Discretely innovating: Barriers to entry, contestability and innovation

Steven Bond-Smith*

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Abstract

This paper considers the effect of a discrete entry barrier (i.e. only an integer number of firms is permitted) on innovation in an endogenous growth model to draw conclusions about the relationship between contestability and innovation under Cournot and Betrand oligopoly. Sector-specific workers become a measure of contestability and provide a tool for calibration. The paper finds that sectors with low contestability have lower innovation. In particular innovation under Cournot oligopoly is always less than under Bertrand with particularly pronounced effects when contestability is low. With increased contestability, innovation increases towards the limit in the continuous entry model as its impact diminishes. Wage inequality varies depending on the extent that the barrier to entry is binding upon a marginal entrant.

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1 Introduction

While high income countries have pursued economic liberalisation in the quest for economic prosperity, inequality has increased and median wages remain stagnant, generating a push-back against the liberalisation and globalisation agenda. Market liberalisation reforms imply that contestability is the key driver of prosperity and growth. Despite this policy agenda, it has become apparent that entrepreneurship and the contribution of start-ups to growth are declining (Decker et al., 2014; 2016; Hathaway & Litan, 2014; Guzman & Stern, 2016) and the labour market is becoming less dynamic (Haltiwanger et al., 2013; Molloy et al., 2016). Innovation is rewarded in the market while a firm holds a technological advantage, but if markets are not contestable then whomever is granted access to the market can already extract such a premium without being innovative. In particular, many industries rely on access to a specific resource which represents a *structural* barrier to entry. These may be traditional barriers to entry such as taxi licences, but other resource barriers can also act as a barrier to entry and these particular resources attract a premium for their unique scarcity. For example, Uber recently poached 40 researchers and scientists from Carnegie Mellon University offering "bonuses of hundreds of thousands of dollars and doubling the

*Curtin Business School, Bankwest Curtin Economics Centre, Curtin University, GPO Box U1987, Perth WA 6845, Australia; E-mail address: steven.bond-smith@curtin.edu.au

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salaries to staff." [1] Understanding the effect of market structure on innovation, growth and inequality is vitally important to developing policies that support long-run prosperity.

Growing inequality suggests that existing reforms have not been successful at generating inclusive growth. If recent public discourse leads to undoing these reforms it may reduce contestability, yet entry barriers could already be causing declining economic dynamism and a lack of contestability. The relationship between market structure and innovation is nuanced and liberalised markets may not be as contestable as first thought. While economists have often examined the relationship between trade barriers and growth, little attention has been paid to understanding the properties of the relationship between barriers to entry, contestability, imperfect competition or market structure more generally and economic growth. By using a discrete entry barrier (i.e. only an integer number of firms is permitted) as a tool to model contestability, this paper develops a generalised understanding of how contestability affects innovation, can influence inequality and how innovation and growth are nuanced by the mode of competition.

Market liberalisation has removed many regulatory and trade barriers, but economic and institutional barriers to entry may still hamper innovation. If firms require a minimum viable scale to participate there may be room in the market for only a few firms, even once regulations are liberalised. In designing markets, policy makers should be careful that liberalisation does not inadvertently leave or create barriers to entry. When barriers to entry remain, whether caused by regulatory, institutional or economic factors, the impact on innovation is negative and inequality may increase. Similarly, economic models should be careful to consider the dynamics of market entry and competition. Real world markets are not the continuous functions used in most economic growth models with infinitely small firms. In order to boost profits, products are highly differentiated and firms often compete with only a few rivals. Therefore, markets may not be as contestable as assumed in most existing theoretical models, particularly those that require scale or otherwise for entry.

Empirical evidence examining the relationship between market structure and innovation suggests that the important factor driving innovation is technological opportunity (Scherer, 1967; Levin et al., 1985; Hashmi, 2013), yet recent theoretical developments have typically focused on competition (Aghion et al., 2009, 2005) rather than contestability. Models of the relationship between competition and innovation or growth may overlook an alternative perspective where barriers to entry prevent markets from being contestable. The opportunity for a new firm to enter, contest and (eventually) replace an incumbent, incentivises innovation by both entrants and incumbents.

In deed, empirical research on specific barriers to entry and the impact of deregulation has highlighted how removal or simplification of regulatory barriers to enable entry has stimulated investment (Djankov et al., 2002; Bertrand & Kramarz, 2002; Alesina et al., 2005; Djankov, 2008; Holmes, 1998). Similarly, the trade literature highlights the effect of international entry due to trade liberalisation on the investment behaviour of firms (Tybout et al., 1991; Pavcnik, 2002; Melitz, 2003; Trefler, 2004; Melitz & Ottaviano, 2008; Verhoogen, 2008; Navas & Licandro, 2011). While liberalisation of industries and trade has sometimes enabled new entry, barriers to entry may still prevent markets from being fully contestable. While these literatures focus on the effect of removing specific entry barriers such as trade or

[1] Source: Wall Street Journal (2015) Carnegie Mellon reels after Uber lures away researchers, http://www.wsj.com/articles/is-uber-a-friend-or-foe-of-carnegie-mellon-in-robotics-1433084582. Accessed November 2016.

industry regulation, this paper examines a model that provides a general understanding of the impact of barriers to entry on innovation by using a novel, discrete entry approach.

Often the endogenous growth literature uses a distance to frontier or technological catch-up approach (Acemoglu et al., 2006; Aghion et al., 2009). This means entrants require a higher entry cost than incumbents in order to match existing technology. While higher entry costs may usually be thought of as a barrier to entry, the typical assumptions of entry costs in endogenous growth models simply set an arbitrary investment cost for entrants who expect to earn a return on their investment, with otherwise entirely contestable markets. A true *structural* barrier to entry occurs where there is a limit on some factor of production that is required for entry, enabling only a limited number of firms. Whether this factor is taxi licensing, viable commercial real estate locations or any other factor of production, the limitation enables the lucky few firms with access to sufficient quantities of the relevant factor of production to enter the market and restricts contestability from further firms. This is the unique and novel approach taken in this paper to understand how barriers to entry affect innovation and growth.

The simplest of entry barriers is assumed: firms can enter a market if there is enough space for an entire profitable firm. Market participants can use incremental innovation to block market entry by a discrete marginal entrant and thereby ignore potential entrants when choosing how much to invest in innovation. As a result, firms make strategic decisions based on the actions of actual rivals rather than potential entrants because marginal entry is not contestable. There are no assumptions about markets being served by a continuum of firms or a portion of a firm. The simplicity of this approach means that the size of the resource base (the supply of specialist labour in this case) becomes an inverse measure of barriers to entry and can be used to calibrate the level of contestability in each market, similar to Krugman's (1982) approach to studying comparative advantage. With this approach, one specialised factor of production that is required for entry pins down the number of entrants while the price of all residual factors of production falls in order to clear the market. While using only one factor of production in this paper simplifies the analysis, the technique may also be extended to examining specific barriers related to other factors of production such as licensing requirements, network externalities, high exit costs, quotas or otherwise. As a result, innovation is lower than in endogenous growth models, where competition is a continuous variable and markets are assumed to be contestable, particularly when sector parameters limit contestability in that sector.

This approach is also grounded by anecdotal evidence from real markets. In many sectors, entry at a viable scale may only be available to a few firms who may be able to maintain dominant market positions and deter potential new entrants, despite limited investment in innovation or minimal competition on price. For example, in many countries retail grocers are typically dominated by only a few firms. Despite some cost-saving innovations in the form of self-checkouts or online shopping, these innovations were not brought about by new entrants and have appeared slower than similar innovations in many other industries. Competition appears intense on price, but this is related to market power in supply relationships rather than competitive pressure that reduces profit margins (Mills, 2003; Bonanno & Lopez, 2012). For supermarkets, the factor of production that limits entry to one or a few firms could be the stores' supply chain networks and store locations such that supermarkets hold market power for other factors of production which fall in price in order to clear the market. Similarly, a few airlines typically dominate each geographical market around the world (Berry, 1992) and, until the entry of "low cost carriers", airlines were able to deter entrants with little improvement to quality or reduction in costs. Incumbent airlines may be able to restrict entry by limiting access for other firms to use airport gates or offer convenient flight times. Firms in the airline and supermarket industries are characteristically large scale enterprises with a particular factor of production (supply chain and store locations or airport gate access) that can be restricted to only a few firms. All of these industries may have high set-up costs, economies of scale, some network effects and perhaps they are industries where firms' reputations have a significant role. These may be characteristics which contribute to preventing entrants from developing innovations in order to enter the market and perpetuate the dominant market positions of incumbents. A minimum viable scale may limit the potential for entry to play its assumed contestability role in general equilibrium models such as those used by endogenous growth theory and competition regulators (Shapiro, 2012).

Alternatively, explicit barriers to entry such as taxi licensing requirements, local hotel room taxes and zoning regulations work in the same way, as a specific factor of production required for entry such that a marginal entrant is prevented from contesting the market. These entry barriers may have allowed both the taxi and hospitality industries to pursue less innovation than would have occurred in a fully-contestable environment. It is only recent innovations in information technology that has invaded other industries allowing Uber and Airbnb to bypass these regulatory barriers (Ranchordas, 2014). The internet has also reduced barriers to entry in the music industry (Lewis et al., 2005) and enabled a contest to re-emerge evidenced by the entry of Spotify, iTunes and Tidal. These innovations were largely unexpected by the incumbent taxi, hospitality or recording industries because the threat of entry was previously able to be ignored. Transport, hospitality and music businesses will no doubt now also respond with new innovations of their own such as improved booking and dispatch systems or be replaced altogether by other businesses that develop better quality products. It is the ability to contest, not competition, that encourages innovation.

Many areas of economics have contributed to understanding the effect of barriers to entry on innovation. The industrial organisation literature has long considered the incentives for investing in innovation, particularly examining the role of rivals (both actual and potential).[2] These microfoundations are fundamental to general equilibrium models of long-run growth. When making the leap from microeconomic models to aggregate models of endogenous growth, academics often rely on the role of potential rivals by assuming the market is contestable, free and continuous. However, this approach ignores the strategic aspects of competition when markets are not contestable and therefore under-appreciates the wider economic implications of barriers to entry for innovation and growth.

Endogenous growth theory allows incumbents to prevent entry by using innovation to keep ahead of a potential rival, but true barriers to entry may allow market participants to ignore potential entrants. Peretto (1996; 1999; 2003) considers the effect of oligopoly on innovation and growth but these models do not consider the direct relationship between barriers to entry or the form of competition with innovation and endogenous growth outcomes. Research closely related to the model here is found in Acemoglu & Cao (2015)

[2] Examples of industrial organisation literature on the incentives of firms to invest in innovation include but are not limited to Gilbert & Newbery (1982); Fudenberg et al. (1983); Reinganum (1983); Salant (1984); Harris & Vickers (1985); Vickers (1985); Reinganum (1989).

with a model of rivalry between entrants and incumbents in a quality ladders model of growth where the market is contestable for entrants with a drastic innovation. Similarly, Klette & Kortum (2004) develop a model of innovation that accounts for the persistence of incumbent innovation. A particularly notable example is Acemoglu & Akcigit (2012) which finds that an optimal intellectual property rights system should offer greater protection for technology leaders that are far ahead of their rivals than those that have close followers because this adds to the incentive for innovation investment to both contest and expand the technology threshold.

