# Ownership of the Means of Production[*]

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## Abstract

Private ownership creates monopoly power, harming the dynamic efficiency of asset allocation. Common ownership can improve allocative efficiency, but eliminates incentives to invest in the common use value of assets. For a class of assets such as publicly owned natural resources, both allocative and investment efficiency are important, thus licenses should be intermediate between public and private ownership to trade off these goals. In this paper, we propose using a system of self-assessed property taxes with a universal right to force a sale at the self-assessed price, as first proposed by Harberger (1965) as a simple implementation of partial property rights. Within a dynamic overlapping-generations model of trade, these *Harberger taxes* decrease monopoly markups, increasing trade frequency and the efficiency of asset allocation in the stationary trading equilibrium of the model. We propose a simple rule-of-thumb policy for license design: tax rates should be set to about half the observed rates of asset trade in existing markets. In calibrations, a 2.5% annual tax rate is near optimal, increasing the steady state value created by assets by approximately 4%.

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*What is common to the greatest number gets the least amount of care. Men pay most attention to what is their own; they care less for what is common; or at any rate they care for it only to the extent to which each is individually concerned.*

– Aristotle, *The Politics*, Book XI, Chapter 3

*Property is only another name for monopoly.*

– William Stanley Jevons, preface to the second edition of *The Theory of Political Economy*

# 1 Introduction

Private ownership of the means of production is perhaps the oldest continually maintained doctrine in mainstream economic thought, dating back to the Greek prehistory of the field, and continues to pervade contemporary thought. Jacobs (1961) and de Soto (2003) argue the undermining or lack of property rights discourages investment in rich and poor countries equally, and Acemoglu and Robinson (2012) to consistently list property rights as the leading example of the "inclusive institutions" they argue foster economic development. Economic theory suggests that private property is necessary to incent agents to invest in maintaining the common use value of assets; this idea is developed in detail by Grossman and Hart (1986) and Hart and Moore (1990).

On the other hand, private property can adversely affect the efficient allocation of assets, by inhibiting the transfer of assets to individuals who value them most. The founders of contemporary economic analysis (Jevons, 1879; Walras, 1896) believed that private property was inherently in conflict with the market principle at the heart of their systems, succinctly expressed in our second leading quote. This argument was formalized by Myerson and Satterthwaite (1981), who show that fully efficient trade of privately owned assets under asymmetric information is impossible. Conversely, if assets are initially owned by a benevolent government, Vickrey (1961) showed that full allocative efficiency can be achieved by auctioning the asset to potential buyers Cramton, Gibbons and Klemperer (1987) and Segal and Whinston (2011) further explore these ideas, showing that partial property rights can improve allocative efficiency. This literature suggests that, in settings where allocative efficiency is a first-order concern, private property may be inferior to partially shared ownership.

For a variety of assets, both allocative and investment efficiency are important concerns. Consider, for example, government-owned natural resources such as fisheries or oilfields. Different firms may have a range of costs of extracting these resources, and these costs may evolve over time, implying that dynamic asset allocation is a first-order concern. If the government runs a one-time auction to sell long-lasting or perpetual ownership rights over the resource to a firm, bargaining frictions in aftermarkets will inhibit efficient trade of the asset to lower-cost firms.

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The government can improve allocative efficiency by maintaining ownership over the resource, and running annual auctions to usage rights to firms. However, under such a system, firms would have no incentives to maintain the common value of assets that they win usage rights over; firms that wins these auctions may tend to overfish the fisheries, or extract too much oil from the oilfields, relative to the social optimum. In such contexts, one might think that a system of partial property rights, trading off the investment incentives from private property with the allocative benefits from common ownership, might in theory dominate both the extremes of full private and full common ownership. However, such partial property systems are rarely used in practice.

In this paper, we propose *self-assessed taxation* as a simple implementation of partial property rights. Because Harberger (1965) first proposed this form of self-assessed taxation, we will refer to this system *Harberger taxation*. Under this system, the government designates certain assets as being owned under *Harberger licenses*, rather than the perpetual ownership licenses associated with private property. Any individual who owns an asset under a Harberger license must periodically announce to the government her valuation for the license, and must pay taxes equal to some fraction of her announced value to the government. The government maintains a ledger of all assets owned under Harberger licenses, and the value announcements of their current owners; potential buyers can purchase any license on the ledger from its current owner at the owner's most recent self-assessed valuation.

If individuals were asked to announce their values for assets without paying taxes, they would tend to announce prices higher than their true values to exploit their monopoly power, inefficiently inhibiting transfer of the asset to buyers with higher values. Under Harberger taxation, license holders effectively repurchase a fraction of their assets from the government each period at the valuations that they announce, reducing their incentives to announce high prices. We show that license owners set prices higher or lower than their true values depending on whether the probability of asset sale is higher or lower than the Harberger tax rate. Thus, when the tax rate is set equal to the probability that the asset is sold, license owners on average announce prices equal to their values, leading to approximate allocative efficiency. However, since license owners pay taxes on the capital value of any assets that they own, positive tax rates decrease license owners' incentives to make common-valued investments in their assets.

We study the effects of Harberger taxation within a dynamic overlapping-generations model of asset trade. An agent holds a Harberger license for an asset; each period, new buyers interested in purchasing the license arrive to the market. Agents' use values for the asset evolve as Markov processes. We solve the model, showing that there exists a unique Markovian equilibrium of the trading game for any Harberger tax level. We numerically calibrate the model to loosely match moments of existing markets for durable assets. We find that annual Harberger taxes set at roughly 2.5%, half the current turnover rate of assets in private markets, are near optimal and always welfare enhancing for a wide range of parameter values relative to pure

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private ownership. This increases the net utility generated by the asset in the stationary trading equilibrium of the model by approximately 4% for our focal parameter values, or roughly a quarter of a standard deviation in willingness-to-pay of different potential asset buyers. We suggest a simple rule-of-thumb for setting approximately optimal Harberger taxes for different classes of assets: tax rates should be set at half the observed turnover rates of similar assets in existing markets. Extrapolated to all capital in the economy, this would generate gains of roughly 1% of gross domestic product annually.

Harberger taxation is fairly simple to implement in practice, and similar systems have been used in a variety of settings dating as far back as ancient Rome.[1] To our knowledge, we are the first to propose using self-assessed taxation to improve allocative efficiency in asset markets.[2] We believe that Harberger licenses with rule-of-thumb tax rates are a robust practical proposal for improving the dynamic efficiency of allocation for a large of assets; in particular, we suggest that Harberger licenses are well-suited as an alternative license design for usage rights over a class of publicly owned natural resources, such as oilfields, fishing rights, and radio spectrum.

In Section 2, we illustrate the basic intuitions behind Harberger taxation in a simple two-stage model. In Section 3, we introduce the general dynamic model and characterize its equilibria. In Section 4, we calibrate the model to existing asset markets. In Section 5, we discuss various extensions, such as the effect of community observability of investment, the effect of taxation on private-value investments, and relaxing our assumptions on the nature of buyers and sellers. In Section 6, we discuss our proposal's relationship to other work on mechanism design, asset taxation and intellectual property. In Section 7 we discuss potential applications of Harberger taxation to different classes of assets. We conclude in Section 8. We present longer and less instructive calculations, proofs, and calibration details in an appendix following the main text.

## 2   Two-stage model

We illustrate the intuitions behind Harberger taxation in a simple two-stage model. Agent S holds a Harberger license for an asset. She first makes some common-valued investment in the asset, then announces a valuation to the government and pays a fraction of this valuation as a Harberger tax. Arriving buyers can then purchase the license from the seller at her announced valuation. In Subsection 2.2, we show that the Harberger tax gives S an incentive to announce lower prices, counteracting her monopolistic incentive to charge prices above her value to buyers. However, in Subsection 2.3, we show that Harberger taxes reduce S's incentives to make

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[1]See Epstein (1997) for a detailed history. The largest scale recent experiment we are aware of was in Taiwan based on a proposal by the founder of modern China, Sun Yat-sen (Sun, 1924; Niou and Tan, 1994), though in that case only the government was allowed to force a sale. However, on the private side, related institutions exist in horse racing (Hall, 1986), partnership dissolutions (Brooks, Landeo and Spier, 2010) and insurance (Cabrales, Calvó-Armengol and Jackson, 2003).

[2]The closest prior idea is Tideman (1969)'s demonstration that Harberger taxes tend to increase the probability of sale, but Tideman does not explicitly model the consequent welfare effects.

investments in the common use value of the asset. In Subsection 2.4, we analyze the tradeoff between the allocative and investment welfare effects of Harberger taxation.

## 2.1 Setup

There are two agents, S and B. There is a single asset, and S initially owns a *Harberger license* for the asset. Values of S and B for the asset are, respectively,

$$v_S = \eta + \gamma_S,$$

$$v_B = \eta + \gamma_B.$$

$\gamma_S$ represents S's idiosyncratic value component for the asset; it is fixed and known to S at the beginning of the game. $\gamma_B \sim F(\cdot)$ is a random variable representing heterogeneity in B's value, which is not observed by S. $\eta$ is a common-value component; S chooses $\eta \geqslant 0$, incurring a convex cost $c(\eta)$ to herself. Both agents are risk neutral.[3]

For a given $\eta$, let $1_S, 1_B$ be indicators, which respectively represent whether S and B hold the license at the end of the game, and let $y$ be any net transfer B pays to S. Final payoffs for S and B respectively are

$$U_S = (\eta + \gamma_S) 1_S - c(\eta) + y$$

$$U_B = (\eta + \gamma_B) 1_B - y.$$

Prior to the beginning of the game, the government decides on a *Harberger tax* level $\tau$. Then, S and B play a two-period game. In period 1, S chooses $\eta$. In period 2, S announces a price $p$ for the license, pays taxes $p\tau$ to the government, and then B can decide whether to buy the license by paying $p$ to S. The revenue that the government raises is distributed to the broader community in a manner we do not specify here.

We solve the game by backwards induction. First, fixing $\eta$ and $\tau$, we analyze the behavior of S in the period 2 price offer game.

## 2.2 Allocative efficiency

For any price $p$, B's optimal strategy is to buy the license if her value is greater than $p$, that is, if $\eta + \gamma_B > p$. Let $m \equiv p - \eta$ be the markup S chooses to set over the common value $\eta$. The probability of sale under markup $m$ is then $1 - F(m)$. Fixing common value $\eta$, and Harberger tax level $\tau$, S's optimal price offer solves:

$$\max_m (1 - F(m))(\eta + m) + F(m)(\eta + \gamma_S) - \tau(\eta + m) - c(\eta)$$

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[3]See Tideman (1969) for a partial analysis of the allocative problem that allows for risk aversion.

We can change variables to work in terms of sale probabilities. Define $q \equiv (1 - F(m))$, and $M(q) \equiv F^{-1}(1 - q)$. S then solves:

$$\max_q (\eta + M(q)) q + (\eta + \gamma_S)(1 - q) - \tau(\eta + M(q)) - c(\eta)$$

Note that the socially efficient outcome corresponds to setting $M(q) = \gamma_S$, or equivalently $q = 1 - F(\gamma_S)$. We can rearrange S's optimization problem to:

$$\max_q (M(q) - \gamma_S)(q - \tau) + (\eta + \gamma_S)(1 - \tau) - c(\eta)$$

Only the *variable profit* term $(M(q) - \gamma_S)(q - \tau)$ depends on the sale probability $q$. Thus, S's optimal choice of sale probability if her value is $\gamma_S$ and the tax rate is $\tau$ can be written as:

$$q^*(\gamma_S, \tau) \equiv \arg\max_q (M(q) - \gamma_S)(q - \tau)$$

We can think of the objective function as the net trade profits of an agent who sells share $q$ of the asset to buyers, and buy share $\tau$ of the asset from the government, both at price $M(q)$. In the following Theorem, we show that the relationship between $\tau$ and $q$ summarizes license owners' incentives to markup or markdown prices.

**Theorem 1.** *(Net trade property)*

- *If $\tau = 1 - F(\gamma_S)$, then $q^*(\gamma_S, \tau) = \tau$ and $M(q^*(\gamma_S, \tau)) = \gamma_S$.*

- *If $\tau < 1 - F(\gamma_S)$, then $q^*(\gamma_S, \tau) \geqslant \tau$ and $M(q^*(\gamma_S, \tau)) \geqslant \gamma_S$.*

- *If $\tau > 1 - F(\gamma_S)$, then $q^*(\gamma_S, \tau) \leqslant \tau$ and $M(q^*(\gamma_S, \tau)) \leqslant \gamma_S$.*

*Proof.* First, suppose $\tau = 1 - F(\gamma_S)$.

- If S chooses sale probability $q = \tau$, she makes no net trades, and receives 0 variable profits. Moreover, the markup is $M(q) = F^{-1}(1 - \tau) = \gamma_S$, so that also $M(q) - \gamma_S = 0$.

- If S chooses a higher sale probability, so that $q - \tau > 0$, we have $M(q) \leqslant \gamma_S$, so variable profits $(M(q) - \gamma_S)(q - \tau) \leqslant 0$. In words, S becomes a net seller at a price lower than her value.

- Symmetrically, if S chooses a lower sale probability $q - \tau < 0$, she becomes a net buyer at a price higher than her value, and once again variable profits $(M(q) - \gamma_S)(q - \tau) \leqslant 0$.

Now suppose that $\tau < 1 - F(\gamma_S)$.

- By the first part of the Theorem, license owners with higher values $\gamma'_S = F^{-1}(1-\tau)$ have $\tau = 1 - F(\gamma'_S)$, hence choose $q^*(\gamma'_S, \tau) = 1 - F(\gamma^*_S)$. By construction, $\gamma_S \leqslant \gamma'_S$; since the variable profit function is supermodular in q and $-\gamma_S$, $q^*(\gamma_S, \tau) \geqslant q^*(\gamma'_S, \tau) = \tau$.

- By the first part of the Theorem, if we set a lower tax $\tau' = 1 - F(\gamma_S)$, we have $q^*(\gamma_S, \tau') = 1 - F(\gamma_S)$ and $M(q^*(\gamma_S, \tau')) = \gamma_S$. By construction, $\tau \leqslant \tau'$. Since $M(q)$ is a decreasing function, the variable profit function is supermodular in q and $\tau$, hence $q^*(\gamma_S, \tau) \leqslant q^*(\gamma_S, \tau') = 1 - F(\gamma_S)$. This implies that $M(q^*(\gamma_S, \tau)) \geqslant M(q^*(\gamma_S, \tau')) = \gamma_S$.

An analogous argument shows that $\tau > 1 - F(\gamma_S)$ implies that $q^*(\gamma_S) \leqslant \tau$ and $M(q^*(\gamma_S, \tau)) \leqslant \gamma_S$. $\qquad \square$

Theorem 1, along with its generalizations which we explore below, is the main theoretical result of the paper. This theorem shows that the net effect of Harberger taxation on sellers' price-setting incentives is linked to an observable quantity: $\tau - q$, the difference between the Harberger tax rate and the probability of sale that it induces. If Harberger taxes are lower than equilibrium sale probabilities, license owners can be thought of as selling to buyers a larger share of the asset than they are buying from the government, hence have net incentives to set prices higher than their values. Likewise, if taxes are higher than sale probabilities, license owners are net buyers of their assets, and set prices below their values. The optimal tax rate is equal to the probability of sale it induces. If the tax rate is below the probability of sale it induces, it is lower than the optimal level, and likewise if the tax rate is above the probability of sale it is higher than optimal.

In Theorem 2 of Section 3, we show that the net trade property generalizes to our full dynamic model. In a setting with many license owners with heterogeneous values, no single tax level can give all owners incentives to truthfully reveal their values; however, we show in our calibration of Section 4 that Harberger taxes equal to average sale probabilities across sellers are close to allocatively optimal.

We proceed to quantify the comparative statics of allocative welfare with respect to the tax rate. Assuming $F(\cdot)$ is twice continuously differentiable, S's first-order condition is

$$M'(q)(q-\tau) + (M(q) - \gamma_S) = 0,$$

so that by the Implicit Function Theorem,

$$\frac{\partial q^\star}{\partial \tau} = \frac{M'(q^\star)}{2M'(q^\star) + M''(q^\star)(q^\star - \tau)} = \frac{1}{2 + \frac{M''(q^\star)(q^\star-\tau)}{M'(q^\star)}} = \frac{1}{2 - \frac{M''(M-\gamma_S)}{(M')^2}},$$

where the last equality invokes the first-order condition and drops arguments. Cournot (1838) showed that this quantity equals the *pass-through* rate $\rho(q^\star)$ of a specific commodity tax into

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price; see Weyl and Fabinger (2013) for a detailed discussion and intuition. $\rho$ is closely related to the curvature of the value distribution; it is large for convex demand and small for concave demand. It is strictly positive for any smooth value distribution and is finite as long as S is at a strict interior optimum.[4]

The marginal gain to social welfare from a unit increase in the probability of sale is equal to the gap between $\gamma_B$ and $\gamma_S$, because the tax raised is simply a transfer. This gap is, by construction, $(M(q^\star) - \gamma_S)$. Thus, the marginal allocative gain from raising $\tau$ is $(M(q^\star) - \gamma_S)\rho(q^\star)$ or $(M - \gamma_S)\rho$ for short. Note that $(M - \gamma_S)\rho$ is 0 at $q = 1 - F(\gamma_S)$, so the first-order social welfare gain from taxation approaches 0 as we approach the allocatively optimal tax of $1 - F(\gamma_S)$. On the other hand, when $\tau = 0$, we have $(M - \gamma_S)\rho > 0$, hence increasing $\tau$ creates a first-order welfare gain.

