=\; \widetilde{\Delta x} + \sigma \sqrt{\frac{\kappa}{1-\kappa}} \tilde{\epsilon}_0 \qquad \text{where} \qquad \tilde{\epsilon}_0 \sim \mathrm{N}(0,\, 1) \quad \text{and} \quad \kappa \neq 1.$$

For a given pair $(\widetilde{\Delta x}; \tilde{\epsilon}_0)$, the distressed trader's equilibrium expected value is

$$\begin{aligned}
V^d(\widetilde{\Delta x}; \tilde{\epsilon}_0) &= \mathrm{E}^B \left\{ \int_0^T - \left[F_t + \lambda(Y^d + Y^\ell) \right] Y^d dt \right\} \\
&= V^{d,0}(\widetilde{\Delta x}) - \lambda T \, Y^\ell Y^d \\
&= V^{d,0}(\widetilde{\Delta x}) + \frac{\lambda}{2T} \left[\kappa\mu\widetilde{\Delta x} + (1-\kappa)(\widetilde{\Delta x})^2 + \sigma\sqrt{\kappa(1-\kappa)}\tilde{\epsilon}_0 \widetilde{\Delta x} \right]
\end{aligned}$$

where the expectation is taken with respect to the Brownian motion B_t and $V^{d,0}(\widetilde{\Delta x})$ is the distressed trader's equilibrium expected value in the absence of the potential predator. Define $\tilde{Y}$ as

$$\tilde{Y} \;=\; \tilde{Y}_0 \widetilde{\Delta x} \qquad \text{where} \qquad \tilde{Y}_0 \;\equiv\; \kappa\mu + (1-\kappa)\widetilde{\Delta x} + \sigma\sqrt{\kappa(1-\kappa)}\tilde{\epsilon}_0.$$

The probability of liquidity provision occurring is the same as $\mathrm{P}[\tilde{Y} > 0]$. Clearly,

$$\mathrm{P}[\tilde{Y} > 0] \;=\; 1 \qquad \text{if} \qquad \kappa = 0.$$

Assume that $\kappa \neq 0$.

$$\begin{aligned}
\mathrm{P}[\tilde{Y} > 0] &= \mathrm{P}[\tilde{Y}_0 > 0, \widetilde{\Delta x} > 0] + \mathrm{P}[\tilde{Y}_0 < 0, \widetilde{\Delta x} < 0] \\
\mathrm{P}[\tilde{Y}_0 > 0, \widetilde{\Delta x} > 0] &= \mathrm{P}[\tilde{Y}_0 > 0 | \widetilde{\Delta x} > 0]\mathrm{P}[\widetilde{\Delta x} > 0] \\
&= \mathrm{P}\left\{\tilde{\epsilon}_0 > -\frac{1}{\sigma}\left[\sqrt{\frac{\kappa}{1-\kappa}}\mu + \sqrt{\frac{1-\kappa}{\kappa}}\widetilde{\Delta x}\right] | \widetilde{\Delta x} > 0\right\}\Phi\left(\frac{\mu}{\sigma}\right) \\
&= \Phi\left(\frac{\mu}{\sigma}\right)\int_0^\infty \Phi\left(\frac{1}{\sigma}\left[\sqrt{\frac{\kappa}{1-\kappa}}\mu + \sqrt{\frac{1-\kappa}{\kappa}}x\right]\right)\frac{\phi(\frac{x-\mu}{\sigma})}{1-\Phi(-\frac{\mu}{\sigma})}dx \\
&= \int_0^\infty \Phi\left(\frac{1}{\sigma}\left[\sqrt{\frac{\kappa}{1-\kappa}}\mu + \sqrt{\frac{1-\kappa}{\kappa}}x\right]\right)\phi\left(\frac{x-\mu}{\sigma}\right)dx \\
\mathrm{P}[\tilde{Y}_0 < 0, \widetilde{\Delta x} < 0] &= \int_{-\infty}^0 \Phi\left(-\frac{1}{\sigma}\left[\sqrt{\frac{\kappa}{1-\kappa}}\mu + \sqrt{\frac{1-\kappa}{\kappa}}x\right]\right)\phi\left(\frac{x-\mu}{\sigma}\right)dx.
\end{aligned}$$

Appendix B. Numerical methods

Appendix B.1. Numerical solutions to differential equations

Appendix B.1.1. Arbitrary C

We first consider and arbitrary constant C. We shall set

$$C \;=\; 1$$

when solving the system numerically. Let

$$H_2 \;\equiv\; \frac{1}{2\lambda}\bar{a}_2'(t) \;=\; \frac{1}{2}\left[\rho a_2(t) + a_2'(t)\right].$$

We can use Equation (A.39) to derive a differential equation satisfied by H_2. For numerical simplicity, we transform the system of equations (A.34) — (A.40) into the following system of ordinary first-order differential equations:

$$
\begin{aligned}
0 &= \delta'(t) + 4\lambda^2\kappa\sigma^2 H_2^2(t)\delta^2(t) && \text{(B.1)}\\
0 &= H_0'(t) - \frac{[1-\kappa\delta(t)]\, c_2(t)}{2+\rho(C+1)\rho(T-t)} && \text{(B.2)}\\
0 &= H_1'(t) - \frac{\mu\kappa\delta(t)c_2(t) + c_3(t)}{2+\rho(C+1)(T-t)} && \text{(B.3)}\\
0 &= c_2'(t) + c_1(t)c_2(t) + H_2(t) && \text{(B.4)}\\
0 &= c_3'(t) + c_1(t)c_3(t) + \frac{2}{3}\rho a_1(t) + \frac{1}{3}\Big[\rho^2(C+1)H_1(t) - \rho\left(\mu\kappa\delta(t)c_2(t) + c_3(t)\right)\\
&\qquad + 4\lambda^2\mu\kappa^2\sigma^2 H_2(t)^2\delta^2(t)c_2(t) + \mu\kappa\delta(t)H_2(t)\Big] && \text{(B.5)}\\
0 &= a_2'(t) + \frac{1}{2}\Big[-\rho^2(C+1)H_0(t) + \rho[1-\kappa\delta(t)]c_2(t) + 4\lambda^2\kappa^2\sigma^2 H_2^2(t)\delta^2(t)c_2(t)\\
&\qquad - [1-\kappa\delta(t)]H_2(t)\Big] && \text{(B.6)}\\
0 &= a_1'(t) - \frac{1}{3}\rho a_1(t) - \frac{2}{3}\Big[\rho^2(C+1)H_1(t) - \rho\left(\mu\kappa\delta(t)c_2(t) + c_3(t)\right)\\
&\qquad + 4\lambda^2\mu\kappa^2\sigma^2 H_2(t)^2\delta^2(t)c_2(t) + \mu\kappa\delta(t)H_2(t)\Big] && \text{(B.7)}\\
0 &= H_2'(t) + \frac{d_2(t)+d_2'(t)}{2d_2(t)H_2(t)+4-d_1(t)}H_2^2(t) - \frac{d_1(t)+d_1'(t)}{2d_2(t)H_2(t)+4-d_1(t)}H_2(t) + \frac{d_0(t)+d_0'(t)}{2d_2(t)H_2(t)+4-d_1(t)} && \text{(B.8)}
\end{aligned}
$$

with boundary conditions

$$
\begin{array}{llll}
H_0(0) = 0; & a_2(0) = \mu_{a_2} - \frac{1}{2}(1-\kappa)c_2(0); & c_2(T) = -\frac{1}{2}a_2(T); & \\
H_1(0) = 0; & a_1(0) = \mu_{a_1} - \frac{1}{2}\left[\mu\kappa c_2(0) + c_3(0)\right]; & c_3(T) = -\frac{1}{2}a_1(T); & \delta(0) = 1.
\end{array}
$$

The functions d_0, d_1, and d_2 are

$$
\begin{aligned}
d_0(t) &= -\rho^2(C+1)H_0(t) + \rho[1-\kappa\delta(t)]c_2(t);\\
d_2(t) &= 4\lambda^2\kappa^2\sigma^2\delta^2(t)c_2(t); \qquad\qquad d_1(t) = 1-\kappa\delta(t).
\end{aligned}
$$

The function *odeint* from the Scipy library for Python can be used to solve the system of first order equations. However a difficulty arises because *odeint* handles Initial Value Problems (IVP) and we have a Boundary Value Problem (BVP). We use the standard shooting method to solve this issue.

