Endogenous Regime Shifts in a New Keynesian Model with a Time-varying Natural Rate of Interest[1]

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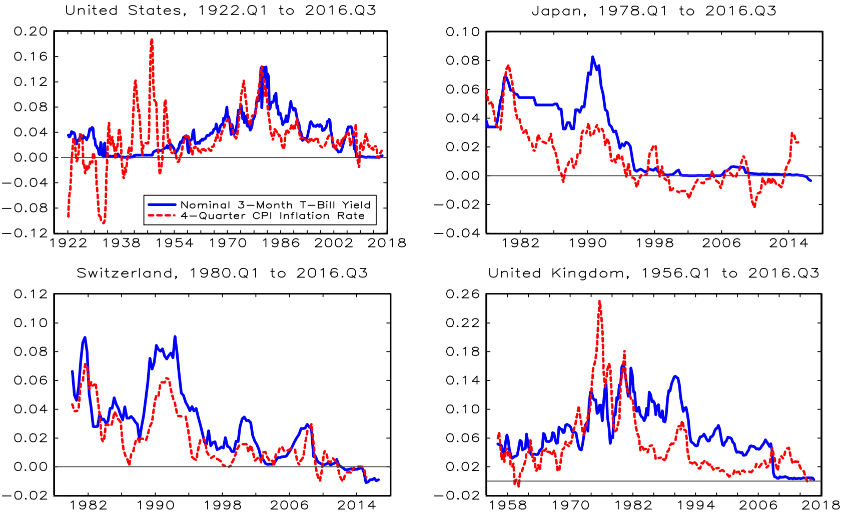
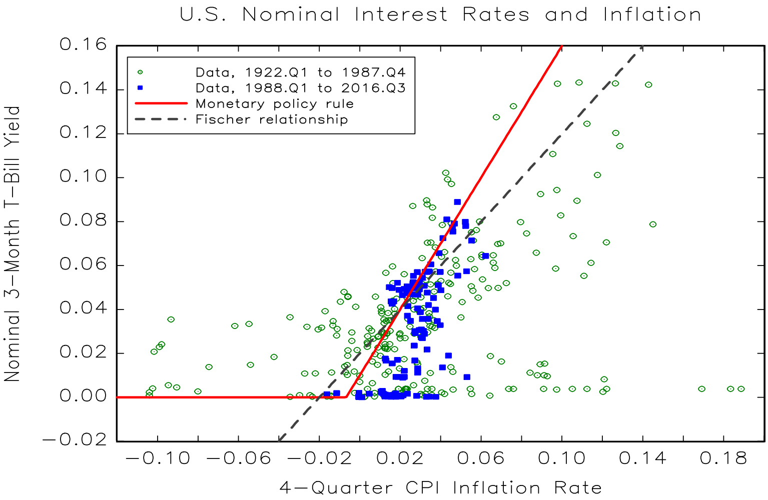
[1] Any opinions expressed here do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

Numerous ZLB (or ELB) episodes in global data



U.S. data: ZLB binding 2008.Q4 to 2015.Q4

"Promising to remain at zero for a long time is a double-edged sword." (Bullard 2010).



Standard NK model has multiple RE equilibria

- Taylor rule + Fisher Eqn. + ZLB $\Rightarrow$ Two steady states. (Benhabib, Schmitt-Grohé & Uribe *AER*, *JET* 2001a,b).
- r^* = "natural rate of interest" (also called "equilibrium" or "neutral" rate). The real rate consistent with full utilization of resources and steady inflation at central bank's target π^*. Evidence: r^* shifts over time (Laubach & Williams 2003, 2015).
- Two long-run endpoints (steady states): (1) targeted where $i = r^* + \pi^*$ and (2) deflation where $i = 0$ and $\pi = -r^*$.
- Two local RE solutions: (1) targeted equilibrium is locally unique, and (2) deflation equilibrium allows for sunspot shocks (focus on MSV solution here; no sunspots).

