Coauthorship and the Measurement of Individual Productivity

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Abstract. We propose a new productivity index, *CoScore*, that disentangles individual from collaborative productivity. We apply it to formally account for coauthorship in quantifying individual scientific productivity. In contrast to existing measures, *CoScore* reflects the complete coauthorship network, not only the publication record of the author being ranked. It uses the varying levels of success of all coauthor partnerships to infer, simultaneously, an author's productivity and her credit on each of her papers. Crucially, the productivities of all authors are determined endogenously via the solution of a fixed point problem. We show that *CoScore* is well defined and provide axiomatic foundations for the associated rule used to allocate credit on coauthored papers.

Keywords: Coauthorship, Productivity indices, Ranking methods, Networks.

1. Introduction

Over the last 50 years, teams have come to dominate the production of scientific research, both in terms of quantity and impact (Wuchty et al., 2007). The pattern is particularly striking in science and engineering, where more than eighty percent of the papers are now coauthored, but is also significant in the social sciences. In economics, the average number of authors per article has increased from 1.3 to 2.2 (Card and DellaVigna, 2013), while articles with more than three authors, very rare before the nineties, now represent over forty percent (Hamermesh, 2013). Research has become truly collective. Yet, scientists still have to be evaluated on an *individual* basis for important decisions such as hiring, tenure or funding. It is therefore

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essential to properly account for coauthorship in quantifying individual scientific productivity, disentangling individual from collaborative productivity. Unfortunately, existing indices either ignore coauthorship, assigning full credit for a joint paper to each of its coauthors (Garfield, 1972; Hirsch, 2005; Lehmann et al., 2006; Perry and Reny, 2016), or only correct for it ad hoc, dividing credit equally among coauthors (Marchant, 2009).

The biases of these approaches are well known. Assigning full credit to all coauthors, artificially inflates their publication record by overrating the value of coauthored papers (Price, 1981). This favors researchers with multiple collaborators or belonging to large teams and gives perverse incentives for artificial coauthorship. Dividing credit uniformly corrects this bias but dilutes the credit due to the intellectual leaders of a paper, ignoring that authors' contributions may differ substantially (Hirsch, 2007; Hagen, 2008; Shen and Barabási, 2014). The same concern applies to the various proposals to allocate credit when authors are listed by order of importance or by other field-specific conventions (Hagen, 2008; Stallings et al., 2013).

The deeply collaborative nature of modern research means that individual productivity cannot be accurately quantified without assigning credit for joint research. At the same time, credit cannot be allocated without evaluating the productive input of the coauthors involved on a paper. That is, the twin problems of measuring individual productivity and allocating credit for joint research can only be solved jointly.

We propose a new index of individual productivity, *CoScore*, capturing the relationship between credit allocation and individual productivity. A researcher's score is defined as her average yearly productivity, the sum of her credit on all of her papers divided by her academic age. In turn, the credit for each paper is distributed proportionally to a fixed power of its coauthors' scores. Crucially, the scores of all authors are determined endogenously and simultaneously as the solution of a fixed point problem. *CoScore* favors researchers who have proved their ability independently of their coauthors or network, by successfully publishing in multiple groups or individually. Conversely, *CoScore* is detrimental to authors who have not been able to produce outside a given team, for instance, only in collaboration with a PhD adviser.

Formally, an academic database consists of a collection of papers C and a group of authors N. Each paper p is described by its group of coauthors $S(p)$ and a cardinal measure of scientific worth $w(p)$. The worth of a paper can be thought of as the number of citations or any other measure of impact comparable across papers. Each author i is described by her list of papers C_i and her academic age a_i. For any such database, the ***CoScore*** $s \in \mathbb{R}_+^N$ is such that, for each author i,

$$s_i = \frac{1}{a_i} \sum_{p \in C_i} w(p) \frac{{s_i}^{\alpha}}{\sum\limits_{j \in S(p)} {s_j}^{\alpha}} \tag{1}$$

where $\alpha \in [0, 1]$ is a fixed parameter that determines how much credit should be allocated to more productive authors. *CoScore* is well defined because the system of equations (1) has a unique solution. The existence of a solution follows from Brouwer's fixed point theorem; uniqueness comes from the fact that a solution of (1) maximizes a strictly concave function over an open and convex set, an optimization problem admitting at most one solution (see Theorem 3 in the appendix for a proof).[1] When α equals 1, credit is allocated proportionally to each coauthor's endogenous productivity score. We call the corresponding index the proportional *CoScore*. As α falls, the credit allocation becomes more egalitarian. At the extreme, when α equals 0, credit on joint papers is allocated equally. We call the corresponding index the egalitarian score.

When α is positive, an author's *CoScore* reflects the structure of the entire coauthorship network and the research records of possibly all scientists in the database.[2] The central insight is that allocating credit on a given paper requires assessing the relative productivity of all of its authors. Therefore, the credit assigned to a scientist on a paper depends on the credit assigned to all of her coauthors on all of their papers, which is itself determined by their coauthors' credit on all their respective papers, and so forth. Hence, the scientist's score and her assigned credit depends on the *identity* of her coauthors. This is in sharp contrast to existing indices that can be computed for each scientist in isolation and can only control for coauthorship anonymously, based solely on the number of coauthors in a publication (Marchant, 2009) or, if available, their position in the author order (Sekercioglu, 2008).

[1]In order for *CoScore* to be well defined, we assume throughout that the aggregate solo contribution of each author, however small, is positive. In practice, this amounts to assigning a single individual citation to each author in the database.

[2]Here, we take the coauthorship network to be the weighted hypergraph where nodes represent different scientists and hyperedges represent partnerships of varying weights (the number of citations). This is in contrast with Newman (2001, 2004), where the coauthorship network is defined as the undirected graph where nodes represent different scientists and two nodes are connected whenever the corresponding scientists have coauthored.

The endogenous formulation of *CoScore* as a fixed point and its intensive use of the coauthorship network are reminiscent of various measures of network centrality used to rigorously quantify socioeconomic phenomena such as reputation, importance, influence, popularity, and the quality of scholarly papers.[3] Most closely, eigenvector centrality (Bonacich, 1972) defines the centrality of an individual to be proportional to the sum of the centralities of the individuals connected to her. Such measures have proved extremely useful in many different contexts, as shown in particular with the PageRank algorithm used by Google to rank the relevance of web pages (Page et al., 1998) or the recursive methods to rank journals (Pinski and Narin, 1976; Palacios-Huerta and Volij, 2004). *CoScore* relies on similar insights but differs substantially from network centrality because it is not defined on a network but on a weighted hypergraph, the database of papers.

Although we formulate *CoScore* for the problem of measuring individual scientific productivity, our approach applies to any situation where teams produce an observable output but individual contributions cannot be objectively quantified. What is the contribution of a football player to the success of her team? What is the contribution of a manager to her firm's profits? What is the contribution of an actor to the box-office revenue of a movie? *CoScore* can be used to answer any of these questions, disentangling individual from collective productivity.

The productivity of economists We illustrate *CoScore* for a large database (all papers published in 33 major economics journals since 1970) using citations as a measure of scientific worth. As expected, we observe a sharp divide between the proportional *CoScore* and the egalitarian score: accounting for the identity of coauthors in a given paper, not only their number, yields significant differences both in rankings and scores. Although *CoScore* cannot be computed independently for each author, since it requires looking at the whole database of papers, its implementation is straightforward. Like PageRank, it can be applied to extremely large databases (possibly all scientific papers) in a systematic way, and provides a concrete alternative to measure individual productivity.

[3] For a survey on network centrality measures see Jackson (2008) and Newman (2010).

Axiomatic foundations We provide axiomatic foundations for *CoScore* in the benchmark case where credit is allocated proportionally to each coauthor's endogenous productivity. More precisely, we show that the associated rule to allocate credit on each paper, which we call the *CoScore* rule, is uniquely characterized by three logically independent axioms: *consistency*, *invariance to merging papers*, and *invariance to merging coauthors* (Theorem 1).

Consistency, requires the distribution of credit to remain invariant when an author is taken away from the database, and the value of every paper she has contributed to is reduced by the amount she was previously allocated. Similar properties, relating databases of different sizes, have been used in the literature on the measurement of intellectual influence (Palacios-Huerta and Volij, 2004), network centrality (Dequiedt and Zenou, 2014) and, more extensively, in resource allocation and game theory (Thomson, 2011).

The second property, invariance to merging papers, requires authors not to benefit from either merging or splitting papers with the same coauthors into papers of equal total value. The allocation of credit on joint papers should only depend on the *aggregate* value produced by each group of coauthors, not on the particular breakdown of that value in papers of different worth. The axiom is related to a property proposed in Perry and Reny (2016), depth relevance, which prevents authors from increasing their score by splitting a paper into two papers of equal total value.

The third property, invariance to merging coauthors, requires an author's credit on a paper to depend only on the strength of her own research record and the *aggregate* research records of her coauthors on the paper. Thus, an author's credit in a paper depends only on her own productivity and the aggregate productivity of all of her coauthors in that paper. What matters for how much credit an author receives is not how many coauthors the paper has but how strong these coauthors are as a whole.

We also provide a second characterization (Theorem 2) replacing invariance to merging coauthors by a simple *proportionality* condition: the credit assigned to each author should be proportional to her single authored contribution when the database only involves two authors, and both authors have exactly one paper on their own and one paper together.

