Technological Spillovers and Dynamics of Comparative Advantage.

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Abstract

This paper builds a dynamic model of international trade in the presence of cross-sector technological spillovers. It investigates the impact of spillovers on uniqueness and multiplicity of balanced growth paths (BGP) and describes the forces that govern the dynamics of sector productivity and comparative advantage. Under isolated clusters of sectors (zero spillovers across clusters) there exists a continuum of BGPs. In this case the number of newly generated technologies inside each cluster is proportional to the amount of labor allocated to the cluster. This allows larger clusters to generate proportionally more technologies and grow at the same rate as smaller ones. As a result, the initial relative productivity and comparative advantage of clusters are preserved and the balanced growth path depends on the initial productivities of sectors. Under connected clusters (each cluster sends or receives technologies from other clusters) technologies that are generated by larger clusters spills to smaller ones and allow the latter to grow faster. As a consequence, less productive sectors are catching up and the comparative advantage of the initially large sectors diminishes. In this case the BGP is unique, characterized by the absence of comparative advantage and has zero inter-sectoral trade flows. The paper describes conditions under which the welfare-improving industrial policy is possible. It requires the presence of inter-sector spillovers. Under only intra-sector spillovers the economy is isomorphic to the one with equalized Marshallian externalities across sectors and no re-allocation of labor can improve its welfare. The strength of cross-sector technological spillovers is quantified using the US patent data. The calibrated model is used for computing the welfare maximizing policy. As the model shows, the optimal policy can increase the economy-wide productivity by 3.5% comparing to the no-policy case.

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1 Introduction

Between 1980 and 1990 around \$160 billion were spent by the US, Japanese, South Korean and Taiwanese governments to establish semiconductor industry in their countries. In a new wave of subsidies, the US and Chinese governments pledged to spend \$150 billion and \$400 billion respectively on the "green energy" sector[1]. The usual rationale for such policy emphasizes economies of scale and positive externalities generated by targeted sectors. To analyse the long-run welfare implications of the above policies, we need to model these externalities and the resulting dynamics of sectoral productivity. The key theoretical challenge here is the possibility of multiple equilibria. If productivity of a sector depends on its size – for example, through accumulation of best practices that become public knowledge within a country – then any initial comparative advantage of a sector becomes self-reinforcing. Furthermore, multiplicity of equilibria is more likely in open economies, since specialization is not thwarted by downward sloping demand. Several theoretical works as, for instance, Young [1991] investigated balanced growth paths of open economies, yet, the derived long-run predictions in those papers depend on the starting points – multiplicity is their typical feature. A large body of empirical research documents the presence of strong technological spillovers between sectors. Accounting for them makes it even harder to model the evolution of sector-level productivities. With cross-sector spillovers each sector can reinforce not only its own productivity but also productivity of proximate sectors.

In this paper I develop a dynamic model of international trade with cross-sector spillovers which under general conditions demonstrates a unique balanced growth path. As I show, the sufficient condition for the uniqueness is the connectedness of sectoral clusters. In addition to the standard effect of comparative advantage on labor allocation, the model accounts for the effects of labor allocation on sector productivity and comparative advantage. The core mechanism is a combination of an idea-generating process within each sector and technolog-

[1] Source: MacKinsey Global Institute, Breakthrough Institute

ical spillovers across sectors. I establish necessary and sufficient conditions for the existence and uniqueness of a balanced growth path (BGP) and describe the conditions under which a welfare-improving industrial policy is possible. I calibrate the model using the US patent data to parametrize the strength of technological spillovers and to describe the optimal policy.

The model builds on a multi-sector Eaton and Kortum [2002] model, as in Costinot et al. [2012], to link sector productivities to labor allocation across sectors. The reverse link from labor allocation to sector productivity is captured by assuming that employment in a particular sector exogenously generates a mass of new (publicly available) technologies useful in the sector or in others. Specifically, a new technology that emerges in an origin sector can be used in producing any variety in a destination sector with some origin-destination probability. The pattern of how technologies flow across sectors is summarized by a matrix of cross-sector spillover probabilities. The cross-sector spillover matrix can be seen as a way to formalize the idea of proximity between different sectors as in Hausmann and Klinger [2006]. For instance, high values of spillover probabilities between origin and destination sectors mean that both these sectors are using similar technologies and are more likely to produce together in a given country. This approach allows me to describe the dynamics of the economy by a simple system of differential equations.

For the case of frictionless trade, I show that depending on the degree of connectedness between sectors the economy may have a unique or multiple BGPs. An important result is that if there are no isolated clusters, (that is, there are no groups of sectors that generate and adopt technologies only for and from members of the group), then the BGP of the model is unique. This result comes as an outcome of interaction of two forces.

The first force, centripetal, tends to equalize productivities across sectors. To see the workings of this force, assume that a country could buy at the same price a random sample of technologies for any sector. It would buy this sample for the least productive sector because in this sector a larger share of the technologies will have productivity that exceeds the sectoral productivity frontier and, thus, will be used. Buying technologies in this example is equivalent to directing labor to the least productive sectors. The centripetal force may explain why we observe weakening of comparative advantage in the data, as documented in Levchenko and Zhang [2011].

The second force we observe is centrifugal. It comes from the fact that it is actually more expensive to buy technologies for the least productive sectors. Namely, to allow the least productive sectors to catch up, labor should be diverted to them from more productive sectors and, as a result, welfare decreases. If we have isolated clusters, then these two forces can balance each other on multiple BGPs. Under no isolated clusters the cross-sector spillovers provide technologies to the least productive sectors "for free". As a result, centripetal force becomes stronger and the economy ends up in a unique BGP where all countries have the same relative productivities across sectors and, hence, no comparative advantage and no cross-sector trade. The model also provides a description of the transition path and for a 2-sector 2-country case has a simple phase-diagram illustration.

The model can be used to think about the welfare effect of policies that induce a reallocation of labor across sectors. If a vector of sectoral productivities is a result of sectoral labor allocation, then a country can choose its BGP as well as a transition path to it. For example, this choice is implemented by re-allocating labor across sectors and, thus, affecting the process of accumulation of new technologies across sectors. Uniqueness of BGP matters for the outcome of such policy and its information intensity. Namely, if uniqueness holds then the BGP to which a country converges doesn't depend on the initial distribution of productivities and all the policy-maker needs in order to predict the long-run implications of policy is the matrix of spillovers. In contrast, if uniqueness doesn't hold, then the policy-maker should know not only the matrix of spillovers but also the initial distribution of productivities across all sectors and countries. I demonstrate that a policy intervention can improve welfare if the spillover matrix has positive inter-sector spillovers, i.e. that some sectors generate technologies both for varieties inside and outside these sectors. Characteristics of such sectors are consistent with the notion of core sectors as in Greenwald and Stiglitz [2006]. That is, core sectors generate widely applicable technologies and increase productivity of the whole economy, providing a rationale for governments to promote and even subsidize them. I also derive the criteria for defining optimal labor reallocation across sectors. For a symmetric 2-country 2-sector case this criteria has an intuitive interpretation. Namely, labor should be reallocated towards a sector that generates a larger share of technologies for destinations sectors weighted by expenditures on varieties of the destination sectors. Thus, I provide a formal framework for quantifying these technological spillovers and deriving the associated optimal policy.

Finally, I calibrate the model using the US patent data and compute the labor allocation that maximizes welfare in the BGP. The calibration part of the paper contributes to the literature on estimating the strength of technological spillovers[2] and extends it by introducing the corrections not only for the size of the destination sector, but also for the size of the sector of the origin. The computed optimal policy improves the country's productivity in the BGP by 3.5% comparing to the no-policy BGP.

The paper is structured as follows. Section 2 outlines the model, investigates its dynamic properties and provides the intuition for the main mechanisms. Section 3 establishes the possibility for welfare-improving economic policy and the necessary conditions for such policy. Section 4 describes the calibration of the model and estimation procedure for the spillover parameters. Section 5 presents the optimal policy based on the calibrated model. Section 6 concludes.

[2]E.g. Ellison et al. [2010]

2 The model

The mechanism that maps sector productivity to labor allocation across sectors works through trade and comparative advantage. It is based on Ricardian models of trade as in Costinot et al. [2012] and Eaton and Kortum [2001]. The mechanism that generates the feedback from labor allocation to sector productivity is based on the exogenous process of generating new technologies and endogenous spillovers of these technologies across sectors. The flow of technologies is described by a matrix of spillover probabilities. At the aggregate level this mechanism generates equations that describe dynamics of sectors' productivity which are similar to the equations suggested in Hanlon [2013]. More details are provided below.

2.1 Demand

There is a discrete number of sectors and a continuum of varieties of mass normalized to 1 within each sector. The set of sectors is denoted as $\mathcal{S} \equiv \{1, \ldots, S\}$. There is also a representative household with a two-tier utility function. Varieties within each sector are aggregated according to a constant elasticity of substitution (CES) aggregator with elasticity parameter σ. Sectoral aggregates, in turn, enter the utility function as Cobb-Douglas with sector specific parameters α^s. Thus, the utility of a representative household in country i at time period t is

$$U_i(t) = \prod_{s=1}^{S} \left(C_i^s(t)\right)^{\alpha^s}, \; C_i^s(t) = \left(\int_0^1 c_i^s(t,\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $c_i^s(t,\omega)$ denotes consumption of variety ω from sector s by a representative household in country i at time t.

The representative household inelastically supplies an exogenous amount of labor $L_i(t)$ that is allocated among S sectors:

$$L_i(t) = \sum_{s=1}^{S} L_i^s(t) \tag{2}$$

In every period the household spends its whole income $I_i(t) = w_i(t)L_i(t)$, where $w_i(t)$ is the wage rate in country i at time t. The share α^s of the income is spent on varieties from sector s. Expenditure for each variety ω from sector s is equal to

$$x_i^s(t,\omega) = \left(\frac{p_i^s(t,\omega)}{p_i^s(t)}\right)^{1-\sigma} \alpha^s I_i(t), \tag{3}$$

where $p_i^s(t) = \left(\int_0^1 p_i^s(t,\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}$. Each variety ω is bought from only one source – a country that supplies it at the lowest price; the set of countries is discrete and is denoted as $\mathcal{N} \equiv \{1,\ldots,N\}$. Perfect competition among producers results in pricing at marginal costs. Let $c_{ij}^s(t,\omega)$ denote the marginal cost of supplying one unit of variety ω in sector s from country i to country j.[3] The set of goods of sector s supplied from i to j at time t is $\Omega_{ij}^s(t) = \left\{\omega : c_{ij}^s(t,\omega) = \min_{k\in\mathcal{N}}\{c_{kj}^s(t,\omega)\}\right\}$. Expenditure of country j for goods of sector s that are supplied by country i at time t is

$$x_{ij}^s(t) = \sum_{\omega\in\Omega_{ij}^s(t)} x_i^s(t,\omega) \tag{4}$$

Utility per capita is equal to

$$\frac{U_i(t)}{L_i(t)} = \prod_{s=1}^{S}\left(\frac{\alpha^s I_i(t)}{p_i^s(t)L_i(t)}\right)^{\alpha_s} = w_i(t)\prod_{s=1}^{S}\left(\frac{\alpha^s}{p_i^s(t)}\right)^{\alpha_s} \tag{5}$$

[3]In this paper I use the notations commonly used in input-output literature when the first subscript denotes the country of the origin and the second one – destination.

2.2 Supply

As it was mentioned above, free entry and perfect competition among producers is the market setting. Production uses only one input – labor – which is transformed into output according to the function

$$Y_i^s(t,\omega) = Z_i^s(t,\omega)L_i^s(t,\omega), \tag{6}$$

where $Z_i^s(t,\omega)$ is a productivity of technology for producing variety ω in sector s of country i at period t. All potential entrants have access to the same technology.

Labor is homogeneous and firms take wages as given. Goods from sector s are traded between countries i and j at cost d_{ij}^s which is modelled as "iceberg" trade cost. As a result, variety ω in sector s can be supplied from country i to country j at cost

$$c_{ij}^s(t,\omega) = \frac{w_i(t)d_{ij}^s}{Z_i^s(t,\omega).} \tag{7}$$

Now let's turn to the mechanism that defines productivity of each variety. I assume that labor of mass 1 exogenously generates technologies[4] at rate ϕ. Each technology is characterized by a sector of its origin and productivity Q. All new technologies immediately become publicly available. Productivity of each of them is drawn from a Pareto distribution:

$$Pr(Q \leq q) = 1 - q^{-\theta} \tag{8}$$

Once any idea is generated in sector s it can be applied to any variety in this sector with probability p^{ss} and to any variety in sector r with probability p^{sr}. In this setting by time t the number of ideas that has ever been generated for any variety in sector s of country i is

[4]In what follows the words "idea" and "technology" are used interchangeably and mean an invented process which can be used for producing varieties of goods.

a random variable distributed as Poisson with parameter $T_i^s(t)$:

$$T_i^s(t) = \sum_{r=1}^{S} p^{rs} \int_0^t \phi L_i^r(\nu) d\nu + T_i^s(0) \tag{9}$$

The matrix of probabilities $\{p^{rs}\}_{\substack{r\in\mathcal{S} \\ s\in\mathcal{S}}}$ summarizes the information on cross-sector spillovers. The only restriction on its elements is $0 \leq p^{rs} \leq 1 \; \forall r, s \in \mathcal{S}$. Sector s with higher $\{p^{sr}\}_{r\in\mathcal{S}}$'s generates more widely-applicable technologies. Sectors r and s that are characterized by high values of p^{rs} and p^{sr} can be viewed as technologically proximate ones: high productivity in one of them helps increase productivity in the other. Intuitively, a country may want to establish or retain a sector that is characterized by high proximity or, the way it is modelled here, the one that generates more general technologies. The latter would allow the economy in the balanced growth path (BGP) to have a higher number of ideas per capita and, thus, higher welfare.

As is established in Eaton and Kortum [2001], for any variety ω in sector s of country i productivity $Z_i^s(t, \omega)$ is a random variable distributed as Fréchet

$$Pr(Z_i^s(t, \omega) \leq z) = e^{-T_i^s(t) z^{-\theta}}. \tag{10}$$

The share of country i in country j's expenditure for goods of sector s is

$$\pi_{ij}^s(t) = \frac{x_{ij}^s(t)}{\sum_k x_{kj}^s(t)} = \frac{T_i^s(t) \left(w_i(t) d_{ij}^s \right)^{-\theta}}{\sum_l T_l^s(t) \left(w_l(t) d_{lj}^s \right)^{-\theta}}. \tag{11}$$

2.3 Equilibrium

Assuming that trade is balanced in each period, we obtain

$$w_i(t) L_i(t) = \sum_{j=1}^{N} \sum_{s=1}^{S} \pi_{ij}^s(t) \alpha^s w_j(t) L_j(t), \tag{12}$$

where the left-hand side is the total income of households in country i and the right-hand side are the total expenditures of all countries for goods produced in country i. Given the total labor supply in each country at time t, $\{L_i(t)\}_{i\in\mathcal{N}}$, and the level of technology for each sector-country which in the current setting is summarized by $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$, one can solve Equation (12) for equilibrium wages, $\{w_i(t)\}_{i\in\mathcal{N}}$.

Since in perfect competition firms earn zero profits, total revenue of each sector in every country is equal to total costs, i.e. to labor income earned in this sector:

$$w_i(t)L_i^s(t) = \sum_{j=1}^{N} \pi_{ij}^s(t)\alpha^s w_j(t)L_j(t), \tag{13}$$

Having solved Equation (12) for wages $\{w_i(t)\}_{i\in\mathcal{N}}$, one can solve Equation (13) for sector labor demand $\{L_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$. Notice from Equations (11) and (12) that, conditional on the relative size of countries in terms of total labor supply, what matters for wages and labor allocation is the relative productivity across countries within sectors, $\left\{\frac{T_i^s(t)}{T_j^s(t)}\right\}_{\substack{i,j\in\mathcal{N}\\ s\in\mathcal{S}}}$. Other things being equal, countries that are relatively more productive observe larger shares of expenditures for their goods and higher welfare.

Thus, the static part of the model tells us how the state of technology $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ and total labor supply $\{L_i(t)\}_{i\in\mathcal{N}}$ affect the equilibrium allocation of labor across sectors $\{L_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$. What the dynamic part of the model adds is the evolution of technologies: given the equilibrium allocation of labor across sectors within each country, productivity of sectors evolves according to Equation (9) or its differential counterpart

$$\dot{T}_i^s(t) = \frac{dT_i^s(t)}{dt} = \phi\left(\sum_{r=1}^{S} p^{rs}L_i^r(t)\right). \tag{14}$$

Thus, the model can be summarized by Equations (11), (12), (13) and (14), where Equations (11), (12), (13) describe the static equilibria of the model, while (14) – describes its dynamics.

At any point in time the model is in static equilibrium.

Definition. **Static equilibrium** of the model at time t is a set of non-negative vectors of total labor supply $\{L_i(t)\}_{i\in\mathcal{N}}$, sector productivity $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ and sector labor allocation $\{L_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ that satisfy Equations (11), (12) and (13).

2.4 Balanced growth path

The model is characterized by semi-endogenous growth. Thus, it can have an equilibrium in which all variables are growing at a constant rate – balanced growth path – only if the total labor supply in each country is growing at some constant rate $g > 0$.

$$L_i(t) = e^{gt} L_i(0) \tag{15}$$

The state of the system is characterized by two vectors – productivities $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ and total labor supply $\{L_i(t)\}_{i\in\mathcal{N}}$. The former changes endogenously, while the latter – exogenously.

Definition. **Balanced growth path** (BGP) of the model is a sequence of static equilibria that satisfies Equation (14) and along which each element of the vector of sector productivities $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ grows at a constant rate.

Definition. Balanced growth path is **locally stable** if the economy converges to it once it starts at any point in some ε-neighborhood of it.

Since one can show that both T's and L's along the BGP are growing at the same rate, their ratios $\left\{\frac{T_i^s}{L_i}\right\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ remain constant.

