Examining the Sources of Excess Return Predictability: Stochastic Volatility or Persistent Investor Forecast Errors?

(Preliminary)

Kevin J. Lansing*
Federal Reserve Bank of San Francisco

Stephen F. LeRoy†
UC Santa Barbara and Federal Reserve Bank of San Francisco

Jun Ma‡
University of Alabama

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Abstract

This paper shows that realized excess returns on stocks relative to bonds in a consumption-based asset pricing model can be represented by an additive combination of the representative investor's percentage forecast errors. As a result, predictability of realized excess returns can arise from only two sources: (1) persistent stochastic volatility of the model's fundamental driving variables, or (2) persistent investor forecast errors, implying a departure from fully-rational expectations. This is a general result that holds for any stochastic discount factor, any consumption or dividend process, and any stream of bond coupon payments. From an empirical perspective, we investigate whether excess returns on stocks can be predicted using the previous period's excess returns, while controlling for persistent stochastic volatility in past returns and persistent stochastic volatility in the growth rates of consumption and dividends. We find evidence of predictability of excess returns from both of the above-named sources using both annual and quarterly data.

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* Corresponding author. Research Department, Federal Reserve Bank of San Francisco, P.O. Box 7702, San Francisco, CA 94120-7702, (415) 974-2393, email: kevin.j.lansing@sf.frb.org, homepage: www.frbsf.org/economics/economists/klansing.html

†Department of Economics, University of California, Santa Barbara, CA 93106, (805) 689-2344, email: sleroy@econ.ucsb.edu, homepage: http://www.econ.ucsb.edu/~sleroy/webpage/

‡ Department of Economics, Finance and Legal Studies, University of Alabama, Box 870224, 200 Alston Hall, Tuscaloosa, AL 35487, (205) 348-8985, email: jma@cba.ua.edu

1 Introduction

A vast literature, pioneered by Fama and French (1988), examines the so-called "predictability" of excess returns, i.e., the ability of observable variables to reliably forecast future excess returns on stocks relative to default-free government bonds. Predictability is typically measured by the size of a slope coefficient and the adjusted R-squared in forecasting regressions over varying time horizons. This paper examines the predictability question from both a theoretical and empirical perspective.

From a theoretical perspective, we show that realized excess returns in a standard consumption-based asset pricing model can be represented by an additive combination of the representative investor's percentage forecast errors. As a result, we show that predictability of realized excess returns can arise from only two sources: (1) persistent stochastic volatility of the model's fundamental driving variables, or (2) persistent investor forecast errors, implying a departure from fully-rational expectations. This is a general result that holds for any stochastic discount factor, any consumption or dividend process, and any stream of bond coupon payments.

Regarding the first source of predictability, we demonstrate analytically that the conditional variance of excess returns provides us with a measure (up to a constant multiplier) of the stochastic volatility terms that drive predictability under rational expectations.

Regarding the second source of predictability, we provide a simple analytical example to show how an investor who employs a misspecified forecast rule can introduce predictability into his own percentage forecast errors. Specifically, we show that this can occur if the investor employs a simple AR(1) law of motion for consumption growth that ignores the stochastic volatility component or if the investor employs a naive random walk forecast for consumption growth.

From an empirical perspective, we investigate whether excess stock returns (i.e., percentage forecast errors) can be predicted using the previous period's excess returns (i.e., previous percentage forecast errors), while controlling for persistent stochastic volatility in past returns and persistent stochastic volatility in consumption growth and dividend growth.

Guo (2006) finds that the predictability of excess returns can be improved by including a measure of past stock market volatility. Our measure of the volatility of past returns is the variable "svar" from Welch and Goyal (2008), defined as the variance of daily stock market returns over the most recent quarter or year, depending on the data frequency used in the predictability regressions. Another way to measure stock market volatility is the implied volatility from options on the S&P 500, i.e., the VIX. Data on the VIX are only available from 1990 onwards. We therefore construct a synthetic measure of VIX for the pre-1990 sample period by running an in-sample regression of VIX on the contemporanous value of svar and its lagged value. The R-squared of the regression is about 90%. By including the variable svar together with the actual and synthetic VIX in the predictability regressions, we capture the two elements that make up the "variance risk premium," as defined by Bollerslev, Tauchen, and Zhou (2009). These authors define the difference between implied volatility from options and realized volatility on the S&P 500 index as the variance risk premium and find that this variable is a useful predictor of future excess returns. Christensen and Prabhala (1998) show that past implied volatility and past realized volatility are useful for predicting future realized volatility. Our theoretical results show that predicting future volatility should help to predict future excess returns.

We further control for persistent stochastic volatility of fundamental driving variables by including additional predictor variables, namely, the trailing standard deviations of consumption growth and dividend growth computed over the last 5 years of data. Results are not particularly sensitive to small adjustments in the length of the moving windows that are used to compute the trailing standard deviations.

Our predictability regressions also include the lagged price-dividend ratio (motivated by Campbell-Shiller return identity) and a measure of lagged inflation to capture the possibility of inflation illusion. We find evidence of predictability of excess returns from both of the above-named sources. Specifically, the predictor variables that measure the volatility of returns or the volatility of fundamentals are often significant. But even after controlling for these sources of predictability, we find that lagged excess returns can often be significant, suggesting persistence in investor forecast errors that is coming from a departure from fully-rational expectations.

Some other recent studies find evidence of excess return predictability that appears to be linked in some way to departures from fully-rational expectations. Cieslik (2016) shows that investors' real time forecast errors about the short-term real interest rate introduces predictability in the bond risk premium. Katz, Lustig, and Nielsen (2016) find that lagged inflation (a proxy for expected inflation) helps to predict lower real stock returns, suggesting a form of sticky information in stock investors' inflation forecasts. Campbell and Vuolteenaho (2004) find evidence of inflation illusion in stock prices, consistent with the hypothesis originally put forth by Modigliani and Cohn (1979).[1] Our empirical analysis includes a measure of lagged inflation as a predictor variable in order to control for the possibility of inflation illusion. We find that lagged inflation is often significant in the predictability regressions.

1.1 Related Literature

Considering the possibility of departures from fully-rational expectations is justified by empirical evidence from surveys which seek to directly measure investor expectations. With regard to stock returns, studies by Fischer and Statman (2002), Vissing-Jorgenson (2004) and Am-

[1] Lansing (2004) provides a summary of the Modigliani-Cohn hypothesis and some supporting evidence.

romin and Sharpe (2014) all find evidence of extrapolative or procyclical expected returns among investors.

Greenwood and Shleifer (2014) find that measures of investor expectations about future U.S. stock returns (and presumably expectations about future excess stock returns) are positively correlated with past returns and the price-dividend ratio. Interestingly, even though a higher price-dividend ratio in the data predicts lower *realized* returns, the survey evidence shows that investors fail to take this relationship into account; instead they continue to forecast high future returns following a sustained run-up in the price-dividend ratio. Koijen, Schmeling, and Vrugt (2015) find similar evidence in other assets classes, including global equities, currencies, and global fixed income investments. With regard to macroeconomic variables (inflation, output growth, the unemployment rate, and housing starts), Coibion and Gordonichencko (2015) find strong evidence of predictability in the mean ex post forecast errors of professional forecasters, a feature that is not consistent with full-information rational expectations.

Numerous studies raise questions about the reliability of the empirical evidence on return predictability. Nelson and Kim (1993), Stambaugh (1999), Boudoukh, Richardson, and Whitelaw (2008) cite estimation problems arising from small sample bias and persistent regressors. In contrast, other studies argue in favor of predictable excess returns in the data. Lettau and Ludvigson (2010) review the empirical evidence and conclude that excess stock returns can in fact be predicted using the dividend-price ratio as well other variables. Campbell (2014) reviews the empirical evidence on the predictability and states that the predictability of excess returns with rational expectations requires a model with predictable time variation in the volatility of the stochastic discount factor.

Cochrane (2008) argues that the absence of predictability for dividend growth and the risk free rate strengthens the evidence in favor of predictable excess returns on stocks. In contrast, Ang and Bekaert (2007) find no evidence of excess stock return predictability at long horizons using the dividend yield, unlike Cochrane (2008). They find evidence of dividend growth predictability in shorter (1952 onwards) sample periods and in international data. They also find that the risk free rate has predictive power for excess stock returns. Chen (2009) finds that dividend growth, when properly measured, is predictable by the dividend yield in pre-WWII sample period but this predictability disappears in the post-WWII period. Correspondingly, he finds that stocks returns are not predictable by the dividend yield in pre-WWII period, but are predictable in the post-WWII period.

Fama and Bliss (1987), Campbell and Shiller (1991), and Cochrane and Piazzesi (2005) all find evidence that excess bond returns (returns on longer term bonds relative to a 1-year bond) are predictable.

