

Macropudential Policy in a Knightian Uncertainty Model with Credit-, Risk-, and Leverage Cycles

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Abstract

We attempt to explain two stylized facts of the Great Recession, namely the build-up of high leverage in the household sector in the boom phase, deep busts and protracted recovery as rare systemic events. We extend Boz and Mendoza (2014) by explicitly modeling the credit markets and modifying the learning to an adaptive set-up. We find that in such a set-up, the build-up of leverage and the collateral price cycles take longer than in other DSGE models with financial frictions. The boom-bust cycles occur as rare events, with two systemic crises per century. The model also replicates asymmetric distributions of key macroeconomic and financial variables, with high skewness and fat tails. In addition, we show that a simple LTV-cap regulation is effective in smoothing the leverage cycles by limiting household borrowing in upturns, both via quantity (higher equity participation requirement) and price (lower collateral value) effects, as well as providing a well defined anchor for the agents in their learning process.

Keywords: uncertainty, financial engineering, deregulation, leverage forecasting, macroprudential policy

JEL: G14, G17, G21, G32, E44, E58

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1 Motivation

The Great Recession has been characterised by two features, which made it distinct from any other recession in the post-war era. First, it was preceded by a significant build-up of leverage, in particular in the household sector. Excessive borrowing, in particular in the mortgage market, was in particular facilitated by securitization, which reached its pick activity in 2007. Second, the depth and length of the recession resulted in a substantial deviation of GDP from its long term trend in the US, UK and in the euro area, which cannot be explained in a standard New Keynesian model set-up even after taking account of financial frictions.

We put forward a model that links these two stylized facts. Financial innovation shocks push the economy into a previously unexplored and unmapped state. In this new state, agents do not know the true riskiness of new financial products and therefore optimize under incomplete information. The incompleteness is not caused by households' cognitive limitations, but because they need to learn the true riskiness of the financial products. This learning process requires sufficient number of realisation of the state variable in order for the information set to be complete. As learning takes time, the economy approaches the new steady state only sluggishly.

The core of the model follows Boz and Mendoza (2014). However, we introduce three important modifications. Following Iacoviello and Neri (2009) we first split households (key agents of the model) into patient, who save and produce land, and impatient, who borrow and consume land. In addition, we introduce a financial intermediary and explicitly model the credit market.

The major friction in the model is uncertainty about new state of the economy after financial innovation shock. In this way, financial innovation interacting with credit/margin constraints can lead to underpricing of the risk associated with a new financial environment. This in turn can lead to the accumulation of leverage and surges in asset prices. Because of limited enforceability of financial contracts, households are required to provide collateral for their loans, and so the relationship between the bank and household is tightened for many periods ahead. Once the agents observe sufficient number of realisations of the new state of the economy and realise that they are overlevered, this can lead to a sudden stop a la Mendoza (2010). More formally, sudden stop is caused by the uncertainty regarding the transition probability of such events. Since systemic crises are rare events, agents inherently misprice the occurrence of such events (see for instance Zeira (1999), Caballero and

Krishnamurthy (2008) or Boz and Mendoza (2014)). Uncertainty coupled with Fisherian deflation mechanism leads to highly volatile and asymmetric distributions in asset prices, consumption, debt, loan- and deposit rates. Our approach is loosely linked to the rational inattention theory (Sims, 2010), which recognises that people have finite information-processing capacity that explains well some of the frictions.

We find that early realizations of the new state result in a much higher (lower) debt, consumption, price of collateral and risk accumulation (de-leveraging) during upturns (downturns) compared to standard financial friction models. Moreover, the loan-to-deposit ratio of banks is rapidly increasing at the onset of the financial innovation phase, and remains very high until sudden stop has materialized for a few periods. We also demonstrate that sluggish learning can explain why the economy can diverge from its long term trend for an extended period of time.

Next, we evaluate the efficacy of standard macroprudential tools, such as a cap on the loan-to-value (LTV) in reducing the leverage of the household sector. We also discuss the role for a new macroprudential policy in reducing the information incompleteness related to financial innovation by generating information that helps the agent learn faster the new environment, or provide a smoother transition to the new economic environment.

The remainder of this paper is organised as follows. In Section 2 we introduce the model set-up. Section 3 is devoted to the main friction of the model - it discusses uncertainty and describes the mechanism of learning. Section 4 presents a strategy for solving the model, while Section 5 presents first order conditions and discusses their implications. [Finally, we present results in section 6 and conclude in section 7].

2 Model

2.1 An overview

The backbone of the model is a standard New Keynesian setup, extended to include uncertainty, learning and credit market frictions. We extend the work of Boz and Mendoza (2014) by endogenizing credit production, modify learning mechanism into an adaptive set-up, as well as include financial and monetary policies. In particular, we endogenize both the quantity and prices of deposits, loans, bank equity, make

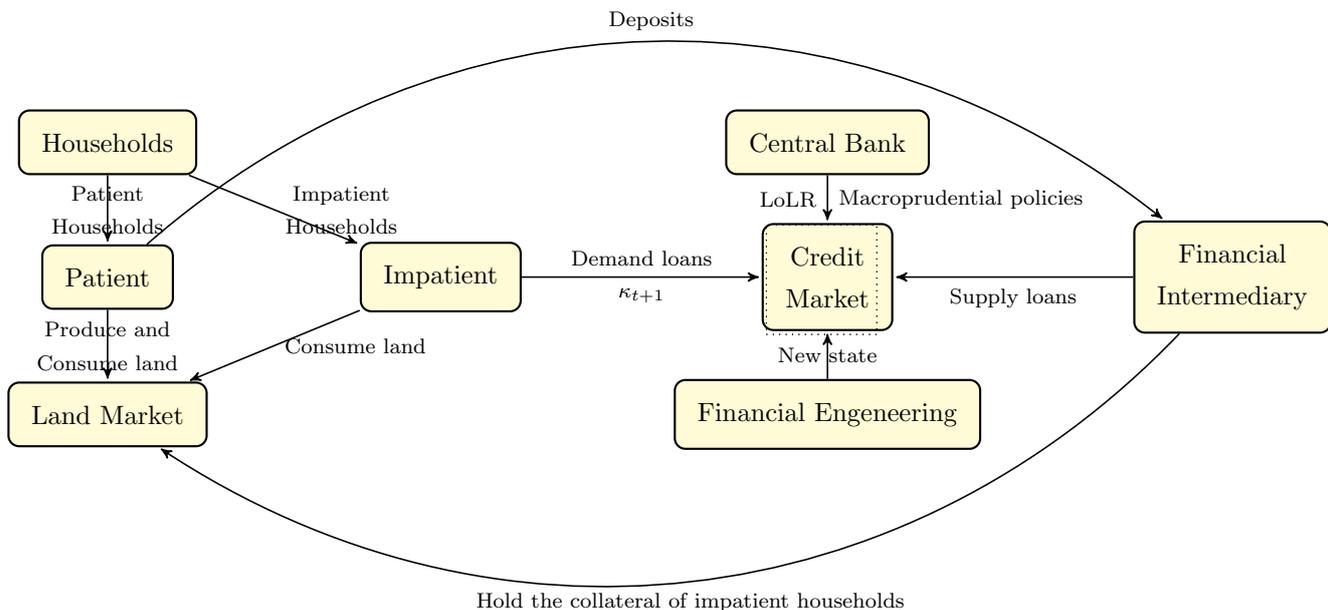
1 agents learn according to (adaptive) heuristics, and include a central bank who
2 simultaneously sets a (time-varying) policy rate and a macroprudential rule.

3 Our model economy is populated by three agents: households, financial interme-
4 diaries, and government. Moreover, we divide households into two categories: the
5 patient and the impatient types. What differentiates them is the degree of patience.
6 The discount factor β of patient households is higher than those of the impatient.
7 This forces the latter to complement their internal funds with loans from the credit
8 market. While patient households both produce and consume land, impatient only
9 consume it. Therefore we explicitly model two markets: market for land and market
10 for credit.

11 Nevertheless, what differentiates this model from most other financial friction
12 frameworks is that this one incorporates uncertainty. Financial sector developments
13 such as financial engineering, de-regulation of markets, and increased competition
14 amongst financial intermediaries has meant that the new market structure is un-
15 known and unexplored to the participants in financial transactions. As a result,
16 agents do not know the true risks, leverage and price of collateral in the ‘new’ econ-
17 omy and therefore optimize under incomplete information. Our take on uncertainty
18 is that agents are (intrinsically) rational insofar that they efficiently optimize over
19 time, but do so under incomplete information regarding two variables in the model:
20 the leverage ratio, and the price of collateral. One is exogenous while the other is
21 endogenous, but dependent on the realization of the first. They engage in adaptive
22 learning and learn about the ‘true’ values of leverage and asset prices only after
23 observing a sufficiently long set of realizations of both variables. Note that this
24 learning is, however, slow since they only learn from their practical experiences.

25 With respect to Boz and Mendoza (2014), the learning in this framework is more
26 active, since one of the learning variables is endogenous. However, this variable is
27 dependent on the exogenous (the shadow value of collateral constraint) variable
28 which facilitates the tractability of the dynamic solution. Therefore, while agents
29 can partially benefit from experimenting with the dynamic optimization to induce
30 the endogenous land price, the exogenous component of this price will make such
31 experimenting slow and costly. In other words, the values of the ‘learning variables’
32 cannot be directly deduced by recursively solving the remaining part of the model.
33 Moreover, we will make their learning contingent on two rules, which will make the
34 learning dynamics even more tractable.

35 Figure 1 provides an overview of the model.



2.2 Households

The consumption sector is populated by two types of infinitely lived households, each with a unit mass and they act atomistically in competitive markets.¹ Both types optimize under uncertainty. The key factor which differentiates them is the degree of impatience. The discount factor β of impatient households (I) is lower than the one of patient (P). This will ‘force’ the impatient households to engage in external credit market. For the sake of simplicity and tractability, we explicitly omit the labour supply decision of households which means that they only derive income from land and saving/borrowing.²

2.2.1 Patient Households

The representative patient risk-averse household chooses consumption c_t , land holdings l_{t+1} , and deposits d_t , taken as given the price of land q_t , the deposit rate R_t^d , and the gross real interest rate R_t so as to maximize a standard CRRA utility function:

¹One could equivalently assume that in each period households die and are born with a constant probability so that on aggregate there is a unit mass of households.

²It would be straightforward to extend the model to include a labour market, as in for instance Gerali et al (2010).

$$E_0^s[\sum_{t=0}^{\infty} \beta^t u(c_t)] \quad (1)$$

1 where $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, and σ is the parameter of relative risk aversion of households.
 2 Because of their relative patience, these households are made natural lenders, and
 3 face the following budget constraint:

$$d_{t+1} + c_t^P \leq z_t g(l_t) + q_t l_t - q_t l_{t+1} + (1 + R_t^d) d_t + e_t \quad (2)$$

4 The share of patient households in the population is θ (and is time invariant).
 5 The production function $g(l_t) = l^\alpha$ is a standard neoclassical one and is subject
 6 to a stochastic productivity shock z_t , which is known to all agents.³ Because we are
 7 interested in uncertainty regarding financial frictions, we omit from imperfect beliefs
 8 regarding the productivity shock. However, an immediate extension could be to also
 9 introduce macroeconomic uncertainty.

10 It is crucial to note that E_t^s in the utility function above represents expectations
 11 subject to agents' (subjective) beliefs using information available up to period t
 12 (inclusive). These beliefs will differ from the ones formulated under rational expect-
 13 tations.

14 2.2.2 Impatient Households

15 The impatient risk-neutral households (with the share of the total population equal
 16 to $1 - \theta$) maximize the same type of CRRA utility function:⁴

$$E_0^s[\sum_{t=0}^{\infty} \beta^t u(c_t)] \quad (3)$$

17 where $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, but face a different budget constraint due to their impatient
 18 nature:

³As is standard in this literature, we will assume that the TFP shock has an autoregressive process. However, we could have equivalently assumed the TFP shock to follow a Markov process, without changing much (or even at all) the results.

⁴Note that we depart here from the representative agent assumption and make the impatient households heterogeneous by subjecting them to different initial land holding (or wealth). Aside from this initial wealth heterogeneity, which will generate a wealth distribution in period $t = 0$, the constrained optimization problem is equal for all agents within this category. We simply need this initial heterogeneity to motivate the endogenous learning dynamics within this group, and the (possible) reason for switches between one rule and the other. The learning dynamics will be explained in further detail at a later stage.

$$c_t^I \leq q_t l_t - q_t l_{t+1} - \frac{b_{t+1}}{R_t^b} + b_t \quad (4)$$

where b_t are the holdings of one-period discount loans (or bonds). Because of imperfections in the credit market (due to limited state-verification a la Townsend), impatient households face restrictions in the quantity of external financing obtained and must provide a collateral as a security.⁵ Therefore, the LTV that the agent must satisfy limits the value of credit $\frac{b_{t+1}}{R_t^b}$ to a time-varying ratio of the market value of their land holdings, κ_t according to:

$$E_t^s[\kappa_{t+1}]q_t l_{t+1} \leq -\frac{b_{t+1}}{R_t^b} \quad (5)$$

From a microeconomic perspective, κ can be seen as the proportional cost of collateral repossession (or liquidation share) in case of default. Debt contract with margin clauses are also captured by this relation (Mendoza, 2010). A relaxation (tightening) of this constraint can either come from an increase (decrease) in the borrowing capacity κ_t or from an increase (decrease) in the value or quantity of the collateral $q_t l_{t+1}$. From a macroeconomic perspective, this relation can be interpreted as the LTV ratio (or leverage) set by the macroprudential authority. This interpretation will become evident later on when we study the impact of macroprudential policies on the model dynamics.

The random variable κ_t is continuous with an upper bound at 1 and a non-negative lower-bound. It is also time-varying. The framework is flexible enough to capture asymmetric regime-switching probabilities between high and low leverage capacities. This is one of the variables that impatient agents have incomplete information about, and which they will need to forecast.

Notice, again that the expectations operator is dependent on beliefs regarding state s . This uncertainty (or ‘ignorance’) regarding the *true* state applies to the entire population equally. Therefore, agents are rational in the sense that they use all available information (and models) at time t , but form subjective beliefs because they act under (evenly distributed) incomplete information.⁶ However, agents engage

⁵See, for instance, Bernanke, Gertler and Gilchrist (1999), or Christiano, Motto and Rostagno (2013) for background information and microfoundations of the state-verification problem in external lending. We use the outcomes from that problem to motivate our collateral constraint, but because of the similarity with the aforementioned frameworks, we abstain from providing full microfoundations of that problem.

⁶This is very different from model settings where one agents has more information than the other (asymmetric), or where agents use heterogeneous information and/or models (due to their

1 in (aggregate) learning and become fully aware of the *true* transition probabilities
 2 as they approach time $t = T$. We will describe the learning mechanism in further
 3 detail once we have outlined the full model.⁷

4 **2.3 Financial intermediary**

5 The representative financial intermediary operates in a perfectly competitive market
 6 and uses deposits from patient households to give out as loans to impatient house-
 7 holds. As in Gerali et al (2010) they are owned by patient households (captured by
 8 the patients' discount factor $\beta_p^t \lambda_t^p$), and maximize the discounted sum of cash flows:

$$E_0^s \sum_{t=0}^{\infty} \beta_p^t \lambda_t^p [(1+R^b)B_t - B_{t+1} + D_{t+1} - (1+R_t^d)D_t + (E_{t+1}^b - E_t^b) - \frac{\kappa_{E^b}}{2} [\frac{E_t^b}{B_t} - \nu^b]^2 E_t^b] \quad (6)$$

9 subject to the balance sheet constraint: $B_t = D_t + E_t^b$. B_t is the total amount
 10 of loans issued at time t , D_t the aggregate number of deposits received from patient
 11 households, E_t^b the bank capital, and ν^b is the long-term capital-to-asset ratio. The
 12 last term in the above maximization problem represents the cost of operating the
 13 financial intermediary. To motivate an undesirable social cost (externality) from
 14 excessive intermediary leverage from the point of view of the macroprudential policy
 15 maker, we impose a quadratic cost function whenever the intermediary's capital-to-
 16 asset ratio $\frac{E_t^b}{B_t}$ moves away from the target value ν^b . Because of the high number of
 17 competitors in the banking industry, the individual intermediary takes the deposit
 18 R_t^d and the loan rates R_t^b as given when maximizing its profits.⁸

19 The aggregate bank capital evolves according to:

$$E_{t+1}^b = (1 - \delta^b)E_t^b + \pi_t^b \quad (7)$$

cognitive restrictions) to infer the true states (irrationality).

⁷Preston (2005) pointed out that infinite horizon microfounded learning models fail to produce optimal dynamic consumption allocations while violating their intertemporal budget constraint, resulting in an inconsistency in the microfoundations. In defense, Hokaohja and Mittra (2011) showed that the intertemporal accounting consistency holds along the (infinite) sequence of temporary equilibria and that this model can be derived as a special case of Preston's framework.

⁸The intermediary also acts under incomplete information. That is why we have conditioned its expectations on the state s beliefs. However, their beliefs are of second order importance since they do not optimize with respect to κ_t nor do they engage in learning. κ_t is instead assumed to be out of direct control by either household or intermediary, and plays a key role only for the optimization of households. Therefore we will omit intermediary's subjective beliefs and in what follows, approximate its beliefs with the RE expectations operator.

where δ^b measures the resources used in managing bank capital and π_t^b are overall real profits made by the financial intermediary at date- t . These are described by the following relation:

$$\pi_t^b = R_t^b B_t - R^d D_t - \frac{\kappa_{E^b}}{2} \left[\frac{E_t^b}{B_t} - \nu^b \right]^2 E_t^b - Adj_t^b \quad (8)$$

with Adj_t^b denoting the adjustment costs for changing interest rates on deposits. This definition of profits is a narrow one as it coincides with the net interest rate margin. It does not include any other items from the income statement in order to maintain a closed-form solution for intermediary's optimization problem while keeping it simple.

2.4 Credit Market

Next we need to derive the lending and deposit rates that financial intermediaries charge. Iterating the balance-sheet constraint of financial intermediaries at date t and $t + 1$ and inserting it into the cash-flow expression in equation 6, we get that the intermediary's objective is to maximize:

$$R_t^b B_t - R_t^d D_t - \frac{\kappa_{E^b}}{2} \left[\frac{E_t^b}{B_t} - \nu^b \right]^2 E_t^b \quad (9)$$

Taking first-order conditions with respect to B_t and D_t and combining them, we get that the spread charged on loans is equal to:

$$R_t^b = R_t^d - \kappa_{E^b} \left[\frac{E_t^b}{B_t} - \nu^b \right] \left[\frac{E_t^b}{B_t} \right]^2 \quad (10)$$

Since patient households are risk-averse, they will ask for a safe rate on their deposits, that by no-arbitrage condition, will equal to the real rate R_t (see for instance Bernanke, Gertler and Gilchrist (1999) or Christiano, Motto and Rostagno (2013) for a microfoundation behind this result).⁹We can thus re-write the above expression as:

$$R_t^b - R_t = \kappa_{E^b} \left[\frac{E_t^b}{B_t} - \nu^b \right] \left[\frac{E_t^b}{B_t} \right]^2 \quad (11)$$

This expression represents the trade-offs that the financial intermediary faces in

⁹Following Gerali et al (2010), we could equivalently assume that the financial intermediary has continuous and risk-free access to central bank liquidity at the safe rate R_t , which by arbitrage would make the deposit rate equal to the safe rate.

1 setting the lending rate. The left-hand side represents the marginal benefit from
 2 increasing lending meanwhile the right-hand side represents the costs of increasing
 3 leverage (by deviating from the ν^b target). The final lending rate will be set where
 4 the two are equal.

