What Is Wrong With Representative Agent Equilibrium Models?

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Abstract

Despite the ability to match an increasing number of data moments, equilibrium models, employing the widely used Representative Agent simplifying construct, do not properly fit the data, why? Using a flexible econometric model I provide, as a function of a set of key drivers capturing violations to the models' key assumptions, a joint model-free test for the Representative Agent Equilibrium Models (RAEMs) in the literature as well as the conditional probability of their failures over time. I find such probabilities to be counter-cyclical, right-skewed and on average high (around 47%). RAEMs are rejected in periods of high illiquidity (market frictions) where the investors' level of disagreement (asymmetric information) is above the median and aggregate expectations are irrationally downward biased. These periods are followed by a decreasing demand to hold the market and, inconsistently with RAEMs predictions, low realized returns. During economic recessions, models are more likely to fail due to market frictions, while in normal times by the asymmetry in information.

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1 Introduction

The Representative Agent and the Rational Expectation paradigm are two pillars of the neo-classical equilibrium models based on micro foundation and individual rationality. From a theoretical perspective, the simplifying Representative Agent device is an easy way out from complex equilibria derived from the interactions of many different economic agents while Rational Expectations (the fact that agents' beliefs align with models' predictions) are a consistency requirement to close such models.

Ever since Lucas (1978), passing through Campbell-Cochrane (1999), Bansal-Yaron (2004), Bansal et al. (2014), Campbell et al. (2016), Barro (2006) and Wachter (2913) just to mention a few, rational representative agent exchange economies have been widely adopted to design equilibrium models matching by simulations an increasing number of empirical moments observed in the financial markets. Nonetheless, their performances on actual data remain rather poor, that is, either unreasonable parameter values are required (e.g. the Mehra-Prescott (1985) equity premium puzzle) or by using enough instruments one can always reject the given model (e.g. through the GMM J-test),[1] *why?*

I develop a unified framework to answer this question: conditioning on a set of key drivers capturing dimensions that go *against* the key assumptions common to the class of models under analysis and exploiting a novel asset pricing restriction (Martin, 2016), I design a joint model-free test for the Representative Agent Equilibrium Models (RAEMs) and derive the conditional probability of their failures at each point in time in the main sample. Illiquidity, as a member of the family of market frictions' proxies (measured by the negative of the Pasor-Stambaugh (2003) index), investors' disagreement as a way of measuring information asymmetries (proxied by the Ludvigson et al. (2016) financial uncertainty index and the Rapach et al. (2016) short interest index), and irrationally downward-biased expectations are the key fragilities of RAEMs. Sub-samples within which the models fail, mostly driven by a negative risk premium, are characterized by levels of illiquidity above the 75-th percentile, disagreement intensity over the median threshold

[1] See for example Hansen-Singleton 1982, Epstein-Zin 1991 and Savov 2011.

and followed by subsequent periods of low market demand and negative realized returns. A poor RAEMs fit of the data is justified by a conditional probability to jointly reject these models that is counter-cyclical, right skewed and quite high: it never goes below 33.18% and has a mean of 46.82%. The probabilities to reject the RAEMs *exclusively* due to illiquidity (market frictions) and disagreement (asymmetry in information) are separately tracked over time, explain all the main spikes in the overall conditional probabilities and unravel the different motivations for which the RAEMs fail over time. In particular, during recessions it is the illiquidity component, or more generally the presence of substantial market frictions, that causes the models to fail, while during normal times the disagreement proxies, or the failure of the symmetric information assumption, are most predominant.

According to the equilibrium models the representative agent, in order to hold the market portfolio in bad times, requires a high risk premium and under rational expectations the (conditional) sample mean of the realized (ex-post) excess market return is a good estimator for the ex-ante market expectations. Therefore the RAEMs predict high average realized market returns following bad periods. Contrary to this prediction, periods of RAEMs' rejections which include all major economic recessions and financial crises for a total of 25.43% of the main sample, are followed by negative realized market returns. The evidence gathered in this study rather suggests that in bad times irrationally pessimistic investors (short) sell the market pushing its price down, thus generating negative returns.

This paper is not the first to document empirical inconsistencies with existing equilibrium models: the list is long and features Hansen-Singleton (1982)'s or Gallant-Tauchen (1989)'s type of model-specific GMM tests, incompatibility of rational expectations and equilibrium models (e.g. Greenwood-Schleifer (2014) and Amromin-Sharpe (2014)) and simulation-based critiques according to which some relevant data features cannot be replicated even in the idealized models' frameworks (e.g. Martin (2016) and Moreira-Muir (2016)). Nonetheless, this paper is the first to (i) give a formal and constructive model-free joint test for the entire class of models and analyzes the systematic drivers behind the rejection in both a static and a dynamic fashion, (ii) provide a test for rational expectations able to detect the direction of the bias and (iii) disentangle the different motives that in different periods are behind the models' rejections. Left for future research is the objective of assembling the new stylized facts into a new alternative theory model.

The rest of the paper is structured as follows: Section 2 sets the framework of the study explaining the logic behind the model free test and the empirical design, Section 3 describes the data and motivates the choice of the drivers, Section 4 shows the results analyzing why and when the RAEMs fail giving a detailed explanation of the findings, Section 5 contains some robustness checks and Section 6 concludes. All the proofs, derivations and extra-analysis are in the Appendix (Section 7).

2 Framework

2.1 The logic behind the joint model free test

In a recent paper, Martin (2016) proposes a new asset pricing restriction linking the conditional risk premium on the market to observables in the marginal investor's information set under weak assumptions. I exploit this relation to provide a model free test for the pricing equation in the context of the RAEMs

$$1 = \mathbb{E}[M \times R^i] \tag{1}$$

where M is the representative agent equilibrium inter-temporal marginal rate of substitution, R^i is the gross return on asset i and $\mathbb{E}[\cdot]$ is the expectation operator, which under rational expectations can be substituted by the (sub)sample mean. Each model differs in term of dynamics and functional forms attached to M and R^i, assumes rational expectations and mainly focuses on the market return R^{mkt}.

By the Fundamental Theorem of Asset Pricing (FTAP),[2] the existence of the pricing equation (1) such that $M > 0$ and an equivalent risk-neutral measure Q such that $R_f = \mathbb{E}^Q[R^i]$, where R_f is the risk-free return, is guaranteed by the Law of One Price, under the assumption of no-arbitrage,

[2]Ross (1973,1978), Harrison and Kreps (1979), Dybvig-Ross (1987).

the absence of market frictions and by modeling uncertainty through the existence of a potentially very large but finite set Ω of states of the worlds. It is than straightforward, in the spirit of Martin (2016), to derive the following proposition

Proposition 1. *In an arbitrage free market where there exists a strictly positive stochastic discount factor, M, satisfying the pricing equation and the Negative Covariance Condition (NCC)*

$$Cov_t(M_{t+1} \times R_{t+1}^{mkt}, R_{t+1}^{mkt}) \leq 0 \tag{2}$$

it is possible to construct a real time conditional lower bound, $LB_t \equiv \frac{Var_t^Q(R_{t+1}^{mkt})}{R_{t,f}}$, on the market risk premium $\mathbb{E}_t[R_{t+1}^{mkt} - R_{t,f}]$ by

$$LB_t = 2\left(\frac{DY_t}{\hat{S}_t}\right)^2 \left(\int_0^{\hat{F}_t} \hat{put}_t(k)dk + \int_{\hat{F}_t}^{\infty} \hat{call}_t(k)\right) \geq 0 \tag{3}$$

by setting $DY_t = 1$ the original Martin (2016) measure is recovered.[3]

Proof. See Appendix ■

The quantities with hats are *ex-dividend*, DY_t is the gross dividend yield on the market portfolio with respect to the period $[t, t+1]$ assumed *known*[4] at t, $\hat{S}_t$ is the closing market level at time t, $\hat{F}_t$ is the forward contract on the market with tenor $1 = (t+1) - t$ and finally $\hat{put}_t(k)$ and $\hat{call}_t(k)$ are European options on the market with unity tenor and strike k. By the Put-Call parity[5] the forward contract $\hat{F}_t \equiv \hat{F}_t(k^*)$ is the unique point $(k^*, \hat{F}_t(k^*))$ at which the call and put functions intersect so that LB_t is just a function of $DY_t, \hat{S}_t, \{\hat{put}_t(k_i), \hat{call}_t(k_i)\}_{k_i \in \mathcal{K}_t}$ where $\mathcal{K}_t$ is the set of observable strikes with unit tenor at time t.

As the next proposition points out, applying the logic of contraposition to Proposition 1 delivers a test for the pricing equation.

[3]In the Robustness section I show how the two measures are empirically identical.

[4]This assumption is empirically without loss of generality given that it's impact in the data, as shown in the Robustness section, is absent.

[5]Adjusted for dividends, i.e. $\hat{call}_t(k) = \hat{put}_t(k) + \hat{S}_t - PV(D_{t+1}) - \frac{k}{R_{t,f}}$.

Proposition 2. *Given a violation of the lower bound measure* (3) *if* $M > 0$ *and the Negative Covariance Condition in* (2) *holds the pricing equation is rejected.*

Because $M > 0$ and the Negative Covariance Condition in (2) holds for the RAEMs class,[6] following Proposition 2 a test for violations of the lower bound measure (3) is a joint model-free test for the RAEMs. At this point, the only missing element for the formulation of such a test is an operational definition of lower bound violations which is given next:

Definition 1. *Given a time series for the lower bound on the market premium at horizon* $1 = (t+1) - t$, $\{LB_t\}_t$, *computed through* (3), *and the associated*[7] *time series for the excess (realized) market return,* $\{R_{t+1} - R_{t,f}\}_t$, *a lower bound violation is a subsample over which the mean of the excess market return is* below *that of the lower bound series.*

2.2 Empirical Design

Armed with the model-free logic this section details how to design a framework to analyze the RAEMs rejections in both a static and a dynamic fashion: statically through the design of a formal test and dynamically through the derivation of a conditional probability to reject the RAEMs based on the information available up to time t and the realization π_{t+1}.

The starting point of both analysis is an econometric model to forecast the excess market return $\pi_{t+1} \equiv R^{mkt}_{t+1} - R_{t,f}$

$$\pi_{t+1} = f_t(D) + e_{t+1} \tag{4}$$

as a function of a set of key drivers D. The flexible modeling choice of the present study is a polynomial of degree 2 able to capture the linear as well as the non-linear impact of the set of key drivers D with a vector of parameters θ_t iteratively re-estimated at each time t in the main sample to capture the time-varying impact of the drivers.

[6] Including Epstein-Zin (1989) with unity coefficient of relative risk aversion and arbitrary elasticity of intertemporal substitution, Campbell and Cochrane (1999), Bansal and Yaron (2004), Bansal et al. (2014), Campbell et al. (2016), Barro (2006), Wachter (2013) and Merton (1973) in the Campbell-Viceria (1999) formulation, see Section III of Martin 2016.

[7] Lagged one period back so to match the forward looking expectations contained in LB_t.

2.2.1 The RAEMs joint model-free test

Define $y_{t+1} \equiv \pi_{t+1} - LB_t$ and assume y_{t+1} to be independent over time. The independence assumption states that once we subtract the lower bound LB_t, computed through (3), from the excess market return process, π_{t+1}, we are left with noise. Note that we are not restricting such noise to be identically distributed. Given any process π_{t+1}, the independence assumption can be justified either by thinking that the lower bound (3) is a good measure for the risk premium, in which case subtracting a good proxy for the conditional mean of π_{t+1} from π_{t+1} just leaves a random disturbance, or on the contrary by viewing LB_t as a bad proxy containing enough noise to offset any predictable pattern in π_{t+1}.

We can now formally state the RAEMs joint model-free test

Definition 2. *A joint model free test for the class of representative agent equilibrium models (RAEMs) is a one-sided t-test*

$$H_0 : \mathbb{E}[y_{t+1} I_t^v(\hat{\pi}_{t+1}, LB_t)] = 0 \textit{ vs. } H_1 : \mathbb{E}[y_{t+1} I_t^v(\hat{\pi}_{t+1}, LB_t)] < 0 \tag{5}$$

with the nonnegative time t function $I_t^v(\hat{\pi}_{t+1}, LB_t) \equiv 1_{[\hat{\pi}_{t+1} < LB_t]}$ capturing joint RAEMs violations in case of rejection at the $1 - \alpha$ confidence level, $\hat{\pi}_{t+1}$ representing the time t forecast of π_{t+1} according to model (4) and $y_{t+1} \equiv \pi_{t+1} - LB_t$ being independent over time.

Note that, according to Definition 1, the lower bound violations can be written as $\mathbb{E}[y_{t+1}|\mathcal{F}_t] < 0$ for some filtration $\mathcal{F}_t$ and $\mathbb{E}[y_{t+1}|\mathcal{F}_t] < 0$ if and only if $\mathbb{E}[y_{t+1} I_t] < 0$ for any nonnegative function I_t. Definition 2 sets $I_t \equiv I_t^v(\hat{\pi}_{t+1}, LB_t)$. Also, the i.d. assumption on the process y_{t+1} guarantees that y_{t+1} and $I_t^v(\hat{\pi}_{t+1}, LB_t)$, given information up to time t, are independent so that the test does not suffer from any kind of sample selection bias.