Nonetheless, these models all rely on a modelling assumption that the market is contestable. The competition and growth literature is grounded in technology race or distance-to-frontier models where the threat to incumbents from frontier entrants influences the innovation investment responses of incumbents to escape entry or escape competition (Aghion et al., 2001; Howitt & Mayer-Foulkes, 2005; Aghion et al., 2009; Acemoglu et al., 2006; Griffith et al., 2009). As a result, the inverted-U relationship between competition and innovation is now widely understood (Aghion et al., 2005). A re-interpretation of the inverted-U could imply a correlation between levels of competition and degrees of contestability such that contestability would be at least partially responsible for a positive relationship between competition and innovation. In effect, these distance-to-frontier models are really measuring the relationship between innovation and a specific barrier to entry: access to technology. Interpreted this way, entrants face an arbitrary entry cost but markets are still contestable. *This paper develops a more generalised understanding of how innovation is affected when markets are not contestable, which until now has remained unclear.*

The model here is also related to the large industry deregulation and trade liberalisation literatures (including but not limited to Tybout et al. (1991); Holmes (1998); Pavcnik (2002); Bertrand & Kramarz (2002); Djankov et al. (2002); Neary (2002); Melitz (2003); Trefler (2004); Alesina et al. (2005); Melitz & Ottaviano (2008); Verhoogen (2008); Djankov (2008)) from which some findings are comparable to the model's results and add robustness to the conclusions. In particular, Navas & Licandro (2011) examines the impact of trade liberalisation on cost reducing innovation, finding similarly that increases in competition are positive for innovation and that Bertrand oligopoly provides a stronger incentive for innovation than Cournot. Provided there is positive growth, it follows that policies to increase contestability, either through trade liberalisation (Navas & Licandro, 2011) or by reducing the entry barrier (in the model here), have a greater impact on industries characterised by Cournot competition than Bertrand, as industries characterised by Bertrand are already closer to the freely-contestable limit rate of innovation. However, by using a novel discrete entry approach, the model here is intended to provide a general understanding of the impact of barriers to entry on innovation. Interestingly, the comparison between modes of competition in the model here is more nuanced because the relevant variable is contestability rather than entry or competition. The discrete entry approach modelled in this paper enables a novel direct comparison between barriers to entry and innovation, rather than focusing on the number of competing firms. As shown in this paper, Cournot oligopoly allows many more firms to enter than industries characterised by Bertrand oligopoly for the same resource constraint on entry because Cournot entrepreneurs withhold a larger share of profit by investing less in innovation. Yet the incentive provided by additional entry does not compensate entirely for the effect of Cournot.

The paper is set out as follows. Section 2 fully specifies the model including consumer preferences, technological development, the supply of labour and discrete market entry. Subsequently, Section 3 derives expressions for equilibrium prices, wages and production, steady state rates of innovation in each sector and economy-wide growth rates. In particular, Section 3 compares innovation outcomes under Cournot and Bertrand oligopoly in the presence of barriers to entry providing a proof that Bertrand oligopoly is always more innovative than an equivalent Cournot market. Section 4 describes a series of simulations as a numerical exercise to further examine the relationship between barriers to entry with innovation and growth under and reconciles the model's findings with other literature. Lastly, Section 5 concludes the paper.

2 Model specification

This section specifies the structure of the model and its parameters. The representative agent consumes varieties from all sectors and prefers variety in each sector. Each variety is produced using a sector specific labour supply by the quality leading firm that developed a quality improvement for that specific variety in the previous period. Only a discrete number of firms are permitted entry such that innovation effort can be limited to only those firms that are lucky enough to secure entry. In each period, the quality leader produces variety i, j as a monopolist competing with other differentiated varieties in sector i and, if it is going to enter in the following period, the firm employs entrepreneurial workers to ensure a quality improvement large enough to maintain entry in its niche monopoly position in the following period and prevent entry of a marginal firm producing a new variety. Firms that are permitted entry pay at least a competitive wage to their entrepreneurs for the required innovation effort plus a monopoly profit due to their advantageous market position.

2.1 Preferences

The quality ladders approach of Grossman & Helpman (1991) is extended by using a constant elasticity of substitution (CES) utility function in each sector nested in an upper level Cobb-Douglas function to allow for many industrial sectors of several varieties each. The number of sectors is assumed to be fixed such that there are always N sectors, but the total number of varieties in each sector is determined by sector specific parameters. For simplicity, all consumers are assumed to have the same preferences, even if workers have different wages. The representative consumer has a taste for variety in each sector and consumes products from all sectors. Consumers have the following intertemporal preferences:

$$U = \sum_{t=0}^{\infty} \alpha^t \ln Q_t, \quad \alpha = 1/(1+\rho), \tag{1}$$

where ρ represents the consumers rate of time preference and Q_t is Cobb-Douglas consumption of manufactured goods from N sectors in period t:

$$Q_t = \prod_{i=1}^{N} c_{i,t}^{\frac{1}{N}}. \tag{1a}$$

Varieties in each sector are neither complements nor substitutes (an elasticity of one from Cobb-Douglas utility in Equation 1a) such that a change in the price of a variety in one sector has no effect on demand for varieties in other sectors and each sector has a constant expenditure share of $1/N$. Consumers have a constant elasticity of substitution between varieties (indexed by j) in the same sector (indexed by i):

$$c_{i,t} = \left[\sum_{j \in i} \left(A_{i,j,t} c_{i,j,t} \right)^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}}, \quad \sigma_i > 1 \forall i = 1, ..., N \tag{1b}$$

where $A_{i,j}$ represents the symmetric elasticity of variety j in sector i. The elasticity of substitution between varieties in the same sector is σ_i for industry i. Each sector i and has a fixed number of firms determined by consumer preferences (σ_i), the supply of specialised labour to the sector (L_i) and the criterion that only a discrete number of firms may enter each market. Each variety j is produced by a single firm and there are no economies of scope. For simplicity, variety j in sector i is referred to as variety i, j.

Intertemporal utility optimisation implies the transversality condition and Euler equation $\frac{E_{t+1}}{E_t} = \frac{1+r}{1+\rho}$, where E_t is expenditure in period t, ρ is the rate of time preference and r is the rate of return on savings between period t and $t+1$. Rearranging gives:

$$\frac{E_{t+1}}{1+r} = \frac{E_t}{1+\rho} = \alpha E_t. \tag{2}$$

Expenditure is normalised in each period to $E_t = 1 \forall t$. For the rest of the paper the subscript t is suppressed where the time dimension is the same for all variables and its inclusion is unnecessary.

2.2 Labour

For simplification, labour is the only factor of production. The structural barrier to entry is scarcity of one or more factors of production required for entry in order to limit contestability. Fixed costs or other usual entry costs should not be thought of as barriers to entry unless access to the factor of production is restricted to a few "lucky" firms. By assuming that labour is the only factor of production the discrete entry approach requires an additional assumption that each sector has its own specialised but homogeneous labour supply that restricts the number of firms permitted to enter. The labour supply can therefore be thought of as a parameter that determines contestability or as an inverse measure of barriers to entry. Workers differ by their sector-specific skills and supply their labour inelastically only to firms in the sector that requires their specific skills, working as either entrepreneurs or in manufacturing. Entrepreneurs develop quality improvements for their firm to enter the market and earn a profit in the subsequent period in order to remunerate entrepreneurs for developing an innovation. Entrepreneurship can be thought of as more than R&D because these workers perform both a research and commercialisation role.

Following the approach of Krugman (1982) each industrial sector is assumed to have an exclusive labour supply L_i where workers workers inelastically provide one unit of labour per period. Workers are immobile between sectors, but those workers are mobile between firms or varieties (j) within their sector (i) and mobile between employment in entrepreneurship or manufacturing if there is employment available. This assumption implies workers' skills are industry-specific and it is difficult for workers to re-skill for employment in other sectors. This assumption is not based on an empirical justification but on the discrete entrant method for implementing barriers to entry. This limit could otherwise be placed on any specialised factor of production to limit a marginal firm from entering the market but for simplicity the model assumes labour as the only factor of production.

In each industrial sector, the labour supply may be written as

$$L_{i,t} = \sum_{j\epsilon n_{i,t}, n_{i,t+1}} l_{i,j,t} \quad \forall i = 1, ..., N, \tag{3}$$

where $l_{i,j,t}$ is the labour employed in entrepreneurship and manufacturing in period t by the firm producing variety i, j and $n_{i,t}$ and $n_{i,t+1}$ are the discrete number of firms in sector i in periods t and $t+1$ respectively. Both the number of varieties in sector i in period t ($n_{i,t}$) and period $t+1$ ($n_{i,t+1}$) are required since each firm employs entrepreneurs to produce quality improvements for the following period and workers in manufacturing to produce the current versions. This incorporates the possibility that a firm producing variety i, j could employ only manufacturing workers (withdrawing from the market in period $t+1$), only entrepreneurs (in order to enter the market in period $t+1$) or both (in order to produce in both period t and $t+1$).

Since the focus of this paper is on the relationship between contestability and innovation, rather than the labour market, it is assumed that the wages of workers in a particular industry are not affected by the market power of employers in the employee-employer relationship.[3] However, wages are affected by the nature of competition in product markets as this determines the revenue to the firm. The calibration of each specialised labour supply can be used to calibrate contestability in each sector in order to compare the level of innovation under different competitive outcomes. Industries with a large labour supply will have a greater ability for firms to enter and contest the market. This allows a direct comparison of the level of contestability or barriers to entry and innovation, without altering demand parameters such as elasticities.

Production involves a fixed labour cost $F_{i,j,t-1}$ in the period prior to production and a constant marginal labour cost of β. For all sectors $i = 1, 2, 3, ..., N$ and varieties $j = 1, 2, 3, ..., n_i$, the labour required by each firm in period t is given by:

$$\begin{aligned} l_{i,j,t} &= 0 && if\ c_{i,j,t} = 0\ and\ c_{i,j,t+1} = 0 \\ l_{i,j,t} &= \beta c_{i,j,t} && if\ c_{i,j,t} > 0\ and\ c_{i,j,t+1} = 0 \\ l_{i,j,t} &= F_{i,j,t} && if\ c_{i,j,t} = 0\ and\ c_{i,j,t+1} > 0 \\ l_{i,j,t} &= F_{i,j,t} + \beta c_{i,j,t} && if\ c_{i,j,t} > 0\ and\ c_{i,j,t+1} > 0. \end{aligned} \tag{3a}$$

where $F_{i,j,t}$ is the fixed cost in period t of developing a quality improvement for production in period $t+1$ and $c_{i,j,t}$ is the period t production of variety i, j at its existing quality level that was developed in the previous period $t-1$.

[3]Of course, it is entirely possible that workers who have very specific skills in an industry dominated by only a few employers could face non-competitive wages (Bhaskar et al., 2002), but monopsony or oligopsony pressure on the labour market is beyond the scope of this paper.

2.3 Technology

Growth is modelled using the quality improving innovation approach of Young (1998). This allows a fair comparison of sectors, because the scale effect is removed without assuming an inverse scale assumption.[4] Holding all else constant, innovation in Young (1998) with continuous free entry would have the same rate of quality improvement and growth for all sectors. Differences between sectors in rates of innovation may arise because of differences in the nature of competition, barriers to entry, and consumer preferences that determine the competitive equilibrium, but not because of scale assumptions. This form of growth allows the model to exclusively focus on examining the impact of barriers to entry or contestability upon innovation activity.

Endogenous growth models can either model innovation as stochastic, whereby entrepreneurial effort increases the probability of success, or as deterministic where entrepreneurial effort increases the size of a quality improvement. Reinganum (1983) reconciles the stochastic and deterministic approaches by suggesting that the deterministic model is most appropriate for incremental innovations where the incumbent can minimise risk by aiming for incremental innovations with a high enough probability it can be thought of as a certainty. For simplicity, the model here maintains the deterministic approach for modelling elegance, following the incremental innovation approach but growth models with stochastic innovations could also be examined. In the deterministic model it does not matter whether the quality improvement is developed by an incumbent or entrant, the important factor for this model is that the number of firms that will enter in each period is discrete.