Given initial values of H_2 and c_2, we can solve the IVP consisting of Equations (B.1), (B.2), (B.4), (B.6), and (B.8). Note that

$$
\begin{aligned}
a_2(0) &= \frac{1}{2\rho}\left[4\lambda^2\kappa^2\sigma^2 c_2(0)H_2^2(0) + (3+\kappa)H_2(0) + \rho(1-\kappa)c_2(0)\right].\\
a_2'(0) &= 2H_2(0) - \rho a_2(0).
\end{aligned}
$$

We use the shooting method to find initial values of H_2 and c_2 for which the boundary conditions for a_2 and c_2 are satisfied. We then repeat the exercise, this time selecting initial values of a_1 and c_3 and solving the entire system of equations.

Appendix B.1.2. No time to liquidate excess holding

We consider the equilibrium in the limit

$$
C \longrightarrow \infty \qquad \Longleftrightarrow \qquad \Delta T \longrightarrow 0.
$$

In this limit, the system of equations determining the equilibrium is:

$$
\begin{aligned}
a_1(t) &= -\frac{1}{2}\Big[\Big(-1+\rho(T-t)\Big)H_1(t)+B_1(t)c_2(t)+c_3(t)\Big]+\mu_{a_1}.\\
a_2(t) &= -\frac{1}{2}\Big[\Big(-1+\rho(T-t)\Big)H_0(t)+B_0(t)c_2(t)\Big]+\mu_{a_2}.\\
\mu_{a_1} &= \frac{1}{2T}\int_0^T\Big[\Big(-1+\rho(T-s)\Big)H_1(s)+B_1(s)c_2(s)+c_3(s)\Big]\,ds.\\
\mu_{a_2} &= \frac{1}{T}+\frac{1}{2T}\int_0^T\Big[\Big(-1+\rho(T-s)\Big)H_0(s)+B_0(s)c_2(s)\Big]\,ds.\\
B_0(t) &= 1-\kappa\delta(t).\\
B_1(t) &= \mu\kappa\delta(t).\\
H_0(t) &= \int_0^t\frac{c_2(s)B_0(s)}{T-s}ds.\\
H_1(t) &= \int_0^t\frac{c_2(s)B_1(s)+c_3(s)}{T-s}ds.\\
c_1(t) &= -\frac{1}{T-t}.\\
0 &= c_2'-\frac{1}{T-t}c_2+\frac{1}{2\lambda}\bar{a}_2'.\\
0 &= c_3'-\frac{1}{T-t}c_3+\frac{1}{2\lambda}\bar{a}_1'.\\
c_2(T) &= -\frac{1}{2}a_2(T).\\
c_3(T) &= -\frac{1}{2}a_1(T).
\end{aligned}
$$

The system of equations corresponding to the differential equations (A.34)—(A.40) is:

$$0 = \delta'(t) + \lambda^2\kappa\sigma^2\left[\rho a_2(t) + a_2'(t)\right]^2\delta^2(t) \tag{B.9}$$

$$0 = H_0'(t) - \frac{[1-\kappa\delta(t)]\,c_2(t)}{T-t} \tag{B.10}$$

$$0 = H_1'(t) - \frac{\mu\kappa\delta(t)c_2(t) + c_3(t)}{T-t} \tag{B.11}$$

$$0 = c_2'(t) + c_1(t)c_2(t) + H_2(t) \tag{B.12}$$

$$0 = c_3'(t) + c_1(t)c_3(t) + \frac{1}{2}\left[\rho a_1(t) + a_1'(t)\right] \tag{B.13}$$

$$0 = a_2'(t) + \frac{1}{2}\left[-\rho H_0(t) + \rho[1-\kappa\delta(t)]c_2(t) + 4\lambda^2\kappa^2\sigma^2 H_2(t)^2\delta^2(t)c_2(t) - [1-\kappa\delta(t)]H_2(t)\right] \tag{B.14}$$

$$0 = a_1'(t) - \frac{1}{3}\rho a_1(t) - \frac{2}{3}\left[\rho H_1(t) - \rho\left(\mu\kappa\delta(t)c_2(t) + c_3(t)\right) + 4\lambda^2\mu\kappa^2\sigma^2 H_2(t)^2\delta^2(t)c_2(t) + \mu\kappa\delta(t)H_2(t)\right] \tag{B.15}$$

$$0 = H_2'(t) + \frac{d_2(t) + d_2'(t)}{2d_2(t)H_2(t) + 4 - d_1(t)}H_2^2(t) - \frac{d_1(t) + d_1'(t)}{2d_2(t)H_2(t) + 4 - d_1(t)}H_2(t) + \frac{d_0(t) + d_0'(t)}{2d_2(t)H_2(t) + 4 - d_1(t)} \tag{B.16}$$

with boundary conditions

$$\begin{array}{lllll} H_0(0) = 0; & a_2(0) = \mu_{a_2} - \frac{1}{2}(1-\kappa)c_2(0); & c_2(T) = -\frac{1}{2}a_2(T); & \\ H_1(0) = 0; & a_1(0) = \mu_{a_1} - \frac{1}{2}\left[\mu\kappa c_2(0) + c_3(0)\right]; & c_3(T) = -\frac{1}{2}a_1(T); & \delta(0) = 1. \end{array}$$

The functions d_0, d_1, and d_2 are

$$\begin{array}{ll} d_0(t) = -\rho H_0(t) + \rho[1-\kappa\delta(t)]c_2(t); & \\ d_2(t) = 4\lambda^2\kappa^2\sigma^2\delta^2(t)c_2(t); & d_1(t) = 1 - \kappa\delta(t). \end{array}$$

We solve for the equilibrium in this case numerically and compute the probability of predatory trading occurring in Table 4

[Insert Figure 4 here]

Appendix B.2. Performance of the Numerical Solutions

We evaluate the performance of our numerical solutions. Recall that the distressed trader strategy is of the form

$$Y_t^d = a_1(t) + a_2(t)\Delta x; \quad \forall\ \Delta x\ \in \mathbb{R}.$$

Moreover, Y^d satisfies

$$\int_0^T Y_t^d dt \ = \ \Delta x; \quad \forall\ \Delta x\ \in \mathbb{R}.$$

Therefore,

$$\int_0^T a_1(t)dt \quad = \quad 0 \tag{B.17}$$

$$\int_0^T a_2(t)dt \quad = \quad 1. \tag{B.18}$$

We compute

$$\left|\int_0^T a_1(t)dt\right| \qquad \text{and} \qquad \left|1 - \int_0^T a_2(t)dt\right|$$

for our numerical solutions presented in the body of the paper and present the results in Table 3. Table 3 shows that our numerical solutions perform well, at least as far as conditions (B.17) and (B.18) are concerned.