## 2.3 Investment efficiency

Note that the variable profits defined in the previous subsection were independent of $\eta$. Only the sunk profits, $(1 - \tau)\eta - c(\eta)$, depend on $\eta$. Thus, regardless of what happens in the second stage of the game, S finds it optimal to choose $\eta$ such that:

$$c'(\eta) = 1 - \tau.$$

We can define the investment supply function $\Gamma(\cdot)$ as:

$$\Gamma(s) \equiv c'^{-1}(s).$$

The value of a unit of investment $\eta$ is always 1, so the socially optimal level of investment is $\Gamma(1)$, whereas investment is only $\Gamma(1 - \tau)$ when the tax rate is $\tau$. The social value of investment is always 1, whereas S only invests up to the point where $c' = 1 - \tau$. Thus, the marginal distortion from under-investment is $\tau$. The marginal increase in investment from a rise in $\tau$ is $\Gamma' = \frac{1}{c''}$ by the inverse function theorem. Thus, the marginal social welfare loss from raising $\tau$ is $\Gamma'\tau$, or $\frac{\tau}{1-\tau}\Gamma\epsilon_\Gamma$, where $\epsilon_\Gamma$ is the elasticity of investment supply. Note that as $\tau \to 0$, this investment distortion goes to 0, so that there is no first-order loss in investment welfare when $\tau = 0$. Since there is a first-order gain in allocative welfare from raising the tax when $\tau = 0$, the optimal level of $\tau$ is strictly greater than 0.

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[4]Myerson (1981)'s regularity condition is sufficient but not necessary for this second-order condition, as we show in Appendix A.1.
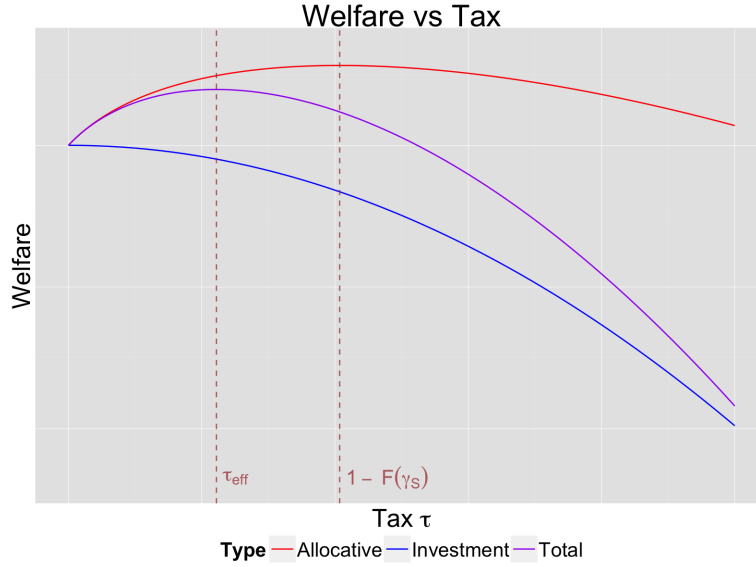
Figure 1: Allocative, Investment, and Total Welfare vs Tax

## 2.4 Tradeoff between allocative and investment welfare

Figure 1 graphically illustrates the tradeoff between allocative and investment welfare. Allocative welfare increases monotonically in $\tau$ on the interval $\tau \in [0, 1 - F(\gamma_S)]$. The marginal gain in allocative welfare from raising the tax is $(M(q^\star) - \gamma_S) \rho(q^\star)$; thus, the marginal allocative gain is 0 when $\tau = 1 - F(\gamma_S)$ and $M(q^\star) = \gamma_S$. Similarly, the marginal investment loss from taxation is $\Gamma'\tau$, which is 0 at $\tau = 0$. These properties hold independently of the cost function and demand distribution; intuitively, this reflects the fact that the marginal trades when $\tau = 1 - F(\gamma_S)$, and the marginal units of investment when $\tau = 0$, both have no social value. Thus, regardless of the underlying cost and demand functions, the efficient tax level $\tau_{\text{eff}}$ lies strictly in the interior of the interval $[0, (1 - F(\gamma_S))]$.

In Figure 1, allocative welfare is concave and investment losses are convex in $\tau$, so total social welfare is a concave function of $\tau$. While this is not true for all cost functions and demand distributions, it tends to hold for well-behaved values of the primitives. Since the markup $M(q^\star)$ is decreasing in $\tau$, allocative marginal gains $(M(q^\star) - \gamma_S) \rho(q^\star)$ tend to be decreasing in $\tau$, and since the marginal investment loss $\Gamma'\tau$ contains a $\tau$ term, marginal investment losses tend to be increasing in $\tau$. Intuitively, as we raise the tax from 0, the first trades that go through are the highest value trades, and the first investment losses are those which are both privately and socially marginal. As we raise the tax, the allocative wedge $M(q^\star) - \gamma_S$ decreases, so new trades caused by the tax are less valuable, and the investment wedge $\tau$ increases, so investment losses caused by the tax are more costly to society. Thus, for relatively smooth demand forms and cost functions, the social optimization problem of maximizing the allocative gain less the investment loss will be concave.

In Appendix A.1, we describe conditions on the cost function $c(\cdot)$ and the inverse demand

function $M(\cdot)$ such that the social optimization problem is concave. Assuming concavity, the following first-order condition, which resembles an optimal tax formula, uniquely characterizes the welfare-maximizing tax level:

$$\frac{\tau_{\text{eff}}}{1 - \tau_{\text{eff}}} = \frac{(M(q^\star(\gamma_S, \tau_{\text{eff}})) - \gamma_S) \, \rho(q^\star(\gamma_S, \tau_{\text{eff}}))}{\Gamma(1 - \tau_{\text{eff}}) \, \epsilon_\Gamma(1 - \tau_{\text{eff}})}. \tag{1}$$

The left-hand side is a monotone-increasing transformation of $\tau$ that appears frequently in elasticity formulas in the optimal tax literature; see, for example, Werning (2007). The right-hand side is the ratio of two terms: the allocative benefit of higher taxes and the investment distortion of higher taxes. The allocative benefit equals the product of the mark-up and the pass-through rate, whereas the investment distortion equals the product of the equilibrium investment size and its elasticity with respect to $1 - \tau$.

## 3  Dynamic model

In this section, we construct a dynamic model of Harberger taxation, and show that the core intuitions of the two-stage model extend to this more general setting. This allows us to study the effect of Harberger taxation on turnover rates and the stationary distribution of values, as well as the influence of Harberger taxation on license prices.

### 3.1  Agents and utilities

Time is discrete, $t = 0, 1, 2 \ldots \infty$. All agents discount utility at rate $\delta$. There is a single asset, which an agent $S_0$ owns at time $t = 0$. In each period, a single buyer $B_t$ arrives to the market and bargains with the period-$t$ owner $S_t$ to purchase the license, through a procedure we detail in Subsubsection 3.2 below. Hence, the set of agents is $\mathscr{A} = \{S_0, B_0, B_1, B_2 \ldots\}$. We will use $S_t$ as an alias for the period-$t$ owner, who may be a buyer $B_{t'}$ from some period $t' < t$. We will often use $A$ to denote a generic agent in $\mathscr{A}$.

In period $t$, agent $A$ has period-$t$ use value $\gamma_t^A$ for the asset. The values of entering buyers $\gamma_t^{B_t}$ are drawn i.i.d. from some distribution $F$. Values evolve according to a Markov process: for any agent $A$ with period-$t$ usage utility $\gamma_t^A$, her use value in the next period $\gamma_{t+1}^A$ is drawn from the transition probability distribution $G(\gamma_{t+1} \mid \gamma_t)$.

**Assumption 1.** $F(M) = 1$ *for some finite* $M$.

**Assumption 2.** $\gamma_t > \gamma_t'$ *implies* $G(\gamma_{t+1} \mid \gamma_t) >_{\text{FOSD}} G(\gamma_{t+1} \mid \gamma_t')$.

**Assumption 3.** $G(\gamma' \mid \gamma)$ *is continuous and differentiable in* $\gamma$ *for any* $\gamma'$.

Assumption 1 implies that agents' values are bounded above. Assumption 2 implies that agents with higher current use values have higher future use values, in the sense of stochastic

dominance. Assumption 3 guarantees that the value decay process is smooth. These three assumptions imply that stationary equilibria of our trading game exist, and that stationary equilibria satisfy our core characterization result, the net trade property of Theorem 2. However, we require an additional assumption in order to prove the uniqueness of stationary equilibrium.

**Assumption 4.** $G(\gamma \mid \gamma) = 1 \; \forall \gamma$, *that is,* $\gamma_{t+1} \leqslant \gamma_t$ *with probability 1.*

In our calibration of Section 4, we attempt to numerically solve versions of the model in which values increase with some probability; we are able to solve for equilibria, the equilibria appear to be unique, and the conclusions of the model are quantitatively similar to models in which values decrease with probability 1. Thus, we view Assumption 4 as inessential for the main results of our paper to hold.

In any period, there is a single user of the asset. Let $1_t^A$ denote agent A being the user of the asset in period A. Agent A's utility for ownership path $1_t^A$ and utility path $\gamma_t^A$, is:

$$\sum_{t=0}^{\infty} \delta^t \left[ 1_t^A \gamma_t^A + y_t^A \right]$$

Where, $y_t^A$ is any net monetary payment made to agent A in period t.

To avoid dealing with repeated strategic interactions, after the period in which agent A arrives to the market as a buyer, we will allow agent A to remain in the market only so long as $1_t^A = 1$; once $1_t^A = 0$, agent A leaves the market forever. Thus, in each period t, only two agents exist in the market: the period t seller $S_t$, and the arriving buyer $B_t$. Any pair of agents interacts at most once.

A (possibly random) *allocation rule* $\Phi(h_t)$ determines in each time t, history $h_t$ whether to allocate the good to $S_t$ or $B_t$. Intuitively, since Assumption 2 states that higher present values imply uniformly higher future values, a social planner aiming to maximize discounted use values should assign the asset to whichever of $S_t, B_t$ has higher current-period value in any given period. This is formalized in the following proposition.

**Proposition 1.** *The socially optimal allocation rule* $\Phi(\cdot)$ *allocates the good to whichever of* $\{S_t, B_t\}$ *has higher use value* $\gamma_t$ *in every period* t.

*Proof.* See Appendix Subsection A.2. □

## 3.2 Game

The community chooses some Harberger tax level $\tau$, constant for all time. For any tax level $\tau$, we will define the following *dynamic Harberger tax game*. At $t = 0$, agent $S_0$ owns the Harberger license, and observes her own use value $\gamma_t^{S_0}$ for the asset. In each period t:

1. **Buyer arrival:** Buyer $B_t$ arrives to the market; his use value $\gamma_t^{B_t}$ is drawn from $F(\cdot)$, and is observed by himself but not the period-t seller $S_t$.

2. **Seller price offer:** The license owner $S_t$ makes a take-it-or-leave-it price offer $p_t$ to buyer $B_t$, and immediately pays tax $\tau p_t$ to the community.

3. **Buyer purchase decision:**

   - If $B_t$ chooses to buy the license, she pays $p_t$ to $S_t$. $B_t$ becomes the period-t asset user, $1_t^{B_t} = 1$, and enjoys period-t use value $\gamma_t^{B_t}$ from the asset. $B_t$ becomes the license owner in period $t+1$, that is, $S_{t+1} \equiv B_t$. Seller $S_t$ receives payment $p_t$ from $B_t$, and seller $S_t$ leaves the market forever, with continuation utility 0.

   - If $B_t$ chooses not to purchase the license, $S_t$ becomes the period-t asset user, $1_t^{S_t} = 1$, and she enjoys period-t use value $\gamma_t^{S_t}$ from the asset. $S_t$ becomes the license owner in period $t+1$, that is, $S_{t+1} \equiv S_t$. Buyer $B_t$ leaves the market forever, with continuation utility 0.

4. **Value updating:** $\gamma_{t+1}^{S_{t+1}}$, the period $t+1$ value for owner $S_{t+1}$, is drawn from $G\left(\gamma_{t+1} \mid \gamma_t^{S_{t+1}}\right)$ according to her period-t value $\gamma_t^{S_{t+1}}$.

## 3.3 Equilibrium

Equilibrium in the dynamic Harberger tax game requires that, in all histories, all sellers make optimal price offers, and all buyers make optimal purchase decisions. Since $\tau$, $F$, $G$ are constant over time, the problem has a Markovian structure: the optimal strategies of buyers and sellers may depend on their types $\gamma_t^{S_t}, \gamma_t^{B_t}$ respectively, but not on the period t. Hence we can apply dynamic programming techniques, characterizing equilibria of the game by a stationary value function $V(\gamma)$ which describes the value of being a type $\gamma$ seller in any given period.

In any period t, we can think of $S_t$ as choosing a probability of sale $q_t$, where buyers in period t make purchase decisions according to the inverse demand function $p(q_t)$. If the continuation value in period $t+1$ for seller type $\gamma_{t+1}$ is $V(\gamma_{t+1})$, the optimization problem that $S_t$ faces is:

$$\max_{q_t} q_t p(q_t) + (1-q_t)\left[\gamma_t + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V(\gamma_{t+1}) \mid \gamma_t\right]\right] - \tau p(q).$$

Simplifying and omitting t subscripts, optimality for sellers requires $V(\gamma)$ to satisfy the following Bellman equation:

$$V(\gamma) = \max_q (q-\tau) p(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V(\gamma') \mid \gamma\right]\right] \tag{2}$$

Buyer optimality pins down the relationship between $p(\cdot)$ and $V(\cdot)$. If buyer $B_t$ with value $\gamma_t$ purchases the license, he receives value $\gamma_t$ in period t, and then becomes the seller in period

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$t+1$, receiving utility $\delta V\left(\gamma_{t+1}^{B_t}\right)$. Hence the period-$t$ willingness-to-pay of buyer type $\gamma_t$ is:

$$WTP\left(\gamma_t\right) = \gamma_t + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma_{t+1}\right)\mid\gamma_t\right]$$

Thus, in equilibrium, optimality for the buyer implies that the inverse demand function $p\left(\cdot\right)$ satisfies:

$$p\left(q\right) = \left\{p:\ P_{\gamma\sim F(\cdot)}\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right)\mid\gamma\right] > p\right] = q\right\} \tag{3}$$

Fixing $\tau$, a value function $V\left(\cdot\right)$ which satisfies Equations 2 and 3 defines an equilibrium of the dynamic Harberger tax game.

Under Assumptions 1–3, we can prove that the net trade property from the two-stage Harberger tax game applies exactly to the dynamic case: net sellers set prices above their continuation values, and net buyers set prices below their continuation values.

**Theorem 2.** *(Dynamic net trade property)*

*Under Assumptions 1–3, in any $\tau$-equilibrium of the dynamic Harberger taxation game, the optimal sale probability function $q^*\left(\gamma\right)$ satisfies:*

- *For type $\gamma$ with $\tau = 1 - F\left(\gamma\right)$, we have $q^*\left(\gamma\right) = \tau$ and $p\left(q^*\left(\gamma\right)\right) = \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right)\mid\gamma\right]$*

- *For types $\gamma$ with $\tau < 1 - F\left(\gamma\right)$, we have $q^*\left(\gamma\right) \geqslant \tau$ and $p\left(q^*\left(\gamma\right)\right) \geqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right)\mid\gamma\right]$*

- *For types $\gamma$ with $\tau > 1 - F\left(\gamma\right)$, we have $q^*\left(\gamma\right) \leqslant \tau$ and $p\left(q^*\left(\gamma\right)\right) \leqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right)\mid\gamma\right]$*

*Proof.* This follows from the stronger Claim 2 in Appendix Subsection A.3. □

If we also impose Assumption 4, we can prove that the dynamic Harberger tax game always admits a unique equilibrium.

**Theorem 3.** *For any $\tau$, $F$, $G$ satisfying our assumptions 1–4, there exists a unique equilibrium of the dynamic Harberger tax game.*

*Proof.* We prove this theorem in Appendix Subsection A.3, where we also describe a numerical procedure that solves for the unique equilibrium for any $\tau$. □

## 3.4 Investment

Suppose that at the beginning of each period $t$, the current license owner $S_t$ can make common-valued investment $\eta_t$ in the asset at cost $c\left(\eta_t\right)$. As before, we assume that all investments are fully observable to all agents. We will allow investments to have long-term effects on common values: suppose that, in period $t' > t$, investment $\eta_t$ increases the common value of the asset for all agents by some $H_{t'-t}\left(\eta_t\right)$. In Appendix A.4, we show that common-valued investment affects the equilibrium of the trading game by shifting all offered prices by some constant.

The social value of investment is the discounted sum:

$$\sum_{t=0}^{\infty} \delta^t H_t (\eta)$$

and, the social FOC sets:

$$c' (\eta) = \sum_{t=0}^{\infty} \delta^t H'_t (\eta).$$

The following proposition shows that the Harberger tax distorts longer term investments more than shorter-term investments. Intuitively, if license owners make investments that pay off t periods in the future, they have to pay taxes for $t+1$ periods on their investments, generating an investment wedge of $(1-\tau)^{t+1}$ relative to the social optimum.

**Proposition 2.** *In any $\tau$-equilibrium of the dynamic Harberger taxation game, all agents choose a constant level of investment $\eta$ such that:*

$$c' (\eta) = \sum_{t=0}^{\infty} \delta^t (1-\tau)^{t+1} H'_t (\eta)$$

*Proof.* See Appendix A.4. □


# 4 Calibration

In this section, we computationally solve our dynamic model under parameters chosen to match moments of various markets for durable assets.