5 2.5 Land Market

6 We can show that the effects of the collateral constraint on asset pricing can be
 7 derived by combining the Euler equations of land for the two households.¹⁰Solving
 8 the equations forward in which the future stream of land dividends is discounted
 9 at the stochastic discount factor and adjusted for the shadow value of the credit
 10 constraint:

$$q_t = E_t^s \sum_{j=0}^{\infty} \left[\prod_{i=0}^j \frac{\beta u'(c_{t+1+i})}{u'(c_{t+i}) - \mu_{t+i} \kappa_{t+i}} \right] z_{t+1+j} g'(l_{t+1+j}) \quad (12)$$

11 This condition equalizes the equilibrium price of land with the marginal cost of
 12 investment. Looking at the denominator of I.22, we see that the collateral constraint
 13 lowers land prices since it increases the rate of return at which future land dividends
 14 are discounted. It is forward-looking since not only will a binding constraint at t
 15 reduce the value of land, but also if agents expect that the constraint can bind at
 16 any future date $E_t^s[\mu_{t+i} \kappa_{t+i}]$ for any $i > 0$, the value of land will fall.

17 If we further define the next period marginal utility of consumption as $\lambda_{t+1} \equiv$
 18 $\beta u'(c_{t+1})$ and return on land as:

$$R_{t+1}^q = \frac{z_{t+1+j} g'(l_{t+1+j}) + q_{t+1}}{q_t} \quad (13)$$

19 we can define the (subjective) premium on land as (Mendoza, 2010):

$$E_t^s [R_{t+1}^q - R_t] = \frac{(1 - \kappa_t) \mu_t - Cov_t^s(\lambda_{t+1}, R_{t+1}^q)}{E_t^s(\lambda_{t+1})} \quad (14)$$

20 The land premium rises in every state in which the collateral constraint binds
 21 because of these three effects:

- 22 • The direct effect, $(1 - \kappa_t) \mu_t$, is due to a rise in the shadow value of the collateral
 23 constraint (with an upper bound determined by κ_t , the amount of the collateral
 24 that can be turned into debt).

¹⁰We follow the method described in Mendoza (2010).

- The indirect effect, represented by a lower $Cov_t^s(\lambda_{t+1}, R_{t+1}^q)$ and a higher $E_t^s(\lambda_{t+1})$.
- Because of the collateral constraint, the household's ability to smoothen her consumption is limited, leading her to transfer the consumption into the future.

To see the effects of this on the price of tangible, we can write the land price as a function of the return according to:

$$q_t = E_t^s \sum_{j=0}^{\infty} \left[\prod_{i=0}^j \frac{1}{E_t^s[R_{t+1}^q]} \right] z_{t+1+j} g'(l_{t+1+j}) \quad (15)$$

since the expected land return satisfies the condition $q_t E_t^s[R_{t+1}^q] = E_t^s[z_{t+1+j} g'(l_{t+1+j})]$. Then, as Aiyagari and Gertler (1999) show, an increase in expected return will lead to lower equity prices in the current period, since the discount rate of future dividends will increase due to the binding collateral constraint, in the current or next period. Hence, if the collateral constraint binds at least occasionally in the stochastic steady state, the entire equilibrium asset pricing function will be distorted by the constraint (independent of whether the constraint binds in the current period or not).

If these effects are at work under rationally formed expectations (with the knowledge of the true state of κ), these effects are further accentuated if we in addition introduce learning into this framework. To understand how, one needs to examine the interactions between the collateral constraint and learning regarding the κ_t variable. Suppose that the constraint was binding at t . In booms (or states with high leverage possibilities), the price of asset is higher, which will relax the LTV constraint. From equation 15 it implies that the land return is lower. So assuming that beliefs are optimistic (pessimistic) in a boom (bust), impatient households will assign a higher probability to lower (higher) future land returns than under RE. This will push land price further up (down), which via the LTV-constraint, will result in higher (lower) indebtedness.

Taking into account the tight and procyclical link between leverage and asset prices, and considering that the the value of κ_t (which is an argument of the land price q_t) is unknown and therefore forecasted, it is reasonable to also make the value q_t uncertain (and state contingent). Hence households will have to forecast the values of κ as well as q .

1 Following Boz and Mendoza (2014), for simplicity we will assume that the aggre-
 2 gate land supply is fixed and equal to 1. Consequently, the market clearing condition
 3 in the land market:

$$1 = \theta l_t^P + (1 - \theta) l_t^I \quad (16)$$

4 implies that the land holdings of the representative household must at each t
 5 satisfy $l_t = 1$, as well as the production function will be reduced to $z_t g(1)$.¹¹

6 2.6 Central Bank

7 To close the model, we separately model the two policies of the central bank. Assum-
 8 ing that the variables without time subscripts denote their steady state values, we
 9 can characterize the monetary policy of the central bank with a standard Taylor-rule
 10 (expressed in deviations-from-the-target terms):

$$\frac{R_t}{R} = \frac{R_{t-1}}{R}^{\gamma_R} \left[\frac{\pi_t}{\pi} \frac{y_t}{y} \right]^{\gamma_y} e^{\epsilon_{R,t}} \quad (17)$$

11 where $\epsilon_{R,t}$ is a monetary policy shock.

12 On the other end, macroprudential policy is modeled as a set of *ex ante* rules
 13 that the intermediary sector must obey to. The first rule is a cap on the LTV ratio
 14 (independent of the state):

$$\kappa_t = \bar{\kappa} \quad (18)$$

15 Alternatively, we will test a more elaborate version of the above LTV-rule. Re-
 16 cently, several papers (Lambertini et al (2013), Angelini et al (2014)) have proposed
 17 Taylor-type macroprudential rules as a good approximation of the Basel II/III-style
 18 of regulatory requirements. We will therefore perform an alternative scenario where
 19 the central bank uses:

$$\kappa_t = \rho_\kappa * \kappa_{t-1} + (1 - \rho_\kappa) * \kappa^* + (1 - \rho_\kappa) * (b_t - b_{t-1}) \quad (19)$$

20 ,where κ^* is the steady state value for the LTV-ratio. We calibrate it to 2 in line
 21 with the above rule in order to facilitate the comparison between a static (state-
 22 independent) and a dynamic (state-dependent) version.

¹¹Hence all the variation in land will come in its value, which is a function of the intertemporal consumption smoothing of households, as well as the shadow value of collateral constraint.

3 Uncertainty and Learning

Now that we outlined the key decision makers in the model, we need to devote some attention to the non-standard aspects of our model. In particular, we wish to describe the environment and the processes that govern the learning of our agents.

3.1 The general outline

Following Boz and Mendoza (2011, 2014), we model a situation in which financial engineering and market de-regulation lead to an increase in credit, leverage and risks. Agents know therefore that the environment (and the value of all these variables) has changed, but they don't know exactly by how much. Thus, the uncertainty concerns the 'true' values of the LTV-ratio κ , and the land price q .¹²Therefore, in contrast to Boz and Mendoza (2011, 2014), we assume that there are more than two possible future regimes as the values of land and leverage can have many different realizations. Moreover, in our framework agents are adaptive learners and use simple heuristics to forecast the two variables. In the (very) long-run, their beliefs converge to rational expectations. In the short run, however, their beliefs will be different from the equilibrium with full information. They learn only from past experience and fully 'understand' the riskiness of the new financial environment only after they have observed a sufficiently large sample of realizations. As a result, agents are slow learners and their learning process is strongly history dependent.¹³

Cecchetti et al (2000) and Cogley and Sargent (2008b) show that CRRA utility functions with Markov process for the consumption growth can generate asymmetric behaviour in consumption. High-growth states in consumption are persistent and common. However, once a low-growth state has been reached, the contractions are severe, with a mean decline of 6.785% p.a. Moreover, once the economy is in the low-growth state, there is a certain positive probability of running into a sequence of contractions, with a total decline in consumption amounting to 25% (assuming the

¹²Equivalently, and using the approach by Boz and Mendoza (2014), one could say that the uncertainty is regarding the transition probability to a new state. This state is a subset to a bounded set between 0 and 1.

¹³In contrast, agents are Bayesian learners in Boz and Mendoza (2014) and their learning space is constrained to only two realizations of the 'learning variable': High or Low leverage states. In addition, the uncertainty concerns leverage only (and not land prices, despite the fact that uncertainty will enter the land price function via the shadow value of collateral constraint.) Therefore the speed of learning and convergence is expected to be higher in their model compared to ours once we acknowledge that the probability space of the (learning) variables in their model is much smaller.

1 contraction lasts for 4 years with a probability of 7.1%). We will use this threshold
2 to identify *ex post* severe contractions (or systemic crises) in our model.

3 The current learning set-up means that agents learn quickly about the lever-
4 age/land price states that occur more frequently. Therefore, taking into account
5 that severe contractions are rare, learning about them will also be slower and asym-
6 metric with respect to expansions. Moreover, because the ergodic probability of
7 a contraction is as small as 0.0434 (Cogley and Sargent, 2008b), the time elapsed
8 before a sufficiently large sample of contractions has been observed is very large.
9 This retards the learning of contractions significantly.

10 The type of uncertainty we model in this paper generates fat tails. In addition,
11 the tails are asymmetric since contractions are more rare than expansions, and so the
12 lower tail (low (or negative) values of consumption, credit and bank equity, or high
13 values of leverage and interest rates) is significantly fatter than the upper tail (as
14 the uncertainty regarding it is higher). This leaves open the possibility for serious
15 downward spirals in contractions.

16 3.2 Specification of the learning process

17 Let us next formalize the learning described above. Our approach is similar to
18 De Grauwe and Macchiarelli insofar that we use the same type of heuristics and
19 updating of beliefs.

20 Under rational expectations, the forecasted variable will equal its realized value
21 in the next period, i.e. $E_t X_{t+1} = X_{t+1}$, denoting generically by X_t any variable
22 in the model. However, as anticipated above, we depart from this assumption in
23 this framework by making the forecast contingent on imperfect information, but
24 allowing the agents to learn. Expectations are replaced by a convex combination
25 of heterogeneous expectation operators $E_t \kappa_{t+1} = E_t^s \kappa_{t+1}$ and $E_t q_{t+1} = E_t^s q_{t+1}$. In
26 particular, agents forecast the LTV-ratio and the land price using two alternative
27 forecasting rules: *fundamentalist* vs. *extrapolative rule*. Under the fundamentalist
28 rule, agents are assumed to use the steady-state value of the LTV-ratio - κ^* , against
29 a naive forecast based on the LTV's latest available observation (extrapolative rule).
30 Equally for the value of land, *fundamentalist* agents are assumed to base their ex-
31 pectations on the steady-state value - q^* against the *extrapolatists* who naively base
32 their forecast on the latest available observable.¹⁴ Defining $i = (\kappa, q)$ we can formally

¹⁴The latest available observation is the best forecast of the future, i.e. a random walk approach

express the fundamentalists as:

$$E_t^{s,f} i_{t+1} = i^* \quad (20)$$

and the extrapolative (or adaptive) rule as:

$$E_t^{s,e} i_{t+1} = \theta i_{t-1} \quad (21)$$

This particular form of adaptive expectations has previously been modelled by Pesaran (1987), Brock and Hommes (1997, 1998), and Branch and McGough (2009), amongst others, in the literature. Setting $\theta = 1$ captures the "naive" agents (as they have a strong belief in history dependence), while a $\theta < 1$ or $\theta > 1$ represents an "adaptive" or an "extrapolative" agent (Brock and Hommes, 1998). For reasons of tractability, we set $\theta = 1$ in this model, but the model dynamics would not be significantly altered with any of the other parameter values.

Note that for the sake of consistency with standard RE DSGE model, all variables here are expressed in gaps. Focusing on their cyclical component makes the model symmetric with respect to the steady state (see Harvey and Jaeger, 1993). Moreover, this facilitates the interpretation of the model as the fundamentalists can be seen as 'benchmarking' the variable values, meanwhile the problem of extrapolists is pinned down to guessing the deviation of these values from their benchmark (or steady state).

Next, agents' preference for one forecast over the other depends on the (historical) performance of the two rules given by a publically available fitness measure, the mean square forecasting error (MSFE). After time $t + 1$ realization is revealed, the two predictors are evaluated *ex post* using MSFE and new fractions of agent types are determined. These updated fractions are used to determine next period (aggregate) forecasts of LTV-and land prices, and so on. Agents' rationality consists therefore in choosing the best-performing predictor using the updated fitness measure. There is a strong empirical motivation for inserting this type of switching mechanism amongst different forecasting rules (see DeGrauwe and Macchiarelli (2015) for a brief discussion of the empirical literature, Frankel and Froot (1990) for a discussion of *fundamentalist* behaviour, and Roos and Schmidt (2012), Cogley (2002), Cogley and Sargent (2007) and Cornea, Hommes and Massaro (2013) for evidence of *extrapolative* behaviour in the context of microeconomic and financial decision-making.).

1 The aggregate market forecasts of the LTV-ratio and land price are obtained as
 2 a weighted average of each rule ($i = \kappa, q$):

$$E_t^s i_{t+1} = \alpha_t^f E_t^{s,f} i_{t+1} + \alpha_t^e E_t^{s,e} i_{t+1} \quad (22)$$

3 where α_t^f is the weighted average of fundamentalists, and α_t^e that of the ex-
 4 trapolists. These shares are time-varying and based on the dynamic predictor se-
 5 lection. The mechanism allows to switch between the two forecasting rules based
 6 on MSFE / utility of the two rules, and increase (decrease) the weight of one rule
 7 over the other at each t . Assuming that the utilities of the two alternative rules
 8 have a deterministic and a random component (with a log-normal distribution as
 9 in Manski and McFadden (1981) or Anderson et al (1992)), the two weights can be
 10 defined based on each period utility $U_{i,t}^x$, $i = (\kappa, q)$, $x = (f, e)$ according to:

$$\alpha_{i,t}^f = \frac{\exp(\gamma U_{i,t}^f)}{\exp(\gamma U_{i,t}^f) + \exp(\gamma U_{i,t}^e)} \quad (23)$$

$$\alpha_{i,t}^e \equiv 1 - \alpha_{i,t}^f = \frac{\exp(\gamma U_{i,t}^e)}{\exp(\gamma U_{i,t}^f) + \exp(\gamma U_{i,t}^e)} \quad (24)$$

11 ,where the utilities are defined as:

$$U_{i,t}^f = - \sum_{k=0}^{\infty} w_k [i_{t-k-1} - E_{t-k-2}^{s,f} i_{t-k-1}]^2 \quad (25)$$

$$U_{i,t}^e = - \sum_{k=0}^{\infty} w_k [i_{t-k-1} - E_{t-k-2}^{s,e} i_{t-k-1}]^2 \quad (26)$$

12 and $w_k = (\rho^k(1 - \rho))$ (with $0 < \rho < 1$) are gemoetrically declining weights
 13 adapted to include the degree of forgetfulness in the model (DeGrauwe, 2012). γ is
 14 a parameter measuring the extent to which the deterministic component of utility
 15 determines actual choice. A value of 0 implies a perfectly stochastic utility. In
 16 that case, each agent decides to be one type or the other simply by tossing a coin,
 17 implying a probability of each type equalizing to 0.5. On the other hand, $\gamma = \infty$
 18 impls a fully deterministic utility, and the probability of using the fundamentalist
 19 (extrapolative) rule is either 1 or 0. Another way of interpreting γ is in terms of
 20 learning from past performance: $\gamma = 0$ impls zero willingness to learn, while it
 21 increases with the size of the parameter, i.e. $0 < \gamma < \infty$.

22 As mentioned above, agents will subject the performance of rules to a goodness-

of-fit measure and choose the one that generates least errors. In that sense, agents are 'boundedly' rational and learn from their mistakes. More importantly, this discrete choice mechanism allows to endogenize the distribution of heterogeneous agents over time with the proportion of each agent using a certain rule (parameter α). The approach is consistent with the empirical studies (Cornea et al, 2012) who show that the distribution of heterogeneous agents varies in reaction to economic or financial volatility (Carroll (2003), Mankiw et al (2004)).

3.3 Learning - stability remark

One valid critique of learning environments in macroeconomics has been that they often become unstable. Unless strict boundaries are imposed on the learning process (i.e. a (high) degree of bounded rationality is imposed which ensures that system will, locally, converge to rational expectations equilibrium), the model often turns explosive.¹⁵ In our framework, however, we do impose strict limits on the learning process and trace the learning dynamics throughout the entire simulations. Moreover, using the results of Bullard and Suda (2011) that macroeconomic systems with Bayesian learning are non-explosive as expectations are locally stable in the sense of Evans and Honkapohja (2001) and the insights from Camerer and Ho (1999) or Payzan-LeNestour and Bossaerts (2014) that adaptive and Bayesian learning are 'close cousins', we can conclude that our framework does not suffer from explosive paths or instability.¹⁶

3.4 The recursive solution method and the numerical set-up

We formulate the model in recursive form and solve using recursive methods (see DeGrauwe (2012) for further details). This allows for non-linear effects and is a more cost-effective alternative to the standard Bellman equation approach since we avoid using aggregate states and iterations to converge on the representative agent condition, matching individual and aggregate laws of motion for credit.

The model has eleven endogenous variables: land price, leverage, consumption, loans, interest rate on loans, deposits, interest rate on deposits, bank profits, bank

¹⁵A very popular example of a bounded rationality environment in the literature has been the recursive least square learning.

¹⁶For future work, however, it would be interesting to computationally test the long-run dynamics of our learning framework, and compare with other (similar) versions of adaptive or Bayesian updating set-ups.

1 equity, land, and the interest rate. The first four are obtained after solving the fol-
 2 lowing reduced equilibrium system that iterates on the policy and pricing functions
 3 using households' FOCs and the forecasting rules:

$$\begin{aligned}
 & \begin{bmatrix} 1 & \mu^{fx} & \bar{c} & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} q_t \\ \kappa_t \\ c_t \\ b_t \end{bmatrix} = \\
 & = \begin{bmatrix} \alpha\nu & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_t^s[q_{t+1}] \\ E_t^s[\kappa_{t+1}] \\ E_t^s[c_{t+1}] \\ E_t^s[b_{t+1}] \end{bmatrix} + \\
 & + \begin{bmatrix} 1 - \nu & -\mu^{fx} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_{t-1} \\ \kappa_{t-1} \\ c_{t-1} \\ b_{t-1} \end{bmatrix} + \\
 & + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 + r_{t-1}^d \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{t-1}^b \\ l_{t-1} \\ d_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \epsilon_t^z \\ \epsilon_t^{Eb} \\ \psi_t \\ \epsilon_t^r \end{bmatrix}
 \end{aligned}$$

7 Using matrix notation, we can write this as: $\mathbf{AZ}_t = \mathbf{B}\tilde{\mathbf{E}}_t\mathbf{Z}_{t+1} + \mathbf{CZ}_{t-1} + \mathbf{DX}_{t-1} + \mathbf{Ev}_t$.
 8 We can solve for \mathbf{Z}_t by inverting: $\mathbf{Z}_t = \mathbf{A}^{-1}(\mathbf{B}\tilde{\mathbf{E}}_t\mathbf{Z}_{t+1} + \mathbf{CZ}_{t-1} + \mathbf{DX}_{t-1} + \mathbf{Ev}_t)$
 9 and assuring \mathbf{A} to be non-singular.

10 Once these optimal values for the policy functions have been found, they are
 11 then inserted into the remaining general equilibrium system and the values of the
 12 remaining model variables are recursively solved. So, the solution for land, the
 13 interest rate on borrowings, deposits, bank profits, bank equity, and the interest
 14 rate are recursively obtained using the solutions obtained for land price, leverage,
 15 consumption and loans iterated above.

16 Expectation terms with an $s E_t^s$ implies that we derive the optimal solution using
 17 the subjective beliefs governed by the learning process specified above.

18 Note that for the forecasts of land price and leverage, the expectation terms in
 19 equations 5, 12 and 15 are substituted by the discrete choice mechanism in 22.

We have four shocks in this model. ϵ_t^z is a standard TFP shock in the land production function. ϵ_t^{Eb} is a shock to bank capital (or equity), ψ_t denotes a shock to income (or collateral value), whereas ϵ_t^r is a standard monetary policy shock. Their parametrizations will be discussed in the next subsection.

3.5 Calibration and simulations

We will divide the discussion in three parts. First, we will discuss the parameters related to the general equilibrium set-up. We will continue with the parameters related to the learning dynamics in the second part, followed by the calibration of the four shocks in the model. A full list of parameters and their values are reported in Table 1.