[Insert Figure 3 here]

Appendix B.3. Simulations

We run 100×100 simulations of the game assuming that each player follows her linear equilibrium strategy. Below is the algorithm describing the simulations

1. Randomly pick a realization Δx using the distribution of $\widetilde{\Delta x}$. Calculate

$$p_{0i} = \Pr\left[\left|\frac{\widetilde{\Delta x} - \Delta x_i}{\sigma}\right| < 0.017\right].$$ [21]

[21] We choose 0.017 in the definition of p_{0i} to ensure that $\sum_{i=1}^{100} p_{0i} \approx 1$ when we randomly select 100 realizations of $\widetilde{\Delta x} \sim \mathrm{N}\left(-10, \sqrt{0.5}\right)$.

2. Simulate 100 paths of the potential predator's equilibrium strategy. Compute p_{1i}, the percentage of paths for which the potential predator engages in predatory trading. Also compute the realized value of both players for each path. Denote the potential predator's (distressed trader's) mean value for the 100 paths v^{ℓ} (v^{d}).
3. Repeat steps one and two 100 times.

We use the ratio

$$\sum_{i=1}^{100} (p_{oi} \times p_{1i}) / \sum_{i=1}^{100} p_{0i}$$

as our proxy for the probability that predatory trading will occur. Similar proxies are made for each player's expected value.

We use the Euler-Maruyama method to solve the system of stochastic differential equations for the state variables numerically in Step 2 above.

Appendix C. Open-Loop Equilibrium

We assume that both traders follow time-dependent strategies. We start by stating the following useful lemmas and definition:

Lemma 4. *Suppose that the distressed trader follows a linear strategy of the form*

$$Y_t^d \;=\; a_1(t) + a_2(t)\Delta x$$

for any realization Δx *of the random variable* $\widetilde{\Delta x}$*. Then,*

$$\int_0^T a_1(t)dt \;=\; 0 \qquad \text{and} \qquad \int_0^T a_2(t)dt \;=\; 1.$$

Proof. The result follows from the requirement that

$$\int_0^T Y_t^d(t)dt \;=\; \Delta x.$$

□

Lemma 5. *The potential predator's estimate of the random liquidation size* $\widetilde{\Delta x}$ *at time* $t = 0$*, denoted*

$$\hat{X}_0 \;=\; \mathrm{E}^{\ell}[\widetilde{\Delta x}] \;=\; \mathrm{E}^{\ell}\left[\widetilde{\Delta x}|\tilde{S}_1\right],$$

is

$$\hat{X}_0 \;=\; \mu + (1-\kappa)(\tilde{S} - \mu). \tag{C.1}$$

The distressed trader's estimate of the random variable $\hat{X}_0$ *is*

$$\mathrm{E}^d[\hat{X}_0] \;=\; \mathrm{E}^d[\hat{X}_0|\Delta x] \;=\; (1-\kappa)\Delta x + \mu\kappa. \tag{C.2}$$

Proof. The proof follows from applying basic conditional expectation formulas for multivariate normal random variables. □

Definition 2. *Let $g : [0, \infty] \to \mathbf{R}$ be an arbitrary integrable function. We define $\bar{g}$ as the function:*

$$\bar{g}(t) \;=\; \int_0^t g(s)ds.$$

Appendix C.1. Best-Response: Potential Predator

Suppose that the distressed trader follows a strategy of the form

$$Y_t^d \;=\; a_1(t) + a_2(t)\Delta x$$

where both a_1 and a_2 are smooth. The the potential predator's best response strategy solves the optimization problem:

$$\begin{array}{ll} \max\limits_{Y^\ell \in \mathcal{Y}} & \mathrm{E}^\ell \left\{ \displaystyle\int_0^T - \left[F_t + \gamma(X_t^\ell + X_t^d) + \lambda(Y_t^\ell + Y_t^d) \right] Y_t^\ell dt \;+\; X_T^\ell \left(F_T + \gamma X_T^d \right) - \frac{C}{2}\gamma (X_T^\ell)^2 \right\} \\ \text{subject to} & \begin{cases} dX_t^\ell \;=\; Y_t^\ell dt \\ X_0^\ell \;=\; 0. \end{cases} \end{array}$$

The Euler-Lagrange equation associated with this problem is

$$\begin{aligned} 0 \;&=\; \gamma \left(\mathrm{E}^\ell[Y_t^d] + Y_t^\ell \right) dt \;+\; \lambda \left(\mathrm{E}^\ell[dY_t^d] + dY_t^\ell \right) \;+\; \lambda dY_t^\ell \;-\; \gamma Y_t^\ell dt \\ &=\; 2\lambda dY_t^\ell \;+\; \gamma \left(a_1(t) + a_2(t)\hat{X}_0 \right) dt \;+\; \lambda \left(a_1'(t) + a_2'(t)\hat{X}_0 \right) dt \end{aligned} \tag{C.3}$$

with transversality condition

$$\begin{aligned} 2\lambda Y_T^\ell + \gamma(1 + C)X_T^\ell \;&=\; -\gamma \mathrm{E}^\ell[X_T^d] - \lambda \mathrm{E}^\ell[Y_T^d] + \gamma \mathrm{E}^\ell[\widetilde{\Delta x}] \\ &=\; -\lambda(a_1(T) + a_2(T)\hat{X}_0). \end{aligned} \tag{C.4}$$

It follows from the Euler-Lagrange equation that

$$\begin{aligned} dY_t^\ell &= -\frac{1}{2}\left[(\rho a_1(t)+a_1'(t))+(\rho a_2(t)+a_2'(t))\hat{X}_0\right]dt \\ \Longrightarrow Y_t^\ell &= Y_0^\ell-\frac{1}{2}\left[f_1(t)+f_2(t)\hat{X}_0\right] \\ \Longrightarrow X_t^\ell &= tY_0^\ell-\frac{1}{2}\left[\bar{f}_1(t)+\bar{f}_2(t)\hat{X}_0\right] \end{aligned} \tag{C.5}$$

where

$$f_i(t) \;=\; \rho\bar{a}_i(t)+a_i(t)-a_i(0).$$

We can derive both terminal values Y_T^ℓ and X_T^ℓ and combine them with the transversality Equation (C.4) to solve for Y_0^ℓ:

$$\begin{cases} Y_T^\ell &= Y_0^\ell-\frac{1}{2}\left[f_1(T)+f_2(T)\hat{X}_0\right] \\ X_T^\ell &= TY_0^\ell-\frac{1}{2}\left[\bar{f}_1(T)+\bar{f}_2(T)\hat{X}_0\right] \end{cases}$$

$$Y_0^\ell=\frac{2f_1(T)+\rho(1+C)\bar{f}_1(T)-2a_1(T)}{2[2+\rho T(1+C)]}+\frac{2f_2(T)+\rho(1+C)\bar{f}_2(T)-2a_2(T)}{2[2+\rho T(1+C)]}\hat{X}_0. \tag{C.6}$$

Therefore, the potential predator's best-response is

$$Y_t^\ell \;=\; \frac{2f_1(T)+\rho(1+C)\bar{f}_1(T)-2a_1(T)}{2[2+\rho T(1+C)]}-\frac{1}{2}f_1(t)+\left[\frac{2f_2(T)+\rho(1+C)\bar{f}_2(T)-2a_2(T)}{2[2+\rho T(1+C)]}-\frac{1}{2}f_2(t)\right]\hat{X}_0. \tag{C.7}$$