2 Excess Returns and Percentage Forecast Errors

The framework for our analysis is a standard consumption-based asset pricing model. For any type of purchased asset and any specification of investor preferences, the first-order condition of the representative investor's optimal consumption choice yields

$$1 = E_t \left[M_{t+1} R^i_{t+1} \right], \tag{1}$$

where E_t is the mathematical expectation operator conditional on information available at time t, M_{t+1} is the investor's stochastic discount factor, and R^i_{t+1} is the gross holding period return on asset type i from time t to $t+1$. For a dividend-paying stock, we have $R^s_{t+1} = \left(d_{t+1} + p^s_{t+1}\right)/p^s_t$, where p^s_t is the ex-dividend stock price and d_{t+1} is the dividend received in period $t+1$. For a default-free bond that pays a stream of coupon payments (measured in consumption units) we have $R^b_t = \left(1 + \delta p^b_{t+1}\right)/p^b_t$, where p^b_t is the ex-coupon bond price and δ is a (possibly stochastic) parameter that governs the decay rate of the coupon payments. A bond purchased in period t yields a coupon stream of 1, δ, δ^2... starting in period $t+1$. When $\delta = 0$, we have a one period discount bound that delivers a single coupon payment of one consumption unit at time $t+1$. In this case, $R^f_{t+1} \equiv 1/p^b_t$ is the risk-free rate of return which is known with certainty at time t. When $\delta = 1$, we have a consol bond that delivers a perpetual stream of coupon payments, each equal to one consumption unit. More generally, the value of δ can be calibrated to achieve a target value for the Macauly duration of the bond, i.e., the present-value weighted average maturity of the bond's cash flows.[2]

With time-separable constant relative risk aversion (CRRA) preferences, we have $M_{t+1} = \beta \left(c_{t+1}/c_t\right)^{-\alpha}$, where β is the subjective time discount factor, c_t is the investor's consumption, and α is the risk aversion coefficient. With recursive preferences along the lines of Epstein and Zin (1989, 1991), we have $M_{t+1} = \beta^{\omega} \left(c_{t+1}/c_t\right)^{-\omega/\sigma} \left(R^c_{t+1}\right)^{\omega-1}$, where $R^c_{t+1} \equiv \left(c_{t+1} + p^c_{t+1}\right)/p^c_t$ is the gross return on an asset that delivers a claim to consumption c_{t+1} in period $t+1$, σ is the elasticity of intertemporal substitution, and $\omega \equiv (1-\alpha)/\left(1-\sigma^{-1}\right)$. In the special case when $\alpha = \sigma^{-1}$, we have $\omega = 1$ such that Epstein-Zin preferences coincide with CRRA preferences. With external habit formation preferences along the lines of Campbell and Cochrane (1999), we have $M_{t+1} = \beta \left[s_{t+1} c_{t+1}/\left(s_t c_t\right)\right]^{-\alpha}$, where $s_t \equiv 1 - x_t/c_t$ is the surplus consumption ratio and x_t is the external habit level.

For stocks, equation (1) can be written as

$$p^s_t/d_t \;=\; E_t \left[M_{t+1} \left(\frac{d_{t+1}}{d_t}\right) \left(1 + p^s_{t+1}/d_{t+1}\right) \right], \tag{2}$$

where p^s_t/d_t is the price-dividend ratio (the inverse of the dividend yield) and d_{t+1}/d_t is the gross growth rate of dividends. At this point, it is convenient to define the following nonlinear

[2] See, for example, Lansing (2015).

change of variables:

$$z_t^s \equiv M_t \left(\frac{d_t}{d_{t-1}}\right)(1+p_t^s/d_t), \tag{3}$$

where z_t^s represents a composite variable that depends on the stochastic discount factor, the growth rate of dividends, and the price-dividend ratio.[3] The investor's first-order condition (2) becomes

$$p_t^s/d_t = E_t\, z_{t+1}^s, \tag{4}$$

which shows that the equilibrium price-dividend ratio is simply the investor's rational forecast of the composite variable z_{t+1}^s.

The gross stock return can now be written as

$$R_{t+1}^s = \frac{d_{t+1}+p_{t+1}^s}{p_t^s} = \left(\frac{1+p_{t+1}^s/d_{t+1}}{p_t^s/d_t}\right)\frac{d_{t+1}}{d_t} \tag{5}$$

$$= \left(\frac{z_{t+1}^s}{E_t\, z_{t+1}^s}\right)\frac{1}{M_{t+1}}, \tag{6}$$

where we have eliminated p_t^s/d_t using the first-order condition (4) and eliminated $p_{t+1}^s/d_{t+1}+1$ using the definitional relationship (3) evaluated at time $t+1$.

Starting again from equation (1) and proceeding in a similar fashion, the bond price is determined by the following first-order condition

$$p_t^b = E_t\, z_{t+1}^b, \tag{7}$$

where $z_t^b \equiv M_t\left(1+\delta p_t^b\right)$. The gross bond return can be written as

$$\begin{aligned} R_{t+1}^b &= \frac{1+\delta p_{t+1}^b}{p_t^b} \\ &= \left(\frac{z_{t+1}^b}{E_t\, z_{t+1}^b}\right)\frac{1}{M_{t+1}}. \end{aligned} \tag{8}$$

Notice that the above expression simplifies to $R_{t+1}^b = R_{t+1}^f = 1/\left(E_t M_{t+1}\right)$ when $\delta = 0$ such that $z_{t+1}^b = M_{t+1}$.

Combining equations (6) and (8) yields the following ratio of the gross stock return to the gross bond return:

$$\frac{R_{t+1}^s}{R_{t+1}^b} = \frac{z_{t+1}^s}{E_t\, z_{t+1}^s}\frac{E_t\, z_{t+1}^b}{z_{t+1}^b}. \tag{9}$$

[3] This nonlinear change of variables technique is also employed by Lansing (2010, 2016) and Lansing and LeRoy (2014).

Taking logs of both sides of equation (9) yields the following compact expression for the excess stock return, i.e., the realized equity premium:

$$\log\left(R^s_{t+1}\right) - \log\left(R^b_{t+1}\right) \;=\; \log\left[\frac{z^s_{t+1}}{E_t\, z^s_{t+1}}\right] \;-\; \log\left[\frac{z^b_{t+1}}{E_t\, z^b_{t+1}}\right], \tag{10}$$

where the second term simplifies to $-\log\left[M_{t+1}/\left(E_t M_{t+1}\right)\right]$ when $\delta = 0$.

In the special case of CRRA utility, *iid* consumption growth, and $c_t = d_t$, the equilibrium price-dividend ratio is constant. The realized equity premium under rational expectations with $\delta = 0$ is $\log\left(R^s_{t+1}/R^f_{t+1}\right) = \varepsilon_{t+1} + (\alpha - 0.5)\,\sigma^2_{\varepsilon}$, where ε_{t+1} is the innovation to consumption/dividend growth and σ^2_{ε} is the associated variance.[4] In this special case, realized excess returns are *iid* and hence are not predictable. More elaborate models that introduce stochastic volatility are needed to generate predictability under rational expectations.

Similarly, we can compute the excess bond return , i.e., the realized term premium, which compares the return on a longer-term bond $(\delta > 0)$ to the risk free rate $(\delta = 0)$. In this case, we have

$$\log\left(R^b_{t+1}\right) - \log\left(R^f_{t+1}\right) \;=\; \log\left[\frac{z^b_{t+1}}{E_t\, z^b_{t+1}}\right] \;-\; \log\left[\frac{M_{t+1}}{E_t\, M_{t+1}}\right]. \tag{11}$$

Equations (10) and (11) are striking. The realized equity premium and the realized term premium are nothing more than additive combinations of the representative investor's percentage forecast errors. This is a general result that holds for any stochastic discount factor, any dividend process, and any stream of bond coupon payments. Given that $\log(a/b) \simeq (a-b)/b$, the assumption of rational expectations might seem to imply that these percentage forecast errors should be uncorrelated over time, making excess returns unpredictable. However, as we show below, predictability can arise under rational expectations if the model's fundamental driving variables exhibit persistent stochastic volatility. According to this class of models, an empirical finding of predictable excess returns can be evidence of (1) persistent stochastic volatility of the model's fundamental driving variables, such as consumption growth or dividend growth, or (2) persistent investor forecast errors, implying a departure from fully-rational expectations.

From equation (10), we can see how a departure from rational expectations, in the form of pessimism about stocks, could serve to magnify the average equity premium. This is a mechanism explored by Abel (2002). Specifically, if the mean realization of z^s_{t+1} systematically exceeds the investor's mean conditional forecast $E_t\, z^s_{t+1}$, then this type of forecast misspecification would serve to increase the mean excess return on stocks.

[4] For the derivation, see Abel (1994), p. 353.

3 Predictability from Persistent Stochastic Volatility

When solving consumption-based asset pricing models, it is common to employ approximation methods that deliver conditional log-normality of the relevant endogenous variables. If a random variable x_t is conditionally log-normal, then

$$\log\left(E_t\, x_{t+1}\right) \;=\; E_t\left[\log\left(x_{t+1}\right)\right] \;+\; \tfrac{1}{2}Var_t\left[\log\left(x_{t+1}\right)\right], \tag{12}$$

where Var_t is the mathematical variance operator conditional on information available to the investor at time t.