For the calibration of parameters related to the general equilibrium, we use the parameters calibrated or estimated in a number of closely related DSGE models. In particular, the (constant) risk aversion coefficient σ in households' utility function is, following Boz and Mendoza (2014), set to 2. We set the share of impatient households in the total economy to 0.61, as in Brzoza-Brzezina et al (2014), in order to match the micro data on the share of liquidity constrained consumers reported in the Survey of Consumer Finances (SCF). This is also in line with the number reported in Justiniano et al (2015). The discount factor β of patient households is higher and set to 0.9943 in order to obtain an annualized average real interest rate of slightly below 3%. This is in line with much of the literature, including Gerali et al (2010) and Brzoza-Brzezina et al (2014). The discount factor of the impatient types is lower and set to 0.975, as in Gerali et al (2010).

For parameters related to financial intermediaries, we use the estimation results from Gerali et al (2010) and De Grauwe and Macchiarelli (2015). In particular, we set the share of bank profits in bank equity equation ω^b to 1, the cost for managing banks' capital position δ^k to 0.1049, the adjustment costs of changing the interest rate on deposits Adj^b to 0 (since the unlimited access to liquidity from the central bank makes this process costless) and the target capital-to-loans ratio ν^b (or the inverse of the leverage target ratio) to 0.09. In order to make the deviation from this target value costly, we calibrate the cost parameter κ^{Eb} to 11.49, which is the value obtained from estimations in Gerali et al (2010).

Turning to the land market, we use the values obtained in Boz and Mendoza (2014). In particular, we calibrate the factor share of land in the production α to

1 0.025, and we set the supply of land l fixed at 1. The (fixed) Lagrange multiplier
2 μ^{fx} in the credit constraint, which is used to derive the shadow value of collateral
3 in the land price function in equation 12, is set to 0.30.

4 Following Boz and Mendoza (2014), we set the consumption-GDP ratio in the
5 aggregate resource constraint to 0.670, or two-thirds of the total output. Meanwhile,
6 the remaining third is split between land and bank equity, where land-GDP ratio is
7 set to 0.20 and bank equity-GDP to 0.13.

8 For the Taylor-rule parameters, we use the values estimated in Gerali et al (2010).
9 In particular, the interest rate smoothing (AR) coefficient is set to 0.77, the response
10 to inflation in the Taylor rule to 2.01, meanwhile the response to output is set to
11 0.35. Equally, for macroprudential policy, we set the target (or cap) on household
12 leverage $\bar{\kappa}$ to 2, and the response of LTV to credit growth ρ^κ in the Taylor-type
13 macroprudential rule to 0.75, as in Lambertini et al (2013).

14 We turn to the parameters governing the learning process. The initial fraction
15 of fundamentalists and extrapolists, α_0^f and α_0^e are each set to 0.5. The switching
16 parameter, γ in equations 23 and 24 is set to 1, as in Brock and Hommes (1998). ρ ,
17 or the geometrically declining weight adapted to include a degree of forgetfulness in
18 the learning dynamics in 25 and 26, is set to 0.5. For fundamentalists, we set the SS
19 value of LTV, κ^* to 0.93 (as in Brzoza-Brzezina et al, 2014), and for the land price
20 q^* simply to 1. To conclude this part, we make the land price highly contingent
21 on its forecasted value by households, and therefore set the weight of the forecasted
22 land price in the land price function ν equal to 0.7. That is in order to capture the
23 uncertainty regarding its future value in the aggregate land dynamics.

24 We are considering four shocks in this model. A shock to TFP (or technology),
25 (bank) capital quality, household income, and a monetary policy shock. The stan-
26 dard deviation of all shocks is normalized to 1 to facilitate the interpretation of the
27 impulse responses. In line with the literature, the TFP and monetary policy shocks
28 include an AR component equal to 0.90. (Bank) capital quality and income shocks,
29 on the other hand, are each modelled as a white noise (with no AR component)
30 since they lack a theoretical grounding for incorporating inertias into their process.

31 We simulate the model for 2000 periods, or 500 years.

Table 1: Parameters in the model and their descriptions

| Parameter | Description | Value |
|-------------------|--|--------------------|
| <u>Calibrated</u> | | <u>Calibration</u> |
| σ | Constant risk aversion parameter in CRRA utility function | 2 |
| ω | Share of impatient households in the economy | 0.61 |
| β^P | Discount factor of patient households | 0.9943 |
| β^I | Discount factor of impatient households | 0.975 |
| ω^b | Share of bank profits in bank equity accumulation | 1 |
| δ^k | Cost for managing banks' capital position | 0.1049 |
| Adj^b | Adjustment cost for changing the deposit rate | 0 |
| ν^b | Target capital-to-asset ratio | 0.09 |
| κ^{Eb} | Cost of deviating from target capital-to-asset ratio | 11.49 |
| α | Factor share of land in production | 0.025 |
| l | Aggregate land supply | 1 |
| μ^{fx} | (Fixed) Lagrangian multiplier of the credit constraint | 0.3 |
| \bar{c} | Consumption-GDP ratio | 0.67 |
| \bar{l} | Land-GDP ratio | 0.20 |
| \bar{c} | Bank equity-GDP ratio | 0.130 |
| γ^r | Interest rate smoothing parameter | 0.77 |
| γ^π | Response to inflation in the Taylor rule | 2.01 |
| γ^y | Response to output in the Taylor rule | 0.35 |
| $\bar{\kappa}$ | Cap on household LTV-ratio | 2 |
| ρ^{κ} | Response of LTV to credit growth | 0.75 |
| α_0^f | Initial fraction of fundamentalists | 0.5 |
| α_0^e | Initial fraction of extrapolators | 0.5 |
| γ | Switching parameter in MSFE | 1 |
| κ^* | SS LTV-ratio | 0.93 |
| q^* | SS land price | 1 |
| ν | Weight of forecasted land price in the land price function | 0.7 |
| \tilde{c} | SS consumption parameter in CRRA geometric series | 0.125 |
| ϵ^z | Standard deviation of the TFP shock | 1 |
| ϵ^{Eb} | Standard deviation of the capital quality shock | 1 |
| ψ | Standard deviation of the income quality shock | 1 |
| ϵ^r | Standard deviation of the monetary policy shock | 1 |
| ρ_z | AR parameter in the TFP shock process | 0.9 |
| ρ_{Eb} | AR parameter in the capital quality shock process | 0 |
| ρ_ψ | AR parameter in the income shock process | 0 |
| ρ_r | AR parameter in the monetary policy shock process | 0.9 |

4 Quantitative results

Our analysis consists of four parts. The first part is a depiction and analysis of the nature of the model variables over the business cycle, with a particular focus on the systemic events. The second part is an examination of the (model generated) second-, and higher-order moments. In the third part, we will analyze (model consistent) impulse responses to the four shocks described above. To end, in the final part we will examine the effects of a macroprudential policy in terms of smoothening

1 the business cycles, reducing the asymmetries and fat tails of model variables, and
 2 improving the welfare.

3 **4.1 Forcing variables**

4 The four shocks we will examine are:

- 5 • (Positive) TFP (or technology) shock, ϵ_{z_t}

$$y_t = z_t \epsilon_{z_t} g(l_t) \quad (27)$$

6 ,where the TFP shock has an AR component, ρ^z calibrated to 0.9:

$$\epsilon_{z_t} = \rho^z + \eta \quad (28)$$

- 7 • (Negative) capital quality shock, ϵ_{Eb_t}

$$E_{t+1}^b = (1 - \delta^b) E_t^b + \pi_t^b + \epsilon_{Eb_t} \quad (29)$$

8 ,where ϵ_{Eb_t} is a white noise shock to the evolution of bank equity stock.

- 9 • (Positive) income shock, ψ_t :

$$\kappa_t q_t l_{t+1} \psi_t \leq -\frac{b_{t+1}}{R_t^b} \quad (30)$$

10 ,where ψ_t is a white noise shock to the collateral constraint of impatient house-
 11 holds. And a

- 12 • Standard (negative) monetary policy shock (ϵ^r):

$$r_t = \gamma^r r_{t-1} + \gamma^\pi \pi_t + \gamma^y y_t + \epsilon^r \quad (31)$$

13 and ϵ^r is a white noise shock to monetary policy. In our simulations, we calibrate
 14 the interest rate smoothing parameter γ^r to 0.9. The standard deviation of all shocks
 15 is normalized to 1.

4.2 The nature of cycles in the model

Figures II.1 to II.4 depict the business cycle evolution of key model variables, such as consumption, price of land, loans to households, interest rate on borrowings, bank equity, nominal interest rate, learning variables, and market sentiment. We are interested in examining two things in this section. On one end, we wish to understand the type of boom-bust cycles that are captured in this model, focusing special attention to whether the model is capable of generating systemic crises as rare events. These are depicted in Figures II.1 to II.3. On the other end, we wish to track the learning process and analyze its impact on the model dynamics. Those are depicted in Figure II.4. Remember that the model is simulated over 2000 quarters (or 500 years).

4.2.1 Macro-financial cycles

In all graphs, the zero-line represents the trend and the area above (below) it represents the positive (negative) cyclical deviation from the trend. The series should be interpreted as the filtered cyclical component of a time-series with an independent time-varying (or time-invariant) trend.

Looking at consumption, the first thing to note is the asymmetric cycles that the model generates. While there are several episodes of heavy boom in consumption (t=100,550,950,1100 or 1600), these are followed by even sharper falls in the level of consumption. So while the heaviest boom in consumption is around 20 % above the trend, the sharpest contractions lie around 30 % below the trend. Moreover, the persistence in booms is higher than the persistence in busts. Hence both the frequency and the amplitude of expansions and contractions are asymmetric.

Next, the model is capable of generating small as well as large boom-bust cycles. While the majority of the cycles are small, several of them can be considered ‘large’. Using Cogley and Sargant’s (2008) definition that a severe (or systemic) crisis is one where consumption contracts by at least 25 %, we find 10 such contractions during the 500 year simulation exercise. They are marked by a grey arrow in Figure II.1. If we take an average over the entire sample, than two systemic crises occur every century. In most of these, the contraction in consumption is higher than the 25%, which makes them clear candidates for a truly systemic contraction. Note, moreoverover that the contractions are preceeded by a heavy surge in consumption. This is in particular true for the second, seventh and tenth contraction, where consumption increases by

1 30-40% before it drastically reverses. Also, the build-up phase is much longer than
2 the subsequent bust. Hence, this allows enough time for risks and credit to build
3 up before they materialize.

4 In addition, there are two key stylized facts that the model captures well. First,
5 Cogley and Sargent (2008b) note that once an economy is in a low-growth state,
6 there is a certain positive probability of running into a sequence of contractions.
7 That is what we see after contraction 2. While the economy tries to recover from
8 the first downfall, in ten years (on average) it runs into the next systemic crisis.
9 As a matter of fact, five systemic contractions occur in less than 300 quarters (75
10 years). The second stylized fact is that a long build-up of stocks, risks and liquidity
11 in the (financial) system makes the entire economy much more unstable and prone
12 to heavy reversals than an economy where the long build-up phase is controlled and
13 shortened. In our model, that is exactly what occurs. Prior to $t=700$, the economy
14 only experiences one systemic crisis. However, after the exceptionally long build-up
15 phase in $t=[180, 700]$, the economy suffers 9 crises in around 1000 quarters (or 6
16 crises in 700 quarters). That is a 6-fold increment during the same time interval.
17 Hence this confirms the fact that a long period with high and sustained build-up of
18 risks, credit, leverage (and speculation via asset prices) with only minor contractions
19 changes the entire structure of the economy over the longer-run. This is because
20 the heavy and sustained accumulation of stocks and optimism make the economy
21 more susceptible to shocks and significantly increases the probability of future sharp
22 downturns.

23 This observation is further confirmed by the business cycle evolution of loans in
24 Figure II.1, where 6 out of the 10 systemic crises result in the historically heaviest
25 contraction in lending. Total lending to households decreases by between 15-20%
26 during those systemic crises, representing a highly significant downturn. Moreover,
27 the preceding build-up of credit before systemic crisis 2, and the following cycle of
28 contractions are clearly visible also on the same graph.

29 The same pattern is observed for land prices. During the same episode that
30 consumption sharply contracts, the land price falls by between 30 and 45%. Via
31 wealth and credit channels, it significantly contracts consumption, which is why we
32 observe the systemic crises above. Moreover, the speculation hypothesis mentioned
33 above is confirmed. Because agents have to forecast the land price, and they can only
34 do it based on the historical evidence on realizations of the same (or on the steady
35 state value normalized in this case to 1), a market sentiment cycle will inherently take

over in the valuation process. The more that the agents observe positive realizations of the land price, the higher the probability of them forecasting a higher land price for the next period. Such sentiments re-inforce themselves even more when a long and sustained series of increasing values have realized and the sample of systemic crisis is small. That is exactly what happens between $t=[180, 700]$. During this period, the land price has an increasing trend and the economy had only experienced one systemic contraction so far. Therefore, while the land price takes a while to start this positive feed-back loop of increases, once in $t=270$, enough positive realizations of land price have occurred and the systemic contraction is sufficiently far behind in order for the agents to turn truly optimistic and put a heavy weight on an increasing future land price at the same time as it puts a negligible probability on a future fall in the price. But, after the second crisis, the weight they put on future contractions is significantly higher, and since we include memory in their learning, the agents also become more ‘insecure’ in their forecast and thus put a higher probability of such events re-occurring in the near future.¹⁷Hence why we see a downward spiral in prices after the second systemic crises has realized.

Turning to bank equity and interest rates on borrowing, we also see that the systemic crises affect the profitability of the bank. Taking into account that bank equity and the interest rate on borrowing have the opposite signs in equation I.13, once the price of household collateral (land) starts to contract, the financial intermediary is obliged to increase its borrowing rate, as the probability of default of impatient households has increased and so it is more risky to lend to them. However, that will reduce the amount of total borrowing, and thus the profits of the bank (since the fall in lending is higher than the rise in the interest rate margin). This will subsequently lead to a fall in bank equity, as governed by equation 7. On contrary, the higher the bank equity that a financial intermediary holds (a feature in upturns), the more leeway the bank has to extend its lending, and so it reduces its lending rate. This is why we see the opposite business cycle evolution for bank equity and the lending rate. Note also how sensitive the interest rate setting is to movements in bank equity. Roughly a 1% drop (rise) in bank equity from its long-term trend generates a 10% rise (drop) in the lending rate from its trend. This is due to the quadratic composite social cost imposed on excessive leverage in equation I.13, which pushes the interest rate up by more than proportionally. Due to

¹⁷Remember that the parameters ρ and γ in the learning set-up in equations 25 and 23 guide the memory of the agents in our learning framework.

1 this heavy (de)-leveraging (or rebalancing) over the cycle that the financial sector
2 becomes a powerful propagator of shocks, originated within the financial sector as
3 well as outside. Thus a sufficiently high de-leveraging can, via the lending channel,
4 cause a severe downturn in the real economy. We will analyze this mechanism in
5 more depth in the impulse response section.

6 **4.2.2 Learning cycles and model dynamics**

7 The model allows us to track the learning dynamics explicitly and examine how
8 imperfect information generates market sentiment in the market for land.

9 The first graph in Figure II.4 depicts the proportion of the households that use
10 the extrapolist rule to forecast the leverage ratio. A value of 1 means that the
11 entire population uses that rule, whereas 0 means that the entire population uses
12 the fundamentalist rule. The same applies for the forecast of land price in the second
13 graph. Finally, the graph on market sentiment depicts the (subjective) valuation of
14 the land price that all agents expect will be in the next period. A value of 1 means
15 that all households believe that the price will rise, i.e. *optimism* while a value of 0
16 means that all households believe the price will fall, i.e. *pessimism*.

17 The first thing to note is that there are cycles, or regime-switches, in the usage
18 of forecast rules. There are, however, more regime-switches in the forecast of land
19 price compared to leverage. Taking into account that the switching parameter γ
20 is calibrated to 1 in both cases, the difference between the two is endogenously
21 generated by the model. This means that the goodness of fit of the fundamentalist
22 rule in the case of leverage is, on average, higher (in relative terms) compared to the
23 same rule in the forecast of land price. We believe this is due to the (more urgent)
24 necessity of households to use a fixed point reference when conditionally optimizing
25 on the leverage ratio. Since leverage is an exogenous process at time $t=0$ and it is
26 dependent on the price of risks (which in this set-up agents do not fully know in
27 real time), a ‘benchmarking’ approach in forecasting becomes much more attractive
28 (and efficient) as agents can use a fixed reference to rationally optimize conditional
29 on this simple heuristics.

30 Nevertheless, note that in both forecast games, there is significantly more inertia
31 in the extrapolative rule. This means that once households use the extrapolative
32 rule, they are very likely to remain under that regime. As a result, most of the time
33 the agents prefer to use history to forecast the future. We should therefore expect
34 to observe strong market sentiment building up over time.

That is confirmed by the third graph in Figure II.4. Most of the time, agents are optimists. However, at some point the sentiment reverts and agents turn pessimists. Just as in Boz and Mendoza (2014), this occurs once they have observed one negative realization of the variable they forecast. Since in period $t=0$ households start in a *tabula rasa* environment, the first observation of the realized land price will determine their forecast of the future price. The more of price rises they observe, the lower the probability they will put on a future price decrease, and therefore they will turn more and more *optimistic* over time. However, as soon as a reverse in price is observed, households will put a very low (if not zero) probability on future price increase, and will therefore forecast a price decrease. Since booms are longer and more sustained than busts, agents will remain in the *optimistic* interval for longer periods of time compared to the *pessimistic* interval. But as agents observe more of the price decreases, they learn about them and put a higher probability of them occurring in the future. Thus that is why we see an increase in the number of sentiment-switches from *optimism* to *pessimism* as time goes by. The speed of this is governed by how much weight they put on the historical realizations (θ parameter) and how far back they remember, (ρ and γ parameters, i.e. how adaptive their learning is). Because we have calibrated the parameters in such a way that households are slow learners, the time elapsed before more frequent switches occur is considerable. Note also that exactly for this reason, persistent *optimism* build-up can also occur after 1000 quarters of simulation since positive observations from recent history outweigh the learning from more distant past if a sufficient sequence of price increases has realized.

Our sequence of beliefs is in many ways similar to the ones obtained in Boz and Mendoza (2011, 2014), but with some important deviations. Comparing the third graph in their Figure 6 to our third graph in Figure II.4, in both frameworks the *optimistic* interval is more persistent. Moreover, as the number of low leverage regimes is observed, the number of switches to *pessimism* increases. However, because the state-space is dichotomous in their world, the reversals are also more abrupt. This should result in sharper turning-points over the business cycle, not because of the model dynamics but because of the constrained model learning construction. In our case, on the other hand, households ‘guess’ a full continuous state-space of values, and so the reversals are more gradual. Hence, if sharp declines are observed in the business cycles, they are entirely generated by the endogenous model dynamics (via the interaction between learning and financial frictions), and not by a demarca-

tion of the state-space. Further to that, learning in their framework is significantly faster than in ours, which means that convergence to a RE model is achieved after a relatively short period of time. Looking at the first two graphs of Figure 6, the subjective transition probability is very close to the actual probability already before 300 quarters (or 75 years). That is possible because the state-space is reduced and because agents engage in restricted Bayesian learning (which has been shown to converge faster). In our model, on the other hand, the environment is more uncertain and learning is slower. Because systemic crises are rare, learning about them is also slow, and that is why uncertainty regarding leverage and land prices remains in the model dynamics for a much longer period of time. Lastly, while in Boz and Mendoza (2014) agents know the land price and forecast only the transition probability of leverage, we extend it to include land prices, since it directly depends on the leverage (via the shadow value of collateral). We think that our approach is more realistic under the asset pricing of Mendoza (2010) since a complete knowledge of the land price would allow households to learn the ‘true’ value of leverage by solving the rest of the model and recursively extract the value of leverage.