Outline The remainder of this paper is organized as follows: Section 2 illustrates *CoScore* for a large database of papers in economics. Section 3 introduces the formal problem and provides two axiomatic characterizations for the rule used by the proportional *CoScore* to allocate credit on coauthored papers. Section 4 discusses various extensions to the *CoScore* indices. Section 5 concludes. All of the proofs are included in the appendix.

2. The productivity of economists

We illustrate *CoScore* for the field of Economics, where authors are usually listed alphabetically, and the specific role of each author in a paper is not specified. We use Thomson Reuters's Web of Science database to extract all published articles with at least one citation from 33 major journals in Economics over the period $1970 - 2015$ (see appendix for details). The database includes 73 732 articles, for a total of 29 226 different authors. As previously observed in other scientific databases (Seglen, 1992), the distributions are extremely skewed, with a small number of authors producing a large fraction of both papers and citations.[4] Authors have on average 4.32 papers, for a total of 168 citations, while the median author only has 2 papers and 22 citations. The share of coauthored papers increases steadily from only 20% in 1970 to almost 80% in 2015, with the average number of coauthors going from 1.25 to 2.31. Papers with more than two authors, a rare occurence before the nineties, represent almost 40% of all papers published in 2015. The increasing prominence of coauthorship also translates in the fraction of coauthored citations, which exhibits the same trend as the fraction of coauthored papers. Figure 1 shows the evolution of the fraction of coauthored papers and citations since 1970.

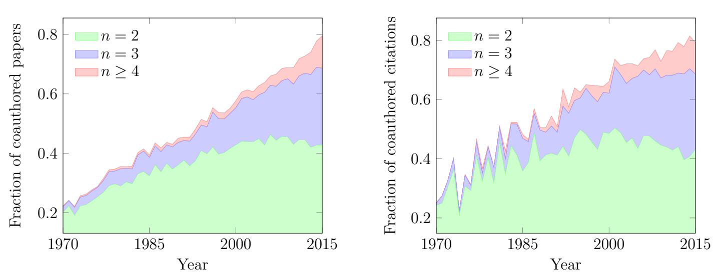

Figure 1. Fraction of coauthored papers (left) and citations (right) with 2, 3 and 4 or more authors from 1970 to 2015.

[4] Here, the database is biased towards more productive researchers because of the restriction to top journals. Since more productive researchers are less likely to publish in lower ranked journals, more of their papers are represented in the database.

We compute the proportional *CoScore* by repeatedly iterating the associated fixed point operator, starting from a vector of equal scores. The process converges quickly to the desired fixed point (see Appendix). As illustrated in Figure 2, we observe a sharp divide between the proportional *CoScore* and the egalitarian score: accounting for the *identity* of coauthors in a given paper, not only their number, yields significant differences both in rankings and scores. For authors with at least 5 papers in the database, scores differ on average by 27.4%, rankings by 7.6%. Compared to the egalitarian score, the proportional *CoScore* tends to favor authors who have proved their individual quality by having published either (i) with multiple groups of coauthors, (ii) with relatively weaker coauthors and/or (iii) single authored papers. Conversely, the proportional *CoScore* tends to be unfavorable to authors who have a limited number of stronger coauthors.

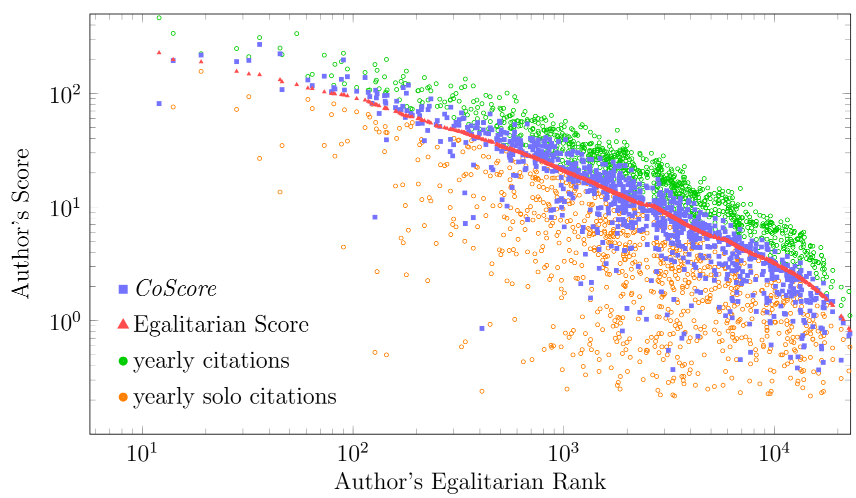

FIGURE 2. Distribution of *CoScore* (blue) and the Egalitarian Score (red) for a random sample of authors with at least 10 papers in the database. Both scores are bounded above by the average yearly number of citations (green) and below by the average yearly number of single-authored citations (yellow). Authors are displayed according to their egalitarian rank. Scales are logarithmic.

The changes are the most dramatic for authors who have a very high fraction of single-authored work, since they have by definition the most to lose and the most to win from credit allocation. R. Blundell, who has 74 papers for a total of 8 113 citations, goes from being ranked 90^{th} with the egalitarian score to being ranked 31^{st} with the proportional *CoScore*, while his score is almost doubled. Despite a very small fraction of single authored citations (0.1%), Blundell benefits from having worked with a large number of coauthors (66 coauthors), thus showing his ability to produce high quality work independently of the strength of his coauthors (some of them being very productive as well). Figure 4 illustrates the coauthorship network of two prominent economists.

In contrast to existing indices, *CoScore* cannot be computed independently for each author, since it requires looking at the whole database of papers. However, as illustrated here for the field of economics, its implementation is very straightforward. Like PageRank, it can be applied to extremely large databases (possibly all scientific papers) in a systematic way, and provides a concrete alternative to measure individual productivity.

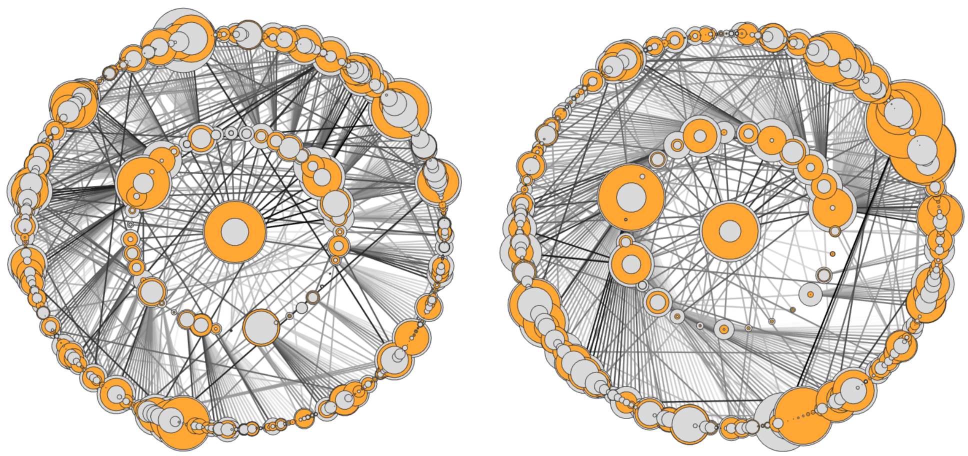

FIGURE 3. Coauthorship networks of J. Stiglitz (left) and J. Tirole (right). Lines represent collaborations between pairs of authors, darker lines reflecting a higher number of joint citations. Nodes represent authors whose degree of separation from Stigliltz and Tirole (at the center) respectively is at most 2. For each author (i) the size of the inner gray circle is proportional to the number of single-authored citations, (ii) the size of the outer gray circle is proportional to the total number of citations and (iii) the size of the orange ring is proportional to the number of jointly authored citations credited to the author by *CoScore*. The scales are specific to each network.

3. Axiomatic foundations

We focus in this section on the simplest case where authors do not differ in their academic age. We provide two axiomatic characterizations of the rule used by the proportional *CoScore* to allocate credit on joint papers.

3.1. **Model.** As before, a database is specified by a collection of papers and each paper is described by its group of coauthors and its scientific worth. We now introduce a general model to study variations in papers, their authors, and worth. The papers constituting a database are drawn from a countable index set $\mathbb{C}$. The collection of finite subsets of $\mathbb{C}$ is denoted by $\mathcal{C}$. Similarly, authors are drawn from a countable set $\mathbb{N}$ and the collection of finite subsets of $\mathbb{N}$ is denoted by $\mathcal{N}$. For each $N \in \mathcal{N}$, a database involving N is a triple $D \equiv (C, w, S)$ where

$$C \in \mathcal{C}, \quad w : C \to \mathbb{R}_+, \quad S : C \to 2^N.$$

Database D is thus described by the collection of papers C and, for each paper p in C, a worth $w(p)$ and a set of authors $S(p)$. For each $i \in N$, let C_i denote the papers in C involving i as an author or coauthor; for each $S \subseteq N$, let $C_S = \{p \in C | S(p) = S\}$ and note that $C_{\{i\}}$ is contained in C_i and that the inclusion is strict whenever i has coauthored. For each $N \in \mathcal{N}$, let $\mathcal{D}^N$ denote the domain of databases involving the authors in N.

