The Role of Population and Human Capital in Determining the Economic Growth Rate

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In an effort to explain why the theoretically relevant growth effect of population growth on economic growth is empirically unobservable, this paper develops a modified idea-based growth model with endogenous human capital and population. Based on the assumption that the number of new ideas created is a positive function of the size of the population and the level of human capital of each person, the model predicts that the growth rate of per capita income is proportional to the growth rates of both population and human capital. The offsetting movement of the growth rates of population and human capital after the demographic transition obscures observation of the growth effect. More interestingly, we find that the stylized facts of economic take-off, demographic transition, increasing human capital investments, and economic convergence can be derived directly from this model. In fact, the model generates an evolution of the growth rates of population, human capital, and per capita income that is consistent with historical and postwar data. (JEL E27 O40)

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Reconciling theoretical predictions with empirical evidence is always a driven force of the evolution of economic growth theory. About two centuries ago, Thomas Malthus (1826) developed a dynamic economic growth model to depict the observations of his era that fertility rises when incomes exceed the equilibrium level, and vice versa. This view has been denied by the demographic transition observed over the last one and a half century that fertility fell rather than rose as incomes grew. In responding to the failure of the Malthusian theory, the neoclassical model shifted attention from population to physical capital. Decreasing returns to physical capital investment, however, imply that long-run growth in the neoclassical model depends crucially on exogenous technological progress.

The subsequent idea-based models put technological change at the heart of economic growth. The early idea-based models, such as Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), typically imply that an increase in the size of the population, other things equal, leads to a higher growth rate of per capita income. Although this implication is generally consistent with the empirical data over most of history (Kremer 1993), twentieth-century empirical evidence from advanced economies is inconsistent with this prediction (Jones 1995b). In the literature, many efforts have been taken to modify the idea-based model to eliminate this counterfactual scale effect prediction.

The modified idea-based models that eliminate the scale effect of the *level* of population still predicted a "growth effect" of the *growth rate* of population: As the rate of growth of population accelerates or retards, so does the rate of growth of per capita income. As summarized in Jones (1999), the modified idea-based models of Jones (1995a), Kortum (1997), and Segerstrom (1998) (J/K/S hereafter) predicted that the economic growth rate is proportional to the population growth rate; the models of Young (1998), Peretto (1998), Aghion et al. (1998), and Dinopoulos and Thompson (1998) (Y/P/AH/DT hereafter) predicts that the economic growth rate is proportional to the growth rate of population and the research effort in each sector. Jones (1999) also illustrated that if the knife-edge assumption that the growth rate of sectors in the economy is exactly equal to the population growth rate is relaxed, the Y/P/AH/DT models lead to the same long-run prediction as in the J/K/S models or still predict the scale effect of the level of population.

The implication of the growth effect of population growth is intuitive and is derived directly from the non-rivalry of ideas. As emphasized by Romer (1986), ideas are non-rivalrous in the sense that the use of an idea by one person does not preclude, at the technological level, the simultaneous use of the idea by another person. A larger population means more ideas can be created, and therefore, more ideas can be used by each person; a higher growth rate of population, other things equal, should lead to a higher growth rate of ideas and therefore a higher growth rate of per capita income.

However, this theoretically relevant prediction of the growth effect of population growth is not well supported by the postwar data. A stylized fact over the last half-century is dramatic declines in population growth rates. Therefore, according to the prediction, we should have observed significant declines in economic growth rates. However, as shown in Figure 1, although most of the 78 sample countries experienced a decline in the growth rate of population from 1951 to 2015 (see Sections A and D of Figure 1), many countries experienced an increase in the growth rate of per capita GDP (see Section A of Figure 1). Additional time-series examinations of this fact are presented in Appendix A.

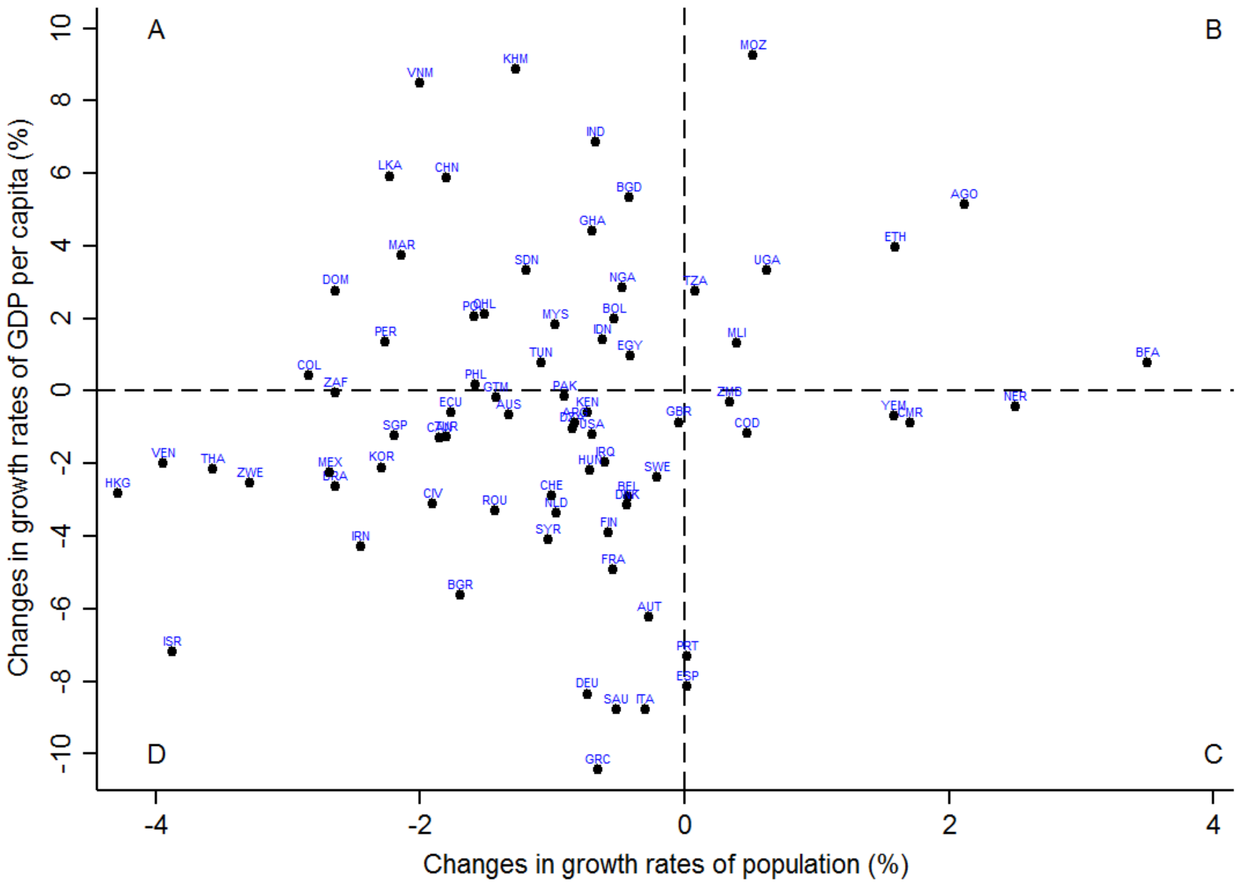

Figure 1: Changes in the growth rate of per capita GDP and the growth rate of population for 78 countries (1950–2015)

Note: Changes in growth rates are calculated as the difference between the 1996–2015 average and the 1951–1970 average. See Appendix A for details of the data source and process.

The inconsistency between theoretical prediction and empirical evidence is not surprising since human capital is treated as exogenous in these idea-based models. The driving force of economic growth is the creation of ideas in the idea-based models. Intuitively, the number of new ideas created is a positive function of the number of researchers and the level of human capital of each researcher. Given the percentage of people employed in the R&D sector, it is likely that the growth rate of ideas is proportional to the growth rates of both population and human capital. Hence, the upward trend in human capital investment that has been widely observed over the last half-century has the potential to explain why the theoretically relevant growth effect of population growth is empirically unobservable; it is possible that the negative growth effect of declines in the population growth rate has been offset by the positive growth effect of increases in human capital investment.[1]

We formally explore this possibility by developing a modified idea-based growth model with endogenous human capital and population. Specifically, we modify the idea-based model of Jones (1995a), which is modified and simplified from the early idea-based models, to allow endogenous growth of population and human capital. We keep most of the assumptions of Jones (1995a) and follow the model to eliminate the scale effect of the level of population. This modification is based on two additional assumptions:

Firstly, we assume that the number of new ideas created is a positive function of the size of the population and the level of human capital of each person. This assumption is in line with the assumption of the unified growth model of Galor and Weil (2000) that the speed of technological progress is a positive function of the size of the population and the level of human capital. A person with a higher level of human capital is more likely to advance the technological frontier by creating new ideas. On the other hand, holding the level of human capital and the proportion of people employed in the R&D sector as a constant, the total number of new ideas created during a given time period is also a positive function of the population size.