4.3 Distributions and statistical moments over the business cycle

The second part of the model evaluation consists of analysing and validating the model-generated distribution and statistical moments over the business cycle. These are generated using the entire sample of 2000 quarters. For our purposes, we will use the data on (auto)-correlations in Table II.1, the data on second and higher moments in Table II.2, as well as histograms of a selection of these variables in Figures II.5 to II.7.

4.3.1 Correlations

The model is capable of generating high persistence in the evolution of key variables. Except for the nominal interest rate, the rest has an autocorrelation coefficient above 0.95. That is much higher than in most DSGE models. Following the discussion in De Grauwe and Macchiarrelli (2015), we believe that the learning framework is the underlying cause of this high persistence in the model since beliefs are self-enforcing and therefore persistent over time. This pattern has also been documented in other medium-size learning DSGE models, such as Slobodyan and Wouters (2012), where

they show that learning improves the fit of the model to the data compared to the benchmark Smets and Wouters (2007) set-up. Moreover, we note that the persistence of the financial friction variables is slightly higher than that of the macroeconomic ones (consumption, land price, policy rate). Hence, there is an additional layer of persistence coming from the credit friction, albeit the difference is small.

Cross-correlations between the model variables is also high. Most variables are positively correlated, except the ones including leverage and the interest rate on borrowing. A higher leverage reduces (future) consumption ($\rho = -0.975$) since it reduces the value of land ($\rho = 0.985$) and it restricts the amount the impatient households can borrow. Similarly, a higher interest rate means that households can borrow less, and therefore consume less ($\rho = -0.99$). Equally, the interest rate on borrowings and the land price are negatively correlated since a lower value of collateral increases the cost of borrowing (via the collateral channel), or a higher borrowing rate increases the discounting of today's value of land, by generating lower consumption ($\rho = -0.99$).

4.3.2 Distributions and statistical moments

The key thing to note in this section is that all model variables captured in Figures II.5 to II.7 and Table II.2 are non-Gaussian, skewed and with fatter tails compared to variables in standard DSGE models. Normality tests have led us to reject the assumption of normal distributions.

The variance of almost all variables (except for bank equity and market sentiment) is very high, which means that there are high fluctuations in these variables over the business cycles (i.e. the amplitude of the cycles is high). Moreover, we see two patterns in the skewness and tails. While consumption, credit, land price, and bank equity have a negative skew (to the left) and with a fatter tails on the left side, leverage, the interest rates and market sentiment have a positive skewness (to the right) and a fatter tail on the right. This means that while there are significant booms (i.e. a high probability mass on the right of the median), with a rise in credit, leverage, consumption and drop in borrowing rate, the reversals are so sharp that the drop in these variables (or rise in borrowing rate) outperforms the booms, and that is why we see those shapes in the distributions. Hence systemic crises drive the probability mass much further from the mean, and the 'true' probability of such events becomes significantly above zero at any point in time. In RE frameworks, households would know all this information which would lead them to

1 price in the probability of systemic shutdowns in their optimizations, which most
2 probably would mean that credit, leverage and consumption would not build-up as
3 much. But, because of uncertainty in our framework, agents have a harder task to
4 define (and understand) these distributions (in particular for leverage and land price
5 and so missprice the ‘true’ tail-risks including an *ex ante* underpricing of systemic
6 crises events.¹⁸

7 **4.4 Impulse response analysis**

8 Figures II.8 to II.9 depict the impulse responses to a positive TFP shock and Figures
9 II.10 to II.11 to an expansionary monetary policy shock.

10 The numbers on the x-axis indicate number of quarters. All the shocks are intro-
11 duced in $t=100$ and we observe the responses over a period of 50 quarters (or 12.5
12 years). Note that in these figures we depict the *median* impulse response in black
13 amongst a distribution of impulse responses generated with different initializations of
14 the learning parameters. The red lines in the Figures represent the 95% confidence
15 intervals, or a full distribution of impulse responses. For the sake of clarity and focus
16 in the discussions, we will only concentrate on the median impulse response, which
17 is a good representation of the overall distribution. Moreover, we will only concen-
18 trate on the standard TFP and monetary policy shocks in the text since these are
19 standard in the literature and are modelled with persistence. However, should you
20 want to see the discussion of the other two shocks (a negative bank capital quality
21 and a positive (financial) wealth shock) do not hesitate to contact the authors.

22 **4.4.1 TFP shock**

23 A 1% TFP shock improves the production of land, and therefore increases the land
24 price by 1.5%. Because quantity of land is fixed, all of the efficiency improvement
25 will go to land price, by improving the intertemporal consumption smoothing of
26 households. Since value of household equity goes up, leverage of impatient house-
27 holds decreases by 1.6%, and their external financing possibilities improves. Via the
28 collateral constraint, impatient households are able to borrow more for the same
29 collateral, which initially pushes up the loans they obtain by 0.4%. For financial
30 intermediaries, this leads to a higher bank equity value (0.04%), which gives them

¹⁸Only after a sufficient number of crises events have occurred, and assuming that the distribu-
tions of these variables do not change over time, they will start to approach their pricing of these
systemic events closer to the ‘true’ probability.

space to extend their credit line even further since their capital-to-asset ratio has increased. Via the interest rate margin equation, they reduce the interest rate on borrowing to households by 0.65%. This lower cost on loan repayment in turn allows impatient households to extend their borrowing even further in $t=102$, resulting in a peak increase in external financing at 0.65% above the pre-shock level. For financial intermediaries, this is an additional increase of bank equity by 0.02%, implying a total of 0.06% expansion in bank equity as a result of the TFP shock. Hence bank can extend its activity and size as a result of an improvement in the real (production) sector.

However, this extension in credit makes the households gradually more leveraged, and the opposite mechanism is then set in motion. The higher leverage raises the value of the left-hand side of equation 5, which reduces the amount of next-period borrowing (because of their negative relation), which in turn reduces their (future) consumption possibility, and therefore the price of land. This opposite mechanism continues until the economy returns to its pre-shock level.

4.4.2 Monetary policy shock

A reduction of 1% in the (risk-free) interest rate reduces the deposit rate by the same amount (since $R_t^d = r_t$). Since this reduces the financing cost for banks, they can therefore reduce their cost of lending in order to extend their asset side and increase their profitability. The resulting rise in bank lending increases the amount of credit that households get, and therefore their (expected) consumption possibility. Via the pricing function of land in equation 12, the land price also increases. This reduces the leverage of (impatient) households, and via the collateral constraint allows them to borrow more. The cost of borrowing therefore reduces even further, and the bank extends its credit even further. As a result, bank equity rises. The total effect of the expansionary monetary policy shock is that the interest rate on borrowings falls by 0.45%, the expansion in credit is 0.45%, the rise in land price is 0.9%, and the fall in leverage 1.55%. The resulting boom in consumption is first 0.8% followed by 1.1%, and the rise in profitability of intermediaries raises its total bank equity by 0.04%. Note that while the economy (including the financial sector) expands following both shocks, the expansion is quantitatively larger for the supply side (or TFP) shock.¹⁹ That is not surprising since our framework lacks

¹⁹Remember that both shocks are calibrated in the same way. The standard deviation of the white noise component is standardized to 1% while AR component is calibrated to 0.9. That is

1 sticky prices or wages which would make the monetary policy transmission more
2 persistent, as in standard NK-models. Therefore in our framework, policy makers
3 should concentrate on supply-side policies to generate sustained booms rather than
4 using (discretionary) monetary policy. Therefore, we also expect (relatively) a high
5 efficiency of macroprudential policy in smoothing the cycles, since the policy can
6 be viewed as a type of supply-side constraint on the “production” in the financial
7 sector.

8 **5 Macroprudential policy**

9 To quantify the (stabilizing) effects that a macroprudential policy can have, in par-
10 ticular on reducing the number (and impact) of systemic crises, we will evaluate
11 one particular type of macroprudential policy. We will concentrate on a cap on
12 (household) LTV, where the central bank will allow households to leverage up to a
13 certain level (but not beyond), and therefore restrict intermediaries to extend their
14 credit supply only up to a certain quantity.

15 Tables II.3 to II.4 report the statistical moments of the model variables with and
16 without macroprudential policy. Figures II.12 to II.14 depict the same moments in
17 terms of distributions for a selection of key model variables. Lastly, Figure II.15
18 compares the number of systemic crises compared to the benchmark model, while
19 Figure II.16 shows the number of switches between the extrapolative and fundamen-
20 talist rule for land price with and without a macroprudential rule.

21 For a list of figures for the remaining model variables, please do not hesitate to
22 contact the authors.

23 In our simulations, we set the LTV cap at 2. The effects are significant.

24 Starting with learning dynamics, because of the benchmark provided by the
25 cap, the learning with respect to the leverage is rapid. It only takes a few periods
26 for households to understand what their maximum limit is, and therefore their
27 subjective expectations converge to rational expectations. Under this setting, the
28 extrapolative rule becomes the best predictor of the future as no variations in that
29 variable will occur.

30 For land price in Figure II.16, the learning dynamics also changes. The funda-
31 mentalist rule becomes more frequently visited compared to the benchmark model
32 setting. We believe the reason is because agents now only need to learn about one

why we can directly compare the two effects.

variable meanwhile the other has a fixed point. Hence, benchmarking the land price q becomes more useful in forecasting since fluctuations in the model are reduced and therefore using a fixed reference point for forecasting becomes more effective.

In terms of the macroeconomic and financial model variables, we also see a significant change. Looking at the statistical moments and distributions, we see that most variables become more Gaussian. The distributions become more symmetric and the fat tails are reduced. In practical terms, it means that sharp rises or drops in these variables are reduced, as well as probabilities of systemic crises. Many (auto)-correlations are reduced, which implies less of the (market) sentiment driven cycles that we observed before. In addition, the volatilities and skewness of the variables are reduced by a factor of 2-4. Meanwhile, the kurtosis increases slightly, which means that the distribution becomes more centred around its mean/median. That is clearly visible in the figures for consumption, land price and credit.

This is also visible in Figure II.15. Comparing the number of systemic crises in the benchmark model with the augmented version, we observe a reduction of 50% in the latter. Instead of the original 10 crises in 500 years, we now get 5 crises over the same time period. In particular, the sequence of systemic crises that occurs after the second are (almost) reduced. Moreover, the losses in each of the crises is also reduced. Noting that the scaling in the second graph is 3 to 4 times smaller, the losses in each of the crises is reduced by, on average, a factor of 3. Thus, an LTV-cap does not only reduce the probability of a systemic crises by 50%, but it also reduces the losses incurred by each. As a result, the business cycles become shorter and the amplitude of each smaller.

While it is clear that the policy smoothens the cycles and reduces the systemic events, we would like to quantify these effects in terms of households' welfare. In standard RE DSGE models, one would value the welfare effects by calculating the (welfare) gains using a second order approximation of households' welfare. However, in our model, RE is substituted with subjective beliefs, which means that the policy maker does not know how to weight these beliefs into a general welfare function. Hence, imperfect information also concerns households' welfare.²⁰

To overcome this problem, we instead value the welfare using utility (or consumption equivalence) measure of an economy with and without the policy. Knowing the parameters in the utility function, and the median consumption of households in

²⁰Recently, Brunnermeier et al (2014) are trying to define 'belief-neutral' welfare functions in models with distorted (or imperfect) beliefs. However, more work is necessary before a robust method can be obtained for loss function derivations.

1 an economy with and without the LTV-cap, we can calculate the utility gains that
2 households will get from imposing the rule.²¹Since household utility only depends
3 on consumption, the gains will be expressed in consumption equivalence terms.

4 We find that the utility gains from using a cap on household LTV is 6.5%. It
5 means that, on average, a household will consume 6.5% more when a central bank
6 imposes a cap compared to an economy without it. Decomposing this gain, we find
7 that 6% out of the 6.5% derives from an increase in the level of consumption, while
8 0.5% comes from a reduction in variability (or volatility) of consumption over the
9 cycle. The reason for this heavy gain in level, we believe, comes from the reduction
10 in the systemic crises. Systemic crises are events when most of the consumption
11 level is reduced. In relative terms, this reduction in level is even higher than the
12 reduction in volatility of consumption, since the *ex ante* probability mass of such
13 event is not big. However, once that state becomes absorbing, the reduction in level
14 is very high.

15 To conclude, we compare our results with a more elaborate version of the LTV-
16 rule. Following the recent literature on macroprudential policy (see, for instance
17 Lambertini et al (2013) or Angelini et al (2014)), we also try a Taylor-type rule
18 specified in equation 19. We find that a more complex LTV-rule generates very
19 similar economic outcomes to the simpler rule we have used before. The only
20 difference is that the Taylor-type rule smoothens the fluctuations in the interest
21 rates by more. We believe that the explanation for this similarity lies in the learning.
22 While a Taylor-type rule increases the information content in the reaction function
23 and allows the central bank to react to more financial variables, it also delays learning
24 since some uncertainty regarding the leverage level remains. In the simpler rule,
25 on the other hand, there are gains from the heavy ‘benchmarking’ that a fixed
26 rule gives to households in their learning process. Having a fixed reference point
27 regarding leverage reduces uncertainty regarding its (future) value compared to a
28 more complex rule. We see that the fluctuations in the usage of extrapolative versus
29 fundamentalist rule for leverage is higher under the Taylor-type rule, which generates
30 additional dynamics. As a result, the simple LTV-rule is, in relative terms, more
31 effective in generating stability at a lower monitoring (or information) cost.

²¹Note that since model variables have asymmetric distributions in the benchmark model, the median is more representative of the centre of the distribution, rather than the mean. That is why we use the median consumption in our calculations. We could, however, trivially re-run the same experiments using the mean consumption values.

6 Discussion and concluding remarks

Deregulation in the financial services industry since 1980's, the increased competition amongst financial intermediaries and the unprecedented expansion in financial engineering since mid-1990's has, in an exceptional manner, increased the size of the financial sector. Their credit lines to the real economy, and the consumption possibility of households has been historically the highest in the period prior to the Great Recession. The US (and to certain extent the EU) economy experienced one of its sharpest booms in early 2000's. On the other end, however, the pricing of risks and leverage became an increasingly difficult task as uncertainty regarding the true accumulation of risks on balance sheets and the true exposure of households increased. The misspricing of risks gave leeway to market speculation and market sentiment-driven cycles. We include these observations in a recent model by Boz and Mendoza (2014), and analyze the effects of dynamic optimization under uncertainty on the macro-financial cycles, and the probability of systemic crises. In particular, we are interested in understanding the role that macroprudential policy plays in reducing the probability of systemic events and the losses generated by these.

Including these facts into a general equilibrium model with adaptive learning result in an increase in the amplitude and frequency of the cycles. The build-up phase of risks, credit, leverage and consumption is much longer and higher than in standard DSGE models. In the same way, once a reversal in lending occurs, the decline in all variables is also much sharper and lasts shorter. The probability of systemic crises is significant, and we find that, on average, 2 such crises occur every century. Moreover, we find that, different from standard boom-bust cycles, a systemic crisis can be followed by a sequence of subsequent contractions, as it makes the economy more unstable. The result is asymmetric distributions of key macroeconomic and financial variables, with high skewness and fat tails.

A simple cap on the LTV-ratio is effective in smoothening the cycles and reducing the effects of a deep contraction on the real-financial variables. The model distributions become much more symmetric and Gaussian. It also reduces the amount of borrowing and leverage in upturns. The number of systemic crises is halved, and the losses at each is reduced by, on average, a factor of 3. The consumption (utility) gains from such a policy are, on average 6.5% compared to an economy without a macroprudential rule. Also the stabilizing role of monetary policy is increased once a macroprudential rule is used. To conclude, a simple LTV-rule is preferred to a

1 more elaborate Taylor-type version because it provides a strong ‘benchmarking’ to
2 agents in their learning process, while generating same welfare (improving) effects
3 at a lower information cost.

4 These are promising results in our understanding of the probability of systemic
5 events, and their destabilizing macroeconomic impacts. While the road in reaching
6 a full understanding of such events is long, these should hopefully be seen as a
7 contribution in the right direction. Future research should therefore try to stretch
8 the framework of this paper in multiple directions.

9 First, a robust comparison is necessary between the learning framework in this
10 model and the Bayesian set-up in Boz and Mendoza (2014). Both are actively used
11 in the literature, and a serious comparison in terms of long-term learning, memory
12 and model dynamics should be welcomed.

13 Equally, the regime-switching in rules in this framework should be compared to
14 homogenous learning set-ups. A lot of the dynamics in this model comes from the
15 regime switching in switching. It would therefore be interesting to see the type of
16 macroeconomic dynamics we would get if agents use only one rule, possibly a more
17 elaborate adaptive rule such as least-square learning.

18 It would also be interesting to conduct a robustness exercise to test the model
19 performance for a larger parameter space of the learning variables. On the same
20 lines, it would be highly relevant for policy purposes to find the optimal LTV-cap
21 whereby gains from such a rule are maximized.

22 Lastly, systemic crises are rare and non-linear events. Therefore, it would be of
23 high interest to zoom-in such periods and only study the dynamics once such event
24 becomes absorbing. In particular, it would be interesting to examine the statistical
25 moments, the distributions and the transmission channels under only such states.
26 That would bring the model closer to the recent but blooming empirical literature
27 on tail-events and hyper correlations.