2.2.2 The conditional probability to reject the RAEMs

The dynamic part of the analysis tackle the issue of *when* the RAEMs are more problematic rather then *why*. Remember that, by the logic of Proposition 2, in the RAEMs class, whenever the risk-premium $\mathbb{E}_t[\pi_{t+1}]$ is below its lower bound LB_t we have a violation. Thus a way to capture, at any given point in time t, the probability of having a lower bound violation and hence a RAEMs rejection, is trhough the following object $P_t(\pi_{t+1} < LB_t)$.

In particular, using model (4) I construct the time-series of such conditional probabilities and further produce the conditional contributions due to specific subset of drivers $d \subset D$.

Model (4) produces the forecast $\hat{\pi}_{t+1}$ at each time t which can be viewed as

$$\pi_{t+1} = \hat{\pi}_{t+1} + \varepsilon_{t+1} \tag{6}$$

thus at each time t retrospectively, the researcher has at disposal the time series $\{\hat{\varepsilon}_{t+1}\}_{t=1}^{t}$ where $\hat{\varepsilon}_{t+1} \equiv \pi_{t+1} - \hat{\pi}_{t+1}$. One can then, according to (6), re-create the conditional distribution of π_{t+1} by bootstrapping[8] $\{\hat{\varepsilon}_{t+1}\}_{t=1}^{t}$ and compute $P_t(\pi_{t+1} < LB_t)$ by subtracting LB_t, counting the number of times $\{\pi_{t+1}^{(s)} - LB_t\}_{s=1}^{Sim}$ is negative and dividing it by Sim, the number of bootstrapping simulations.

Using a similar logic and model (4), the single contribution of subsets of drivers $d \subset D$ on the probability to reject the RAEMs, $P_t(d : \pi_{t+1}(d) < LB_t)$ is also computable, this time *only*[9] using information up to time t. As a matter of facts, model (4) gives us π_{t+1} as a function of D and the model parameters given information up to time t, θ_t. $P_t(d : \pi_{t+1}(d) < LB_t)$ is computed by looking at the joint empirical frequency of d using the sample $\{1, .., t\}$ such that at time t for given θ_t and $d^c \equiv D - d$ fixed at their time t realizations $\pi_{t+1}(d) < LB_t$.

3 Data

The data used in this study is at the monthly frequency and covers the United States Financial Markets over the period $Feb : 1973 - Dec : 2014$. The sample is spitted into a training sample $TS = \{1, ..., T_s\}$ and a main sample $MS = \{T_s + 1, ..., T\}$ with $T_s = Dec : 1989$. Model (4) is initially estimated in the training sample, thus $\hat{\pi}_{t+1}$ with $t+1 = T_s+1$ uses the parameter vector θ_t

[8]Or block-bootstrapping to more precisely take into account potential serial correlation.

[9]I.e. avoiding the usage of realization π_{t+1}.

calibrated exclusively in TS and for each following $t \in MS$ the parameter vector θ_t is re-estimated using information up to time t included. The RAEMs analysis is conducted in MS only and the choice of T_s is due to the unavailability of option data, needed to compute the lower bound LB_t according to (3), in the training sample. That is, $T_s + 1$ is the first date for which option quotes are available. Any observation t reflects the information available at the *beginning* of the t month intended as the first business day of that month.[10] The data is divided into two categories: (i) the Main Variables, the key variable of interest, namely the market return R^{mkt}_{t+1}, the risk-free return $R_{t,f}$ and the lower bound LB_t and (ii) the Drivers D. Each category is detailed next.

3.1 Main Variables

The gross total market return is defined as $R_{t+1} \equiv \frac{\hat{S}_{t+1}}{\hat{S}_t} DY_t$ where $\hat{S}$ represents the daily closing level of the *Standard* & *Poor's* 500 (SP500) index and $DY_t \equiv \frac{D_{t+1}}{\hat{S}_{t+1}}$ is the gross dividend yield with $\{D_t\}$ being the $SP500$ dividend time series (divided by 12) available on Prof. Shiller website.[11] The gross return on a risk-free investment, $R_{t,f}$, is defined as the gross yield to maturity extracted from the Center for Research in Security Prices (CRSP) continuously compounded yield curve computed over liquid secondary market transactions on U.S. Treasuries.

The time-series of the market premium lower bound, $\{LB_t\}$, is computed according to equation (3) in the most conservative way by a cubic spline interpolation[12] on the Chicago Board Options Exchange (CBOE) SPX options *bid* prices; the data from January 1990 trhough December 1995 is provided by Optsum data, while data from January 1996 trough December 2014 is taken from OptionMetrics. For dates t in which the data is not sufficient/absent to deliver LB_t at the exact options maturity of 1 month I linearly interpolate between the contemporaneous t lower bounds with the two closest maturities.

The following table summarize the main variables

[10] For all the ambiguous cases in which it is not clear what is the exact timing of an observation recorded at t we lag it back one period to make sure it is in the investor information set at time t. If anything, this step only makes it harder to find the results of this study.

[11] http://www.econ.yale.edu/ shiller/data.htm

[12] In the Robustness section I show how very similar results are obtained if we use a linear interpolation instead.

[Table 1 goes about here]

3.2 Drivers

The set D of drivers plays an important role in the interpretation of the RAEMs analysis: they represent the conditioning upon which the equilibrium models fail. As such, they are selected with the goal of describing dimensions that go *against* the RAEMs assumptions. In particular, we know that the first order conditions of such models give the pricing equation (1) under the testable[13] assumption of no-arbitrage and the absence of market frictions: thus the first couple of dimensions we want to include should contain proxies for arbitrage opportunities and market frictions. We also know that the class of RAEMs only deals with closed[14] exchange economies thus the impact of money and foreign markets is outside the scope of the models: for this reason the next couple of dimensions we want to have are those which contains proxies for the value of money and the impact of foreign markets on the pricing of the domestic assets. A final important dimension is the one concerning the representative agent and its existence, as the proposition below motivates, an essential (and stringent)[15] assumption in this context is the homogeneity of investors' beliefs

Proposition 3. *If the following hold*

- *The set of intervals t the time period $[0, T]$ can be divided into, the set $\Omega = \{\omega_t\}_{t=0}^T$ of states of the world, and the set of investor types J are finite*
- *Investor type j have homogeneous beliefs and standard[16] von-Neumann Morgenstern utilities over the consumption process $\{c_{j,t}(\omega_t)\}_{t=0}^T$*
- *The Law of One Price hold, the financial market is complete, arbitrage-free and features a finite number, $N + 1$, of primitive securities with ex-dividend price processes, $S_t^T = (S_{0,t}, ..., S_{N,t})$*

[13] As we detailed in Section 2 we also need the Law of One Price and the finiteness of the state space but these are not testable.

[14] Or more generically, the impact of foreign markets is not explicitly modeled.

[15] An entire literature starting form Akerlof (1970), passing through Grossman-Stigliz (1980) studies the effect of asymmetric information.

[16] Strictly increasing, strictly concave, time-additive and state-independent preferences.

- *The space of feasible net trades is linear (markets are frictionless)*

Then for any aggregate endowment process the resulting exchange economies have Pareto optimal competitive equilibria with prices that equivalently sustain a no-trade economy with a single agent, with Inter-temporal Marginal Rate of Substitution (IMRS)

$$M_{t+1} \equiv \beta \frac{u'_{t+1}(C_{t+1})}{u'_t(C_t)}$$

holding the market in equilibrium and optimally consuming the aggregate endowments $C_t \equiv \sum_{j=1}^{J} c_{j,t}$.

Proof. See Appendix. ■

the proposition shows how under the additional requirements of market completeness, vNM preferences and the *testable* assumption of homogeneity in beliefs we can construct a no-trade economy with a single agent, the representative agent,[17] holding the market portfolio, in the RAEMs framework.

In light of these reasoning and in the sake of parsimony I select the following drivers:

$$D = \{F, SII, TAX, ILLIQ, MDI, BM, USDg\} \tag{7}$$

where:

1. F, as a proxy for investors' disagreement[18] , is the Ludvigson et al. (2016) financial uncertainty measure: computed as the cross-sectional average conditional volatility of the 1-month Root Mean Squared Error in predictive regressions over approximately 150 monthly financial time series.

[17] In reality, even if in the modern finance jargon it is called the representative agent, such single agent is an *ex-post* representative agent in that is mainly a device used to explain ex-post a set of observable prices thought to be in equilibrium. Aggregate consumption in equilibrium is a function of the aggregate wealth and the asset prices, this implies that if prices changes than also the (aggregate) endowment and thus the agent holding the market in equilibrium change. Therefore the ex-post representative agent pins-down just a point, the equilibrium one, in the aggregate demand function. A true *ex-ante* agent needs to have the extra additional requirement of preferences that are independent from the aggregate endowment and the prices distributions. Unfortunately such agent can only be derived under very restrictive assumptions. (See Lewbel (1989))

[18] In the Appendix it is shown that 80% of the variability of F can be explained using a number of disagreement proxies only generating an estimate which correlates 0.9038 with the original series.

2. SII, as a proxy for investors' disagreement[19], is the Rapach et al. (2016) short interest index: computed as the log of the equal-weighted mean of short interest (as a percentage of share outstanding) across all publicly listed stocks on U.S. exchanges.

3. TAX, as a proxy for market frictions, is the annual time series of the rate of change on total taxes paid on capital gains as reported by the U.S. Department of the Treasury.

4. $ILLIQ$, as a proxy for market frictions, is the negative of the Pastor-Stambaugh (2003) liquidity index: computed as the (negative of the) aggregate average (over a month) daily response of signed volume to next day return for all individual stocks on the New York Stock Exchange and the American Stock Exchange.[20]

5. MDI, as a proxy for arbitrage opportunities, is the Pasquariello (2014) Market Dislocation Index: computed as a monthly average of hundreds of individual abnormal absolute violations of three textbook arbitrage parities in stocks, foreign exchange and money markets.

6. BM, as a proxy for arbitrage opportunities (through miss-pricing), is the book-to-market ratio taken from Goyal database:[21] book-to-market value ratio for the Dow Jones Industrial Average.

7. $USDg$, as a joint proxy for the value of money and the impact of foreign financial markets[22], is the U.S. Dollar appreciation index: computed as the linear return on the Trade Weighted U.S. Dollar Index available from the Saint Louis Federal Reserve[23]

[19] High values of the index indicate that a sizable portion of investors is betting on the market going down by short-selling stocks. Selling large amounts of stocks is only possible if on the other side of the transactions there are buyers, i.e. investors who presumably think, for whatever reason, that holding the market is better.

[20] The intuition behind the measure is that if we view liquidity as the ability to trade large quantities without moving the price and think of signed volume as a proxy for the order flow then lower liquidity is reflected in a greater tendency for order flow in a given direction on day d to be followed by a price change in the opposite direction on day $d + 1$.

[21] Available at http://www.hec.unil.ch/agoyal.

[22] The latter, as reported by Bertaut-Judson (2014) on behalf of the Board of Governors of the Federal Reserve System, is a consequence of the fact that the U.S. runs a deficit in the current account since 1985 and the excess of imports over export has been funded primarily by foreign acquisitions of U.S. securities.

[23] The index is a weighted (over the volume of bilateral transactions) average of the foreign exchange value of the U.S. dollar against the currencies of a broad group of major U.S. trading partners.

The next table gives the summary statistics of the selected drivers

[Table 2 goes about here]

We conclude this subsection by illustrating, through the correlation matrix below, how the parsimoniously selected drivers, indeed cover a variety of different information sources

[Table 3 goes about here]

The average absolute correlation is 0.1119 with the highest linear association of 0.3743 being the one between F and $ILLIQ$ followed by the 0.3187 between F and MDI. In the appendix we show how, despite a level of correlation of 0.3743, F and $ILLIQ$ are fundamentally different in that only the first one can be replicated by disagreement proxies, while in the Robustness section we document how using a version of MDI orthogonalized from F gives very similar results suggesting the difference in the F and MDI contents is what is driving the result in the main specification.

4 Results

This section contains the detailed analysis of the reason *why* the representative agent equilibrium models do not properly fit the data and *when*, over the last 25 years (the main sample MS) in the U.S. market this is mostly the case. The last subsection provides a comprehensive summary and interpretation of the analyzed results.

4.1 Why the Models fail?

We first tackle the motivations for the RAEMs failures through a static analysis in which we show the result of the joint model-free test of Definition 2 providing us with the subsample, I_t^v, containing the violation periods as a function of the selected drivers D. We then use the violation periods to perform a test for the rational expectations and look at how investors react to the RAEMs failures.