A *score* is a systematic procedure which associates to every database $D \in \mathcal{D}^N$ a profile of individual productivity scores $s(D) \in \mathbb{R}^N_+$. We refer to the ith coordinate of $s(D)$, denoted by $s_i(D)$, as the score of author i when there is no room for confusion.

A problem closely related to assigning scores is that of allocating the credit of a given paper among its coauthors. A *rule* is a systematic procedure which distributes the worth of each paper among its coauthors. Formally, a rule m is a function such that, for each $D = (C, w, S) \in \mathcal{D}^N$,

$$m(D) \in Z(D) \equiv \bigtimes_{p \in C} \left\{ x \in \mathbb{R}^N_+ \middle| \sum_{i \in S(p)} x_i = w(p), \;\; x_{N \setminus S(p)} = 0_{N \setminus S(p)} \right\}.$$

For each $p \in C$ and each $i \in S(p)$, $m^p_i(D)$ denotes the credit attributed to individual i on paper p. A rule allocates credit for a paper among its coauthors so that the credit received by all adds up to the paper's worth, $\sum_{i \in S(p)} m^p_i(D) = w(p)$. Following Liebowitz (2013), we thus rule out the possibility that, for example, in a two-author paper, each author receives more than one half of the credit for the paper. Authors only get credit for papers they have contributed to.

Our main proposal, the *CoScore* rule, allocates credit on each paper proportionally to the individual productivity of each coauthor, as (endogenously) measured by the sum of the author's credits on all of her papers: for each $D = (C, w, S) \in \mathcal{D}^N$, each $p \in C$, and each $i \in S(p)$,

$$(2) \qquad m^p_i(D) = \frac{\displaystyle\sum_{q \in C_i} m^q_i(D)}{\displaystyle\sum_{j \in S(p)} \sum_{q \in C_j} m^q_j(D)} w(p).$$

The *CoScore rule* is closely related to the *CoScore* index since it can equivalently be defined as the rule which allocates the value of each paper proportionally to the proportional *CoScore* of each coauthor.[5] The *CoScore* rule is well defined because the proportional *CoScore* is itself well defined.

We could similarly define credit allocation rules for any of the other *CoScore* indices (1). For example, when $\alpha = 0$, we obtain the *egalitarian rule* that allocates the value of each paper equally among all its coauthors: $m_i^p(D) = \dfrac{w(p)}{|S(p)|}$.

Other alternative rules can also be considered. For example, the *solo rule* allocates the value of each paper proportionally to the value of the (aggregate) solo contribution: for each $D = (C, w, S) \in \mathcal{D}^N$, each $p \in C$, and each $i \in S(p)$,

$$m_i^p(D) = \frac{\sum_{q \in C_{\{i\}}} w(q)}{\sum_{j \in S(p)} \sum_{q \in C_{\{j\}}} w(q)} w(p).$$

When assigning credit on a paper, the solo rule makes inferences on authorship based only on the worth of the individual contributions of its coauthors.

Notation For each $N \in \mathcal{N}$ and each $D = (C, w, S) \in \mathcal{D}^N$, let $\mathcal{C}(D) \subseteq 2^C$ denote the subsets of papers which share exactly the same group of coauthors and let $\mathcal{S}(D) \subseteq 2^N$ denote the subsets of authors which share exactly the same list of coauthored papers:

$$\mathcal{C}(D) = \{C' \subseteq C |\ S(p) = S(q) \ \ \forall\, p, q \in C'\}$$
$$\mathcal{S}(D) = \{N' \subseteq N |\ C_i \setminus C_{\{i\}} = C_j \setminus C_{\{j\}} \ \ \forall\, i, j \in N'\}.$$

3.2. **Characterization.** We now introduce the three axioms characterizing the *CoScore* rule. The first axiom, consistency, concerns the response of the rule to the withdrawal of authors from the database. For example, starting from the database of all papers in economics and upon removing all economic historians, we would like to assess the distribution of credit among the remaining authors. Removing economic historians requires removing papers written *exclusively* by them from the database. Papers written by economic historians and other economists must remain in the database. However, the credit assigned to the economic historians involved in these papers needs to be deducted from these papers before we can reassess the allocation of credit among the

[5] As defined in system of equations (1) for $\alpha = 1$, assuming that $a_i = 1$ for each $i \in N$.

remaining economists. Consistency, requires the distribution of credit among other economists to remain identical after economic historians have been removed and the value of each paper reduced accordingly.

Thus, if the assignment of credit is considered desirable for a group of authors, then it should still be considered desirable if an author is taken away from the database and the worth of every paper is reduced by the amount she was previously assigned. Conversely, if an author were to be added to existing papers, her credit should be equal to the corresponding increase in the value of the paper, while the credit of the other authors would remain unchanged.

Formally, the database obtained from $D = (C, w, S) \in \mathcal{D}^N$ upon the departure of $k \in N$ starting from credit assignment $x \in Z(D)$, $c_k^x(D) = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{D}^{N \setminus \{k\}}$, is such that

i. $\tilde{C} = C \setminus C_{\{k\}}$,

ii. for each $p \in \tilde{C}$, $\tilde{w}(p) = w(p) - x_k^p$ and $\tilde{S}(p) = S(p) \setminus \{k\}$.

Consistency: For each $D \in \mathcal{D}^N$, $k \in N$ and $p \in \tilde{C}$, $m^p_{N \setminus \{k\}}(D) = m^p_{N \setminus \{k\}}(c_k^{m(D)}(D))$.

Thus, a rule m is consistent if the assignment of credit among the authors in $N \setminus \{k\}$ is the same in databases D and $c_k^{m(P)}(D)$. A related property is used in the axiomatic characterization of the "invariant" journal ranking of Palacios-Huerta and Volij (2004).[6] Other conceptually related conditions have also been central in the analysis of resource allocation and game theory.[7] Consistency is satisfied by the egalitarian and the *CoScore* rules but not by the solo rule.

The second axiom, invariance to merging papers, prevents authors from increasing their credit by either fragmenting or consolidating papers with the same group of coauthors. For example, no author should be given incentives to split one of her solo works, say an Econometrica article, into two Journal of Economic Theory articles of equal total worth. Perry and Reny (2016) propose[8] a related property, "depth relevance", which requires that an author's score does not increase upon splitting an article in her publication list into two articles with the same total number of citations.

[6]This ranking relies on the same ideas at the core of PageRank (Page et al., 1998), the procedure used by Google to rank web pages.

[7]See Thomson (2011) for a survey and Thomson (2012) for a discussion of the ethical content of consistency-type requirements.

[8]In the context of ranking authors based only on their citations list (Hirsch, 2005).

In contrast, invariance to merging papers prevents authors from both splitting and merging papers.

Formally, consider a database $D = (C, w, S) \in \mathcal{D}^N$ and a set of papers $C' \subseteq C$ with the same coauthors, $C' \in \mathcal{C}(D)$. The database obtained by merging C' into a single paper r, $c^{C' \to r}(P) = (\tilde{C}, \tilde{w}, \tilde{S})$, is such that

i. $\tilde{C} = [C \setminus C'] \cup \{r\}$,

ii. for each $p \in C \setminus C'$, $\tilde{S}(p) = S(p)$ and $\tilde{w}(p) = w(p)$,

iii. for each $p \in C'$, $\tilde{S}(r) = S(p)$ and $\tilde{w}(r) = \sum_{p \in C'} w(p)$.

Invariance to merging papers: For each $D = (C, w, S) \in \mathcal{D}^N$, $C' \in \mathcal{C}(D)$, and $r \notin C \setminus C'$, $m^r(c^{C' \to r}(D)) = \sum_{p \in C'} m^p(D)$ and $m^p(c^{C' \to r}(D)) = m^p(D)$ for each $p \in C \setminus C'$.

Thus, a rule m satisfies invariance to merging papers if the credit allocated to any author for the merged paper r in the new database $c^{C' \to r}(D)$ is equal to the the total credit that was previously allocated to her for all papers in C' while the allocation of credit on all other papers remains the same. Invariance to merging papers is satisfied by the egalitarian, the solo, and the *CoScore* rules.

The third axiom captures the idea that an author's credit on a paper should depend only on the strength of her own research record and the *aggregate* research records of her coauthors on the paper. More precisely, we consider the implication of merging the research records of a proper sub-group of coauthors in a paper. This means that each author in this group is replaced by a new single author in the database whose research portfolio is comprised of all the articles previously authored by the members of the sub-group. Our axiom, invariance to merging coauthors, requires the allocation of credit among all remaining authors to be the same. To avoid introducing a new coauthor in papers written only by some members of the sub-group, we only impose the property on coauthors who share the exact same list of coauthored articles.

Formally, to define invariance to merging coauthors, consider a database $D = (C, w, S) \in \mathcal{D}^N$ and a group of authors $I \subseteq N$. Suppose that each author in I has the same list of coauthored papers, $I \in \mathcal{S}(D)$. The database obtained by merging authors in I into author k, $c_{I \to k}(D) = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{D}^{N \setminus I \cup \{k\}}$, is such that each paper involving a coauthor in I is authored by k instead, everything else equal:

i. $\tilde{C} = C$,

ii. $\tilde{w} = w$, and

iii. for each $p \in \tilde{C}$, if $I \cap S(p) = \varnothing$, $\tilde{S}(p) = S(p)$, otherwise $\tilde{S}(p) = [S(p) \setminus I] \cup \{k\}$.