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3 Appendices

4 I The FOCs and the full model system

5 I.1 Households

6 Setting up the Lagrangians for the patient households, we get:

$$L_P = E_0 \beta_P^t \frac{(c_t^P)^{1-\sigma}}{1-\sigma} + \lambda_t^P [z_t g(l_t) + q_t l_t - q_t l_{t+1} + (1 + R^d) d_t - d_{t+1} - c_t^P - e_t] \quad (\text{I.1})$$

7 Taking the first order conditions with respect to c_t^P, l_t and d_t , we get:

$$\frac{\partial L_P}{\partial c_t^P} = (1 - \sigma) \frac{(c_t^P)^{-\sigma}}{1 - \sigma} - \lambda_t^P = 0 \Leftrightarrow \lambda_t^P = (c_t^P)^{-\sigma} \quad (\text{I.2})$$

8 where we set $\sigma < 5$ in order to find a reasonable solution.

$$\begin{aligned} \frac{\partial L_P}{\partial l_t} &= \lambda_t^P z_t g'(l_t) + q_t - \beta_P E_t^s [\lambda_{t+1}^P q_{t+1}] = 0 \Leftrightarrow \\ & q_t = \beta_P E_t^s [\lambda_{t+1}^P q_{t+1}] - \lambda_t^P z_t g'(l_t) \Leftrightarrow \\ & q_t = \beta_P E_t^s [(c_{t+1}^P)^{-\sigma} q_{t+1}] - (c_t^P)^{-\sigma} z_t g'(l_t) \end{aligned} \quad (\text{I.3})$$

$$\begin{aligned} \frac{\partial L_P}{\partial d_t} &= \lambda_t^P (1 + R_t^d) - \beta^P E_t^s [\lambda_{t+1}^P] = 0 \Leftrightarrow \\ & (1 + R_t^d) = \frac{\beta^P E_t^s [\lambda_{t+1}^P]}{\lambda_t^P} \Leftrightarrow \\ & (1 + R_t^d) = \frac{\beta^P E_t^s [(c_{t+1}^P)^{-\sigma}]}{(c_t^P)^{-\sigma}} \end{aligned} \quad (\text{I.4})$$

9 Similarly, for impatient households we get the following Lagrangian:

$$L_I = E_0 \beta_I^t \frac{(c_t^I)^{1-\sigma}}{1-\sigma} + \lambda_t^I [q_t l_t - q_t l_{t+1} - \frac{b_{t+1}}{R_t^B} + b_t - c_t^I] + \gamma_t^I [-\kappa_t q_t l_{t+1} - \frac{b_{t+1}}{R_t^B}] \quad (I.5)$$

Taking the first order conditions with respect to c_t^I, l_t and b_t , we get:

$$\frac{\partial L_I}{\partial c_t^I} = (1-\sigma) \frac{(c_t^I)^{-\sigma}}{1-\sigma} - \lambda_t^I = 0 \Leftrightarrow \lambda_t^I = (c_t^I)^{-\sigma} \quad (I.6)$$

where we again set $\sigma < 5$ in order to find a reasonable solution.

$$\begin{aligned} \frac{\partial L_I}{\partial l_t} &= \lambda_t^I q_t - \beta_I E_t^s [\lambda_{t+1}^I q_{t+1} + \gamma^I \kappa_t q_{t+1}] = 0 \Leftrightarrow \\ &\lambda_t^I q_t = \beta_I E_t^s [\lambda_{t+1}^I q_{t+1} + \gamma^I \kappa_t q_{t+1}] \Leftrightarrow \\ &q_t = \frac{\beta_I E_t^s [\lambda_{t+1}^I q_{t+1} + \gamma^I \kappa_t q_{t+1}]}{\lambda_t^I} \Leftrightarrow \\ &q_t = \frac{\beta_I E_t^s [(c_{t+1}^I)^{-\sigma} q_{t+1} + \gamma^I \kappa_t q_{t+1}]}{(c_t^I)^{-\sigma}} \Leftrightarrow \\ &q_t = \frac{\beta_I E_t^s [((c_{t+1}^I)^{-\sigma} + \gamma^I \kappa_t) q_{t+1}]}{(c_t^I)^{-\sigma}} \Leftrightarrow \\ &q_t = \frac{\beta_I ((c_{t+1}^I)^{-\sigma} + \gamma^I \kappa_t - \gamma^I \kappa_t)}{(c_t^I)^{-\sigma} - \gamma^I \kappa_t} \Leftrightarrow \\ &q_t = \frac{\beta_I (c_{t+1}^I)^{-\sigma}}{(c_t^I)^{-\sigma} - \gamma^I \kappa_t} \end{aligned} \quad (I.7)$$

By combining equations I.3 and I.7, we get the expression for land price as in Boz and Mendoza (2014). However, because we differentiate our households, the price of land is determined by the Euler equations of both the patient and impatient households. As a result, the (final) price of land will be a balance of the two, taking into account the differentiated discount factors ($\beta^P < \beta^I$), i.e. $\frac{\beta_I c_{t+1}}{c_t - \gamma^I \kappa_t} = \beta_P E_t^s [c_{t+1} q_{t+1}] - c_t z_t g'(l_t) \equiv q_t$. Because the share of the two household types in the economy is normalized to unti, we can write: $c_t^P = c_t^I = c_t$. That is the same expression as in I.22.

$$\begin{aligned}
\frac{\partial L_I}{\partial b_t} &= \lambda_t^I - \beta_I E_t^s \left[\frac{\lambda_{t+1}^I}{R_{t+1}^B} + \frac{\gamma_{t+1}^I}{R_{t+1}^B} \right] = 0 \Leftrightarrow \\
\lambda_t^I &= \beta_I E_t^s \left[\frac{\lambda_{t+1}^I}{R_{t+1}^B} + \frac{\gamma_{t+1}^I}{R_{t+1}^B} \right] \Leftrightarrow \\
(c_t^I)^{-\sigma} &= \beta_I \frac{1}{R_{t+1}^B} E_t^s [(c_{t+1}^I)^{-\sigma} + \gamma_{t+1}^I]
\end{aligned} \tag{I.8}$$

1 In addition, we include the three constraints (one for impatient and two for
2 patient households) as first order conditions with respect to λ_t^P , λ_t^I and γ_t^I .

3 **I.2 Financial Intermediary**

4 For financial intermediaries, once we recognize that patient households own the
5 intermediaries, we get the following Lagrangian:

$$\begin{aligned}
L_F &= E_0 \sum_{t=0}^{\infty} \beta_P^t \lambda_t^P [(1 + R^b)B_t - B_{t+1} + D_{t+1} - (1 + R_t^d)D_t + (E_{t+1}^b - E_t^b) \\
&\quad - \frac{\kappa_{EB}}{2} \left[\frac{E_t^b}{B_t} - \nu^b \right]^2 E_t^b + \lambda_t^F (D_t + E_t^B - B_t)] \tag{I.9}
\end{aligned}$$

6 Taking the partial derivatives with respect to B_t and D_t gives us:

$$\frac{\partial L_F}{\partial B_t} = \beta_P^t \lambda_t^P [R_t^B - \kappa_{EB} \left[\frac{E_t^B}{B_t} - \nu^B \right] \left[-\frac{E_t^B}{B_t^2} \right] E_t^B - \lambda_t^F] = 0 \Leftrightarrow \lambda_t^F = [R_t^B - \kappa_{EB} \left[\frac{E_t^B}{B_t} - \nu^B \right] \left[\frac{E_t^B}{B_t} \right]^2] / \beta^P \lambda_t^P \tag{I.10}$$

$$\frac{\partial L_F}{\partial D_t} = \beta_P^t \lambda_t^P [-R_t^D + \lambda_t^F] = 0 \Leftrightarrow \lambda_t^F = R_t^D / \beta^P \lambda_t^P \tag{I.11}$$

7 Substituting $\frac{\partial L_F}{\partial B_t}$ into $\frac{\partial L_F}{\partial D_t}$, via the Lagrangian multiplier λ_t^F , and multiplying
8 both sides with $\beta^P \lambda_t^P$ to get:

$$R_t^D = R_t^B - \kappa_{EB} \left[\frac{E_t^B}{B_t} - \nu^B \right] \left[\frac{E_t^B}{B_t} \right]^2 \tag{I.12}$$

9 Using the no arbitrage condition in bank financing, it implies that $R_t^D \equiv r_t$, and
10 we can rewrite the last expression as:

$$R_t^B = r_t + \kappa_{EB} \left[\frac{E_t^B}{B_t} - \nu^B \right] \left[\frac{E_t^B}{B_t} \right]^2 \quad (\text{I.13})$$

This is the same as in expression 11. Moreover, we include the balance sheet constraint $B_t = D_t + E_t^B$ into the list of FOC. We also include the evolution of bank capital in the conditions for financial intermediary, which is:

$$E_t^B = (1 - \delta^k) E_{t-1}^B + \omega^B [R_t^B B_t - R_t^D D_t - \frac{\kappa_{EB}}{2} \left[\frac{E_t^B}{B_t} - \nu^B \right]^2 E_t^B - Adj_t^B] \quad (\text{I.14})$$

I.3 Government

We complement the private sector optimizations with the government policies. In particular, the government sets r_t according to:

$$\frac{r_t}{r} = \frac{r_{t-1}}{r}^{\gamma_R} \left[\frac{\pi_t}{\pi} \frac{y_t}{y} \right]^{1-\gamma_R} e^{\epsilon_{R,t}} \quad (\text{I.15})$$

As a compliment, the government conducts a set of macroprudential policies (caps on LTV-ratios and capital requirements) according to:

$$\kappa_t = \bar{\kappa} \quad (\text{I.16})$$

with the *true* drift equal to:

$$\kappa_t = \gamma_\kappa \kappa_{t-1} + (1 - \gamma_\kappa) \bar{\kappa} + \epsilon_{\kappa,t} \quad (\text{I.17})$$

and a (bank) capital ratio to:

$$\frac{e_t}{b_t} \leq \bar{e} \text{ or } \frac{d_t}{b_t} \geq \underline{d} \quad (\text{I.18})$$

I.4 Aggregation

The clearing condition for the market of goods is:

$$c_t + \frac{b_{t+1}}{R_t^b} + Adj_t^B = b_t + z_t g(l_t) + \delta^k E_t^B \quad (\text{I.19})$$

The aggregate resource constraint in this economy is therefore:

$$rc_t = c_t + q_t [l_t - l_{t-1}] + \delta^k E_t^B + Adj_t^B \quad (\text{I.20})$$

1 where $c_t = c_t^P + c_t^I$ and Adj_t^B is a calibrated cost parameter. In the asset (land)
 2 market, the clearing condition is given by:

$$1 = l_t \quad (I.21)$$

3 I.5 Equilibrium system

4 Next, we need to define the equilibrium conditions and the state variables which will
 5 be the key for solving the learning model. We define the pricing function for land:

$$q_t = E_t^s \left[\sum_{j=0}^{\infty} \left[\prod_{i=0}^{\infty} \frac{1}{E_t^s [R_{t+1+i}^q]} \right] z_{t+1+j} g'(l_{t+1+j}) \right] \quad (I.22)$$

6 where the spread between the return on land and the policy rate is:

$$E_t^s [R_{t+1}^q - R_t] = \frac{(1 - \kappa_t) \gamma_t^I - Cov_t^s(\lambda_{t+1}^I, R_{t+1}^q)}{E_t^s [\lambda_{t+1}^I]}$$

7 and the return on land conditional on date-t belief is defined as:

$$E_t^s [R_{t+1}^q] = \frac{z_{t+1} g'(l_{t+1}) + q_{t+1}}{q_t}$$

8 and the (land) production function is $g(l_t) = l_t^\alpha$.

9 The pricing function for the interest rate on loans (bond) is:

$$R_t^B = r_t + \kappa^{EB} \left[\frac{E_t^B}{B_t} - \nu^B \right] \left[\frac{E_t^B}{B_t} \right]^2 \quad (I.23)$$

10 and for the interest rate on deposits it is:

$$(1 + R_t^D) = \frac{\beta^P E_t^s [c_{t+1}^P]}{c_t^P} \quad (I.24)$$

11 We also have the following policy functions:

$$(c_t^I)^{-\sigma} = \beta^I \frac{1}{R_{t+1}^B} E_t^s [(c_{t+1}^I)^{-\sigma} + \gamma_{t+1}^I] \quad (I.25)$$

$$d_{t+1} + c_t^P \leq z_t g(l_t) + q_t l_t - q_t l_{t+1} + (1 + R_t^d) d_t + e_t \quad (I.26)$$

$$c_t^I \leq q_t l_t - q_t l_{t+1} - \frac{b_{t+1}}{R_t^b} + b_t \quad (I.27)$$

$$\kappa_t q_t l_{t+1} \leq -\frac{b_{t+1}}{R_t^b} \quad (\text{I.28})$$

and the state equation:

$$E_t^B = (1 - \delta^k) E_{t-1}^B + \omega^B [R_t^B B_t - R_t^D D_t - \frac{\kappa_{E^B}}{2} [\frac{E_t^B}{B_t} - \nu^B]^2 E_t^B - Adj_t^B] \quad (\text{I.29})$$

To complete the system we define the shocks that we use. In particular, we define the (known) TFP-shock process z_t as:

$$\ln(z_{t+1}) = \rho \ln(z_t) + \epsilon_z \quad (\text{I.30})$$

the capital quality shock ϵ_{κ^B} as:

$$\epsilon_{\kappa^B} \sim N(0, \sigma_{\epsilon_{\kappa^B}}^2) \quad (\text{I.31})$$

where the same shock could be interpreted as an equity injection by the macro-prudential authority. Lastly, we define an income shock e^ψ according to:

$$\psi \sim N(0, \sigma_\psi) \quad (\text{I.32})$$

The income shock can be interpreted as exogenous movements in land prices, or in return of land.

We solve this system according to the method and algorithm described in section 3.

1 II Figures and Tables

Table II.1: Model (auto)-correlations

| Variables | (Auto)-correlations |
|--------------------------------|---------------------|
| $\rho(b_t, b_{t-1})$ | 0.988 |
| $\rho(c_t, c_{t-1})$ | 0.972 |
| $\rho(E_t^b, E_{t-1}^b)$ | 0.988 |
| $\rho(q_t, q_{t-1})$ | 0.986 |
| $\rho(r_t, r_{t-1})$ | 0.883 |
| $\rho(\kappa_t, \kappa_{t-1})$ | 0.99 |
| $\rho(r_t^b, r_{t-1}^b)$ | 0.988 |
| $\rho(b_t, \kappa_t)$ | 0.99 |
| $\rho(c_t, ms_t)$ | 0.82 |
| $\rho(b_t, c_t)$ | 0.989 |
| $\rho(c_t, \kappa_t)$ | -0.975 |
| $\rho(b_t, q_t)$ | 0.99 |
| $\rho(q_t, \kappa_t)$ | -0.985 |
| $\rho(r_t, y_t)$ | 0.17 |
| $\rho(c_t, r_t^b)$ | -0.99 |
| $\rho(q_t, r_t^b)$ | -0.99 |

Table II.2: Second and higher moments

| Variables | Standard deviation | Skewness | Kurtosis |
|------------|--------------------|----------|----------|
| b_t | 4.76 | -0.09 | 2.65 |
| c_t | 9.55 | -0.09 | 2.68 |
| E_t^b | 0.43 | -0.09 | 2.65 |
| κ_t | 12.03 | 0.1 | 2.68 |
| q_t | 11.24 | -0.06 | 2.64 |
| ms_t | 0.48 | 0.26 | 1.11 |
| r_t | 2.17 | 0.24 | 3.11 |
| r_t^b | 4.76 | 0.09 | 2.65 |

Table II.3: Model (auto)-correlations comparison

| Variables | (Auto)-correlations without macro-pru | (Auto)-correlations with macro-pru |
|--------------------------------|---------------------------------------|------------------------------------|
| $\rho(b_t, b_{t-1})$ | 0.988 | 0.954 |
| $\rho(c_t, c_{t-1})$ | 0.972 | 0.836 |
| $\rho(E_t^b, E_{t-1}^b)$ | 0.988 | 0.954 |
| $\rho(q_t, q_{t-1})$ | 0.986 | 0.952 |
| $\rho(r_t, r_{t-1})$ | 0.883 | 0.88 |
| $\rho(\kappa_t, \kappa_{t-1})$ | 0.99 | 0.99 |
| $\rho(r_t^b, r_{t-1}^b)$ | 0.988 | 0.954 |
| $\rho(b_t, \kappa_t)$ | 0.99 | 0.01 |
| $\rho(c_t, a_s t)$ | 0.82 | 0.76 |
| $\rho(b_t, c_t)$ | 0.989 | 0.888 |
| $\rho(c_t, \kappa_t)$ | -0.975 | -0.01 |
| $\rho(b_t, q_t)$ | 0.99 | 0.97 |
| $\rho(q_t, \kappa_t)$ | -0.985 | -0.01 |
| $\rho(r_t, y_t)$ | 0.17 | 0.30 |
| $\rho(c_t, r_t^b)$ | -0.99 | -0.88 |
| $\rho(q_t, r_t^b)$ | -0.99 | -0.97 |

Table II.4: Second and higher moments comparison

| Variables | Standard deviations (pre/post) | Skewness (pre/post) | Kurtosis (pre/post) |
|------------|--------------------------------|---------------------|---------------------|
| b_t | 4.76/1.46 | -0.09/-0.04 | 2.65/2.72 |
| c_t | 9.55/3 | -0.09/-0.03 | 2.68/2.87 |
| E_t^b | 0.43/0.13 | -0.09/-0.04 | 2.65/2.72 |
| κ_t | 12.03/0.05 | 0.1/-44.69 | 2.68/2000 |
| q_t | 11.24/5.01 | -0.06/0.03 | 2.64/2.7 |
| as_t | 0.48/0.43 | 0.26/0.60 | 1.11/1.55 |
| r_t | 2.17/2.17 | 0.24/0.24 | 3.11/3.11 |
| r_t^b | 4.76/1.46 | 0.09/0.04 | 2.65/2.72 |

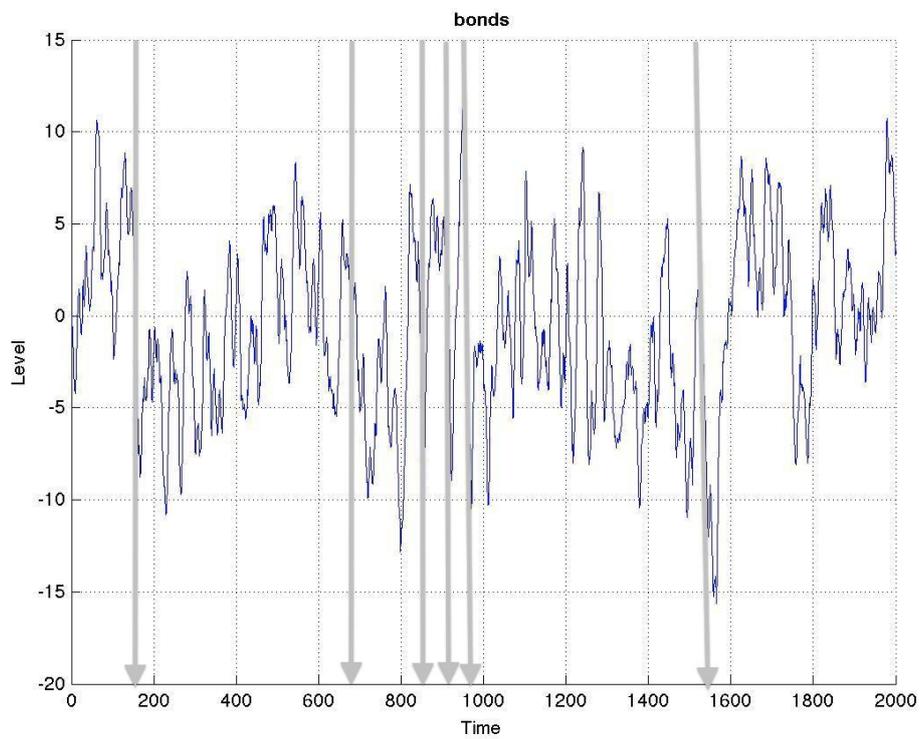
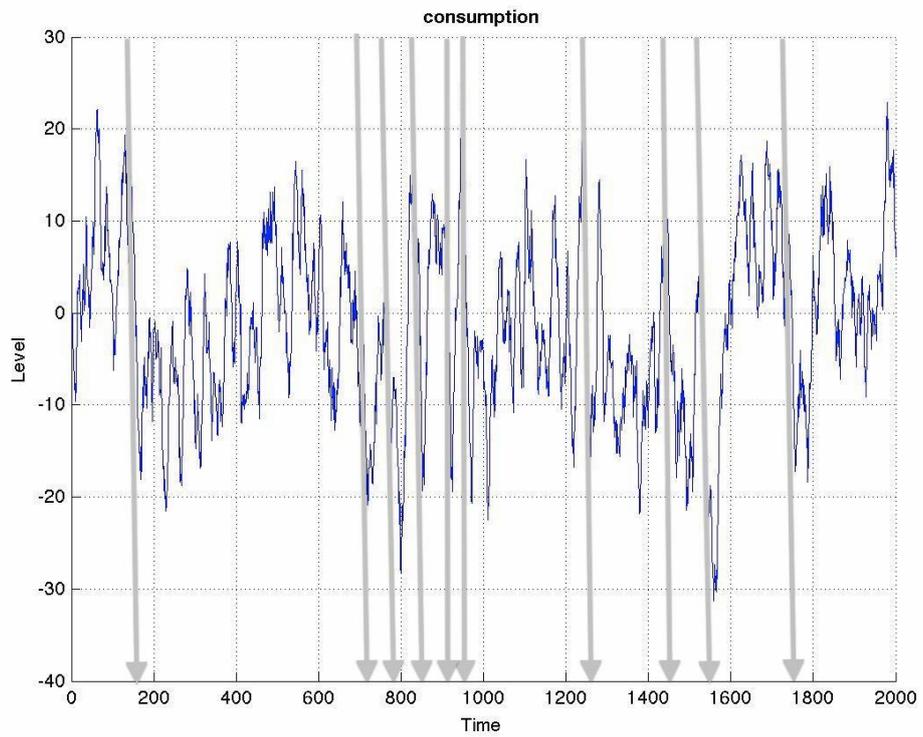


