Frictional Goods Markets: Theory and Applications*

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Abstract

We develop an equilibrium model consistent with several features of retail trade: there is price dispersion; markups are high and variable; there is some random and some directed search; buyers use both money and credit; sellers post prices but may also bargain. Money and credit coexist because the former (latter) bears the inflation tax (transaction costs). Other phenomena arise due to informational heterogeneity and the combination of random plus directed search. Theory delivers sharp qualitative predictions. Calibration yields quantitative results consistent with facts. We discuss policy, and quantify the welfare effects of inflation, information frictions and changes in credit conditions.

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> [Some consumers] receive no information on the price in this market. (It is natural to think of them as tourists, having no local information.) A second type of consumer (resident) receives some information... It would be interesting to develop models with both types of consumers and, I suspect, would result in a different structure of equilibrium. Diamond (1971)

> I think the bilateral monopoly problem has been solved. There are stores that compete. I know where the drug store and the supermarket are, and I take their posted prices as given. If some supermarket offers the same quality of services and charges lower prices, I shop at that lower price supermarket. Prescott (2005)

1 Introduction

This paper develops a general equilibrium model with frictional goods markets meant to resemble actual retail trade. While we do not claim to capture every detail of retail, the framework is consistent with several salient features. These markets display price dispersion. There are high and variable markups. There is some random matching of sellers and buyers, but also some directed search by buyers that are better informed. Retail relies heavily on currency and related instruments, like checks or debit cards, although there is also substantial use of credit. The terms of trade are often posted by sellers, yet there is also some bargaining. We construct a model to match these stylized facts and use it to study several substantive issues, and in particular, to quantify the effects of changes in inflation, information and credit conditions.

Our background environment is provided by the New Monetarist framework, as described in the survey by Lagos et al. (2015), where commitment and information frictions hinder credit and make money useful. We relax these frictions by allowing credit to be used but only at a cost. We also amend the usual random matching specification by incorporating a certain amount of directed search, and amend the bargaining component by allowing sellers to post the terms of trade. Posting with directed search – called competitive search equilibrium since Moen (1997) – is a natural way to think about retail, as suggested above by Prescott. However, as suggested by Diamond, not everyone in every market is so well informed. Hence, we use semi-directed search: as in Lester (2011), some buyers know the terms offered by individual sellers when deciding where to shop; others do not know this and therefore search randomly.[1]

To say more about the formal setup, let us call informed and uninformed buyers locals and tourists (although one should not take this literally, given some people know less than others about certain markets even in their home town, as discussed in fn. 7). An important component of the theory is free entry of sellers, and they can post terms to attract locals, although this does nothing to attract tourists. In equilibrium, some sellers (local shops) cater to informed buyers by posting attractive terms; others (tourist shops) serve only the uninformed. Locals sample exclusively from local shops – or, in the local submarket. Tourists sample randomly, and may end up at a tourist or a local shop. It is convenient, and realistic, to say buyers are not obliged to pay posted prices, and can opt to bargain. In equilibrium, there is bargaining at tourist but not local shops.

Since the use of money is subject to inflation, while credit involves transaction costs, both may be used in equilibrium. This is desirable because we see both in the data, and important for policy analysis because it allows substitution between payment methods. Moreover, incorporating costly credit avoids an indeterminacy of equilibria that plagues related models (see fn. 11). And it is realistic: credit generally involves resources spent on record keeping, screening, enforcement etc. As a narrow example, if credit makes transactions easier to tax, and paying with cash avoids this, the choice of money versus credit involves comparing sales and inflation taxes. More broadly, Nosal and Rocheteau (2011) discuss a body of work modeling money and alternative payment instruments (e.g., credit or bank liabilities) by giving the latter transaction costs, and based on this, it is fair to say we are following a long tradition.[2]

[1]To be clear, Lagos et al. (2015) discuss models with directed search and posting, or undirected search and bargaining, but not both. We nest these. Lester (2011) combines random and directed search, but has no bargaining, does not consider money in the model, does not embed it in dynamic general equilibrium, and does not attempt to calibrate it in order to match facts or measure welfare effects.

[2]A particular cost of credit is *monitoring*. As Wallace (2013) says, "If we want both monetary trade and credit in the same model, we need something between perfect monitoring and no monitoring. As in other areas of economics... extreme versions are both easy to describe and easy to analyze. The challenge is to specify and analyze intermediate situations." Here one can say that monitoring is available but not free, which is not especially deep, but serves the purpose.

We establish existence and provide strong predictions about the effects of parameters. A few results differ from conventional wisdom – e.g., several well-known papers say prices fall if buyers become better informed (Salop and Stiglitz, 1977; Varian, 1980; Burdett and Judd, 1983; Stahl, 1989), but we show the opposite is also possible. A few results are consistent with conventional wisdom but for different reasons – e.g., we can match the finding in Benabou (1992b) of a negative relation between markups and inflation through a channel different from Benabou (1992a) or Head and Kumar (2005). We can also match the positive relation between price dispersion and inflation found by Parsley (1996), Debelle and Lamont (1997) and others.[3] Importantly, even when the inflation tax and cost of credit vanish, the first best may not obtain: the outcome is efficient in local but not tourist shops. Inflation can improve this, in principe, by taxing more heavily the more expensive tourist shops. For some values for the fraction of informed buyers λ, this outweighs the usual cost of inflation, and the Friedman rule can be suboptimal (this is always true in an extension where λ is endogenous).

To say more about the quantitative analysis, first, on inflation, a consensus that emerged from cash-in-advance and related models is that the cost of having 10% instead of an optimal policy is around 0.5% of consumption – see Lucas (2000) for a well-known example and Aruoba et al. (2011) for more references. In New Monetarist models, e.g., Lagos and Wright (2005), the cost of inflation can be an order of magnitude bigger. This is due to random search and bargaining, which some people find objectionable, we think because they think it is unrealistic.[4] Competitive search models partially deflect that critique. They also yield welfare numbers much smaller than random search and bargaining. Intuitively, if optimal policy gives efficiency, moderate inflation entails a small loss, by the Envelope Theorem. With Nash bargaining, e.g., optimal policy gives

[3]This finding is not a universally accepted fact – e.g., Reinsdorf (1994) finds a negative and Caglayan et al. (2008) find a U-shaped relationship.

[4]In addition to Prescott's view in the epigraph, on random matching in particular, consider Hahn (1987): "someone wishing to exchange his house goes to estate agents or advertises – he does not, like some crazed particle, wait to bump into a buyer." Or consider Howitt (2005): "when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer... Few people would think of planning their economic lives on the basis of random encounters."

efficiency iff buyers' bargaining power is 1, which is the Hosios (1990) condition in this context. Competitive search delivers that condition endogenously.

This motivates asking what happens with a combination of posting and bargaining, with a balance disciplined by data. The result is 1.39%. This is closer to models with directed search and posting rather than random search and bargaining, even though calibration delivers plenty of tourists: only a fraction $\lambda = 0.27$ of our buyers are informed. The difference from existing search-and-bargaining models comes from several ingredients. One is that inflation here can have a desirable impact on market composition, i.e., the mix between local and tourist shops. Other factors include our assumption of free entry and our particular bargaining solution. To see how all this matters, we reproduce results from previous studies by shutting down various channels, and provide an accounting of the importance of different factors. Also, again, some inflation can be desirable, in principle, but the benchmark calibration implies the Friedman rule is still optimal. However, that is quite sensitive to parameters.

As for the impact of information on these results, decreasing λ worsens the cost of inflation when inflation is above 7% and mitigates the cost otherwise. For intermediate values of λ a small deviation from the Friedman rule can be desirable, as mentioned above, because of composition effects. As inflation increases, however, eventually the share of tourist shops begins to rise, which compounds the welfare impact. As for the impact of changing information given inflation, increasing λ slightly from 0 lowers welfare – hence, counterintuitively, having more information can backfire on consumers. Yet moving from $\lambda = 0$ to $\lambda = 1$ is beneficial, and worth 3.1% of consumption. Indeed, an unanticipated results is that at λ just above $1/2$, the tourist submarket shuts down, and the economy behaves as if $\lambda = 1$. Thus, a simple majority of local buyers is enough to render this particular information friction innocuous.

Lastly, we explore the importance of credit conditions. Perhaps surprisingly, increasing the use of credit by lowering its cost hurts welfare and output, although the effects are not large – e.g., adjusting the cost to double the use of debt yields a 0.09%

drop in output and has a cost of 0.10% in terms of consumption. This is related Gu et al. (2016), who prove that, when credit is costless, increasing the debt limit is neutral in monetary economies, because it only crowds out real balances to leave total liquidity the same. With costly credit, expanding its use tends to reduce welfare, although there are also composition effects on the mix between local and tourist shops. In general, the net effect can go either way, but at the calibrated parameters it is negative. This stands in stark contrast to a nonmonetary version of our model, where we find that reducing the cost of credit always enhances output and welfare.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 provides analytic results. Section 4 presents quantitative results. Section 5 concludes. All proofs are contained in an Appendix.[5]

2 Model

Each period in discrete time has two subperiods: first there is a decentralized market, or DM, with frictions detailed below; then there is a frictionless centralized market, or CM. This follows Lagos and Wright (2005), and is natural for our purposes because at its core is an asynchronization of expenditures and receipts – obviously crucial for any analysis of money or credit. There is a measure 1 of infinite-lived agents called buyers and a measure N of sellers. Both types can be interpreted as households, or, without changing the equations, sellers can be reinterpreted as retail firms acting on behalf of their owners (see Aruoba et al., 2014). In the CM agents trade goods and labor (x, ℓ), pay taxes, settle debts and adjust money balances. In the DM buyers cannot produce but want to consume a different good q produced by sellers. Period utility is $U(x)+u(q)-\ell$ for buyers and $U(x) - q - \ell$ for sellers, where u and U have the usual properties. All agents discount at $\beta \in (0, 1)$ between the CM and DM.

[5] We mention that this project abstracts from long-term relationships between consumers and retailers, what Gourio and Rudanko (2014) call customer capital. This is not because we think customer capital is unimportant; we just want to focus on other issues for now. We also mention related work by Liu et al. (2015), which also has money and credit, but is technically different (mostly because it uses Burdett–Judd search and price posting) and pursues distinct applications (mainly sticky prices).

Buyers and sellers meet bilaterally in the DM where they trade (p, q). Here p denotes a payment by the buyer measured in units of numeraire x in the next CM. If q represents *quantity*, $P = p/q$ is the unit price. If q represents *quality* that we do not observe, although the agents do, one can say p is the price.[6] In any case, we call $P = p/q$ the markup. The DM partitions sellers into submarkets with the same (p, q). In each submarket, agents match randomly, with arrival rates depending on market tightness, or the buyer-seller ratio, n. A submarket is thus identified by $\Gamma = (p, q, n)$. This much is standard. Less standard is that a buyer with probability λ is informed (a local) and sees Γ in every submarket, but with probability $1 - \lambda$ is uninformed (a tourist) and only knows the distribution across submarkets.[7]

As a benchmark, assume buyers in the CM do not know if they will be informed in the next DM (the other case is available on request). Also, assume that a buyer can either agree to the posted terms (p, q) or can opt to bargain. Anticipating some results, equilibrium involves two submarkets, one with local shops catering to the informed and one with tourist shops serving only the uninformed. Local shops offer favorable terms, informed customers visit them, and accept the posted (p, q). Uniformed customers search randomly, and may find a local shop where they accept (p, q), or a tourist shop where they bargain. See Fig. 1, where N_L and N_T are the measures of local and tourist shops.

If ω_j denotes the *ex ante* probability a buyer goes to submarket j, before knowing if he will be informed, then

$$\omega_T = \frac{(1-\lambda)N_T}{N_L + N_T} \quad \text{and} \quad \omega_L = \frac{N_L + \lambda N_T}{N_L + N_T}. \tag{1}$$

Thus, ω_T is the probability of being uninformed times the probability of finding a tourist shop. As there is a measure 1 of buyers, ω_j is also the measure of buyers in submarket

[6] Consider two wines costing \$10 and \$100 a bottle. If we do not know the latter is higher quality we would say it has a higher price, but it might actually be a better deal. Of course empirical price measures try to control for quality, but may not get it exactly right.

[7] One does not have to take the local and tourist labels literally. Suppose Mr. A knows all the shops with cheap apples but not those with good deals on bananas, and *vice versa* for Ms. B. On days when Mr. A and Ms. B both need apples, he acts like a local and she acts like a tourist, while the opposite is true when they both need bananas. This is formally equivalent to having a generic good and different locations, with individuals randomly transiting between them, and knowing more about some than others.

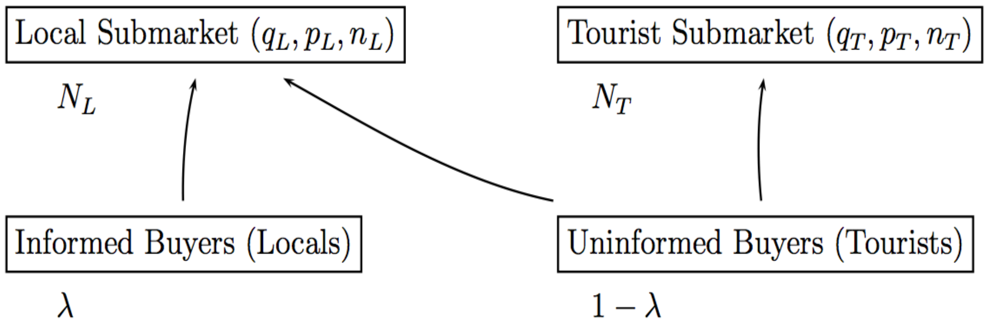

Figure 1: Decentralized market structure

j. Market tightness in each submarket is given by

$$n_T = \frac{1-\lambda}{N_T + N_L} \quad \text{and} \quad n_L = n_T + \frac{\lambda}{N_L}. \tag{2}$$

Within a submarket, buyers and sellers meet according to a CRS matching technology: for a seller, the probability of meeting a buyer is $\alpha(n_j)$; for a buyer the probability of meeting a seller is $\alpha(n_j)/n_j$. Assume $\alpha(n)$ is strictly increasing if $\alpha(n) < 1$ and $\alpha(n)/n$ is strictly decreasing if $\alpha(n) < n$. Also, $\alpha(0) = 0$ and $\alpha(\hat{n}) = 1$ for some $\hat{n} \in (0, \infty]$. Notice from (2) that $n_L \geq n_T$, with $n_L > n_T$ if $\lambda > 0$. This says local submarkets are necessarily tighter.