4.1.1 RAEMs' failures

Remember that a joint model-free test for the RAEMs failures is a one sided t-test

$$H_0 : \mathbb{E}[y_{t+1} I_t^v(\hat{\pi}_{t+1}, LB_t)] = 0 \text{ vs. } H_1 : \mathbb{E}[y_{t+1} I_t^v(\hat{\pi}_{t+1}, LB_t)] < 0$$

with $y_{t+1} \equiv \pi_{t+1} - LB_t$ assumed independent over time. Thus the starting point is an analysis of the empirical properties of y_{t+1}: Figure 1 shows the correlogram of y_{t+1} together with the 95% confidence bands confirming the absence of any linear form of dependence, while the other statistics of interest are summarized in the table below

[Table 4 goes about here]

The statistics, in line with Martin (2016), document the unconditional tightness of the lower bound measure in that the mean of y, 0.0028, is not statistically different from 0 and confirm the usual estimate for the unconditional risk premium, 0.0061 (0.0732 annualized).

We next turn to the (conditional) joint model-free test for the RAEMs. Throughout the study, in order to forecast the excess market return, π, we adopt a shrinkage full quadratic specification for model (4) in which we regress π on a constant, each driver in D, all the drivers' interactions and their squares for a total of 35 regressors. To avoid over-fitting and improve the quality of the forecasts, yielding a good 0.0981 out-of-sample R^2, I use the iterated approach detailed in Lin et al. (2016) which basically amounts to a regression shrinkage in which the out-of-diagonal elements in the regressors' matrix are set to zero and the regressors' coefficients are divided by 35. The results are summarized in Table 5 and graphically in Figure 2.

[Table 5 goes about here]

Table 5 shows how the RAEMs are rejected at the 5% level conditional on periods captured by the nonnegative function $I^v \equiv 1_{[\hat{\pi}_{t+1} < LB_t]}$ with the help of model (4). As a matter of facts, the conditional risk premium is below its average lower bound by a solid monthly 1.65%. Notice that the negative figure is driven by a conditional negative risk premium, on the order of -1%, rather than a very low average lower bound, which instead displays a conditional mean of 0.006 higher than its unconditional counterpart 0.0033. Furthermore, the negativity of the risk premium is in turn driven by an average negative market return, on the order of -0.4%, rather than the average level of the risk free rate, a statistically positive 0.06%. Therefore RAEMs rejections are driven by negative market returns; this insight is further corroborated by the analysis in the Robustness section showing that the same rejections are obtained by fixing the lower bound and the risk free time-series to their unconditional means.

Figure 2 gives a graphical description of the joint test, in dashed red plots the sub-sample of the y_{t+1} process, displayed in blue, selected by the function I^v, the pink areas highlights the National Bureau of Economic Research (NBER) recessions. The violations sub-sample covers 25.43% of the main sample MS, contains the three NBER recessions as well as the 1998 Long-Term Capital Management crisis and the sovereign debt crises in the aftermath of the 2008 Great Recession. Note how the rejection periods mainly incarnates the definition of bad times.

Remember that I^v through model (4) is a function of the selected drivers D, it thus make sense to use such drivers, designed to incorporate relevant non-overlapping dimensions against the assumptions of RAEMs, to explain why these models fail. In order to achive this goal I run the following regression

$$I_t^v = \beta_0 + D_t\beta + u_t \tag{8}$$

and report the result in the next table

[Table 6 goes about here]

Note how the first three drivers F ,SII and $ILLIQ$ are key: all their coefficients are significant at the 1% level, their partial R^2 are at least 3 times those of the remaining drivers and, as highlighted by the last column of the table, running regression (8) only using the first three drivers explains 0.4537 of the violation function I_v, which is 90% of the variability explained by using the whole set of drivers D.

We showed how F ,SII and $ILLIQ$ are the main responsible for the RAEMs rejections; next we illustrate how the violation sub-sample defined by I_v can indeed be characterized in terms of the main detected drivers. Consider the following model

$$d_t = \alpha_1 I_t^v + \alpha_2(1 - I_t^v) + w_t, \text{ with } d_t \in \{F_t, SII_t, ILLIQ_t\} \tag{9}$$

model (9) compares the conditional mean of the dependent variable d in the violation periods with the one computed in the rest of the main sample. Table 7 reports the results of model (9)

[Table 7 goes about here]

The key drivers F, SII and $ILLIQ$ are substantially different in periods of RAEMs failures, as a matter of facts, the difference in their means in the violations' periods and non-violation' periods are statistically significant at the 1%. All drivers have higher values in rejections' periods, in particular, the disagreement proxies (F and SII) are above their unconditional median while illiquidity is above its unconditional 75-th percentile. Thus we conclude that rejections' periods are characterized by substantial investors disagreement and high illiquidity.

4.1.2 Irrational Expectations

One of the key pillars of the RAEMs is the rational expectations assumption: as briefly mentioned in the introduction this is a consistency requirement on the agents' expectations so that they are aligned with the models' predictions. In other words, investors' expectations have to be correct, at least on average and over time (Muth, 1961). Recently Greenwood-Shleifer (2014) and Amromin-Sharpe (2014) documented how equilibrium-based required returns and investors' expectations display a counter-intuitive negative correlation casting doubts on the compatibility of rational equilibrium models and actual data. In this subsection I test whether we can detect systematic biases in the investors' expectations in the presence of RAEMs rejections and in order to do so I run the following regression

$$z_{t+1} - \mathbb{E}_t[z_{t+1}] = \gamma_1 I_t^v + \gamma_2(1 - I_t^v) + \eta_{t+1} \tag{10}$$

The random variable z_{t+1} is the quantity over which investors, using information up to time t included, form expectations $\mathbb{E}_t[z_{t+1}]$. Following the logic of model (9), I compare the conditional mean of the forecast error, $z_{t+1} - \mathbb{E}_t[z_{t+1}]$, in the presence of RAEMs rejections, captured by γ_1, and in the rest of the sample, detected by γ_2. Note that specification (10), as specification (9) before, suffers from the errors-in-variable problem in that the regressor I_t^v is itself an estimate.[24] Such bias in linear regressions deflate the real (unobservable) coefficients γ_1 and γ_2 towards zero, so that any significant result we find is robust to this problem.[25] The next table reports the result of this analysis

[Table 8 goes about here]

I use four different popular financial indicators as random variables over which investors form expectations and three different methods to capture such expectations. In the first three column z represents the return of the market in excess of the risk-free rate and the expectations are collected from survey data (Gallup survey, American Association of Individual Investors and Shiller's survey) validated in Greenwood-Shleifer (2014). z in the fourth column represent inflation, $Infl$ and the expectations are the market implied (and priced) ones from the difference in the yield of 5-year inflation indexed treasury bounds and the yield of 5-year nominal treasury bonds. In the last two columns z captures a key economic indicator, the U.S unemployment rate, UR, and a core financial indicator, the spread between the BAA rated corporate bonds and the federal funds rate, SP; expectations in this case are computed as forecasts through the specification of an econometric model following the Box-Jenkins (1970) procedure (See Appendix for the details on the specification procedure).

Under the null of rational expectations $\gamma_1 = \gamma_2 = 0$, that is, there is no systematic bias in the time series of forecasted errors $z_{t+1} - \mathbb{E}_t[z_{t+1}]$. In 5 out of 6 cases covering the three different methodologies implemented for inferring the investors' expectations, $\gamma_1 > 0$ with a significance level of 5% for the Gallup market return expectations and the model-based unemployment rate expectations. Furthermore, in 4 out of 6 cases covering the survey and model-based expectations,

[24] We are in fact sure at the 95% that it contains RAEMs violations not at the 100%.

[25] This is the reason why I did not mention the issue while describing model (9).

$\gamma_1 \neq \gamma_2$ at the 5% and 1% level. These results suggest a consistently irrationally sizable downward bias in the expectations during periods of RAEMs rejections. Note also that estimates for γ_2, the mean of the forecasting errors in the periods in which we do not reject the REEMs, are economically negative in 5 out of 6 specifications and in 2 cases, covering the survey and model-based expectations, are statistically significant at the 1% level. This last set of results suggest that the rational expectations assumption on its own, even when we cannot reject the RAEMs, is problematic.

4.1.3 Low Demand for the Market Portfolio

The last piece of evidence I gather in the static analysis of the RAEMs failures concerns the investors' aggregate reaction: we already documented that the RAEMs rejections periods are characterized by times of high market illiquidity, disagreement and irrationally downward biased expectations followed by negative market returns (which realize in $t+1$) and that the negativity of the market return drives the risk premium below the average lower bound causing the RAEMs failures. This subsection wants to provide a link between the former and the latter set of evidence. Specifically I test whether or not rejections times are associated with a lower demand for the market portfolio which, if present, justifies the negativity of the realized market returns. The usual logic of model (9) and (10) is implemented, this time yielding the following model specification

$$q_{t+1} = \delta_1 I_t^v + \delta_2 (1 - I_t^v) + \psi_{t+1} \tag{11}$$

where q_{t+1} is a proxy for the demand of the market portfolio. The next table illustrates the results for this subsection

[Table 9 goes about here]

Three proxies for the market portfolio demand are used: the de-trended[26] log volume of SPDR $SP500$ ETF (measured as the log of the number of shares sold), Vol, the Rapach et al. (2016)

[26] The result still hold without the de-trending but time-series graphs (available upon request) show it might wrongly pick up some time effects.

short interest index SII and the the net purchase position (purchases-sales) in U.S. equity from foreign investors, $NetEquityPurch$. δ_1 is statistically different from zero (positive) at the 1% level in all the specifications. Furthermore, δ_1 is statistically different (grater) than δ_2 when the demand proxies are the (log) number of $SP500$ ETFs sold, Vol, and the aggregate equity volume shorted SII while the opposite occur for the case in which the demand proxy is the net purchase of U.S. equities from foreign investors. Overall these evidence document how investors' demand for the market portfolio is lower following RAEMs rejections periods.

4.2 When the Models fail?

The second part of the analysis of the RAEMs failures is dynamic: having certified, at 95% confidence, that such models fail and attempted to explain the reason why we now turn to the study of when it is more probable that this happens.

Figure 3 plots the conditional probability distribution $\hat{P}_t(\pi_{t+1} < LB_t)$ of rejecting the RAEMs at each point in time in the main sample MS against the negative of the GDP growth[27] and shows its empirical distribution. The probability to reject the RAEMs is right-skewed, well approximated by a lognormal distribution, and always quite high: it has a mean of approximately 47% (median of 41.08%) and never goes below 33.18%. The time-series is counter-cyclical, having a negative correlation with respect to the U.S. GDP growth of 0.4817, and very high during the Great Recession period. Other notable spikes occur in periods of financial distress such as the 1998 long term capital management crises or in the sovereign debt crises in the aftermath of the Great Recession. Thus, unsurprisingly, models perform worst in periods of high financial distress, however more interestingly, due to the counter-cyclical nature, the pattern generalizes to all periods of economic contractions. As a matter of facts, in these periods the average probability to reject the RAEMs is 48.17%, statistically greater than the analog probability, 45.63%, in periods of economic expansions. Also, the fact that the probability to reject is always quite high justifying the documented poor empirical fit.

[27] The pink areas represent the NBER recessions.

Next I investigate the contribution of the main drivers $d \subset D$ on the conditional probability to reject the RAEMs. In the Robustness section I show how the ranking found in the static analysis is the same and the first three most important drivers are still the disagreement proxies F, SII and the illiquidity measure $ILLIQ$ explaining 82.92% of the variability explained by all drivers. The results are shown in Figure 4: in the upper graph the joint contribution of the disagreement proxies F, SII, which can be viewed as a new structural index, is plotted in the form of a dashed red line, the contribution of the illiquiity index $ILLIQ$, which can also be regarded as a novel structural proxy, is represented by a dotted green line, while the overall conditional probability of rejecting the RAEMs is still a solid blue line as in Figure 3. Note how the new indexes explain all the most notable spikes in the overall rejection probabilities. The novel structural proxies, even if, as expected[28], are highly correlated (with a coefficient of 0.7160), carry nonetheless different information as displayed by the bottom graph. The solid light blue line tracks the difference between the disagreement and the illiquidity series; positive values indicate an higher contribution of structural disagreement while negative values a predominant contribution of structural illiquidity. The emerging pattern is interesting, the RAEMs are impaired over time for different reasons: around NBER recessions the probability that models fail is mostly due to the illiquidity (or market frictions) component, while in normal times is mostly the disagreement (thus the failure of the symmetry-in-information assumption) part that drives the failures' likelihood.

4.3 What Is Wrong With Representative Agent Equilibrium Models?

In this last subsection we give a summary of the findings and a potential explanation.