Secondly, we assume that rates of return on investments in human capital rise rather than decline as the stock of human capital increases, at least until the stock becomes large. This assumption has been widely used in growth models in which accumulation of human capital serve as the engine of economic growth (see, for example, Becker, Murphy, and Tamura 1990, Morand 1999, Kalemli-

[1] There are at least other two potential explanations for the unobservable growth effect of population growth. First, it is possible that the technological spillover from developed countries, which are more likely located in the technological frontier, to developing countries helps the developing countries offset the negative effects of declines in the population growth rate. Second, based on the predictions of the Y/P/AH/DT models, the increase in research efforts in each sector also have the potential to offset the negative effect of the population growth rate decline and therefore obscure the growth effect. However, we tend to believe that the explanation of the trade-off between population and human capital is more theoretically relevant and empirically testable.

Ozcan 2002). The human capita is seen as knowledge and skills embodied in physical labors through education or training. This assumption is supported by the fact that parents with higher levels of human capital can provide a better home environment for the learning of children and that teachers with higher levels of human capital can teach more efficiently. In addition, the benefit from embodying additional knowledge in a person may depend positively rather than negatively on the knowledge he or she already has.

In the model, we follow Becker, Murphy, and Tamura (1990) to endogenize human capital and population in the way that altruistic parents choose the number of children and the human capital investment in each child to maximize a dynastic utility function. Based on the first assumption, the model implies that the growth rate of ideas is proportional to the growth rates of both population and human capital. Therefore, the growth rate of ideas and hence the growth rate of per capita income is determined by the utility-maximizing behavior of parents.

The model predicts that the growth rates of population, human capital, and per capita income increase together over time when per capita income and the level of human capital are low. Specifically, the rise in the growth rates of population and human capital push up the growth rate of per capita income by increasing the growth rate of ideas. In addition, when per capita income is low, the income effect of increases in per-capita income eases parents' budget constraints, allowing them to choose to have more children. Furthermore, according to the second assumption, the growth rate of human capital rises over time when the level of human capital is low because the returns of investment in human capital rise with the level of human capital.

However, because the production and rearing of children are very time intensive, the substitution effect of increases in the wage rate will eventually induce parents to choose to have a smaller number of children and therefore trigger a demographic transition. On the other hand, parents may continue to raise the rate of investment in the human capital of each child, considering the higher returns of investment due to the higher levels of human capital. The trade-off between the "quantity" and "quality" of children made by parents results in the offsetting movement of the growth rates of population and human capital after demographic transition.[2] This obscures the observation of the growth effect of population growth.

The growth effect of population growth will reappear in the empirical data with further economic development. Diminishing marginal returns imply that the marginal positive effect of an increase in human capital investment declines over time while the marginal negative effect of a decline in the population growth rate increases over time. Eventually, the negative effect dominates the positive effect, and the growth rate of per-capita income starts to decline. The growth effect reappears in the empirical data in the form that a decline in the population growth rate corresponds to a decline in the economic growth rate.

The model explains why the empirical evidence of the growth effect, shown in Figure 1, is ambiguous. In countries with very low (very high) per capita income, the growth rate of per capita income shows the same upward (downward) trend as the growth rate of population. In other countries, the growth rate of per capita income rises when the growth rate of population declines. The prediction of the differences in the growth effect among countries with different income levels is supported by cross-sectional data from 78 developed and developing countries. The prediction of the long-run dynamics of growth rates of per capita income, population, and human capital is also supported by time-series data from twelve Western Europe countries over a period of two centuries.

Similar to the unified growth model of Galor and Weil (2000), the model in this study generates an evolution of population and per capita income that is largely consistent with the long-run

[2] The offsetting movement of the "quantity" and "quality" of children can also be modeled by other mechanisms. For example, it is possible that increasing life expectancy at birth resulting from mortality reduction raises wages and the returns from each child, which induces parents to choose to have fewer children and to invest more in each child (Soares 2005). Similarly, as the relative wages of women increase with the stock of physical capital, economic development will raise the opportunity cost of children and raise the returns from each child, which may also have the potential to cause the trade-off (Galor and Weil 1996).

historical process of development (see Figure 2). The main distinctions are that, first, we model explicitly the technological progress, population growth, and human capital accumulation within the framework of an idea-based model, which enables us to explain why the growth effect of population growth is unobservable after the demographic transition and enables us to provide a more testable prediction for the determinants of long-run economic growth rates. Second, we follow Jones (1995a) to eliminate the scale effect at the level of population and therefore focus on the dynamic of the growth rates of population, human capita, and per capita income.

In the remainder of this paper, Section I summarizes the growth models from which the model of this paper is developed. Section II develops the model and discusses its steady-state equilibrium. Section III analyzes the transition dynamics of the model. Section IV presents the empirical evidence. The last section is concluding remarks.

I. Literature

In this section, the growth models from which the model of this paper is developed are briefly summarized. This paper depends heavily on the model of Jones (1995a), which was modified from early idea-based models to eliminate the counterfactual prediction of scale effects. As detailed in Jones (1995b), abstracting from many of the important insights, the basic elements of early idea-based models, such as Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), can be simplified to the following "reduced-form" model:

$$Y_t = \left(A_t L_{yt}\right)^{\alpha} K_t^{1-\alpha} \ , \tag{1}$$

$$\dot{A}_t = \delta L_{At} A_t \ . \tag{2}$$

The final output Y_t is produced using ideas A_t, labor L_{Yt}, and physical capital K_t. New ideas $\dot{A}_t$ are produced using labor L_{At} and the existing stock of ideas A_t. The total amount of labor in the economy is $L = L_{Yt} + L_{At}$. In the steady-state, the growth rate of per capita income g_y equals the growth rate of ideas g_A and the growth rate of physical capital g_K, and is proportional to the share of the labor input in the R&D sector $s = L_{At}/L$ and the size of labor force L:

$$g_y = g_A = g_k \equiv \delta sL \ . \tag{3}$$

Therefore, the prediction is that an increase in the size of the population, other things equal, leads to a proportional increase in the growth rate of per capita income. Jones (1995b) provided time-series evidence from advanced economies to reject this prediction and argued that this counterfactual scale effect prediction resulted from the strong assumption that the growth rate of ideas is linear to the input in innovation. To eliminate this counterfactual prediction, Jones (1995a) relaxed this assumption and replaced the production function for new ideas as shown in equation (2) with

$$\dot{A}_t = \delta L_{At}^{\lambda} A_t^{\phi} \ , \tag{4}$$

where $\phi < 1$ and $0 < \lambda \leq 1$. $\phi < 0$ corresponds to the case of "fishing out," and the rate of innovation decreases with the level of knowledge; $0 < \phi < 1$ corresponds to the case of positive external returns.

The modified model eliminates the scale effect of the *level* of population on economic growth rates but predicts a growth effect of the *growth rate* of population. An increase in the exogenous growth rate of population (n), other things equal, leads to a proportional increase in the economic growth rates:

$$g_y = g_A = g_k \equiv \frac{\lambda}{1-\phi} n \ . \tag{5}$$

However, although the prediction of the growth effect of the population growth rate is theoretically relevant, it is not well supported by empirical data. To explain this inconsistency, this paper extends the idea-based model by endogenizing the population and human capital following the method of Becker, Murphy, and Tamura (1990). We use a simplified version of the model of Becker, Murphy, and Tamura (1990) to present the basic elements that will be incorporated in the model of this paper. The basic structure of the model of Becker, Murphy, and Tamura (1990) is summarized in the following:

In an overlapping-generations economy, everyone is identical and lives for two periods, childhood and adulthood. Each child consumes only a fixed quantity e of his parent's time and spends all of his childhood accumulating human capital. Adults are endowed with T hours of working time and choose the number of offspring n and the time spent on teaching each child z_t to maximize the dynastic utility function:

$$V_t = u\left(c_t\right) + a\left(n_t\right) n_t V_{t+1} \ . \tag{6}$$

The dynastic utility of a parent V_t depends on his consumption c_t, the degree of altruism toward each child $a\left(n_t\right)$, the number of children n_t, and the utility of each child V_{t+1}. Diminishing marginal utility implies that the degree of parental altruism toward each child declines as the number of children increases (i.e. $a' < 0$). The consumable y_t is produced using labor l_t and human capital h_t:

$$y_t = A l_t h_t \ , \tag{7}$$

in which A measures the exogenous technology. The human capital of a child h_{t+1} is accumulated according to a learning technique with increasing returns to the parents' human capital level h_t:

$$h_{t+1} = B h_t z_t \ , \tag{8}$$

in which B measures the productivity of the investments. Parents maximize the dynastic utility function subjected to the time constraint:

$$T = l_t + n_t\left(e + z_t\right) \ . \tag{9}$$

The model assumes that the rate of return to human capital increases with the level of the parents' human capital at least for a while. With economic development, parents choose to invest more in the human capital of each child as the rise of the stock of human capital. On the other hand, because the production and rearing of children are time intensive, the substitution effect of the increase in the wage rate induces parents to choose to have a small number of children. Therefore, this model generates a trade-off between the "quality" and "quantity" of children.