Invariance to merging coauthors: For each $D \in \mathcal{D}^N$, $I \in \mathcal{S}(D)$, $k \notin N \setminus I$, and $p \in C$, $m^p_{N\setminus I}(c_{I\to k}(D)) = m^p_{N\setminus I}(D)$.

Thus, a rule m satisfies invariance to merging coauthors if the allocation of credit for authors in $N \setminus I$ remains identical after authors in I have been merged into author k. As an implication, the credit allocated to author k must be equal to the the total credit that used to be allocated to all individuals in I. Invariance to merging coauthors is satisfied by the solo and the *CoScore* rules, but is violated by the egalitarian rule.

Theorem 1. *The only rule satisfying consistency, invariance to merging papers, and invariance to merging coauthors is the CoScore rule.*

The properties in Theorem 1 are logically independent. See appendix C.

Our last property, proportionality, requires credit to be allocated proportionally to each coauthor's single authored contribution when the database only involves two authors, and both authors have exactly one paper on their own and one paper together.

Proportionality: For each $\{i, j\} \in \mathcal{N}$ and each $D = (C, w, S) \in \mathcal{D}^{\{i,j\}}$ where $C = \{p_i, p_j, p\}$, $S(p_i) = \{i\}$, $S(p_j) = \{j\}$, and $S(p) = \{i, j\}$, $m^p_i(D)w(p_j) = m^p_j(D)w(p_i)$.

In contrast to the *CoScore* and solo rules, the egalitarian rule does not satisfy proportionality. The *CoScore* rule is uniquely characterized by consistency, invariance to merging papers and proportionality.

Theorem 2. *The only rule satisfying consistency, invariance to merging papers, and proportionality is the CoScore rule.*

The properties in Theorem 2 are logically independent. See appendix C.

4. Discussion

Non-alphabetical authorship We have formulated *CoScore* for academic fields where authors are listed alphabetically, such as economics or mathematics. However, in many other fields, the order of authors is meaningful, reflecting either the importance or the nature of their contribution to the paper. In the biological sciences, for example, most recognition typically goes to the the first and last authors, while the remaining authors' contributions can be harder to determine. Increasingly, journals also require the capacity of each coauthor (research design, research, data analysis, writing, etc) to be specified (Allen et al., 2014). Although conventions vary widely, even across sub-fields, this information is indicative of the contribution of each author and should play a role in the allocation of credit. *CoScore* can naturally be extended to account for ordered authorship and the capacity of each coauthor. A first alternative is allocating relatively more credit to authors who have a higher rank or capacity, by introducing paper-specific weights in the right hand side of the *CoScore* formula. A second alternative is separating the allocation of credit across groups of authors who have contributed to the paper in similar capacities.[9] Each paper is subdivided into several artificial papers, one for each group, where the number of citations assigned to each paper (or group) depends on the relative importance of the associated capacity.[10] *CoScore* can then be applied to the resulting enlarged database.

Topic-specific experience The issue of quantitatively allocating credit on a paper was recently considered by Shen and Barabási (2014) who suggest a simple algorithm based on citation patterns. Their assumption is that authors with more experience on the topic of the paper should receive more credit; their algorithm determines this topic-specific experience and allocates credit accordingly. A researcher, however, may often contribute more despite having relatively less topic-specific experience. This is particularly true for intellectual leaders, who usually have broad interests, and would therefore lose most of the credit for their coauthored work.[11] Such a procedure would thus favor specialized researchers, discouraging interdisciplinary work and atypical collaborations, which have been shown to play an important role in the production of scientific knowledge (Uzzi et al., 2013). In contrast, *CoScore* exploits the information contained in the whole database of papers to infer the quality of each scientist, independently of the considered topic. As in the case of ordered authorship, it can be extended to incorporate topic-specific experience, for example, by allocating relatively more credit to authors who have more experience on the topic of the paper, as

[9] It is common for a paper to state that several of its authors contributed in the same capacities and for its authors not to be ranked strictly, for example, having two or more lead authors. A paper's authors can thus be ordered into contribution classes, whereby authors in the highest class are considered the most significant contributors, authors in the second highest class are the second most significant, and so forth.

[10] The credit allocation across groups of authors could follow the proposals of Stallings et al. (2013) or Hagen (2008).

[11] Jean Tirole (Nobel 2014), for instance, has coauthored in topics ranging from game theory, macroeconomics, industrial organization, finance to psychology.

measured by Shen and Barabási's algorithm.

Beyond citations Following a long tradition (Garfield, 1972), we measure the quality of a scientific paper by its total number of citations. Citations provide an objective metric that is based on observable data and can be used systematically on a large scale. However, citations have been shown to suffer from various shortcomings: (i) citation patterns and intensities differ substantially across fields and even sub-fields, making inter field comparisons problematic (Radicchi et al., 2008; Ellison, 2013), (ii) citations take time to accumulate, underestimating the value of recent papers (Wang et al., 2013), and (iii) citation counts treat each citation equally, ignoring that citations originating from more influential papers reflect a higher value. In response, various alternative metrics have been proposed, respectively: (i) normalizing the citation count by the average per-paper citations in the field (sub-field) to insure comparability (Radicchi et al., 2008), (ii) measuring the quality of the journal where the paper was published (Palacios-Huerta and Volij, 2014) or discounting the citation count by the age of the paper, and (iii) giving more weight to citations originating from more influential papers, as measured by recursive network centrality measures (Pinski and Narin, 1976; Palacios-Huerta and Volij, 2004). *CoScore* can be computed with any of these alternative measures of paper quality, for which it provides complementary assessments of a scientist's productivity.

5. Conclusion

CoScore contributes to an array of indices that have thus far not been able to properly account for the growth of coauthorship. As all existing indices, *CoScore* summarizes a scientist's record into a one dimensional measure of scientific productivity, which inevitably entails some loss of information. Since authors' contributions are not observable, it must necessarily involve some degree of approximation. However, scientists still need to be evaluated and compared by universities and government agencies for hiring, tenure and funding; this should be done according to objective aggregation indices, reflecting the increasing level of coauthorship. As discussed in the previous section, *CoScore* is flexible and can be adjusted to capture additional information such as ordered authorship, topic specific experience, and alternative measures of paper quality. Although we focus here on the benchmark case where credit is allocated proportionally to each coauthor's productivity, *CoScore* may also be computed for other, possibly more appropriate, values of parameter α. An important objective for future research will thus be to determine which of the *CoScore* indices performs best, either in terms of predicting future productivity (Hirsch, 2007) or emulating job market outcomes (Ellison, 2013).[12] Another important avenue of research will be to investigate the application of *CoScore* to other forms of joint production (boards of directors, team sports, artistic collaborations, etc.), or to the valuation of different attributes in consumption products (conjoint analysis).

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[12]Note that such a calibration will require a very comprehensive dataset (possibly all papers in economics) as the score of each author depends on the entire database. Focusing on the complete cv of a subset of authors, as is done in Hirsch (2007) or Ellison (2013), would not be completely meaningful for investigating the performance of *CoScore*.

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Appendix A. *CoScore* is well defined

Theorem 3. *Let $\alpha \in [0,1]$, $a \in \mathbb{R}^N_{++}$, and suppose that each individual has at least one single authored contribution. Then, the system of equations* (1) *has a unique solution.*

Proof. We first consider the case where $\alpha > 0$. Note that by assumption, since for each $i \in N$ there exists $p \in C$ such that $S(p) = \{i\}$ and $w(p) > 0$, any solution of system (1) is necessarily in $\mathbb{R}^N_{++}$. Let $\phi : \mathbb{R}^N_{++} \to \mathbb{R}^N_{++}$ denote the function defined by setting, for each $x \in \mathbb{R}^N_{++}$ and each $i \in N$,

$$\phi_i(x) = \frac{1}{a_i} \sum_{p \in C_i} w(p) \frac{{x_i}^\alpha}{\displaystyle\sum_{j \in S(p)} {x_j}^\alpha}. \tag{3}$$

We now show that ϕ has a unique fixed point. Let $W = \sum_{p \in C} w(p)$ and, for each $i \in N$, let $W_i = \sum_{p \in C | S(p) = \{i\}} w(p)$. Thus, W denotes the total scientific worth in the database and W_i denotes the the total independent contribution of author i. Note that, for each $x \in \mathbb{R}^N_{++}$ and each $i \in N$, $a_i \phi_i(x) \geq W_i > 0$ and $\sum_{i \in N} a_i \phi_i(x) = W$. Thus, letting

$$K = \left\{ x \in \mathbb{R}^N \middle| \forall i \in N, \;\; W_i \leq a_i x_i \leq W \right\},$$

we can redefine ϕ as a function mapping K into K. Note that K is compact and convex. Furthermore, ϕ is continuous on K. Thus, by Brouwer's fixed point theorem, ϕ has at least one fixed point. We now prove that this fixed point is unique. Let x^* denote any one of such fixed points. By (3) and since the coordinates of x^* are positive, for each $i \in N$,

$$\sum_{p \in C_i} w(p) \frac{x_i^{*\alpha-1}}{\sum_{j \in S(p)} x_j^{*\alpha}} = a_i. \tag{4}$$

