# Aggregate Recruiting Intensity[*]

*Alessandro Gavazza*[‡]   *Simon Mongey*[§]   *and*   *Giovanni L. Violante*[¶]

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## Abstract

We develop a model of firm dynamics with random search in the labor market where hiring firms exert recruiting effort by spending resources to fill vacancies faster. Consistent with micro evidence, in the model fast-growing firms invest more in recruiting activities and achieve higher job-filling rates. In equilibrium, individual decisions of hiring firms aggregate into an index of economy-wide recruiting intensity. We use the model to study how aggregate shocks transmit to recruiting intensity, and whether this channel can account for the dynamics of aggregate matching efficiency around the Great Recession. Productivity and financial shocks lead to sizable pro-cyclical fluctuations in matching efficiency through recruiting effort. Quantitatively, the main mechanism is that firms attain their employment targets by adjusting their recruiting effort as labor market tightness varies. Shifts in sectoral composition can have a sizable impact on aggregate recruiting intensity. Fluctuations in new-firm entry, instead, have a negligible effect despite their contribution to aggregate job and vacancy creations.

**Keywords:** Aggregate Matching Efficiency, Firm Dynamics, Macroeconomic Shocks, Recruiting Intensity, Unemployment, Vacancies.

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[‡]London School of Economics and CEPR
[§]New York University
[¶]New York University, CEPR, IFS, IZA, and NBER.

# 1  Introduction

A large literature documents cyclical changes in the rate at which the US macroeconomy matches job seekers and employers with vacant positions. Aggregate matching efficiency, measured as the residual of an aggregate matching function that generates hires from inputs of job seekers and vacancies, epitomizes this crucial role of the labor market.

The Great Recession represents a particularly stark episode of deterioration in aggregate matching efficiency. Our reading of the data, displayed in Figure 1, is that this decline contributed to a depressed vacancy yield, to a collapse in the job finding rate and to persistently higher unemployment following the crisis. Identifying the deep determinants of aggregate matching efficiency is therefore necessary for a full understanding of the labor market dynamics during that period.

A number of explanations have been offered for the decline in aggregate matching efficiency around the recession, virtually all of which have emphasized the worker side.[1] A shift in the composition of the pool of job seekers towards the long-term unemployed, by itself, goes a long way towards explaining the drop (Hall and Schulhofer-Wohl, 2013); however as documented by Mukoyama, Patterson, and Şahin (2013), workers' job search effort is counter-cyclical and tends to compensate compositional changes. Hornstein and Kudlyak (2015) include both margins in their rich measurement exercise and conclude that they offset each other almost perfectly, leaving the entire drop in match efficiency from unadjusted data to be explained. A rise in occupational mismatch shows more promise, but it can account for at most one third of the drop and for very little of its persistence (Şahin, Song, Topa, and Violante, 2014).

The alternative view we set forth in this paper is that fluctuations in the effort with which firms try to fill their open positions affect aggregate matching efficiency. When aggregated over firms, we call this factor *aggregate recruiting intensity*. Our goal is to investigate whether this is an important source of the dynamics of aggregate matching efficiency, and to study the economic forces that shape how it responds to macroeconomic shocks.

Our main motivation is the empirical analysis of recruiting intensity at the firm level in Davis, Faberman, and Haltiwanger (2013) (henceforth DFH)—the first paper to rigorously use

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[1] A notable exception is the model in Sedláček (2014) that generates endogenous fluctuations in match efficiency through time-varying firms' hiring standards.

Figure 1: Labor market dynamics around the Great Recession (2008:01 - 2014:01)
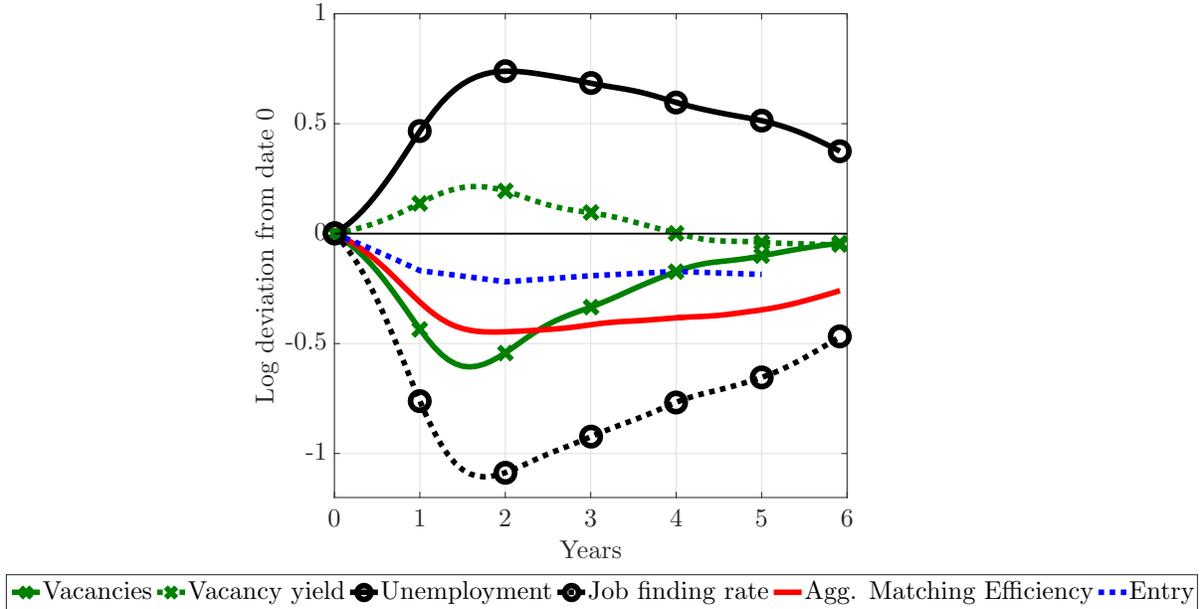


Notes *(i)* Vacancies $V_t$, and hires $H_t$ (used to compute vacancy-yield $H_t/V_t$) taken from monthly JOLTS data. Hires exclude recalls. *(ii)* Unemployment $U_t$ is from the BLS and exclude workers on temporary layoffs. *(iii)* The job finding rate is $H_t/U_t$. *(iv)* Aggregate matching efficiency is equal to $H_t/\left(V_t^\alpha U_t^{1-\alpha}\right)$ with $\alpha = 0.5$. *(v)* These first five series are measured from January 2001 to January 2014, expressed in logs and then HP-filtered. We plot level differences of these series from January 2008. *(vi)* (Firm) entry is taken from Census *Business Dynamics Statistics* and computed annually as the number of firms aged less than or equal to one year old at the time of survey and is available from 1977 to 2007. To this we fit and remove a linear trend. We plot log differences of this series from 2007.

JOLTS micro-data to examine what factors are correlated with vacancy-yields at the firm-level. The robust finding of DFH is that firms that grow faster fill their vacancies at a faster rate.[2] The corollary of this fact is that if an aggregate negative shock depresses firm growth rates, aggregate recruiting intensity—and, thus, aggregate match efficiency—declines because hiring firms use lower recruiting effort to fill their posted vacancies. We call this transmission channel, whereby the macro shock affects the growth rate distribution of hiring firms, the *composition effect*. Macro shocks also induce movements in equilibrium labor market tightness. When a negative shock hits the economy, job seekers become more abundant relative to vacancies, so firms meet workers more easily and can therefore exert less recruiting effort to reach a given hiring target. We call this second transmission channel the *slackness effect*, in reference to aggregate labor market conditions.

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[2]The numerous exercises in DFH show that this finding is not in any way spurious. For example, by definition, a firm that luckily fills a large amount of its vacancies will have both a higher vacancy yield and a higher growth rate. The authors show that luck does not drive their main result.

Both mechanisms seem potentially relevant in the context of the Great Recession. As evident from Figure 1, the data display a collapse in market tightness indicating the potential for a strong slackness effect. The figure also shows that the rate at which firms entered the economy fell dramatically in the aftermath of the recession. The dominant narrative is that the crisis was associated with a sharp reduction in borrowing capacity, and start-up creation as well as young firm growth are particularly sensitive to financial shocks (Chodorow-Reich, 2014; Davis and Haltiwanger, 2015; Mehrotra and Sergeyev, 2015; Siemer, 2013). Combining this observation with the fact that much of job creation (and thus hires) are generated by young firms (Haltiwanger, Jarmin, and Miranda, 2010) paves the way for a sizable composition effect.

Our approach is to develop a model of firm dynamics in frictional labor markets that can guide us to inspect the transmission mechanism of two common macroeconomic impulses—productivity and financial shocks—on aggregate recruiting intensity. The model is consistent with the stylized facts that are salient to an investigation of the interaction between macro shocks and recruiting activities: (i) it matches the DFH finding that increases in firm hiring rates are realized chiefly through increases in vacancy yields rather than increases in vacancy rates; (ii) it allows for credit constraints that hinder the birth of start-ups and slow the expansion of young firms; and (iii) it is set in general equilibrium, since the recruiting behavior of hiring firms depends on labor market tightness which fluctuates strongly in the data (Shimer, 2005).

Our model is a version of the canonical Diamond-Mortensen-Pissarides random matching framework with decreasing returns in production and non-convex hiring costs (Cooper, Haltiwanger, and Willis, 2007; Elsby and Michaels, 2013; Acemoglu and Hawkins, 2014). The model simultaneously features a realistic firm life-cycle, consistent with its classic competitive setting counterparts (Jovanovic, 1982; Hopenhayn, 1992), and a frictional labor market with slack on both demand and supply sides. We augment this environment in three dimensions.

First, we allow for endogenous entry and exit of firms. This is a key element for understanding the effects of macroeconomic shocks on the growth rates of hiring firms, since it is well documented that young firms account for a disproportionately large fraction of job creation, grow faster than old firms, and are more sensitive to financial conditions.

Second, we introduce a recruiting intensity decision at the firm level: besides the number of open positions that they are willing to fill in each period, hiring firms choose the amount of re-

Figure 2: Breakdown of spending on recruiting activities. Source: Bersin & Associates (2011)
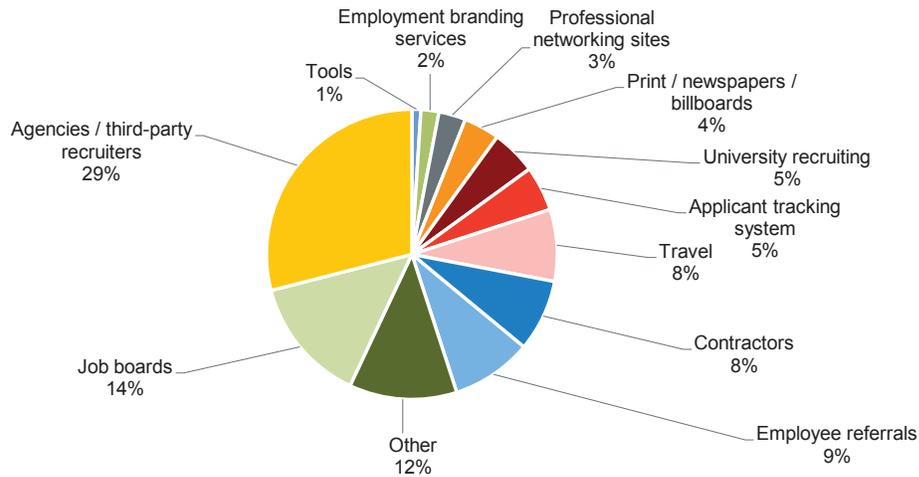


sources that they devote to recruitment activities. This endogenous recruiting intensity margin generates heterogeneous job filling rates across firms. In turn, the sum of all individual firms' recruitment efforts, weighted by their vacancy share, aggregates to the economy's measured matching efficiency.

Third, we introduce financial frictions: incumbent firms cannot issue equity, and a constraint on borrowing restricts leverage to a multiple of collateralizable assets, as in Evans and Jovanovic (1989).[3]

We parameterize our model to match a rich set of aggregate labor market statistics and firm-level cross-sectional moments. In choosing the recruiting cost function, we 'reverse-engineer' a specification that allows the model to replicate DFH's empirical relation between the job-filling rate and the growth rate at the establishment level from the Job Openings and Labor Turnover Survey (JOLTS) micro data. Our parameterization of this cost function is based on a novel source of data, a survey of recruitment cost and practices based on over 400 firms representative of the US economy. Figure 2 gives a breakdown of spending on all recruitment activities in which firms engage in order to attract workers and quickly fill their open positions, as reported by the survey. Our hiring cost function is meant to summarize all such components.

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[3]Other papers that consider various forms of financial constraints in frictional labor market models include Wasmer and Weil (2004), Petrosky-Nadeau and Wasmer (2013), Eckstein, Setty, and Weiss (2014), and Buera, Jaef, and Shin (2015). Though none of these models displays endogenous fluctuations in match efficiency. An exception is Mehrotra and Sergeyev (2013) where a financial shock has a differential impact across industries and induces sectoral mismatch between job-seekers and vacancies.

We find that both productivity and financial shocks—modelled as shifts in the collateral parameter—generate substantial pro-cyclical fluctuations in aggregate recruiting intensity. However, the financial shock generates movements in firms entry, labor productivity and borrowing consistent with those observed during the 2008 recession, whereas the productivity shock does not. The credit tightening accounts for approximately half of the drop in aggregate matching efficiency observed in the Great Recession through a decline in aggregate recruiting intensity. Notably, our model is consistent with a key cross-sectional fact documented by Moscarini and Postel-Vinay (2016): the vacancy yield of small establishments spiked up as the economy entered the downturn, whereas that of large establishments was much flatter. The reason is that the financial shock impedes the growth of a segment of very productive, already large, but relatively young, firms with much of their growth potential still unrealized. These firms drastically cut their hiring effort.

Our examination of the transmission mechanism indicates that the slackness effect is the dominant force: aggregate recruiting intensity falls mainly because the number of available job seekers per vacancy increases, allowing firms to attain their recruitment targets even by spending less on hiring costs. Surprisingly, the impact of the shock through the the shift in the distribution of firm growth rates (and, in particular, the decline in firm entry and young firm expansion) on aggregate recruiting intensity is quantitatively small. Two counteracting forces weaken this composition effect: (i) hiring firms are selected, thus relatively more productive than in steady-state; and (ii) the rise in the abundance of job seekers, relative to open positions, allows productive units —especially those financially unconstrained— to grow faster.

In an extension of the model, we augment the composition effect with a sectoral component by allowing permanent heterogeneity in recruiting technologies across industries. As Davis, Faberman, and Haltiwanger (2013) document, Construction and a few other sectors stand out in terms of their frictional characteristics by systematically displaying higher than average vacancy filling rates. In addition, these are the industries that were hit hardest by the crisis. In agreement with Davis, Faberman, and Haltiwanger (2012b), our measurement exercise concludes that, in the context of the Great Recession, the shift in composition of labor demand away from these high-yield sectors played a nontrivial role in the decline of aggregate recruiting intensity.

We conclude the paper by making use of our theory to propose a rule-of-thumb index of

aggregate recruiting intensity that is easy to compute from available labor market aggregates and can be updated in real time, as new JOLTS and BLS data gets released. We compare our index to that put forward by DFH, which is based on a distinct derivation entirely rooted in their 'generalized matching function'. We find that the two measures track each other quite closely in the downturn, however our indicator displays a faster recovery. This result tentatively leads us to conclude that the protracted atrophy of US aggregate match efficiency is caused by factors other than a persistent cutback in the recruiting effort of employers.

To the best of our knowledge, only two other papers have developed models of recruiting intensity. Leduc and Liu (2016) extend a standard Diamond-Mortensen-Pissarides model to one in which a representative firm chooses search intensity per vacancy. Without firm heterogeneity, they are unable to speak to the cross-sectional empirical evidence that recruiting intensity is tightly linked to firm growth rates, a key observation that we use to discipline our framework and assess the magnitude of the composition effect. Kaas and Kircher (2015) is the only other paper that focuses on heterogeneous job filling rates across firms. In their directed search environment, different firms post distinct wages that attract jobseekers at differential rates, whereas we study how firms' costly recruiting activities determine differential job filling rates. One would expect both factors to be important determinants of the ability of firms to grow rapidly. For example, from Austrian data, Kettemann, Mueller, and Zweimuller (2016) document that job filling rates are higher at high-paying firms but. However, after controlling for the firm component of wages, they remain increasing in firms' growth rates implying that wages are not the whole story: employers use other instruments besides the compensation package to hire quickly.

Moreover, while they (and Leduc and Liu, 2016) study aggregate productivity shocks—as we do, as well—we further analyse financial shocks, showing that the dynamics of macroeconomic variables during the Great Recessions are consistent with financial, rather than productivity, shocks. Finally, while in both our and their model aggregate recruiting intensity drops after a negative aggregate shock, the reasons fundamentally differ. Kaas and Kircher (2015) argue that the drop depends on recruiting intensity being a concave function of firms' hiring policies, whose dispersion across firms increases after a negative shock. Our decomposition of the transmission mechanism linking macroeconomic shocks and aggregate recruiting intensity allows us to infer that the main source of the drop is the increase in the number of available job

seekers per vacancy, which allows firms to scale back their recruiting effort.

The rest of the paper is organized as follows. Section 2 formalizes the nexus between firm-level recruiting intensity and aggregate match efficiency. Section 3 outlines the model economy and the stationary equilibrium. Section 4 describes the parameterization of the model, and highlights some cross-sectional features of the economy. Section 5 describes the dynamic response of the economy to macroeconomic shocks, explains the transmission mechanism, and outlines the main results of the paper. Section 6 discusses two extensions of the model (i) sectoral heterogeneity in vacancy filling rates and (ii) on-the-job search. Section 7 proposes a novel empirical measure of aggregate recruiting intensity based on our model, and illustrates its behavior over time. Section 8 concludes.

## 2 Recruiting Intensity and Aggregate Matching Efficiency

We briefly describe how we can aggregate hiring decisions at the firm level into an economy-wide matching function with an efficiency factor that has the interpretation of average recruiting intensity. This derivation follows DFH.