The model predicts that the steady state growth rate of per capita income g_y is proportional to the parental investment in the human capital accumulation of each child (z):

$$1+g_y=\frac{c_{t+1}}{c_t}=\frac{h_{t+1}}{h_t}=Bz \quad . \tag{10}$$

To sum up, as shown in equation (5), the idea-based model that assumes exogenous human capital and population predicts the growth effect of population growth on economic growth. On the other hand, as shown in equation (10), the growth model that treats ideas as exogenous but models population and human capital as endogenous implies that the growth rate of per capita income is proportional to the rate of human capital investment but is not proportional to the population growth rate. The growth effect of population growth does not show up in the model of Becker, Murphy, and Tamura (1990) because ideas are treated as exogenous and human capital, unlike ideas, is rivalrous.

Intuitively, by assuming that the number of new ideas that can be created is a positive function of the size of the population and the level of human capital, an idea-based model with endogenous population and human capital may predict that the growth rate of per capita income is an increasing function of the growth rate of population and the rate of investment in human capital. In addition, the trade-off between the "quality" and "quantity" of children as modeled in Becker, Murphy, and Tamura (1990) implies offsetting movements of the population growth rate and the rate of investment in human capital. Therefore, the unobservable growth effect of population growth can be potentially explained by a modified idea-based growth model that includes the basic elements of the model of Becker, Murphy, and Tamura (1990). This mechanism is formally presented in the next section.

II. An idea-based model with endogenous human capital and population

This section combines the basic elements of the model of Jones (1995a) and the model of Becker, Murphy, and Tamura (1990) to develop an idea-based model with endogenous human capital and population. To present the long-run dynamics of the growth rates of population, human capital, and per capital income in the clearest fashion, we abstract from many of the important insights of the previous idea-based models, such as the decentralized model specification with intermediate sectors, vertical and horizontal product differentiation, and the uncertainty of innovation.

A. The model

Consider an overlapping-generations economy in which everyone lives for two periods: childhood and adulthood. In this paper, it is convenient to think of society as divided into a number of groups. All adults of any one group are identical, live in the same geographic area, and share the same ideas about production, but groups differ in the stocks of ideas and human capital. Children are identical across groups at birth. For simplicity, assume there is no technology spillover among groups.[3] In most of the following analyses, we focus on a representative group before we generalize the conclusion to the whole economy with multiple groups.

[3] The main conclusions of this paper are not subject to the assumption of groups and the assumption of no technological spillover among groups; we can keep most of the key implications of this paper by assuming an economy with identical adults who use the same production technology. However, assuming only adults within each group are identical is more realistic and is helpful in understanding the true scale effect of the size of an economy.

An adult chooses the number of children n_t at the beginning of adulthood.[4] The production and rearing of children are expensive and time-intensive. We assume each child consumes fixed hours e of his parent's working time and consumes fixed units f of goods. Each adult is endowed with T hours of working time that can be spent on producing consumer goods, rearing children, and investing in the human capital of children. Children spend all their time on learning.

A single consumption good Y_t is produced using ideas A_t, labor L_t, and physical capital K_t. Physical capital is accumulated consumer goods that do not wear out. We assume the consumer good is produced according to a Cobb-Douglas production function in which ideas are "labor-augmenting":

$$Y_t = A_t^{\beta} L_t^{\beta} K_t^{1-\beta} = C_t + \dot{K}_t + N_t n_t f \quad , \tag{11}$$

where C_t is the total consumption of generation t, $\dot{K}_t$ is the net investment in physical capital, and N_t is the number of adults. The assumption of $0 < \beta < 1$ implies constant returns to labor and physical capital together, and increasing returns to ideas, labor, and physical capital as a whole. The production function can be measured in per capita terms by dividing both sides by the number of adults (N_t):

$$Y_t / N_t = y_t = c_t + \dot{k}_t + n_t f = A_t^{\beta} l_t^{\beta} k_t^{1-\beta} \quad , \tag{12}$$

in which y_t is the per capita output, l_t is the per capita time spent on production, and k_t is the per capita physical capital.

The creation of ideas is the driving force of the long-run growth of per capita income. We modified the growth function of ideas of Jones (1995a), as shown in equation (4), by assuming that the number of new ideas created in each period $\dot{A}_t$ is an increasing function of the total amount of human capital spent on creating ideas H_t:

[4] We prefer to explain n_t as the expected number of children who will live through childhood and adulthood.

$$\dot{A}_t = \delta H_t^{\mu} A_t^{\phi} = \delta \left(h_t N_t sT \right)^{\mu} A_t^{\phi} \ , \tag{13}$$

with $\phi < 1$, $0 < \mu \leq 1$, and $\delta > 0$. $\phi < 0$ corresponds to the case that the rate of innovation decreases with the level of knowledge; $0 < \phi < 1$ corresponds to the case of positive external returns. Assume each identical adult of a group spends a constant share s of working time on creating new ideas.[5] As shown on the far-right side of equation (13), the total amount of human capital spent on creating new ideas is a function of the level of human capital of each adult h_t, the number of adults N_t, and the time each adult spends on searching for new ideas sT.

Underlying the growth function (13) is the intuitive assumption of the unified growth model of Galor and Weil (2000): The speed of technological progress is a positive function of the size of the population (N_t) and the level of human capital (h_t). Adults with high levels of human capital are more likely to advance the technological frontier by creating new ideas. On the other hand, holding the level of human capital, the total number of new ideas created during a given time period is also a positive function of the population size.

The growth rate of ideas is

$$g_{At} = \frac{\dot{A}_t}{A_t} = \delta \frac{H_t^{\mu}}{A_t^{1-\phi}} \ . \tag{14}$$

The stock of ideas can be written as a function of the growth rate of ideas, human capital, and number of adults:

$$A_t = b \left(h_t N_t \right)^{\kappa} g_{At}^{-\kappa\mu^{-1}} \ , \tag{15}$$

where the constants $\kappa = \mu / (1 - \phi)$ and $b = \delta^{\kappa/\mu} \left(sT \right)^{\kappa}$.

[5] A similar assumption is that a constant percentage of adults work in the R&D sector and that they receive the same wages as those work in the production sector.

We follow Becker, Murphy, and Tamura (1990) to see human capital as ideas embodied in physical labor through education or training and to assume that the human capital of children is accumulated according to a learning technique with positivity externality of the parental level of human capital:

$$h_{t+1} = \upsilon h_t^{\gamma} z_t + h_0 \ , \tag{16}$$

with constants $0 < \gamma \leq 1$ and $\upsilon > 0$. The human capital of a child h_{t+1} depends on the parental level of human capital h_t, the time z_t a parent invests in the human capital accumulation of each child, and the endowed human capital at birth h_0.

Altruistic parents choose the number of children (n_t) and the human capital investment of each child (z_t) to maximize the dynastic utility function:

$$V_t = u(c_t) + a(n_t) n_t V_{t+1} \ . \tag{17}$$

The dynastic utility of a parent V_t depends on his consumption c_t, the degree of altruism per child $a(n_t)$, the number of children n_t, and the utility of each child V_{t+1}. The dynastic utility function is simplified with

$$u(c_t) = \frac{c_t^{\sigma}}{\sigma}, \quad a(n_t) = \alpha n_t^{-\varepsilon} \ , \tag{18}$$

where $0 < \sigma < 1, 0 \leq \varepsilon < 1$, and $\alpha > 0$. Thus, the discount rate applied by the present generation to the per capita consumption of subsequent generations $a(n_t)$ depends negatively on the number of children of the present generation. Parents maximize the utility function subject to the following time and budget constraints:

$$(1-s)T = l_t + n_t(e + z_t) \ , \tag{19}$$

$$c_t = A_t^{\beta} l_t^{\beta} k_t^{1-\beta} - \dot{k}_t - n_t f \ . \tag{20}$$

B. Steady-state growth of population, human capital, and per capita income

This section focuses on the steady-state with positive human capital investments; the corner solution with zero investment in human capital is discussed in Section III and Appendix C.