## 4.1 Functional forms and moment matching

To complete the dynamic Harberger taxation game, we need to specify the distribution of entering buyer values $F(\gamma)$ and the transition probability distribution $G(\gamma' \mid \gamma)$. In addition, we need to choose the discount rate $\delta$, and the investment cost function $c(\eta)$ and benefit functions $H_t(\eta)$.

We use the standard choice of annual discount rate $\delta = 0.95$. We assume that the distribution of entering buyer values $F(\cdot)$ is log-normal, with log mean normalized to 0. The log standard deviation $\sigma$ serves the role of a spread parameter, controlling the amount of idiosyncratic dispersion in values. While this $F$ does not satisfy boundedness, as required by Assumption 1, in our computation we will approximate $F$ using a bounded grid distribution.

We will use two different choices for the transition distribution $G(\gamma' \mid \gamma)$. For our baseline specification, we use a smooth stochastic decay process. If an agent has value $\gamma_t$ in period t, her period $t+1$ value is $\chi\gamma$, where $\chi$ has a beta distribution with mean $\beta$. Thus, values decay by a

factor β in expectation in each period. We might think of this process as modeling, for example, the evolution of firms' idiosyncratic costs for utilizing natural resources, which should change gradually over time.

In some contexts, we might think that value decay is more volatile than our smooth decay model suggests; for example, we might think that private individuals' values for housing may be usually roughly constant, unless they decide to move, at which point their values jump downwards by a large amount. To model these contexts, we use a specification in which values drift downwards in expectation, but also have some probability ω of jumping to 0 in each period. We discuss these two transition processes in more detail, and report results from two other alternative specifications in Appendix B.3.

The parameters to be determined in the model are the log standard deviation σ of F, and either the decay rate β for the smooth transition process or the jump rate ω for the jump transition process. To determine these parameters, we will aim to match two empirical moments. First, we match the dispersion of buyer valuations to the dispersion of bids in various static auction settings. A large empirical literature studies static auctions for various usage rights for government resources; these papers tend to find fairly high dispersion in the willingness-to-pay of different buyers for identical assets. In particular, the ratio of the standard deviation of idiosyncratic buyer values to its mean is found to be roughly 0.5 for timber auctions (Athey, Levin and Seira, 2011), 0.18 for highway procurement contracts (Krasnokutskaya and Seim, 2011), and roughly 0.2 for oil drilling rights (Li, Perrigne and Vuong, 2000). We will thus require, conservatively for our estimates of welfare gains, that in our model that the standard deviation of the equilibrium willingness-to-pay of entering buyers is 0.20 times its mean.

Second, we match the average turnover rate of assets in private markets. We were unable to find appropriate data on the turnover rates of the resource usage rights we use for the valuation dispersion moment. However, Maksimovic and Phillips (2001) finds that roughly 5% of manufacturing plants change owners each year, and Emrath (2013) finds that private housing changes hands on average once per 13 years. We believe that private housing and manufacturing plants are likely to be traded infrequently relative to resource usage rights; hence, we view these turnover rates as likely to be lower than the corresponding numbers for assets such as resource usage rights, which are the main focus of our analysis. We will choose parameters to produce an annual turnover rate of 5% when the tax rate is set at $\tau = 0$.

Increasing the rate of value decay by either lowering β or increasing ω should increase the efficient probability of sale, the equilibrium probability of sale, and the efficient tax level. Increasing the lognormal standard deviation σ should increase the dispersion of values, the dispersion of prices, and the total achievable allocative welfare gains. Thus, intuitively, the $saleprob$ moment should be matched mostly by the decay parameters β or ω, while the $sdmean$ moment should be matched mostly by σ. We confirm these intuitions in Figure 4, which we discuss further in Subsection 4.2. However, since both parameters simultaneously influence the
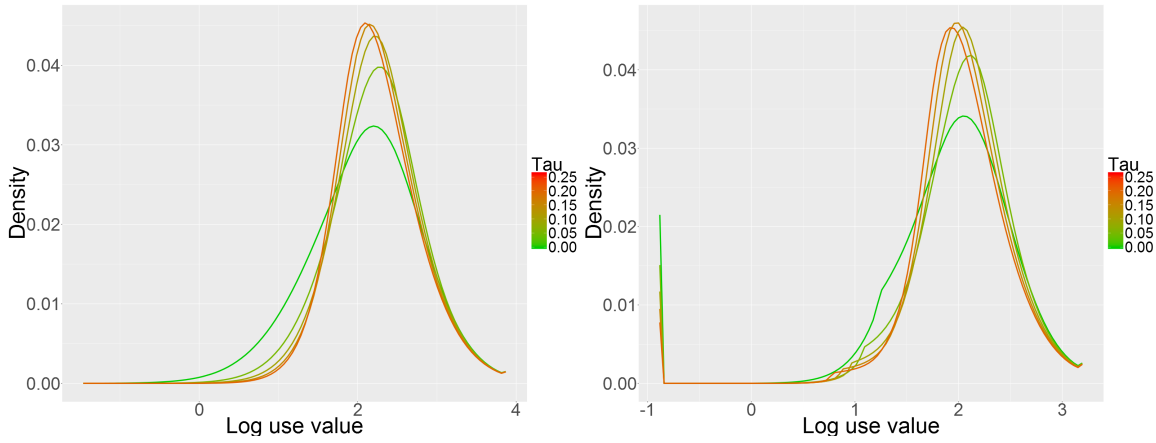
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Figure 2:  Stationary Use Value Distributions vs $\tau$, smooth G (left) and jump G (right)

probability of sale and the dispersion of prices, in each specification, we jointly choose $\sigma$ and either $\beta$ or $\omega$ to match the $\mathtt{saleprob}$ and $\mathtt{sdmean}$ moments.

We will assume that investment has geometrically depreciating value over time: $H_t(\eta) = \theta^t\eta$, $\theta < 1$. For our calibrations, we will set $\theta = 0.85$, which is similar to depreciation rates from in the literature on capital depreciation (Nadiri and Prucha, 1996). To pin down the total value of investment, we will use results from a large literature using either cross-sectional or regression-discontinuity to study the effects of property rights enforcement on asset values. This literature has found effects of property rights on either asset value or total productivity of roughly 0.2 for water usage rights (Leonard and Libecap, 2016), between 0.01 and 0.7 for agricultural land (Goldstein and Udry, 2008; Jacoby, Li and Rozelle, 2002), and roughly 0.4 for private housing (Galiani and Schargrodsky, 2010). In our baseline specification, we will assume that investment value constitutes a fraction 0.4 of total average asset value. We will also report quantitative results assuming that the fraction of investment value is either 0.1 or 0.7. In Appendix B.1, we derive analytical expressions for the equilibrium investment level and investment welfare for any value of $\tau$. In Appendix B.2, we describe further details of the numerical procedure we use to analyze the game and solve for equilibria.

## 4.2   Results

In Figure 2, we plot the equilibrium stationary distribution of use values for different values of $\tau$ under each transition process. For both transition distributions, as we increase $\tau$ from 0 to 0.15, probability mass moves from relatively low F-quantiles towards higher values, as a result of lower markups and increased frequency of sales to high-value entering buyers. However, starting at around .05, increasing $\tau$ also moves mass from the highest values towards somewhat lower values, though this effect does not become pronounced until $\tau$ reaches .15. This is likely because the highest value license owners set prices below their values, causing the license to occasionally be purchased by buyers with values lower than that of the owner. The main difference between

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the transition processes is that the smooth process leads to roughly hump-shaped stationary distributions, whereas the jump process leads to stationary distributions with a spike at use value 0. Intuitively, Harberger taxation should perform well when most agents have similar values, as a time-constant tax rate is near-optimal for most agents in steady state. For the jump process stationary distribution, implementing fully efficient trade would require charging a 100% tax to low-value agents and a much smaller tax to high value agents; this will not be approximated well by any constant Harberger tax rate, and we might expect Harberger taxation to perform less well in such settings, though a small tax should still have first-order welfare gains, which we observe from the large reduction of mass at very low values when the tax rate moves from 0% to 5%.

In Figure 3, we show the behavior of various quantities in stationary equilibrium as functions of $\tau$ for the smooth transition process, assuming investment is 40% of total asset value. The topmost panel shows allocative, investment and total welfare. Allocative welfare is maximized at approximately $\tau = 0.07$, which optimizes the trade-off between moving mass away from low value quantiles and away from the highest quantiles. The horizontal line labelled $eff\_alloc\_welfare$ represents the max possible allocative welfare, calculated by finding the steady-state distribution of use values assuming that the asset is always transferred to the agent with higher value in any period. The allocatively optimal tax rate $\tau_{alloc}$ achieves over 70% of the total possible allocative welfare gains.

If we take into account investment welfare, the optimal tax rate is $\tau = 0.035$, and this increases total welfare by 4.1%. As in Figure 1 in the two-stage model, total welfare is concave in the tax rate. Investment losses are not globally convex, likely due to the complex interactions of taxation with persistent investment. However, allocative welfare is still concave, and thus total social welfare is also concave in $\tau$ for tax rates below and near the efficient tax rate.

The second panel shows the equilibrium sale frequency and the average quantile markup set by sellers, as well as a line of slope 1 representing the tax $\tau$ itself. When the tax is set equal to the efficient probability of trade, the equilibrium trade probability is also equal to the average tax rate, and the average quantile markup is near 0. However, the optimal tax rate is lower than the efficient probability of trade, likely because the right-skew of the lognormal distribution means that the losses from excessive turnover by high value sellers outweigh the gains from eliminating inefficiently low turnover rates by low value sellers.

In the third panel of Figure 3 we show the behavior of various stock/flow quantities as we vary $\tau$. License prices rapidly decrease and tax revenues rapidly increase as we increase $\tau$. Intuitively, if agents have to pay tax $\tau$ every period, this is roughly equivalent to discounting by rate $\delta(1-\tau)$; thus, increasing $\tau$ has a similar effect to increasing discounting, and rapidly lowers license prices.

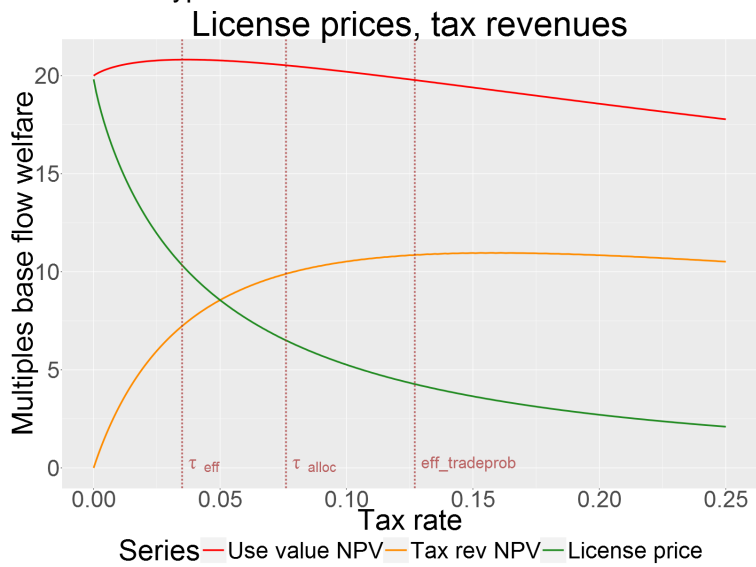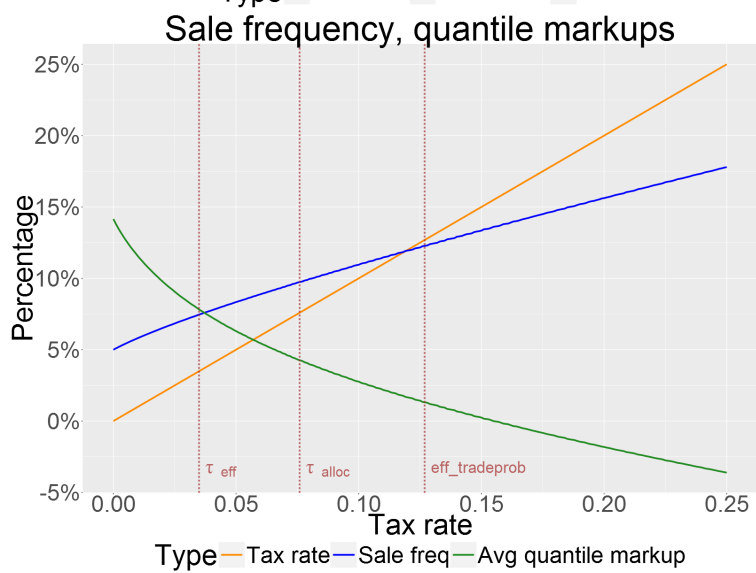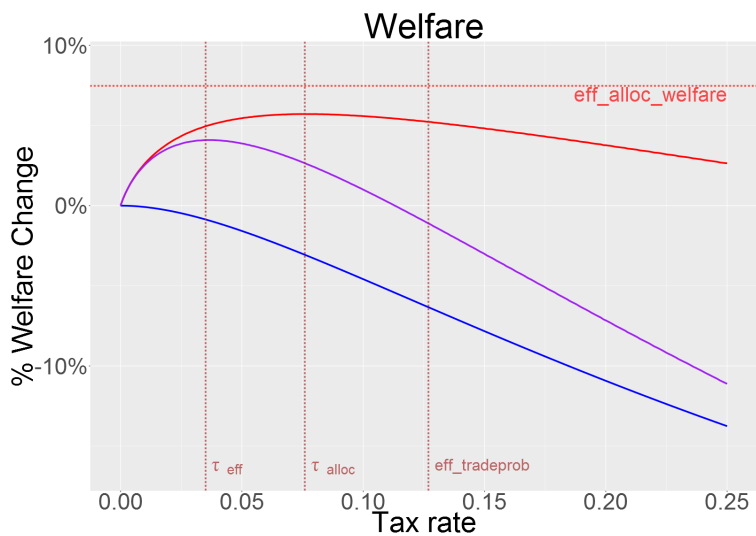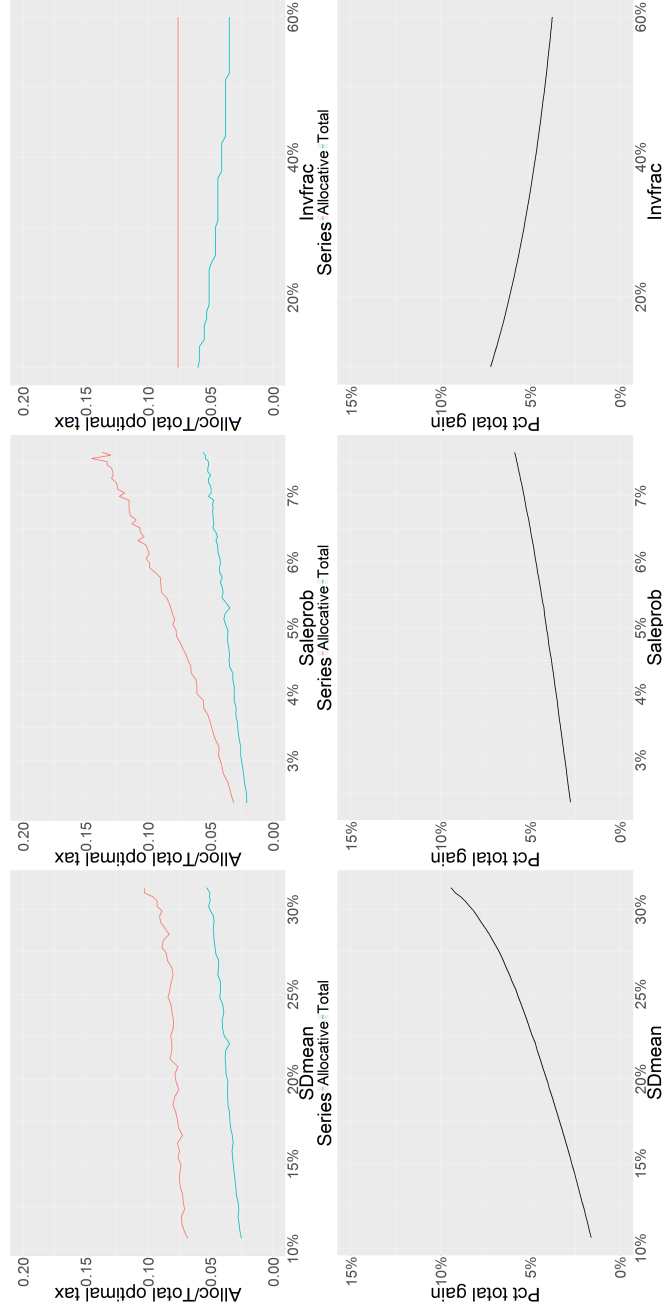Figure 3:  Comparative statics vs τ

Figure 4: Sensitivity to moments

Table 1: Calibration results

| Transition process | Invfrac | Optimal $\tau$ | Total gain | Alloc gain | Inv loss | 2.5% tax gain |
|---|---|---|---|---|---|---|
| Smooth | 0 | 0.076 | 8.69% | 8.69% | 0.00% | 6.60% |
| Smooth | 0.1 | 0.063 | 7.38% | 7.92% | -0.54% | 5.96% |
| Smooth | 0.4 | 0.035 | 4.09% | 4.95% | -0.86% | 3.87% |
| Smooth | 0.7 | 0.02 | 1.51% | 2.11% | -0.59% | 1.45% |
| Jump | 0 | 0.054 | 5.61% | 5.61% | 0.00% | 4.79% |
| Jump | 0.1 | 0.046 | 4.79% | 5.11% | -0.32% | 4.30% |
| Jump | 0.4 | 0.029 | 2.69% | 3.31% | -0.62% | 2.68% |
| Jump | 0.7 | 0.015 | 1.01% | 1.35% | -0.35% | 0.81% |

*Notes.* All gains are in units of percentages of total welfare at $\tau = 0$. All columns show welfare changes from the optimal $\tau$, except for the last column, which shows the total welfare gain from imposing a 2.5% tax.

