FINANCIAL CYCLES WITH HETEROGENEOUS INTERMEDIARIES

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Abstract

This paper develops a dynamic macroeconomic model with heterogeneous financial intermediaries and endogenous entry. It features time-varying endogenous macroeconomic risk that arises from the risk-shifting behaviour of financial intermediaries combined with entry and exit. We show that when interest rates are high, a decrease in interest rates stimulates investment and increases financial stability. In contrast, when interest rates are low, further stimulus can increase systemic risk and induce a fall in the risk premium through increased risk-shifting. In this case, the monetary authority faces a trade-off between stimulating the economy and financial stability. *JEL Codes:* E32, E44, E52, G21.

Keywords: Banking, Macroeconomics, Monetary Policy, Risk-shifting, Leverage, Financial cycle.

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1 Introduction

The recent crisis has called into question our modeling of the macroeconomy and of the role of financial intermediaries. It has become more obvious that the financial sector, far from being a veil, plays a key role in the transmission of shocks and in driving fluctuations in aggregate risk. The precise mechanisms by which this happens are to a large extent still unknown. In particular, the underlying forces driving endogenous systemic risk and even a precise and empirically relevant definition of systemic risk remains elusive. This is where our paper attempts to make a contribution.

Macroeconomic models have long recognized the importance of capital market frictions for the transmission and the amplification of shocks. In the literature featuring a collateral constraint (see e.g. Bernanke and Gertler (1989), Kiyotaki and Moore (1997)), agency costs between borrowers and lenders introduce a wedge between the opportunity cost of internal finance and the cost of external finance: the external finance premium. Any shock lowering the net worth of firms, households or banks can cause adverse selection and moral hazard problems to worsen, as the borrowers stake in the investment project varies, increasing the size of the external finance premium. As a result this leads to a decrease in lending and a fall in economic activity. Other recent models where financial market frictions play a key amplifying role are Mendoza (2010), Mendoza and Smith (2014), Gertler and Kiyotaki (2015), Gertler and Karadi (2011) who use a collateral constraint[1]; Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013) where an intermediary cannot raise more than a fixed amount of equity; Adrian and Shin (2010), Coimbra (2016) and Adrian and Boyarchenko (2015) where intermediaries face Value-at-Risk constraints. Some papers discuss endogenous fluctuations in macroeconomic risk. In Brunnermeier and Sannikov (2014), for example, the economy may spend time in suboptimal low asset price and low investment states. As a consequence of the existence of such paths, macroeconomic risk may increase and will do so in periods where asset prices tend to be depressed and financial intermediaries underinvest. Similarly, He and Krishnamurthy (2014) develop a model to quantify systemic risk, defined as the risk that financial constraints bind in the future.

This paper develops a simple general equilibrium model of monetary policy transmission with a risk-taking channel, in which systemic risk increases in periods of low volatility, low interest rate, high investment and compressed spreads, as observed during the pre-crisis period between 2003 and 2007. Unlike most of the previous literature, systemic risk is defined in terms of default risk of financial intermediaries and not

[1]See also Gertler et al. (2012), Curdia and Woodford (2010), Farhi and Werning (2016), Aoki et al. (2016).

simply the risk that financial constraints bind in the future. We provide a precise definition of systemic risk as a state that would trigger generalized solvency issues in the financial sector. In the model, financial crises tend to happen after periods of credit booms, a pattern observed in the data as documented by Gorton (1988) and Jorda et al. (2011). This is achieved by building a novel framework with a moral hazard friction due to limited liability that leads to risk-shifting in a model with a continuum of financial intermediaries heterogeneous in their Value-at-Risk constraints. The literature (for example Gertler and Kiyotaki (2015), Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013)) has traditionally modelled the financial sector as one representative bank, so that heterogeneity in financial intermediaries characteristics plays no role and information on the time variations of the cross-section of balance sheet data cannot be exploited. An important exception is Boissay et al. (2016) which feature intermediaries heterogenous in their abilities. In their set up, low ability intermediaries become active in boom times and adverse selection plays an important role in credit collapses.[2]

Value-at-Risk constraints are realistic features of the regulatory environment; they are embedded in Basel II and Basel III. They also reflect the practice of internal risk management in financial intermediaries, whether as a whole or for specific business lines within financial firms.[3] Their heterogeneity may reflect heterogenous risk attitudes by the boards of financial intermediaries or different implementations of regulatory constraints across institutions. Like us, Fostel and Geanakoplos (2012) emphasize financial frictions and heterogeneity in investors to generate fluctuations in asset prices. Furthermore, we assume deposit guarantees which are a widespread institutional feature.[4] The importance of risk-shifting by financial intermediaries for asset prices has been highlighted in a number of papers. Allen and Gale (2000) have shown that current and future credit expansion can increase risk shifting and create bubbles in asset markets, while Nuño and Thomas (2017) show that the presence of risk-shifting

[2]Their modelling strategy and ours are however very different and so are the implications of the two models. In particular in their set-up there is a backward bending demand curve for loans; not in ours. Other major differences are that they do not model monetary policy, nor do they model the cross section of banks leverage, which is a key variable for us. Another recent attempt to introduce heterogeneity using an evolutionary approach is Korinek and Nowak (2017). Koijen and Yogo (2016) develop an empirical asset pricing model with heterogeneity across investors.

[3]Value-at-Risk constraints to model financial intermediaries have been used in a number of papers (see for example Danielsson, Shin and Zigrand (2010), Adrian and Shin (2010), Adrian and Shin (2014) who provide microfoundations and Adrian and Boyarchenko (2015)).

[4]We are therefore abstracting from the important literature on bank runs (see e.g. Diamond and Dybvig (1983), Diamond and Kashyap (2016), Gertler and Kiyotaki (2015), Angeloni and Faia (2013)). Kareken and Wallace (1978) point out that an important side effect of deposit insurance is excessive risk-taking.

creates a link between asset prices and bank leverage. Malherbe (2015) also presents a model with excessive build-up of risk during economic booms as the lending of an individual bank exerts a negative externality on other banks. In Martinez-Miera and Suarez (2014), bankers determine their exposure to systemic shocks by trading-off the risk-shifting gains due to limited liability with the value of preserving their capital after a systemic shock. Angeloni et al. (2015) show that monetary policy expansions can induce banks to increase leverage which increases their probability of default risk via a bank run. In the present paper, a more risk-taking intermediary will have a higher willingness-to-pay for risky assets[5], which endogenously leads to higher relative probabilities of default. This is an important distinction as competition among intermediaries for risky financial assets is what will drive fluctuations in aggregate risk-taking.

Different levels of leverage across financial intermediaries and the presence of risk-shifting play an important role in our model. They jointly generate heterogeneous willingness to pay for risky assets and therefore a link between aggregate risk-taking and the distribution of leverage. Our model provides therefore a different and complementary view of financial fragility from Gennaioli et al. (2012). In their model, excess risk-taking comes from a thought process bias ("local thinking"): bankers neglect to take into consideration the probability that some improbable risk materializes. Finally, although our modeling strategy is very different, our paper is related to the growing literature on the risk-taking channel of monetary policy (Borio and Zhu (2012), Bruno and Shin (2015), DellAriccia et al. (2014) and Acharya and Plantin (2016)). Challe et al. (2013) describe a two-period model with heterogeneous intermediaries and limited liability which, like ours, features a link between interest rates and systemic risk.[6]

One important contribution of our paper is to analyse the joint dynamics of economics and financial variables in a model with risk-shifting and a pool of heterogenous intermediaries. This generates endogenous macroeconomic risk fluctuations and movements in the risk premium. We relate the macroeconomic dynamics to the cross sectional shifts in the distribution of leverage of financial intermediaries. Another contribution is to provide an intuitive and clearly defined measure of systemic risk within a standard dynamic macroeconomic model. We now describe our model briefly.

Financial intermediaries collect deposits from households and invest and hold shares in the aggregate capital stock, which provides a risky return. Realistically, financial intermediaries have limited liability, which introduces a risk-shifting motive for invest-

[5] Arising from the interaction between a looser leverage constraint and limited liability.

[6] They focus on portfolio choice and heterogeneity in equity of intermediaries while we emphasize aggregate uncertainty and differences in risk-taking. Another important difference is that, unlike them, we embed the financial sector in an otherwise standard DSGE model.

ment and mispricing of risk. Deposits are guaranteed by the government. Intermediaries with a looser Value-at-Risk constraint have a higher option value of default, which will generate pricing effects of entry and exit in risky financial markets. Both the aggregate capital stock and the risk premium of the economy are determined by an *extensive margin* (which financial intermediaries lever) and an *intensive margin* (how much does each lever). This is a key novel feature of the model. Having variation in the intensive and the extensive margins generates both movements in aggregate leverage and asset pricing implications which are unusual in our models but seem to bear some resemblance with reality. Contemporaneously, output and consumption vary monotonically with the interest rate while the underlying financial structure (and systemic risk) is non-monotonic. We explain here the basic economic intuition behind the workings of the model.

Our model features an endogenous non-linearity in the trade-off between monetary policy (which affects the funding costs of intermediaries) and financial stability. When the level of interest rates is high, a fall in interest rates leads to entry of less risk-taking intermediaries into the market for risky projects. The average intermediary is then less risky, so a fall in interest rates (i.e. a monetary expansion) has the effect of reducing systemic risk and expanding the capital stock. There is no trade-off in this case between stimulating the economy and financial stability. However, when interest rates are very low, a monetary expansion leads to the exit of the least risk-taking active intermediaries, which are priced out of the market by a large increase in leverage of the more risk-taking ones. This increases systemic risk in the economy despite positive effects on the aggregate capital stock, which is always increasing with a fall in interest rates. For this region, the intensive margin growth in leverage dominates the extensive margin fall as interest rates are reduced. In other words, the most risk-taking intermediaries increase their leverage so much that they more than compensate for the exit of the least risk-taking ones. There seems to be a clear trade-off between stimulating the economy and financial stability. Stimulating the economy shifts the distribution of assets towards the more risk-taking intermediaries, which have a higher default risk and increases aggregate risk-shifting. Of course, the level of the interest rate is itself an outcome of the general equilibrium model and therefore a fixed point problem has to be solved. This non-monotonicity constitutes a substantial difference from the existing literature and is a robust mechanism coming from the interplay of the two margins. It provides a novel way to model the risk-taking channel of monetary policy analysed in Borio and Zhu (2012), Challe et al. (2013) and Bruno and Shin (2015). Recent empirical evidence on the risk-taking channel of monetary policy for loan books has been provided by Dell'Ariccia et al. (2013) on US data, Jimenez et al. (2014) and Morais et al. (2015), exploiting registry data on millions of loans of the Spanish and Mexican Central Banks respectively, and also by Faia and Karau (2017) using a panel of global systemically important banks. Moreover, using Turkish level data, Baskaya et al. (2017) highlight the importance of bank heterogeneity in the transmission of global risk shocks.

There are several important advantages of this novel set up to model financial intermediation. First, it takes seriously the risk-taking channel in general equilibrium and therefore allows the joint study of the usual expansionary effect of monetary policy - via a boost in investment - and of the macroeconomic financial stability risk, which is endogenous. Monetary policy is modeled as a reduction in the real funding costs of financial intermediaries[7] and an extension of the model featuring nominal variables is left for future work. Second, it is able to generate periods of low risk premium which coincide with periods of high endogenous macroeconomic risk. This happens when the market is dominated by more risk-taking intermediaries which also feature high levels of leverage. These periods also correspond to high levels of investment and inflated asset prices due to stronger risk-shifting motives. Thirdly, the model is crafted in a way such that the financial intermediation building block, although rich, can be easily inserted in a general equilibrium macroeconomic model. Fourthly, because the model introduces a simple way to model financial intermediary heterogeneity, it opens the door to a vast array of empirical tests based on microeconomic data on banks, shadow banks, asset managers, and so on[8]. Indeed the heterogeneity can be in principle matched in the data with actual companies or business lines within companies and with their leverage behaviours.

Section 2 of the paper describes the model. Section 3 presents the main results in partial equilibrium, thereby building intuition. Section 4 shows the general equilibrium results and the response to monetary policy shocks. Section 5 looks at some empirical evidence for the cross-sectional implications of the model. The case of financial crises with costly intermediary default is analyzed in section 6 and section 7 concludes.

2 The Model

The general equilibrium model is composed of a representative risk-averse household who faces an intertemporal consumption saving decision, a continuum of risk-neutral financial intermediaries, and a stylized Central Bank and government. There are only aggregate shocks, in the form of productivity and monetary policy shocks. Given the heterogeneity in bank balance sheets that the model features, this will still lead to

[7]Any change in regulation that affects funding costs would have similar implications.

[8]Our model attemps to perform in macro-finance something similar to what Melitz (2003) has done in international trade by relating aggregate outcomes to underlying microeconomic heterogeneity. We are not aware of any other paper in the macro-finance literature that pursues a similar aim.

idiosyncratic risks of default in the intermediation sector.

2.1 Households and the production sector

The representative household has an infinite horizon and consumes a final good C_t^H. She finances her purchases using labour income W_t and returns from a savings portfolio. We assume that the household has a fixed labour supply and does not invest directly in the capital stock K_t.[9] It can either save using a one-to-one storage technology S_t^H and/or as deposit D_t^H with financial intermediaries at interest rate r_t^D. The return on deposits $R_t^D \equiv 1 + r_t^D$ is risk-free and guaranteed by the government. Intermediaries use deposits, along with inside equity ω_t, to invest in capital and storage. In Section 4 we will introduce monetary policy as a source of wholesale funding. Monetary policy will therefore affect the weighted average cost of funds for intermediaries.

The production function combines labour and capital in a typical Cobb-Douglas function. Since labour supply is fixed, we normalize it to 1. Output Y_t is produced according to the following technology:

$$Y_t = Z_t K_{t-1}^{\theta} \tag{1}$$

$$\log Z_t = \rho^z \log Z_{t-1} + \varepsilon_t^z \tag{2}$$

$$\varepsilon_t^z \sim N(0, \sigma_z) \tag{3}$$

where Z_t represents total factor productivity. θ is the capital share, while ε_t^z is the shock to the log of exogenous productivity with persistence ρ^z and standard deviation σ_z. Let $F(\epsilon_t^z)$ be the cumulative distribution function (cdf) of $\exp(\epsilon_t^z)$, a notation which will be convenient later. Firm maximization implies that wages $W_t = (1-\theta) Z_t K_{t-1}^{\theta-1}$ and returns on a unit of capital $R_t^K = \theta Z_t K_{t-1}^{\theta-1} + (1-\delta)$.

The household program can be written as follows:

$$\max_{\{C_t, S_t^H, D_t^H\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^H) \qquad \text{s.t.} \tag{4}$$

$$C_t^H + D_t^H + S_t^H = R_t^D D_{t-1}^H + S_{t-1}^H + W_t - T_t \qquad \forall_t \tag{5}$$

[9]Given households are risk-averse and intermediaries are risk neutral (and engage in risk-shifting), relaxing the assumption households cannot invest directly would make no difference in equilibrium unless all intermediaries are active and constrained. There are also little hedging properties in the asset, since the correlation of the shock to returns with wage income is positive. In the numerical exercises, it is never the case that all intermediaries are active and constrained, so to simplify notation and clarify the household problem we assume directly that only intermediaries can invest in the risky capital stock.

where β is the subjective discount factor and $u(.)$ the period utility function. T_t are lump sum taxes and S_t^H are savings invested in the one-to-one storage technology. Note that the return on deposits is risk-free despite the possibility of intermediary default. The reason is that deposits are guaranteed by the government, which may need to raise taxes T_t in the event intermediaries cannot cover their liabilities. Households understand that the higher the leverage of intermediaries, the more likely it is for them to be taxed in the future. However, they do not internalize this in their individual portfolio decisions since each household cannot by itself change aggregate deposits nor the expectation of future taxes.