The arbitrage condition between per capita consumption in periods t and $t+1$ is

$$\frac{u'(c_t)}{au'(c_{t+1})} = \alpha^{-1} n_t^{\varepsilon} \left(\frac{c_{t+1}}{c_t} \right)^{1-\sigma} \geq R_{zt} = 1 + r_{zt} \tag{21}$$

where r_{zt} is the rate of return on investment in human capital, and equality holds when investments are positive. The rate of return is determined from[6]

$$R_{ht} = \upsilon n_t \left(l_{t+1} + n_{t+1} z_{t+1} \right) \ . \tag{22}$$

Since the rate of return measures the effect on c_{t+1} of increasing h_{t+1}, it depends on the productivity of greater h_{t+1}, which depends on n_t, l_{t+1}, z_{t+1}, and n_{t+1} according to the production functions of the ideas, consumption good, and human capital.

By differentiating the utility function with respect to n_t, we get the first-order condition for maximizing the utility with respect to the number of children:

$$\left(1-\varepsilon\right) \alpha n_t^{-\varepsilon} V_{t+1}\left(h_{t+1}\right) = u'\left(c_t\right) \left[\beta A_t^{\beta} l_t^{\beta-1} k_t^{1-\beta} \left(e + z_t\right) + f \right] . \tag{23}$$

The marginal utility from an additional child is given on the left-hand side of equation (23), while the right-hand side gives the total costs of producing and rearing a child. Costs depend on the productivity of labor ($\beta A_t^{\beta} l_t^{\beta-1} k_t^{1-\beta}$), the fixed time ($e$) and goods ($f$) inputs, and the endogenous time (z_t) spent investing in each child.

[6] See Appendix B for the calculation, in which we apply the simplified assumption of $\kappa = \gamma = 1$.

For the non-corner solution with a positive investment in human capital, we get the first-order condition for maximizing utility with respect to the investment in human capital accumulation by differentiating the utility function with respect to z_t:

$$\alpha n_t^{-\varepsilon} \upsilon h_t \frac{dV_{t+1}}{dh_{t+1}} = \beta u'(c_t) A_t^{\beta} l_t^{\beta-1} k_t^{1-\beta} \ . \tag{24}$$

The marginal utility of an additional unit of time spent investing in children's human capital is given on the left-hand side of equation (24), while the right-hand side gives the marginal costs of time.

The economy converges to a steady-state growth path with a constant time (z^*) invested in each child's human capital, a constant number of children (n^*), and a constant growth rate of per capita income (g_c^*). The steady-state values z^* and n^* are determined from the first-order conditions as shown in equations (23) and (24). In the steady-state equilibrium, the time spent investing in each child's human capital is

$$z^* = \frac{\beta\sigma e}{1-\varepsilon-\beta\sigma} \tag{25}$$

The equilibrium education level of a child rises with the labor share of total output β, the elasticity of consumption σ, the fixed time cost of children rearing e, and the elasticity of altruism per child ε.

The steady-state number of children is found by substituting into equations (21) and (22):

$$\alpha^{-1} n^{*\varepsilon-1} \left(1+g^*\right)^{1-\delta} = \upsilon\left[\left(1-s\right)T - en^*\right] \ . \tag{26}$$

The steady-state growth rate of per capita income is equal to the growth rate of physical capital and the growth rate of ideas:

$$1+g^* = \frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{A_{t+1}}{A_t} = \upsilon z^* n^* \ . \tag{27}$$

The equilibrium growth rate is proportional to the investments in children's learning ($z*$) and the number of children of each parent ($n*$). The constant equilibrium ratio of k to A is determined by the condition:

$$\upsilon n*\left[(1-s)T - en*\right] = R_h = R_k = \alpha^{-1} n*^{\varepsilon} \left(1+g*\right)^{1-\delta} \quad . \tag{28}$$

C. Convergence in the growth rate but in not the level of per capita income

The model of this paper is built on an environment in which the economy is divided into a number of groups; all adults of any one group live in the same geographic area and share the same ideas about production, but the groups differ in the stock of ideas and perhaps the population size. If there are I groups in an economy and the groups have the same coefficients (i.e., α, β, σ, e, ε, υ, b, s, T), according to equations (25), (26), and (27), the groups will converge to the same steady-state growth rates of population, human capital, and per capita income (i.e., $g_{ic}^{*} = g_{c}^{*}$, $g_{ih}^{*} = g_{h}^{*}$, $g_{iN}^{*} = g_{N}^{*}$).

However, the groups will not converge in per capita income as long as the ideas created by one group cannot be perfectly used by another group.[7] In the steady-state, the per capita income of group i is

$$c_{it} = \left(A_{it} l^{*}\right)^{\beta} k_{it}^{1-\beta} - \dot{k}^{*} - n^{*} f = \left(b h_{it} N_{it} l^{*}\right)^{\beta} k_{it}^{1-\beta} - \dot{k}^{*} - n^{*} f \quad . \tag{29}$$

Therefore, given the level of human capital, a group with a larger population in the steady-state will have a higher per capita income.

Nevertheless, this group-level conclusion cannot be applied directly to the whole economy. We cannot say a country with a larger population will have a higher per capita income in the steady-state. The average per capita income across groups is:

[7] The inefficiency of technology spillover among groups is generally true even when there is no intellectual property protection. For example, it is hard for a new idea created in medical science to be directly used in computer science.

$$\bar{c}_t = \frac{\left(bh_t l^*\right)^\beta k_t^{1-\beta} \sum_{i=1}^{I} N_{it}^{1+\beta}}{\sum_{i=1}^{I} N_{it}} - \left(\dot{k}^* + n^* f\right) \ . \qquad (30)$$

For a country with a given population size, the average per capita income across groups is an increasing function of the size of each group and a decreasing function of the number of groups. In the steady-state equilibrium, it is possible that a country with a larger population has a lower per capita income if the society of this country is divided into many more groups due to geographic, cultural, or other reasons or if the groups of this country are less efficient in sharing production ideas.

III. Transitional dynamics and the growth effect

This section shows how the endogenous movement of the population growth rate (g_{Nt}) and the rate of human capital investment (z_t) results in the long-run movement of the growth rate of per capita income (g_{yt}) from a Malthusian regime through a post-Malthusian regime to a modern growth regime. The Malthusian regime is characterized by stagnant growth in population and per capita income, while the modern growth regime is characterized by significant growth in per capita income but a declining population growth rate. The post-Malthusian regime, which occurs between the Malthusian and modern growth regimes, is characterized by significant growth in the population and per capita income.

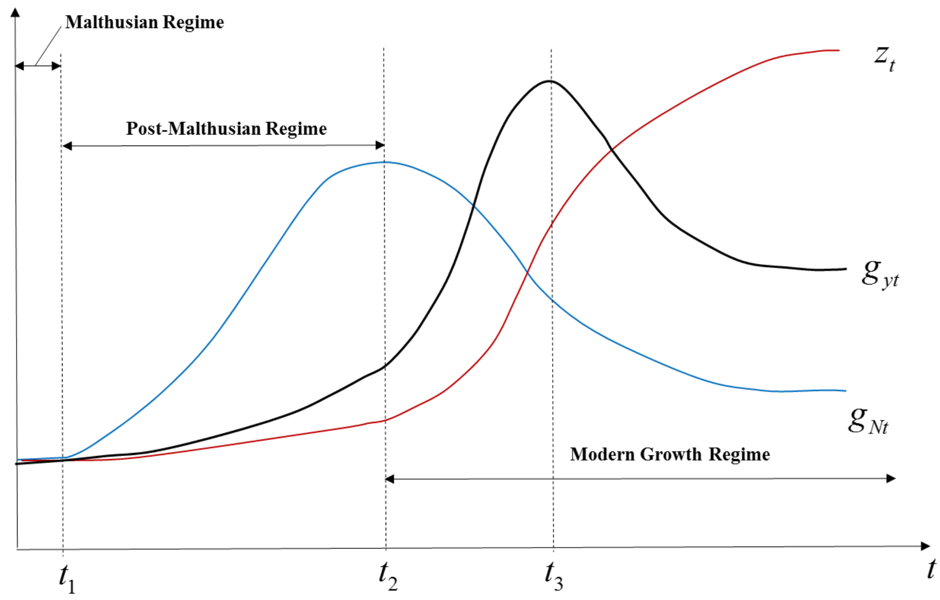

Figure 2. Model predictions for the long-run movement of the population growth rate, human capital investment rate, and growth rate of per capita income

The model predicts the relationships among g_{yt}, g_{Nt}, and z_t as presented in Figure 2. The period before time t_1 represents the thousands of years of Malthusian stagnation during which g_{yt}, g_{Nt}, and z_t are extremely small. The period between t_1 and t_2 represents the post-Malthusian regime with persistent increases in g_{yt}, g_{Nt} and z_t. The period after t_2 is the modern growth regime during which g_{Nt} declines and approaches a constant level, z_t increases and approaches a constant level, and g_{yt} first increases and then declines and approaches a constant level.