In Figure 4, we vary the input moments used in the calibrations for the smooth transition process, and show that optimal tax rates and total welfare gains depend on input moments in intuitive ways. Changing the sdmean moment moves the total welfare gain, with a relatively small effect on the optimal tax rate. Changing the saleprob moment moves the optimal tax rate, with a smaller effect on welfare gains. Changing the invfrac moment lowers the total welfare-optimal tax rate relative to the allocatively optimal tax.

In Table 1, we show results from the smooth and jump transition processes, for different values of the total investment fraction. The optimal Harberger tax rate ranges from 1.5% to 7.6%. Total gains from Harberger taxation range from 1% to 7%. Harberger taxation performs better under the smooth transition process than the jump process; again, this is likely because the increased dispersion in the stationary distribution of values under the jump process makes a constant Harberger tax level less effective. In all cases, setting the tax rate equal to 2.5%, or half the asset turnover rate in existing markets, achieves most of the welfare gains (always at least 75%) from the optimal tax and achieves positive and economically quite large welfare gains under all parameter settings we tried.

We suggest two main takeaways from our calibrations. First, Harberger taxation works best in settings where use values evolve smoothly over time, and thus most asset owners in stationary equilibrium have values which are fairly close in quantile space. In such settings, Harberger taxes give most asset owners approximate incentives for truthful value revelation, and thus can achieve a large fraction of all possible allocative welfare gains. In contrast, when the stationary distribution of values is more disperse, any constant Harberger tax rate is fairly far from optimal for most asset owners in stationary equilibrium.

Thus, we suggest that Harberger taxation is more suitable for corporate assets such resource usage rights, for our smooth decay process might be an appropriate model, rather than assets such as private housing or other personally owned goods, for which our jump process may be more appropriate. However note that even in the case to which it is less well-suited, Harberger

taxation of 2.5% robustly achieves large gains in welfare, just a smaller fraction of the gains the could be achieved by an optimal mechanism than in the smoother case. Furthermore, because research on housing suggests that investment plays a smaller role in housing than in many corporate assets, total gains from Harberger taxation may not be too different between these contexts. Glaeser and Shapiro (2003) find roughly a 25% investment value share in their analysis of the US housing market, which would imply welfare gains of 3.5% in the housing context to which, for other reasons (such as liquidity constraints and pre-existing ownership), Harberger taxation may be less easily suited.

Second, we suggest the following rule of thumb for designing Harberger tax systems: tax rates should be set at about half the observed trade rates in markets for similar privately owned assets. From Table 1, across most specifications, the optimal tax rate is at or slightly below the private market trade rate of 5%, but a rate half this achieves most welfare gains and robustly achieves positive gains; a 5% tax leads to very small welfare losses for some extreme parameter values. Such a rule is likely to be very conservative in terms of allocative welfare gains from Harberger taxation – since the probability of trade in tax-free equilibrium is lower than the socially optimal probability of trade, this rule will tend to choose values of $\tau$ lower than the allocatively efficient tax rate and by halving this we are very likely to therefore more than fully adjust for the trade-off with investment incentives. Since total welfare is increasing and concave in $\tau$ for tax rates between 0 and the optimal tax rate, any tax level smaller than the optimal value is welfare-improving relative to pure private ownership, and fairly small taxes can capture a large fraction of all possible welfare gains from Harberger taxation.

# 5 Extensions

In this section, we consider several extensions that investigate the robustness of our analysis and enrich it in various directions. For simplicity, all these extensions build off of the two-stage model of Section 2. In Subsection 5.1, we show that if the community is able to observe and directly incent the seller to make common-valued investments, it can alleviate the investment efficiency losses from Harberger taxation, leading to higher optimal tax levels. In Subsection 5.2, we show that increasing competition on the buyer side lowers the optimal level of the Harberger tax. In Subsection 5.3, we show that Harberger taxation does not distort pure private-valued investments.

## 5.1 Partial observability

In some cases, governments may be able to observe and directly reward capital investments. After all, the leading property of common-valued investments is that they affect the objective value of the asset to all individuals, and not just the idiosyncratic value of the seller. If some mix

of objective appraisal and various incentive-compatible elicitation mechanisms could provide at least a noisy signal of capital value, the public may be able to directly reward investments through a tax deduction for investment value, thus counteracting some of the investment distortions from Harberger taxation. As a result, the optimal Harberger tax rate will rise towards the allocatively efficient tax level. In this subsection, we formalize these intuitions by following the analysis of Baker (1992) to construct optimal direct property subsidies to mitigate the negative effects of Harberger taxation on investment incentives.

As in Section 2, suppose that $S$ chooses common-valued investment $\eta$ at cost $c(\eta)$. However, now suppose common value $\upsilon$ is determined by

$$\upsilon = \zeta \eta,$$

where $\zeta$ is a random variable representing the value of investment in different states of the world. There is *local information* about $\zeta$; that is, $S$ and $B$ both observe $\zeta$ prior to investment, but the public does not.[5]

The community can observe $\eta$ and a signal $\xi$. Prior to period 1, the community can choose an *incentive scheme* $\eta \psi(\xi)$, meaning that if the community observes $\xi$, it will pay $S$ some amount $\psi(\xi)$ for each unit $\eta$ of investment $S$ makes. This policy is simply a negative property (property subsidy) based on an objective appraisal of $\upsilon$.

Fix a realization of the signal value $\xi$. If the community chooses reward function $\psi(\xi) \eta$, and the Harberger tax rate is $\tau$, investment level $\Gamma(\psi(\xi) + (1-\tau)\zeta)$ will be induced. For expositional simplicity, we now focus on the case when costs of investment are quadratic and thus $\Gamma$ is linear: $\eta(\zeta) = g\zeta$ for some $g > 0$, or, equivalently, that cost is $c(\gamma) = \frac{\gamma^2}{2g}$. Baker (1992) uses Taylor approximations to show that similar conclusions hold for more general investment cost functions.

Given any choice of $\psi(\xi)$, $S$ chooses investment:

$$\Gamma(\psi(\xi) + (1-\tau)\zeta) = g(\psi(\xi) + (1-\tau)\zeta).$$

Hence, for a fixed $\tau$, the optimal $\psi$ solves pointwise over realizations of $\xi$ the maximization problem:

$$\max_{\psi} \mathbb{E}\left[ g(\psi(\xi) + (1-\tau)\zeta)\zeta - \frac{(g(\psi(\xi) + (1-\tau)\zeta))^2}{2g} \mid \xi \right].$$

---

[5]An alternative formulation of this model is that the cost of investment is uncertain and is known to the seller and buyer, but the community only observes a noisy signal of the cost. Although this interpretation is more natural in many settings, we focus on the investment value interpretation to stay closer to Baker's analysis and because it is simpler to present.

This program has the simple linear solution $\psi(\xi) = \tau \mathbb{E}(\zeta \mid \xi)$, which induces investment

$$g(\tau \mathbb{E}(\zeta \mid \xi) + (1 - \tau)\zeta) = g(\mathbb{E}(\zeta \mid \xi) + (1 - \tau)(\zeta - \mathbb{E}(\zeta \mid \xi))).$$

Investment is thus equal to the conditional expectation of investment value conditional on the signal $\xi$, plus a multiple $1 - \tau$ of the deviation $\zeta - \mathbb{E}(\zeta \mid \xi)$ from the conditional mean. For higher values of $\tau$, investment is closer to the conditional mean.

The social welfare loss from this noisy estimation is

$$\text{IVL} = \frac{\tau^2}{2} g \mathbb{E}\left[\zeta^2 - (\text{E}(\zeta \mid \xi))^2\right] = \frac{\tau^2}{2} g \left(1 - r^2\right) \text{Var}(\zeta),$$

where $r^2 = \mathbb{E}\left[(\mathbb{E}(\zeta \mid \xi))^2\right]$ is the fraction of the variance in $\zeta$ that is predictable by $\xi$. If we take the derivative with respect to $\tau$, we get

$$\frac{\text{dIVL}}{\text{d}\tau} = -\tau g \left(1 - r^2\right) \text{Var}(\zeta). \tag{4}$$

As $r^2$ increases, $\frac{\text{dIVL}}{\text{d}\tau}$ decreases in magnitude, and the socially optimal choice of the Harberger tax given by equation

$$\frac{\tau^\star}{1 - \tau^\star} = \frac{(M - \gamma_S)\rho}{\tau g (1 - r^2) \text{Var}(\zeta)}$$

moves closer to the allocatively efficient level $(1 - F(\gamma_S))$. Thus, to the degree that the public can observe and reward capital investment, the detrimental effect of Harberger taxes on investment efficiency diminishes, and the optimal Harberger tax level is higher.

These arguments capture some of Hayek (1945)'s intuition that local knowledge is what limits the prospects of common ownership, and is consistent with Lange (1967)'s argument that the improvement of observation and computation through the improvement of information technology would increasingly make common ownership of the means of production feasible.

## 5.2   Many buyers

Our analysis above assumes that there is a single potential buyer of the license. In many realistic cases, several bidders may be competing to buy the license. In such settings, we might implement Harberger taxation by allowing potential buyers to participate in an auction for the asset, with reserve price equal to the value announcement of the current asset owner. Here, we analyze optimal Harberger tax rates in such an auction model.

Suppose the license belongs to the $S$, and assume for simplicity that $\gamma_S = 0$. There are multiple potential buyers $B_1 \ldots B_n$, with values drawn i.i.d. from distribution $F$. The license is sold in a second-price auction, where $S$ can set a reserve price $p$. $S$ pays a tax on the reserve price $p$. Let $y_1$ represent the highest bid, and let $y_2$ represent the second-highest bid. $S$'s objective

function is

$$\pi_S = y_2 1_{y_1,y_2>p} + p 1_{y_1>p>y_2} + \eta 1_{y_1,y_2<p} - p\tau.$$

Taking expectations over $y_1, y_2$ and then taking derivatives with respect to $p$ yields

$$\frac{d\mathbb{E}\left[\pi_S\right]}{dp} = P\left(y_1 > p > y_2\right) - \tau - m\frac{dP\left(y_1, y_2 < p\right)}{dp},$$

where, as in Section 2, we define the markup $m \equiv p - \eta$. Substituting for the probability expressions, the derivative becomes

$$\frac{d\mathbb{E}\left[\pi_S\right]}{dp} = nF^{n-1}\left(p\right)\left(1 - F\left(p\right)\right) - \tau - mnF^{n-1}\left(p\right)f\left(p\right).$$

If we set this to 0, we get

$$\tau = nF^{n-1}\left(p\right)\left(1 - F\left(p\right)\right) - mnF^{n-1}\left(p\right)f\left(p\right). \tag{5}$$

Allocative efficiency is achieved when $p = \eta$ and thus $m = 0$, which requires

$$\tau = nF^{n-1}\left(\eta\right)\left(1 - F\left(\eta\right)\right).$$

As $n \to \infty$, this expression goes exponentially to 0. Thus, the allocatively optimal Harberger tax goes to 0 as competition grows. This conclusion is intuitive, given the follow-up to Jevons (1879)'s quote in our epigraph:

> *But when different persons own property of exactly the same kind, they become subject to the important Law of Indifference...that in the same open market...there cannot be two prices for the same kind of article. Thus monopoly is limited by competition, and no owner, whether of labour, land, or capital, can, theoretically speaking, obtain a larger share of produce for it than what other owners of exactly the same kind of property are willing to accept.*

Larsen (2015) confirms this intuition empirically, showing that the welfare loss from asymmetric information in a fairly competitive auction market for used automobiles is only 2%-4% of first-best allocative efficiency. Thus optimal Harberger tax rates will tend to be lower in more competitive environments.

## 5.3 Selfish investments

Thus far, we have assumed the seller's investment only affects the common value of the good. Here, we show that if the seller can make private-value investments, which affect only her own value for the good, these investments are efficient conditional on the final probability that the

owner keeps the license. This efficiency guarantee is true regardless of the level of the tax, which is why we largely ignore such selfish investments in our analysis above.

Suppose that S can invest in increasing her private value for the good: she can increase her own use value for the good by $\lambda$ at cost $c(\lambda)$. As before, B has value $\eta + \gamma_B$ for the good, and all other features of the game are identical to those in Section 2. Fixing $\gamma_S$ and $\lambda$, S's second stage profits are once again:

$$\pi_S(\lambda, \gamma_S, \tau) \equiv \max_q p(q) q + (\eta + \gamma_S + \lambda)(1 - q) - p(q) \tau.$$

Let $q^*(\tau, \gamma_S)$ represent S's choice of $q$ for any given $\tau, \gamma_S$. In the investment stage, S chooses $\lambda$ to maximize $\pi_S(\lambda, \gamma_S, \tau) - c(\lambda)$. But, using the envelope theorem, we have that

$$\frac{d\pi_S(\lambda, \gamma_S, \tau)}{d\gamma} = \frac{\partial}{\partial \lambda}\left[p(q^*(\tau, \gamma_S)) q^*(\tau, \gamma_S) + (\eta + \gamma_S + \lambda)(1 - q^*(\tau, \gamma_S)) - p(q^*(\tau, \gamma_S)) \tau\right]$$

$$= 1 - q^*(\tau, \gamma_S).$$

Hence, the first-order condition for S's choice of private-valued investment $\lambda$ is

$$c'(\lambda) = 1 - q^*(\tau, \gamma_S).$$

This equation defines the constrained efficient level of investment, conditional on S keeping the license with probability $1 - q^*(\tau, \gamma_S)$. Thus, while Harberger taxes do decrease the propensity of license owners to make private-valued investments, they do so efficiently. Harberger taxation causes license owners to set lower prices and sell their licenses more often; license owners correspondingly reduce private-valued investments, in a manner which is both privately and socially desirable.

A few more informal observations are in order:

1. We might also consider investments by license owners that affect the value of the asset to potential buyers, but not to the owners themselves. A natural objection to self-assessed taxation is that license owners who are unwilling to sell their assets will set low prices to minimize tax payments, but then purposefully damage their assets, making them less attractive to buyers in order to deter purchase. However, at optimal or rule-of-thumb tax rates, $\tau$ is smaller than the probability of sale $q^*(\tau, \gamma_S)$ for most license owners. Thus, the majority of license owners are net sellers of their assets in any given period; if marginal buyers' values increase, license owners gain more from increased sale prices than they lose from increased Harberger tax payments. Thus most license owners have net positive incentives to make investments which increase the value of the asset only for potential buyers. Another implication of this fact is that, under reasonable Harberger tax rates, most license owners set prices above their values, and thus receive higher total utility

from selling their assets than keeping them. Thus appropriately designed Harberger taxes induce very few regretful sales of assets – in market equilibrium, most trades make both buyers and sellers better off.

2. Certain classes of assets, such as jewelry or other personal items, may have relatively homogeneous low values to all potential owners ex ante, deriving most of their value from owners' private-valued emotional attachments developed over time. In such contexts, both the socially efficient probability of asset trade and the turnover rates of assets in market equilibrium will tend to be low. Rule-of-thumb choices of Harberger tax rates will be correspondingly low. While Harberger taxation cannot greatly improve allocative efficiency in such contexts, it will not lead to serious adverse effects on the efficiency of asset trade. Rule-of-thumb Harberger taxes thus naturally adapt to the primitives of asset markets, playing a large role only when high market turnover rates suggest that efficient dynamic reallocation is a first-order concern.

3. In a world in which Harberger licenses and common ownership were ubiquitous, the overall level of private-valued investments that individuals make in assets that they use may decrease significantly. For example, if most cars were rented or held under Harberger licenses, individuals would tend to expend less resources personalizing the cars that they use, and to grow less emotionally attached to their cars. We have argued that this is an efficient response to the lowered probability of long-term usage of assets. Many religious and social thinkers, especially from the Buddhist tradition, have suggested that capitalism leads individuals to develop excessive attachments to material possessions. Our arguments suggest that these attachments are not fundamental to market-based systems, but are instead tied to the private ownership of property and the frictions that it creates for efficient exchange. The relationship between individuals and the material assets that they use may develop quite differently in market economies augmented with alternative systems of property ownership.

# 6 Connections

In this section, we relate our proposal to previous economic analysis and practices related to property rights in mechanism design, asset taxation, and intellectual property.

## 6.1 Other mechanisms

As we discuss in the introduction, this paper is inspired by a body of work that analyzes the role of property rights in asymmetric information bargaining problems (Cramton, Gibbons and Klemperer, 1987; Segal and Whinston, 2011). The mechanisms discussed in this literature

tend to be complex, involving entry fees from both sellers and buyers calculated based on the unobserved distributions of agents' values followed by payments based on the rule proposed by Vickrey (1961), Clarke (1971) and Groves (1973). We view the main benefit of Harberger taxation relative to this body of work as its practicality. Harberger taxation is fairly simple to describe and implement; it takes the form of a simple tax based on announced values, and thus requires license administrators only to interact directly with current license owners, rather than all potential license buyers. Harberger license design only involves a single tuning parameter, the tax rate, and the license designer can choose this rate appropriately using observed asset turnover rates independently of the unobserved distributions of buyer and seller values. We note, however, that Harberger taxation does not achieve the full efficiency that is achieved by the more complex bargaining protocols described by Cramton, Gibbons and Klemperer and Segal and Whinston. We view Harberger taxation as a first step towards designing practical partial property rights systems; we leave the question of whether more sophisticated schemes admit similarly simple implementations to future research.