The static analysis builds on the formal joint model-free test for the RAEMs class: the critical identifying assumption of the test, the fact that the process $y_{t+1} = \pi_{t+1} - LB_t$ is independent, is supported by the data in that no linear dependence is spotted in the series and, at least unconditionally and in line with Martin (2016), LB_t seems a good proxy for the risk-premium. We thus certifies that, at the 95% confidence, the RAEMs class is rejected in the data conditioning,

[28] By construction they are both intimately linked to the overall rejection probability time-series.

according to the objective rule $I^v(\hat{\pi}, LB)$, mainly on periods of high illiquidity where the level of investors' disagreement is above the median, covering 25% of the main sample MS and containing all the three NBER recessions as well as other periods of financial instability such as the 1998 Long Term Capital Management crisis and the sovereign bond crisis in the aftermath of the Great Recession. In the rejection periods, the conditional risk-premium (under rational expectations) or simply the following period average excess realized market return is negative and systematically lower than the mean lower bound (which by construction is nonnegative) causing the RAEMs rejections as per Proposition 2. In spite of the attenuation bias caused by the usage of a proxy capturing the RAEMs rejections with a Type I error of 5%, we are able to detect violations of the Rational Expectations hypothesis: aggregate expectations, recorded through three different methodologies, systematically under-estimate the true variables of interest in periods in which equilibrium models are rejected[29] and instead overshoot in the rest of the sample where either we do not have enough power to reject or supposedly the models hold. This seems to suggest that rational expectations on their own, beyond their role in RAEMs, are problematic.

According to the equilibrium models, the representative agent, in order to hold the market portfolio in bad times, requires an high risk premium, which, under rational expectations, should coincide with high average realized excess market returns in following periods. This is not what we observe: periods following bad times, substantially captured by the rejection sub-sample, are characterized by a falling aggregate market demand and negative realized market returns and hence[30] excess market returns, rather suggesting the presence of pessimistic investors who, afraid of the poor financial-economic conditions they observe in t, decide to (short) sell the market pushing the demand down and causing negative realized returns in $t + 1$.

The dynamic analysis detects an high conditional probability to reject the RAEMs at any point in time in the main sample MS, a fact that justifies the overall poor fit of this models to the data: it never goes below 33.18% and on average is 47%. More importantly, it shows how the relative importance of the main reasons behind the rejections, which are the same as the ones we detected

[29] In the case the variable of interest is the market return this evidence suggests investors' pessimism.

[30] Given the risk-free rate is almost never negative.

statically, changes over time: in periods of economic recessions the more severe issue is the one of market frictions (illiquidity) while in normal time it is more a matter of asymmetric information.

5 Robustness

Any of the subsection below is independent and can be read on its own.

5.1 Linear versus cubic spline lower bound

In order to compute the lower bound measure at time t, LB_t, according to equation (3) we use the SPX option (Put and Call) bid quotes at horizon 1 month for the different available strikes as at the end of day t from Optsum and Optionmetrics. In order to compute the integral in (3) we first need to interpolate the functions $\hat{put}(k)$ and $\hat{call}(k)$ over a continuum of strikes. Because theoretically we know of the convexity of these functions, in the study so far we have used a cubic-spline interpolation. Another obvious interpolant option is the linear one; Figure 5 shows the time-series of lower bounds in the main sample MS computed with the linear as well as the cubic-spline method

[Figure 5 goes about here]

the upper graph plots the two time series while the bottom one shows, in percentage, the absolute difference in terms of the cubic-spline approximation. The two time-series are overall very similar, the mean absolute difference is 2.4358% with most of the differences in the periods pre-Optionmetrics (i.e until 1996). All the results in the paper are unaffected by the way we compute the bounds.

5.2 The impact of dividends on the lower bound measure

Martin (2016) derives a lower bound for the risk premium, LB_M, which is an implicit function of the market dividends. In his formulation dividends are assumed known and part of the SP500 index.[31] Following this assumption all the contracts on the SP500 are to be considered *as if* written on the total value of the index rather than the ex-dividend one, an expedient which simplify the derivations and it is equivalent to the assumption that there are no dividends at all: as a matter of fact in my derivation $LB \equiv LB_M$ if and only if the gross dividend yield DY_t is equal to 1. I argue that, more realistically, one should account for the fact that such contracts are written on the *ex-dividend* level of the SP500 so that dividends (or divided yields), even if assumed known, should become an explicit input in the lower bound derivation. Empirically whether they are a function of the dividends or not and whether dividends are indeed to be considered deterministic or stochastic turns out to be irrelevant in the current analysis. However, the realization of such a convenient simplification, would have been otherwise impossible to detect if no such formula, namely equation (3), for the bound as a function of dividend had been derived. I now make the argument concrete by showing Table 10 which compares the key moments of the lower bound empirical distributions under the Martin, LB_M^m, and the current, LB^m setup for the linear $m \equiv l$ interpolation as well as the cubic-spline $m \equiv cs$

[Table 10 goes about here]

the four distributions are virtually the same: it is evident how the empirical role of *deterministic* dividends be negligible. Nonetheless, the conclusion in the current framework is even more general: if dividends were stochastic and the correlation between the gross dividend yield and the ex-dividend market return was zero, $\rho \equiv corr(DY_t, \hat{R}_t^{mkt}) = 0$, then $Var^Q(R_{t+1}^{mkt}) \approx Var^Q(\hat{R}_{t+1}^{mkt})$ so LB_M^m would still be a good *overall* measure. The overall in-sample correlation is $\hat{\rho} = -0.0515$ with a p-value of 0.2334. I thus conclude that the impact of dividends is empirically irrelevant.

5.3 RAEMs' rejections driven by negative market returns

In the Results section I show that the joint model-free test for the RAEMs rejects the models at the 95% confidence and that the result seems to be due to the negativity of the market return. In this subsection I confirm this evidence by running, and rejecting, an analog joint test where I

[31] This way the stochastic component of S only comes from the ex-dividend level $\hat{S}$.

keep the lower bound measure and the risk-free rate at their unconditional mean. Since we know from Table 4 that unconditionally the lower bound is quite tight and *below* the risk premium, it follows that the rejection mainly steams from the dynamic of the market return.

[Table 11 goes about here]

note how the results are very similar to those of the main joint model-free test displayed in Table 5.

5.4 Explaining RAEMs' failures via the rejection probabilities

In section 4.1.1 (Table 6) we show that the key drivers in the rejection of the RAEMs are the disagreement proxies F and SII as well as the (negative) of the Pastor-Stambaugh (2003) illiquidity index $ILLIQ$, a similar conclusion can be reached if instead of explaining the joint rejections, captured by the indicator function $I_t^v(\hat{\pi}_{t+1}, LB_t) \equiv 1_{[\hat{\pi}_{t+1} < LB_t]}$, we regress the drivers D on the conditional probabilities to reject the RAEMs according to the model

$$P_t(\pi_{t+1} < LB_t) = \beta_0 + D_t\beta + u_t \tag{12}$$

the results are reported in the following table

[Table 12 goes about here]

note that, according to the partial R^2, F, SII and $ILLIQ$ are still the first most important variables capturing $82.82\% = \frac{0.6876}{0.83022} \times 100$ of the variation explained by all drivers.

5.5 Drivers: purging the MDI index

When we first introduced the drivers D in section 3.2 we showed how the correlation between MDI and F, 0.3187 as reported in Table 3, is the second highest. In this subsection I argue that the two variables still carry fundamentally different information. As a matter of fact, I construct a new MDI variable, MDI^O, as the residual from the regression

$$MDI_t = b_0 + b_1 F_t + e_t \tag{13}$$

MDI_t^O is by construction orthogonal to F_t, nonetheless substituting it to the original MDI in the specification of the drivers' matrix D I still find all the results[32] detailed in section 4. I conclude that the difference in the F and MDI contents seems to be what is driving the results in the main specification.

6 Conclusions

In this paper I investigate the reasons why the popular consumption-based equilibrium framework of the representative agent, featuring exchange economies and rational expectations, does not properly fit the data despite the growing number of empirical moments that it is now able to match by simulations.[33] In order to do so, I use a flexible econometric model to predict the market risk premium, function of a set of drivers capturing testable dimensions against the assumptions of the models, in conjunction with a novel restriction linking the conditional risk premium on the market to observables in the marginal investor's information set (Martin, 2016). The selected drivers cover four dimensions with proxies for the failure of the symmetric information assumption, the absence of market frictions and arbitrage as well as proxies for the impact of money and foreign markets on the pricing of domestic assets.

I find that irrationally downward-biased aggregate expectations in periods of substantial asymmetric information and high market frictions cause the Representative Agent Equilibrium Models (RAEMs) to have a poor fit to the data. These findings are corroborated by a counter-cyclical conditional probability to reject the models at any point in time in the main sample, featuring a high average of approximately 47% which never goes below 33%. During bad times equilibrium models

[32] Available upon request.

[33] See for example Campbell-Cochrane (1999), Bansal-Yaron (2004), Bansal et al. (2014), Barro (2006) and Wachter (2913).

require the rational investor to demand higher risk premia, in contrast, data suggest pessimistic investors depress market prices pushing the conditional risk-premium below its lower bound by selling the market portfolio. Tracking the contribution of the main drivers to the RAEMs' rejection probability over time reviles the RAEMs' probability to fail during economic recessions is mostly due to market frictions while in normal times to asymmetric information.

Left to future research is the task of assimilating these new documented facts into a theory model able to justify them.

7 Appendix

7.1 Proof of Proposition 1

First I show why LB_t is a lower bound for the market risk premium $\mathbb{E}_t[R^{mkt}_{t+1} - R_{t,f}]$ then I derive equation (3).

Suppose markets are arbitrage free and there exist a stochastic discount factor M, satisfying the pricing equation (1) then by the FTAP $M > 0$ and there exist an equivalent risk-neutral measure Q such that $R_f = \mathbb{E}[R^i]$ for any gross return R^i.

By definition the conditional risk neutral variance for the market return at horizon $t+1$ can be written as

$$Var^Q_t(R^{mkt}_{t+1}) \equiv E^Q_t[R^{mkt2}_{t+1}] - E^Q_t[R^{mkt}_{t+1}]^2$$

where R^{mkt}_{t+1} is the gross cum-dividend market return. Still from FTAP we can go back and forth from the physical probability measure and the risk-neutral one, thus $E^Q_t[R^{mkt2}_{t+1}] = E_t[R_{t,f}M_{t+1}R^{mkt2}_{t+1}]$ and by the definition of risk-neutral measure, $E^Q_t[R^{mkt}_{t+1}]^2 = {R_{t,f}}^2$, hence

$$Var^Q_t(R^{mkt}_{t+1}) = E_t[R_{t,f}M_{t+1}R^{mkt2}_{t+1}] - R^2_{t,f}$$

dividing the above equation by the gross risk-free return $R_{t,f}$ and rearranging

$$\frac{Var_t^Q(R_{t+1}^{mkt})}{R_{t,f}} = E_t[R_{t+1}^{mkt} - R_{t,f}] + Cov_t(M_{t+1}R_{t+1}^{mkt}, R_{t+1}^{mkt})$$

if $Cov_t(M_{t+1}R_{t+1}^{mkt}, R_{t+1}^{mkt}) \leq 0$, the NCC, then $LB_t \equiv \frac{Var_t^Q(R_{t+1}^{mkt})}{R_{t,f}}$ is a lower bound for $RP_t \equiv E_t[R_{t+1}^{mkt} - R_{t,f}]$.

