Some Unpleasant Central Bank Balance Sheet Arithmetic *

Saroj Bhattarai†
University of Texas at Austin

Abstract

I introduce maturity and currency mismatches in the central bank balance sheet. When such a central bank's remittances to the Treasury is constrained, balance sheet arithmetic shows that it loses freedom in its policy actions. The expected future change of the short-term nominal interest rate or the nominal exchange rate get determined by balance sheet considerations: if they increase today, they have to decrease in future. I embed the balance sheet constrained central bank in a standard dynamic general equilibrium model and study monetary policy transmission mechanisms. Following a positive short-term nominal interest rate shock, central bank balance sheet considerations lead, dynamically, to a drop in the short-term interest rate and a positive correlation between it and inflation and a negative correlation between it and the real interest rate. This holds even though the model has sticky prices. Central bank balance sheet considerations make forward guidance less effective. Following news of a negative short-term nominal interest rate shock, while inflation and output increase initially, they do so by a diminished amount, and are in fact followed by deflation and contraction in economic activity in future. This is because the real interest rate remains persistently high even though the nominal interest rate is low.

Keywords: Central Bank Balance Sheet, Balance Sheet Constrained Central Bank, Central Bank Balance Sheet Mismatch, Central Bank Remittances to Treasury, Monetary Policy Transmission, Forward Guidance

JEL Classification: E52, E58, E61, E63, E31

---

*I am grateful to Oli Coibion, Mohammad Davoodalhoseini, Nobu Kiyotaki, Virgiliu Midrigan, Steve Williamson, and seminar and conference participants at the Canadian Macroeconomic Study Group Meetings, Keio University, Bank of Japan, Annual Dynare Conference, and University of Texas-Austin, for helpful comments and suggestions. I thank Choongryul Yang for superb research assistance. First version: December 2016; This version: Nov 2017.

†2225 Speedway, Stop C3100, Department of Economics, University of Texas at Austin, Austin, TX 78712, U.S.A. Email: saroj.bhattarai@austin.utexas.edu.

1 Introduction

The size and composition of many major central bank balance sheets have changed dramatically recently, primarily as a result of unconventional monetary policy that was undertaken to combat the financial crisis. In particular, maturity and currency mismatches between assets and liabilities have appeared in central bank balance sheets. Many advanced economy central banks now hold large amounts of long-term domestic currency assets or foreign currency assets while they have short-term domestic currency liabilities.[1] Such mismatches, combined with large gross asset and liability positions, open up the possibility of non-trivial fluctuations in the net interest income of the central bank. These fluctuations can arise due to interest rate changes that affect returns on assets and liabilities of different maturities differentially, or due to exchange rate changes that affect returns on assets and liabilities of different currency denominations differentially, or simply due to interest rate expense on repurchases agreements. These movements then in turn can lead to volatility in the central bank remittances to the government.

In this paper, I present simple closed and open economy models that feature a central bank with maturity and currency mismatch between assets and liabilities on its balance sheet. In particular, motivated by the discussion above and the stylized facts on various central bank balance sheets that I present in more detail below, in the model, the central bank holds long-term domestic or short-term foreign currency assets while it issues short-term domestic currency liabilities. Using the flow budget constraint of the central bank together with just equilibrium asset pricing conditions allows me to undertake a present value analysis in the spirit of Sargent and Wallace (1981). In particular, I show with such balance sheet arithmetic that the outstanding net asset position of the central bank is equal, in equilibrium, to the difference between the present value of its transfers to the Treasury and net interest income of the central bank.[2] One key result of the simple model is that the net interest income is given by essentially the expected change in the short-term nominal interest rate in the closed economy model and the expected change in the nominal exchange rate in the open economy model.

I then consider a *balance sheet constrained central bank*, one which faces a constraint in the path of its remittances to the Treasury. This is similar in spirit to the central bank under fiscal dominance in Sargent and Wallace (1981). For simplicity, consider a case where remittances to the Treasury are fixed. Then, the present value analysis implies that an increase (decrease) in interest income *today* must lead to a decrease (increase) in *future*.[3] Critically, since the interest income depends on the change in the short-term nominal interest rate or the nominal exchange rate, this means that *interest rate policy* or *exchange rate policy* get constrained in this case. Thus, the central bank cannot freely choose the path of its policy instruments in future as it is bound by balance sheet considerations. Precisely this constitutes *unpleasant central bank balance sheet arithmetic*.

In contrast, a *balance sheet unconstrained central bank* does not face a constraint in the path of its remittances to the Treasury. In particular, it can freely choose the path of its instrument, the short-term nominal interest rate or the nominal exchange rate, as remittances to the Treasury adjust endogenously to the state of its balance sheet. In such a case, the present value analysis implies that an increase (decrease) in transfers to the Treasury *today* must lead to a decrease (increase) in *future*.

I embed the maturity mismatched central bank balance sheet in a fully specified, otherwise standard,

[1] Major central banks now have interest bearing short-term liabilities, such as reserves.

[2] Net interest income is the one-period interest on the net asset position of the central bank.

[3] In Sargent and Wallace (1981), the income of the central bank consists of classical seigniorage revenues, while in my model, it consists of net interest income from the portfolio composition of the central bank balance sheet. In Sargent and Wallace (1981), under fiscal dominance, an increase (decrease) in seigniorage today must lead to a decrease (increase) in future.

non-linear closed economy equilibrium model with nominal rigidities. This allows me to assess the generality of the results from the simple model, such as precisely how central bank behavior gets constrained, as well as study the effects on endogenously determined inflation and output dynamics. The model features policy rules for the determination of the short-term nominal interest rate and for transfers to the Treasury. The two key analyses I consider are the effects of a transitory current innovation to the interest rate rule of the central bank and a transitory future innovation to the interest rate rule of the central bank. The first is a standard monetary policy shock while the second is a policy news or a forward guidance shock.

For the balance sheet unconstrained central bank, the dynamics following a current surprise monetary policy shock are standard: nominal interest rate increases, inflation falls, and the two are negatively correlated. Given the transitory shock and no state variables, the dynamics, including the negative effect on output, are completely transitory. In sharp contrast, the model with balance sheet constrained central bank leads to very different dynamics of the short-term interest rate and its correlation with inflation. On impact, the short-term nominal interest rate still rises. The net interest income falls on impact, as is to be expected given a direct rise in interest paid on central bank liabilities. After the first period and dynamically however, the short-term nominal interest rate falls. This is essential because of balance sheet considerations that require the net interest income to rise in future. Thus, after falling in the second period, the short-rate transitions back to steady-state, rising along the transition. Inflation throughout is lower and slowly transitions back to steady-state. This means that after the initial period of the shock, the short-term interest rate and inflation are positively correlated, even though the model has sticky prices.

One key intuition for the dynamic correlation between inflation and short-term interest rate is that when the central bank is balance sheet constrained, it has to forego following the Taylor principle. In its interest rate rule, while the short-rate responds positively to inflation, it does so by less than one-for-one. This positive response in the policy rule then leads to a positive correlation over time between inflation and the short-term interest rate. In other words, along the transition, while the nominal interest rate is below steady-state and increasing, the real interest rate is above steady-state and decreasing. These persistent dynamics arise, even though the shock is transitory, as central bank balance sheet variables are now endogenous state variables that affect the dynamics of interest rates and inflation. This also implies that when the central bank is balance sheet constrained, output declines persistently following a one-time contractionary monetary policy shock, with the real interest rate persistently higher. Thus monetary policy shock is more contractionary than when the central bank is not balance sheet unconstrained and leads to a stronger and more persistent effect on both inflation and output.

I next model effects of forward guidance policy by introducing a monetary policy news shock. In particular, in the current period, there is an anticipated negative shock to the interest rate rule in future. For the balance sheet unconstrained central bank, the dynamics following such a monetary policy news shock are standard: inflation and output increase on impact, and as a result given the feedback interest rate rule, the short-term nominal interest rate does as well. When the news shock is actually realized, the interest rate falls, and as the shock is transitory and the model completely forward looking, output, inflation, and the short-term rate all go back to steady-state the period after. All throughout the transition, inflation and output are both positively affected and do not go below steady-state.

In sharp contrast, forward guidance leads to a very different dynamics of inflation and output when the central bank is balance sheet constrained. Inflation and output are still positively affected on impact, but by a lower amount. More importantly, dynamically, both are affected negatively and there is in future both deflation and a contraction in economic activity. After the monetary policy news shock is realized, these variables do not go back to steady-state immediately next period but instead transition back slowly as central bank balance sheet variables are now state variables in the model. Along this transition after the shock is realized, nominal interest rate and inflation are both negative and, again, positively correlated. Moreover, the nominal interest rate and the real interest rate are negatively correlated.

What drives this lower stimulative effect of forward guidance initially as well as deflation and contraction in output in future? When the central bank is balance sheet constrained, the net interest income of the central bank is constrained in its path. In particular, consider the period when the negative news shock is realized. The short-term nominal interest rate cannot immediately go back to steady-state next period, but instead transitions slowly back so that the net interest income is positive for several future periods. Only this dynamics ensures a stationary equilibrium, as before the news shock is realized, net interest income is negative with the short-term interest rate falling over time. In this regime however, a negative short-term nominal interest rate leads to a negative effect on inflation as they are positively correlated as I described above. Moreover, a negative nominal interest rate is associated with a positive real interest rate along the transition. Nominal rigidities ensure that output falls after the news shock is realized as well. Finally, forward looking inflation dynamics in fact lead to inflation, and because of that also the nominal interest-rate, falling even many periods before the news shock is realized.

This paper is related to several strands of the literature. My analysis is clearly close to Sargent and Wallace (1981), whose focus was on the dynamics of the classical seigniorage revenues of the central bank and how the central bank might lose freedom in its path of inflation/seigniorage when its remittances to Treasury are constrained. I focus here on the dynamics of the net interest income of the central bank that arises from the portfolio composition of, and mismatch on, its balance sheet, a situation that has become relevant recently for central banks of even advanced countries. Their case of unpleasant monetarist arithmetic when a central bank is under fiscal dominance motivates my analysis of unpleasant central bank balance sheet arithmetic when a central bank is balance sheet constrained.[4]

On emphasizing the connections between fiscal and monetary policy, and in highlighting that inflation determination depends on the full general equilibrium of the model and requires a complete specification of policy rules, my paper also shares the theme of the fiscal theory of the price level, for example, as developed in Sims (1994) and Woodford (1994). I use policy rules in the general equilibrium model that are motivated in their formulation by this literature. One difference regarding mechanisms is that the revaluation of government liabilities through inflation does not play a role in my model. Rather, a key role is played by adjustment in the short-term nominal interest rate, the policy instrument, in future.[5]

My paper is also clearly related to recent work that has assessed how central bank balance sheet dynamics might affect monetary policy. Prior to the crisis and unconventional policy by central banks, Sims (2004), Sims (2005), Zhu (2004), and Berriel and Bhattarai (2009) analyze various implications of imposing a central bank budget constraint on the conduct of monetary policy. Recently, Hall and Reis (2013), Reis (2013), Bassetto and Messer (2013), and Del Negro and Sims (2015) have provided several important insights related to consequences of central banks issuing interest bearing liabilities, the reliability of stable money demand at high inflation, and whether the central bank might lose control of inflation because of solvency concerns.[6] My

[4] I do set up a general model with cash, but eventually abstract from seigniorage considerations for sharp focus. Bhattarai, Lee, and Park (2014) sets up the fiscal dominance regime in an equilibrium model with nominal rigidities and cash.

[5] Canzoneri, Cumby, and Diba (2011) is a recent survey of both the unpleasant monetarist arithmetic and the fiscal theory of the price level literature. See also Sims (2013).

[6] See also Ennis and Wolman (2010) for a discussion of possible implications of a large amount of interest bearing excess reserves. Carpenter et al (2013) empirically assess how income of the Federal Reserve and its transfers to the Treasury might get affected in future, depending on increases in short-term interest rates.

relative focus is on explaining precisely how central bank behavior might get affected, in terms of dynamics of net interest income that constrains interest rate and exchange rate policy, under maturity and currency mismatches in its balance sheet. Additionally, I use a complete non-linear dynamic equilibrium sticky price model to assess the differences in the real effects of a monetary policy shock, and the comovement of inflation and (nominal and real) interest rates following such a shock, when a central bank is balance sheet constrained. The distinction between, and co-movement (or lack thereof) of nominal and real interest rates for instance is a key aspect of my analysis, which is possible because of nominal rigidities. Finally, I am able to study the effects of forward guidance, in a model where inflation dynamics affect economic activity.[7]

I focus on issues of monetary policy transmission when central bank policy is constrained by balance sheet considerations, while taking the mismatches in the balance sheet as given. It is however related to recent normative work that breaks the neutrality of open market operations of Wallace (1981) when the central bank cannot commit and cares about its balance sheet. In particular, in a specific zero lower bound situation, the constraints that I highlight in terms of interest rate and exchange rate policy could in fact be beneficial when the central bank cannot credibly commit to future interest rate policy. Bhattarai, Eggertsson, and Gafarov (2016) show this in a closed economy model with optimal monetary policy when the central bank balance sheet has a maturity mismatch while Jeanne and Svensson (2007) show this in an open economy model when the central bank balance sheet has a currency mismatch.[8] My results here show that generally, such balance sheet constraints can lead to a non standard transmission of monetary policy shock, both a surprise shock and a news shock to interest rates.[9] Moreover, in such a case, innovations to the balance sheet variables transmit to the rest of the macroeconomy by inducing responses from monetary policy.

My result on the transmission of a current shock to the interest rate rule when the central bank is balance sheet constrained is related to some recent results in the fiscal theory of the price level and the Neo Fisherism literature. Under the fiscal theory of the price level, in a sticky price model with one-period government debt, Bhattarai, Lee, and Park (2014) show analytically that a positive inflation target shock leads to a decrease in inflation while in a sticky price model with long-term government debt, Sims (2011) shows that a positive interest rate shock increases inflation after a delay. These results arise because of the wealth effect arising from government debt holdings that is inherent in the transmission mechanism of monetary policy in this theory. Bhattarai, Lee, and Park (2014) moreover emphasize that in this case, the inflation target and inflation are negatively correlated, in sharp contrast to the standard case where they are positively correlated.[10] Additionally, under Neo Fisherism, for instance as described in Williamson (2016), raising interest rates raises inflation. In my model when the central bank is balance sheet constrained, in contrast to the traditional text book case, nominal interest rates and inflation are indeed dynamically positively correlated while nominal and real interest rates are negatively correlated. At the same time however, a positive interest rate shock on impact, does lead to an increase in the short-term nominal and real interest rate, and more importantly, a negative response of inflation and output throughout.

Finally, my paper is related to the recent literature on the effectiveness of forward guidance in the New Keynesian model. Del Negro, Giannoni, and Patterson (2015) have established that news about future expansionary monetary policy has quite large, and unrealistic, effects in a standard New Keynesian model.

[7]Although not the focus of this paper, such a model also allows me to consider a situation where the zero lower bound on the short-term nominal interest rate might bind and lead to deflation.

[8]See also Benigno and Nistico (2015) and Berriel and Mendes (2015).

[9]I discuss later some key differences that arise from taking a feedback rules based approach, as in this paper, compared to an optimal policy approach, as in Bhattarai, Eggertsson, and Gafarov (2016), as these two imply a different equilibrium/solution concept.

[10]Bhattarai, Lee, and Park (2016) show this result in an estimated model as well.

They have dubbed this the forward guidance puzzle and various resolutions to this puzzle have been proposed. In particular, the literature has proposed reducing the forward looking behavior of either households or firms in the model. For instance, Del Negro, Giannoni, and Patterson (2015) use a finite life-time model; Carlstrom, Fuerst, and Paustian (2015) and Kiley (2016) use a sticky information friction instead of the usual sticky price friction; and Mckay, Nakamura, and Steinsson (2016) use a model with borrowing constraints. Forward guidance is also less effective in my model when the central bank is balance sheet constrained, but households and firms are completely forward looking in an infinite horizon environment, as in the standard New Keynesian model. In fact, I show that the entire dynamics of inflation and output when the central bank is balance sheet constrained looks very similar to those in models that reduce forward looking behavior of consumers due to incomplete risk-sharing. Central bank balance sheet considerations can thus be another channel through which the effectiveness of forward guidance gets reduced.

2 Central Bank Balance Sheet Data

Before presenting the model, for more context and motivation, in Figures 1-4, I present descriptive data on evolution of assets and liabilities as well as net income and transfers to the government of a few major central banks. These data highlight the rapid expansion of central bank balance sheets, compared to the pre-crisis levels, together with changes in composition that have led to maturity and/or currency mismatches in the balance sheets. In another departure from the pre-crisis era, these central banks now also pay interest on liabilities/reserves. As is obvious from Figures 1-4, all these central bank balance sheets have expanded substantially, with several increasing three to four times their pre-2008 levels.

Panel (a) of Figure 1 shows that for the U.S., the maturity of assets purchased has increased substantially, which was funded mostly by issuing interest bearing short-term reserves. The maturity mismatch on the Federal Reserve balance sheet is quite substantial. Panel (b) of Figure 1 shows that until now, the Federal Reserve has been making profits and remitting large amounts to the Treasury, in spite of interest expense related to payment on reserves as well as those due to repurchases agreements. As short-term interest rates rise, and interest expense on liabilities increase, this situation can change in future.

In fact, some other prominent central banks have already made losses and transfers to the government have already gone close to or even exactly to zero at times. Panel (a) of Figure 2 shows that for Japan, assets are predominantly domestic government bonds or foreign currency denominated assets. Moreover, the government bonds held by Bank of Japan are primarily long-term, with average maturity of around 7 years. These asset holdings are financed mostly by reserves and some by repurchase agreements. Thus, the Bank of Japan's balance sheet features both maturity and currency mismatch. Panel (b) of Figure 2 shows that there have been major fluctuations in income of Bank of Japan, and in fact, there have been years with losses due to exchange rate fluctuations. Similarly, panel (a) of Figure 3 shows that for Switzerland, the major component of assets are foreign currency investments, which have been funded mostly by reserves. These reserves are domestic currency denominated. This massive currency mismatch has led to rapid fluctuations in the income of Swiss National Bank, with net losses made in several years recently, as shown in panel (b) of Figure 3. For both the Bank of Japan and the Swiss National Bank, transfers to the government have in turn varied, moving with net income, and in some years, declined close to or exactly to zero.

Finally, Figure 4 presents data on the European Central Bank, where I present the data for ECB and the Eurosystem (ECB and all national central banks consolidated) separately, in panels (a) and (c) respectively. Panel (c) shows that for the Eurosystem as a whole, assets are mostly securities purchased, those accruing from different kinds of lending/refinancing programs, and claims denominated on foreign currency. The securities purchased as a result of recent unconventional monetary policy actions are primarily long-term. For instance, those purchased under the public sector purchase program and securities market program have remaining maturity of around 8 and 4 years respectively. These asset holdings are financed, recently, by non-currency liabilities denominated in Euro, in particular reserves. Thus, the Eurosystem balance sheet features maturity, and to some extent also currency, mismatch.[11] As shown in panel (b) of Figure 4, there have also been non-trivial fluctuations in the net income of the ECB, with in fact losses accruing in several years even in the pre-crisis period. In such years, the major sources of losses have been either write-downs or valuation changes arising from exchange rate or interest rate changes. Following the fluctuations in net income, transfers to the national central banks have in turn varied, with several years of zero transfers.[12]

3 Simple Model

In this section, I keep the model deliberately simple with the goal of focusing simply on the central bank balance sheet. I consider two important, and recently empirically relevant, mismatches between assets and liabilities of the central bank: maturity mismatch and currency mismatch. I model maturity mismatch in a closed economy and currency mismatch in an open economy separately to clarify the main results. The dynamics of the central bank balance sheet, together with some standard asset pricing conditions only, allows me to derive several insights on when and how monetary policy actions might get constrained by balance sheet considerations. Moreover, the analysis here is also deliberately in the spirit of Sargent and Wallace (1981)'s analysis of the implications of the present-value government budget constraint.

3.1 Maturity mismatch in a closed economy

I first consider a general, closed economy central bank balance sheet with nominal assets and liabilities. The central bank issues non-interest bearing and interest bearing short-term liabilities while it holds long-term assets. Without loss of generality, the short-term liabilities are one-period bonds while the long-term assets are two-period bonds. The model is thus one of maturity mismatch.

3.1.1 Unpleasant central bank balance sheet arithmetic

The nominal flow budget constraint of the central bank is given by

$$Q_{1t}B_{t-1} - L_{t-1} - M_{t-1} - P_tT_t = Q_{2t}B_t - Q_{1t}L_t - M_t$$

where M_t is non-interest bearing liabilities (e.g. cash), L_t is one-period liabilities (e.g. interest-bearing reserves) with Q_{1t} its price, and B_t is two-period assets with Q_{2t} its price. Moreover, P_t is the nominal price

[11] I also present the data on the ECB balance sheet separately for completeness. Several variables have similar dynamics in panel (a) and (c), but there are some differences. For instance, intra-Euro System claims and liabilities appear in panel (a), but not panel (c). Additionally, for currency in circulation, ECB has 8% of the Euro Area currency in circulation on its balance sheet, the income from which is distributed to the national central banks.