The entrepreneurial labour requirement to achieve the targeted quality level $A_{i,j,t}$ for variety i, j and the fixed cost incurred in the previous period $t-1$ for production in period t is given by:

$$F_{i,j,t-1}(A_{i,j,t}, \bar{A}_{i,j,t-1}) = \begin{cases} \gamma e^{\eta A_{i,j,t}/\overline{A}_{i,j,t-1}} & \text{if } A_{i,j,t} \geqslant \overline{A}_{i,j,t-1} \\ \gamma e^{\eta} & \text{otherwise} \end{cases} . \tag{4}$$

The parameters γ and η are constants used for calibration. $\overline{A}_{i,j,t-1}$ is an index of technological opportunity for variety i, j, representing the intertemporal spillover of knowledge available to variety i, j entrepreneurs. The index of technological opportunity is simply the highest existing quality level for variety i, j. Each sector has symmetric firms such that all varieties in a single sector have the same quality level. The fixed cost can be thought of as two components: a standard fixed cost of γe^{η} irrespective of quality improvement and a research cost of $\gamma e^{\eta A_{i,t}/\bar{A}_{i,j,t-1}} - \gamma e^{\eta}$.

For an entrepreneur to develop a greater quality improvement further entrepreneurial effort must be employed as governed by Equation 4, although it requires sharing profits with these additional entrepreneurial partners, as they too would have the option of developing a quality improvement in an alternative product if there is enough space in the market for an additional firm. Therefore, entrepreneurs face a trade-off between sharing profits with additional partners or allowing more rival firms to profitably enter the market.

[4]Examination of typical endogenous growth models reveals a distortion from scale effects and the inverse scale assumptions that are typically used to remove the scale effect (Bond-Smith et al., 2016). Young (1998) removes scale effects without making an inverse scale assumption. See (Jones, 1999) for a survey and comparison of endogenous growth models with scale effects, semi-endogenous growth and fully-endogenous growth models without scale effects.

Entrepreneurs may earn greater wages than manufacturing workers as discrete entry creates a barrier to workers participating in the labour market as entrepreneurs, with manufacturing wages falling until the sector's labour market clears. While the firm earns a monopoly profit, this profit is not retained within the firm but shared between entrepreneurs as a premium above the competitive wage. In models that extend this approach to other barriers to entry, this premium would be assumed to apply to whomever controls the relevant factor of production required for entry.

2.4 Barriers to entry

The model here uses a simple discrete entry barrier that allows entry by only an integer number of firms that meet a requirement for positive profits. A firm can only enter the market if there is enough space for an additional discrete firm to profitably enter. Prior to specifying how potential entry for new firms may occur, I first consider the concept of modelling barriers to entry.

In an economic modelling sense, *free* entry can be thought of as an expectation that potential entrepreneurs will pursue profitable opportunities and enter profitable segments of the market. This potential for entry means a market is contestable, even if entrants face varying or possibly higher entry costs than incumbents. A *structural barrier to entry* in economic modelling terms represents a barrier to even the potential for additional entry. With this understanding of barriers to entry, the existing literature may be inadequate to understand the relationship between contestability and innovation.

The discrete entry assumption is relatively straightforward and common to the industrial organisation literature (Bain, 1956). Examples from the industrial organisation literature on the incentives of firms to invest in innovation include but are not limited to Gilbert & Newbery (1982); Fudenberg et al. (1983); Reinganum (1983); Salant (1984); Harris & Vickers (1985); Vickers (1985); Reinganum (1989). Reiss & Spiller (1989) and Berry (1992) both examine price competition in airline markets where the incumbent firms price strategically to deter discrete entrants. These microfoundations are an important element of understanding macroeconomic growth that may be missing from existing endogenous growth models. Models with discrete entry are also not completely new to macroeconomics or international trade.[5] For example, Eaton et al. (2013) connect the microeconomic characteristic of discrete entry within an international trade model to explain international trade patterns. Similarly, discrete entry is a characteristic of a series of trade models that investigate export subsidies to a domestic producer to deter entry of a foreign business (Spencer & Brander, 1983; Brander & Spencer, 1985). To the best of the author's knowledge, this paper presents the first model that examines a generalised relationship between barriers to entry and innovation using an approach relying on discrete entry. The approach is well-grounded in the industrial organisation and trade literature so extending it to macroeconomic growth based on the same microfoundations is therefore an intuitive and relevant step that yields interesting and elegant insights.

The discrete entry barrier is made up of two key assumptions: (1) Only an integer number

[5]The idea for discrete entry in relation to growth as developed in this paper was developed following a discussion with Prof Sam Kortum who used a trade model with an integer number of firms to explain the zeros commonly found in trade data while maintaining other desirable model characteristics (Eaton et al., 2013), even though typical trade models would predict at least some minuscule level of trade with every country.

of firms may participate in any sector; and (2) that some specific factor of production is *required* as part of the cost of entry. The second assumption means the factor of production for entry cannot substituted for an alternative. It is this factor of production that pins down the number of entrants in a particular sector, such that once entry is determined, market participants do not need to consider the threat of a marginal entrant. In the model here, the scarce factor of production is the specialised labour force for each sector. Extensions of the model could include a different entry-limiting factor of production or specific barrier to entry but the same implications for contestability would arise. In the simplest example, it may be a regulatory requirement for firms to hold a license to enter a particular market. In this case, the cost of a license would attract a premium as part of a firm's fixed cost of production, pinning down the number of firms to the number of available licenses in each sector. The price of other factors of production would then fall until each factor market clears. A less obvious example is a heterogeneous labour market where entrepreneurial skills are scarce but required for profitable entry. This would allow a few firms to enter with those lucky entrepreneurs that find success attracting a substantial premium while the wage rate for unlucky entrepreneurs and the rest of the labour supply diminishes until the market clears. In the model here a specialised but homogeneous labour supply is the only factor of production but the implication is the same. Once entry is determined, those workers with the luck to be entrepreneurs earn a premium while the equilibrium wage for other workers diminish until the market clears. The results of the model here can therefore be understood as applying to barriers to entry in general.

To facilitate the potential for new entrant firms in new varieties that did not produce in the previous period, a comparable index of technological opportunity is assumed. If there is space in the market, it is assumed that the new entrant creates a new variety, where the knowledge input to innovation is an average of quality levels for its particular industry. This maintains symmetry in each industrial sector even if a new variety is introduced. The index of technological opportunity for a firm with a new variety is given by $\overline{A}_{i,j,t-1} = \frac{\sum_{j \in n_{i,t-1}} A_{i,j,t-1}}{n_{i,t-1}}$. If participants were to fail to innovate by enough (for any arbitrary reason) such that a marginal discrete firm could profitably enter the market it is possible that the new disequilibrium would result in negative profits for participating firms and income for entrepreneurs that is less than the market clearing wage for workers in manufacturing. Therefore, participants would be unable to employ the additional entrepreneurs required as workers would find themselves with higher wages by offering their labour in manufacturing. The disequilibrium would be corrected when a firm would inevitably exit the market by being unable to develop a sufficiently high quality level for entry because of a scarcity of entrepreneurs. This characteristic can be thought of as the market having partially free entry, in that (partial) contestability and entry are still characteristics, but the extent of contestability is determined by the availability of the entry-limiting factor of production, in this case the sector specific labour supply. It is this mechanism that blocks the marginal entrant.

3 Equilibrium and the steady state

Equilibrium wages, prices and production follow from typical optimisation of CES sectors adjusted for the number of firms permitted entry in each sector. In equilibrium the specialised labour supply pins down the number of firms in each sector. While the exact number of firms is yet to be determined, all model parameters are solved as functions of the number of entrants and subsequently the number of entrants can be solved and defined. Profits must be positive, but are diminished to zero not by competitive pressure, but by entrepreneurs appropriating all profit in the entrepreneurial "dividend". Those workers who are lucky enough to be employed as entrepreneurs can extract a "dividend" above the competitive wage while the wages of all remaining workers employed in manufacturing only falls in order to clear the market.

Firms choose prices or the quantity supplied to the market and a level of investment in quality-improving innovations based on the representative consumer's taste for quality, the expected actions of other participating firms and the form of competition, Cournot or Bertrand. Usually in CES models, assuming free entry means both Cournot and Bertrand competition yield the same results, because the number of firms is assumed very large or continuous. However, with discrete entry, outcomes differ by the form of competition. Cournot and Bertrand approaches are also extended to determine equilibrium innovation investment. If the profit maximising output is determined by Cournot competition, it is assumed that firms choose both a level of quality improvement and output given consumers' willingness to pay for higher quality products and to clear the market. Alternatively, if the profit maximising price is determined by Bertrand competition, it is assumed that firms choose both a level of quality improvement and prices given consumers' quantity demanded in response to higher quality products and proposed prices in order to clear the market.

3.1 Demand

Consumers allocate expenditure across sectors and varieties subject to the budget constraint $\sum_{i\in N} P_i c_i \leq E$, where P_i is the price index of sector i (to be defined in Equation 6a) and c_i is a demand for all varieties in sector i (defined by Equation 1b). From Cobb-Douglas utility across sectors, expenditure per sector is:

$$c_i P_i = \frac{E}{N}, \tag{5}$$

such that the consumer spends a $\frac{1}{N}$ share of her expenditure on varieties in each sector i. Direct demand is required for determining equilibrium using Bertrand oligopoly while inverse demand is required for determining equilibrium using Cournot. Maximising the utility function finds that the direct demand function for each variety i, j is given by:

$$c_{i,j} = \frac{E}{N} A_{i,j}^{\sigma_i - 1} P_{i,j}^{-\sigma_i} P_i^{\sigma_i - 1} \tag{6}$$

where $P_{i,j}$ and $A_{i,j}$ are the price and quality level of variety i, j respectively. P_i is the sector i index of price and quality defined by:

$$P_i = \left[\sum_{j\in i,} A_{i,j}^{\sigma_i - 1} P_{i,j}^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}}. \tag{6a}$$

Alternatively, the inverse demand function can be found by optimising for the indirect utility function such that inverse demand in each sector is given by:

$$P_{i,j} = A_{i,j}^{\frac{\sigma_i - 1}{\sigma_i}} c_{i,j}^{\frac{-1}{\sigma_i}} \frac{E}{N} \left[c_i^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{-1}, \tag{7}$$

where c_i is specified in consumer preferences (Equation 1b). In either form of competition, the definition of the price index is the same for both Bertrand and Cournot competition, since it is defined by the budget constraint $\sum_{i \epsilon N} c_i P_i \leq E$.

The usual free entry condition, which implies that competition diminishes profits to zero due to free entry, does not apply. As there is discrete entry, an additional entrant is only permitted if there is enough space in the market for an entire profitable firm. Those firms that are permitted entry will earn positive profits and entrepreneurs withdraw profits in their own entrepreneurial earnings, if doing so does not permit an additional marginal firm to enter the market at the quality level required for entry. Therefore, the usual free entry condition is modified to an alternative "entrepreneurial profit condition" such that *entrepreneurs appropriate all profit and their dividends are always positive*. While the firm may make considerable profit with imperfect competition and barriers to entry, all profit is paid to entrepreneurs through the entrepreneur's "dividend" $d_{i,t}$ over and above the standard manufacturing wage that all workers earn, including entrepreneurs, such that no profit remains with the firm. The firm's profit function to be paid as dividends to entrepreneurs is given by:

$$\pi_{i,j,t} = \frac{\left(P_{i,j,t} - \beta w_{i,t}\right) c_{i,j,t}}{(1 + r_t)} - w_{i,t-1} F_{i,j,t-1} = \frac{d_{i,t}}{(1 + r_t)} F_{i,j,t-1}, \tag{8}$$

subject to the appropriate variety i, j demand function and the condition that $d_{i,t} \geq 0$. The sector i earnings in period $t - 1$ for each entrepreneur who developed the current quality level $A_{i,j,t}$ is $y_{i,E,t-1} = w_{i,t-1} + \frac{d_{i,t}}{(1+r_t)}$ and $w_{i,t}$ is the sector i wage of workers in period t in order to clear the sector i labour market. Period t revenues and costs are discounted one period, since investment and innovation effort occurs in the period prior to production and the entrepreneur's dividend is paid in the period when profits are earned.