A. The growth effect before the demographic transition

During the thousands of years of Malthusian stagnation, low labor productivity implies that parents can afford to have only a small number of children. According to equation (22), a small population growth rate corresponds to a small return on human capital investment. As discussed in Appendix C, the economy is locked in Malthusian stagnation with no human capital investment when the population growth rate is very low.[8]

However, in this idea-based model, the Malthusian equilibrium is unstable. A positive shock that increases the population growth rate temporarily leads to a higher growth rate of ideas, which, in turn, can support a higher population growth rate. Therefore, a large enough positive shock on population growth or several small shocks over time may be enough for an economy to emerge from Malthusian stagnation (see Appendix C). Parents start to invest in the human capital of children, and the economy enters the post-Malthusian regime (start from time t_1 of Figure 2).

Positive investments in human capital lead to increased human capital investments because the rates of return on human capital investment rise with the level of human capital at least when the stock of human capital is not too high. The rise in the rate of investment in human capital leads to a higher growth rate of ideas. From equation (15), the growth rate of ideas can be written as:

$$g_{At} = g_{ht} + g_{Nt} - \mu^{-1} \frac{\dot{g}_{At}}{g_{At}} \quad . \tag{31}$$

Solving this first-order linear nonhomogeneous differential equation, we get that the growth rate of ideas (g_{At}) is an increased function of the growth rate of human capital (g_{ht}) and the growth rate of the population (g_{Nt}):

$$g_{At} = \frac{\int \left(g_{ht} + g_{Nt}\right) de^{\mu t}}{e^{\mu t}} \quad . \tag{32}$$

An increase in human capital investment leads to a higher growth rate of human capital and therefore a higher growth rate of ideas, which implies accelerated growth in per capita income.[9]

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[8] It is worthwhile to stress the difference between the number of children and fertility. During Malthusian stagnation, the fertility rate is actually quite high. However, the high mortality rate of children at the same time leads to a small number of surviving children for each parent. Here, the relevant measure of the parents' decision is the number of surviving children because rational parents should take mortality into account when they make decisions about fertility.

[9] It is straightforward to show that the growth rate of per-capita income is proportional to the growth rate of ideas. Along the growth path, the ratio $A_t l_t / k_t$ is constant and equal to $\beta / (1-\beta)$. Since there are no adjustment costs in this model,

The increase in per capita income has a positive income effect and a negative substitution effect on the demand for children. Initially, the positive income effect dominates, and the growth rate of the population rises over time. When an economy first emerges from Malthusian stagnation, the opportunity cost of time (e) spent on children rearing is small because labor productivity is low, and the main cost is the fixed good cost (f). The growth in per capita income eases parents' budget constraints, allowing them to spend more resources on raising children. Thus, the growth rate of the population rises over time when the economy first emerges from Malthusian stagnation.

However, the negative substitution effect of the increase in per capita income on the demand for children grows over time. Increases in the growth rates of both human capital and population accelerate the growth of ideas and therefore lead to higher labor productivity. The opportunity cost of time spent on rearing children increases over time with the increase in labor productivity. Eventually, the substitution effect dominates the income effect, and a demographic transition is triggered: The population growth rate starts to fall after peaking at time t_2 as shown in Figure 2. The start of the demographic transition marks the end of the post-Malthusian regime and the beginning of the modern growth regime.

The growth effect of population growth on economic growth is observable before the demographic transition. During the Malthusian regime, as shown in Appendix C, the growth rate of per capita income is proportional to the population growth rate. During the post-Malthusian regime, the growth rate of per capita income is proportional to the growth rates of both population and human capital. If we examine time-series data from the Malthusian regime and the post-Malthusian

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the economy will instantaneously adjust the initial amounts of k so that this ratio is achieved. Relative to the stock of ideas (A_t) and physical capital (k_t), the per-capita labor input can be approximately seen as constant. According to equation (12), per-capita income can be written as an increasing function of the stock of ideas; thus, the growth rate of per-capita income is approximately proportional to the growth rate of ideas.

regime, we should be able to find a growth effect in the form that an increase in the population growth rate corresponds to an increase in the growth rate of per capita income.

B. The unobservable growth effect during the early stage of the modern growth regime

The demographic transition marks the beginning of the modern growth regime, during which the rate of human capital investment (z_t) continues to rise as a result of the increase in the stock of human capital, while the number of children of each parent (n_t) continues to fall due to the substitution effect of increases in labor productivity. A trade-off between the "quality" and "quantity" of children is observed after the demographic transition.

The growth rate of per capita income is likely to continue to rise at least for a while before starting to fall. From equation (31), we get the first-order derivative of g_{At}:

$$\frac{dg_{At}}{dt} = \frac{dg_{ht}}{dt} + \frac{dg_{Nt}}{dt} - \mu^{-1} \frac{d\left(\dot{g}_{At}/g_{At}\right)}{dt} \ . \tag{33}$$

At $t = t_2$ (see Figure 2), we have $dg_{ht_2}/dt_2 > 0$ and $dg_{Nt_2}/dt_2 = 0$. If the growth path of g_{At} is not strictly convex at t_2, we have $d\left(\dot{g}_{At}/g_{At}\right)/dt \le 0$. Therefore, we have $dg_{At}/dt > 0$ at t_2. The inequality $dg_{At}/dt > 0$ must hold for at least some small increases in time from t_2.[10] Since g_{yt} is proportional to g_{At}, it will increase at least for a while after time t_2.

Since the growth rate of per capita income continues to rise just after the population growth rate starts to decline, the growth effect of population growth on economic growth is unobservable when an economy first enters the modern growth regime. If we examine time-series data for a country during this period, we find only a negative correlation between the population growth rate and the economic growth rate. Therefore, during the early stage of the modern growth regime, the growth

[10] The condition that g_{At} is not strictly convex at t_2 is a sufficient but not necessary condition.

effect of population growth is obscured by the offsetting movement of the population growth rate and the rate of human capital investment.

C. Approaching steady-state and the reappearance of the growth effect

The increasing trend of the economic growth rate will be reversed with further declines in the population growth rate. The growth rates of ideas can be written as:

$$g_{At} = \frac{A_{t+1}}{A_t} - 1 = \upsilon n_t z_t \left(\frac{\dot{g}_{At}}{g_{At}} + 1 \right)^{-\mu^{-1}} - 1 \ . \tag{34}$$

The functional form of n_t and z_t in equation (34) implies that the marginal positive contribution of an increase in z_t to the growth rate of ideas falls as n_t declines, while the marginal negative effect of a decline in n_t rises as z_t increases.[11] The negative effect of declines of n_t might eventually overcome the positive effect of increases of z_t, and the growth rate of ideas starts to decline ($dg_{At}/dt < 0$) after arriving at its highest point. Since the growth rate of per capita income is proportional to the growth rate of ideas, the per capita income growth rate will start to decline after arriving at its highest point as shown at t_3 as shown in Figure 2.[12]

After the growth rate of per capita income starts to decline, the growth effect of population growth on economic growth reappears in the empirical data in the form that a decline in the population growth rates corresponds to a decline in the economic growth rates. If we examine the time-series data for a country after time t_3, we should be able to find a positive correlation between the population growth rate and the economic growth rate.

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[11] The multiplier relation between n_t and z_t is derived directly from the assumption that the number of new ideas created is a positive function of both the size of the population and the level of human capital of each person.