Standard NK model has multiple RE equilibria

- This paper: NK model with shifting r_t^*. Agent employs weighted-average of the two local forecast rules. Weights depend on past forecast performance, i.e., *RMSFE*.
- Forecast rules from deflation equilibrium induce more volatility in π_t and y_t in response to r_t^* shocks.
- Results: Negative $r_t - r_t^* \Rightarrow$ more weight on deflation forecast rules $\Rightarrow$ deflation can become self-fulfilling. Episode accompanied by severe recession (highly negative output gap) with nominal rate at ZLB. Similar to 2007-09 Great Recession.
- But even in normal times, agent may place nontrivial weight on deflation forecast rules, causing central bank to consistently undershoot π^* (like now: $\pi_t^{\text{U.S.}} < 0.02$ since mid-2012).

Related literature (partial list)

- Infrequent but long-lived ZLB episodes in global data
  Dordal-i-Carreras, Coibion, Gorodnichenko & Wieland (2016)
- Transition between regimes driven by sunpots
  Aruoba, Cuba-Borda, & Schorfheide (2014, WP)
  Aruoba & Schorfheide (2015, WP)
- Adaptive learning to select among multiple equilibria
  Evans & Honkapohja (2005, *RED*),
  Eusepi (2007, *JME*)
  Evans, Guse, & Honkapohja (2008, *EER*)
  Benhabib, Evans & Honkapohja (2014, *JEDC*)
- Optimal monetary policy with shifting natural rate
  Eggertsson and Woodford (2003, BPEA)
  Evans, Fisher, Gourio & Krane (2015, *BPEA*)
  Hamilton, Harris, Hatzius, & West (2016. *IMF Econ. Rev.*)
  Gust, Johannsen, López-Salido (2015, WP)
  Basu & Bundick (2015, NBER WP 21838)

New Keynesian model with zero lower bound (ZLB)

$$
\begin{aligned}
y_t &= E_t\, y_{t+1} - \alpha \overbrace{\left[i_t - E_t\, \pi_{t+1} - r_t\right]}^{\text{Fisher relationship}} + v_t, \qquad v_t \sim N\left(0,\, \sigma_v^2\right) \\
\pi_t &= \beta E_t\, \pi_{t+1} + \kappa y_t + u_t, \qquad u_t \sim N\left(0,\, \sigma_u^2\right) \\
i_t^* &= \rho i_{t-1}^* + (1-\rho)\left[E_t r_t^* + \pi^* + g_\pi\left(\overline{\pi}_t - \pi^*\right) + g_y\left(y_t - y^*\right)\right] \\
\overline{\pi}_t &= \omega\, \pi_t + (1-\omega)\, \overline{\pi}_{t-1}, \qquad \overline{\pi}_t \simeq \tfrac{1}{4}\left(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right) \\
i_t &= \max\left\{0,\ i_t^*\right\}
\end{aligned}
$$

Natural rate of interest (exogenous):

$$
r_t \equiv -\log \underbrace{\left[\beta \exp\left(v_t\right)\right]}_{\text{Discount factor}} + \underbrace{E_t \Delta \bar{y}_{t+1}}_{\text{Expected potential output growth}}
$$

$$
\begin{aligned}
r_t &= \rho_r\, r_{t-1} + \left(1-\rho_r\right) r_t^* + \varepsilon_t, \qquad \varepsilon_t \sim N\left(0,\, \sigma_\varepsilon^2\right) \\
r_t^* &= r_{t-1}^* + \eta_t, \qquad \eta_t \sim N\left(0,\, \sigma_\eta^2\right)
\end{aligned}
$$

Two long-run endpoints (steady states)

Targeted Endpoint

$$\pi_t = \pi^*$$
$$y_t = y^* \equiv \pi^* (1-\beta)/\kappa$$
$$i_t^* = r_t^* + \pi^*$$
$$i_t = i_t^*$$

Deflation Endpoint

$$\pi_t = -r_t^*$$
$$y_t = -r_t^* (1-\beta)/\kappa$$
$$i_t^* = (r_t^* + \pi^*) \left[1 - g_\pi - \frac{g_y (1-\beta)}{\kappa}\right]$$
$$i_t = 0$$