An agent's state in the CM is net worth A. For a buyer, $A = \phi m - d - \gamma\left(d - \bar{d}\right) + \tau$, where m is money brought from the previous DM, ϕ is the price of m in terms of CM numeraire x, d is debt from the previous DM, $\gamma\left(d - \bar{d}\right)$ is a transaction cost incurred by using debt above some exogenous level $\bar{d}$, and τ is a lump sum transfer that can be used to inject currency.[8] For a seller, A is similar. For either, the CM problem is

$$W(A) = \max_{x,\ell,\hat{m}} \left\{U(x) - \ell + \beta V(\phi_+ \hat{m})\right\} \text{ st } x = w\ell - \phi\hat{m} + A,$$

where w is the wage, $\hat{m}$ is money taken out of the CM, and V is the DM value function, depending on real balances at next period's prices, $z \equiv \phi_+ \hat{m}$. We focus on stationary equilibrium, where W and V are independent of time.

[8]While $\gamma\left(d - \bar{d}\right)$ is borne by buyers, the results are the same if it is borne by sellers, as in standard tax-incidence theory. Indeed, one interpretation of $\gamma\left(d - \bar{d}\right)$ is a sales tax on paying more that $\bar{d}$, which can be avoided by using cash, as in Gomis-Porqueras et al. (2014).

To ease notation, assume a CM technology $x = \ell$, so that $w = 1$. Then, eliminating ℓ, we get

$$W(A) = A + \max_x \{U(x) - x\} + \max_z \{-(1+\pi) z + \beta V(z)\},$$

where $1+\pi = \phi/\phi_{+1}$ is inflation, the same as the growth of the money supply stationary equilibrium. For buyers, the FOC for $z > 0$ is $1 + \pi = \beta V'(z)$. For sellers, $z = 0$ since they have no need for liquidity in the DM. For both, z is independent of A, and $W'(A) = 1$, due to quasi-linear utility. A seller's DM trade surplus is $W(A + p) - W(A) - q$. Using $W'(A) = 1$, this is net revenue, $R = p - q$. A buyer's DM surplus is $S = u(q) - p - \gamma\left(d - \bar{d}\right)$, with $d = p - z$. Below we consider two cases: a pure credit (nonmonetary) equilibrium with $z = 0$; and a monetary equilibrium with $z > 0$. In monetary applications we often set $\bar{d} = 0$ to reduce notation. Also, let the costly part of debt be $D = p - \bar{d} - z \geq 0$, and assume $\gamma(0) = \gamma'(0) = 0$ and $\gamma'(D), \gamma''(D) > 0$ $\forall D > 0$. Given $\gamma'(0) = 0$, even in monetary equilibrium buyers use some credit.

Sellers choose whether to participate in the DM at cost k, and if they participate, the submarket to enter, $j \in \{L, T\}$. For the market to open, impose

$$k < (1 - \underline{\eta})[u(q^*) - q^*], \tag{3}$$

where $u'(q^*) = 1$, and $\underline{\eta}$ is the min of $\eta \equiv n\alpha'(n)/\alpha(n)$, the elasticity of matching. As in Lester (2011), a seller either posts terms to attract locals, or gets only tourists. By constant returns, for generic parameters we can say there is at most 1 submarket of each type. We let tourist shops post terms to extract the entire surplus, but this does not actually matter because, in equilibrium, buyers bargain in these shops.

Sellers' expected surplus in submarket j is $\Pi_j = \alpha(n_j) R_j - k$, where $R_j = p_j - q_j$ for $j \in \{L, T\}$. In the local submarket, as is standard with free entry, the terms posted are determined by maximizing buyers' surplus subject to $\Pi_j = 0$:

$$\max_{p,q,n} \left\{ \frac{\alpha(n)}{n} \left[u(q) - p - \gamma\left(p - \bar{d} - z\right)\right] \right\} \text{ st } \alpha(n)(p - q) = k. \tag{4}$$

Then $\Gamma_L = (p_L, q_L, n_L)$ solves the constraint and the FOC's wrt q and n,

$$\begin{aligned} u'(q_L) &= 1 + \gamma'\left(p_L - \bar{d} - z\right) && (5) \\ p_L - q_L &= \frac{(1-\eta_L)\left[u(q_L) - q_L - \gamma(p_L - \bar{d} - z)\right]}{\eta_L u'(q_L) + 1 - \eta_L}. && (6) \end{aligned}$$

By (6), the seller gets a fraction $(1-\eta_L)/[\eta_L u'(q) + 1 - \eta_L]$ of the total trade surplus.[9]

Now consider tourist shops. Here we adopt the Kalai bargaining solution, which, as Aruoba et al. (2007) argue, has several advantages over Nash in models with liquidity considerations. Kalai's solution in this context can be found by maximizing the surplus subject to the seller getting a share $1-\theta$, where θ is the buyer's bargaining power:

$$\max_{p,q}\{u(q) - q - \gamma(p - \bar{d} - z)\} \text{ st } p - q = (1-\theta)[u(q) - q - \gamma(p - \bar{d} - z)]. \quad (7)$$

This leads to

$$\begin{aligned} u'\left(q_T\right) &= 1 + \gamma'\left(p_T - \bar{d} - z\right) && (8) \\ p_T - q_T &= (1-\theta)[u(q_T) - q_T - \gamma(p_T - \bar{d} - z)]. && (9) \end{aligned}$$

Then $\Gamma_T = (p_T, q_T, n_T)$ solves (8), (9) and $\Pi_T = \alpha\left(n_T\right) R_T - k = 0$. Note Γ_T solves the same conditions as Γ_L, except $(1-\eta_L)/[\eta_L u'(q) + 1 - \eta_L]$ is replaced by $1-\theta$. Hence, if θ is not too big, buyers prefer to not bargain at local shops.[10]

Buyers' DM payoff is their value in the next CM plus the expected DM surplus,

$$\begin{aligned} V(z) = {} & W\left(z+\tau\right) + \omega_L \frac{\alpha(n_L)}{n_L}\left[u(q_L) - p_L - \gamma\left(p_L - \bar{d} - z\right)\right] \\ & + \omega_T \frac{\alpha(n_T)}{n_T}\left[u(q_T) - p_T - \gamma(p_T - \bar{d} - z)\right]. \end{aligned}$$

Notice trade does (does not) depend on z at tourist (local) shops, where customers bargain (accept the posted terms). From this it is easy to derive

$$V'\left(z\right) = 1 + \omega_L \frac{\alpha(n_L)}{n_L}\gamma'\left(D_L\right) + \omega_T \frac{\alpha(n_T)}{n_T}\left[u'(q_T)q_T' - p_T' - \gamma'(D_T)(p_T' - 1)\right], \quad (10)$$

[9] This formulation differs slightly from Rocheteau and Wright (2005), where sellers do not take z as given but post terms to induce buyers to bring a particular z to the DM. With buyers' information realized after leaving the CM, it is more natural to have them take (Γ_L, Γ_T) as given when choosing z.

[10] To be clear, they do not bargain at local shops on or off the equilibrium path – i.e., it is not a profitable deviation to bring $z' \neq z$ and bargain (as long as i is not too big, as we assume below).

where q_T' and p_T' are derivatives wrt to z, and $D_j = p_j - \bar{d} - z$ is the part of credit subject to the transaction cost.

Inserting (10) into buyers' FOC from the CM, $(1+\pi) = \beta V'(z)$, we arrive at

$$i = \omega_L \frac{\alpha(n_L)}{n_L} \gamma'(D_L) + \omega_T \frac{\alpha(n_T)}{n_T} \left[u'(q_T) q_T' - p_T' - \gamma'(D_T)(p_T' - 1) \right], \qquad (11)$$

where the Fisher equation is used to define a nominal interest rate by $1+i = (1+\pi)/\beta$. As usual, i is the return agents require in the next CM to give up a dollar in the current CM, and we can price this trade whether or not it occurs in equilibrium. Based on this, the LHS of (11) can be understood as the cost of carrying an extra dollar, while the RHS is the expected marginal benefit.

We now show $V(z)$ is concave and buyers cash out in at least some trades. Proofs of these and other nonobvious results are in the Appendix.

Lemma 1 *In monetary equilibrium* $V''(z) < 0 \; \forall z < \hat{p} = \max\{p_L, p_T\}$.

Lemma 2 *In monetary equilibrium* $0 < z < \hat{p}$*.*

We show below that $\hat{p} = p_T > p_L$. Thus, Lemma 2 implies buyers cash out at tourist shops for sure, while they may or may not cash out at local shops. Lemma 1 implies the solution to (11) is unique and generates a well-behaved demand for money.[11]

Definition 1 *A pure credit equilibrium is a nonnegative list* $\langle \Gamma_L, \Gamma_T \rangle$ *such that* Γ_j *solves the relevant conditions in each submarket with* $z = 0$*.*

Definition 2 *A monetary equilibrium is a nonnegative list* $\langle \Gamma_L, \Gamma_T, z \rangle$ *such that* Γ_j *solves the relevant conditions in each submarket given* $z > 0$*, and* z *solves the money demand problem given* (Γ_L, Γ_T)*.*

[11] Notice V'' exists due to a smooth cost of credit $\gamma(D)$. This avoids an indeterminacy of monetary steady states in a series of papers following Green and Zhou (1998). See Jean et al. (2010) for more discussion, but consider the case of indivisible goods. If sellers think all buyers bring $m = X$ to market, they all post $p = X$ as long X is not too small; and if they all post $p = X$ buyers bring $m = X$ as long as X is not too big. So $p = m = X$ is an equilibrium for any X in some range. Without credit, a similar indeterminacy arises here, because $V(z)$ is discontinuous, but adding $\gamma(D)$ resolves the problem.

3 Analytic Results

We now establish existence and uniqueness of equilibrium where at least submarket L is open, and compare submarkets when both are open. Pure credit and money are considered in turn.

3.1 Pure Credit

Proposition 1 *Pure credit equilibrium exists uniquely if $\bar{d}$ is not too small.*

Proposition 2 *In pure credit equilibrium with $N_L, N_T > 0$, the local submarket has greater tightness $n_L > n_T$, a lower payment $p_L < p_T$, a lower markup $P_L < P_T$, lower net revenue per trade $R_L < R_T$, and higher output $q_L \geq q_T$, with $q_L > q_T$ as long as $q_T < q^*$, where $u'(q^*) = 1$.*

In Proposition 1, the requirement that $\bar{d}$ is not too small can bind, as otherwise the gains from trade would not justify sellers' fixed cost k. In Proposition 2, submarket L is tighter because locals only go there and tourists search randomly. Since $\Pi_L = \Pi_T$ and submarket L is tighter, $R_T > R_L$. Indeed, $p_T > p_L$, and $q_L > q_T$ as long as $q_T < q^*$. Now $N_L > 0$ if $\bar{d}$ is not too small, k is not too big and $\lambda > 0$, since sellers are always willing to cater to informed buyers. Stronger assumptions are necessary for $N_T > 0$, but it is always true when λ is small.

The next result establishes Γ_L and Γ_T are each unique and continuous in $\bar{d}$.[12] In terms of substance, as credit gets easier, sellers produce more, charge more, earn more per trade, and enter more.

Proposition 3 *In pure credit equilibrium, as $\bar{d}$ increases, p_j and q_j increase while n_j and D_j decrease continuously in both submarkets. The effect on P_j is ambiguous. Moreover, Γ_T is differentiable in $\bar{d}$, and if $n_L < \hat{n}$ then Γ_L is differentiable in $\bar{d}$.*

[12] In directed search it is usually difficult to say much on parameter changes, as Γ_L may not be unique or continuous, as even if $u(q)$ and $\alpha(n)$ are concave the product $\alpha(n) u(q)$ in the objective function may not be. In the spirit of Choi (2015), we make progress using methods of monotone comparative statics. Although the argument in the Appendix is lengthy, it delivers clean results that may be considered a contribution to the pure theory of directed search.

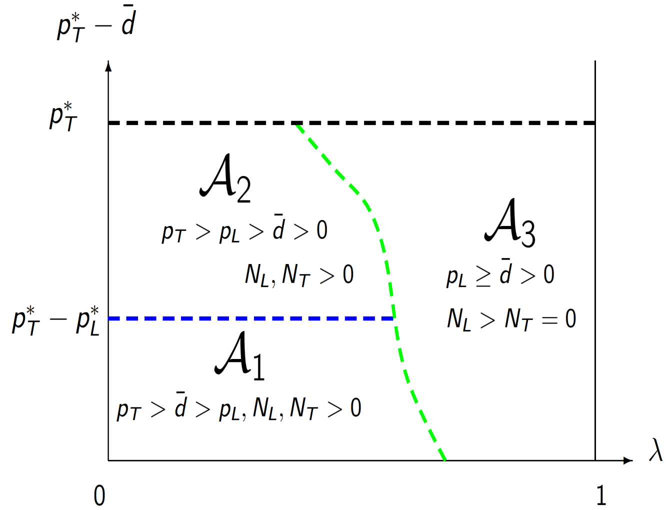

Figure 2: Equilibrium at different $\bar{d}$ and λ.

There are different types of equilibria. First, $p_T > p_L$ means buyers are more likely to be constrained by $\bar{d}$ in submarket T. Let p_j^* be the payment required to get q^* in submarket j. If $p_j^* \leq \bar{d}$, buyers get q^* in submarket j; if $p_j^* > \bar{d}$, although they could get q^* using costly credit, they choose $q_j < q^*$ (by the Envelope Theorem). In Fig. 2, in area $\mathcal{A}_1$ both submarkets are active, $N_L, N_T > 0$, and buyers use costly credit in tourist but not local shops, $p_L^* < \bar{d} < p_T^*$. In $\mathcal{A}_2$, again, $N_L, N_T > 0$, but now buyers use credit in both submarkets, $p_T > p_L > \bar{d}$. In $\mathcal{A}_3$, submarket T shuts down, and buyers use credit in local shops iff $\bar{d} < p_L^*$.