Starting from equation (10), we make the assumption that the composite variables z^s_{t+1} and z^b_{t+1} are both conditionally log-normal. Making use of equation (12) to eliminate $\log\left(E_t z^s_{t+1}\right)$ and $\log\left(E_t z^b_{t+1}\right)$ yields the following alternate expression for the realized excess return

$$\begin{aligned}\log\left(R^s_{t+1}\right) - \log\left(R^b_{t+1}\right) \;&=\; \left[\log\left(z^s_{t+1}\right) - E_t\log\left(z^s_{t+1}\right)\right] \;-\; \left[\log\left(z^b_{t+1}\right) - E_t\log\left(z^b_{t+1}\right)\right] \\ &\quad -\tfrac{1}{2}Var_t\left[\log\left(z^s_{t+1}\right) - \log\left(z^b_{t+1}\right)\right]\end{aligned} \tag{13}$$

where $z^b_{t+1} = M_{t+1}$ for a 1 period bond which has $\delta = 0$. Notice that the first two terms in equation (13) are the investor's forecast errors for $\log\left(z^s_{t+1}\right)$ and $\log\left(z^b_{t+1}\right)$. These terms cannot be a source of predictability under rational expectations. However, the last two terms in equation (13) show that predictability can arise under rational expectations if the laws of motion for $\log\left(z^s_{t+1}\right)$ and $\log\left(z^b_{t+1}\right)$ exhibit stochastic volatility that is persistent from one period to the next.

Assuming rational expectations and taking the conditional variance of both sides of equation (13) yields

$$Var_t\left[\log\left(R^s_{t+1}\right) - \log\left(R^b_{t+1}\right)\right] = Var_t\left[\log\left(z^s_{t+1}\right) - \log\left(z^b_{t+1}\right)\right]. \tag{14}$$

Hence the conditional variance of excess returns provides us with a measure (up to a constant multiplier) of the stochastic volatility terms in equation (13). In our empirical analysis, we approximate $Var_t\left[\log\left(R^s_{t+1}\right) - \log\left(R^b_{t+1}\right)\right]$ using the variable "svar" from Welch and Goyal (2008), defined as the variance of daily returns over the most recent month, quarter, or year, depending on the data frequency used in the predictability regressions. Another way to measure market volatility is the implied volatility from options on the S&P 500, i.e., the VIX. Data on the VIX are only available from 1990 onwards. We therefore construct a synthetic measure of VIX for the pre-1990 sample period by running an in-sample regression of VIX on svar and lagged svar. The R-squared of the regression is about 90%. By including svar together with and the actual and synthetic VIX in the predictability regressions, we capture the two elements that make up the "variance risk premium," as defined by Bollerslev, Tauchen, and Zhou (2009). These authors define the difference between implied volatility from options and realized volatility on the S&P 500 index as the variance risk premium and find that this variable is a useful predictor of future excess returns.

Attanasio (1991) undertakes a related approximation to equation (1) and concludes (p. 481) "that predictability of excess returns constitutes direct evidence against the joint hypothesis that markets are efficient and second moments are constant." In other words, he says that a finding of predictable excess returns in the data would not necessarily rule out market efficiency (i.e., rational expectations), provided that one could find the appropriate evidence of time-varying second moments. Ludvigson (2013) states (p. 872) "There is a growing interest in the role of [persistent] stochastic volatility in consumption growth as a mechanism for explaining the predictability of [excess] stock returns."

In the rational external habit model of Campbell and Cochrane (1999), persistent stochastic volatility is achieved via a nonlinear sensitivity function that determines how innovations to consumption growth influence the logarithm of the surplus consumption ratio. In the rational long-run risks model of Bansal and Yaron (2004), persistent stochastic volatility is achieved directly by assuming an AR(1) process for the volatility of innovations to both consumption growth and dividend growth. Despite these features, subsequent analysis has shown that these fully-rational models fail to deliver predictability results for excess stock returns that are anywhere close to that found in the data. Li (2001) extends the model of Campbell and Cochrane (1999) to allow for AR(1) consumption growth. He finds (p. 895) "The fraction of stock [excess] return variance that can be explained by surplus consumption is economically small." Kirby (1998) had previously shown that the habit model of Abel (1990) and the recursive preference model of Epstein and Zin (1989, 1991) both fail to generate any significant predictability in excess stock returns. Chen and Hwang (2014) extend Kirby's analysis to the models of Campbell Cochrane (1999) and Bansal and Yaron (2004) and find that neither model can generate any significant predictable excess returns. Using simulated data, Beeler and Campbell (2012) show that the long-run risks model of Bansal and Yaron (2004) fails to deliver any appreciable predictability of excess returns, even at longer horizons.

The failure of the leading rational asset pricing models to produce empirically realistic predictability of excess returns lends support to the idea of pursuing alternative sources of predictability, in the form of departures from fully-rational expectations. If we allow for departures from fully-rational expectations, then equation (10) can be rewritten as follows:

$$\log\left(R^s_{t+1}\right) - \log\left(R^b_{t+1}\right) \;=\; \log\left[\frac{z^s_{t+1}}{\widehat{E}_t\, z^s_{t+1}}\right] \;-\; \log\left[\frac{z^b_{t+1}}{\widehat{E}_t\, z^b_{t+1}}\right], \tag{15}$$

where we use the symbol $\widehat{E}_t$ to represent the agent's subjective conditional forecast computed using the agent's perceived law of motion for the variable in question. Under rational expectations, the perceived law of motion coincides with the actual law of motion such that $\widehat{E}_t = E_t$. But when $\widehat{E}_t \neq E_t$, the agent's misspecified forecast rule can generate persistent forecast errors that in turn can and contribute to the predictability of excess returns. We provide a simple analytical example of this mechanism in Section 4.

3.1 Realized versus Expected Excess Returns

Most empirical studies of predictability take the form of regressions in which the left hand side variable is *realized* excess returns over a particular holding period. The regression results are often interpreted as shedding light on whether there exists predictability in *expected* excess returns. The maintained assumption is that investors' expectations are rational such that expected returns and realized returns differ only by a rational forecast error that is uncorrelated with information dated t or earlier.[5]

Following Campbell (2014), an expression for the expected excess return can be derived by decomposing the conditional expectation in equation (1) as follows

$$\underbrace{E_t\left[M_{t+1}R^s_{t+1}\right]}_{=1} \;=\; \underbrace{E_tM_{t+1}}_{=1/R^f_{t+1}}E_tR^s_{t+1} \;+\; Cov_t\left[M_{t+1},\; R^s_{t+1}\right]. \tag{16}$$

Solving the above expression for $E_tR^s_{t+1}/R^f_{t+1}$ and then taking logs yields

$$\log\left(E_tR^s_{t+1}\right) - \log\left(R^f_{t+1}\right) \;=\; \log\left\{1 - Cov_t\left[M_{t+1},\; R^s_{t+1}\right]\right\}, \tag{17}$$

which shows that expected excess returns can be predictable under rational expectations if $Cov_t\left[M_{t+1},\; R^s_{t+1}\right]$ is stochastic and persistent.

4 Predictability from a Misspecified Forecast Rule

The theory of rational expectations is based on strong assumptions about investors' information. Specifically, the theory assumes that investors know the true stochastic processes for all relevant fundamental driving variables such as consumption growth and dividend growth. Here we provide a simple example to show how a departure from fully-rational expectations, in the form a misspecified forecast rule for consumption growth, can give rise to persistent percentage forecast errors.

[5] For example, the abstract of Cochrane and Piazzesi (2005) reads "We study time variation in expected excess bond returns. We run regressions of one-year [realized] excess returns on initial forward rates..." Ludvigson (2013, p. 810) states "Predictability of [realized] excess returns implies that the conditional expectation [of excess returns] varies." Menzly, Santos, Veronesi (2004) develop a theoretical model that exhibits predictability in expected excess returns, but their empirical application employs realized excess returns.

Suppose the investor's stochastic discount factor given by

$$
\begin{aligned}
M_{t+1} &= \beta\left(c_{t+1}/c_t\right)^{-\alpha} = \beta\exp\left(-\alpha x_{t+1}\right), && (18)\\
x_{t+1} &= \overline{x} + \rho\left(x_t - \overline{x}\right) + \sigma_t\varepsilon_{t+1}, \qquad |\rho| < 1, \qquad \varepsilon_t \sim NID\left(0,1\right), && (19)\\
\sigma_{t+1}^2 &= \overline{\sigma}^2 + \gamma\left(\sigma_t^2 - \overline{\sigma}^2\right) + u_{t+1}, \qquad |\gamma| < 1, \qquad u_t \sim NID\left(0,\sigma_u^2\right), && (20)
\end{aligned}
$$

where $x_{t+1} \equiv \log\left(c_{t+1}/c_t\right)$ is consumption growth that evolves as an AR(1) process with mean $\overline{x}$ and persistence parameter ρ. The consumption growth innovation ε_{t+1} is normally and independently distributed (NID) with mean zero and variance of one. We allow for time-varying fundamental uncertainty along the lines of Bansal and Yaron (2004), where γ governs the persistence of volatility and u_{t+1} is the innovation to volatility.