The initial entrepreneur determines the firm's strategic decision to hire further entrepreneurs in order to achieve a higher quality improvement. Therefore the optimisation decision is modified such that these initial entrepreneurs attempt to maximise their income from both wages and dividends. Rearranging the entrepreneurial profit condition, the firms output, wage, innovation and pricing decisions are governed by the optimisation problem:

$$\max y_{i,E,t-1} = \frac{\left(P_{i,j,t} - \beta w_{i,t}\right) c_{i,j,t}}{(1 + r_t) F_{i,j,t-1}} \tag{8a}$$

subject to market demand and positive dividends to entrepreneurs.

Workers will choose to be entrepreneurs if the opportunity is available either from an existing entrant hiring more workers to join as entrepreneurs or by creating a new variety if there is space for an entire profitable firm. If no further entrepreneurial position is possible with a positive dividend, remaining workers are employed in manufacturing with manufacturing wages falling until the sector i labour market clears. As a result, workers who are entrepreneurs may have higher earnings than workers employed in manufacturing.

3.1.1 Cournot prices

In sectors characterised by Cournot oligopoly, the firm makes strategic choices of output and quality improvement on the basis of consumers' willingness to pay for that quantity and quality in order to clear the market. Entrepreneurs maximise their dividend given the inverse demand function (Equation 7). Differentiating Equation 8a with respect to the decision variables of output and quality respectively, the first order conditions for maximising entrepreneurial income are given by:

$$\frac{\partial y_{i,E,t-1}}{\partial c_{i,j,t}} = \frac{P_{i,j,t} - \beta w_{i,t}}{(1+r_t)\, F_{i,j,t-1}} + \frac{c_{i,j,t}}{(1+r_t)\, F_{i,j,t-1}} \frac{\partial P_{i,j,t}}{\partial c_{i,j,t}} = 0 \qquad \text{and} \tag{9}$$

$$\frac{\partial y_{i,E,t-1}}{\partial A_{i,j,t}} = \frac{c_{i,j,t}}{(1+r_t)} \left(\frac{\frac{\partial P_{i,j,t}}{\partial A_{i,j,t}} F_{i,j,t-1} - \left(P_{i,j,t} - \beta w_{i,t}\right) \frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}}}{\left(F_{i,j,t-1}\right)^2} \right) = 0. \tag{9a}$$

Rearranging Equation 9 gives:

$$P_{i,j,t} = \frac{1}{\left(1 + \frac{c_{i,j,t}}{P_{i,j,t}} \frac{\partial P_{i,j,t}}{\partial c_{i,j,t}}\right)} \beta w_{i,t}. \tag{9b}$$

Price is a function of marginal cost and the elasticity of substitution as in typical Dixit-Stiglitz models.

While the exact number of firms in each sector (n_i) is still to be determined, symmetry in each sector means the inverse demand function (Equation 7) for variety i, j can be written as:

$$P_{i,j} = \frac{E}{N} A_{i,j}^{\frac{\sigma_i - 1}{\sigma_i}} c_{i,j}^{\frac{-1}{\sigma_i}} \left(A_{i,j}^{\frac{\sigma_i - 1}{\sigma_i}} c_{i,j}^{\frac{\sigma_i - 1}{\sigma_i}} + (n_i - 1)\, A_{i,k}^{\frac{\sigma_i - 1}{\sigma_i}} c_{i,k}^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{-1}, \tag{9c}$$

where variety i, k represents each of the other symmetrical varieties in sector i. In contrast to typical Dixit-Stiglitz optimisation where n_i is assumed very large or a continuous function is used, it is no longer assumed that the number of firms is large enough for the firm producing variety i, j to not notice the effect of its own prices on the sector's price index. Differentiating with respect to consumption of variety i, j and simplifying the differential using symmetrical i, k varieties, the equilibrium price of variety i, j is evaluated by substituting the differential and the demand function into Equation 9b:

$$P_{i,j}(n_i) = \left(\frac{n_i \sigma_i}{(n_i - 1)(\sigma_i - 1)} \right) \beta w_i, \qquad n_i \geq 2. \tag{9d}$$

As n_i increases, the price of a firm's own variety has less effect on the sector's price index and therefore on the perceived elasticity of substitution until the familiar CES pricing rule is reached that is common to both Cournot and Bertrand competition $P_{i,j} = \beta w_i \left(\frac{\sigma_i}{\sigma_i - 1}\right)$.

In the case of $n_i = 1$ the firm would receive the same revenue irrespective of the if they are monopolists with Cobb-Douglas utility between sectors because consumers allocate a specific portion of expenditure on that sector as defined by Equation 5 so the equation breaks down. This limitation requires that $n \geq 2$. To include the $n = 1$ scenario would require the upper-level of the utility function to be an alternative such as a CES function. While this still provides elegant solutions for the relationship between price and the number of firms even for $n_i = 1$, it makes labour market clearing less elegant, because determining wages in one sector would require including the prices and elasticities of all sectors. For simplicity, the Cobb-Douglas version is used because clearing the labour market only requires consideration of prices in the same sector but there is no loss of generality for the conclusions regarding the effects of competition.

3.1.2 Bertrand prices

In sectors characterised by Bertrand oligopoly, entrepreneurs make strategic choices regarding price and quality improvement on the basis of quantity demanded by consumers at the price and quality in order to clear the market. Entrepreneurs maximise their own income given the direct demand function in Equation 6. Differentiating Equation 8a with respect to the decision variables of price and quality respectively, the first order conditions are given by:

$$\frac{\partial y_{i,E,t-1}}{\partial P_{i,j,t}} = \frac{c_{i,j,t}}{(1+r_t)\,F_{i,j,t-1}\left(A_{i,j,t}\right)} + \frac{\left(P_{i,j,t} - \beta w_{i,t}\right)}{(1+r_t)\,F_{i,j,t-1}} \frac{\partial c_{i,j,t}}{\partial P_{i,j,t}} = 0 \qquad \text{and} \tag{10}$$

$$\frac{\partial y_{i,E,t-1}}{\partial A_{i,j,t}} = \frac{\left(P_{i,j,t} - \beta w_{i,M,t}\right)}{(1+r_t)} \left(\frac{\frac{\partial c_{i,j,t}}{\partial A_{i,j,t}} F_{i,j,t-1} - c_{i,j,t} \frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}}}{\left(F_{i,j,t-1}\right)^2} \right) = 0. \tag{10a}$$

Rearranging Equation 10 gives:

$$P_{i,j,t} \left(\frac{c_{i,j,t}}{P_{i,j,t}} \frac{1}{\frac{\partial c_{i,j,t}}{\partial P_{i,j,t}}} + 1 \right) = \beta w_{i,t}. \tag{10b}$$

With discrete firms and symmetry in each sector, the direct demand function can be written as:

$$c_{i,j}\left(n_i\right) = \frac{E}{N} A_{i,j}^{\sigma_i - 1} P_{i,j}^{-\sigma_i} \left[P_{i,j}^{1-\sigma_i} A_{i,j}^{\sigma_i - 1} + \left(n_i - 1\right) P_{i,k}^{1-\sigma_i} A_{i,k}^{\sigma_i - 1} \right]^{-1}. \tag{10c}$$

Differentiating demand with respect to the price of variety i, j and simplifying the differential using symmetrical varieties, price can be evaluated by substituting the differential and direct demand function into Equation 10b:

$$P_{i,j}(n) = \left(\frac{\sigma_i n_i - (\sigma_i - 1)}{(n_i - 1)\,(\sigma_i - 1)} \right) \beta w_i, \qquad n \geq 2. \tag{10d}$$

As with Cournot competition, when n_i increases, the price of a firm's own variety has less effect on the sector's price index and therefore on the perceived elasticity of substitution. As the number of firms tends to infinity, the familiar CES pricing rule $P_{i,j} = \beta w_i \left(\frac{\sigma_i}{\sigma_i - 1} \right)$ is also reached with Bertrand competition.

3.2 Labour market clearing

Since β units of labour are required for each unit of production, the labour used in manufacturing in sector i simply equals the number of units consumed. Labour required can therefore be found by dividing consumer expenditure in sector i by the symmetrical price per unit and multiplying by β:

$$l_{i,m,t} = \frac{E_t}{NP_{i,j,t}}\beta. \tag{11a}$$

Labour employed as entrepreneurs in period t equals the number of entrants in the coming period multiplied by the investment in quality improvement per firm,

$$l_{i,I,t} = n_{i,t+1}F_{i,j,t}. \tag{11b}$$

Labour market clearing requires that the total amount of labour used in period t in manufacturing and entrepreneurship in each sector is equal to the total amount of specialised sectoral labour available. Total labour in each sector is therefore equal to:

$$L_{i,t} = n_{i,t+1}F_{i,j,t} + \frac{E_t}{NP_{i,j,t}}\beta. \tag{11c}$$

3.2.1 Manufacturing labour

Substituting Cournot prices into Equation 11c, the labour employed in sector i, characterised by Cournot oligopoly, is given by:

$$L_{i,t} = n_{i,t+1}F_{i,t} + \frac{E_t\left(n_{i,t}-1\right)\left(\sigma_i-1\right)}{Nn_{i,t}\sigma_i w_{i,t}}. \tag{12}$$

Solving for Cournot manufacturing wages gives:

$$w_{i,t} = \frac{E_t\left(n_{i,t}-1\right)\left(\sigma_i-1\right)}{Nn_{i,t}\sigma_i\left(L_{i,t}-n_{i,t+1}F_{i,t}\right)}. \tag{12a}$$

Alternatively substituting Bertrand prices into Equation 11c, the labour employed in sector i, characterised by Bertrand oligopoly, is given by:

$$L_{i,t} = n_{i,t+1}F_{i,t} + \frac{E_t\left(n_{i,t}-1\right)\left(\sigma_i-1\right)}{N\left(\sigma_i n_i-\left(\sigma_i-1\right)\right)w_{i,t}}. \tag{13}$$

Rearranging, Bertrand manufacturing wages are given by:

$$w_{i,t} = \frac{E_t\left(n_{i,t}-1\right)\left(\sigma_i-1\right)}{N\left(\sigma_i n_i-\left(\sigma_i-1\right)\right)\left(L_{i,t}-n_{i,t+1}F_{i,t}\right)}. \tag{13a}$$

After paying for manufacturing costs all remaining profits are paid to entrepreneurs.

Workers are mobile between employment in manufacturing or as an entrepreneur and there are no barriers to entering the labour market as a manufacturing worker. However, there are barriers to working as an entrepreneur, since each sector is restricted to a discrete number of firms and being an entrepreneur either requires a business to hire more entrepreneurs or space in the market for an additional firm to enter. Therefore, entrepreneurs dividends must be positive, such that their earnings are never less than the wages of manufacturing workers, because workers will switch to the manufacturing role if dividends were negative, but the reverse process may only be possible if there is space for an additional discrete firm. In equilibrium, labour market clearing and discrete entry requires that $d_{i,t} \geq 0$.