We can simplify the expression for Y_0^d using Lemma 4:

$$f_1(T)=a_1(T)-a_1(0); \qquad \bar{f}_1(T)=\rho\bar{\bar{a}}_1(T)-Ta_1(0);$$

$$f_2(T)=\rho+a_2(T)-a_2(0); \qquad \bar{f}_2(T)=\;\rho\bar{\bar{a}}_2(T)+1-Ta_2(0).$$

Appendix C.2. Best-Response: Distressed Trader

Suppose that the potential predator follows a strategy of the form

$$Y_t^\ell = c_1(t) + c_2(t)\hat{X}_0$$

where both c_1 and c_2 are smooth. The distress trader Euler-Lagrande equation yields

$$\begin{aligned} dY_t^d &= -\frac{1}{2}\left[\rho\left(c_1(t) + c_2(t)\Big[(1-\kappa)\Delta x + \mu\kappa\Big]\right) + \left(c_1'(t) + c_2'(t)\Big[(1-\kappa)\Delta x + \mu\kappa\Big]\right)\right] && \text{(C.8)} \\ \Longrightarrow Y_t^d &= Y_0^d - \frac{1}{2}\Big[e_1(t) + \mu\kappa e_2(t) + (1-\kappa)e_2(t)\Delta x\Big] && \text{(C.9)} \end{aligned}$$

where

$$e_i(t) = \rho\bar{c}_i(t) + c_i(t) - c_i(0); \quad i = 1, 2$$

and we made used of Equation (C.2). The boundary condition

$$\int_0^T Y_t^d dt = \Delta x$$

together with Equation (C.9) yield Y_0^d:

$$\begin{aligned} \Delta x &= TY_0^d - \frac{1}{2}\Big[\bar{e}_1(T) + \mu\kappa\bar{e}_2(T) + (1-\kappa)\bar{e}_2\Delta x\Big] \\ Y_0^d &= \frac{\bar{e}_1(T) + \mu\kappa\bar{e}_2(T)}{2T} + \frac{1}{T}\left[1 + \frac{(1-\kappa)\bar{e}_2(T)}{2}\right]\Delta x. && \text{(C.10)} \end{aligned}$$

Hence,

$$Y_t^d = \frac{\bar{e}_1(T) + \mu\kappa\bar{e}_2(T)}{2T} - \frac{e_1(t) + \mu\kappa e_2(t)}{2} + \left[\frac{1}{T} + \frac{(1-\kappa)\bar{e}_2(T)}{2T} - \frac{(1-\kappa)e_2(t)}{2}\right]\Delta x. \quad \text{(C.11)}$$

Appendix C.3. Equilibrium

We showed that each trader's best-response strategy to a linear strategy by her opponent is also a linear strategy. We now solve for the linear equilibrium. Equations (C.8) and (C.5) imply that

$$\begin{cases} dY_t^d &= -\frac{1}{2}\Big[\rho c_1(t) + c_1'(t) + \mu\kappa(\rho c_2(t) + c_2'(t)) + (1-\kappa)(\rho c_2(t) + c_2'(t))\Delta x\Big] dt \\ dY_t^\ell &= -\frac{1}{2}\Big[(\rho a_1(t) + a_1'(t)) + (\rho a_2(t) + a_2'(t))\hat{X}_0\Big] dt \end{cases}$$

$$\implies \begin{cases} 2a_1'(t) &= -\rho c_1(t) - c_1'(t) - \mu\kappa(\rho c_2(t) + c_2'(t)) \\ 2a_2'(t) &= -(1-\kappa)(\rho c_2(t) + c_2'(t)) \\ 2c_1'(t) &= -(\rho a_1(t) + a_1'(t)) \\ 2c_2'(t) &= -(\rho a_2(t) + a_2'(t)) \end{cases}$$

We use Equations (C.6) and (C.10) to get the boundary conditions of the system above:

$$\begin{cases} a_2(0) &= \frac{1}{T}\left[1 + \frac{1-\kappa}{2}\Big(\rho\bar{\bar{c}}_2(T) + \bar{c}_2(T) - Tc_2(0)\Big)\right] \\ c_2(0) &= \frac{1}{2[2+\rho T(1+C)]}\left[2\Big(\rho - a_2(0)\Big) + \rho(1+C)\Big(\rho\bar{\bar{a}}_2 + 1 - Ta_2(0)\Big)\right] \end{cases} \tag{C.12}$$

$$\begin{cases} a_1(0) &= \frac{1}{2T}\left[\rho\bar{\bar{c}}_1(T) + \bar{c}_1(T) - Tc_1(0) + \mu\kappa\Big(\rho\bar{\bar{c}}_2(T) + \bar{c}_2(T) - Tc_2(0)\Big)\right] \\ c_1(0) &= \frac{1}{2[2+\rho T(1+C)]}\left[-2a_1(0) + \rho(1+C)\Big(\rho\bar{\bar{a}}_1 - Ta_1(0)\Big)\right] \end{cases}. \tag{C.13}$$

We rewrite the system of first-order differential equations above as two matrix equations:

$$\begin{bmatrix} 2 & 1-\kappa \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a_2'(t) \\ c_2'(t) \end{bmatrix} = -\rho \begin{bmatrix} 0 & 1-\kappa \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_2(t) \\ c_2(t) \end{bmatrix} \tag{C.14}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a_1'(t) \\ c_1'(t) \end{bmatrix} = -\rho \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_1(t) \\ c_1(t) \end{bmatrix} - \mu\kappa \begin{bmatrix} \rho c_2(t) + c_2'(t) \\ 0 \end{bmatrix}. \tag{C.15}$$

The matrices on the LHS of Equations (C.14) and (C.15) are both invertible. Therefore, we have

$$\begin{bmatrix} a_2'(t) \\ c_2'(t) \end{bmatrix} = -\frac{\rho}{3+\kappa} \begin{bmatrix} -1+\kappa & 2(1-\kappa) \\ 2 & -1+\kappa \end{bmatrix} \begin{bmatrix} a_2(t) \\ c_2(t) \end{bmatrix} \tag{C.16}$$

$$\begin{bmatrix} a_1'(t) \\ c_1'(t) \end{bmatrix} = -\frac{\rho}{3} \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a_1(t) \\ c_1(t) \end{bmatrix} - \frac{\mu\kappa}{3} \begin{bmatrix} 2(\rho c_2(t) + c_2'(t)) \\ -(\rho c_2(t) + c_2'(t)) \end{bmatrix}. \tag{C.17}$$

The matrix in Equation (C.16) satisfies

$$\begin{bmatrix} -1+\kappa & 2(1-\kappa) \\ 2 & -1+\kappa \end{bmatrix} = \begin{bmatrix} \sqrt{1-\kappa} & \sqrt{1-\kappa} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \lambda_1(\kappa) & 0 \\ 0 & \lambda_2(\kappa) \end{bmatrix} \begin{bmatrix} \frac{1}{2\sqrt{1-\kappa}} & \frac{1}{2} \\ \frac{1}{2\sqrt{1-\kappa}} & \frac{-1}{2} \end{bmatrix}$$

where

$$\lambda_1(\kappa) \;=\; -(1-\kappa)+2\sqrt{1-\kappa} \qquad \text{and} \qquad \lambda_2(\kappa) \;=\; -(1-\kappa)-2\sqrt{1-\kappa}.$$