One can also show that in the BGP

$$\frac{T_i^s}{L_i} \equiv t_i^s = \frac{\phi \sum_r p^{rs} l_i^r}{g} = \frac{\phi_i^s}{g}, \tag{16}$$

where $l_i^r \equiv \frac{L_i^r}{L_i} \in [0;1]$, $\sum_r l_i^r = 1$ and $\phi_i^s \equiv \phi \sum_r p^{rs} l_i^r$. Dynamics of $t_\cdot^\cdot$'s outside the BGP is described by the differential equation[5]:

$$\dot{t}_i^s(t) = \phi \sum_r p^{rs} l_i^r(t) - g t_i^s(t), \ \forall i = 1, ..., N, \ \forall s = 1, ..., S. \tag{17}$$

Clearly, if $p^{sr} = p \ \forall s, r$ then the first term on the right-hand side of Equation (17) turns into ϕp and it is easy to see that the BGP is unique and stable: $t_i^s = \phi p / g$. Another observation is that the first term on the right-hand side of Equation (17) is bounded both from above and from below – with boundaries $\phi \max_r(p^{rs}) \geq 0$ and $\phi \min_r(p^{rs}) \geq 0$ correspondingly, while $g t_i^s(t) \in [0; \infty)$, thus, the steady state level $t_i^s \in [\phi \min_r(p^{rs})/g; \phi \max_r(p^{rs})/g]$. The last inequality guarantees that even if there exists an unstable steady state it has a stable steady state in its neighbourhood to which the system will converge. In other words it can not be that $t_i^s \to \infty$. If all spillovers are non-zero – $p^{rs} > 0 \ \forall r, s$ – then we can also exclude the cases where $t_i^s \to 0$. Graphical illustration of the latter argument is provided in Figure (1). Two thick lines – one solid and one dashed – show two possible patterns of relation between $\phi_i^s(t)$ and $t_i^s(t)$. Although gross substitutability between sectors guarantees that more productive sectors attract more labor, the relation between $t_\cdot^\cdot$'s and $l_\cdot^\cdot$ is non-linear, so, potentially there might be multiple BGPs.

One observation from Figure (1) is that ϕ_i^s is in limit approaching ϕp^{ss} – as sector s in country i becomes more productive, more labor is allocated to it with limit $\phi_i^s = \phi p^{ss}$ attained when sector s employs all the labor in the country, $L_i^s = L_i$. As I mentioned above, potentially, there is possibility for both stable (points A, B and D) and unstable (point C)

[5] To clarify notations – t without sub-/superscripts denotes time, while $t_i^s(t) \equiv \frac{T_i^s(t)}{L_i(t)}$ denotes the number of ideas per capita in country i sector s at time t.

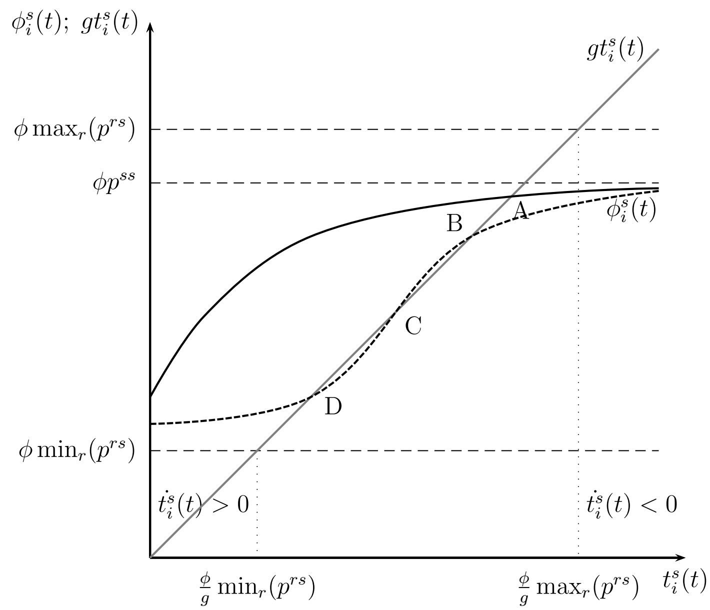


Figure 1: Dynamics of technological endowment per capita, $t_i^s(t)$

BGPs.

In general it is not trivial to show that Equations (17) have a unique and stable solution, thus, in what follows I will consider the above described model for some particular cases.

Autarky. Under autarky $\pi_{ij}^s = 0 \ \forall i \neq j \in \mathcal{N}$ and $\pi_{ii}^s = 1 \ \forall s \in \mathcal{S}$. It follows from Equation (13) that labor is allocated across sectors proportional to the shares of expenditure:

$$L_i^s(t) = \alpha^s L_i(t) \tag{18}$$

As I stated before, along the BGP all elements of $\{T_i^s(t)\}_{\substack{i=1,\dots,N \\ s=1,\dots,S}}$ are growing at the same rate g, thus, the ratios of T's across sectors and countries along the BGP remain the same:

$$\frac{T_i^s}{T_i^r} = \frac{\sum_{q=1}^S l_i^q p^{qs}}{\sum_{q=1}^S l_i^q p^{qr}} = \frac{\sum_{q=1}^S \alpha^q p^{qs}}{\sum_{q=1}^S \alpha^q p^{qr}} \tag{19}$$

Utility per capita (under wages normalized to 1) can be expressed as

$$\frac{U_i(t)}{L_i(t)} = \frac{1}{L_i(t)}\prod_{s=1}^{S}\left(C_i^s(t)\right)^{\alpha^s} = \frac{1}{L_i(t)}\prod_{s=1}^{S}\left(\frac{\alpha^s L_i(t)}{p_i^s(t)}\right)^{\alpha^s} = \frac{\prod_s \alpha^{s\alpha^s}}{\gamma}\prod_s\left(\frac{T_i^s(t)}{L_i(t)}\right)^{\frac{\alpha^s}{\theta}} \cdot L_i(t)^{\frac{1}{\theta}}, \tag{20}$$

where $\gamma \equiv \left(\Gamma\left(\frac{1-\sigma}{\theta}+1\right)\right)^{\frac{1}{1-\sigma}}$. As follows from Equations (16) and (18), $\frac{T_i^s(t)}{L_i(t)} = \frac{\phi\sum_r p^{rs}\alpha^r}{g} = const$. So, one can conclude that: 1) any economy on the BGP in autarky grows at rate $\frac{g}{\theta}$; 2) utility per capita for symmetric countries (i.e. countries with equal total labor supply) is the same; 3) relative productivity of sectors and labor allocation doesn't depend on the initial conditions. As a result, autarky is characterized by a unique and stable BGP. Another observation is that economies with more interrelated sectors (higher p^{rs}'s) have higher T/L and, thus, higher utility per capita under the same level of total labor supply.

Costless trade. Now let's consider another extreme – the case of costless trade: $d_{ij}^s = 1\ \forall i,j\in\mathcal{N},\ \forall s\in\mathcal{S}$. The immediate implication of this case is that the share of country i in expenditures for goods of sector s is the same across all destinations: $\pi_{ij}^s = \pi_{ii}^s \equiv \pi_{i\cdot}^s$. Hence, from Equation (13) one can obtain ratios of labor allocated across sectors r and s:

$$\frac{L_i^s(t)}{L_i^r(t)} = \frac{\alpha^s\pi_{i\cdot}^s(t)}{\alpha^r\pi_{i\cdot}^r(t)} = \frac{\alpha^s T_i^s(t)\sum_l T_l^r(t)(w_l(t))^{-\theta}}{\alpha^r T_i^r(t)\sum_l T_l^s(t)(w_l(t))^{-\theta}} \tag{21}$$

For any pair of countries i and j and any pair of sectors s and r

$$\frac{L_i^s(t)}{L_i^r(t)}\frac{T_i^r(t)}{T_i^s(t)} = \frac{L_j^s(t)}{L_j^r(t)}\frac{T_j^r(t)}{T_j^s(t)} \tag{22}$$

The second system of equations relating ratios of productivity parameters T's and labor allocation is the system for the steady state (thus, time index t is omitted):

$$\frac{T_i^r}{T_i^s} = \frac{\sum_q p^{qr} l_i^q}{\sum_q p^{qs} l_i^q}, \tag{23}$$

where $l_i^s \equiv \frac{L_i^s}{L_i},\ s \in \mathcal{S}$.

Before I formulate a Proposition on uniqueness of BGP, let me introduce and briefly explain the definitions used in the Proposition.

Definition. Sector $s \in \mathcal{S}$ is **stagnant** if $\sum_r p^{rs} = 0$.

A sector is defined as stagnant if it doesn't have possibilities for growth. Namely, neither technologies generated in other sectors, nor the ones generated in the sector itself can be used in a stagnant sector. Note, that this definition doesn't preclude a stagnant sector from generating technologies for other sectors.

Definition. BGP is **interior** if all elements of a sequence of vectors $\{T_i^s(t)\}_{\substack{i\in\mathcal{N}\\ s\in\mathcal{S}}}$ are positive. BGP is **corner** if at least one element of this sequence of vectors is equal to zero.

Interior equilibrium means an equilibrium in which every country has non-zero productivity in every sector. In contrast, a corner equilibrium is the one in which some country has zero productivity in some sectors.

Definition. Matrix of spillovers $\{p^{rs}\}_{r,s\in\mathcal{S}}$ has **no isolated clusters** if its digraph is connected.

Absence of isolated clusters means that there are no groups of sectors that generate and receive technologies only for and from the members of the group. A simplest example of a spillover matrix with isolated clusters is a diagonal matrix – the case in which each sector generates technologies only for itself. Another way to define a matrix without isolated clusters is to say that such matrix can not be represented as a block-diagonal one by permuting rows and columns in the same order.

Finally, Proposition 1:[6]

Proposition 1 *Under zero trade costs, no isolated clusters and no stagnant sectors, the model has a* ***unique and stable*** *interior balanced growth path in which labor allocation vectors are the same across countries:* $L_i^s = \alpha^s L_i \; \forall i \in \mathcal{N}, \; \forall s \in \mathcal{S}$.

To provide some intuition behind Proposition 1 let's build a phase-diagram for a simplified version of the model with 2 countries (i and j) and 2 sectors (s and r). For convenience I'll re-write the main equations of the model here. Time variable t can be omitted for brevity because we are going to consider a BGP. First, having relative productivities $t^s \equiv \frac{T_j^s}{T_i^s}$, $t^r \equiv \frac{T_j^r}{T_i^r}$ and relative size of the countries $\frac{L_j}{L_i}$ one can find relative wages $\frac{w_j}{w_i}$ from the trade balance equation:

$$1 = \left(\frac{\alpha^s}{1 + \frac{T_j^s}{T_i^s}\left(\frac{w_j}{w_i}\right)^{-\theta}} + \frac{\alpha^r}{1 + \frac{T_j^r}{T_i^r}\left(\frac{w_j}{w_i}\right)^{-\theta}} \right) \left(1 + \frac{w_j}{w_i}\frac{L_j}{L_i}\right)$$

Next, labor allocation across sectors – $l_i^r \equiv \frac{L_i^r}{L_i}$ – can be obtained from

$$\frac{l_i^r}{1-l_i^r} = \frac{L_i^r}{L_i^s} = \frac{\alpha^r}{\alpha^s} \frac{1 + \frac{T_j^s}{T_i^s}\left(\frac{w_j}{w_i}\right)^{-\theta}}{1 + \frac{T_j^r}{T_i^r}\left(\frac{w_j}{w_i}\right)^{-\theta}} \text{ and } \frac{l_j^r}{1-l_j^r} = \frac{L_j^r}{L_j^s} = \frac{\alpha^r}{\alpha^s} \frac{1 + \frac{T_i^s}{T_j^s}\left(\frac{w_j}{w_i}\right)^{\theta}}{1 + \frac{T_i^r}{T_j^r}\left(\frac{w_j}{w_i}\right)^{\theta}}.$$

Finally, the dynamics and BGP level of relative productivities can be described by

$$\dot{t^s} = 0 : t^s \equiv \frac{T_j^s}{T_i^s} = \frac{\phi L_j (p^{rs} l_j^r + p^{ss} l_j^s)}{\phi L_i (p^{rs} l_i^r + p^{ss} l_i^s)} \text{ and } \dot{t^r} = 0 : t^r \equiv \frac{T_j^r}{T_i^r} = \frac{\phi L_j (p^{rr} l_j^r + p^{sr} l_j^s)}{\phi L_i (p^{rr} l_i^r + p^{sr} l_i^s)}.$$

If the left-hand side of the latter equations is below their right-hand side, then the current labor allocation contributes more to relative productivity of country j than country i in a given sector, hence, $\frac{T_j^s}{T_i^s}$ increases if $\frac{T_j^s}{T_i^s} < \frac{\phi L_j (p^{rs} l_j^r + p^{ss} l_j^s)}{\phi L_i (p^{rs} l_i^r + p^{ss} l_i^s)}$. The same holds for sector r. Now, one can see that the BGP is characterized by two endogenous state variables – relative productivities of countries within each sector, $t^s \equiv \frac{T_j^s}{T_i^s}$ and $t^r \equiv \frac{T_j^r}{T_i^r}$, which can be viewed as

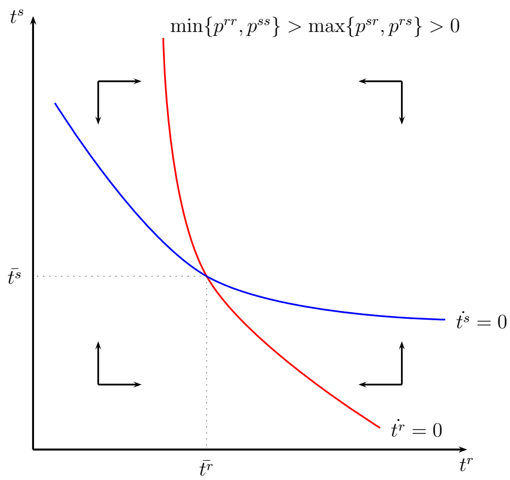
[6] All proofs are provided in Appendix A.

measures of comparative advantage of country j in sectors s and r correspondingly.

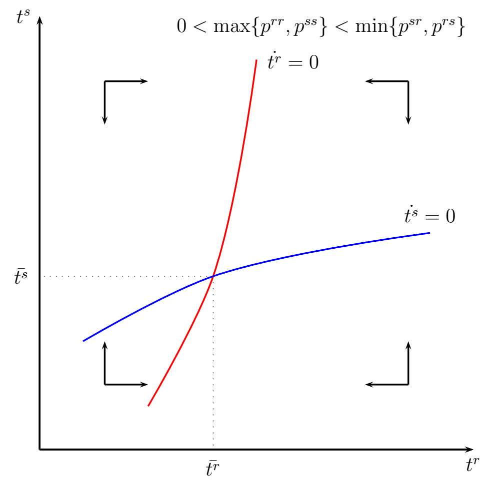
Figure (2a) illustrates Proposition 1 for this simple 2-sector 2-country case. Sub-figures (a) and (b) depict the phase diagrams for the considered economy when all spillovers are positive, $p^{rs} > 0 \; \forall r, s \in \mathcal{S}$, i.e. there are no isolated clusters. Sub-figure (a) corresponds to a more realistic case when intra-sector spillovers are stronger than cross-sector. Sub-figure (b) illustrates the opposite case. As Proposition 1 states, when there are no isolated clusters the curves $\dot{t^s} = 0$ and $\dot{t^r} = 0$ intersect only once at point $(\bar{t^r}, \bar{t^s})$ in such a way that $(\bar{t^r}, \bar{t^s})$ is a stable BGP. If the two sectors are isolated – $p^{sr} = p^{rs} = 0$ – then curves $\dot{t^s} = 0$ and $\dot{t^r} = 0$ merge into one curve, any point on which is a BGP, e.g. both points $(\bar{t^r}, \bar{t^s})$ and $(\bar{t^r}', \bar{t^s}')$ on Figure (2c) are on the BGP. Thus, there exist infinitely many equilibria and the initial conditions define to which BGP the economy converges.

Notice that under zero cross-sectoral spillovers there also exist unstable balanced growth paths outside the downward sloping curve $\dot{t^s} = 0, \; \dot{t^r} = 0$. If, say, country j doesn't have sector s, i.e. $T_j^s = 0$, then referring to Figure (2c) the state of the system can be described by a point on the horizontal axis ($t^s = 0$) that shifts over time to the right. As a result countries' production and trade patterns approach complete specialization: country j always produces only goods r, while country i produces both s and r, yet, its share in sector r is ever-shrinking. The path is unstable since any transfer of technology that turns T_j^s into a positive number will bring the system to an equilibrium on the downward-sloping curve $\dot{t^s} = 0, \; \dot{t^r} = 0$.

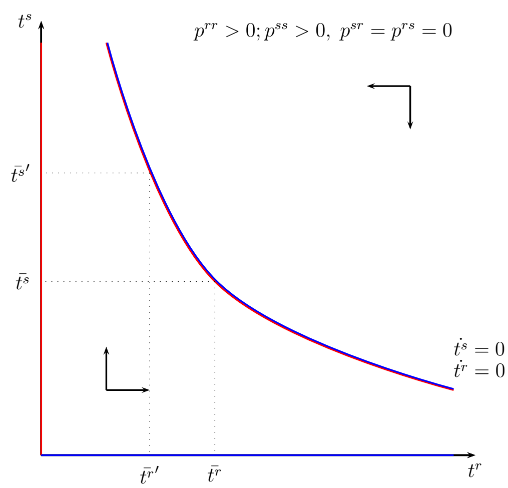
Now let's consider the forces that govern the dynamics of relative productivities and, hence, comparative advantage. The first force – let's name it the **country size force** – prevents the relative productivities of country j in both sectors from going to either (∞, ∞) or $(0, 0)$. This force dominates in the North-East and South-West quadrants of Figures (2a)–(2c). The explanation for it comes from the fact that each unit of labor in both countries i and j can



(a) Within sectors spillovers stronger than between



(b) Between sectors spillovers stronger than within



(c) Positive "within" and zero "between" spillovers.

Figure 2: Dynamics of relative productivity for a 2×2 model.

generate technologies at the same rate ϕ, hence, the ratio of productivities on the BGP will be finite and proportional to the ratio of sizes of the two countries.

The second force – **comparative advantage centripetal force** – prevents the self-reinforcing specialization and causes a decline in comparative advantage of each country. This force dominates in the North-West and South-East quadrants of Figures (2a) and (2b). To explain the mechanism that creates it let's consider a country that can buy at the same price some mass dT of technologies with productivities drawn from the same Pareto distribution. For which sector would it buy these technologies? For the least productive one! To see why, let's assume that the country in this example is a small open economy with 2 sectors with equal shares in consumption expenditures, $\alpha^A = \alpha^B = 0.5$. If we normalize the income of the rest of the World to 1 then the income of this country can be approximated by $w_i L_i \approx 0.5\pi_i^A + 0.5\pi_i^B$. The country will invest the mass of technologies dT in a sector for which $\frac{d\pi_i}{dT}$ is the largest. One can show that the expenditure share π_i^X is an increasing and concave function in the corresponding T_i^X. Concavity comes from the fact that the more productive is the sector and the larger is its share in expenditures the harder it is to come up with a technology that would excel the existing high productivity level in this sector. Thus, the investment in the same mass dT of technologies would have the highest return in the least productive sector[7]. The comparative advantage centripetal force can explain why we observe productivity of sectors with initial comparative disadvantage growing faster and, as a result, decreasing comparative advantage across countries. The latter salient feature of the data is documented in Levchenko and Zhang [2011].