3.2.2 Entrepreneurial income and Cournot innovation

The first order condition for entrepreneurs choosing the quality improvement that maximises earnings was given in Equation 9a. Rearranging and dividing both sides by $A_{i,j,t}$ gives:

$$\frac{F_{i,j,t-1}}{A_{i,j,t}} \frac{1}{\frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}}} = \frac{\left(P_{i,j,t} - \beta w_{i,t}\right)}{A_{i,j,t}} \frac{1}{\frac{\partial P_{i,j,t}}{\partial A_{i,j,t}}} \quad . \tag{14}$$

As in Young (1998) entrepreneurs select a quality level where the elasticity of the research cost with respect to quality equals the elasticity of inverse demand (in terms of willingness to pay) with respect to quality. However, with a few discrete firms, the firm's perceived elasticity of substitution includes its own effects on the sector's price-quality index. Differentiating Equations 4 and 9c, substituting the differentials into Equation 14, evaluating $\beta w_{i,t}$ using Cournot prices (Equation 9d) and substituting the demand and cost of innovation functions, the entrepreneur's income-maximising quality target is given by:

$$A_{i,j,t} = \overline{A}_{i,j,t-1} \frac{(\sigma_i - 1)\left(n_{i,t} - 1\right)}{\eta\left(n_{i,t} + (\sigma_i - 1)\right)}. \tag{14a}$$

As in Young (1998) parameters are assumed such that there is always growth in the fully competitive market. That is $\frac{\sigma_i - 1}{\eta} > 1$.

However, it is possible under Cournot competition that strategically responding to a few discrete rivals would result in no growth in the steady state because $\frac{(\sigma_i-1)(n_{i,t}-1)}{\eta(n_{i,t}+(\sigma_i-1))} \leq 1$ for some low levels of n_i. Solving for n_i, growth will not occur if $n_{i,t} \leq \frac{(\eta+1)(\sigma_i-1)}{(\sigma_i-1)-\eta}$. That is, just by entering the market at the minimum fixed cost, participating firms are able to deter the marginal entrant, even without developing a quality-improvement. In these cases, the quality target is given by the existing quality level $\overline{A}_{i,j,t-1}$ and there is zero growth.

Substituting the quality target in Equation 14a into the innovation function (Equation 4), the entrepreneurial effort required per firm is given by:

$$F_{i,j,t-1} = \begin{cases} \gamma e^{\frac{(\sigma_i-1)(n_{i,t}-1)}{n_{i,t}+(\sigma_i-1)}} & \text{for } n_{i,t} > \frac{(\eta+1)(\sigma_i-1)}{(\sigma_i-1)-\eta} \\ \gamma e^{\eta} & \text{otherwise.} \end{cases} \tag{14b}$$

As n_i increases, the innovation target and investment in innovation tends towards the results in the continuous free entry model in Young (1998).

Substituting the research cost into Equation 8a, including workers time preferences (Equation 2), wages defined by prices and taking the revenue of all firms in sector i as symmetric, the income of entrepreneurs in period $t-1$ from both wages and future dividends in sectors characterised by Cournot competition are given by:

$$y_{i,E,t-1} = \begin{cases} \frac{(n_{i,t}+\sigma_i-1)\alpha E_{t-1}}{N n_{i,t}^2 \sigma_i \gamma e^{\frac{(\sigma_i-1)(n_{i,t}-1)}{n_{i,t}+(\sigma_i-1)}}} & \text{for } n_{i,t} > \frac{(\eta+1)(\sigma_i-1)}{(\sigma_i-1)-\eta} \\ \frac{(n_{i,t}+(\sigma_i-1))\alpha E_{t-1}}{N n_{i,t}^2 \sigma_i \gamma e^{\eta}} & \text{otherwise.} \end{cases} \tag{15}$$

3.2.3 Entrepreneurial income and Bertrand innovation

The first order condition was given by Equation 10a. Rearranging and dividing both sides by $A_{i,j,t}$ gives:

$$\frac{F_{i,j,t-1}}{A_{i,j,t}} \frac{1}{\frac{\partial F_{i,j,t-1}}{\partial A_{i,j,t}}} = \frac{c_{i,j,t}}{A_{i,j,t}} \frac{1}{\frac{\partial c_{i,j,t}}{\partial A_{i,j,t}}}. \tag{16}$$

Differentiating Equations 4 and 10c, substituting the differentials and equations into Equation 16 and rearranging, the entrepreneur's earnings-maximising quality target is given by:

$$A_{i,j,t} = \overline{A}_{i,j,t-1} \frac{(\sigma_i - 1)(n_{i,t} - 1)}{\eta n_{i,t}}. \tag{16a}$$

As long as $\sigma_i > 2\eta + 1$, Bertrand competition always results in growth for any level of competition $n_i \geq 2$. For $1 < \sigma_i < 2\eta + 1$, the supply of labour must be large enough that $n_i > \frac{\sigma_i - 1}{(\sigma_i-1)-\eta}$ for growth to occur. Substituting the quality target into Equation 4, the entrepreneurial effort required per firm is given by the function:

$$F_{i,j,t-1} = \begin{cases} \gamma e^{\frac{(\sigma_i-1)(n_{i,t}-1)}{n_{i,t}}} & \text{for } n_{i,t} > \frac{\sigma_i-1}{(\sigma_i-1)-\eta} \\ \gamma e^{\eta} & \text{otherwise.} \end{cases} \tag{16b}$$

Similar to Cournot innovation, as n_i increases, the innovation target and investment in innovation tends toward the competitive level found in Young (1998).

Substituting into Equation 8a, including workers time preferences (Equation 2), wages as defined by prices and taking the revenue of all firms in sector i as symmetric the income of entrepreneurs in period $t-1$ from both wages and future dividends in sectors characterised by Bertrand oligopoly are given by:

$$y_{i,E,t-1} = \begin{cases} \frac{\alpha E_{t-1}}{N(\sigma_i n_{i,t} - (\sigma_i-1))\gamma e^{\frac{(\sigma_i-1)(n_{i,t}-1)}{n_{i,t}}}} & \text{for } n_{i,t} > \frac{\sigma_i-1}{(\sigma_i-1)-\eta} \\ \frac{\alpha E_{t-1}}{N(\sigma_i n_{i,t} - (\sigma_i-1))\gamma e^{\eta}} & \text{otherwise.} \end{cases} \tag{17}$$

3.2.4 Endogenous variety

As described above, entry by discrete firms and labour market mobility for entrepreneurs to pursue employment in manufacturing requires:

$$d_i = \geq 0. \tag{18}$$

Essentially, workers will enter the market as entrepreneurs, either with an existing firm or an entrant, until adding additional entrepreneurial effort or a marginal firm would result in the entrepreneurial wage falling below the manufacturing wage. Substituting Cournot manufacturing wages and entrepreneurial income (Equations 12a and 15 respectively), the positive dividend requirement (Equation 18) can be rearranged to:

$$L_{i,t} \geq \left(\frac{(n_{i,t} - 1)(\sigma_i - 1) n_{i,t+1}^2}{n_{i,t}(n_{i,t+1} + \sigma_i - 1)\alpha} + n_{i,t+1} \right) F_{i,t} \tag{19}$$

In the steady state, the number of firms is unchanging such that the largest integer $n_i \geq 2$ that satisfies

$$L_i \geq n_i \left(\frac{(n_i - 1)(\sigma_i - 1)}{(n_i + \sigma_i - 1)\alpha} + 1 \right) F_i \tag{19a}$$

implicitly defines the steady state number of firms in a sector characterised by Cournot competition. The Cournot model is now complete. The n_i which satisfies Equation 19a can be used to solve any variable.

Similarly, the positive dividend requirement in Equation 18 can be rearranged by substituting Bertrand manufacturing wages and entrepreneurial income (Equations 13a and 17) such that:

$$L_{i,t} \geq \left(\frac{(n_{i,t} - 1)(\sigma_i - 1)(\sigma_i n_{i,t+1} - (\sigma_i - 1))}{\alpha(\sigma_i n_{i,t} - (\sigma_i - 1))} + n_{i,t+1} \right) F_{i,t} \tag{20}$$

With a constant number of firms in the steady state, the largest integer $n_i \geq 2$ that satisfies

$$L_i \geq \frac{(n_i - 1)(\sigma_i - 1) + \alpha n_i}{\alpha} F_i \tag{20a}$$

implicitly defines the steady state number of firms in a Bertrand market. In the non-growth scenario for Bertrand competition, the number of firms can be explicitly defined as the largest integer $n_i \geq 2$ which satisfies:

$$n_i \leq \frac{\alpha L_i + (\sigma_i - 1)\gamma e^\eta}{((\sigma_i - 1) + \alpha)\gamma e^\eta} \tag{20b}$$

The n_i which satisfies Equation 20a (or for $n_i < \frac{\sigma_i - 1}{(\sigma_i - 1) - \eta}$ also satisfies 20b) can be used to solve for any variable.

3.2.5 Cournot versus Bertrand Innovation

In comparable markets with the same parameters, it can quickly be seen that the difference between quality targets for sectors characterised by Cournot or Bertrand oligopoly converges to zero as the number of firms tend to infinity because each converges to the steady state level of innovation that is found in the continuous free entry model. Similarly, Cournot and Bertrand models converge towards the equal outcomes as σ tends to one. However, the relationship between innovation targets for Cournot and Bertrand competition is not so clear with discrete entry for contestability below the continuous free entry model, given that varieties are substitutes ($\sigma > 1$). A direct comparison between the quality targets cannot be made because the number of firms is implicitly defined by the size of each specialised labour type L_i.

Intuition could imply that typically higher margins in a Cournot market would be an attractive prize for market participants and result in greater incentives to innovate. Furthermore, the larger denominator in Equation 19a compared to Equation 20a suggests that a Cournot market could be expected to have more firms in equilibrium and additional competition would also incentivise firms to pursue a higher quality target. Nonetheless, the presence of a fixed cost for all firms, γe^{η} and expectations of fewer firms in a Bertrand market, would mean that firms in the Bertrand market would produce quality improvements more efficiently by enabling a larger share of research labour to actually contribute to each quality improvement but it also enables a larger share of labour to be devoted to manufacturing than quality improvement. This section seeks to establish and understand the comparison of innovation between Cournot and Bertrand oligopoly:

Theorem. *Let all parameters be equal in two sectors characterised by Cournot and Bertrand oligopoly where varieties within each sector are substitutes. Participating firms in a sector characterised by Bertrand oligopoly always set a higher quality target than participants in a comparable sector characterised by Cournot oligopoly for all possible calibrations.*

Before proving the theorem we need the following Lemmas:

Lemma 1: *The difference between quality targets for two equal sectors governed by Bertrand and Cournot oligopoly is zero only if and only if* $\sigma_i = \frac{n_C-1}{n_B-1}$.