The return on storage is also risk-free, which implies that households will be indifferent between deposits and storage if and only if $R_t^D = 1$. Therefore, they will not save in the form of deposits if $R_t^D < 1$ and will not invest in storage if $R_t^D > 1$. In equilibrium, the deposits rate will be bounded from below by the unity return on storage, implying that $R_t^D \geq 1$. In the case $R_t^D = 1$, the deposit quantity will be given by financial intermediary demand, with the remaining household savings being allocated to storage.

2.2 Financial intermediaries

The financial sector is composed of two-period financial intermediaries which fund themselves through inside equity and household deposits[10]. They use these funds to invest in the aggregate risky capital stock and/or in the riskless one-to-one storage technology. They benefit from limited liability. Intermediaries are risk neutral agents who maximize expected second period consumption subject to a Value-at-Risk constraint. To capture the diversity of risk attitudes among financial intermediaries, we assume that they are heterogeneous in α^i, the maximal probability their return on equity is negative. α^i is exogenously given and the key parameter in the VaR constraint. This probability varies across intermediaries and is continuously distributed according to the measure $G(\alpha^i)$ with $\alpha^i \in [\underline{\alpha}, \overline{\alpha}]$.

The balance sheet of intermediary i at the end of period t is as follows:

Assets	Liabilities
k_{it}	ω_t^i
s_{it}	d_{it}

[10]We will extend the funding options to include wholesale funding, whose cost is influenced by monetary policy, in section 4. By assumption the economy does not feature an interbank market or other funding possibilities.

where k_{it} are the shares of the aggregate capital stock held by intermediary i, s_{it} the amount of storage held, d_{it} the deposit amount contracted at interest rate r_t^D, and ω_t^i the inside equity. At the beginning of the next period, R_{t+1}^K is revealed and the net cash flow $\pi_{i,t+1}$ is:

$$\pi_{i,t+1} = R_{t+1}^K k_{it} + s_{it} - R_t^D d_{it} \tag{6}$$

2.2.1 Value-at-Risk constraint

Financial intermediaries are assumed to be constrained by a Value-at-Risk condition. This condition imposes that intermediary i invests in such a way that the probability its return on equity is negative must be smaller than an exogenous intermediary-specific parameter α^i.[11] The VaR constraint for intermediary i can then be written as:

$$\Pr(\pi_{i,t+1} < \omega_t^i) \leq \alpha^i \tag{7}$$

The probability that the net cash flow is smaller than starting equity must be less or equal than α^i. This constraint follows the spirit of the Basel Agreements, which aim at limiting downside risk and preserving an equity cushion. Furthermore, Value-at-Risk techniques are used by banks and other financial intermediaries (for example asset managers) to manage risk internally. When binding, it also has the property of generating procyclical leverage, which can be observed in the data for some intermediaries as described in Geanakoplos (2011) and Adrian and Shin (2014) when equity is measured at book value. Using a panel of European and US commercial and investment banks Kalemli-Özcan et al. (2012) provide evidence of procyclical leverage but also emphasize important cross-sectional variations across types of intermediaries.

In Figure (5) we also show that leverage behaves heterogeneously in the cross-section. Heterogeneity in the parameter of the Value-at-Risk constraint can be rationalized in different ways. It could be understood as reflecting different risk management practices or differentiated implementation of regulatory requirements. For example, the Basel Committee undertook a review of the consistency of risk weights used when calculating how much capital global banks put aside for precisely defined portfolio. When given a diversified test portfolio the global banks surveyed produced a wide range of results in terms of modeled Value-at-Risk and gave answers ranging from 13 million to 33 million euros in terms of capital requirement with a median of about 18 million (see Basel Committee on Banking Supervision (2013) p.52). Some of the differences are due

[11] Alternatively we could posit that the threshold is at a calibrated non-zero return on equity. There is a mapping between the distribution $G(\alpha^i)$ and such a threshold, so for any value we could find a $\tilde{G}(\alpha^i)$ that would make the two specifications equivalent given expected returns. We decide to use the current one as it reduces the parameter space.

to different models used, some to different discretionary requirements by supervisors and some to different risk appetites, as "Basel standards deliberately allow banks and supervisors some flexibility in measuring risks in order to accommodate for differences in risk appetite and local practices" (p.7).

2.2.2 Intermediary investment problem

We assume that our risk neutral intermediaries live for two periods, receiving an endowment of equity $\omega_t^i = \omega$ in the first and consuming their net worth in the second, if it is positive. This assumption of constant equity is a simplifying assumption and we find indeed that book value equity is very sticky in the data[12]. We show in Figure (10) in the appendix the almost one-for-one correlation between changes in the size of debt and assets at book value for a sample of banks, as well as the stickiness of book value equity. Balance sheet expansions and contractions tend to be done through changes in debt and not through movements in equity. Other papers in the literature, which feature a representative intermediary, assume that a maximum amount of equity can be raised (Brunnermeier and Sannikov (2014), He and Krishnamurthy (2014)) or that dividend payouts are costly as in Jermann and Quadrini (2012).

Net worth consumed by financial intermediary i is denoted by c_{it}.[13] When the net cash flow is negative, $c_{it} = 0$ and the government repays depositors as it upholds deposit insurance. This is a pure transfer, funded by a lump sum tax on households. Hence, in our model, households are forward-looking and do intertemporal optimization while most of the action in the intermediation sector comes from heterogenous leverage and risk-taking in the cross-section. This two-period modeling choice is made for simplicity[14] and allows us to highlight the role of different leverage responses across financial intermediaries.

Each intermediary will have to decide whether it participates or not in the market for risky assets or invests in the storage technology (*participating intermediary* versus *non-participating intermediary*) and, conditionally on participating whether it uses deposits to lever up (*risky intermediary*) or just invests its own equity (*safe*

[12]We make the opposite assumption of the literature which often assumes a representative bank and focuses on the dynamics of net worth (see e.g. Gertler and Kiyotaki (2015)). In contrast, we assume constant equity but allow for heterogeneous intermediaries.

[13]When intermediary j is inactive, then $c_{jt} = \omega$ as the return of the storage technology is one.

[14]Other papers in the literature have used related assumptions, for example exogenous death of intermediaries in Gertler and Kiyotaki (2015) or difference in impatience parameters in Brunnermeier and Sannikov (2014).

intermediary). In Appendix D, we show that an alternative model where intermediaries can choose to lend to each other as an outside option has very similar implications.[15]

Intermediaries are assumed to be (constrained) risk-neutral price takers, operating in a competitive environment. Each maximizes consumption over the next period by picking k_{it} (investment in risky assets) and s_{it} (investment in the storage technology), under the VaR constraint, while taking interest rates on deposits r_t^D and asset return distributions $R_{t+1}^K(\varepsilon)$ as given. The program of each intermediary i is given by:

$$
\begin{aligned}
V_{it} = & \max \ \mathbb{E}_t(c_{i,t+1}) && (8)\\
\text{s.t.} \ & \Pr(\pi_{i,t+1} < \omega_t^i) \leq \alpha^i && (9)\\
& k_{it} + s_{it} = \omega_t^i + d_{it} && (10)\\
& c_{i,t+1} = \max\left(0, \pi_{i,t+1}\right) && (11)\\
& \pi_{i,t+1} = R_{t+1}^K k_{it} + s_{it} - R_t^D d_{it}
\end{aligned}
$$

where α^i is the Value-at-Risk threshold (the maximum probability of not being able to repay stakeholders fully) and $\pi_{i,t+1}$ the net cash flow.

Intermediaries can also choose to stay out of risky financial markets and not participate. In this case, they have the outside option of investing all their equity in the storage technology and collect it at the beginning of the next period. The value function of a non-participating intermediary investing in the outside option is:

$$V_{it}^O = V^O = \omega \tag{12}$$

2.2.3 Limited liability

The presence of limited liability truncates the profit function at zero, generating an option value of default that intermediaries can exploit. For a given expected value of returns, a higher variance increases the option value of default as intermediaries benefit from the upside but do not suffer from the downside. For a given choice of k_{it} and d_{it} we have that:

$$\mathbb{E}_t\left[\max(0, \pi_{i,t+1})\right] \geq \mathbb{E}_t\left[\pi_{i,t+1}\right] \tag{13}$$

with the inequality being strict whenever the probability of default is strictly positive. Deposit insurance transfers t_t^i happen when net cash flow is negative and are given by:

$$t_{t+1}^i = \max\left(0, -\pi_{t+1}\right) \tag{14}$$

[15]In appendix D, we consider a standard centralized market for intermediary deposits. For a model of financial stability issues arising from banking networks see Aldasoro et al. (2017)

The max operator selects the appropriate case depending on whether intermediary i can repay its liabilities or not. If it can, then deposits repayments are lower than return on assets and deposit insurance transfers are zero. Total intermediary consumption C_t^I and aggregate transfers/taxes T_t are given by integrating over the mass of intermediaries:

$$C_t^I = \int c_{it} \, dG(\alpha^i) \tag{15}$$

$$T_t = \int t_t^i \, dG(\alpha^i) \tag{16}$$

For now we assume default is costless in the sense that there is no deadweight loss when the government is required to pay deposit insurance. In section 6, we will drop the assumption of costless default by having a more general setup that allows for a lower return on assets held by distressed intermediaries.

2.3 Investment strategies and financial market equilibrium

Financial intermediaries are price takers, therefore the decision of each one depends only on the expected return on assets[16] and the cost of liabilities. Since the mass of each intermediary is zero, individual balance sheet size does not affect returns on the aggregate capital stock. Intermediary i will be a *participating intermediary* in the market for risky assets whenever $V_{it} \geq V^O$. This condition determines entry and exit into the market for risky capital endogenously.

There is however another important endogenous decision. Intermediaries which participate in the market for risky assets have to choose whether to lever up and, if they do, by how much. We will refer to the decision to lever up or not, i.e. to enter the market for deposits as the *extensive margin*. We will refer to the decision regarding how much to lever up as the *intensive margin*. Financial intermediaries which lever up are called *risky* intermediaries. Financial intermediaries which participate in the market for risky capital but do not lever up are called *safe* intermediaries.

Proposition 2.1 *When $\mathbb{E}[R_{t+1}^K] \geq 1$, participating intermediary i will either lever up to its Value-at-Risk constraint or not raise deposits at all.*

Proof: See Appendix B.

Proposition 2.1 states that if the return to risky capital is higher in expectation than the return on the storage technology then whenever an intermediary decides to

[16] Taking into account limited liability.

lever up, it will do so up to its Value-at-Risk constraint and will not invest in storage. Hence all *risky* intermediaries will be operating at their constraint.

When expected return on risky capital is smaller than return on storage: $\mathbb{E}[R^K_{t+1}] < 1$, it might be the case that storage is preferred to capital in equilibrium by some intermediariess. We then have equilibria in which some intermediaries invest in storage and possibly some of the most risk-taking ones leverage up a lot taking advantage of the option value of default. In what follows we focus on cases where $\mathbb{E}[R^K_{t+1}] \geq 1$ which is always the case in our simulations.

2.3.1 Intensive margin and investment of risky intermediaries

Let $Z^e_{t+1} \equiv \mathbb{E}_t(Z_{t+1}) = Z_t^{\rho^z}$ where Z_t is Total Factor Productivity. For a participating intermediary i deciding to lever up, the VaR condition will bind (see Proposition 2.1):

$$\Pr\left[\pi^i_{t+1} \leq \omega\right] \leq \alpha^i \tag{17}$$

Hence, after some straightforward algebra, we obtain the following:

$$\Pr\left[e^{\varepsilon^z_{t+1}} \leq \frac{r^D_t + \delta - \frac{\omega}{k_{it}} r^D_t}{\theta Z^e_t K^{\theta-1}_t}\right] = \alpha^i \tag{18}$$

The leverage λ_{it} of an active intermediary is given by:

$$\lambda_{it} \equiv \frac{k_{it}}{\omega} = \frac{r^D_t}{r^D_t - \theta Z^e_{t+1} K^{\theta-1} F^{-1}(\alpha^i) + \delta} \tag{19}$$

where we defined leverage as assets over equity and $F^{-1}(\alpha^i)$ as the inverse cdf of the technology shock $e^{\varepsilon^z_{t+1}}$ evaluated at probability α^i.

Proposition 2.2 *For an intermediary i, the leverage λ_{it} has the following properties: it is increasing in α^i, increasing in expected marginal productivity of capital $\theta Z^e_{t+1} K^{\theta-1}$. Furthermore, $\frac{\partial \lambda_{it}}{\partial r^D_t} < 0$, $\frac{\partial^2 \lambda_{it}}{\partial (r^D_t)^2} > 0$ and $\frac{\partial^2 \lambda_{it}}{\partial r^D_t \partial \alpha^i} < 0$.*

Proof: Immediate from Equation (19) and given the monotonicity of the cdf and the shape of $F^{-1}()$.

Proposition 2.2 implies that, from the perspective of an individual intermediary (i.e. absent general equilibrium effects on K_t), leverage will be decreasing in r^D_t. A fall in the interest rate will lead to a larger increase, the lower is the level of r^D_t to begin with. Moreover, the more risk-taking is the intermediary, the larger the increase in leverage following a fall in interest rates. Generally, intermediary leverage will also be decreasing in the volatility of productivity shocks σ_z. This will be true whenever $F(\alpha_i)$ is increasing in σ_z, implying realistically that the probability of a negative return on equity is (ceteris paribus) increasing in the volatility of returns.

2.3.2 Extensive margin and endogenous leverage

We now focus on the *extensive margin* that is to say whether intermediaries who participate in risky capital markets choose to lever up using deposits or not.[17]

Let V^L denote the value function of *risky* intermediaries who decide to lever up using deposits and V^N the value function of the *safe* ones who only invest their equity in the risky capital stock. We denote by $\mathbb{E}_t^i$ the expectation of a financial intermediary taking into account limited liability (expectation truncated at zero).

$$V_{it}^L = \mathbb{E}_t^i[R_{t+1}^K k_{it} - R_t^D d_{it}] \tag{20}$$

$$V_{it}^N = \mathbb{E}_t[R_{t+1}^K] k_{it}^N + \omega - k_{it}^N \tag{21}$$

with $k_{it}^N \in [0, \omega]$. Since there is no risk of defaulting on deposits if you have none, there is no option value of default for non-levered intermediaries. This N group includes intermediaries who invest all their equity in capital markets ($k_{it}^N = \omega$) and intermediaries who do so only partially. This occurs only if the intermediary has a sufficiently tight VaR constraint.

We can then use the condition $V_{it}^L = V_{it}^N$ to find the cutoff value $\alpha_t^L = \alpha_t^j$ for which intermediary j is indifferent between leveraging up or not. Above α_t^L (looser Value-at-Risk constraints), all intermediaries will be levered up to their respective constraints and do not invest in storage as shown in Proposition 2.1. For any levered intermediary i, we have:

$$\mathbb{E}_t^i \left[k_{it} R_{t+1}^K - R_t^D d_{it} \right] \geq \omega \ \mathbb{E}_t \left[R_{t+1}^K \right] \tag{22}$$

where the left hand side is the expected payoff on the assets of intermediary i and the right hand side is the expected payoff when it invests only its equity ω in capital markets. Using the balance sheet equation $k_{it} = d_{it} + \omega$, we can substitute for deposits, which leads to the following condition:

$$\mathbb{E}_t^i \left[k_{it} \left(R_{t+1}^K - R_t^D \right) + R_t^D \omega \right] \geq \omega \ \mathbb{E}_t \left[R_{t+1}^K \right] \tag{23}$$

[17] Remember that intermediaries can also decide not to invest in risky capital markets and instead to use the storage technology. If they do so, then their value function is $V^O = \omega$ given the unit return to storage.