Shifting Endpoint Time Series Model (Kozick-Tinsley, JMCB 2012)

$$E_t r_t^* = \lambda \left[\frac{r_t - \rho_r r_{t-1}}{1-\rho_r}\right] + (1-\lambda) E_{t-1} r_{t-1}^*$$

Kalman gain

$$\lambda = \frac{-(1-\rho_r)^2 \phi + (1-\rho_r)\sqrt{(1-\rho_r)^2 \phi^2 + 4\phi}}{2}, \qquad \phi \equiv \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$$

$$E_t (r_{t+k} - r_{t+k}^*) = (\rho_r)^k (r_t - E_t r_t^*), \qquad \rho_r = 0.857$$

Two local RE equilibria

Targeted Equilibrium (Unique) assumes $i_t^* = i_t > 0$

$$\begin{aligned}
\pi_t &= \ldots + \mathbf{A}_{11}\left(r_t - E_t r_t^*\right) + \mathbf{A}_{12}\left(\overline{\pi}_{t-1} - \pi^*\right) + \mathbf{A}_{13} u_t + \mathbf{A}_{14} v_t \\
y_t &= \ldots + \mathbf{A}_{21}\left(r_t - E_t r_t^*\right) + \mathbf{A}_{22}\left(\overline{\pi}_{t-1} - \pi^*\right) + \mathbf{A}_{23} u_t + \mathbf{A}_{24} v_t \\
i_t^* &= \ldots + \mathbf{A}_{31}\left(r_t - E_t r_t^*\right) + \mathbf{A}_{32}\left(\overline{\pi}_{t-1} - \pi^*\right) + \mathbf{A}_{33} u_t + \mathbf{A}_{34} v_t
\end{aligned}$$

Deflation Equilibrium (MSV) assumes $i_t^* \leq 0, \;\; i_t = 0$

$$\begin{aligned}
\pi_t &= \ldots + \mathbf{B}_{11}\left(r_t - E_t r_t^*\right) + u_t + \kappa v_t \\
y_t &= \ldots + \mathbf{B}_{21}\left(r_t - E_t r_t^*\right) + v_t \\
i_t^* &= \ldots + \mathbf{B}_{31}\left(r_t - E_t r_t^*\right) + \mathbf{B}_{32}\left(\overline{\pi}_{t-1} - \pi^*\right) + \mathbf{B}_{33} u_t + \mathbf{B}_{34} v_t
\end{aligned}$$

Solution coefficients when β, $\omega \to 1$ and $g_y \to 0$:

$$\frac{\mathbf{B}_{11}}{\mathbf{A}_{11}} = \frac{\mathbf{B}_{21}}{\mathbf{A}_{21}} = \frac{\mathbf{B}_{31}}{\mathbf{A}_{31}} = 1 + \underbrace{\frac{(1-\rho)g_\pi}{(\rho_r - \rho)} \frac{\rho_r \alpha \kappa}{\left[(1-\rho_r)^2 - \rho_r \alpha \kappa\right]}}_{>>\,1}$$

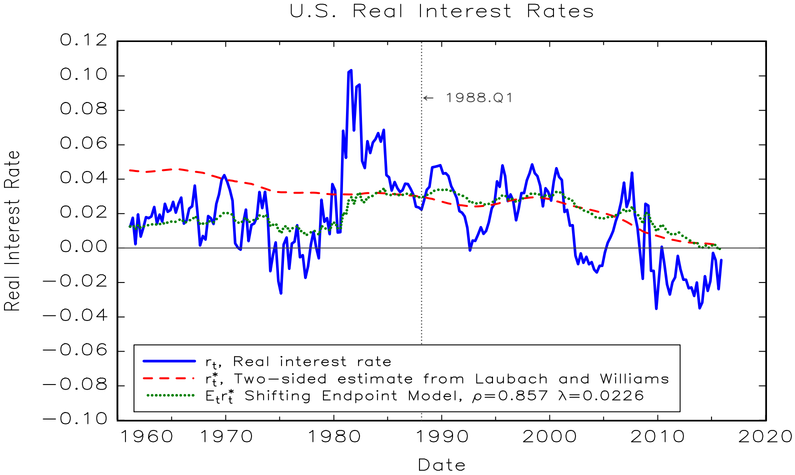
$\Rightarrow$ Deflation equilibrium exhibits much more volatility.