Our model also differs from other dynamic extensions of bilateral trade models (Athey and Miller, 2007; Skrzypacz and Toikka, 2015) which focus on repeated strategic interactions between a fixed set of individuals. In contrast, our overlapping-generations trade model allows us to study dynamic properties in stationary trading equilibrium, such as license prices and turnover rates, while abstracting away from the technical difficulties associated with repeated strategic interactions. An important benefit of this approach is that our model is quite tractable, giving point predictions about stationary trading equilibria, in contrast to previous work on dynamic trade, which in the style of mechanism design, characterizes a set of feasible outcomes. We believe that our model is well-suited to markets for a number of assets, such as natural resources or radio spectrum, where there are enough agents in the market that it is reasonable to abstract away from repeated interactions between any given pair of agents. However, studying the effects of Harberger taxation in settings where repeated strategic interactions are important appears to us to be a valuable direction for further research.

Another mechanism that could avoid market power distortions and thus obviate the need for common ownership is an approximation to the "counter-speculation" subsidies that Vickrey (1961) proposed. In particular, a subsidy in the amount $\frac{M'(1-F(\gamma_S))(1-F(\gamma_S))}{2}$ that is paid to the seller if and only if a sale takes place has the same effect as as an allocatively efficient Harberger tax. Such a subsidy could conceivably be implemented without distorting investment incentives and thus could potentially be superior to an optimal Harberger tax.

We are concerned, however, that such a scheme would be impracticable for a variety of informational reasons. First, it requires knowing the value distribution, but without a simple means to iteratively calculate the value, because it also depends on the value of $M'$. Second, $M'$ is particularly difficult to measure and requires a lot of the planner, especially given its value could be significantly context dependent in a way known to the seller but not to the cadastral

authorities; see Weyl and Tirole (2012) for a detailed related discussion. Third, and perhaps most importantly, the scheme would be open to tremendous manipulation. Two-way sales could take place in succession and generate net subsidies to the participants. Finally, and perhaps most importantly, although this scheme would avoid common ownership in some sense, it would involve much more discretionary official intervention than would a Harberger tax. We therefore do not consider it a credible or less radical alternative.

## 6.2   Asset taxation

Discussion of self-assessment systems for taxation and other purposes dates back to at least ancient Rome (Epstein, 1993). However, through most of its history, self-assessment has been thought of a mechanism for assessing the value of goods for the purposes of equitably raising revenue. It thus resembles a number of other schemes for using market outcomes to assess the values of assets for tax purposes; a similar example is the common practice of setting property taxes based on the most recent sale price of a house.

A problem common to this class of market-based valuation schemes is that nontrivially large taxes based on market outcomes influence the behavior of market participants. For example, setting property taxes based on previous sale prices creates incentives for agents to hold on to property inefficiently long when asset values are rising, in order to avoid paying increased taxes. In the case of self-assessed taxation, previous authors have noted that higher tax rates lower value announcements and increase asset turnover rates. Viewed as an instrument for raising revenue, this appears to be an undesirable side effect of self-assessment. Levmore writes that "It is perhaps unfortunate that these... effects to self-assessment [on turnover rate] exist," and other critiques of self-assessed taxation (Epstein, 1993; Chang, 2012) also highlight the undesirable effects of self-assessment on value assessments and market outcomes.

Our proposal essentially inverts the classical argument for self-assessment. Rather than using information from market transactions to more effectively tax assets we propose to apply taxes on assets to increase the efficiency of the markets. We show, through variants of the net trade property, that the side effects of self-assessment on value announcements can be thought of in terms of the price-setting incentives of a net trade who buys share $\tau$ and sells share $q$ of an asset in each period. Thus, market participants on average announce prices above or below their values depending on whether the Harberger tax rate is lower or higher than the probability of asset turnover that it induces. Harberger taxation is thus difficult to use as an instrument for value revelation, as it cannot simultaneously incent all license owners to announce their valuations truthfully; however, any positive Harberger tax rate lower than the average turnover rate it induces will tend to reduce markups and increase allocative efficiency in stationary trading equilibrium. Thus, we suggest that self-assessed taxation is better suited, and easier to apply, as a tool for increasing the efficiency of asset allocation, compared to its historical role as

a system for assessing asset values for tax purposes.

We are not the first to suggest that certain forms of taxation can increase the efficiency of asset allocation, although such proposals are fairly scarce. To our knowledge, Jevons (1879) and Walras (1896) were the first to suggest that common ownership could improve allocative efficiency. This idea was further explored by George (1879), who argued that common ownership of land could be implemented by taxing away rents. Beyond the revenue it raised, he suggested this would also improve the efficiency of allocation by breaking up wasteful "feudal" land holdings. More recently, Tideman (1969) highlights how self-assessed taxes might increase turnover of property. However, he does not consider the impact of self-assessed taxation on investment or explicitly model allocative efficiency. To our knowledge, we are the first to explicitly study the effects of self-assessed taxation on the efficiency of asset allocation and investment in stationary trading equilibrium.

Throughout the paper, we have largely ignored the revenue raised by self-assessed taxation. Since the primary motive for Harberger taxation is to increase allocative efficiency, the manner in which the government uses the revenues thus raised is largely inconsequential. Thus, our analysis is largely orthogonal to most other rationales for capital taxation explored in the literature. These include local property taxes as a source of funds for local public goods provision (Lindahl, 1919; Bergstrom, 1979; Arnott and Stiglitz, 1979), a mechanism for dynamic redistribution (Judd, 1985; Chamley, 1986; Golosov and Tsyvinski, 2015), a mechanism for governments without commitment power to avoid the temptation of appropriative redistribution (Farhi et al., 2012; Piketty, 2014; Scheuer and Wolitzky, 2016) and a tax on wealth as a consumption good Saez and Stantcheva (2016). In some cases, capital taxation using Harberger taxation can simultaneously accomplish some of the goals outlined in these works; in other cases, our tax will tend to create an excessive wedge along these dimensions that should be compensated by a subsidy (e.g., on savings) to ensure wedges are of optimal size.

## 6.3   Intellectual property

Since intellectual property is non-rivalrous in consumption, the investment-allocation tradeoff from property rights is particularly clear: the socially optimal allocation is to allow all parties to use all innovations at no cost, but such a system gives agents no inventives to invest in developing innovations. A fairly large literature has addressed the question of optimal ownership rights over intellectual property, largely finding that partial ownership systems, such as limited-term patents, are optimal. In a sense, our argument in this paper is that a similar allocative-investment tradeoff from property rights is relevant for many assets which are rivalrous in consumption.

Directly applying systems like Harberger taxation to intellectual property is challenging for a number of reasons. First, rewarding investment is substantially more complicated in the nonrival case. In this paper, we only considered the impact of Harberger taxation on a simple

type of investment, one which induces a uniform increase in values across agents. By contrast, Weyl and Tirole (2012) argue that inventions differ along multiple dimensions in terms of the market for the products they produce. Although physical property usually has a tangible value and easily observable investments, the value of intellectual property is usually not apparent until a long process of marketing, adoption, and market testing has sorted out its value-added.

To make matters worse, charging a Harberger tax as a fraction of the total value of intellectual property requires knowing the total size of the market for the product if it were offered for free, so that the tax can be applied as a fraction of this total size. We have shown that the equilibrium asset turnover rate, which is bounded between 0 and 1, serves as a robust observable signal for the extent of allocative distortions for rival goods. Unlike the turnover rate the value of market sizes will vary by orders of magnitude for observably similar products. For example, many apps on Apple's App Store receive only a few downloads, whereas others "go viral" and are downloaded a billion times. Without knowledge of this market size (which almost solves the problem itself, because a prize could be given directly), a Harberger tax would likely be laughably small for some markets while leaching all profits out of others.[6] Thus while Harberger taxation shares motivational similarities with temporal and breadth limitations on intellectual property rights, it is not easily applicable to intellectual property as such. More appropriate are more centralized schemes that rely more heavily on central administrators, such as those proposed by Kremer (1998) and Weyl and Tirole (2012).

# 7  Applications

In this section, we discuss three categories of applications of Harberger taxation. The first concerns license design for a class of government-owned natural resources, many of which are currently assigned by auctioning off term-limited use licenses. The second is within the private sector, concerning partial property systems "sharing economy" platforms as well as systems for asset sharing within firms. Finally, we discuss a broader implementation that could be applied economy-wide and would trade off investment incentives and allocative efficiency as in our calibrations above. Our discussion here is relatively brief; see Posner and Weyl (Forthcoming) for a detailed discussion of these applications and the relationship to existing legal institutions.

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[6]If a public authority knew the efficient market size (call it $\sigma$) for a good, our scheme would be equivalent to setting a tax equal to $p\sigma\tau$, where $\tau$ is the Harberger tax as previously and $p$ is the price chosen by the monopolist. The required level of Harberger taxation to achieve taxation would be $\tau = 1$ (but would all eliminate monopoly profits and thus innovation incentive). A lower tax would still incent lower prices than pure intellectual property, but the innovator would be left with some rents, implementing the trade-off we analyzed above.

Even if such a scheme could be implemented, it would involve a substantially worse trade-off than with physical property, because the allocatively efficient tax is so high. However, such a system seems impractical given the difficulty of estimating $\sigma$. For example, a $\sigma$ estimated at 1,000 would have no appreciable impact on the bottom line, and therefore prices of a product that ended up having a mass market. On the other hand, it would drive out all profits even at a modest 10% tax rate for a product with niche appeal to only a hundred clients.

## 7.1 Resource rights

We believe that Harberger taxation is well-suited as a license system for usage rights over publicly owned natural resources. Usage licenses for many kinds of natural resources, such as timber (Baldwin, Marshall and Richard, 1997; Athey and Levin, 2001), oil and gas drilling rights (Porter, 1995), and wireless spectrum (Milgrom, 2000), are currently allocated to private firms using auctions. A large body of recent work has been devoted to improving the design of static auctions for efficiently allocating these resource licenses to firms; comparatively little work has analyzed the optimal design of these usage licenses. At present, most usage rights over resources take the form of fixed-term licenses. While auctions of such licenses may lead to statically optimal allocations, these licenses can inefficiently inhibit future trade, distorting the dynamic efficiency of asset allocation. These distortions are quite visible in practice; the complexity of the recent FCC incentive auction (Milgrom and Segal, 2015) derives in large part from the fact that it must simultaneously purchase licenses from existing spectrum owners and resell them to new buyers. If governments instead sold resource usage rights under Harberger licenses, firms that win licenses in static auctions would set lower markups in future periods, increasing trade frequency and thus dynamic allocative efficiency without further centralized organization.

An alternative method to Harberger taxation is to shorten the length of term limits and run auctions more frequently. While Harberger licenses behave qualitatively similar to shorter term limits, we highlight a few advantages of our proposal. Firstly, term limits function by periodically removing all future ownership stake in the asset from license owners. At the end of her term limit, the license owner has no incentives to maintain the common value of the asset. Consider a firm who owns a relatively short term-limit license for a fishery or oilfield; towards the end of the term limit, it has no ownership stake in the asset, and thus will tend to inefficiently overextract fish or oil relative to the social optimum. In contrast, even allocatively optimal Harberger tax rates are fairly low – 7% annually in our calibration – so Harberger license owners retain fairly large incentives for making common-value investments.

Secondly, Harberger licenses induce property rights which are stationary over time, which tends to outperform systems such as term limits which function by infrequently reducing property rights by large amounts. We discussed in Subsection 2.4 that social welfare tends to be concave in the Harberger tax level in the simple two-stage model, and Figure 1 shows that welfare is concave in tax in our dynamic calibration. Intuitively, if we set high taxes in one period and lower taxes for the next few periods, markups are high in most periods; while most trades will happen in the few high-tax periods, we will lose many valuable trades from high-value buyers and low-value sellers who happen to meets during low-tax, high-markup periods. With time-constant lower taxes, in each period some less valuable trades are lost, but buyers of sufficiently high value and sellers of sufficiently low value will be able to trade regardless of when they arrive to the market. Similar arguments show that small stationary

distortions to investment incentives are generally preferable to occasional large distortions.[7]

Harberger licensing also smooths the income that governments receive from resource rights. Instead of a large lump-sum payment from the auction of term-limit licenses, governments receive a smaller lump-sum from the auction of Harberger licenses, and a flow of Harberger tax payments over time. In our calibration, we showed that the total net present value of the license price and tax payments are not significantly affected by the tax rate. From the perspective of potential license buyers, Harberger licensing also decreases the budget and liquidity requirements for participating in markets for resource licenses, relative to long-term or perpetual licenses. In markets for perpetual ownership or long term-limit licenses over assets, license prices are approximately equal to the discounted sum of use values from the asset, hence firms that may want to use the asset only for short periods of time must nonetheless have large amounts of capital at hand to purchase and then resell these licenses. Harberger licenses have much lower prices than perpetual ownership for the same assets, so the capital outlay required to purchase Harberger licenses for short-term use is significantly lower.

For certain kinds of nonphysical resources, such as radio spectrum or internet domain names, there may be relatively few ways in which agents can affect the common value of the resource to all potential owners. For such assets, in the context of our model, full allocative efficiency can be achieved by running auctions to rent the asset in each period, thus Harberger licensing is dominated. Our argument in Subsection 5.3 above suggests that private-valued investment incentives should not be distorted by this system of frequent rental auctions. Nonetheless, Harberger licenses might be simpler to use in practice, given the practical difficulties of frequently running centralized auctions. Moreover, concerns about common value may have some relevance even for nonphysical resources – for example, if an owner of a valuable internet domain name invests in advertising, the persistent increase in website traffic may also benefit future owners of the domain name.

## 7.2 Private sector

While we have framed most of this paper in terms of the sale of government-owned assets, another natural class of applications of efficiency-enhancing Harberger taxation lies within the private sector. We discuss two such categories of applications.

The first is a partial private property system for "sharing economy" platforms, such as Zipcar and AirBnB. Such platforms currently operate by renting assets out to users for short-term use; as a result, users have no incentives to maintain the common value of these assets, and platforms engage in intensive and costly monitoring to prevent agents from damaging rented assets. In

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[7] Our argument resembles that of Gilbert and Shapiro (1990), who show in the context of intellectual property rights that, since the social welfare loss from increased monopoly power in any period is likely to be convex, policies which grant innovators time-invariant decreased monopoly power over intellectual property are often preferable to policies granting full monopoly power for a limited period of time.

such contexts, platforms could instead allocate assets by establishing marketplaces for Harberger licenses over the assets they own.

Under such a system, a user who wishes to rent an asset would be required to purchase the Harberger license for the asset from its previous user; from the user's perspective, the cost of the Harberger license can be thought of like a deposit payment. While assets are in use, users periodically pay some fraction of their value announcements as Harberger taxes to the platform; these tax payments resemble self-assessed rental fees. While she uses the asset, the user sets a price at which it may be reclaimed from her and transferred to the next user; she would presumably set this price high while maximally exploiting the asset, and then lower it as she begins utilizing it less. Since users expect to resell their Harberger licenses, they have partial incentives to maintain the common value of the asset, so as to keep the resale value of the asset high. Thus, partial ownership systems such as Harberger taxation may be used by sharing economy platforms to supplement costly monitoring in incenting agents to maintain the common value of shared assets. However, because the benefits of Harberger taxation relative to private ownership accrue entirely to potential future buyers, the platform would have to charge an upfront fee to members to access assets on the platform. Persuading users to pay such a fee might be challenging until the platform has a large number of assets up and running, creating a potential for coordination problems familiar from the platforms literature (Caillaud and Jullien, 2003).

Another potential class of applications in the private sector is a system of Harberger taxation for managing asset usage rights within corporations. Coase (1937) famously argued that an important reason for the existence of firms was the "transaction costs of the market." While a large literature since his work has analyzed what these transaction costs comprise, Coase highlighted costs of bargaining avoided within firms. Such costs may naturally be interpreted as the monopoly distortion the Harberger tax addresses, as in the double marginalization problem of Cournot (1838) and Spengler (1950) or as wasteful investments in reducing asymmetric information to avoid these distortions. Thus, firms may be seen as a form of private common property aimed at increasing allocative efficiency within the firm. Groves and Loeb (1979) take this interpretation to its logical extreme by arguing that a Vickrey auction should be used to allocate resources within firms.

However, as emphasized by Grossman and Hart (1986) and subsequent work in the property rights literature surveyed by Segal and Whinston (2013), such common ownership can reduce investment incentives of various stakeholders within the firm, just as a Harberger tax does. This suggest that a formalized institution of Harberger taxation may be an effective way to formalize and optimize internal markets for resources within firms that are often managed through informal relational contracts (Baker, Gibbons and Murphy, 2002; Levin, 2003). Because the relevant taxes would be collected by the firm and all individuals participating in the market employed by it, the necessity of gate-keeping to capture the associated efficiency benefits

which limits the applicability of the platform model described above would be unlikely to be a significant concern.

## 7.3 Economy-wide implementation

In the previous subsections, we considered the simplest cases for applying Harberger taxation. Now we consider how broad its scope should be in the long run. For the most liquid forms of currency, government bonds and stocks, Harberger taxation is neither harmful nor beneficial: these assets carry no market power, but also require no investment. One should therefore be indifferent to Harberger taxation of them except for the issues raised our discussion in Subsection 6.2 above about avoiding savings distortions. On the other hand, our argument suggests the application of Harberger taxation to essentially all other forms of assets. While we typically think of much of the capital stock as perfectly liquid, substantial recent evidence suggests otherwise (Syverson, 2011). Syverson (2004) finds a standard deviation of productivity across narrowly defined industries of about 25%, suggesting substantial idiosyncratic value for assets.