[12] Concerns about the central bank making large losses and remitting nothing to the Treasury have previously been discussed mostly for emerging markets and developing countries. See for instance, Sims (2004), Stella (2005), and Berriel and Bhattarai (2009) for some empirical facts and pre-crisis policy discussions of how central banks are worried about the state of their balance sheet for political economy/independence reasons. Moreover, see also the more recent contributions in Milton and Sinclair (2015).

level and T_t is the central bank transfer to the Treasury.[13] The flow budget constraint can be written in real terms as

$$Q_{1t}b_{t-1}\Pi_t^{-1} - l_{t-1}\Pi_t^{-1} - m_{t-1}\Pi_t^{-1} - T_t = Q_{2t}b_t - Q_{1t}l_t - m_t \tag{1}$$

where $b_t = \frac{B_t}{P_t}$, $l_t = \frac{L_t}{P_t}$, $m_t = \frac{M_t}{P_t}$, and $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation. Finally, under complete markets, the two bond prices are determined by standard asset-pricing conditions

$$Q_{1t} = E_t\left[\zeta_{t,t+1}\Pi_{t+1}^{-1}\right],\ Q_{2t} = E_t\left[\zeta_{t,t+1}Q_{1t+1}\Pi_{t+1}^{-1}\right] \tag{2}$$

where E_t is the mathematical conditional expectation operator and $\zeta_{t,t+1}$ is the unique stochastic discount factor.[14]

To get insights on how the maturity mismatch between assets and liabilities can constrain the central bank, it is useful to approximate the central bank budget constraint around a non-stochastic steady-state. The analysis in this case will be very similar to the one in Sargent and Wallace (1981). Log-linearizing (1) gives

$$b_{N,t} = \frac{1}{\beta}b_{N,t-1} + \frac{1}{\beta}s_t - \frac{1}{\beta}\hat{T}_t + b\left[\hat{Q}_{1t} - \beta E_t\hat{Q}_{1t+1}\right] \tag{3}$$

where $b_{N,t} \equiv \beta\hat{b}_t - \hat{l}_t$ is the (real) net asset position of the central bank in terms of difference in the stock of interest bearing assets and liabilities, $s_t \equiv m[\hat{m}_t + \pi_t - \hat{m}_{t-1}]$ is revenue from issuing non-interest bearing liabilities or seigniorage, and β is the discount factor.[15]

There are several aspects of (3) that require some discussion. First, while deriving it, (2) log-linearized has been imposed, which is a no-arbitrage condition that illustrates the expectation hypothesis

$$\hat{Q}_{2t} = \hat{Q}_{1t} + E_t\hat{Q}_{1t+1} \tag{4}$$

and links the two-period bond price with the current and expected future one-period bond prices. Second, other than through the seigniorage term, inflation does not appear directly as both assets and liabilities are nominal.[16] Third, in this approximated budget constraint, only the net asset position, and not the separate gross asset and liability positions, appears.[17]

Fourth, and perhaps more important, the dynamics of net asset of the central bank get affected by the net interest income on its position, $\hat{Q}_{1t} - \beta E_t\hat{Q}_{1t+1}$. Why is this the flow net interest income of the central bank?

[13]Through out the paper, I write flow budget constraints in terms of market values and define flow transfers to the Treasury accordingly. The transfers here can be interpreted as net of some operations costs of the central bank. For discussion of use of market prices compared to hold to maturity, see Bassetto and Messer (2013). Hall and Reis (2013) and Del Negro and Sims (2015) discuss issues related to the accounting standards that are used in determining central bank transfers to the Treasury. A detailed modeling of these conventions is not the focus of my paper. In terms of timing convention for stock variables, I use X_t to represent the value of X in the beginning of period $t+1$.

[14]$\zeta_{t,t+1}$ in a completely specified general equilibrium model, such as the one presented in the next section, will be given by $\beta\frac{U_{c,t+1}}{U_{c,t}}$ where β is the discount factor and U_c is the marginal utility of the representative household.

[15]In terms of approximation, I log-linearize all variables, except for asset and liability positions and transfers to the Treasury, which are expressed in terms of deviations from steady-state. The steady-state is a zero inflation steady-state where the net position is zero. In terms of notation, an approximated variable is represented by a circumflex above the variable while the variable in steady-state is represented without a time subscript.

[16]In addition, for this result, the steady-state values of assets and liabilities have to be equal, as is the assumption for simplicity here.

[17]This is a feature of log-linearization and in the general non-linear model presented in the next section, gross positions will need to be determined separately. Moreover, this is not a feature specific to my model and is a general property of any model with different assets. For instance, it is feature of international finance models that have portfolio choice, such as Berriel and Bhattarai (2013).

It is useful to go back to the original budget constraint (1). On its one-period liability, the central bank pays the one-period interest rate $\frac{1}{Q_{1t}}$. On its two-period asset, the effective one-period interest rate earned is $\frac{Q_{1t}}{Q_{2t}}$. With a first-order approximation, the difference between the two, after accounting for discounting in the definition of the net asset position, is given by $\hat{Q}_{1t} - \beta E_t \hat{Q}_{1t+1}$.[18] In particular, note that this is not equal to the interest rate spread, the difference between the interest rate on the two-period and one-period bonds. Finally, the effect of the net interest income on the dynamics of net asset position depends on the steady-state gross position of the central bank balance sheet. In this sense, a large size of the balance sheet can magnify the effects of fluctuations in interest rates.

To get implications similar to Sargent and Wallace (1981) while focusing on the maturity mismatch, I will now simplify by assuming that seigniorage revenues are negligible. This will help make the new implications of this paper distinct.[19] Then, (3) is given by

$$b_{N,t} = \frac{1}{\beta} b_{N,t-1} - \frac{1}{\beta} \hat{T}_t + b R_{N,t} \tag{5}$$

where $R_{N,t} \equiv \hat{Q}_{1t} - \beta E_t \hat{Q}_{1t+1}$ is the one-period net interest income of the central bank discussed above. The flow budget constraint can then be expressed in a present value form after imposing a terminal condition on net asset of the central bank as

$$b_{N,t-1} = \sum_{t=0}^{\infty} \beta^t \hat{T}_t - \beta \sum_{t=0}^{\infty} \beta^t b R_{N,t} \tag{6}$$

where I will take the initial net position $b_{N,t-1}$ as given.[20] Now, using (6), consider how any outstanding/initial net position of the central bank needs to be financed. In particular, if $b_{N,t-1} < 0$, then adjustment must come, in a present value sense, either from lower central bank transfers to the Treasury $\left(\hat{T}_t\right)$ or from higher net interest income of the central bank $(R_{N,t})$.[21]

Even more importantly, consider a case similar to Fiscal dominance in Sargent and Wallace (1981) where now the present value of central bank transfers to the Treasury is fixed (or exogenous generally). This describes the *closed economy balance sheet constrained central bank*. Then it is clear that an increase (decrease) in interest income *today* must lead to a decrease (increase) in *future*. In particular, since the interest income depends on the one-period interest rate, the policy instrument of the central bank, this means that the central bank's *interest rate policy* gets constrained in this case. Thus for example, it cannot freely choose the path of its policy instrument in future. This constitutes *unpleasant central bank balance sheet arithmetic* in a closed economy with a maturity mismatch in the central bank balance sheet.[22]

Now consider again the case of $b_{N,t-1} < 0$ for the closed economy balance sheet constrained central bank. In this case, as adjustment does not come from central bank transfers to the Treasury (for intuition, we can think of there being no response of transfers at all), it must be the case that adjustment comes from higher net interest income of the central bank. Additionally, this unpleasant balance sheet arithmetic implies that if transfers to the Treasury increase (decrease) today with no future off-setting decreases (increases), then net interest income must increase (decrease) in future, in order to satisfy (6). Again, in both these cases, the policy instrument of the central bank must adjust and respond appropriately to support the required

[18]Alternatively for intuition, note that under risk-neutrality, as $Q_{2t} = Q_{1t} E_t [Q_{1t+1}]$, the difference between the two would be $\frac{1}{E_t[Q_{1t+1}]}$-$\frac{1}{Q_{1t}}$.

[19]See Hall and Reis (2013), Reis (2013), and Del Negro and Sims (2015) for insights on the role played by seigniorage.

[20]One can focus here on a perfect foresight environment without loss of generality.

[21]This is similar to the Sargent and Wallace (1981) analysis on how outstanding debt of the government needs to be financed.

[22]This is an analogue to the unpleasant monetarist arithmetic in Sargent and Wallace (1981) where with the present value of tax revenues fixed, it implied that if seigniorage increased today, it must decrease in future.

adjustment in net interest income. How exactly does the policy instrument respond? In both cases, the nominal short-term interest rate must fall, thereby ensuring a direct fall in interest paid on (short-term) liabilities, which in turn in equilibrium leads to an increase in the net interest income.[23]

If the present value of central bank transfers to the Treasury is not fixed, and in particular, if it adjusts to the state of the balance sheet to satisfy (6), then that describes the *closed economy balance sheet unconstrained central bank*. For such a central bank, as the path of net interest income, and thereby its policy instrument, is not constrained, it can freely pursue its objectives regarding the dynamics of the one-period interest rate. On the flip side, this then implies that if transfers to the Treasury increase (decrease) today, they must decrease (increase) in future, in order to satisfy (6).

3.1.2 Policy rules, transition dynamics, and responses to shocks

To get further insights and to make the mechanisms described above even more concrete, I now consider explicit monetary and balance sheet/transfer policy rules that can be used to model the two types of central banks. A complete equilibrium analysis of the transition dynamics associated with $b_{N,t-1} < 0$ can now be undertaken in a straightforward way. Moreover, I introduce exogenous shocks to the interest rate and transfers to the Treasury in order to trace out the transmission of policy changes to model variables in a standard impulse response function analysis. This framework also allows a clear illustration of the basic logic discussed above regarding which term in (6) adjusts and which variable is set freely by the central bank.

Policy rules for the two central banks For the *closed economy balance sheet unconstrained central bank*, as it is free to set the path of the short-term interest rate ($i_t \equiv -\hat{Q}_{1t}$) and transfers to the Treasury can respond appropriately to the state of its balance sheet, the rules are given by

$$\hat{Q}_{1t} = \varepsilon_t, \; \hat{T}_t = \psi_T b_{N,t-1} + \xi_t \tag{7}$$

where $\psi_T > 1 - \beta$, which ensures that the central bank does not have to use its policy instrument to ensure that it satisfies (6). That is, in this case, transfers to the Treasury will co-move sufficiently positively with the net asset of the central bank. Additionally, ε_t in this simple model is simply an *iid* shock and just illustrates that the central bank is free to set the path of the short-term interest rate while ξ_t is an *iid* shock in the transfer rule.[24]

For the *closed economy balance sheet constrained central bank*, the path of its transfers to the Treasury is constrained (or fixed/exogenous) and as a result the net interest income will have to respond to the state of its balance sheet. This implies that it cannot freely set the path of its policy instrument. The rules are then given by

$$R_{N,t} = \phi_R b_{N,t-1} + \varepsilon_t, \, \hat{T}_t = \xi_t \tag{8}$$

[23] I illustrate below using an equilibrium analysis the full dynamic path through which the policy instrument responds to these two cases. There will be dynamics because of the state variable, the net asset position of the central bank, that matters for the evolution of the nominal short interest rate.

[24] Because the model here is very simple, I consider a somewhat unconventional interest rate rule to clarify the main point. In the fully specified equilibrium model in the next section, I will consider a standard interest rate feedback rule that responds to inflation. As I noted above, here, inflation does not appear in the approximated central bank budget constraint and specifying a rule like this is useful both to keep the model very parsimonious, as well as, in clarifying the central mechanism of the paper. Moreover, for now I use simple feedback rules for transfers to Treasury with two exogenous central bank models, as opposed to some other alternatives, such as lower bound on transfers and endogenous switch of the two types of central banks, as that leads to a very straight forward analysis and makes clear the mechanisms. I do consider later in this section a model with bounds on transfers to Treasury and show that the main insights remain the same. See Hall and Reis (2013) and Del Negro and Sims (2015) in addition for using bounds on transfers to the Treasury.

where $\phi_R < b\left(1-\beta^{-1}\right)$, which ensures that (6) is satisfied in the face of exogenous transfers to the Treasury. That is, in this case, net interest income of the central bank will co-move sufficiently negatively with the net asset of the central bank.[25] Here, ξ_t is an *iid* shock in the transfer rule, which just illustrates that the path of transfers is exogenous.

Transition dynamics and responses to shocks How do variables respond to the two shocks, to the interest rate/interest income and transfers to the Treasury? How does the transition dynamics associated with $b_{N,t-1} < 0$ look like?

Closed economy balance sheet unconstrained central bank First, consider the *closed economy balance sheet unconstrained* central bank. The model here simply consists of (5) and (7). In this case, the solution of the model is similar to conventional text-book models, at least for interest rate shock and resulting dynamics. When there is an *iid* shock to the interest rate rule, say a shock that increases the short-term interest rate, then the short-term will increase on impact and then go back to steady-state next period. The central bank thus has complete control of its policy instrument, as in conventional models. Because of a temporary, one-period increase in the short-term interest rate, the net interest income of the central bank decreases by the same amount on impact. This means that the net asset position of the central bank and after a period, transfers to the Treasury, also decrease. The balance sheet variables (and not interest rate and central bank net interest income) have endogenous dynamics as the variables evolve over time, as given clearly in (5). The result for the interest rate shock for the *closed economy balance sheet unconstrained* central bank is illustrated in panel (a) of Figure 5.[26]

Now I can analyze transition dynamics associated with $b_{N,t-1} < 0$. Here, all the adjustment comes from transfers to the Treasury, which must decline. Given that we are looking at stationary dynamics, the net asset position slowly reverts back to steady-state, as does transfers to the Treasury. The short-term nominal interest rate or net interest income of the central bank is not affected at all. The results for the transfer shock for the *closed economy balance sheet unconstrained* central bank is illustrated in panel (b) of Figure 5.

Finally, consider an *iid* shock to the transfer rule, say a shock that increases transfers to the Treasury. Then the transfers will increase on impact. Following the present-value logic described above however, in order to satisfy (6), transfers will decrease in future. The net asset position of the central bank will decrease on impact, as transfers have increased, and then slowly transition back to steady-state. This shock does not affect interest rates or the net interest income of the central bank. Thus, other than in terms of implications for balance sheet variables, there is no effect of this shock on other variables in the model. The results for the transfer shock for the *closed economy balance sheet unconstrained* central bank is illustrated in panel (c) of Figure 5.

[25] Note again that the net interest income is given by $R_{N,t} \equiv \hat{Q}_{1t} - \beta E_t \hat{Q}_{1t+1}$, which is a quasi first-difference of the short-term interest rate. Here, it is instructive to directly specify a rule for net interest income as that is the main object of analysis. Bhattarai, Lee, and Park (2014), to illustrate the original Sargent and Wallace (1981) mechanism, use directly a rule for seigniorage for a central bank under fiscal dominance. In the unconstrained central bank case as well, I can use a rule for net interest income without any loss of generality for the iid shock case. Because in that case, since as we will see later, interest rates do not depend on central bank balance sheet variables, the response of the interest rate is completely transitory, thereby implying a one-to-one negative correspondence between interest income and the short-term interest rate. Finally, the analytical boundaries for the two types of central bank regimes are quite straightforward to derive, based on ruling out explosive dynamics of net assets of the central bank. The policy rules (where shocks can be ignored) can simply be plugged into (5) and then the derivation requires that the coefficient on lagged net asset position is less than 1. The signs of the policy rule coefficients are particularly intuitive as they appropriately ensure that (6) is satisfied.

[26] There are very few parameters in the model. I use for illustration $\beta = 0.99$, $b = 1$, and $\psi_T = 0.02$.

Closed economy balance sheet constrained central bank In sharp contrast, next, consider the *closed economy balance sheet constrained* central bank. The model here simply consists of (5) and (8). In this case, the solution of the model is quite different from the case above that was more in line with conventional text-book models. When there is an *iid* shock to the net interest income, say a shock that decreases it, then on impact it will decrease. But, by the logic presented above, in this case, the net interest income in future must adjust as transfers do not. In particular, net interest income must increase in future in order to satisfy (6). Thus next period, it increases and then slowly transitions back to steady-state. What is the resulting equilibrium dynamics of the short-term interest rate? The short-term interest rate does not move initially, by construction, and then it falls, before rising back to steady-state, as the net interest income is essentially the first-difference of the short-term interest rate. The result for the net interest income shock for the *closed economy balance sheet constrained* central bank is illustrated in panel (a) of Figure 6.[27] The dynamics of policy rate in this case are thus clearly determined by central bank balance sheet considerations and are in sharp contrast to those in panel (a) of Figure 5.

Next, I can analyze transition dynamics associated with $b_{N,t-1} < 0$. Here, again, all the adjustment comes from net interest income of the central bank and none from central bank transfers to the Treasury. Thus, net interest income must increase to satisfy (6). Given that we are looking at stationary dynamics, the net asset position slowly reverts back to steady-state, as does net interest income of the central bank. The short-term nominal interest rate is now clearly affected as it has to be consistent with the dynamics of the net interest income. Specifically, the short-term nominal interest rate must fall initially, so that there is a direct reduction in the interest paid on the (short-term) liabilities, and then it reverts back to steady-state, increasing along the transition (as is consistent with the net interest income, which is essentially the slope of this transition path, being positive). The results for the transition dynamics for the *closed economy balance sheet unconstrained* central bank is illustrated in panel (b) of Figure 6 and they are in sharp contrast to those in panel (b) of Figure 5.

Finally, consider an *iid* shock to the transfer rule, say a shock that increases transfers to the Treasury. Then the transfers will increase on impact. Since this is a one-time shock, transfers go back to steady-state next period. In contrast to the *closed economy balance sheet unconstrained* central bank, now interest rate dynamics get affected as the net interest income of the central bank must respond in equilibrium. In particular, here, the net interest income must increase in a present value sense. Initially however, because of the rule in (8), as initial outstanding net asset position has not changed (making this different in this respect from the transition dynamics experiment above), net interest income does not change either. In future however, net asset decreases, and consistently, net interest income will increase. The equilibrium short-term interest rate path implies that first it decreases, again, to ensure a direct fall in the interest paid on (short-term) liabilities. It also decreases further next period, with an overshooting, before rising and transitioning back to steady-state.[28] Why does the short-term interest rate decrease further for one period? The key reason is that the net interest income in the first-period does not change. For this to be consistent with interest rate dynamics, it must be that the next period interest rate is slightly more negative as $\beta < 1$ (note that $R_{N,t} \equiv \hat{Q}_{1t} - \beta E_t \hat{Q}_{1t+1}$). The results for the transfer shock for the *closed economy balance sheet constrained* central bank is illustrated in panel (c) of Figure 6. These results are in sharp contrast to those in panel (c) of Figure 5, where the shock to the transfer rule for instance did not affect interest rate or the net interest income of the central bank at all.

[27] There are very few parameters in the model. I use for illustration $\beta = 0.99$, $b = 1$, and $\phi_R = -0.02$.

[28] Note that the short-term interest rate cannot rise on impact. There will be no stationary dynamics in that case.

3.1.3 Alternate model of central bank balance sheet constraints

I now consider a slightly different, but related, model of central bank balance sheet constraints. Instead of modeling the two cases where the central bank is constrained or not separately in an exogenous way, I now present results where this regime change occurs endogenously. Moreover, instead of analyzing an exogenous central bank transfer rule, I consider a lower bound on central bank transfers. Thus, there are now two regimes in the model, where the switch happens endogenously.

In particular, let the lower bound on central bank transfers be given by

$$\hat{T}_t \geq -\chi \tag{9}$$

where $\chi > 0$. Then, if the central bank transfer lower bound is not reached, the policy rules are given by

$$\hat{Q}_{1t} = \varepsilon_t, \ \hat{T}_t = \psi_T b_{N,t-1} + \xi_t$$

where $\psi_T > 1 - \beta$. This is the case of a closed economy balance sheet unconstrained central bank. If the central bank transfer lower bound is reached, then the policy rules are given by

$$R_{N,t} = \phi_R b_{N,t-1} + \varepsilon_t \, \hat{T}_t = -\chi$$

where $\phi_R < b\left(1 - \beta^{-1}\right)$. This is now the case of a closed economy balance sheet constrained central bank. Again, given a shock, whether and how long the bound on central bank transfers binds, and thus for how long the central bank is balance sheet constrained, is determined endogenously in equilibrium.[29] The central bank budget constraint is still given by (5).

Figure 7 shows the results for the net interest income shock in this variant of the model. For ease of comparison, two cases are shown at the same time in the Figure. First, there is a balance sheet unconstrained central bank case, where there is no bound on the central bank transfers. This is essentially the same as the results in panel (a) of Figure 5. Next, a lower bound on the central bank transfers is introduced ($\chi = 0.055$), so that it binds after the first period. Moreover, this lower bound is binding for 10 periods. In this case, it is clear that the model dynamics are different from the unconstrained case. In fact, they are exactly the same as in panel (a) of Figure 6. This makes it clear that this alternate model, where transfers from the central bank do not permanently follow an exogenous process, but instead could be constrained by a lower bound for a finite number of periods, has the same effects on endogenous model variables. The key component again is that the behavior of interest rates and net interest income get determined by central bank balance sheet considerations.