Model parameter values

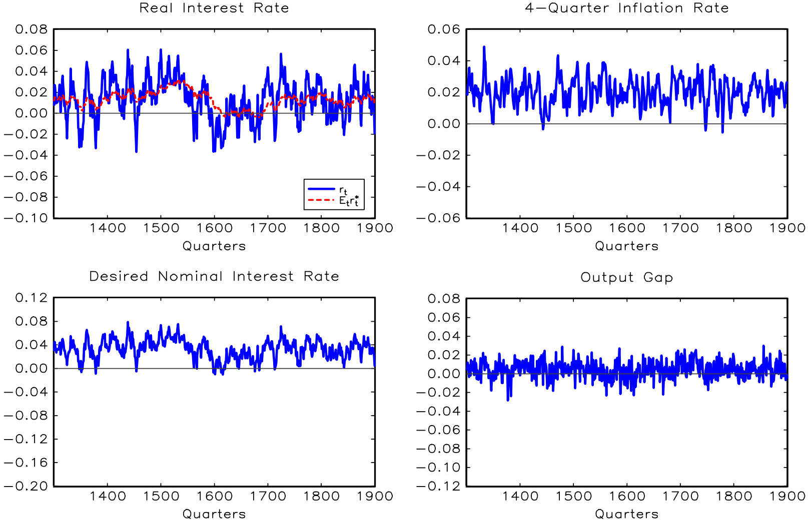
Parameter	Value	Description/Target
α	0.2	Interest rate coefficient in Euler equation.
β	0.995	Discount factor in Phillips curve.
κ	0.025	Output gap coefficient in Phillips curve.
π^*	0.02	Central bank inflation target.
ω	0.684	$\overline{\pi}_t \simeq$ 4-quarter inflation rate.
g_π	1.5	Policy rule response to inflation.
g_y	0.5	Policy rule response to output gap.
ρ	0.80	Interest rate smoothing parameter.
ρ_r	0.857	Persistence parameter for natural rate.
σ_ε	0.0099	Std. dev. temporary shock to natural rate.
σ_η	0.0016	Std. dev. permanent shock to natural rate.
λ	0.0226	Optimal Kalman gain for $E_t r_t^*$.
σ_v	0.008	Std. dev. of aggregate demand shock.
σ_u	0.016	Std. dev. of cost push shock.
RE solution coefficients:		$\mathbf{B}_{11}/\mathbf{A}_{11} \simeq \mathbf{B}_{21}/\mathbf{A}_{21} \simeq \mathbf{B}_{31}/\mathbf{A}_{31} \simeq 5.1$

Natural rate process approximates Laubach-Williams r-star

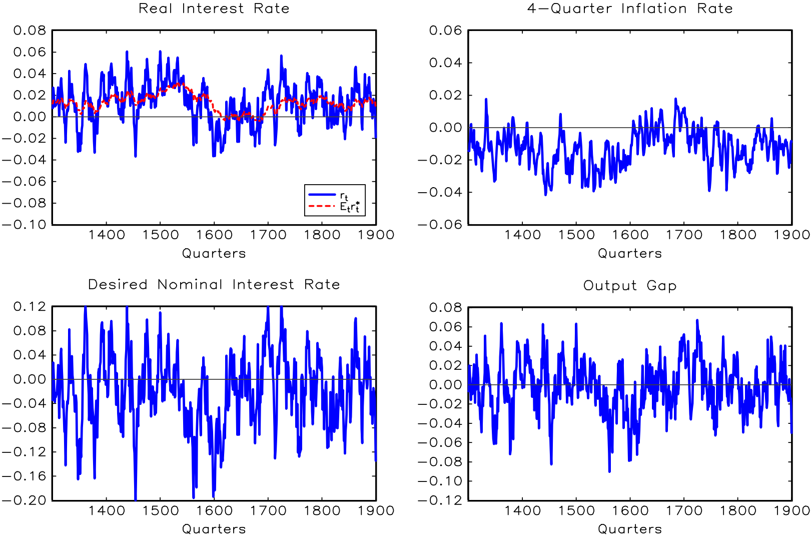
Bounds for simulations: $0.002 \leq r_t^* \leq 0.0298$ (1988.Q1 to 2015.Q4).