[12] Depending on the coefficients, it is also possible that the growth rate of per-capita income levels off when it arrives at the highest point. However, since the target of parents is to maximize the family utility but not the growth rate, this case is not quite possible.

In the long-run, the increasing trend of z_t will finally level off because of the parents' time constraint and because of the increase in the opportunity cost of time invested in the human capital of children. The declining trend of n_t will also finally level off because the negative marginal effect of a decline in n_t increases over time. Eventually, the economy converges to a steady-state growth path with constant time invested in each child's human capital (z^*), a constant number of children of each parent (n^*), and a constant growth rate of ideas (g_A^*) and per capita income (g_y^*).

IV. Empirical evidence

This section employs time-series data from twelve Western Europe countries over a period of two centuries to show the actual movements of the population growth rate, the level of human capital investment, and the growth rate of per capita income. We also use time-series data from 78 countries over the past half-century to show the growth effect of population growth for countries that belong to different growth regimes.

A. Long-run movement of economic growth rates, population growth rates, and education

This section uses empirical data from twelve Western Europe countries to show the historical movements of population growth rates, years of schooling, and growth rates of per capita GDP. The sample countries are Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Sweden, Switzerland, and the United Kingdom. We first collect data for each country within its present borders, and then sum up the population and GDP across the twelve countries and calculate the growth rates of population and per capita GDP for these countries as a whole. Years of schooling are the simple average across countries.

We prefer to use the average measures for these twelve countries for two reasons. First, the average measures from these geographically connected countries help reduce the measurement error that may arise due to migration and border changes. Second, these countries probably belong to the same growth regime for any historical period. This fact helps avoid the confusion that may arise from combining data from countries in different growth regimes, because, as predicted in the theoretical model, growth trends of population and per capita GDP vary across growth regimes.

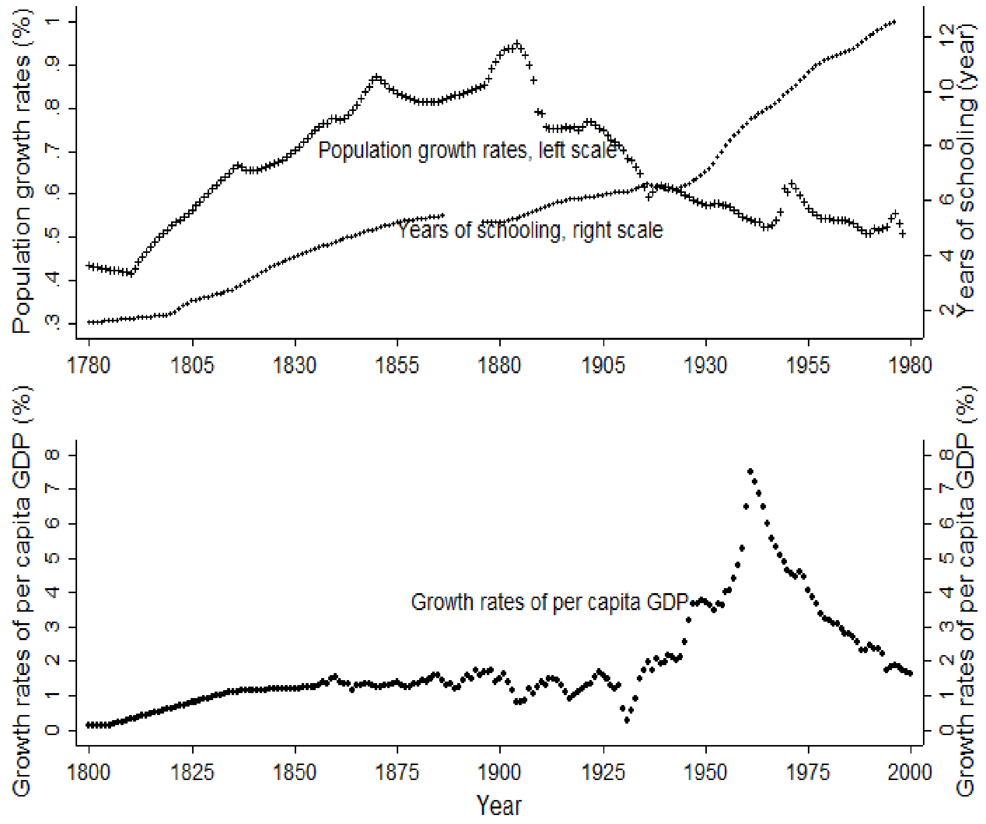

Figure 3. Long-run movement of population growth rates, years of schooling by birth cohort, and growth rates of per capita GDP for twelve Western Europe countries

Note: The data for the growth rates of per capita GDP and population are derived from Maddison (2007) and the Conference Board Total Economy Database. The years of schooling by birth cohort are derived from the dataset of Barro and Lee (2013) and Matthews, Feinstein, and Odling-Smee (1982). See the text for details.

As presented in Figure 3, the long-run movements of population growth rates, years of schooling, and growth rates of per capita GDP for these countries are quite similar to the predictions in Figure 2. Specifically, these three variables increased together starting from very low initial levels. After a long-run rise for about a century, the population growth rate peaked at the end of the nineteenth century and then began to fall. The growth rate of per capita GDP peaked about half-century later than that of the population, while years of schooling continues to rise. This figure does not include the thousands of years of Malthusian stagnation because the data for years of schooling are not available before the nineteenth century. Nevertheless, the stagnation of GDP per capita and population before the nineteenth century are supported by the dataset: During AD 1–1800, the average yearly growth rate is only 0.04 percent for GDP per capita and 0.09 percent for the population.

In this empirical examination, we measure the inter-generation growth rate of income per capita (g_{yt}) by the yearly growth rate of per capita GDP, measure the inter-generation population growth rate (g_{Nt}) by the yearly population growth rate, and measure the investment in human capital (z_t) by the average years of total schooling by the birth cohort. As shown in the x-axis of the top panel in Figure 3, we delay the population growth rates and years of schooling by 20 years, which are approximately the years for an infant to become an adult, to reflect the delayed effects of population growth and education on economic growth.[13]

The data for per capita GDP and population before 2003 are derived from Maddison (2007), and the data after 2004 come from the Conference Board Total Economy Database.[14] For some of the sample countries, the data before 1870 are available only for some years; continuous yearly measures are generated by linear interpolation. The per capita GDP is in 1990 International Geary-Khamis dollars. The growth rates of per capita GDP and population are calculated as 30-year moving averages in order to eliminate short-term variations associated with business cycles and other disturbances, such as wars and plagues.

[13] As shown in Figure 3, delaying these two variables by 10 or 30 years instead does not change the conclusions.

[14] Source: The Conference Board. 2015. The Conference Board Total Economy Database™, September 2015, http://www.conference-board.org/data/economydatabase/

Years of schooling by birth cohort from 1876 to 1975 are derived from the dataset of Barro and Lee (2013) for each country. They are calculated from years of schooling by each 5-year age group. For example, if the years of schooling for age group 50–54 are 7.3 years in 1950, then the years of schooling by the 1896–1900 birth cohort are 7.3 years. Years of schooling by birth cohort before 1876 are not available from this dataset. We use the dataset for England and Wales from 1770 to 1866 instead, which is available in Matthews, Feinstein, and Odling-Smee (1982). We assign the cohort years of schooling to the middle year of each cohort and generate continuous yearly data by linear interpolation.

B. The growth effect for countries with different income levels

As predicted in Figure 2, if we examine time-series data from a low-income country that is likely to belong to the growth regime before the demographic transition ($t < t_2$), we should observe that an increase in the population growth rate corresponds to an increase in the economic growth rate. If the country is a middle-income country that belongs to the stage just after the demographic transition but before the decline of the economic growth rate ($t_2 < t < t_3$), we should observe a negative correlation between the population growth rate and the economic growth rate. If the country is a high-income country that belongs to the stage after the decline of the economic growth rate ($t > t_3$), we should be able to observe the growth effect in the form that a decline in the population growth rate corresponds to a decline in the economic growth rate.

In this section, time-series data from 78 countries for the period from 1950 to 2015 are used to provide empirical support for the predictions on the growth effect of population growth.[15] The sample includes high-income, middle-income, and low-income countries, so the data have the potential to show the growth effect for countries in different stages of economic development. In this examination, we first regress the yearly growth rates of per capita GDP against the yearly growth

[15] Yearly data before 1950 are not available for most of the developing countries.

rates of the population for each country and collect the regression coefficient of the population growth rate. Then we plot the coefficient against the country-level per capita GDP. In the regression, the yearly population growth rate is delayed by 20 years to reflect the delayed effect of population growth on economic growth.