For the marginal intermediary j, equation (23) holds with equality:

$$\mathbb{E}_t^j \left[k_{jt} \left(R_{t+1}^K - R_t^D \right) + R_t^D \omega \right] = \omega \, \mathbb{E}_t \left[R_{t+1}^K \right] \tag{24}$$

Since all *risky* intermediaries will be at the constraint, we can combine equation (24) with equation (19) evaluated at the marginal intermediary (whose Value-at-Risk parameter is α_t^L). Moreover, $\mathbb{E}_t \left[R_{t+1}^K \right]$ is a function of Z_{t+1}^e and K_t therefore equation (24) and equation (19) jointly define an implicit function of the threshold VaR parameter α_t^L $(= \alpha^j)$ with variables (r_t^D, Z_{t+1}^e, K_t).

Hence we have the following result:

Proposition 2.3 *There exists a cutoff value α_t^L in the distribution of Value-at-Risk parameters such that all intermediaries with Value-at-Risk constraints looser than the cut-off will use deposits to leverage up to their constraint. All intermediaries with Value-at-Risk constraints tighter than the cut-off will not leverage up. Equations (24) and (19) define an implicit function of the threshold $\alpha_t^L = A(r_t^D, Z_{t+1}^e, K_t)$.*

2.3.3 Financial market equilibrium and deposit demand curve

To close the financial market equilibrium, we need to use the market clearing condition. The aggregate capital stock of the economy is equal to the total investment in risky projects by all intermediaries.

$$K_t = \int_{\underline{\alpha}}^{\overline{\alpha}} k_{it} \; dG(\alpha^i) \tag{25}$$

The integral has potentially three main blocks corresponding to *risky* levered intermediaries (above α_t^L), *safe* intermediaries who do not lever up but invest all their equity in the capital stock (between α_t^N and α_t^L) and *safe* intermediaries who invest in the capital stock only a fraction (possibly zero) of their equity, the remainder being in storage (below α_t^N).

For non-levered *safe* intermediaries who invest all their equity in capital shares, the VaR constraint is given by $F\left(\frac{\delta K_t^{1-\theta}}{\theta Z_{t+1}^e}\right) \leq \alpha^i$. Let α_t^N be the marginal intermediary for whom the constraint binds exactly.

$$F\left(\frac{\delta K_t^{1-\theta}}{\theta Z_{t+1}^e}\right) = \alpha_t^N \tag{26}$$

As long as $\mathbb{E}[R_{t+1}^K \geq 1]$, then for $\alpha^i \in [\alpha_t^N, \alpha_t^L]$, we have that $k_{it} = \omega$. On the other hand, intermediaries $\alpha^i \in [\underline{\alpha}, \alpha_t^N]$ will invest up to their VaR constraint, leading to the following asset holdings:

$$k_{it} = \frac{\omega K_t^{1-\theta}}{\theta Z_{t+1}^e F^{-1}(\alpha^i) + 1 - \delta} \tag{27}$$

By plugging in the expressions for asset purchases k_{it} and using the expression for α_t^N in equation (26), equation (25) defines an implicit function of $(\alpha_t^L, r_t^D, Z_{t+1}^e, K_t)$. Since Z_{t+1}^e is determined at t by state variables and intermediaries are price takers, this financial market clearing function together with the implicit function[18] $A(r_t^D, Z_{t+1}^e, K_t)$ pin down the aggregate capital stock K_t and the marginal levered intermediary α_t^L, for a given deposit rate r_t^D and expected productivity Z_{t+1}^e. Together they determine the *aggregate demand curve for deposits* as a function of deposit rates and expected productivity. In general equilibrium, the deposit rate r_t^D will be determined in conjunction with the *aggregate deposit supply demand curve* coming from the recursive household problem described in section 4.

2.3.4 Systemic Risk

Importantly, we can now give a precise and intuitive definition of systemic risk in our model.

Definition 1: Systemic crisis and systemic risk. A systemic crisis can be defined as a state of the world where all levered intermediaries are unable to repay in full their stakeholders (deposits and equity). Systemic risk is defined by the probability of a systemic crisis occuring and can be directly measured by the cut-off α_t^L.

Since all the risk in the model is aggregate, the probability of a systemic crisis is the probability that the least risk-taking levered intermediary is distressed, which is simply given by the cut-off α_t^L. Hence in the model, a fall in α_t^L (meaning that the marginal entrant has a tighter Value-at-Risk constraint) is isomorphic to a decrease in systemic risk since it is equivalent to a decrease in the probability that the entire leveraged financial sector is distressed.

There are other possible definitions that could have been used. The fact that we can describe the whole cross-sectional distribution of leverage and intermediary risk allows for a wide range of potential alternatives. We highlight this point by providing some other alternative measures, such as calculating an asset-weighted mean of active α^i (putting more weight on larger intermediaries) or the probability that a fraction of the capital is held by distressed intermediaries (50%, rather than 100% as in the baseline

[18] Extensive margin, see Proposition (2.3).

case). We also calculate the expected share of capital held by defaulting intermediaries at $t+1$, a measure that relates to the costly default described in the general equilibrium section.

The baseline measure that is defined in Definition 1 has the advantage of not only describing the risk of the whole sector but also tracking the marginal buyer in financial markets, an important concept in leverage cycles as highlighted by Geanakoplos (2011).

3 Partial equilibrium results

To provide a better illustration of the financial sector mechanics in the model, we first show a set of partial equilibrium results taking as given the deposit rate before moving on to general equilibrium in section 4 where the household problem will close the model. From now on we study the properties of the model using numerical simulations.[19]

We begin by analysing the distribution of intermediary leverage conditional on the deposit rates r_t^D and on expected productivity Z_{t+1}^e. In Figure (1), we show an example of the cross-sectional distribution of leverage for three different values of the deposit rate. The calibration of the model is discussed in more detail in section 4.

In the three cases, the area below each line[20] is proportional to the aggregate capital stock $K_t = \int k_{it}\, dG(\alpha^i)$. The vertical line showing a large drop in leverage identifies the marginal levered intermediary α_t^L. To the left of the cutoff α_t^L, intermediaries are not levered, which corresponds to the more conservative VaR constraints. They are *safe* intermediaries. To the right of the cutoff, leverage and balance sheet size k_{it} increase with α_t^i. That is, the more risk-taking is the intermediary, the larger will be its balance sheet for a given r_t^D and Z_{t+1}^e. Those are *risky* intermediaries.

The graph illustrates how the *intensive* and *extensive* margins affect leverage and the aggregate capital stock as the deposit interest rate changes. For the three cases displayed, as deposit rates fall, the *intensive* margin is always increasing. That is, for every intermediary that is levered up, the balance sheet grows when the cost of leverage falls. This is because a lower rate reduces the probability of default for a given balance sheet size, as a lower rate reduces the cost of liabilities that needs repaying

[19]We performed many different calibrations but only report a few. Results (available upon request) are qualitatively robust across simulations.

[20]Assuming a uniform distribution for $G(\alpha^i)$ as in the baseline calibration. The details of the numerical method to solve the model are given in Appendix A. The fact that the financial bloc of the model is self contained taking as given the interest rate on deposits renders the solution simple.

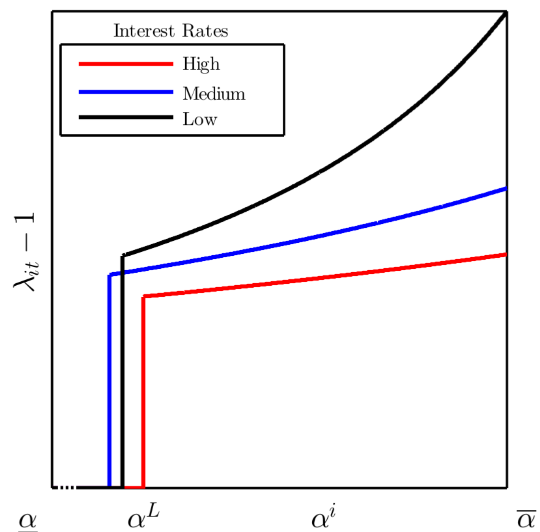

Figure 1: *Cross-sectional distribution of leverage λ_{it} as a function of the VaR parameter α^i*

next period. Intermediaries expand their balance sheet up to the new limit and levered intermediaries grow in size.

Perhaps less intuitively, the effect on the *extensive* margin is ambiguous. One would expect that a fall in interest rate would lead to entry by more risk averse intermediaries. This is what happens when one goes from a high level of interest rate to a medium level of interest rate (the cutoff moves to the left). But this is no longer the case when one moves from a medium level of interest rate to a low level of interest rate: the cutoff moves to the right! Depending on the level of interest rates, a fall in interest rates can lead to more or fewer intermediaries choosing to lever up. We explain below this strong non-linearity of the effect of interest rates on systemic risk.

3.1 Non-linear trade-off between increased output and systemic risk

Following a fall in interest rates on deposits, intermediaries expand their asset holdings raising the aggregate capital stock. This lowers the return on risky asset holdings due to decreasing returns to capital in the aggregate. As seen in the graph above, we have very interesting asymmetries depending on the *level* of the interest rate.

When the interest rate level is high, the lower cost of liabilities reduces the probability of default for a given balance sheet size. Hence all intermediaries with a risky business model can lever more (*intensive* margin). In this case, there are also positive returns for the (previously) marginal intermediary due to the now lower cost of leverage. More intermediaries can lever up and enter the market for deposits (*extensive* margin), reducing the cutoff α^L. In this case, the system becomes less risky since newly entered intermediaries have a stricter Value-at-Risk constraint. There is therefore no trade-off between using lower interest rates to stimulate investment and financial stability.

When the interest rate level is low, the *intensive* margin effect of a decrease in the interest rate is strong (see Proposition 2.2), leverage and investment are high and the curvature of the production function leads to a decrease in expected asset returns which is large enough to price out of the market the most risk averse intermediaries. The sign of the effect on α^L depends on whether the fall in asset returns is stronger than the fall in the cost of liabilities. In the case of initially low interest rates, a further fall (in those rates) leads to fewer intermediaries choosing to lever up. Those intermediaries are larger and more risk-taking on average. There is therefore a clear trade-off between a lower interest rate (which corresponds in equilibrium to an expansionary monetary policy) and financial stability.

In order to gain some intuition, think of two polar cases. In the first, aggregate capital is infinitely elastic and return distributions $R^K_{t+1}(\varepsilon)$ are fixed. In this case, a decrease in the cost of funding can only lead to entry as the (previously) marginal intermediary will now make positive profits. The cutoff falls and there is no trade-off. In the second example, aggregate capital is fixed and returns adjust to clear the market[21]. If a fall in the cost of funding allows more leverage from the more risk-taking intermediaries, then it must be that the (previously) marginal intermediary no longer holds capital and returns fall enough to price him out. In this case, there is always a trade-off. In intermediate cases, the strength of the intensive margin effect is important as it determines the extent to which returns fall due to decreasing returns in the aggregate capital stock. The stronger is this effect (i.e. the more leverage increases following a fall in interest rates or the more *interest-elastic* the banks are), the more likely a trade-off will be present. As stated in Proposition 2.2, leverage increases faster as the interest rate falls (conditional on being levered). This means the *intensive* margin effect is particularly strong when interest rates are low.

[21]In this case the price of capital will adjust, as it is no longer pinned down by the investment technology. For recent macroeconomic models in which extensive and intensive margin have interesting interactions (albeit in very different contexts) see Martin and Ventura (2015) and Bergin and Corsetti (2015).

Hence, as shown in Figure (1), when interest rates fall from high to medium to low, balance sheets become more heterogeneous in size and the difference between the most leveraged and the least leveraged intermediary rises. In the left panel of Figure (6) we report skewness as a function of interest rates for 3 different productivity levels. We can state the following implication of our model:

Implication 1: Heterogeneity and skewness of leverage. The lower is the interest rate, the more heterogeneous is leverage across intermediaries. For low level of interest rates, there is an increased concentration of assets as the most risk-taking intermediaries leverage up a lot, leading to a higher cross-sectional skewness of leverage.

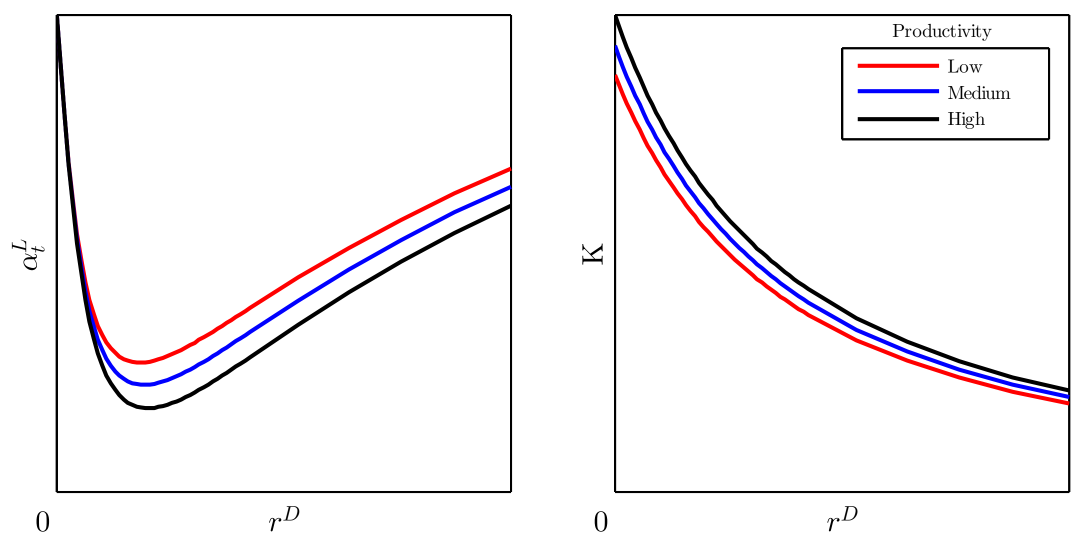

Figure 2: *Cut-off level α_t^L and aggregate capital stock as a function of deposit rates r_t^D*

In Figure (2), the left graph plots the cutoff α_t^L as a function of deposit rates r_t^D for three different productivity levels, while the right graph does the same for the aggregate capital stock K_t. As we can see, K_t is monotonically decreasing with r_t^D. As expected, the lower is the interest rate, the higher will be aggregate investment and we have a standard deposit supply curve. However, the change in *financial structure* underlying the smooth response in the capital stock is non-monotonic. As we can see from the left graph, the cutoff α_t^L first decreases when we go from high interest rates to lower ones and then goes up sharply as we approach zero.

Implication 2: Trade-off between financial stability and economic activity. When interest rates are high, a fall in interest rates leads to entry by less risk-taking intermediaries (a fall in the cutoff α_t^L) into levered markets. But when interest rates are low, a fall in interest rates leads to a rise in the cutoff α_t^L, which means the least risk-taking intermediaries drop off the market while more risk-taking intermediaries increase their balance sheet size and leverage.

Therefore, unlike in the earlier literature, there is a potential trade-off between financial stability and monetary policy when interest rates are low, but not when they are high. The level of the interest rate matters. During a monetary expansion, the cost of liabilities is reduced and the partial equilibrium results described above follow. The fact that risk-taking intermediaries are able to lever more can increase the capital stock while still pricing out less risk-taking ones. This means that the financial sector becomes less stable, with risky assets concentrated in very large, more risk-taking financial institutions. There is also potentially large mispricing of risk[22], since the active intermediaries are those who engage the most in risk-shifting. As a result, the effects of risk-shifting on investment are amplified through the change in the extensive margin. As can be seen in Figure (19), other measures of systemic risk also highlight the presence of an important systemic risk trade-off which occurs only at low levels of the interest rate.