Figure II.1: Evolution of the key aggregate variables

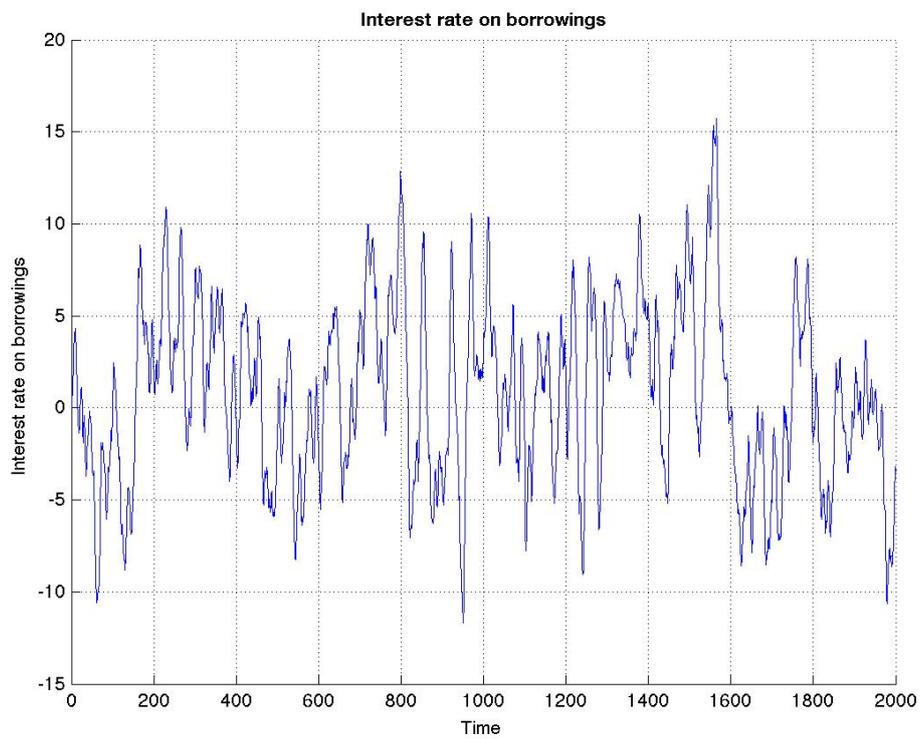
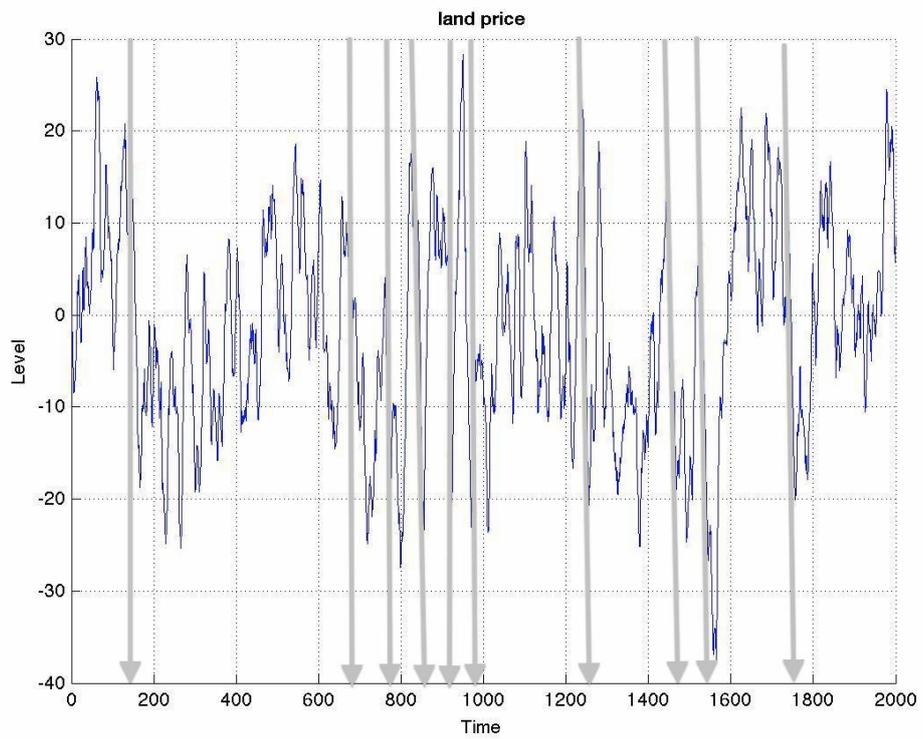


Figure II.2: Evolution of the key aggregate variables 2

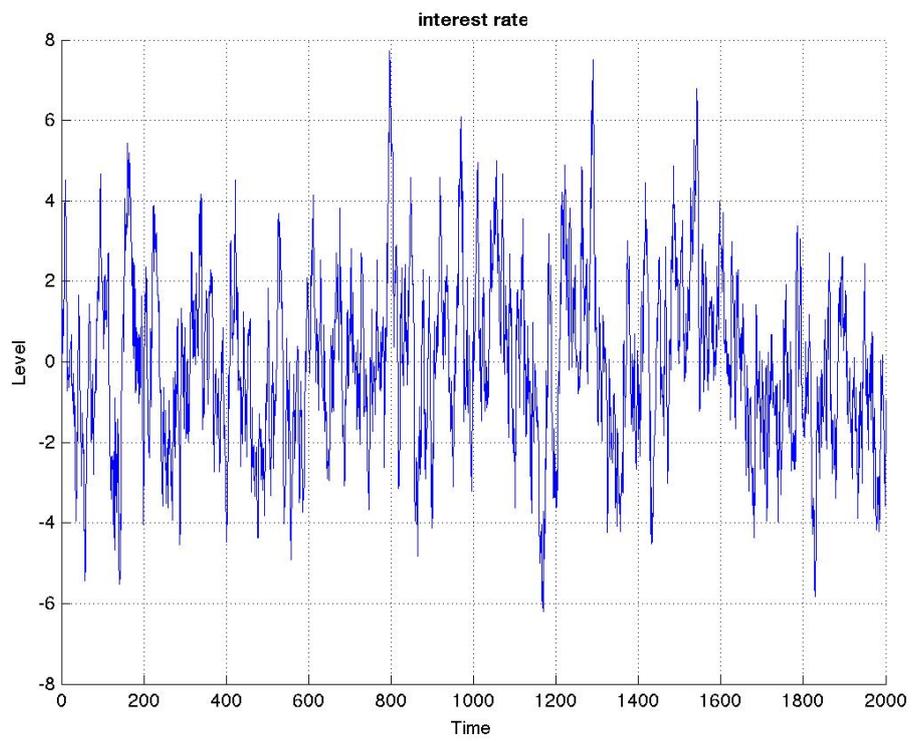
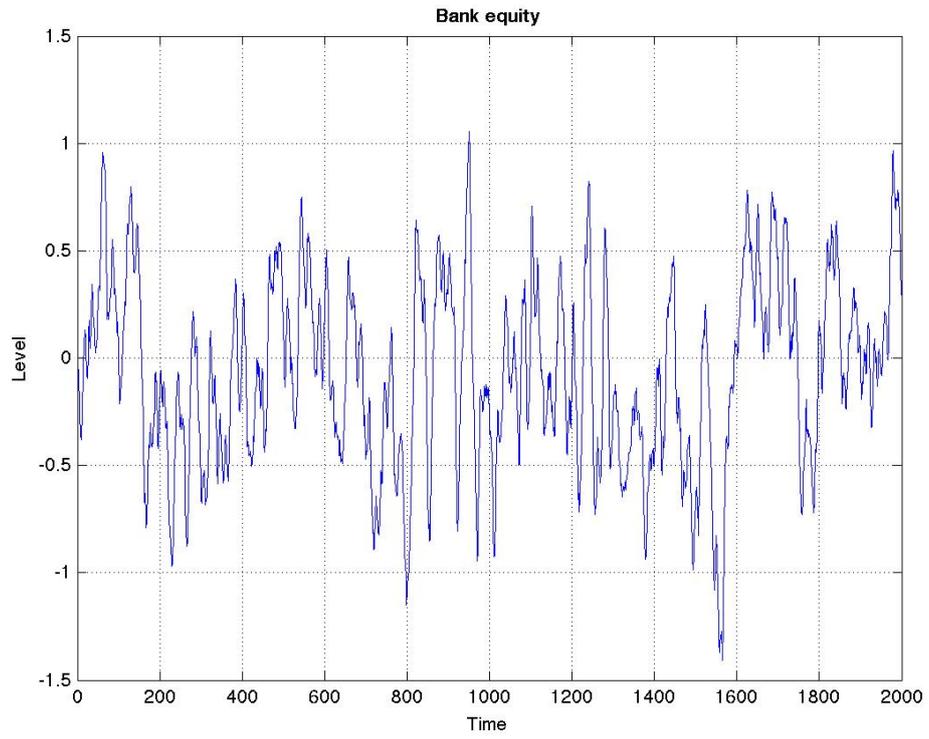
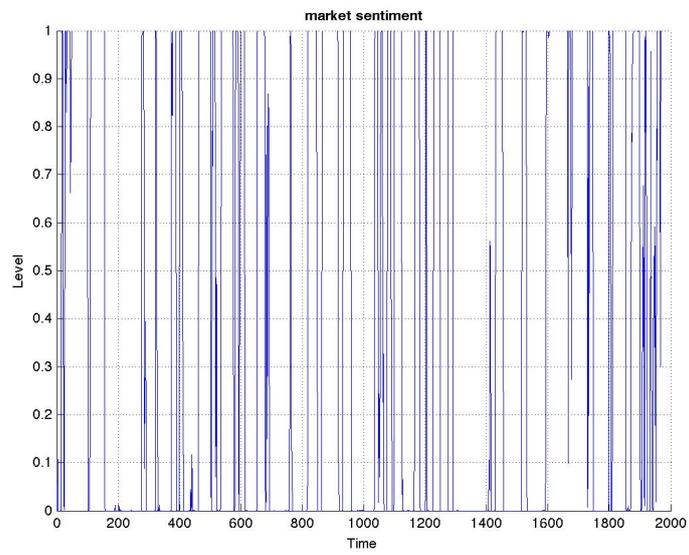
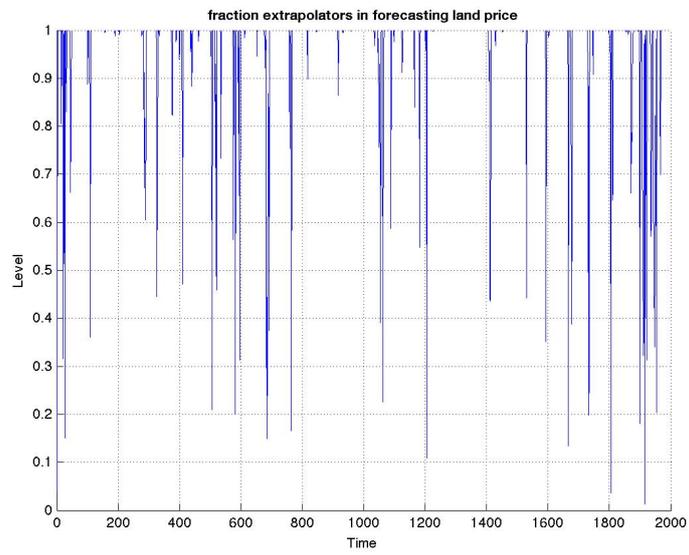
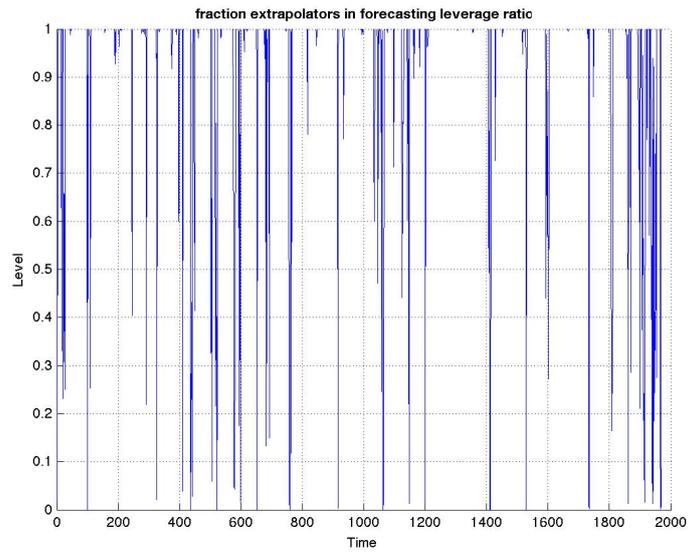


Figure II.3: Evolution of the key aggregate variables



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Figure II.4: Learning dynamics

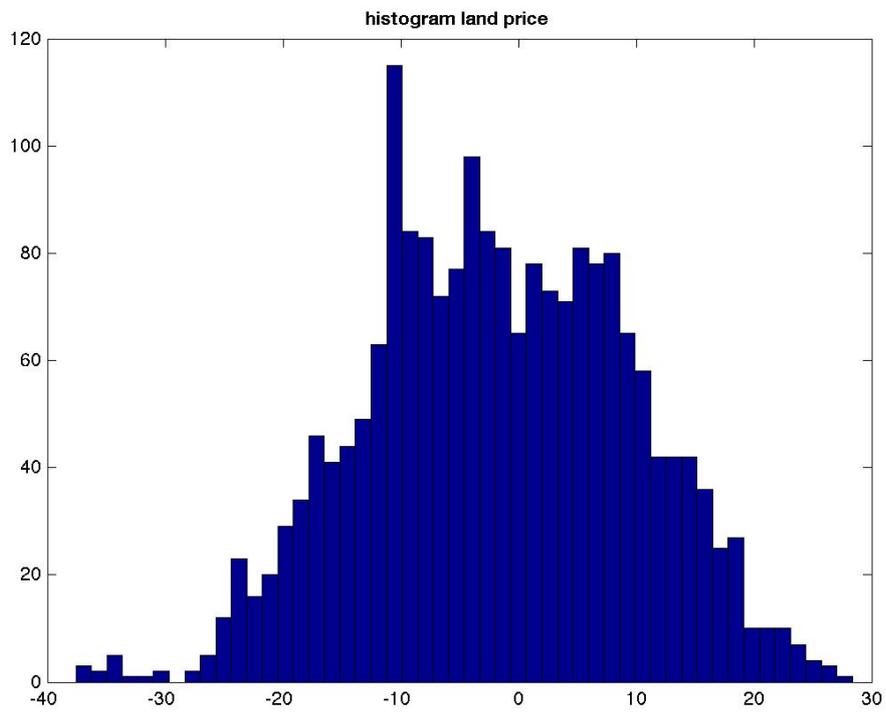
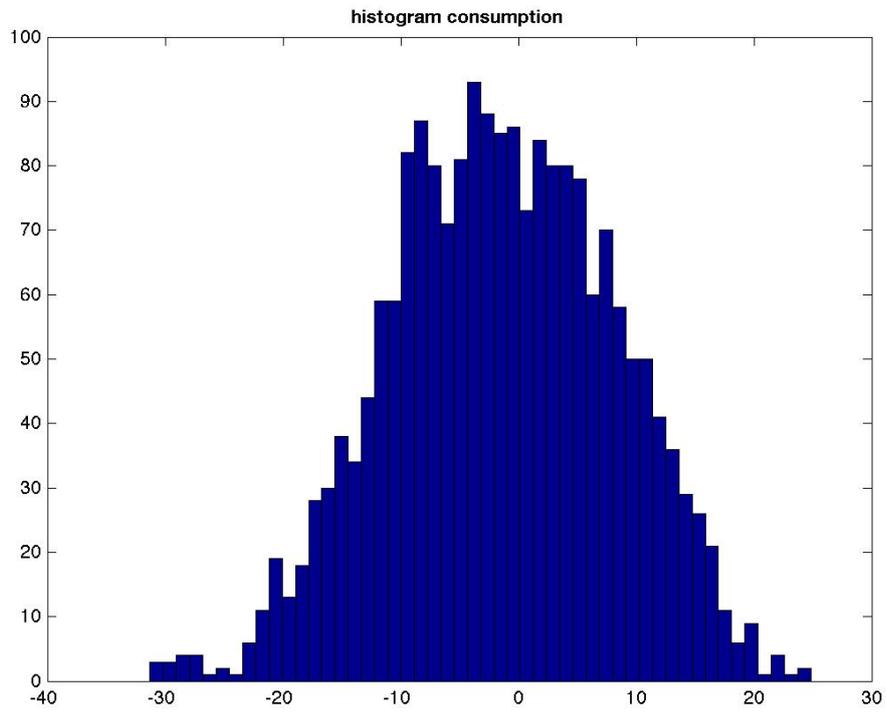