The third force that plays an important role in the described dynamics of comparative advantage is the **comparative advantage centrifugal force**. This force emerges from the fact that, using the wording of the simple example above, it actually costs more to invest

[7]If sectors have different shares in final consumption expenditures then the changes in expenditure shares $d\pi/dT$ should be weighted using the corresponding expenditure shares when defining the return on investment in dT.

dT in the least productive sector than in the most productive. This happens because to invest dT in the least productive sector the country should divert some mass of labor to it from more productive sectors. Under no cross-sector spillovers there are multiple BGPs in which the comparative advantage centrifugal and centripetal forces equalize each other – costs or reallocating some marginal amount of labor across sectors are equal to benefits from such reallocations. Cross-sector spillovers allow to make the "investments" in the least productive sectors less costly, essentially providing technologies for them from the most productive sectors "for free" – now there is no need to divert labor to least productive sectors in order to allow them to catch up. As a result the centripetal force becomes stronger and all economies converge to the BGP without any comparative advantage. In this paper I do not model international technological spillovers, but their presence would work in the same way as domestic spillovers – they will weaken the centrifugal force and allow the least productive sectors to catch up without diverting labor from the more productive ones; the centripetal force will remain the same.

An alternative way to convey the intuition behind Proposition 1 is to consider a simple example of 2×2 economy with and without cross-sector spillovers. Assume that we have two countries (i and j) and two sectors (r and s) for which $L_i = L_j$ and $\alpha^s = \alpha^r$ and $\frac{T_i^s}{T_i^r} = \frac{T_j^r}{T_j^s} = 10$. If $p^{ss} = p^{rr} = 1$ and $p^{sr} = p^{rs} = 0$ then the equilibrium in which $\frac{L_i^s}{L_i^r} = \frac{L_j^r}{L_j^s} = 10$ will be a BGP. Indeed, in this case sector s in i is 10 times more productive than r, it employs 10 times more labor and generates 10 times more ideas per unit of time than r. Hence, s grows at the same rate as r (productivity of sector s in i is proportional to $(T_i^s)^{1/\theta}$). So, we have illustrated that such an equilibrium is a BGP and because number "10" can be replaced by any other positive number, there is a continuum of such BGPs. Now let's modify the assumption of zero inter-sector spillovers and set $p^{sr} = p^{rs} = 0.1$. What will happen now is that labor allocation $\frac{L_i^s}{L_i^r} = \frac{L_j^r}{L_j^s} = 10$ will generate a mass of technologies equal to 2 for sector r in country i and only 10.1 – for sector s. This will allow sector r to catch up with sector

s. The opposite will happen in country j. The international spillovers will act in the same manner as inter-sector spillovers: even under $p^{ss} = p^{rr} = 1$ and $p^{sr} = p^{rs} = 0$ sector r in i disproportionally more technologies than from j than s, thus, r will grow faster up until the point when relative sizes of both sectors in both countries become the same.

3 Economic policy

Technology in the current model is a public good. Sectors differ in terms of how strong are the externalities that each of them generates. So, some sectors can generate technologies that are more widely used and, thus, such sectors can be considered core sectors as in Greenwald and Stiglitz [2006]. As a result, there may exist a room for a welfare-improving economic policy when the government promotes the core sectors to increase productivity in the whole economy. The current section describes the necessary conditions for such policy and also gives an example of it in a form of sector-specific taxes. In what follows I assume zero discount rates, so that it is only welfare on the BGP that is taken into account as a criteria for policy optimality. This assumption allows me to provide some closed form results to describe the optimal policy.

Let's modify the above mentioned model in the following way – the government in country i taxes producers in sector s at rate τ_i^s. Thus, unit costs of producers of variety ω in sector s of country i at time t is $\frac{\tau_i^s w_i(t)}{Z_i^s(\omega,t)}$. Collected tax revenue is distributed among households as a lump-sum transfer. With this modification Equation (11) turns into

$$\pi_{ij}^s(t) = \frac{T_i^s(t)\left(w_i(t)\tau_i^s d_{ij}^s\right)^{-\theta}}{\sum_l T_l^s(t)\left(w_l(t)\tau_l^s d_{lj}^s\right)^{-\theta}} \tag{24}$$

Revenue of all producers in sector s of country i now becomes $L_i^s(t)\tau_i^s w_i(t)$. What matters for the allocation of labor in the current setting and, hence, for utility per capita, are the ratios of taxes across sectors within each country, $\left\{\frac{\tau_i^r}{\tau_i^s}\right\}_{s,r\in\mathcal{S}}$, but not absolute values of taxes

$\{\tau_i^r\}_{r\in\mathcal{S}}$.

There are several explanations that justify the use of namely this policy tool. First, it is an indirect and viable tool of economic policy unlike some direct tools such as direct labor allocation across sectors. Second, in the context of an open economy this tool seems preferable to any trade policy instruments because it can be easily implemented and it doesn't discriminate between . Finally, in the context of the model, introducing sector-specific taxes is isomorphic to introducing an exogenous component of productivity for a particular sector in a particular country[8]. So, the insights obtained from modelling the impact of taxes on the equilibrium outcomes are identical to those that would be obtained under the presence of exogenous components in sectoral productivity and comparative advantage.

3.1 Autarky

To proceed, let's again consider two extreme regimes of trade and start with **autarky**. Countries are isolated, so country indices can be dropped. Normalizing wages in a country to $w = 1$, income per capita can be written down as $\frac{I(t)}{L(t)} = \sum_s \tau^s l^s(t)$. Because of Cobb-Douglas utility at the level of sector aggregates we have $\alpha^s I(t) = \tau^s L^s(t)\ \forall s \in \mathcal{S}$, so the equilibrium labor allocation depends only on the expenditure shares $\{\alpha^s\}_{s\in\mathcal{S}}$ and taxes $\{\tau^s\}_{s\in\mathcal{S}}$. Labor demand is homogeneous in taxes of degree zero, so one can normalize taxes in one sector to 1, say $\tau^S \equiv 1$. The resulting labor allocation can be obtained as a solution of the system of equations

$$\tau^r = \frac{L^S}{L^r}\frac{\alpha^r}{\alpha^S},\ \forall r \in 1,\dots,S-1, \tag{25}$$

subject to the total labor supply constraint $\sum_{s\in\mathcal{S}} L^s = \bar{L}$.

Price level in sector r is equal to $p^r(t) = \tau^r\gamma\left(T^r(t)\right)^{-\frac{1}{\theta}}$, while the number of ideas per capita along the BGP is $\frac{T^r}{L} = \frac{\phi}{g}\sum_q p^{qr} l^q$. Using these two expressions the BGP level of

[8]Indeed under a country-sector specific productivity shifter A_i^s – exogenous component of productivity – the unit costs becomes $\frac{w_i d_{ij}^s}{A_i^s Z_i^s(\omega)}$ which is equal to the unit costs under taxation $\frac{\tau_i^s w_i d_{ij}^s}{Z_i^s(\omega)}$ if $\tau_i^s = 1/A_i^s$.

utility per capita can be written down as

$$\frac{U}{L}=\prod_{r}\left(\frac{\alpha^{r}}{p^{r}}\frac{I}{L}\right)^{\alpha^{r}}=\frac{I}{\gamma L}\left(\frac{\phi L}{g}\right)^{\frac{1}{\theta}}\prod_{r}\left(\frac{\alpha^{r}}{\tau^{r}}\left(\sum_{q}p^{qr}l^{q}\right)^{\frac{1}{\theta}}\right)^{\alpha^{r}} \tag{26}$$

To find the optimal taxes I maximize U/L w.r.t τ's for any given level of L. For a 2-sector economy one can show that under equal shares in expenditures – $\alpha^s=\alpha^r=0.5$ – sector r should be taxed at a higher rate in order to re-allocate labor to sector s if $p^{ss}p^{sr}>p^{rr}p^{rs}$, i.e. if sector s generates more widely applicable technologies than sector r. Besides, the optimal tax[9] $\frac{\tau^r}{\tau^s}$ can not be infinitely large – which would result in a collapse of the more heavily taxed sector r – first, because of love for variety and, second, because having "donor" sectors that generate more general technologies makes sense only if there are "recipient" sectors that can adopt those technologies.

An alternative way to think about industrial policy is to consider direct labor re-allocation across sectors. In this case the social planner solves the following constrained optimization problem

$$\max_{\{L^s\}_{s\in\mathcal{S}}}\gamma\left(\frac{\phi}{g}\right)^{\frac{1}{\theta}}\prod_{s\in\mathcal{S}}\left(L^{s}\left(\sum_{r\in\mathcal{S}}p^{rs}L^{r}\right)^{\frac{1}{\theta}}\right)^{\alpha^{s}},\quad\text{s.t.}\ \sum_{r\in\mathcal{S}}L^{r}=\bar{L}, \tag{27}$$

the first order conditions for which are

$$\frac{\alpha^{r}}{L^{r}}+\sum_{s\in\mathcal{S}}\frac{\alpha^{s}}{\theta}\frac{p^{rs}}{\sum_{q\in\mathcal{S}}p^{qs}L^{q}}-\lambda=0\ \forall r\in\mathcal{S}\ \text{and}\ \sum_{r\in\mathcal{S}}L^{r}=\bar{L} \tag{28}$$

For any pair of sectors r and v the optimality requires

$$\frac{1}{L^{r}}\left(\alpha^{r}+\frac{1}{\theta}\sum_{s\in\mathcal{S}}\frac{\alpha^{s}L^{r}p^{rs}}{\sum_{q\in\mathcal{S}}L^{q}p^{qs}}\right)=\frac{1}{L^{v}}\left(\alpha^{v}+\frac{1}{\theta}\sum_{s\in\mathcal{S}}\frac{\alpha^{s}L^{v}p^{vs}}{\sum_{q\in\mathcal{S}}L^{q}p^{qs}}\right) \tag{29}$$

[9]The optimal tax in a 2-sector economy $\tau\equiv\frac{\tau^s}{\tau^r}$ solves the FOC of maximization of the BGP level of $\frac{U}{L}$ for any level of L: $\frac{\alpha^r}{\theta}\frac{p^{rr}\alpha^r}{p^{rr}\alpha^r\tau+p^{sr}\alpha^s}+\frac{\alpha^s}{\theta}\frac{p^{rs}\alpha^r}{p^{rs}\alpha^r\tau+p^{ss}\alpha^s}+\frac{1-\alpha^s}{\tau}-\left(1+\frac{1}{\theta}\right)\frac{\alpha^r}{\alpha^r\tau+\alpha^s}=0$.

From Equation (25) one can find the schedule of taxes that results in any particular labor allocation $\{L^s\}_{s \in \mathcal{S}}$, including the one described by Equation (29).

Equation (29) provides a criteria for optimal re-allocation of labor within any pair of sectors. If under the free market labor allocation ($L^s/\bar{L} = \alpha^s \; \forall s \in \mathcal{S}$) the right-hand side of Equation (29) is larger than the left-hand side then it means that sector r under free market generates a larger share of technologies for more important sectors than sector v and, thus, labor should be re-allocated from v to r. Clearly, if both sectors generate technologies of the same applicability ($p^{vs} = p^{rs} \; \forall s \in \mathcal{S}$) then labor should not be re-allocated across these two sectors comparing to the free market outcome when $\frac{L^r}{L^v} = \frac{\alpha^r}{\alpha^v}$. One can also notice that for a diagonal matrix of spillovers ($p^{qs} = p^q$ if $q = s$ and $p^{qs} = 0$ if $q \neq s$) the first order conditions turn into $L^r = \alpha^r \bar{L} \; \forall r \in \mathcal{S}$ which describes exactly the labor allocation that would took place without any policy interventions. Thus, as follows from this simple 2×2 example, the labor re-allocating policy in autarky can be welfare-improving only if there exist positive inter-sector spillovers – intra-sector spillovers alone are not enough to create a room for such policy. In fact, under diagonal matrix of spillovers the described autarkic economy is isomorphic to the one with Marshallian externalities within each sector where output in each sector s is proportional to $(L^s)^{1+\frac{1}{\theta}}$. Finally, notice that asymmetry either in expenditure shares across sectors or in spillovers is required. If $\alpha^s = 1/S \; \forall s \in \mathcal{S}$ and $p^{rs} = p^{sr} \; \forall r, s \in \mathcal{S}$ then, again, there is no room for welfare-improving policy.

3.2 Open economy

Under **costless trade** the asymmetric taxation re-allocates labor in the same manner as in autarky, yet, now this mechanism involves some additional factors. First, love for variety no longer impedes re-allocation, so the responsiveness of labor demand to taxes should be higher. Second, economic policy of trade partners comes into play. E.g. if country i through taxation re-allocates labor from a sector with narrowly-applicable technologies to a sector with widely-applicable technologies then, absent any economic policy in country j, labor in j will be re-allocated in the opposite direction. As a result, in the BGP economy i will have higher number of ideas per capita (T/L) in each sector than country j. So, i will be characterized by higher welfare, though, both economies will be growing at the same rate g/θ.

To provide a more detailed description on the role of spillovers for the implications of the outlined policy let's first consider the 2×2 model with costless trade and **zero inter-sector spillovers**: $p^{sr} = 0$, $p^{ss} > 0$. Zero inter-sector spillovers imply that on the BGP $T_i^s = \frac{\phi}{g} p^{ss} L_i^s$ which together with sector labor demand $w_i \tau_i^s L_i^s = \alpha^s \pi_i^s I$ results in either $\frac{\tau_i^s}{\tau_j^s} = \frac{\tau_i^r}{\tau_j^r}$ or some $L_\cdot^\cdot = 0$ – corner solution. In order to describe the behaviour of the model under different taxes we find all $\{t^r, t^s\}$ that characterize $\dot{t}^s = 0$ and $\dot{t}^r = 0$. For $\dot{t}^s = 0$ these loci are described by $t^s = 0$ and

$$t^s = \frac{L_j^s}{L_i^s} = \frac{L_j(A + 1 + Ft^r) - L_i BFt^r}{L_i(BFt^r + 1 + Ft^r) - L_j A}, \tag{30}$$

while for $\dot{t}^r = 0$ – by $t^r = 0$ and

$$t^r = \frac{L_j^r}{L_i^r} = \frac{L_j(C + 1 + Ht^s) - L_i DHt^s}{L_i(DHt^s + 1 + Ht^s) - L_j C}, \tag{31}$$

where $A \equiv \frac{\alpha^r}{\alpha^s} \frac{\tau_i^s}{\tau_i^r} \equiv C^{-1}$, $B \equiv \frac{\alpha^r}{\alpha^s} \frac{\tau_j^s}{\tau_j^r} \equiv D^{-1}$, $F \equiv \left(\frac{\tau_j^r}{\tau_j^s} \frac{\tau_i^s}{\tau_i^r} \right)^{-\theta} \equiv H^{-1}$.

It can be shown that symmetric taxes do not eliminate multiplicity of BGPs and result in a "corner" BGP only is the economy starts with absent sectors in some countries. To be more clear let's consider symmetric countries $L_i = L_j$ with symmetric consumption shares $\alpha^s = \alpha^r$ and symmetric taxes $\frac{\tau_i^s}{\tau_i^r} = \frac{\tau_j^s}{\tau_j^r} = \frac{\tau^s}{\tau^r} > 1$ (w.l.o.g). Figure (3) illustrates a new equilibrium – blue lines for $\dot{t}^s = 0$ and red lines for $\dot{t}^r = 0$ – and the old equilibrium – a downward sloping grey dashed line. Both loci go through point (1,1) because symmetric

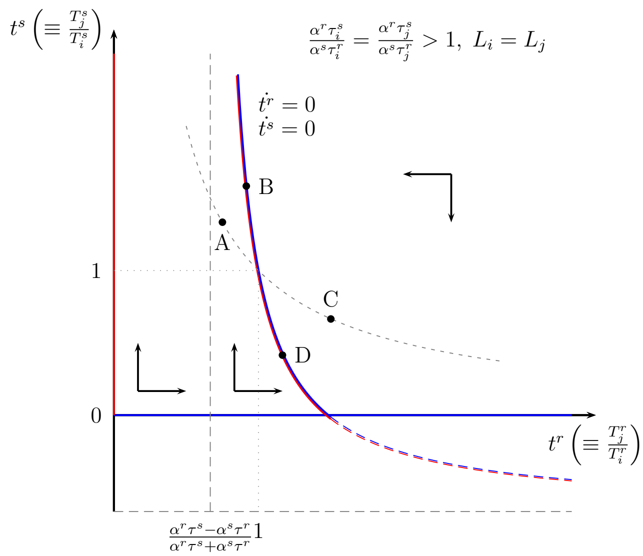


Figure 3: Symmetric taxes under zero inter-sector spillovers.

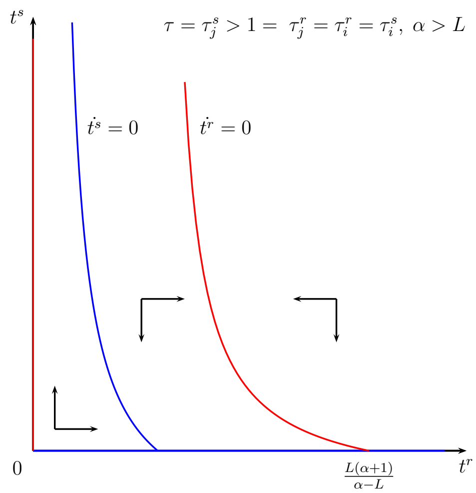
taxes do not introduce any asymmetry if initially productivities were the same. Downward sloping curve is described by equation $t^s = \frac{(1+A)+t^r(1-A)}{t^r(1+A)+(1-A)}$, $A \equiv \frac{\alpha^r \tau^s}{\alpha^s \tau^r} > 1$. As one can see, there exist infinitely many interior solutions when both countries produce in both sectors. Besides, unlike in the case of no taxes, now there exist four corner solutions – two with complete specialization as before and two new, with incomplete specialization. The corner solution with complete specialization emerges when country j starts without sector r; with incomplete specialization – if j starts without s. The other two corner solutions are the symmetric cases of the above mentioned ones but when i starts either without s or r.

When both countries start with non-zero productivity in each sector the symmetric tax on sector s (equivalently, subsidy of r) has asymmetric impact on specialization and welfare. First, each point on the curve $\dot{t^s} = 0$, $\dot{t^r} = 0$ is characterized by the same total utility $U_i + U_j$ which decreases whenever $\frac{\tau^s}{\tau^r}$ deviates from 1. Second, the allocation of welfare along the curve is not the same – as the BGP point $(t^r,\ t^s)$ moves upwards U_j increases while U_i declines. Thus, if the economy started at point A where j specialized more in s while i – in r and both countries had equal welfare per capita $\left(\frac{U_i}{L_i} = \frac{U_j}{L_j}\right)$, the tax on s (subsidy for r) will improve j's relative productivity in both sectors and make it better off comparing to i, yet, it will also shrink the total size of the "pie" so that the aggregate welfare will decline. The intuition for changes in relative productivity across countries can again be described in terms of "returns" to an additional mass of technologies in a more versus less productive sectors. The logic is similar to what we saw before: the same mass of technologies will be applied to a greater number of varieties in a less productive sector than in a more productive one. Symmetric taxes on a more productive sector s in country j re-allocate labor towards less productive sector r, increasing productivity of r by a larger factor than are the losses in productivity in s. On the contrary, country i re-allocates labor from a less productive s to a more productive r, losing a significant portion of productivity in s and gaining disproportionately less in productivity in r. As a result, relative productivity of country j increases in both sectors. It can be shown that under zero taxes $\frac{d \log U_j}{d \log \tau^s/\tau^r} = \frac{l_i^r - l_j^r}{2}$, thus, if country j specializes more in sector s it will gain (and country i will lose) from some positive tax on s/subsidy for r conditional on this tax/subsidy being symmetric across countries. Since the relation between U_j and τ^s/τ^r is not monotonic there exists an optimal level of τ^s/τ^r after which U_j will decrease.