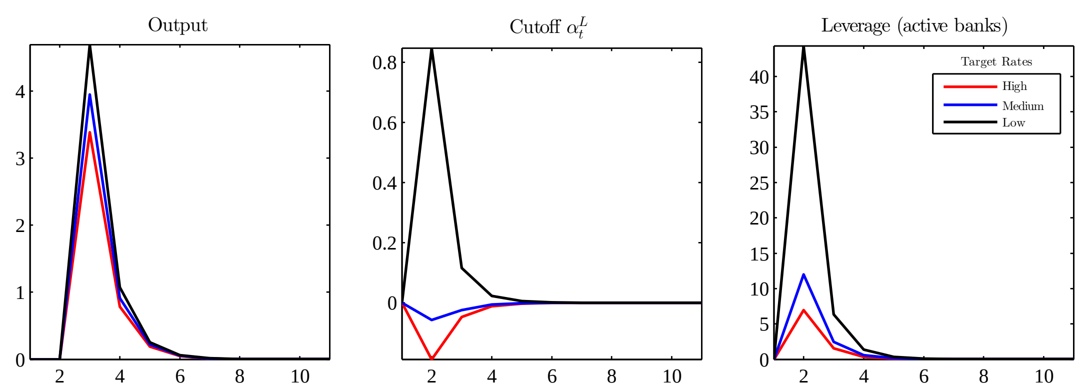

Figure 3: *Partial equilibrium IRF to a 100 basis points fall in deposit rates. Scale in percentage point deviations from the baseline*

We illustrate this point in our partial equilibrium setting by doing a 100 basis points monetary expansion for different target rates. For this experiment, we assume a very

[22]Defined here as the difference between the market price and the price investors would be willing to pay in the absence of limited liability.

simple monetary policy rule:

$$R_t = R_{t-1}^{\nu} \bar{R}^{1-\nu} \varepsilon_t^R \tag{28}$$

where $R_t = 1 + r_t^D$ is the return on deposit or the cost of leverage for intermediaries. ε_t^R is a monetary policy shock, $\bar{R}$ is the long-run level of interest rates and ν the persistence of the shock, calibrated[23] to 0.24. For simplicity, in this simple partial equilibrium exercise, we assume that the monetary authority can directly affect the deposit rate. We relax this assumption in section 4 and show how it can be mapped into this exercise.

Results can be seen in Figure (3), plotted as percentage changes from their respective values at target rates $\bar{R}$.[24] The time period corresponds to one year and the state of the economy when the shock hits is the one corresponding to the target rates. In the left graph we see that the rise in output seems to be slightly larger when rates are low, with the rise ranging from 3.2% to 4.5%. The monotonicity of K_t with respect to r_t^D ensures, as expected, that a monetary policy expansion stimulates investment and the capital stock in all cases. The behaviour of the cutoff α_t^L is, however, very differentiated. When the target rates are high, there is a small negative effect of a monetary expansion on the cutoff. That means that less risk-taking intermediaries enter risky markets and the average probability of intermediary default falls. In this case, there is no trade-off between financial stability and monetary expansion. This is definitely not the case when target interest rates are low. In that case, average leverage increases massively by 43% and the cutoff also rises. The large increase in leverage by very risk-taking intermediaries then prices out the less risk-taking ones at the margin, raising the average probability of default among levered intermediaries. This large effect on leverage is a combination between both the intensive margin effect, and a composition effect due to exit of the most risk averse intermediaries. For intermediate levels, we see that this effect is muted, with leverage increasing slightly more than in the first case and the effects on the systemic risk (cutoff) being negative but only marginally so. This leads us to state another implication of our model regarding links between macroeconomic variables and underlying financial structures, which we believe is quite novel in the literature.

Implication 3: Similar aggregate investment outcomes can be supported by very different underlying financial structures.

To sum up, in some cases, lowering interest rates may well stimulate the economy but also contribute to an increase in systemic risk. This happens through a change in

[23]Annualized value as estimated by Curdia et al. (2015)

[24]Note that there is no truly dynamic aspect in the partial equilibrium model and it can be seen as a sequence of static problems. The general equilibrium model of section 4 will feature a fully dynamic household problem which affects the banking problem via demand for deposits.

the composition of intermediaries. Less risk-taking intermediaries exit levered markets and decrease their asset holdings as they are priced out by more risk-taking institutions due to decreasing returns to capital. The latter use low interest rates to increase their leverage significantly. Given that risk-shifting is larger in riskier intermediaries, this also generates more risk-taking on aggregate. But these effects happen only for low levels of interest rates. At higher levels, there is no such trade-off between monetary policy and financial stability. Our framework, appropriately enriched[25], should ultimately help us quantify the importance of the risk-taking channel of monetary policy.

We note that even in the absence of monetary policy, the effects described above have implications for the cyclicality of leverage, systemic risk and aggregate risk-shifting. The cyclicality of the savings behaviour and its effect on equilibrium deposit rates will also lead to cyclical movements in leverage and investment. To understand this more fully, we now close the general equilibrium model by adding the intertemporally optimizing household sector to determine the deposit rate endogenously.

4 General Equilibrium

In this section, we solve the model in general equilibrium by joining the household and intermediary problems. We show that the financial sector equilibrium can be easily integrated in a standard dynamic stochastic general equilibrium framework, with monetary policy and productivity shocks, as well as costly default.

4.1 Monetary policy as a change in the cost of external funds

In this section we allow intermediaries to fund themselves through wholesale funding l_{it}. We assume that the monetary authority can control the rate of wholesale funding relative to deposits, by providing funds at a spread γ_t from deposits.[26]. Wholesale funding is remunerated at rate $R_t^L = 1 + r_t^L$ and we denote the deposit rate r_t^D as before. We assume that:

$$R_t^L = R_t^D(1 - \gamma_t) \tag{29}$$

[25] In particular by introducing nominal rigidities, which is left to another paper

[26] The monetary authority is assumed to be a deep-pocketed institution which can always fund wholesale funding. Like deposits, wholesale funds are always repayed (by bailout if necessary). To avoid dealing with the monetary authority's internal asset management, we assume the cost of fund is a deadweight loss (or gain).

Monetary policy is exogenous, akin to a funding subsidy γ_t which follows a simple AR(1) process in logs.

$$\log \gamma_t = (1-\rho^\gamma)\mu^\gamma + \rho^\gamma \log \gamma_{t-1} + \varepsilon_t^\gamma \tag{30}$$
$$\varepsilon_t^\gamma \sim N(0, \sigma_\gamma) \tag{31}$$

where μ^γ is the central bank target subsidy, ρ^γ the subsidy's persistence and ε^γ are monetary policy shocks with σ_γ standard deviation.

If the central bank were to provide unlimited funds to intermediaries at this rate, they would leverage using only wholesale funding. We assume that wholesale funding is given in a fixed proportion χ of other liabilites, which in this case are simply deposits. Total wholesale funding for intermediary i is then:

$$l_{it} = \chi d_{it} \tag{32}$$

The balance sheet of an intermediary i is then:

Assets	Liabilities
k_{it}	ω
s_{it}	d_{it}
	l_{it}

Given our assumptions, we can then define R_t^F as the total cost of a unit of funding and f_{it} as total external funds of bank i.

$$R_t^F = \frac{1+\chi(1-\gamma_t)}{1+\chi} R_t^D \tag{33}$$
$$f_{it} = (1+\chi) d_{it} \tag{34}$$

We can then write the balance sheet as:

Assets	Liabilities
k_{it}	ω
s_{it}	f_{it}

With external funds being remunerated at rate R_t^F. We obtain the same banking problem as before, replacing deposits by total funds f_{it} and the deposit rate by the unit cost of funds R_t^F. We can solve analogously as before, and map f_{it} and R_t^F easily to deposits d_{it} and their rate R_t^D. By moving γ_t the central bank will be able to change R_t^F as long as changes in equilibrium R_t^D do not offset perfectly the changes in the spread on the total cost of funding.

4.2 Solving the dynamic model

The financial sector equilibrium can be seen as a sequence of static problems given funding costs R_t^F. We can then solve for the aggregate capital stock K and cutoff α_t^L as a function of R_t^F and expected productivity Z^e.

$$K = K^*(R^F, Z^e) \tag{35}$$

$$\alpha^L = \alpha^{L,*}(R^F, Z^e) \tag{36}$$

By integrating balance sheet equations, we obtain an expression for total funds F_t and deposit supply D_t:

$$F_t = \int_{\alpha_t^L}^{\overline{\alpha}} k_{it}^L \, dG(\alpha^i) - [1 - G(\alpha_t^L)]\omega \tag{37}$$

$$D_t = \int_{\alpha_t^L}^{\overline{\alpha}} d_{it}^L \, dG(\alpha^i) = \frac{F_t}{1+\chi} \tag{38}$$

where $F_t = \int f_{it} \, dG(\alpha^i)$ are total liabilities held by *leveraged* intermediaries and D_t is aggregate deposit demand. Market clearing in the deposit market requires supply and demand to be equal.

$$D_t^H = D_t \tag{39}$$

Goods market clearing requires that output is used in consumption of intermediaries and households, investment and the accumulation of storage. The investment good is the consumption good and there are no capital or investment adjustment costs[27]. Aggregate investment I_t is given by the law of motion of the capital stock $K_t = (1-\delta)K_{t-1} + I_t$.

$$S_{t-1}^H + S_{t-1}^I + Y_t = C_t^H + C_t^I + S_t^H + S_t^I + I_t + T_t \tag{40}$$

where $C_t^I = \int c_{it} \, dG(\alpha^i)$ and $T_t = \int t_t^i \, dG(\alpha^i)$. Note that taxes here are equal to the deposit insurance repayments, which require real resources. S_t^H are the holdings of storage held by households and $S_t^I = \int s_{it} \, dG(\alpha^i)$ are aggregate storage holdings held by financial intermediaries at t.

Definition 2: Equilibrium.

Let $\mathcal{S} = \{D_{t-1}, S_{t-1}^H, S_{t-1}^I, K_{t-1}, Z_{t-1}, \gamma_{t-1}, \varepsilon_t^z, \varepsilon_t^\gamma\}_{t=0}^\infty$ be the vector of state variables and shocks. Given a sequence of rates $\{r_t^D\}_{t=0}^\infty$ and financial market rules

[27] We also do not constrain new capital to be larger than the stock of undepreciated capital

$K(\mathcal{S}), \alpha^L(\mathcal{S}), S(\mathcal{S})$, let us define the optimal decisions of the representative household as $C^H(\mathcal{S}), D^H(\mathcal{S}), S^H(\mathcal{S})$.

An equilibrium is a sequence of rates $\{r_t^D\}_{t=0}^{\infty}$, and policy rules $C^H(\mathcal{S}), D^H(\mathcal{S}), S^H(\mathcal{S}), S^I(\mathcal{S}), K(\mathcal{S}), \alpha^L(\mathcal{S})$, such that:

- $C(\mathcal{S}), D^H(\mathcal{S}), S^H(\mathcal{S}), S^I(\mathcal{S}), K(\mathcal{S}), \alpha_t^L(\mathcal{S})$ are optimal given $\{r_t^D\}_{t=0}^{\infty}$
- Asset and goods markets clear at every period t

In equilibrium, we need to find a deposit rate which, conditional on exogenous variables and the financial sector equilibrium, is consistent with the household problem. We proceed by iterating on r_t^D, imposing the financial market equilibrium results. For a given deposit rate r_t^D, we can find the law of motion for household wealth and consumption and use the Euler equation errors to update the deposit rate. A more detailed explanation of the algorithm used for our global solution method can be seen in Appendix A.

4.3 Calibration

To solve the model numerically, we need to specify the period utility function, the shape of the distribution of the Value-at-Risk probabilities and calibrate the remaining parameters. Given the interaction between extensive and intensive margin effects, the mass of intermediaries in a given section of the distribution could have an important role in determining which of the two effects dominates. To highlight that the results described are not a consequence of this distribution, we assume that $G(\alpha^i)$ is uniform between $[0, \overline{\alpha}]$. For the utility function, we assume a standard CRRA representation.

$$u(C) = \frac{C^{1-\psi} - 1}{1 - \psi} \tag{41}$$

The calibration can be seen in Table (1). For the utility function parameters, risk aversion ψ, the subjective discount factor β, the TFP parameters ρ^z and σ_z we use standard values from the literature. Similarly for θ, the capital share of output, and for δ the depreciation rate of the capital stock. To calibrate the monetary policy parameters, we calculate the subsidy as the difference between the Effective Fed funds Rate and $1/\beta$, the long-run deposit rate. We then fit an AR(1) process to get the parameters used.

The wholesale funding percentage used to calibrate χ was calculated from the time series mean of the cross-sectional asset-weighted average in Bankscope data[28] for the

[28] Bankscope contains a large panel of banks balance sheet data; see Appendix C

Table 1: Calibration of selected parameters

Parameter	Value	Description
ψ	4	Risk aversion parameter
β	0.95	Subjective discount factor
ρ^z	0.9	AR(1) parameter for TFP
σ_z	0.028	Standard deviation of TFP shock
μ^γ	0.023	Target spread over deposit rates
ρ^γ	0.816	Spread persistence
σ_γ	0.0128	Standard deviation of spread
$\frac{\chi}{1+\chi}$	0.41	Wholesale funding percentage
θ	0.35	Capital share of output
δ	0.1	Depreciation rate
ω	0.51	Equity of intermediaries
$\overline{\alpha}$	0.1	Upper bound of distribution $G(\alpha^i)$

period 1993-2015. For the purpose of this calibration, wholesale funding was assumed to be all non-deposit liabilities of each financial intermediary.

$\overline{\alpha}$ is the probability of default of the riskiest intermediary in activity. Bali, Brown and Caglayan (2014) report that the median lifespan of a hedge fund is slightly less than 5 years, which would imply a value of approximately 0.2 for $\overline{\alpha}$. Using FDIC data on failed banks, we find that the median age of failed banks in the US was around 20.5 years. The full sample distribution of ages at failure can be seen in Figure (9). Given our specification of a uniform distribution and the fact that equity size is fixed in our model, we use a more conservative value of 0.1 for $\overline{\alpha}$. This implies an average lifespan of 10 years for the most risky of financial intermediaries and 20 years for the median intermediary. Although we do not believe the uniform distribution is realistic, if the underlying distribution in the data was uniform, and defaulting intermediaries replaced with identical ones, we should see higher frequencies of default for lower ages than for longer ones in a manner not too disimilar with the FDIC data.

ω is chosen to fit average leverage at steady-state. Some of the intermediaries are leveraged and others are not, so we cannot use only Bankscope data (which contains mostly leveraged banks) to calibrate average leverage. According to the "broad measure" of Other Financial Institutions (OFIs) in the Global Shadow Banking Report (Financial Stability Board (2015)), non-levered intermediaries hold about 137 trillions of assets while banking assets are around 135 trillion. We use these figures to calculate an asset-weighted average of leverage of 7.3, which is reached by combining the Bankscope asset-weighted average leverage of 13.5 for 2015 and assuming a leverage of 1 for the OFIs. We target our calibration of ω so the model matches this average leverage[29].