Under rational expectations, denoted by the subscript "re," the investor's percentage forecast error err_{t+1}^{re} is given by

$$
\begin{aligned}
err_{t+1}^{\text{re}} &= \log\left[\frac{M_{t+1}}{E_t\,M_{t+1}}\right] = \log\left[\frac{\beta\exp\left(-\alpha x_{t+1}\right)}{\beta\exp\left(-\alpha\overline{x} - \alpha\rho\left(x_t - \overline{x}\right) + \alpha^2\sigma_t^2/2\right)}\right],\\
&= -\alpha\sigma_t\,\varepsilon_{t+1} - \alpha^2\sigma_t^2/2, && (21)
\end{aligned}
$$

which shows that err_{t+1}^{re} will be predictable only when $\gamma \neq 0$ such that σ_t^2 is predictable. Notice that if the investor is risk neutral $(\alpha = 0)$, then this mechanism is not effective in generating predictability. Notice also that equation (20) does not allow for time-varying volatility-of-volatility. Allowing such a specification would not change the functional form of err_{t+1}^{re}, but it would imply that a forecasting equation obtained by regressing err_{t+1}^{re} on σ_t^2 for a given sample period might deliver poor out-of-sample forecasting performance. This is because the forecasting equation omits a hidden state variable, namely the time-varying volatility of σ_t^2.

Now consider an investor who ignores stochastic volatility and employs the following perceived law of motion for consumption growth

$$
x_{t+1} = \widehat{x} + \widehat{\rho}\left(x_t - \widehat{x}\right) + v_{t+1}, \qquad v_t \sim NID\left(0,\widehat{\sigma}_v^2\right), \tag{22}
$$

where $\widehat{x} = E\left(x_t\right), \widehat{\rho} = Corr\left(x_t,\, x_{t-1}\right)$ and $\widehat{\sigma}_v^2 = Var\left(x_t\right)\left(1-\widehat{\rho}^2\right)$ are the parameter values that the investor would estimate by simply matching the moments of observable data generated by the true law of motion (19). Under subjective expectations, denoted by the subscript "se," the investor's percentage forecast error is given by

$$
\begin{aligned}
err_{t+1}^{\text{se}} &= \log\left[\frac{M_{t+1}}{\widehat{E}_t\,M_{t+1}}\right] = \log\left[\frac{\beta\exp\left(-\alpha x_{t+1}\right)}{\beta\exp\left(-\alpha\widehat{x} - \alpha\widehat{\rho}\left(x_t - \widehat{x}\right) + \alpha^2\widehat{\sigma}_v^2/2\right)}\right],\\
&= -\alpha\sigma_t\,\varepsilon_{t+1} - \alpha^2\widehat{\sigma}_v^2/2 + \alpha\left(1-\widehat{\rho}\right)\left(\widehat{x} - \overline{x}\right) + \alpha\left(\widehat{\rho} - \rho\right)\left(x_t - \overline{x}\right), && (23)
\end{aligned}
$$

where $\widehat{E}_t\, M_{t+1}$ is the investor's subjective forecast based on the perceived law of motion (22). The above expression shows that the investor's forecast error can exhibit persistence due to the last term involving $x_t - \overline{x}$. In this case, err^{se}_{t+1} would be predictable using the previous period's percentage forecast error err^{SE}_t because the previous percentage forecast error includes a persistent term involving $x_{t-1} - \overline{x}$.

Similar results obtain for other types of misspecified forecast rules. Suppose, for example, that the investor does not know the precise law of motion for consumption growth and therefore employs a naive random-walk forecast such that $\widehat{E}_t\, M_{t+1} = M_t$. In this case, we have

$$\begin{aligned} err^{\mathrm{se}}_{t+1} &= \log\left[\frac{M_{t+1}}{\widehat{E}_t\, M_{t+1}}\right] = \log\left[\frac{\beta\exp\left(-\alpha x_{t+1}\right)}{\beta\exp\left(-\alpha x_t\right)}\right], \\ &= -\alpha\sigma_t\,\varepsilon_{t+1} + \alpha\left(1-\rho\right)\left(x_t - \overline{x}\right), \end{aligned} \tag{24}$$

which again shows that the investor's percentage forecast error will exhibit persistence due to the last term involving $x_t - \overline{x}$.

5 Predictability Regressions

Annual and quarterly data on nominal stock prices, nominal returns, nominal dividends, inflation, and realized stock market volatility are from Welch and Goyal (2008, updated).[6] Data on real personal consumption expenditures and population are from the Bureau of Economic Analysis (BEA), NIPA Table 2.3.5. Pre-1929 consumption data are from Robert Shiller's website. Data on implied volatility since 1990 comes from CBOE, and we construct a synthetic measure for VIX before 1990 by regressing VIX on realized volatility and lagged realized volatility.[7]

Table 1 (annual data) and Table 2 (quarterly data) provide summary statistics for the excess return series and the various predictor variables. Note that the annual excess returns series typically have negative serial correlations while the quarterly excess returns series have positive serial correlations. The log price-dividend ratio is very persistent, consistent with results previously documented in the literature (Cochrane, 2008). The trailing standard deviations of consumption growth and dividend growth are intended to capture persistent stochastic volatility in fundamental driving variables. These variables are all highly persistent,

[6] Updated data through 2014Q4 are available from www.hec.unil.ch/agoyal/.

[7] Specifically, we run the regression $\mathbf{VIX}_t = c_0 + c_1\sqrt{\mathbf{svar}_t} + c_2\sqrt{\mathbf{svar}_{t-1}}$ for the sample period 1990.Q1 to 2014.Q4. The variable $\mathbf{svar}_t$ is from Welch and Goyal (2008), defined as the variance of daily stock market returns over the most recent quarter or year, depending on the data frequency used in the predictability regressions. The estimated coefficients are used to construct a synthetic measure of $\mathbf{VIX}_t$ for the pre-1990 sample period.

exhibiting first-order autocorrelation statistics in the range of 0.75 to 0.95.[8]

The realized volatility of past stock market returns, as measured by **svar,** is much less persistent than the other predictive variables including the implied volatility measure **VIX.** There are several interesting cross correlations in Tables 1 and 2. In particular, excess returns on stocks are negatively correlated with both **svar** and **VIX** (consistent with the findings of Lettau and Ludvigson, 2010), but positively correlated with the trailing standard deviations of consumption growth **cgrowsd** and dividend growth **dgrowsd**. There is a substantial negative correlation between the trailing standard deviation of consumption growth and the log price-dividend ratio. Finally, the two return volatility measures, **svar** and **VIX,** are strongly correlated.

Our general predictability regression takes the following form for the excess return on stocks relative to the risk free rate:

$$\begin{aligned}\mathbf{ersf}_{t+1} &= c_0 + c_1\,\mathbf{pd}_t + c_2\,\mathbf{cgrowsd}_t + c_3\,\mathbf{dgrowsd}_t + c_4\,\mathbf{svar}_t + c_5\,\mathbf{VIX}_t \\ &\quad + c_6\,\mathbf{ersf}_t + c_7\,\mathbf{ersb}_t + c_8\,\mathbf{ersf}_t^2 + c_9\,\mathbf{infl}_t, \end{aligned} \tag{25}$$

and the following form for the excess return on stocks relative to long term government bonds:

$$\begin{aligned}\mathbf{ersb}_{t+1} &= c_0 + c_1\,\mathbf{pd}_t + c_2\,\mathbf{cgrowsd}_t + c_3\,\mathbf{dgrowsd}_t + c_4\,\mathbf{svar}_t + c_5\,\mathbf{VIX}_t \\ &\quad + c_6\,\mathbf{ersb}_t + c_7\,\mathbf{ersf}_t + c_8\,\mathbf{ersb}_t^2 + c_9\,\mathbf{infl}_t, \end{aligned} \tag{26}$$

where $\mathbf{ersf}_{t+1} \equiv \log(R^s_{t+1}/R^f_{t+1})$ is the excess return on stocks relative to the risk free rate, $\mathbf{pd}_t$ is the logarithm of the price-dividend ratio for S&P 500 stock index, $\mathbf{cgrowsd}_t$ and $\mathbf{dgrowsd}_t$ are the trailing standard deviations (computed over 5-years for annual data or 20-quarters for quarterly data) of real per capita consumption growth and real dividend growth, respectively. The variable $\mathbf{svar}_t$ is the realized volatility of equity returns, $\mathbf{VIX}_t$ is the implied volatility based on the stock option index, $\mathbf{ersb}_t \equiv \log(R^s_t/R^b_t)$ is the excess return on stocks relative to long-term government bonds, and $\mathbf{infl}_t$ is the trailing mean CPI inflation rate (computed over 5-years for annual data or 20-quarters for quarterly data). Results are not particularly sensitive to small adjustments in the length of the moving windows that are used to compute the trailing standard deviations and means. We do not perform long-horizon predictability regressions because the empirical reliability of such results have been called into question by Boudoukh, Richardson, and Whitelaw (2008).

The price dividend ratio $\mathbf{pd}_t$ is a standard variable that often appears in predictability regressions. As originally shown by Campbell and Shiller (1988), a log-linear approximation of

[8] Note that part of the persistence in these volatility measures is due to the overlapping observations in the calculations. However, the potentially exaggerated persistence in these volatility measures works in favor of our argument as we will show that persistent stochastic volatility in the fundamental driving variables is not sufficient to explain all of the predictability in realized excess returns.

the stock return identity implies that the variance of the log price-dividend ratio must equal the sum of the ratio's covariances with: (1) future dividend growth rates, (2) future risk-free rates, and (3) future excess returns on stocks. The magnitude of each covariance term is a measure of the predictability of each component when the current price-dividend ratio is employed as the sole regressor in a forecasting equation.[9] Ma (2013) shows that the long-run risk model of Bansal and Yaron (2004) implies that the log price-dividend ratio is a linear function of the volatilities of consumption growth and dividend growth. The log price-dividend ratio therefore serves as a proxy variable for the volatility of fundamental driving variables when these latent variables are not directly observable. In this sense, including $\mathbf{pd}_t$ as a regressor complements $\mathbf{cgrowsd}_t$ and $\mathbf{dgrowsd}_t$ as a way of capturing persistent stochastic volatility of the fundamental driving variables.