At date $t$, any given hiring firm $i$ chooses $v_{it}$, the number of open positions, ready to be staffed, and costly to create, as well as $e_{it}$, an indicator of recruiting intensity. Let $v_{it}^* = e_{it}v_{it}$ be the number of *effective* vacancies in firm $i$. Integrating over all firms we obtain:

$$V_t^* = \int e_{it}v_{it}di, \tag{1}$$

the aggregate number of effective vacancies. Under our maintained assumption of a constant returns to scale Cobb-Douglas matching function, aggregate hires equal:

$$H_t = (V_t^*)^\alpha U_t^{1-\alpha} = \Phi_t V_t^\alpha U_t^{1-\alpha}, \text{ with } \Phi_t = \left(\frac{V_t^*}{V_t}\right)^\alpha = \left[\int e_{it}\left(\frac{v_{it}}{V_t}\right)di\right]^\alpha, \tag{2}$$

which corresponds to DFH's generalized matching function. Therefore, measured aggregate matching efficiency $\Phi_t$ is an average of firm-level recruiting intensity weighted by individual vacancy shares, raised to the power of $\alpha$, the economy-wide elasticity of hires to vacancies.

Finally, consistency requires that each firm $i$ faces hiring frictions, implying that

$$h_{it} = q\left(\theta_t^*\right) e_{it} v_{it}, \tag{3}$$

where $\theta_t^* = V_t^*/U_t$ is effective market tightness.[4] Thus, $q\left(\theta_t^*\right) = H_t/V_t^* = \left(\theta_t^*\right)^{\alpha-1}$ is the aggregate job filling rate per effective vacancy, constant across all firms at date $t$.

# 3   Model

Our point of departure is an equilibrium random-matching model of the labor market in which firms are heterogeneous in productivity and size, and the hiring process occurs through an aggregate matching function. As discussed in the Introduction, we augment this model in three dimensions—all of which are essential to develop a framework that can address our question. First, our framework features endogenous firm entry and exit. Second, beyond the number of positions to open (vacancies), hiring firms optimally choose their recruiting intensity: by spending more on recruitment resources, they can increase the rate at which they meet job seekers. Third, once in existence, firms face financial constraints.

   In what follows, we present the economic environment in detail, outline the model timing, then describe the firm, bank, and household problems. Finally, we define a stationary equilibrium for the aggregate economy. Since our experiments will consist of perfect foresight transition dynamics, we do not make reference to aggregate state variables in agents' problems. We use a recursive formulation throughout.

## 3.1   Environment

Time is discrete and the horizon is infinite. Three types of agents populate the economy: firms, banks, and households.

**Firms**. There is an exogenous measure $\lambda_0$ of potential entrants each period, and an endogenous measure $\lambda$ of incumbent firms. Firms are heterogeneous in their productivity $z \in Z$, stochastic

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[4]Throughout we are faithful to the notation in this literature and denote *measured* labor market tightness $V_t/U_t$ as $\theta_t$.

and i.i.d. across all firms, and operate a decreasing-returns-to-scale (DRS) production technology $y(z, n', k)$ that uses inputs of labor $n' \in N$ and capital $k \in K$. The output of production is a homogeneous final good, whose competitive price is the numeraire of the economy.

All potential entrants receive an initial equity injection $a_0$ from households. Next, they draw a value of $z$ from the initial distribution $\Gamma_0(z)$ and, conditional on this draw, decide whether to enter and become an incumbent by paying the set-up cost $\chi_0$. Those that do not enter return the initial equity to the households.[5] This is the only time when firms can obtain funds directly from households—throughout the rest of their lifecycle they must rely on debt issuance.

Incumbents can exit exogenously or endogenously. With probability $\zeta$, a destruction shock hits an incumbent firm, forcing it to exit. Surviving firms observe their new value of $z$, drawn from the conditional distribution $\Gamma(dz', z)$, and choose whether to exit or continue production. Under either exogenous or endogenous exit, the firm pays out its positive net-worth $a$ to households. Those incumbents that decide to stay in the industry pay a per-period operating cost $\chi$ and then choose labor and capital inputs.

The labor decision involves either firing some existing employees or hiring new workers. Firing is frictionless, but hiring is not: a hiring firm chooses both vacancies $v$ and recruitment effort $e$ with associated hiring cost $\mathcal{C}(e, v, n)$, which also depends on initial employment. Given $(e, v)$, the individual hiring function (3) determines current period employment $n'$ used in production. To simplify wage setting, we assume firms' owners make take-it-or-leave it offers to workers, so the wage rate equals $\omega$, the individual flow value from non-employment.

The capital decision involves borrowing capital from financial intermediaries (banks) in intraperiod loans. Due to imperfect contractual enforcement frictions, firms can appropriate a fraction $1/\varphi$ of the capital received by banks, with $\varphi > 1$. To pre-empt this behavior, a firm renting $k$ units of capital is required to deposit $k/\varphi$ units of their net worth with the bank. This guarantees that, ex-post, the firm does not have an incentive to abscond with the capital. Thus, a firm with current net worth $a$ faces a collateral constraint $k \leq \varphi a$. This model of financial frictions is based on Evans and Jovanovic (1989).

**Banks.** The banking sector is perfectly competitive. Banks receive household deposits, freely transform them into capital, and rent it to firms. The one-period contract with households pays

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[5]Without loss of generality, we could have assumed that a fraction of the initial equity is sunk to develop the blueprint, i.e., attain the draw of $z$, and in case of no entry only the remaining fraction is returned to the financier.
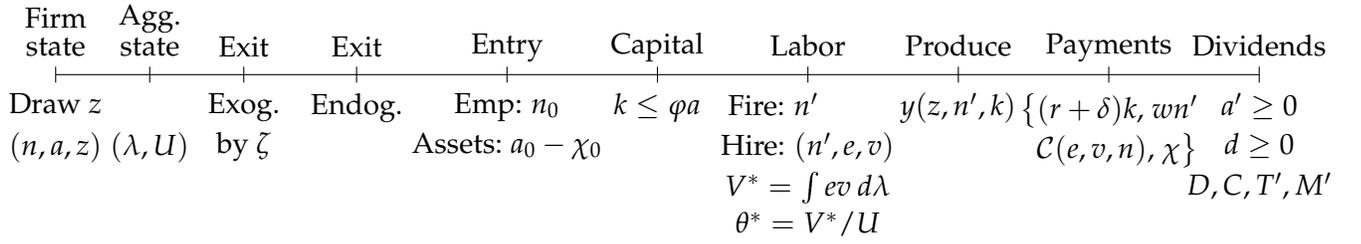
| Firm state | Agg. state | Exit | Exit | Entry | Capital | Labor | Produce | Payments | Dividends |
|---|---|---|---|---|---|---|---|---|---|
| Draw $z$ | | Exog. | Endog. | Emp: $n_0$ | $k \leq \varphi a$ | Fire: $n'$ | $y(z, n', k)$ | $\{(r+\delta)k, wn'$ | $a' \geq 0$ |
| $(n, a, z)$ | $(\lambda, U)$ | by $\zeta$ | | Assets: $a_0 - \chi_0$ | | Hire: $(n', e, v)$ | | $\mathcal{C}(e, v, n), \chi\}$ | $d \geq 0$ |
| | | | | | | $V^* = \int ev \, d\lambda$ | | | $D, C, T', M'$ |
| | | | | | | $\theta^* = V^*/U$ | | | |

Figure 3: Timeline of the model

a risk-free interest rate of $r$. Capital depreciates at rate $\delta$ in production, and so the price of capital charged by banks to firms is $(r + \delta)$.

**Households**. We envision a representative household with $\bar{L}$ family members, $U$ of which are unemployed. The household is risk-neutral with discount factor $\beta \in (0, 1)$. It trades shares $M$ of a mutual fund comprised of all firms in the economy and makes bank deposits $T$. It earns interest $r$ on deposits, the total wage payments that firms make to employed family members, and $D$ dividends per share held in the mutual fund. Moreover, unemployed workers produce $\omega$ units of the final good at home. Household consumption is denoted by $C$.

Before describing the firm's problem in detail, we outline the precise timing of the model, summarized in Figure 3. Within a period, the events unfold as follows: (i) realization of the productivity shocks for incumbent firms; (ii) endogenous and exogenous exit of incumbents; (iii) realization of initial productivity and entry decision of potential entrants; (iv) borrowing decisions by incumbents; (v) hiring/firing decisions and labor market matching; (vi) production and revenues from sales; (vii) payment of wage bill, costs of capital, hiring and operation expenses; firm dividend payment/saving decisions, and household consumption/saving decisions.

To be consistent with our transition dynamics experiments in Section 5, it is useful to note that we record aggregate state variables—the measures of incumbent firms $\lambda$ and unemployment $U$—at the beginning of the period, between stages (i) and (ii). Moreover, even though the labor market opens after firms exit or fire, workers who separate in the current period can only start searching in the next one.

## 3.2 Firm Problem

We first consider the entry and exit decisions, then analyze the problem of incumbent firms.

**Entry.** A potential entrant who has drawn $z$ from $\Gamma_0(z)$ solves the following problem

$$\max \left\{ a_0, \, \mathbb{V}^i(n_0, a_0 - \chi_0, z) \right\}, \tag{4}$$

where $\mathbb{V}^i$ is the value of an incumbent firm, a function of $(n, a, z)$. The firm enters if the value to the risk-neutral shareholder of becoming an incumbent with one employee ($n_0 = 1$), initial net worth equal to the household equity injection $a_0$ minus the entry cost $\chi_0$, and productivity $z$ exceeds the value of returning $a_0$ to the household. Let $i(z) \in \{0, 1\}$ denote the entry decision rule, which depends only on the initial productivity draw, since all potential entrants share the same entry cost, initial employment and ex-ante equity injection. As $\mathbb{V}^i$ is increasing in $z$, there is an endogenous productivity cut-off $z^*$ such that for all $z \geq z^*$ the firm chooses to enter. The measure of entrants is therefore

$$\lambda_e = \lambda_0 \int_Z i(z) d\Gamma_0 = \lambda_0 \left[1 - \Gamma_0(z^*)\right]. \tag{5}$$

**Exit.** Firms exit exogenously with probability $\zeta$. Conditional on survival the firm then chooses to continue or exit. An exiting firm pays out its net worth $a$ to shareholders. The firm's expected value $\mathbb{V}$ before the destruction shock equals

$$\mathbb{V}(n, a, z) = \zeta a + (1 - \zeta) \max \left\{ \mathbb{V}^i(n, a, z), \, a \right\}. \tag{6}$$

We denote by $x(n, a, z) \in \{0, 1\}$ the exit decision.

**Hire or Fire.** An incumbent firm $i$ with employment, assets, and productivity equal to the triplet $(n, a, z)$ chooses whether to hire or fire workers to solve

$$\mathbb{V}^i(n, a, z) = \max \left\{ \mathbb{V}^h(n, a, z), \mathbb{V}^f(n, a, z) \right\}. \tag{7}$$

The two value functions $\mathbb{V}^f$ and $\mathbb{V}^h$ associated with firing $(f)$ and hiring $(h)$ are described below.

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**The Firing Firm.** A firm that has chosen to fire some of its workers (or not to adjust its work force) solves

$$\mathbb{V}^f(n,a,z) = \max_{n',k,d} d + \beta \int_Z \mathbb{V}(n',a',z')\Gamma(dz',z) \tag{8}$$

$$s.t.$$

$$n' \leq n,$$

$$d + a' = y(n',k,z) + (1+r)a - \omega n' - (r+\delta)k - \chi,$$

$$k \leq \varphi a,$$

$$d \geq 0.$$

Firms maximize shareholder value and, because of risk-neutrality, use $\beta$ as their discount factor. The change in net-worth $a' - a$ is given by revenues from production and interest on savings net of the wage bill, rental and operating costs, and dividend payouts $d$. The last two equations in (8) reiterate that firms face a collateral constraint on the maximum amount of capital they can rent and a non-negativity constraint on dividends.

To help understand the budget constraint and preface how we take the model to the data, define firm debt by the identity $b \equiv k - a$, with the understanding that $b < 0$ denotes savings. Making this substitution reveals an alternative formulation of the model in which the firm owns its capital and faces a constraint on leverage. With state vector $(n, k, b, z)$, the firm faces the following budget and collateral constraints

$$d + \underbrace{[k' - (1-\delta)k]}_{\text{Investment}} = \underbrace{[y(n',k,z) - \omega n' - \chi - rb]}_{\text{Operating Profit}} + \underbrace{[b' - b]}_{\Delta \text{ Borrowing}},$$

$$b/k \leq (\varphi - 1)/\varphi.$$

This makes clear that the firm can fund equity payouts and investment in capital through either operating profits or expanding borrowing/reducing saving.

**The Hiring Firm.** The hiring firm additionally chooses the number of vacancies to post $v \in \mathbb{R}_+$ and recruitment effort $e \in \mathbb{R}_+$, understanding that, by a law of large numbers, its new hires $n' - n$ equal the firm's job-filling rate $qe$ of each of its vacancies times the number of vacancies

$v$ created: $n' - n = q(\theta^*)ev$.[6] Note that the individual job-filling rate depends on the aggregate meeting rate $q$, which is determined in equilibrium and the firm takes as given, as well as its recruiting effort $e$. The firm faces a variable cost function $\mathcal{C}(e, v, n)$, increasing and convex in $e$ and $v$.

A firm's continuation value depends on $n'$, not on the mix of recruiting intensity $e$ and vacant positions $v$ that generates it. As a result, one can split the problem of the hiring firm in two stages. First, the choice of $n'$, $k$ and $d$. Second, given $n'$, the choice of the optimal combination of inputs $(e, v)$. The latter reduces to a static cost-minimization problem:

$$C^* (n, n') = \min_{e,v} \mathcal{C}(e, v, n) \tag{9}$$
$$s.t. \quad e \geq 0, \quad v \geq 0, \quad n' - n = q(\theta^*)ev.$$

yielding the lowest cost combination $e(n, n')$ and $v(n, n')$ that delivers $h = n' - n$ hires to a firm of size $n$, and implied cost function $C^* (n, n')$.

The remaining choices of $n'$, $k$ and $d$ require solving the dynamic problem

$$\mathbb{V}^h(n, a, z) = \max_{n',k,d} d + \beta \int_Z \mathbb{V}(n', a', z')\Gamma(dz', z) \tag{10}$$
$$s.t.$$
$$n' > n,$$
$$d + a' = y(n', k, z) + (1 + r)a - \omega n' - (r + \delta)k - \chi - C^* (n, n'),$$
$$k \leq \varphi a,$$
$$d \geq 0.$$

The solution of this problem includes the decision rule $n'(n, a, z)$. Using this function in the solution of (9), we obtain decision rules $e(n, a, z)$ and $v(n, a, z)$ for recruitment effort and vacancies in terms of firm state variables.

Given the centrality of the hiring cost function $\mathcal{C}(e, v, n)$ to our analysis, we now discuss its

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[6]The linearity of the individual hiring function in vacancies is one of the key empirical findings of DFH.

specification. In what follows, we choose the functional form

$$\mathcal{C}\left(e,v,n\right)=\left[\frac{\kappa_1}{\gamma_1}e^{\gamma_1}+\frac{\kappa_2}{\gamma_2+1}\left(\frac{v}{n}\right)^{\gamma_2}\right]v, \tag{11}$$

with $\gamma_1 \geq 1$ and $\gamma_2 \geq 0$ being necessary conditions for convexity of the maximization problem (9). This cost function implies that the average cost of a vacancy, $\mathcal{C}/v$, has two separate components. The first is increasing and convex in recruiting intensity per vacancy $e$. The idea is that, for any given open position, the firm can choose to spend resources on recruitment activities (recall Figure 2) to make the position more visible or the firm more attractive as a potential employer, or to assess more candidates per unit of time, but all such activities are increasingly costly on a per-vacancy basis. The second component is increasing and convex in the vacancy rate, and captures the fact that expanding productive capacity is costly in *relative* terms: for example, creating 10 new positions involves a more expensive reorganization of production in a firm with 10 employees than in a firm with 1000 employees.

In Appendix A we derive several results for the static hiring problem of the firm (9) under this cost function and derive the exact expression for $\mathcal{C}^* \left(n, n'\right)$ used in the dynamic problem (10). We show that, by combining first-order conditions, we obtain the optimal choice of $e$

$$e\left(n,n'\right) = \left[\frac{\kappa_2}{\kappa_1}\left(\frac{\gamma_1}{\gamma_1-1}\right)\right]^{\frac{1}{\gamma_1+\gamma_2}}q\left(\theta^*\right)^{-\frac{\gamma_2}{\gamma_1+\gamma_2}}\left(\frac{n'-n}{n}\right)^{\frac{\gamma_2}{\gamma_1+\gamma_2}}, \tag{12}$$

and, hence, the firm-level job filling rate $f\left(n,n'\right) \equiv q\left(\theta^*\right)e\left(n,n'\right)$, as well as the optimal vacancy-rate:

$$\frac{v}{n}=\left[\frac{\kappa_2}{\kappa_1}\left(\frac{\gamma_1}{\gamma_1-1}\right)\right]^{\frac{1}{\gamma_1+\gamma_2}}q\left(\theta^*\right)^{-\frac{\gamma_1}{\gamma_1+\gamma_2}}\left(\frac{n'-n}{n}\right)^{\frac{\gamma_1}{\gamma_1+\gamma_2}}. \tag{13}$$

Equation (12) demonstrates that the model implies a log-linear relation between the job filling rate and employment growth at the firm level, with elasticity $\gamma_2/(\gamma_1 + \gamma_2)$. This is the key empirical finding of DFH, who estimate this elasticity to be 0.82. In fact, one could interpret our functional choice for $\mathcal{C}$ in equation (11) as a 'reverse-engineering' strategy in order to obtain, from first principles, the empirical cross-sectional relation between firm-level job-filling rate and firm-level hiring rate uncovered by DFH. Put differently, micro data sharply discipline the

Figure 4: Cross-sectional relationships between monthly employment growth $(n'-n)/n$ and the vacancy rate $v/n$ and the job filling rate $eq$. Data from DFH online supplemental materials.
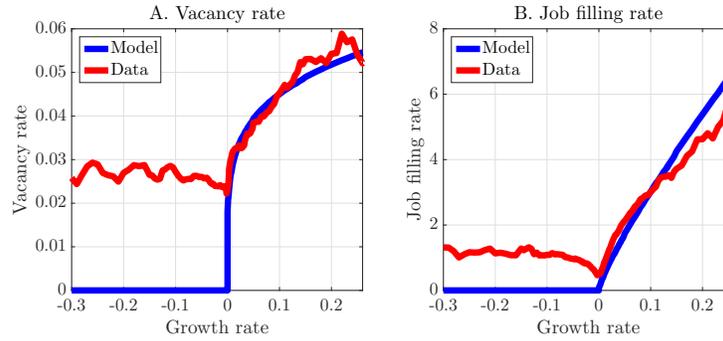


recruiting cost function of the model.[7]

Why does firm optimality imply that the job filling rate increases with the growth rate with elasticity $\gamma_2/(\gamma_1 + \gamma_2)$? Recruiting intensity $e$ and the vacancy rate $(v/n)$ are substitutes in the production of a target employment growth rate $(n'-n)/n$—see the last equation in (9). Thus, a firm that wants to grow faster than another will optimally create more positions and, at the same time, spend more in recruiting effort. However, the stronger the convexity of $\mathcal{C}$ in the vacancy rate $(\gamma_2)$, relative to its degree of convexity in effort $(\gamma_1)$, the more an expanding firm finds it optimal to substitute away from vacancies into recruiting intensity to realize its target growth rate. In the special case when $\gamma_2 = 0$, all the adjustment occurs through vacancies and recruiting effort is irresponsive to the growth rate and to macroeconomic conditions, as in the canonical model of Pissarides (2000).