The solution to Equation (C.16) is thus

$$\begin{aligned}
\begin{bmatrix} a_2(t) \\ c_2(t) \end{bmatrix} &= \exp\left\{ -\frac{t\rho}{3+\kappa} \begin{bmatrix} -1+\kappa & 2(1-\kappa) \\ 2 & -1+\kappa \end{bmatrix} \right\} \begin{bmatrix} a_2(0) \\ c_2(0) \end{bmatrix} \\
&= \begin{bmatrix} \sqrt{1-\kappa} & \sqrt{1-\kappa} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} e^{-\frac{t\rho\lambda_1(\kappa)}{3+\kappa}} & 0 \\ 0 & e^{-\frac{t\rho\lambda_2(\kappa)}{3+\kappa}} \end{bmatrix} \begin{bmatrix} \frac{1}{2\sqrt{1-\kappa}} & \frac{1}{2} \\ \frac{1}{2\sqrt{1-\kappa}} & \frac{-1}{2} \end{bmatrix} \begin{bmatrix} a_2(0) \\ c_2(0) \end{bmatrix} \\
&= \begin{bmatrix} \frac{e^{-\frac{t\rho\lambda_1(\kappa)}{3+\kappa}} + e^{-\frac{t\rho\lambda_2(\kappa)}{3+\kappa}}}{2} & \frac{\sqrt{1-\kappa}\left(e^{-\frac{t\rho\lambda_1(\kappa)}{3+\kappa}} - e^{-\frac{t\rho\lambda_2(\kappa)}{3+\kappa}}\right)}{2} \\ \frac{e^{-\frac{t\rho\lambda_1(\kappa)}{3+\kappa}} - e^{-\frac{t\rho\lambda_2(\kappa)}{3+\kappa}}}{2\sqrt{1-\kappa}} & \frac{e^{-\frac{t\rho\lambda_1(\kappa)}{3+\kappa}} + e^{-\frac{t\rho\lambda_2(\kappa)}{3+\kappa}}}{2} \end{bmatrix} \begin{bmatrix} a_2(0) \\ c_2(0) \end{bmatrix}.
\end{aligned} \tag{C.18}$$

We integrate Equation (C.18) to obtain $\bar{a}_2(t)$, $\bar{c}_2(t)$, $\bar{\bar{a}}_2(t)$, and $\bar{\bar{c}}_2(t)$. All four functions are linear in $a_2(0)$ and $c_2(0)$. We then plug $a_2(T)$, $c_2(T)$, $\bar{a}_2(T)$, $\bar{c}_2(T)$, $\bar{\bar{a}}_2(T)$, and $\bar{\bar{c}}_2(T)$ into Equations (C.10) and (C.6) to solve for both $a_2(0)$ and $c_2(0)$.

We use the same approach to solve Equation (C.17) and then for both $a_1(0)$ and $c_1(0)$. :

$$\begin{aligned}
\begin{bmatrix} a_1(t) \\ c_1(t) \end{bmatrix} &= \exp\left\{ -\frac{t\rho}{3} \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \right\} \left(\begin{bmatrix} a_1(0) \\ c_1(0) \end{bmatrix} - \frac{\mu\kappa}{3} \int_0^t \exp\left\{ \frac{s\rho}{3} \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \right\} \begin{bmatrix} 2(\rho c_2(s) + c_2'(s)) \\ -(\rho c_2(s) + c_2'(s)) \end{bmatrix} \right) \\
&= \begin{bmatrix} \frac{e^{-\frac{t\rho}{3}} + e^{t\rho}}{2} & \frac{e^{-\frac{t\rho}{3}} - e^{t\rho}}{2} \\ \frac{e^{-\frac{t\rho}{3}} - e^{t\rho}}{2} & \frac{e^{-\frac{t\rho}{3}} + e^{t\rho}}{2} \end{bmatrix} \left(\begin{bmatrix} a_1(0) \\ c_1(0) \end{bmatrix} - \frac{\mu\kappa}{3} \int_0^t \begin{bmatrix} \frac{e^{\frac{s\rho}{3}} + e^{-s\rho}}{2} & \frac{e^{\frac{s\rho}{3}} - e^{-s\rho}}{2} \\ \frac{e^{\frac{s\rho}{3}} - e^{-s\rho}}{2} & \frac{e^{\frac{s\rho}{3}} + e^{-s\rho}}{2} \end{bmatrix} \begin{bmatrix} 2(\rho c_2(s) + c_2'(s)) \\ -(\rho c_2(s) + c_2'(s)) \end{bmatrix} \right).
\end{aligned} \tag{C.19}$$

The distressed trader's equilibrium *surplus* returns for a given path of the Brownian motion B, a realization Δx of $\widetilde{\Delta x}$, and a realization S of $\tilde{S}$ relative to the case when the potential predator is absent is

$$\Delta V^d = -\int_0^T \Big[F_t + \gamma\Big(\bar{a}_1(t) + \bar{c}_1(t) + \bar{a}_2(t)\Delta x + \bar{c}_2(t)\hat{X}_0\Big)$$
$$+\lambda\Big(a_1(t) + c_1(t) + a_2(t)\Delta x + c_2(t)\hat{X}_0\Big)\Big]\Big(a_1 + a_2\Delta x\Big)dt + \frac{\Delta x}{T}\int_0^T F_t dt + \left(\frac{\gamma}{2} + \frac{\lambda}{T}\right)\Delta x^2. \tag{C.20}$$

The potential predator's value is

$$V^\ell = -\int_0^T \Big[F_t + \gamma\Big(\bar{a}_1(t) + \bar{c}_1(t) + \bar{a}_2(t)\Delta x + \bar{c}_2(t)\hat{X}_0\Big)$$
$$+\lambda\Big(a_1(t) + c_1(t) + a_2(t)\Delta x + c_2(t)\hat{X}_0\Big)\Big]\Big(c_1(t) + c_2(t)\hat{X}_0\Big)dt$$
$$+ \Big(\bar{c}_1(T) + \bar{c}_2(T)\hat{X}_0\Big)\left(F_T + \gamma\Delta x\right) - \frac{C}{2}\gamma\Big(\bar{c}_1(T) + \bar{c}_2(T)\hat{X}_0\Big)^2. \tag{C.21}$$

Appendix C.4. Numerical Analysis

The constant C affects the equilibrium only through its effects on the initial values $a_1(0), c_1(0), c_2(0)$ and $a_2(0)$. We proceed by presenting, for each limit, the system of equations corresponding to systems (C.12) and (C.13).

In the case

$$C = 1,$$

the systems (C.12) and (C.13) become:

$$\begin{cases} a_2(0) &= \frac{1}{T}\Big[1 + \frac{1-\kappa}{2}\Big(\rho\bar{\bar{c}}_2(T) + \bar{c}_2(T) - Tc_2(0)\Big)\Big] \\ c_2(0) &= \frac{1}{2[1+\rho T]}\Big[\Big(\rho - a_2(0)\Big) + \rho\Big(\rho\bar{\bar{a}}_2 + 1 - Ta_2(0)\Big)\Big] \end{cases}$$
$$\begin{cases} a_1(0) &= \frac{1}{2T}\Big[\rho\bar{\bar{c}}_1(T) + \bar{c}_1(T) - Tc_1(0) + \mu\kappa\Big(\rho\bar{\bar{c}}_2(T) + \bar{c}_2(T) - Tc_2(0)\Big)\Big] \\ c_1(0) &= \frac{1}{2[1+\rho T]}\Big[-a_1(0) + \rho\Big(\rho\bar{\bar{a}}_1 - Ta_1(0)\Big)\Big] \end{cases}.$$

In the limit

$$C \longrightarrow \infty \qquad \Longleftrightarrow \qquad \Delta T \longrightarrow 0,$$

the systems (C.12) and (C.13) become:

$$\begin{cases} a_2(0) &= \frac{1}{T}\left[1 + \frac{1-\kappa}{2}\Big(\rho\bar{\bar{c}}_2(T) + \bar{c}_2(T) - Tc_2(0)\Big)\right] \\ c_2(0) &= \frac{1}{2T}\left[\rho\bar{\bar{a}}_2 + 1 - Ta_2(0)\right] \end{cases}$$
$$\begin{cases} a_1(0) &= \frac{1}{2T}\left[\rho\bar{\bar{c}}_1(T) + \bar{c}_1(T) - Tc_1(0) + \mu\kappa\Big(\rho\bar{\bar{c}}_2(T) + \bar{c}_2(T) - Tc_2(0)\Big)\right] \\ c_1(0) &= \frac{\rho\bar{\bar{a}}_1 - Ta_1(0)}{2T} \end{cases}.$$

We compute the probability of predatory trading occurring in the case

$$C = 1$$

and present the results in Table 5.