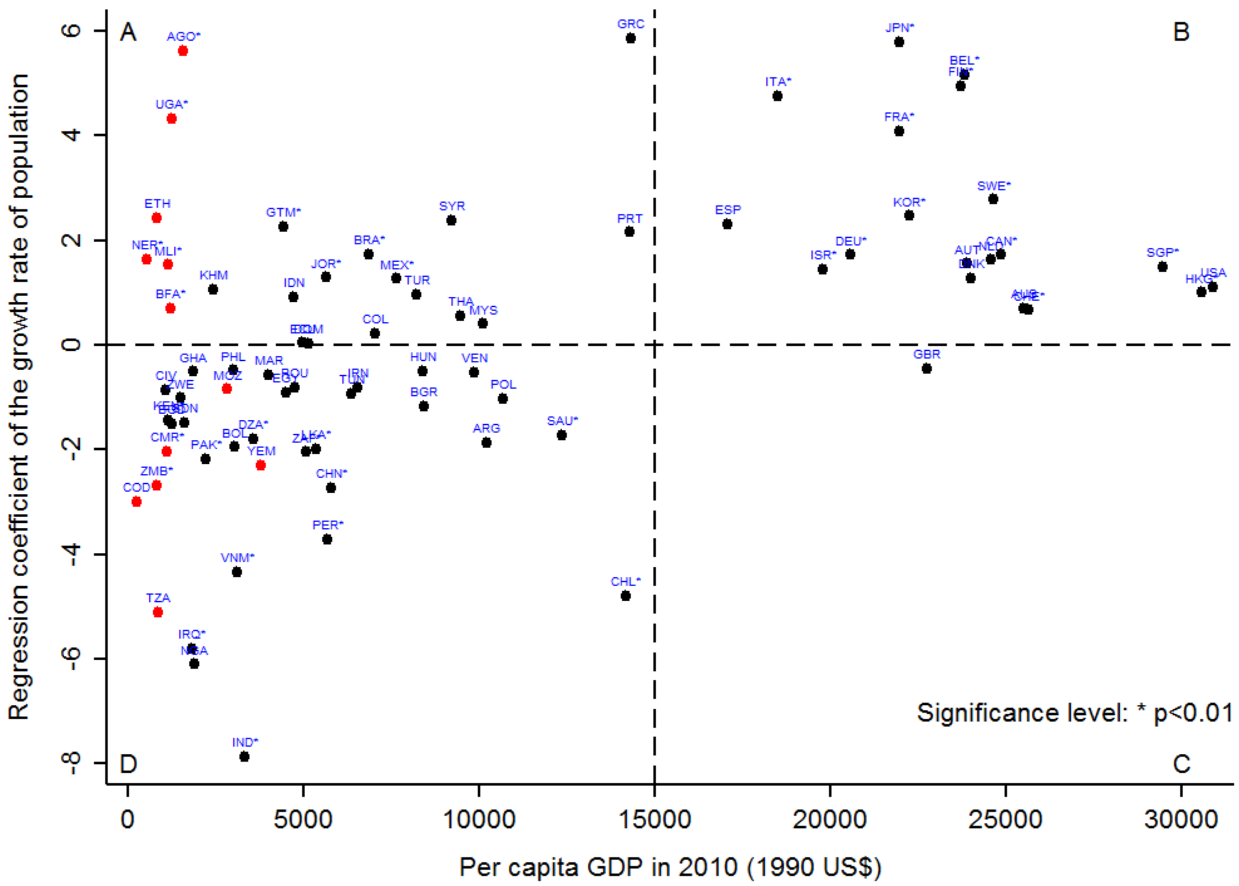

Figure 4: The difference in the growth effect of population growth across countries that are in different stages of economic development

Note: The black circle marks the countries with a declining trend in population growth rate during the sample period (the countries in sections A and D of Figure 1), while the red circle marks the countries with an increasing trend in the population growth rate (the countries in sections B and C of Figure 1). See Appendix A for details of the data source and process.

As shown in sections B and C of Figure 4, almost all countries with per capita GDP higher than US$15000 in 2010 show a positive and statistically significant correlation between the growth rate of per capita GDP and the population growth rate; the only exception is that GBR presents a negative but statistically insignificant correlation (at the 1 percent significance level). Since all the countries in Section B experienced a decline in the population growth rate, this result supports the prediction that for countries in the high-income stage (i.e., $t > t_3$ in Figure 2), a decline in the population growth rate corresponds to a decline in the economic growth rate.

As shown in sections A and C of Figure 4, the scale effect is ambiguous for countries with a per capita GDP lower than US$15000; positive and negative correlations are observed. These countries can be divided into two groups according to the trend of the population growth rate: The first group, marked with a *black circle*, experienced a decline in the population growth rate during the period from 1950 to 2015; the second group, marked with a *red circle*, experienced an increase in the population growth rate during this period.

The countries marked with a *black circle* and with a per capita GDP of less than US$15000 may have been moving from the middle-income stage (i.e., $t_2 < t < t_3$ in Figure 2) to the high-income stage (i.e., $t > t_3$ in Figure 2) during the period from 1950 to 2015. For countries mainly in the $t_2 < t < t_3$ stage during this period, we should find a negative correlation between the growth rate of per capita GDP and the population growth rate; these are countries in Section D of Figure 4 marked with a black circle. For countries mainly located in the $t > t_3$ stage during this period, we should find a positive correlation; these are countries in Section A of Figure 4 marked with a black circle.

The twelve countries that have experienced an increase in population growth rate during the sample period (marked with a red circle) all have an extremely low per capita GDP. These countries should belong to the $t < t_2$ stage (see Figure 2), and we should observe a positive correlation between the growth rate of per capita GDP and the population growth rate in these countries. However, only seven show a positive correlation, and five show a negative correlation. The empirical evidence from these five countries is inconsistent with the model predictions, although only the negative correlation in two of them are statistically significant (i.e., CMR and ZMB). A possible explanation is that the political crises and wars that occurred during the sample period in these five extremely poor countries have disturbed the normal path of economic development.[16]

V. Concluding Remarks

Building upon a number of early insights, this article uses an idea-based model to explain why the theoretically relevant growth effect of population growth on economic growth is empirically unobservable for some countries over the past half-century. The model predicts that the growth rate of per capita income is proportional to the population growth rate and the rate of human capital investment; the offsetting movement of population growth rates and human capital investments after the demographic transition obscures the observation of the growth effect.

The model also provides a simple way to model the transition from the thousands of years of Malthusian stagnation through a post-Malthusian regime to a modern growth regime. The model predicts that an economy is locked in Malthusian stagnation when the population growth rate is very low and parents do not invest in the human capital of children. A large enough positive shock on the population growth rate can significantly increase the rate of returns on human capital investment and therefore induce positive human capital investment, which enables an economy to escape the Malthusian trap and enter the post-Malthusian regime.

Since returns on human capital investments rise with the level of human capital, once an economy escapes the Malthusian trap, human capital investments increase over time. A larger human

[16] These five countries are Cameroon, Democratic Republic of the Congo, Tanzania, Zambia, and Yemen. They have been involved in serious political crises or wars in 1971, 1996, 2008, 1970, and 2011, respectively. For example, continental and civil wars in the Democratic Republic of the Congo have continued for two decades (1996–present) and have devastated the country.

capital investment results in a higher economic growth rate, which initially raises the population growth rate due to the income effect. A higher population growth rate, in turn, pushes up the economic growth rate. Therefore, the growth rates of human capital, population, and per capita income reinforce each other during the post-Malthusian regime.

However, since the production and rearing of children are time intensive, the substitution effect of income growth on the demand for children will eventually dominate the income effect, and a demographic transition is triggered. The economy enters the modern growth regime in which population growth rates decline over time, while human capital investments and per capita income growth rates continue to rise at least for a while.

Eventually, the increasing trend of human capital investment will level off because the opportunity cost of time invested in human capital accumulation increases with economic development. The declining trend of the population growth rate will also eventually level off because the negative marginal effect on the economic growth of a decline in the population growth rate increases over time. During the modern growth regime, the growth rate of per capita income first increases and then declines and finally approaches a constant level as defined by the steady-state growth rates of population and human capital.