Let $f : \mathbb{R}^N_{++} \to \mathbb{R}$ denote the function defined by setting, for each $x \in \mathbb{R}^N_{++}$,

$$f(x) \equiv -\sum_{i \in N} a_i x_i + \frac{1}{\alpha} \sum_{p \in C} w(p) \ln \left(\sum_{j \in S(p)} x_j{}^{\alpha} \right).$$

We first show that f is strictly concave. Let $g : \mathbb{R}^N_{++} \to \mathbb{R}$ be defined, for each $x \in \mathbb{R}^N_{++}$, by $g(x) = \frac{1}{\alpha} \sum_{i \in N} W_i \ln (x_i{}^{\alpha})$. The function g is strictly concave since its Hessian is negative definite and, for each $i \in N$, $W_i > 0$. Let $D \subseteq C$ consist of all of the papers in C with at least two coauthors: $p \in D$ if and only if $|S(p)| \geq 2$. Let $h : \mathbb{R}^N_{++} \to \mathbb{R}$ be defined, for each $x \in \mathbb{R}^N_{++}$, by:

$$h(x) = -\sum_{i \in N} a_i x_i + \frac{1}{\alpha} \sum_{p \in D} w(p) \ln \left(\sum_{i \in S(p)} x_i{}^{\alpha} \right).$$

As the sum and composition of concave functions, h is concave. Note that $f = g + h$. Since the sum of a strictly concave function and a concave function is strictly concave, f is strictly concave.

By the strict concavity of f and the fact that $\mathbb{R}^N_{++}$ is open and convex, x^{**} is the unique maximizer of f over $\mathbb{R}^N_{++}$ if and only if $\nabla f(x^{**}) = 0$. That is, for each $i \in N$,

$$0 = \frac{\partial f}{\partial x_i}(x^{**}) = -a_i + \sum_{p \in C_i} w(p) \frac{x_i^{**\alpha-1}}{\sum_{j \in S(p)} x_j^{**\alpha}}.$$

Thus, by (4), $x^* = x^{**}$. Recall that x^* is an arbitrary fixed point of ϕ. By uniqueness of x^{**}, we conclude that the fixed point of ϕ is indeed unique.

It remains to consider the case where $\alpha = 0$. The system of equations (1) reduces to, for each $i \in N$,

$$s_i = \frac{1}{a_i} \sum_{p \in C_i} \frac{w(p)}{|S(p)|}.$$

The system thus (trivially) has a unique solution. ∎

Appendix B. Proof of Theorem 2

Lemma 1. *The CoScore rule satisfies Consistency, Invariance to merging papers, Invariance to merging coauthors, and proportionality.*

Proof. Let $\check{m}$ denote the *CoScore* rule and $\check{s}$ denote the score defined in (1) for $\alpha = 1$ and $a_i = 1$ for each $i \in N$. Let $N \in \mathcal{N}$ and $D = (C, w, S) \in \mathcal{D}^N$.

Consistency

Let $k \in N$, $x \equiv \check{m}(D)$, and $y \equiv \check{m}(c_k^x(D))$. Recall that $c_k^x(D) = (\tilde{C}, \tilde{w}, \tilde{S})$ is such that:

- $\tilde{C} = C \setminus C_{\{k\}}$
- For each $p \in \tilde{C}$ such that $p \in C_k$, $\tilde{w}(p) = w(p) - x_k^p$.
- For each $p \in \tilde{C}$ such that $p \notin C_k$, $\tilde{w}(p) = w(p)$.
- For each $p \in \tilde{C}$, $\tilde{S}(p) = S(p) \setminus \{k\}$.

Then, for each $p \in C_k \setminus C_{\{k\}}$, and each $i \in S(p) \setminus \{k\}$,

$$\begin{aligned} y_i^p &= [w(p) - x_k^p] \frac{\sum\limits_{q \in C_i} y_i^q}{\sum\limits_{j \in S(p)\setminus\{k\}} \sum\limits_{q \in C_j} y_j^q} \\ &= w(p) \left[1 - \frac{\sum\limits_{q \in C_k} x_k^q}{\sum\limits_{j \in S(p)} \sum\limits_{q \in C_j} x_j^q} \right] \frac{\sum\limits_{q \in C_i} y_i^q}{\sum\limits_{j \in S(p)\setminus\{k\}} \sum\limits_{q \in C_j} y_j^q} \qquad (5) \\ &= w(p) \left[\frac{\sum\limits_{j \in S(p)\setminus\{k\}} \sum\limits_{q \in C_j} x_j^q}{\sum\limits_{j \in S(p)} \sum\limits_{q \in C_j} x_j^q} \right] \frac{\sum\limits_{q \in C_i} y_i^q}{\sum\limits_{j \in S(p)\setminus\{k\}} \sum\limits_{q \in C_j} y_j^q} \end{aligned}$$

while for each $p \in C \setminus C_k$, and each $i \in S(p)$,

$$y_i^p = w(p) \frac{\sum_{q \in C_i} y_i^q}{\sum_{j \in S(p)} \sum_{q \in C_j} y_j^q}. \tag{6}$$

On the other hand, for each $p \in C_k \setminus C_{\{k\}}$ and each $i \in S(p) \setminus \{k\}$,

$$x_i^p = w(p) \left[\frac{\sum_{q \in C_i} x_i^q}{\sum_{j \in S(p)} \sum_{q \in C_j} x_j^q} \right] = w(p) \left[\frac{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q}{\sum_{j \in S(p)} \sum_{q \in C_j} x_j^q} \right] \frac{\sum_{q \in C_i} x_i^q}{\sum_{j \in S(p) \setminus \{k\}} \sum_{q \in C_j} x_j^q}. \tag{7}$$

while for each $p \in C \setminus C_k$, and each $i \in S(p)$,

$$x_i^p = w(p) \frac{\sum_{q \in C_i} x_i^q}{\sum_{j \in S(p)} \sum_{q \in C_j} x_j^q}. \tag{8}$$

Let $z \in Z(r_k^x(D))$ be such that, for each $p \in \tilde{C}$ and each $i \in N \setminus \{k\}$, $z_i^p = x_i^p$. Note that, by (7) and (8), z satisfies the system of equations in (5) and (6). By (2), (5) and (6) uniquely define y. Thus, $z = y$. Thus, for each $i \in N \setminus \{k\}$, $\check{m}_i(c_k^x(D)) = \check{m}_i(D)$, so $\check{m}$ satisfies consistency.

Invariance to merging papers

Let $C' \in \mathcal{C}(D)$ and recall that $c^{C' \to r}(P) = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{D}^N$ is such that:

- $\tilde{C} = [C \setminus C'] \cup \{r\}$ where $r \notin C \setminus C'$.
- $\tilde{w}(r) = \sum_{q \in D} w(q)$ and, for each $q \in C \setminus C'$, $\tilde{w}(q) = w(q)$.
- For each $q \in \tilde{C}$, $S(q) = \tilde{S}(q)$.

From (1), $\check{s}(c^{C' \to r}(D)) = \check{s}(D)$. By definition of $\check{m}$, for each $p \in C'$ and each $i \in S(p)$,

$$\sum_{q \in D} \check{m}_i^q(D) = \sum_{q \in D} w(q) \frac{\check{s}_i(D)}{\sum_{j \in S(p)} \check{s}_j(D)} = \tilde{w}(r) \frac{\check{s}_i(c^{C' \to r}(P))}{\sum_{j \in S(p)} \check{s}_j(c^{C' \to r}(D))} = \check{m}_i^r(c^{C' \to r}(D))$$

while for each $\hat{p} \in C \setminus C'$ and each $i \in S(\hat{p})$,

$$\check{m}_i^{\hat{p}}(D) = w(p) \frac{\check{s}_i(D)}{\sum_{j \in S(p)} \check{s}_j(D)} = \tilde{w}(\hat{p}) \frac{\check{s}_i(c^{C' \to r}(P))}{\sum_{j \in S(p)} \check{s}_j(c^{C' \to r}(D))} = \check{m}_i^{\hat{p}}(c^{C' \to r}(D)).$$

Thus, $\check{m}$ satisfies invariance to merging papers.

Invariance to merging coauthors

Let $I \in \mathcal{S}(D)$, $k \in \mathbb{N} \setminus N$, and recall that $c_{I\to k}(D) = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{D}^{[N\setminus I]\cup\{k\}}$ is such that:

- $\tilde{C} = C$,
- for each $p \in \tilde{C}$, $\tilde{w}(p) = w(p)$,
- for each $p \in \tilde{C}$, $\tilde{S}(p) = [S(p) \setminus I] \cup \{k\}$ if $I \cap S(p) \neq \varnothing$ and $\tilde{S}(p) = S(p)$ otherwise.