[Insert Table 5 here]

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Table 1. Probability of Predatory Trading Occurring in the Presence of Information Asymmetry.

We run 100×100 simulations of the game assuming that each player follows her linear equilibrium strategy and estimate the probability that predatory trading occurs. See Appendix Appendix B.3 for more details on the simulations. Parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$ and $T = 1.0$.

(a) Fixed $\lambda = 1$.

κ \ γ	1	2	3	4	5
0.0	0.0	1.0	1.0	1.0	1.0
0.12	0.0	0.97	1.0	1.0	1.0
0.25	0.0	0.74	0.99	1.0	1.0
0.5	0.0	0.1	0.88	0.93	0.97
0.75	0.0	0.0	0.32	0.5	0.44
0.95	0.0	0.0	0.0	0.12	0.08

(b) Fixed $\gamma = 2.5$.

κ \ λ	0.5	1	1.25	2.5	5
0.0	1.0	1.0	1.0	0.0	0.0
0.12	1.0	1.0	0.74	0.0	0.0
0.25	1.0	0.89	0.22	0.0	0.0
0.5	0.94	0.42	0.13	0.0	0.0
0.75	0.76	0.1	0.01	0.0	0.0
0.95	0.40	0.01	0.0	0.0	0.0

Table 2. Degree of Uncertainty and Welfare.
We run 100×100 simulations of the game assuming that each player follows her linear equilibrium strategy and estimate each player's expected value. We present the potential predator's wealth. We also present the distressed trader's wealth as a percentage change relative to her wealth in the absence of the potential predator. Finally, we present the strategic traders' aggregate wealth as a percentage change relative to their aggregate wealth in the absence of the potential predator. See Appendix Appendix B.3 for details of the estimation procedure. Parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$, $\lambda = 1$, and $T = 1$.

(a) Distressed Trader.

κ \ γ	1	2	3	4	5
0.0	0.37	-0.43	-1.31	-2.28	-3.32
0.12	0.38	-0.42	-1.26	-2.19	-3.18
0.25	0.38	-0.42	-1.1	-1.89	-3.0
0.5	0.39	0.03	-0.66	-1.04	-1.53
0.75	0.44	0.16	0.1	0.01	0.01
0.95	0.49	0.24	0.08	0.05	0.04

(b) Potential Predator.

κ \ γ	1	2	3	4	5
0.0	6.9	14.26	31.1	52.33	75.21
0.12	6.86	14.09	30.44	52.05	74.81
0.25	6.79	14.08	30.19	50.85	74.16
0.5	6.83	12.26	25.69	40.59	55.92
0.75	6.7	10.55	18.23	25.55	31.4
0.95	6.25	8.74	13.44	17.29	20.14

(c) Aggregate Strategic Trader.

κ \ γ	1	2	3	4	5
0.0	0.54	-0.05	-0.48	-0.88	-1.26
0.12	0.56	-0.04	-0.44	-0.88	-1.22
0.25	0.56	0.01	-0.3	-0.52	-1.1
0.5	0.57	0.17	0.05	0.0	0.01
0.75	0.61	0.34	0.22	0.21	0.11
0.95	0.66	0.39	0.31	0.28	0.16

Table 3. Performance of the Numerical Solutions.

We numerically solve for each player's linear equilibrium strategy and estimate both $A_0 = \left|\int_0^T a_1(t)dt\right|$ and $A_1 = \left|1 - \int_0^T a_2(t)dt\right|$. See Appendix Appendix B.1 for details of the numerical solutions. Parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, 0.5)$ and $T = 1$.

(a) Fixed $\lambda = 1$; A_0.

κ \ γ	1	1.8	2	3	5
0.12	1.1×10^{-7}	3.8×10^{-8}	1×10^{-7}	3.8×10^{-7}	1.5×10^{-6}
0.25	5.9×10^{-7}	4.4×10^{-7}	6×10^{-7}	1.8×10^{-7}	2.8×10^{-6}
0.5	3×10^{-6}	4.9×10^{-6}	6.1×10^{-7}	3.3×10^{-6}	8.7×10^{-6}
0.75	8×10^{-6}	1.2×10^{-5}	1.2×10^{-5}	1.1×10^{-5}	7.4×10^{-6}
0.95	1.3×10^{-5}	2×10^{-5}	2.1×10^{-5}	2.6×10^{-5}	4×10^{-5}

(b) Fixed $\lambda = 1$; A_1.

κ \ γ	1	1.8	2	3	5
0.12	5.3×10^{-7}	3.5×10^{-7}	6.8×10^{-7}	3.8×10^{-6}	1.8×10^{-5}
0.25	5×10^{-7}	3.6×10^{-7}	6.9×10^{-7}	3.6×10^{-6}	1.7×10^{-5}
0.5	2.8×10^{-7}	5.2×10^{-7}	8×10^{-7}	3.3×10^{-6}	1×10^{-5}
0.75	1.3×10^{-6}	6.5×10^{-7}	7.8×10^{-5}	1.2×10^{-6}	2×10^{-6}
0.95	5.1×10^{-7}	6.8×10^{-7}	6.3×10^{-5}	3.9×10^{-7}	6.4×10^{-7}

(c) Fixed $\gamma = 2.5$; A_0.

κ \ λ	1	1.8	2	3	5
0.12	2.5×10^{-6}	1.7×10^{-7}	5.5×10^{-8}	1.5×10^{-7}	8.5×10^{-8}
0.25	7.3×10^{-6}	3.3×10^{-7}	1.6×10^{-6}	1×10^{-6}	6.4×10^{-7}
0.5	1.8×10^{-5}	4.9×10^{-6}	6×10^{-6}	5.5×10^{-6}	3.5×10^{-6}
0.75	3×10^{-5}	1.3×10^{-5}	1.4×10^{-5}	1.1×10^{-5}	6.9×10^{-6}
0.95	1.4×10^{-5}	2.4×10^{-5}	2.3×10^{-5}	1.6×10^{-5}	4.2×10^{-4}

(b) Fixed $\gamma = 2.5$; A_1.