3.2 Money and Credit

We now set $\bar{d} = 0$ and look for monetary equilibrium. This can be reduced to a solution $T(z) = i$, where $T(z)$ is given by the RHS of (11), taking into account that (Γ_L, Γ_T) depend on z. Write this as

$$T(z) = \frac{\omega_L \alpha(n_L)}{n_L} \gamma'(p_L - z) + \frac{\omega_T \alpha(n_T)}{n_T} \left[u'(q_T) q_T' - p_T' - \gamma'(p_T - z)(p_T' - 1) \right] \quad (12)$$

for $N_T(z) > 0$, and

$$T(z) = \frac{\alpha(n_L)}{n_L} \gamma'(p_L - z) \quad (13)$$

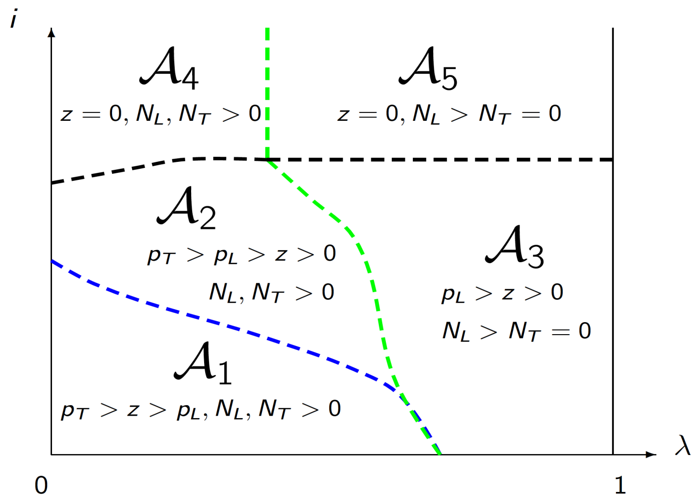

Figure 3: Equilibrium at different i and λ.

for $N_T(z) = 0$. If i is not too high, as is true in the calibration, then $T'(z) < 0$, and there is a unique solution to $T(z) = i$. Moreover, if $N_T > 0$ then $q_T < q^*$, while $q_L < q^*$ or $q_L = q^*$ are both possible, and if $N_T = 0$ then $q_L < q^*$, since buyers must be constrained in some trades. Here are analogs to the results with pure credit:

Proposition 4 *Monetary equilibrium exists uniquely if i is not too big.*

Proposition 5 *In monetary equilibrium the results in Proposition 2 all hold.*

Proposition 6 *In monetary equilibrium, as i increases, z, p_j and q_j decrease, while n_j and D_j increase, continuously. The effect on P_j is ambiguous.*

There are again different types of equilibria, as shown in Fig. 3, this time in (λ, i) space. In area $\mathcal{A}_1$, we have $N_L, N_T > 0$, and buyers use credit in tourist but not local shops. In $\mathcal{A}_2$, we have $N_L, N_T > 0$ and buyers use credit in both. In $\mathcal{A}_3$, $N_T = 0$ and buyers use credit in local shops. When i is too high there is no monetary equilibrium, and agents trade using only credit in $\mathcal{A}_4$ and $\mathcal{A}_5$. Again, submarket T shuts down for big λ. By way of preview, the calibration below puts us in $\mathcal{A}_2$, where buyers use money and credit in both submarkets, but if i were to fall sufficiently, we would move into $\mathcal{A}_1$, with no credit used in local shops.

Although this is discussed more in the context of the quantitative work, recall that some research finds a negative relation between markups and inflation. By Proposition 6, the impact of i on P_j is ambiguous, but even if $\partial P_T/\partial i > 0$ and $P_L/\partial i > 0$, the average DM markup can fall with i because $\partial N_T/\partial i < 0$, $\partial N_L/\partial i > 0$ and $P_T > P_L$. Moreover, the aggregate markup averages this with the CM markup, which is 1 because the CM is competitive. Since the DM shrinks as i rises, the aggregate markup can easily fall. So at least in theory we can match the evidence on inflation and markups.

3.3 Welfare and Policy

Having discussed some technical results and observable implications, let us turn to welfare, measured by the sum of buyers' and sellers' DM payoffs (CM payoffs are constant for the interventions considered). Net of entry costs, this is given by

$$\begin{aligned}\Omega \;=\;& \omega_L \left\{ \frac{\alpha(n_L)}{n_L} \left[u(q_L) - q_L - \gamma\left(p_L - z\right)\right] - \frac{k}{n_L} \right\} + \\ & (1-\omega_L) \left\{ \frac{\alpha(n_T)}{n_T} \left[u(q_T) - q_T - \gamma(p_T - z)\right] - \frac{k}{n_T} \right\}.\end{aligned} \tag{14}$$

With pure credit, tourist shops are bad for two reasons. First, the buyer-seller ratios are different across submarkets, $n_L > n_T$. With $\alpha\left(n\right)$ concave, this does not maximize the number of trades. Second, as usual with competitive search, Γ_L is efficient; in general, however, Γ_T is not. So if feasible a regulator should eliminate tourist shops, but not intervene in the local submarket, where entry and the terms of trade are efficient.

Things are less obvious in monetary equilibrium.[13] It can be checked that $q_j \to q^*$ as $i \to 0$ by (15), (5) and (8), but this does not necessarily mean equilibrium is efficient, due to entry. For i close to 0 it would be desirable to regulate tourist shops out of existence.

[13] To summarize known results, with random search and Nash bargaining, Lagos and Wright (2005) show $i = 0$ is optimal and achieves q^* iff $\theta = 1$. For random search and Kalai bargaining, Aruoba et al. (2007) show $i = 0$ is optimal and implies q^* $\forall\theta$. For directed search and posting, Rocheteau and Wright (2005, 2009) show $i = 0$ is optimal and achieves q^*. With random search and mechanism design, Hu et al. (2009) and Gu et al. (2016) can sometimes support q^* even at $i > 0$. Those models are without entry. With entry, for random search and bargaining, $i = 0$ is optimal but does not generally deliver the first best, as discussed in Berentsen et al. (2007). For posting and directed search with entry, Rocheteau and Wright (2005, 2009) show $i = 0$ may or may not deliver the first best, depending on details.

For $i > 0$, however, this need not be the case, because real balances are inefficiently low. Since $p_T > p_L$, the presence of tourist shops actually encourages money demand, and it is possible that $N_T > 0$ can partially offset the tendency for z to be too low, by the usual theory of the second best.

In practice, it may be hard for regulators to identify tourist shops or dictate their terms of trade. Suppose we focus on controlling i, and let equilibrium determine the market's composition and terms. When $\lambda = 1$ or $\lambda = 0$, inflation is always bad, but we claim that for intermediate λ it can be good. For $(\lambda, i) \in \mathcal{A}_1$, an increase in i has two opposing effects: (i) z and the surplus in submarket T fall; and (ii) N_T falls, so buyers are less likely to end up at tourist shops. There is an non-empty area $\mathcal{A}_1^*$ in the lower right part of $\mathcal{A}_1$ such that the net effect is positive. To see why, notice N_T is small in this area, so the total impact of reducing the surplus in submarket T is small and dominated by the gain from reducing N_T:

Proposition 7 *In monetary equilibrium* $(\lambda, i) \in \mathcal{A}_1^* \Rightarrow \partial\Omega/\partial i > 0$*, where* $\mathcal{A}_1^* \neq \varnothing$*.*

The calibration below puts us in $\mathcal{A}_2$, but reducing i moves us into $\mathcal{A}_1$. Whether or not we move into $\mathcal{A}_1^*$ is sensitve – this does not happen at the point values for parameters, so $i = 0$ is optimal, but fairly minor variations can make it suboptimal. This highlights the importance of partially directed search: the optimality of $i = 0$ is guaranteed for $\lambda \in \{0, 1\}$, but not for $\lambda \in (0, 1)$. And this is less trite than some results – e.g., if people smoke too much and cigarettes tend to be purchased with cash, high i is desirable for health reasons. That is trite unless one explains why cigarettes cannot be taxed directly or why they are purchased with cash in the first place. Here it may not be easy to identify and tax tourist shops directly, and it is an endogenous outcome that they use more cash. To be clear, we are not saying tourist shops are more cash intensive, but since their prices are higher, buyers in submarket T tend to use more cash and more credit.

Now consider the interaction between money and credit. One might expect a higher cost of credit reduces welfare, but that need not be true. From an individual's point of view, both money and credit are costly, but from the social perspective only the latter is costly (given seigniorage revenue is rebated via transfers). Hence, agents use too much credit, and raising its cost might increase Ω. To make this precise, define a ranking of cost functions as follows: $\gamma_2(D)$ is more costly than $\gamma_1(D)$ if $\gamma_2^{-1}(x)$ is weakly flatter than $\gamma_1^{-1}(x)$ $\forall x > 0$. Thus, if $\gamma_2(D)$ is more costly than $\gamma_1(D)$ then $\gamma_2(D) > \gamma_1(D)$ $\forall D \geq 0$. The next result allows us to make the discussion rigorous:

Lemma 3 *For any cost functions γ_2 and γ_1, $\gamma_2(\gamma_2'^{-1}(a)) < \gamma_1(\gamma_1'^{-1}(a))$ $\forall a \geq 0$ if γ_2 is more costly than γ_1.*

Consider first pure random or pure directed search, so only one submarket is open, with $\Gamma = (p, q, n)$ and $D = p - z$. Then making credit more costly is good.

Proposition 8 *Assume $\lambda \in \{0, 1\}$. As γ becomes more costly in the sense defined above, n and D fall while p, q, z and Ω rise.*

Intuitively, when $\gamma(D)$ is more costly, z increases, and so does the surplus per transaction. Hence, given either $\lambda \in \{0, 1\}$, welfare is maximized at $D = 0$ when credit is prohibitively expensive. This is related to Gu et al. (2016), where the cost of credit is 0 for $D \leq \bar{d}$, and ∞ otherwise. There an increase in $\bar{d}$ has no impact – it simply crowds out z, leaving total liquidity the same. Here, since it is costly, increasing the use of credit lowers welfare because it uses up resources without increasing liquidity.

Now consider $\lambda \in (0, 1)$, where the results depend on parameters. If $(\lambda, i) \in \mathcal{A}_1$ then, as $\gamma(D)$ becomes more costly, the surplus in submarket L is unchanged while the surplus in submarket T rises. Hence more sellers enter submarket T and buyers trade at tourist shops with higher probability. Also recall that for $\lambda \in \{0, 1\}$ the optimal policy is $i = 0$, but for other values of λ welfare is higher with $i > 0$ because it shrinks submarket T. Similarly, when $\lambda \in \{0, 1\}$ a lower cost of credit hurts, but for some λ it can raise Ω by discouraging tourist shops:

Proposition 9 *For $(\lambda, i) \in \mathcal{A}_1$, as γ becomes more costly, Γ_L stays the same, p_T, q_T and ω_T rise, and n_T falls. For $(\lambda, i) \in \mathcal{A}_1^*$, Ω falls as γ becomes more costly.*

3.4 The Impact of Information

Here we consider exogenous changes in λ, as well as the implications of making λ endogenous. For the first, recall that standard nonmonetary models predict that prices fall when there are more informed buyers. This is also true in our model with pure credit: an increase in λ does not affect Γ_T or Γ_L, but increases N_L/N_T, and hence average p and $P = p/q$ fall. However, in a monetary economy, a change in λ affects not only information, it endogenously affects z and hence sellers' strategies.

To pursue this, recall that Γ_j does not depend on λ directly, but only through z, and p moves in the same direction as z. Then the next result is useful:

Proposition 10 *In monetary equilibrium, z is monotone in λ. For some $\tilde{\imath} > 0$, $\partial z/\partial\lambda \leq 0 \; \forall i < \tilde{\imath}$; for $i > \tilde{\imath}$, $\partial z/\partial\lambda > 0$ is possible.*

This is illustrated by example in the right panel of Fig. 4, where $\partial z/\partial\lambda > 0$ for some i. The left panel is a different example, using parameters calibrated below, where $\partial z/\partial\lambda < 0 \; \forall i$; but one does not have to stray far from the calibration to get $\partial z/\partial\lambda > 0$. Again, when z rises, so does p. If q represents unobserved quality, we can call p the price and say it rises with λ, counter to conventional wisdom. We are less sure about $P = p/q$, since q goes up along with p, but in examples P can go up with λ. The bottom line is that prices need not fall when information improves in monetary economies.[14]

Now suppose buyers can acquire information if they pay a fixed cost $s > 0$, and are uninformed with probability 1 otherwise. We claim that as long as s is not too big, so that some buyers become informed, and increases in i around 0 improves welfare. To verify this, first, note the informed and uninformed now choose different real balances, z_I and z_U. Given this, the expected payoff for an uninformed buyer in submarket j is

[14]Here is the intuition: As λ increases, buyers more often buy at local shops. Then z rises iff the marginal value of money is higher in submarket L than T. If i is small, z is almost enough to get q^* at local shops, so an extra dollar is marginally more valuable in submarket T. As λ increases agents reduce z and in response p_L and p_T fall. If i is large, however, an extra dollar can be more valuable in submarket L. In this case z, p_L and p_T can increase with λ. Of course, for very big λ submarket T shuts down and further increases have no impact, as shown by the flat segments in Fig. 4.

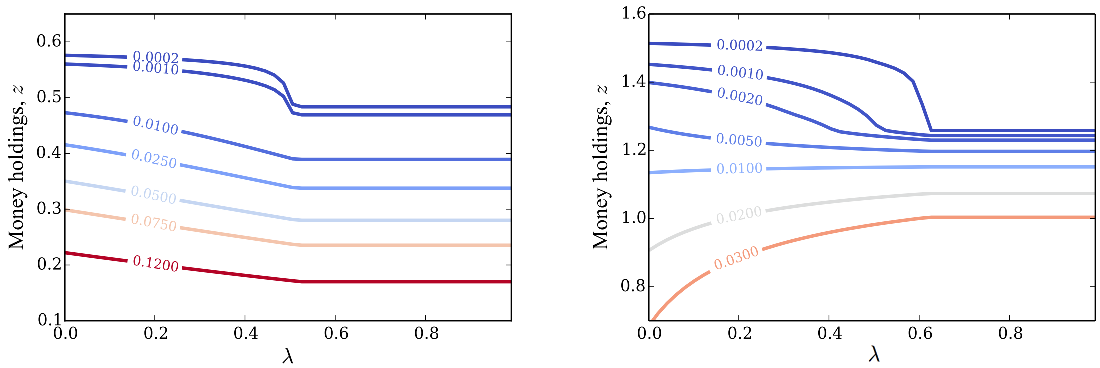

Figure 4: Effect of λ on z for different i in examples.