Model simulation: Targeted Equilibrium



Model simulation: Deflation Equilibrium



Endogenous forecast rule switching

Discrete choice framework along the lines of Brock and Hommes (1997, 1998)

$$\begin{aligned}
\widehat{E}_t\, y_{t+1} &= \mu_t E_t^{\text{targ}}\, y_{t+1} + (1-\mu_t)\, E_t^{\text{defl}}\, y_{t+1} \\
\widehat{E}_t\, \pi_{t+1} &= \mu_t E_t^{\text{targ}}\, \pi_{t+1} + (1-\mu_t)\, E_t^{\text{defl}}\, \pi_{t+1} \\
\widehat{E}_t\, i^*_{t+1} &= \mu_t E_t^{\text{targ}}\, i^*_{t+1} + (1-\mu_t)\, E_t^{\text{defl}}\, i^*_{t+1}
\end{aligned}$$

$$\mu_t = \frac{\exp\left[\psi\left(RMSFE_{t-1}^{\text{defl}} - RMSFE_{t-1}^{\text{targ}}\right)\right]}{1 + \exp\left[\psi\left(RMSFE_{t-1}^{\text{defl}} - RMSFE_{t-1}^{\text{targ}}\right)\right]} \qquad \psi = 75$$

"Intensity of choice"

Forecast fitness measure for i = targ, defl:

$$RMSE_{t-1}^{\text{i}} = \frac{1}{8}\sum_{i=1}^{8}\Big[\left(y_{t-j} - E_{t-j-1}^{\text{i}}\, y_{t-j}\right)^2 + \left(\pi_{t-j} - E_{t-j-1}^{\text{i}}\, \pi_{t-j}\right)^2 + \left(i^*_{t-j} + E_{t-j-1}^{\text{i}}\, i^*_{t-j}\right)^2\Big]^{0.5}$$

Given current forecasts, solve for equilibrium variables

$$
\begin{aligned}
i_t^* &= \frac{1}{\rho}\left\{\widehat{E}_t\, i_{t+1}^* - (1-\rho)\left[E_t r_{t+1}^* + \pi^* + g_\pi\, \omega\left(\widehat{E}_t\, \pi_{t+1} - \pi^*\right)\right.\right. \\
&\qquad \left.\left. + (1-\omega)\, g_\pi\left(\overline{\pi}_t - \pi^*\right) + g_y\left(\widehat{E}_t\, y_{t+1} - y^*\right)\right]\right\}
\end{aligned}
$$

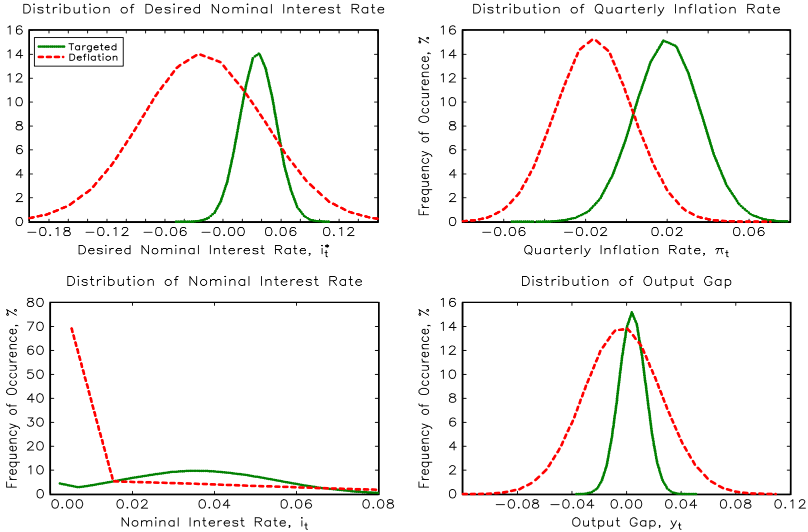
$$i_t = \max\left\{0,\ i_t^*\right\}$$

$$y_t = \widehat{E}_t\, y_{t+1} - \alpha\left[i_t - \widehat{E}_t\, \pi_{t+1} - r_t\right] + v_t$$