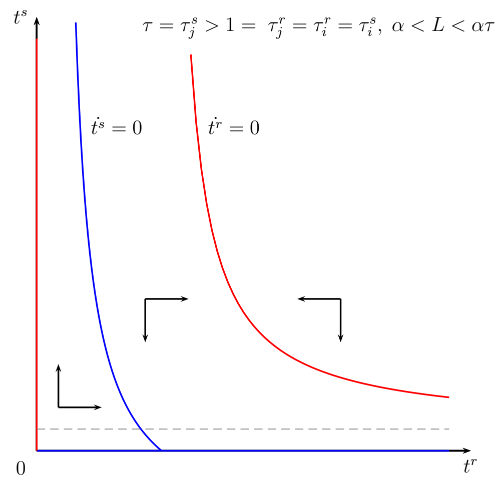
Symmetric taxes across countries is a very particular case of policy[10], so now we consider a more general case of asymmetric taxes. For simplicity and w.l.o.g. assume $\tau \equiv \tau_j^s > 1 = \tau_i^s = \tau_i^r = \tau_j^r$, yet, avoid the assumption on symmetry in sectors' shares and country size because depending on these characteristics the outcomes of taxation will be different. Equations (30) and (31) turn into $t^s = \frac{L(\alpha+1)+t^r(L\tau^{\theta}-\alpha\tau^{\theta+1})}{t^r(\tau^{\theta}+\alpha\tau^{\theta+1})+1-L\alpha}$ and $t^r = \frac{L(\alpha^{-1}+1)+t^s(L\tau^{-\theta}-\alpha^{-1}\tau^{-\theta-1})}{t^s(\tau^{-\theta}+\alpha^{-1}\tau^{-\theta-1})+1-L\alpha^{-1}}$, where $L \equiv \frac{L_j}{L_i}$, $\alpha \equiv \frac{\alpha^r}{\alpha^s}$. Figures (4a)-(4c) depict the corresponding loci for $\dot{t}^s = 0$ and $\dot{t}^r = 0$ and the resulting patterns of specialization. Asymmetric taxes under zero cross-sector spillovers result in corner BGPs: one sector in one country collapses. Namely, if relative size of country j is smaller than the relative share of sector r – $L < \alpha$ – then taxation of s in j will result in

[10]Although a particular case of policy, the symmetric taxes exemplify well the case of industrial policy motivated as a "response to foreign targeting" as it is described in Krugman [1983]. This example shows that country i is strictly worse off under such response to the policy of j and, as we will see later, under zero inter-sector spillovers country j would not unilaterally initiate any policy if it didn't expect a symmetric response from country i.

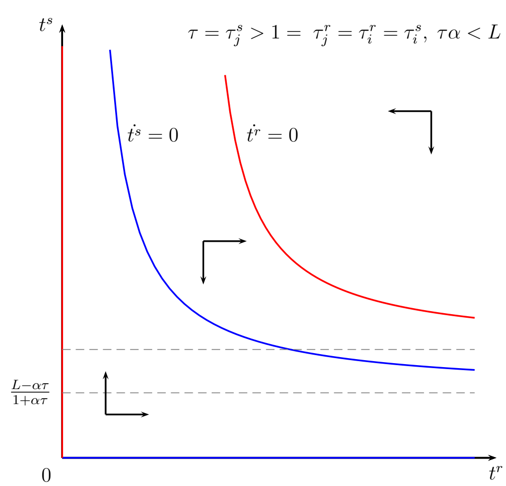
j's complete specialization in r while i will produce both r and s. This BGP is illustrated by Figure (4a) where the equilibrium relative productivity stabilizes at $t^s = 0$ and $t^r = \frac{L(\alpha+1)}{\alpha-L}$. On the contrary, if j is large enough – $L > \alpha\tau$ – then the reallocation of labor from s to r inside country j will be associated with more labor allocated to s in i, so that ultimately i will produce only s while j will produce both. As Figure (4c) shows, in this case the relative productivities t^s and t^r approach $\frac{L-\alpha\tau}{1+\alpha\tau}$ and ∞ correspondingly. In the intermediate case – $\alpha < L < \alpha\tau$ – complete specialization will be observed: j will produce only r and i – only s: in Figure (4b) $t^s \to 0$ and $t^r \to \infty$.



(a)



(b)



(c)

Figure 4: Asymmetric taxes under zero inter-sector spillovers.

Now let's turn to the welfare implications of each of these resulting specialization patterns. As one can show asymmetric taxes in the current setting can not make country j better off: under $\frac{L_j}{L_i} < \frac{\alpha^r}{\alpha^s}$ its utility per capita remains the same as under zero taxes, while under $\frac{L_j}{L_i} > \frac{\alpha^r}{\alpha^s}$ it decreases with τ. So we can conclude that under zero cross-sector spillovers no unilateral industrial policy can make country j better off. Coordinated symmetric policy can improve country j welfare only at a cost of country i. To summarize this section on economic policy in the open economy under zero inter-sector spillovers: we saw that no labor re-allocating policy can improve the total welfare of the World and no countries have incentives to implement such policy unilaterally. Symmetric economic policy under the diagonal matrix of spillovers can redistribute welfare across trading partners, but will unambiguously decrease the total welfare. Thus, positive inter-sector spillovers are necessary for the possibility of welfare-improving policy. It follows from the latter that under zero inter-sector spillovers it really doesn't matter in what products the country specializes.[11]

The welfare implications of trade openness may be different in the case of **positive inter-sector spillovers**. This comes from the fact that under positive spillovers labor re-allocation can improve welfare. Trade openness also leads to labor re-allocation which is not necessarily aligned with the optimal one. As before, let's start with the case of symmetric taxes on sector s, $\tau_i^s = \tau_j^s > 1$. As Figure (5) shows, under symmetric tax both loci $\dot{t^r} = 0$ and $\dot{t^s} = 0$ will rotate clockwise, yet, symmetric taxes will not introduce any asymmetry to the BGP – it will remain on the 45-degree line meaning no comparative advantage in either of the countries in the long-run. What will change is the allocation of labor within each country, relative productivity between sectors within countries and, hence, intensity of spillover flows between sectors. In the same manner as re-allocation of labor towards core sectors in the autarky helped to increase welfare, symmetric taxes that favor core sectors in the open economy

[11] This result, as I mentioned above, if isomorphic to the result with equal Marshallian externalities across sectors. Yet, it crucially depends on the equality of θ across sectors. If the latter are allowed to vary across sectors then then even under diagonal matrix of spillovers it would make sense to allocate labor to sectors with lower θ, i.e. to those with thicker tail of distribution of productivity.

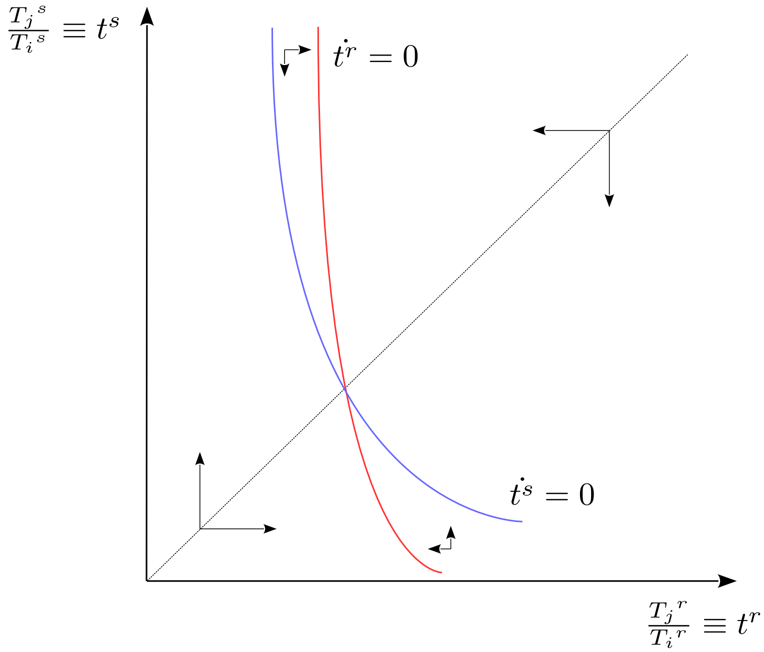
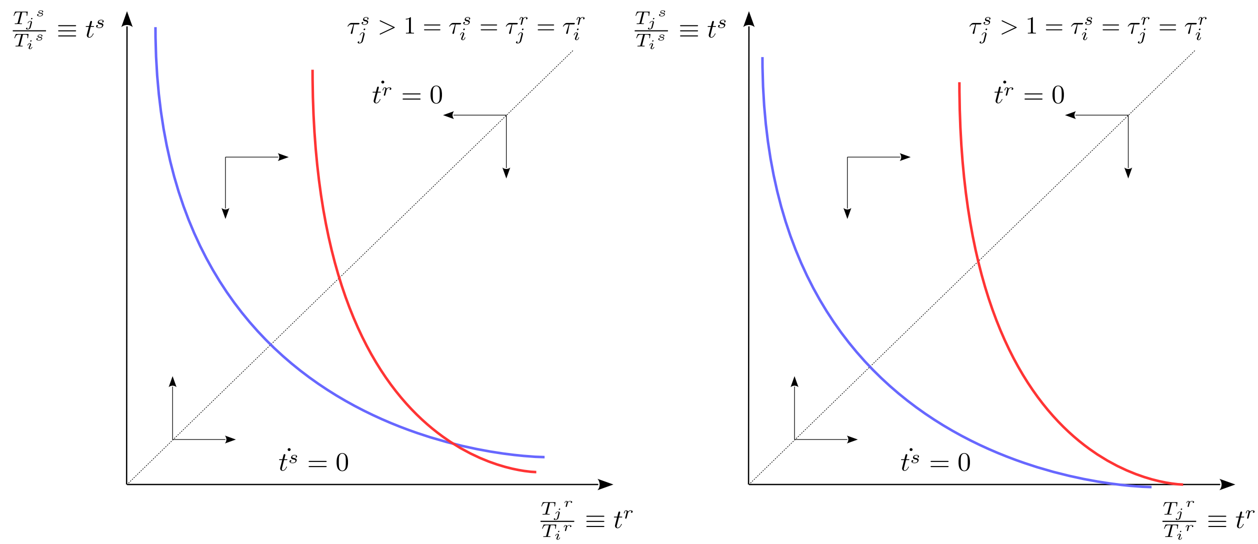


Figure 5: Symmetric taxes under positive inter-sector spillovers.

increase the welfare in each country and the World as a whole (the whole World can be treated as an autarky).

For asymmetric taxes let's again consider the case of country j taxing sector s: $\tau_j^s > 1 = \tau_i^s = \tau_j^r = \tau_i^r$. Now sector s has a comparative disadvantage in country j, while r – comparative advantage. This tax-wedge will shift the BGP downwards, so that in the long-run country j has a comparative advantage in sector r, while country i – in sector s. This outcome is illustrated by Figure (6a). The straightforward result of taxation is that the employment in the taxed sectors decreases comparing to the no-tax scenario and this labor allocation is preserved on the BGP. This is especially important if we think about exogenous factors of comparative advantage (e.g. deposits of natural resources) as taxes or subsidies on particular sectors. Trade openness without any policy interventions will result in lower employment in sectors that are more heavily "taxed", i.e. those that originally are at a disadvantaged position due to exogenous factors. If the disadvantaged sectors are the core sectors, then productivity in the whole economy will decline under trade openness comparing to what it would be under the autarkic labor allocation. This mechanism is similar to the one



(a) Small tax (b) Large tax

Figure 6: Asymmetric taxes under positive inter-sector spillovers.

described in Greenwald and Stiglitz [2006], yet, the consequences of core sectors' shrinkage here is not a zero growth rate of the economy (in the long-run all economies are growing at rate $g/\theta > 0$ regardless of their sectoral composition), but a lower productivity and, potentially, lower welfare on the BGP.

To illustrate the last point let's add some more details to out 2-country 2-sector example. For simplicity let's consider i and j of equal size ($L_i = L_j$) with sector r and s with equal expenditure shares ($\alpha^r = \alpha^s$). Assume that sector r is a core sector, i.e. it generates more widely applicable technologies than sector s: $p^{rr} = 0.9 = p^{ss}$, $p^{rs} = 0.7$, $p^{sr} = 0.1$. Let's also assume that country i has some exogenous comparative advantage in a non-core sector s which is expressed by an equivalent exogenous subsidy $\tau_i^s < 1$. Figure (7) illustrates the utility per capita of country i on the BGP under autarky and frictionless trade depending on the magnitude of exogenous comparative advantage in s. The horizontal axis shows the extent of exogenous comparative advantage of s: small $\tau_i^s 1$ means that sector s receives a large exogenous "subsidy" comparing to sector r. In other words, exogenous "subsidy" means that for the same number of technologies accumulated in s and r in country

i, sector s will be more productive than r by factor $1/\tau_i^s$. When country i opens to trade it observes two forces affecting its welfare. First, it observes lower prices for varieties in which country j is more productive than i. This is a standard force of comparative advantage and specialization in varieties with higher productivity which unambiguously leads to an increase in welfare of i with opening to trade. Second, the labor is reallocated to the non-core sector s with an exogenous comparative advantage: now varieties of sector r can be imported and demand for them no longer leads to the previous relatively high employment in r. This force results in a decreased productivity of economy i and a decrease in its welfare. For this example the second force dominates in the interval of $\tau_i^s \in (0.53;\ 0.76)$ which means that country i with τ_i^s in this interval may be better off in autarky than under frictionless trade. An interesting observation is the non-monotonicity of gains from trade in the strength of exogenous comparative advantage. Countries with either very low (τ_i^s close to 1) or very high (τ_i^s close to 0) exogenous comparative advantage in sector s would gain from trade. The former ones observe weak forces that pull their labor from r to s with trade openness because the exogenous advantage of s is weak. The latter ones have a significant share of labor allocated to s even in autarky, thus, trade openness doesn't have that much labor to re-allocate from r to s. It is the countries with intermediate levels of exogenous advantage in a non-core sector that may lose from openness to trade.

Unlike for the autarky, criteria for the optimal policy in the open economy does not have a closed form expression. To show this let's write down the welfare maximization problem of a social planner. Let's define the optimal tax schedule of country i as a set of taxes $\{\tau_i^s\}s \in \mathcal{S}$ that maximizes utility per capita in country i on the BGP, $u_i \equiv \frac{U_i}{L_i}$.

$$u_i \equiv \frac{U_i}{L_i} = \prod_{r \in \mathcal{S}} \left(\frac{\alpha^r}{p^r} \frac{I_i}{L_i} \right)^{\alpha^r}, \ p^r = \gamma \left(\sum_{j \in \mathcal{N}} T_j^r (w_j \tau_j^r)^{-\theta} \right)^{-\frac{1}{\theta}}, \ \frac{I_i}{L_i} = \sum_{r \in \mathcal{S}} \tau_i^r l_i^r w_i, \tag{32}$$

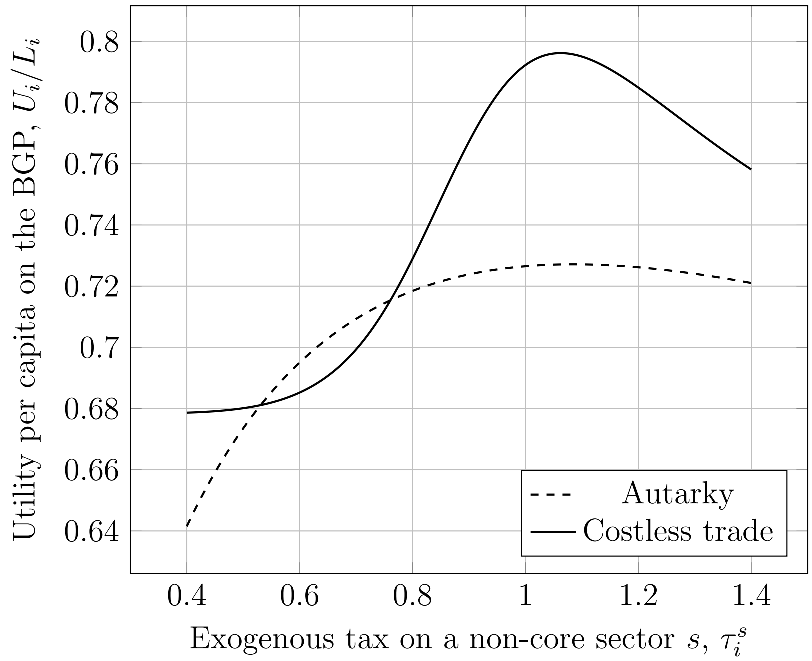


Figure 7: Exogenous comparative advantage and gains from trade.