Figure 4 plots the cross-sectional relationship between the vacancy rate and employment growth (panel A) and the job filling rate and employment growth (panel B) in the model and in the DFH data, with the elasticity of the job filling rate to firm's growth $\gamma_2/(\gamma_1 + \gamma_2) = 0.82$.[8] Since the individual hiring function is linear in vacancies, the elasticity of the vacancy rate to firm's growth equals $\gamma_1/(\gamma_1 + \gamma_2) = 0.18$.

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[7]Appendix A also shows that, once the optimal choice of $e$ is substituted into (11), $\mathcal{C}$ can be stated solely in terms of the vacancy rate and becomes equivalent to one of the hiring cost functions that Kaas and Kircher (2015) use in their empirical analysis.

[8]In Figure 4, the model implies zero hires for firms with negative growth rates, whereas in the data time aggregation and replacement hires leads to positive vacancy rates and vacancy yields also for shrinking firms.

## 3.3 Household Problem

The representative household solves

$$\mathbb{W}(T, M) \quad = \quad \max_{T', M', C > 0} \quad C + \beta \mathbb{W}(T', M') \tag{14}$$

$$s.t.$$

$$C + \bar{Q}T' + PM' \quad = \quad \omega \bar{L} + (D + P)M + T,$$

where $C$ denotes household consumption; $T$ are bank deposits; $M$ are shares of the mutual fund composed of all firms in the economy, with the aggregate number of shares normalized to one; $\bar{L}$ denotes the number of household members. The share price is $P$ and owning shares entitles the household to dividends $D$, the sum of all firm dividends.[9] Since the return from working in the market and working at home are the same, total income is simply $\omega \bar{L}$ (which is also the reason why unemployment $U$ is not a state variable in the household's problem).

The total wage bill is the integral over all wage payments from firms, while workers that are idle this period and begin next period as unemployed job seekers produce $\omega$ units of the final good via home production.

From the first-order conditions for deposits and share holdings, we obtain $\bar{Q} = \beta$ and $P = \beta (P + D)$ which imply a constant return of $r = \beta^{-1} - 1$ on both deposits and shares and, thus, the household is indifferent over portfolios. Since the household is risk neutral, it is also indifferent over the timing of consumption.

## 3.4 Stationary Equilibrium and Aggregation

Let $\Sigma_N$, $\Sigma_A$, and $\Sigma_Z$ be the Borel sigma algebras over $N$ and $A$, and $Z$. The state space for an incumbent firm is $S = N \times A \times Z$, and we denote with $s$ one of its points $(n, a, z)$. Let $\Sigma_S$ be the sigma algebra on the state space, with typical set $\mathcal{S} = \mathcal{N} \times \mathcal{A} \times \mathcal{Z}$, and $(S, \Sigma_S)$ be the corresponding measurable space. Denote with $\lambda : \Sigma_S \to [0, \infty)$ the stationary measure of incumbent firms at the beginning of the period, following the draw of firm level productivity, before the exogenous exit shock.

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[9]The initial equity injections into successful start-ups are treated as negative dividends.

To simplify the exposition of the equilibrium, it is convenient to use $s \equiv (n, a, z)$ and $s_0 \equiv (n_0, a_0 - \chi_0, z)$ as the argument for incumbents' and entrants' decision rules.

A stationary recursive competitive equilibrium is a collection of firms' decision rules $\{i(z), x(s), n'(s), e(s), v(s), a'(s), d(s), k(s)\}$, value functions $\{\mathbb{V}, \mathbb{V}^i, \mathbb{V}^f, \mathbb{V}^h\}$, a measure of entrants $\lambda_e$, share price $P$ and aggregate dividends $D$, wage $\omega$, a distribution of firms $\lambda$, and a value for effective labor-market tightness $\theta^*$ such that: (i) the decision rules solve the firms problems (4)-(10), $\{\mathbb{V}, \mathbb{V}^i, \mathbb{V}^f, \mathbb{V}^h\}$ are the associated value functions, and $\lambda_e$ is the mass of entrants implied by (5); (ii) the market for shares clears at $M = 1$ with share price

$$P = \int_S \mathbb{V}(s) d\lambda + \lambda_0 \int_Z i(z) \mathbb{V}^i(s_0) d\Gamma_0$$

and aggregate dividends

$$D = \zeta \int_S a d\lambda + (1 - \zeta) \int_S \{[1 - x(s)] d(s) + x(s) a\} d\lambda - \lambda_0 \int_Z i(z) a_0 d\Gamma_0;$$

(iii) the stationary distribution $\lambda$ is the fixed point of the recursion:

$$\lambda(\mathcal{N} \times \mathcal{A} \times \mathcal{Z}) = (1 - \zeta) \int_S [1 - x(s)] \mathbf{1}_{\{n'(s) \in \mathcal{N}\}} \mathbf{1}_{\{a'(s) \in \mathcal{A}\}} \Gamma(\mathcal{Z}, z) d\lambda \tag{15}$$
$$+ \lambda_0 \int_Z i(z) \mathbf{1}_{\{n'(s_0) \in \mathcal{N}\}} \mathbf{1}_{\{a'(s_0) \in \mathcal{A}\}} \Gamma(\mathcal{Z}, z) d\Gamma_0,$$

where the first term refers to existing incumbents and the second to new entrants; (iv) effective market tightness $\theta^*$ is determined by the balanced flow condition

$$\bar{L} - N(\theta^*) = \frac{F(\theta^*) - \lambda_e(\theta^*) n_0}{p(\theta^*)}, \tag{16}$$

where $p(\theta^*)$ is the aggregate job finding rate, $N(\theta^*)$ is aggregate employment

$$N(\theta^*) = (1 - \zeta) \int_S [1 - x(s)] n'(s) d\lambda + \lambda_0 \int_Z i(z) n'(s_0) d\Gamma_0, \tag{17}$$

and $F(\theta^*)$ are aggregate separations

$$F(\theta^*) = \zeta \int_S n d\lambda + (1 - \zeta) \int_S x(s) n d\lambda + (1 - \zeta) \int_S [1 - x(s)] (n - n'(s))^- d\lambda, \tag{18}$$

which include all employment losses from firms exiting exogenously and endogenously, plus all the workers fired by shrinking firms, which we have denoted by $(n - n'(s))^-$.[10] In equations (16)-(18), the dependence of $\lambda_e$, $N$ and $F$ on $\theta^*$ comes through the decision rules and the stationary distribution, even though, for notational ease, we have omitted $\theta^*$ as their explicit argument.

The left-hand side of (16) is the definition of unemployment—labor force minus employment—whereas the right-hand side is the steady-state Beveridge curve, i.e., the law of motion for unemployment

$$U' = U - p(\theta^*)U + F(\theta^*) - \lambda_e(\theta^*)n_0 \tag{19}$$

evaluated in steady state. As in Elsby and Michaels (2013), the two sides of (16) are independent equations determining the same variable—unemployment—and, combined, they yield equilibrium market tightness $\theta^*$.[11] Note that equations (16) and (19) account for the fact that every new firm enters with $n_0$ workers hired 'outside' the frictional labor market (e.g., the firm founders).

Clearly, once $\theta^*$ and $\lambda$ are determined, so is $U$ from either side of (16) and, therefore, $V^*$. Finally, we note that measured aggregate matching efficiency, in equilibrium, is $\Phi = (V^*/V)^\alpha$, where measured and effective vacancies are respectively

$$
\begin{aligned}
V &= (1 - \zeta) \int_S [1 - x(s)] \, v(s) \, d\lambda \quad\quad + \lambda_0 \int_Z i(z) \, v(s_0) \, d\Gamma_0, \\
V^* &= (1 - \zeta) \int_S [1 - x(s)] \, e(s) \, v(s) \, d\lambda + \lambda_0 \int_Z i(z) \, e(s_0) \, v(s_0) \, d\Gamma_0.
\end{aligned}
$$

Appendix C provides details on the computation of the decision rules and the stationary equilibrium.

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[10]Entrant firms never fire, as they enter with the lowest value on the support for $N$, $n_0$ normalized to 1.

[11]Our computation showed that, typically, $N(\theta^*)$ is decreasing in its argument and the right-hand side of (16) is always positive and decreasing. Thus, the crossing point of left- and right-hand side is unique, when it exists. However, an equilibrium may not exist. For example, for very low hiring costs, $N(\theta^*)$ may be greater than $\bar{L}$. Conversely, for large enough operating or hiring costs, no firms will enter the economy. In this case, there is no equilibrium with market production (albeit there is always some home-production in the economy).

Table 1: Externally set parameter values

| Parameter | | Value | Target |
|---|---|---|---|
| Discount factor (monthly) | $\beta$ | 0.9967 | Annual risk-free rate = 4% |
| Mass of potential entrants | $\lambda_0$ | 0.02 | Measure of incumbents = 1 |
| Size of labor force | $\bar{L}$ | 24.6 | Average firm size = 23 |
| Elasticity of matching function wrt $V_t$ | $\alpha$ | 0.5 | JOLTS |

# 4 Parameterization

## 4.1 Externally Calibrated

We begin from the subset of parameters that are calibrated externally. The model period is one month. We set $\beta$ to replicate an annualized risk-free rate of 4 percent. Since the measure of potential entrants $\lambda_0$ scales $\lambda$—see equation (15)—we choose $\lambda_0$ to normalize the total measure of incumbent firms to one. We normalize the size of the labor force $\bar{L}$ so that, given a measure one of firms, under our target unemployment rate of 7 percent, the average firm size will be 23 consistent with Business Dynamics Statistics (BDS) data over the period 2001-2007.[12] In line with empirical studies, we set $\alpha$, the elasticity of aggregate hires to aggregate vacancies in the matching function, to 0.5. Table 1 summarizes these parameter values.

## 4.2 Internally Calibrated

Table 2 lists the remaining 19 parameters of the model that are set by minimizing the distance between an equal number of empirical moments and their equilibrium counterparts in the model.[13] Table 2 lists the targeted moments, their empirical values, and their simulated values from the model. Even though every targeted moment is determined simultaneously by all parameters, in what follows we discuss each of them in relation to the parameter for which,

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[12]The unemployment rate is $u = \bar{L}/N(\theta^*) - 1$, and with a unit mass of firms the average firm size is simply $N(\theta^*)$. Hence given $u = 0.07$, $\bar{L}$ determines average firm size.

[13]Specifically, the vector of parameters $\Psi$ is chosen to minimize the minimum-distance-estimator criterion function

$$f(\Psi) = (\mathbf{m}_{data} - \mathbf{m}_{model}(\Psi))' \mathbf{W} (\mathbf{m}_{data} - \mathbf{m}_{model}(\Psi))$$

where $\mathbf{m}_{data}$ and $\mathbf{m}_{model}(\Psi)$ are the vectors of moments in the data and model, and $\mathbf{W} = \text{diag}\left(1/\mathbf{m}_{data}^2\right)$ is a diagonal weighting matrix.

Table 2: Parameter values estimated internally

| Parameter | | Value | Target | Model | Data |
|---|---|---|---|---|---|
| Flow of home production | $\omega$ | 1.000 | Monthly separ. rate | 0.033 | 0.030 |
| Scaling of match. funct. | $\bar{\Phi}$ | 0.208 | Monthly job finding rate | 0.411 | 0.400 |
| Prod. weight on labor | $\nu$ | 0.804 | Labor share | 0.627 | 0.640 |
| Midpoint DRS in prod. | $\sigma_M$ | 0.800 | Employment share $n$: 0-49 | 0.294 | 0.306 |
| High-Low spread in DRS | $\Delta\sigma$ | 0.094 | Employment share $n$: 500+ | 0.430 | 0.470 |
| Mass - Low DRS | $\mu_L$ | 0.826 | Firm share $n$: 0-49 | 0.955 | 0.956 |
| Mass - High DRS | $\mu_H$ | 0.032 | Firm share $n$: 500+ | 0.004 | 0.004 |
| Std. dev of $z$ shocks | $\vartheta_z$ | 0.052 | Std. dev ann emp growth | 0.440 | 0.420 |
| Persistence of $z$ shocks | $\rho_z$ | 0.992 | Mean Q4 emp / Mean Q1 emp | 75.161 | 76.000 |
| Mean $z_0 \sim Exp(\bar{z}_0^{-1})$ | $\bar{z}_0$ | 0.390 | $\Delta \log z$: Young vs. Mature | -0.246 | -0.353 |
| Cost elasticity wrt $e$ | $\gamma_1$ | 1.114 | Elasticity of vac yield wrt $g$ | 0.814 | 0.820 |
| Cost elasticity wrt $v$ | $\gamma_2$ | 4.599 | Ratio vac yield: <50/>50 | 1.136 | 1.440 |
| Cost shifter wrt $e$ | $\kappa_1$ | 0.101 | Hiring cost (100+) / wage | 0.935 | 0.927 |
| Cost shifter wrt $v$ | $\kappa_2$ | 5.000 | Vacancy share $n < 50$ | 0.350 | 0.370 |
| Exogenous exit probability | $\zeta$ | 0.006 | Survive $\geq 5$ years | 0.497 | 0.500 |
| Entry cost | $\chi_0$ | 9.354 | Annual entry rate | 0.099 | 0.110 |
| Operating cost | $\chi$ | 0.035 | Fraction of JD by exit | 0.210 | 0.340 |
| Initial wealth | $a_0$ | 10.000 | Start-up Debt to Output | 1.361 | 1.280 |
| Collateral constraint | $\varphi$ | 10.210 | Aggregate debt-to-Net worth | 0.280 | 0.350 |

intuitively, that moment yields the most identification power.

We set the flow of home production of the unemployed $\omega$ to replicate a monthly separation rate of 0.03. We choose the shift parameter of the matching function (a normalization of the value of $\Phi$ in steady state) in order to pin down a monthly job finding rate of 0.40. Together, these two moments yield a steady-state unemployment rate of 0.07.

We assume the revenue function $y(z, n', k) = z\left[(n')^{\nu}k^{1-\nu}\right]^{\sigma}$ and introduce a small degree of permanent heterogeneity in the scale parameter $\sigma$.[14] Specifically we consider a three-point distribution with support $\{\sigma_L, \sigma_M, \sigma_H\}$—symmetric about $\sigma_M$—leaving four parameters to choose:

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[14]Since we specify the *revenue* function, we do not take a stand on whether $z$ represents demand or productivity shocks, or whether $\sigma$ represents DRS in production or the interaction of a production function with a downward sloping demand curve. Given this understanding we discuss the revenue function as if it were a production function: $\sigma$ represents span of control and $z$ is total factor productivity. Sedlacek and Sterk (2014) solve a firm dynamics model where scale heterogeneity arises because different producers face demand curves with different elasticities.

(i) the value of $\sigma_M$; (ii) the spread $\Delta \sigma \equiv (\sigma_H - \sigma_L)$; and (iii)-(iv) the fractions of low and high DRS firms $\mu_L, \mu_H$. In the same spirit as the use of permanent heterogeneity in productivity in the quantitative applications of Elsby and Michaels (2013) and Kaas and Kircher (2015), heterogeneity in the scale of production allows us to match the firm size distribution and to generate, within the model, small old firms alongside young large firms, thus decoupling age and size which tend to be too strongly correlated in standard firm dynamics models with mean reverting stochastic productivity. In addition, the assumption of heterogeneity in $\sigma$ captures the appealing idea that there exist some very productive, but small, businesses simply because the optimal scale of production for many goods or services is small. The values of these four parameters allow the model to match the BDS statistics on employment and establishment shares of firms of size 0-49 and 500+.[15]

Firm productivity $z$ follows an AR(1) process in logs: $\log z' = \log Z + \rho_z \log z + \varepsilon$, with $\varepsilon \sim \mathcal{N}(-\vartheta_z^2/2, \vartheta_z)$. We calibrate $\rho_z$ and $\vartheta_z$ to match two measures of employment dispersion, one in growth and one in levels: the standard deviation of annual employment growth for continuing establishments in the Longitudinal Business Database (Elsby and Michaels, 2013), and the ratio of the mean size of fourth to first quartile of the firm distribution (Haltiwanger, 2011a).[16]

The initial productivity distribution for entrants $\Gamma_0$ is Exponential, with mean $\bar{z}_0$ chosen to match the productivity gap between entrants and incumbents, specifically the differential in revenue productivity between firms older than 10 and younger than 1 year old (Foster, Haltiwanger, and Syverson, 2016).

We now turn to hiring costs. The cost function (11) has four parameters: the two elasticities $(\gamma_1, \gamma_2)$ and the two cost shifters $(\kappa_1, \kappa_2)$. Recall, from the discussion surrounding equations (11) and (12), that the cross-sectional elasticity of job filling rates to employment growth rates, estimated to be 0.82 by DFH, is a function of the ratio of these two elasticities.[17] The second moment used to separately identify the two elasticities is the ratio of vacancy yields of small ($< 50$ employees) to large ($> 50$ employees) firms from JOLTS data on hires and vacancies by

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[15]In terms of the description of the model and stationary equilibrium, one should add $\sigma$ to the firm's state vector $s$, but nothing substantial in the firm problem and the definition of equilibrium would change.

[16]In the numerical solution and simulation of the model, $z$ remains a continuous state variable.

[17]We cannot map $\gamma_2/(\gamma_1 + \gamma_2)$ directly into this value since in DFH, and in the model's simulations for consistency, the growth rate is the Davis-Haltiwanger growth rate normalized in $[-2, 2]$. In practice, as seen in Table 2, the discrepancy between structural and estimated parameter is very small.

firm size. Intuitively, when $\gamma_2 = 0$, recruiting effort is constant across firms and this ratio is one.