Next, I derive equation (3). From the definition of variance, using hats to denotes ex-dividend quantities and letting S be the cum-dividend market level

$$\begin{aligned} Var_t^Q(R_{t+1}^{mkt}) \equiv & \quad E_t^Q\left[\left(\frac{S_{t+1}}{S_t}\right)^2\right] - E_t^Q\left[\frac{S_{t+1}}{S_t}\right]^2 \\ = & \quad E_t^Q\left[\left(\frac{\hat{S}_{t+1}}{\hat{S}_t}DY_t\right)^2\right] - {R_{t,f}}^2 \\ = & \quad \frac{(DY_t)^2 R_{t,f}}{(\hat{S}_t)^2} E_t^Q\left[\frac{\hat{S}_{t+1}^2}{R_{t,f}}\right] - {R_{t,f}}^2 \end{aligned}$$

by no arbitrage (see Martin 2016), since the options are written on $\hat{S}_t$

$$E_t^Q\left[\frac{\hat{S}_{t+1}^2}{R_{t,f}}\right] = 2\int_0^\infty \hat{call}_t(k)dK = 2\left(\int_0^{\hat{F}_t} \hat{call}_t(k)dK + \int_{\hat{F}_t}^\infty \hat{call}_t(k)dK\right)$$

since deep-in-the-money call options are neither liquid in practice nor intuitive to think about, it is convenient to split the range of integration for $E_t^Q\left[\frac{\hat{S}_{t+1}^2}{R_{t,f}}\right]$ into two and use the put-call parity to replace in-the-money call prices with out- of-the-money put prices. Assume that Market Dividends are paid as lump sums D_{t+1} at the and of the period $[t : t+1]$ but before $t+1$, then the following is true

$$max(S_{t+1} - D_{t+1} - k, 0) = max(k - S_{t+1} + D_{t+1}, 0) + (S_{t+1} - D_{t+1}) - k$$

since $\hat{S}_{t+1} = S_{t+1} - D_{t+1}$

$$max(\hat{S}_{t+1} - k, 0) = max(k - \hat{S}_{t+1}, 0) + (S_{t+1} - D_{t+1}) - k$$

by linearity of the pricing equation

$$\hat{call}_t(k) = \hat{put}_t(k) + \hat{S}_t - PV(D_{t+1}) - \frac{k}{R_{t,f}}$$

where $PV(D_{t+1}) = \mathbb{E}_t^Q\left[\frac{D_{t+1}}{R_{t,f}}\right] = (1 - DY_t)\mathbb{E}_t^Q\left[\frac{\hat{S}_{t+1}}{R_{t,f}}\right] = \frac{DY_t - 1}{DY_t}\hat{S}_t$ and the last equality comes from $R_{t,f} = \mathbb{E}_t^Q\left[\frac{S_{t+1}}{S_t}\right]$. Applying the put-call parity

$$\begin{aligned}
\int_0^{\hat{F}_t} \hat{call}_t(k)dK = & \int_0^{\hat{F}_t} \hat{put}_t(k)dK + \hat{F}_t\left(\hat{S}_t - \frac{DY_t - 1}{DY_t}\hat{S}_t\right) - \frac{\hat{F}_t^2}{2R_{t,f}} \\
= & \int_0^{\hat{F}_t} \hat{put}_t(k)dK + \hat{F}_t\left(\frac{\hat{S}_t}{DY_t} - \frac{\hat{F}_t}{2R_{t,f}}\right)
\end{aligned}$$

which implies

$$E_t^Q\left[\frac{\hat{S}_{t+1}^2}{R_{t,f}}\right] = 2\left[\int_0^{\hat{F}_t} \hat{put}_t(k)dK + \hat{F}_t\left(\frac{\hat{S}_t}{DY_t} - \frac{\hat{F}_t}{2R_{t,f}}\right) + \int_{\hat{F}_t}^{\infty} \hat{call}_t(k)dK\right]$$

plugging $E_t^Q\left[\frac{\hat{S}_{t+1}^2}{R_{t,f}}\right]$ in $Var_t^Q(R_{t+1}) = \frac{(DY_t)^2 R_{t,f}}{(\hat{S}_t)^2}E_t^Q\left[\frac{\hat{S}_{t+1}^2}{R_{t,f}}\right] - {R_{t,f}}^2$ delivers equation (3)

$$LB_t = 2\frac{(Q_t)^2}{(\hat{S}_t)^2}\left(\int_0^{\hat{F}_t} \hat{put}_t(k)dK + \hat{call}_t(k)dK\right)$$

7.2 Proof of Proposition 3

Denote homogeneous agents' beliefs as $\{\{p_t(\omega_t)\}_{\omega_t}\}_{t=0}^T$ with $p_0(\omega_0) = p_0 = 1$. Define a Lucas type economy where each asset pays dividends $D_t^T = (D_{0,t}, ..., D_{N,t})$ at t. Since the space of feasible net trades is linear agent j at time t can trade (buy and sell) any asset in any (even infinitesimal) quantity $\alpha_{j,t}^T = (\alpha_{j,t}^0, ..., \alpha_{j,t}^N)$. The problem that investor j faces is

$$\max_{\{\{c_{j,t}(\omega_t),\alpha_{j,t}(\omega_t)\}_{\omega_t}\}_t} \quad \sum_t \beta_j^t \sum_{\omega_t} p_t(\omega_t)u_{j,t}(c_{j,t}(\omega_t))$$

subject to

$$c_{j,t}(\omega_t) + \alpha_{j,t}(\omega_t)^T S_t(\omega_t) \leq \alpha_{j,t}(\omega_t)^T (S_t(\omega_t) + D_t(\omega_t)) \text{ for every } t \text{ and } \omega_t$$

where β_j^t is the subjective time discount factor of agent j and $u_{j,t}$ is strictly incising and strictly concave. The market is required to clear in the aggregate meaning

$$C_t(\omega_t) \equiv \sum_j c_{j,t}(\omega_t) = \sum_i D_{i,t}(\omega_t) \text{ for every } t \text{ and } \omega_t$$

From the F.O.C. of agent j problem with respect to $\alpha_{j,t}$

$$S_t(\omega_t) = \sum_{\omega_{t+1}} \beta_j \frac{u'_{j,t+1}(c_{j,t+1}(\omega_{t+1}))}{u'_{j,t}(c_{j,t}(\omega_t))} \frac{p_{t+1}(\omega_{t+1})}{p_t(\omega_t)} (S_{t+1}(\omega_{t+1}) + D_{t+1}(\omega_{t+1}))$$

Define for every t, $|\omega_t| \equiv \Omega_t$, then the market payoff matrix that can be reached from time t at state ω_t is characterized by

$$Y_{t+1}(\omega_t) = \begin{bmatrix} S^0_{t+1}(1) + D^0_{t+1}(1) & \cdots & S^0_{t+1}(\Omega_{t+1}) + D^0_{t+1}(\Omega_{t+1}) \\ \vdots & \ddots & \vdots \\ S^N_{t+1}(1) + D^N_{t+1}(1) & \cdots & S^N_{t+1}(\Omega_{t+1}) + D^N_{t+1}(\Omega_{t+1}) \end{bmatrix}$$

because the market is complete $rank(Y_{t+1}(\omega_t)) = \Omega_{t+1}$ and $N+1$ is large enough such that $N+1 = \Omega_{t+1}$. Further define

$$z^j_{t+1}(\omega_t) = \begin{bmatrix} \beta_j \frac{u'_{j,t+1}(c_{j,t+1}(1))}{u'_{j,t}(c_{j,t}(\omega_t))} \frac{p_{t+1}(1)}{p_t(\omega_t)} \\ \vdots \\ \beta_j \frac{u'_{j,t+1}(c_{j,t+1}(\Omega_{t+1}))}{u'_{j,t}(c_{j,t}(\omega_t))} \frac{p_{t+1}(\Omega_{t+1})}{p_t(\omega_t)} \end{bmatrix}$$

thus the F.O.C. can be rewritten as

$$S_t(\omega_t) = Y_{t+1}(\omega_t) z^j_{t+1}(\omega_t)$$

and the payoff matrix $Y_{t+1}(\omega_t)$ is invertible and $z^j_{t+1}(\omega_t)$ is uniquely determined. That is for any agent j and i

$$\beta_j \frac{u'_{j,t+1}(c_{j,t+1}(\omega_{t+1}))}{u'_{j,t}(c_{j,t}(\omega_t))} \frac{p_{t+1}(\omega_{t+1})}{p_t(\omega_t)} = \beta_i \frac{u'_{i,t+1}(c_{i,t+1}(\omega_{t+1}))}{u'_{i,t}(c_{i,t}(\omega_t))} \frac{p_{t+1}(\omega_{t+1})}{p_t(\omega_t)} \equiv \frac{p_{t+1}(\omega_{t+1})}{p_t(\omega_t)} M_{t+1}(\omega_{t+1}) \equiv m_{t+1}(\omega_{t+1})$$

note that the state contingent claim that pays 1 unit of consumption in state ω_{t+1} only can now be obtained through the asset allocation $\alpha_t(\omega_t)^T = (\alpha^0_t(\omega_t), ..., \alpha^N_t(\omega_t))$ such that

$$(\alpha^0_t(\omega_t), ..., \alpha^N_t(\omega_t)) = (1, 0, ..., 0) Y_{t+1}(\omega_t)^{-1}$$

thus in a complete market any state contingent claim at any time t is attainable. Define $\phi_0(\omega_{t+1})$ as the time 0 price of the contingent claim that at $t+1$ delivers 1 unit of consumption if state ω_{t+1} realizes, then by the Law of One Price

$$\phi_0(\omega_{t+1}) = price_0((\alpha^0_t(\omega_t), ..., \alpha^N_t(\omega_t)) = (1, 0, ..., 0) Y_{t+1}(\omega_t)^{-1})$$

the set $\{\{\phi_0(\omega_t)\}_{\omega_t}\}_t$ contains all the state prices of the economy, where by definition $\phi_0(\omega_0) = \phi_0 = 1$.

The fact that the market is complete enable to re-state the problem of agent j as follows

$$\max_{\{\{c_{j,t}(\omega_t)\}_{\omega_t}\}_t} \sum_t \beta^t_j \sum_{\omega_t} p_t(\omega_t) u_{j,t}(c_{j,t}(\omega_t))$$

subject to

$$\sum_t \sum_{\omega_t} \phi_0(\omega_t) c_{j,t}(\omega_t) \leq \sum_t \sum_{\omega_t} \phi_0(\omega_t) e_{j,t}(\omega_t)$$

where $e_{j,t}(\omega_t)$ is the agent (exogenous) endowment at time t in state ω_t. From the F.O.C. of this problem

$$\phi_0(\omega_t) = \beta^t_j \frac{u'_{j,t}(c_{j,t}(\omega_t))}{u'_{j,0}(c_{j,0})} p_t(\omega_t) \text{ for every } t \text{ and } \omega_t$$

where $u'_{j,0}(c_{j,0}) = \delta_j$ and δ_j is the Lagrange multiplier, note that

$$\phi_0(\omega_t) = m_t(\omega_t) \times m_{t-1}(\omega_{t-1}) \times ... \times m_1(\omega_1) \text{ for every } t \text{ and } \omega_t$$

I next show that the competitive equilibrium allocations $\{\{c_{1,t}\}_t, ..., \{c_{J,t}\}_t\}$ are Pareto optimal. A Pareto optimal allocation is a feasible allocation, that is

$$\sum_j c_{j,0} = C_0$$

and

$$\sum_j c_{j,t}(\omega_t) = C_t(\omega_t) \text{ for every } t \text{ and } \omega_t$$

such that it does not exist any other allocation which is feasible and can strictly increase at least one individual's utility without decreasing the utilities of the others. From the classical second welfare theorem (see e.g. Varain (1978)), it is known that corresponding to every Pareto optimal allocation, there exist a set of non-negative numbers, $\{\lambda_j\}_j$, such that the same allocation can be achieved by a social planner solving the following problem

$$\max_{\{\{\{c_{j,t}(\omega_t)\}_{\omega_t}\}_t\}_j} \quad \sum_i \lambda_j \sum_t \beta_j^t \sum_{\omega_t} p_t(\omega_t) u_{j,t}(c_{j,t}(\omega_t))$$

subject to

$$\sum_j c_{j,0} = C_0$$

and

$$\sum_j c_{j,t}(\omega_t) = C_t(\omega_t) \text{ for every } t \text{ and } \omega_t$$

where in order to avoid the trivial (and unrealistic) case of Pareto Optima where only some investor get something I require the Pareto weights to be strictly positive. It is then easy to show that the F.O.C of this problem are the same of these of last problem provided we set the Lagrange multipliers of this problem, $\gamma_t(\omega_t)$, equal to the state prices, i.e. $\gamma_t(\omega_t) = \phi_0(\omega_t) > 0$ and we set the Pareto weights such that $\lambda_j = \frac{1}{\delta_j}$ where δ_j was the j-th Lagrange multiplier in the previous problem. Thus the competitive equilibrium allocations $\{\{c_{1,t}\}_t, ..., \{c_{J,t}\}_t\}$ are Pareto optimal.