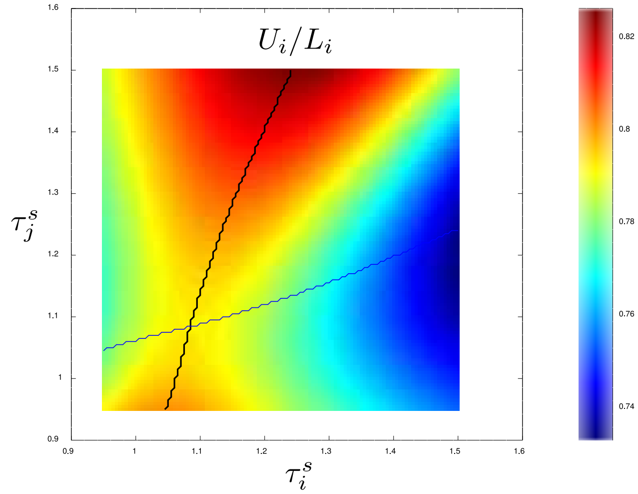
where $l_i^s \equiv L_i^s/L_i$. Substituting the last two expressions into the first one and taking logs of both sides of the resulting expression we obtain

$$\log u_i = \sum_{r\in\mathcal{S}} \alpha^r \log \alpha^r - \gamma + \log \sum_{r\in\mathcal{S}} \tau_i^r l_i^r w_i + \frac{1}{\theta} \sum_{r\in\mathcal{S}} \alpha^r \log \left(\sum_{j\in\mathcal{N}} L_j (w_j \tau_j^r)^{-\theta} \frac{\phi}{g} \sum_{s\in\mathcal{S}} p^{sr} l_j^s \right), \quad (33)$$

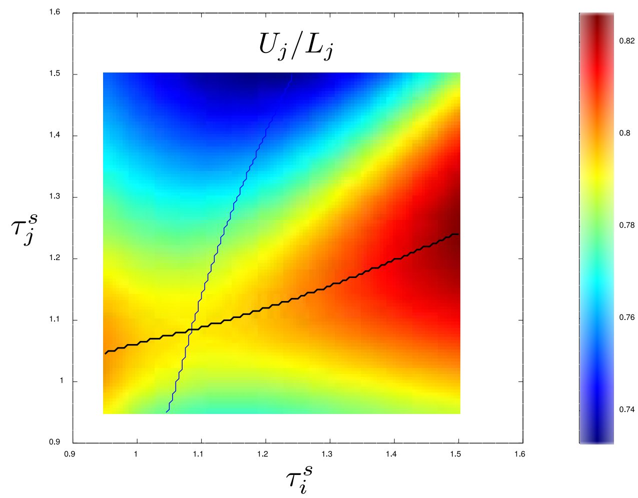
where I also made use of $\frac{T_j^r}{L_j} = \frac{\phi}{g} \sum_q p^{qr} l_j^q$. To find the optimal taxes we need to know the responses of wages and labor allocations across all sectors and countries to changes in taxes in country i – $\left\{\frac{\partial l_j^r}{\partial \tau_i^s}\right\}$, $\left\{\frac{\partial w_j}{\partial \tau_i^s}\right\}$, $i, j \in \mathcal{N}$, $r, s \in \mathcal{S}$. To find these derivatives at BGP one can use the implicit function theorem for the system of equations in l's and w's: $\tau_i^s l_i^s w_i = \pi_i^s \alpha^s I$, where $I = \sum_j \sum_s \tau_j^s l_j^s w_j$, and $\pi_j^s = \frac{T_j^s (w_j \tau_j^s)^{-\theta}}{\sum_k T_k^s (w_k \tau_k^s)^{-\theta}}$. This exercise of computing the optimal policy for an open economy will be completed in the next section using the calibrated model for the US.

To close the current section I would like to provide a numerical example that illustrates the welfare implications of industrial policy under positive inter-sector spillovers in an open economy. Consider the same two symmetric economies as in the example above. But now let's assume that both countries start with zero exogenous factors of comparative advantage and can choose different tax rates for sector s: τ_i^s and τ_j^s.[12] Figures (8a)–(8c) illustrate the BGP levels of utility per capita that can be attained by countries i and j depending on taxes τ_i^s (horizontal axis) and τ_j^s (vertical axis) that each of them imposes. Solid black curves in Figures (8a) and (8b) illustrate the optimal responses of countries i and j correspondingly to taxes imposed by their trade partner (blue lines denote the optimal responses of their trade partners). As Figures (8a) and (8b) show, a given country, conditional on no taxes introduced by its trade partner, has an incentive to subsidize the core sector r: given $\tau_j^s = 1$ the optimal $\tau_i^s > 1$. Same is true for country j. As one can see from Figure (8c) the total welfare is maximized at some $\tau_j^s > 0$, $\tau_i^s > 0$ which is similar to the prediction obtained for autarky if we consider the whole world to be an autarky – re-allocation of labor by all countries towards the core sector r increases the welfare globally. For this particular example one can compute that under autarky the optimal labor allocation is $l^r = 52\%$, $l^s = 48\%$ for both countries which is attained by tax $\tau^s = 1.0844$. If both economies can trade at zero cost and only i implements the industrial policy, then its optimal labor allocation in i turns into $l_i^r = 63\%$ and $l_i^s = 38\%$, yet, the required tax is now smaller – $\tau_i^s = 1.0622$. In other words, under costless trade country i doesn't need to produce all varieties of the non-core sector r itself and can allocate even more labor to the core sector s than in autarky. The labor demand is now more responsive to industrial policy, so the larger re-allocation is achieved with smaller taxes. One interesting question for further research is what would happen if country j can respond to taxes introduced in i and whether this game of subsidizing the core sectors has a Nash equilibirum. The second question is – provided that the Nash equilibrium exists, does it result in an optimal level of global welfare or is some coordination between the countries required for attaining the maximum of global welfare.

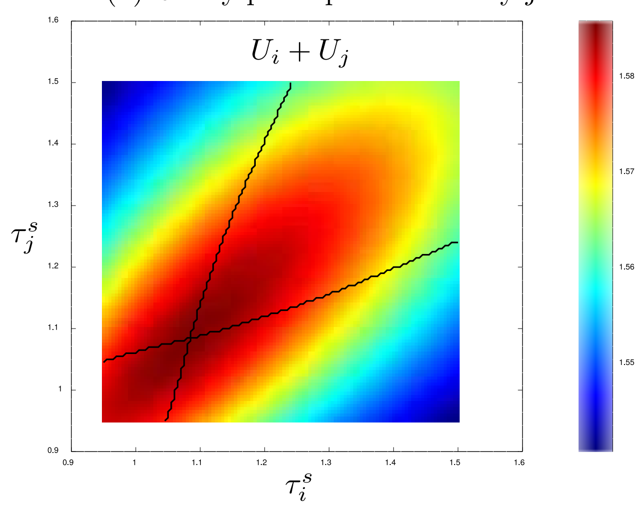
[12] Taxes on sector r in both countries can be normalized to 1.



(a) Utility per capita in country i



(b) Utility per capita in country j



(c) Total utility

Figure 8: Utility per capita under positive inter-sector spillovers and sector-specific taxes. Black lines depict optimal responses of a given country to taxes imposed by its trade partner. Blue lines – optimal responses of trade partners

4 Calibration

In this section I quantify the matrix of spillovers and discuss calibration of other parameters. As Equations (29) and (32) show, one need parameters θ, $\{\alpha^s\}_{s\in\mathcal{S}}$, $\{p^{qs}\}_{q,s\in\mathcal{S}}$ and $\{L_i\}_{i\in\mathcal{N}}$ to characterize the optimal industrial policy (the last set is needed for the open economy case, but not for the autarky).

The parameter that describes spillovers from sector q to sector s has a straightforward interpretation – p^{qs} is a probability of an event that a random technology created in sector q is used in producing any randomly picked variety in sector s. There exists a vast literature in urban economics and economic geography that estimates the strength of technological spillovers. The most recent example is the paper by Ellison et al. [2010]. The authors of that paper measured the strength of spillovers between sectors s and q as a share of citations generated by patents in sector s that are attributed to sector q:

$$p_{EGK}^{qs} = \frac{C^{qs}}{\sum_{k\in\mathcal{S}} C_{ks}}, \tag{34}$$

where C^{qs} is the number of citations sent from s to q and, hence, flows of ideas from q to s.[13] Although this metric quantifies the importance of sector q as a source of ideas for sector s, it also reflect the size of sector q, not only the extent of applicability of ideas from q. As an illustration, let's assume that sector q has 99% of all available ideas (patents) while sector s – only 1%. Let's also assume that ideas from q have the same probability of being used and cited by any patent in s as ideas from s ($p^{qs} = p^{sq}$). If we measure the extent of applicability of ideas from q and from s in sector s with $p_{EGK}^{\cdot}$ we will obtain $p_{EGK}^{qs} = 0.99$ and $p_{EKG}^{sq} = 0.01$.

To obtain the estimates of $\{p^{qs}\}$ that reflect only probabilities of cross-sector spillovers but not size of sectors I derive two estimators for $\{p^{qs}\}$ which I use with the US and Japan

[13]The subscript in p_{EGK} stands for the initials of the authors of Ellison et al. [2010].

patents data.[14] Referring to my model, I treat each patent as a technology – when it is cited, and as a variety which receives a new technology – when it cites other patents. For estimation of $\{p^{qs}\}$ I follow two approaches that use somewhat different dimensions of the patent data and, thus, allow me to check the consistency of the estimates. The first approach – name it a **"cohort approach"** – splits all patents into cohorts based on the year of issuance and the assigned sector: patents issued in year t in sector q enter cohort (q, t). Each cohort (q, t) is characterized by the total number of patents in it $Q(q, t)$, by the number of citations sent to any previous cohort (s, t'), $C(sq, t', t)$, and received from any subsequent cohort (r, t''), $C(qr, t, t'')$, where $t' < t < t''$. Let's consider two cohorts (q, t) and (r, t'') where $t < t''$. Each idea from (q, t) can be applied to any variety in (r, t'') with probability p^{qr}. If the total number of ideas and varieties in q and r are $Q(q, t)$ and $Q(r, t'')$ correspondingly, then the number of citations from (r, t'') to (q, t) will be distributed as $C(qr, t, t'') \sim Poisson\left(p^{qr}Q(q, t)Q(r, t'')\right)$. Figure (9) illustrates this example. Considering only the origin and the destination cohorts

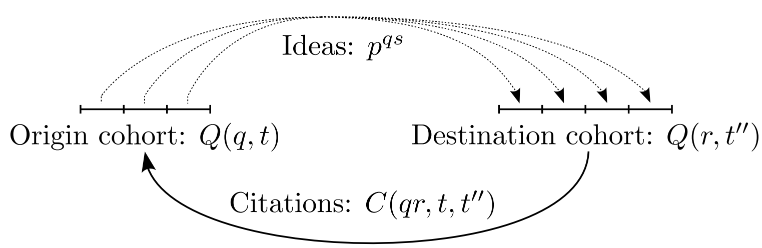


Figure 9: Cohort approach to estimating the $\{p^{qs}\}$ matrix

that are separated by some constant time interval Δt I obtain the MLE estimator for p^{qr}

$$p^{qr} = \frac{\sum_t C(qr, t, t + \Delta t)}{\sum_t Q(q, t)Q(r, t + \Delta t)}, \tag{35}$$

where $C(\dots)$'s and $Q(\dots)$'s are directly observable. When implementing this method I consider all patent registered within one year in an origin sector q as the origin cohort and patents registered in a destination sector r within 2-11 years after the origin cohort – as the destination cohort. Using pairs of cohorts separated by the same time interval I disregard

[14] For a more detailed description of the data, please, see Appendix B.

some citation data, yet, it allows me to exclude the impact of time patterns of citations arrivals on the estimates.

In the second approach, which I name a **"sequence approach"**, I use the information on the order in which patents were issued (it follows immediately from the patent numbers). Let's consider a patent ι originating in sector q. Denote the number of patents in a destination sector r that were issued after ι as $N_\iota(r)$, out of which $K_\iota(r)$ actually cited ι. One can treat the issuance of each of the $N_\iota(r)$ patents as a trial in which a positive outcome that ι is cited has probability p^{qr}. Then the total number of citations received by ι is a random variable $K_\iota(r) \sim \; Poisson(p^{qr} N_\iota(r))$. The number of total citations received by all patents in sector q by patents in sector r is $\sum_\iota K_\iota(r) \sim \; Poisson(\sum_\iota p^{qr} N_\iota(r))$ from where the MLE estimator for p^{qr} is equal to

$$p^{qr} = \frac{\sum_\iota K_\iota(r)}{\sum_\iota N_\iota(r)}, \tag{36}$$

where the sums are computed across all patents ι that have ever been registered in sector q. The sequence approach uses all the available data on citations, but is likely to give lower estimates than cohort method because the cohort method considers only the part of patent life-cycle in which patents receive citations at highest rates. Yet, what matters for the optimal policy exercise is the relative size between spillover probabilities and not their absolute values, so that the absolute values of $\{p^{qs}\}$ can be scaled either upwards or downwards.

For calibrating the probabilities of spillovers I used the US patent data for patents issued in 1976–2006. Out of the whole pool of patents I consider the ones that excel the 50% threshold of citations for patents of a given age (which each patent had in 2006) and from a given sector of origin. Although this truncation shifts absolute estimates of spillover probabilities upwards, it allows to consider a pool of more homogenous patents in terms of their significance. For the estimation procedure the data is aggregated into 93 sectors following the BLS-NAICS classification. For matching the international patent categories (IPC) to NAICS codes I use the probabilistic concordance matrices from Lybbert and Zolas [2014]. One obvious downside of the existing concordance schemes is that they allow to match patents to the fields of economic activity that employ roughly 30-40% of labor. Namely, concordances exist for manufacturing, agriculture, utilities, mining and construction, but not for retail and wholesale trade, transportation and all kinds of services.

As Figure (10) shows, both above described methods for estimating $\{p^{qs}\}$ produce very similar results – the fitted line (black) is very close to the 45-degree line (red). As an

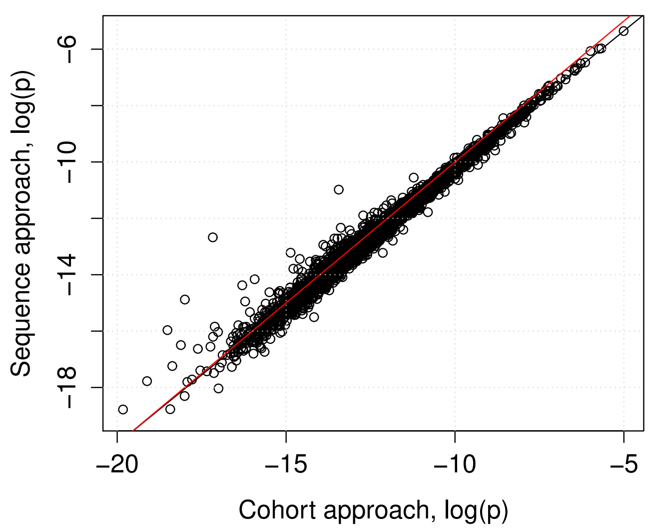
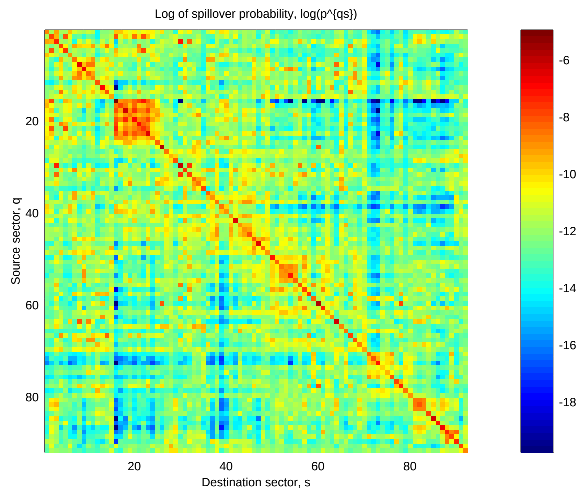
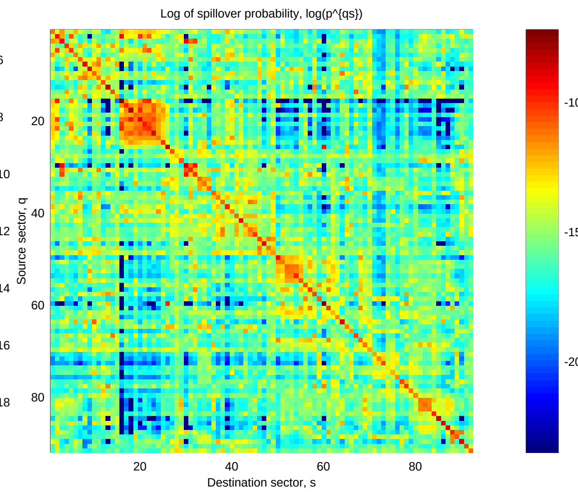


Figure 10: Logs of estimated spillover probabilities under cohort and sequence approaches.

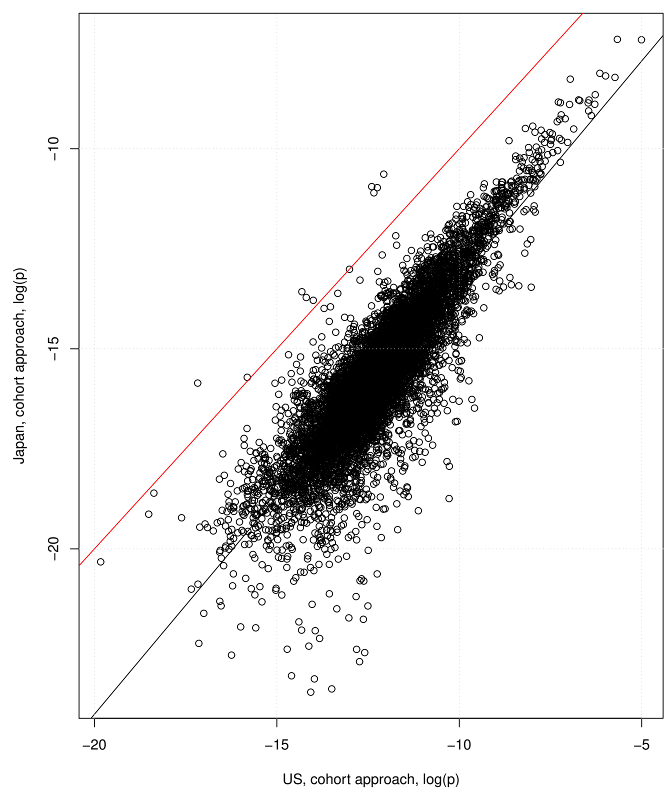
additional check, I compare the estimated log-probabilities of spillovers for the USA to those of Japan. Figures (11a) and (11b) depict the estimates for the two countries. Visually, the heat-maps look similarly, though, for Japan absolute values of estimates are on average lower. Figure (11c) confirms both the high correlation between the estimates and the difference in the average values (red line is, again, a the 45-degree line, while the black one is a fitted line). The difference in absolute values might be attributed to differences in the procedures of patenting and citing across the two countries.



(a) Log-probabilities of spillovers, the USA.



(b) Log-probabilities of spillovers, Japan.



(c) Estimated log-probabilities of spillovers for the USA and Japan

Figure 11: Estimated log-probabilities of spillovers

In the model what matters for spillovers is the rate at which a unit of labor in sector q generates technologies for sector s, i.e. ϕp^{qs}. In the data this rate may differ across sectors both due to variation in the probabilities $\{p^{qs}\}$ and in intensity of idea-generating process across sectors, $\{\phi^q\}$. To address this issue I calibrate the vector of intensities $\{\phi^q\}$ (ϕ^q – a number of patents generated in sector q per 1 million hours of working time) and use it to normalize the matrix of spillovers. Namely, the correspondence between parameters in the model and the data is $\phi p^{qs} = \frac{\phi^q}{\max_{r\in\mathcal{S}}\{\phi^r\}}\hat{p}^{qs}$, where $\hat{p}^{qs}$ are the above described estimates of spillover probabilities.