$B_j(z_U)$, where

$$\begin{aligned} B_T(z_U) &\equiv \frac{\alpha(n_T)}{n_T} \{u[q_T(z_U)] - p_T(z_U) - \gamma[p_T(z_U) - z_U]\} \\ B_L(z_U) &\equiv \frac{\alpha(n_L)}{n_L}[u(q_L) - p_L - \gamma(p_L - z_U)]. \end{aligned}$$

Then the expected DM payoffs for uninformed and informed buyers are

$$\begin{aligned} V_U &= \max_{z_U}\{W(0) - iz_U + \frac{N_L}{N_T + N_L}B_L(z_U) + \frac{N_T}{N_T + N_L}B_T(z_U)\} \\ V_I &= \max_{z_I}\{W(0) - iz_I + B_L(z_I)\}. \end{aligned}$$

A buyer is willing to pay for information iff $s \leq V_I - V_U$.

Consider equilibrium where $\lambda \in (0, 1)$ and $s = V_I - V_U$. For small s and i, such an equilibrium exists uniquely.[15] The Appendix shows this:

Proposition 11 *With endogenous information,* $\lambda \in (0, 1) \Rightarrow i > 0$ *is optimal.*

In terms of economics, when i increases from near 0, buyers want to carry less cash, and hence are more willing to pay s to avoid tourist shops. Lower money balances entail a second-order welfare loss, by the Envelope Theorem, but since there are more informed buyers we get fewer tourist shops, and that is a first-order gain. This reminiscent of

[15]We only prove uniqueness for small i, but there should be no obvious presumption of multiplicity, because when more buyers are informed there are fewer tourist shops and that makes information less valuable. This is different from Lester et al. (2012), e.g., where sellers pay for information, while buyers choose asset positions, and that makes multiplicity natural.

Benabou (1992a) and Head and Kumar (2005), although the economics is different. They say that inflation increases price dispersion, which increases the return to search, which increases competition across sellers and hence buyers' surplus. Here inflation directly discourages tourist shops, which is beneficial whether or not it increases price dispersion. To say more about these kinds of issues, the next step is to calibrate the model.

4 Quantitative Analysis

Here we discuss the quantitative impact of changing inflation, information and credit conditions, and show the model can match certain facts. For this exercise, a period is a year, with $1 + r = 1/\beta = 1.03$. However, in this type of model period length is not critical, and can change with minimal effect, if we scale variables like r and the trading probabilities (see, e.g., Aruoba et al. (2011)).

4.1 Calibration

The CM and DM utility functions are $U(x) = \log(x)$ and $u(q) = Bq^{1-b}/(1-b)$, with (b, B) set to match the aggregate money demand curve, i.e. the empirical relationship between nominal interest rates and some measure of money scaled by output, M/PY. With $U(x) = \log(x)$, real CM output is $x^* = 1$ (a normalization), while DM output in the same units from submarket j is $\alpha(n_j)N_j[p_j - \gamma(p_j - z)] - N_j k$. Aggregate Y sums these, while M/P is given by z, and hence M/PY is determined in equilibrium as a function of i. As usual, for our measure of M we want some notion of $M1$.[16]

[16]Some economists, e.g. Lucas (2000), express a tension between what in theory looks like currency but in practice, in their empirical work, includes demand deposits. We do not consider this a problem. In many recent papers in the area, various assets can be more or less available for use in the DM (see Venkateswaran and Wright (2013) for an extended discussion). As rationale for $M1$, consider these points: (i) Checks and debit cards are heavily used in retail, where they are almost as liquid as currency, and backed by demand deposits paying about the same interest. (ii) It is not really relevant for our theory whether one's money is in one's pocket or checking account. (iii) A key feature of credit is that it allows buyers to pay for DM goods by working in the *next* CM, while cash, check and debit purchases all require working in the *previous* CM, which matters especially when DM trade is random. (iv) Using $M1$ in the macro data is consistent with measuring money and credit usage in the micro data discussed below.

The best available information on money demand comes from Lucas and Nicolini (2012), a sample of annual observations between 1919 and 2008 on $M1J$ and the nominal interest rate on 3 month T-bills. Their $M1J$ series augments the usual $M1$ series by including money market accounts after regulatory amendments in 1980 made these about as liquid as checking accounts, and Lucas and Nicolini (2012) argue that with this correction the empirical money demand relationship is stable over the sample. To fit the data with $u(q) = Bq^{1-b}/(1-b)$, intuitively, changing B shifts the curve up and down and is set to match a mean M/PY of 0.27, while b captures the elasticity and is set to minimize the sum of squared residuals between the model and data. A version of this procedure is used in most quantitative monetary economics.

Let the use of credit in the aggregate, as opposed to in a given buyer-seller meeting, be Δ. The cost of credit function is $\gamma(D) = CD^c$, where D is total credit in a meeting because here we set $\bar{d} = 0$. Consumers' willingness to substitute between money and credit as i changes is captured by c, and the share of purchases made with credit by C. Mimicking the procedure for money demand, we set (c, C) to match the empirical relationship between Δ/Y and i or, equivalently, given we match z/Y, the empirical relationship between z/Δ and i. We use Federal Reserve Board data on credit for household, family and other personal expenditure, exclusive of loans secured by real estate (see FRB's G.19 consumer credit release, FRED Series: TOTALSL). This is appropriate since such credit largely supports retail trade. In annual observations between 1943 and 2008, on average $z/\Delta = 2.3$, which implies about 30% of DM transactions by value are made with credit, not far from the micro data.[17]

Estimated credit and money demand curves are shown in Fig. 5. The fit for the latter (top left) is good, even at low interest rates, which is a challenge for some models

[17] Note that 30% is not targeted, but can be considered a consistency check. Liu et al. (2015) discuss the micro data and primary sources in more detail, but as a quick summary, in both Boston Fed and Bank of Canada studies, credit is used in 20% of transactions by volume. By value rather than volume, American data say it is still around 20% while Canadian data say it is closer to 40%. It is a puzzle why they agree by volume but not value, but those who collect the data stand by their numbers. One can say the Canadian findings are more in line with the conventional view that cash is used for smaller purchases. In any event, our 30% by value falls precisely midway between the American and Canadian numbers.

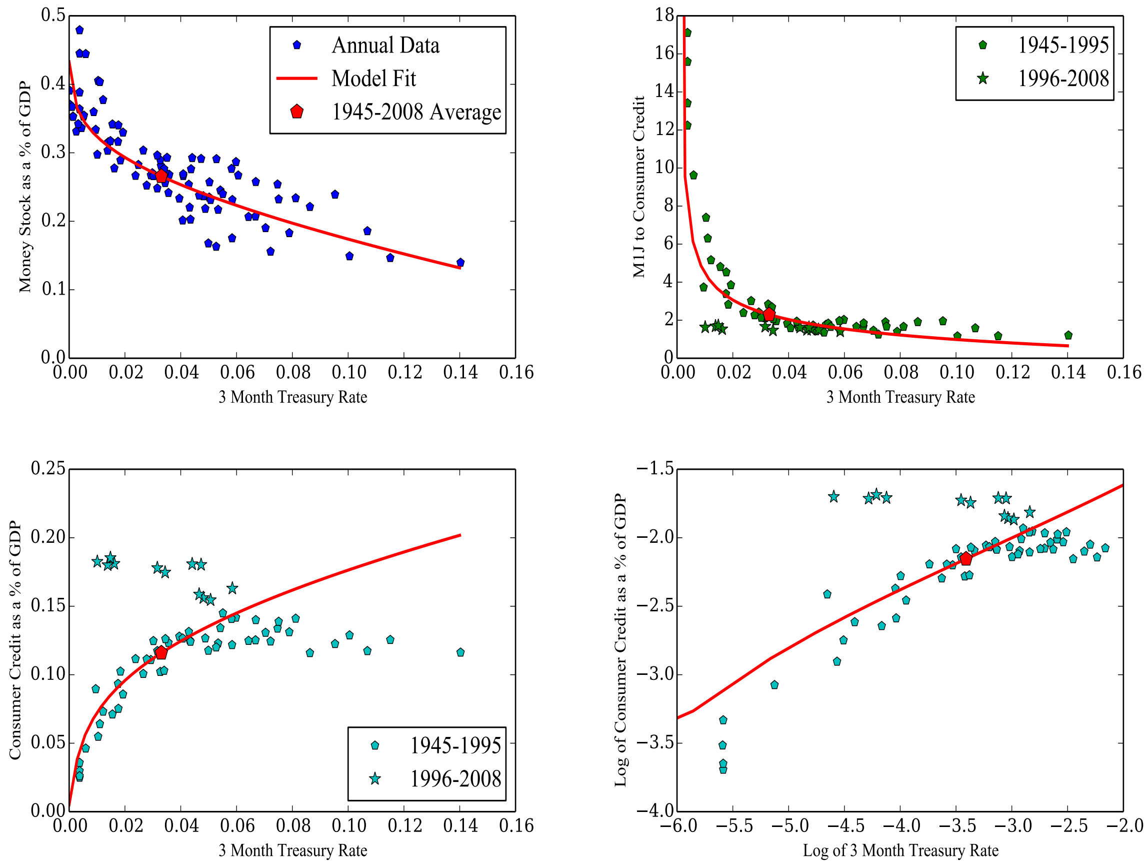

Figure 5: Estimated money and credit demand curves.

(see Lucas, 2000). The fit for credit in levels (bottom left) or logs (bottom right) is reasonable, although apparently there is a structural break in the 1990s. However, the relationship between z/Δ and i (top right) seems stable and the fit is remarkably good, displaying clear substitution from money to credit when i rises. Understanding the break in the credit series is beyond the scope of this paper, but perhaps future work can try to capture this by changes in, e.g., lending practices. While similar issues are relevant for monetary data, Lucas and Nicolini (2012) try to control for this, which is not the case for the credit data. Still, the key factor for us is substitutability between money and credit, and this we evidently capture quite well.[18]

[18] If an upward-sloping credit demand curve seems puzzling, note that standard theory concerns borrowing and *real* interest rates, while i is the *nominal* rate. Credit usage increases with i since it is the relative cost of money and credit in their roles as alternative payment instruments.

The DM matching technology is $\alpha(n) = n/(1+n)$, following search models of money going back to Kiyotaki and Wright (1993). Buyers' bargaining power is θ, and with this matching technology, their effective bargaining power in local shops is endogenously given by $\eta = n/\left[u'(q)+n\right]$. The degree of price dispersion depends on $\theta - \eta$, which depends on the measure of informed buyers λ. We calibrate θ to match the relative standard deviation of retail store prices in the Kaplan and Menzio (2014) data. However, using their standard deviation of 19% would attribute all price dispersion to informational heterogeneity, which seems extreme. Hence, our benchmark is the dispersion they report due to differences across stores for a given good at a given time, and differences within a store for a given good over time, which accounts for 55% of their total variability. This gives a relative standard deviation of 14%, and implies $\theta = 0.73$.

Then λ is set so the DM markup matches the average in retail trade survey data (see https://www.census.gov/retail). The average ratio of gross margins to sales in retail between 1992-2008 is 0.28, implying an average markup of $1 + 0.28/(1-0.28) = 1.39$. Our markups are 1.55 and 1.27 in submarkets T and L, given the latter contributes about 53% of DM output. This is quite close to differences in markups in the data where the low end includes, e.g., Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, while the high end has Specialty Foods, Clothing, Footwear and Furniture. We do not push this too hard, as our variability is not due to heterogeneous goods but information (although Warehouse Clubs and Superstores are arguably excellent examples of local shops). Finally, entry cost k is set to get an aggregate (across the CM and DM) markup of 1.1 based on Basu and Fernald (1997). The size of the DM depends on k. Since our DM markup is 1.39 while the CM markup is 1.0, k pins down the average trade-weighted markup, and makes the DM share of total output 25%.[19]

Table 1 summarizes the discussion. As one can see there are not too many parameters, considering the specification has money and credit, heterogeneous information,

[19] We do not calibrate the CM and DM output shares – like the shares of local and tourist shops in the DM, these emerge from targeting observables. Conveniently, at least for remembering the numbers, the DM contributes $1/4$ of total output, with $1/2$ of that coming from tourist shops and $1/2$ from local shops.

bargaining plus posting, etc. Moreover, they are all naturally tied to simple and economically reasonable targets.

Description	Value	Source/Target
DM utility curvature, b	0.65	relationship between z/Y and i
DM utility level, B	0.55	average z/Y
Credit cost curvature, c	4.01	relationship between z/Δ and i
Credit cost level, C	2.06	average z/Δ
Informed buyers, λ	0.27	average retail markup
Sellers' DM entry cost, k	0.015	average aggregate markup
Buyer's bargaining power, θ	0.73	average price dispersion

Table 1: Calibrated parameter values

4.2 Inflation

We use a standard measure of welfare: first, compute the equilibrium payoff at a given π; then compute the percentage reduction in total (CM plus DM) consumption that agents would accept to reduce inflation to a new level, like $\pi = 0$, or the Friedman rule $i = 0$. Fig. 6 reports results using $i = 0$, chosen because that is optimal at the calibrated parameters. In the benchmark specification, the cost of having 10% inflation, e.g., is 1.39% of consumption. Compared to models with pure random search and bargaining 1.39% is low. One might guess this is due to a large fraction of informed buyers, but since calibration yields only $\lambda = 0.27$, there is more involved.

Figure 6 also gives results for $\lambda = 0$ and 1, i.e., pure random and pure directed search.[20] Even at $\lambda = 0$ our results are significantly lower than existing search-and-bargaining models, due various factors, including the bargaining solution, costly credit and free entry. Suppose we try to replicate the results in Lagos and Wright (2005), e.g., at $\lambda = 0$ by picking similar values of θ and shutting down entry. With Nash bargaining and $\theta = 0.315$, their cost of 10% inflation is 6.9% while we get 6.0%; and at $\theta = 0.5$ they get 3.6% while we get 3.3%. These numbers are close but not exactly the same because

[20]The models are re-calibrated using a similar procedure, except when $\lambda \in \{0, 1\}$ we drop the target for DM price dispersion, since there isn't any, and when $\lambda = 1$ we drop the retail markup, since we lose the parameter θ.