3.2 Currency mismatch in an open economy

I consider next a general, open economy central bank balance sheet with nominal assets and liabilities. The central bank issues non-interest bearing and interest bearing domestic currency liabilities while it holds foreign currency assets. To focus on the implications of the currency mismatch as opposed to the maturity mismatch, I here consider only one-period assets and liabilities. The model is thus one of currency mismatch.

[29] A less constraining rule than this lower bound on the flow remittances of the central bank would be to consider a "deferred asset" constraint, as explained in Hall and Reis (2013). Using such a rule, by providing an opportunity to smooth remittances over time, would lessen the effects of the constraint on the endogenous variables. Qualitatively at least however, especially for large shocks, it would lead to similar results as the ones in this paper.

3.2.1 Unpleasant central bank balance sheet arithmetic

The nominal flow budget constraint of the central bank is then given by

$$S_t B^*_{t-1} - L_{t-1} - M_{t-1} - P_t T_t = S_t Q_{B^*t} B^*_t - Q_{Lt} L_t - M_t$$

where M_t is non-interest bearing domestic currency liabilities, L_t is one-period domestic currency liabilities with $Q_{L,t}$ its price, and B^*_t is one-period foreign currency assets with Q_{B^*t} its price. Moreover, P_t is the home nominal price level, T_t is the central bank transfer to the Treasury, and S_t is the nominal exchange, that is the price of the foreign currency in terms of home currency.[30] The flow budget constraint can be written in real terms as

$$\varsigma_t b^*_{t-1} \Pi^{*-1}_t - l_{t-1} \Pi^{-1}_t - m_{t-1} \Pi^{-1}_t - T_t = \varsigma_t Q_{B^*t} b^*_t - Q_{Lt} l_t - m_t \tag{10}$$

where $b^*_t = \frac{B^*_t}{P^*_t}$, $l_t = \frac{L_t}{P_t}$, $m_t = \frac{M_t}{P_t}$, and $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross home inflation while $\Pi^*_t = \frac{P^*_t}{P^*_{t-1}}$ is gross foreign inflation. Moreover, distinctly in this open economy environment, ς_t is the real exchange rate, defined as $\varsigma_t \equiv S_t \frac{P^*_t}{P_t}$, that is, the price of the foreign good in terms of the home good.

Under complete markets, the two bond prices are determined by standard open economy asset-pricing conditions

$$Q_{Lt} = \beta E_t \left[\zeta_{t,t+1} \Pi^{-1}_{t+1} \right], \ \varsigma_t Q_{B^*t} = \beta E_t \left[\zeta_{t,t+1} \varsigma_{t+1} \Pi^{*-1}_{t+1} \right] \tag{11}$$

where E_t is the mathematical conditional expectation operator and $\zeta_{t,t+1}$ is the unique stochastic discount factor. Finally, while this is not necessary, for simplicity of exposition, I will assume that purchasing power parity holds in the model, so that the real exchange rate is unity

$$\varsigma_t = S_t \frac{P^*_t}{P_t} = 1. \tag{12}$$

To get insights on how the currency mismatch between assets and liabilities can constrain the central bank, it is again useful to approximate the central bank budget constraint around a non-stochastic steady-state. The analysis in this case will again be very similar to the one in Sargent and Wallace (1981). Then, (10) log-linearized gives

$$b_{N,t} = \frac{1}{\beta} b_{N,t-1} + \frac{1}{\beta} s_t - \frac{1}{\beta} \hat{T}_t + b^* \left[\hat{S}_t - \beta E_t \hat{S}_{t+1} \right] \tag{13}$$

where $b_{N,t} \equiv \hat{b}^*_t - \hat{l}_t - b^* \hat{S}_t$ is the net asset position of the central bank in terms of difference in the stock of assets and liabilities after appropriately accounting for currency differences, $s_t \equiv m[\hat{m}_t + \pi_t - \hat{m}_{t-1}]$ is revenue from issuing non-interest bearing liabilities or seigniorage, and β is the discount factor.[31]

There are several aspects of (13) that require some discussion. First, while deriving it, (11) log-linearized has been imposed, which is a no-arbitrage condition that illustrates uncovered nominal interest rate parity

$$\hat{Q}_{B^*t} - \hat{Q}_{Lt} = E_t \left[\hat{S}_{t+1} - \hat{S}_t \right] \tag{14}$$

[30] An increase in S_t is therefore a depreciation of the home currency.

[31] In terms of approximation, I again log-linearize all variables, except for asset and liability positions and transfers to the Treasury, which are expressed in terms of deviations from steady-state. The steady-state is a zero inflation, unit real exchange rate steady-state where net position is zero.

and links the home bond price with the foreign bond price and expected change in the nominal exchange rate. Moreover, (12) log-linearized has also been imposed as $\pi_t - \pi_t^* = \hat{S}_t - \hat{S}_{t-1}$. These manipulations have the advantage of focusing attention directly on the nominal exchange rate, a key variable in an open economy, and which in fact, is at the source of balance sheet mismatch. Second, other than through the seigniorage term, home inflation does not appear directly, as purchasing power parity has been imposed. Third, as in the closed-economy model, in this approximated budget constraint only the net asset position, and not the separate gross asset and liability positions, appear.

Fourth, and perhaps more important, the dynamics of net asset of the central bank get affected by the net interest income on its position, $\hat{S}_t - \beta E_t \hat{S}_{t+1}$. Why is this the flow net interest income of the central bank? It is useful to go back to the original budget constraint (10) and for simplicity, impose purchasing power parity. On its home-currency liability, the central bank pays the home interest rate $\frac{1}{\Pi_t Q_{Lt}}$. On its foreign currency asset, the interest rate earned is $\frac{1}{\Pi_t^* Q_{Bt}^*}$. With a first-order approximation, the difference between the two, accounting for discounting in the definition of the net asset position, is given by $\hat{S}_t - \beta E_t \hat{S}_{t+1}$. Finally, the effect of the net interest income on the dynamics of net asset position depends on the steady-state gross position of the central bank balance sheet. In this sense, a large size of the balance sheet can magnify the effects of fluctuations in exchange rates.

Like before, to get implications similar to Sargent and Wallace (1981) while focusing on the currency mismatch, I will now simplify by assuming that seigniorage revenues are negligible. Then, (13) is given by

$$b_{N,t} = \frac{1}{\beta} b_{N,t-1} - \frac{1}{\beta} \hat{T}_t + b^* R_{N,t} \tag{15}$$

where $R_{N,t} \equiv \hat{S}_t - \beta E_t \hat{S}_{t+1}$ is the one-period net interest income of the central bank described above. The flow budget constraint can then again be expressed in a present value form after imposing a terminal condition on net asset of the central bank as

$$b_{N,t-1} = \sum_{t=0}^{\infty} \beta^t \hat{T}_t - \beta \sum_{t=0}^{\infty} \beta^t b^* R_{N,t} \tag{16}$$

where I will take the initial net position $b_{N,t-1}$ as given. Now, using (16), consider how the any outstanding/initial net position of the central bank needs to be financed. In particular, if $b_{N,t-1} < 0$, then adjustment must come, in a present value sense, either from lower central bank transfers to the Treasury $\left(\hat{T}_t\right)$ or from higher net interest income of the central bank $(R_{N,t})$.

Even more importantly, consider a case similar to Fiscal dominance in Sargent and Wallace (1981) where now the present value of central bank transfers to the Treasury is fixed (or exogenous generally). This describes the *open economy balance sheet constrained central bank*. Then it is clear that an increase (decrease) in interest income *today* must lead to a decrease (increase) in *future*. In particular, since the interest income depends on the nominal exchange rate, here directly considered as the policy instrument of the central bank, this means that the central bank's *exchange rate policy* gets constrained in this case. Thus for example, it cannot freely choose the path of the nominal exchange rate in future. This constitutes *unpleasant central bank balance sheet arithmetic* in an open economy with a currency mismatch in the central bank balance sheet.

Now consider again the case of $b_{N,t-1} < 0$ for the *open economy balance sheet constrained central bank*. In this case, as adjustment does not come from central bank transfers to the Treasury (for intuition, can think of there being no response of transfers at all), it must be the case that adjustment comes from higher net interest incomes of the central bank. Additionally, this balance sheet arithmetic implies that if transfers to the Treasury increase (decrease) today with no future off-setting decreases (increases), then net interest income must increase (decrease) in future, in order to satisfy (16). Again, in both these cases, the nominal exchange rate, the effective policy instrument here, must adjust and respond appropriately to support the required adjustment in net interest income. How exactly does the policy instrument respond? In both cases, the nominal exchange rate must depreciate, thereby ensuring a direct fall in the return on (domestic currency) liabilities, which in turn in equilibrium leads to an increase in the net interest income.[32]

If the present value of central bank transfers to the Treasury is not fixed, and in particular, if it adjusts to the state of the balance sheet to satisfy (16), then that describes the *open economy balance sheet unconstrained central bank*. For such a central bank, as the path of net interest income, and thereby its policy instrument, is not constrained, it can freely pursue its objectives regarding the dynamics of the nominal exchange rate. On the flip side, this then implies that if transfers to the Treasury increase (decrease) today, they must decrease (increase) in future, in order to satisfy (16).

Thus, the insights of the closed economy model extend in a natural way also to an open economy. Moreover, given the similarly of the models, all the results on impulse responses to policy shocks as well as transition dynamics are also isomorphic. To conserve space, I present them in the Appendix. I now move to a familiar general equilibrium model where prices and output are endogenously determined.

4 Non-linear General Equilibrium Model

I now present a fully specified non-linear general equilibrium closed economy model to illustrate the results highlighted above using the simple model, as well as show additional results on forward guidance effects and explore thoroughly the monetary policy transmission mechanisms.[33] In this set-up I use standard interest rate policy rule specifications and will evaluate inflation and output dynamics in equilibrium. Moreover, I specify clearly both the central bank and Treasury balance sheet policies and budget constraints. Relatedly, as the model is non-linear, the central bank balance sheet rules that I specify will need to make clear how gross, and not just net, positions are determined.[34]

4.1 Private sector

I first describe the environment and the maximization problems faced by households and firms.

4.1.1 Households

A representative household maximizes expected discounted utility over the infinite horizon

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t\right) \tag{17}$$

[32] I illustrate in the Appendix using an equilibrium analysis the full dynamic path through which the nominal exchange rate responds to these two cases. There will be dynamics because of the state variable, the net asset position of the central bank, that matters for the evolution of the nominal exchange rate.

[33] In the interest of space, I do not separately present an open economy model. The insights from the simple open economy model however will go through in a fully specified general equilibrium model. The small open economy non-linear monetary model in Bhattarai and Egorov (2016) for instance can be used to show those results.

[34] The non-linear model also helps make the point that the results in the previous section are not specific to a first-order approximated model, and in particular, one where the central bank budget constrained is linearized and the asset pricing conditions imply the expectation hypothesis. The baseline results I present are based on a fully non-linear deterministic solution method. I also present results in an extension based on a stochastic, third-order approximation solution method.

where $0 < \beta < 1$ is the discount factor, c_t is household consumption of the composite good, and h_t is hours supplied by the household. E_0 is the mathematical expectation operator conditional on period-0 information and $U(c_t, h_t)$, the period utility, is increasing in c_t and decreasing in h_t. The composite consumption good c_t is an aggregate of a continuum of differentiated varieties indexed by $i \in [0,1]$. The consumption good is defined as $c_t = \left[\int_0^1 c_t(i)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}}$, where $\nu > 1$ is the elasticity of substitution among the varieties.

Before presenting the flow budget constraint, it is useful to discuss the static expenditure minimization problem faced by households. Let P_t be the nominal price of the aggregate consumption good and $P_t(i)$ the nominal price of the varieties. Then, formally, the household chooses $\{c_t(i)\}_{t=0}^{\infty}$ to minimize

$$\int_0^1 c_t(i) P_t(i) di$$

subject to

$$\left[\int_0^1 c_t(i)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}} \geq c_t \tag{18}$$

while taking as exogenously given $\{P_t(i)\}_{t=0}^{\infty}$. Then, the shadow price on (18) is equal to P_t, the nominal price of the home consumption good.[35] The demand function for the variety of goods takes the standard downward-sloping form and is obtained from the optimality condition of the problem above.

The household's flow budget constraint (in real terms) is then given by

$$l_{t-1}^H \Pi_t^{-1} - Q_{1t} b_{t-1}^H \Pi_t^{-1} = Q_{1t} l_t^H - Q_{2t} b_t^H + c_t - w_t h_t - \varphi_t + \tau_t \tag{19}$$

where $b_t^H = \frac{B_t^H}{P_t}$ with B_t^H the nominal two-period debt of the household and $l_t^H = \frac{L_t^H}{P_t}$ with L_t^H the nominal one-period asset held by the household. Here, $\Pi_t = \frac{P_{t-1}}{P_t}$ is gross inflation, Q_{1t} is the price of the one-period asset, and Q_{2t} is the price of the two-period debt. Moreover, w_t is real wages, φ_t is profits from firms, and τ_t is net lump-sum taxes from the Treasury.[36]

Given the first-stage, static expenditure minimization problem discussed above, the problem of the home household then is to choose $\{c_t,\, h_t,\, l_t^H,\, b_t^H\}_{t=0}^{\infty}$ to maximize (17) subject to a sequence of budget constraints (19), while taking as exogenously given initial wealth and $\{\Pi_t,\, \varphi_t,\, w_t,\, Q_{1t},\, Q_{2t},\, \tau_t\}_{t=0}^{\infty}$.

4.1.2 Firms

There is a continuum of monopolistically competitive firms that produce differentiated varieties. The firms are of measure 1 and indexed by $i \in [0,1]$. Firm i produces output y_t using labor as input

$$y_t(i) = F(h_t(i)) \tag{20}$$

where the production function $F(h_t(i))$ is increasing in $h_t(i)$. Firms hire labor in a common, perfectly competitive factor market.

Firm i sets prices $P_t(i)$ for its good. I introduce nominal rigidities following Rotemberg (1982). Thus, firms face a cost of adjusting prices given by $d\left(P_t(i)/P_{t-1}(i)\right)$ where $d(.)$ is a convex function. Moreover,

[35] P_t is thus the minimum-expenditure price index.

[36] Note that the flow budget constraint is written in terms of real values, where the deflator is the common price level of the aggregate consumption good. Also, while I make this clear later, I use debt and asset holdings of the household in terms of terminology to pre-empt the market clearing conditions on the two-types of assets and the assumptions on monetary and fiscal policy. The household is subject to a no-Ponzi game condition.

the demand function for variety i is derived from the cost-minimization problem of the household over differentiated varieties discussed above and given by

$$\frac{y_t(i)}{y_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\nu} \tag{21}$$

where y_t is aggregate demand that is taken as given by the firms.

Firms maximize expected discounted profits over the infinite horizon

$$E_0 \sum_{t=0}^{\infty} \zeta_{0,t} \varphi_t\left(i\right) \tag{22}$$

where they use the stochastic discount factor of the household $\zeta_{0,t} = \beta^t \frac{U_c(c_t,h_t)}{U_c(c_0,h_0)}$ to discount future profits and where (real) flow profits $\varphi_t(i)$ are given by

$$\varphi_t(i) = F(h_t(i)) - d\left(\frac{P_t(i)}{P_{t-1}(i)}\right) - w_t h_t(i).$$

The problem of firm i is then to choose $\{h_t(i),\, P_t(i)\}_{t=0}^{\infty}$ to maximize (22) subject to a sequence of production functions (20) and demand functions (21), while taking as exogenously given $\{P_t,\, y_t,\, \zeta_{0,t},\, w_t\}_{t=0}^{\infty}$. As is standard, I will focus on a symmetric equilibrium where all firms choose the same price and produce the same amount of output.

4.2 Government

I now describe how monetary and fiscal policy is determined and the balance sheet structure of the central bank and the Treasury. This is a key aspect of the analysis. Similar to the simple model in the previous section, I also define under what policy rule configurations the central bank is balance sheet constrained.

4.2.1 Monetary policy

The central bank faces the flow budget constraint written in real terms as

$$Q_{1t} b_{t-1} \Pi_t^{-1} - l_{t-1} \Pi_t^{-1} - T_t + K = Q_{2t} b_t - Q_{1t} l_t \tag{23}$$

where $b_t = \frac{B_t}{P_t}$ with B_t the nominal two-period asset of the central bank and $l_t = \frac{L_t}{P_t}$ with L_t the nominal one-period liability of the central bank. Moreover, T_t is the central bank transfer to the Treasury and K its any other revenue net of operations cost (constant for simplicity). The budget constraint takes the same form as in the simple model of the previous section.[37]

Central bank's short-term interest rate policy is determined according to a simple feedback rule

$$\beta Q_{1t}^{-1} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \exp(\varepsilon_{It}) \tag{24}$$

where $\phi_\pi \geq 0$ is the feedback parameter, Π is the steady-state value of gross inflation, and ε_{It} is an interest rate shock that follow a stationary process.[38]

[37] In this model since I directly abstract from cash, I interpret this revenue as coming from seigniorage, which net of operations cost, I assume to be constant.

[38] As I explain below, I consider both a current/surprise and future/news monetary policy shock.

Next, I have to specify the central bank's policy that determines the evolution of its balance sheet. Unlike in the simple linearized model, it is not enough to posit a rule that only determines how total transfers to the Treasury evolves as a function of the net asset position of the central bank. Here, I will need to model separate transfer policy rules for both asset and liability positions (that is, equivalently, we also need to determine the gross positions and not just net positions, in order to complete the model).[39] I assume that the central bank uses the following rules for determining transfers to the Treasury

$$T_t^b - \frac{T}{2} = \psi_{bT}\beta\left(b_{t-1} - b\right), \; T_t^l - \frac{T}{2} = -\psi_{lT}\left(l_{t-1} - l\right) \tag{25}$$

where $\psi_{bT} \geq 0$, $\psi_{lT} \geq 0$ are the feedback parameters, b is the steady-state value of central-bank assets, l is the steady-state value of central-bank liabilities, and T is the steady-state value of transfers.[40] Here, the two transfers T_t^b and T_t^l add up to the total transfers to the Treasury as

$$T_t = T_t^b + T_t^l$$

and so in terms of total transfers to the Treasury, these rules can be written as

$$T_t - T = \psi_{bT}\beta\left(b_{t-1} - b\right) - \psi_{lT}\left(l_{t-1} - l\right).$$

Then, for determination of gross positions, I assume that they will then always be in steady-state, or equivalently, $T_t^b = T_t^l$.[41]

Finally, notice that generally, I need not restrict the feedback parameters to be the same. But in that specific case, when $\psi_{bT} = \psi_{lT} = \psi_T$, there is a natural interpretation in terms of the response of the total central bank transfers to the Treasury as being determined by the net asset position

$$T_t - T = \psi_T\left[\beta\left(b_{t-1} - b\right) - \left(l_{t-1} - l\right)\right]$$

exactly as in the simple linearized model of the previous section.[42] I therefore, consider this to be the baseline case in the analysis later.

4.2.2 Fiscal policy

I keep fiscal policy deliberately very simple to focus the analysis on monetary policy. In particular, I assume that the Treasury follows essentially a balanced budget policy. For simplicity, there is no government spending, and the Treasury turns over all remittances from the central bank to the household. Moreover, for simplicity, it pursues a zero holdings policy of the two nominal bonds. Thus, formally

$$T_t = \tau_t \tag{26}$$

[39] Thus even if I were to assume a deterministic framework, as I do for the baseline analysis, the non-linearity matters.

[40] Like in the simple model, I thus use a feedback rule for transfers to the Treasury and study the two central bank models separately. This baseline approach leads to a more straightforward analysis. But like in the simple model, I consider later in this section a model with a lower bound for transfers, which could bind endogenously, for a finite number of periods.

[41] Another alternative to determine gross positions is to specify a rule for central bank asset holdings separately, as in Del Negro and Sims (2015) and Benigno and Nistico (2015). For instance, Del Negro and Sims (2015) posit that assets are fixed over time. Here, I choose the rules in terms of remittances of the central bank depending on assets and liabilities separately, as these rules can be interpreted more easily and directly with models (such as the linearized case) where only the net asset position of the central bank matters and remittances are a function of the net asset position.

[42] For conciseness, in this general equilibrium model, I only study the implications of an interest rate shock. It is however, straightforward to also introduce shocks in the central bank balance sheet, as in the simple model, here as well.

and

$$L_t^T = 0, \; B_t^T = 0 \tag{27}$$

where L_t^T is the Treasury holding of central bank issued one-period nominal liabilities and B_t^T is the Treasury's supply of two-period nominal bonds. Note that the assumption of zero holdings from the Treasury is for simplification and generally, it can be any fixed amount.[43]

4.2.3 Central bank balance sheet constraints

Given the description of the balance sheet and policy rules of the central banks and Treasury, I now define, in a manner similar to the simple model, a balance sheet unconstrained and a balance sheet constrained central bank. Generally, a balance sheet unconstrained central bank is free to adjust interest rates to focus on inflation stabilization without being constrained by balance sheet considerations and in particular, the path of transfers to the Treasury. A stable dynamics of its assets and liability positions is thus assured through appropriate changes in the path of its transfers to the Treasury.