We use two targets to pin down the cost shift parameters. The first is the total hiring cost as a fraction of monthly wage per hire, a standard target for the single vacancy cost parameter that usually appears in vacancy posting models. We have a new source for this statistic. The consulting company Bersin and Associates runs a periodic survey of recruitment cost and practices based on over 400 firms—all with more than 100 employees. Once the firms are re-weighted by industry and size, the sample is representative of this size segment of the US economy. They compute that, on average, annual spending on all recruiting activities (including internal staff compensation, university recruiting, agencies/third-party recruiters, professional networking sites, job boards, social media, contractors, employment branding services, employee referral bonuses, pay-per-click media, travel to interview candidates, applicant tracking systems, print/media/billboards, other tools/technologies) divided by the number of hires in 2011 was $3,479 (see Table 3 in O'Leonard 2011). Given average annual earnings of roughly $45,000 in 2011, in the model we target a ratio of average recruiting cost to average monthly wage (in firms with more than 100 employees) of 0.928. The second target is the vacancy share of small ($n < 50$) firms from JOLTS: $\kappa_2$ determines the size of hiring costs for small (low $n$) firms and, thus, the amount of vacancies they create.

The parameters $\chi$ and $\zeta$ have large effects on firm exit. The operation cost $\chi$ mostly impacts exit rates of young firms; therefore, we target the five-year survival rate found in BDS data, which is approximately 50 percent. The parameter $\zeta$ contributes to the exit of large and old firms; hence we target the fraction of total job destruction due to exit. To pin down the set-up cost $\chi_0$, we target the annual entry-rate of 11 percent from the BDS.[18]

The remaining two parameters are the size of the initial equity injection $a_0$ and the collateral parameter $\varphi$. To inform their calibration, we target the debt-output ratio of start-up firms computed from the Kauffman Survey (Robb and Robinson, 2014), and the aggregate debt to total assets ratio from the Flow of Funds.[19]

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[18]When computing moments designed to be comparable to their counterparts in the BDS, we carefully time-aggregate the model to an annual frequency. For example, the entry-rate in the BDS is measured as the number of age zero firms in a given year divided by the total number of firms. Computing this statistic in the model requires aggregating monthly entry and exit over 12 months. See Appendix C for details.

[19]Robb and Robinson (2014) report $68,000 of average debt (credit cards, personal and business bank loans, and credit lines) and $53,000 of average revenue for the 2004 cohort of start-ups in their first year, see their Table 5. From

Table 3: Non-targeted moments

| Moment | Model | Data | Source |
|---|---|---|---|
| Aggregate dividend / profits | 0.411 | 0.400 | NIPA |
| [1]Employment share: $growth \in [-2.00, -0.20)$ | 0.070 | 0.076 | Davis et al. (2010) |
| Employment share: $growth \in (-0.20, -0.20]$ | 0.828 | 0.848 | Davis et al. (2010) |
| Employment share: $growth \in (0.20, 2.00]$ | 0.102 | 0.076 | Davis et al. (2010) |
| Employment share: $Age \leq 1$ | 0.013 | 0.020 | BDS |
| Employment share: $Age \in (1, 10)$ | 0.309 | 0.230 | BDS |
| Employment share: $Age \geq 10$ | 0.678 | 0.750 | BDS |

(1.) *Firm growth rates are annual and are interior to $[-2, 2]$ so do not include entering and exiting firms*

## 4.3   Cross-Sectional Implications

We now explore the main cross-sectional implications of the calibrated model, at its steady-state equilibrium.

Table 3 reports some empirical moments that we did not target in the calibration and their model-generated counterparts. The fact that the ratio of dividend payments to profits in the model is close to its empirical value reinforces the view that our collateral constraint is neither too tight nor too loose. The model can also replicate well the distribution of employment by growth rate and by firm age, neither of which was explicitly targeted.

Figure 5 shows that the model is also able to replicate satisfactorily the observed distribution of hires and vacancies by size class from the JOLTS data.

In Figure 6 we plot the average firm size, job creation and destruction rates, fraction of constrained firms and leverage (debt/saving over net worth, $b/a$) for firms from birth through to maturity. Panel A shows that $\sigma_H$-firms, those with closer to constant returns in production, account for the upper tail in the size and growth-rate distributions. On average, though, firm size grows by much less over the life cycle, since these 'gazelles'—as they are often referred to in the literature—are only a small fraction of the total. On average, the model and the data line up well: average size grows by a factor of 3 between ages 1-5 and 20-25 in the model and 3.4 in

the Flow of Funds 2005, we computed total debt as the sum of securities and loans and total assets as the sum of all nonfinancial assets plus financial assets net of trade receivables, FDIs and miscellaneous liabilities (Tables L.103 and L.104, Liabilities of Nonfinancial Corporate and Noncorporate Business), and divided by the sum of corporate and noncorporate net worth (Tables B.103 and B.104, Balance Sheet of Nonfinancial Corporate and Noncorporate Business).

Figure 5: Hire and vacancy shares by size class. Model in blue, JOLTS data 2002-2007 in red.
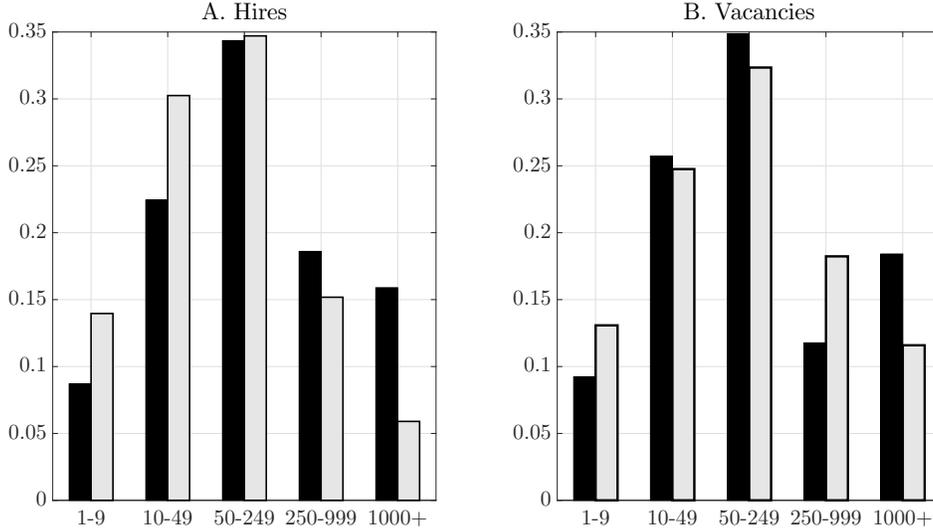
the BDS data. Convex recruiting costs and collateral constraints slow down growth: most firms reach their optimal size around age 10, and $\sigma_H$-firms keep growing for much longer.

Panel B plots job creation and destruction rates by age. It is a stark representation of the 'up-or-out' dynamics of young firms documented in the literature (Haltiwanger, 2011b). Panel C depicts the fraction of constrained firms (defined as those with $k = \varphi a$ and $d = 0$) over the life cycle. In the model, financial constraints bind only for the first few years of a firm's life, when net worth is insufficient to fund the optimal level of capital. Panel D illustrates that leverage declines with age and after age 10 the median firm is saving (i.e., $b < 0$). Much like in the classical household 'income fluctuation problem,' in our model firms have a precautionary saving motive due to the simultaneous presence of three elements: (i) a concave payoff function because of DRS; (ii) stochastic productivity; and (iii) the collateral constraint.

Panel A of Figure 7 shows that recruiting intensity and the vacancy rate are sharply decreasing with age. These features arise because our cost function implies that both optimal hiring effort and optimal vacancy rates are increasing in the growth rate, and young firms are those with the highest desired growth rates. Moreover, the stronger convexity of $\mathcal{C}$ in the vacancy rate ($\gamma_2$), relative to its degree of convexity in effort ($\gamma_1$) implies that a rapidly expanding firm prefers to substitute away from vacancies into recruiting intensity to realize its target growth rate. Thus, young firms find it optimal to limit the number of new positions, but recruit very aggressively for the ones that they open. As firms age, growth rates fall and this force weakens.

24

Figure 6: Average life cycle of firms in the model
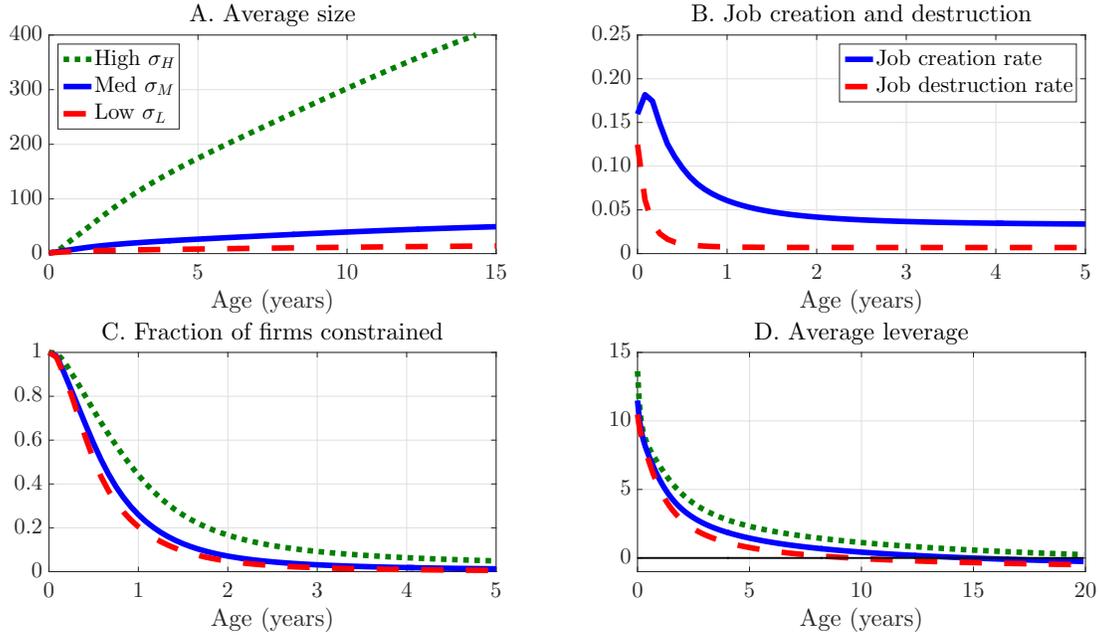


Panel B plots the fraction of total recruiting effort, vacancies and hiring firms by age. It shows that, relative to the steady-state age distribution of hiring firms, the effort distribution is skewed towards young firms, whereas the vacancy distribution is skewed towards older firms. In the model the age-distribution of vacancies is almost uniform: young firms grow faster than old ones and, thus, post more vacancies per worker; however, they are smaller and, thus, they post fewer vacancies for a given growth rate. These two forces counteract each other and the ensuing vacancy distribution over ages is nearly flat. Figure 7 highlights that the JOLTS notion of vacancy as 'open position ready to be filled' is a good metric of hiring effort for old firms, for whom recruiting intensity is nearly constant, whereas it is quite imperfect for young firms aged 0-5, whose average recruiting intensity, as well as its variance, are much higher than those of mature firms.[20]

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[20]Unfortunately, the JOLTS does not report the age of the firm, so there are no U.S. data on vacancies and recruiting intensity by firm age we can directly compare to our model. Kettemann, Mueller, and Zweimuller (2016) find that, in Austrian data, after controlling for firm fixed effects job filling rates are decreasing with firm age.

Figure 7: Vacancy and Effort Distributions by Age
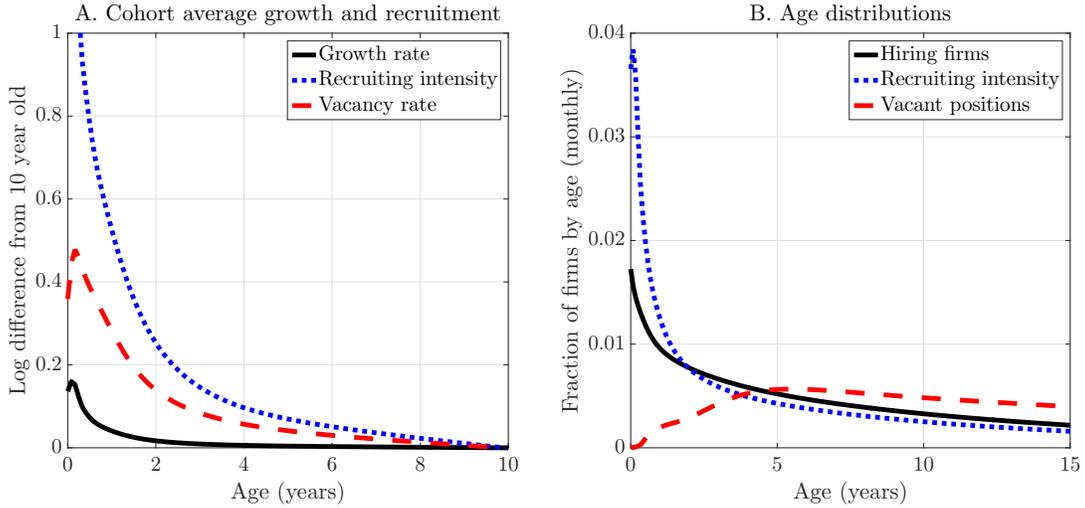


# 5  Aggregate Recruiting Intensity and Macroeconomic Shocks

Our main experiments examine the equilibrium of the economy along perfect foresight paths for shocks to aggregate productivity $Z$ and to the financial constraint parameter $\varphi$. Appendix C provides details on the solution of the model along these transitional dynamics away from, and back to, the steady state.
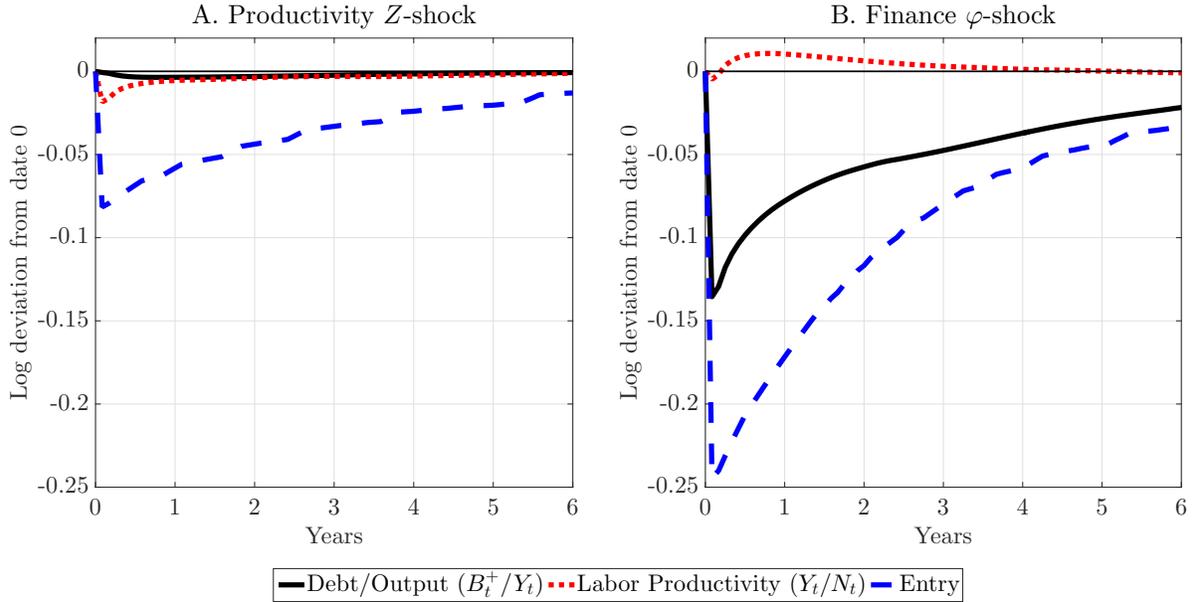
We frame these experiments in the context of the Great Recession. Specifically, we consider mean-reverting AR(1) shocks, choosing their size so that the model matches the maximum deviation of detrended output over 2008-2012 from its value in 2007, a value of -10 percent (Fernald, 2015). Their persistence is set so that the half-life of output dynamics is three years under both shocks. This strategy results in a 4-percent shock to $Z$, and a 75-percent shock to $\varphi$.[21]

Figure 8 plots the dynamics of some key aggregate variables. The financial shock displays three features that are absent from the macroeconomic transition under the productivity shock, but present in the data. First, a sizable drop in the debt-output ratio of magnitude and persistence comparable to the data.[22] Second, an endogenous *rise* in aggregate labor productivity of 1.5 percent, close to the 2 percent rise over 2008-10 measured by McGrattan and Prescott (2012). Labor productivity rises because more severe financial frictions prevent the expansion of firms,

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[21]The implied (monthly) persistence parameters are 0.990 for $Z$ and 0.976 for $\varphi$. Figure B1 in Appendix B displays the —almost identical by construction— paths for output in the two experiments.

[22]In the US since 2008, the debt-output ratio drops by nearly 10 percent points and five years later is still 4 percent below its pre-recession level.

Figure 8: Dynamics of some macroeconomic variables



A. Productivity $Z$-shock      B. Finance $\varphi$-shock

Legend: Debt/Output $(B_t^+/Y_t)$ · · · Labor Productivity $(Y_t/N_t)$ — Entry

especially the high-$\sigma$ ones with large scale of production, as we will show more in detail below. As firm size falls, because of DRS, average labor productivity increases. Third, a 24 percent decline in entry which, again, matches well its empirical counterpart of 22 percent.[23] Specifically, young-firm values decline sharply, since a large fraction of them are constrained (recall Figure 6), leading to a decline in start-ups. Overall, we conclude that the differential responses of these three variables clearly identify a financial shock in the 2008 recession.

Figure 9 displays the dynamics of the key labor market variables under the two shocks. Overall, in both experiments the labor-market response to the shock is close to its empirical counterpart of Figure 1.[24] The financial shock induces bigger and more persistent movements in vacancies, unemployment, and the job finding rate. Under both scenarios, the drop in aggregate recruiting intensity is sizable, but its magnitude and persistence are, again, larger under the financial shock: $\Phi_t$ falls by 25 percent at impact (20 percent under the productivity shock) and five years later it is still 10 percent below its initial value (5 percent under the productivity shock).[25] We conclude that, in the model, the financial shock—the more promising candidate

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[23]Entry in the data is measured as the number of firms reporting an age of zero divided by the total number of firms in the LBD. The survey is in March and so this measure excludes firms which enter and exit between surveys.

[24]In the data, labor market variables move more slowly, but recall that the shocks we fed are AR(1) designed to match the peak-trough drop in output but not its slow recovery.

[25]We note that the persistence of $\Phi_t$ is higher under the financial tightening in spite of the fact that the financial shock itself is less persistent than the productivity shock.