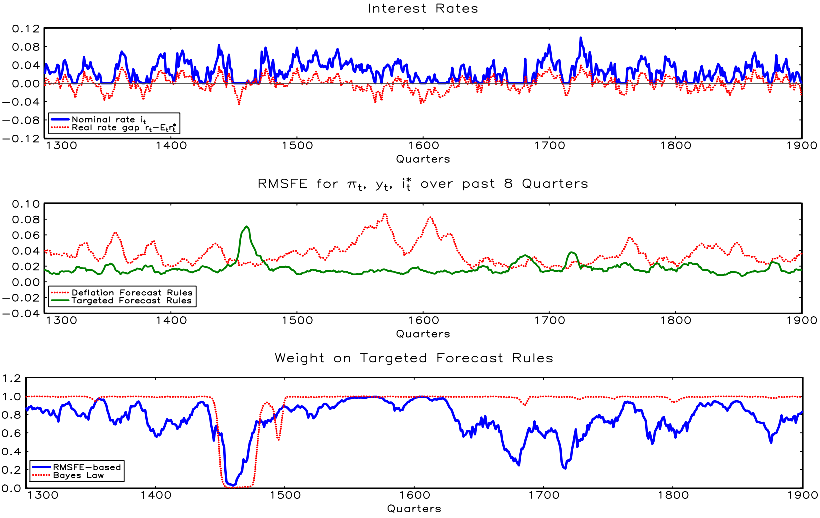
$$\pi_t = \beta \widehat{E}_t\, \pi_{t+1} + \kappa y_t + u_t$$

$$\overline{\pi}_t = \omega\, \pi_t + (1-\omega)\, \overline{\pi}_{t-1}$$

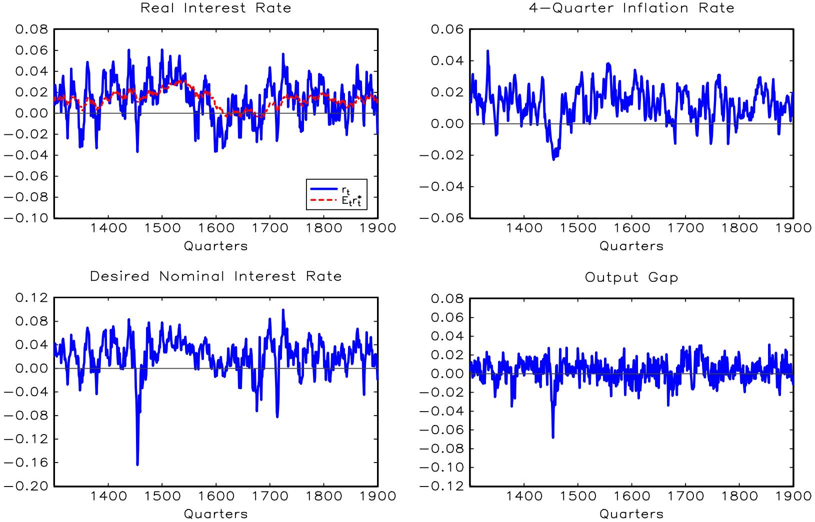
Overlapping distributions induce endogenous regime shifts



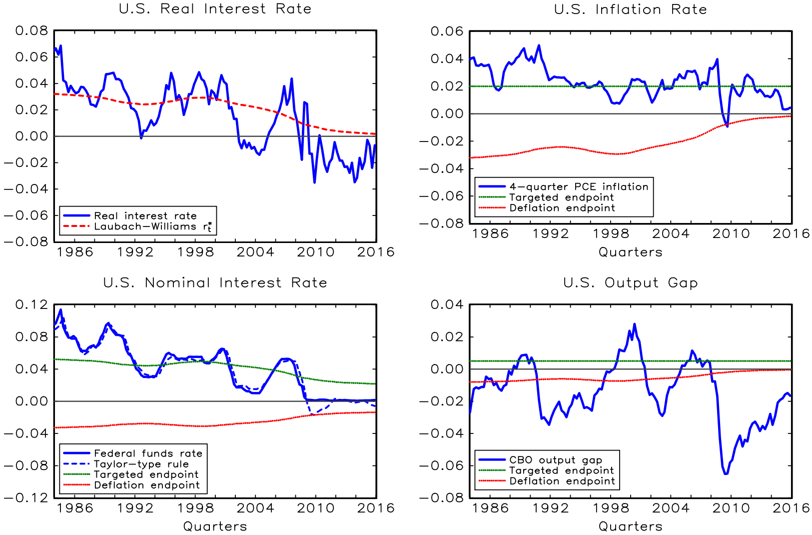
Weight on targeted forecast rules can decline rapidly