A balance sheet constrained central bank has to adjust its interest rate path, in equilibrium, as the path of its transfers to the Treasury is constrained. Only doing so will ensure that the dynamics of its asset and liability positions are stable/stationary. Again, a balance sheet constrained central bank in my model is similar in spirit to the central bank under fiscal dominance in the Sargent and Wallace (1981) model as its path of remittances to the Treasury is constrained. One can thus interpret the constraint on the central bank, in terms of its remittances to the Treasury, as coming from fiscal dominance.[44]

Specifically, here, with the policy rules I use, a balance sheet unconstrained central bank will follow the Taylor principle in setting interest rate policy ($\phi_\pi > 1$) and whose path of transfers to the Treasury respond strongly enough to asset and liability positions (high enough ψ_{bT} and ψ_{lT}) such that a stable path of asset and liability dynamics is assured. With the policy rules I use, a balance sheet constrained central bank in contrast will not follow the Taylor principle in setting interest rate policy ($\phi_\pi < 1$). Moreover, its path of transfers to the Treasury do not respond strongly enough to asset and liability positions (low ψ_{bT} and ψ_{lT}) such that a non-explosive path of asset and liability dynamics is not assured through adjustment in the transfers automatically. These are the two policy parameter configurations that will lead to a unique stationary/stable equilibrium in the model.[45]

[43] I avoid more completely endogenizing the spending and debt policies of the Treasury in order to keep the focus of the paper sharply on the balance sheet considerations of the central bank in this general equilibrium model. My goal here is to keep the model relatively simple to bring out the new results as transparently as possible. Allowing for a detailed specification of rules by the Treasury, including possibly "active" fiscal policy, would add another layer to the analysis. See Benigno and Nistico (2015) for an analysis that features this additional specification of fiscal policy rules.

[44] In fact, given the simple model of fiscal policy I use, where there are no other sources of taxes and the Treasury has zero debt/asset holdings, if one consolidates the Treasury and central bank budget constraints into one, the interpretation will be exactly the same as in Sargent and Wallace (1981).

[45] Del Negro and Sims (2015) study the possibility of self-fulfilling multiple equilibria due to central bank solvency concerns. I abstract from solutions that feature explosive inflation. Moreover, these policy parameter configurations are based on the convention and insights from models that feature the fiscal theory of the price level such as Sims (1994). I note however that an exogenous process, such as a peg, for the nominal interest rate will not lead to a determinate equilibrium in my model, unlike the fiscal theory of the price level.

4.3 Market clearing and exogenous processes

The goods, factor, and bonds markets clear in equilibrium.[46] In particular, the social resource constraint, at the variety level, is given by

$$y_t(i) = c_t(i) + d\left(\frac{P_t(i)}{P_{t-1}(i)}\right)$$

where I incorporate the resource cost of adjusting prices. Given the assumptions on Treasury's debt policy, the bond market clearing conditions are then given by

$$b_t - b_t^H = b_t^T = 0,\ l_t^H - l_t = l_t^T = 0. \tag{28}$$

Finally, I assume that the interest rate shock ε_{It} follows an AR(1) process for the current, surprise shock experiment

$$\varepsilon_{It} = \rho\varepsilon_{It-1} + \exp(\sigma)\epsilon_t \tag{29}$$

where the innovation $\epsilon_t \sim N(0,1)$, $\rho \in [0,1)$, and $\exp(\sigma) > 0$. Then for the news shock experiment, which models forward guidance policy, I assume that the interest rate shock ε_{It} follows an AR(1) process

$$\varepsilon_{It} = \rho\varepsilon_{It-1} + \exp(\sigma)\epsilon_{t-k} \tag{30}$$

where the innovation $\epsilon_t \sim N(0,1)$, $\rho \in [0,1)$, $\exp(\sigma) > 0$, and $k > 0$ governs the number of time-periods in advance of which the shock is anticipated. Note, especially for the news shock experiment, that I model policy in terms of the nominal interest rate, which is the true monetary policy instrument in the model. While it is also common to specify forward guidance policy directly in terms of the real interest rate, I cannot take that approach here as the distinction and correlation between the nominal and real interest rate is a critical aspect of my analysis of the two types of central banks.

4.4 Equilibrium

I now define the equilibrium of the model and discuss the policy rules and aggregate optimality and feasibility conditions that characterize it.

4.4.1 Definition

An equilibrium is a collection of allocations (of goods varieties and aggregates) for the household, $\{c_t(i),\, c_t,\, h_t,\, l_t^H,\, b_t^H\}_{t=0}^{\infty}$, allocations and goods prices for the firms $\{h(i),\, p_{H,t}(i)\}_{t=0}^{\infty}$, a sequence of aggregate prices $\{\Pi_t,\, Q_{2t},\, w_t,\, \rho_{0,t}\}_{t=0}^{\infty}$ and output $\{y_t\}_{t=0}^{\infty}$, and monetary and fiscal policy instruments $\{Q_{1t}^{-1}, b_t,\, l_t,\, T_t,\, \tau_t,\, l_t^T,\, b_t^T\}_{t=0}^{\infty}$ such that

(i) Given prices and monetary and fiscal policy, the allocations are such that they satisfy the maximization problems of the household,

(ii) Given aggregate prices, aggregate output, and monetary and fiscal policy, the goods prices and allocations are such that they satisfy the maximization problem of the firms,

(iii) The allocations and goods prices across firms are symmetric,

(iv) The nominal interest rate is determined by the interest rate policy rule and central bank assets and liabilities by the central bank balance sheet policy rule,

[46] The notation on hours/labor already imposes that factor markets clear in equilibrium.

(v) Household taxes are determined by the Treasury tax rule and Treasury debt and asset holdings by the Treasury debt policy rule, and

(vi) Goods, factor, and bonds markets clear,

given the initial household and central bank balance sheet positions and an exogenous process for $\{\varepsilon_{It}\}_{t=0}^{\infty}$. As I discussed above while defining a balance sheet constrained and unconstrained central bank, I will focus on two types of stationary equilibria, depending on the configurations of the central bank policy parameters.

4.4.2 Equilibrium conditions

The aggregate non-linear equilibrium conditions of the model are fairly standard. To simplify even further, I assume log-linear specifications for preferences and technology

$$U(c_t, h_t) = \log C_t - Ah_t,\ y_t = F(h_t) = h_t$$

and a standard quadratic price-adjustment cost function

$$d(\Pi_t) = \frac{\kappa}{2}(\Pi_t - 1)^2.$$

Given these specifications, the aggregate private sector equilibrium conditions are given by

$$Q_{1t} = \beta E_t\left[\frac{C_t}{C_{t+1}}\Pi_{t+1}^{-1}\right],\ Q_{2t} = \beta E_t\left[\frac{C_t}{C_{t+1}}Q_{1t+1}\Pi_{t+1}^{-1}\right] \tag{31}$$

$$\left[\left(\frac{\nu-1}{\nu}\right) - AC_t\right]\nu Y_t + \kappa(\Pi_t - 1)\Pi_t = \beta E_t\left[\frac{C_t}{C_{t+1}}\kappa(\Pi_{t+1} - 1)\Pi_{t+1}\right] \tag{32}$$

$$C_t + \frac{\kappa}{2}(\Pi_t - 1)^2 = Y_t \tag{33}$$

where (31) are the two asset pricing conditions, (32) is the Phillips curve, and (33) is the resource constraint.[47] Then, the policy block is given by the central bank budget constraints and the rules for setting monetary policy and balance sheet variables

$$Q_{1t}b_{t-1}\Pi_t^{-1} - l_{t-1}\Pi_t^{-1} - T_t + K = Q_{2t}b_t - Q_{1t}l_t$$

$$\beta Q_{1,t}^{-1} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\exp(\varepsilon_{It})$$

$$T_t - T = 2\psi_{bT}\beta(b_{t-1} - b),\ T_t - T = -2\psi_{lT}(l_{t-1} - l).$$

Note that the above eight equations determine the dynamics of the eight endogenous variables $\{Q_{1,t}, Q_{2,t}, C_t, Y_t, \mathit{\Pi}_t, T_t, b_t, l_t\}$ given initial conditions, exogenous processes, and appropriate restrictions on the policy parameters that ensure stationary dynamics of the endogenous variables above.[48]

In addition to these variables, for comparison with the simple linear model presented in the last section, I will define the net interest income $R_{N,t}$ and net asset position $b_{N,t}$ of the central bank in a similar manner

[47] Note that the specific functional forms for preferences and technology are useful to reduce the number of parameters in the model, but the private sector equilibrium conditions can be reduced to such three conditions generally, even without the assumptions. The derivations of all the optimality conditions in the model are very standard and are omitted for brevity.

[48] Note that we do not need to keep track of either the household budget constraint (due to Walras law) or the Treasury policy rules separately as well (as they have essentially already been imposed above in terms of market clearing conditions). Moreover, here, I also impose the rule determining gross positions.

in this non-linear model as well

$$R_{N,t} \equiv \frac{Q_{1t}}{Q_{2t}} - \frac{1}{Q_{1t}}$$

$$b_{N,t-1} \equiv \left[\beta \left(b_{t-1} - b\right) - \left(l_{t-1} - l\right)\right].$$

While in this non-linear model, these variables are not necessarily always sufficient statistics to ascertain the behavior of the balance sheet constrained central bank, I will show in the next section that they continue to be very helpful in interpreting the results.[49]

4.5 Results

I solve the model non-linearly for two kinds of monetary policy shock. The first one is a standard monetary policy shock experiment, where in period 0, an unexpected shock to the interest rate rule hits the economy, and then the economy evolves deterministically thereafter. The second one is a standard forward guidance shock experiment, where there is an anticipated shock to the interest rate rule, which realizes at a certain period in future. This is a fully deterministic experiment. In the initial period, the economy is assumed to be in the non stochastic steady-state.[50] Then, I solve for a stationary equilibrium such that in the long-run, variables transition back to the same non-stochastic steady-state.[51] All results are shown in terms of deviations from this steady-state. As I noted above, I will consider two different types of central bank regimes under which such stationary equilibria exist.

4.5.1 Parameterization

As the main objective of the model simulation results is to show qualitative properties, I use parameter values that are common in the literature and similar to the simple model. The parameter values used in the numerical analysis are presented in Table 1. The assumption on preferences and technology described above already imposes several parameter values. Like the simple model, I then use $\beta = 0.99$ and $b = 1$. For the steady-state central bank transfers to the Treasury, I use a value of 0.1. For the price stickiness parameter κ I use a value of 150 and a standard value of 7 for the elasticity of substitution across varieties of goods.[52]

In terms of the policy parameters, for the baseline results here, I impose the same values on the two balance sheet feedback policy parameters and an *iid* process for the interest rate policy shocks. The transitory nature of the shocks is very useful to illustrate clearly when central bank balance sheet variables are endogenous state variables for inflation and output. For the two types of central banks, I consider feedback parameters that are useful to illustrate the differences between the two cases. For the balance sheet unconstrained central

[49]These variables will be sufficient as long as a linear approximation is a good enough approximation, which is the case here with small shocks. Again, note that even with a deterministic analysis, to close the model a determination of gross balance sheet variables is essential as the model is non-linear. In a deterministic framework, the asset pricing conditions give $Q_{2t} = Q_{1t}Q_{1t+1}$, which can be used in the central bank budget constraint to yield

$$\left(Q_{1t}b_{t-1} - l_{t-1}\right)\Pi_t^{-1} - T_t + K = Q_{1t}\left(Q_{1t+1}b_t - l_t\right).$$

This simplifies but does not reduce the budget constraint to one state variable like $b_{N,t-1}$.

[50]The non-stochastic steady-state is a standard one where I assume zero net inflation and equal asset and liability positions of the central bank (such that Transfers to the Treasury are equal to the revenue net of operations cost of the central bank). Moreover, I normalize steady-state output to be 1 by picking a value for A appropriately.

[51]I use a non-linear solver to compute this perfect foresight solution. In particular, I use the solver csolve written by Chris Sims. In an extension, I consider a stochastic solution based on a third-order approximation.

[52]The value for price stickiness is higher than sometimes used in the literature, such as 50 in Ireland (2000). The implied slope of the Phillips curve however will not be very different from other calibrations as I use log-linear preference and a linear production technology.

bank, I use a value for the interest rate feedback parameter that is slightly higher than the one implied by the Taylor principle (ϕ_π=1.05) combined with a sufficiently large value for the transfer feedback parameter ($\psi_T = 0.012$) while for the balance sheet unconstrained central bank, I use a value for the interest rate feedback parameter that is lower than 1 (ϕ_π=0.95) combined with a just low enough value for the transfer feedback parameter ($\psi_T = 0.009$).[53]

4.5.2 Surprise Current Monetary Policy Shock

I start by considering dynamics following an unexpected monetary policy shock. I discuss the balance sheet unconstrained central bank first and then contrast it with the balance sheet constrained central bank.

Balance sheet unconstrained central bank Equilibrium responses of variables to a current positive interest rate shock (of 1%) for the balance sheet unconstrained central bank are shown in Figure 8.[54] The responses of the short-term interest rate, inflation, and output are standard and follow textbook analysis. With a rise in the short-term interest rate, and with the interest rate rule satisfying the Taylor principle, inflation decreases. Moreover, in this model with nominal rigidities, the fall in inflation leads to a fall in output as well. Since the shock is transitory and the private sector equilibrium conditions are all purely forward looking, all three of these variables go back to steady state next period. The balance sheet variables do not affect the dynamics of these standard macro variables at all, as in the simple model.[55]

For the balance sheet variables, a transitory increase in the short-term nominal interest rate leads to a decline in the net interest income by essentially the same amount. As a result, net asset position falls next period. Since this is the balance sheet unconstrained model, transfers to the Treasury adjust by enough to ensure that in the long-run, balance sheet variables revert back to steady-state. Here, as in the simple model, transfers to the Treasury decline as net asset position of the central bank deteriorates.

Balance sheet constrained central bank Equilibrium responses of variables to a current positive interest rate shock (of 1%) for the balance sheet constrained central bank are shown in Figure 9. The responses of the short-term interest rate, inflation, and output are now very different from the standard case and are determined by balance sheet considerations of the central bank.[56] First, note that on impact, the short-term interest rate increases. Moreover, like for the unconstrained central bank case, net interest income of the central bank decreases on impact, as is to be expected given an increase on interest paid on liabilities. This however now has important consequences for the determination of future short-term interest rate. Because the central bank here is balance sheet constrained, while transfers to Treasury do decrease as the net asset position of the central bank falls, they do not decrease by enough to ensure stationary dynamics.[57] Thus, net interest income will have to increase in future. This is exactly what we see in Figure 9 and in fact the dynamics is qualitatively the same as in the simple model. Thus, the intuition can be obtained again from a present value analysis of the central bank budget constraint.

[53] The boundary for the balance sheet constrained model, in the linearized case, would be exactly 1-β= 0.01. Note that the precise value of ψ_T used for the balance sheet unconstrained model is irrelevant for inflation, output, and interest rate dynamics and just affects the dynamics of the balance sheet variables.

[54] The responses are shown in terms of deviations from the non-stochastic steady-state.

[55] That is, the model is block-recursive, and balance sheet variables are not state variables for inflation, interest rate, and output.

[56] Thus, the model is no longer block recursive and the central bank balance sheet variables are state variables for inflation, interest rate, and output.

[57] One way to make this even starker would have been, like in the simple model, to make transfers to Treasury completely unresponsive to the state of the central bank balance sheet. I take a more reasonable modeling of the central bank policy here.

What are the implications of this equilibrium dynamics of net interest income for the short-term interest rate? For interest rate dynamics to be consistent with the net interest income behavior, it must be that in future, the short-term interest rate falls, and then it transitions back to steady-state, rising along the transition.[58] Now I can also assess the implications for inflation and output as they are both endogenously determined in this model and depend on monetary policy in a non-trivial way. Because of the positive relationship between the short-term interest rate and inflation implied by the monetary policy rule (24), inflation then follows the same path as the short-term interest rate.[59] Moreover, on impact as well, inflation must fall, because of forward looking dynamics implied by the Phillips curve in the model. Finally, given nominal rigidities, output also falls along the entire path, basically following the path of inflation. These persistent dynamics of interest rates, inflation, and output arise, in spite of a transitory shock, as now central bank balance sheet variables are endogenous state variables in the model.[60]

There are thus critical differences between the balance sheet unconstrained and constrained central bank in terms of the central bank's control of the monetary policy instrument. Unlike the balance sheet unconstrained central bank, the constrained one cannot freely set the path of its policy instrument. In fact, here, while a positive interest rate shock today increases the short-term interest rate on impact, starting with next period, the short-term interest rate falls, and then slowly rises back to steady-state. This path is the only one consistent with the required equilibrium path of net interest income of the central bank. This is thus the clearest illustration of unpleasant central bank balance sheet arithmetic in this fully specified general equilibrium model.

Finally, in this case, dynamically, the short-term interest rate and inflation are positively correlated along the transition. This is in sharp contrast to the standard case, where they are negatively correlated. Thus, in this respect, the model has so called Neo Fisherian properties. At the same time however, a positive interest rate shock still leads to an increase in the short-term interest rate on impact, and more importantly, to a consistent decline in inflation. Thus, it does not follow from this model that a positive short-term interest rate shock actually leads to higher inflation. At the same time, however, there is an important difference between the two cases in terms of real interest rate dynamics, which I discuss next.

Real interest rate comparison Equilibrium responses of the real interest rate to a current positive nominal interest rate shock (of 1%) for the balance sheet constrained and unconstrained central banks are shown together in Figure 10. This comparison illustrates a key difference between the two cases as the real interest rate is a key variable underlying the monetary transmission in the sticky price model.

When the central bank is not balance sheet constrained and thus, the interest rate rule satisfies the Taylor principle, the real interest rate increases on impact. Next period, as the shock is iid, the real interest rate

[58] Because the short-term interest rate needs to follow this path as a function of the state variables, in my model, unlike in the fiscal theory of the price level, a purely exogenous path for the interest rate does not lead to a determinate equilibrium. This is quite straightforward to prove in the simple linear model.

[59] Of course, the coefficient on the policy rule is positive even for the balance sheet unconstrained central bank. In that case however, the coefficient is greater than 1. Price level determination then happens through the Taylor Principle channel, which leads to inflation falling and interest rates rising when a positive interest rate shock hits the economy. Thus along the transition back to steady-state, the two are negatively correlated for the balance sheet unconstrained central bank (as I discuss later below, the Taylor Principle channel also implies a positive correlation between the nominal and real interest rates along the transition, while for the balance sheet constrained central bank, the correlation is negative). This will also be generally the case if the monetary policy shock is not iid and has persistence so that there are transition dynamics beyond the impact period. In the balance sheet constrained case, the transition dynamics, even with an iid monetary policy shock, are induced by the balance sheet variables which are state variables in the model.

[60] One way to put this is that the one-time transitory surprise positive interest rate shock has bigger and much more persistent negative effects on inflation and output when the central bank is balance sheet constrained. Perversely, by inducing a response of interest rate in future, as well as a positive relationship between interest rates and inflation, it actually increases monetary non-neutrality. This is different in the monetary policy news shock (or forward guidance) case that I discuss below.

goes back to steady-state. Thus nominal and real rates are positively correlated. When the central bank is balance sheet constrained, while the real interest rate still increases on impact, it continues to be above steady-state throughout the transition back to steady-state. Thus, while the nominal interest rate is below steady-state along the transition, as shown in Figure 9 and discussed above, the real interest rate is above steady-state. Nominal and real interest rates are then negatively correlated. Thus, in this respect, the model has so called Neo Fisherian properties.

4.5.3 Anticipated Future Monetary Policy Shock

I now model effects of forward guidance by considering dynamics following an anticipated monetary policy shock. In particular, in the current period, there is an anticipated negative shock to the interest rate rule in future. I discuss the balance sheet unconstrained central bank first and then contrast the results with the balance sheet constrained central bank. I consider a negative shock to the interest rate rule that is announced today and will realize 20 quarters later.[61]

Balance sheet unconstrained central bank Equilibrium responses of variables to an anticipated negative interest rate shock (of 0.5%) that will be realized 20 quarters later, for the balance sheet unconstrained central bank, are shown in Figure 11. The dynamics following such a monetary policy news shock are standard. Given news of an expansionary monetary policy shock in future and forward-looking behavior, inflation and output increase on impact, and given the feedback interest rate rule, the short-term nominal interest rate does as well. When the news shock is actually realized at 20 quarters, as to be expected, the interest rate falls. Then, as the shock is transitory and the model completely forward looking, output, inflation, and the short-term rate all go back to steady-state the period after the realization of the shock. Also importantly, all throughout the transition, both inflation and output are positively affected and do not go below steady-state. This effectiveness of forward guidance policy in the standard New Keynesian model, even one that occurs very far out in the future, has been dubbed the forward guidance puzzle by Del Negro, Giannoni, and Patterson (2015).

The central bank balance sheet variables do not affect the dynamics of output, inflation, and interest rate. In particular, the dynamics of net interest income is essentially given by first-difference of the short-term interest rate, with it declining until the period before the realization of the news shock and where in the period following the realization of the news shock, it going back to steady-state. Net asset position of the central bank and transfers to the Treasury decline initially and then later increase along the transition back to steady-state. Here, transfers to the Treasury adjust by enough to lead to stable dynamics of the net asset position.