Figure II.5: Histograms

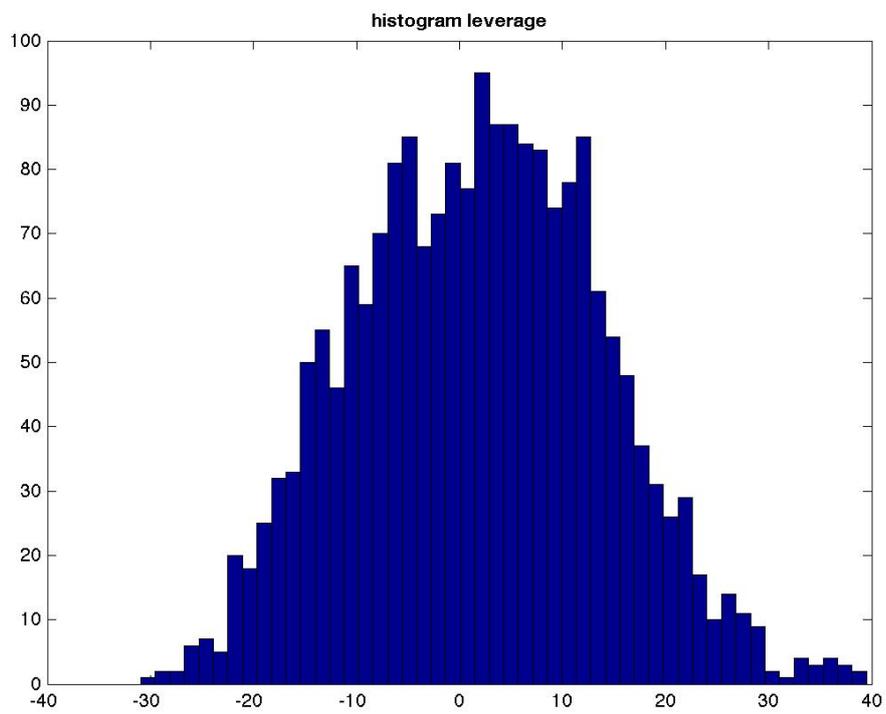
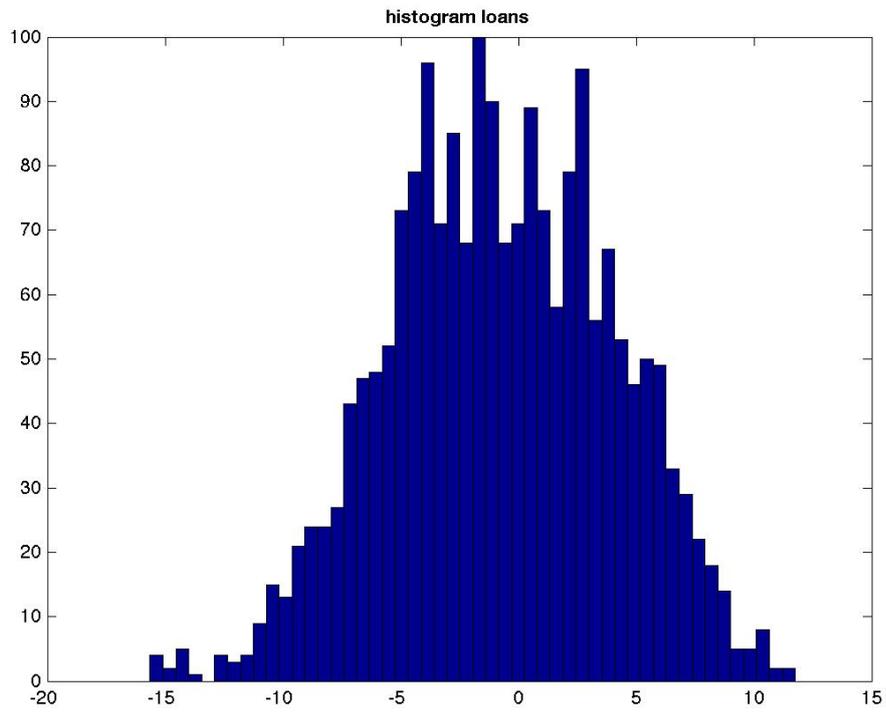
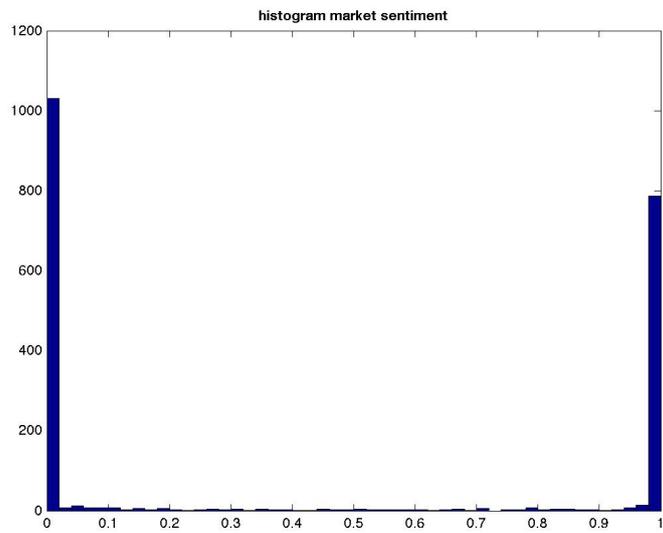
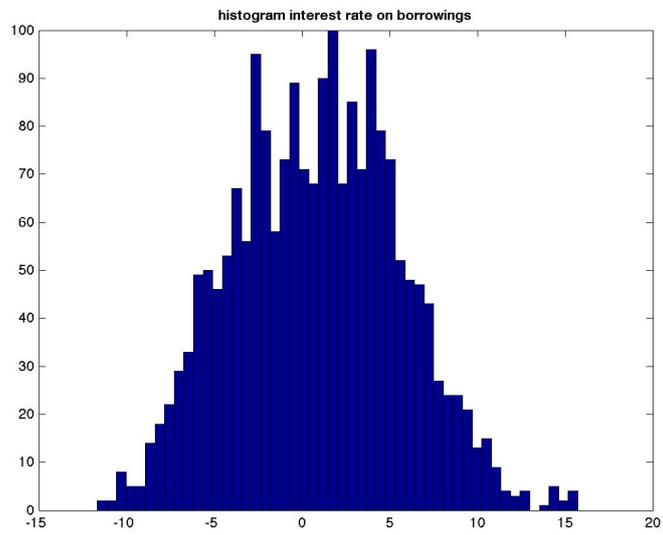
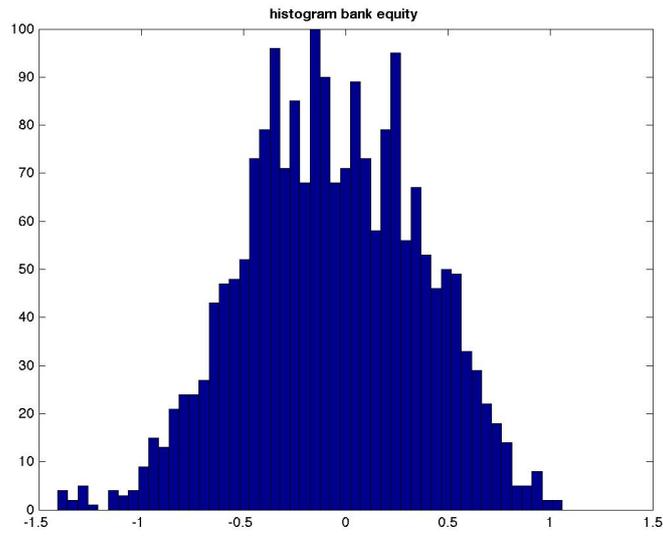


Figure II.6: Histograms 2



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Figure II.7: Histograms 3

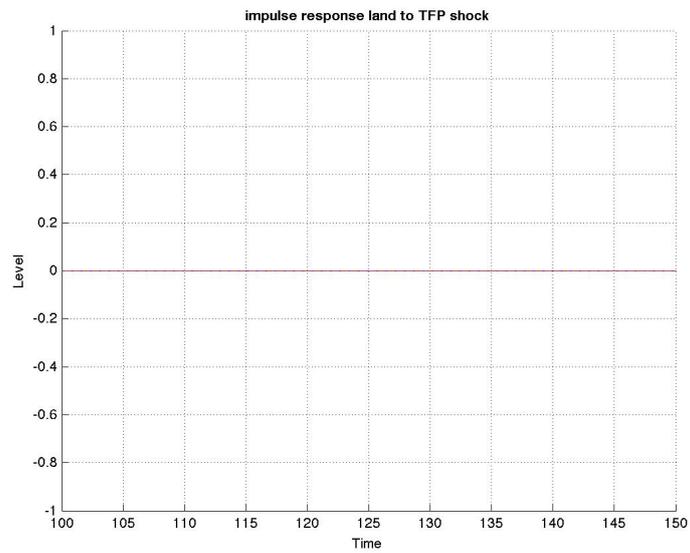
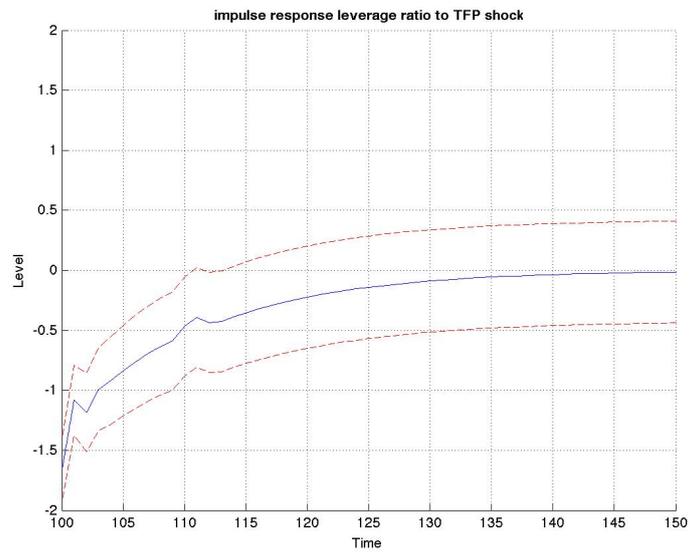
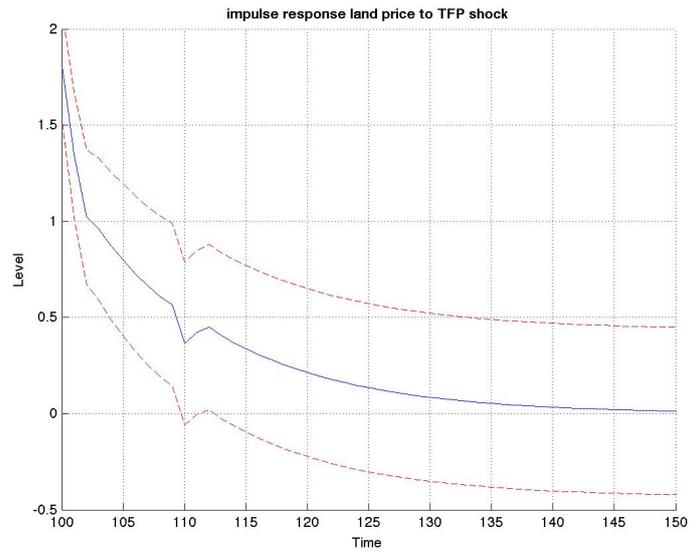


Figure II.8: Impulse responses to a positive TFP shock in $t=100$

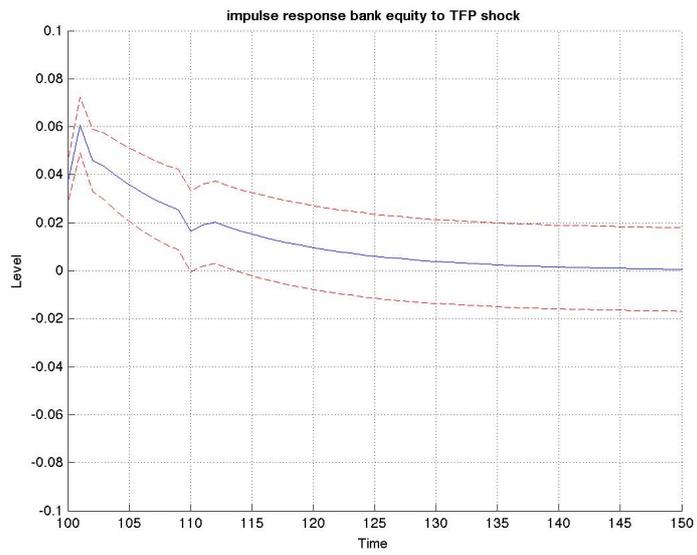
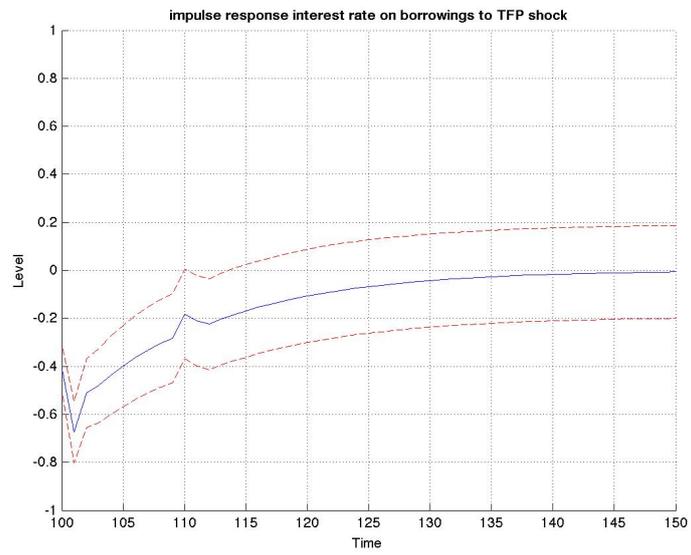
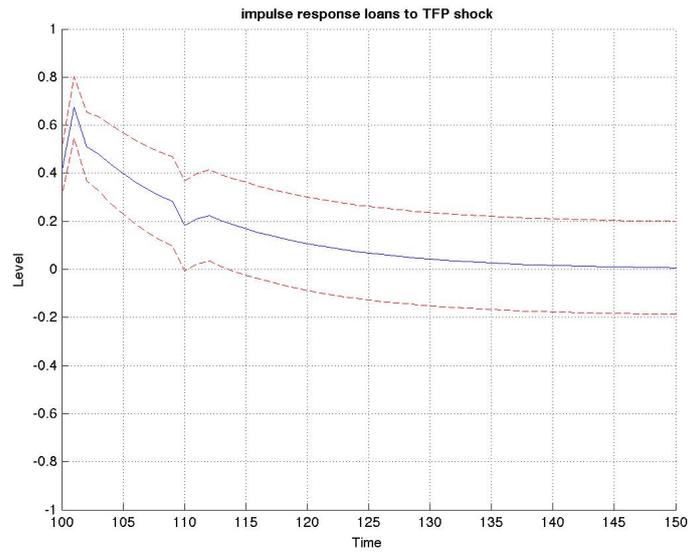


Figure II.9: Impulse responses 2⁵⁷ to a positive TFP shock in t=100

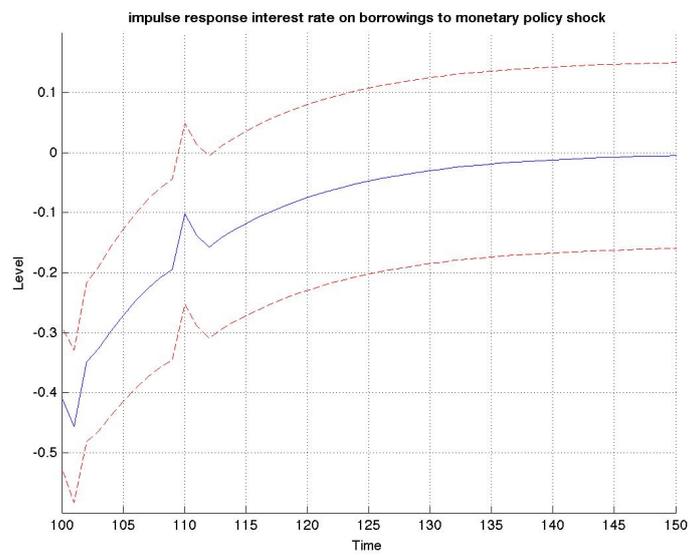
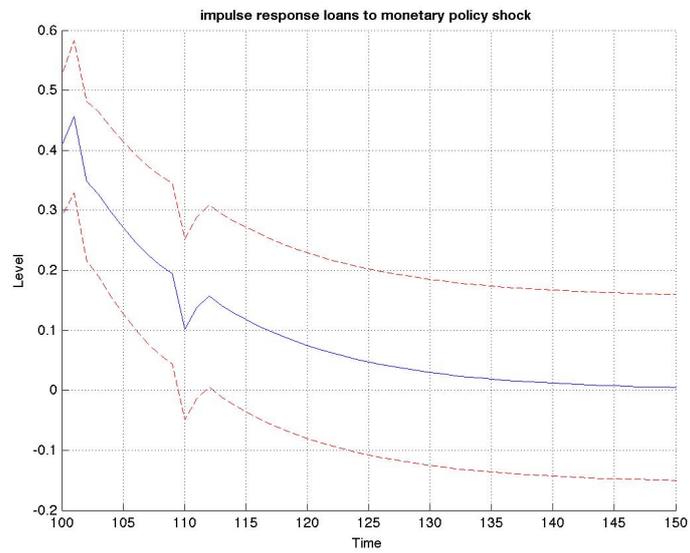
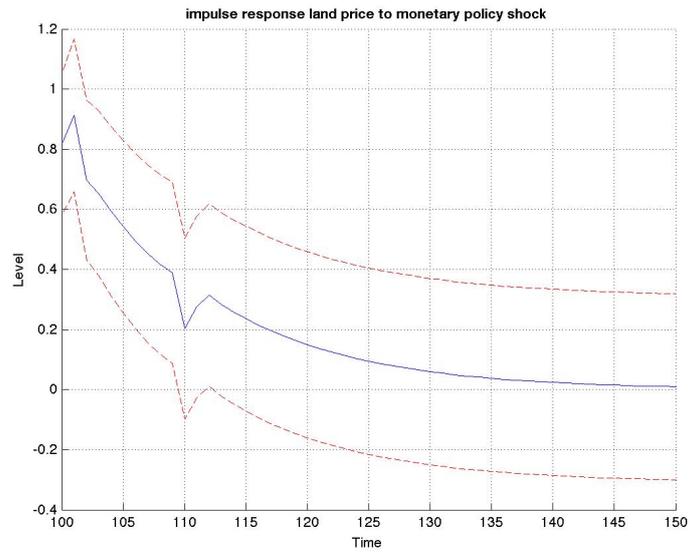


Figure II.10: Impulse responses to an expansionary monetary policy shock in $t=100$

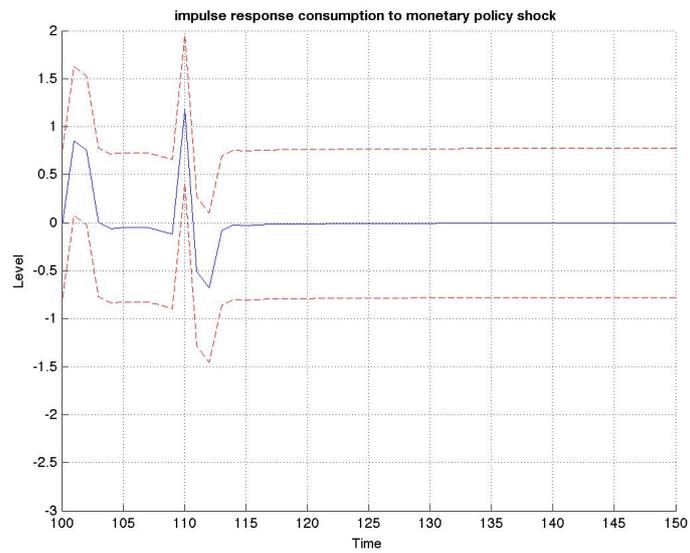
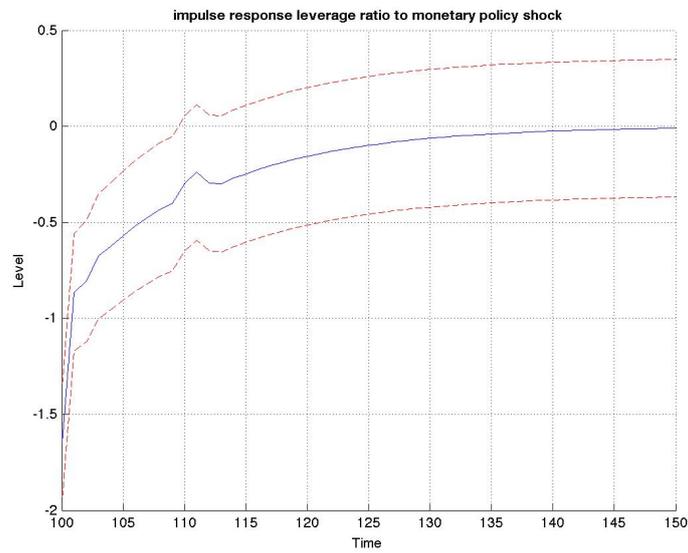
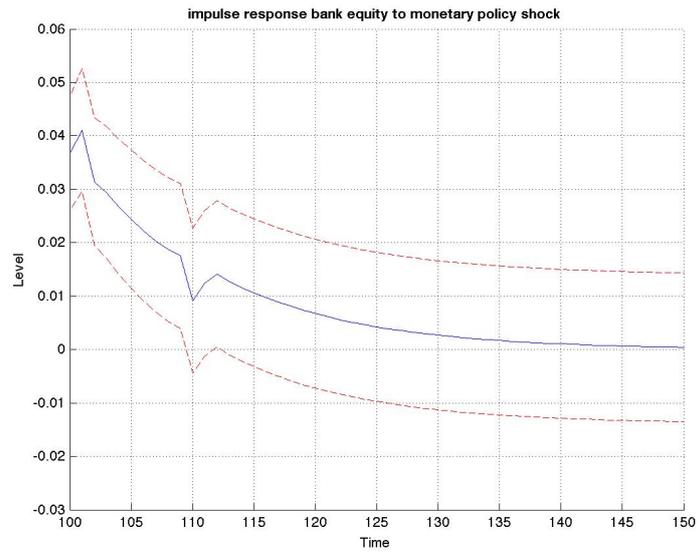


Figure II.11: Impulse responses 2 to an expansionary monetary policy shock in $t=100$