Figure 9: Dynamics of labor market variables



A. Productivity $Z$-shock       B. Finance $\varphi$-shock

Vacancies   Vacancy yield   Unemployment   Job finding rate   Agg. recruiting intensity
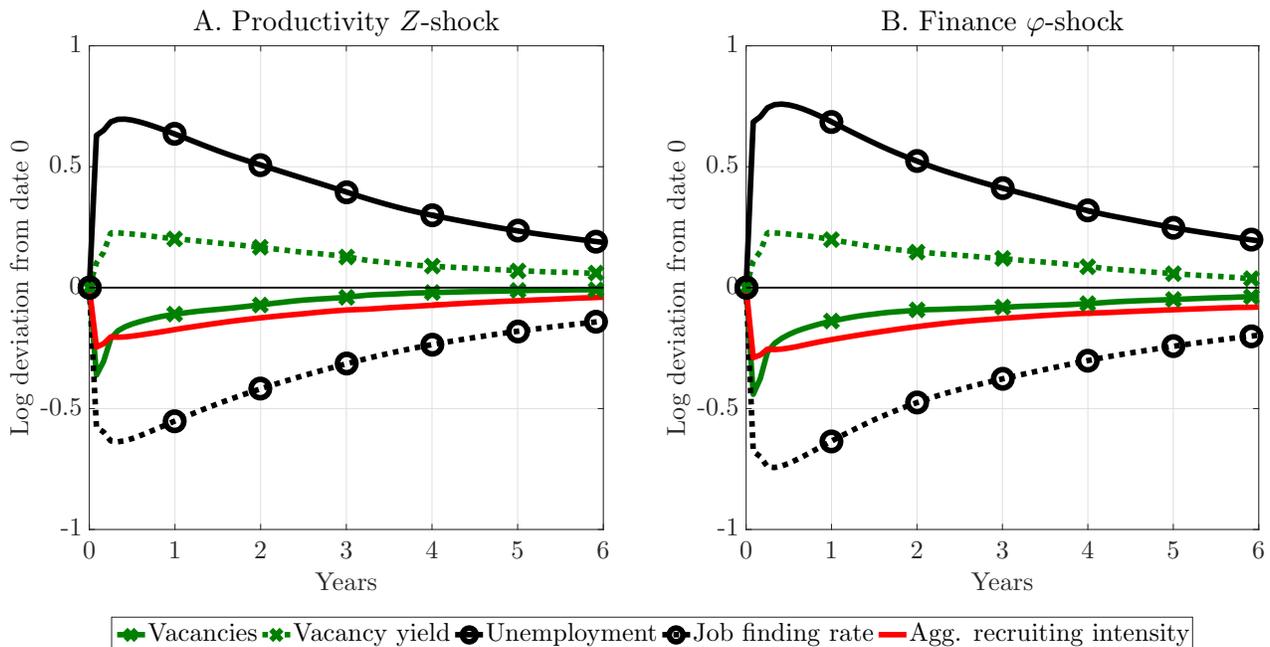
to rationalize the Great Recession based on our discussion of Figure 8—can explain around half of the observed decline in aggregate match efficiency (recall the empirical path in Figure 1).

At first sight, it may be surprising that the response of aggregate recruiting intensity is not too dissimilar across the two macro shocks although the entry rate of new firms—which accounts for a disproportionate share of job creation—remarkably differs under the two experiments. In what follows, we explain this apparent puzzle.

## 5.1 The Transmission Mechanism

To understand how macro shocks transmit to aggregate recruiting intensity, we return to our expression for $\Phi_t$, using $\lambda^h$ to denote the distribution of hiring firms:

$$\Phi_t = \left(\frac{V_t^*}{V_t}\right)^\alpha = \left[\int e_{it}\left(\frac{v_{it}}{V_t}\right) d\lambda_t^h\right]^\alpha. \tag{20}$$

Substituting the policy function for recruitment effort (12) into the above equation and taking log differences, we obtain:

$$\Delta \log \Phi_t = \underbrace{-\alpha \frac{\gamma_2}{\gamma_1 + \gamma_2} \Delta \log q(\theta_t^*)}_{\text{Slackness effect}} + \underbrace{\alpha \Delta \log \left[ \int g_{it}^{\frac{\gamma_2}{\gamma_1 + \gamma_2}} \left( \frac{v_{it}}{V_t} \right) d\lambda_t^h \right]}_{\text{Composition effect}}. \tag{21}$$

We call the two elements of this equation the *slackness* and *composition* effect, respectively.

**The Slackness Effect.** The slackness effect is the change in aggregate recruiting intensity $\Phi_t$ due to firms changing effort in response to movements in labor market slackness $q(\theta_t^*)$, holding constant growth rates $g_{it}$, vacancies $v_{it}$ and the distribution of hiring firms $\lambda_t^h$.

In a recession, labor market slackness increases, as the reduction in expected profitability reduces firms' vacancy creation and a spike in job separations increases the pool of unemployed workers. This surge in slackness raises the probability $q(\theta_t^*)$ that any vacancy matches with a job seeker. Therefore, given the hiring technology $g_{it} = q(\theta_t^*)e_{it}v_{it}/n_{it}$, a growing firm with a target growth rate $g_{it}$ now reoptimizes its combination of recruiting inputs $e_{it}$ and $v_{it}$ and decreases both: a slack labor market makes it easier for employers to hire, so employers spend less to attract workers. Since recruiting effort is more sensitive than vacancies to $q(\theta_t^*)$—recall the decision rules (12) and (13)—the slackness effect is always stronger for the first margin and, in the aggregate, $V_t^*$ declines more than $V_t$ or, equivalently, $\Phi_t$ falls in recessions.
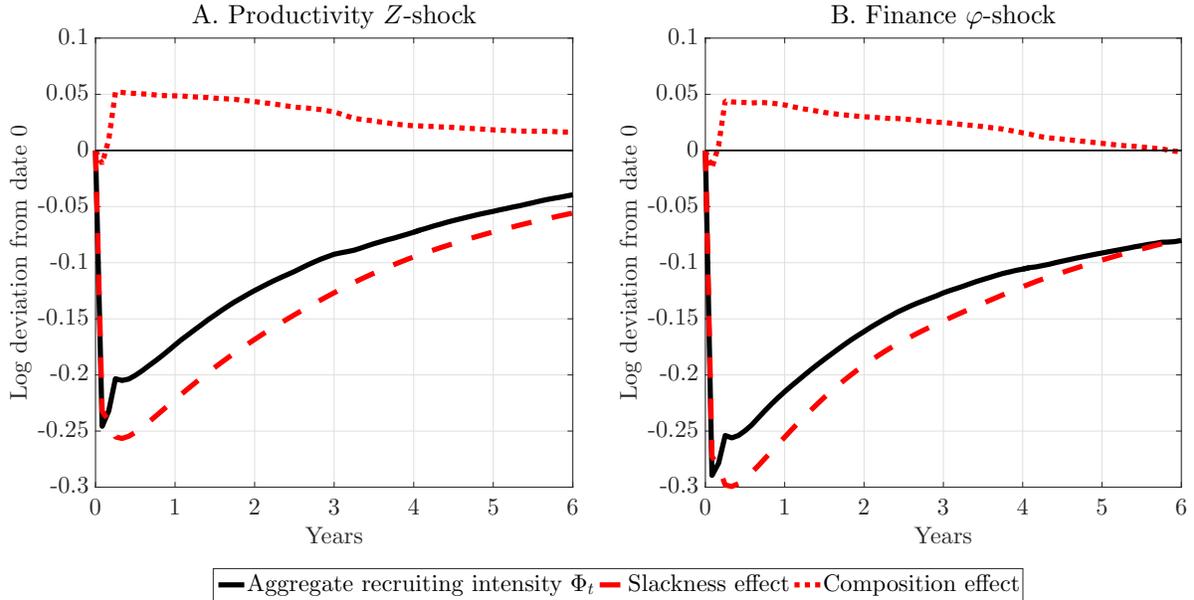
**The Composition Effect.** We define the composition effect residually, thereby including the impact on aggregate recruiting intensity of changes in the distribution of growth rates $g_{it}$ and vacancy policies $v_{it}$ among all hiring firms.

Figure 10 shows how these two components of aggregate recruiting intensity respond to the shocks. These figures reveal that the slackness effect (dashed line) is quantitatively the largest one, accounting for almost all the decline in aggregate recruiting intensity (solid line).

The large magnitude of the slackness effect was, perhaps, expected. Market tightness plunges and the elasticity of firm-level recruiting intensity with respect to $q$ is high, nearly one.[26] What is more surprising is that the composition effect is so small and, in particular, after

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[26]We chose to express the slackness effect as a function of $\theta_t^*$ because this is a sufficient statistic for aggregate labor market conditions in the firm's hiring problem. One can also obtain an expression for the slackness effect that is a function of the more common measure of tightness $\theta$. Substituting the relationship $q(\theta_t^*) =$

Figure 10: Decomposition of Aggregate Recruiting Intensity



a drop at impact it becomes positive, i.e., it induces a small countercyclical movement in $\Phi_t$.

### 5.1.1 Inspecting the Composition Effect

It is useful to split the composition effect into its two main elements, which we plot in Figure 11.[27] The first is a *direct composition effect*: the response to the shock in a partial-equilibrium economy, keeping $\theta_t^*$ at its steady state level, denoted $\bar{\theta}^*$. The second is the *indirect composition effect*: the response in an economy under the equilibrium path for $\theta_t^*$ induced by the shock, while keeping $\varphi_t$ at its steady-state value $\bar{\varphi}$.

The direct effect reduces aggregate recruiting intensity on impact, since the drop in the collateral parameter lowers firm growth rates and reallocates hiring away from young, fast-growing firms that account for the bulk of recruiting intensity in the economy. Note that the direct component reverts rapidly towards zero. The reason is that the decline in $\varphi_t$ induces positive selection among the hiring firms. The fraction of firms hiring drops from 55 percent in

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$q(\theta_t)\Phi_t^{-\frac{1-\alpha}{\alpha}}$ in (21) and collecting the terms in $\Phi_t$ yields the alternative representation of the slackness effect $\frac{-\alpha[\gamma_2/(\gamma_1+\gamma_2)]}{1-(1-\alpha)[\gamma_2/(\gamma_1+\gamma_2)]}\Delta \log q(\theta_t)$. The denominator is less than one and captures a 'multiplier': when $\Phi$ is low in the aggregate, firms exert less effort $e$. This alternative decomposition gives very similar results: if anything, the slackness effect is somewhat stronger.

[27]We illustrate this decomposition only for the tightening of the collateral constraint. Results for the productivity shock are almost identical.

Figure 11: Unpacking the composition effect
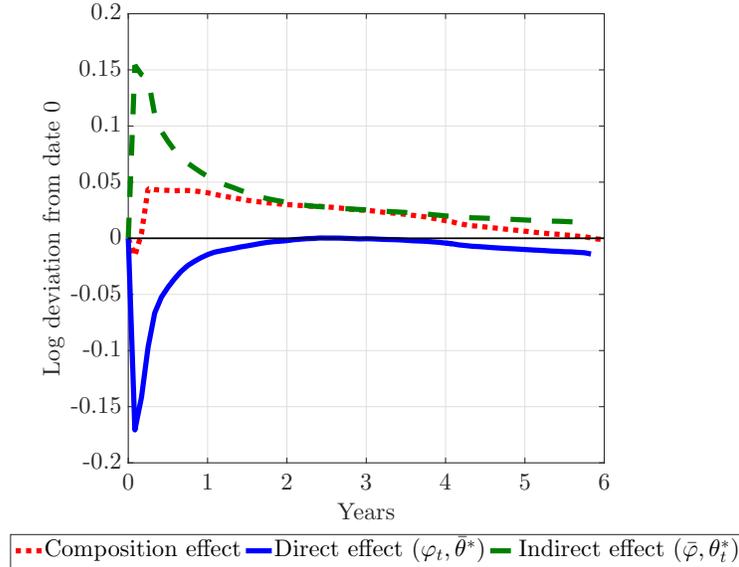
steady state to 22 percent following the shock, so these firms are, on average, better and thus grow slightly more—a force that pushes aggregate recruiting intensity back up.

The indirect effect increases aggregate recruiting intensity on impact, since firms grow faster when $q(\theta_t^*)$ rises, as they meet job seekers more easily. Selection of hiring firms on productivity tempers this effect as well: the increase in $q(\theta^*)$ reduces the average productivity of hiring firms, since some firms that did not hire in steady-state do hire after the shock, thereby generating a force towards lower aggregate recruiting intensity.

Overall, the direct and indirect components show large movements, but these movements offset each other and the composition effect remains small throughout the transition.

Another way to appreciate why the slackness effect is bound to dominate the composition effect is through Figure 12. The left-panel shows the (unweighted) distribution of growth rates in steady-state ($t = 0$) and right after the shock hits ($t = 2$). The distribution shows that firing firms contract faster and that hiring firms expand slightly faster in $t = 2$ relative to $t = 0$ (thus, the dispersion of growth rate increases, as we discuss in some detail below).[28] The right-panel shows how the slackness effect contributes to lower recruiting intensity at any given hiring rate (recall eq. 12). It is apparent from these two panels, that compositional changes in the pool of

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[28]Figure B2 plots the employment-weighted kernel density function of the distribution of firm-level growth rates in the model. This distribution reproduces well its data counterpart, Figure 5 in Davis, Faberman, and Haltiwanger (2012a).

Figure 12: Growth rate distribution and recruitment policies before and after the shock
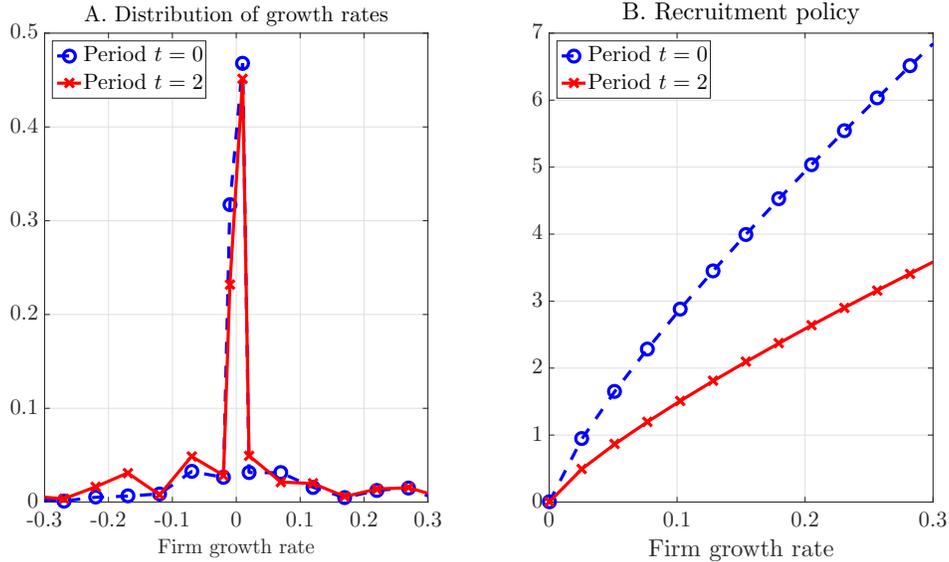
hiring firms will always be dominated by the choice of hiring firms to exert less (more) effort in recruiting when market tightness is lower (higher).

The analysis in this section highlights the role of general equilibrium feedbacks in the dynamics of aggregate recruiting effort of firms. A casual look at the microeconomic relation between the job filling rate and the hiring rate may induce one to conclude that economywide recruiting intensity declines after a negative macro shock because the shock curtails the speed at which hiring firms expand. While such force is present, this logic ignores that the adjustment of equilibrium market tightness following a macro shock sets in motion the slackness effect and the indirect composition effect, two essential—and quantitatively large—pieces of the transmission mechanism.

### 5.1.2 When Can the Composition Effect Be Large?

The magnitude of the composition effect is sensitive to the value of $\alpha$, the elasticity of hires with respect to vacancies. Figure 13 plots the response of aggregate recruiting intensity (panel A) and the composition effect (panel B) for three values of $\alpha$ in the neighborhood of existing estimates. In the range below 0.5, our baseline value, the total composition effect is small at

Figure 13: Size of the composition effect under different values of $\alpha$
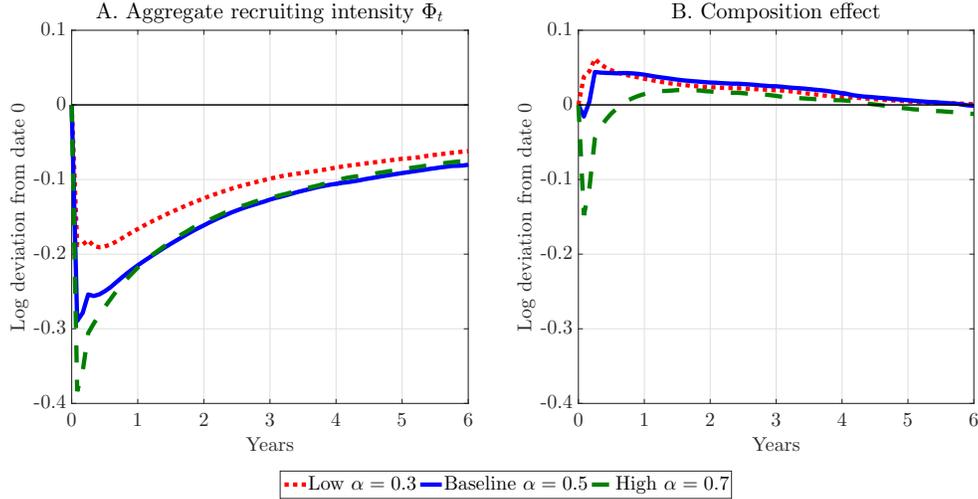
impact and turns positive quickly, as its indirect component takes over. However, for $\alpha = 0.7$, the composition effect becomes sizable at impact and remains negative for almost a year after the shock.

To understand this result, note that the strength of the indirect component of the composition effect (which facilitates firm's growth in a recession as labor market tightness falls) is determined by how much $\log(q_t) = -(1-\alpha)\log\theta_t^*$ rises in a downturn. Clearly, the closer is $\alpha$ to 1, the smaller this effect. Hence, a large value of $\alpha$ mutes the indirect component and induces bigger pro-cyclical movements in the composition effect. A stronger composition effect also explains the deeper drop in $\Phi_t$, as shown in panel A.[29]

We conclude by noting that our theory of recruiting intensity has implications for the estimation of the elasticity parameters in the aggregate matching function. When the equation taken to the data is $\log(H_t/U_t) = \beta_0 + \beta_1 \log\theta_t + \varepsilon_t$, our model implies that the error term—which contains $\Phi_t$—is positively correlated with the regressor, inducing an upward bias in the OLS estimate of $\alpha$.[30] Appendix D develops this point in detail and argues that our choice of $\alpha = 0.5$ for the baseline model is indeed conservative with respect to the model's ability to account for

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[29]Note that the slackness effect is not too sensitive to $\alpha$ because, as seen in equation (21), $\log q$, which contains the term $1 - \alpha$ is also multiplied by $\alpha$.