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Appendix for online publication

A. The unobservable growth effect of population growth

To test the growth effect of population growth on economic growth, we collected the yearly growth rates of per capita income and population for 78 countries from 1950 to 2015. The yearly growth rates of per capita GDP and population are derived from the Conference Board Total Economy

Database.[17] From the countries or regions (called countries for simplicity) with available data during 1950–2015, we drop 19 countries with a population of less than 5 million because this paper is interested in endogenous economic and population growth, while small countries are much more vulnerable to exogenous shocks.

We regress the yearly growth rates of per capita income against the yearly population growth rates for each country and report the estimate of the coefficient of population growth rate in Table A1. In the regression, the population data are delayed by 20 years because it takes about 20 years for a child to become an adult. Thus, the regression captures the correlation between the current population growth rates and the per capita income growth rates 20 years later. We also tried regressions with no population delay or delayed by 5, 10, 15, or 25 years, and find similar results. Among the 78 sample countries, only 31 report positive and statistically significant correlations between population growth rates and economic growth rates; the other 47 countries report a negative or statistically insignificant correlation. Therefore, the growth effect of population growth rates on economic growth rates is supported only by data from fewer than half of the sample countries over the past half-century.

[17] Source: The Conference Board. 2015. The Conference Board Total Economy Database™, September 2015, http://www.conference-board.org/data/economydatabase/

Table 1: The correlation between per capita income growth rates and population growth rates (1950–2015)

Significant and Positive		Insignificant		Significant and Negative	
Country or region	Coefficient	Country or region	Coefficient	Country or region	Coefficient
Switzerland	0.665***	Dominican Republic	0.026	Venezuela	-0.520*
Australia	0.696**	Ecuador	0.04	Côte d'Ivoire	-0.856**
Burkina Faso	0.699***	Madagascar	0.222	Tunisia	-0.932*
Turkey	0.955**	Colombia	0.223	Zimbabwe	-1.017*
Hong Kong	1.000***	Malaysia	0.399	Bulgaria	-1.175*
Cambodia	1.058**	Thailand	0.554	Kenya	-1.432***
United States	1.113**	Indonesia	0.911	Sudan	-1.486*
Mexico	1.260***	Austria	1.55	Saudi Arabia	-1.719***
Denmark	1.273*	Portugal	2.15	Algeria	-1.792***
Jordan	1.286***	Spain	2.3	Sri Lanka	-1.995***
Israel	1.445***	United Kingdom	-0.449	Cameroon	-2.042***
Singapore	1.485***	Philippines	-0.488	South Africa	-2.047***
Mali	1.528***	Hungary	-0.506	Pakistan	-2.188***
Netherlands	1.622**	Ghana	-0.515	Yemen	-2.303**
Niger	1.640***	Morocco	-0.572	Zambia	-2.679***
Germany	1.717***	Iran	-0.811	China	-2.731***
Canada	1.722***	Romania	-0.82	DR Congo	-2.994**
Brazil	1.729***	Mozambique	-0.847	Peru	-3.718***
Guatemala	2.253***	Egypt	-0.904	Vietnam	-4.335***
Syria	2.372*	Poland	-1.02	Chile	-4.791***
Ethiopia	2.422**	Bangladesh	-1.5	Iraq	-5.809***
South Korea	2.462***	Argentina	-1.87	Nigeria	-6.106**
Sweden	2.778***	Bolivia	-1.95	India	-7.866***
France	4.086***			Myanmar	-24.71***
Uganda	4.325***				
Italy	4.759***				
Finland	4.944***				
Belgium	5.160***				
Angola	5.610***				
Japan	5.794***				
Greece	5.860**				

Note: We regress the yearly growth rates of per capita income against the yearly population growth rates for each country and report the coefficient of the population growth rate in this table. In the regression, the population data are delayed by 20 years. Significance levels are *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

B. The rate of return on the investment in human capital

To calculate the rate of return on investments in human capital, we rewrite the Bellman equation using the learning technology (16), the time constraint (19), and the budget constraint (20) to yield

$$V_t\left(h_t\right)=\max\left\{\begin{array}{l}\sigma^{-1}\left[\left(bh_tN_t\right)^{\beta}\left[\left(1-s\right)T-n_te-n_t\left(\upsilon h_t\right)^{-1}h_{t+1}-n_t\left(\upsilon h_t\right)^{-1}h_0\right]^{\beta}k_t^{1-\beta}-\dot{k}_t\right]^{\sigma}\\+\alpha n_t^{1-\varepsilon}V_{t+1}\left(h_{t+1}\right)\end{array}\right\}\ . \qquad (35)$$

Here we apply the simplification assumption of $\kappa=\gamma=1$. Differentiating with respect to h_{t+1} leads to:

$$-c_t^{\sigma-1}\beta\left(bh_tN_t\right)^{\beta}\left(\upsilon h_t\right)^{-1}l_t^{\beta-1}k_t^{1-\beta}+\alpha n_t^{-\varepsilon}V'_{t+1}\leq 0\ . \qquad (36)$$

Using the envelope theorem provides:

$$V'_{t+1}=c_{t+1}^{\sigma-1}\beta l_{t+1}^{\beta-1}k_{t+1}^{1-\beta}\left(bh_{t+1}N_{t+1}\right)^{\beta}\left(h_{t+1}\right)^{-1}\left(l_{t+1}+n_{t+1}z_{t+1}\right)\ . \qquad (37)$$

Combine (36) and (37) to get the arbitrage condition:

$$\alpha^{-1}n_t^{\varepsilon}\left(\frac{c_{t+1}}{c_t}\right)^{1-\sigma}\geq R_{ht}=\upsilon n_t\left(l_{t+1}+n_{t+1}z_{t+1}\right)\ . \qquad (38)$$

C. A corner solution and emergence from Malthusian stagnation

During the thousands of years of Malthusian stagnation (the period before t_1 in Figure 2), labor productivity as measured by the stock of ideas (A_t) is quite low. Parents can afford to have only a small number of children because the production and rearing of children are expensive and time intensive. Parents do not invest in the human capital of children ($z = 0$), because a small number of children may lead to the strict inequality of equation (21):

$$\alpha^{-1} n_u^{\varepsilon} \left(\frac{c_{t+1}}{c_t} \right)^{1-\sigma} > \upsilon n_u l_u \ , \tag{39}$$

with n_u the steady-state number of children each parent chooses, and l_u the steady-state working time of each adult. The inequality (39) holds for a proper combination of coefficients and a sufficiently small n_u. The underlying reason is that, according to equation (13), a small n_t means low productivity of h_{t+1} in creating new ideas in the next period (A_{t+1}). Since the rate of return on human capital investment measures the effect of increasing h_{t+1} on c_{t+1}, which is proportional to A_{t+1}, a small n_t means a low rate of return on human capital investment. Parents do not invest in the human capital of children if the return is sufficiently small.

In the Malthusian equilibrium, the stock of ideas is

$$A_t = b\left(h_0 N_t\right)^{\kappa} \ . \tag{40}$$

The growth rate of ideas is proportional to the population growth rate: $g_{Au} = \kappa\left(n_u - 1\right)$. Since the population growth rate is quite small, ideas grow slowly with the population over time during Malthusian stagnation. Per capita income also grows over time but at an even smaller rate than that of ideas and population: $g_{yu} = \beta g_{Au}$. Empirical evidence supports that the growth rates of population and living standard during Malthusian stagnation are extremely small. For example, the Maddison

(2007) dataset shows that during AD 1–1800, the average yearly growth rate is only 0.04 percent for GDP per capita and 0.09 percent for the population.

However, the steady-state of Malthusian stagnation is unstable. A temporally higher population growth rate may result in a higher growth rate of ideas, which, in turn, can support a higher population growth rate. Thus, a positive shock on n_t may move the economy up to a higher steady-state than with higher growth rates in ideas, population, and consumption. Therefore, a large enough positive shock on n_t or several small shocks over time may be enough to reverse the inequality (39), and then parents start to invest in the human capital of children. Once parents start to invest in the human capital of children, the economy emerges from the thousands of years of Malthusian stagnation and enters the post-Malthusian regime (the period between t_1 and t_2 of Figure 2). The coming into play of human capital investment makes the dynamic of the growth rates of population and per capita income in the post-Malthusian regime significantly differ from that in Malthusian stagnation.

The conclusion that increases in the population growth rate result in the emergence from Malthusian stagnation is slightly different from the conclusion of Galor and Weil (2000), in which they argue that increases in the population size is the reason for escape from the Malthusian trap. The differences occur because this paper follows Jones (1995a) to eliminate the scale effect of the population size and keeps the growth effect of the population growth rate.