The last step of the proof concern the construction of the single agent economy which, given the stream of endowments $\sum_t \sum_{\omega_t} e_{j,t}(\omega_t)$ for each agent j, is sustained by the *same* set $\{\{\phi_0(\omega_t)\}_{\omega_t}\}_t$ of state prices that sustains the competitive equilibrium in the multi-agent economies that we have defined in this proof. Define

$$\beta^t = \sum_j \frac{\lambda_j}{\sum_j \lambda_j} \beta_j^t$$

$$u_0(W_0) = \max_{\{w_{j,0}\}_j} \quad \sum_j \lambda_j u_{j,0}(w_{j,0})$$

subject to

$$\sum_j w_{j,0} = W_0$$

$$u_t(W_t(\omega_t)) = \max_{\{w_{j,t}(\omega_t)\}_j} \quad \frac{1}{\beta^t} \sum_j \lambda_j \beta_j^t u_{j,t}(w_{j,t}(\omega_t)) p_t(\omega_t)$$

subject to

$$\sum_j w_{j,t}(\omega_t) = W_t(\omega_t)$$

note then from the feasibility constraints it follows that

$$u_0'(C_0) = J$$

and

$$u_t'(C_t(\omega_t)) = J \frac{\phi_0(\omega_t)}{\beta^t}$$

now consider an agent whose utility function and endowments are $\sum_t \beta^t \sum_{\omega_t} p_t(\omega_t) u_t(C_t(\omega_t))$ and $\{\{C_t(\omega_t)\}_{\omega_t}\}_t$ respectively where $C_t(\omega_t) = \sum_j e_{j,t}(\omega_t)$ for every t and ω_t so that the market clears. Then the state prices must be set so that the agent optimal consumption is to hold the aggregate endowments. Therefore, using the time 0 consumption good as the numeraire, the ratio of state prices $\frac{\phi_0(\omega_{t+1})}{\phi_0(\omega_t)}$ must be equal to the single agent's marginal rate of substitution between time t in state ω_t and time $t+1$ in state ω_{t+1}, a necessary condition which is indeed satisfied

$$M_{t+1}(\omega_{t+1}) \equiv \beta \frac{u'_{t+1}(C_{t+1}(\omega_{t+1}))}{u'_t(C_t(\omega_t))} \frac{p_{t+1}(\omega_{t+1})}{p_t(\omega_t)} = \frac{\phi_0(\omega_{t+1})}{\phi_0(\omega_t)}$$

It is straightforward to show that the set $\{\{\phi_0(\omega_t)\}_{\omega_t}\}_t$ of state prices are indeeed equilibrium prices in the economy of the single agent. As a matter of fact the agent solves

$$\max_{\{\{C_t(\omega_t)\}_{\omega_t}\}_t} \quad \sum_t \beta^t \sum_{\omega_t} p_t(\omega_t) u_t(C_t(\omega_t))$$

subject to

$$\sum_t \sum_{\omega_t} \phi_0(\omega_t) C_{j,t}(\omega_t) = \sum_t \sum_{\omega_t} \phi_0(\omega_t) (\sum_j e_{j,t}(\omega_t))$$

from the F.O.C

$$\phi_0(\omega_{t+1}) = \beta^{t+1} \frac{u'_{t+1}(C_{t+1}(\omega_{t+1}))}{u'_0(C_0)} p_{t+1}(\omega_{t+1})$$

thus

$$\frac{\phi_0(\omega_{t+1})}{\phi_0(\omega_t)} = \beta \frac{u'_{t+1}(C_{t+1}(\omega_{t+1}))}{u'_t(C_t(\omega_t))} \frac{p_{t+1}(\omega_{t+1})}{p_t(\omega_t)} \equiv M_{t+1}(\omega_{t+1})$$

As a last important remark notice that the single agent utility is a function of the Pareto optimal weights $\{\lambda_j\}_j$ and that for each j $\lambda_j = \frac{1}{\gamma_j}$ and γ_j is the Lagrange multiplier for $(\sum_t \sum_{\omega_t} (\phi_0(\omega_t)(e_{j,t}((\omega_t)) - c_{j,t}((\omega_t)))))$ so that by changing the (exogenous) endowment distribution $\{\{e_{j,t}((\omega_t))\}_{\omega_t}\}_t$ or, in general, the aggregate endowment distribution $\{\{\sum_j e_{j,t}((\omega_t))\}_{\omega_t}\}_t$ the equilibrium point changes and also potentially the agent that at the new equilibrium point holds the market. Because in general endowments as well as the ex-dividend asset prices S_t are functions of a set of state variables $\{Z_t^1, ..., Z_t^s\}$ by changing the state variables both prices and endowments changes leading to a potential change in the single agent who holds the market.

7.3 The Ludvigson et al. 2016 financial uncertainty index F as disagreement

In this subsection I show how the financial uncertainty index F, designed to capture "the conditional volatility of a disturbance that is unforeseeable from the perspective of economic agents",[34] can be viewed as a proxy for disagreement. Since, unlike the classical disagreement proxies available in the literature, the monthly data for F dates back to the sixties, it is particularly convenient for my study which uses an overall sample starting in February 1973.

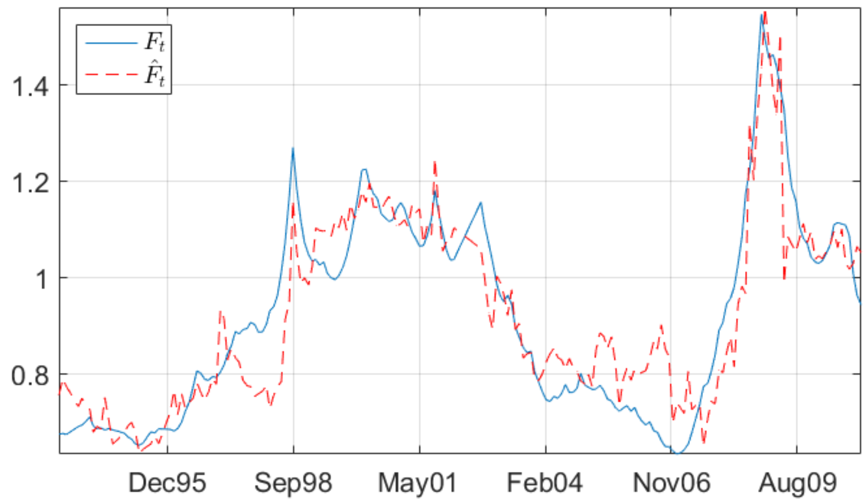
The reason why we can think of F as a proxy for disagreement is due to the fact that its time-series can be almost replicated by a nonlinear regression which is only a function of the classical disagreement proxies available in the literature and the Rapach et al. (2016) short interest index SII (also being a proxy for disagreement). In particular, on top of SII I use the standard deviation of the $I/B/E/S$ time-series of 1-year $SP500$ top.down earning-per-share analysts' forecasts (available from January 1992), EPS^{TD},the Yu (2011) bottom-up disagreement measure computed by aggregating disagreements regarding the individual assets in the SP500 portfolio (available from January 1982 to December 2011), EPS^{BU}, and the Carlin et al. (2014) disagreement measure calculated as the level of disagreement among Wall Street mortgage dealers about prepayment speeds (available from January 1993 to December 2012), CLM.

The following graph shows the time-series of F and $\hat{F}_t$, the estimate of F from the model

$$F_t = \beta_0 + f(EPS^{TD}, EPS^{BU}, CLM, SII) + u_t$$

where $f(\cdot)$ is a full second order polynomial in its arguments.

[34] See Ludvigson et al. 2016



The adjusted R^2 of the regression is 0.8028 while the correlation between the two time-series is 0.9038.

7.4 The fundamental difference between F and $ILLIQ$

This subsection is basically an extension of the previous one: in order to show the difference in nature of the two indexes despite a correlation of 0.3743, I repeat the analysis conducted on F to $ILLIQ$. The model is

$$ILLIQ_t = \beta_0 + f(EPS^{TD}, EPS^{BU}, CLM, SII) + u_t$$

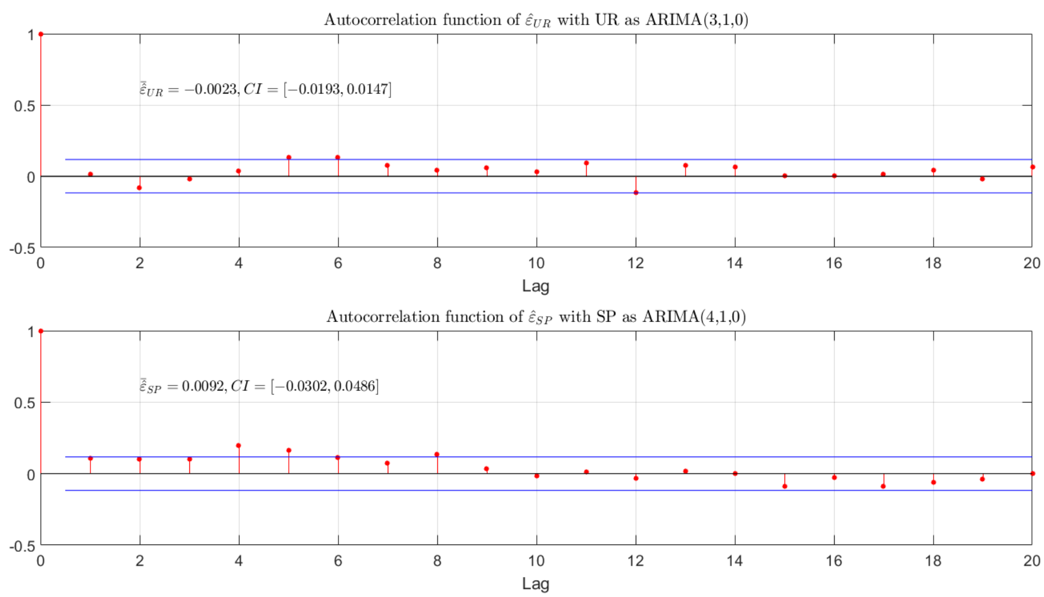
where $f(\cdot)$ is a full second order polynomial in its arguments. This time, very differently from the case of F I find a regression adjusted R^2 of approximately 2%. I conclude that $ILLIQ$, unlike F, cannot be replicated by disagreement proxies, thus containing fundamentally different information.

7.5 Forecasting the Unemployment and the Spread between BAA corporate yields and the federal funds rate

In this subsection I show how I specified, following the Box-Jenkins (1970) procedure, the forecasting models for the Unemployment rate, UR, and the spread between the BAA rated corporate bonds and the federal funds rate, SP, yielding the time-series for the conditional expectations $\mathbb{E}_t[UR_{t+1}]$ and $\mathbb{E}_t[SP_{t+1}]$ respectively.

For both time-series I used the autocorrelation and partial autocorrelation functions and plotted the first differences in order to generate a set of candidate parameters for the ARIMA class of time-series model to be used, then I exploited the AIC and BIC criteria to select the optimal set of parameters and finally performed an Augmented Dickey-Fuller test to check for stationarity. The time-series of conditional expectations, $\mathbb{E}_t[y_{t+1}]$ with $y \in \{UR, SP\}$, are computed as iterative out-of-sample one-step ahead forecasts using the best specified stationary ARMIA model. If the model is correctly specified, the innovations $\varepsilon_{t+1} = y_{t+1} - \mathbb{E}_t[y_{t+1}]$ should be independent over time and have zero mean.[35]

The results are displayed in the graphs below



as shown in the upper autocorrelation plot for ε_{UR}, the best selected ARIMA model, calibrated in the sample $Jan : 1948 - Dec : 1989$, features a first difference in UR to which an AR process of order 3 has been applied and generates out-of-sample innovations with no systematic (linear) dependence and a mean not statistically significant from zero. The bottom graph reports the analogous analysis for the case of SP; results are similar to those of UR except that the best

[35] Which is not guaranteed by construction since the forecast are out of sample.

selected model, calibrated in the sample $July : 1954 - Dec : 1989$, is an ARIMA(4,1,0), i.e. the first difference of SP is modeled through an AR process of order 4.

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9 Tables

Table 1: Statistics on Main Variables

Variable	Mean	Std.Dev.	Min	Max	N. Obs.
$R_{t+1} - 1$	0.0093	0.0457	-0.2162	0.1705	494
$R_{t,f} - 1$	0.0042	0.0029	0.000	0.0138	494
LB_t	0.0033	0.0032	0.000	0.0347	291

The table summarizes the main variables: $R_{t+1} - 1$ is the total linear return on the $SP500$, $R_{t,f} - 1$ is the 1-month yield to maturity on U.S. Treasuries and LB_t is the market premium lower bound measure computed through (3). Observations are at the monthly frequency (not annualized). The lower bound statistics are computed in the main sample $Jan : 1990 - Dec : 2014$ while the market and the risk-free returns' ones are computed over the entire sample $Feb : 1973 - Dec : 2014$.

Table 2: Statistics on the selected drivers D

Variable	**Mean**	**Std.Dev.**	**Min**	**Max**	**N. Obs.**
F	0.9187	0.1755	0.6336	1.5464	494
SII	0.0114	0.9990	-2.1931	2.9358	494
TAX	0.1211	0.2999	-0.4984	0.9998	494
$ILLIQ$	0.0300	0.0647	-0.2010	0.4610	494
MDI	-0.0271	0.1611	-0.6519	1.4715	494
BM	0.4948	0.2935	0.1205	1.2065	494
$USDg$	0.0022	0.0130	-0.0409	0.0663	494

The table summarizes the selected drivers D over the entire sample $Feb: 1973 - Dec: 2014$. F is the Ludvigson et al. (2016) financial uncertainty measure: computed as the cross-sectional average conditional volatility of the 1-month Root Mean Squared Error in predictive regressions over approximately 150 monthly financial time series. SII is the Rapach et al. (2016) short interest index: computed as the log of the equal-weighted mean of short interest (as a percentage of share outstanding) across all publicly listed stocks on U.S. exchanges. TAX is the annual time series of the rate of change on total taxes paid on capital gains as reported by the U.S. Department of the Treasury. $ILLIQ$ is the negative of the Pastor-Stambaugh (2003) liquidity index: computed as the (negative of the) aggregate average daily response over a month of signed volume to next day return for all individual stocks on the New York Stock Exchange and the American Stock Exchange. MDI is the Pasquariello (2014) Market Dislocation Index: computed as a monthly average of hundreds of individual abnormal absolute violations of three textbook arbitrage parities in stocks, foreign exchange and money markets. BM is the book-to-market value ratio for the Dow Jones Industrial Average. $USDg$ is the U.S. Dollar appreciation index: computed as the linear return on the Trade Weighted U.S. Dollar Index available from the Saint Louis Federal Reserve; the index is a weighted (over the volume of bilateral transactions) average of the foreign exchange value of the U.S. dollar against the currencies of a broad group of major U.S. trading partners.