The variables $\mathbf{ersf}_t$ in equation (25) and $\mathbf{ersb}_t$ in equation (26) represent the previous period's excess return, representing the investor's previous percentage forecast error according to our theoretical framework. These variables are intended to capture persistence in investor forecast errors that may arise due to departures from fully-rational expectations, after controlling for persistent stochastic volatility in fundamental driving variables. We also include the squared value of the previous period's excess return ($\mathbf{ersf}_t^2$ or $\mathbf{ersb}_t^2$) to account for possible asymmetry or nonlinearity in the investor's forecast errors. For example, prospect theory suggests that investors react differently to losses versus gains (Kahneman and Tversky, 1979). Investors may also react differently to negative versus positive forecast errors. Finally, the complimentary variables $\mathbf{ersb}_t$ in equation (25) and $\mathbf{ersf}_t$ in equation (26) are included to capture the possibility that investors employ a joint forecasting algorithm for excess returns on stocks and long-term bonds, such as a vector autoregression.[10]

Tables 3 and 4 show the results of 1-year ahead predictability regressions using annual data. Tables 5 and 6 show the results of 1-quarter ahead predictability regressions using quarterly data. We consider excess returns on stocks relative to the risk free rate (Tables 3 and 5) and excess returns on stocks relative to long-term government bonds (Tables 4 and 6). In each table, we examine the sensitivity of the regression results to different sample periods. In addition to the full set of predictive variables, we show results for several restricted specifications that include subsets of the predictive variables in equations (25) and (26).

The log price-dividend ratio is consistently significant with a negative coefficient in most of the regressions. This result indicates that a higher valuation ratio tends to be followed by a lower excess returns on stocks, consistent with previous results in the literature (Campbell and Shiller 1988, Cochrane 2008). In Regression 2 we add the set of fundamental driving variables

[9] Details of the variance decomposition are contained in the appendix.

[10] Table 3 through 6 report results for estimating equations (25) and (26) separately. Similar results are obtained when we estimate equations (25) and (26) as a system using the method of seemingly unrelated regressions.

such as $\mathbf{cgrowsd}_t$ and $\mathbf{dgrowsd}_t$ together with the returns volatility variables $\mathbf{svar}_t$ and $\mathbf{VIX}_t$. We find that $\mathbf{cgrowsd}_t$ is usually associated with a negative, albeit insignificant regression coefficient most of the time, except for the last regression in Table 4. In contrast, $\mathbf{dgrowsd}_t$ is more likely to be significant, but the sign is typically positive. The realized volatility measure $\mathbf{svar}_t$ is usually significant and associated with a negative regression coefficient, consistent with the findings of Lettau and Ludvigson (2010). The implied volatility measure $\mathbf{VIX}_t$ has a significantly positive coefficient in most cases, highlighting the potentially distinct roles of these two volatility measures. In Regression 3 we add the lagged excess returns and lagged squared excess returns as well as the trailing mean inflation rate to the regression that includes the log price-dividend ratio. Regression 3, which is intended to capture the persistence of investor forecast errors, yields a better fit compared to the first two regressions. Regression 4, which includes all predictive variables, yields the best fit as measure by the adjusted R-squared. In most cases, either the lagged excess return variable or the lagged squared excess return variable (or both) is significant. We find that lagged squared excess returns are particularly important, highlighting a role for asymmetric or nonlinear type of forecast errors.

The regression coefficients associated with the trailing mean inflation rate are mostly significant and negative, clearly indicating a role for inflation illusion. Specifically, higher inflation predicts lower excess returns on stocks.[11] This finding is consistent with what has been documented about the negative relationship between the inflation rate and the real equity returns (Nelson and Schwert, 1977).

Figures 1 through 4 show the actual and predicted values of the excess returns on stocks using annual data (Figures 1 and 2) and quarterly data (Figures 3 and 4).

Overall, we find that the lagged excess return variables enter the regressions in a significant way, even after we control for the persistent stochastic volatility of the fundamental driving variables. Hence, findings of predictability in excess stock returns appear to be partly driven by persistence in investors' forecast errors. This result strongly suggests that the investors' forecasts are not fully rational.

6 Conclusion

This paper shows that the realized excess returns on stocks relative to bonds in a standard consumption-based asset pricing model can be represented by an additive combination of the representative investor's percentage forecast errors. As a result, predictability of realized excess returns can arise from only two sources: (1) persistent stochastic volatility of the model's fundamental driving variables, or (2) persistent investor forecast errors, implying a departure

[11] Recall that we define excess returns as $\log(R^s_{t+1}/R^f_{t+1})$ or $\log(R^s_{t+1}/R^b_{t+1})$. where R^s_{t+1}, R^f_{t+1}, and R^b_{t+1} are nominal returns, each of which is influenced by inflation . The computation of excess returns causes the effects of inflation to drop out, providing a measure of excess real returns.

from fully-rational expectations. This is a general result that holds for any stochastic discount factor, any consumption or dividend process, and any stream of bond coupon payments.

From an empirical perspective, we find evidence of predictability of excess returns from both of the above-named sources. In particular, we find that lagged excess returns and lagged squared excess return are important in explaining movements in realized excess returns even after controlling for a variety of fundamental driving variables, including the price-dividend ratio, the recent volatilities of consumption growth and dividend growth, and measures of the realized and implied volatility of past stock market returns. Overall, our results suggest that departures from fully-rational expectations are an important contributor to the predictability of excess returns on stocks.

A Appendix: Variance Decomposition of p/d Ratio

Campbell and Shiller (1988), Campbell (1991), and Cochrane (1992, 2005) show that a log-linear approximation of the equity return identity implies that the variance of the log price-dividend ratio must equal the sum of the ratio's covariances with: (1) future dividend growth rates, (2) future risk-free rates, and (3) future excess returns on equity.

The definition of the log return on stocks (5) can be approximated as follows:

$$\begin{aligned}\log(R^s_{t+1}) &\equiv \log\left(p^s_{t+1}/d_{t+1}+1\right)+\log\left(d_{t+1}/d_t\right)-\log\left(p^s_t/d_t\right), \\ &\simeq \kappa_0+\kappa_1\log\left(p^s_{t+1}/d_{t+1}\right)+\log\left(d_{t+1}/d_t\right)-\log\left(p^s_t/d_t\right), \qquad \text{(A.1)}\end{aligned}$$

where κ_0 and κ_1 are Taylor-series coefficients.[12] Solving equation (A.1) for $\log\left(p^s_t/d_t\right)$ and then successively iterating the resulting expression forward to eliminate $\log(p^s_{t+1+j}/d_{t+1+j})$ for $j=0,1,2...$ yields the following approximate identity:

$$\begin{aligned}\log\left(p^s_t/d_t\right) &\simeq \frac{\kappa_0}{1-\kappa_1}+\sum_{j=0}^{\infty}(\kappa_1)^j\left[\log\left(d_{t+1+j}/d_{t+j}\right)-\log(R^s_{t+1+j})\right], \\ &\simeq \frac{\kappa_0}{1-\kappa_1}+\sum_{j=0}^{\infty}(\kappa_1)^j\left[\log\left(d_{t+1+j}/d_{t+j}\right)+\log(R^f_{t+1+j})-\log(R^s_{t+1+j}/R^f_{t+1+j})\right], \\ & \qquad \text{(A.2)}\end{aligned}$$

where the second version of the expression breaks up future log stock returns into two parts: future risk free rates, denoted by $\log(R^f_{t+1+j})$, and future excess stock returns, as given by $\log(R^s_{t+1+j}/R^f_{t+1+j})$. Equation which shows that movements in the log price-dividend ratio must be accounted for by movements in either future dividend growth rates, future risk free rates, or future excess returns.

The variables in the approximate identity (A.2) can be expressed as deviations from their unconditional means while the means are consolidated into the constant term. Multiplying both sides of the resulting expression by $\log\left(p^s_t/d_t\right)-E\left[\log\left(p^s_t/d_t\right)\right]$ and then taking the unconditional expectation of both sides yields

$$\begin{aligned}Var\left[\log\left(p^s_t/d_t\right)\right] &= Cov\left[\log\left(p^s_t/d_t\right),\ \sum_{j=0}^{\infty}(\kappa_1)^j\log\left(d_{t+1+j}/d_{t+j}\right)\right] \\ &\quad -Cov\left[\log\left(p^s_t/d_t\right),\ \sum_{j=0}^{\infty}(\kappa_1)^j\log(R^f_{t+1+j})\right] \\ &\quad -Cov\left[\log\left(p^s_t/d_t\right),\ \sum_{j=0}^{\infty}(\kappa_1)^j\log(R^s_{t+1+j}/R^f_{t+1+j})\right]. \qquad \text{(A.3)}\end{aligned}$$

[12] The Taylor series coefficients are given by $\kappa_1=\exp\left[E\log\left(p^s_t/d_t\right)\right]/\left\{1+\exp\left[E\log\left(p^s_t/d_t\right)\right]\right\}$ and $\kappa_0=-\log\left(1-\kappa_1\right)$.