4.4 Monetary policy shocks

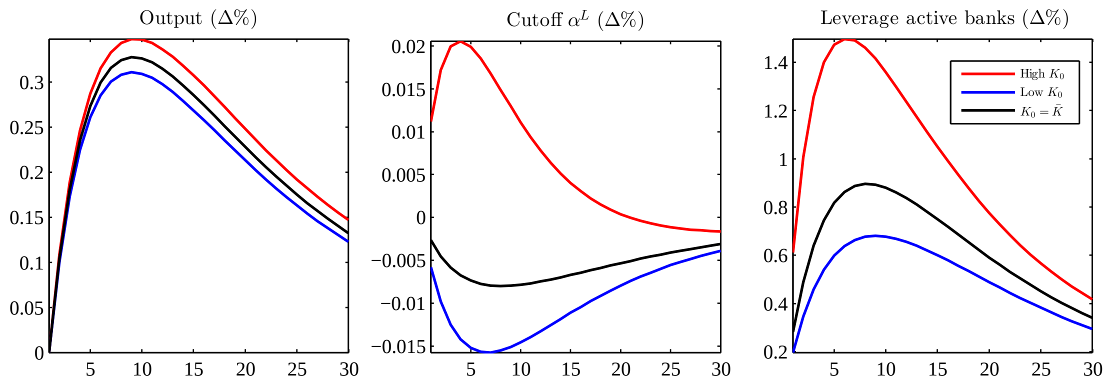

Figure 4: *Monetary policy shock of 100 basis points to* γ_t

We now look at the impact of positive subsidy shock, which we will refer to as an expansionary monetary policy shock. In Figure (4) we see the impact of a 100 basis points to the subsidy[30] in three different scenarios to illustrate the non-linear effects of monetary policy on systemic risk. Impulse response functions are expressed as deviations from the respective scenario in the absence of the shock. This monetary policy loosening decreases the funding rate of the banks as can be seen in the top right panel of Figure (11). Scenario 1 (blue line) features a low initial capital stock (corresponding to high equilibrium levels of the interest rate), where there is no trade-off

[29]The value of $\overline{\alpha}$, the shape of the distribution and ω all contribute to determine the financial sector reaction to changes in deposit rates. For that reason, we also conducted some comparative statics on both $\overline{\alpha}$ and ω to see how the model changes with those parameter calibrations. There is very little effect on the first moments of real variables such as output and consumption but there are changes on equilibrium leverage and systemic risk when we vary ω and/or $\overline{\alpha}$. We leave for future work to perform a (technically challenging) estimation of the model where distributions $G(\alpha)$ or of ω could potentially be backed out from the data and focus here on undertanding the mechanics of the model

[30]Note that this translates in a lower reduction in the total cost of funds (see Figure (11)). This is due to the fact that the cost of funds is a composite of deposits and wholesale funds, but also due to endogenous movements in the deposit rate.

between monetary policy and financial stability. Scenario 2 (red line) is for a larger capital stock (corresponding to a low level of equilibrium interest rate) where we are in the trade-off zone. Scenario 3 (black line) is at the risky steady-state[31]. As in Coeurdacier et al. (2011) we define the risky steady-state as the steady-state in which there are no shocks but economic agents take into account the full stochastic structure of the model when they optimize (unlike in the deterministic steady-state where they expect no shocks).

We can easily relate the general equilibrium results to the partial equilibrium intuitions developed above. In the case of a low initial capital stock (associated with a high equilibrium funding rate), a positive monetary policy shock expands output, increases aggregate leverage and at the same time it decreases systemic risk, due to the entry of less risk-taking intermediaries in deposit markets. We are in the "no trade-off zone of monetary policy" where a decrease in the interest rate increases investment and financial stability. In the case of a high initial capital stock (associated to a low funding cost for intermediaries), an expansionary shock has a larger positive effect on output and leverage but this time, risk averse intermediaries at the margin choose not to lever, reducing their balance sheet size significantly. More risk-taking intermediaries leverage a lot and financial stability is affected negatively.

Implication 4. There exists a "trade-off zone for monetary policy" where there is a conflict between stimulus and financial stability. This is a very different trade-off from the traditional Phillips curve which has been the benchmark model driving monetary policy analysis for many years. Aggregate economic variables such as consumption, wealth or capital behave smoothly as evidence in Figure (12) but the underlying change in financial structure supporting these macroeconomic outcomes can be significant depending on the level of the interest rate.

Implication 5. The sensitivity of leverage to the cost of funding is larger for intermediaries who are in the upper range of the risk-taking distribution. So the more concentrated is capital in this upper range, the more sensitive will aggregate leverage be to interest rate changes.

Implication 6. The risk-premium decreases with a monetary policy loosening. The marginal intermediary pricing the risk changes with the level of the interest rate due to the extensive margin (see Figure (11)). When the interest rate is lower, the

[31]These three scenarios were chosen to illustrate the parallel with the partial equilibrium setting, since the solution of the model is such that there is, ceteris paribus, a negative correlation between the capital stock and the funding rate as can be seen in Figure (2).

intermediaries pricing risk are the most risk-loving ones, hence the lower risk premia. Note that risk premium is defined here as the wedge between expected return to capital and the cost of funds. The decline of the risk premium is also stronger when the level of interest rate is lower.

5 Empirical evidence on the cross-section of intermediary balance sheets

We do not present here a test of our model but a number of important new stylized facts on the distribution of intermediaries balance sheets over the cycle. These facts escaped the previous literature as it did not feature intermediary heterogeneity. In contrast our model has strong predictions.

First, the model implies that the time correlation between leverage and interest rate is different across quantiles of the leverage distribution, even as aggregate leverage is monotonically decreasing with the interest rate. In the top quantiles, leverage and interest rates are negatively related. In the bottom quantiles, some intermediaries may stop leveraging as interest rate goes down so the correlation sign may flip. For intermediaries that remain levered, Proposition 2.2 indicates that the covariance between interest rates and leverage should be larger for the most risk-taking (and most levered) intermediaries.

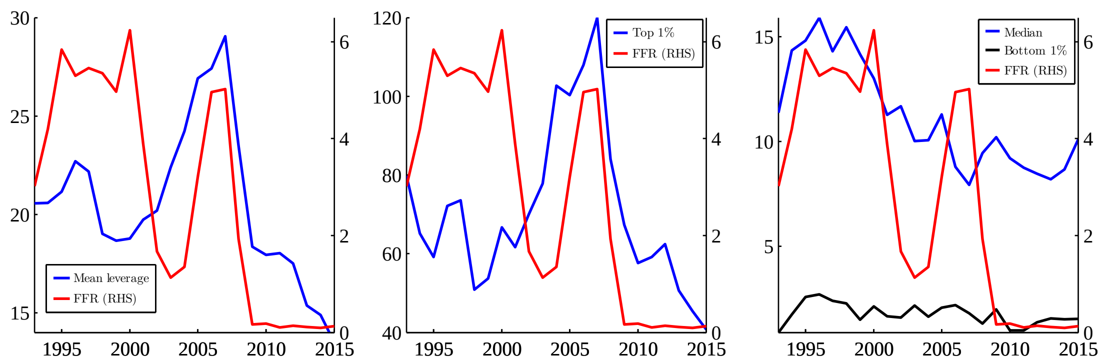

Figure 5: *Mean and selected quantiles of asset-weighted leverage of intermediaries (blue, LHS scale) and the Effective Fed Funds Rate (red, RHS scale, pp).*

We use balance sheet data of financial intermediaries from Bankscope (see Appendix C) to compute leverage at the intermediary level. Leverage is defined as the ratio of assets over equity at book value. In Figure (5) we show the time series of leverage weighted by intermediary assets for different quantiles of the distribution and for the aggregate.[32] As can be seen in the middle and right panels, there is a very significant heterogeneity within the financial sector in terms of time variation of leverage and its correlation with the interest rate. As predicted by the model, there is a clear dichotomy between the two parts of the distribution. The top quantile is negatively correlated with interest rates (correlation of -0.26 up to 2007), with the more leveraged intermediaries increasing leverage sharply as interest rates fall to low levels in the early 2000s. This large increase in leverage is not apparent in neither the median nor the bottom 1% of the distribution where if anything there is a decrease. According to the model this is due to the less risk-taking intermediaries exiting the risky leveraged market as they are priced out by the most risk-taking ones. Aggregate leverage, shown in the left panel is also negatively correlated with interest rate (correlation of -0.11 up to 2007) and increases abruptly when the rates decreased markedly, reproducing to some extent the interest rate sensitivity of the top quantile. The post-2008 period is of course very special with large state interventions and changes in regulation as well as unconventional monetary policy at the Zero Lower Bound (ZLB), all elements which are absent for our model.

Second, low levels of the interest rate are associated with an increased skewness of leverage: risk gets concentrated in the (endogenously) larger, more risk-taking players. This is a very distinction implication of our model. As with aggregate leverage, when rates are low, skewness is more sensitive to interest rate movements. In the left panel of Figure (6), we show the shape of cross-sectional skewness as a function of r_t^D for three different levels of productivity. It is apparent that the direct impact of productivity on skewness, although positive, seems second-order relative to the impact of interest rates[33]. We use the same data as before to compute the time series of the skewness of leverage. In the right panel of Figure (6), we present the time series of asset-weighted skewness in parallel with the movements of the Effective Fed Funds Rate. There is a strong negative correlation between the Effective Fed Funds Rate and skewness as predicted by the model (correlation of -0.58 up to 2007), with a large spike in skewness as the interest rate is very low.[34] These results are striking

[32]Results are very similar if we use instead CPI-deflated real rates, as can be seen in Figure (16). Up to 2007, the correlation between the top 1% and the real efffective Fed Funds rate is -0.5 and it is -0.31 for the average leverage.

[33]In general equilibrium, productivity will also affect skewness indirectly via its impact on deposit rates.

[34]Admittedly, the situation after 2008 when monetary policy is at the ZLB is (as explained above) quite unusual with large state interventions in the banking sector and changes in regulation, including leverage caps. Again, results are similar if we use CPI-deflated real rates instead, as can be seen in Figure (15). The correlation between skewness and the real effective Fed Funds rate is -0.45. We also computed skewness using only US bank data, results were very similar and are available on request.

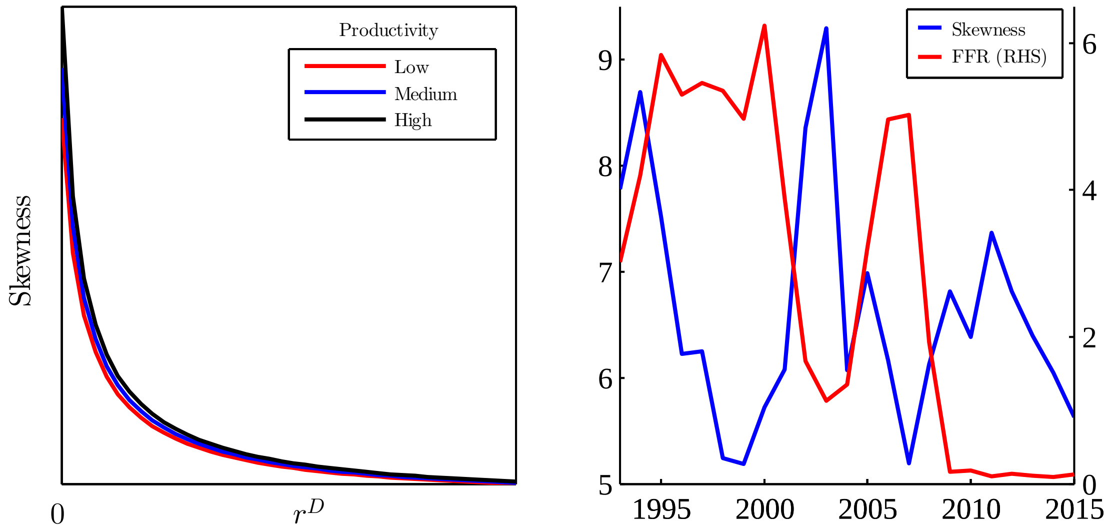

Figure 6: *Left:* Cross-sectional skewness as a function of deposit rates in the model *Right:* Time series of asset-weighted skewness of intermediary leverage (blue, LHS scale) and the Effective Fed Funds Rate (red, RHS scale) in pp.

and very encouraging for the mechanism of our model. We are not aware of any paper studying the distribution and skewness of leverage and linking it to monetary policy.

Third, the model implies that in the cross-section (endogenously) larger more leveraged intermediaries make higher profits in good states of the world but are more exposed to aggregate risk. Accordingly we analyse the returns of financial intermediaries in the run up to the crisis and look at its correlation with leverage and with the exposure to aggregate risk (measured by the world market beta). Figure (7) shows a positive correlation between pre-crisis betas and bank returns. We find a positive correlation between returns and leverage, confirming the results of Miranda-Agrippino and Rey (2015), who also show that the higher beta banks tended to do worse in the crisis as they were more exposed to aggregate risk.

Although none of this constitutes a formal test, we view those facts as supporting the relevance of the main mechanism of our model. This underlines the importance of looking at cross-sectional dynamics of the balance sheets of financial intermediaries, in order to understand macroeconomic developments.

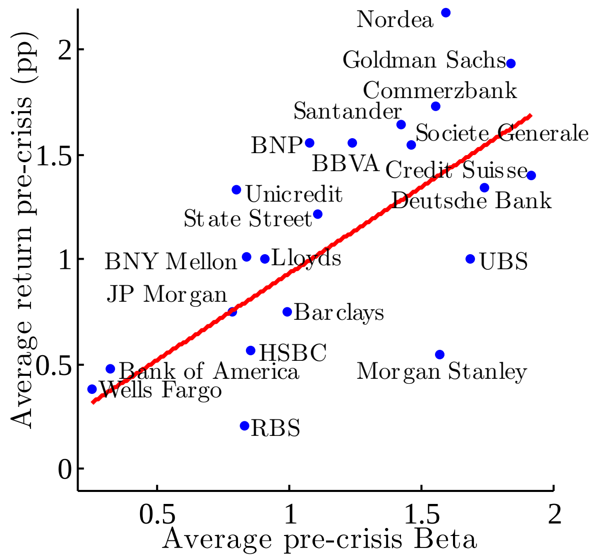

Figure 7: *Market beta and pre-crisis returns for SIFIs (Systemically Important Financial Institutions)*

6 Costly intermediary default

In this section, we relax the assumption of costless intermediary default. As in the previous section, leveraged intermediaries active in risky financial markets can potentially default on depositors if the realisation of the productivity shock is low enough. This requires intervention by the government to pay deposit insurance, which is now less benign than previously assumed as there is a deadweight loss[35].

We parameterize the cost of intermediary bailouts by assuming that capital held by defaulting intermediaries suffers a proportional productivity loss $\overline{\Delta}$ relative to the productivity of capital held by non-defaulting intermediaries. This disruption can affect financial markets in the following periods by creating an efficiency loss Δ_t which is proportional to the mass of capital held by defaulting banks μ_t^d.[36]

The loss of productivity is intermediary-specific during default (it affects only the defaulting intermediaries, not the others), but it can affect the whole economy moving

[35] As before, deposit guarantees will be financed by lump sum taxation of households. The welfare analysis of our set up is left for future work.

