

Selling with Evidence

Frédéric KOESSLER

Vasiliki SKRETA

Paris School of Economics – CNRS

University College London, CEPR

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Setting

A seller has private information about his product characteristics

A consumer has private information about his taste

Seller can certify product characteristics

Seller chooses a selling procedure at *interim* (**KNOWING** his type)

Questions

What is an **equilibrium selling procedure**?

Is it **ex-ante profit maximizing**?

Does seller's private info increase profit vis a vis **full information**?

Or do we get product information **unraveling**?

Contributions of the Paper

- Formulation of the **informed-principal problem with certifiable information** for the principal
- **Equilibrium characterization** under own type certifiability
 - ▶ Ex-ante profit maximizing
 - ▶ No information unravelling

Some Background

- **profit-maximizing selling procedures:** Myerson (1981), Riley and Zeckhauser (1983), Yilankaya (1999), Koessler and Skreta (2016) ...
- **mechanism design by an informed principal:** Myerson (1983), Maskin and Tirole (1990), Maskin and Tirole (1992), Mylovanov and Tröger (2014)...
- **mechanism design with certifiable info:** Green and Laffont (1986), Forges and Koessler (2005), Bull and Watson (2007), Deneckere and Severinov (2008), Strausz (2016) ...
- **info disclosure, advertizing:** Johnson and Myatt (2006), Anderson and Renault (2006), Eső and Szentes (2007)...
- **bargaining, selling with certifiable info:** De Clippel and Minelli (2004), Koessler and Renault (2012)...

Model

- One **seller**, privately known type (**product characteristic**) $s \in S$; (fully) certifiable at zero cost (**for talk**)
- One **buyer**, privately known type (**taste**) $t \in T$; uncertifiable
- Types are independently distributed. Priors $\sigma \in \Delta(S)$ and $\tau \in \Delta(T)$
- **Buyer's valuation**: $u(s, t) \in \mathbb{R}$
- **Seller's reservation value (or cost)**: $v(s, t) \in \mathbb{R}$

Payoffs

Allocation: $(p, x) : S \times T \rightarrow [0, 1] \times \mathbb{R}$

$$\begin{cases} p(s, t) : & \text{probability of trade} \\ x(s, t) : & \text{price, transfer from buyer to seller} \end{cases}$$

Seller's profit: $V(s, t) \equiv x(s, t) - p(s, t)v(s, t)$

Buyer's utility: $U(s, t) \equiv p(s, t)u(s, t) - x(s, t)$

Interim: $V(s) \equiv \sum_t \tau(t)V(s, t) \quad U(t) \equiv \sum_s \sigma(s)U(s, t)$

Mechanism-Proposal Game

- 1 Nature draws seller's type s and buyer's type t
- 2 Seller knowing s proposes a mechanism (M_T, m) where

$$m : S \times M_T \rightarrow [0, 1] \times \mathbb{R}$$

- 3 Each seller type certifies s to mechanism; simultaneously, buyer decides whether or not to participate & sends a message $m_T \in M_T$

An allocation is implemented as a function of mechanism m and reporting and participation strategies

Expectational Equilibrium (Myerson, 1983)

An allocation (p, x) is an **Expectational Equilibrium** (or strong Perfect Bayesian Equilibrium) iff

- (i.) It is **feasible** for the **prior** (WLOG an **inscrutable** mechanism proposed along the equilibrium path Myerson, 1983)
- (ii.) There is **no profitable mechanism deviation**: for every mechanism \tilde{m} , **there exists a belief** $\tilde{\pi} \in \Delta(S)$ for the buyer, reporting and participation strategies that form a **continuation Nash equilibrium** given \tilde{m} and $\tilde{\pi}$, with outcome (\tilde{p}, \tilde{x}) , such that

$$V(s) \geq \tilde{V}(s) \quad \text{for every } s$$

Feasible Allocations

Definition

An allocation is **feasible if and only if** the following incentive compatibility and participation constraints are satisfied:

$$V(s) \geq 0, \text{ for every } s \in S \quad (\text{S-PC})$$

$$U(t) \geq U(t' \mid t), \text{ for every } t, t' \in T \quad (\text{B-IC})$$

$$U(t) \geq 0, \text{ for every } t \in T \quad (\text{B-PC})$$

Remark (Partial Certifiability)

*In the paper we show that the above conditions **together with an appropriately defined seller-IC condition**, are necessary and sufficient conditions for feasibility under general partial certifiability structures.*

What are the effects of certifiability?