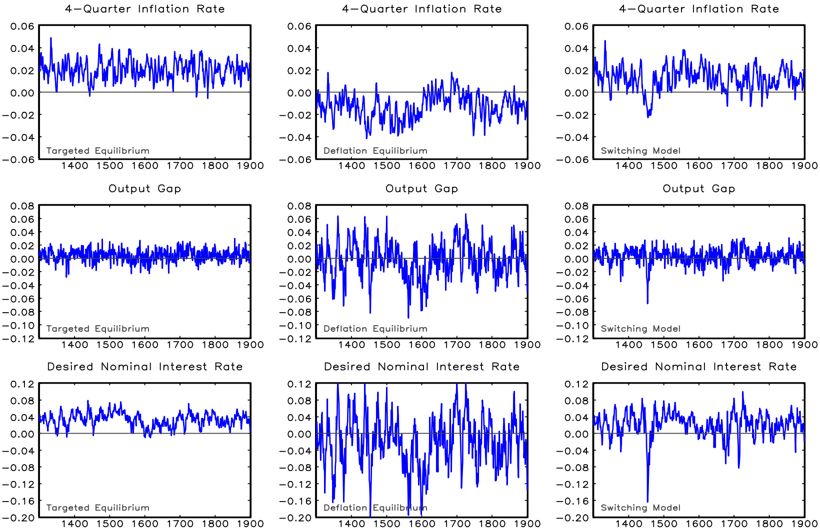
Switching model: Severe recession, deflation, ZLB binding



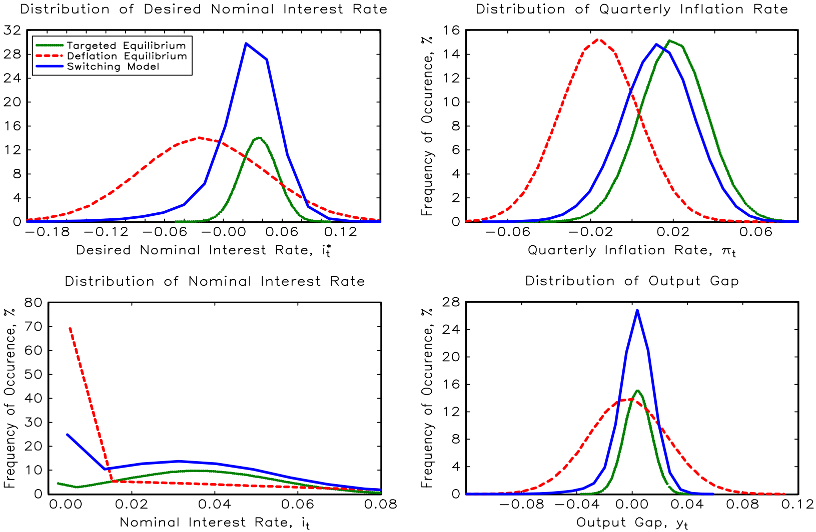
U.S. data: Severe recession, deflation, ZLB binding