Under such a system all property would be registered in a cadaster (a property registry) with a regularly updated value. The cadaster would be made available, perhaps through a smartphone app, and the standard right of property would be replaced with a right to property that has not been purchased at the cadastral value, combined with a right to appropriate any property of another at its cadastral value. Cadastral proceeds would fund the enforcement of this system as taxes fund present-state enforcement of property law. Excess revenue could be returned to the community in any desired fashion, but it would likely be most desirable to use the revenues to reduce or eliminate existing distortionary means of public funds or to provide some part of the value of the asset to agents other than the owner who might influence its value through externalities, as this would further increase the social value created by the Harberger tax.[8]

Our calibrations suggest that Harberger taxation would eventually make a large impact on aggregate welfare. To see this we briefly consider the magnitude of revenue in a world in which all capital is subject to Harberger taxation. Suppose, following our dynamic calibration of Section 3, the entire capital stock of the world is subject to a 2.5% annual Harberger tax. This would generate annual revenue of approximately a third of capital income. According to Piketty and Zucman (2014), capital's share is roughly 30% of national income in G7 countries, implying revenue of approximately 10% of national income annually. This would account for half of the

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[8]As we discuss above, raising such large revenues need not lead to a large redistribution of those resources, because the revenue collected could be distributed in a manner specified by rules or given back as savings or property subsidies to offset other distortions. However, once such a large revenue stream is collected, a community would not necessarily wish to disperse it with the same inequality with which capital ownership itself is presently distributed. Because the Harberger tax offers a non-distortive (actually efficiency enhancing) means of generating a large pool of revenue, some communities might use it for redistributive purposes even if its justification is not redistributive.

tax burden in the United States and would also eliminate almost a third of the value of private capital, reducing its levels almost to the lows of the middle twentieth century.

To consider the magnitude of welfare gains, note that we found 3.5%-4% welfare gains as a fraction of asset values. Multiplying this by capital's share gives roughly welfare gains of 1% of national income, or roughly $150 billion in the United States or $1 trillion globally at purchasing power parity. We suspect this figure is a significant underestimate, as Harberger taxation should optimally be applied not only to final assets but to financial assets written on top of these as well, such as stock, bonds, derivatives etc. and while its value there would be more limited given the greater liquidity of these assets it would not be trivial.

# 8 Conclusion

In this paper, we argue the means of production should generally be owned neither in common nor privately, but rather through a mixed system that trades off the allocative benefits of common ownership against the investment incentives created by private ownership. We then show that a simple proposal for self-assessed capital taxes put forward by Harberger (1965) (for a very different purpose) can implement this system. Finally, using the results from our calibration, we suggest a simple rule of thumb for Harberger license design: Harberger tax rates should be set at about half of the existing rates of trade for assets in private markets.

Our analysis thus far considers only inanimate and not human capital. However, human capital receives a larger fraction of national income than inanimate capital and is likely as important a source of market power, given the unique talents many individual workers possess and the distortions to these talents, caused by labor income taxes. Indeed, most societies that have practiced common ownership of inanimate capital (e.g., the Israeli *kibbutzim* and the Soviet Union) have also socialized earning capacity to a significant extent. In these societies, human capital was largely directed according to social needs, rather than the choice of the human capitalist. Of course, these arrangements famously undermined human capital accumulation (Abramitzky and Lavy, 2014). Nonetheless, many methods exist for objectively assessing human capital that could be used to offer human capital subsidies to overcome this problem. In any case, partial common ownership would be a far smaller deterrent to investment than full common ownership. A fascinating question for future research is thus whether a workable system of more partial common ownership of human capital could be devised along the lines above.

Such a system would have to deal with, among other challenges, the differing amenities of different workplaces (Sorkin, 2016) that make human capitalists far from indifferent across competing purchasers of their labor. However, on the upside, it could be used not only to address distortions to labor but also to various environmental externalities impacting human life by giving a market basis for the valuation of such externalities. It would also be interesting to develop variants of Harberger taxation that could be applied to market power over variable-

production goods markets, rather than just over assets, given that existing solutions to market power have proven inadequate especially in developing countries (Berquist, 2016).

In this paper, we have abstracted away from a number of issues which are relevant in many asset markets. We have assumed that the common value of assets is fully observed by all participants; for many assets, asymmetric information about common values and adverse selection are first-order concerns. Our model abstracts away from repeated strategic interactions between agents. Assets such as internet domain names or radio spectrum may be of interest only to a small and temporally persistent set of agents; in such settings, we might think of these agents repeatedly bargaining over several periods over the allocation of the asset. We have assumed that only the current user of the asset can make investments that affect the common value of the asset. For many kinds of land use rights, agents who do not own the land use license may be able to take actions, such as polluting the surrounding area, which affect the value of the land. It is conceivable that systems resembling Harberger taxation could give partial ownership stakes, and thus common-valued investment incentives, to all agents who can take actions that affect the value of a given asset. We have modeled repeated trade of a single asset; certain assets such as radio spectrum display high degrees of complementarity, which may cause much greater losses from market power than occur with single assets because of hold-out problems (Cournot, 1838; Mailath and Postelwaite, 1990; Kominers and Weyl, 2012). Studying the interactions of Harberger taxation with these effects seems to us to be a valuable area for further research. Finally, as we discuss in Subsection 6.1 above, Harberger taxation does not exhaust the possibilities for mitigating market power by changing the nature of property license, and we hope that future research continues to explore the space of practical partial property rights systems and their effects on the efficiency of asset allocation.

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# Appendix

## A   Proofs and derivations

### A.1   Baseline model

Here, we prove our statement in Subsection 2.2 that Myerson (1981)'s regularity condition is sufficient for $\frac{\partial q^\star}{\partial \tau}$ to be finite for all tax values below the efficient probability of sale $\tau = 1 - F(\gamma_S)$. Myerson's regularity condition states that marginal revenue is monotone. A monopolist seller with value $\gamma_S$ for the good has revenue $(M(q) - \gamma_S)q$. Taking a derivative yields $M'(q)q + (M(q) - \gamma_S)$. Taking the second derivative, we have

$$2M'(q) + M''(q)q < 0.$$

Now consider the monopolist's problem under a Harberger tax $\tau < 1 - F(\gamma_S)$. By Theorem 1, $q(\tau) > \tau$; hence, $0 < q(\tau) - \tau < q(\tau)$. We want to show the following quantity exists:

$$\frac{\partial q^*}{\partial \tau} = \frac{M'(q)}{2M'(q) + M''(q)(q - \tau)}.$$

So we have to show that the denominator is bounded away from 0. From our full-support assumptions on $\epsilon$, $M'(q)$ exists and is negative for all $q$. If $M''(q) \leqslant 0$, we know $q - \tau > 0$, so $M''(q)(q - \tau) \leqslant 0$, and the numerator and denominator are both strictly negative; hence, their ratio is positive and nonzero and $\frac{\partial q^*}{\partial \tau}$ exists. So suppose $M''(q) > 0$. Then

$$2M'(q) + M''(q)(q - \tau) < 2M'(q) + M''(q)q < 0$$

Where we first use that $0 < q(\tau) - \tau < q(\tau)$, and then apply Myerson regularity. Hence, the denominator $2M'(q) + M''(q)(q - \tau)$ is strictly negative, and the ratio $\frac{\partial q^*}{\partial \tau}$ exists and is positive.

Now we turn to conditions on the cost and demand functions such that social welfare is a concave function of the tax rate. From the text, the marginal benefit of increased Harberger taxation is $M(q^\star(\tau))\rho(q^\star(\tau))$ and the marginal cost is $\Gamma'(1-\tau)\tau$. Recalling that $\rho = \frac{\partial q^\star}{\partial \tau}$, the second-order condition is

$$M'\rho^2 + \rho'M\rho + \Gamma''\tau - \Gamma'.$$

The first term is always negative ($\rho > 0 > M'$) and represents the quadratic nature of the allocative distortion discussed in the text. The final term is always negative as $\Gamma' > 0$ and represents the quadratic nature of the investment distortion. The two central terms are more ambiguous. However, Fabinger and Weyl (2016) argue $\rho'$ is typically negative for most plausible demand forms (those with a bell-shaped distribution of willingness to pay, as we assume in most calibrations) and thus, given that $M, \rho > 0$, the second term is likely to be negative as well. The third term is more ambiguous. By the inverse function theorem, given that $\Gamma = (c')^{-1}$,

$$\Gamma'' = -\frac{c'''}{(c'')^3}.$$

Assuming a convex cost function, this quantity is negative if and only if $c''' > 0$. Thus, a grossly sufficient condition (assuming $\rho'$) for the first-order conditions to uniquely determine the optimal tax is that $c''' > 0$. However, note this term is multiplied by $\tau$, which is typically on the order of 10% in our calibrations. Thus $c'''$ would have to be quite negative indeed to cause the problem to be nonconvex.

## A.2  Proof of Proposition 1

*Proof.* The single-period value $\gamma_t$ is trivially increasing in value $\gamma_t$. This fact, together with our Assumption 2 that higher $\gamma_t$'s imply uniformly higher $\gamma_{t+1}$, implies that the social planner's V is component-wise increasing. For completeness, we sketch the fairly standard proof of this result. Similar arguments can be found in, for example, Stokey and Lucas (1989) and Smith and McCardle (2002).

Define the social planner's Bellman operator $\mathcal{S}$:

$$\mathcal{S}\left(W\left(\gamma_t^{S_t}, \gamma_t^{B_t}\right)\right) =$$
$$\max\left[\gamma_t^{S_t} + \delta\mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{S_t}\right), \gamma_t^{B_t} + \delta\mathbb{E}\left(W\left(\gamma_{t+1}^{B_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{B_t}\right)\right].$$

Because, by assumption, $\gamma_t$ is uniformly bounded above, this expression is a bounded discounted problem, and by standard arguments, $\mathcal{S}$ is a contraction mapping with a unique fixed point.

Suppose W is componentwise increasing in each component. Then, supposing $\hat{\gamma}_t^{S_t} > \gamma_t$, by

the FOSD property of G, we have

$$\mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \hat{\gamma}_t^{S_t}\right) > \mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{S_t}\right).$$

Hence,

$$\hat{\gamma}_t^{S_t} + \mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \hat{\gamma}_t^{S_t}\right) > \gamma_t^{S_t} + \mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{S_t}\right),$$

and likewise for $B_t$ and $\gamma_t^{B_t}$. Hence $\mathcal{S}\left(W\left(\gamma_t^{S_t}, \gamma_t^{B_t}\right)\right)$ is componentwise increasing. Hence $V$, the unique fixed point of $\mathcal{S}$, must be componentwise increasing.

Because $V$ is componentwise increasing and $G$ is FOSD-increasing in $\gamma_t$, we have that $\gamma_t^{S_t} > \gamma_t^{B_t}$ implies

$$\gamma_t^{S_t} + \delta\mathbb{E}\left(V\left(\gamma_{t+1}^{S_{t+1}}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{S_t}\right) > \gamma_t^{B_t} + \delta\mathbb{E}\left(V\left(\gamma_{t+1}^{B_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{B_t}\right).$$

Hence, the social planner's optimal strategy in each period is to assign the asset to the agent with higher $\gamma_t$. □

## A.3 Dynamic Harberger taxation proofs

In this section, using Assumptions 1–3, we prove Theorem 2, the net trade property for the dynamic Harberger taxation game. Then, adding Assumption 4, we prove Theorem 3, that there is a unique equilibrium for the dynamic Harberger tax game.

### A.3.1 $V(\cdot)$ is strictly increasing

To begin with, we show, that any equilibrium $V(\cdot)$ must be increasing. This will allow us to consider only increasing candidate $\hat{V}(\cdot)$ functions for the remainder of the proof.

*Claim* 1. In any stationary equilibrium, $V(\gamma)$ is strictly increasing.

*Proof.* The proof is essentially the same as that of Subsection A.2. Consider a stationary equilibrium described by value function $V(\cdot)$. This defines an inverse demand function $p_{V(\cdot),F(\cdot)}(q)$. We will define the following Bellman operator for candidate value functions $\hat{V}(\cdot)$ for the seller's optimization problem:

$$\mathcal{R}\left[\hat{V}(\cdot)\right] \equiv \max_q (q - \tau) p_{V(\cdot),F(\cdot)}(q) + (1 - q)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]\right]. \tag{6}$$

Note that $\mathcal{R}$ fixes the demand distribution $p_{V(\cdot),F(\cdot)}$ at the true equilibrium value function $V(\cdot)$, and only depends on $\hat{V}$ through the seller's continuation value $\delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$. As a result, $\mathcal{R}$ is a standard Bellman equation satisfying Blackwell's sufficient conditions for a contraction mapping.

Consider a candidate value function $\hat{V}(\cdot)$ which is nondecreasing in $\gamma$. Supposing $\tilde{\gamma} > \gamma$, the single-period value $(q - \tau) p_{V(\cdot),F(\cdot)}(q) + (1 - q)\gamma$ is strictly higher under $\tilde{\gamma}$ relative to $\gamma$ for all $q$, and the continuation value $\delta \mathbb{E}_{G(\cdot|\cdot)}[\hat{V}(\gamma') \mid \gamma]$ is weakly higher under $\tilde{\gamma}$, since $G(\gamma' \mid \tilde{\gamma}) >_{\text{FOSD}} G(\gamma' \mid \gamma)$. Hence, $\mathcal{R}[\hat{V}(\cdot)](\tilde{\gamma}) > \mathcal{R}[\hat{V}(\cdot)](\gamma)$, hence $\mathcal{R}[\hat{V}(\cdot)]$ is strictly increasing in $\gamma$. Hence $\mathcal{R}$ takes nondecreasing $\hat{V}$ functions to strictly increasing $\hat{V}$ functions; hence the true value function $V$, which is the unique fixed point of $\mathcal{R}$, must be strictly increasing in $\gamma$. $\qquad\square$

### A.3.2 The pseudo-Bellman operator $\mathcal{T}$

As we discuss in Subsection 3.3, stationary equilibria of the dynamic Harberger tax game must satisfy two conditions. First, the sellers' value function must be satisfied for any $\gamma$:

$$V(\gamma) = \max_q (q - \tau) p_{V(\cdot),F(\cdot)}(q) + (1 - q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma') \mid \gamma]\right].$$

Second, the WTP distribution $p_{V(\cdot),F(\cdot)}$ must be consistent with the value function $V(\gamma)$, that is,

$$p_{V(\cdot),F(\cdot)}(q) = \left\{ p : \ P_{V(\cdot),F(\cdot)}\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma') \mid \gamma] > p\right] = q \right\}.$$

We will define the following *pseudo-Bellman operator* $\mathcal{T}$:

$$\mathcal{T}[\hat{V}(\cdot)](\gamma) \equiv \max_q (q - \tau) p_{\hat{V}(\cdot),F(\cdot)}(q) + (1 - q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}[\hat{V}(\gamma') \mid \gamma]\right] \qquad (7)$$

The operator $\mathcal{T}$ is similar to the seller's Bellman operator $\mathcal{R}$ in Equation 6. The difference is that $\mathcal{R}$ fixes the inverse demand function $p_{V(\cdot),F(\cdot)}(\cdot)$ at its true equilibrium value, whereas $\mathcal{T}$ calculates the inverse demand distribution $p_{\hat{V}(\cdot),F(\cdot)}(\cdot)$ assuming that buyers also act according to continuation value $\hat{V}(\cdot)$. We will likewise define the "candidate optimal sale probability function" $q_{\mathcal{T}}^*(\gamma; \hat{V}(\cdot))$ assuming continuation value $\hat{V}(\cdot)$, as:

$$q_{\mathcal{T}}^*(\gamma; \hat{V}(\cdot)) \equiv \arg\max_q (q - \tau) p_{\hat{V}(\cdot),F(\cdot)}(q) + (1 - q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}[\hat{V}(\gamma') \mid \gamma]\right]$$

In words, $\mathcal{T}[\hat{V}(\cdot)]$, $q_{\mathcal{T}}^*(\gamma; \hat{V}(\gamma_t))$ and $p_{\hat{V}(\cdot),F(\cdot)}(q)$ describe the values and optimal behavior of buyers and sellers, assuming that the continuation value of a license owner of type $\gamma$ in the next period is $\hat{V}(\gamma)$. Equilibria of the Harberger tax game are fixed points of the $\mathcal{T}$ operator.

Since $\mathcal{T}$ characterizes the equilibrium of a game rather than a single-agent optimization problem, it is not necessarily a contraction mapping, and the standard contraction-based proofs of uniqueness in bounded discounted dynamic programs do not apply. However, equilibrium existence is not a problem – Assumptions 1, 2 and 3, imply that $\mathcal{T}$ is a smooth function of $\hat{V}$, hence Brouwer's fixed point theorem implies that $\mathcal{T}$ must have a fixed point in the convex compact set of bounded $\hat{V}$ functions.

### A.3.3 𝒯 **Net trade property**

In Claim 2, we show that, for any increasing candidate $\hat{V}$ function, the corresponding candidate optimal sale probability $q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right)$ respects the net trade property of Theorem 1. Since Claim 2 also applies to the true policy function $q^*\left(\cdot\right)$, this proves Theorem 2, the dynamic net trade property.