The above equation states that the variance of the log price-dividend ratio must be accounted for by the covariance of the ratio with future dividend growth rates, future risk free rates, or future excess returns. The magnitude of each covariance term is a measure of the predictability of each component when the current price-dividend ratio is employed as the sole regressor in a forecasting equation.

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Table 1: Summary Statistics for Annual Data, 1935 to 2014

Variable	Mean	Std. Dev.	Skewness	Min.	Max.	Autocorrelation
ersf	0.067	0.174	−0.748	−0.455	0.402	−0.049
ersb	0.047	0.198	−0.802	−0.685	0.417	−0.122
erbf	0.021	0.098	0.322	−0.184	0.263	−0.090
pd	3.404	0.457	0.344	2.505	4.478	0.891
cgrowsd	0.021	0.014	1.343	0.004	0.076	0.737
dgrowsd	0.071	0.079	1.899	0.010	0.357	0.879
svar	0.026	0.027	2.801	0.003	0.168	0.464
VIX	18.422	5.207	1.285	10.971	35.235	0.702

				Cross Correlations				
	ersf	**ersb**	**erbf**	**pd**	**cgrowsd**	**dgrowsd**	**svar**	**VIX**
ersf	1.000							
ersb	0.869	1.000						
erbf	0.014	−0.482	1.000					
pd	0.142	0.050	0.150	1.000				
cgrowsd	0.114	0.115	−0.032	−0.574	1.000			
dgrowsd	0.094	0.055	0.055	−0.376	0.627	1.000		
svar	−0.400	−0.449	0.199	0.025	0.151	0.325	1.000	
VIX	−0.284	−0.334	0.173	0.056	0.248	0.429	0.923	1.000

Notes: ersf = excess return on S&P 500 index relative to the risk free rate, as measured by the return on 3-month Treasury bills, erbf = excess return on S&P 500 index relative to return on 20-year U.S. government bonds
erbf = excess return on 20-year U.S. government bonds relative to the risk free rate
pd = log price-dividend ratio for S&P 500, cgrowsd = 5-year trailing standard deviation of real per capita consumption growth. dgrowsd = 5-year trailing standard deviation of real dividend growth for S&P 500 index.
svar = realized volatility, VIX = implied volatility.

Table 2: Summary Statistics for Quarterly Data, 1955Q1 to 2014Q4

Variable	Mean	Std. Dev.	Skewness	Min.	Max.	Autocorrelation
ersf	0.013	0.080	−0.894	−0.309	0.193	0.091
ersb	0.008	0.094	−1.134	−0.436	0.233	0.056
erbf	0.004	0.053	0.518	−0.194	0.201	-0.036
pd	3.554	0.391	0.414	2.779	4.493	0.973
cgrowsd	0.007	0.002	1.067	0.002	0.018	0.920
dgrowsd	0.015	0.009	1.224	0.006	0.040	0.972
svar	0.006	0.010	7.232	0.0004	0.114	0.416
VIX	17.935	6.013	2.732	10.384	58.067	0.755

Cross Correlations

	ersf	**ersb**	**erbf**	**pd**	**cgrowsd**	**dgrowsd**	**svar**	**VIX**
ersf	1.000							
ersb	0.829	1.000						
erbf	0.034	−0.531	1.000					
pd	0.120	0.055	0.084	1.000				
cgrowsd	0.021	0.068	−0.091	−0.752	1.000			
dgrowsd	0.102	0.107	−0.037	0.121	0.148	1.000		
svar	−0.428	−0.517	0.275	0.062	−0.116	−0.062	1.000	
VIX	−0.316	−0.387	0.212	0.143	−0.167	−0.065	0.871	1.000

Notes: ersf = excess return on S&P 500 index relative to the risk free rate, as measured by the return on 3-month Treasury bills, erbf = excess return on S&P 500 index relative to return on 20-year U.S. government bonds
erbf = excess return on 20-year U.S. government bonds relative to the risk free rate
pd = log price-dividend ratio for S&P 500, cgrowsd = 5-year (20-quarter) trailing standard deviation of real per capita consumption growth. dgrowsd = 5-year (20-quarter) trailing standard deviation of real dividend growth for S&P 500 index. svar = realized volatility, VIX = implied volatility.

Table 3: Predicting Excess Returns on Stocks vs. Risk Free Rate (Annual Data)

ersf$_{t+1}$	**pd**$_t$	**cgrowsd**$_t$	**dgrowsd**$_t$	**svar**$_t$	**VIX**$_t$	**ersf**$_t$	**ersb**$_t$	**ersf**$_t^2$	**infl**$_t$	adj. R^2
					1935 to 2014					
1	**−0.095**									0.050
	(−2.421)									
2	**−0.154**	−2.049	−0.223	−0.846	0.011					0.044
	(−2.815)	(−1.058)	(−0.455)	(−0.613)	(1.100)					
3	**−0.108**					0.201	−0.235	**0.589**	−0.671	0.064
	(−2.137)					(1.022)	(−1.498)	(2.020)	(−0.860)	
4	**−0.249**	−4.265	−0.485	**−3.080**	**0.020**	0.310	−0.270	**0.993**	**−1.721**	0.122
	(−3.383)	(−1.824)	(−0.996)	(−2.149)	(2.088)	(1.406)	(−1.653)	(2.643)	(−2.267)	
					1955 to 2014					
1	−0.075									0.017
	(−1.499)									
2	**−0.161**	−4.686	**1.100**	0.029	0.007		0.038			0.050
	(−3.123)	(−1.436)	(2.529)	(0.026)	(0.925)					
3	**−0.136**					0.302	−0.286	**0.859**	**−1.471**	0.090
	(−2.472)					(1.457)	(−1.674)	(3.149)	(−2.254)	
4	**−0.265**	−6.687	**0.885**	**−3.029**	**0.020**	0.304	**−0.314**	**0.949**	**−2.034**	0.138
	(−3.815)	(−1.846)	(2.022)	(−2.546)	(2.651)	(1.333)	(−2.055)	(2.317)	(−2.524)	

Notes: Newey-West corrected t-statistics in parentheses. Boldface indicates significant at 5% level.

ersf = excess return on S&P 500 index relative to the risk free rate, as measured by the return on 3-month Treasury bills

erbf = excess return on 20-year U.S. government bonds relative to the risk free rate, pd = log price-dividend ratio for S&P 500

cgrowsd = five-year trailing standard deviation of real per capita consumption growth, svar = realized volatility, VIX = implied volatility

dgrowsd = five-year trailing standard deviation of real dividend growth for S&P 500 index, infl = inflation rate

Table 4: Predicting Excess Returns on Stocks vs. Long-Term Bonds (Annual Data)

$\mathbf{ersb}_{t+1}$	$\mathbf{pd}_t$	$\mathbf{cgrowsd}_t$	$\mathbf{dgrowsd}_t$	$\mathbf{svar}_t$	$\mathbf{VIX}_t$	$\mathbf{ersb}_t$	$\mathbf{ersf}_t$	$\mathbf{ersb}_t^2$	$\mathbf{infl}_t$	adj. R^2
					1935 to 2014					
1	**−0.132** (−2.875)									0.080
2	**−0.184** (−3.245)	−1.570 (−0.810)	−0.326 (−0.663)	1.936 (1.131)	−0.004 (−0.413)					0.085
3	**−0.159** (−2.690)					**−0.380** (−2.343)	0.413 (1.872)	**0.766** (4.500)	−0.510 (−0.579)	0.146
4	**−0.314** (−4.312)	−4.025 (−1.738)	−0.604 (−1.379)	−1.951 (−1.006)	0.009 (0.931)	**−0.478** (−2.573)	**0.592** (2.516)	**1.048** (3.841)	**−1.974** (−2.451)	0.211
					1955 to 2014					
1	**−0.141** (−2.221)									0.062
2	**−0.213** (−2.940)	−5.392 (−1.485)	**1.247** (2.321)	**3.526** (2.651)	−0.012 (−1.566)					0.097
3	**−0.255** (−3.614)					**−0.478** (−2.651)	**0.569** (2.459)	**0.763** (4.917)	**−2.311** (−2.964)	0.215
4	**−0.371** (−3.884)	**−9.353** (−2.311)	0.844 (1.315)	−1.974 (−0.768)	0.009 (0.779)	**−0.529** (−3.136)	**0.599** (2.524)	**1.017** (2.849)	**−2.339** (−2.352)	0.214

Notes: Newey-West corrected t-statistics in parentheses. Boldface indicates significant at 5% level.

ersf = excess return on S&P 500 index relative to the risk free rate, as measured by the return on 3-month Treasury bills

erbf = excess return on 20-year U.S. government bonds relative to the risk free rate, pd = log price-dividend ratio for S&P 500

cgrowsd = five-year trailing standard deviation of real per capita consumption growth, svar = realized volatility, VIX = implied volatility

dgrowsd = five-year trailing standard deviation of real dividend growth for S&P 500 index, infl = inflation rate