By definition of the *CoScore* rule, for each $p \in C$ and each $r \in S(p)$:

$$\check{m}^p_r(D) = w(p)\frac{\displaystyle\sum_{q\in C_r} \check{m}^q_r(D)}{\displaystyle\sum_{t\in S(p)}\sum_{q\in C_t} \check{m}^q_t(D)}. \tag{9}$$

Let $z \in Z(c_{I\to k}(D))$ be such that, for each $p \in \tilde{C}$,

$$z^p_k = \sum_{i\in I} \check{m}^p_i(D) \quad \text{and, for each } r \in \tilde{S}(p) \setminus \{k\}, \quad z^p_r = \check{m}^p_r(D).$$

Then, by (9), for each $p \in \tilde{C}$ and each $r \in \tilde{S}(p)$,

$$z^p_r = \tilde{w}(p)\frac{\displaystyle\sum_{q\in \tilde{C}_r} z^q_r}{\displaystyle\sum_{t\in \tilde{S}(p)}\sum_{q\in \tilde{C}_t} z^q_t}$$

which means z satisfies the system of equations (2) for database $c_{I\to k}(D)$. By (2), since *CoScore* is well defined, $\check{m}(c_{I\to k}(D)) = z$. Thus, $\check{m}$ satisfies invariance to merging coauthors.

Proportionality

Suppose that $N = \{i, j\}$, $C = \{p_i, p_j, p\}$, $S(p_i) = \{i\}$, $S(p_j) = \{j\}$, and $S(p) = \{i, j\}$. Then, it follows from the system of equations (2) that:

$$\frac{\check{m}^p_i(D)}{\check{m}^p_j(D)} = \frac{w(p_i) + \check{m}^p_i(D)}{w(p_j) + \check{m}^p_j(D)}$$

and thus $m^p_i(D)w(p_j) = m^p_j(D)w(p_i)$. Thus, $\check{m}$ satisfies proportionality. ∎

We introduce two additional properties that will be useful in the proof of Theorem 2. The first of these reflects the requirement that the indexing of papers is irrelevant to the credit assignment. Only the worth of papers and the coauthorship relations are taken into consideration:

Neutrality: For each $N \in \mathcal{N}$ and each pair $D = (C, w, S), D' = (C', w', S') \in \mathcal{D}^N$, if there is a bijection $\sigma : C \to C'$ such that, for each $p \in C$, $w'(\sigma(p)) = w(p)$ and $S'(\sigma(p)) = S(p)$, then, for each $p \in C$ and each $i \in S'(\sigma(p))$, $m_i^{\sigma(p)}(D') = m_i^p(D)$.

The second property specifies that the name of authors bears no influence on the credit assignment. Again, only the worth of papers and the coauthorship pattern are taken into consideration:

Anonymity: For each pair $N, N' \in \mathcal{N}$, each $D = (C, w, S) \in \mathcal{D}^N$, and each $D' = (C', w', S') \in \mathcal{D}^{N'}$, if $C = C'$, $w = w'$, and there is a bijection $\pi : N \to N'$ such that, for each $p \in C$, $S'(p) = \pi(S(p))$, then, for each $p \in C$ and each $i \in S(p)$, $m_i^p(D) = m_{\pi(i)}^p(D')$.

Remark 1. *Invariance to merging papers implies neutrality. Invariance to merging coauthors implies anonymity.*

Proof of Theorem 2. Let m denote a rule satisfying consistency, invariance to merging papers, and proportionality. Let $\check{m}$ denote the *CoScore* rule. By Lemma 1, it suffices to prove that $m = \check{m}$.

Step 1: *For each $N \in \mathcal{N}$ such that $|N| = 2$ and each $D \in \mathcal{D}^N$, $m(D) = \check{m}(D)$.*

Let $\{i, j\} \in \mathcal{N}$ and $D = (C, w, S) \in \mathcal{D}^N$. Let K denote the total value of the joint papers in D, $K = \sum_{p \in C_{\{i,j\}}} w(p)$. Let $\mathcal{Q} \subseteq \mathcal{D}^N$ consist of all databases $Q = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{D}^N$ such that:

- $\tilde{C}_{\{i\}} = C_{\{i\}}$ and $\tilde{C}_{\{j\}} = C_{\{j\}}$.
- for each $p \in \tilde{C}_{\{i\}} \cup \tilde{C}_{\{j\}}$, $\tilde{w}(p) = w(p)$.
- $\sum_{p \in \tilde{C}_{\{ij\}}} \tilde{w}(p) = K$.

Note that $D \in \mathcal{Q}$. The argument proceeds in three steps.

Step 1A: *For each $D' = (C', w', S'), D'' = (C'', w'', S'') \in \mathcal{Q}$, each $p' \in C'_{\{i,j\}}$, and each $p'' \in C''_{\{i,j\}}$, $w'(p') = w''(p'')$ implies $m^{p'}(D') = m^{p''}(D'')$.*

Let all of the notation be as in the statement of Step 1A. Without loss of generality, suppose that $|C'_{\{i,j\}}| \geq 2$ and $|C''_{\{i,j\}}| \geq 2$.[13] Let $\hat{D} = (\hat{C}, \hat{w}, \hat{S}) \in \mathcal{Q}$ be such that $\hat{C}_{\{i,j\}} = \{p', \hat{p}\}$, $\hat{w}(p) = w'(p')$, and $\hat{w}(\hat{p}) = K - w'(p')$. By invariance to merging papers, merging all the papers in $C'_{\{ij\}} \setminus \{p'\}$ into $\hat{p}$, $m^{p'}(D') = m^{p'}(\hat{D})$. Similarly, let $\tilde{D} = (\tilde{C}, \tilde{w}, \tilde{S}) \in \mathcal{Q}$ be such that $\tilde{C}_{\{ij\}} = \{p'', \tilde{p}\}$, $\tilde{w}(p) = w''(p'')$, and $\tilde{w}(\tilde{p}) = K - w''(p'')$. By invariance to merging papers, merging all the papers in $C''_{\{ij\}} \setminus \{p''\}$ into $\tilde{p}$, $m^{p''}(D'') = m^{p''}(\tilde{D})$. By neutrality, which is implied by invariance to merging papers (Remark 1), since the databases $\hat{D}$ and $\tilde{D}$ differ only in the indexing of papers, $m^{p'}(D') = m^{p'}(\hat{D}) = m^{p''}(\tilde{D}) = m^{p''}(D'')$.

Step 1B: *For each pair* $p, q \in C$, $\dfrac{m_i^p(D)}{m_i^q(D)} = \dfrac{w(p)}{w(q)}$.

Let $f : [0, K] \to \mathbb{R}$ be such that

$$f(x) = m_i^p(Q) \quad \forall x \in [0, K]$$

where $Q = (\mathring{C}, \mathring{w}, \mathring{S}) \in \mathcal{Q}$ and p is any paper in $\mathring{C}$ such that $\mathring{w}(p) = x$ and $\mathring{S}(p) = \{i, j\}$. By Step $1A$, f is well defined. Furthermore, by invariance to merging papers,

$$f(x + y) = f(x) + f(y) \quad \forall x, y \in [0, K] \text{ such that } x + y \leq K.$$

Thus, f satisfies the Cauchy functional equation and there is a constant c_K such that, for each $x \in [0, K]$, $f(x) = c_K x$ (see Theorem 3 on page 48 of Aczél, 2006). Since $D \in \mathcal{Q}$, for any $p, q \in C$, $m_i^p(D) = f(w(p)) = c_K w(p)$ and $m_i^q(D) = f(w(q)) = c_K w(q)$. This establishes Step $1B$.

Step 1C: *Let $w_{\{i\}}$ and $w_{\{j\}}$ denote the total worth of i and j's individual contributions, respectively.*[14] *For each $q \in C_{\{ij\}}$, $m_i^q(D) = \dfrac{w_{\{i\}}}{w_{\{i\}} + w_{\{j\}}} w(q)$.*

Let $D^* = (C^*, w^*, S^*) \in \mathcal{D}^N$ be such that

$$\begin{aligned}
&C^* = \{p_i, p_i, p_{ij}\} \\
&S^*(p_i) = \{i\}, \quad S^*(p_j) = \{j\}, \quad S^*(p_{ij}) = \{i, j\}, \\
&w^*(p_i) = w_{\{i\}}, \quad w^*(p_j) = w_{\{j\}}, \quad w^*(p_{ij}) = \sum_{p \in C_{\{i,j\}}} w(p).
\end{aligned}$$

By invariance to merging papers,

[13] Otherwise, we necessary have $|C'_{\{i,j\}}| = |C''_{\{i,j\}}| = 1$ and the result simply follows from neutrality which, by Remark 1, is implied by invariance to merging papers.

[14] Thus, $w_{\{i\}} = \sum_{p \in C_{\{i\}}} w(p)$.

(10) $$m_i^{p_{ij}}(D^*) = \sum_{p \in C_{\{i,j\}}} m_i^p(D).$$

By proportionality,

(11) $$m_i^{p_{ij}}(D^*) = \frac{w^*(p_i)}{w^*(p_i) + w^*(p_j)} w^*(p_{ij}).$$

By Step 1B, for each pair $p, q \in C$,

$$m_i^p(D) = \frac{w(p)}{w(q)} m_i^q(D)$$

Thus, by (10),

$$m_i^{p_{ij}}(D^*) = \sum_{p \in C_{\{i,j\}}} \frac{w(p)}{w(q)} m_i^q(D) = \frac{m_i^q(D)}{w(q)} \sum_{p \in C_{\{i,j\}}} w(p) = \frac{m_i^q(D)}{w(q)} w^*(p_{ij}).$$

Thus, by (11),

$$\frac{w^*(p_i)}{w^*(p_i) + w^*(p_j)} = \frac{m_i^q(D)}{w(q)}.$$

Thus, for each $q \in C_{\{i,j\}}$,

$$m_i^q(D) = \frac{w^*(p_i)}{w^*(p_i) + w^*(p_j)} w(q) = \frac{w_{\{i\}}}{w_{\{i\}} + w_{\{j\}}} w(q) = \check{m}_i^q(D).$$

We conclude that $m(D) = \check{m}(D)$.