Proof: Examine the difference between the two quality targets for Bertrand and Cournot sectors given in Equations 16a and 14a respectively, with n_B and n_C describing the number of firms in Bertrand and Cournot sectors:

$$A_{Diff} = A_{Bertrand} - A_{Cournot} = \overline{A}_{i,j,t-1} \frac{(\sigma_i - 1)}{\eta} \left(\frac{(n_B - 1)}{n_B} - \frac{(n_C - 1)}{(n_C + (\sigma_i - 1))} \right). \qquad (21)$$

Setting this difference equal to zero and rearranging finds that in order for the difference to be zero, $\sigma_i = 1$ or:

$$\sigma_i = \tfrac{n_C-1}{n_B-1}. \qquad (22)$$

As the first of these options breaches the assumption that varieties in the same sector are substitutes, $\sigma_i > 1$, it can be seen that the difference in quality targets is only zero if the elasticity is equal to the ratio of $n_C - 1$ to $n_B - 1$. ||

Lemma 2: *Let* $\sigma_i = \frac{n_C-1}{n_B-1} + \varepsilon$. *A positive* ε *results in a sector with Bertrand oligopoly setting a higher quality target than a sector with Cournot oligopoly.*

Proof: Assuming that a Bertrand sector has the higher quality target so the difference between the two quality targets for Cournot and Bertrand sectors is positive and rearranging finds the two solutions for ε are both positive:

$$A_{Diff}\left(\sigma_i = \frac{n_C-1}{n_B-1}+\varepsilon\right) = \overline{A}_{i,j,t}\frac{\left(\frac{n_C-1}{n_B-1}+\varepsilon-1\right)}{\eta}\left(\frac{(n_B-1)}{n_B} - \frac{(n_C-1)}{\left(n_C+\left(\frac{n_C-1}{n_B-1}+\varepsilon-1\right)\right)}\right) > 0$$

$$\overline{A}_{i,j,t}\frac{\left(\frac{n_C-1}{n_B-1}+\varepsilon-1\right)}{\eta} > 0 \; or \; \frac{(n_B-1)}{n_B} - \frac{(n_C-1)}{\left(n_C+\left(\frac{n_C-1}{n_B-1}+\varepsilon-1\right)\right)} > 0 \tag{23}$$

$$\varepsilon > \frac{n_B-n_C}{n_B-1} > 0 \; or \; \varepsilon > 0$$

||

Proof of Theorem: n_C and n_B are not independent because they are both implicitly defined by the same parameters according to equations 19a and 20a. Making these definitions equal (based on the same L_i, thereby holding aside the integer requirement on n), substituting $n_B = \frac{1}{\sigma_i}(n_C+\sigma_i-1)$ (based on Lemma 1) and rearranging finds that $\sigma_i = \frac{n_C-1}{n_B-1}$ is only possible when:

$$\frac{(n_C-1)(\sigma_i-1)+\alpha(n_C+\sigma_i-1)}{\sigma_i\alpha} = n_C\left(\frac{(n_C-1)(\sigma_i-1)}{(n_C+(\sigma_i-1))\alpha}+1\right)$$
$$\sigma_i = 1, \tag{24}$$

which again breaches the assumption that varieties in the same sector are substitutes. Given varieties are substitutes ($\sigma_i > 1$) Lemma 2 always applies and ε must be positive. In comparable markets with the same parameters, σ_i is always greater than the ratio of $n_C - 1$ to $n_B - 1$. Combined with the proof of Lemma 2, this completes the proof.||

This result is comparable to Navas & Licandro (2011), but applies generally to barriers to entry, not just specific barriers such as trade. The result can be extended to understand the impact of barriers to entry in other models of innovation, competition or growth. The intuitive explanation is that firms in a Cournot market withhold investment in innovation in order to save the additional markup that is possible in a Cournot market compared to a Bertrand market. In doing so, firms allow additional competition to enter the market under the discrete entry barrier, but this competitive pressure never compensates for the effect of retaining the Cournot markup.

3.3 Steady-state growth

The measure of growth for the economy as a whole is the rate at which total output increases. In this model, total output is made up of two components: production of manufactured goods and production of quality-improving innovations. Output increases at the rate that real incomes increase because all profits are the income of workers and real income reflects what incomes are actually worth in consumption.

Real wages are defined by $\omega_{i,t} = \frac{w_{i,t}}{P}$, where P describes the perfect price index such that it recognises the changes in costs and quality across all sectors. Since nominal wages are unchanging in the model, real wages are increasing at the rate that the economy-wide price index declines. So far, price indices have been defined for each sector only. The perfect price index describes the overall cost of living with spending across all sectors. It is the P that buys one unit of Q:

$$P = \frac{1}{N} \prod_{i=1}^{N} \left(P_i^{\frac{1}{N}} \right), \tag{25}$$

where P_i is each sector's price index given by Equation 6a.

The first step to determine growth in consumption, is to first determine how the price index changes between periods. Defining technology growth for each industrial sector i as the rate that the quality level increases, the steady-state technology growth rate in a sector with Cournot competition is given by:

$$g_{i,A} = \frac{A_{i,j,t} - A_{i,j,t-1}}{A_{i,j,t-1}} = \frac{\varepsilon (\sigma_i - 1)(n_i + \varepsilon - 1)}{\eta (\sigma_i (\varepsilon + 1) + n_i - 1)} - 1. \tag{26}$$

The technology growth rate in a sector with Bertrand competition is given by:

$$g_{i,A} = \frac{\varepsilon (\sigma_i - 1)(n_i + \varepsilon - 1)}{\eta (n_i + \varepsilon)} - 1. \tag{27}$$

The sector i price index is falling at a rate of $(g_{i,A})^{\sigma-1} \frac{1}{1-\sigma}$. Therefore the growth rate of consumption/production is given by the rate that the perfect price index declines:

$$g_Q = \frac{1}{N(1-\sigma)} \prod_{i=1}^{N} (g_{i,A})^{\sigma-1} \tag{28}$$

Since production/consumption is one portion of the economy and the other portion of the economy, innovation, is constant in this model, to find the overall growth rate of GDP the growth rate of each sector must be multiplied by the proportion of the workforce employed in manufacturing. Growth in consumption/production and GDP growth depends upon the make-up of the economy. In particular, the form of competition in each market, the size of each sector and the level of contestability determine the share of employment in innovation or manufacturing, growth in technology in each sector and if these vary between sectors and countries, so will long-run growth rates.

4 Simulations and discussion

The simulations in this section vary the supply of specialised labour as a proxy for contestability in each sector to understand the relationship between contestability and innovation. This conveniently isolates the impact on innovation due to contestability under the discrete entry barrier by adjusting only the labour supply on a sector basis and also allows for a fair comparison between Bertrand and Cournot competition. Furthermore, the use of a growth model without scale or inverse scale assumptions means any differences between sectors' investment in innovation is purely a result of contestability effects under the discrete entry barrier. Simulations are a numerical exercise to demonstrate the impact of differing barriers to entry on innovation. This approach compares innovation outcomes in relation to mode of competition and extent of contestability, drawing implications for different economies, industries, regions and countries, and is not than calibrated estimates of actual specific economic outcomes.[6]

4.1 Calibration

These calibrations are based on typical calibrations in other models that use the CES formulation. Krugman (1991) uses an elasticity of $\sigma = 4$ and Baldwin & Forslid (2000) uses $\sigma = 5$. Since the model here is disaggregated to a much greater extent than other CES models, this justifies a higher elasticity of substitution between varieties in the same sector because the elasticity of substitution for varieties in different sectors is exactly one (i.e. varieties are neither substitutes nor complements for varieties in other sectors). However, an extremely high σ means varieties in the same sector are strong substitutes for each other so consumers are more responsive to changes in quality. That is, a high σ overwhelms other factors in determining the quality target. Alternatively, a very low σ does not sufficiently demonstrate differences between Cournot and Bertrand models of oligopoly because models converge as σ tends towards zero. Therefore, a fixed elasticity of $\sigma = 10$ is chosen for all simulations. While the parameter is higher than Krugman (1991) or Baldwin & Forslid (2000), it is not so high as to overwhelm calculations and is sufficiently high to demonstrate clear differences between models of oligopoly. Furthermore, a sensitivity analysis of σ does not affect the the conclusions drawn.

The rate of time preference is set by following Baldwin & Forslid (2000) with $\alpha = \frac{1}{2}$, which implies an annual discount rate of approximately 7% when periods represent 10 years. η is calibrated such that the expected free entry growth rate equals the rate of time preference, i.e. $\eta = \frac{(\sigma-1)}{2} = 12$. γ only has the effect of adjusting the scale in L and it's calibration does not affect results. Initial values for A_i are set to one such that all growth in technology can be easily compared to initial technology levels.

4.2 Number of firms and innovation

The following figures describe the relationship between the number of firms and innovation with discrete firms on a sector basis. As the number of firms in a sector rises, innovation increases with each additional firm due to competitive pressure, as firms respond less to their own effect on the price index, because they become a smaller overall share of the sector. The relationship between innovation and contestability can also be considered in the figures as supply of the factor of production (labour) is directly related to the number of firms. As the availability of the factor of production rises, contestability increases if it leads to an additional discrete firm. As a result, innovation increases stepwise, with each step due to the addition of another discrete firm.

With the innovation rate of one being the expected innovation rate under the fully competitive limit model, Figure 1 describes the innovation rates for $n_i = 2$ to 50 firms in a sector under Cournot and Bertrand competition respectively. As the number of firms increases, innovation rates tend towards the expected rate under the fully competitive or continuous model. With the assumption of discrete entry, innovation rates never quite reach the continuous limit such that there are always costs to the market from the discrete barrier to entry. Under Cournot competition, for low to moderate levels of contestability, there is

[6] Calibrating the model to a specific real world example is beyond the scope of this paper as it requires a detailed empirical foundation for calibration.

zero growth, as simply paying the minimum fixed cost without any quality improvement is enough to deter entry of the marginal firm. Notably under this comparable calibration, it is not until there are 12 firms in the sector before there is positive growth. Notably this is an absurd calibration as 12 firms in a Cournot sector appears to be unnecessarily large and extremely competitive on price, but the comparable calibration is maintained to fairly compare Bertrand and Cournot competition.

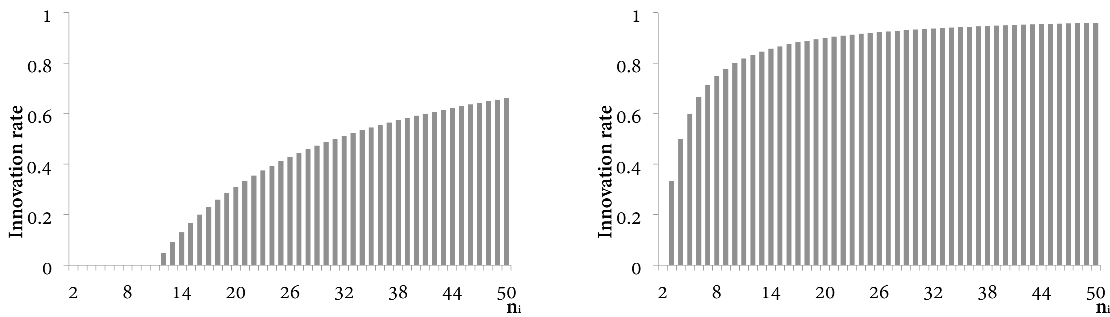

Figure 1: Number of discrete firms vs innovation rate under Cournot and Bertrand competition

4.3 Barriers to entry and innovation

However, this does not tell the full story. The comparison on a firm level basis can be misleading, because firms in Bertrand and Cournot sectors require different levels of the factor of production for entry. When a single firm employs fewer entrepreneurs in Cournot competition, the resource constraint barrier allows many more entrants under Cournot competition than under Bertrand competition. For example, using the same calibration as above, a labour supply that results in 5 firms (and an innovation rate at 0.33) in a Bertrand sector results in 12 firms (and an innovation rate of 0.05) if it were a Cournot sector.

Using the same calibration as above, the minimum and maximum labour supply is calculated that would result in 2, 3, and up to 50 firms in a sector for both Cournot and Bertrand innovation and the figures below consider the innovation rates in relation to the labour supply. With one being the expected innovation rate for the continuous model, Figure 2 describes how innovation rates rise with increases in the supply of labour for Cournot and Bertrand innovation respectively. The scale in L_i is not shown because it is simply a scalar that is dependent upon the calibration of γ, but both charts have the same scale.

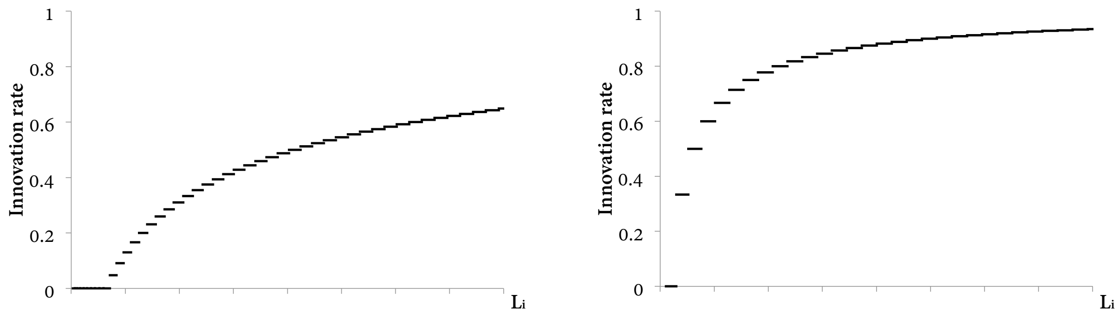

Figure 2: Labour supply vs innovation rate under Cournot and Bertrand competition

Innovation rates increase stepwise as additional labour allows an additional firm to contest the market at each "step". Each step is discrete and tends towards the growth rate of the continuous model. These steps upwards are initially larger in Bertrand competition resulting in higher innovation rates for the same labour supply (keeping all other parameters the same). But these higher rates occur with substantially fewer firms in Bertrand competition, meaning the two alternatives are not as substantially different as it appears in Figure 1. Comparing the innovation rates under Bertrand and Cournot competition on a labour supply basis, Figure 3 combines the two charts in Figure 2. The lower line represents Cournot innovation and the upper line represents Bertrand innovation. Notably the trend towards the continuous innovation rate is much closer when compared on a labour supply basis rather than a number of firms basis.