[36] For example, if defaulting intermediares held 3% of total capital during default at $t-1$, then if the crisis persists $\Delta_t = 0.03\overline{\Delta}$.

forward (the allocative process of the whole economy is impaired). We call this the *crisis* state. We model the persistence of the crisis state through a Poisson process, with a constant probability p of exiting the crisis at each period. Depending on the process, variable ξ_t takes the value of one if the crisis carries on to the next period or zero if it does not. Our specification nests both the case of costless default ($\overline{\Delta} = 0$) and the case where there is no disruption of financial markets in subsequent periods ($p = 1$). We have:

$$\mu_t^d = \frac{\int k_{it} \ \mathbb{1}_{(\pi^i<0)} \ dG(\alpha^i)}{K_t} \tag{42}$$

$$\Delta_t = \xi_{t-1} \max(\mu_{t-1}^d \ \overline{\Delta}, \Delta_{t-1}) \tag{43}$$

$$\tag{44}$$

where the indicator function takes the value of 1 if intermediary i is in default or 0 if not. If there are also defaults during a crisis state, then the max operator ensures that the largest penalty applies going forward. Whenever the economy is in crisis, productivity for all financial intermediaries is scaled down by a factor μ^d proportional to the percentage of total capital held by defaulting intermediaries. ξ_{t-1} is known to agents when they make their investment decisions at period $t-1$, so the uncertainty on the returns on their capital investment is only on the realization of the exogenous productivity process[37]. This timing assumption allows us to keep tractability as the main difference in the financial sector block is that now $Z_{t+1}^e = (1 - \Delta_t) Z_t^{\rho^Z}$. Since both Δ_t and Z_t are state variables, we can still solve for the financial sector equilibrium as before.

This set up is tractable and allows us to parameterize crises of different severity and length. Reinhart and Rogoff (2009) presents a classic description of the characteristics of crises across history, and evidence that crises associated with banking crises are more severe. Borio et al. (2016) and Laeven and Valencia (2012) present empirical evidence showing that there can be substantial and long lasting productivity drops after financial crises. To calibrate these parameters we refer to the database of Laeven and Valencia (2012), setting $p = 0.5$ to target an average crisis length of 2 years as in the data, and $\overline{\Delta} = 0.11$ implying a maximal efficiency loss of 11% per year[38].

6.1 Productivity shocks and financial crises

In this section we study the impact of a financial crisis on the path of the economy, following a large productivity shock. Figure (8) shows the impact of a large productivity

[37]There is still uncertainty on asset returns if the intermediary defaults but this is not considered in the intermediary problem due to limited liability truncating the profit functions at zero in those states.

[38]In the database of Laeven and Valencia (2012), the average cumulative output loss is 23%

shock in 3 possible scenarios[39].

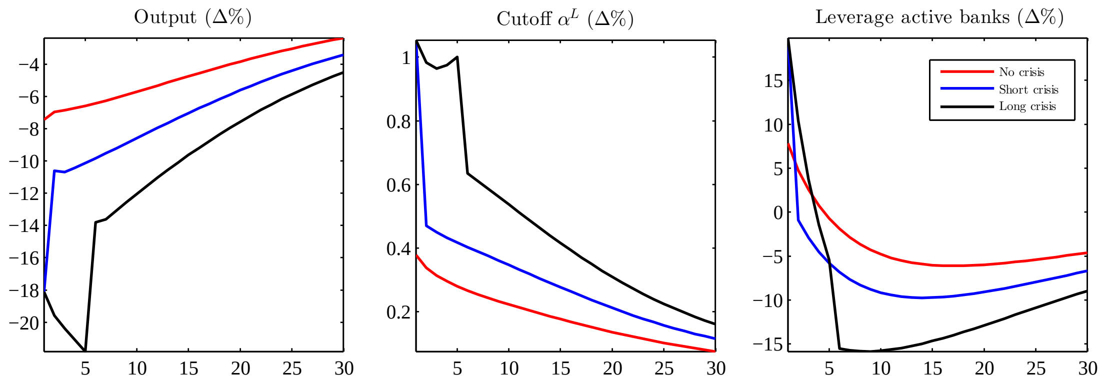

Figure 8: *Large shock to exogenous productivity*

In scenario 1 (red line) the economy at the risky steady-state is hit at period t by the largest possible shock that does not trigger any defaults. In scenarios 2 (blue line) and 3 (black line) the economy is hit with the smallest shock such that all *levered* intermediaries default. The difference between scenarios 2 and 3 is in the length of the crisis. Scenario 2 is the "lucky" scenario, where the crisis does not carry on to the next periods: $\xi_t = 0$. Scenario 3 is the "unlucky" scenario, where the crisis carries on for an additional 4 periods: $\xi_s = 1$ for $s = t$ to $t+3$. The length of the crisis is unknown beforehand to the agents in the economy, although as mentioned before they observe the value of ξ_t when they make their investment decisions at t. Not surprisingly, when the crisis hits there is a large decline in output. As productivity is low, only the intermediaries with the looser Value-at-Risk constraints can operate. The average leverage of active intermediaries first shoots up but then decreases and falls below the pre-crisis state as more intermediaries find it worthwhile to lever up when productivity improves. Note that this initial rise in leverage (in the subset of banks which are borrowing) is a pure composition effect since total sector leverage falls as evidenced in the bottom right panel of Figure (13).

The length of the crisis also has very interesting dynamic effects on financial variables, as can be seen in Figure (13). Deposit and funding rates decrease on impact as productivity drops. So does the risk premium because of the low expected return to capital. Given that households expect to exit the crisis state with probability p, when exit fails to materialize in Scenario 3 they are running down their wealth and their

[39]Impulse response functions expressed in basis points deviations for rates or otherwise in percent deviations from the risky-steady state

consumption dips down (see Figure (14). As wealth falls, the deposit rate grows, and along with it funding costs, as it becomes more costly for the household to save and fund bank leverage. When eventually the economy exits the crisis state, household wealth is low and demand for leverage rises, leading to a jump in deposit rates to compensate households for decreased consumption today. This leads also to a higher risk premium as expected return to capital jumps up. Total leverage, which had massively declined goes up again. This effect is also present with a short crisis, but is particularly stark for the longer crisis.

7 Conclusion

This paper develops a novel framework for modeling a financial sector with heterogeneous financial intermediaries and aggregate risk. The heterogeneity in the Value-at-Risk constraints coupled with limited liability generates not only endogeneous entry and exit in risky capital markets, but also time variation in leverage, risk-shifting and systemic risk.

The interaction between the intensive and the extensive margins of investment creates a rich set of non-linear dynamics where the level of interest rates plays a key role. When interest rates are high, a monetary expansion (defined here as a decrease in the cost of funding for intermediaries) increases both the intensive margin (the amount of leverage) and the extensive margin (which intermediaries decide to lever). The intensive margin grows because active intermediaries are able to lever more and the fall in the cost of funding leads to increased participation by less risk-taking institutions which enter levered markets. The monetary authority is able to stimulate the economy, while at the same time decreasing systemic risk.

However, when interest rates are already low, a further reduction can lead to large increases in leverage by the most risk-taking institutions, pricing out -due to decreasing aggregate returns to capital- previously active intermediaries despite the fall in the cost of funding. As before, the intensive margin grows but there is a fall in the extensive margin as intermediaries at the margin exit levered markets. Importantly, the intermediaries who stop levering and decrease their balance sheet size have lower probabilities of default than those that remain levered, leading to an increase in systemic risk. Hence our model, unlike the existing literature, generates a tradeoff between economic activity and financial stability depending on the level of the interest rate.

Because our framework has heterogeneity at its heart, it allows us to make use of cross-sectional data on intermediary balance sheets. For example, we derive novel implications linking the times series of the skewness of leverage and monetary policy. These implications are strikingly borne out in the data. We believe we are the first paper able to link changes in the distribution of leverage in the cross section, macroeconomic developments and fluctuations in systemic risk. We show that similar macroeconomic outcomes can be supported by very different underlying financial structures. This has important implications for the transmission of monetary policy and the sensitivity of the economy to interest rate movements.

A major advantage of our framework is that our financial block is easy to embed in a standard dynamic stochastic general equilibrium framework. The rich dynamics that arise from fluctuations in the composition of active financial intermediaries can be described simply by tracking changes in the aggregate capital stock K_t and the cutoff α_t^L. We plan to extend our model to environments with sticky prices and a more complex portfolio choice on the bank side as well as to study boom and bust cycles in emerging markets. We also plan to apply it to explain the dynamics of the real estate market, using detailed data, as well as the endogenous dynamics of the VIX.

The model could also be calibrated to fit a distribution of financial intermediaries characteristics, as one could in practice back out the distribution of α^i from leverage data and map it to the ergodic distribution of leverage in the model. Given the numerical integration approach, it is also possible to extend the model to have a distribution of intermediary-specific equity ω^i. That said, allowing for time variation in equity would require the introduction of an additional state-variable in the financial sector problem which would make the solution more computationally intensive.[40] We leave these issues, as well as the welfare implications of our model, for future research.

[40] And having together time-varying and intermediary-specific equity could require an infinitely dimensional state-space without additional assumptions.

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Figures

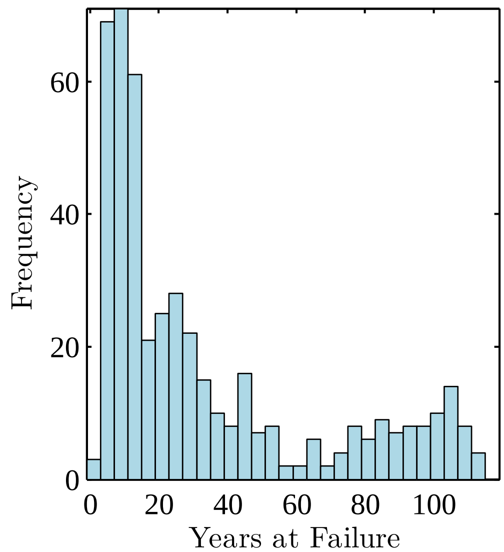

Figure 9: *Histogram of age of banks at closing date (in years). Data for failures in the US since October, 2000. Source: FDIC.*

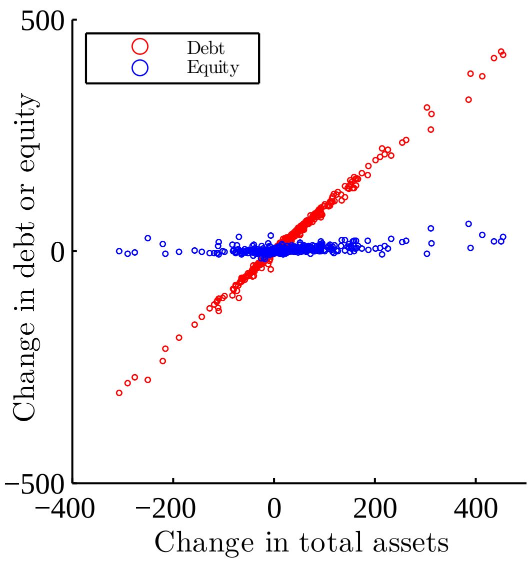

Figure 10: *Yearly changes in total asset against yearly changes in equity or debt from 1993 to 2015. Billions of USD. Source: Bankscope*

Monetary policy shock

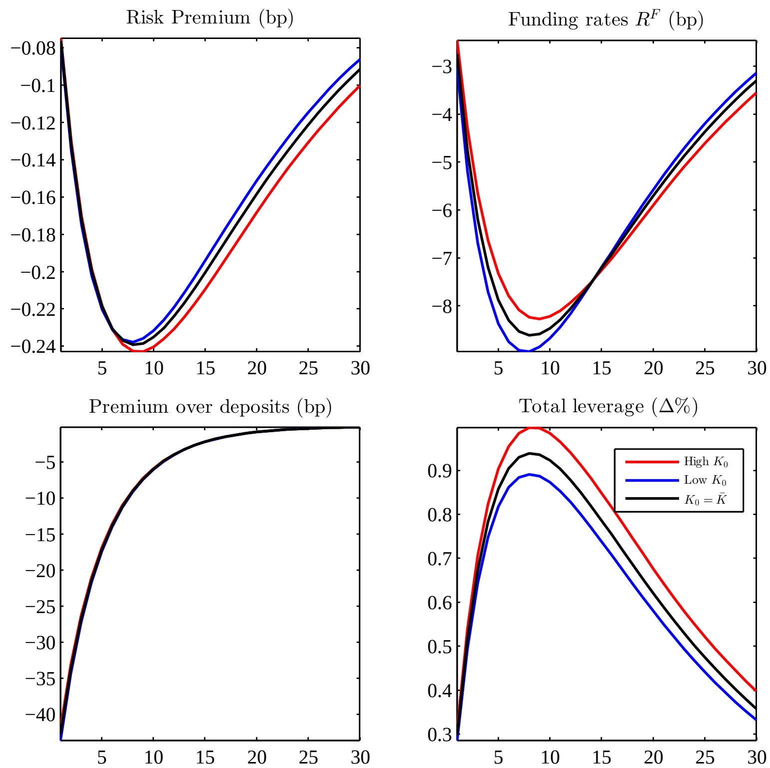

Figure 11: *Monetary policy shock of 100 basis points to γ_t: Financial variables*