Balance sheet constrained central bank Equilibrium responses of variables to an anticipated negative interest rate shock (of 0.5%) that will be realized 20 quarters later, for the balance sheet constrained central bank, are shown in Figure 12. The results are in sharp contrast to those in Figure 11 as forward guidance leads to a very different dynamics of inflation and output when the central bank is balance sheet constrained. Inflation and output are still positively affected on impact, but by a lower amount. The same applies to the nominal short-term interest rate. Even more importantly, dynamically, both are affected negatively and there is in future both deflation and a contraction in economic activity. After the monetary policy news shock is

[61] I use this particular horizon simply for direct comparison with some results in the literature. My results, which pertain to the comparison of the balance sheet unconstrained and constrained central banks, are robust to the horizon that I consider. I later also present some results on initial impact given different forward guidance horizons.

realized, these variables do not go back to steady-state immediately next period but instead transition back slowly as central bank balance sheet variables are now state variables in the model. Along this transition, nominal interest rate and inflation are again positively correlated, which is a key feature of the model when the central bank is balance sheet constrained.

What drives this lower stimulative effect of forward guidance initially as well as deflation and contraction in output in future? Again, when the central bank is balance sheet constrained, it is critically important to understand the dynamics of the net interest income of the central bank, as it is constrained in its path. While transfers to Treasury do adjust negatively as the net asset position is declining, it does not adjust by enough and thus the net interest income has to adjust at least partially. In particular, consider the period when the negative news shock is realized. The short-term nominal interest rate cannot immediately go back to steady-state next period, unlike when the central bank is balance sheet unconstrained, but instead transitions slowly back. This ensures that the net interest income is positive for several future periods.[62] Only this dynamics ensures a stationary equilibrium, as before the news shock is realized, net interest income is negative with the short-term interest rate falling over time. In this model however, a negative short-term interest rate leads to a negative effect on inflation as they are positively correlated.

Nominal rigidities ensure that output falls after the news shock is realized as well. That is, unlike the balance sheet unconstrained case, where the real interest rate goes back to steady-state after the news shock is realized, here, the real interest rate is in fact positive in future.[63] Finally, forward looking inflation dynamics lead to inflation falling even many periods before the news shock is realized. Moreover, with inflation falling and given the monetary policy reaction function, the nominal interest-rate also falls.

There are thus critical differences between the balance sheet unconstrained and constrained central bank in terms of the effectiveness of the central bank's forward guidance policy. Again, the main driving force of the differences is that the central bank has an imperfect control over the path of its policy instrument when it is balance sheet constrained.

Net interest income, inflation, output, and real interest rate comparison I now plot directly the dynamics of net interest income, inflation, output, and real rate responses in the central bank balance sheet unconstrained and constrained models. These will help highlight even more directly the critical differences between the two regimes. Moreover, it will also help place the dynamics in the context of the previous literature on forward guidance, which I discuss next.

First, Figure 13 shows clearly that the main difference between the two cases occurs in net interest income dynamics after the realization of the news shock. It is precisely this need to keep net interest income high for additional periods that in turn leads to the short-term nominal interest rate staying below steady-state for several periods when the central bank is balances sheet constrained. Figures 14-15 show the dynamics of inflation and output for the two cases. It is clear that the response of output and inflation is lower when central bank balance sheet considerations are at work and that in fact both inflation and output fall in future.

Finally, as an alternative interpretation for the results and to highlight even more starkly the difference between the two models, Figure 16 shows the dynamics of the real interest rate. The key difference between the two cases is in dynamics of the real interest rate after the news shock is realized: in the central bank balance sheet unconstrained model, it goes back to steady-state next period while in the balance sheet

[62] Some of the dynamics might be hard to see because of scale issues, but I show them in detail next.

[63] I show in more detail real interest rate dynamics next.

constrained model it is above steady-state persistently. Moreover, this positive response of the real interest rate in the central bank balance sheet constrained model goes along with the negative response of the nominal interest rate that is shown in Figure 12. Thus, again, the nominal and real interest rate, after the shock is realized, are negatively correlated.

Comparison to the literature I next compare the results above in terms of previous findings in the literature. One approach to reducing the effects of forward guidance in the standard sticky price model has been to introduce incomplete markets such that the forward looking nature of the Euler equation, which constitutes the monetary policy transmission mechanism in such a model, gets diminished. This was illustrated in Mckay, Nakamura, and Steinsson (2016). In terms of the comparison with this approach, Figures 14-15 above, in fact look very similar to the comparison of the complete markets model (which is the same essentially as my balance sheet unconstrained central bank) with the incomplete markets model in Figures 3 and 4 of Mckay, Nakamura, and Steinsson (2016).[64] The similarities extend both to the overall shape of the dynamic responses, as well as the fact that both inflation and output go negative in future. This highlights that even in a fully forward looking model, central bank balance sheet considerations by themselves can decrease the power of forward guidance.[65]

Finally, the results above on the real interest rate dynamics when the central bank has maturity mismatch and cares about transfers to the Treasury also help connect to the findings in Bhattarai, Eggertsson, and Gafarov (2016). In Bhattarai, Eggertsson, and Gafarov (2016), when the central bank has maturity mismatch and cares about transfers to the Treasury, it also provides an incentive to the central to keep nominal interest rates low. In that way, quantitative easing acts as a commitment device to keep nominal interest rates low when the central bank cannot directly commit to time-inconsistent future policy. The model in Bhattarai, Eggertsson, and Gafarov (2016) is based on an optimal policy framework while here I use feedback rules with policy shocks. Regardless, the key reason the results are different is that in Bhattarai, Eggertsson, and Gafarov (2016), lower nominal interest rates along the transition also lead to lower real interest rates under optimal discretionary policy. This highlights how results can depend on the way policy is formulated and the equilibrium/solution concept used in the analysis.[66]

Different forward guidance horizons I now consider different forward guidance horizons, both to show that the results are robust in terms of differences across the two central bank regimes, as well as to connect further to the effectiveness of the forward guidance policy in standard models.

In the literature, the large effects on output of forward guidance policies, even those very far out in the

[64]The forward guidance horizon is 20 quarters in both cases. One small difference is that Mckay, Nakamura, and Steinsson (2016) consider forward guidance in terms of the real interest rate directly, which makes the output response flat in the complete markets model, as the real interest rate is constant before the forward guidance shock. In my model, as I emphasized before, forward guidance is in terms of the nominal interest rate and there is an interest rate feedback rule. Therefore, before the news shock, both nominal and real interest rate have dynamics, even in the balance sheet unconstrained central bank case. Moreover, the endogenous determination of the real interest rate in the two different cases is critical to my analysis, as I have shown above.

[65]As I discuss in the introduction, another approach suggested in the literature is to reduce the forward looking nature of firms by adopting sticky information instead of sticky prices. I undertake a comparison with this approach in the next section as the comparison is much less direct for reasons I discuss there.

[66]Bhattarai, Eggertsson, and Gafarov (2016) use a Markov-perfect equilibrium concept in a model of optimal policy without commitment, while here I model policy using feedback rules with commitment to those rules and policy shocks. Thus, in Bhattarai, Eggertsson, and Gafarov (2016), central bank concerns about its balance sheet lead to change in nominal interest rate behavior (but through an optimal "targeting rule"), without changing fundamentally the equilibrium dynamics of inflation and real interest rates. Here, central bank concerns about its balance sheet necessitates an interest rate policy that does not satisfy the Taylor Principle, thereby changing drastically the equilibrium. Such a change fundamentally affects the dynamic correlation of the nominal rate with inflation and the real rate. These differences are similar to those that might arise in any model based on optimal policy versus feedback policy rules when modeling monetary and fiscal policy interactions.

future, has been dubbed the forward guidance puzzle. From this perspective, I find that the forward guidance puzzle is also mitigated in the central bank balance sheet constrained model. To show this clearly, I present in Figure 17 a standard analysis in this context, which is the initial response of output (scaled by the response to a current, surprise shock) at different forward guidance horizons for the balance sheet unconstrained and constrained central banks. For the balance sheet unconstrained central bank, which constitutes the standard model that is used implicitly in the literature, indeed forward guidance is quite effective, even when the expansionary news shock is realized 20 quarters in the future.[67] For the balance sheet constrained central bank, the initial effects are more diminished, with a rapid decline in the response of output just for a one period forward guidance horizon.[68]

Finally, I show that the comparison between the balance sheet constrained and unconstrained central bank for the anticipated policy shock case is robust to the horizon. In Figure 18 I plot the results when the forward guidance horizon is 4 periods. It is clear that the results are robust. This horizon also helps compare the results with those of the different policy rule and regimes exercise next, where I focus on a 4 period horizon.

4.6 Alternate model of central bank balance sheet constraints

I now consider, like in the simple model, an alternate model of central bank balance sheet constraints and policy rules. Thus, instead of modeling the two cases where the central bank is constrained or not separately in an exogenous way, I now present results where this regime change occurs endogenously as a result of a lower bound on central bank transfers. The private sector part of the model as well as the calibration of all common parameters remain the same as before.

In particular, let the bound on central bank transfers be given by

$$T_t - T \geq -\chi \tag{34}$$

where $\chi > 0$. Then, if the central bank transfer lower bound is not reached, the policy rules are given by

$$\beta Q_{1,t}^{-1} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \exp(\varepsilon_{It}),\ T_t - T = \psi_T \left[\beta\left(b_{t-1} - b\right) - \left(l_{t-1} - l\right)\right]$$

where the interest rate rule satisfies the Taylor Principle (ϕ_π>1) and central bank transfers adjust to net asset position strongly. If the central bank transfer lower bound is reached, then the policy rules are given by

$$\beta Q_{1,t}^{-1} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \exp(\varepsilon_{It}),\ T_t - T = -\chi$$

where the interest rate rule does not satisfy the Taylor Principle (ϕ_π<1). Given a shock, whether and how long the bound on central bank transfers binds and thus for how long the central bank is balance sheet

[67] Note here that the response of output is decreasing with horizon even in the standard balance sheet unconstrained case. If one simulates the model with the zero lower bound binding while the forward guidance is in effect, then output could positive be increasing in horizon as inflation increases with forward guidance horizon at positive interest rates. Sometimes such an analysis, while imposing the zero lower bound, is presented in the literature (Carlstrom, Fuerst, and Paustian (2015), Kiley (2016)).

[68] This comparison again yields similar results to the comparison between complete and incomplete markets model in Mckay, Nakamura, and Steinsson (2016). In Mckay, Nakamura, and Steinsson (2016), since forward guidance is in terms of the real interest rate, at positive interest rates, the level of output response is flat in forward guidance horizon for the standard complete markets model. Also, I show the initial responses normalized by the initial response to a surprise shock. As I describe above, in fact the surprise shock has a bigger effect on output in the model where the central bank is balance sheet constrained. Even if I look at the level of initial output response starting with horizon one however, the response is still lower in the model where the central bank is balance sheet constrained.

constrained is determined endogenously in equilibrium.

I start with results from the surprise monetary policy shock, which are shown in Figure 19. For ease of comparison, two cases are shown at the same time in the Figure. First, there is a balance sheet unconstrained central bank case, where there is no bound on the central bank transfers. This leads to essentially the same results as in Figure 8. Next, a lower bound on the central bank transfers is introduced ($\chi = 0.012$), so that it binds after the first period. Moreover, this lower bound is binding for 34 periods, which is determined endogenously. In this case, it is clear that the model dynamics are different from the unconstrained case. In particular, the dynamics are essentially the same as those described in Figure 9, with the nominal interest rate going below steady-state after the period of the shock and with persistent negative effects on inflation and output. Thus even when central bank transfers are fixed only for a finite number of periods, and are not permanently non-responsive to the state of the balance sheet, it suffices to produce macroeconomics implications similar to the exogenous, permanently balance sheet constrained central bank.

I next present results from the anticipated monetary policy shock, which are shown in Figure 20. For this forward guidance example, I present results based on a 4 quarter horizon and a 1% shock.[69] Again for ease of comparison, two cases are shown at the same time in the Figure. First, the case where there are no bound on the central bank transfers and second, where a lower bound on the central bank transfers is introduced ($\chi = 0.012$). This constraint binds for 31 periods, starting with the period where the news shock is realized. When the lower bound on central bank transfers does not bind, the results are standard as described above while explaining the results in Figure 11. When the lower bound on central bank transfers does bind for some time, the results instead are as described above while explaining the results in Figure 12 for the always constrained central bank. Most importantly, the nominal interest rate stays low for an extended period after the shock is realized, which here is associated with a negative effect on inflation and output. Thus, again, the stimulative effects of forward guidance are reduced in the model when the central bank is balance sheet constrained, even for a finite number of periods.

4.7 Sensitivity Analysis and Extensions

While the purpose of the numerical results above is to highlight the qualitative aspects of the model, it is nevertheless important to assess the robustness of the results to various parameter values and assumptions. I also present results on when a zero lower bound on the short-term interest rate binds. Finally, I also provide a comparison with the sticky information based model to connect further to the literature on effectiveness of forward guidance policies. All the results of this section are in the Appendix.

4.7.1 Price stickiness comparative statics

I first consider a comparative static experiment to further highlight the differences between the balance sheet unconstrained and constrained central bank cases in this general equilibrium model. Additionally, this exercise also serves to highlight the robustness of the results described above for various parameter values, especially for the balance sheet constrained central bank. In particular, I consider different degrees of price stickiness, that is various values for the price adjustment cost parameter.

For the unconstrained central bank, as shown in Figure 23, and as is to be expected given the Taylor rule used by the central bank, higher price stickiness (that is, a greater value of κ) leads to a smaller equilibrium response of inflation when an interest rate shock hits the economy. This also implies a larger response of the

[69] I do this instead of a 20 period horizon, purely to not make the number of periods very long in the graphs. The main points behind the results do not depend on the horizon.

short-term interest rate in equilibrium. For a positive interest rate shock, inflation drops by less when prices are more sticky.

In sharp contrast, for the constrained central bank, as shown in Figure 24, higher price stickiness leads to a larger equilibrium response of inflation when an interest rate shock hits the economy. What drives this result? When prices are more sticky, in response to the interest rate policy shock, the short-term interest rate rises by more on impact, as is the case for the unconstrained central bank case. Thus, just on impact, inflation drops less for more sticky prices. But following the impact period, the short-term interest rate falls more, throughout the rest of the transition back to steady-state. Since short-term interest rate and inflation are positively correlated along the transition in this model, inflation also responds more strongly with greater degree of price stickiness. This stronger response of the inflation when prices are more sticky is a unique feature of my model, one that is driven by the restriction imposed on future interest rate policy due to central bank balance sheet considerations. For instance, in the fiscal theory of the price level model, when prices are more sticky, inflation responds by less (note that a positive interest rate shock in that model raises inflation).[70]

4.7.2 Policy parameter comparative statics

I consider now several values for the policy feedback parameters, both in the interest rate rule and transfer to Treasury rule. Since the balance sheet unconstrained central bank model is well understood, for concreteness, I focus on the balance sheet constrained case. Moreover, I focus on the standard monetary policy shock experiment. Figures 25 and 26 show the results for various values of the two policy feedback parameters for the balance sheet constrained central bank, where I include the baseline parameterization for straightforward comparison. It is clear that the shape of the responses, including critically, the dynamics of the net interest income of the central bank, are robust.

4.7.3 Extensions

I next consider two extensions to the baseline analysis where I focus again on showing results for the balance sheet constrained central bank. First, I allow the two balance sheet feedback rule parameters to be different. Thus, I now use ψ_{bT} =0.009 and ψ_{lT}=0.005, whereas in the baseline they were both equal to 0.009. The equilibrium responses of variables to a positive interest rate shock for the balance sheet constrained central bank for this different model and calibration are shown in Figure 27. It is clear that the results are robust to this extension.

Second, I use a different non-linear solution method, in particular, a stochastic third-order perturbation solution method. Compared to the baseline non-linear deterministic solution method, this method in principle incorporates stochastic aspects such as a term premium. The equilibrium responses of variables to a positive interest rate shock for the balance sheet constrained central bank for this different solution method are shown in Figure 28. It is again clear that the results are robust to this alternate solution method.

[70] Additionally, here output responds more negatively when prices are more sticky for both cases. The model and mechanism is thus completely different from the standard sticky price model in Bhattarai, Eggertsson, and Schoenle (2014), where output and inflation both can respond less negatively when prices are more sticky, even with the Taylor principle holding. This happens in the standard sticky price model if the weight on output in the Taylor rule is high enough to satisfy the Taylor principle, while the weight on inflation is less than the persistence of the monetary policy shock, and thus necessarily less than one. Bhattarai, Eggertsson, and Schoenle (2014) show that in such a case, the aggregate demand curve becomes upward sloping, thereby leading to a smaller response of both output and inflation for a monetary policy shock when the aggregate supply curve becomes steeper. Here, in the Taylor rule, there is only a response to inflation in both the balance sheet constrained and unconstrained cases and the Taylor principle does not hold in the balance sheet constrained central bank case.

4.7.4 Zero lower bound

I present now an analysis where the zero lower bound might bind in the model. The motivation for this analysis is to assess if the extent to which the zero lower bound is a constraint on monetary policy depends on whether the central bank is balance sheet constrained or not. Even though the balance sheet constrained central bank in the model does not satisfy the Taylor principle in its interest rate rule and the propagation of a demand shock is very different as I show below, which makes the issue of negative effects typically associated with a binding zero lower bound hard to directly compare, it is perhaps still be of some interest to compare the two central bank models.[71] As is standard in the literature, I will consider the possibility of a liquidity trap situation due to a negative demand/preference shock. Thus, I change preferences to allow for a discount factor shock that follows an AR(1) process.[72]

In order to help interpret the results at the zero lower bound, as well as to illustrate the very different propagation mechanisms, in Figure 29 I first show the equilibrium response of the variables to a persistent negative demand shock (with persistence parameter 0.8 and a size of 1%) for the balance sheet unconstrained and constrained central banks. The dynamics for the balance sheet unconstrained central bank are standard: the central bank decreases the nominal interest rate to counter the negative demand shock, while both inflation and output fall. For the balance sheet constrained central bank however, while the nominal interest rate still falls for a few period, later it rises persistently. This reversal in the path of the nominal interest rate can be traced to balance sheet considerations, that is, the dynamics of the net interest income of the central bank, as it is constrained in its path. Because the net interest income of the central bank has to adjust in future, it necessitates that the short-term interest rate go above steady-state in future and then slowly transition back.[73] In this model though, as I have emphasized several times above, nominal interest rates and inflation are positively correlated along the transition back to steady-state. Thus, here, we see a positive effect on inflation and a boom in output along the transition.

These results at positive interest rates for a demand shock already provide hints that the zero lower bound is likely to be a much less severe concern for the balance sheet constrained central bank and that a comparison with the balance sheet unconstrained central bank is not straightforward. First, the nominal interest rate drops by less, which will require a much larger shock to make the zero lower bound bind in the initial period. Moreover, given the large and persistent positive response in future, if one does not insist on a binding zero lower bound initially, then in future, it can bind for a *positive* demand shock.[74] Second, since the transition dynamics are such that the nominal interest rate goes above steady-state before transitioning back, which lead to a positive effect on inflation, it suggests that the drop in inflation and output are likely to be less. This is indeed what I show in Figure 30 where a large enough persistent negative demand shock (with persistence parameter 0.8 and a size of 50%) makes the zero lower bound bind in both models initially. For by now well understood reasons, for the balance sheet unconstrained central bank, there is a large drop in inflation and output. For the balance sheet constrained central bank, while inflation and output does drop

[71]It is for this reason that I do not consider the zero lower bound in most the analysis and instead focus on the monetary policy transmission mechanism only.

[72]Period utility is now given by $U(c_t, h_t) = \xi_t (\log C_t - A h_t)$ where ξ_t is a preference shock. The optimality conditions change due to this shock, but in very obvious ways, by essentially just changing the stochastic discount factor in the model. I thus omit a detailed derivation of the optimality conditions.

[73]It is exactly this kind of reversal in the path of the nominal interest rate that I highlighted for the surprise monetary policy shock in Figure 9. For a transitory preference shock, the response of the nominal interest rate would show a reversal of exactly the same kind: it drops initially, but then rises above steady-state immediately in the next period. Bhattarai, Lee, and Park (2014) show a similar reversal in the path of inflation/interest rate in a one-period debt model when the monetary and fiscal policy regime is such that the fiscal theory of the price level is in operation. Thus, these dynamics are similar, even though the models and the key mechanisms are very different.

[74]This feature, in particular, makes the two models very different while studying a zero lower bound situation.

while at the zero lower bound, it drops by less than for the balance sheet unconstrained central bank, and moreover, inflation later increases above steady-state.

4.7.5 Comparison with the sticky information model

Carlstrom, Fuerst, and Paustian (2015) and Kiley (2016) use a sticky information friction, as introduced to the literature in Mankiw and Reis (2002) instead of the usual sticky price friction and propose that such a model can reduce the effectiveness of forward guidance. I provide the details of this model, which differs only on the firm side and leads to a different Phillips curve than the model here, in the appendix. For parameterization I use the same parameter values for the parameters that are common between the models and for the sticky information parameter, I calibrate it such that the slope of the Phillips curve is the same across the two models. In Figures 31-32 I show the equilibrium response of the variables to an anticipated negative interest rate shock (of 0.5%) that will be realized 20 quarters later, where I compare the sticky information model with the balance sheet constrained and unconstrained models respectively.