[30]The endogeneity problem in matching function estimation is well understood, see e.g. Borowczyk-Martins, Jolivet, and Postel-Vinay (2013). Our contribution here is to offer a microfoundation for one potential source of endogeneity.

fluctuations in aggregate matching efficiency.

### 5.1.3  Relationship with Kaas and Kircher (2015)

In Kaas and Kircher's model of competitive search, aggregate recruiting intensity can be expressed as an average of meeting rates in each market, where each meeting rate is a concave function of market tightness. In terms of our notation, $\Phi_t^{KK} = \int q(\theta_{mt})(v_m/V)dm$, where $m$ indexes markets. The authors find that, during productivity-driven recessions, the dispersion of tightness across markets increases, leading to a decline in $\Phi_t^{KK}$. They ascribe the procyclicality of aggregate recruiting intensity chiefly to this mechanism.[31]

A version of this mechanism is present in our model, as well. This source of fluctuations in $\Phi_t$ enters into the composition effect because the second term in (21) is concave in $g_{it}$, since $\gamma_2/(\gamma_1 + \gamma_2) < 1$. A rise in the dispersion of growth rates therefore has a negative effect on $\Phi_t$.

How quantitatively important is this mechanism in our model? To begin with, we note that our steady-state calibration matches the empirical standard deviation of growth rates. Moreover, the financial shock generates a 45 percent increase in the standard deviation of growth rates, which compares well to the 39 percent increase reported by Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) over the same period.[32] To gauge the importance of this mechanism, we assess the strength of Jensen's inequality by measuring the gap:

$$\alpha \Delta \log \left[ \int g_{it}^{\frac{\gamma_2}{\gamma_1+\gamma_2}} \left( \frac{v_{it}}{V_t} \right) d\lambda_t^h \right] - \alpha \Delta \log \left[ \int g_{it} \left( \frac{v_{it}}{V_t} \right) d\lambda_t^h \right]^{\frac{\gamma_2}{\gamma_1+\gamma_2}}.$$

The first term is the composition effect and the second is its Jensen's counterpart where we raise the entire integral (not the integrand) to the exponent. We find that, between $t = 0$ and $t = 2$, this gap equals $-4$ percent: this is the magnitude of the fall in aggregate recruiting intensity that can be ascribed to this mechanism. Its contribution is limited by the fact that because $\gamma_2/(\gamma_1 + \gamma_2)$ is close to 1 empirically, and thus the degree of concavity of the integrand

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[31]This mechanism is explained on pages 3053-3054 of their article.

[32]This is a notable feature of our model in response to a financial shock. Even without a shock to the dispersion of firm level productivity growth we attain a significant increase in the dispersion of employment growth: as explained above, the shock adversely affects some firms reducing growth rates, while some other firms respond to the surge in labor market slackness by growing faster.

Figure 14: Vacancy Yield by Size in the Model and the Data
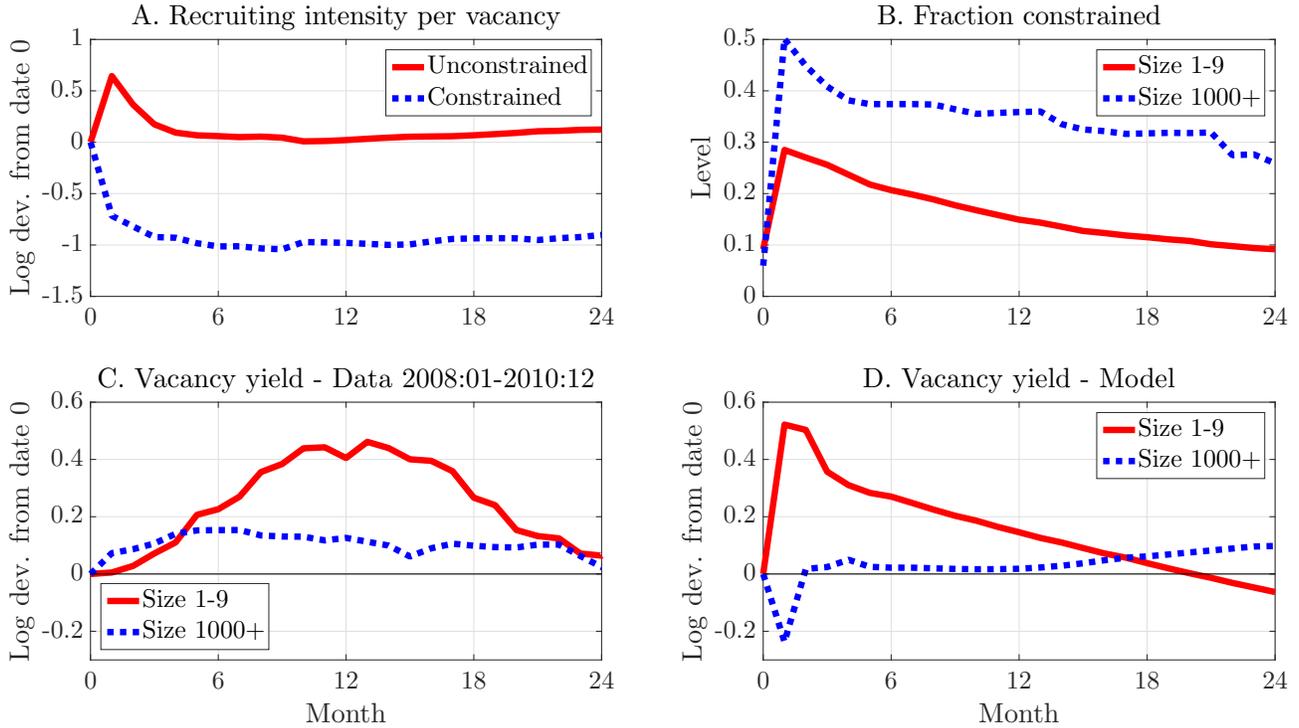
function in the composition effect is small. We conclude that the key transmission mechanism of our model, the slackness effect, is different from that emphasized by Kaas and Kircher (2015).

## 5.2 Dynamics of Vacancy Yields by Size: the Role of Financial Constraints

We use our decomposition of aggregate recruiting intensity to understand the cross-sectional dynamics of vacancy yields following a macroeconomic shock that Moscarini and Postel-Vinay (2016) document in the context of the Great Recession. We begin by splitting firms into those financially constrained (for whom both the collateral and dividend constraints bind) and those unconstrained. Panel A of Figure 14 shows that recruiting intensity dynamics differ markedly between the two types of firms.

The financial shock causes a sharp drop in the growth rate (and recruiting effort) of the constrained firms, whereas many of the unconstrained ones increase their hiring in response to the surge in labor market slackness. The constrained firms are, thus, driving the direct component of the composition effect, whereas unconstrained firms are driving the indirect component.

Panel B shows that, in the aftermath of the shock, the fraction of constrained firms rises

significantly across all sizes, but it does so in particular among some of the large firms, the young fast-growing firms with high span of control parameter ($\sigma$).

Panel D illustrates that the vacancy yield of large firms is flat, as they reduce recruitment effort to offset the higher aggregate filling rate; meanwhile, the vacancy yield of small firms increases, as they receive the full effect of higher $q\left(\theta_t^*\right)$ to grow. Panel C shows that the narrative implied by our model is borne out in the data.

# 6    Extensions

In this section, we outline two extensions of the baseline model. First, we analyze how permanent heterogeneity in vacancy filling rate across industries, due for example to different recruiting methods, affects the size of the composition effect. Second, we reflect on how the inclusion of on the job search could affect our main results.

## 6.1    Sectoral Heterogeneity in Recruiting Technology

Ours is a one-sector model of the aggregate economy in which all firms face the same recruiting technology. DFH document that different sectors of the economy display consistently different vacancy yields. To the extent that such discrepancies in vacancy yields stem from systematic differentials in growth rates across sectors, then our model will capture these.[33] If, however, they are due to permanent characteristics of the recruiting technology across sectors, then a macro shock that changes the sectoral composition of hiring firms will affect aggregate match efficiency.[34]

In the context of the Great Recession, this point is especially relevant because the Construction sector is an outlier in terms of its frictional characteristics (its vacancy yield is about 2.5 times as large as in the economy as a whole), and it was hit particularly hard in the recession. One would therefore expect Construction to play a significant role in the national movement of aggregate recruiting intensity, in spite of its small size relative to the aggregate economy (Davis, Faberman, and Haltiwanger, 2012b).

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[33]Indeed DFH Fig 5B shows that the cross-sector variation in average growth rates is strongly correlated with the cross-sector variation in vacancy-yields.

[34]We thank Steve Davis for suggestions which lead to the inclusion of this section.

A fully specified multi-sector model is beyond the scope of this paper, but we can nevertheless estimate the size of this *sectoral composition effect* using the structure of our model and industry-level data on vacancy-yields and vacancy shares from JOLTS.[35] Suppose that the firm-level hiring technology in each sector $s = 1, \ldots, S$ is subject to a sector-specific recruitment efficiency shifter $\phi_s$, i.e.

$$h_{ist} = \phi_s q\left(\theta_t^*\right) e_{ist} v_{ist},$$

leading to a modified expression for aggregate recruiting intensity:

$$\Phi_t = \left[\int_i \phi_s e_{ist} \frac{v_{ist}}{V_t} di\right]^\alpha, \tag{22}$$

and the optimal choice of firm-level recruiting intensity:

$$e_{ist} = Constant \times \phi_s^{-\frac{\gamma_2}{\gamma_1+\gamma_2}} q\left(\theta_t^*\right)^{-\frac{\gamma_2}{\gamma_1+\gamma_2}} g_{ist}^{\frac{\gamma_2}{\gamma_1+\gamma_2}}. \tag{23}$$

Firm-level recruiting intensity depends negatively on sector-specific efficiency since firms belonging to sectors with a high recruiting efficiency can use less effort to realize any desired growth rate.

To decompose aggregate recruiting intensity, we can again substitute the optimal policy (23) into (22) to arrive at:

$$\Phi_t = Constant \times q\left(\theta_t^*\right)^{-\frac{\alpha\gamma_2}{\gamma_1+\gamma_2}} \times \left[\sum_{s=1}^S \phi_s^{\frac{\gamma_1}{\gamma_1+\gamma_2}} \left(\frac{v_{st}}{V_t}\right) \int_{\{i\in s\}} \left[g_{ist}^{\frac{\gamma_2}{\gamma_1+\gamma_2}} \frac{v_{ist}}{v_{st}}\right] di\right]^\alpha.$$

The essence of the effect that we are trying to determine comes from the interaction of permanent differences in match efficiencies across sectors $\phi_s$ and the sectoral composition of hiring firms given by the vacancy share $v_{st}/V_t$. Therefore, we assume that the distribution of growth rates and vacancies is identical within each sector and, thus, the integral term is constant across sectors. Under this assumption, we obtain a counterpart to our previous decomposition of aggregate recruiting intensity, with an additional term characterizing the sectoral composition

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[35] In what follows, we maintain the assumption that all firms hire in the same labor market. Accordingly, one could read our exercise as the counterpart to one conducted on the worker side, in which different groups of job-seekers enter the same labor market, but are weighted by some fixed level of search efficiency. For example, see Hall and Schulhofer-Wohl (2013), and Hornstein and Kudlyak (2015).

## Figure 15: Sectoral composition effect



A. Sector component: $\hat{\phi}_s^{\frac{\gamma_1}{\gamma_1+\gamma_2}} \frac{v_{st}}{V_t}$

B. Sectoral composition effect
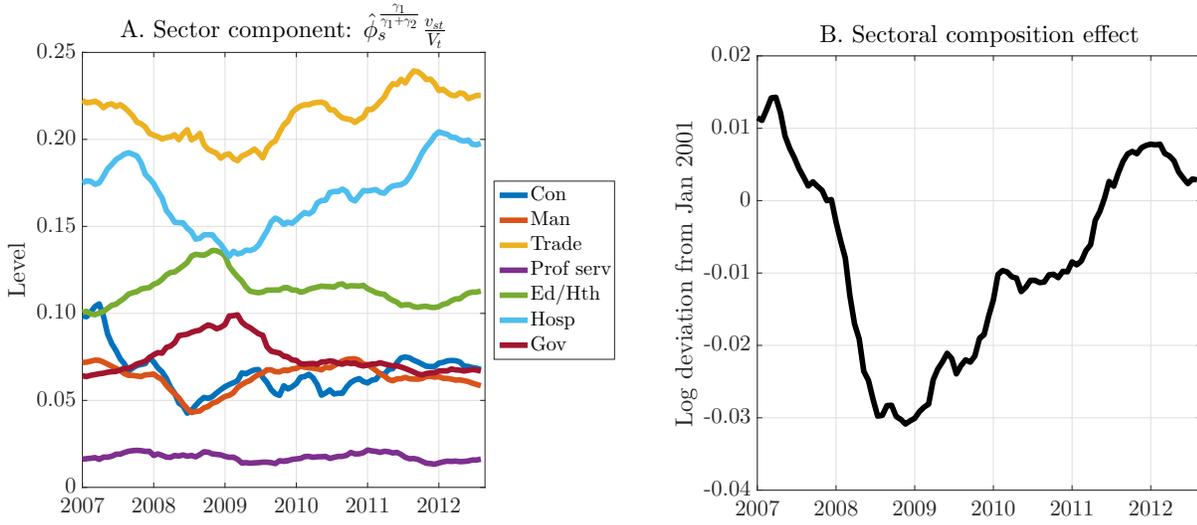
**Note** (i) Panel A plots the vacancy yield in the 7 largest industries. In Panel B, we have normalized the log of the sectoral composition effect to zero in January 2008.

effect

$$\Delta \log \Phi_t = -\alpha \frac{\gamma_2}{\gamma_1 + \gamma_2} \Delta \log q\left(\theta_t^*\right) + \alpha \Delta \log \left[\int_i g_{it}^{\frac{\gamma_2}{\gamma_1+\gamma_2}} \frac{v_{it}}{V_t} di\right] + \underbrace{\alpha \Delta \log \left[\sum_{s=1}^{S} \phi_s^{\frac{\gamma_1}{\gamma_1+\gamma_2}} \frac{v_{st}}{V_t}\right]}_{\text{Sectoral composition effect}}. \quad (24)$$

Note that $\phi_s$ enters with exponent $\frac{\gamma_1}{\gamma_1+\gamma_2}$, which is less than one. A sector with a higher $\phi_s$ will be more productive in creating matches, increasing the measure of recruiting intensity with an elasticity of one with respect to its vacancy share; however, the firms in that sector will also decrease effort with an elasticity of $\frac{\gamma_2}{\gamma_1+\gamma_2}$, leaving the net elasticity of $\frac{\gamma_1}{\gamma_1+\gamma_2}$.

Computing the last term in (24) requires data on vacancy shares by sector, readily available from JOLTS and data on sectoral match efficiency. Under our assumptions, it is easy to see that:

$$\phi_s^{\frac{\gamma_1}{\gamma_1+\gamma_2}} = \frac{H_{st}/V_{st}}{H_{kt}/V_{kt}},$$

where we can normalize match efficiency of the baseline sector $k$ to one without loss of gener-

ality.[36]

Using data on all eleven 2-digit industries from JOLTS, we plot the sectoral component $\phi_s^{\frac{\gamma_1}{\gamma_1+\gamma_2}} \frac{v_{st}}{V_t}$ for the largest seven sectors in Panel A of Figure 15, and the total sectoral composition effect in Panel B. We find that this component accounts for an additional 4 percent drop in aggregate recruiting intensity around the Great Recession— a fall mostly due to the decline in the vacancy shares of Construction, Manufacturing, and Hospitality and Leisure. Even though adding this mechanism shifts the decomposition slightly more towards the composition effect, it does not modify our conclusion that the slackness channel dominates it.

## 6.2 Robustness to the Inclusion of On-the-Job Search

In an economy where firms make take-it-or-leave-it offers to risk-neutral workers, modelling search on the job is relatively simple once it is assumed that, when an employed worker receives an outside offer, the firm does not respond to the poaching competitor, and the worker—who is indifferent between staying and going—quits.

Search on the job can be relevant in understanding the dynamics of aggregate recruiting intensity for two reasons. First, it induces separations of workers from the firm that will have to be replaced if the firm is not going to shrink. Such 'replacement hires' are associated with firms that have smaller growth rates, on average, compared to the expanding units. Since recruiting intensity is linked to growth rates, fluctuations in replacement hires could impact the composition effect. Second, when a portion of job-seekers is employed, the response of market tightness to spikes in layoffs—like those following financial and productivity shocks—would be smaller. This mechanism has the potential to weaken the slackness effect.

Adding on-the-job search requires making the following minimal amendments to the model: (i) all employed workers search with a relative search intensity of $s$ determining the effective units of search of an employed worker relative to an unemployed worker (whose intensity is normalized to 1); (ii) the matching function is modified to take the total measure of effective search units $S_t = U_t + sN_t$ as an input, where $N_t = \bar{L} - U_t$ is the measure of employed workers. The firm-level hiring technology remains $h_{it} = q_t e_{it} v_{it}$, but the law of motion for firm employ-

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[36]To estimate of $\phi_s$, we use ratios of average sectoral vacancy yields from 2005 to 2006. We take Professional Business Services as the normalizing sector, since its average vacancy-yield of 1.30 is the sectoral median. We use data for all 9 sectors available in JOLTS.

ment is now

$$n_{it+1} = n_{it} + h_{it} - f_{it} - sp\left(\theta_t^*\right)n_{it},$$

where $p\left(\theta_t^*\right)$ is the job finding rate of the unemployed. By constant returns to scale in the matching functions, $sp\left(\theta_t^*\right)$ is the job finding rate of employed workers. As a result, the law of motion for unemployment becomes:

$$U_{t+1} = U_t + F_t - \left[\frac{U_t}{U_t + sN_t}\right]H_t$$

where $U_t / \left(U_t + sN_t\right)$ is the fraction of total hires that come from unemployment.

In choosing a value for on the job search intensity, note that $s$ is equal to the ratio of employment-employment (EE) to unemployment-employment (UE) transition rates. Following Fujita and Moscarini (2013) and, thus, excluding recalls and workers on temporary layoffs from UE, we obtain $s = 0.09$ for the pre-recession period. We now discuss the effect of this extension on the model.