Table 5: Predicting Excess Returns on Stocks vs. Risk Free Rate (Quarterly Data)

ersf$_{t+1}$	**pd**$_t$	**cgrowsd**$_t$	**dgrowsd**$_t$	**svar**$_t$	**VIX**$_t$	**ersf**$_t$	**ersb**$_t$	**ersf**$_t^2$	**infl**$_t$	adj. R^2
					1955Q1 to 2014Q4					
1	−0.018									0.003
	(−1.238)									
2	**−0.059**	−5.957	**1.648**	**−2.731**	**0.006**					0.063
	(−2.388)	(−1.504)	(3.352)	(−2.221)	(3.206)					
3	**−0.039**					**0.274**	−0.120	**1.193**	**−2.248**	0.051
	(−2.564)					(2.374)	(−0.997)	(3.219)	(−2.393)	
4	**−0.092**	−7.598	**1.116**	**−3.607**	**0.006**	**0.293**	**−0.179**	**1.170**	**−3.022**	0.114
	(−3.623)	(−1.949)	(2.268)	(−2.945)	(2.269)	(2.739)	(−2.103)	(2.365)	(−2.795)	
					1975Q1 to 2014Q4					
1	−0.019									0.005
	(−1.208)									
2	**−0.079**	−11.530	**1.107**	**−2.903**	**0.006**					0.068
	(−2.569)	(−1.965)	(1.995)	(−2.213)	(2.817)					
3	**−0.092**					0.159	−0.051	0.719	**−6.398**	0.062
	(−3.538)					(1.432)	(−0.426)	(1.331)	(−3.228)	
4	**−0.124**	−6.770	0.633	**−4.535**	**0.006**	0.137	−0.158	**1.776**	**−6.060**	0.128
	(−3.555)	(−1.086)	(1.065)	(−3.483)	(2.857)	(1.212)	(−1.659)	(2.200)	(−2.149)	

Notes: Newey-West corrected t-statistics in parentheses. Boldface indicates significant at 5% level.
ersf = excess return on S&P 500 index relative to the risk free rate, as measured by the return on 3-month Treasury bills
erbf = excess return on 20-year U.S. government bonds relative to the risk free rate, pd = log price-dividend ratio for S&P 500
cgrowsd = five-year trailing standard deviation of real per capita consumption growth, svar = realized volatility, VIX = implied volatility
dgrowsd = five-year trailing standard deviation of real dividend growth for S&P 500 index, infl = inflation rate

Table 6: Predicting Excess Returns on Stocks vs. Long-Term Bonds (Quarterly Data)

$\mathbf{ersb}_{t+1}$	$\mathbf{pd}_t$	$\mathbf{cgrowsd}_t$	$\mathbf{dgrowsd}_t$	$\mathbf{svar}_t$	$\mathbf{VIX}_t$	$\mathbf{ersb}_t$	$\mathbf{ersf}_t$	$\mathbf{ersb}_t^2$	$\mathbf{infl}_t$	adj. R^2
	1955Q1 to 2014Q4									
1	−0.029									0.010
	(−1.660)									
2	**−0.058**	−3.629	**1.627**	**−2.539**	**0.006**					0.045
	(−2.125)	(−0.961)	(2.506)	(−2.067)	(2.674)					
3	**−0.060**					−0.169	**0.381**	0.402	**−2.718**	0.053
	(−3.061)					(−1.470)	(2.917)	(1.390)	(−2.563)	
4	**−0.107**	−5.764	0.888	−3.118	**0.006**	−0.198	**0.397**	0.304	**−3.661**	0.092
	(−3.511)	(−1.543)	(1.145)	(−1.612)	(2.723)	(−1.958)	(3.179)	(0.636)	(−2.769)	
	1975Q1 to 2014Q4									
1	−0.028									0.009
	(−1.523)									
2	**−0.087**	−10.692	**1.294**	**−3.179**	**0.007**					0.059
	(−2.442)	(−1.676)	(1.557)	(−2.305)	(2.773)					
3	−0.090					−0.129	**0.318**	**0.250**	**−5.218**	0.039
	(−2.672)					(1.002)	(2.111)	(0.771)	(−2.114)	
4	**−0.125**	−9.003	0.909	−3.813	**0.007**	−0.210	**0.346**	0.365	−3.746	0.078
	(−3.133)	(−1.287)	(0.982)	(−1.716)	(2.544)	(−1.754)	(2.264)	(0.634)	(−1.200)	

Notes: Newey-West corrected t-statistics in parentheses. Boldface indicates significant at 5% level.
ersf = excess return on S&P 500 index relative to the risk free rate, as measured by the return on 3-month Treasury bills
erbf = excess return on 20-year U.S. government bonds relative to the risk free rate, pd = log price-dividend ratio for S&P 500
cgrowsd = five-year trailing standard deviation of real per capita consumption growth, svar = realized volatility, VIX = implied volatility
dgrowsd = five-year trailing standard deviation of real dividend growth for S&P 500 index, infl = inflation rate

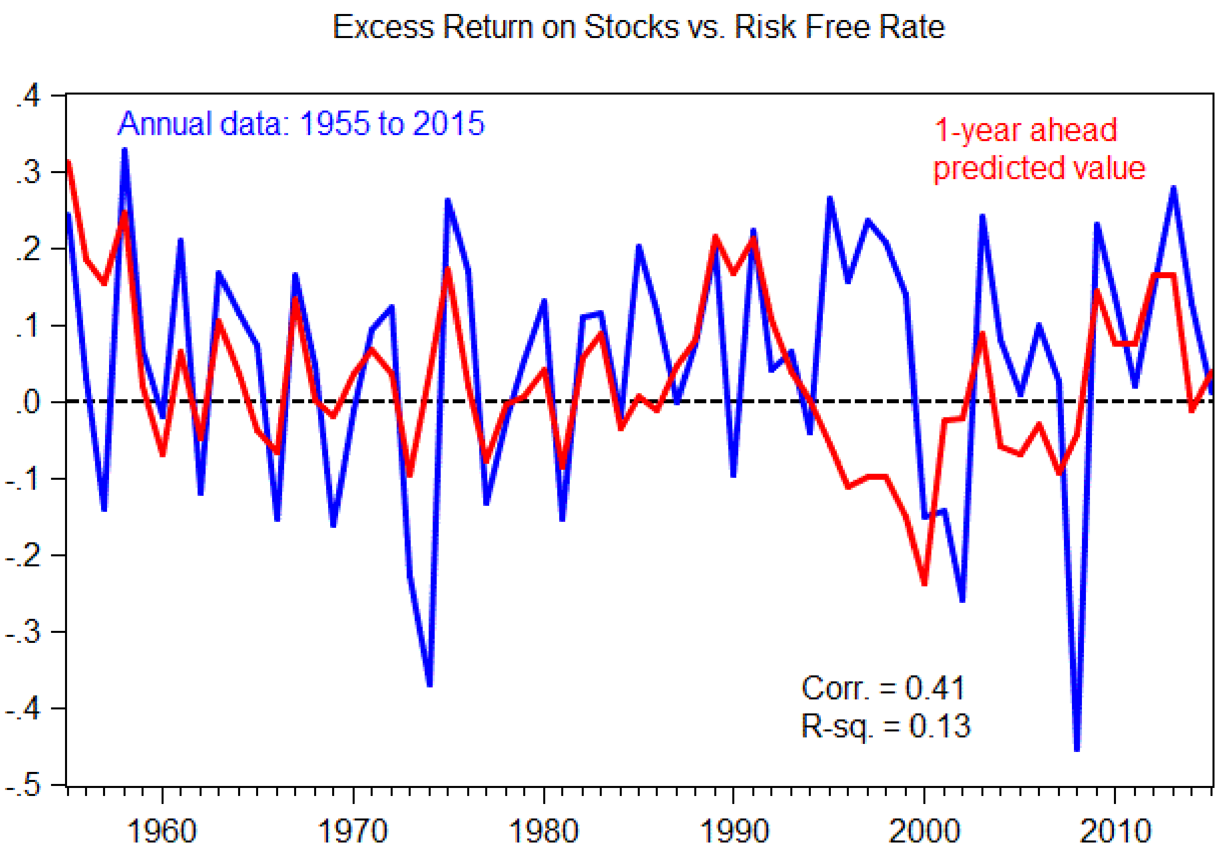

Figure 1: The predicted value is constructed using equation (25). The adjusted R-squared is 0.13.