*Claim* 2. (𝒯 net trade property) Suppose that $\hat{V}\left(\cdot\right)$ is strictly increasing. Then $q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right)$ satisfies:

- If $\tau = 1 - F\left(\gamma\right)$, we have $q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right) = \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right)\right) = \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$

- If $\tau < 1 - F\left(\gamma\right)$, we have $q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right) \geqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right)\right) \geqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$

- If $\tau > 1 - F\left(\gamma\right)$, we have $q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right) \leqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right)\right) \leqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$

*Proof.* We prove this by constructing an analogy to a two-stage Harberger tax problem. Fixing any increasing candidate value function $\hat{V}$, the optimization problem for a license owner with value $\gamma$ is:

$$q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right) = \arg\max_q \left(q - \tau\right) p_{\hat{V}(\cdot),F(\cdot)}\left(q\right) + \left(1 - q\right)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]\right]$$

Note that, by definition, we also have $p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right) = \gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$. In words, $p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)$ is the WTP of buyer quantile $F\left(\gamma\right)$, which is just the use value $\gamma$ plus the continuation value $\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$. Hence, we can write $q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right)$ as:

$$q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right) = \arg\max_q \left(q - \tau\right) p_{\hat{V}(\cdot),F(\cdot)}\left(q\right) + \left(1 - q\right) p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right).$$

Subtracting the term $\left(1 - \tau\right) p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)$, which does not depend on $q$, we get:

$$q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right) = \arg\max_q \left(q - \tau\right)\left(p_{\hat{V}(\cdot),F(\cdot)}\left(q\right) - p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right).$$

This can be interpreted as the optimization of a variable profit function from a two-stage Harberger taxation game, for a seller with use value $p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)$ for keeping the asset, faced with buyer values distributed as $p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)$, $q \sim U\left[0,1\right]$. Let $H_{\hat{V}(\cdot),F(\cdot)}\left(\cdot\right)$ represent the distribution of $p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)$; Theorem 1 implies that:

- If $\tau = 1 - H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right)$, we have $q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right) = \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right)\right) = \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$

- If $\tau < 1 - H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right)$, we have $q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right) \geqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q_{\mathcal{T}}^*\left(\gamma;\hat{V}\left(\cdot\right)\right)\right) \geqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$

- If $\tau > 1 - H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right)$, we have $q_{\mathcal{T}}^*\left(\gamma; \hat{V}\left(\cdot\right)\right) \leqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q_{\mathcal{T}}^*\left(\gamma; \hat{V}\left(\cdot\right)\right)\right) \leqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]$

To complete the proof, we must show that $H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right) = F\left(\gamma\right)$. Since $p_{\hat{V}(\cdot),F(\cdot)}\left(\cdot\right)$ is an increasing function, for a seller of value $\gamma$,

$$p_{\hat{V}(\cdot),F(\cdot)}\left(q\right) \leqslant p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right) \iff q \leqslant F\left(\gamma\right),$$

hence,

$$H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right) = P\left[p_{\hat{V}(\cdot),F(\cdot)}\left(q\right) \leqslant p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right] = P\left[q \leqslant F\left(\gamma\right)\right] = F\left(\gamma\right).$$

While the notation is somewhat cumbersome, the intuition behind this sequence of equalities is straightforwards. The WTP function $p\left(\cdot\right)$ is an increasing function of the $F$-quantile $q$. Thus, for a license owner of type $\gamma$, the arriving buyer's willingness to pay $p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)$ is lower than the license owner's own continuation value $p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)$ if and only if the arriving buyer has $F$-quantile lower than the license owner's quantile $F\left(\gamma\right)$. Thus, the probability $H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right)$ that the arriving buyer's WTP is lower than the license holder's WTP is exactly $F\left(\gamma\right)$.

$\square$

### A.3.4 Proof of equilibrium uniqueness

If we impose Assumption 4, we can prove uniqueness of the dynamic Harberger tax equilibrium. In Claim 4 we will show that $\mathcal{T}$ is a contraction mapping for any seller types $\gamma$ for which $\gamma < F^{-1}\left(1 - \tau\right)$; that is, for all sellers with values below the $1 - \tau'$th buyer quantile. In Claim 3 we show that the value function for these seller types can be solved for without reference to the value function above $F^{-1}\left(1 - \tau\right)$. Thus, Claims 3 and 4 show that $\mathcal{T}$ uniquely pins down $V\left(\cdot\right)$ on $\gamma \in \left[0, F^{-1}\left(1 - \tau\right)\right]$. Then, in Claim 5, we show that for any type $\tilde{\gamma} \geqslant F^{-1}\left(1 - \tau\right)$, the derivative $V'\left(\tilde{\gamma}\right)$ can be calculated using only knowledge of $V\left(\cdot\right)$ on values $\gamma \in [0, \tilde{\gamma}]$ lower than $\tilde{\gamma}$. Thus, once we know $V\left(\cdot\right)$ on the interval $\gamma \in \left[0, F^{-1}\left(1 - \tau\right)\right]$, we can integrate $V'\left(\tilde{\gamma}\right)$ upwards from $F^{-1}\left(1 - \tau\right)$ to recover the entire unique equilibrium $V\left(\cdot\right)$ function.

Consider the distribution of entering buyer values $F\left(\gamma\right)$. We will define $\rho$-*quantile truncations* of $F\left(\cdot\right)$ as follows:

**Definition.** $\tilde{F}\left(\gamma; \rho\right)$ is the $\rho-$*quantile truncation* of $F\left(\gamma\right)$, defined as:

$$\tilde{F}\left(\gamma; \rho\right) = \begin{cases} F\left(\gamma\right) & F\left(\gamma\right) \leqslant \rho \\ 1 & F\left(\gamma\right) > \rho \end{cases}$$

In words, $\tilde{F}\left(\gamma; \rho\right)$ takes all probability mass above the $\rho$th quantile of $F$, and puts it on the

ρth quantile. In the two-stage Harberger tax game, sellers at quantiles below $1 - \tau$ set quantile markups between their quantile $1 - F^{-1}(\gamma)$ and $1 - \tau$. Intuitively, then, the demand distribution at quantiles above $1 - \tau$ should not affect the behavior of these sellers; in particular, we can place all probability mass above buyer quantile $1 - \tau$ at the $(1 - \tau)$'th quantile, and this will not affect the behavior of sellers below the $(1 - \tau)$'th quantile. This is formalized in the following claim.

*Claim* 3. (Truncation property) Suppose that $V(\cdot)$ is a stationary equilibrium value function for the Harberger tax game under tax $\tau$, with entering buyer distribution $F$. Then, $V(\cdot)$ restricted to the interval $\gamma \in \left[0, F^{-1}(\rho)\right]$ is a stationary equilibrium value function for the Harberger tax game under tax $\tau$, with entering buyer distribution $\tilde{F}(\gamma; \rho)$, for any $\rho \geqslant 1 - \tau$.

*Proof.* An equilibrium of $V(\cdot)$ is a fixed point of the pseudo-Bellman operator $\mathcal{T}$, that is, it satisfies, for all $\gamma$:

$$V(\gamma) = \max_q (q - \tau) p_{V(\cdot), F(\cdot)}(q) + (1 - q)\left[\gamma + \delta \mathbb{E}_{G(\cdot | \cdot)}\left[V(\gamma') \mid \gamma\right]\right].$$

We want to show that, for any such $V$, we also have, for any $\rho \geqslant 1 - \tau$,

$$V(\gamma) = \max_q (q - \tau) p_{V(\cdot), \tilde{F}(\gamma; \rho)}(q) + (1 - q)\left[\gamma + \delta \mathbb{E}_{G(\cdot | \cdot)}\left[V(\gamma') \mid \gamma\right]\right] \ \forall \gamma \leqslant F^{-1}(\rho).$$

In other words, we want to show that the truncation of $F$ and $V(\cdot)$ does not affect the optimization problem of any type with $\gamma < F^{-1}(\rho)$.

Recall the definition of the WTP function:

$$WTP(\gamma) \equiv \gamma + \delta \mathbb{E}_{G(\cdot | \cdot)}\left[V(\gamma') \mid \gamma\right].$$

We have from Assumption 4 that $G(\gamma' \mid \gamma)$ satisfies $\gamma' \leqslant \gamma$ with probability 1, so evaluating the WTP function at $\gamma$ only requires evaluating $V$ on the interval $[0, \gamma]$. Thus, for all $\gamma \in \left[0, F^{-1}(\rho)\right]$, we can still evaluate $WTP$ using $V$ truncated to the interval $\left[0, F^{-1}(\rho)\right]$. Under $\tilde{F}(\gamma; \rho)$, the inverse demand function becomes:

$$\tilde{p}(q; \rho) = \begin{cases} p(q) & 1 - q \leqslant \rho \\ p(1 - \rho) & 1 - q > \rho \end{cases}$$

Thus, by construction, the modified inverse demand function agrees with $p$ on the inverval $[0, \rho]$, that is:

$$\tilde{p}(q; \rho) = p(q) \ \forall \{q : 1 - q \in [0, \rho]\}.$$

From Claim 2, under any increasing candidate $\hat{V}$ function, any seller with quantile $F^{-1}(\gamma) \in [0, 1 - \tau]$ chooses some $1 - q \in [F(\gamma), 1 - \tau]$. It must then be that the behavior of the inverse demand function $p(q)$ outside the range $1 - q \in \left[F^{-1}(\gamma), 1 - \tau\right]$ does not affect these sellers'

optimization problem (as long as $p(q)$ is derived from an increasing candidate $\hat{V}$ function, i.e. is monotone). Likewise, from Claim 2, any seller quantile $F(\gamma) \in [0, 1]$ chooses some $1 - q \in [1 - \tau, F(\gamma)]$, hence the behavior of $p(q)$ outside the range $1 - q \in [1 - \tau, F(\gamma)]$ does not affect the optimization problem of seller value $F^{-1}(\gamma)$.

In the $\rho$-truncated problem, sellers with values $F(\gamma) \leqslant 1 - \tau$ care about $p(q)$ in the range $[0, 1 - \tau]$, and sellers with values $1 - \tau \leqslant F(\gamma) \leqslant \rho$ care about $p(q)$ in the range $[1 - \tau, \rho]$. Since $\tilde{p}(q; \rho) = p(q)$ on the interval $[0, \rho]$, $\tilde{p}(q; \rho)$ is identical to $p(q)$ from the perspective of all sellers types with quantiles $F^{-1}(\gamma) \in [0, \rho]$. Hence there is no seller type in the $\rho$-truncated problem whose optimization problem is affected by the truncation of $p(q)$. Thus, any optimal policy $q^*(\gamma)$ and value function $V(\gamma)$ in the original problem remains optimal in the truncated problem, proving the claim. $\qquad \square$

We will now consider the most extreme possible truncation, $\rho = 1 - \tau$. Under this truncation, there are no types with values strictly above the $(1 - \tau)$th quantile; thus, all seller types in the truncated interval are net sellers. In the following claim, we use the net seller property to show that $\mathcal{T}$ is a contraction mapping on the $(1 - \tau)$-truncated problem.

*Claim* 4. (Contraction property) For any $\tau, F(\cdot)$, consider the $(1 - \tau)$-truncated problem, with entering buyer distribution $\tilde{F}(\gamma, 1 - \tau)$. $\mathcal{T}$ is a contraction mapping on this problem, hence admits a unique fixed point. Moreover, the unique fixed point $V(\cdot)$ of $\mathcal{T}$ must be continuous.

*Proof.* Once again, $\mathcal{T}$ is:

$$\mathcal{T}\left[\hat{V}\right](\gamma) = \max_q (q - \tau) \, p_{\hat{V}(\cdot), F(\cdot)}(q) + (1 - q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]\right].$$

Consider $\hat{V}, \tilde{V}$ s.t. $\sup_\gamma \left|\hat{V}(\gamma) - \tilde{V}(\gamma)\right| \leqslant a$. We want to bound the sup norm difference between $\mathcal{T}\left[\hat{V}\right](\gamma)$ and $\mathcal{T}\left[\tilde{V}\right](\gamma)$. First, note that from the definition of $p_{\hat{V}(\cdot), F(\cdot)}(\cdot)$,

$$p_{\hat{V}(\cdot), F(\cdot)}(q) \equiv \left\{p : \; P_{\hat{V}(\cdot), F(\cdot)}\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right] > p\right] = q\right\},$$

hence we have that

$$\left|p_{\hat{V}(\cdot), F(\cdot)}(q) - p_{\tilde{V}(\cdot), F(\cdot)}(q)\right| \leqslant \left|\delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right] - \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\tilde{V}(\gamma') \mid \gamma\right]\right| \leqslant \delta a.$$

Now, writing $\mathcal{T}\left[\hat{V}\right]$:

$$\mathcal{T}\left[\hat{V}\right](\gamma) = \left[q_{\mathcal{T}}^*(\gamma; \hat{V}) - \tau\right] p_{\hat{V}(\cdot), F(\cdot)}\left(q_{\mathcal{T}}^*(\gamma; \hat{V})\right) + \left(1 - q_{\mathcal{T}}^*(\gamma; \hat{V})\right)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]\right].$$

We will show that, if under $\tilde{V}$ we fix the sale probability at $q_{\mathcal{T}}^*(\gamma; \hat{V})$, we lose at most $\delta a$. Hence the sup norm difference between $T\left[\hat{V}\right], T\left[\tilde{V}\right]$ is at most $\delta a$.

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We can separately deal with the "buyer" inverse demand and "seller" continuation value terms. For the seller's continuation value term, note that:

$$\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right] \leqslant \gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\tilde{V}\left(\gamma'\right) \mid \gamma\right] + \delta a.$$

Hence,

$$\left(1 - q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right] \leqslant$$
$$\left(1 - q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\tilde{V}\left(\gamma'\right) \mid \gamma\right]\right] + \left(1 - q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right)\delta a$$

For the buyer inverse demand term,

$$\left[q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right) - \tau\right] p_{\hat{V}(\cdot),F(\cdot)}\left(q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right) \leqslant$$
$$\left[q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right) - \tau\right] p_{\tilde{V}(\cdot),F(\cdot)}\left(q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right) + \left|q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right) - \tau\right| \delta a$$

Adding these inequalities, we have that:

$$\mathcal{T}\left[\hat{V}\right](\gamma) =$$
$$\left[q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right) - \tau\right] p_{\hat{V}(\cdot),F(\cdot)}\left(q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right) + \left(1 - q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right] \leqslant$$
$$\mathcal{T}\left[\tilde{V}\right](\gamma) + \left(1 - q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right)\delta a + \left|q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right) - \tau\right| \delta a.$$

By Claim 2, we know that all sellers in the truncated range are net sellers, that is, $1 - q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right) \leqslant 1 - \tau$, or $q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right) \geqslant \tau$. Hence $\left|q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right) - \tau\right| < \left|q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right|$, hence we have:

$$\left(1 - q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right)\delta a + \left|q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right) - \tau\right| \delta a \leqslant \left|\left(1 - q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right)\right| \delta a + \left|q_{\mathcal{T}}^*\left(\gamma; \hat{V}\right)\right| \delta a \leqslant \delta a. \quad (8)$$

We have thus shown that, for all $\gamma$,

$$\mathcal{T}\left[\hat{V}\right](\gamma) \leqslant \mathcal{T}\left[\tilde{V}\right](\gamma) + \delta a.$$

$\hat{V}$ and $\tilde{V}$ were arbitrary, so by switching their roles we get:

$$\mathcal{T}\left[\tilde{V}\right](\gamma) \leqslant \mathcal{T}\left[\hat{V}\right](\gamma) + \delta a$$

$$\implies \sup_{\gamma}\left[\mathcal{T}\left[\tilde{V}\right](\gamma) - \mathcal{T}\left[\hat{V}\right](\gamma)\right] \leqslant \delta a.$$

Hence $\mathcal{T}$ is a contraction mapping of modulus $\delta$.

To show that the unique fixed point $V(\cdot)$ must be continuous, we will show that for an increasing but possibly discontinuous candidate value function $\hat{V}(\cdot)$, $\mathcal{T}\left[\hat{V}\right]$ must be continuous.

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Once again, $\mathcal{T}$ is:

$$\mathcal{T}\left[\hat{V}\right] = \max_q (q - \tau) \, p_{\hat{V}(\cdot), F(\cdot)}(q) + (1 - q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]\right].$$

We need only to show that $\mathbb{E}_{G(\cdot|\cdot)}[V(\gamma') \mid \gamma]$ is continuous in $\gamma$, since all other components of the maximand are continuous in $\gamma$. Since $\hat{V}$ is strictly increasing, the generalized inverse function $\hat{\Gamma}(v) \equiv \arg\min_\gamma |V(\gamma) - v|$ is everywhere well-defined. Since $\hat{V}(\cdot)$ is bounded, the "layer cake" representation of its expected value obtains. Letting $\bar{V} \equiv \max_\gamma \hat{V}(\gamma)$, we have:[9]

$$\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right] = \int_0^{\bar{V}} 1 - G\left(\hat{\Gamma}(v) \mid \gamma\right) dv.$$

Since $G(\gamma' \mid \gamma)$ is continuous in $\gamma$ for any $\gamma'$, the integral $\int_0^{\bar{V}} G\left(\hat{\Gamma}(v) \mid \gamma\right) dv$ is also continuous in $\gamma$.[10] $\qquad \square$

*Remark.* Equation 8 is the step where the contraction property fails in general, and is why we need this truncation argument. Suppose for example $\tau = 1$, so that $\tau > q$. Then the continuation value term has modulus $(1 - q)\delta a$ and the buyer price term has modulus $|q - \tau|\delta a = |1 - q|\delta a$. So the total modulus bound is $2(1 - q)\delta a$, and we can't guarantee that $\mathcal{T}$ is a contraction.