Table 3: Pearson correlation matrix for the drivers D

Variable	F	SII	TAX	$ILLIQ$	MDI	BM	$USDg$
F	1						
SII	0.0703	1					
TAX	-0.2139	-0.0594	1				
$ILLIQ$	0.3743	0.1240	-0.0422	1			
MDI	0.3187	-0.0200	-0.0549	0.1442	1		
BM	0.0982	-0.2427	0.0141	0.1213	-0.0128	1	
$USDg$	0.0005	-0.1036	0.0191	-0.0139	0.0772	0.1055	1

The table displays Pearson correlation coefficients for the selected drivers D, described in the notes to Table 2, over the entire sample $Feb:1973-Dec:2014$.

Table 4: Statistics on y and its components

Statistic	y	π	R^{mkt}	R_f	LB
Mean	0.0028				
Mean		0.0061**			0.0033***
Mean			0.0086***	0.0025***	

The table summarizes the statistics of $y_{t+1} \equiv \pi_{t+1} - LB_t$ and its components ($\pi_{t+1} \equiv R^{mkt}_{t+1} - R_{t,f}$ being the excess market return and LB_t the lower bound measure for the risk premium $\mathbb{E}_t[\pi_{t+1}]$ computed through (3)) over the main sample $Jan:1990-Dec:2014$. One star symbols the statistic is significantly different from zero at the 10% level, two stars at the 5% and three stars at the 1%.

Table 5: Joint model-free test for the RAEMs: statistics on $y|I^v$ and its components

Statistic	$y\|I^v$	$\pi\|I^v$	$R^{mkt}\|I^v$	$R_f\|I^v$	$LB\|I^v$
Cond.Mean	**-0.0165****				
Cond.Mean		-0.010			0.006***
Cond.Mean			-0.004	0.006***	

The table summarizes the statistics of $y_{t+1} \equiv \pi_{t+1} - LB_t$ and its components ($\pi_{t+1} \equiv R^{mkt}_{t+1} - R_{t,f}$ being the excess market return and LB_t the lower bound measure for the risk premium $\mathbb{E}_t[\pi_{t+1}]$ computed through (3)) conditional on the nonnegative function $I^v \equiv 1_{[\hat{\pi}_{t+1} < LB_t]}$ isolating the periods in which the RAEMs are rejected at the 5% level (as shown in the first entry of the second column) over the main sample $Jan:1990 - Dec:2014$. One star symbols the statistic is significantly different from zero at the 10% level, two stars at the 5% and three stars at the 1%.

Table 6: Explaining the RAEMs rejections

Variable	β	**Partial** R^2	**Adj.** R^2	**Adj.** R^2 $(F, SII, ILLIQ)$
F	1.1557***	0.1224	0.5039	0.4537
SII	0.1295***	0.0931	0.5039	0.4537
$ILLIQ$	1.9897***	0.0829	0.5039	0.4537
BM	0.9655***	0.0298	0.5039	
$USDg$	4.6158**	0.0158	0.5039	
TAX	0.1910*	0.0121	0.5039	
MDI	-0.0921	0.0008	0.5039	

The table reports the result (omitting the constant term) from the regression $I_t^v = \beta_0 + D_t\beta + u_t$ ranked by partial R^2 on the β coefficients over the main sample $Jan: 1990 - Dec: 2014$. I_t^v is a non-negative step function $I^v \equiv 1_{[\hat{\pi}_{t+1} < LB_t]}$ isolating the periods in which the RAEMs are rejected at the 5% level, while D_t is the matrix of selected drivers (For a description of the drivers see notes to Table 2). The last column shows the adjusted R^2 of the regression when D_t only includes the first three most important drivers (i.e. F ,SII and $ILLIQ$). One star symbols the statistic is significantly different from zero at the 10% level, two stars at the 5% and three stars at the 1%.

Table 7: RAEMs' rejections in terms of the main drivers' characteristics

Coeff.	F	SII	$ILLIQ$
α_1	1.0646***	0.7926***	0.0765***
$Sig.atMed.$	YES	YES	YES
$Sig.at75pc.$	NO	NO	YES
α_2	0.8469***	-0.1419	0.0045
$\alpha_1 - \alpha_2$	0.2177***	0.9345***	0.0720***

The table shows the result from the regression $d_t = \alpha_1 I_t^v + \alpha_2(1 - I_t^v) + w_t$ over the main sample $Jan : 1990 - Dec : 2014$. $d_t \in \{F_t, SII_t, ILLIQ_t\}$ is one among the main drivers while I_t^v is a non-negative step function $I^v \equiv 1_{[\hat{\pi}_{t+1} < LB_t]}$ isolating the periods in which the RAEMs are rejected at the 5% level. Rows three and four report whether or not the estimate for α_1 is statistically grater than the unconditional median and 75-th percentile. One star symbols the statistic is significantly different from zero at the 10% level, two stars at the 5% and three stars at the 1%.

Table 8: Irrational Expectations Tests

Coeff.	$\pi_{t+1} - \mathbb{E}_t^{Ga}[\pi_{t+1}]$	$\pi_{t+1} - \mathbb{E}_t^{AA}[\pi_{t+1}]$	$\pi_{t+1} - \mathbb{E}_t^{Sh}[\pi_{t+1}]$	$Infl_{t+1} - \mathbb{E}_t[Infl_{t+1}]$	$UR_{t+1} - \mathbb{E}_t[UR_{t+1}]$	$SP_{t+1} - \mathbb{E}_t[SP_{t+1}]$
γ_1	0.7489**	0.2535	-0.0024	0.3806	0.0488**	0.0910*
γ_2	0.4242***	-0.1403	-0.1384	0.1291	-0.0197***	-0.0187
$\gamma_1 - \gamma_2$	1.1732***	0.3938**	0.1360	0.2515	0.0685***	0.1097***

The table shows the result from the regression $z_{t+1} - \mathbb{E}_t[z_{t+1}] = \gamma_1 I_t^v + \gamma_2(1 - I_t^v)$ over the main sample $Jan: 1990 - Dec: 2014$. z is the random variable according to which investors form expectations $\mathbb{E}_t[z_{t+1}]$, while I_t^v is a non-negative step function $I^v \equiv 1_{[\hat{\pi}_{t+1} < LB_t]}$ isolating the periods in which the RAEMs are rejected at the 5% level. In the first three column z is the return of the market in excess of the risk-free rate and the expectations are collected from survey data (Gallup survey, American Association of Individual Investors and Shiller's survey) validated in Greenwood-Shleifer (2014). z in the fourth column represent inflation, $Infl$ and the expectations are the market implied (and priced) ones from the difference in the yield of 5-year inflation indexed treasury bounds and the yield of 5-year nominal treasury bonds. In the last two columns z defines the U.S unemployment rate, UR, and the spread between the BAA rated corporate bonds and the federal funds rate, SP; expectations in this case are computed as forecasts through the specification of an econometric model following the Box-Jenkins (1970) procedure. One star symbols the statistic is significantly different from zero at the 10% level, two stars at the 5% and three stars at the 1%.

Table 9: Market portfolio demand

Coeff.	Vol_{t+1}	SII_{t+1}	$NetEquityPurch_{t+1}$
δ_1	0.3055***	0.7617***	3069***
δ_2	-0.1054*	-0.1481	5545
$\delta_1 - \delta_2$	0.4109***	0.9097***	-2476**

The table shows the result from the regression $q_{t+1} = \delta_1 I_t^v + \delta_2(1 - I_t^v) + \psi_{t+1}$ over the main sample $Jan : 1990 - Dec : 2014$. q_{t+1} is a proxy for the demand for the market portfolio in $t+1$ and I_t^v is a non-negative step function $I^v \equiv 1_{[\hat{\pi}_{t+1} < LB_t]}$ isolating the periods in which the RAEMs are rejected at the 5% level. Three proxies for q_{t+1}, corresponding to the different columns, are used: the de-trended log volume of SPDR $SP500$ ETF (measured as the log of the number of shares sold), Vol, the Rapach et al. (2016) short interest index SII and the the net purchase position (purchases-sales) in U.S. equity from foreign investors, $NetEquityPurch$. One star symbols the statistic is significantly different from zero at the 10% level, two stars at the 5% and three stars at the 1%.

Table 10: The role of dividends

Measure	**Mean**	**Std.**	**Min.**	**Qtl.** 0.25	**Qtl.** 0.5	**Qtl.** 0.75	**Max.**
LB_M^l	0.3279	0.3181	0.0702	0.1527	0.2505	0.3943	3.4812
LB^l	0.3293	0.3198	0.0706	0.1532	0.2512	0.3956	3.5023
LB_M^{cs}	0.3296	0.3178	0.0687	0.1475	0.2536	0.3925	3.4501
LB^{cs}	0.3311	0.3196	0.0691	0.1481	0.2552	0.3940	3.4710

The table shows the summary statistic of the empirical distribution in the main sample $Jan : 1990 - Dec : 2014$ of the lower bound measures computed through (3). LB_M^m, with $m \in \{l, cs\}$, corresponds to the case the dividend yield DY is set to 1, which is the Martin (2016) formulation, $m \in \{l, cs\}$ being the measure calculated via the linear and the cubic-spline approximation. LB^m, represents the measure which uses the $SP500$ dividends from Shiller.

Table 11: RAEMs' rejections driven by negative market returns

Statistic	$y\|I^v$	$\pi\|I^v$	$R^{mkt}\|I^v$	$\bar{R_f}$	$\bar{LB}$
Cond.Mean	**-0.0133****				
Cond.Mean		-0.010			0.003
Cond.Mean			-0.004	0.003	

The table summarizes the statistics in the main sample $Jan:1990-Dec:2014$ concerning the joint model free test for the RAEMs detailed in Definition 2 and Table 5. Differently from the main test reported in Table 5, this one fixes the risk-free rate and the lower bound measure to their unconditional mean, $\bar{R_f}$, and $\bar{LB}$ respectively. One star symbols the statistic is significantly different from zero at the 10% level, two stars at the 5% and three stars at the 1%.

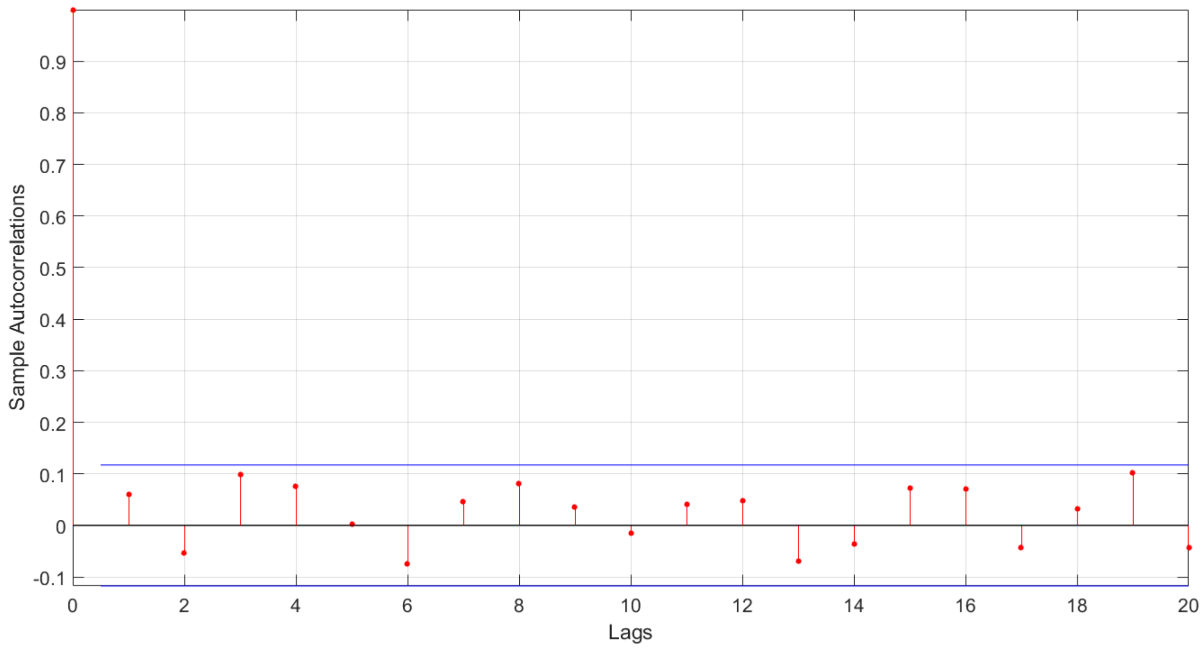
Table 12: Explaining the RAEMs' rejection probabilities

Variable	β	**Partial** R^2	**Adj.** R^2	**Adj.** R^2 $(F, SII, ILLIQ)$
SII	0.0461***	0.2102	0.8302	0.6876
$ILLIQ$	0.5315***	0.1052	0.8302	0.6876
F	0.2055***	0.0689	0.8302	0.6876
$USDg$	1.9219***	0.0489	0.8302	
MDI	0.1416***	0.0323	0.8302	
BM	0.1694***	0.0163	0.8302	
TAX	0.0233	0.0032	0.8302	

The table reports the result (omitting the constant term) from the regression $P_t(\pi_{t+1} < LB_t) = \beta_0 + D_t\beta + u_t$ ranked by partial R^2 on the β coefficients over the main sample $Jan : 1990 - Dec : 2014$. $P_t(\pi_{t+1} < LB_t)$ is the conditional probability to reject the RAEMs at time t introduced in section 2.2.2, while D_t is the matrix of selected drivers (For a description of the drivers see notes to Table 2). The last column shows the adjusted R^2 of the regression when D_t only includes the first three most important drivers (i.e. F ,SII and $ILLIQ$). One star symbols the statistic is significantly different from zero at the 10% level, two stars at the 5% and three stars at the 1%.