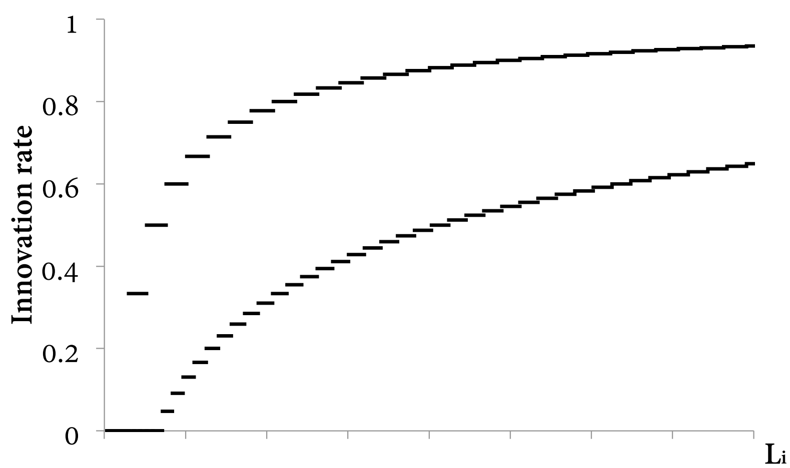

Figure 3: Labour supply vs innovation rate under Cournot and Bertrand competition

For policy makers, the mode of competition is particularly important when considering the effect of competition on innovation rates. Firms that compete fiercely on price in sectors under Bertrand competition may be particularly innovative, even with only a few firms, while Cournot sectors could contain many firms but not achieve high rates of innovation. While these Cournot markets may appear competitive based on traditional competition analysis and the number of market participants, a lack of competition may be more visible if competition authorities and policy makers examine rates of innovation.

4.4 Innovation and wage inequality

Continuing with this calibration, it is possible to also consider how the relationship between innovation and wage inequality is affected by varying levels of contestability. The impact of the discrete entry barrier on wage inequality varies according to the extent that the marginal firm is prevented from entering by the limited supply of specialised labour. When the labour supply only just lets a marginal firm enter, entrepreneurs earn exactly the same as workers in manufacturing and the entrepreneurial dividend diminishes to zero. However, if the labour supply just prevents entry of a marginal firm, those workers who are lucky enough to be employed as entrepreneurs can earn substantially more than manufacturing workers through their entrepreneurial dividend. The extent of this range of wage inequality declines as the labour supply increases.

4 describes the ratio of entrepreneurs' income including dividends to the wage of manufacturing workers' wages as the labour supply increases under both Cournot and Bertrand competition respectively. While the barrier to entry enables those firms permitted entry to exercise market power over prices and innovative effort, the extent that this is reflected in the entrepreneurial wage or shared with all workers depends upon the extent that the barrier to entry is binding upon a marginal entrant. In this way there are two processes affecting wage inequality. Firstly, the ability to appropriate monopoly rents and limit innovation declines as contestability increases. But secondly, the extent that the payoff is shared with other employees decreases until a step change occurs with an additional discrete firm.

For very low contestability (or labour supply), the resulting level of inequality varies substantially, but the range diminishes as contestability increases. Notably, the range of wage inequality increases again at a single step in the Cournot sector which occurs at the contestability level when growth commences. This is because the labour supply committed to innovation given by Equation 15 is constant when there is no growth at low levels of contestability, but is increasing when innovation results in a quality improvement. The increase in the fixed cost requirement for entry increases each subsequent step size, although the range of wage inequality returns to the decreasing trend. Notably the steps are larger under Bertrand competition than under Cournot because Cournot allows many more firms to enter for the same resource constraint. The scale in L_i is increased in the charts to examine each step in closer detail.

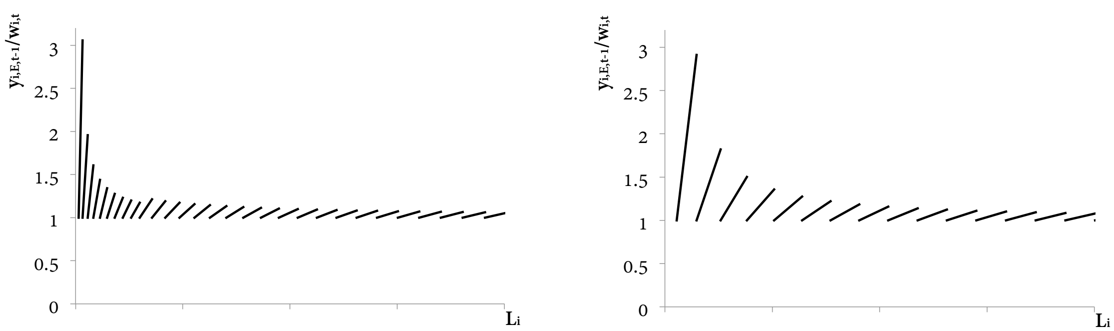

Figure 4: Labour supply vs ratio of entrepreneurial income to the market wage under Cournot and Bertrand competition

Using this comparison, it is ambiguous whether the range of inequality is larger under one form of competition or the other. For example, the range of wage inequality for Cournot competition with only two firms is greater than for Bertrand competition with only two firms, but, this direct comparison is also misleading because the potential size of the labour supply over that range is substantially different. With a higher labour supply, the range of inequality diminishes faster for Cournot competition than for Bertrand, as the impact of additional competition in Cournot has a greater impact on entrepreneurial income.

Alternatively, Figure 5 compares innovation rates and wage inequality at various levels of the labour supply under Cournot and Bertrand competition respectively. While the rate of quality improvement in both Cournot and Bertrand eventually converges to one, this does not quite occur within the 2 to 400 firms used in these figures. Notably, the upper limits of wage inequality are negatively correlated with the rate of quality improvement because high inequality and low innovation are both a result of barriers to entry or low contestability.

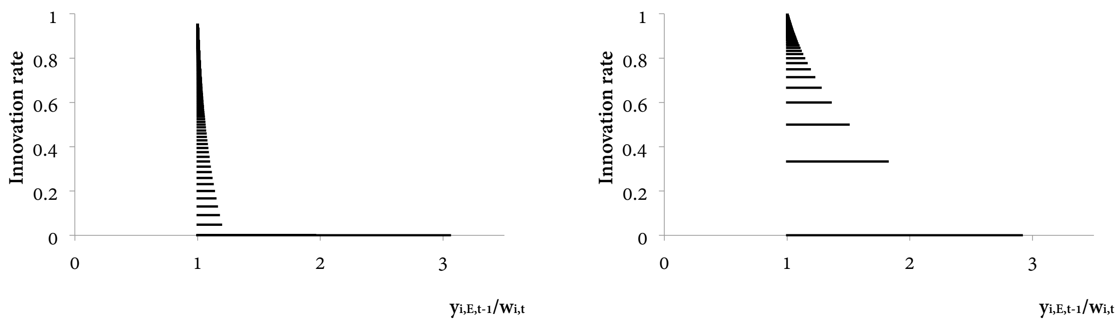

Figure 5: Innovation rate vs ratio of entrepreneurial income to the market wage under Cournot and Bertrand competition

4.5 Discussion of the model

The discrete entry barrier results in firms investing in innovation in response to actual competitors but not responding to potential innovations by entrants because the marginal entrant would not be profitable and is blocked from entry. Notably, the existence of a few small markets does not condemn an economy to lower innovation and growth under Bertrand competition because each sector only makes up a $1/N$ proportion of an economy. If the economy is dominated by small markets then there would be a significantly greater effect. Furthermore, the exact distribution of sector size is important such that average market size is not the important factor, but to what extent the overall market is made up of small sectors. Also real world markets would have a more realistic labour supply where at least some skills of the specialised workforce would be transferable to other markets. Nonetheless, low contestability strongly perverts the incentive for market participants to invest in innovation. Reducing barriers to entry and expanding the transferable skills of the labour force is likely to lead to improved innovation performance. This section discusses the model in relation to the existing literature and addressing policy implications.

Low contestability from discrete entry as modelled in this paper may be a more common characteristic for small or isolated economies. The size of these markets may not be attractive to a multi-national or new entrant such that these markets can remain isolated from the competitive forces that occur in larger agglomerated economies. In this way, the model developed by this paper provides further evidence for localised and industry-specific innovation and growth policy, particularly in small, isolated, non-tradables markets or markets with a significant economies of scale effect where discrete entry is more likely to be an issue. There is no one-size-fits-all approach to stimulating innovation and growth. Economic policy must be tailored to an industry, country and region's specific characteristics.

4.5.1 The additional market failure in innovation from barriers to entry

There is a market failure for innovation in endogenous growth models such that firms underinvest in innovation activities because of inter-temporal spillovers (Aghion & Howitt, 1992; Grossman & Helpman, 1991; Romer, 1990). Investors in R&D do not invest at the socially optimal level of innovation, because they cannot keep all of the future benefits from their investment in generating new knowledge. Those benefits accrue to future innovators when the value of past knowledge is appropriated by new innovators. This feature is also present in the model here, but there is an additional market failure such that firms under-invest in innovation because they respond to actual competitors rather than potential competitors since the marginal entrant is blocked. The market failure is evident in both lower innovation rates and labour market distortions that lead to wage inequality.

Examining Figures 1 to 3, the size of the additional market failure for innovation in each sector is the difference between the level of quality improvement achieved and the quality improvement in the model with continuous free entry at one. While the continuous free entry model assumes that a large number of entrants has made this negligible, in the disaggregated discrete model, this market failure can be significant. The market failure appears most clearly through prices as is already well understood in the industrial organisation literature. What may have been less clear up until now is the extent to which market failure from imperfect competition could be present in market participants' incentives to develop innovations and how the extent of the market failure is affected by the mode of competition.

The size of the market failure is determined by the extent of contestability under the discrete barrier to entry and by the mode of competition. The extent of contestability determines how the firm's observed elasticity of substitution is affected by the firm's own prices and quality levels. In Bertrand competition, even if a firm is one of only a few competitors, its effect on the perceived elasticity may be limited. Adding firms to the sector results in the perceived elasticity quickly tending towards the CES elasticity that is observed when there is continuous free entry. However, in Cournot competition, the perceived elasticity tends towards the growth outcome in the continuous free entry model at a much slower rate as contestability increases. The effect is mitigated because many more firms can enter the market under Cournot competition than under Bertrand competition using the same factors of production. Nonetheless, pressure from additional entry is not enough to entirely mitigate the Cournot effect on innovation. As proven in this paper, innovation is always lower under Cournot competition than Bertrand competition in comparable sectors. Therefore, the size of the innovation market failure is greater under Cournot competition.