The second set of parameters that is required for the optimal policy exercise are the expenditure shares, $\{\alpha^q\}_{q\in\mathcal{S}}$. I calibrate these shares using the BLS input-output tables:

$$\alpha^s = \frac{X_i^s}{\sum_{q\in\mathcal{S}} X_i^q}, \tag{37}$$

where X_i^s stands for total expenditures of country i for output of sector s. For calibrating the distribution of technologies I use the estimate of $\theta = 8$ which is in line with the estimates suggested in Eaton and Kortum [2002]. Finally, for computing the optimal policy in the open economy one needs the vector of labor force distribution across countries, $\{L_i\}_{i\in\mathcal{N}}$. I calibrate the latter using the data on economically active population provided by International Labour Organization (ILO). A more detailed description of the data will be provided in Appendix B.

5 Optimal policy.

In this section I describe the policy that maximizes country's welfare on a balanced growth path. The first exercise uses the calibrated model and computes the vector of optimal labor allocation for the autarky. For the ease of computation and interpretation I aggregate the data to 24 sectors. The first 23 sectors, results on which I report, correspond to those for which the IPC-NAICS concordances exist. The 24th sector is a composite of services for which there are no such concordances, hence, it is treated as a stagnant one. To solve the optimization problem (27) I use the first order conditions (29). Taxes that result in the optimal labor allocation are computed using Equation (25).

The main result of this exercise is presented in Table (1). The first three columns of Table (1) compare the free-market labor allocation in BGP (in which, according to Proposition 1, labor is allocated proportional to sector's share in expenditure, α^q) to the welfare-maximizing labor allocation of autarky (l^{q*}) and the actual labor allocation in the US in 1990-2006 (l^q). The column with $l^{q*} - \alpha^q$ shows the wedge between the free-market la-

Table 1: Actual (l_i^q), autarky-optimal BGP (l_i^{q*}), free market BGP (α^q) labor allocations and optimal taxes ($\tau^q - 1$).

Sector	α^q	l^{q*}	l^q	$l^{q*} - \alpha^q$	$\tau^q - 1$
Agriculture, forestry, fishing and hunting	3.9%	3.7%	8.4%	-7.2%	0.0%
Mining	8.6%	8.6%	2.3%	-0.5%	-6.6%
Utilities	5.4%	5.3%	2.3%	-0.7%	-6.3%
Construction	15.8%	14.4%	27.1%	-9.2%	2.0%
Food manufacturing	6.6%	6.3%	5.4%	-5.1%	-2.2%
Beverage and tobacco	2.0%	2.0%	0.7%	0.1%	-7.0%
Textile and leather products	3.2%	3.3%	4.4%	2.4%	-9.2%
Wood products	1.6%	1.4%	2.1%	-8.4%	1.1%
Paper products	2.3%	2.2%	2.2%	-3.5%	-3.6%
Printing and related support activities	1.4%	1.3%	2.8%	-9.4%	2.2%
Petroleum and coal products	6.2%	6.4%	0.5%	4.4%	-11.0%
Chemical manufacturing	7.6%	8.8%	3.6%	14.1%	-19.2%
Plastic and rubber products	2.6%	2.5%	3.1%	-3.3%	-3.8%
Nonmetallic mineral products	1.6%	1.7%	1.9%	5.6%	-12.1%
Primary metal	3.1%	3.0%	2.2%	-3.0%	-4.2%
Fabricated metal products	4.0%	3.8%	5.9%	-3.6%	-3.6%
Machinery	3.9%	4.0%	5.0%	2.7%	-9.4%
Computer and electronic products	5.2%	6.3%	5.9%	19.3%	-23.3%
Electrical equipment	1.8%	1.9%	2.0%	1.4%	-8.3%
Transportation equipment	9.7%	9.5%	7.3%	-2.0%	-5.1%
Furniture and related products	1.2%	1.2%	2.3%	-4.3%	-2.9%
Medical equipment and supplies	0.8%	0.9%	1.1%	15.1%	-20.1%
Other miscellaneous manufacturing	1.4%	1.4%	1.5%	-0.4%	-6.6%

bor allocation in BGP and the optimal one. Larger values in this column correspond to sectors that generate more widely applicable technologies than others. The core sectors are "Computer and electronic products" (+19.3%), "Medical equipment and supplies" (+15.1%), "Chemical manufacturing" (+14.1%). Among sectors that generate technologies for others at lowest rates are "Construction" (-9.2%), "Printing and related support activities" (-9.4%) and "Wood products" (-8.4%). The last column with $\tau^q - 1$ presents taxes that allow to attain the optimal labor allocations. Taxes on "Agriculture, forestry, fishing and hunting" are normalized to 1; negative taxes mean subsidies relative to the agricultural sector. The result in the last column supports the intuition that the core sectors (those with larger $l^{q*} - \alpha^q$) should be subsidized more intensively that those whose contribution to the overall productivity of the economy is low (sectors with low $l^{q*} - \alpha^q$ values).

The outlined exercise also allows me to consider the welfare implications of different labor allocations:

$$\log\left(\frac{U}{L}\right)^* - \log\left(\frac{U}{L}\right) = \sum_q \alpha^q \log\left(\frac{l^{q*}}{l^q}\right) + \sum_n \frac{\alpha^n}{\theta}\left(\frac{\sum_q p^{qn} l^{q*}}{\sum_q p^{qn} l^q}\right) \tag{38}$$

According to this formula, shifting the employment structure from the actual to the autarky-optimal one can raise productivity in the US in the BGP by 3.5% (the second summand). The whole increase in welfare is estimated at 15.5%. This number is large, yet, 12.5% out of it comes from the assumption that the labor allocation and production defines the consumption structure. The last assumption is true for the autarky, but not for an open economy, so the second exercise in this section is to find the optimal sector labor allocation for the US as an open economy. This exercise will be added in the next version of the paper. The result of it is expected to be in line with the above outlined intuition from a 2×2 example. Namely, conditional on absent industrial policy in the rest of the World, the US as an open economy will reallocate more labor to core sectors than in autarky, yet, using lower taxes/subsidies.

6 Conclusion.

In this paper I build a dynamic trade model with technological spillovers and show that under general conditions it is characterized by a unique balanced growth path. The model provides a framework for predicting the long-run consequences of trade and industrial policies. I derive the conditions that allow to identify the core sectors and design the optimal industrial policy in autarky and open economy. For quantifying the probabilities of spillovers I suggest and use two approaches consistent with the modelled mechanism of technology-generating process. I use the calibrated model for computing the optimal vector of labor allocation and show that in the balanced growth path such policy can provide a 3.5% increase in productivity of the whole economy.

There are several immediate extensions for all three parts of the paper – model, data and the optimal policy. The first extension to the model can be a possibility of international technological spillovers. In my intuition such spillovers will strengthen the centripetal forces of comparative advantage and will lead to a faster convergence towards the BGP. A simple example that supports this intuition is outlines at the end of Section 2. Presence of input-output linkages and positive trade costs are among other interesting theoretical extensions.

The calibration and optimal policy parts will benefit a lot from new concordance schemes that allow to quantify the rates of flows of ideas across all fields of economic activity, including services. Precise estimates of absolute levels of such rates would allow to talk about the speed of convergence and take the transition paths into account when designing the optimal policy. Finally, the strategic interaction between countries in the game of subsidizing the core sectors brings in the question of the existence of the Nash equilibrium in this game and its optimality for the global welfare.

Appendix A1

The first part of Appendix A1 proves Proposition 1 for a simple 2x2 case.

Proposition 1 considers the BGP, so all the equations below are considered for the BGP, thus, time variable t is omitted. First, let's show the **uniqueness** of the BGP under $p^{rs} > 0$, $p^{sr} > 0$. Combining the equations for BPG level of productivity parameters

$$\frac{T_i^r}{T_i^s} = \frac{p^{rr} + p^{sr}\frac{l_i^s}{l_i^r}}{p^{rs} + p^{ss}\frac{l_i^s}{l_i^r}} \text{ and } \frac{T_j^r}{T_j^s} = \frac{p^{rr} + p^{sr}\frac{l_j^s}{l_j^r}}{p^{rs} + p^{ss}\frac{l_j^s}{l_j^r}}$$

with labor allocation equilibrium condition

$$\frac{T_i^r}{T_i^s}\frac{l_i^s}{l_i^r} = \frac{T_j^r}{T_j^s}\frac{l_j^s}{l_j^r}$$

we obtain

$$\left(\frac{l_i^s}{l_i^r} - \frac{l_j^s}{l_j^r}\right)\left(p^{rr}p^{rs} + p^{sr}p^{rs}\left(\frac{l_i^s}{l_i^r} + \frac{l_j^s}{l_j^r}\right) + p^{sr}p^{ss}\frac{l_i^s}{l_i^r}\frac{l_j^s}{l_j^r}\right) = 0$$

which can be equal to zero under $p^{rs} > 0$, $p^{sr} > 0$ only if $\frac{l_i^s}{l_i^r} = \frac{l_j^s}{l_j^r}$ since the second multiplier is > 0 (case with $\frac{l_i^s}{l_i^r} = \frac{l_j^s}{l_j^r} = 0$ can be disregarded since it is possible only under $T_i^s = T_j^s = 0$ which in turn is not an equilibrium outcome – under $p^{rs} > 0$ some ideas from r will always be applicable to s, thus, in equilibrium $T_i^s > 0$, $T_j^s > 0$). From $\frac{l_i^s}{l_i^r} = \frac{l_j^s}{l_j^r}$ it follows that $\frac{T_i^r}{T_i^s} = \frac{T_j^r}{T_j^s}$ and, hence, $t^s \equiv \frac{T_j^s}{T_i^s} = \frac{T_j^r}{T_i^r} \equiv t^r$. Finally, plugging this result into the equations for labor allocation $\frac{L_i^r}{L_i^s} = \frac{\alpha^r}{\alpha^s}\frac{1+\frac{T_j^s}{T_i^s}\left(\frac{w_j}{w_i}\right)^{-\theta}}{1+\frac{T_j^r}{T_i^r}\left(\frac{w_j}{w_i}\right)^{-\theta}}$ and $\frac{L_j^r}{L_j^s} = \frac{\alpha^r}{\alpha^s}\frac{1+\frac{T_i^s}{T_j^s}\left(\frac{w_j}{w_i}\right)^{\theta}}{1+\frac{T_i^r}{T_j^r}\left(\frac{w_j}{w_i}\right)^{\theta}}$ we obtain $\frac{L_i^r}{L_i^s} = \frac{L_i^r}{L_i^s} = \frac{\alpha^r}{\alpha^s}$ and $t^s \equiv \frac{T_j^s}{T_i^s} = \frac{T_j^r}{T_i^r} \equiv t^r = \frac{L_j}{L_i}$. As it was mentioned in the main text, t^s and t^r fully describe the system in the BGP. Thus, we can conclude that the BGP is unique.

Multiplicity of equilibria under $p^{rs} = p^{sr} = 0$ can be demonstrated in the following way. From $w_i L_i^r = \alpha^r \pi_i^r (w_i L_i + w_j L_j)$ and BGP equation $\frac{T_i^r}{T_j^r} = \frac{p^{rr}L_i^r + p^{sr}L_i^s}{p^{rr}L_j^r + p^{sr}L_j^s}$ one obtains

$\frac{L_i^r}{L_j^r} = \frac{p^{rr}L_i^r + p^{sr}L_i^s}{p^{rr}L_j^r + p^{sr}L_j^s}\left(\frac{w_i}{w_j}\right)^{-\theta-1}$ and $\frac{L_i^s}{L_j^s} = \frac{p^{rs}L_i^r + p^{ss}L_i^s}{p^{rs}L_j^r + p^{ss}L_j^s}\left(\frac{w_i}{w_j}\right)^{-\theta-1}$. From the last two equations under $p^{rs} = p^{sr} = 0$ we can get $\frac{w_i}{w_j} = 1$ and using this result in the trade balance equation we obtain $\frac{L_i}{L_i+L_j} = \frac{\alpha^r}{1+\frac{T_j^r}{T_i^r}} + \frac{\alpha^s}{1+\frac{T_j^s}{T_i^s}}$. Thus, after having used all equations of the system we obtained the expression for all combinations of $t^s \equiv \frac{T_j^s}{T_i^s}$ and $t^r \equiv \frac{T_j^r}{T_i^r}$ that characterize the BGP. Figure (2c) depicts all such combinations of (t^r, t^s) as a downward-sloping curve. Notice, that since the relative wages are equal to 1 and prices are equalized across countries (because of zero trade costs) utility per capita is equalized across the countries and remains the same regardless in which BGP the economy ends. The last part follows from the fact that price level depends on the allocation of labor across sectors, $L_i^s + L_j^s$ and $L_i^r + L_j^r$, which in turn remains the same – $L_i^s + L_j^s = \alpha^s(L_i + L_j),\ L_i^r + L_j^r = \alpha^r(L_i + L_j)$.

To prove **stability** of the BGP let's return to Equation (17) which can be re-written in matrix form as

$$\underbrace{\begin{bmatrix} \dot{t}_i^s \\ \dot{t}_i^r \\ \dot{t}_j^s \\ \dot{t}_j^r \end{bmatrix}}_{\dot{X}} \underbrace{= \phi \begin{bmatrix} p^{ss} & p^{rs} & 0 & 0 \\ p^{sr} & p^{rr} & 0 & 0 \\ 0 & 0 & p^{ss} & p^{rs} \\ 0 & 0 & p^{sr} & p^{rr} \end{bmatrix} \begin{bmatrix} l_i^s \\ l_i^r \\ l_j^s \\ l_j^r \end{bmatrix} - g \begin{bmatrix} t_i^s \\ t_i^r \\ t_j^s \\ t_j^r \end{bmatrix}}_{F(X)}$$

$F(X)$ is a non-linear function, so to prove stability of the above mentioned unique BGP we need to show that all eigenvalues of Jacobian $DF(X)$ estimated at that BGP are negative. Jacobian $DF(X)$ can be written down as

$$DF(X) = \phi \begin{bmatrix} p^{ss} & p^{rs} & 0 & 0 \\ p^{sr} & p^{rr} & 0 & 0 \\ 0 & 0 & p^{ss} & p^{rs} \\ 0 & 0 & p^{sr} & p^{rr} \end{bmatrix} \begin{bmatrix} \frac{\partial l_i^s}{\partial t_i^s} & \frac{\partial l_i^s}{\partial t_i^r} & \frac{\partial l_i^s}{\partial t_j^s} & \frac{\partial l_i^s}{\partial t_j^r} \\ \frac{\partial l_i^r}{\partial t_i^s} & \frac{\partial l_i^r}{\partial t_i^r} & \frac{\partial l_i^r}{\partial t_j^s} & \frac{\partial l_i^r}{\partial t_j^r} \\ \frac{\partial l_j^s}{\partial t_i^s} & \frac{\partial l_j^s}{\partial t_i^r} & \frac{\partial l_j^s}{\partial t_j^s} & \frac{\partial l_j^s}{\partial t_j^r} \\ \frac{\partial l_j^r}{\partial t_i^s} & \frac{\partial l_j^r}{\partial t_i^r} & \frac{\partial l_j^r}{\partial t_j^s} & \frac{\partial l_j^r}{\partial t_j^r} \end{bmatrix} - \begin{bmatrix} g & 0 & 0 & 0 \\ 0 & g & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & g \end{bmatrix}$$

Eigenvalues of $DF(X)$ will be equal to eigenvalues of the first summand of $DF(X)$ minus g. Using implicit function differentiation for Equations (13) and (12) we obtain expressions for $\frac{\partial l_\cdot^\cdot}{\partial t_\cdot^\cdot}$: $\frac{\partial l_i^r}{\partial t_i^s} = -\frac{\partial l_i^r}{\partial t_j^s} = -\frac{\alpha^r\alpha^s l}{t_i^s(1+l)}$, $\frac{\partial l_i^r}{\partial t_i^r} = \frac{\partial l_i^r}{\partial t_j^r} = -\frac{\alpha^r\alpha^s l}{t_i^r(1+l)}$, $\frac{\partial l_j^r}{\partial t_i^s} = -\frac{\partial l_j^r}{\partial t_j^s} = \frac{\alpha^r\alpha^s l}{t_i^s(1+l)}$ and $\frac{\partial l_j^r}{\partial t_i^r} = \frac{\partial l_j^r}{\partial t_j^r} = -\frac{\alpha^r\alpha^s l}{t_i^r(1+l)}$, where $l \equiv \frac{L_j}{L_i}$ and $t_i^s = t_j^s = \frac{\phi(p^{rs}\alpha^r + p^{ss}\alpha^s)}{g}$, $t_i^r = t_j^r = \frac{\phi(p^{rr}\alpha^r + p^{sr}\alpha^s)}{g}$. Besides, $\frac{\partial l_i^s}{\partial t_a^b} = -\frac{\partial l_i^r}{\partial t_a^b}$ since $l_i^s + l_i^r = l_j^s + l_j^r = 1$.

The first summand of $DF(X)$ can be represented using Kronecker product (operator $\otimes$) as

$$\phi\alpha^s\alpha^r\left(\underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{A}\otimes\underbrace{\begin{bmatrix}p^{ss} & p^{rs}\\ p^{sr} & p^{rr}\end{bmatrix}}_{B}\right)\left(\underbrace{\begin{bmatrix}\frac{-l}{1+l} & \frac{l}{1+l}\\ \frac{1}{1+l} & \frac{-1}{1+l}\end{bmatrix}}_{C}\otimes\underbrace{\begin{bmatrix}\frac{-1}{t_i^s} & \frac{1}{t_i^r}\\ \frac{1}{t_i^s} & \frac{-1}{t_i^r}\end{bmatrix}}_{D}\right) = \phi\alpha^s\alpha^r(AC\otimes BD)$$

Now, we'll use the property of eigenvalues and Kronecker product – eigenvalues of the first summand of $DF(X)$ are equal to $\phi\alpha^s\alpha^r\mu_1\eta_1$, $\phi\alpha^s\alpha^r\mu_1\eta_2$, $\phi\alpha^s\alpha^r\mu_2\eta_1$ and $\phi\alpha^s\alpha^r\mu_2\eta_2$, where μ_1, μ_2 are eigenvalues of AC and η_1, η_2 are eigenvalues of BD. It is straightforward to see that $\mu_1 = 0$, $\mu_2 = -1$, $\eta_1 = 0$ and $\eta_2 = \frac{p^{rs}-p^{ss}}{t_i^s} + \frac{p^{sr}-p^{rr}}{t_i^r}$. Thus, three eigenvalues of $DF(X)$ are equal to $-g < 0$. The only eigenvalue of $DF(X)$ the sign of which we need to check is $\phi\alpha^s\alpha^r\mu_2\eta_2 - g = \phi\alpha^s\alpha^r\frac{g}{\phi}\left[\frac{p^{ss}-p^{rs}}{p^{ss}\alpha^s+p^{rs}\alpha^r} + \frac{p^{rr}-p^{sr}}{p^{rr}\alpha^r+p^{sr}\alpha^s}\right] - g = g\left(\alpha^s\alpha^r\left[\frac{p^{ss}-p^{rs}}{p^{ss}\alpha^s+p^{rs}\alpha^r} + \frac{p^{rr}-p^{sr}}{p^{rr}\alpha^r+p^{sr}\alpha^s}\right] - 1\right)$. It is relatively easy to see that this eigenvalue is also negative. Namely,

$$\alpha^s\alpha^r\frac{p^{ss}-p^{rs}}{p^{ss}\alpha^s+p^{rs}\alpha^r} \le \alpha^s\alpha^r\frac{p^{ss}}{p^{ss}\alpha^s+p^{rs}\alpha^r} \le \alpha^s\alpha^r\frac{p^{ss}}{p^{ss}\alpha^s} = \alpha^r$$

$$\alpha^s\alpha^r\frac{p^{rr}-p^{sr}}{p^{rr}\alpha^r+p^{sr}\alpha^r} \le \alpha^s\alpha^r\frac{p^{rr}}{p^{rr}\alpha^r+p^{sr}\alpha^r} \le \alpha^s\alpha^r\frac{p^{rr}}{p^{rr}\alpha^r} = \alpha^s,$$

thus,

$$\alpha^s\alpha^r\left[\frac{p^{ss}-p^{rs}}{p^{ss}\alpha^s+p^{rs}\alpha^r} + \frac{p^{rr}-p^{sr}}{p^{rr}\alpha^r+p^{sr}\alpha^s}\right] \le \alpha^s + \alpha^r = 1,$$

where the last inequality becomes strict if sectors are not isolated, i.e. if either $p^{rs} > 0$ or $p^{sr} > 0$. Thus, under no isolated sectors the above mentioned unique interior equilibrium is stable.