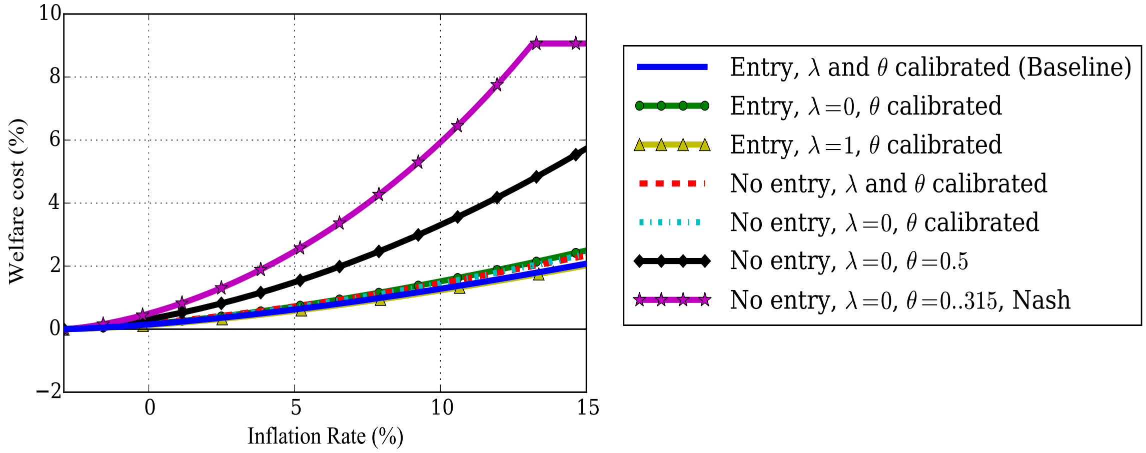

Figure 6: Cost of inflation: Model comparisons.

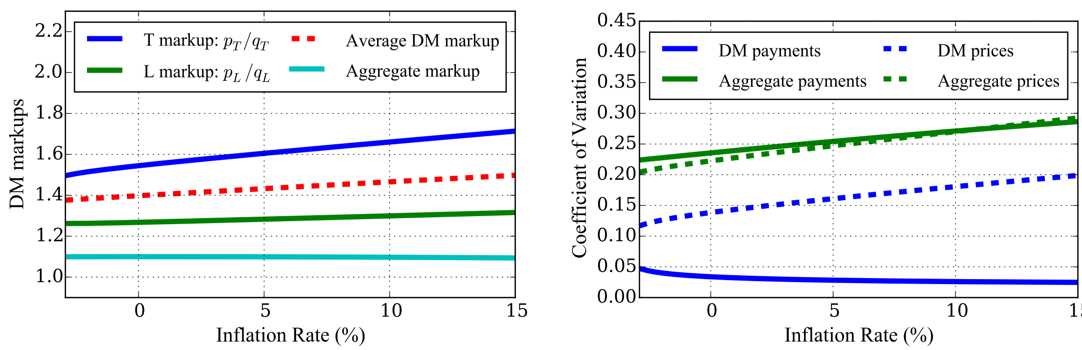

Figure 7: Effects of inflation on markups and price dispersion.

our calibration procedure and data are slightly different, we use Kalai bargaining, and we have credit. Still, starting from this approximation, one can measure the contribution of each factor. Our calibration yields $\theta = 0.73$, higher than what they had because the way we use markup data is different and arguably better. Using this instead of $\theta = 0.5$ lowers the welfare cost from 3.3% to 1.6%. Allowing entry brings it down further to 1.5%. And finally, using $\lambda = 0.27$ instead of 0 delivers 1.39%, our baseline result.

Inflation affects not only welfare, but observables like markups and price dispersion, as discussed above. The left panel of Fig. 7 shows DM markups are increasing in π, counter to some empirical findings. But because the DM shrinks with π, the aggregate markup averaged over the CM and DM is falling in π. This makes it clear how the choice of sample is important in empirical work – both positive and negative relationships are consistent with theory, in general, and consistent with the calibration, depending on what we measure. A similar issue applies to price dispersion, shown in the right panel of Fig. 7: dispersion in aggregate prices increases while dispersion in DM (retail) prices decreases with π.

4.3 Information

Consider now the consequences of information frictions. In Fig. 8, the top-left panel shows the cost (benefit if negative) of moving away from the calibrated $\lambda = 0.27$. There are two forces. As λ increases the *ex ante* probability of trading at a tourist shop falls, as shown in the bottom left. Then, since local shops are less expensive, buyers carry less cash. As Proposition 6 says, lower z implies lower surplus in both submarkets, suggesting Ω may fall as λ increases; but the gain from having more local shops dominates, and on net Ω goes up. Precisely, increasing λ from 0 to 1 is worth a sizable 3.32% of consumption. Indeed, Ω peaks around $\lambda = 0.53$, where submarket T shuts down. The good news is that having just over half of the buyers informed renders this friction innocuous. The bad news is that calibration only delivers $\lambda = 0.27$.

We can also ask how information affects markups and price dispersion. In Fig. 9, as λ increases the markup in each submarket rises, but the average DM markup falls as a result of changing composition. Also, the aggregate markup across the CM and DM falls since the latter shrinks. Again, the effect on markups depends on which market we consider. In terms of price dispersion, for low (high) values of λ it increases (decreases) with information. So both positive and negative effects of information on dispersion can be consistent with the theory. Further, for low values of λ, aggregate price dispersion falls while retail dispersion rises with information.

We can also ask how the cost of inflation changes with λ. As λ falls from 1 to 0, submarket T increases, and since inflation discourages tourist shops this can be good for welfare. Inflation also hurts consumer surplus in both local and tourist shops, but as

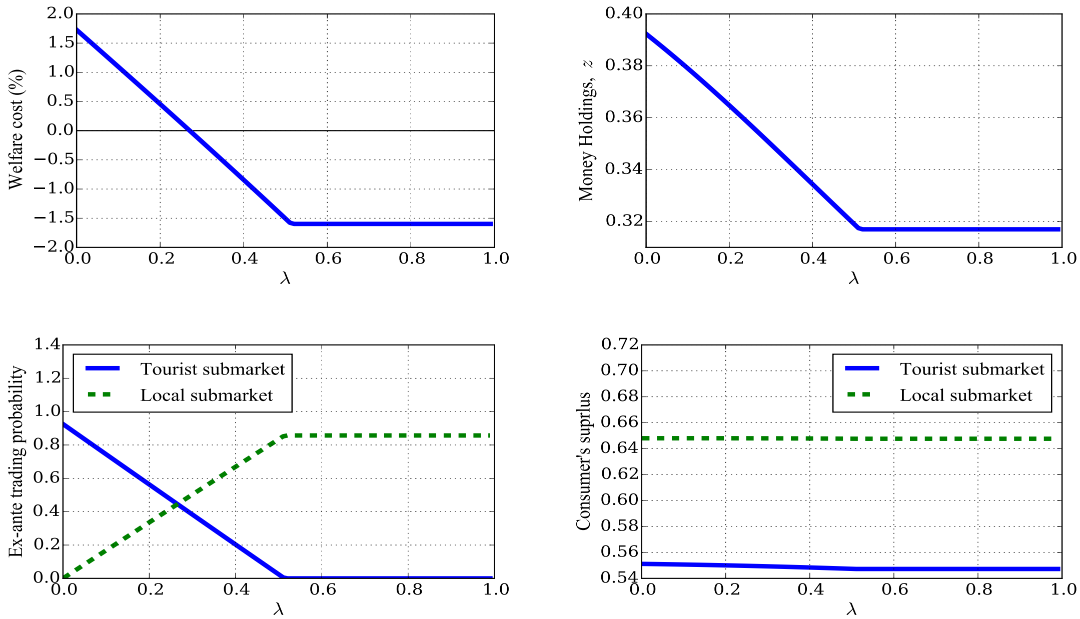

Figure 8: Effects of changing λ.

discussed above, for intermediate values of λ the first effect dominates. This is shown in Fig. 10 for $\lambda \in \{0.01, 0.27, 0.50, 0.99\}$. In the left panel, with $\pi > 7\%$, the cost of inflation increases as λ falls, in line with work showing inflation is more costly with random search and bargaining. However, for $\pi < 7\%$, the cost decreases as λ falls due to composition effects. For different λ, inflation can hurt more than with either $\lambda = 0$ or $\lambda = 1$, or can be welfare improving. The right panel of Fig. 10 shows that at the calibrated $\lambda = 0.27$, moderate inflation hurts welfare more than when $\lambda \in \{0, 1\}$.

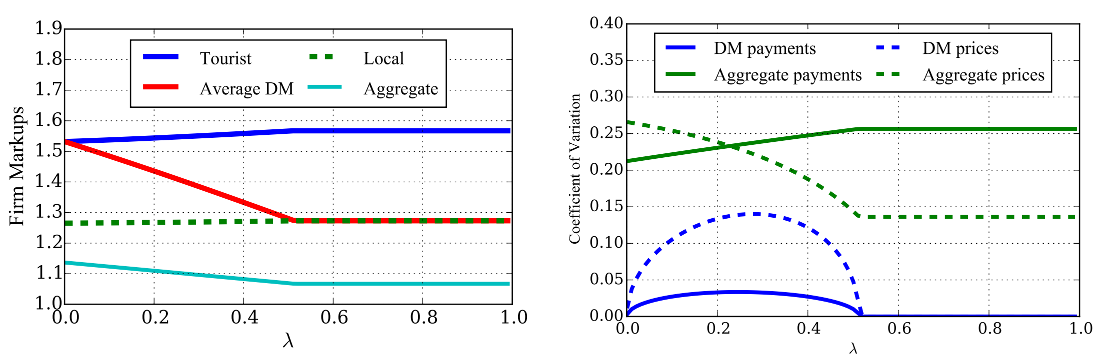

Figure 9: Effects of λ on markups and dispersion.

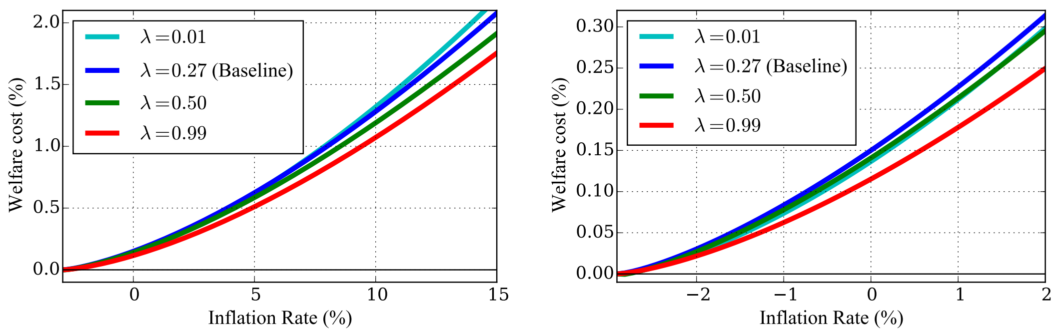

Figure 10: Cost of high (left) and low inflation (right) for alternative λ.

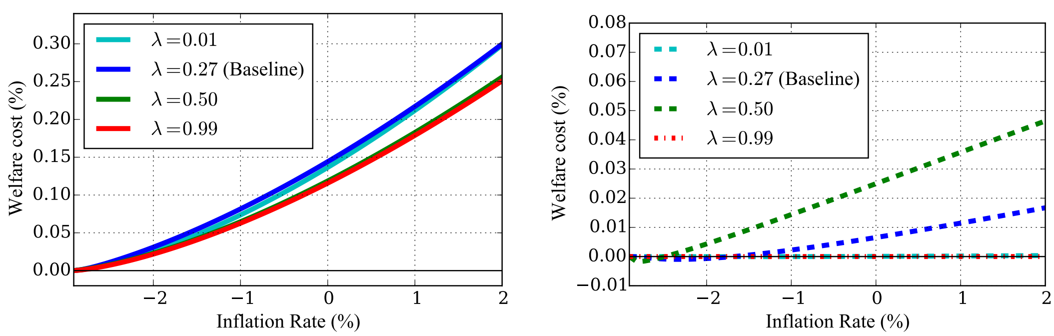

Figure 11: Cost of inflation from surplus (left) and composition (right).

How exactly does information affect the cost of inflation? To investigate this, consider decomposing welfare into two components:

$$\Omega = \sum_{j=L,T} \underbrace{\omega_j}_{\text{composition}} \underbrace{\frac{\alpha(n_j)}{n_j} \left[u(q_j) - p_j - \gamma\left(p_j - z\right)\right]}_{\text{expected surplus}}.$$

Fig. 11 recalculates the cost of 10% inflation instead of the Friedman rule holding the expected surplus (right) and composition (left) constant. In the left panel, inflation lowers the surpluses, and the impact is larger at $\lambda = 0.27$ than $\lambda = 0$. Hence, reducing information can help welfare on this dimension at low inflation rates. In the right panel, only composition changes. When $\lambda \in \{0, 1\}$, composition is fixed, but when $\lambda \in (0, 1)$ small deviations from $i = 0$ discourage tourist shops and improve welfare. As i continues to rise, it discourages both local and tourist shops, but the former at a faster rate. From this effect better information can compound the cost of inflation.

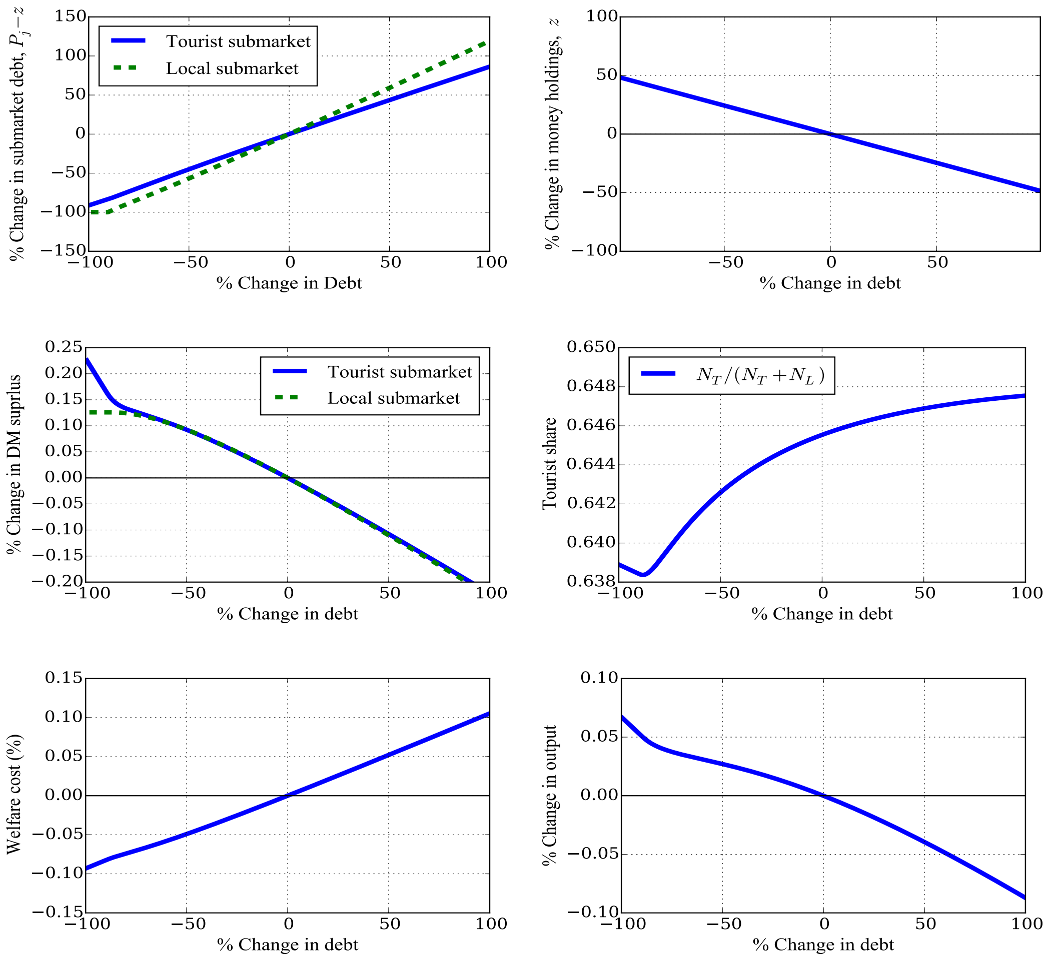

Figure 12: Effects of changes in debt induced by changes in the cost of credit.