κ \ λ	1	1.8	2	3	5
0.12	1.8×10^{-5}	2.2×10^{-6}	6.9×10^{-7}	5.3×10^{-7}	3.2×10^{-7}
0.25	1.7×10^{-5}	2.2×10^{-6}	7.1×10^{-7}	4.5×10^{-7}	2.7×10^{-7}
0.5	1.2×10^{-7}	1.9×10^{-6}	7.5×10^{-7}	5.5×10^{-6}	2.2×10^{-8}
0.75	8.4×10^{-6}	1×10^{-6}	2×10^{-7}	1.7×10^{-7}	3.1×10^{-7}
0.95	1.4×10^{-5}	5×10^{-7}	4.3×10^{-7}	5.4×10^{-7}	3.8×10^{-4}

Table 4. Probability of Predatory Trading Occurring in the Presence of Information Asymmetry: No Time to Liquidate.

We run 100×100 simulations of the game assuming that each player follows her linear equilibrium strategy and estimate the probability that predatory trading occurs when the potential predator has no time to liquidate her excess holding at the end of the game. In effect, she has zero excess holding at the end of the game. See Appendix Appendix B.3 for more details on the simulations. Parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$, $\lambda = 1$ and $T = 1.0$.

κ \ γ	1	2	3	4	5
0.0	0.0	1.0	1.0	1.0	1.0
0.12	0.0	1.0	1.0	1.0	1.0
0.25	0.0	0.83	1.0	1.0	1.0
0.5	0.0	0.36	0.81	0.97	0.91
0.75	0.0	0.10	0.39	0.62	0.73
0.95	0.0	0.0	0.20	0.12	0.23

Table 5. Probability of Predatory Trading Occurring in the Presence of Information Asymmetry: Open-Loop Strategies.

We run 100×100 simulations of the game assuming that each player follows her open loop equilibrium strategy and estimate the probability that predatory trading occurs. See Appendix Appendix B.3 for more details on the simulations. Parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{10})$, $\lambda = 1$ and $T = 1.0$.

κ \ γ	1	2	3	4	5
0.0	0.0	1.0	1.0	1.0	1.0
0.12	0.0	0.94	0.96	0.97	0.99
0.25	0.0	0.32	0.88	0.91	0.93
0.5	0.0	0.0	0.45	0.63	0.77
0.75	0.0	0.0	0.23	0.5	0.44
0.95	0.0	0.0	0.0	0.1	0.16

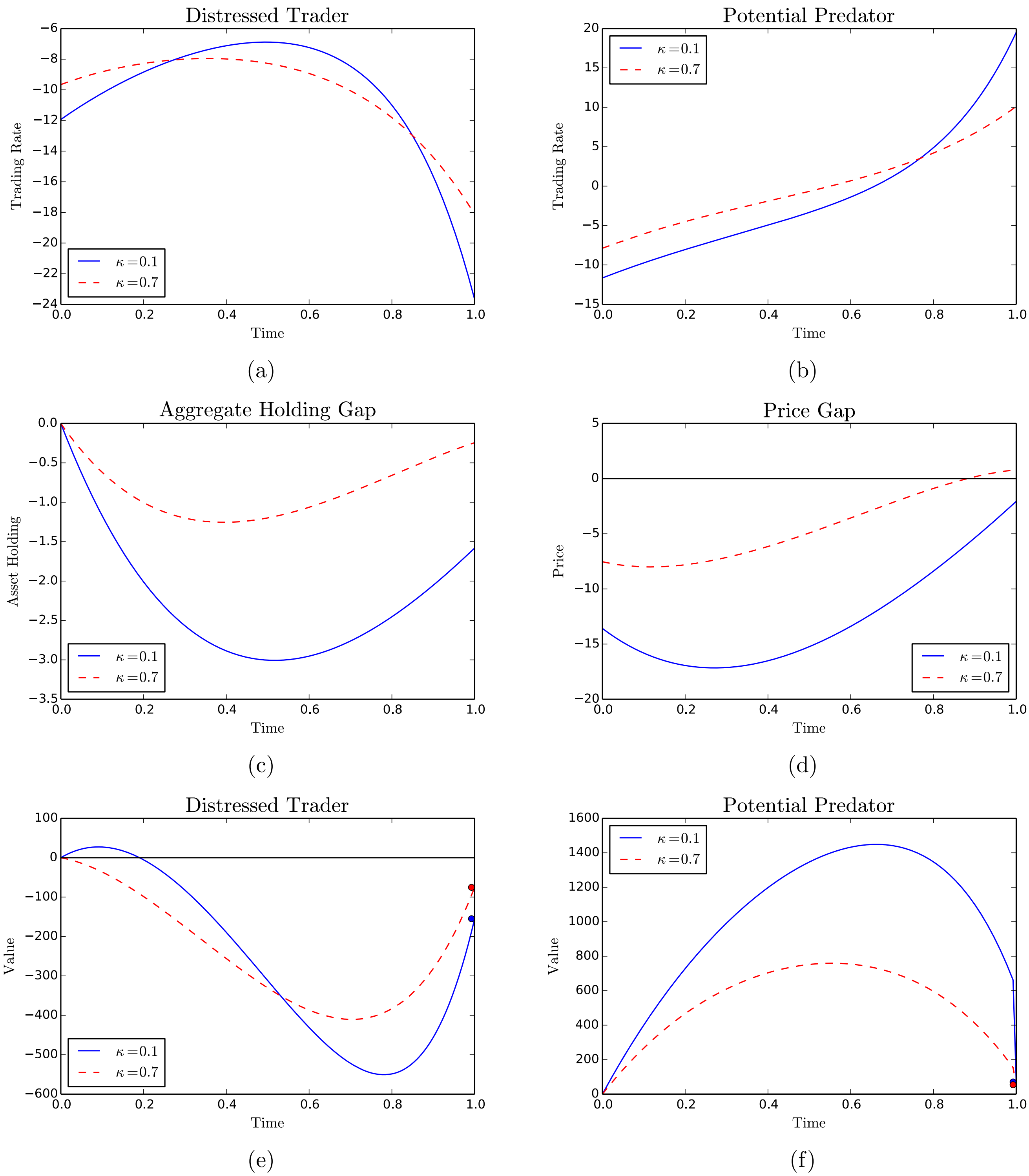


Figure 1.
The Effects of Information Asymmetry.
We simulate 100×100 equilibrium paths of the game for 100 realizations of the liquidation size $\widetilde{\Delta x}$ and 100 paths of the risky asset's fundamental value for two values of the degree of information asymmetry κ. We plot the *average* equilibrium strategies for both strategic traders (Panel (a) and Panel (b)). We also plot the corresponding aggregate holding gap for the strategic traders and the price gap (Panel (c) and Panel (d)). Finally, we plot each trader's expected value gap (Panel (e) and Panel (f)). A quantity's gap is the difference between that quantity in equilibrium and the same quantity when the potential predator is not in the market. Other parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$, $\gamma = 5$, $\lambda = 1$, and $T = 1$.