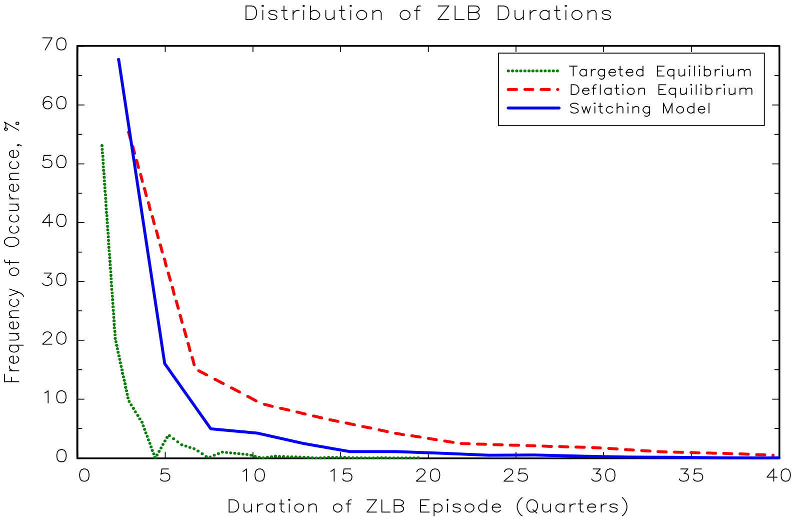
Comparing simulations: Targeted, Deflation, Switching



Switching model: Inflation distribution shifts left



Switching model: Infrequent but long-lived ZLB episodes



Quantitative Comparison

Statistic	U.S. Data 1988.Q1-2015.Q4	Model Simulations Targeted	Deflation	Switching
Mean $\pi_{t-3\rightarrow t}$	2.20%	$\boxed{1.99\%}$	-1.60%	$\boxed{1.21\%}$
Std. Dev.	1.09%	0.81%	1.27%	1.08%
Corr. Lag 1	0.89	0.75	0.90	0.86
Mean y_t	-1.51%	0.40%	-0.32%	0.24%
Std. Dev.	2.02%	0.97%	2.83%	1.34%
Corr. Lag 1	0.96	0.27	0.78	0.55
Mean i_t^*	3.45%	3.59%	-2.15%	2.42%
Std. Dev.	2.84%	1.84%	6.35%	3.46%
Corr. Lag 1	0.99	0.88	0.85	0.89
% periods $i_t = 0$	25.9%	2.59%	63.3%	17.5%
Mean ZLB duration	29 qtrs.	2.2 qtrs.	7.6 qtrs.	4.0 qtrs.
Max. ZLB duration	29 qtrs.	20 qtrs.	96 qtrs.	67 qtrs.

Notes: ZLB in U.S. data: 2008.Q4 through 2015.Q4. Model results computed from a 300,000 period simulation.

Effect of Raising the Inflation Target

Statistic	Switching Model $\pi^*= 0.02$	$\pi^*= 0.03$	$\pi^*= 0.04$	$\pi^*= 0.05$
Std. Dev. $\pi_{t-3\to t}$	1.08%	1.04%	0.91%	0.83%
Std. Dev. y_t	1.34%	1.12%	1.01%	0.98%
Std. Dev. i_t^*	3.46%	2.72%	2.14%	1.92%
% periods $i_t = 0$	17.5%	5.72%	0.99%	0.11%
Mean ZLB duration	4.0 qtrs.	3.3 qtrs.	2.9 qtrs.	3.1 qtrs
Max. ZLB duration	67 qtrs.	55 qtrs.	38 qtrs.	32 qtrs

Note: Model results computed from a 300,000 period simulation.

- Higher π^* can prevent switching to volatile deflation equilibrium where recessions are more severe.
- Numerous papers examine benefits of higher π^* using models that ignore deflation equilibrium. This methodology likely understates the benefits of a higher π^*.

Conclusion

- Standard NK model with shifting r_t^* and occasionally binding ZLB. Two RE equilibria. Endogenous forecast rule switching based on past *RMSFE* performance.

- Model can produce Great Recessions when $r_t - E_t r_t^*$ is negative, causing agent to place significant weight on deflation forecast rules. Escape from ZLB occurs endogenously when $r_t - E_t r_t^*$ eventually starts rising.

- In normal times, non-trivial weight on deflation forecast rules may cause central bank to undershoot π^* (like today?).

- When $\pi^* = 0.04$, probability of ZLB episode is small $\simeq 1\%$ and average duration of ZLB episode is only 3 quarters.