Unlike the comparison with the incomplete markets sticky price model, the comparison with the sticky information model is not as direct and clear. First, Figure 31 shows that when compared to the balance sheet constrained model, there are important differences in that there is no contraction in economic activity in future and while inflation response is muted overall in the sticky information model, it falls below steady-state only after the monetary policy news shock is realized.[75] Even when compared to the balance sheet unconstrained model in Figure 32, which is essentially just a comparison between the sticky information and sticky price model, the sticky information model does clearly mute the effects on inflation, especially after the shock is realized, but again, the key difference is that after the shock is realized, there is no contraction in output. In fact here, the effect on output on impact are higher under the sticky information model compared to the sticky price model regardless of whether the central bank is balance sheet constrained or not. In contrast, the effect on inflation on impact are lower under the sticky information model compared to the sticky price model regardless of whether the central bank is balance sheet constrained or not.[76]

Why is the behavior of output different, with additional expansionary effect, once the news shock is realized? In the sticky information model, as is well known, there are dynamics in the model and Phillips curve due to dependence of current inflation on past expectations of inflation/output. Thus, once the news shock is realized, the nominal interest rate reverts slowly back to steady-state. Since the Taylor principle holds here, nominal interest rate below steady-state is accompanied by the real interest rate being below steady-state. This in-turn then implies that output is in fact above steady-state, as the Euler equation applies in this model. Moreover, this dynamics after the shock is realized also leads to output being above steady-state throughout the transition. For the balance sheet unconstrained model, which is essentially just the standard sticky price model that is completely forward looking, the nominal and real interest rate revert back to steady-state immediately after the monetary policy news shock is realized. Thus, there is no positive effect on output due to real interest rate being below steady-state.

[75] As some details of this comparison, especially on level effects, can depend on precisely how one calibrates the model, I focus here mostly on the most general differences, which have to do with the dynamics of output, especially once the news shock is realized.

[76] But again, note that inflation goes below steady-state in the sticky information model only after the new shock is realized, unlike the balance sheet constrained model.

5 Conclusion

As a result of unconventional monetary policy, maturity and/or currency mismatches have appeared in many central bank balance sheets. I analyze the implications of such developments in the central bank balance sheet in an equilibrium model. In the model, the central bank holds long-term domestic or short-term foreign currency assets while it issues short-term domestic currency liabilities. Following Sargent and Wallace (1981), I consider a *balance sheet constrained central bank*, one which faces a constraint in the path of its remittances to the Treasury.

Overall, my results suggest that whether a central bank is affected by balance sheet constraints or not can drastically affect the monetary policy transmission mechanisms and the equilibrium dynamics and comovement of interest rates (exchange rates) and inflation. I show using simple balance sheet arithmetic that *a balance sheet constrained central bank* central bank loses freedom in its policy actions. In particular, *interest rate policy (or exchange rate policy)* get constrained by balance sheet considerations. If the expected future change of the short-term nominal interest rate (or the nominal exchange rate) increases today, it has to decrease in future. This constitutes *unpleasant central bank balance sheet arithmetic*.

I embed such a central bank in a non-linear dynamic general equilibrium model to study monetary policy transmission. For a standard surprise monetary policy shock, the model with balance sheet constrained central bank leads to a very different dynamics of the short-term interest rate and its correlation with inflation. Central bank balance sheet considerations make forward guidance less effective. Following news of a negative short-term nominal interest rate shock, while inflation and output do increase initially, they do so by a smaller amount than in the standard case, and are in fact followed by deflation and contraction in economic activity in future. The key reason is that in the model, real interest rate remains persistently high even though the nominal interest rate is (promised to be) low.

In future work, it will be interesting to fully assess the quantitative significance of these results, including when the zero lower bound can be a concern. Moreover, since the equilibrium dynamics of inflation, interest rates, and exchange rates can be very different when the central bank is balance sheet constrained compared to when it is not, it might be worthwhile to empirically assess whether some historically puzzling behavior of these variables, either for advanced or emerging market countries, can be explained in terms of the model presented in this paper.

References

[1] Bassetto, Marco, and Todd Messer (2013), "Fiscal Consequences of Paying Interest on Reserves," *Fiscal Studies*, 34, 413-436.

[2] Benigno, Pierpaolo, and Salvatore Nistico (2015), "Non-Neutrality of Open-Market Operations," working paper.

[3] Berriel, Tiago C., and Saroj Bhattarai (2009), "Monetary Policy and Central Bank Balance Sheet Concerns," *The B.E. Journal of Macroeconomics*, 9, 1-33.

[4] Berriel, Tiago C., and Saroj Bhattarai (2012), "Hedging Against the Government: A Solution to the Home Asset Bias Puzzle," *American Economic Journal: Macroeconomics*, 5, 102-34.

[5] Berriel, Tiago C., and Arthur Mendes (2015), "Central Bank Balance Sheet, Liquidity Trap, and Quantitative Easing," working paper.

[6] Bhattarai, Saroj, and Konstantin Egorov (2016), "Optimal Monetary and Fiscal Policy at the Zero Lower Bound in a Small Open Economy," working paper.

[7] Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park (2014), "Inflation Dynamics: The Role of Public Debt and Policy Regimes," *Journal of Monetary Economics*, 67, 93-108.

[8] Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park (2016), "Policy Regimes, Policy Shifts, and U.S. Business Cycles," *Review of Economics and Statistics*, 98, 968-983.

[9] Bhattarai, Saroj, Gauti Eggertsson, and Bulat Gafarov (2016), "Time-Consistency and the Duration of Government Debt: A Signalling Theory of Quantitative Easing," working paper.

[10] Bhattarai, Saroj, Gauti Eggertsson, and Raphael Schoenle (2014), "Is Increased Price Flexibility Stabilizing? Redux," working paper.

[11] Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian (2015), "Inflation and Output in New Keynesian Models with a Transient Interest Rate Peg," *Journal of Monetary Economics*, 76, 230-243.

[12] Canzoneri, Matthew, Robert Cumby, and Behzad Diba (2011), "The Interaction between Monetary and Fiscal Policy," in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, 3B, 935-999.

[13] Carpenter, Seth B., Jaen. E. Ihrig, Elizabeth C. Klee, Daniel W. Quinn, and Alexander H. Boote (2013), "The Federal Reserve's Balance Sheet and Earnings: A Primer and Projections," working paper.

[14] Del Negro, Marco, and Christopher A. Sims (2015), "When Does a Central Bank's Balance Sheet Require Fiscal Support?" *Journal of Monetary Economics*, 73, 1-19.

[15] Del Negro, Marco, Marc Giannoni, and Christina Patterson (2015), "The Forward Guidance Puzzle," working paper.

[16] Ennis, Huberto M., and Alexander L. Wolman (2010), "Excess Reserves and the New Challenges for Monetary Policy," *Economic Brief*, March, 1-3.

[17] Hall, Robert E. and Ricardo Reis (2013), "Maintaining Central-Bank Solvency under New-Style Central Banking," working paper.

[18] Ireland, Peter N. (2000), "Interest Rates, Inflation, and Federal Reserve Policy since 1980," J*ournal of Money, Credit, and Banking*, 32, 416-34.

[19] Kiley, Michael (2016), "Policy Paradoxes in the New-Keynesian Model," *Review of Economic Dynamics*, 21, 1-15.

[20] Mankiw, Gregory N., and Ricardo Reis (2002), "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Philips Curve," *Quarterly Journal of Economics*, 117, 1295-1328.

[21] Mckay, Alisdair, Emi Nakamura, and Jon Steinsson (2016), "The Power of Forward Guidance Revisited," *American Economic Review*, 106, 3133-58.

[22] Milton, Sure, and Peter Sinclair (2015), *The Capital Needs of Central Banks*, London: Taylor and Francis.

[23] Reis, Ricardo (2013), "The Mystique Surrounding the Central Bank's Balance Sheet, Applied to the European Crisis," *American Economic Review*, 103, 135-40.

[24] Rotemberg, Julio J. (1982), "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies*, 49, 517-531.

[25] Sargent, Thomas J., and Neil Wallace (1981), "Some Unpleasant Monetarist Arithmetic," *Quarterly Review*, 1-17.

[26] Sims, Christopher A. (1994), "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4, 381-99.

[27] Sims, Christopher A. (2004), "Fiscal Aspects of Central Bank Independence," in Hans-Werner Sinn, Mika Widgren, and Marko Kothenburger, eds., *European Monetary Integration*, Cambridge: MIT Press, 103-116.

[28] Sims, Christopher A. (2005), "Limits to Inflation Targeting," in Ben S. Bernanke and Michael Woodford, eds., *The Inflation Targeting Debate*, Chicago: University of Chicago Press, 283-310.

[29] Sims, Christopher A. (2011), "Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the 1970s," *European Economic Review*, 55, 48-56.

[30] Sims, Christopher A. (2013), "Paper Money," *American Economic Review*, 103, 563-84.

[31] Stella, Peter (2005), "Central Bank Financial Strength, Transparency, and Policy Credibility," *IMF Staff Papers*, 52, 335-365.

[32] Jeanne, Olivier, and Lars E. O. Svensson (2007), "Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank," *American Economic Review*, 97, 474-490.

[33] Wallace, Neil (1981), "A Modigliani-Miller Theorem for Open-Market Operations," *American Economic Review*, 71, 267-274.

[34] Williamson, Stephen (2016), "Neo Fisherism: A Radical Idea, or the Most Obvious Solution to the Low-Inflation Problem," *Regional Economist*, July, 4-9.

[35] Woodford, Michael (1994), "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy," *Economic Theory*, 4, 345-80.

[36] Zhu, Feng (2004), "Central Bank Balance Sheet Concerns, Monetary and Fiscal Rules, and Macroeconomic Stability," working paper.

6 Tables and figures

Table 1: Parameterization of the model

Parameter	Value	Parameter	Value
b	1	ϕ_π	1.05; 0.95
κ	150	ψ_T	0.012; 0.009
T	0.1	ν	7
β	0.99	ρ	0

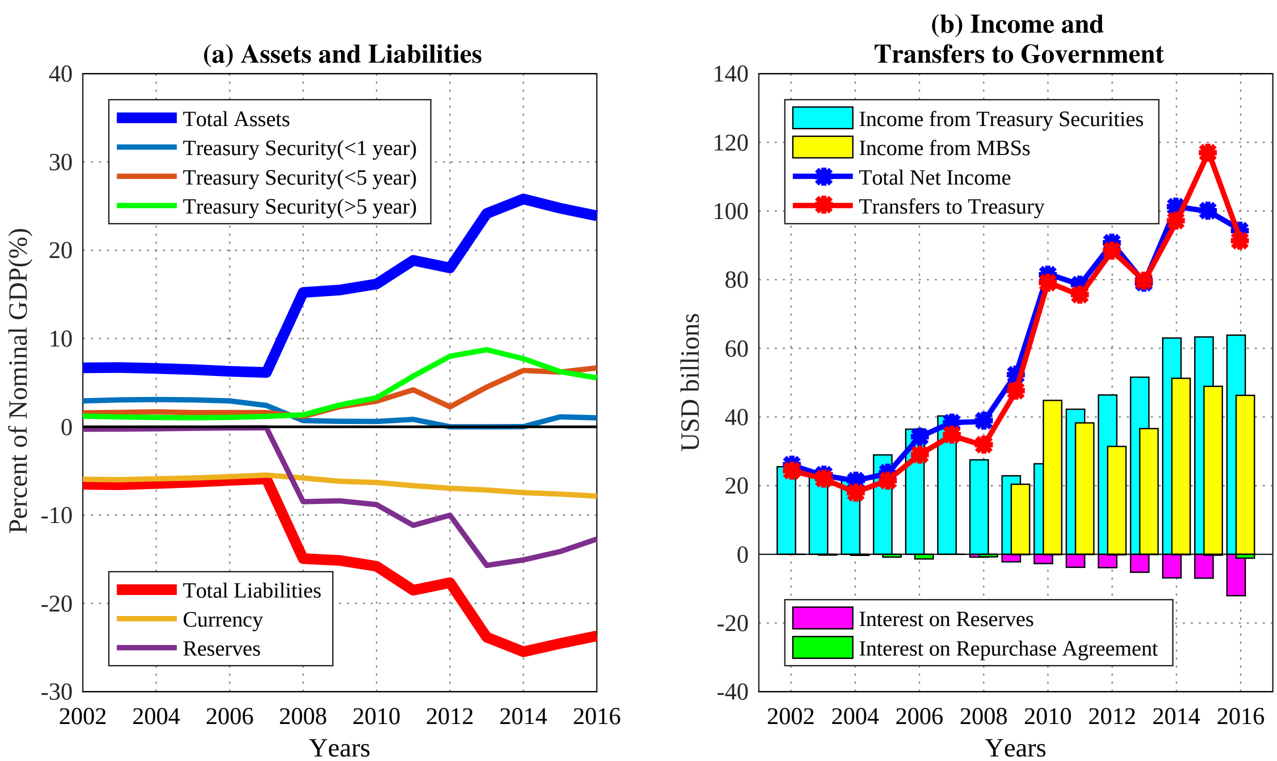

Figure 1: Assets, Liabilities, and Income of the Federal Reserve

Notes: Source of data is Federal Reserve Board. For assets, other than Treasuries, the rest of holdings are primarily Agency debt securities and MBSs, which are also long-term.

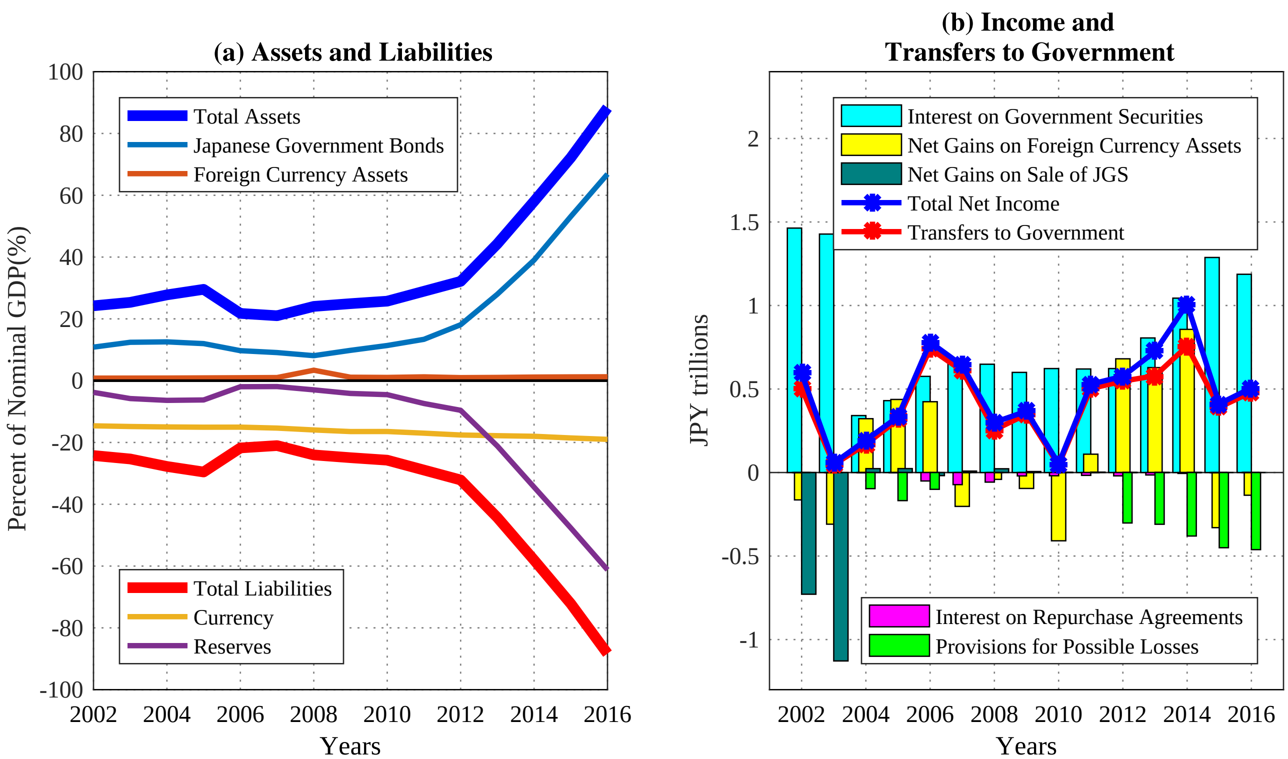

Figure 2: Assets, Liabilities, and Income of the Bank of Japan

Notes: Source of data is Bank of Japan. For assets, the other two important categories are Treasury discount bills and loans. The total net income measure is the sum of net operational and special income.

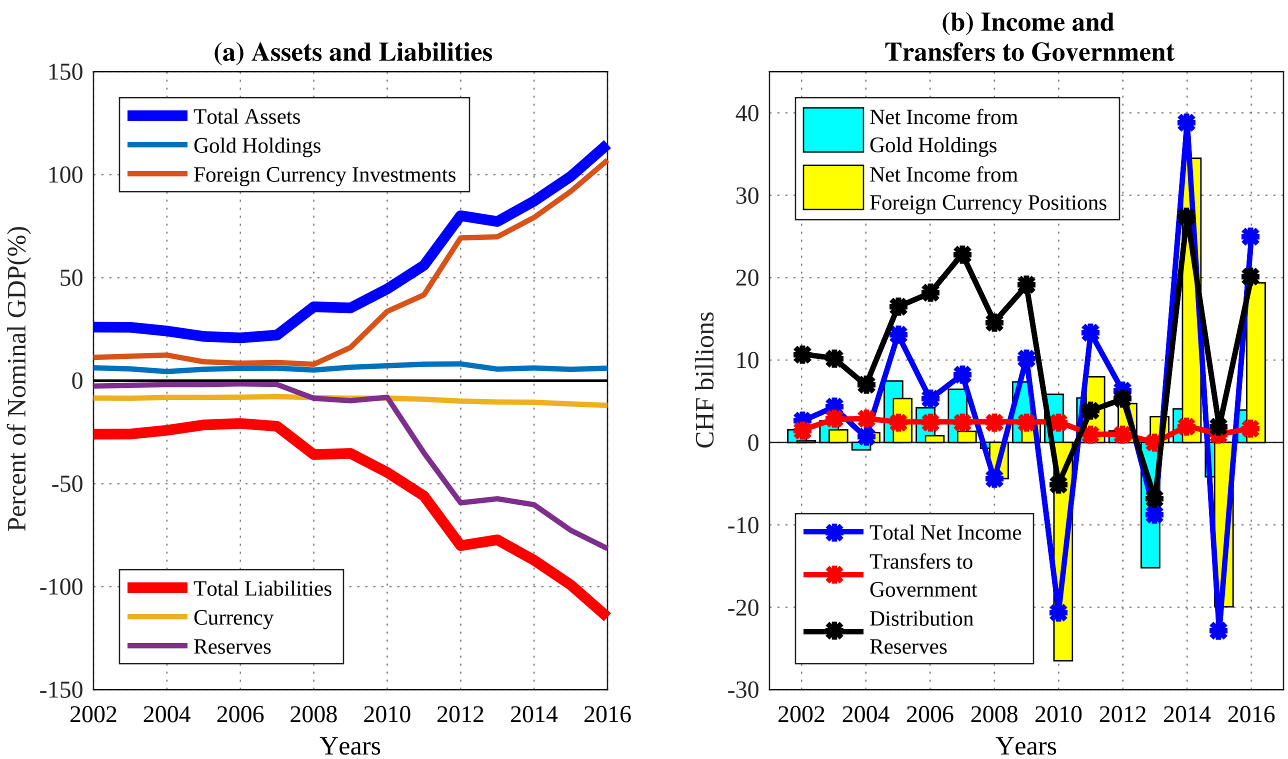

Figure 3: Assets, Liabilities, and Income of the Swiss National Bank

Notes: Source of data is Swiss National Bank. The transfers to government for SNB is fixed over the medium term using projections with an aim of smoothing the flow. SNB has a distribution reserve which is shown in the figure as well.