Generalization to the N-country S-sector case. Similarly to the 2×2 case we start with **uniqueness** – under no isolated clusters of sectors there exists a unique BGP with positive amounts of labor allocated to each sector in each country in which sector allocation of labor is identical across countries: $l_i^s = l_j^s > 0 \; \forall i \in \mathcal{N}, \; \forall s \in \mathcal{S}$, where $\mathcal{N} \equiv \{1, \ldots, N\}$ and $\mathcal{S} \equiv \{1, \ldots, S\}$. In what follows we'll consider only $\mathbb{R}_{++}^{N \times S}$, i.e. only cases in which $l_i^s > 0 \; \forall i, s$. Let's start with a system of equations that describes the sector labor allocation and the ratio of BGP productivities across sectors:

$$
\begin{aligned}
\frac{l_i^s}{l_i^r}\frac{T_i^r}{T_i^s} &= \frac{l_j^s}{l_j^r}\frac{T_j^r}{T_j^s} \; \forall i, j \in \mathcal{N}, \; \forall s, r \in \mathcal{S} \\
\sum_q l_i^q &= 1 \; \forall i \in \mathcal{N}, \; \forall q \in \mathcal{S} \\
\frac{T_i^r}{T_i^s} &= \frac{\sum_q p^{qr} l_i^q}{\sum_q p^{qs} l_i^q} \; \forall i \in \mathcal{N}, \; \forall q \in \mathcal{S}
\end{aligned}
$$

Plugging the last equation into the left hand side and right hand side parts of the first one, taking logs of it and combining with the log of the second equation we obtain the following vector-valued function:

$$
F = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_{S-1} \\ F_S \end{bmatrix} = \begin{bmatrix} \log l^1 - \log l^2 + \log \sum_q p^{q2} l^q - \log \sum_q p^{q1} l^q \\ \log l^2 - \log l^3 + \log \sum_q p^{q3} l^q - \log \sum_q p^{q2} l^q \\ \vdots \\ \log l^{S-1} - \log l^S + \log \sum_q p^{q\,S} l^q - \log \sum_q p^{q\,S-1} l^q \\ \log \sum_q l^q \end{bmatrix}
$$

For simplicity of the following steps we'll treat F as a vector-valued function of $\{\log l_i^q\}$ – since $l_i^s > 0 \; \forall i, s$ the function is well-defined and differentiable at each point of its domain[15]. In equilibrium F has the same value for each country: $F(\{\log l_i^q\}) = F(\{\log l_j^q\})$. If F is

[15]Function F has the same expression for each country i, thus, country subscript i can be omitted

an injective function then $F(\{\log l_i^q\}) = F(\{\log l_j^q\})$ implies that the equilibrium vectors $\{l_i^q\}$ should be equalized across countries[16]. Thus, showing the conditions under which F is injective we show the conditions under which the equilibrium has $l_i^q = l_j^q \ \forall i, q$.

To show injectivity of F we use the sufficient condition of injectivity stated in Coomes [1989], namely, that a differentiable function $F: \ G \to R^M$ on an open convex subset of $G \in R^M$ is injective if a convex hull of $\{\nabla F(x) : x \in G\}$ contains only non-singular matrices. Using for brevity new notation $\bar{t^s} = \sum_q p^{qs} l^q$ the Jacobian ∇F (derivatives w.r.t $\log l^q$) can be written down as

$$\nabla F = \begin{bmatrix} 1 + \frac{p^{12}l^1}{\bar{t^2}} - \frac{p^{11}l^1}{\bar{t^1}} & -1 + \frac{p^{22}l^2}{\bar{t^2}} - \frac{p^{21}l^2}{\bar{t^1}} & \cdots & \frac{p^{S2}l^S}{\bar{t^2}} - \frac{p^{S1}l^S}{\bar{t^1}} \\ \frac{p^{13}l^1}{\bar{t^3}} - \frac{p^{12}l^1}{\bar{t^2}} & 1 + \frac{p^{23}l^2}{\bar{t^3}} - \frac{p^{22}l^2}{\bar{t^2}} & \cdots & \frac{p^{S3}l^S}{\bar{t^3}} - \frac{p^{S2}l^S}{\bar{t^2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{\bar{t^S}} - \frac{p^{1S-1}l^1}{\bar{t^{S-1}}} & \frac{p^{2S}l^2}{\bar{t^S}} - \frac{p^{2S-1}l^2}{\bar{t^{S-1}}} & \cdots & -1 + \frac{p^{SS}l^S}{\bar{t^S}} - \frac{p^{S\,S-1}l^S}{\bar{t^{S-1}}} \\ l^1 & l^2 & \cdots & l^S \end{bmatrix},$$

where the condition $\sum_q l_i^q = 1$ was used for simplifying the expression in the last row. Considering the value of Jacobian at two different points $\{\log l_i^q\}_{i,q}$ and $\{\log l_i^{q'}\}_{i,q}$ and taking any value $z = z \in [0;1], \ z' \equiv (1-z)$ we can write down a convex combination of Jacobians at these two arbitrary points as $z\nabla F + z'\nabla F'$. Next, we'll describe the applied matrix operations with mentioning if the operations can be applied to the corresponding convex combination and if any claim about ∇F is valid for the convex combination as well. First, notice that sum of elements by rows in ∇F is equal to zero in each row except the last one in which it is 1. Thus, by adding all columns to the last one and using Laplace expansion we can claim that determinant of the matrix obtained from ∇F by deleting the last row and the last column is the same as the determinant of ∇F. The same is true for the combination of ∇F and $\nabla F'$. The next operation that allows us to remove -1 from the second diagonal

[16] Here I use the property of injective functions that if $f \circ g$ is injective then g is injective, so if F is injective the the original system (before using logs) is also injective.

of the obtained $(S-1)\times(S-1)$ matrix is the addition of rows: $(S-1)$th to $(S-2)$th, the resulting $(S-2)$th – to $(S-3)$th and so on. The resulting matrix J will again have the same determinant as ∇F. The same holds for $z\nabla F + z'\nabla F'$. J:

$$\underset{(S-1)\times(S-1)}{J} = \begin{bmatrix} 1+\frac{p^{1S}l^1}{t^S}-\frac{p^{11}l^1}{t^1} & \frac{p^{2S}l^2}{t^S}-\frac{p^{21}l^2}{t^1} & \dots & \frac{p^{S-1,S}l^{S-1}}{t^S}-\frac{p^{S-1,1}l^{S-1}}{t^1} \\ \frac{p^{1S}l^1}{t^S}-\frac{p^{12}l^1}{t^2} & 1+\frac{p^{2S}l^2}{t^S}-\frac{p^{22}l^2}{t^2} & \dots & \frac{p^{S-1,S}l^{S-1}}{t^S}-\frac{p^{S-1,2}l^{S-1}}{t^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{t^S}-\frac{p^{1S-1}l^1}{t^{S-1}} & \frac{p^{2S}l^2}{t^S}-\frac{p^{2S-1}l^2}{t^{S-1}} & \dots & 1+\frac{p^{S-1,S}l^{S-1}}{t^S}-\frac{p^{S-1,S-1}l^{S-1}}{t^{S-1}} \end{bmatrix}$$

For the convex combination $z\nabla F + z'\nabla F'$ each entry $\frac{p^{qS}l^q}{t^S}-\frac{p^{qi}l^q}{t^i}$ in J should be replaced with a corresponding convex combination $z\left(\frac{p^{qS}l^q}{t^S}-\frac{p^{qi}l^q}{t^i}\right)+z'\left(\frac{p^{qS}l^{q\prime}}{t^{S\prime}}-\frac{p^{qi}l^{q\prime}}{t^{i\prime}}\right)$. Next, augment J (and it's convex combination counterpart) to a new matrix K of size $S\times S$ by attaching one column from the right and one row from the bottom so that the attached row and column comply with the general pattern of entries in J. Namely,

$$\underset{S\times S}{K} = \begin{bmatrix} 1+\frac{p^{1S}l^1}{t^S}-\frac{p^{11}l^1}{t^1} & \frac{p^{2S}l^2}{t^S}-\frac{p^{21}l^2}{t^1} & \dots & \frac{p^{S,S}l^{S}}{t^S}-\frac{p^{S,1}l^{S}}{t^1} \\ \frac{p^{1S}l^1}{t^S}-\frac{p^{12}l^1}{t^2} & 1+\frac{p^{2S}l^2}{t^S}-\frac{p^{22}l^2}{t^2} & \dots & \frac{p^{S,S}l^{S}}{t^S}-\frac{p^{S,2}l^{S}}{t^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{t^S}-\frac{p^{1S}l^1}{t^{S}} & \frac{p^{2S}l^2}{t^S}-\frac{p^{2S}l^2}{t^{S}} & \dots & 1+\frac{p^{S,S}l^{S}}{t^S}-\frac{p^{S,S}l^{S}}{t^{S}} \end{bmatrix}$$

Determinant of K will be the same as that of J: to see this notice that each element of the attached row is 0 except the last one which is 1, thus, using again Laplace expansion we can see that $det(K) = det(J)$. Next, we represent matrix K as a sum of three matrices:

$$\underset{S\times S}{K} = \underset{S\times S}{I} + \underbrace{\begin{bmatrix} \frac{p^{1S}l^1}{t^S} & \frac{p^{2S}l^2}{t^S} & \dots & \frac{p^{S,S}l^S}{t^S} \\ \frac{p^{1S}l^1}{t^S} & \frac{p^{2S}l^2}{t^S} & \dots & \frac{p^{S,S}l^S}{t^S} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{t^S} & \frac{p^{2S}l^2}{t^S} & \dots & \frac{p^{S,S}l^S}{t^S} \end{bmatrix}}_{\equiv A} - \underbrace{\begin{bmatrix} \frac{p^{11}l^1}{t^1} & \frac{p^{21}l^2}{t^1} & \dots & \frac{p^{S,1}l^S}{t^1} \\ \frac{p^{12}l^1}{t^2} & \frac{p^{22}l^2}{t^2} & \dots & \frac{p^{S,2}l^S}{t^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}l^1}{t^S} & \frac{p^{2S}l^2}{t^S} & \dots & \frac{p^{S,S}l^S}{t^S} \end{bmatrix}}_{\equiv B}.$$

Again, for the convex combination counterpart of K each entry $\frac{p^{ij}l^i}{\bar{t}^j}$ should be replaced with $z\left(\frac{p^{ij}l^i}{\bar{t}^j}\right)+z'\left(\frac{p^{ij}l^{i'}}{\bar{t}^{j'}}\right)$. Now we notice several regularities about matrices A and B. First, all rows of A are the same and equal to the last row of B. Second, rows in both A and B sum up to 1 and each entry of them $\in [0;1]$ so that both matrices can be considered stochastic transition matrices. The last observation allows us to use the well established fact that spectral radius of both matrices is equal to 1. Third, since $l_i^q \in (0;1)$ then every entry (q,r) of B can take a zero value only if the corresponding $p^{qr}=0$. Finally, eigenvalues of $A-B$ are equal to $\{0(=\lambda_{A,1}-\lambda_{B,1}),\ -\lambda_{B,2},\ -\lambda_{B,3},\dots,-\lambda_{B,S}\}$, where $\lambda_{B,1}=1,\ \lambda_{B,2},\ \dots,\ \lambda_{B,S}\in[-1;1]$ are eigenvalues of B and $\lambda_{A,1}=1$ $\lambda_{A,2}=\dots=\lambda_{A,S}=0$ – eigenvalues of A.

The last claim is least obvious, so here are the details. Matrix A has rank 1 so it can have at most 1 non-zero eigenvalue, besides $tr(A)=1$, so A has eigenvalues $\lambda_{A,1}=1$ of multiplicity 1 and $\lambda_{A,2}=\dots=\lambda_{A,S}=0$ of multiplicity $S-1$. For matrix B – as a stochastic transition matrix – we know from Perron-Frobenius theorem that all its eigenvalues belong to the interval $[-1;1]$ and at least one is equal to 1. Now, we derive eigenvalues of $A-B$. The first obvious eigenvalue is 0 with corresponding eigenvector $(1,1,\dots,1)^T$ (this follows immediately from equal row sums in both A and B). The other $S-1$ eigenvalues of $(A-B)$ are $\{-\lambda_{B,2},\ -\lambda_{B,3},\dots,-\lambda_{B,S}\}$. To see this let's consider the matrix equation $(A-B-(-\lambda_B I))X=(A+(I\lambda_B-B))X$, where λ_B is one of eigenvalues $\{\lambda_{B,2},\ \lambda_{B,3},\dots,\lambda_{B,S}\}$ of B matrix. To show that $-\lambda_B$ is an eigenvalue of $A-B$ it suffices to show that $\det(A+(I\lambda_B-B))=0$ or – which is equivalent – that $rank(A+(I\lambda_B-B))\leq S-1$. Here I use the result from Marsaglia [1964]: if C_1, C_2 are column spaces of matrices U and V and R_1 and R_2 are row spaces, $c=dim(C_1\cap C_2)$, $d=dim(R_1\cap R_2)$ then $rank(U+V)\leq rank(U)+rank(V)-max(c,d)$. Before proceeding, one key observation – if each row of a matrix sums up to the same number then vector $l\equiv(1,1,\dots,1)^T$ belongs to the column space of this matrix (indeed, adding up all columns and dividing the resultant vector by the row sum of the matrix we obtain vector l). Now back to $A+(I\lambda_B-B)$: 1) A has identical rows that sum up to the same number, so its column space contains vector l – in fact, the whole column space of A consists only of vectors collinear to l; 2) $rank(A) = 1$; 3) $I\lambda_B - B$ is singular, so $rank(I\lambda_B - B) \leq S-1$; 4) each row of $I\lambda_B - B$ sums up to $1-\lambda_B$, so the columns space of this matrix also contains l. From 1) and 4) it follows that $c = dim(C_1 \cap C_2) \geq 1$, thus $rank(A + I\lambda_B - B) \leq rank(A) + rank(I\lambda_B - B) - max(c, d) \leq S-1$ which proves that $\det(A + (I\lambda_B - B)) = 0$ and $\{-\lambda_{B,2},\ -\lambda_{B,3}, \dots, -\lambda_{B,S}\}$ are the other $S-1$ eigenvalues of $A - B$ except the initially mentioned 0. So, $\{0, -\lambda_{B,2},\ -\lambda_{B,3}, \dots, -\lambda_{B,S}\}$ are the eigenvalues of $A - B$.

Before proceeding to the next step, let's state another key observation that will be used shortly: if a stochastic transition matrix P describes an irreducible Markov chain then it has a unique stationary distribution vector $\underset{1\times S}{X}$ that corresponds to a unique eigenvalue equal to 1: $XP = X$ or $P^T X^T = X^T$. On the contrary, if the chain is not irreducible (has several closed sets of states) then there exists multiple stationary distributions and matrix P has eigenvalue 1 with multiplicity > 1. A detailed explanation of this statement can be found in Grimmett and Stirzaker [2001] on p.229. In our case matrix B is the analogy of the above mentioned Markov chain transition matrix P – and presence of closed sets of states in P corresponds to presence of isolated clusters of sectors in B. In essence it means that under no isolated clusters of sectors B has only one eigenvalue equal to 1, while if there are isolated clusters then $\lambda_B = 1$ has multiplicity > 1. Now, returning to matrix $K = I + (A - B)$. Under no isolated clusters of sectors all eigenvalues of $(A - B)$ $\{0, -\lambda_{B,2},\ -\lambda_{B,3}, \dots, -\lambda_{B,S}\}$ belong to $(-1; 1)$ ($\lambda_{B,1} = 1$ cancels out with $\lambda_{A,1} = 1$), hence, all eigenvalues of $K = I + A - B$ belong to $(0; 2)$ and $det(K) > 0$. On the other hand, if matrix of spillovers has isolated clusters then $\lambda_{B,1} = 1$ has multiplicity greater than 1, thus, at least one eigenvalue of $(A - B)$ – $\{0, -\lambda_{B,2},\ -\lambda_{B,3}, \dots, -\lambda_{B,S}\}$ – is equal to -1, which will turn into a 0-eigenvalue for K and, hence, $det(K) = 0$. Thus, absence of isolated clusters in the matrix of spillovers is a necessary and sufficient condition for non-singularity on ∇F.

Clearly, all the above mentioned arguments are applicable to a convex combination analogy of matrix K since the counterparts of matrices A and B have the same properties as A and B themselves (including the property that matrix B has zero entries only if the underlying matrix of spillovers $\{p^{rs}\}$ has corresponding entries equal to 0). Thus, non-singularity of ∇F at any point of $\mathbb{R}_+^{N\times S}$ is equivalent to non-singularity of $z\nabla F + z'\nabla F'$ between any two points in $\mathbb{R}_+^{N\times S}$ and, hence, injectivity of F. The opposite is also true – if ∇F is singular at each point of the domain of F then it means that at each point there exists a non-zero vector ΔX such that $\nabla F \Delta X = 0$ – so moving along such a sequence of vectors ΔX we obtain a sequence of different points characterized by the same value of function F, hence, F is non-injective. (ELABORATE the following argument... "Line integral" might be a good starting point! Say, we start at some point a and move along such vector ΔX_1, so that value of F remains the same – $\nabla F \Delta X_1 = 0$ – at a new point $a + \Delta X_1$ there exists another vector ΔX_2 such that $F(a + \Delta X_1 + \Delta X_2) = F(a + \Delta X_1) = F(a)$, yet $a + \Delta X_1 + \Delta X_2 \neq a$: the last follows from the fact that if $\Delta X_2 = -\Delta X_1$ if would mean that choosing the opposite direction for ΔX_2 – same as ΔX_1 – will move us further away from a, yet will keep F the same since if $\nabla F \Delta X_2 = 0$ so is $\nabla F(-\Delta X_2) = 0$.)