4.4 Credit

One reason to introduce credit is to give agents an alternative to money that allows them to partially avoid the inflation tax. However, credit is not costless and, as has become apparent over the last decade, credit conditions fluctuate. We now use the model to examine the implications of changes in debt resulting from changes in the cost of credit. As in Section 3.3, we vary the cost of credit $\gamma(D) = CD^c$ by changing C, and compute the impact on endogenous variables, as summarized in Fig. 12.

In the spirit of Proposition 8, lower C raises debt in a submarket whenever it is used, and this crowds out z, but less than one for one (top-right). The effect on the surplus

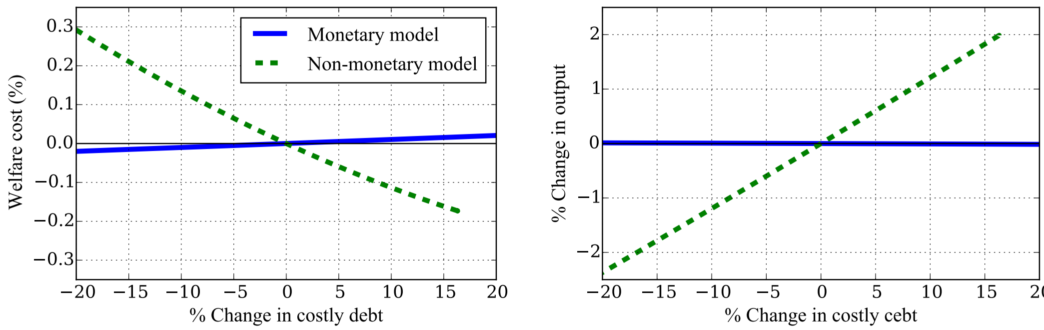

Figure 13: Effects of credit in monetary and nonmonetary models.

differs in submarkets L and T. For high C, debt is not used in submarket L (top-left). In this case, as C falls, q_T increases and p_T decreases, with the net effect on the surplus negative. As a result, tourist shops exit, so total output and welfare fall. As C falls further, credit gets used in both submarkets. In this range a decrease in C benefits the more expensive tourist shops, so that p_T and q_T rise while p_L and q_L fall. However, the net effect on each submarket, including the cost of credit, is negative. There is then a shift in composition towards tourist shops, and Ω falls. Indeed, Ω is monotonically decreasing with the use of debt.

Many recent studies of credit are void of cash. To see how this matters, consider changing credit conditions in the nonmonetary equilibrium of our model. For this experiment we keep the parameters in Table 1 unchanged, but add a costless credit limit $\bar{d}$, set to z from monetary equilibrium (this means we do not have to recalibrate and ensures differences in results are not due to differences in parameters). As Fig.13 shows, in nonmonetary equilibrium, tighter credit is quite bad: changing C to get a 20% reduction in debt has a welfare cost of 0.3% and reduces output by 2.0%. In monetary equilibrium the effects are reversed. Now, real-world credit markets are complicated and varied, and we do not claim, e.g., ours is a good model of mortgages, student loans or other forms of borrowing over the life cycle. Still, the results provide a word of caution to those trying to measure the importance of credit conditions in environments where agents have nothing in the way of alternatives like money.

5 Conclusion

This paper has reported results from our study of frictional goods markets. The theory was shown to display the following features: there is price dispersion; markups are high and variable; there is some random and some directed search; buyers use both money and credit; and while sellers typically post the terms of trade, there is also some bargaining. This resembles actual retail trade (abstracting from other features, e.g., long-term relationships). We derived sharp analytic results on the existence of equilibria with various properties and on the effects of parameter changes. The model can capture the behavior of observables like price dispersion and markups. Some novel results emerged, such as the possibility of higher prices when consumers are better informed in monetary equilibrium. In terms of policy, deviations from the Friedman rule can be optimal, and although that is not true at the calibration, it is for small changes in parameters. The economic channel involves composition effects, given that inflation is a tax, and that it hits expensive, inefficient, tourist shops harder.

Calibration involved information on retail and aggregate markups, price dispersion, and the empirical credit and money demand relations. This was used to quantify several effects. The implied cost of 10% inflation is 1.39% of consumption, which can be decomposed into components related to bargaining, entry and information. Our stylized decentralized market accounted for $1/4$ of output, with about equal shares coming from local and tourist shops, and the fraction of informed consumers was $\lambda = 0.27$. Compared to an economy where buyers are all uninformed – i.e., pure random search – this is half way to the λ required to put tourist shops out of business. We also analyzed the quantitative impact of information frictions and credit conditions. Reducing the cost, and hence increasing the use, of credit was shown to be bad. Although this effect was not large, it is certainly different than one would estimate if one ignored money broadly defined. We think we learned a lot from these exercises, and that much more can be done in terms of theory, measurement and policy analysis; that is left for future work.

Appendix: Proofs

Lemma 1: In monetary equilibrium, we can rewrite (10) using $u\left(q_T\right)-p_T-\gamma\left(D_T\right)=\theta\left[u\left(q_T\right)-q_T-\gamma\left(D_T\right)\right]$ (from Kalai bargaining) as

$$\begin{aligned}
V'\left(z\right) &= 1+\omega_L\frac{\alpha(n_L)}{n_L}\gamma'\left(D_L\right)+\omega_T\frac{\alpha(n_T)}{n_T}\theta\left[u'(q_T)q_T'-q_T'-\gamma'(D_T)\left(p_T'-1\right)\right] \\
&= 1+\omega_L\frac{\alpha(n_L)}{n_L}\gamma'\left(D_L\right)+\omega_T\frac{\alpha(n_T)}{n_T}\theta\gamma'\left(D_T\right)\left(q_T'-p_T'+1\right) \\
&= 1+\omega_L\frac{\alpha(n_L)}{n_L}\gamma'\left(D_L\right)+\omega_T\frac{\alpha(n_T)}{n_T}\frac{\theta\gamma'(D_T)}{1+\theta\gamma'(D_T)}. \qquad (15)
\end{aligned}$$

The second line uses $u'\left(q_T\right)=1+\gamma'\left(D_T\right)$ by (8). The third line uses $q_T'-p_T'+1=1/[1+\theta\gamma'(D_t)]$ by differentiating (9) wrt z. Since ω_j and n_j are independent of z and γ is convex, $V''\left(z\right)\leq 0$ as long as $D_j=p_j-z$ decreases in z for $j=L,T$, and D_L falls in z, because p_L is not a function of z. Next, D_T falls in z by (8) and (9). Finally, $V''\left(z\right)<0$ if either D_L or D_T falls strictly in z, and D_j falls strictly in z as long as it is strictly positive. We know either $D_L>0$ or $D_T>0$ because $z<\max\{p_L,p_T\}$. ■

Lemma 2: In monetary equilibrium, the FOC holds at equality, namely $1+i=V'\left(z\right)$. Hence, $i>0$ implies either $\gamma'(D_L)>0$ or $\gamma'(D_T)>0$ by (15). This implies either $D_L>0$ or $D_T>0$ as $\gamma'(D)>0$ iff $D>0$, and thus $z<\max\left\{p_L,p_T\right\}$. ■

Proposition 1: Given $\bar{d}$, from (8)-(9), the solution for $q_T(\bar{d})$ and $p_T(\bar{d})$ are unique and $p_T'(\bar{d})-q_T'(\bar{d})\geq 0$ (see the proof of Proposition 3). Given (3) and $\theta<\underline{\eta}$, if $\bar{d}$ is big enough then $p_T(\bar{d})-q_T(\bar{d})>k$. In this case there is a unique $n_T(\bar{d})>0$ solving $\alpha(n_T(\bar{d}))(p_T(\bar{d})-q_T(\bar{d}))=k$, and submarket T is active at $\Gamma_T=\left(p_T(\bar{d}),q_T(\bar{d}),n_T(\bar{d})\right)$. One can also show $n_L(\bar{d})$ and $q_L(\bar{d})$ are unique (see the proof of Proposition 3). By free entry $p_L(\bar{d})$ is also unique, and submarket L is active at Γ_L if the surplus for consumers is nonnegative. Since the terms of trade in submarket L are better than submarket T, if the latter is active so is the former. Hence, a unique pure credit equilibrium exists when $\bar{d}$ is not too small. ■

Proposition 2: From $N_L>0$, $\lambda>0$ (there cannot be local shops if everyone is uninformed). Immediately (2) and $\lambda,N_L>0$ imply $n_L>n_T$. Since n_L solves (4), $n_L\leq\hat{n}$ where $\alpha\left(\hat{n}\right)=1$. For $n_L>\hat{n}$ the objective function in (4) can be increased by lowering n_L. Hence, $\alpha(n_L)>\alpha(n_T)$, and $\Pi_L=\Pi_T$ implies $p_L-q_L<p_T-q_T$. The

results for P and R are obvious once we check q and p, so it remains to show $q_L \geq q_T$ and $p_L < p_T$. There are different cases. First suppose $p_L, p_T \geq \bar{d}$. By (5)-(8), if q_j is big then p_j is small, so $p_L - q_L < p_T - q_T$ implies $p_L < p_T$ and $q_L > q_T$. Second suppose $p_L < \bar{d} \leq p_T$. By (5)-(8), $q_L = q^* \geq q_T$. Third suppose $p_L \geq \bar{d} > p_T$. By (5)-(8), $q_T = q^* \geq q_L$, but then $\Pi_L = \Pi_T$ implies $p_T \geq p_L$, contradicting the supposition $p_L \geq \bar{d} > p_T$; so this case cannot occur. Finally suppose $p_L, p_T < \bar{d}$. Then $q_L = q_T = q^*$, so $\Pi_L = \Pi_T$ implies $p_L < p_T$. If $q_T < q^*$, then either the first or the second case holds, and $q_L > q_T$. ■

Proposition 3: Consider first tourist shops, with bargaining, which is easy compared to directed search with posting. If $p_T^* \leq \bar{d}$, then no costly credited is needed and thus all derivatives wrt $\bar{d}$ are 0. If $p_T^* > \bar{d}$, we can apply the Implicit Function Theorem to (8)-(9) to define the bargaining solution $p_T(\bar{d})$ and $q_T(\bar{d})$. Then differentiate them wrt $\bar{d}$ to get

$$\begin{aligned} p_T'(\bar{d}) &= 1 - \left\{ 1 - \frac{\gamma''(D_T)}{u''(q_T)} \left[1 + (1-\theta)\gamma'(D_T) \frac{\gamma''(D_T) - u''(q_T)}{\gamma''(D_T)} \right] \right\}^{-1} \in [0,1) \\ q_T'(\bar{d}) &= \left[p_T'(\bar{d}) - 1 \right] \frac{\gamma''(D_T)}{u''(q_T)} > 0. \end{aligned}$$

Then $D_T'\left(\bar{d}\right) = p_T'(\bar{d}) - 1 < 0$ when $D_T > 0$ because $p_T'(\bar{d}) \in [0,1)$. Next, the RHS of (9) rises in $\bar{d}$ because $q_T'(\bar{d}) > 0$ and $D_T'\left(\bar{d}\right) < 0$, and thus $R_T'\left(\bar{d}\right) > 0$. This implies $n_T'\left(\bar{d}\right) < 0$ by free entry.

Now consider local shops' problem (4). Making a change of variables by defining $y = p - \bar{d} - q$ and using the constraint to eliminate p and $\bar{d}$, (4) becomes

$$\max_{n,q,y} \left\{ \frac{\alpha(n)}{n} \left[u(q) - q - \gamma(y+q) \right] - \frac{k}{n} \right\} \text{ st } \alpha(n)(y+\bar{d}) \geq k. \tag{16}$$

We can first choose q independent of n. Define $G(y) \equiv \max_q\{u(q) - q - \gamma(y+q)\}$. The solution for q satisfies $u'(q) - 1 = \gamma'(y+q)$ because u is concave and γ is convex. Thus, q decreases continuously and is differentiable in y. By the Envelope Theorem, $G'(y) = -\gamma'(y+q) < 0$ and $G''(y) = \gamma''(y+q)u''(q)/[\gamma''(y+q) - u''(q)] < 0$. Eliminating y using the constraint from (16), we get

$$\max_n F(n,\bar{d}) \equiv \max_n \left\{ \frac{\alpha(n)}{n} G\left[\frac{k}{\alpha(n)} - \bar{d} \right] - \frac{k}{n} \right\}.$$

Since $F(0, \bar{d}) = -\infty$, $n = 0$ is not a solution. Since $\alpha(n) = 1$ and $\alpha'(n) = 0 \; \forall n > \hat{n}$, the solution is $n(\bar{d}) \leq \hat{n}$, as otherwise we can lower n to increase the objective function. Thus $n(\bar{d}) \in (0, \hat{n}]$. If $n(\bar{d})$ is interior then $\partial F(n, \bar{d})/\partial n|_{n=n(\bar{d})} = 0$; otherwise $n(\bar{d}) = \hat{n}$.