- 1 **Extends set of feasible allocations** Seller-IC automatically satisfied
- 2 **Extends the set of off-path continuation equilibrium outcomes:**
High quality seller deviates to a mechanism specifying a high price for high quality (this ability drives the unravelling result under posted prices).
 - ▶ such a deviation not possible if info soft since low quality can mimic
- 3 Each seller type gets **at least full-information profit** in equilibrium

Example

Two product types $\{s_1, s_2\}$, uniform prior

Two consumer types $\{t_1, t_2\}$, uniform prior

The seller only cares about revenue ($v(s, t) = 0$)

Buyer's valuation for the product:

$$u(s, t) =$$

	t_1	t_2
s_1	5	3
s_2	1	2

s_1 : high quality; the seller can certify the quality at no cost

t_1 cares more about quality than t_2

Posted Prices and Direct Certification

$$u(s, t) =$$

	t_1	t_2
s_1	5	3
s_2	1	2

- 1 Seller and buyer privately observe their types, s and t
- 2 Each seller type s certifies his type and proposes a price $x(s)$
- 3 Buyer observes $x(s)$ and certified information, decides whether or not to accept

Unravelling: The **Full-Information allocation** is the *unique* equilibrium outcome (u is “pairwise monotonic”, Koessler and Renault, 2012)

$x(s_1) = 3$, $x(s_2) = 1$, all buyer types accept \Rightarrow interim revenues
($V(s_1)$, $V(s_2)$) = (3, 1), **not** ex-ante profit maximizing

A Better Selling Procedure

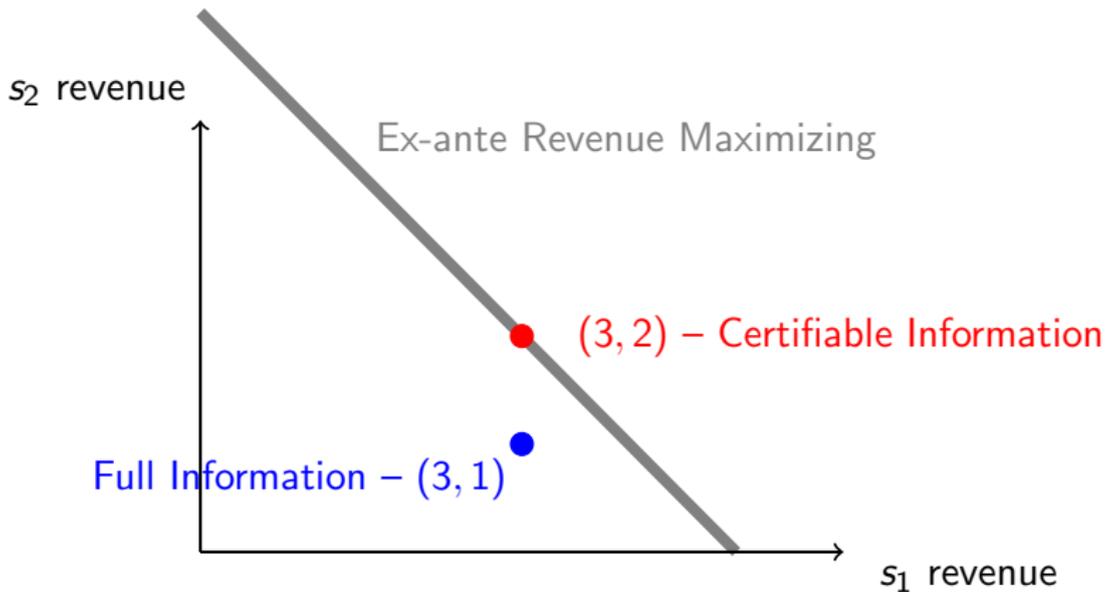
Seller commits to an “Evidence-conditional” contract: Buyer has to pay a price of 3 if seller certifies s_1 and otherwise must pay 2