Summing up: if there are no isolated clusters of sectors then determinant of Jacobian of function F , ∇F, is positive at each point of $\mathbb{R}_+^{N\times S}$, so is the determinant of convex hull $z\nabla F + z'\nabla F'$, which means that F is injective on this subspace. Injectivity of F, in turn, implies that if there exists a BGP in terms of $\{l_i^s\}_{i\in\langle N\rangle, s\in\langle S\rangle}$ on $\mathbb{R}_+^{N\times S}$ then it is symmetric across countries by sectors: $l_i^s = l_j^s \ \forall i, j \in \langle N\rangle \ \forall s \in \langle S\rangle$. Translating the latter into the language of Economics: under no isolated clusters of sectors if there exists an equilibrium in which each country has non-zero productivity in each sector it is the equilibrium in which the same share of labor is allocated to each sector across countries.

Having derived this important characteristic of an interior equilibrium – $l_i^s = l_j^s \ \forall i, j, s$ – we can show now that there exists only one such equilibrium. First, ratio of productivities for a pair of countries i, j and any sector s is the same and equal to the ratio of country labor supply:

$$\frac{T_j^s}{T_i^s} = \frac{\phi L_j \sum_q p^{qs} l_j^q}{\phi L_i \sum_q p^{qs} l_i^q} = \{l_i^q = l_j^q\} = \frac{L_j}{L_i}.$$

From the last equality it follows immediately that shares of expenditures on coutry i's products is the same in each sector: $\pi_i^s = \frac{T_i^s (w_i)^{-\theta}}{\sum_j T_j^s (w_j)^{-\theta}} = \pi_i^r \; \forall r, s$. Considering ratio of sector labor demand we obtain that actual equilibrium labor allocation is $l_i^s = \alpha^s \; \forall i, s$: $\frac{w_i L_i^q}{w_i L_i^r} = \frac{l_i^q}{l_i^r} = \frac{\alpha^q \pi_i^q}{\alpha^r \pi_i^r} = \frac{\alpha^q}{\alpha^r}$. Clearly, there exists only one BGP along which labor in each country is allocated across sectors proportional to consumption shares α's. Finally, the considered interior equilibrium is characterized by equal wages across countries – from sector labor demand for the same sector and different countries:

$$\frac{L_i^q w_i}{L_j^q w_j} = \frac{\pi_i^q \sum_k L_k w_k}{\pi_j^q \sum_k L_k w_k} = \frac{L_i w_i}{L_j w_j} = \frac{T_i^q (w_i)^{-\theta}}{T_j^q (w_j)^{-\theta}} \Rightarrow \frac{w_i}{w_j} = \left(\frac{w_i}{w_j}\right)^{-\theta} \Rightarrow \frac{w_i}{w_j} = 1$$

Now we proceed to demonstrate **stability** of the above derived unique interior equilibrium. An easy way to demonstrate it is to refer to Figure (1). As it was proved above, the bounded curve $\phi_i^s(t)$ intersects $gt_i^s(t)$ in the region with $t_i^s > 0$ at only one point. On Figure (1) this case corresponds to the solid upward sloping curve $\phi_i^s(t)$ and intersection at point A, which should be a stable equilibrium. For the sake of rigorousness let's mention that Proposition 1 makes a statement only about the uniqueness of the interior equilibrium, yet, there may exist multiple boundary equilibria even under the conditions of Proposition 1 (no isolated clusters). Figure (1) admits a possibility of another unstable equilibrium on the boundary – if $\phi \min_r(p^{rs}) = 0$ (sector s doesn't receive any technologies from sector r), $t_i^s = 0$ (sector s starts with zero productivity) and all labor at the beginning is allocated to such sector r, then an upward sloping curve $\phi_i^s(t)$ intersects line $gt_i^s(t)$ both at 0 and at A. To elaborate this intuition on stability let's replicate the same argument that was used for the 2×2 case and show that all eigenvalues of Jacobian of $G : \dot{t}_i^s = G(t) = \phi \sum_q p^{qs} l_i^q - gt_i^s$, $t \equiv \{t_i^s\}_{\substack{i \in \langle N \rangle \\ s \in \langle S \rangle}}$ are negative at the considered interior equilibrium. The Jacobian can be written down in a matrix form as:

$$G(t) = \begin{bmatrix} G_1^1(t) \\ G_1^2(t) \\ \vdots \\ G_1^S(t) \\ G_2^1(t) \\ \vdots \\ G_S^S(t) \end{bmatrix} \Rightarrow \nabla G = \underset{NS\times NS}{\left[\left\{\frac{\partial G_i^s}{\partial t_j^r}\right\}\right]} = \phi \left[\underset{N\times N}{I} \otimes \underset{S\times S}{P^T}\right] \underset{NS\times NS}{\left[\left\{\frac{\partial l_i^q}{t_j^r}\right\}\right]} - g \underset{NS\times NS}{I},$$

where P^T is a transposed matrix of cross-sectoral spillovers (reminder: in P sectors in rows are donors and in columns – recipients of ideas). For each pair of countries (i, j) and sectors (s, r) it can be re-written as

$$\frac{\partial G_i^s}{\partial t_j^r} = \phi \sum_q p^{qs} \frac{\partial l_i^q}{\partial t_j^r} - g \frac{\partial t_i^s}{\partial t_j^r}.$$

The set of derivatives $\left\{\frac{\partial G_i^s}{\partial t_j^r}\right\}$ need to be calculated at the equilibrium. As a reminder, in equilibrium we have $\pi_i^q = \frac{L_i}{\bar{L}} \equiv l_i \ \forall q \in \langle S \rangle$, where $\bar{L} = \sum_j L_j$ is total population in all countries; $w_i = 1 \ \forall i \in \langle N \rangle$, $l_i^q = \frac{L_i^q}{L_i} = \alpha^q \ \forall i \in \langle N \rangle;\ q \in \langle S \rangle;\ t_i^s = \frac{\phi}{g} \sum_q p^{qs} \alpha^q = t^s \ \forall i \in \langle N \rangle$. The difficulty is that each l_i^s is a non-linear function of the whole vector t and can not be expressed explicitly.

To obtain the derivative of labor shares employed w.r.t. the level of technology – $\frac{\partial l_i^s}{\partial t_j^r}$ – we use the equations for sector labor demand.

$$F_i^s : \ l_i^s w_i \left[\sum_k t_k^s \frac{L_k}{L_i} (w_k)^{-\theta}\right] - \alpha^s t_i^s (w_i^{-\theta}) \left[\sum_k \frac{L_k}{L_i} w_k\right] = 0$$

and the implicit differentiation to obtain $\frac{\partial l_i^s}{\partial t_j^r} = -\frac{\frac{\partial F_i^s}{\partial t_j^r}}{\frac{\partial F_i^s}{\partial l_i^s}}$. While the expression for denominator can be obtained immediately as $\frac{\partial F_i^s}{\partial l_i^s} = \frac{t^s}{L_i}\bar{L}$, the expression for the numerator requires some additional steps since it also contains the derivatives of wages w.r.t. the level of technology, $\frac{\partial w_\cdot}{\partial t_j^r}$. In the considered version of the model wages are defined as a solution to the system of trade balance equations

$$B_i: \; L_i w_i - \sum_q \pi_i^q \alpha^q \left(\sum_k L_k w_k \right) = 0 \; \forall i \in \langle N \rangle.$$

Treating w's as variables and t's as parameters we differentiate each of these equations w.r.t. some t_j^r – $\sum_k \frac{\partial B_i}{\partial w_k} \frac{\partial w_k}{\partial t_j^r} + \frac{\partial B_i}{\partial t_j^r} = 0$ – we obtain the system which can be solved for $\frac{\partial w_\cdot}{\partial t_j^r}$:

$$\begin{bmatrix} \frac{\partial w_1}{\partial t_j^r} \\ \vdots \\ \frac{\partial w_{N-1}}{\partial t_j^r} \end{bmatrix} = - \begin{bmatrix} \frac{\partial B_1}{\partial w_1} & \cdots & \frac{\partial B_1}{\partial w_{N-1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial B_{N-1}}{\partial w_1} & \cdots & \frac{\partial B_{N-1}}{\partial w_{N-1}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial B_1}{\partial t_j^r} \\ \vdots \\ \frac{\partial B_{N-1}}{\partial t_j^r} \end{bmatrix}$$

Since the model has only $N-1$ independent wages, so we normalized $w_N \equiv 1$. Here we will skip some algebra, but mention that for finding the inverse of matrix with $\frac{\partial B_\cdot}{w_\cdot}$ we used the result from Miller [1981]: if matrices G and $G+E$ are non-singular and E is of rank one then $(G+E)^{-1} = G^{-1} - \frac{1}{1+g} G^{-1} E G^{-1}$, where $g = tr(EG^{-1})$. To summarize this part:

$$\frac{\partial w_i}{\partial t_j^r} = \begin{cases} 0, \;\; \text{if } i \neq j < N \text{ or } \; i = j = N \\ \frac{\alpha^r}{(1+\theta)t^r}, \;\; \text{if } i = j < N \\ \frac{-\alpha^r}{(1+\theta)t^r}, \;\; \text{if } i \neq j = N \end{cases},$$

where, again, w_N is normalized to 1, thus, remains constant.

Now we can return to the expressions for $\frac{\partial F_i^s}{\partial t_j^r}$ and $\frac{\partial l_i^s}{\partial t_j^r}$. Again, skipping some tedious, yet, uninvolved algebra, we'll report the derived expressions for $\frac{\partial l_i^s}{\partial t_j^r}$:

$$\frac{\partial l_i^s}{\partial t_j^r} = \begin{cases} \frac{\alpha^s(1-\alpha^s)}{t^r}(1-l_i), & \text{if } i=j,\ r=s \\ \frac{-\alpha^s\alpha^r}{t^r}(1-l_i), & \text{if } i=j,\ r\neq s \\ \frac{-\alpha^s(1-\alpha^s)}{t^r}l_j, & \text{if } i\neq j,\ r=s \\ \frac{\alpha^s\alpha^r}{t^r}l_j, & \text{if } i\neq j,\ r=s \end{cases},$$

Next, we can write down the matrix of partial derivatives of labor shares l_i^s w.r.t. the level of technology t_j^r using Kronecker product as

$$\left[\left\{\frac{\partial l_i^s}{\partial t_j^r}\right\}\right] = (\mathbb{1}_N l - I_N) \otimes \left((\alpha \mathbb{1}_S - I_S)\alpha t^{-1}\right),$$

where I_S is the identity matrix of dimensionality $S \times S$ and

$$\mathbb{1}_S = \underset{S\times S}{\begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix}},\ \alpha = \underset{S\times S}{\begin{bmatrix} \alpha^1 & 0 & \dots & 0 \\ 0 & \alpha^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha^S \end{bmatrix}},\ t = \underset{S\times S}{\begin{bmatrix} t^1 & 0 & \dots & 0 \\ 0 & t^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & t^S \end{bmatrix}},\ l = \underset{N\times N}{\begin{bmatrix} l_1 & 0 & \dots & 0 \\ 0 & l_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & l_N \end{bmatrix}}.$$

Finally, we can assemble the matrix for the Jacobian of the non-linear dynamic system:

$$\nabla G = g\left[\mathbb{1}_N l - I_N\right] \otimes \left[P^T(\alpha \mathbb{1}_S - I_S)\alpha\tau^{-1}\right] - gI_{NS},$$

where $\underset{S\times S}{\tau} = \frac{g}{\phi}t$. We need to show that all eigenvalues of ∇G are negative under the condition of no isolated clusters. Let's consider the first term of Kronecker product in the expression above. One can easily see that eigenvalues of $\mathbb{1}_N l$ – matrix all repeated rows that sum up to 1 – are 0 and 1 with multiplicity $N-1$ and 1 correspondingly. Thus, eigenvalues of $\mathbb{1}_N l - I_N$ are correspondingly -1 and 0 with multiplicity $N-1$ and 1. Now, to the second term – using simple matrix operations one can obtain the following expression for it $\left[P^T(\alpha\mathbb{1}_S - I_S)\alpha\tau^{-1}\right] =$

$$= \begin{bmatrix} \tau^1 & 0 & \dots & 0 \\ 0 & \tau^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \tau^S \end{bmatrix} \left[\underbrace{\begin{matrix} \alpha^1 & \alpha^2 & \dots & \alpha^S \\ \alpha^1 & \alpha^2 & \dots & \alpha^S \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^1 & \alpha^2 & \dots & \alpha^S \end{matrix}}_{\equiv M} - \underbrace{\begin{matrix} \frac{p^{11}\alpha^1}{\tau^1} & \frac{p^{21}\alpha^2}{\tau^1} & \dots & \frac{p^{S1}\alpha^S}{\tau^1} \\ \frac{p^{12}\alpha^1}{\tau^2} & \frac{p^{22}\alpha^2}{\tau^2} & \dots & \frac{p^{S2}\alpha^S}{\tau^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{p^{1S}\alpha^1}{\tau^S} & \frac{p^{2S}\alpha^2}{\tau^S} & \dots & \frac{p^{SS}\alpha^S}{\tau^S} \end{matrix}}_{\equiv N} \right] \begin{bmatrix} \frac{1}{\tau^1} & 0 & \dots & 0 \\ 0 & \frac{1}{\tau^2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{\tau^S} \end{bmatrix}$$

The last expression has a form $\tau(M-N)\tau^{-1}$, where τ is an invertible matrix, thus, $\tau(M-N)\tau^{-1}$ is similar to $(M-N)$ and has same eigenvalues as $(M-N)$. Now if we recall that $\tau^s = \sum_q p^{qs}\alpha^q$ we can see that $(M-N)$ has exactly same properties as matrix $(A-B)$ in the part of the proof devoted to uniqueness: M as well as A consists of repeated rows that sum up to 1 while matrix N sums up to 1 by rows, yet, has different rows and can be treated as a stochastic transition matrix. In exactly the same manner as for A – elements of N can be zero only it the corresponding elements of P-matrix are zero. So, in the same way as for $A-B$ above, we can claim that eigenvalues of $M-N$ are $\{0, -\lambda_{N,1}, \dots, -\lambda_{N,S-1}\}$. If matrix P is characterized by absent isolated clusters, all eigenvalues of $M-N$ (and of $\left[P^T(\alpha\mathbb{1}_S - I_S)\alpha\tau^{-1}\right]$) are in $(-1,1)$ interval (the eigenvalue equal to 1 that corresponds to the only stationary distribution of Markov chain represented by N is canceled out with eigenvalue equal to 1 of M). Same is true for the eigenvalues of $[\mathbb{1}_N l - I_N] \otimes \left[P^T(\alpha\mathbb{1}_S - I_S)\alpha\tau^{-1}\right]$ (since those are cross products of eigenvalues of $M-N$ and eigenvalue of $[\mathbb{1}_N l - I_N]$ which are $\{0,-1\}$) and, hence, eigenvalues of ∇G are in $(-2g; 0)$ interval – strictly negative – so, the considered internal equilibrium is locally stable! On the contrary, if P has isolated clusters then N has eigenvalue 1 of multiplicity larger than 1, thus, at least one of eigenvalues of $M-N$ is equal to -1 and at least one of eigenvalues of ∇G is equal to zero – in this case stability of equilibrium can not be guaranteed.

Appendix A2

This appendix outlines a proof of Proposition 1 for a 2×2 economy with trade costs and a diagonal matrix of spillovers, i.e. under intra-sector but not inter-sector spillovers: $p^{AB} = 0$ if $A \neq B$ and $p^{AB} > 0$ if $A = B$. In this case it is easy to see that $\frac{T_i^A}{T_i^B} = \frac{p^{AA} L_i^A}{p^{BB} L_i^B}$ and, hence, $\frac{L_i^A T_i^B}{T_i^A L_i^B} = \frac{L_j^A T_j^B}{T_j^A L_j^B}$. Plugging the expressions for labor demand $w_i L_i^A = \alpha^A \sum_k \pi_{ik}^A L_k w_k$ (and similar for sector B and country j) into this ratio and substituting the the corresponding expressions for π's, one obtains

$$\frac{L_i w_i \left(T_i^A (dw_i)^{-\theta} + T_j^A (w_j)^{-\theta}\right) + L_j w_j \left(T_i^A (w_i)^{-\theta} + T_j^A (dw_j)^{-\theta}\right) d^{-\theta}}{L_i w_i \left(T_i^B (dw_i)^{-\theta} + T_j^B (w_j)^{-\theta}\right) + L_j w_j \left(T_i^B (w_i)^{-\theta} + T_j^B (dw_j)^{-\theta}\right) d^{-\theta}} = \frac{L_i w_i \left(T_i^A (dw_i)^{-\theta} + T_j^A (w_j)^{-\theta}\right) d^{-\theta} + L_j w_j \left(T_i^A (w_i)^{-\theta} + T_j^A (dw_j)^{-\theta}\right)}{L_i w_i \left(T_i^B (dw_i)^{-\theta} + T_j^B (w_j)^{-\theta}\right) d^{-\theta} + L_j w_j \left(T_i^B (w_i)^{-\theta} + T_j^B (dw_j)^{-\theta}\right)}$$

from where, taking into account $d > 1$, one can derive $T_i^A T_j^B (w_i w_j)^{-\theta} = T_i^B T_j^A (w_i w_j)^{-\theta}$ and, as a result, $\frac{T_i^A}{T_j^A} = \frac{T_i^B}{T_j^B}$. Combining the last expression with the above mentioned $\frac{L_i^A T_i^B}{T_i^A L_i^B} = \frac{L_j^A T_j^B}{T_j^A L_j^B}$ one obtains $\frac{L_i^A}{L_i^B} = \frac{L_j^A}{L_j^B}$ which means that the interior BGP (with positive output in each country-sector) is characterized by the same sectoral labor allocation within each country. From $\frac{T_i^A}{T_j^A} = \frac{T_i^B}{T_j^B}$ it follows that $\pi_{ii}^A = \pi_{ii}^B$, $\pi_{ij}^A = \pi_{ij}^B$ and same for country j from where $\frac{L_i^A}{l_i^B} = \frac{L_j^A}{l_j^B} = \frac{\alpha^A}{\alpha^B}$, hence, unique interior BGP.

Appendix B

This part will describe the details on the aggregation schemes and concordance matrices, approaches I used to deal with patents assigned to several sectors, and cutoffs that I used to remove patents with the number of citations below X-percentile for a given sector.

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