Next we verify the SOC and show the solution for n is unique. Consider

$$\begin{aligned}\frac{\partial F(n, \bar{d})}{\partial n} &= \left[\frac{\alpha'(n)}{n} - \frac{\alpha(n)}{n^2}\right] G\left[\frac{k}{\alpha(n)} - \bar{d}\right] - \frac{k\alpha'(n)}{\alpha(n)n} G'\left[\frac{k}{\alpha(n)} - \bar{d}\right] + \frac{k}{n^2} \\ &= \frac{1}{n^2}\left\{[n\alpha'(n) - \alpha(n)]\, G\left[\frac{k}{\alpha(n)} - \bar{d}\right] - \frac{k\alpha'(n)n}{\alpha(n)} G'\left[\frac{k}{\alpha(n)} - \bar{d}\right] + k\right\}.\end{aligned}$$

This derivative vanishes at an interior solution. To verify the SOC, differentiate the expression in the braces wrt n to derive

$$\alpha''(n)nG\left[\frac{k}{\alpha(n)} - \bar{d}\right] - \frac{k\alpha''(n)n}{\alpha(n)} G'\left[\frac{k}{\alpha(n)} - \bar{d}\right] + \frac{\alpha'(n)^2 nk^2}{\alpha(n)^3} G''\left[\frac{k}{\alpha(n)} - \bar{d}\right].$$

Since $G[k/\alpha(n) - \bar{d}] > 0$ at $n = n(\bar{d})$, this is strictly negative at $n = n(\bar{d})$ because $\alpha'', G', G'' < 0$, so the SOC is satisfied.

Next we argue the solution is unique. Since $G'(y) < 0$ and $\alpha' \geq 0$, $G[k/\alpha(n) - \bar{d}] > 0 \; \forall n \geq n(\bar{d})$. This implies the expression is strictly negative $\forall n \geq n(\bar{d})$ and thus the solution to $\partial F(n, \bar{d})/\partial n = 0$ is unique whenever it exists. Moreover, $n = \hat{n}$ cannot be optimal when an interior solution exists because $\partial F(n, \bar{d})/\partial n < 0 \; \forall n$ that exceeds the interior solution. This proves the solution is in general is unique. Now we show $n(\bar{d})$ falls in $\bar{d}$. Since

$$\frac{\partial^2 F(n, \bar{d})}{\partial \bar{d} \partial n} = \frac{\alpha'(n)k}{\alpha(n)n} G''\left[\frac{k}{\alpha(n)} - \bar{d}\right] + \frac{\alpha(n)}{n^2}\left[1 - \frac{\alpha'(n)n}{\alpha(n)}\right] G'\left[\frac{k}{\alpha(n)} - \bar{d}\right] \leq 0,$$

any interior $n(\bar{d})$ is differentiable with $n'(\bar{d}) \leq 0$. In any equilibrium with $q_L < q^*$, we have $G'[k/\alpha(n) - \bar{d}] < 0$ and hence $\partial^2 F(n, \bar{d})/\partial \bar{d} \partial n < 0$ and $n'(\bar{d}) < 0$.

Finally, we show q rises in $\bar{d}$. If $n(\bar{d}) = \hat{n}$, it is easy to see the solution for y in (16) decreases in $\bar{d}$ and hence $q(\bar{d})$ increases in $\bar{d}$ continuously. For interior $n(\bar{d})$, write (16) as a Lagrangian

$$\max_{n,y}\left\{\frac{\alpha(n)}{n} G(y) - \frac{k}{n} + \zeta[\alpha(n)(y + \bar{d}) - k]\right\},$$

with ζ the multiplier for the free entry condition. The FOC's wrt n and y are

$$0 = \left[\frac{\alpha'(n)}{n} - \frac{\alpha(n)}{n^2}\right] G(y) + \frac{k}{n^2} + \zeta\alpha'(n)(y+\bar{d}) \quad (17)$$

$$0 = G'(y) + \zeta n. \quad (18)$$

Now eliminate ζ from (17) using (18) to get a differentiable equation linking y, n and $\bar{d}$. Since $n'(\bar{d})$ exists, $y'(\bar{d})$ exists. Hence $q'(\bar{d})$ exists.

To show $q'(\bar{d}) > 0$, it is sufficient to show $y'(\bar{d}) < 0$ because we already argue the solution for q falls and is differentiable in y. Consider (17). The RHS rises in $\bar{d}$ or y. At the solution, $y + \bar{d} = p - q > 0$ because sellers must get a positive surplus, and so the RHS increases in ζ. Also, the SOC holds at the solution and hence the RHS falls with n. Imagine a small increase in $\bar{d}$ such that the RHS increases. Since $n'(\bar{d}) < 0$, either y falls or ζ falls with $\bar{d}$ so that the RHS stays 0. If $y'(\bar{d}) < 0$ we are done. Suppose ζ falls. Then ζn falls with $\bar{d}$, by (18) and $G'' < 0$, $y'(\bar{d}) < 0$. ■

Proposition 4: We first show $T(z)$ is continuous. By Proposition 2, $q_j(z)$, $n_j(z)$, $N_j(z)$ and $p_j(z)$ are continuous, and thus (12) and (20) are continuous in z. When $\lambda = 1 - n_T(z)/n_L(z)$, (12) and (20) are identical, therefore $T(z)$ is continuous. At $i = 0$, by (3), the solution for z is big enough to sustain an equilibrium with positive number of sellers, namely $N_T + N_L > 0$. Since $T(z)$ decreases in z, a unique monetary equilibrium exists, when i is small, by continuity. ■

Proposition 5: By Lemma 2 and Proposition 2, buyers always cash out in submarket T. Therefore, by (8), $q_T < q^*$. By Proposition 2, $n_T > n_L, q_T < q_L, p_T > p_L, P_L < P_T$ and $R_L < R_T$. ■

Proposition 6: By Proposition 3, $p_j(z)$ and $q_j(z)$ rise in z and $n_j(z)$ and $D_j(z)$ fall in z. Since $T(z)$ is continuous and $T(z)$ decreases in z given the maintained assumptions, we know the solution for $T(z) = i$ decreases continuously in i and the results follow. ■

Proposition 7: An alternative way to write (14) is

$$\Omega = \frac{\omega_L \alpha(n_L)}{n_L}\left[u(q_L) - p_L - \gamma\left(p_L - z\right)\right] + \frac{\omega_T \alpha(n_T)}{n_T}\left[u(q_T) - p_T - \gamma(p_T - z)\right],$$

where the entry costs do not show up because the terms in brackets are the buyer's (not total) surplus, and entry costs cancel with the seller's surplus. Using (6), (9) and free entry, then using (1) and (2), we simplify this to

$$\Omega = \frac{\omega_L}{n_L}\left(\frac{\eta_L k}{1-\eta_L}\right)+\frac{\omega_T}{n_T}\left(\frac{\theta k}{1-\theta}\right) = \frac{\lambda}{n_L - n_T}\left(\frac{\eta_L k}{1-\eta_L}\right)+\frac{(1-\lambda)n_L - n_T}{n_T(n_L-n_T)}\left(\frac{\theta k}{1-\theta}\right).$$

The RHS depends on i only through n_T and n_L. Since $\partial n_T/\partial i > \partial n_L/\partial i = 0$ in $\mathcal{A}_1$, $\partial\Omega/\partial i$ has the same sign as $\partial\Omega/\partial n_T$. Differentiating the RHS wrt n_T yields

$$\frac{\partial\Omega}{\partial n_T} = \frac{k}{(n_L-n_T)^2}\left\{\frac{\lambda\eta_L}{1-\eta_L}+\frac{\theta}{1-\theta}\left[(1-\lambda)\frac{n_L}{n_T}\left(2-\frac{n_L}{n_T}\right)-1\right]\right\}. \tag{19}$$

The sign of the RHS depends on the term in braces, call it Φ. An equilibrium in $\mathcal{A}_1$ is in $\mathcal{A}_1^*$ iff $\Phi \geq 0$. At the intersection point of $\mathcal{A}_1$ and $\mathcal{A}_3$, $N_T = 0 \Leftrightarrow (1-\lambda)n_L = n_T$. In this situation, inflation enhances welfare:

$$\begin{aligned}
\Phi &\equiv \frac{\lambda\eta_L}{1-\eta_L}+\frac{\theta}{1-\theta}\left[(1-\lambda)\frac{n_L}{n_T}\left(2-\frac{n_L}{n_T}\right)-1\right] \\
&= \frac{\lambda\eta_L}{1-\eta_L}-\frac{\theta\lambda}{1-\theta}\frac{n_L}{n_T} \\
&= \frac{\lambda n_L}{k}\left\{\frac{\alpha(n_L)}{n_L}\left[u(q_L)-p_L-\gamma\left(D_L\right)\right]-\frac{\alpha(n_T)}{n_T}\left[u(q_T)-p_T-\gamma(D_T)\right]\right\} > 0.
\end{aligned}$$

The second equation uses $(1-\lambda)n_L = n_T$. The third equation uses (6), (9) and free entry. The last inequality is true because a buyer receives a higher expected payoff in submarket L than in submarket T. By continuity, there is an interval for λ where $\Phi > 0$ at $i = 0$. ■

Lemma 3: Since $\gamma_2^{-1}(b)$ is flatter than $\gamma_1^{-1}(b)$ and $\gamma_1'(b), \gamma_2'(b) > 0\ \forall b > 0$,

$$\frac{\partial\gamma_2^{-1}(b)}{\partial b} < \frac{\partial\gamma_1^{-1}(b)}{\partial b} \Leftrightarrow \frac{1}{\gamma_2'(\gamma_2^{-1}(b))} < \frac{1}{\gamma_1'(\gamma_1^{-1}(b))} \Leftrightarrow \gamma_1'(\gamma_1^{-1}(b)) < \gamma_2'(\gamma_2^{-1}(b)).$$

Since $\gamma'(D)$ and $\gamma^{-1}(b)$ are increasing, $\gamma'(\gamma^{-1}(b))$ rises with b. Thus, the last inequality implies $\forall b_1, b_2$, $\gamma_2'(\gamma_2^{-1}(b_2)) = \gamma_1'(\gamma_1^{-1}(b_1)) \Rightarrow b_1 > b_2$. In other words, $a = \gamma_2'(\gamma_2^{-1}(b_2)) = \gamma_1'(\gamma_1^{-1}(b_1)) \Rightarrow \gamma_2(\gamma_2'^{-1}(a)) = b_2 < b_1 = \gamma_1(\gamma_1'^{-1}(a))$. Therefore, $\gamma(\gamma'^{-1}(a))$ falls strictly as γ^{-1} grows flatter. ■

Proposition 8. *Part (a):* $\lambda = 0$. Since $\gamma(D)$ is strictly increasing, strictly convex and differentiable $\forall D \geq 0$, $\gamma'(D)$ exists and rises in D. For any given $a = \gamma'(D)$, one can write $\gamma(D)$ as an implicit function $\gamma(\gamma'^{-1}(a))$. When $\lambda = 0$, only submarket T exists and by (12) and (15) money demand is

$$i = \frac{\alpha(n_T)}{n_T} \frac{\theta\gamma'(D)}{1+\theta\gamma'(D)} = \frac{\alpha(n_T)}{n_T} \frac{\theta a}{1+\theta a}. \tag{20}$$

By (8), q_T falls continuously in $\gamma'(D)$. Thus, one can also write q_T as an implicit function $q(a) \equiv u'^{-1}(1+a)$ where $a = \gamma'(D)$. The inverse function exists since we assume $u''(q) < 0$ for $q < q^*$. It is easy to verify $q(0) = q^*$. By (9) and free entry,

$$\frac{k}{(1-\theta)\alpha(n_T)} = u(q(a)) - q(a) - \gamma(\gamma'^{-1}(a)). \tag{21}$$

Using (20) one can define an implicit function $n_T = n_1(a)$ for any a because $\alpha(n_T)/n_T$ falls in n_T. Similarly, one can define $n_T = n_2(a)$ by (21). Any a solving $n_1(a) = n_2(a)$ pins down the terms of trade in equilibrium. Since the equilibrium is unique by Proposition 4, the solution for $n_1(a) = n_2(a)$ is unique. To characterize it, note that (20) and (21) imply $n_1(a)$ and $n_2(a)$ rise with a. At $a = 0$, $n_1(0) = 0$ by (20) and $n_2(0) = \alpha^{-1}(k/(1-\theta)[u(q^*) - q^*]) > 0$ by (21). Thus $n_2(a)$ cuts $n_1(a)$ from above once as a rises.

As the cost of credit increases, the implicit function $\gamma(\gamma'^{-1}(a))$ falls $\forall a$ by Lemma 3. In this case, $n_2(a)$ falls $\forall a$ by (21). This implies a and n fall, so there are more sellers and $\alpha(n_T)/n_T$ rises. Also, $q_T(a)$ rises as a falls because $u'(q_T(a)) = 1 + a$ and u is concave. Moreover, the surplus for sellers $p_T - q_T$ rises by free entry, and thus p_T rises. The surplus for buyers $u(q_T) - q_T - \gamma(D_T)$ rises in $p_T - q_T$, by the bargaining solution. So buyers get a larger surplus per transaction and a higher matching probability, therefore welfare Ω rises. Finally, as the cost of using credit rises, the total expenditure on credit $\gamma(D_T) = \gamma({\gamma'}^{-1}(a))$ falls because a falls and $\gamma(\gamma'^{-1}(a))$ falls $\forall a$. Then debt D_T falls because $\gamma(d)$ rises $\forall d > 0$ and $\gamma(D_T)$ falls in equilibrium. Since p_T rises and D_T falls, $z = p_T - D_T$ rises.