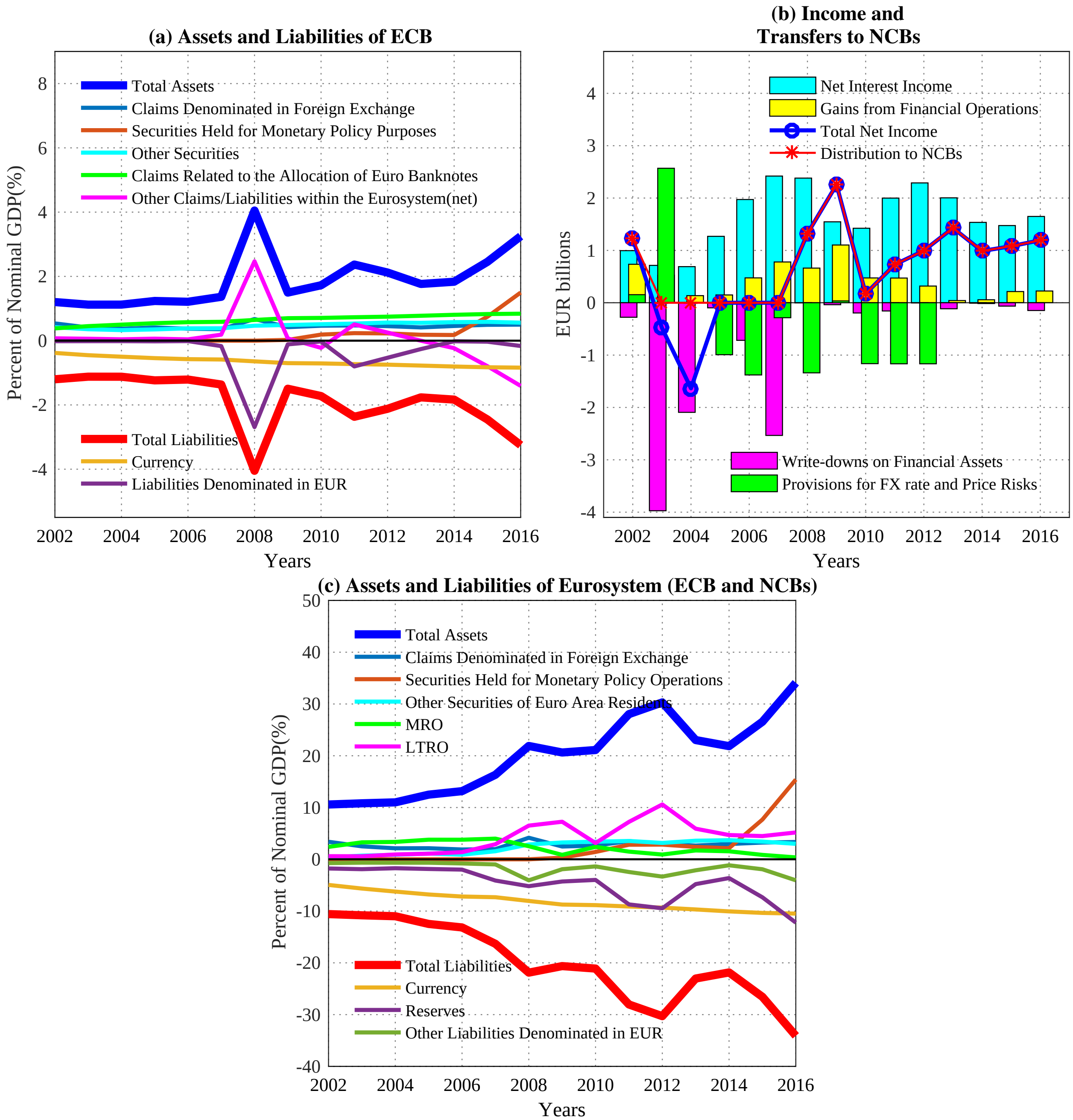

Figure 4: Assets, Liabilities, and Income of the European Central Bank and the Consolidated Euro System

Notes: Source of data is European Central Bank. Securities held for monetary policy purposes include asset purchases under the public sector purchase program, securities market program, asset-backed securities purchase program, corporate sector purchase program, and covered bond purchase program. Other liabilities denominated in Euro are liabilities to Euro Area and non-Euro Area residents. The transfers to national central banks is the sum of the "interim" profit distribution and profit distribution and equals total net income.

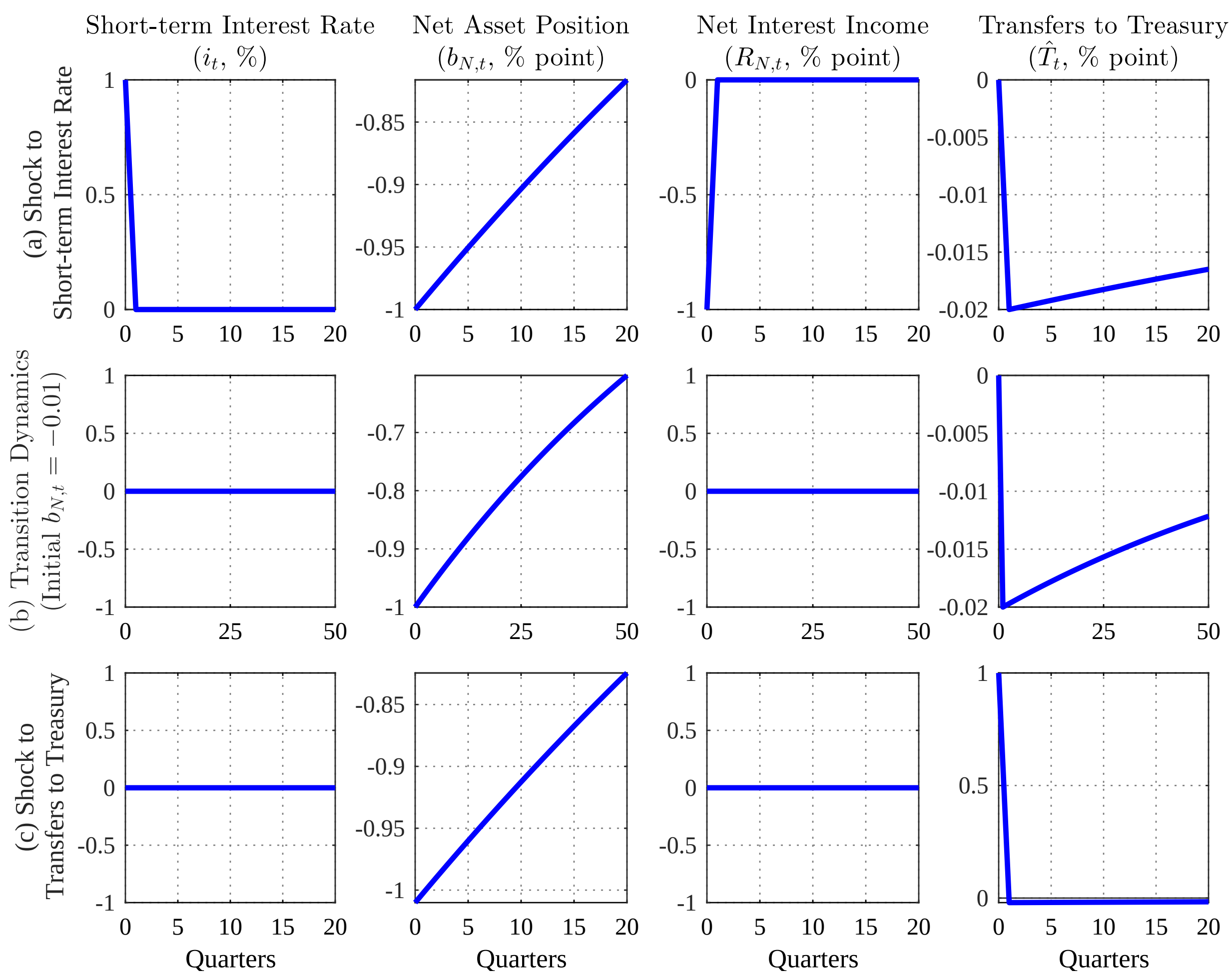

Figure 5: Impulse responses and transition dynamics in the closed economy balance sheet unconstrained central bank model

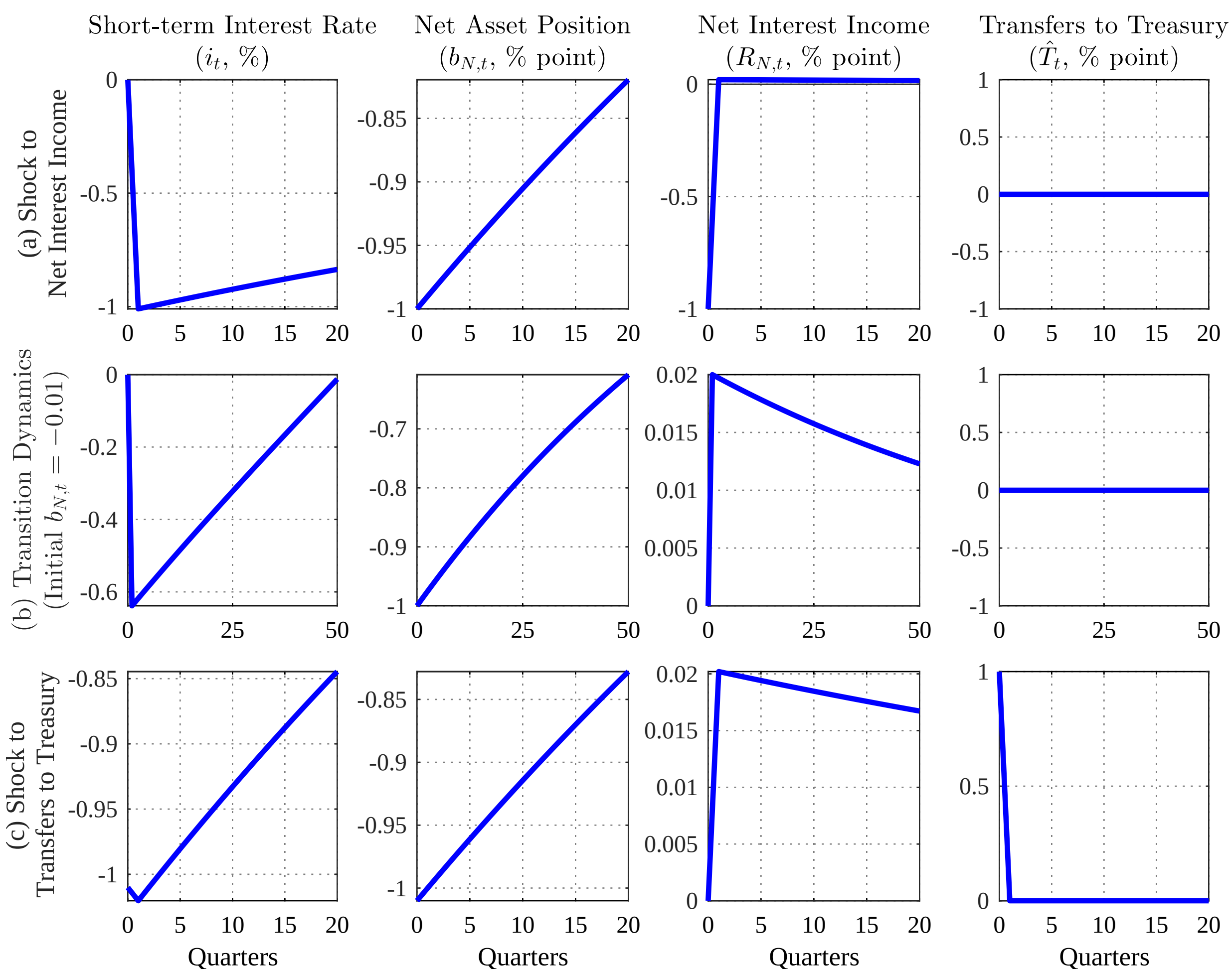

Figure 6: Impulse responses and transition dynamics in the closed economy balance sheet constrained central bank model

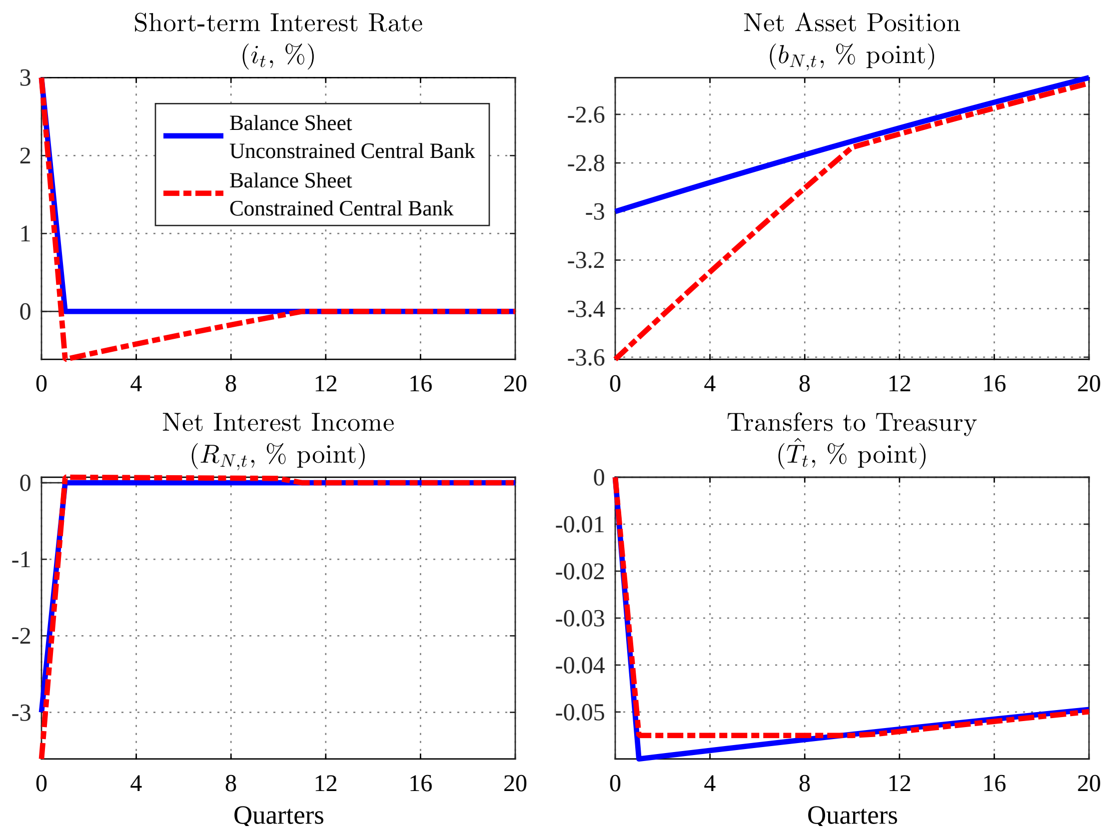

Figure 7: Impulse responses to a shock to net interest income in the closed economy balance model (different transfer rule and policy regime for the balance sheet constrained central bank)

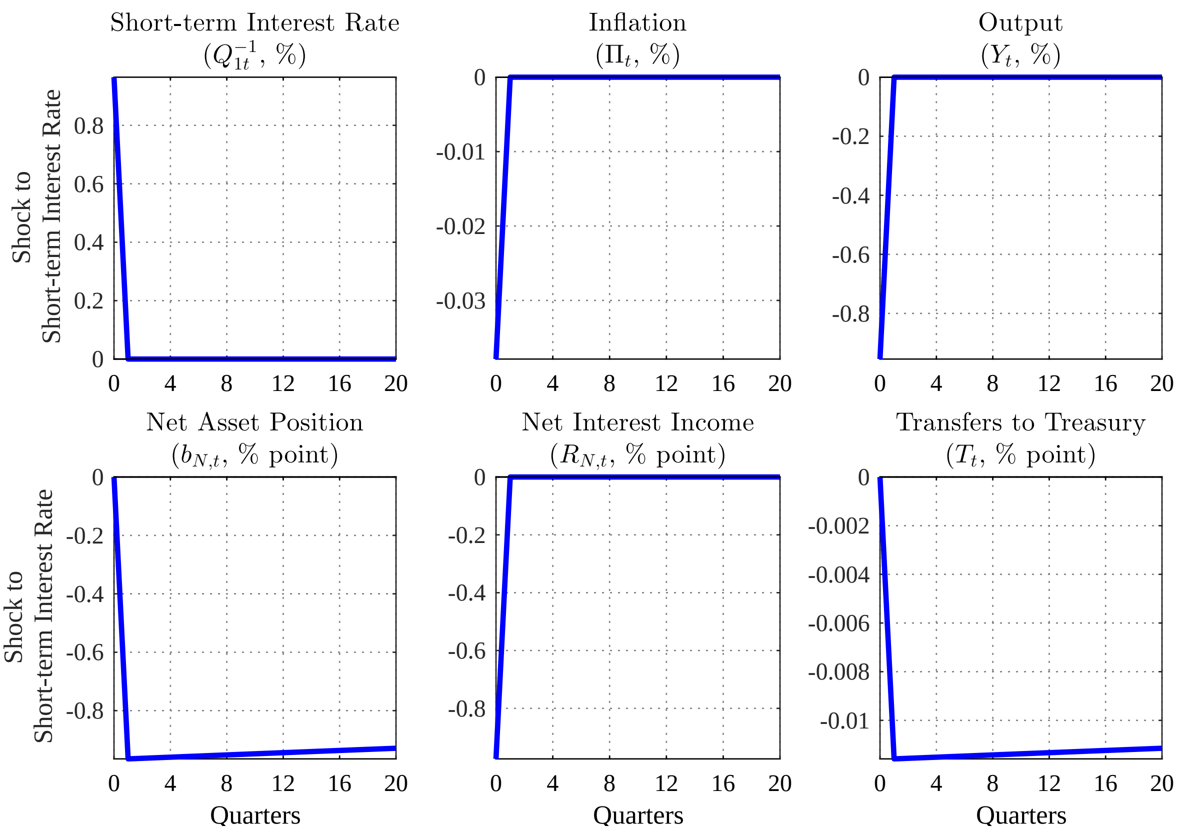

Figure 8: Non-linear impulse responses in the closed economy general equilibrium balance sheet unconstrained central bank model

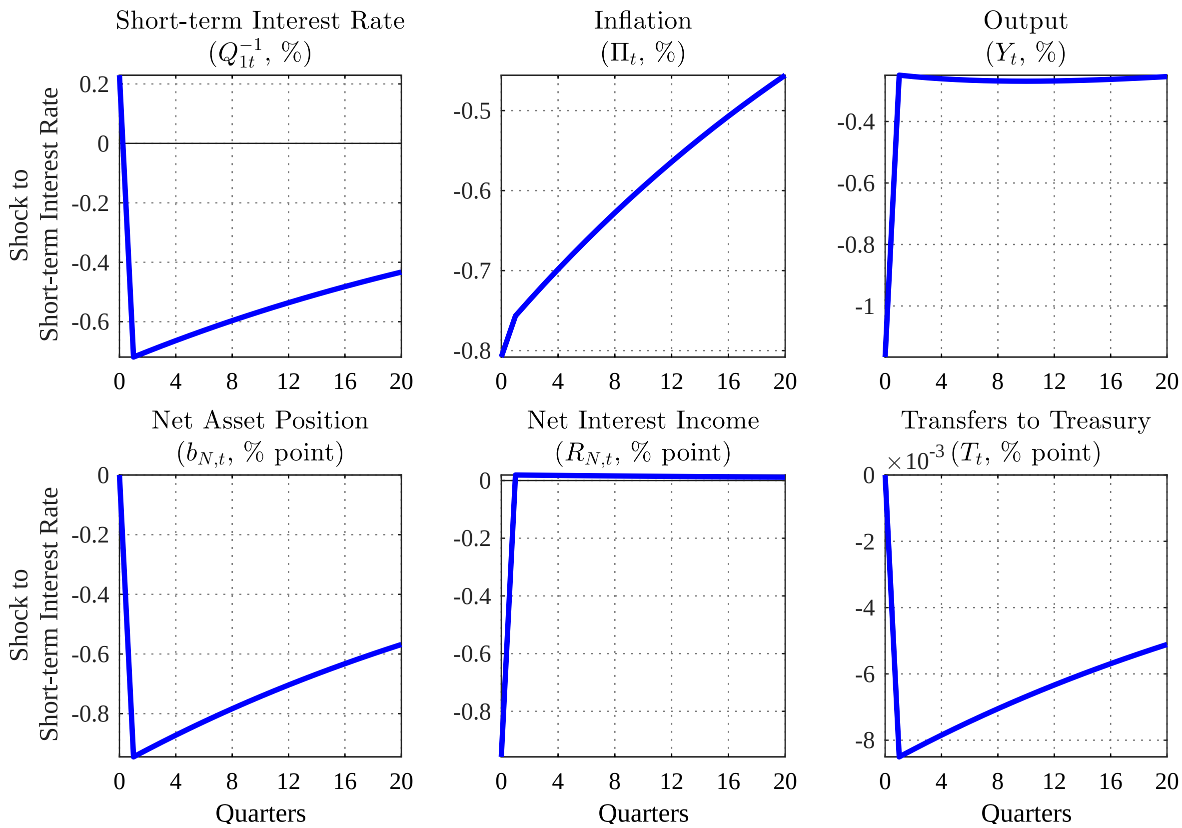

Figure 9: Non-linear impulse responses in the closed economy general equilibrium balance sheet constrained central bank model