We begin with the role of replacement hires. In the presence of on the job search, firms lose employees to other poaching firms, and must make some replacement hires. For example, a large firm with 1,000 employees that has 2 percent of the workforce quit every period must make 20 new hires to maintain its size, i.e. the same number of hires of a firm of 20 employees that doubles its size. Therefore, relative to the baseline model, this force shifts the distribution of hires towards older, larger firms with close to zero net growth rates. We have solved the model under various rates of exogenous quits, between one and three percent per month, and found our results to be quantitatively very robust. The reason is that, as in the data, the bulk of hires are still made by expanding firms whose decisions are well described by our model.

We now turn to the effect of on the job search on the dynamics of labor market slackness. Consider an increase in the firing rate due to a negative macro shock. In the baseline model, the monthly firing rate is $F_t/N_t = 0.03$. Suppose that this ratio were to spike in a recession, doubling. In the baseline model without on-the-job search, the mass of effective search units increases nearly one for one, by 0.03. In the model with on the job search, $S_t = (1-s)U_t + s\bar{L}$, so although the number of unemployed workers rises by 0.03, the measure of total job seekers increases by $(1-s) \times 0.03 = 0.027$. Therefore, labor market tightness falls by less and

the slackness effect is somewhat weakened, as expected, but this correction is quantitatively small. The reason is that, although the *stock* of employed workers is large, their average search intensity is low relative to that of the unemployed. Moreover, if one were also to allow *s* to vary over the cycle, and match the data, then the relative intensity of the employed would be countercyclical.[37] This force would partially counteract the initial correction, thus making the total effect of on-the-job search on the dynamics of market tightness even smaller.

Frictional models of the labor market with both a realistic firm size distribution induced by DRS in production, and a rich job ladder whereby high-productivity high-wage firms can poach workers more easily from other firms —and thus the vacancy filling rate is increasing in the firm type because it is further up the ladder— have not yet been developed.[38] Whether such class of models has novel forces at work relative to those emphasized here remains to be established.

# 7 A New Aggregate Recruiting Intensity Index

We conclude the paper by proposing how to construct a model-based rule-of-thumb index of aggregate recruiting intensity which is easy to compute from observable labor market aggregates and can be updated in real time, as new data on unemployment, vacancies, and hires are released.

The transition dynamics of our model indicate that the slackness effect accounts for most movements in aggregate recruiting intensity. Thus, we propose a back-of-the-envelope measure of aggregate recruiting intensity that focuses on this component. Specifically, as per equation (21), we set $d \log \Phi_t = -\alpha \frac{\gamma_2}{\gamma_1 + \gamma_2} d \log q(\theta_t^*)$. Using the relationship $q(\theta_t^*) = \left( \Phi_t^{\frac{1}{\alpha}} \theta_t \right)^{-(1-\alpha)}$ into this equation, we arrive at our empirical index of changes in aggregate recruiting intensity:

$$ d \log \Phi_t^{GMV} = \underbrace{\frac{\alpha(1-\alpha)\left[\gamma_2/(\gamma_1 + \gamma_2)\right]}{1 - (1-\alpha)\left[\gamma_2/(\gamma_1 + \gamma_2)\right]}}_{\approx 0.35} d \log \theta_t. \tag{25} $$

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[37]Figure B3 in the Appendix documents the cyclicality of relative search intensity of the employed.

[38]A notable exception is Lentz and Mortensen (2008). Even though they do not analyze the determinants of and the transmission of shocks to aggregate recruiting intensity, like we do, their framework lends naturally to addressing such questions as well.

Figure 16: Index of implied aggregate recruiting intensity (Jan 2007 = 0)
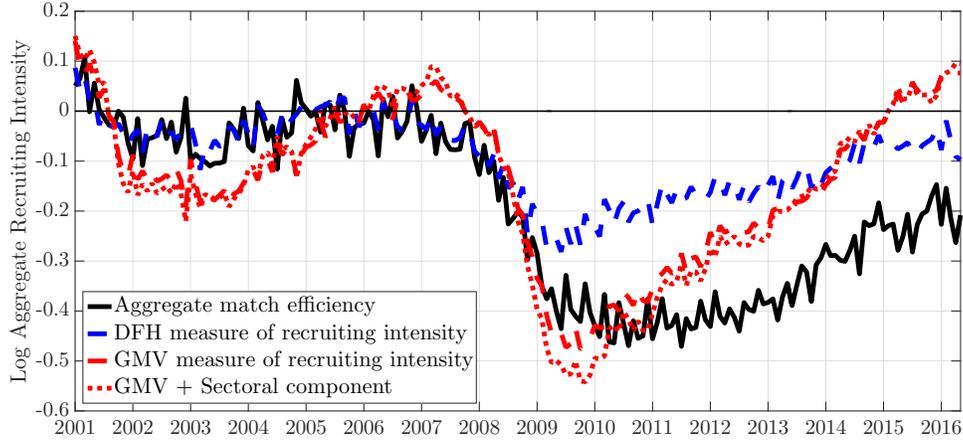


The derivation of our empirical index is grounded in our theory, and thus differs from the one that DFH set forth, which is based on their 'generalized matching function.' We reproduce here their derivation to emphasize the distinctions between the two indicators. Rearranging the aggregate matching function, one obtains $\log(H_t/V_t) = \log \Phi_t + \log q_t$, a relationship which links the aggregate vacancy-yield, recruiting intensity, and matching rate. Totally differentiating this expression with respect to (i) the aggregate hiring rate $H_t/N_t$, and (ii) the matching rate $q_t$, we obtain:

$$\frac{d \log H_t/V_t}{d \log H_t/N_t} + 1 \;\; = \;\; \frac{d \log \Phi_t}{d \log(H_t/N_t)} + \frac{d \log \Phi_t}{d \log q_t} + \frac{d \log q_t}{d \log(H_t/N_t)} + 1. \tag{26}$$

The working hypothesis of DFH is that (i) the second and third terms on the right-hand side are zero; and (ii) the term on the left-hand side—the macro-elasticity of the vacancy-yield to the hiring-rate—is the same as the estimated micro-elasticity. These assumptions deliver the DFH measure of aggregate recruiting intensity:

$$d \log \Phi_t^{DFH} = \frac{\gamma_2}{\gamma_1 + \gamma_2} d \log(H_t/N_t). \tag{27}$$

Figure 16 replicates Figure X of DFH, adding our index $\Phi_t^{GMV}$, as well as aggregate matching efficiency estimated from the data (the unfiltered version of the series in Figure 1). The figure also plots an augmented version of our index based on the industry-level analysis of Section 6.1 where we have added the sectoral composition effect to the right-hand side of equation (25).

We should point out that the *causal* interpretations of the two indices differ. Our index $\Phi_t^{GMV}$ relies on the role of market-tightness, as our general-equilibrium model highlights that this is the primary driver of aggregate recruiting intensity. The DFH index $\Phi_t^{DFH}$ instead relies on the aggregate hiring rate. The two indices comove closely due to the strong correlation between market tightness and hiring rates in the time-series. However, our index displays larger fluctuations and tracks well the drop in aggregate match efficiency in the recession.[39] At the same time, it shows a faster recovery in the post-recession years compared to the DFH measure, indicating that the protracted deterioration of US aggregate match efficiency is likely to be caused by factors other than the diminished recruiting effort of employers.

We end this section with two observations. First, an even more refined version of this index could be constructed by obtaining access to the JOLTS microdata. These data would allow one to compute the within-sector composition effect that we have set to zero in light of the predictions of our model. Second, while this empirical index is useful to gauge the degree of recruiting effort exerted by US firms at fairly high frequency, it cannot, alone, answer questions of more immediate interest to macroeconomists such as the propagation of macro shocks or the effect of policies on matching efficiency.

# 8 Conclusions

The existing literature on the cyclical fluctuations of aggregate match efficiency has focused almost exclusively on explanations involving the worker side of the labor market, such as occupational mismatch, shifts in job-search intensity of the unemployed over the cycle, and compositional changes among the pool of job seekers. In this paper we have shifted the focus to the firm side and, building on the microeconomic evidence in Davis, Faberman, and Haltiwanger (2013), developed a macroeconomic model of aggregate recruiting intensity.

The model, parameterized to replicate a range of cross-sectional facts about firm dynamics and hiring behavior, indicates that a financial shock of plausible magnitude is consistent with several features of the US macroeconomy around the Great Recession and is able to explain

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[39]We note that the calculations in this section imply that the reason why our model of Section 5 accounts only partially for the observed drop in match efficiency is that it only generates half of the observed fall in market tightness.

about half of the collapse in aggregate match efficiency over the same period through a sharp decline in firms' recruiting intensity. Our analysis of the transmission mechanism of the shock points towards the importance of general equilibrium forces: aggregate recruiting intensity declined mainly because the number of available job seekers per vacancy increased (i.e., labor market tightness declined) making it easier for firms to achieve their recruitment targets without having to spend as much on recruitment costs. Changes in within-sector composition of the pool of hiring firms, due for example to the fall in new firm entry that is well matched by the model, did not play a large role. The shift in sectoral composition—in particular the bust in Construction and other sectors with structurally high job-filling rates, did instead contribute to the measured deterioration in aggregate recruiting effort.

Besides its contribution to understanding the determinants of movements in match efficiency, and thus the job finding rate—a key object for labor market analysis—our theory has broader implications for macroeconomics. First, as for example Faberman (2016) discusses, making progress in understanding how firms' hiring decisions respond to macroeconomic conditions is important since job creation policies that fail to recognize the determinants of employer's recruitment effort may fall short in achieving their goal. Our model predicts that subsidizing firm hiring (abstracting from offsetting effects from higher tax rates) will increase the average firm growth rate and induce a rise in recruiting intensity, whereas a subsidy to workers' job search that decreases market tightness will induce a decline in recruiting intensity, through the slackness effect discussed in the paper. Second, a richer model of employer recruiting behavior can lead to better estimates of the true marginal cost of labor and, therefore, result in improved measures of the labor wedge and of the relative importance of labor and product market wedges (Bils, Klenow, and Malin, 2014). In this respect, our model suggests that the price of labor faced by firms may be more procyclical than what would appear from naively using wages as a proxy.

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# APPENDIX FOR ONLINE PUBLICATION

This Appendix is organized as follows. Section A contains the derivations of the hiring cost function that we introduced in Section 4. Section B provides additional figures referenced in the main text. Section C details the algorithms for the computation of the stationary equilibrium, transitional dynamics, and estimation of model's parameters. Section D argues that our theory of recruiting intensity implies that the common OLS estimates of the elasticity of hires to vacancies in the aggregate matching function are biased.

## A   The hiring cost function

In this section we show that, once we postulate the hiring cost function

$$\mathcal{C}(n, e, v) = \left[ \frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left( \frac{v}{n} \right)^{\gamma_2} \right] v, \tag{A1}$$

then, through firm's optimization, we obtain a log-linear cross-sectional relationship between the job-filling rate and the employment growth rate that is consistent with the empirical findings in DFH. Next, by substituting the firm FOCs into $(A1)$, we derive a formulation of the cost only in terms of $(n, n')$ that we use in the intertemporal problem $(10)$ in the main text.

As we explained in Section $(3.1)$, the firm solves a static cost minimization problem: given a choice of $n'$, it determines the lowest cost combination of $(e, v)$ that can deliver $n'$. The hiring firm's cost minimization problem is

$$
\begin{aligned}
\mathcal{C}(n, n') \;=\; &\min_{e,v} \; \left[ \frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left( \frac{v}{n} \right)^{\gamma_2} \right] v \\
&s.t. \quad : \quad n' - n \le q\left(\theta^*\right) ev \\
&\qquad\qquad\quad v \ge 0
\end{aligned}
\tag{A2}
$$

Convexity of the cost function $(A1)$ in $(e, v)$ requires $\gamma_1 \ge 1$ and $\gamma_2 \ge 0$. After setting up the Lagrangian, one can easily derive the two FOCs with respect to $e$ and $v$ that, combined together, yield a relationship between the optimal choice of $e$ and the optimal choice of the vacancy rate $v/n$ :

$$e = \left[ \frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1}} \left( \frac{v}{n} \right)^{\frac{\gamma_2}{\gamma_1}}. \tag{A3}$$

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Note that, if $\gamma_2 = 0$, as in Pissarides (2000), recruiting intensity is equal to a constant for all firms and it is independent of aggregate labor market conditions —both counterfactual implications. The following changes in parameters (ceteris paribus) result in a substitution away from vacancies and towards effort: $\uparrow \kappa_2, \downarrow \kappa_1, \uparrow \gamma_2$, and $\downarrow \gamma_1$. The effect of the cost shifter is obvious. A higher curvature on the vacancy rate in the cost function ($\uparrow \gamma_2$) makes the marginal cost of creating vacancies rising faster than the marginal cost of recruiting effort; since the gain in terms of additional hires from a marginal unit of effort or vacancies is unaffected by $\gamma_2$, it is optimal for the firm to use relatively more effort.

Now, substituting the law of motion for employment at the firm level into $(A3)$, we obtain the optimal recruitment effort choice, expressed only as a function of the firm-level variables $(n, n')$:

$$e\left(n, n'\right) = \left[\frac{\kappa_2}{\kappa_1}\left(\frac{\gamma_1}{\gamma_1 - 1}\right)\right]^{\frac{1}{\gamma_1 + \gamma_2}} q\left(\theta^*\right)^{-\frac{\gamma_2}{\gamma_1 + \gamma_2}}\left(\frac{n' - n}{n}\right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}} \tag{A4}$$

which, in turn implies, for the job filling rate,

$$f\left(n, n'\right) = q\left(\theta^*\right) e\left(n, n'\right) = \left[\frac{\kappa_2}{\kappa_1}\left(\frac{\gamma_1}{\gamma_1 - 1}\right)\right]^{\frac{1}{\gamma_1 + \gamma_2}} q\left(\theta^*\right)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}}\left(\frac{n' - n}{n}\right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}. \tag{A5}$$

This equation demonstrates that the model implies a log-linear relation between the job filling rate and employment growth at the firm level, with elasticity $\gamma_2 / (\gamma_1 + \gamma_2) < 1$ as in the data.

Finally, substituting $(A5)$ into the firm-level law of motion for employment yields an expression for the vacancy rate

$$\frac{v}{n} = \left[\frac{\kappa_2}{\kappa_1}\left(\frac{\gamma_1}{\gamma_1 - 1}\right)\right]^{\frac{1}{\gamma_1 + \gamma_2}} q\left(\theta^*\right)^{-\frac{\gamma_1}{\gamma_1 + \gamma_2}}\left(\frac{n' - n}{n}\right)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}}. \tag{A6}$$

Now, note that, by substituting the optimal choice for recruitment effort $(A3)$ into $(A1)$, we obtain the following formulation for the cost function

$$C\left(n, v\right) = \left[\kappa_2 \frac{\gamma_1 + \gamma_2}{(\gamma_1 - 1)(\gamma_2 + 1)}\left(\frac{v}{n}\right)^{\gamma_2}\right] v, \tag{A7}$$

which is one of the specifications invoked by Kaas and Kircher (2015).

Finally, if we use $(A6)$ in $(A7)$, we obtain a version of the cost function only as a function of

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$(n, n')$ that we can use directly in the dynamic problem (10):

$$C^* \left( n, n' \right) = \kappa_2 \left[ \frac{\gamma_1 + \gamma_2}{(\gamma_1 - 1)(\gamma_2 + 1)} \right] \left\{ \left[ \frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q \left( \theta^* \right)^{-\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left( \frac{n' - n}{n} \right)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}} \right\}^{1 + \gamma_2} n.$$

# B   Additional figures

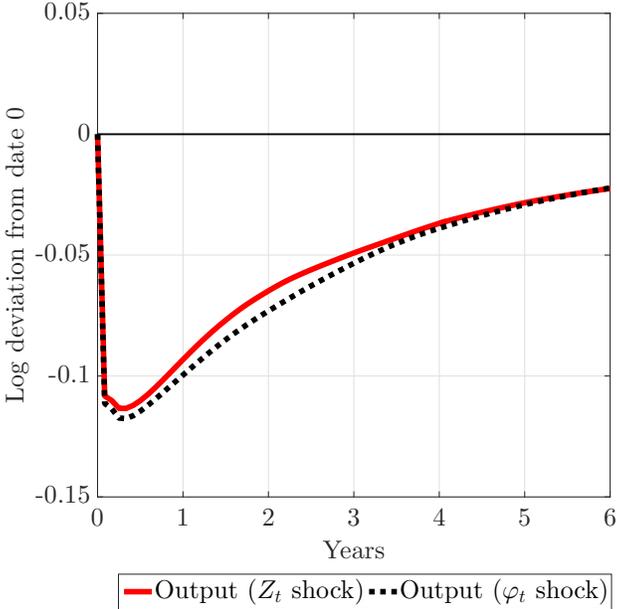Figure B1: Dynamics of output under the two shocks



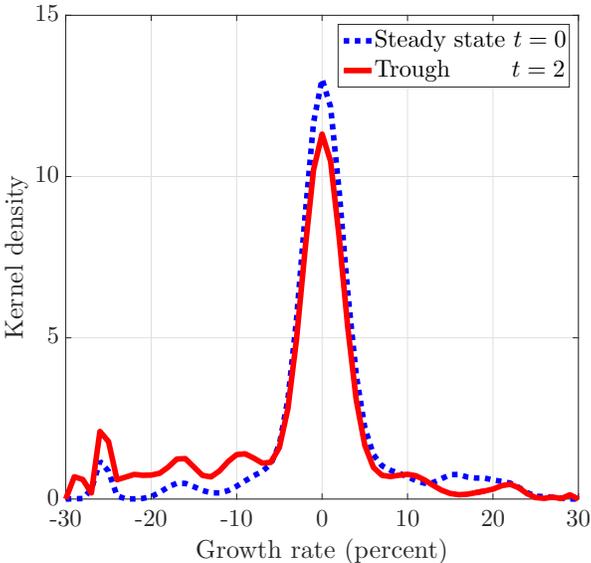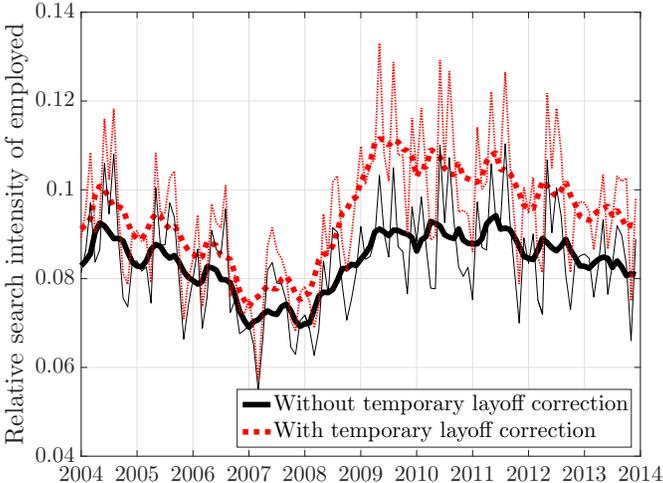Figure B2: Employment-weighted growth rate distribution in the model

Figure B3: Relative search intensity of employed workers

**Note** Note: The two series are constructed as the ratio of the EE flow rate to UE flow rate. In one case the UE rate is taken directly from the BLS, whereas in the other case we adjust the UE rate by subtracting unemployed workers on temporary layoff from the denominator and rehires from temporary layoffs from the numerator.