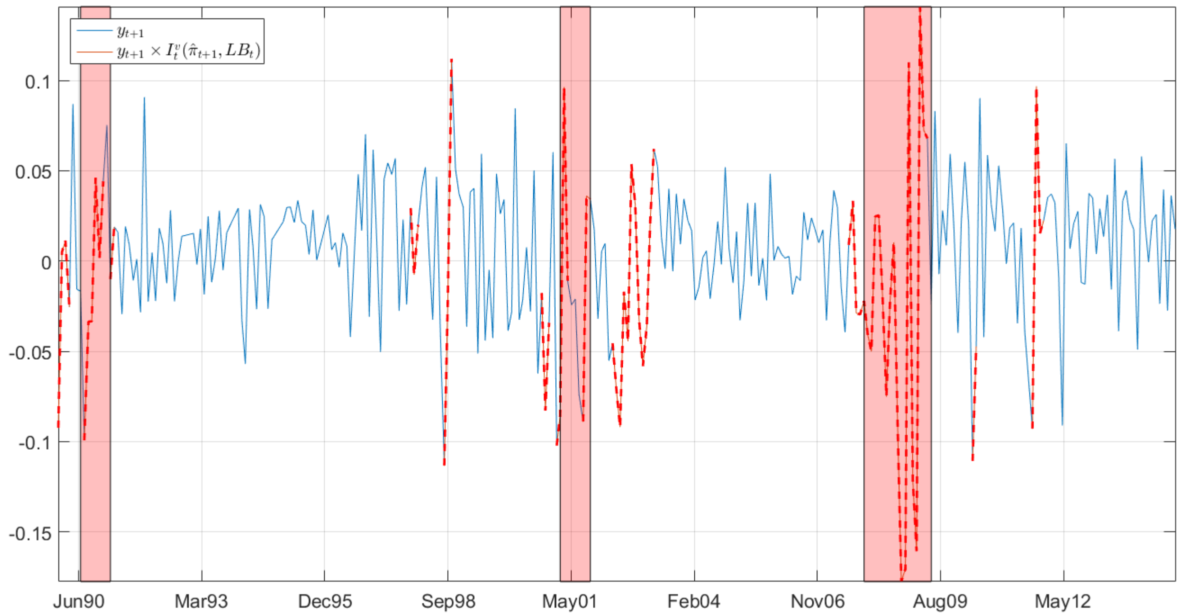
10 Figures

Figure 1: autocorrelation function of y_{t+1}



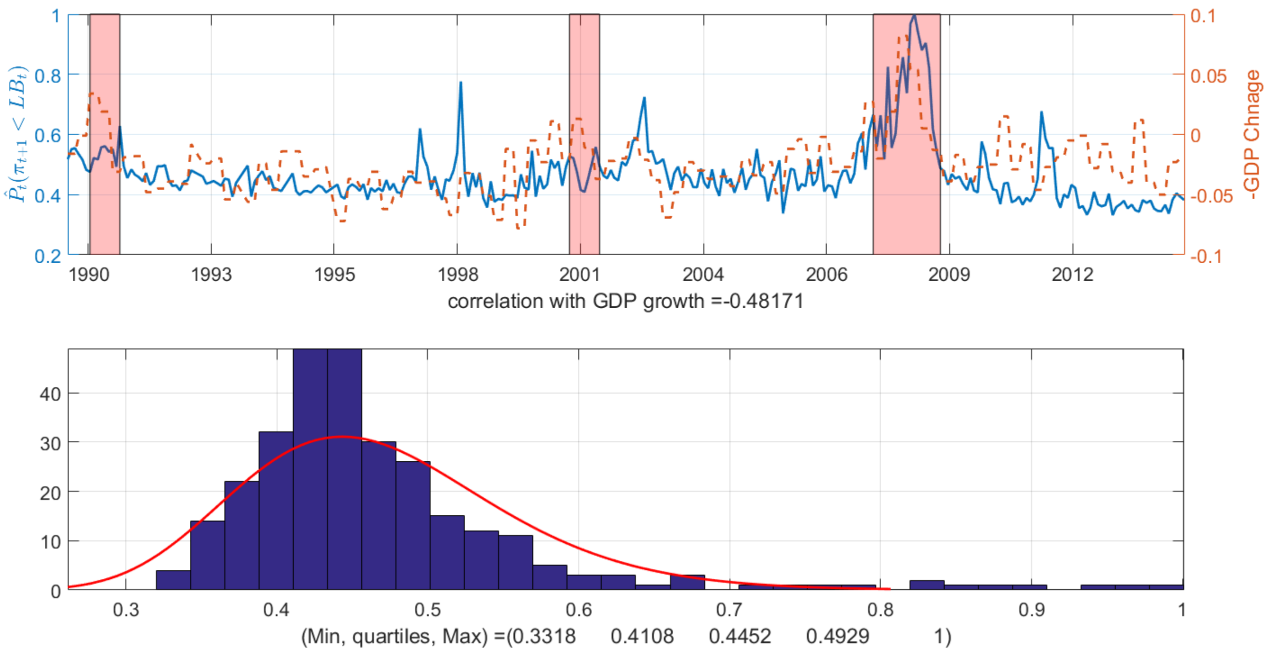
The autocorrelation function of $y_{t+1} \equiv \pi_{t+1} - LB_t$ ($\pi_{t+1} \equiv R^{mkt}_{t+1} - R_{t,f}$ being the excess market return and LB_t the lower bound measure for the risk premium $\mathbb{E}_t[\pi_{t+1}]$ computed through (3)) together with the 95% confidence bands.

Figure 2: joint model-free test for the RAEMs



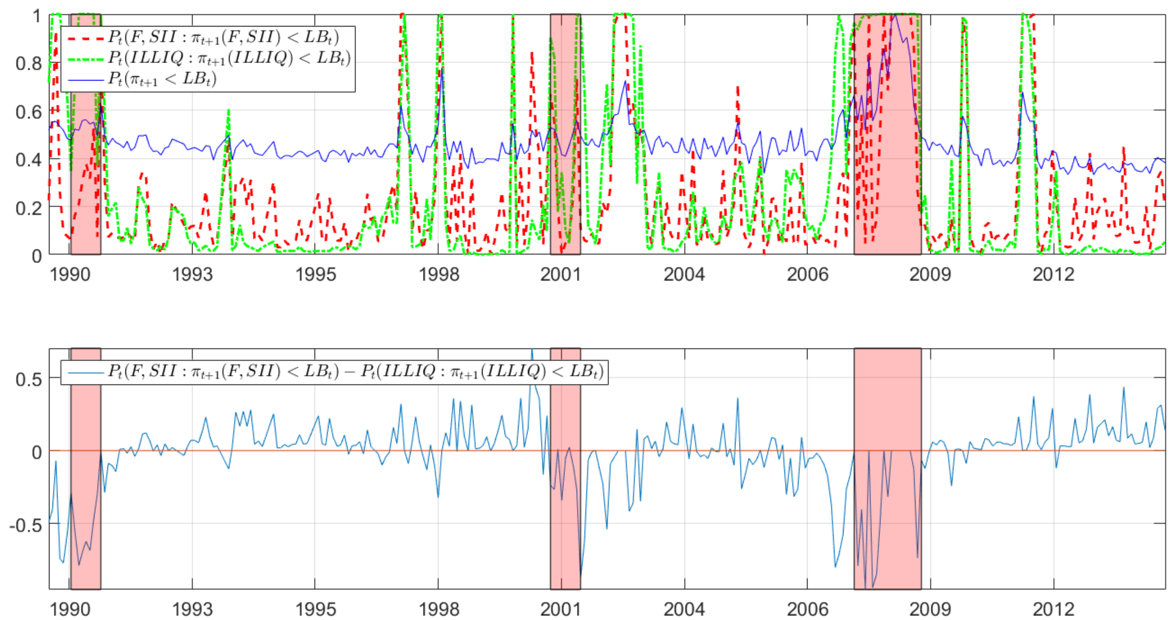
The figure displays in solid blue the time series of $y_{t+1} \equiv \pi_{t+1} - LB_t$ ($\pi_{t+1} \equiv R_{t+1}^{mkt} - R_{t,f}$ being the excess market return and LB_t the lower bound measure for the risk premium $\mathbb{E}_t[\pi_{t+1}]$ computed through (3)) while in dashed red the time series highlighting the sub-sample in which the RAEMs are jointly rejected at the 5% level. The pink shaded areas emphasize the NBER recessions.

Figure 3: conditional probability to reject the RAEMs



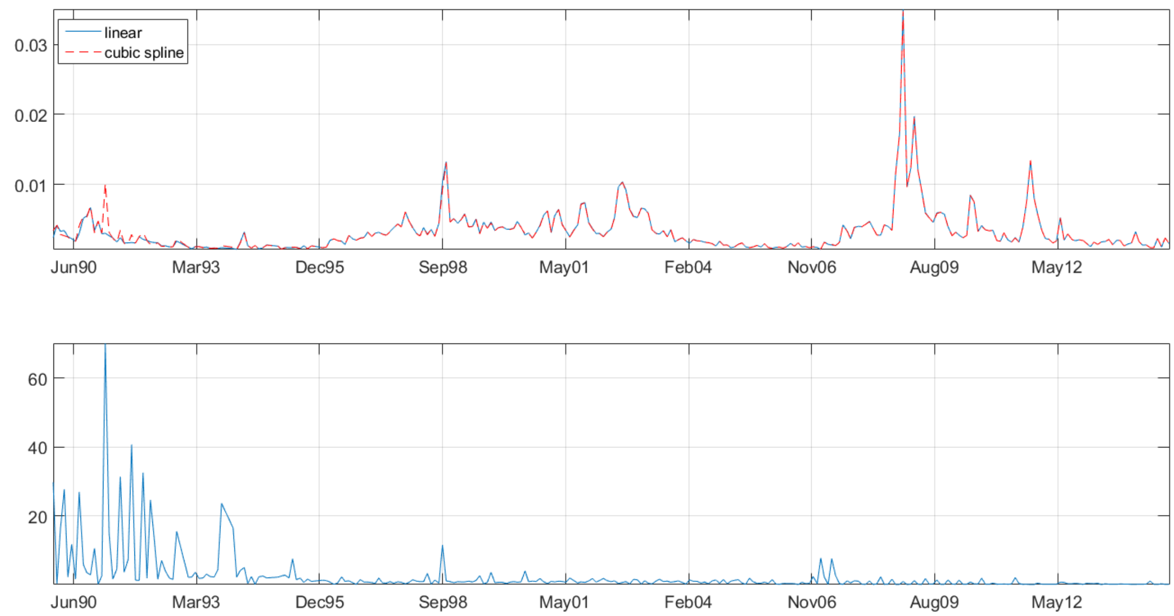
The figure displays the conditional probability to reject the RAEMs: the upper graph plots the time-series, solid blue line, against the negative of the U.S. GDP growth, dashed line, and the pink areas represents the NBER recession over the main sample $Jan : 1990 - Dec : 2014$. The lower graph illustrates the empirical distribution against the lognormal benchmark and reports the minimum the 25-th, the 50-th, the 75-th quantiles and the maximum.

Figure 4: different motivations behind RAEMs' failures over time



The figure displays the contribution to the conditional probability to reject the RAEMs of the main drivers: in the upper graph the joint contribution of the disagreement proxies F, SII is plotted in the form of a dashed red line, the contribution of the illiquiity index $ILLIQ$, is represented by a dotted green line, while the overall conditional probability of rejecting the RAEMs is still a solid blue line as in Figure 3. In the bottom graph the solid light blue line tracks the difference between the disagreement and the illiquidity series; positive values indicate an higher contribution of disagreement (asymmetric information) while negative values a predominant contribution of illiquidity (market frictions).

Figure 5: The lower bound measure: linear versus cubic-spline interpolation



The figure displays the two different interpolation scheme adopted in the study to compute the lower bound measure according to equation (3). The upper graph plots the two time-series of lower bounds under the different interpolations, while the bottom one shows, in percentage, the absolute difference in terms of the cubic-spline approximation.