***Step 2**: For each $N \in \mathcal{N}$ and each $D \in \mathcal{D}^N$, $m(D) = \check{m}(D)$.*

Induction hypothesis: Let n denote a positive integer and suppose that, for each $N \in \mathcal{N}$ such that $|N| \leq n$ and each $D \in \mathcal{D}^N$, $m(D) = \check{m}(D)$.

Let $N \in \mathcal{N}$ be such that $|N| = n + 1$, $D = (C, w, S) \in \mathcal{D}^N$, and $x \equiv m(D)$. For each $k \in N$, let $N_k = N \setminus \{k\}$. Let $k \in N$ and consider database $c_k^x(D)$. By consistency,

(12) for each $p \in C \setminus C_{\{k\}}$ and each $i \in S(p) \setminus \{k\}$, $m_i^p(c_k^x(D)) = x_i^p$.

By the induction hypothesis, since $|N_k| = n$, for each $p \in C \setminus C_{\{k\}}$ and each $i \in S(p) \setminus \{k\}$,

$$m_i^p(c_k^x(D)) = \check{m}_i^p(c_k^x(D)).$$

Thus, by the definition of $\check{m}$, for each $p \in C \setminus C_{\{k\}}$ and each $i \in S(p) \setminus \{k\}$,

$$m_i^p(c_k^x(D)) = (w(p) - x_k^p) \frac{\displaystyle\sum_{q \in C_i} m_i^q(c_k^x(D))}{\displaystyle\sum_{j \in S(p)\setminus\{k\}} \sum_{q \in C_j} m_j^q(c_k^x(D))}$$

where, abusing notation, if $p \in C_k$, we let $x_k^p = 0$. Thus, by (12), for each $p \in C \setminus C_k$ and each $i \in S(p) \setminus \{k\}$,

$$x_i^p = (w(p) - x_k^p) \frac{\displaystyle\sum_{q \in C_i} x_i^q}{\displaystyle\sum_{j \in S(p)\setminus\{k\}} \sum_{q \in C_j} x_j^q}$$

where, again, abusing notation, if $p \in C_k$, we let $x_k^p = 0$. Thus, for each $p \in C \setminus C_{\{k\}}$ and each pair $i, j \in S(p) \setminus \{k\}$,

$$\frac{x_j^p}{x_i^p} = \frac{\displaystyle\sum_{q \in C_j} x_j^q}{\displaystyle\sum_{q \in C_i} x_i^q}.$$

Repeating the same argument for each $k \in N$, for each $p \in C$ and each pair $i, j \in S(p)$,

$$x_j^p = x_i^p \frac{\displaystyle\sum_{q \in C_j} x_j^q}{\displaystyle\sum_{q \in C_i} x_i^q}.$$

Thus, for each $p \in C$ and each $i \in S(p)$, summing over $j \in S(p)$,

$$w(p) = \sum_{j \in S(p)} x_j^p = x_i^p \frac{\displaystyle\sum_{j \in S(p)} \sum_{q \in C_j} x_j^q}{\displaystyle\sum_{q \in C_i} x_i^q}.$$

Thus, for each $p \in C$ and each $i \in S(p)$,

$$x_i^p = w(p) \frac{\displaystyle\sum_{q \in C_i} x_i^q}{\displaystyle\sum_{j \in S(p)} \sum_{q \in C_j} x_j^q}.$$

Hence, x satisfies the system of equations (2) defining $\check{m}(D)$. However, $\check{m}(D)$ is the only solution to the system of equations (2). Thus, in fact, $\check{m}(D) = x = m(D)$, as desired. $\blacksquare$

Lemma 2. *If m satisfies neutrality and invariance to merging coauthors, then it satisfies proportionality.*

Proof. Suppose that m satisfies neutrality and invariance to merging coauthors. By Remark 1, m also satisfies anonymity. For each $N \in \mathcal{N}$, let $\mathcal{Q}^N \subseteq \mathcal{D}^N$ consist of all databases such that each author $i \in N$ has exactly one single-authored paper, denoted by p_i, and one joint paper coauthored by every author in N, denoted by p. Let $\{i, j\} \in \mathcal{N}$ and $D = (C, w, S) \in \mathcal{Q}^{\{i,j\}}$. For each $a \in [0, w(p_i) + w(p_j)]$, let $D_a = (C^a, w^a, S^a) \in \mathcal{Q}^{\{i,j\}}$ be such that

$$w^a(p_i) = a, \quad w^a(p_j) = w(p_i) + w(p_j) - a \quad \text{and} \quad w^a(p) = w(p).$$

Note that $D = D_{w(p_i)}$. Let $a', a'' \in [0, w(p_i) + w(p_j)]$ be such that $a' + a'' = a$. Let $i', i'' \in \mathbb{N} \setminus \{i, j\}$ and $D'_a = (C', S', w') \in \mathcal{Q}^{\{i',i'',j\}}$ be such that

$$w'(p_{i'}) = a', \quad w'(p_{i''}) = a'', \quad w'(p_j) = w^a(p_j), \quad w'(p) = w(p).$$

Since $w'(p_{i'}) + w'(p_{i''}) = w^a(p_i)$, by invariance to merging coauthors,

$$m^p_i(D_a) = m^p_{i'}(D'_a) + m^p_{i''}(D'_a). \tag{13}$$

Similarly, let $j', j'' \in \mathbb{N} \setminus \{i, j\}$ and $D''_a = (C'', S'', w'') \in \mathcal{Q}^{\{i,j',j''\}}$ be such that

$$w''(p_i) = a', \quad w''(p_{j'}) = a'', \quad w''(p_{j''}) = w^a(p_j), \quad w''(p) = w(p).$$

Since $w''(p_{j'}) + w''(p_{j''}) = w^{a'}(p_j)$, by invariance to merging coauthors,

$$m^p_i(D_{a'}) = m^p_i(D''_a).$$

By anonymity and neutrality,

$$m^p_i(D''_a) = m^p_{i'}(D'_a).$$

Combining the two previous equations, $m^p_i(D_{a'}) = m^p_{i'}(D'_a)$. Similarly, $m^p_i(D_{a''}) = m^p_{i''}(D'_a)$. Thus, by (13) and since $a = a' + a''$,

$$m^p_i(D_{a'+a''}) = m^p_i(D_{a'}) + m^p_i(D_{a''}).$$

Define $f : [0, w(p_i) + w(p_j)] \to [0, w(p)]$ by setting, for each $a \in [0, w(p_i) + w(p_j)]$, $f(a) = m^p_i(D_a)$. Then, $f(a' + a'') = f(a') + f(a'')$. Thus, f satisfies the Cauchy functional equation and there is a constant c_D such that, for each $a \in [0, w(p_i)+w(p_j)]$, $f(a) = c_D a$ (Theorem 3, page 48, Aczél, 2006). Since $D = D_{w(p_i)}$, $m^p_i(D) = c_D w(p_i)$.

Furthermore, by anonymity and neutrality , $m_j^p(D) = m_j^p(D_{w(p_j)}) = c_D w(p_j)$. We conclude that

$$m_i^p(D)w(p_j) = m_j^p(D)w(p_i)$$

That is, m satisfies proportionality. ∎

Proof of Theorem 1. By Lemma 1, the *CoScore* rule satisfies the properties in Theorem 1. Conversely, let m denote a rule satisfying these properties. By Remark 1, m satisfies neutrality. Thus, by Lemma 2, m satisfies proportionality. Thus, by Theorem 2, m is the *CoScore* rule. ∎

Appendix C. Logical independence

Proposition 1. *Consistency, invariance to merging papers, and proportionality are logically independent.*

Proof. First note that the egalitarian rule satisfies consistency and invariance to merging papers but not *proportionality* while the solo rule satisfies invariance to merging papers and proportionality but not consistency. We conclude the proof by exhibiting a rule that satisfies consistency and proportionality but not invariance to merging papers.

Let $\check{m}$ denote the *CoScore* rule. Let $i \in \mathbb{N}$ and $x : \mathbb{C} \to \{0, 1\}$. Let $N \in \mathcal{N}$ and $D = (C, w, S) \in \mathcal{D}^N$. If $i \in N$, let $z(D) \in Z(D)$ be such that, for each $p \in C$, $z_i^p(D) = x(p)w(p)$. Define the rule $\mathring{m}$ as follows:

1^{st} Case: i appears in at most 2 papers in C. Then, for each paper $p \in C$,

$$\mathring{m}^p(D) = \check{m}^p(D).$$

2^{nd} Case: i appears in at least 3 papers in C. Then, for each paper $p \in C$,

$$\mathring{m}_i^p(D) = x(p)w(p)$$
$$\mathring{m}_j^p(D) = \check{m}_j^p(c_i^{z(D)}(D)) \quad \text{for each } j \neq i.$$

First note that proportionality only applies in Case 1. Therefore, by Lemma 1, $\mathring{m}$ satisfies proportionality. Let $D = (C, w, S) \in \mathcal{D}^N$ be such that $i \in N$ and that there are two papers p and q in C_i with $i \in S(p) = S(q)$ and $w(p) = w(q)$. Then, since $x(\cdot)$ may be chosen in an arbitrary way, we can write:

$$\mathring{m}_i^p(D) = x(p)w(p) \neq x(q)w(q) = \mathring{m}_i^q(D)$$

thus violating invariance to merging papers (which implies that two identical papers are shared in the same way).