4.5.2 Barriers to entry and innovation

Despite the different curve shape, the results here are consistent with Aghion et al. (2005) which examined the relationship between competition and innovation, finding an inverse U-relationship. The upward sloping portion of the inverted-U can be observed in the increasing rate of innovation in response to a greater number of firms and the diminishing size of the increase at higher growth rates. Aghion and Griffith's (2005) research describes a relationship between technology-leading and following firms where competition discourages following firms from innovating, but encourages leading firms who are attempting to "escape competition". In this model, firms are also more innovative as they respond to greater competition, but following firms are unable to make a profit, as an entrant could incrementally innovate by a greater amount and maintain its market position ahead of a following firm as a discrete participant. Escaping competition in the Aghion et al. (2005) model comes in two forms. Firstly, firms develop innovations to ensure that they are the technology leading firm in the coming period. This is known as "escaping competition" *for* the market. Contestability as in the continuous growth model stimulates innovation by both incumbents and potential entrants. The model here also includes the "escape competition" effect but it is only in its second form, that is, competition *in* the market, because of the discrete entry criteria. Discrete entry eliminates the escape competition *for* the market effect, but maintains the escape competition *in* the market effect. The downward sloping portion of the inverted U-relationship is not seen because discrete entry prevents the escape competition *for* the market effect from eventually dominating the escape competition *in* the market effect.

Similarly the model here is consistent with the approach taken in Desmet & Parente (2010) whereby larger markets increase competition and facilitate innovation. As in the model here a larger market supports increased product variety and increases the observed price elasticity of demand. Examining the results closer reveals a similar conclusion that firm size is larger in the more competitive markets in order to amortise R&D costs over greater levels of production. Our results could be extended to the model in Desmet & Parente (2010) where research costs are also amortized over more varieties. Alternatively, Desmet & Parente (2010) could be expanded as is done here to understand the differences between Cournot and Bertrand models of oligopoly.

The model highlights how the mode of competition is a particularly important market characteristic for innovation. Cournot sectors have significantly less innovation per firm, although this allows many more firms to enter the market, if innovation is required for entry. Additional firms do not necessarily lead to high growth rates, because Cournot models require significantly more firms to enter the market before innovation tends close to the continuous free entry growth rate. In considering the effect of market entry and competition characteristics on innovation, each region and industry will have unique characteristics which determine the overall effect on growth and the appropriate policy response to support innovation.

Low contestability in a Cournot sector results in a significant reduction to economic growth. When comparing Cournot and Bertrand innovation on a production factor basis (labour), Cournot sectors have considerably lower innovation rates, even though Cournot competition allows significantly more firms to enter the market. As a result, policy-makers examining competition and innovation policy need to consider both the number of firms and the mode of competition. In response to barriers to entry such as the discrete entry assumption used in this paper, policy makers should focus on removing or reducing these barriers, perhaps by offering assistance to new entrants and greater assistance for entry in Cournot markets. This would increase the threat of entry and also motivate incumbents to innovate. Opening markets to international competition and integrating markets across regions will also increase the threat of entry.

By disaggregating the model into many sectors, introducing a barrier to entry and examining different models of competition, this paper helps to understand the positive relationship between contestability, innovation and growth by reinterpreting the relationship between market structure and innovation. When a market is only partially contestable, as with a barrier to entry, the additional market power enables participants to underinvest in innovation and retain a higher margin for entrepreneurs.

4.5.3 Barriers to entry and growth

Over an entire economy, it may be possible for a few or many (or somewhere in between) sectors to have low levels of competition but so long as these markets are contestable by potential entrants, there is no effect on innovation. However, when barriers to entry are present, firms only respond strategically to actual participants allowing the firm owners to take home greater earnings by underinvesting in innovation. The overall effect on economy-wide growth depends upon the portion of the economy made up of sectors with barriers to entry.

Increasing the resource constraint to reduce barriers to entry clearly leads to higher growth rates. This provides an additional theoretical explanation for the upward sloping (and diminishing slope) portion of the inverted U relationship between competition and innovation (Aghion et al., 2005) but one driven by barriers to entry. The inverted U shape owes itself to the escape competition effect being dominant for competitors that have "neck and neck" technology levels. Each firm's "distance to frontier" (Acemoglu et al., 2006) technology determines the extent to which competition has an increasing or diminishing effect on innovation. The model here is simpler by assuming symmetry, but a similar "escape competition" in the market effect is still present such that firms have to invest more in innovation when there is an apparent threat from a marginal entrant at lower barriers to entry.

However the existence of some low competition sectors does not always have a substantial effect on the economy-wide growth rate. Each sector may makes up a small portion of expenditure and unless the economy is dominated by sectors with barriers to entry, the effect on growth is not necessarily substantial. There are implications for innovation and growth policy at a sector level. Hence, it is important to not only consider growth policy as a macroeconomic problem that requires economy-wide solutions, but growth policy should incorporate microeconomic reform in individual sectors. As can be seen that market characteristics which impede entry (such as discrete entry) have a negative effect on innovation and consequently on economic growth. With Cournot competition, there are considerably more firms per sector, but the additional competition results in lower growth due to the nature of innovation under Cournot competition.

4.5.4 Policy implications

The intuitive policy response is to examine the factors affecting the innovation cost function and policies that adjust these factors can stimulate innovation. For example, contestability can be expanded by increasing the particular factor of production which is constraining discrete entry. For example, if taxi licenses were the particular factor of production that blocks a marginal entrant, expanding the number of taxi licenses could be expected to increase contestability in the market and encourage innovation by market participants. Similarly, increasing investment in R&D infrastructure or otherwise to reduce individual firm requirements for that particular factor of production that constrains entry would also expand contestability and encourage innovation.

However, neither of these two intuitive policy solutions address the particular problem of a discrete entry barrier. The entry barrier still exists but its effects are diminished by these "weak form" policies. There are other "strong form" policies which break the discrete nature of entry, allowing smaller firms to enter, foreign entry or other ways to expand contestability by removing the discrete entry barrier altogether. For example, the removal of license requirements, trade barriers or other minimum scale requirements enables new and smaller firms to enter to compete with incumbents.

Alternatively, industries such as telecommunications or electricity have overcome the economies of scale required for each discrete entrant by regulation. Telecommunications typically fosters a "ladder of investment" approach (Cave, 2004, 2006) that enables contestability by regulating access to unbundled services such that entrant firms can be established with a minimal level of investment and gradually increase their involvement with higher order levels of investment over time. Alternatively, the regulated separation of retail and wholesale electricity services from network services enables contestability for some elements of the electricity supply chain without the scale required for firms to investment in a network. These types of policy approaches break the discrete entry requirement by allowing market entry in other ways to foster contestability and thereby encourage innovation.

Regulatory interventions such as these are common in network industries such as telecommunications or electricity where the barriers to entry are more obvious. These regulations typically target the natural monopoly nature of those industries but may also have the positive unintended consequence of removing the discrete entry barrier in the retail market. However, it is possible that other markets also suffer from the discrete entry barrier as modelled in this paper. Policy approaches that enable new forms of market entry are therefore likely to improve contestability and stimulate innovation in a variety of other industries.

Alternatively, markets may find a way to develop their tools on their own, for alternate forms of market entry in order to overcome the discrete entry barrier. For example, eBay and Amazon have enabled even very small firms to establish a national or even global online sales presence in a number of retail markets. Amazon and Google both enable small businesses to utilise cloud computing without the investment required for their own data centres. Similarly, other firms such as AirBnB or Uber have bypassed the entry barrier altogether by developing products in a way that does not require the particular factor of production that causes the constraint on entry. Markets with a discrete entry barrier are likely to be an attractive target for innovation to overcome these barriers. Nonetheless, these barriers constrain entry and encourage investment in innovations that would otherwise be unnecessary if the original market design actually enabled contestability. While markets can sometimes overcome entry barriers in new ways, regulators and policy makers should still consider whether the barrier or factor of production itself is unnecessary and therefore whether investment in these types of innovations should be required or not in order to overcome any contestability issues.

5 Conclusion

Using a simple discrete entry barrier (i.e. only an integer number of firms is permitted) as a tool to model contestability, this paper develops a generalised understanding of how barriers to entry reduce investment in innovation, can increase inequality and how sectors characterised by Cournot competition face a greater constraint on innovation from barriers to entry than sectors characterised by Bertrand oligopoly. When markets are not contestable then those lucky enough to gain entry can extract monopoly rents with limited competition on price or innovation. However, the ability of entrepreneurs to appropriate rents in their entrepreneurial dividend depends upon the extent that the barrier to entry is binding upon a marginal entrant. The model here provides a unique insight into this ambiguous and nuanced relationship between market structure and innovation.

The model here combines the partial equilibrium characteristics of individual sectors such as barriers to entry, Cournot or Bertrand oligopoly, and imperfect markets with the broader general equilibrium features present in endogenous growth theory to understand the relationship between market structure and innovation. This encourages a revision of economists understanding of endogenous growth theory to also consider local, regional or industry characteristics. The model shows that when contestability is limited by a discrete entry barrier, innovation is constrained because the entry barrier leaves only the "escape competition *in* the market effect". As a result, firms investing in incremental innovations are able to invest in innovation only in response to actual market participants contesting the market rather than all potential entrants. This re-examination of the relationship between contestability and innovation considers the causes of market structure in equilibrium which then have flow-on effects for innovation investment and income inequality.

Models that examine the relationship between competition and innovation will benefit from understanding the causes of competition. The ability to contest the market cannot always be assumed. Perhaps future research can examine whether the inverted-U relationship found in Aghion et al. (2005) is instead related to the extent of contestability in markets with low levels of competition. This research would imply that barriers to entry lead to the upward sloping portion of the inverted-U while competition finally leads to the downward sloping portion of the inverted-U. As such, the impact of the "escape competition *for* the market effect" could be overestimated if market structure and contestability are not considered. Lower innovation because of discrete entry has a flow on effect to overall economic growth, even when it occurs in just a few sectors. Contestability levels are a characteristic of individual sectors and therefore policies to stimulate growth should also focus on policies that target innovation or firm entry in these individual sectors. Innovation and growth policy is not necessarily an economy wide generic policy approach, but a localised, industry- and economy-specific policy problem.

The paper finds that sectors with low contestability have lower levels of investment in innovation and potential for greater disparity between the wages of manufacturing workers and the income of entrepreneurs. Notably, the modelling technique of discrete entry, such that continuous varieties, infinitesimally small firms or even free entry, results in discontinuous outcomes that could be difficult to examine empirically. This could be a valuable area for future research on innovation and growth, to consider the market dynamics of discrete market participants on market outcomes, similar to efforts in international trade, to explain zero trade with some countries (Eaton et al., 2013).

It is possible that the type of entry barriers described are more common in small or isolated markets where competition from imported goods is relatively more expensive due to economies of scale in transport, a small market has only the capacity for a few firms at a viable scale or where cultural and nationalistic barriers may make entry by foreign firms difficult. The model therefore provides new and interesting implications for innovation and economic growth policy that are particularly relevant to small, isolated or peripheral regions and countries. Future research on understanding innovation and its relationship with contestability or market structure should therefore also focus on localised, industry or economy-specific characteristics that may affect competition levels, firm entry, market contestability, industry churn or the ability to produce innovations in individual sectors. Fields such as industrial organisation, labour economics, economic geography, and regional and urban economics have much to contribute to the study of local or industry factors affecting firm entry, contestability, innovation and subsequently economic growth. Innovation is much more complex than endogenous growth theories have suggested and the growth model here adds to the growing body of evidence that growth and innovation policy should focus on localised and industry-specific factors of innovation.

This approach, to understand the causes of market structure as part of an endogenous growth model, yields elegant new insights on the relationship between barriers to entry, competition, innovation and growth. Notably, the model here points to significant differences for competition and innovation outcomes under Cournot and Bertrand competition such that policy makers and competition regulators must also pay close attention to the mode of competition. While Cournot may enable more firms to contest the market, holding all else constant, sectors characterised by Bertrand oligopoly have greater investment in innovation than sectors characterised by Cournot. The model suggests that the relationship between competition and innovation is really about the ability for firms to contest the market with innovation: something that cannot always be assumed.

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