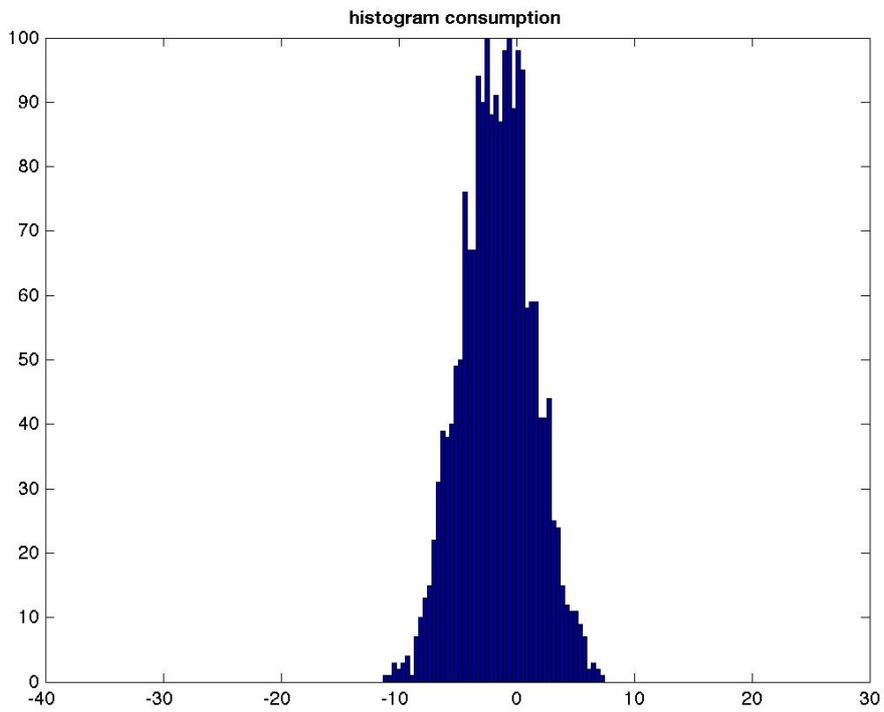
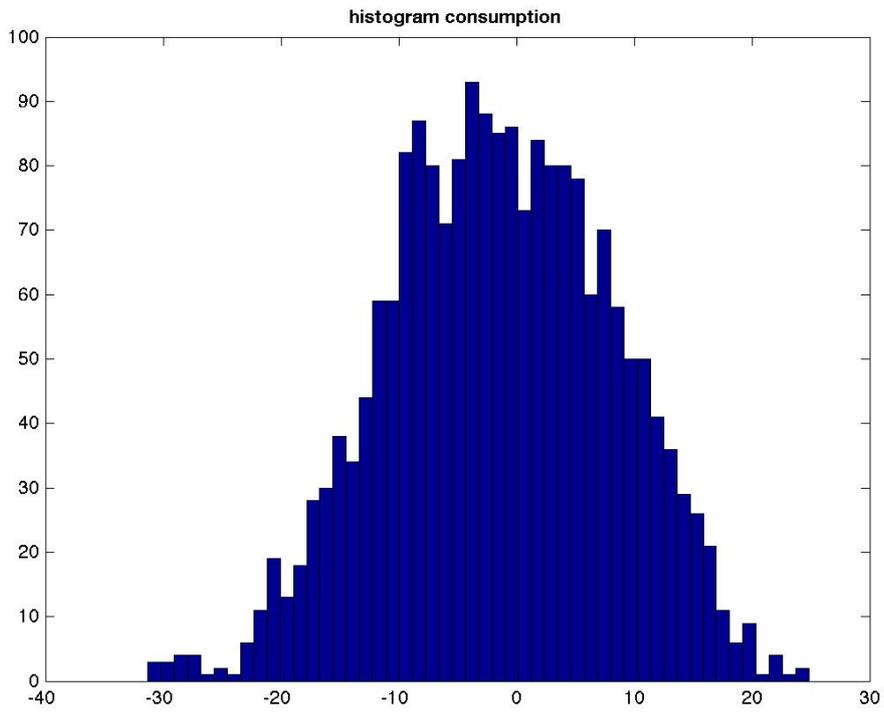


Figure II.12: Macroprudential policy effects

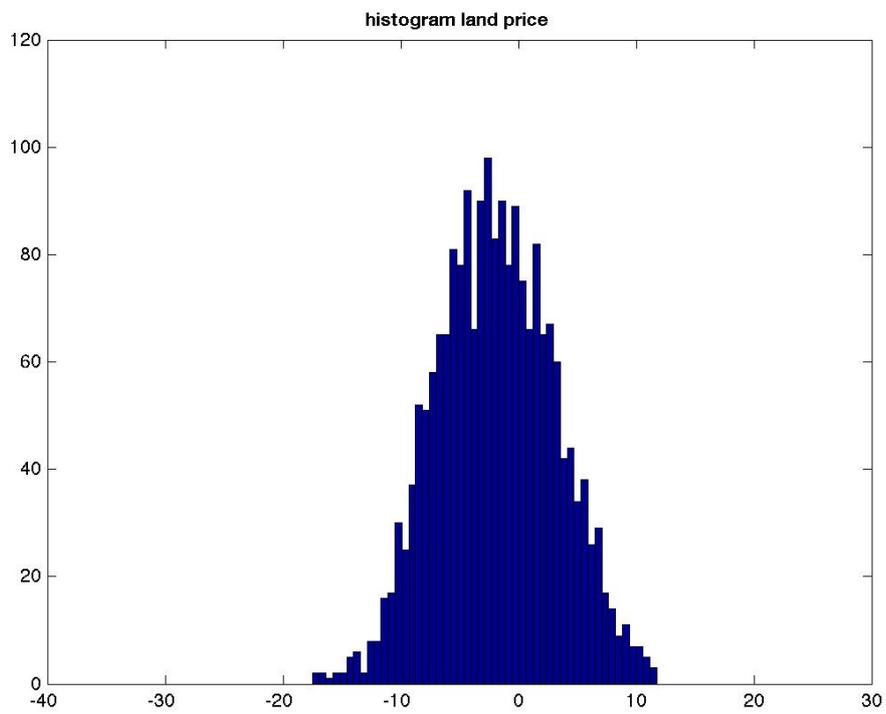
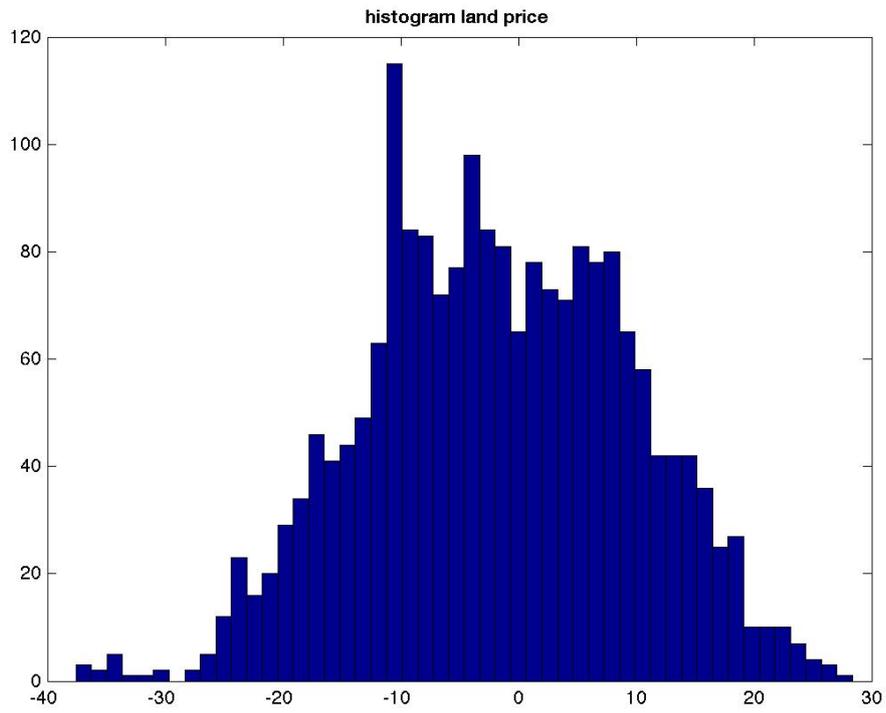


Figure II.13: Macroprudential policy effects 2

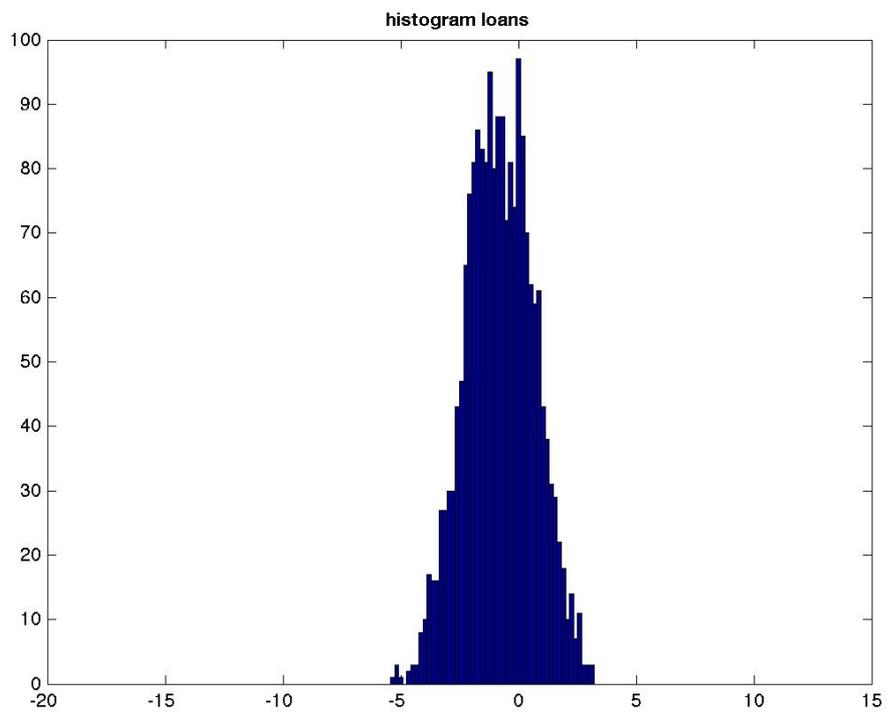
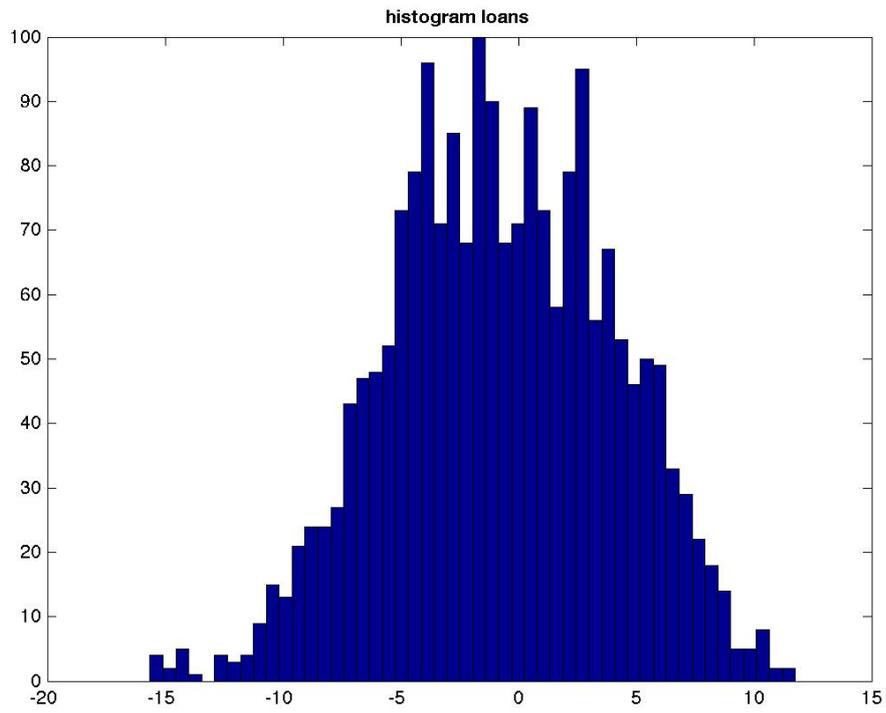


Figure II.14: Macroprudential policy effects 3

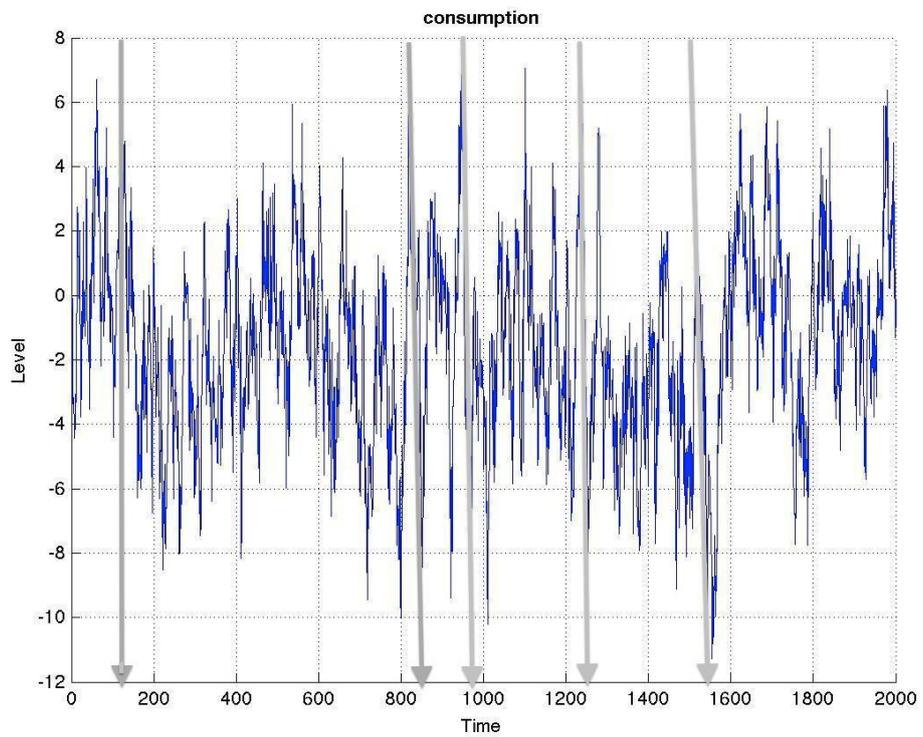
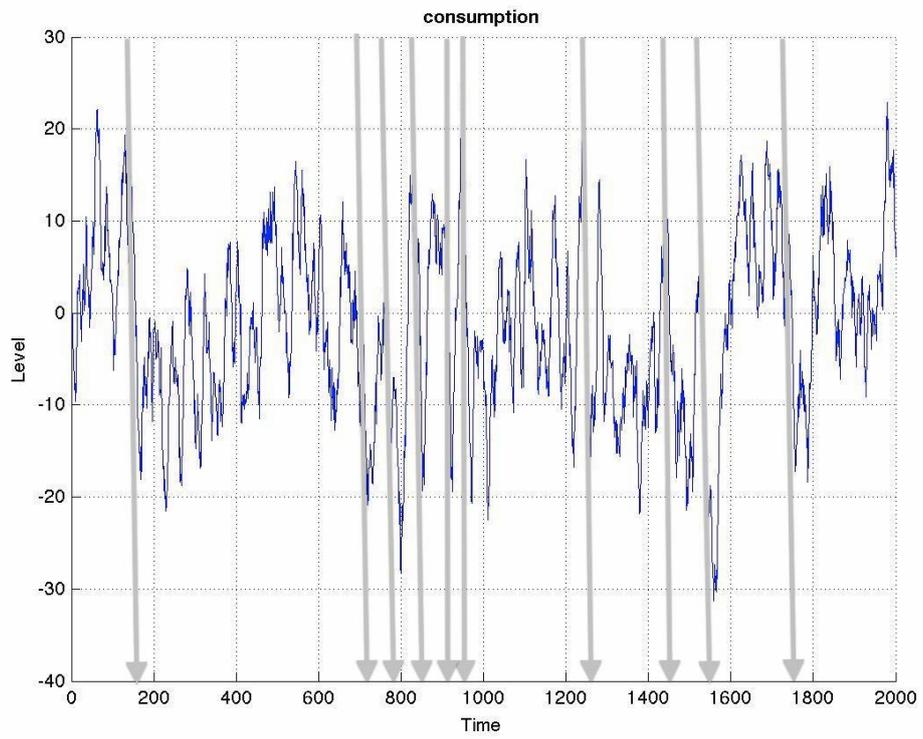


Figure II.15: Macroprudential policy effects 4

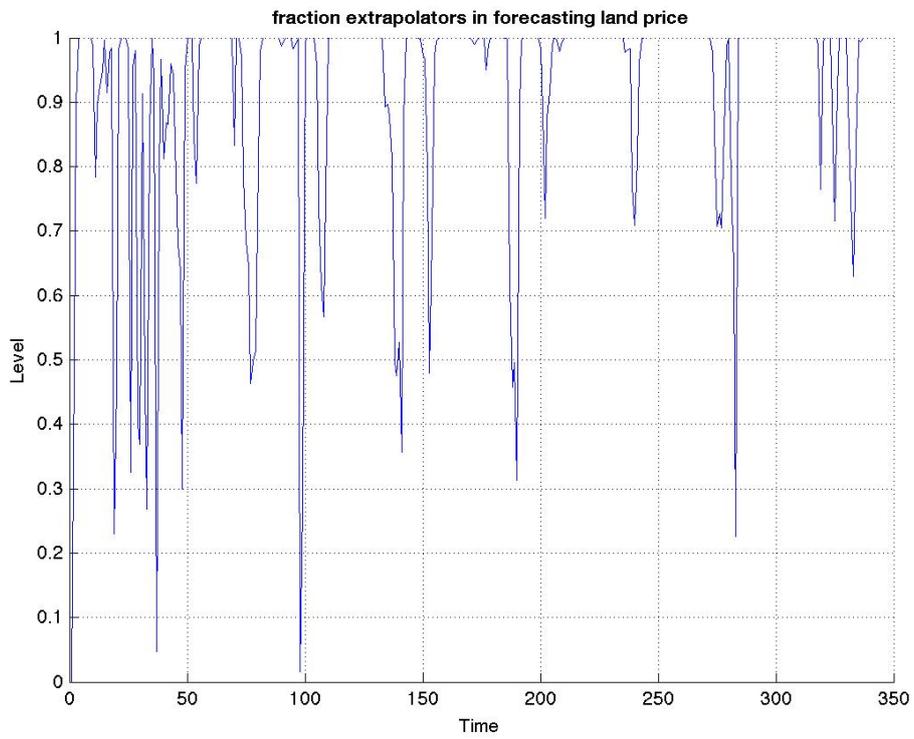
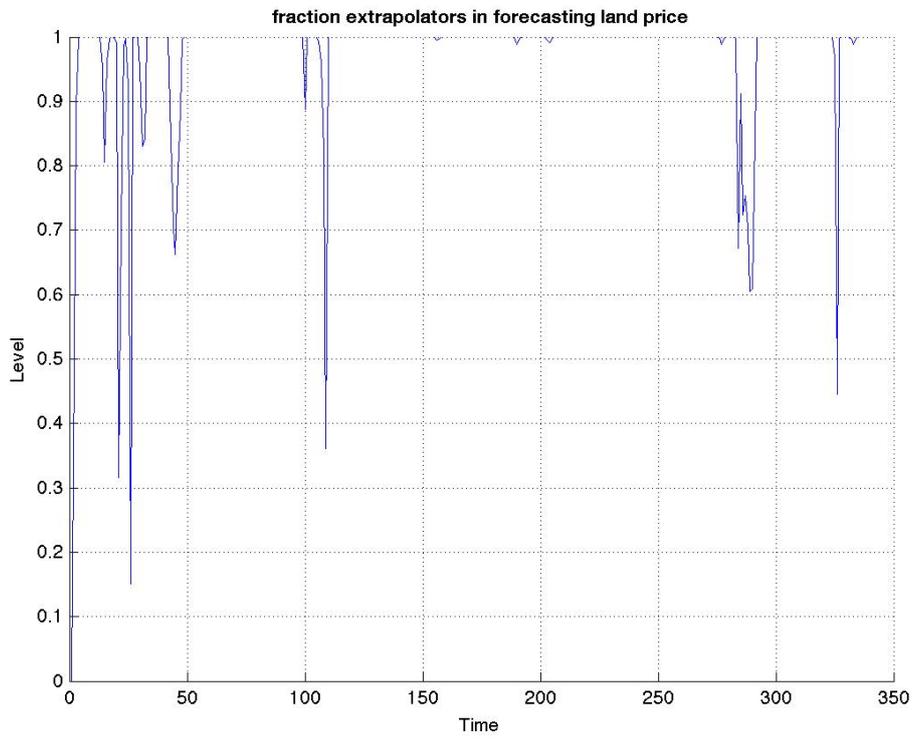


Figure II.16: Macroprudential policy effects 5