Finally, we show that $\mathring{m}$ satisfies consistency. Let $N \in \mathcal{N}$ and $D = (C, w, S) \in \mathcal{D}^N$. Let $k \in N$ and consider the database $c_k^{\mathring{m}(D)}(D) = (\tilde{C}, \tilde{w}, \tilde{S})$.

1^{st} Case: i appears in at most 2 papers in C. Then, whether $k = i$ (i is taken out) or $k \neq i$, i still appears in at most 2 papers in $\tilde{C}$. Therefore, for each $p \in C$ and each $j \in N \setminus \{k\}$:

$$\mathring{m}_j^p(D) = \check{m}_j^p(D) = \check{m}_j^p(c_k^{\check{m}(D)}(D)) = \mathring{m}_j^p(c_k^{\mathring{m}(D)}(D))$$

since the *CoScore* rule $\check{m}$ satisfies consistency.

2^{nd} Case: i appears in at least 3 papers in C.

2.1: $k = i$. Then, since i doesn't appear in any paper in $\tilde{C}$ (i has been taken out), we have $\mathring{m}_j^p(c_k^{\mathring{m}(D)}(D)) = \check{m}_j^p(c_k^{\mathring{m}(D)}(D))$. Furthermore, since $c_k^{\mathring{m}(D)}(D) = c_k^{z(D)}(D)$, we also have: $\mathring{m}_j^p(D) = \check{m}_j^p(c_i^z(D)) = \check{m}_j^p(c_k^{\mathring{m}(D)}(D))$ so that $\mathring{m}_j^p(D) = \mathring{m}_j^p(c_k^{\mathring{m}}(D))$. We conclude that $\mathring{m}$ satisfies consistency.

2.2: $k \neq i$. Then i still appears in at least 3 papers in $\tilde{C}$. Therefore, for each $p \in C$,

$$\mathring{m}_i^p(c_k^{\mathring{m}(D)}(D)) = x(p)\left(w(p) - \mathring{m}_k^p(D)\right) = x(p)w(p) = \mathring{m}_i^p(D)$$

For each $j \in N \setminus \{i, k\}$,

$$\mathring{m}_j^p(c_k^{\mathring{m}(D)}(D)) = \check{m}_j^p(c_i^{z(c_k^{\mathring{m}(D)}(D))}(c_k^{\mathring{m}(D)}(D)))$$

Furthermore, since i is allocated the same share of paper p in databases $c_k^{\mathring{m}}(D)$ and D:

$$c_i^{z(r_k^{\mathring{m}(D)}(D))}(c_k^{\mathring{m}(D)}(D)) = c_i^{z(D)}(c_k^{\mathring{m}(D)}(D))$$

and since (a) reducing D first by taking out k (according to $\mathring{m}(D)$) then by taking out i (according to $z(D)$) is equivalent to (b) reducing D first by taking out i (according to $z(D)$), then by taking out k (according to $\mathring{m}(c_i^{z(D)}(D))$), which is also equivalent to (c) reducing D first by taking out i (according to $z(D)$), then by taking out k (according to $\check{m}(c_i^{z(D)}(D))$):

$$c_i^{z(D)}(c_k^{\mathring{m}(D)}(D)) = c_k^{\mathring{m}(c_i^{z(D)}(D))}(c_i^{z(D)}(D)) = c_k^{\check{m}(c_i^{z(D)}(D))}(c_i^{z(D)}(D))$$

We conclude that

$$\mathring{m}_j^p(c_k^{\mathring{m}(D)}(D)) = \check{m}_j^p(c_k^{\check{m}(r_i^{z(D)}(D))}(c_i^{z(D)}(D))) = \check{m}_j^p(c_i^{z(D)}(D)) = \mathring{m}_j^p(D),$$

using the fact that $\check{m}$ satisfies consistency. Thus, $\mathring{m}$ satisfies consistency. ∎

Proposition 2. *Consistency, invariance to merging papers, and invariance to merging coauthors are logically independent.*

Proof. First note that the egalitarian rule satisfies consistency and invariance to merging papers but not invariance to merging coauthors while the solo rule satisfies invariance to merging papers and invariance to merging coauthors but not consistency. We conclude the proof by exhibiting a rule that satisfies consistency and invariance to merging coauthors but not invariance to merging papers.

Let $q \in \mathbb{C}$. For each $D = (C, w, S) \in \mathcal{D}^N$, define the following associated databases:

- $D_{-q} = (C', w', S') \in \mathcal{D}^N$ such that $C' = C \setminus \{q\}$ and, for each $p \in C'$, $w'(p) = w(p)$ and $S'(p) = S(p)$. Thus, D_{-q} denotes the database obtained from D by removing paper q.
- $D_{+q} = (C', w', S') \in \mathcal{D}^N$ such that $C' = \{q\} \cup \bigcup_{i \in N} C_{\{i\}}$ and, for each $p \in C'$, $w'(p) = w(p)$ and $S'(p) = S(p)$. Thus, D_{+q} denotes the database obtained from D by removing all joint papers except perhaps for q.

Define the rule $\dot{m}$ as follows: for each $D = (C, w, S) \in \mathcal{D}^N$,

$$\dot{m}^q(D) = \check{m}^q(D_{+q}) \quad \text{and} \quad \dot{m}^p(D) = \check{m}^p(D_{-q}) \quad \text{for each } \ p \in C \setminus \{q\}.$$

Rule $\dot{m}$ violates invariance to merging papers because two papers with the same worth and the same group of authors (one of them being q) may be shared in different ways. It satisfies invariance to merging coauthors and consistency because the *CoScore* rule does. ∎

Appendix D. Data

The data was retrieved from Thomson Reuters's Web of Science database on February 10th 2016 (http://ipscience.thomsonreuters.com/product/web-of-science/). We extracted all published articles with at least one citation from the following 33 major journals in Economics over the period $1970 - 2015$:

Econometrica, Quarterly Journal of Economics, Journal of Economic Literature, American Economic Review, Journal of Political Economy, Review of Economic Studies, Journal of Monetary Economics, Journal of Economic Theory, Games and Economic Behavior, Journal of Economics Perspectives, Journal of Econometrics, Rand Journal of Economics, Economic Theory, Journal of Labor Economics, Journal of Human Resources, Journal of Public Economics, Review of Economics and Statistics, Econometric Theory, Journal of Risk and Uncertainty, International Economic Review, Journal of Applied Econometrics, European Economic Review, International Journal of Game Theory, Social Choice and Welfare, Journal of Environmental Economics and Management, Economic Journal, Journal of International Economics, Journal of Economic Dynamics and Control, Journal of Mathematical Economics, Economic Inquiry, Scandinavian Journal of Economics, Economics Letters, Oxford Bulletin of Economics and Statistics.

The Web of Science database is not entirely consistent regarding the name of the authors. Authors are sometimes identified with their first initial only, all their initials and in some cases their full name. We choose here to retain the first initial only, which prevents the same author from being identified under two or more labels but means that authors with the exact same name and first initial are collapsed together. Note that the number of citations provided by Web of Science is usually smaller than the one of Google Scholar.

We compute *CoScore* by repeatedly iterating the fixed point operator ϕ (see Appendix A), starting from a vector of equal scores. We assign an extra 10 single authored citations to all authors in the database to compensate for the restricted set of journals. In doing so, we ensure that every author has at least one single-authored citation, so that *CoScore* is indeed well defined. The process converges quickly to the desired fixed point. After a hundred iterations, $\|\phi(x) - x\|_\infty = 0.0332 \leq 0.001\|x\|_\infty$, after a thousand iterations, $\|\phi(x) - x\|_\infty = 4.10^{-8} \leq 3.10^{-9}\|x\|_\infty$. Table 1 gives the top 15 economists according to *CoScore*.

Rank	Scholars	# Papers	Solo cit. (%)	# Coauthors	*CoScore*	Eg. Score
1	Shleifer, A	77	0,05	41	564,1	231,8
2	Kahneman, D	23	0,04	20	559,0	314,8
3	Heckman, J	115	0,50	68	508,5	369,4
4	Acemoglu, D	101	0,19	51	425,7	230,7
5	Stiglitz, J	137	0,18	43	419,8	267,6
6	Engle, R	52	0,31	43	368,6	277,4
7	Bollerslev, T	41	0,51	29	365,8	273,2
8	Barro, R	75	0,59	23	331,0	279,2
9	Fehr, E	56	0,001	47	310,1	146,8
10	Tirole, J	113	0,12	29	304,9	199,8
11	Lucas, R	59	0,80	18	295,1	272,1
12	White, H	65	0,81	43	290,2	266,4
13	Alesina, A	59	0,06	38	275,1	156,4
14	Romer, P	20	0,91	9	272,3	260,5
15	Stock, J	46	0,08	27	270,0	146,0

TABLE 1. Top 15 economists according to *CoScore*. Solo cit. gives the fraction of single authored citations.