# C   Computational details

## C.1   Value and policy functions

We use collocation methods to solve the firm's value function problem (4)-(7). Let $s = (n, a, z)$ be the firm's idiosyncratic state, abstracting from heterogeneity in $\sigma$ since this is fixed. We solve for an approximant of the *expected* value function $\mathbb{V}^e(n', a', z)$ which gives the firm's expected value conditional on current decisions for net-worth and employment

$$\mathbb{V}^e(n', a', z) = \int_{\mathcal{Z}} \mathbb{V}(n', a', z') d\Gamma(z, z'),$$

where the integrand is the value given in (6).

We set up a grid of collocation nodes $S = N \times A \times Z$ where $N = \{n_1, \dots, n_{N_n}\}$, with $N_n = N_a = N_z = 10$. We construct $Z$ by first creating equi-spaced nodes from 0.001 to 0.999, which we then invert through the cdf of the stationary distribution implied by the AR(1) process for $z$ to obtain $Z$. This ensures better coverage in the higher probability regions for $z$. We choose $A$ and $N$ to have a higher density at lower values. The upper bound for employment, $\bar{n}$, is chosen so that the optimal size of the highest productivity firm $n^*(\bar{z})$ is less than $\bar{n}$. We choose the upper bound for assets, $\bar{a}$, so that the maximum optimal capital $k^*(\bar{z})$ can be financed, that is $k^*(\bar{z}) < \varphi\bar{a}$. Note that $N$, $A$, and $Z$ are parameter dependent, therefore recomputed for each new vector of parameters considered in estimation.

We approximate $\mathbb{V}^e(s)$ on $S$ using a linear spline with $N_s = N_n \times N_a \times N_z$ coefficients. Given a guess for the spline's coefficients we iterate towards a vector of coefficients that solve the system of $N_s$ Bellman equations, which are linear in the $N_s$ unknown coefficients. Each iteration proceeds as follows. Given the spline coefficients we use golden search to compute the optimal policies for all states $s \in S$, and the value function $\mathbb{V}(s)$. We then fit another spline to $\mathbb{V}(s)$ which facilitates integration of productivity shocks $\varepsilon \sim \mathcal{N}(0, \vartheta_z)$. To compute $\mathbb{V}^e(s)$ on $S$ we approximate the integral by

$$\mathbb{V}^e(n, a, z) = \sum_{i=1}^{N_\varepsilon} w_i \mathbb{V}\left(n, a, \exp(\rho_z \log(z) + \varepsilon_i)\right).$$

Here $N_\varepsilon = 80$ and the values of $\varepsilon_i$ are constructed by creating a grid of equi-spaced nodes between 0.001 to 0.999, then using the inverse cdf of the shocks (normal) to create a grid in $\varepsilon$.

The weights $w_i$ are given by the probability mass of the normal distribution centered around each $\varepsilon_i$. Note that this differs from quadrature schemes in which one is trying to minimize the number of evaluations of the integrand, usually with $N_\varepsilon$ around four. Since $\mathbb{V}(s)$ is already given by an approximant at this step, and the integral is only computed once each iteration, this is not a concern and we compute the integral very precisely. We then fit an updated vector of coefficients to $\mathbb{V}^e(s)$ and continue.[40]

## C.2  Stationary distribution

To construct the stationary distribution we use the method of non-stochastic simulation from Young (2010), modified to accommodate a continuously distributed stochastic state. We create a new, fine grid of points $S^f$ on which we approximate the stationary distribution using a histogram, setting $N_n^f = N_n^f = N_z^f = 100$. Given our approximation of the expected continuation value we solve for the policy functions $n'\left(s^f\right)$ and $a'\left(s^f\right)$ on the new grid and use these to create two transition matrices $\mathbf{Q}_n$ and $\mathbf{Q}_a$ which determine how mass shifts from points $s^f \in S^f$ to points in $N^f$ and $A^f$, respectively. We construct $\mathbf{Q}_x$ as follows for $x \in \{a, n\}$

$$\mathbf{Q}_x[i,j] = \left[ \mathbf{1}_{x'\left(s_i^f\right) \in \left[X_{j-1}^f, X_j^f\right]} \frac{x'\left(s_i^f\right) - X_j^f}{X_j^f - X_{j-1}^f} + \mathbf{1}_{x'\left(s_i^f\right) \in \left[X_j^f, X_{j+1}^f\right]} \frac{X_{j+1} - x'(s_i)}{X_{j+1}^f - X_j^f} \right],$$

for $i = 1, \ldots, N_s^f$ and $j = 1, \ldots, N_x^f$.[41] This approach ensures that aggregates computed from the stationary distribution will be unbiased. For example if $x'(s) \in (X_j, X_j + 1)$, then masses $w_j$ and $w_{j+1}$ are allocated to $X_j$ and $X_{j+1}$ such that $w_j X_j + w_{j+1} X_{j+1} = x'(s)$. The transition matrix for the process for $z$ is computed by $\mathbf{Q}_z = \sum_{i=1}^{N_\varepsilon} w_i \mathbf{Q}_z^i$, where $\mathbf{Q}_z^i$ is computed as above under $z'(s^f) = \exp(\rho_z \log z + \varepsilon_i)$. Finally the overall incumbent transition matrix $\mathbf{Q}$ is the tensor product $\mathbf{Q} = \mathbf{Q}_z \otimes \mathbf{Q}_a \otimes \mathbf{Q}_n$.

To compute the stationary distribution we still need the distribution of entrants. To allow for entry cut-offs to move smoothly we compute entrant policies on a dense grid of $N_z^0 = 500$ productivities. This is clearly important for us since it ensures that entry does not jump in the

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[40]In practice, instead of this simple iterative approach to solve for the coefficients, we follow a Newton algorithm as in Miranda and Fackler (2002), which is two orders of magnitude faster. The Newton algorithm requires computing the Jacobian of the system of Bellman equations with respect to the coefficient vector. The insight of Miranda and Fackler (2002) is that this is simple to compute given the linearity of the system in the coefficients.

[41]If exit is optimal on grid point $s_i^f$ then we set row $i$ of $\mathbf{Q}_x$ to zero.

transition dynamics or across parameters in calibration. The grid $Z^0$ is constructed by taking an equally spaced grid in cumulative probabilities and inverting it through the cdf of potential entrant productivities (exponential). Let the corresponding vector of weights be given by $\mathbf{P}_0$. Given the approximation of the continuation value $\mathbb{V}^e$ we can solve the potential entrant's policies $n'_0(s_0)$ and $a'_0(s_0)$, conditional on entry. We can then solve the firm's discrete entry decision. Finally we compute an equivalent transition matrix $\mathbf{Q}_0$ using these policies, where non-entry results in a row of zeros in $\mathbf{Q}_0$.

The discretized stationary distribution $\mathbf{L}$ on $\mathcal{S}^f$ is then found by the following approximation to the law of motion (15)

$$\mathbf{L} = (1 - \zeta)\mathbf{Q}'\mathbf{L} + \lambda_0\mathbf{Q}'_0\mathbf{P}_0,$$

which is a contraction on $\mathbf{L}$, solved by iterating on a guess for $\mathbf{L}$. The final stationary distribution is found by choosing $\lambda_0$ such that $\sum_{i=1}^{N_s^f} L_i = 1$.

## C.3   Computation of moments

We compute an aggregate moment $X$ by integrating $\lambda$ over firm policies $x(s)$. Using the above approximation this is simply $X = \mathbf{L}'x(s)$.

For age based statistics, our moments in the data refer to firm ages in years. We therefore generate an 'age zero' measure of firms by allowing for 12 months of entry. We then iterate this distribution forward to compute age statistics such as average debt to output for age 1 firms, or the distribution of vacancies by age.

For statistics such as the average annual growth rate conditional on survival we need to simulate the model. In this case we draw 100,000 firms on $\mathcal{S}^f$ in proportion to $\mathbf{L}$ and simulate these forwards solving (rather than interpolating) firm policies each period and evolving productivity with draws from the continuous distribution of innovations $\varepsilon$. To remove the effect of the starting grid, we simulate for 36 months and compute our statistics comparing firms across months 24 and 36.

## C.4   Estimation

The model has a large number of unknown parameters and a criterion function that is potentially non-smooth. Furthermore the model does not have an equilibrium for large regions of the

parameter space.[42] For these reasons, using a sequential optimizer that takes the information from successive draws from the parameter space and updates its guess is prohibitive. For example, a Nelder-Mead optimizer both needs to be returned values for the objective function at each evaluation and needs to make many evaluations of the function when taking each 'step'.

Our solution is to use an algorithm that we can very easily parallelize, that efficiently explores the parameter space, and for which we can ignore cases with no equilibrium. We set up a hyper-cube in the parameter space and then initialize a Sobol sequence to explore it. A Sobol sequence is a quasi-random low-discrepancy sequence that maintains a maximum dispersion in each dimension and far out-performs standard random number generators. We then partition the sequence and submit each partition to a separate CPU on a High Performance Computer (HPC). From each evaluation of the parameter hyper-cube we save the vector of model moments, and regularly splice these together, choosing one that minimizes the criterion function. Starting with wide bounds on the parameters we run this procedure a number of times, shrinking the hypercube each time.

This procedure has a number of benefits. First, we trade in the optimization steps associated with a traditional solver for scale. Instead of using a 10 CPU machine to run a Nelder-Mead algorithm, we can simultaneously solve the model on 300+ CPUs. Second, the output of the exercise gives a strong intuition for the identification of the model. From an optimizer one may retrieve the moments of the model along the path of the parameter vector chosen by the algorithm. In our case, we retrieve thousands of evaluation knowing that the low-discrepancy property of the Sobol sequence implies that for an interval of any one parameter, the remaining parameters are drawn uniformly. Plotting moments against parameters therefore shows the effect of a parameter on a certain moment, conditional on local draws of all other parameters. Plotting a histogram of the moments returned, as in Figure C1, gives a strong indication as to which moments may be difficult to match for the current bounds of the parameter space.

## C.5 Transition dynamics

We solve for transition dynamics as follows. Consider the case of a shock to aggregate productivity $Z$. We specify a path for $\{Z_t\}_{t=0}^{T}$ with $Z_0 = Z_T = \bar{Z}$. Given a conjectured path for

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[42]For example, if the value of home production is very low then unemployment derived from the labor demand condition may be *negative*. Wages are so low that labor demand eclipses the fixed supply $\bar{L}$.

Figure C1: Histograms of selected moments from final calibration exercise
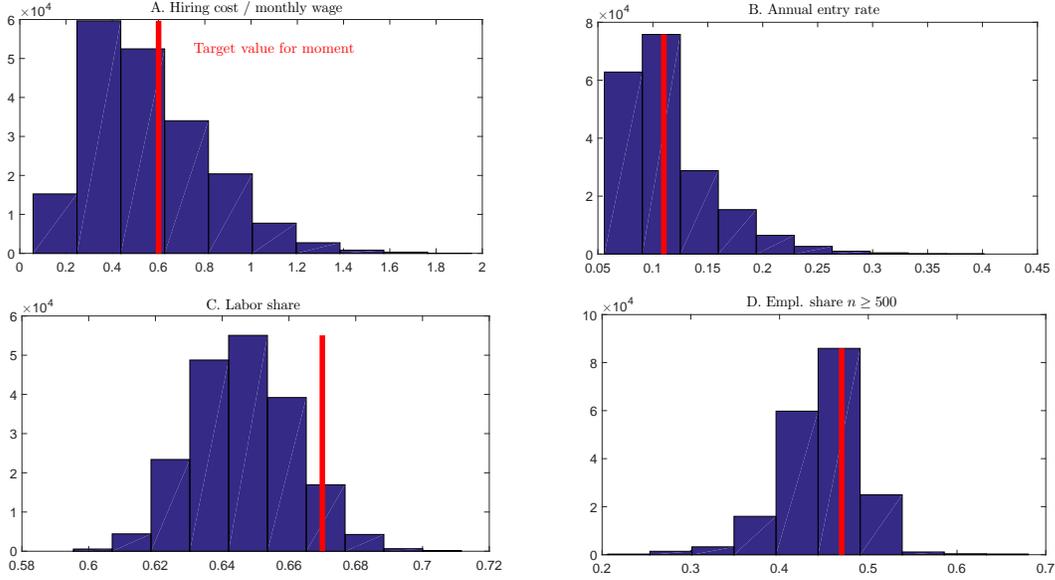


equilibrium market-tightness $\left\{\tilde{\theta}_t^*\right\}_{t=0}^T$ and the assumption that the date $T$ continuation values of the firm are the same as in steady state, one can solve *backwards* for expected value functions $\mathbf{V}_t^e$ at all dates $T-1, T-2, \ldots, 1$. Setting the aggregate states $U_0 = \bar{U}$ and $\lambda_0 = \bar{\lambda}$, and using the conjectured path $\theta_t^*$, the shocks and continuation values one can then solve *forwards* for a new market-clearing $\theta_t^{*\prime}$ that equates unemployment from labor demand $U_t^{demand}$ and worker flows $U_t^{flows}$ in every period using the labor demand and evolution of unemployment equations

$$U_{t+1}^{flows} = U_t - H(\theta_t^*) + F(\theta_t^*) - \lambda_{e,t} n_0$$
$$U_{t+1}^{demand} = \bar{L} - \int n'(s, \theta_t^*, A_t, \mathbf{V}_t^e) d\lambda_t$$

Once we reach $t = T$ we set $\tilde{\theta}_t^* = \theta_t^*$ and iterate until the proposed and equilibrium paths for market tightness converge.

# D   Estimation of matching function elasticity

As mentioned in Section 5.1.2, our main finding that recruiting intensity strongly co-moves with market tightness has implications for the estimation of the elasticity of the aggregate matching function.

Consider the true matching function

$$H_t = \Phi_t V_t^\alpha U_t^{1-\alpha}.$$

To estimate the elasticity parameter $\alpha$, a common strategy in the literature is to divide by unemployment, take logs and estimate the following equation by OLS

$$\log JFR_t = \beta_0 + \beta_1 \log \theta_t + \epsilon_t, \tag{D1}$$

where $JFR_t$ is the job finding rate. Our theory implies that a component of the error term $\epsilon_t$ is positively correlated with $\theta_t$: in recessions markets are slack (lower $\theta_t$) and recruiting intensity is low (lower $\varepsilon_t$). This will result in an *upward* bias in the estimate of $\hat{\beta}_1 > \alpha$.

We can get a sense of the magnitude of this bias using our approximate measure of recruiting intensity that abstracts from the composition effect (small in the model's simulations) and takes a first-order approximation of the function $q(\theta^*)$:

$$\log \Phi_t \simeq \text{Constant} + \frac{\gamma_2}{\gamma_1 + \gamma_2} \alpha(1-\alpha)\varepsilon_{\theta^*,\theta} \log \theta_t$$

where $\varepsilon_{\theta^*,\theta}$ is the equilibrium elasticity of $\theta^*$ to $\theta$.

Substituting this formula into an empirical equation for the matching function where $\eta_t$ contains other components orthogonal to labor market tightness, we obtain:
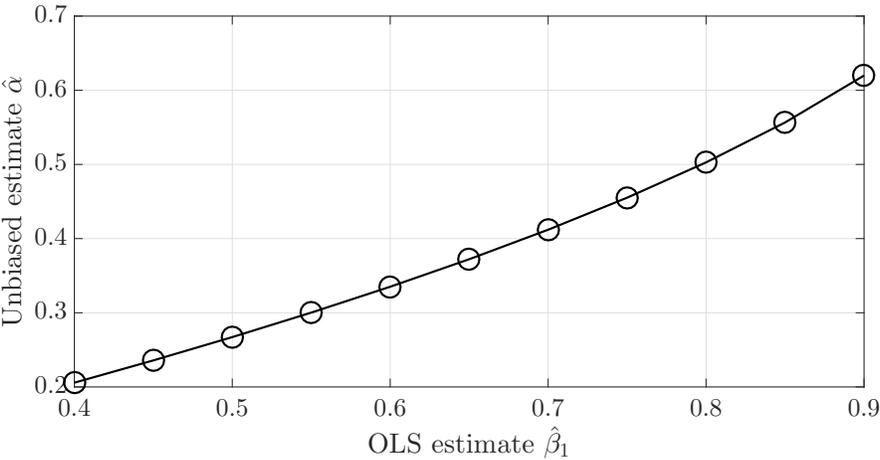
$$\log JFR_t = \text{Constant} + \alpha \left[ 1 + \frac{\gamma_2}{\gamma_1 + \gamma_2}(1-\alpha)\varepsilon_{\theta^*,\theta} \right] \log \theta_t + \log \eta_t .$$

For a given value of $\hat{\beta}_1$ obtained from estimating D1, $\hat{\alpha}$ which is the unbiased estimator of $\alpha$ is the solution to the quadratic equation

$$\hat{\beta}_1 = \hat{\alpha} \left[ 1 + \frac{\gamma_2}{\gamma_1 + \gamma_2}(1-\hat{\alpha})\varepsilon_{\theta^*,\theta} \right] .$$

Figure D1 plots these solutions for different values of $\hat{\beta}_1$.

Figure D1: Relationship between biased and unbiased estimates of the matching function elasticity of hires with respect to vacancies ($\alpha$)



We note that Borowczyk-Martins, Jolivet, and Postel-Vinay (2013) compare OLS estimates of $\alpha$ and GMM estimates corrected for endogeneity and do indeed find evidence of an upward bias. Their OLS estimate of 0.84 (their Table 1) would correspond to a true value, under our model, of $\alpha = 0.55$. As a baseline, we chose the slightly lower value of 0.5 to be conservative on the fraction of the decline in aggregate matching efficiency that can be explained by the model.