Claim 4 shows that the equilibrium value function $V(\cdot)$ is uniquely pinned down in the $(1 - \tau)$-truncated problem, and Claim 3 shows that the equilibrium $V(\cdot)$ functions from the original and truncated problems must agree. Hence we have shown that the equilibrium $V(\cdot)$ is unique at least in the truncated interval $\left[0, F^{-1}(1 - \tau)\right]$. In Claim 5, we show that, in any $\rho$-truncated equilibrium, we can calculate the derivative of the value function $\frac{dV}{d\gamma}\big|_{\gamma = F^{-1}(\rho)}$ at the boundary type $F^{-1}(\rho)$.

*Claim 5.* (Envelope theorem) The envelope theorem applies to any equilibrium:

$$\frac{dV}{d\gamma} = \frac{\partial}{\partial\gamma}\left[(q^*(\gamma) - \tau)\, p_{V(\cdot), F(\cdot)}(q^*(\gamma)) + (1 - q^*(\gamma))\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[V(\gamma') \mid \gamma\right]\right]\right]$$

$$= (1 - q^*(\gamma))\left[1 + \delta\frac{\partial\mathbb{E}_{G(\cdot|\cdot)}\left[V(\gamma') \mid \gamma\right]}{\partial\gamma}\right]. \tag{9}$$

*Proof.* Following Milgrom and Segal (2002), we need to show that the conjectured derivative:

$$(1 - q)\left[1 + \delta\frac{\partial\mathbb{E}_{G(\cdot|\cdot)}\left[V(\gamma') \mid \gamma\right]}{\partial\gamma}\right]$$

---

[9]This definition of the layer-cake integral is slightly wrong, failing if $\hat{V}(\cdot)$ and $G(\cdot)$ have discontinuities at the same value of $\gamma$. This can be fixed by redefining $G$ such that that probability mass falls at the correct side of each $\hat{V}$ discontinuity.

[10]This follows even without assuming differentiability of $G$, from a "set excision" argument: for any $\epsilon$, there is a $\delta$ satisfying continuity for $G\left(\hat{\Gamma}(v) \mid \gamma\right)$ except on a set of arbitrarily small $v$-measure, and $G$ is bounded between $[0, 1]$ on the excised set.

is finite for any choice of q. Using the layer cake representation once again:

$$\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]=\int_0^{\bar{V}}1-G\left(\hat{\Gamma}\left(v\right)\mid\gamma\right)dv.$$

Leibniz' formula implies that

$$\frac{\partial\mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right)\mid\gamma\right]}{\partial\gamma}=-\int_0^{\bar{V}}\frac{\partial G\left(\hat{\Gamma}\left(v\right)\mid\gamma\right)}{\partial\gamma}dv.$$

We have from Assumption 3 that $\frac{\partial G(\hat{\Gamma}(v)|\gamma)}{\partial\gamma}$ exists for any $v$, hence this quantity is finite for any q, and thus the envelope theorem applies. $\qquad\square$

Thus, in any $\rho$-truncated equilibrium, we can evaluate $q^*\left(\gamma\right)$ for the boundary type $\gamma = F^{-1}\left(\rho\right)$. Moreover, since Assumption 4 the transition probability distribution G satisfies that $\gamma_{t+1}<\gamma_t$ a.s., the expectation $\mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right)\mid\gamma\right]$ puts positive probability only on values $\gamma'<\gamma$. Hence, in any $\rho$-truncated equilibrium, we can evaluate the derivative $\frac{dV}{d\gamma}$ for the boundary type $\gamma=F^{-1}\left(\rho\right)$ using only knowledge of the equilibrium V on the truncated interval $\left[0,F^{-1}\left(\rho\right)\right]$. Thus, after solving for V on the interval $\left[0,F^{-1}\left(1-\tau\right)\right]$ using the contraction mapping of Claim 4, we canintegrate the envelope formula 9 to recover the rest of the equilibrium $V\left(\cdot\right)$ function:

$$V\left(\gamma\right)=\int_{F^{-1}(1-\tau)}^{F^{-1}(\gamma)}\left(1-q^*\left(\tilde{\gamma}\right)\right)\left[1+\delta\frac{\partial\mathbb{E}_{G(\tilde{\gamma}'|\tilde{\gamma})}\left[V\left(\tilde{\gamma}'\right)\mid\tilde{\gamma}\right]}{\partial\tilde{\gamma}}\right]d\tilde{\gamma}\;\forall\gamma>F^{-1}\left(1-\tau\right).$$

We have thus proved Theorem 3: for any $\tau$, $F\left(\gamma\right)$, $G\left(\gamma'\mid\gamma\right)$ satisfying our assumptions, there is a unique equilibrium of the dynamic Harberger taxation game.

*Remark.* We used Assumption 4 at two points. First, the proof of Claim 3 requires that the $\mathcal{T}$ operator applied to the truncated value distribution does not reference values of $\gamma$ above the truncation quantile. Second, to evaluate the envelope derivative formula in Claim 5, we once again need to be able to reference only values of $\gamma$ below the truncation quantile.

## A.4 Persistent investment

In order to accomodate investment, we need a nonstationary definition of equilibria in the Harberger tax game. Let $\zeta=\left(\zeta_0,\zeta_1,\zeta_2\ldots\right)$ represent the path of common use values over time, and suppose that this is common knowledge. The use value for any agent $A_t$ in any period is thus $\zeta_t+\gamma_t^{A_t}$. We will define the nonstationary value function $V_t\left(\gamma_t,\zeta\right)$ as the value of being a seller with type $\gamma_t$ in period t, if the path of common use values is $\zeta$. Analogously to above, we will define the inverse demand function in period t as:

$$P_{t,V_t(\cdot,\zeta),F(\cdot)}\left(q_t\right)=\left\{p_t:\;P_{V_{t+1}(\cdot,\zeta),F(\cdot)}\left[\gamma_t^{B_t}+\zeta_t+\delta\mathbb{E}_{G(\cdot|\cdot)}\left[V_{t+1}\left(\gamma_{t+1}^{B_t},\zeta\right)\mid\gamma_t\right]>p_t\right]=q_t\right\}.$$

Equilibrium then requires that, in each history,

$$V_t(\gamma_t, \zeta) = \max_{q_t} (q_t - \tau) \, p_{V_{t+1}(\cdot, \zeta), F(\cdot)}(q_t) + (1 - q_t) \left[ \gamma_t + \zeta_t + \delta \mathbb{E}_{G(\cdot|\cdot)} [V_{t+1}(\gamma_{t+1}, \zeta) \mid \gamma_t] \right]. \quad (10)$$

We conjecture an equilibrium of this game of the following form:

$$V_t(\gamma_t, \zeta) = V(\gamma) + \sum_{t'=0}^{\infty} \delta^{t'} (1 - \tau)^{t'+1} \zeta_{t+t'}.$$

One can verify that if $V(\cdot)$ satisfies the "allocative" equilibrium Equation 7, then $V_t(\gamma_t, \zeta_t)$ satisfies Equation 10. Intuitively, as in the two-stage case, if the tax is $\tau$ agent $A_t$ only owns $(1 - \tau)$ of the asset in period t. However, if the asset has some common value in period $t + t'$, agent $A_t$ has to pay taxes $t'$ times on the asset before enjoying its use value; hence she effectively only owns $(1 - \tau)^{t'+1}$ of any common value of the asset in period $t'$.

For simplicity, we analyze the investment decision of the $t = 0$ agent; the problem is additive and identical for all agents in all periods, hence all agents make the same choice of investment in each period. Suppose investment level $\eta_0$ produces common value $\zeta_t = H_t(\eta)$ in the future. Agents' FOC for investment is:

$$c'(\eta_0) = \frac{\partial V_0(\gamma_t, \zeta(\eta_0))}{\partial \eta_0}.$$

This implies that

$$c'(\eta_0) = \sum_{t=0}^{\infty} \delta^t (1 - \tau)^{t+1} H_t'(\eta_0), \quad (11)$$

proving Proposition 2.

# B  Calibration details

## B.1  Persistent investment algebra

In our calibrations, we assume that investment decays geometrically at rate $\theta < 1$; that is, persistent investment $\eta_0$ generates period t value:

$$H_t(\eta_0) = \theta^t \eta_0.$$

Hence, following Equation 11 in Appendix Subsection A.4, the present value of a unit of investment is:

$$\sum_{t=0}^{\infty} \eta_0 \delta^t \theta^t (1 - \tau)^{t+1} = \frac{\eta_0 (1 - \tau)}{1 - \delta\theta (1 - \tau)},$$

and agents' investment FOCs are thus:

$$c'(\eta_0) = \frac{1 - \tau}{1 - \delta\theta(1 - \tau)}.$$

We will suppose that the cost function is:

$$c(\eta) = \frac{\eta^2}{2(1 - \delta\theta)g},$$

for some value of parameter $g$. This is a convenient functional form which leads to a simple analytical solution. Total social investment welfare for investment level $\eta_0$ is

$$\frac{\eta_0}{(1 - \delta\theta)} - \frac{\eta_0^2}{2(1 - \delta\theta)g}.$$

Since we measure allocative efficiency in terms of flow value, we will normalize by a factor $1 - \delta$ to convert investment value into equivalent flow value:

$$\text{Investment Welfare} = \left(\eta_0 - \frac{\eta_0^2}{2g}\right)\left(\frac{1 - \delta}{1 - \delta\theta}\right). \tag{12}$$

The socially optimal level of investment is $\eta_0 = g$. The maximum possible investment NPV, in flow value terms, is thus:

$$\frac{g}{2}\frac{1 - \delta}{1 - \delta\theta}.$$

As we discuss in Subsection 4.1, we choose $g$ such that the maximum possible flow value of investment is some target fraction $invfrac$ of the average transaction price.

Given some tax level $\tau$, constant for all time, the seller's FOC for investment is:

$$\frac{\eta}{(1 - \delta\theta)g} = \frac{1 - \tau}{1 - \delta\theta(1 - \tau)},$$

$$\implies \eta = g\frac{(1 - \tau)(1 - \delta\theta)}{1 - \delta\theta(1 - \tau)}.$$

We can plug this into Equation 12 to calculate total investment welfare for any given value of $\tau$.

## B.2 Numerical procedures

As we discuss in Appendix Subsection A.3, the equilibria of the dynamic Harberger taxation game are the unique fixed points of the pseudo-Bellman operator $\mathcal{T}$:

$$\mathcal{T}[\hat{V}] = \max_q (q - \tau)\, p_{\hat{V}(\cdot), F(\cdot)}(q) + (1 - q)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]\right].$$

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While our proof of uniqueness in Subsection A.3 involves a multiple-step procedure involving truncation of the buyer value distribution, in practice, for buyer value distributions $F(\cdot)$ supported on discrete grids, iteratively applying $\mathcal{T}$ rapidly converges to a fixed point V. Thus, we numerically solve our calibrations by iterating $\mathcal{T}$ on grid-supported F distributions.

We use gradient descent with our numerical equilibrium solver function to find moments $\sigma, \beta$ to match the `sdmean` and `saleprob` moments, as we describe in Subsection 4.1 of the text. Given a candidate value function $\hat{V}$ and decay rate $\beta$, we can evaluate the continuation value $\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$ for any type $\gamma$, and thus also the inverse demand function $p_{\hat{V}(\cdot),F(\cdot)}(q)$. Thus, we can find the optimal sale probability $q^*_{\mathcal{T}}(\gamma; \hat{V})$ for any $\hat{V}$, and thus calculate $\mathcal{T}(\hat{V})$. Starting from a linearly increasing $V(\cdot)$ function, we iteratively apply $\mathcal{T}$ until convergence, defined as:

$$\sup_{\gamma} |\mathcal{T}[V](\gamma) - V(\gamma)| < 10^{-3}.$$

Once we have solved for $V(\gamma)$, this gives us equilibrium sale probability functions $q^*(\gamma, V)$ for every type $\gamma$. Together, the equilibrium $q^*(\gamma, V)$, the transition probability distribution $G(\gamma' \mid \gamma)$ and the distribution of entering buyer values $F(\gamma)$ define a ergodic discrete state Markov chain over values $\gamma$ of the period-t owner of the asset $S_t$. We construct this transition probability matrix of this Markov chain, and solve for its unique stationary distribution, which we call $H_\tau(\gamma)$. We plot these stationary distributions for various values of $\tau$ in Figure 2.

Once we have solved for the equilibrium $V(\cdot)$, we can recover the equilibrium sale probability function $q^*(\gamma, V)$ and inverse demand function $p_{V(\cdot),F(\cdot)}(\cdot)$, and we can use these, together with $H_\tau(\gamma)$, to recover the stationary averages of various quantities that we plot in Figure 3. Specifically, these quantities are averages of the following variables with respect to $H_\tau(\gamma)$:

- **Use value**: $\gamma + \eta$

- **Sale probability**: $q^*(\gamma, V)$

- **Quantile markup**: $(1 - q^*(\gamma, V)) - (1 - F(\gamma))$

- **Tax revenue**: $\tau p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))$

There is a minor accounting subtlety for stationary average use values: if sale occurs in period t, we should use the buyer's use value, not the seller's, in calculating the stationary average. This is accomodated by multiplying the stationary distribution $H_\tau(\gamma)$ by a "buyer transition" adjustment matrix, which reflects the probability that seller type $\gamma$ "transitions" through sale of the good to any buyer type $\gamma'$ with $p_{V(\cdot),F(\cdot)}(1 - F(\gamma')) > p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))$.

For **asset prices**, we observe in the real world asset prices only for successful transactions; correspondingly, we would like to take an average of asset prices weighted by the probability of

sale for each seller value $\gamma$. Thus, average asset prices in Figure 3, panel 2 are calculated as:

$$\frac{\int p_{V(\cdot),F(\cdot)}\left(q^*\left(\gamma,V\right)\right)q^*\left(\gamma,V\right)dH_\tau\left(\gamma\right)}{\int q^*\left(\gamma,V\right)dH_\tau\left(\gamma\right)}.$$

We include investment value in the asset price by multiplying investment flow value by a factor $\frac{1}{1-\delta(1-\tau)}$. Note that, since taxes are collected regardless of sale, we do not weight offered prices by $q^*\left(\gamma,V\right)$ when we calculate average tax revenues. Values labelled "NPV" are calculated by taking average flow values and multiplying by $\frac{1}{1-\delta}$.

For the sensitivity graphs in Figure 4 for the sdmean moment, for a grid of values of $\sigma$, we search for a value of $\beta$ which keeps saleprob at its initial calibration value. Likewise, for the saleprob graphs, we use a grid of $\beta$ values, searching for matching $\sigma$ values. Since the invfrac moment is only affected by the investment total value parameter, for the invfrac graphs we simply vary this parameter holding all others fixed. Note from Figure 3, panel 1 that welfare is very flat about the allocative and total welfare maximizing tax values. Thus, it is difficult to precisely pin down the values of optimal taxes, and some numerical error in the range of $\tau \pm 0.01$ or so is visible in Figure 4.

## B.3 Transition distribution details

For the baseline smooth decay specification, we assume the following: if an agent has value $\gamma_t$ in period $t$, her value in period $t+1$ is $\chi\gamma_t$, where $\chi$ has a Beta distribution with shape parameters $10\beta$, $10(1-\beta)$. Thus, the expected value of $\gamma_{t+1}$ is $\beta\gamma_t$. We chose the other shape parameter 10 somewhat arbitrarily; changing this shape parameter has very small effects on the results. In this specification, $\beta$ is chosen using our numerical moment-matching procedure.

For the jump decay specification, with probability $1-\omega$ values decay as a Beta distribution with mean $\beta = 0.97$, and with probability $\omega$ values jump to 0. $\omega$ is chosen using numerical moment-matching.

We also report results from two other specifications. First, we use a smooth decay process with some probability of values increasing; we suppose that $\gamma_{t+1}$ is generated as $1.1\chi\gamma_t$, where $\chi$ is Beta-distributed with shape parameters $10\frac{\beta}{1.1}, 10\left(1-\frac{\beta}{1.1}\right)$. Thus, the mean of $\gamma_{t+1}$ is still $\beta\gamma_t$, but values have some probability of increasing. We will refer to this as Up–drift smooth. While we needed Assumption 4 that values decrease to prove uniqueness of equilibria, we were able to solve for equilibria for this specification using the pseudo-Bellman iteration, and the equilibria appear not to depend on the starting values we chose.

Second, we use a specification in which values jump downwards without any drift component: with probability $1-\omega$, we have $\gamma_{t+1} = \gamma_t$, and with probability $\omega$ we have $\gamma_{t+1} = 0$. We will refer to this as No–drift jump.

We report results from all four specifications in Table 2, assuming throughout that invfrac =

Table 2: Alternative specifications

| Transition process | Invfrac | Optimal $\tau$ | Total gain | Alloc gain | Inv loss | 2.5% tax gain |
|---|---|---|---|---|---|---|
| Smooth | 0.4 | 0.035 | 4.09% | 4.95% | -0.86% | 3.87% |
| Up-drift smooth | 0.4 | 0.034 | 3.73% | 4.54% | -0.81% | 3.58% |
| Jump | 0.4 | 0.029 | 2.69% | 3.31% | -0.62% | 2.68% |
| No-drift jump | 0.4 | 0.015 | 1.46% | 1.63% | -0.17% | 1.20% |

*Notes.* All gains are in units of percentages of total welfare at $\tau = 0$.

0.4. Results from the Up–drift smooth are very similar to those from the baseline specification. For the No–drift jump specification, the welfare gains from Harberger taxation are lower, thus the optimal Harberger tax level is also fairly low. This is likely because the driftless jump process leads to an even more disperse stationary distribution of values, thus no constant Harberger tax level is close to optimal for most agent types in stationary trading equilibrium.