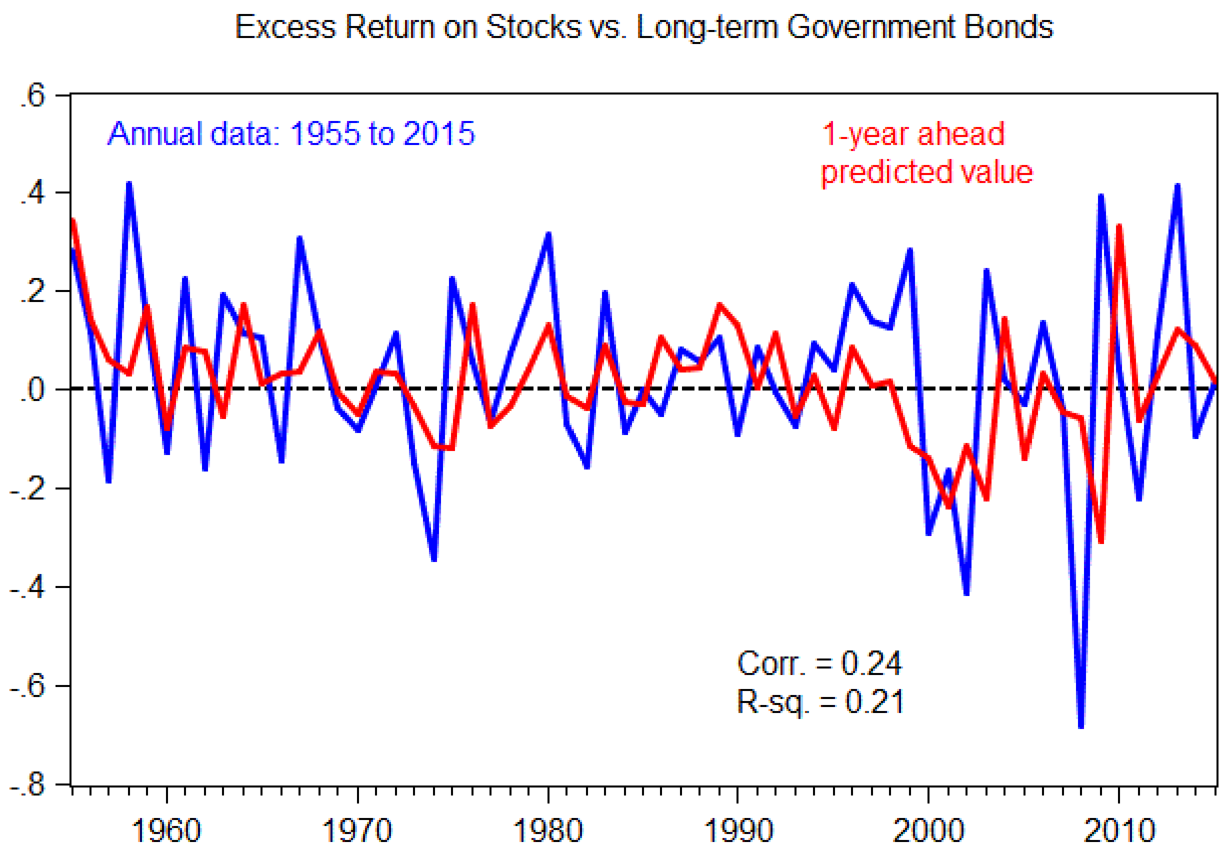

Figure 2: The predicted value is constructed using equation (26). The adjusted R-squared is 0.21.

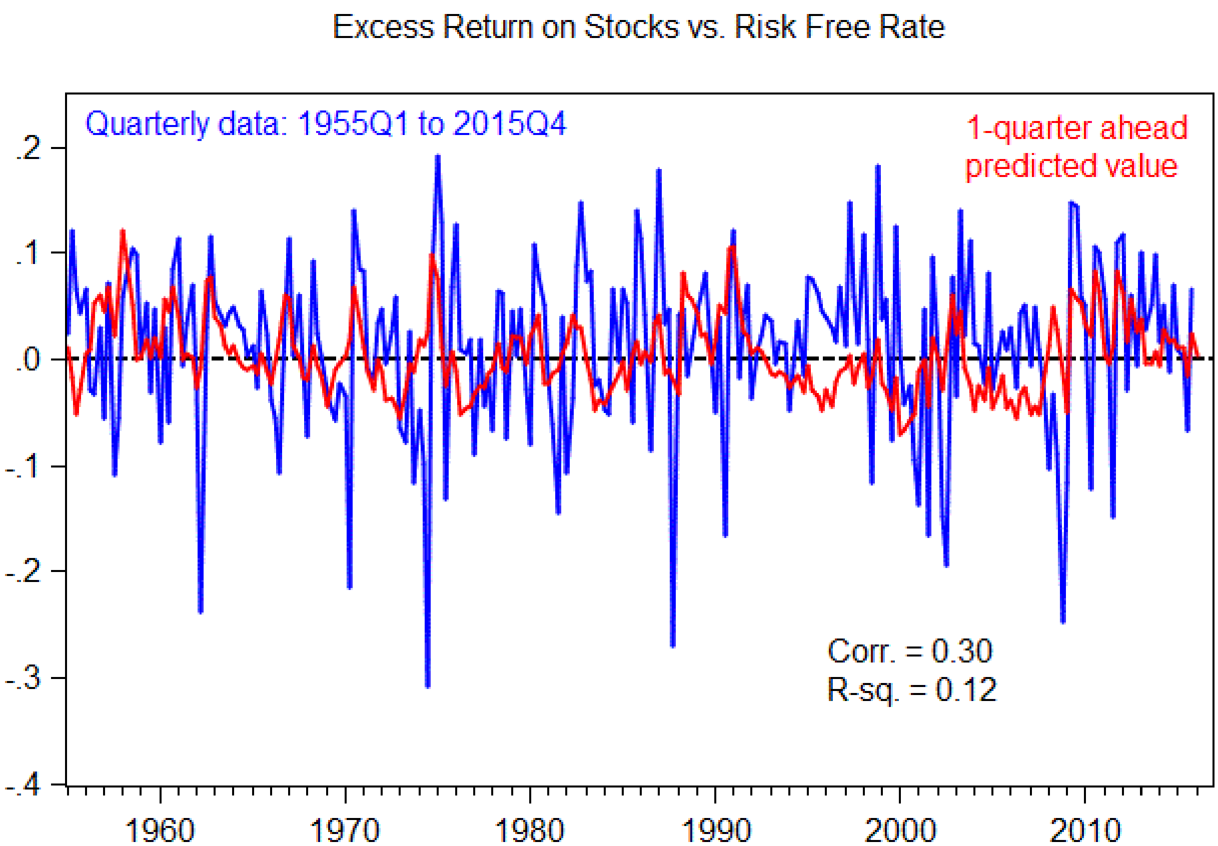

Figure 3: The predicted value is constructed using equation (25). The adjusted R-squared is 0.12.

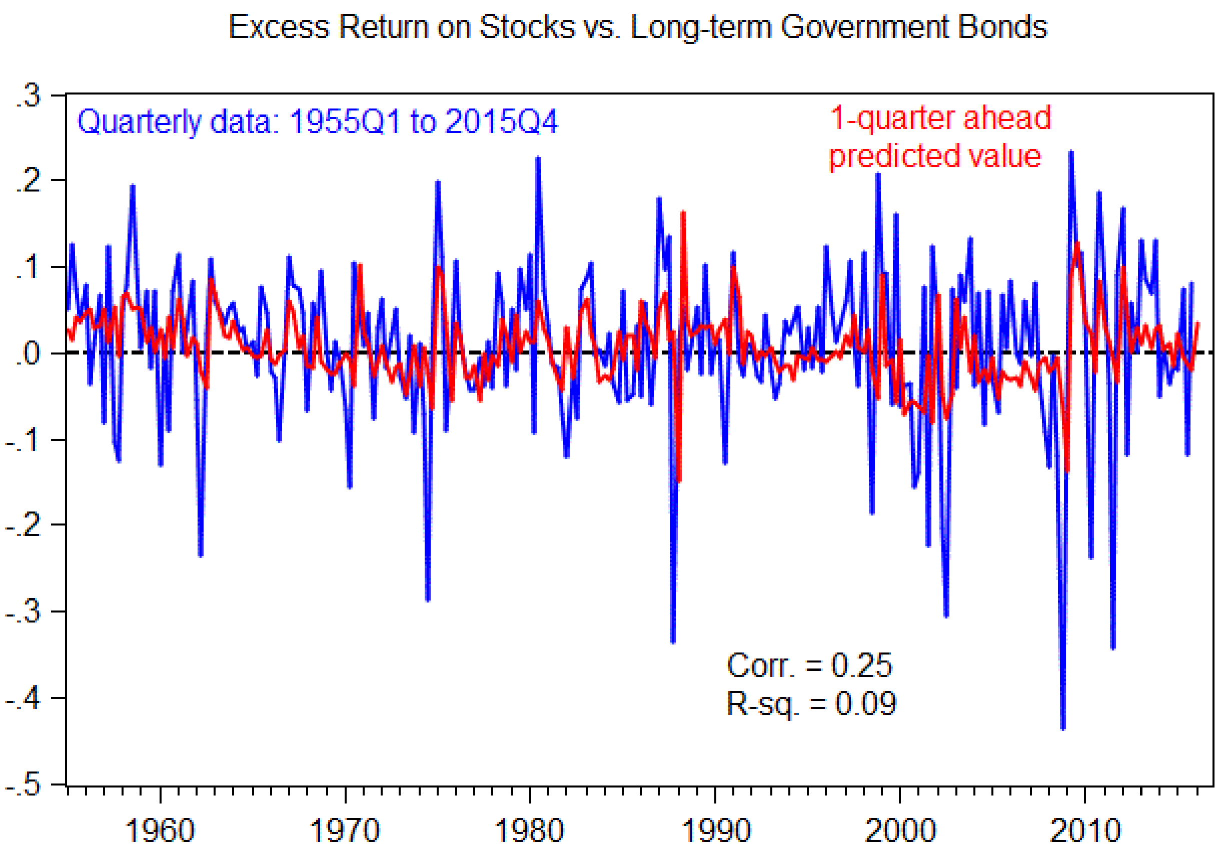

Figure 4: The predicted value is constructed using equation (26). The adjusted R-squared is 0.09.