Implements the allocation

$$(p, x)(s, t) =$$

	t_1	t_2
s_1	1, 3	1, 3
s_2	1, 2	1, 2

Interim revenues: $(3, 2) > (3, 1)$



We now show

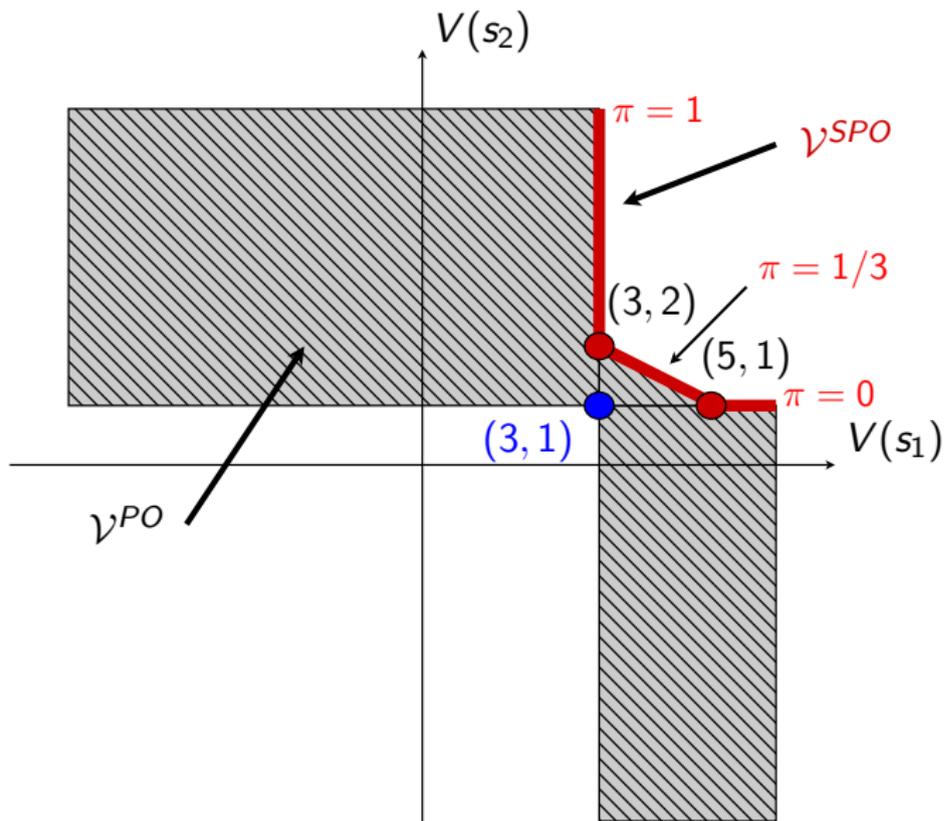
- that $(3, 2)$ is indeed a profile of interim equilibrium revenues
- $(3, 2)$ is the **unique** profile of interim equilibrium revenues
- how to obtain such an equilibrium in general
- that the equilibrium obtained that way is always ex-ante profit-maximizing (optimal)

Finding Expectational Equilibria

Idea:

- Characterize all interim profits, for a buyer beliefs π that satisfy buyer incentive, participation given π ; take union over *all* beliefs
- Take the Pareto frontier of this union set: **SPO**
- Show that SPO profit vector
 - 1 exists for all priors
 - 2 is an expectational equilibrium outcome
 - 3 SPO for prior is **ex-ante profit maximizing**

Illustration in the $\begin{pmatrix} 5 & 3 \\ 1 & 2 \end{pmatrix}$ Example



Main Results

Theorem

Every SPO allocation for the prior is an expectational equilibrium of the mechanism-proposal game.

Proof

Proposition

Every SPO allocation for the prior is ex-ante profit maximizing

Corollary

There exists an ex-ante profit maximizing expectational equilibrium

Only SPO allocations are equilibrium allocations

Back to the example

Consider the following mechanism \tilde{m} as a deviation:

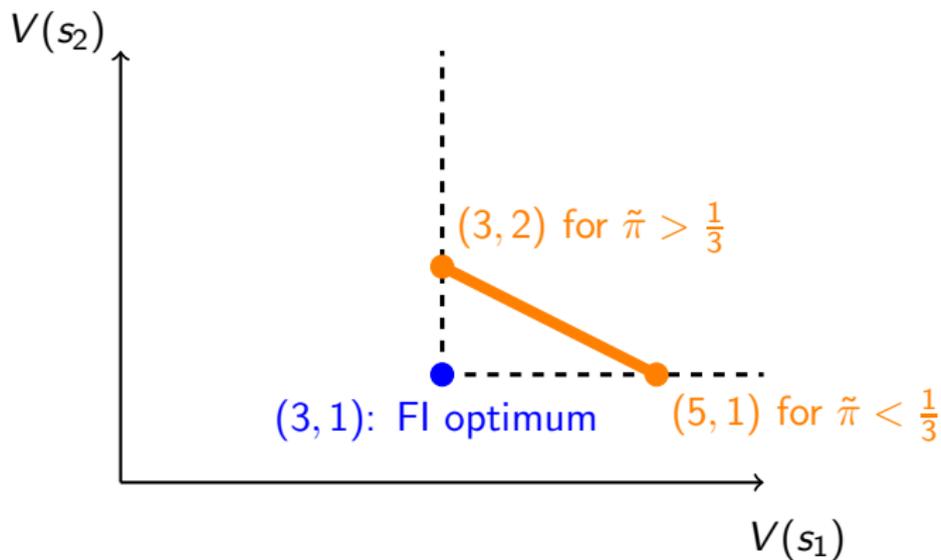
$$\tilde{m} =$$

	Left	Right
s_1	1, 5	1, 3
s_2	1, 1	1, 2

If the buyer's belief about s_1 is $\tilde{\pi}$

- “Left”: expected payment $\tilde{\pi}5 + (1 - \tilde{\pi})1$
- “Right”: expected payment $\tilde{\pi}3 + (1 - \tilde{\pi})2$
- “Left” \succ “Right” iff $\tilde{\pi} < \frac{1}{3}$
- The buyer never rejects whatever $\tilde{\pi}$

Continuation interim equilibrium profits induced by \tilde{m} as off-path belief
 $\tilde{\pi}$ varies



\Rightarrow An expectational equilibrium must be above this line

\Rightarrow Since only $(3, 2)$ is feasible, it is the unique equilibrium outcome

To show that equilibrium allocations are always SPO in general we need further assumptions:

- Add a dummy agent who is rewarded by the seller to report the buyer's belief off the equilibrium path
- Add a tie-breaking rule such that the buyer participates and reports truthfully when indifferent in a direct mechanism off-path

Theorem (Only SPO allocations are equilibrium allocations)

Every expectational equilibrium outcome of the mechanism-proposal game is a SPO allocation for the prior

Partial certifiability

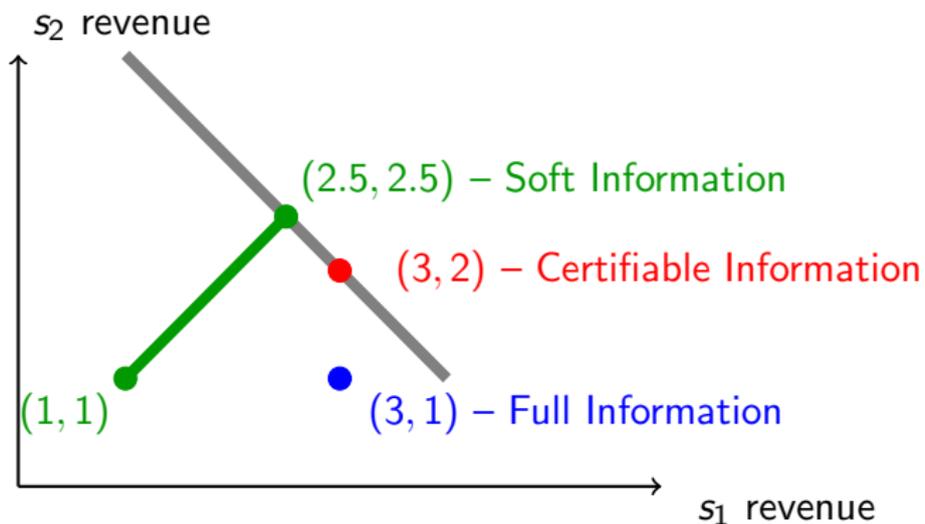
Formulation, feasibility in paper

Observation

If a SPO allocation for the prior is feasible under partial certifiability structure, then it is an equilibrium of the mechanism-proposal game

Proof: Under partial certifiability the set of possible deviations of the seller is smaller than under full certifiability

Equilibria in the Example: Hard versus Soft Info



Thank You!

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