Part (b): $\lambda = 1$. With pure directed search trade (n, q, p) solves

$$\max_{n,q,p,z} \left\{ \frac{\alpha(n)}{n} [u(q) - q - \gamma(p-z)] - \frac{k}{n} - iz \right\} \text{ st } k = \alpha(n)(p-q).$$

Now we make several changes of variables. First, let $a = \gamma'(p-z)$ so $p = \gamma'^{-1}(a) + z$. Second, since the solution satisfies (5), q solves $q(a) \equiv u'^{-1}(1+a)$. Third, by free entry $k = \alpha(n)(p-q)$, $z = p - \gamma'^{-1}(a) = q(a) - \gamma'^{-1}(a) + k/\alpha(n)$. Fourth, from the FOC $i = \gamma'(p_L - z)\alpha(n)/n$, one can express n as an implicit function $n(a)$ where $i = a\alpha(n(a))/n(a)$. Substitute $a, q(a), n(a)$ and $z = q(a) - \gamma'^{-1}(a) + k/\alpha(n)$ into the problem to get

$$\max_a \left\{ \frac{i}{a} \left[u(q(a)) - q(a) - \gamma(\gamma'^{-1}(a)) \right] - \frac{k}{n(a)} - i \left[q(a) - \gamma'^{-1}(a) + \frac{k}{\alpha(n(a))} \right] \right\}.$$

Now we argue the solution for a falls strictly as γ becomes more costly. Let γ_2 be more costly than γ_1, so that γ_2^{-1} is weakly flatter than γ_1^{-1}. Let $F_j(a)$ be the function in braces when the cost function is γ_j for $j = 1, 2$. Differentiate F_j to get

$$\frac{\partial F_2(a)}{\partial a} - \frac{\partial F_1(a)}{\partial a} = \frac{i}{a^2} \left[\gamma_2(\gamma_2'^{-1}(a)) - \gamma_1(\gamma_1'^{-1}(a)) \right] < 0.$$

The last inequality uses Lemma 3. Consequently, a falls strictly in j by standard monotone comparative statics. It follows that n falls as γ becomes more costly by (20). The rest of the proof is identical to the last part of (a).

Finally, we verify Ω is maximized when credit is not used if $\lambda \in \{0, 1\}$. Define $\bar{\gamma}(D) \equiv \gamma(bD)$ for $b > 1$, so $\bar{\gamma}^{-1}$ is flatter than γ^{-1}. As b rises, $\bar{\gamma}$ grows more costly and Ω rises by parts (a) and (b). As $b \to \infty$, $\bar{\gamma}(D) \to \infty \ \forall D > 0$ and the equilibrium converges to one without costly credit. ■

Proposition 9. For the first claim, by (1) and (2), $\omega_T = 1 - \lambda n_L/(n_L - n_T)$. Substitute this into (15) and let $a = \gamma(D_T)$ to get

$$i = \left(1 - \frac{\lambda n_L}{n_L - n_T}\right) \frac{\alpha(n_T)}{n_T} \frac{\theta a}{1 + \theta a}. \tag{22}$$

Any (n_T, a) solving (21) and (22) characterizes Γ_T. Now we argue that if $(\lambda, i) \in \mathcal{A}_1$ we stay in $\mathcal{A}_1$ as γ grows more costly. To show this, we assume we stay in $\mathcal{A}_1$ as γ grows more costly, and then verify it. If we stay in $\mathcal{A}_1$, buyers have enough money to purchase q^* in submarket L, and Γ_L is constant. Using the logic in part (a) of Proposition 8, define $n_1(a)$ and $n_2(a)$ by (22) and (21), so Γ_T is characterized by the solution $n_1(a) = n_2(a)$. One can use part (a) of the proof of Proposition 8 to show that n_T falls and p_T, q_T and z rise as γ grows more costly. Since z rises, buyers have enough z to get q^* in submarket

L, and the equilibrium stays in $\mathcal{A}_1$. Then $\omega_T = 1 - \lambda n_L/(n_L - n_T)$ rises since n_T falls and n_L is constant.

For the second claim, if the equilibrium is in $\mathcal{A}_1^* \subset \mathcal{A}_1$, then n_T rises as γ grows less costly as in the first claim. Moreover, Ω rises in n_T when equilibrium is in $\mathcal{A}_1^*$ by (19). Therefore Ω rises as γ grows less costly in $\mathcal{A}_1^*$. ■

Proposition 10. If $N_T = 0$ then $T(z)$ is given by (20) and λ has no effect on z, so $\partial z/\partial\lambda = 0$. If $N_T > 0$ then $T(z)$ is given by (12). Eliminating N_j using (2) and differentiating, we get

$$\frac{\partial T(z)}{\partial \lambda} \propto \frac{\alpha(n_L)}{n_L}\gamma'\left(p_L - z\right) - \frac{\alpha(n_T)}{n_T}\left[u'(q_T)q_T' - p_T' - \gamma'(p_T - z)(p_T' - 1)\right].$$

The RHS is continuous in z and independent of λ. By the Implicit Function Theorem, $\partial z/\partial\lambda$ and $\partial T(z)/\partial\lambda$ have the same sign because $T'(z)\partial z/\partial\lambda + \partial T(z)/\partial\lambda = 0$ and $T'(z) < 0$. When the RHS is 0, $\partial T(z)/\partial\lambda = 0 \Rightarrow \partial z/\partial\lambda = 0$, so the RHS stays 0 as λ increases further. Hence, $\partial z/\partial\lambda$ never changes sign as λ increases. When $\lambda = 0$ there is $\tilde{\imath} > 0$ such that $\forall i \leq \tilde{\imath}$ an extra dollar is redundant in local shops, $z > p_L^*$. Thus, $\forall i < \tilde{\imath}$, $\partial T(z)/\partial\lambda|_{\lambda=0} < 0 \Rightarrow \partial z/\partial\lambda|_{\lambda=0} < 0$. By earlier analysis, $\partial z/\partial\lambda \leq 0\ \forall\lambda$ and $i \leq \tilde{\imath}$. ■

Proposition 11. We have $\Omega = -\lambda s + \lambda V_I + (1-\lambda)V_U + \tau i$, where $\tau = \lambda z_I + (1-\lambda)z_U$. Substitute τ and $s = V_I - V_U$ to get $\Omega = V_I - s + i[\lambda z_I + (1-\lambda)z_U]$, and derive

$$\frac{\partial\Omega}{\partial i} = (1-\lambda)(z_U - z_I) + i\frac{\partial[\lambda z_I + (1-\lambda)z_U]}{\partial i}.$$

This uses $\partial V_I/\partial i = -z_I$ by the Envelope Theorem. This expression is strictly positive at $i = 0$ because $z_U = p_T^* > p_L^* = z_I$ at $i = 0$. ■

References

Aruoba, S., M. Davis, and R. Wright (2014). Homework in monetary economics: Inflation, home production, and the production of homes. *Review of Economics Dynamics*, forthcoming.

Aruoba, S., G. Rocheteau, and C. Waller (2007). Bargaining and the value of money. *Journal of Monetary Economics 54*(8), 2636–2655.

Aruoba, S., C. Waller, and R. Wright (2011). Money and capital. *Journal of Monetary Economics 58*(2), 98–116.

Basu, S. and J. Fernald (1997). Returns to scale in U.S. production: Estimates and implications. *Journal of Political Economy 105*(2), 249–283.

Benabou, R. (1992a). Inflation and efficiency in search markets. *Review of Economic Studies 59*(2), 299–329.

Benabou, R. (1992b). Inflation and markups: Theories and evidence from the retail trade sector. *European Economic Review 36*(2-3), 566–574.

Berentsen, A., G. Rocheteau, and S. Shi (2007). Friedman meets Hosios: Efficiency in search models of money. *Economic Journal 117*(516), 174–195.

Burdett, K. and K. Judd (1983). Equilibrium price dispersion. *Econometrica 51*(4), 955–69.

Caglayan, M., A. Filiztekin, and M. Rauh (2008). Inflation, price dispersion, and market structure. *European Economic Review 52*(7), 1187–1208.

Choi, M. (2015). Monotone comparative statics for directed search models. *Working paper*.

Debelle, G. and O. Lamont (1997). Relative price variability and inflation: Evidence from U.S. cities. *Journal of Political Economy 105*(1), 132–152.

Diamond, P. (1971). A model of price adjustment. *Journal of Economic Theory 3*(2), 156–168.

Gomis-Porqueras, P., A. Peralta-Alva, and C. Waller (2014). The shadow economy as an equilibrium outcome. *Journal of Economic Dynamics and Control 41*, 1–19.

Gourio, F. and L. Rudanko (2014). Customer capital. *Review of Economic Studies*.

Green, E. and R. Zhou (1998). A rudimentary random-matching model with divisible money and prices. *Journal of Economic Theory 81*(2), 252–271.

Gu, C., F. Mattesini, and R. Wright (2016). Money and credit redux. *Econometrica 84*(1), 1–32.

Hahn, F. (1987). *The Foundations of Monetary Theory*. London: MacMillan.

Head, A. and A. Kumar (2005). Price dispersion, inflation, and welfare. *International Economic Review 46*(2), 533–572.

Hosios, A. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies 57*(2), 279–298.

Howitt, P. (2005). Beyond search: Fiat money in organized exchange. *International Economic Review 46*(2), 405–429.

Hu, T., J. Kennan, and N. Wallace (2009). Coalition-proof trade and the Friedman rule in the Lagos-Wright model. *Journal of Political Economy 117*(1), 116–137.

Jean, K., S. Rabinovich, and R. Wright (2010). On the multiplicity of monetary equilibria: Green-Zhou meets Lagos-Wright. *Journal of Economic Theory 145*(1), 392–401.

Kaplan, G. and G. Menzio (2014). The morphology of price dispersion. *International Economic Review*, forthcoming.

Kiyotaki, N. and R. Wright (1993). A search-theoretic approach to monetary economics. *American Economic Review 83*(1), 63–77.

Lagos, R., G. Rocheteau, and R. Wright (2015). Liquidity: A new monetarist perspective. *Journal of Economic Literature*, forthcoming.

Lagos, R. and R. Wright (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy 113*(3), 463–484.

Lester, B. (2011). Information and prices with capacity constraints. *American Economic Review 101*(4), 1591–1600.

Lester, B., A. Postlewaite, and R. Wright (2012). Information, liquidity, asset prices, and monetary policy. *Review of Economic Studies 79*(3), 1209–1238.

Liu, Q., L. Wang, and R. Wright (2015). Sticky prices and costly credit. *Working paper*.

Lucas, R. (2000). Inflation and welfare. *Econometrica 68*(2), 247–274.

Lucas, R. and J. Nicolini (2012). On the stability of money demand. *Working paper*.

Moen, E. (1997). Competitive search equilibrium. *Journal of Political Economy 105*(2), 385–411.

Nosal, E. and G. Rocheteau (2011). *Money, payments, and liquidity*. MIT press.

Parsley, D. (1996). Inflation and relative price variability in the short and long run: New evidence from the United States. *Journal of Money, Credit and Banking 28*(3), 323–341.

Prescott, E. (2005). Comments on "Inflation, output, and welfare" by Ricardo Lagos and Guillaume Rocheteau. *International Economic Review 46*(2), 523–531.

Reinsdorf, M. (1994). New evidence on the relation between inflation and price dispersion. *American Economic Review 84*(3), 720–31.

Rocheteau, G. and R. Wright (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica 73*(1), 175–202.

Rocheteau, G. and R. Wright (2009). Inflation and welfare in models with trading frictions. In D. Altig and E. Nosal (Eds.), *Monetary Policy in Low-Inflation Economies*. Cambridge University Press.

Salop, S. and J. Stiglitz (1977). Bargains and ripoffs: A model of monopolistically competitive price dispersion. *Review of Economic Studies 44*(3), 493–510.

Stahl, D. (1989). Oligopolistic pricing with sequential consumer search. *American Economic Review 79*(4), 700–712.

Varian, H. (1980). A model of sales. *American Economic Review 70*(4), 651–659.

Venkateswaran, V. and R. Wright (2013). Pledgability and liquidity: A new monetarist model of financial and macroeconomic activity. *NBER Macroeconomics Annual 28*, 227–270.

Wallace, N. (2013). An alternative to New-Keynesian models for the study of optimal (monetary) policy. *mimeo*.

Supplemental Appendix (Not for Publication)

We sketch a version where buyers choose their money holding after learning their information status, and hence informed and uninformed buyers choose different z. For an informed buyer z_I solves $i = [\alpha(n_L)/n_L]\,\gamma'(p_L - z_I)$; for an uninformed buyer z_U solves

$$\begin{aligned} i &= \frac{N_L}{N_L + N_T}\frac{\alpha(n_L)}{n_L}\gamma'(p_L - z_U) \\ &\quad + \frac{N_T}{N_L + N_T}\frac{\alpha(n_T)}{n_T}\left[u'(q_T)q_T' - p_T' - \gamma'(p_T - z_U)(p_T' - 1)\right]. \end{aligned}$$

The equilibrium structure is recursive. We first solve for the terms of trade at local shops. Define

$$T_I(z_I) \equiv \frac{\alpha(n_L)}{n_L}\gamma'(p_L - z_I)$$

where n_L and p_L are functions of z_I. As in the baseline model, there exists z_I that solves $i = T_I(z_I)$ if i is not too big. Then $\Gamma_L = (n_L, p_L, q_L)$ solves (4), and z_U solves

$$\begin{aligned} T_U(z_U) &\equiv \frac{N_L}{N_L + N_T}\frac{\alpha(n_L)}{n_L}\gamma'(p_L - z_U) \\ &\quad + \frac{N_T}{N_L + N_T}\frac{\alpha(n_T)}{n_T}\left[u'(q_T)q_T' - p_T' - \gamma'(p_T - z_U)(p_T' - 1)\right] \end{aligned}$$

where (n_T, p_T, q_T) is a function of z_U. Moreover, N_L and N_T are functions of n_L and $n_T(z_U)$.

As in the version with information acquisition, uninformed buyers do not bargain in submarket L when i is not too big. There exists a solution for z_U to $i = T_U(z_U)$ when i and k are not too big. In equilibrium, $\phi M = \lambda z_I + (1-\lambda) z_U$. In general, one cannot say how n_L/n_T changes with i, but suppose i is small and the elasticity η is constant. Let $g(q) = \gamma \circ \gamma'^{-1}\left[u'(q) - 1\right]$, and consider

$$\frac{p_L - q_L}{p_T - q_T} = \frac{1}{1-\theta}\frac{1-\eta_L}{\eta_L u'(q_L) + 1 - \eta_L}\frac{u(q_L) - q_L - g(q_L)}{u(q_T) - q_T - g(q_T)}.$$

At $i = 0$, this ratio is $(1-\eta_L)/(1-\theta)$. As i increases, surpluses in both submarkets remain unchanged but $u'(q_L)$ increases. Thus $(p_L - q_L)/(p_T - q_T)$ falls. By $\alpha(n_j)(p_j - q_j) = k$, we know $\alpha(n_L)/\alpha(n_T)$ increases in i. Since η is constant, n_L/n_T increases and N_L/N_T decreases with i. Since the surpluses remain constant but uninformed buyers have a lower probability of contacting a local shop, welfare falls in i. Hence, near $i = 0$, with η constant, as i increases the measure of local shops falls and welfare of informed buyers remains constant while that of uninformed buyers falls.