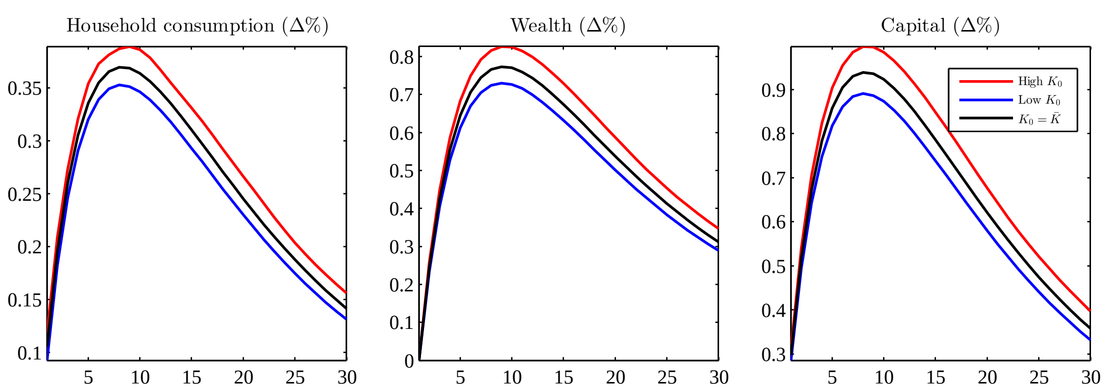

Figure 12: *Monetary policy shock of 100 basis points to γ_t: Real variables*

Productivity shock

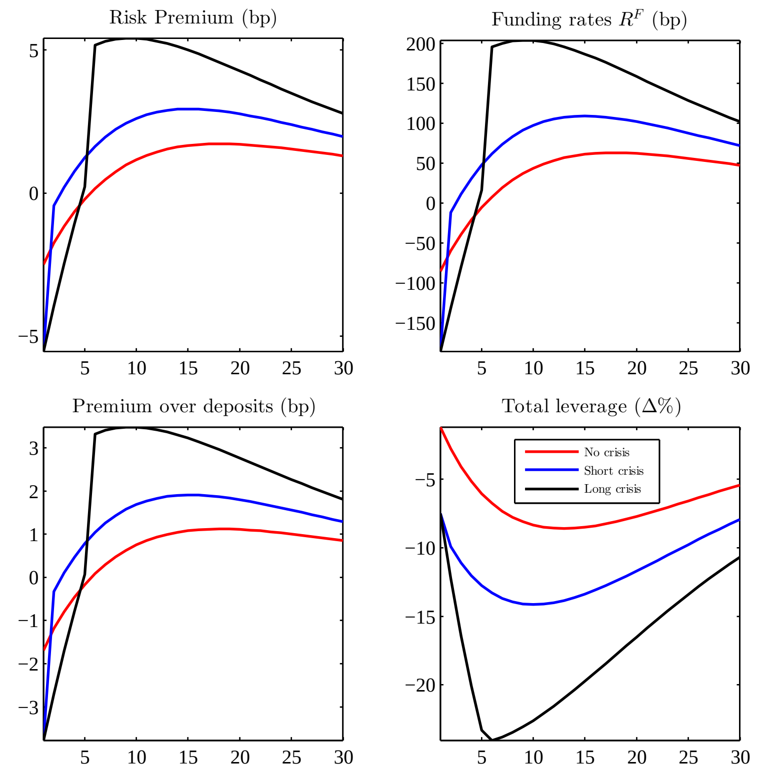

Figure 13: *Large shock to exogenous productivity: Financial variables*

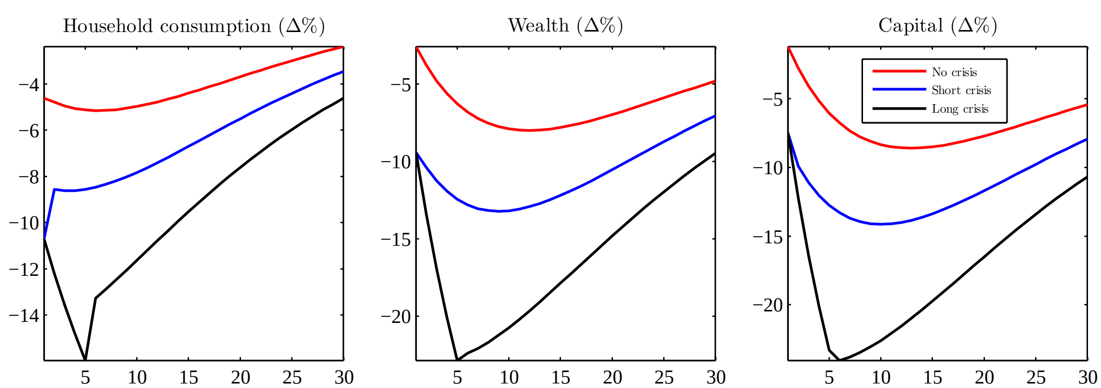

Figure 14: *Large shock to exogenous productivity: Real variables*

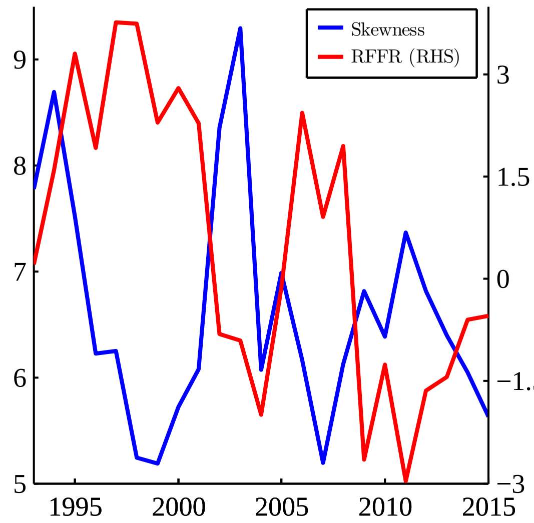

Figure 15: Time series of asset-weighted skewness of intermediary leverage (blue, LHS scale) and real (CPI deflated) Effective Fed Funds Rate (red, RHS scale) in pp.

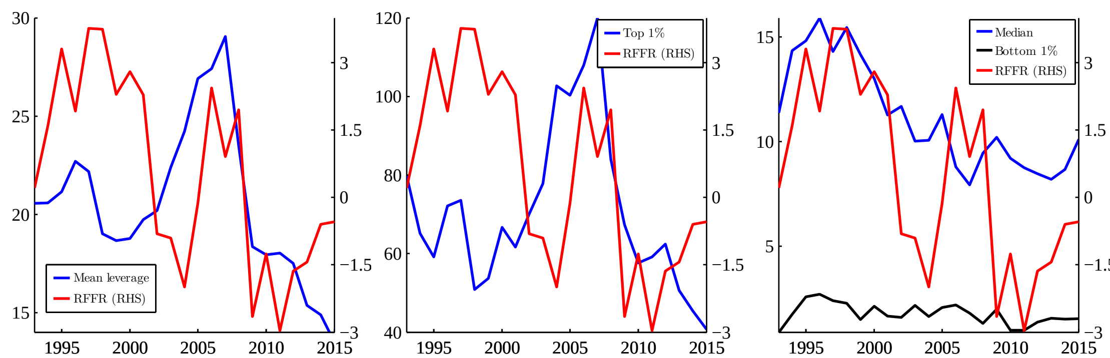

Figure 16: *Mean and selected quantiles of asset-weighted leverage of intermediaries (blue, LHS scale) and real (CPI-deflated) Effective Fed Funds Rate (red, RHS scale, pp).*

Monetary policy shock and interbank market

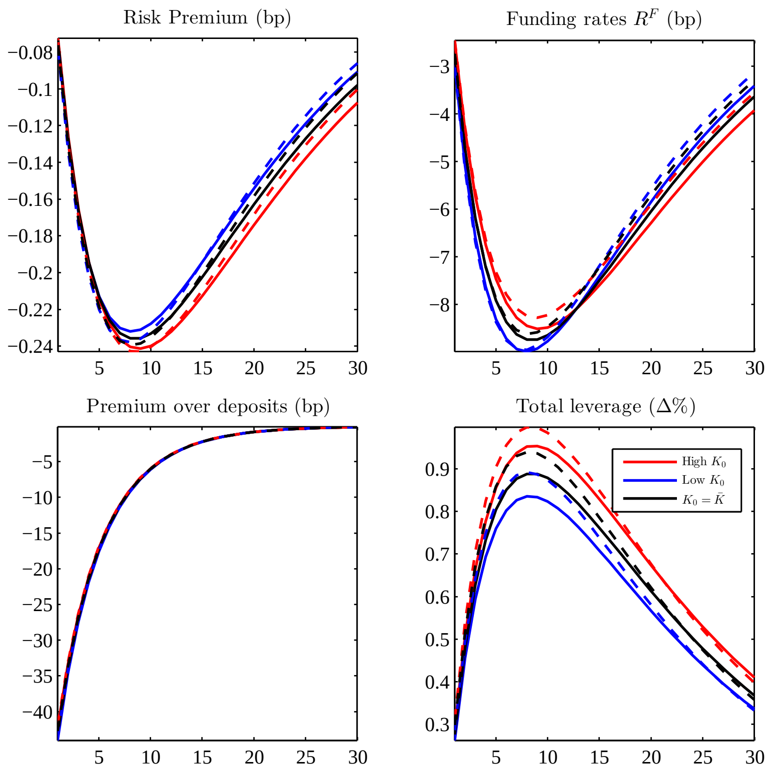

Figure 17: *Monetary policy shock of 100 basis points to γ_t: Financial variables*

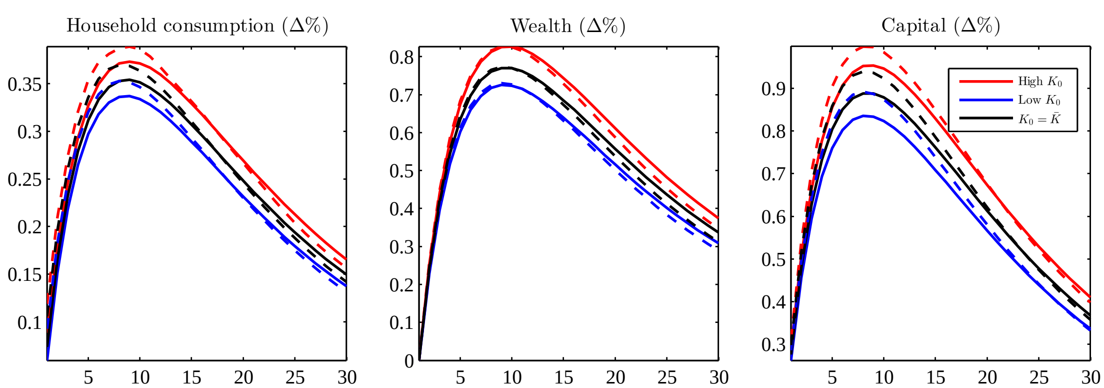

Figure 18: *Monetary policy shock of 100 basis points to γ_t: Real variables*

Alternative measures of systemic risk

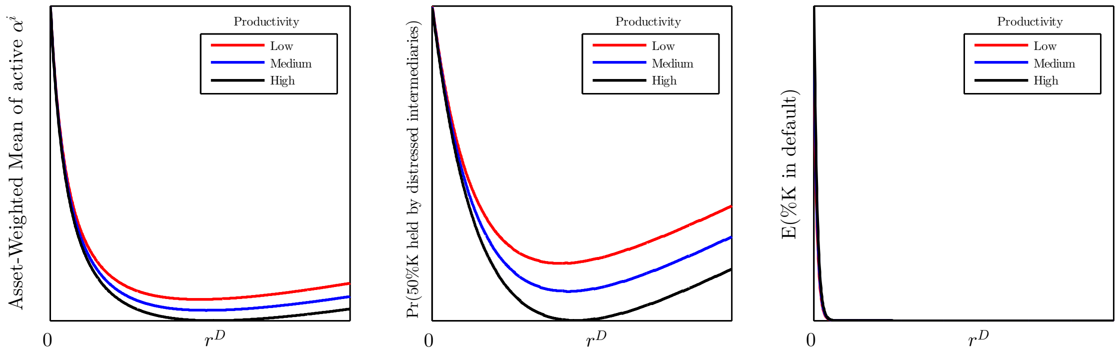

Figure 19: *Alternative measures of systemic risk*

Appendix A. Numerical solution method

The solution method is composed of two main blocks. The first block solves the partial equilibrium problem for a grid of points for variables r^f and Z^e. We discretize the state space using 100 nodes for Z^e and 200 for r^f. Given funding costs r^f and expected productivity Z^e we can solve jointly for equations (23) and (25), plugging in equation (19) in the latter. We also use the property that levered intermediaries never invest in storage. This gives us policy functions $K^*(r^f, Z^e)$ and $\alpha^{L,*}(r^f, Z^e)$.

The second block is the recursive one. First we define the household savings problem as a function of disposable wealth Ω_t, productivity $\tilde{Z}_t$, efficiency adjustment Δ_t and monetary policy γ_t.

$$\Omega_t = (1-\theta)Y_t - T_t + D^H_{t-1} + S^H_{t-1}$$

The procedure entails the following steps

1. Discretize the state space $\mathcal{S}$ for the variables $(\Omega, Z, \Delta, \gamma)$. The process for Z and γ are approximated using a Tauchen and Hussey (1991) quadrature procedure with 11 and 7 nodes respectively. The state space for the variable Ω is discretized using 500 nodes and we use 10 for Δ.

2. Iterate on prices r^D and policy function $C^*(\mathcal{S})$ starting with an initial guess $r^D(\mathcal{S})$ for deposit prices and $C^*(\mathcal{S})$. For every point $\mathcal{S}_j \in \mathcal{S}$:

   (a) Using the state vector and r^d_j, calculate r^f_j and Z^e_j.

   (b) Solve for (K_j, α^L_j) using $K^*(r^f_j, Z^e_j)$ and $\alpha^{L,*}(r^f_j, Z^e_j)$. Back out deposit supply D_j from the balance sheet equations.

   (c) Plug D_j in the budget constraint of the agent. Together with $C_j = C^*(\mathcal{S}_j)$ this pins down S^H_j.

   (d) Calculate expectations of $(\mathcal{S}'|\mathcal{S})$ and update deposit prices and policy functions using the optimality conditions and numerical integration.

   (e) Check for convergence. If $||(r'_j - r_j)|| + ||(C^*_j)' - C^*_j||$ is smaller than a threshold value stop. Else, go back to (a) and repeat.

To numerically integrate intermediary variables, Gauss-Legendre quadrature using 51 points is used. To calculate expectations of future net disposable wealth, we also need to calculate taxes conditional on future shocks. For a given productivity draw $Z'|Z_j$ we identify the threshold intermediary for which no bailout is needed: $(R^K k_i - R^D d_i) = \omega$. We can then calculate the amount T_t of taxes required by numerical integration.

Appendix B. Proof of Proposition 2.1

When $\mathbb{E}[R^K_{t+1}] \geq 1$, participating intermediary i will either lever up to its Value-at-Risk constraint: $d_{it} = \overline{d}^i_t$, or not raise deposits at all : $d_{it} = 0$.

Given the option value of default and the condition $\mathbb{E}[R^K_{t+1}] \geq 1$, participating intermediaries will not invest in storage. The Value-at-Risk constraint bounds the maximum level of leverage of intermediary i, therefore $d_{it} \in [0, \overline{d}^i_t]$. The profits of intermediary i as a function of deposits are:

$$\pi^i_t(d_{it}) = \int_{\varepsilon^i_t(d_{it})}^{\infty} \left[R^K_{t+1}(\omega + d^i_t) - R^D_t d^i_t\right] dF(\varepsilon) \tag{45}$$

where ε^i_t is the max of 0 (the lower bound of the support for ε) and the shock for which profits are zero).

$$\varepsilon^i_t(d^i_t) = \max\left(0, \frac{\frac{R^D_t d^i_t}{\omega + d^i_t} - 1 + \delta}{\theta Z^{\rho^Z} K^{\theta-1}_t}\right) \tag{46}$$

Taking derivatives:

$$\frac{\partial \pi^i_t}{\partial d^i_t} = \int_{\varepsilon^i_t(d^i_t)}^{\infty} \left(R^K_{t+1}(\varepsilon) - R^D_t\right) dF(\varepsilon) - \pi^i_t(\varepsilon^i)\frac{\partial \varepsilon^i}{\partial d^i_t} \tag{47}$$

Lemma 1 *Given equations (45) and (46), then* $\pi^i_t(\varepsilon^i_t)\frac{\partial \varepsilon^i}{\partial d^i_t} = 0$

This is easy to check. For any $d_{it} \geq \frac{\omega(1-\delta)}{R^d_t - 1 + \delta}$, then $\pi^i_t(\varepsilon^i_t) = 0$ by definition of ε^i_t. For $d^i_t < \frac{\omega(1-\delta)}{R^d_t - 1 + \delta}$, then $e^i = 0$ and $\frac{\partial \varepsilon^i}{\partial d^i_t} = 0$ due to the max operator.

We then have as first and second derivative:

$$\begin{aligned} \frac{\partial \pi^i_t}{\partial d_{it}} &= \int_{\varepsilon^i_t(d^i_t)}^{\infty} \left(R^K_{t+1}(\varepsilon) - R^D_t\right) dF(\varepsilon) \\ \frac{\partial^2 \pi^i_t}{\partial d^2_{i,t}} &= -\left[R^K_{t+1}(\varepsilon(d_{it})) - R^D_t\right] \frac{\partial \varepsilon^i_t}{\partial d_{it}} \end{aligned} \tag{48}$$

Given the monotonicity of $R^K_{t+1}(\varepsilon)$, then $\forall \tilde{d}$ such that $\left.\frac{\partial \pi^i_t}{\partial d_{it}}\right|_{\tilde{d}} = 0$, it follows that $R^K_{t+1}(\varepsilon^i_t(\tilde{d})) - R^D_t < 0$ or all elements in the integral are non-negative and it cannot be zero. Since $\frac{\partial \varepsilon^i_t}{\partial d_{it}} > 0$, then $\left.\frac{\partial^2 \pi^i_t}{\partial d^2_{it}}\right|_{\tilde{d}} > 0$ by equation (48). If $\tilde{d}$ exists, it must be a minimum and we therefore conclude that the maximum must be at the bounds: $d_{it} = \arg\max\left(\pi_t(0), \pi^i_t(\overline{d}^i_t)\right)$.

Appendix C. Data Description

Bank balance sheet data uses annual data from the Bankscope database. Bank return data are from Datastream. Market returns were calculated using the MSCI World Index data available from Bloomberg. The Effective Federal Funds Rate and the CPI are from the Federal Reserve Economic Data.