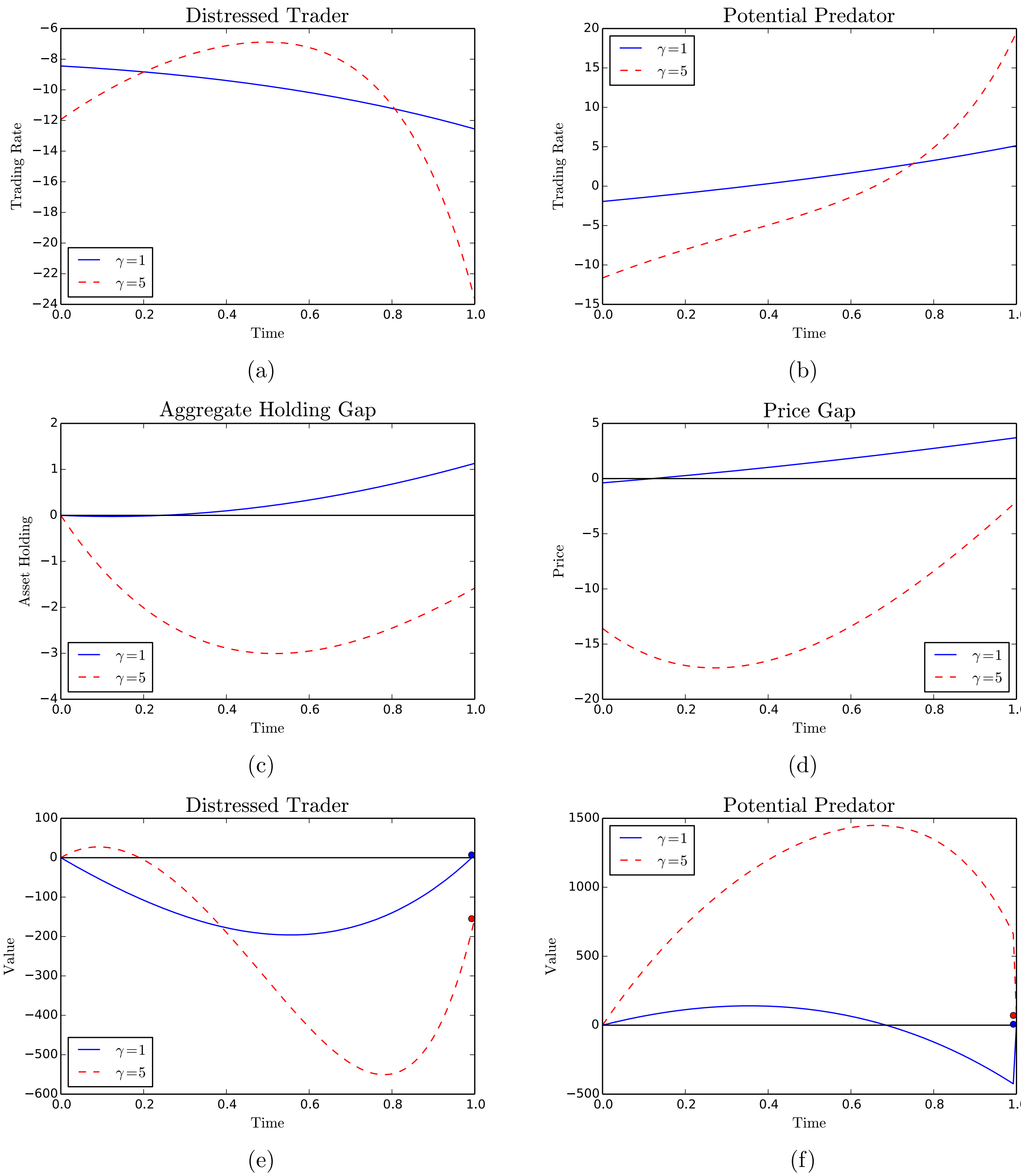
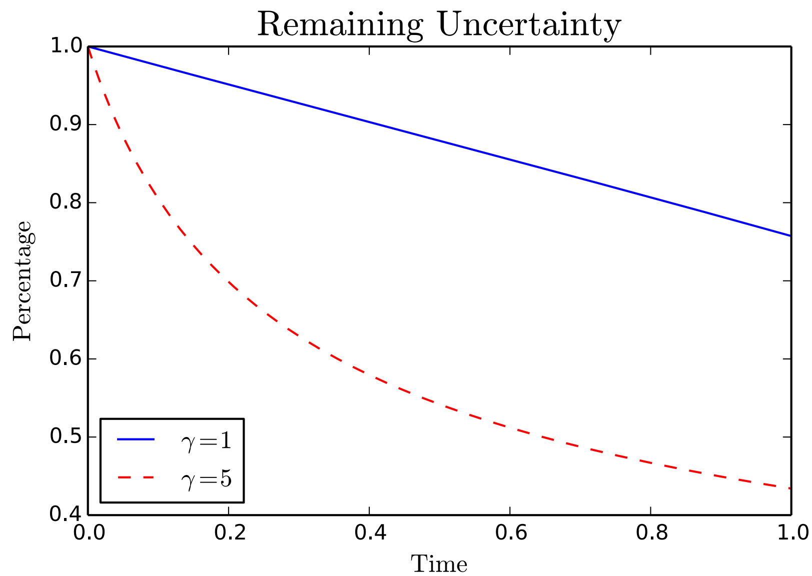
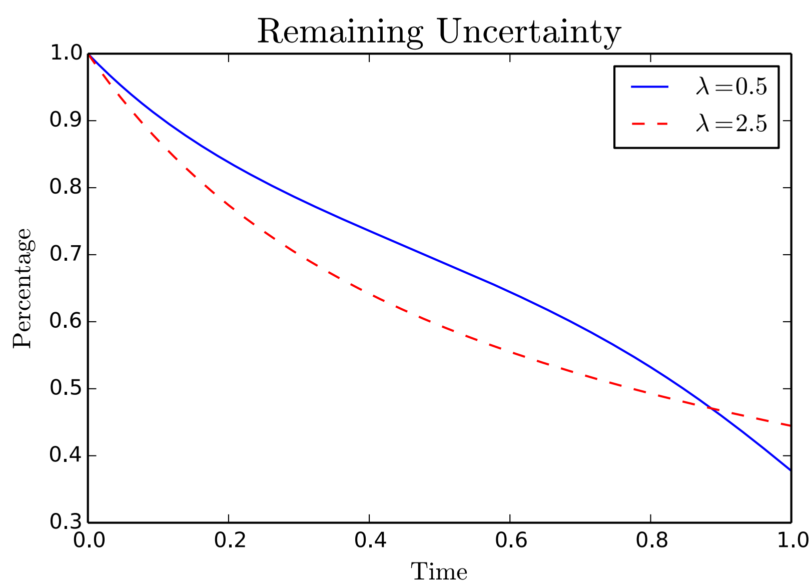


Figure 2. The Effects of the Permanent Price Impact.
We simulate 100×100 equilibrium paths of the game for 100 realizations of the liquidation size $\widetilde{\Delta x}$ and 100 paths of the risky asset's fundamental value for two values of the permanent price impact γ. We plot the *average* equilibrium strategies for both strategic traders (Panel (a) and Panel (b)). We also plot the corresponding aggregate holding gap for the strategic traders and the price gap (Panel (c) and Panel (d)). Finally, we plot each trader's expected value gap (Panel (e) and Panel (f)). A quantity's gap is the difference between that quantity in equilibrium and the same quantity when the potential predator is not in the market. Other parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$, $\kappa = 0.1$, $\lambda = 1$, and $T = 1$.



(a)



(b)

Figure 3. The Effects of the Permanent Price Impact.
The remaining uncertainty about the liquidation size $\widetilde{\Delta x}$ is the ratio

$$\delta(t) \; = \; \frac{\Omega(t)}{\Omega(0)}.$$

$\Omega(t)$ is the variance of $\hat{X}_t$, the random variable representing the potential predator's estimate of $\widetilde{\Delta x}$ conditional on her signal and the realizations of the price process. We solve for the equilibrium $\delta(t)$ numerically. Other parameters: $\widetilde{\Delta x} \sim \mathrm{N}(-10, \sqrt{0.5})$, $\kappa = 0.7$, $T = 1$, $\lambda = 1$ when γ is varying, and $\gamma = 2.5$ when λ is varying.