The leverage ratio is defined as the ratio of total assets to total equity, here defined as common equity. We drop negative equity from the dataset, and institutions with assets worth less than 1 million USD. We also remove institutions that have leverage larger than 1000 at least once across the sample.

For the leverage series, we compute both unweighted and weighted averages of the leverage ratio for each quarter. For the weighted average we use total assets as weights. We checked using total equity as weights and results are qualitatively unchanged. We also compute the 1st and 99th percentiles of both unweighted and weighted leverage.

For the skewness of leverage, we compute the cross-sectional standard deviation and third moment of the leverage ratio for every period. And then compute the cross-sectional sample skewness using a simple approach laid out below.

$$
\begin{aligned}
m_t(3) &= \frac{\sum_{i=1}^{N}\left(x_{it}-\bar{x}_t\right)^3}{N} \\
s_t &= \sqrt{\frac{\sum_{i=1}^{N}\left(x_{it}-\bar{x}_t\right)^2}{N}} \\
S_t &= \frac{m_t(3)}{(s_t)^3}
\end{aligned}
$$

where x_{it} is the leverage ratio of bank i in period t, $\bar{x}_t$ is the period-specific cross-sectional mean of leverage, S_t is the sample cross-sectional in period t, s_t is the period-specific sample cross-sectional variance and $m_t(3)$ the period-specific sample third central moment of the cross-section. We use this approach to the unweighted series or weighted by either total assets or total equity. We ran the same exercise using $s_t = \sqrt{\frac{\sum_{i=1}^{N}(x_{it}-\bar{x}_t)^2}{N-1}}$ for robustness and there was no qualitative difference.

Year	Assets		Equity		Leverage			#Obs
	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.	Skewness	
1993	12419	39962	737	1930	14.71	13.77	10.67	276
1994	13725	44263	825	2179	14.81	10.95	5.47	329
1995	14638	42323	898	2200	14.33	11.16	5.73	349
1996	14278	35918	912	2109	14.82	16.26	9.48	364
1997	16778	42793	1073	2469	14.57	16.96	10.44	369
1998	18661	47154	1290	3158	13.71	10.69	5.67	392
1999	21478	64999	1526	4632	14.30	13.26	7.95	437
2000	24176	77576	1768	5599	14.04	15.11	10.79	456
2001	27648	87629	2079	6843	14.00	17.40	11.10	446
2002	28693	91765	2224	7328	14.09	25.57	13.34	458
2003	32012	103752	2414	8030	14.52	28.92	11.23	454
2004	44022	149190	3160	10731	14.58	17.96	7.65	444
2005	46877	152941	3287	10794	16.28	27.52	7.77	468
2006	56112	193439	3697	12586	15.69	24.76	8.28	422
2007	71684	245968	4458	14634	15.23	19.00	5.78	396
2008	67652	258317	4359	16032	15.59	27.13	13.73	374
2009	63755	240442	4872	17701	14.54	17.88	6.21	401
2010	66402	232329	5191	17920	12.90	11.93	4.77	448
2011	64599	235938	5318	18441	12.96	20.23	13.19	461
2012	65011	238222	5570	19317	12.51	21.56	13.29	467
2013	69559	249110	6468	22352	10.69	7.47	3.18	469
2014	70429	245651	6734	23223	10.58	7.71	4.31	475
2015	67587	228692	6923	23808	10.43	7.31	3.49	439

Table 2: Descriptive cross-sectional statistics by period (unweighted).

Appendix D. Interbank market

In this appendix, we present a version of the baseline model where intermediaries can supply funds to each other through deposits. The main difference in the financial intermediary problem, is that inactive intermediaries will optimally choose to deposit their net worth, thus supplying funds to leveraged banks. These deposits are also guaranteed by the government and therefore the same asset as household deposits from the point of view of the borrowing bank.

Whenever $R_t^D > 1$, storage is dominated by deposits and will never be used. Inactive intermediaries will also optimally prefer to hold deposits over shares of the capital stock. Since intermediaries are risk-neutral, the presence of an option value of default implies that in equilibrium $E(R^K_{t+1}) > R^D_t$. Since inactive intermediaries will not be able to exploit the option value of default, they strictly prefer deposits over shares of the capital stock, implying $\alpha^N_t = \underline{\alpha}$ for all t. The balance sheet of an intermediary i that chooses to lend its net worth is then:

Assets	Liabilities
$-d_{it}$	ω^i_t

where to maintain consistency in notation, deposits held as assets are noted as negative d_{it}. The intermediary program is as before:

$$V_{it} = \max \ \mathbb{E}_t(c_{i,t+1}) \tag{49}$$

$$\text{s.t. } \Pr(\pi_{i,t+1} < \omega^i_t) \leq \alpha^i \tag{50}$$

$$k_{it} + s_{it} = \omega^i_t + d_{it} \tag{51}$$

$$c_{i,t+1} = \max(0, \pi_{i,t+1}) \tag{52}$$

$$\pi_{i,t+1} = R^K_{t+1} k_{it} + s_{it} - R^D_t d_{it}$$

Since borrowing to deposit is revenue neutral, it follows that Proposition 2.1 again holds in this case. Each intermediary will choose to leverage up to its VaR constraint or not raise deposits at all.

Writing the value functions under this case we have

$$V^L_{it} = \mathbb{E}^i_t[R^K_{t+1} k_{it} - R^D_t d_{it}] \tag{53}$$

$$V^N_{it} = R^D_t \omega \tag{54}$$

$$V^O_{it} = \omega \tag{55}$$

The deposit market clearing equation is as before:

$$D_t = \int d_{it} dG(\alpha^i) = D^H_t \tag{56}$$

With the diffence that now D_t is the *net* borrowing from the financial sector as a whole. The market clearing is $D_t = D^H_t$, where D^H_t are total household deposits. We also define $D^L_t = \int_{\alpha^L_t}^{\overline{\alpha}} d_{it} dG(\alpha^i)$ as the total deposit liabilities in levered intermediaries. Equation (38) then becomes:

$$F_t = \frac{D^L_t}{1+\lambda} \tag{57}$$

The rest of the equations of the model are exactly the same, but underlying them are a few key differences. All capital is now held by levered intermediaries, which implies that no fraction of the capital stock is ever free from potential distress at $t+1$. Moreover, the extensive margin now also affects the deposit supply. The more intermediaries drop out from levered markets, the larger is aggregate deposit supply (ceteris paribus). Partial equilibrium results are very similar to the ones without the interbank market, as can be seen in Figure (20). Note that for the aggregate capital stock supply curve, the two models are almost indistinguishable.

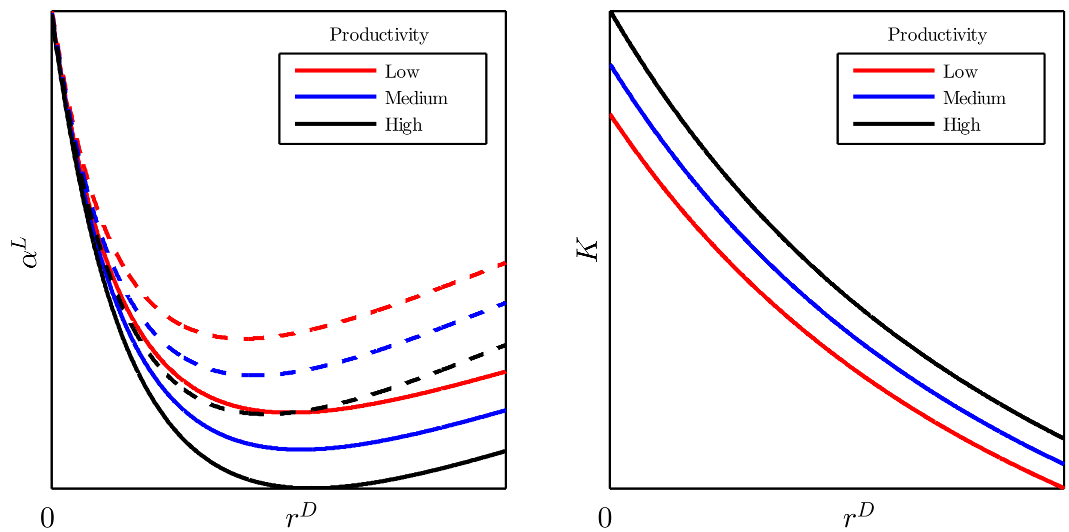

Figure 20: *Cut-off level α_t^L and aggregate capital stock as a function of deposit rates r_t^D in the model with an interbank market (full lines). For comparison, the baseline model is also plotted (dotted lines).*

The main difference in partial equilibrium seems to be that for a given interest rate, the cut-off is now lower. Since non-active intermediaries no longer invest, had leverage and the cut-off remained the same the capital stock would be smaller and returns higher. This leads to both higher leverage from intermediaries above the cut-off (intensive margin) and a lower cut-off (extensive margin). Given that the real effects in partial equilibrium are extremely small, the margin that seems to adjust the most is the extensive one.

In general equilibrium, we can also see that the main results are extremely similar to our baseline model. Other variables also behave very similar across the two models, as

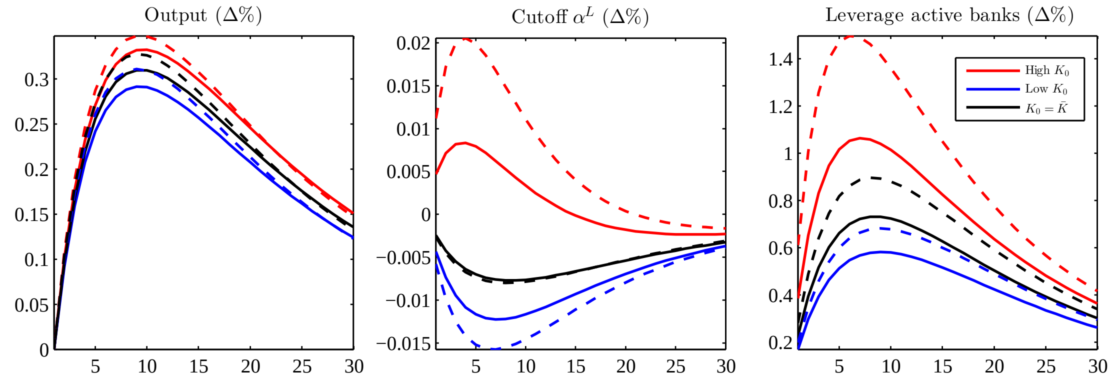

Figure 21: *Monetary policy shock of 100 basis points to* γ_t *in the model with an interbank market (full lines). For comparison, the baseline model is also plotted (dotted lines).*

can be seen in figures (17) and (18). The main difference seems to be in the behaviour of the cut-off where the baseline model seems to have additional amplification, particularly away from the steady-state.

Appendix E. Comparative statics

Here we explore the role of volatility and net worth in the financial block of the model. We perform two exercises. In the first one we change the parameter σ_z, governing the exogenous volatility of the TFP process. As we can see in figure (22), the main change is in the composition of the financial sector. When volatility is higher, the VaR is tighter and therefore the intensive margin is reduced. Leverage from active intermediaries is lower, which leads to both lower capital stock and cutoff α^L. As it turns out, the lower is volatility, the easier it is for more risk-taking intermediaries to capture more of the market due to the loosening of VaR constraints.

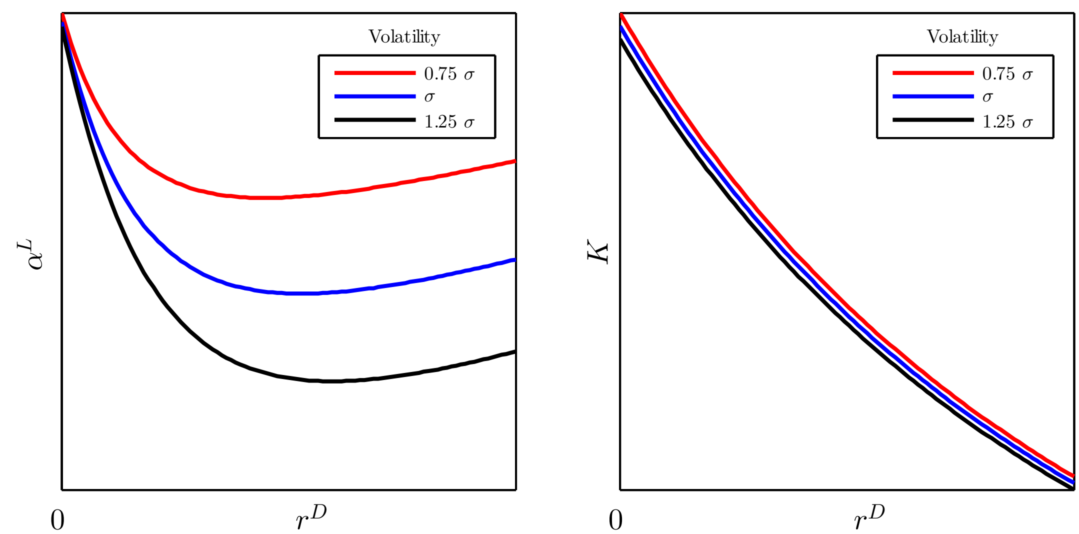

Figure 22: Comparative statics on volatility and interest rates for the financial sector block

We also look at the effect of changing the parameter ω, the endowment of net worth received by intermediaries. As can be seen in Figure (23), the effect is almost purely compositional with almost no effect on the total amount of capital (differences too marginal to show up in the graph). Given that the right hand side of equation (19) is independent of ω, then changing net worth is just allowing the more risk-taking intermediaries to acquire more assets (given aggregate variables). As with lower volatility, the higher ω is the easier it is for more risk-taking intermediaries to capture a larger share of the market.

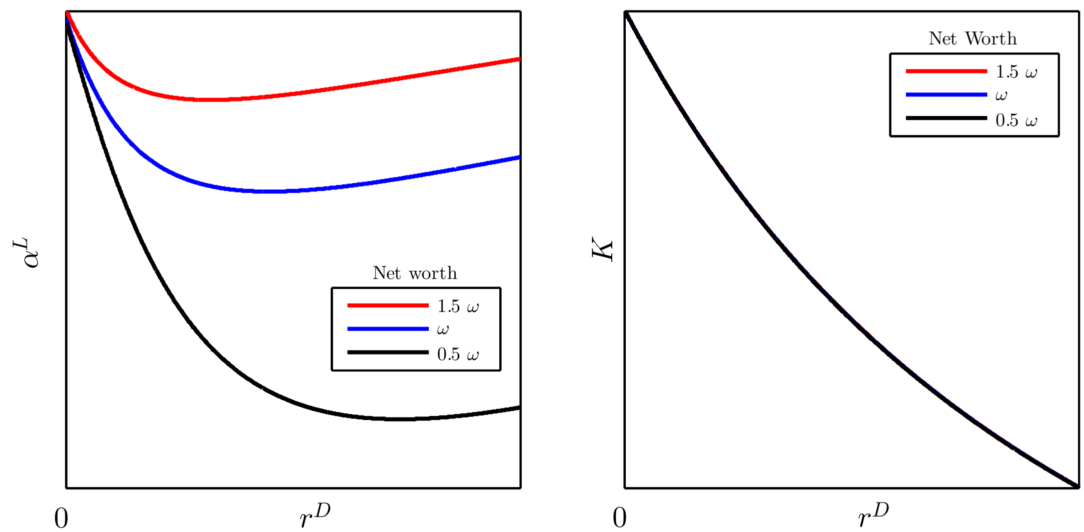

Figure 23: Comparative statics on net worth and interest rates for the financial sector block