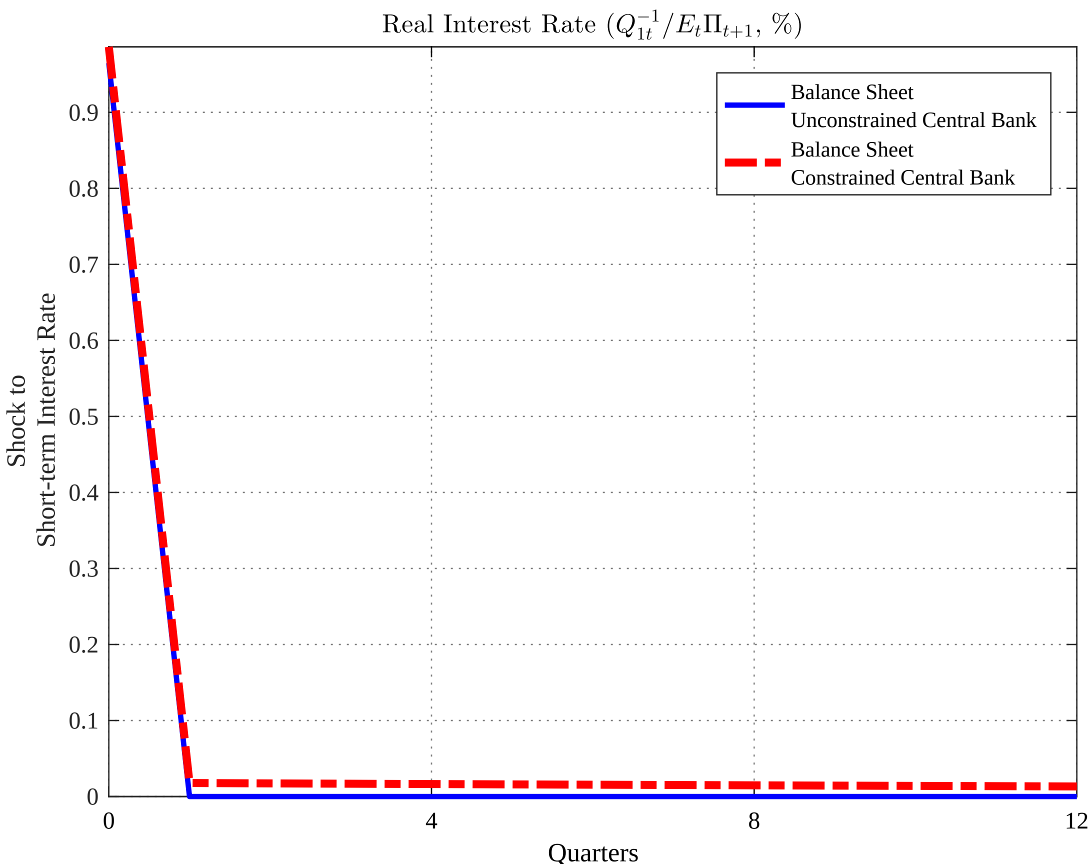

Figure 10: Comparison of response of real interest rate in the closed economy general equilibrium balance sheet unconstrained and constrained central bank models

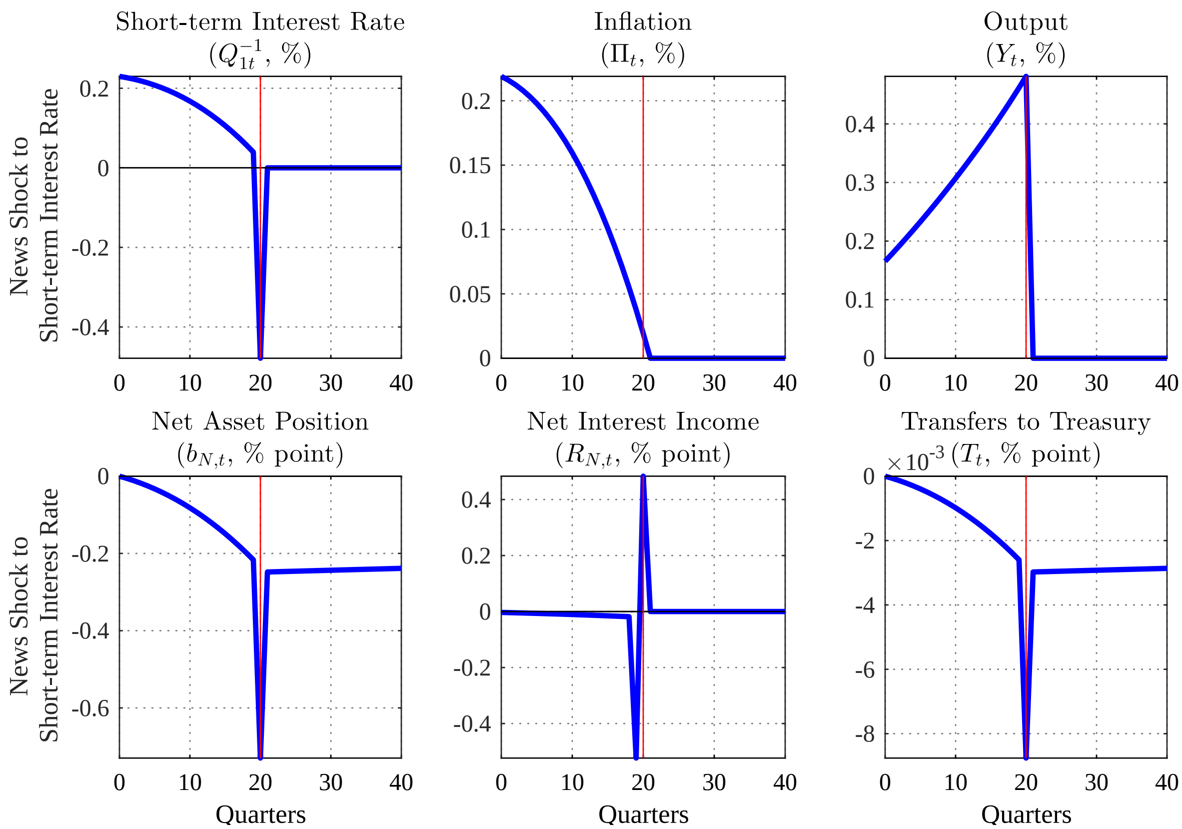

Figure 11: Non-linear impulse responses in the closed economy general equilibrium balance sheet unconstrained central bank model (forward guidance)

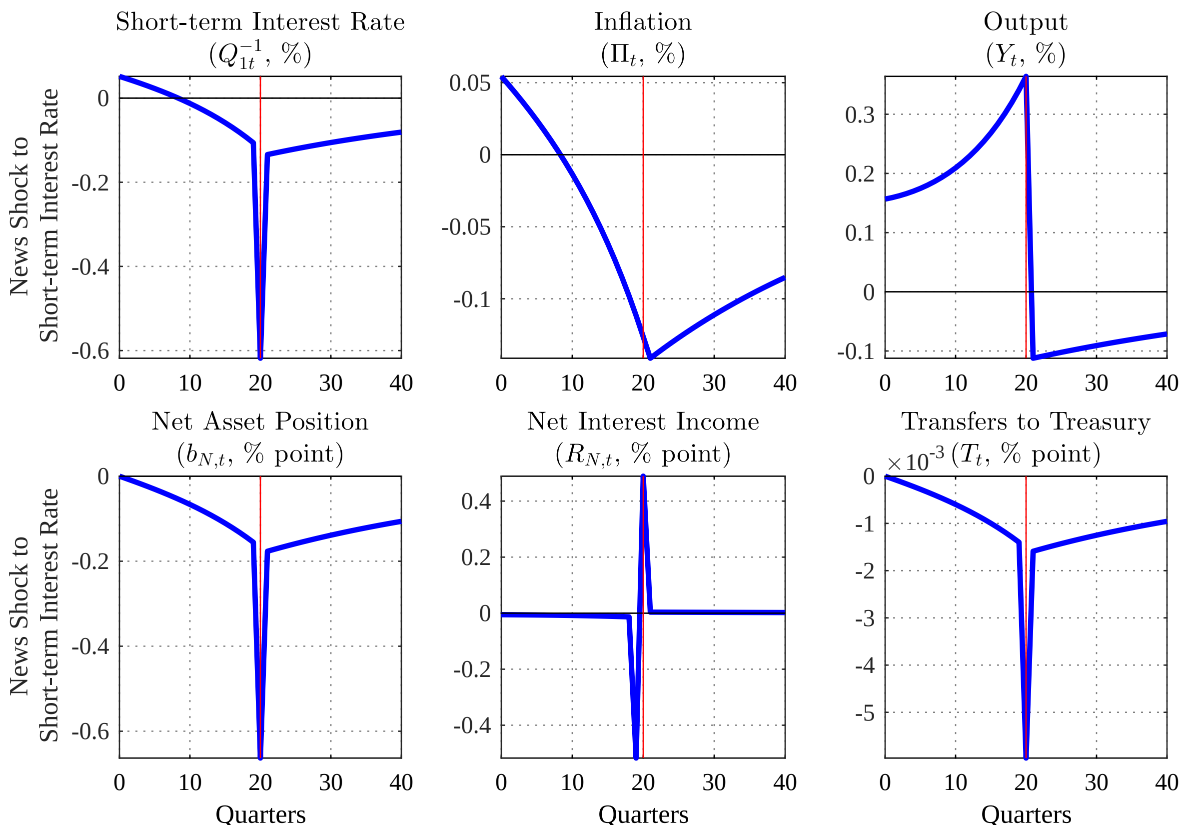

Figure 12: Non-linear impulse responses in the closed economy general equilibrium balance sheet constrained central bank model (forward guidance)

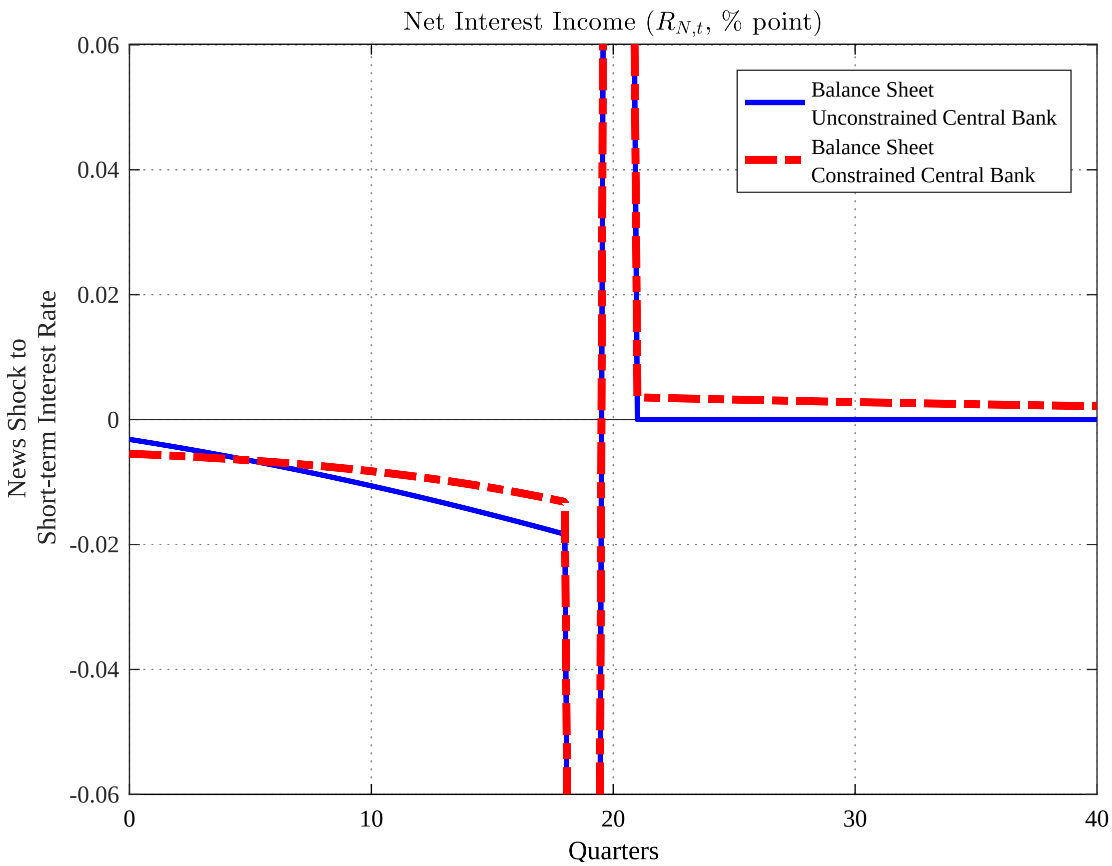

Figure 13: Comparison of response of net interest income in the closed economy general equilibrium balance sheet unconstrained and constrained central bank models (forward guidance)

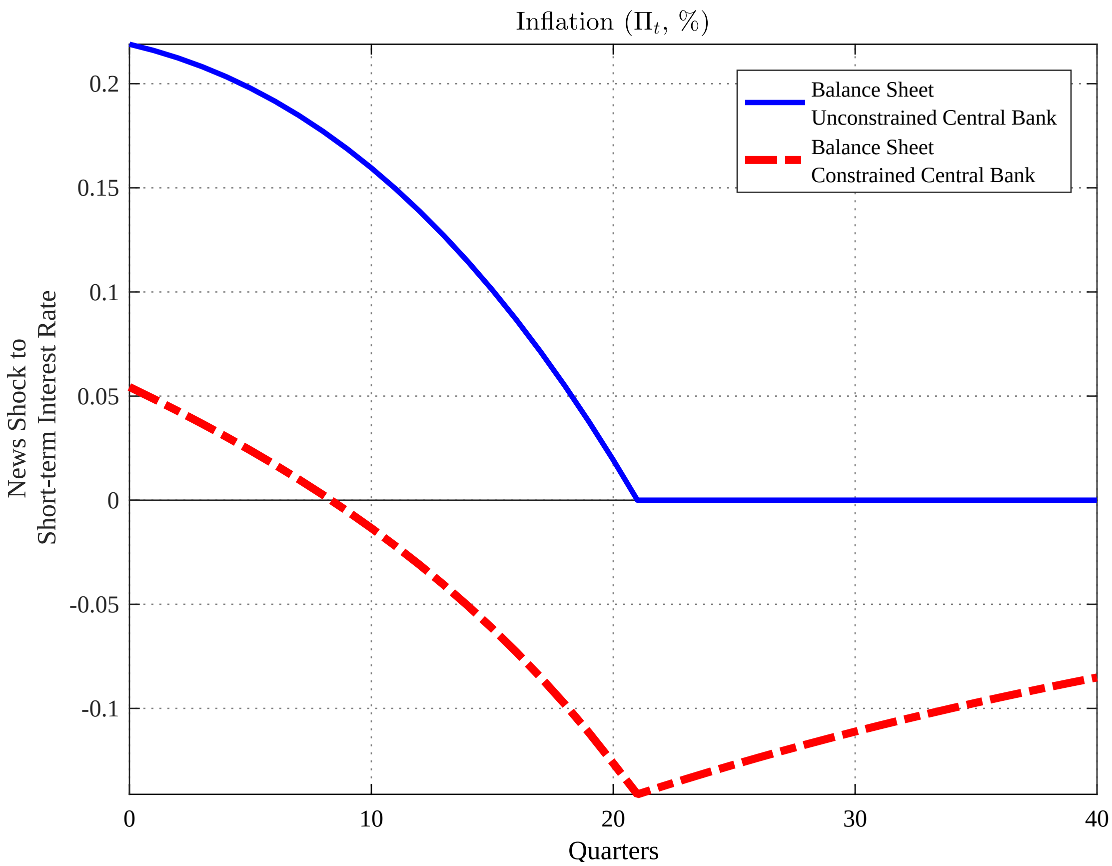

Figure 14: Comparison of response of inflation in the closed economy general equilibrium balance sheet unconstrained and constrained central bank models (forward guidance)

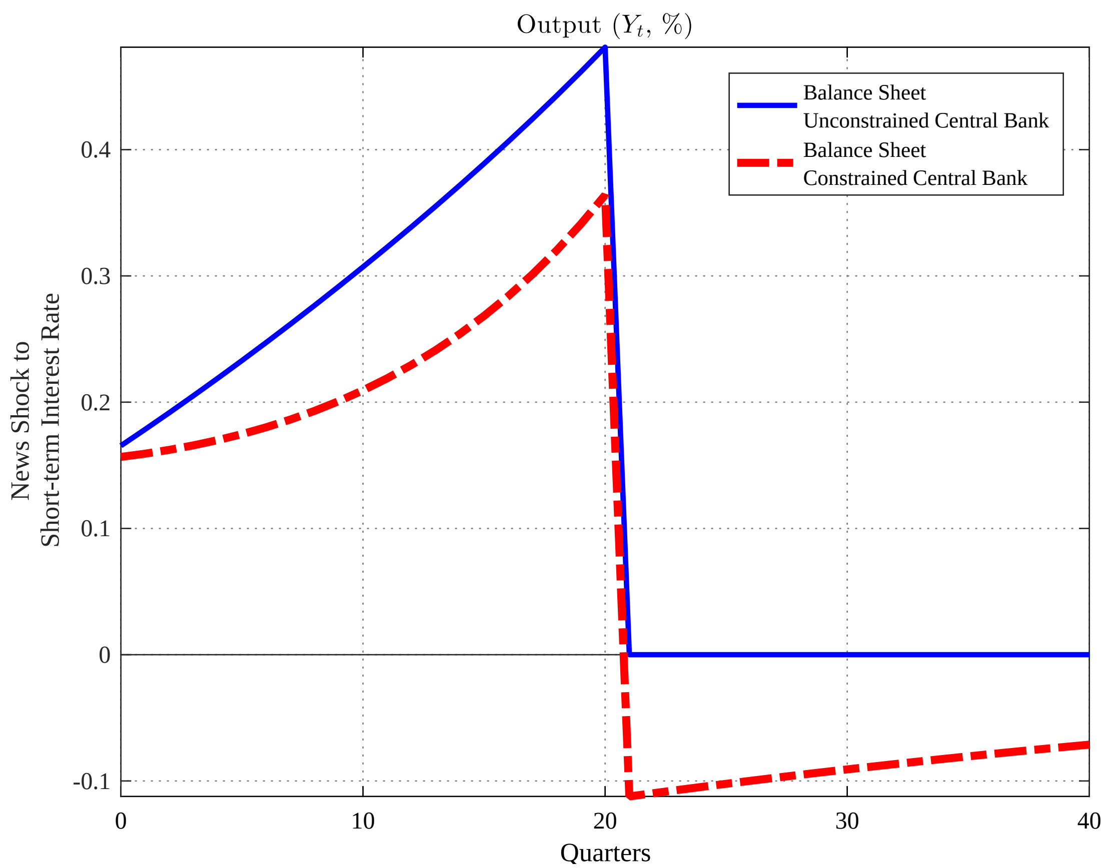

Figure 15: Comparison of response of output in the closed economy general equilibrium balance sheet unconstrained and constrained central bank models (forward guidance)

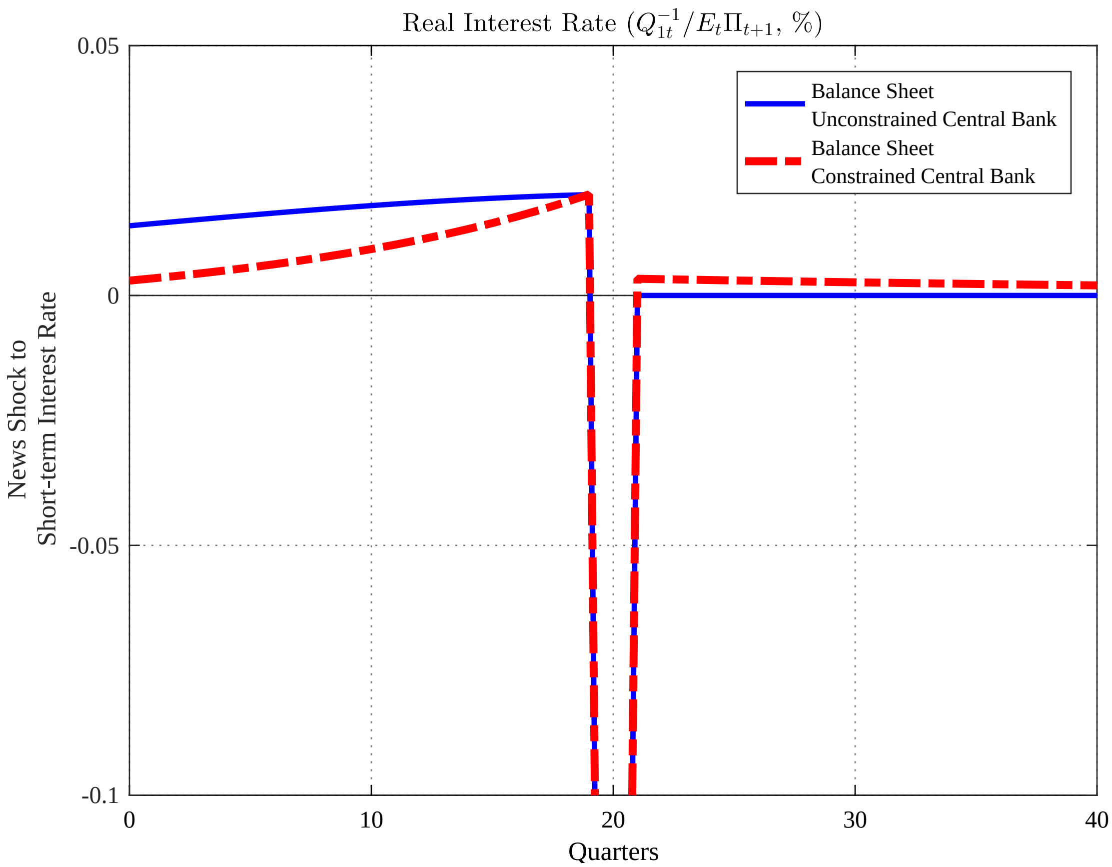

Figure 16: Comparison of response of real interest rate in the closed economy general equilibrium balance sheet unconstrained and constrained central bank models (forward guidance)

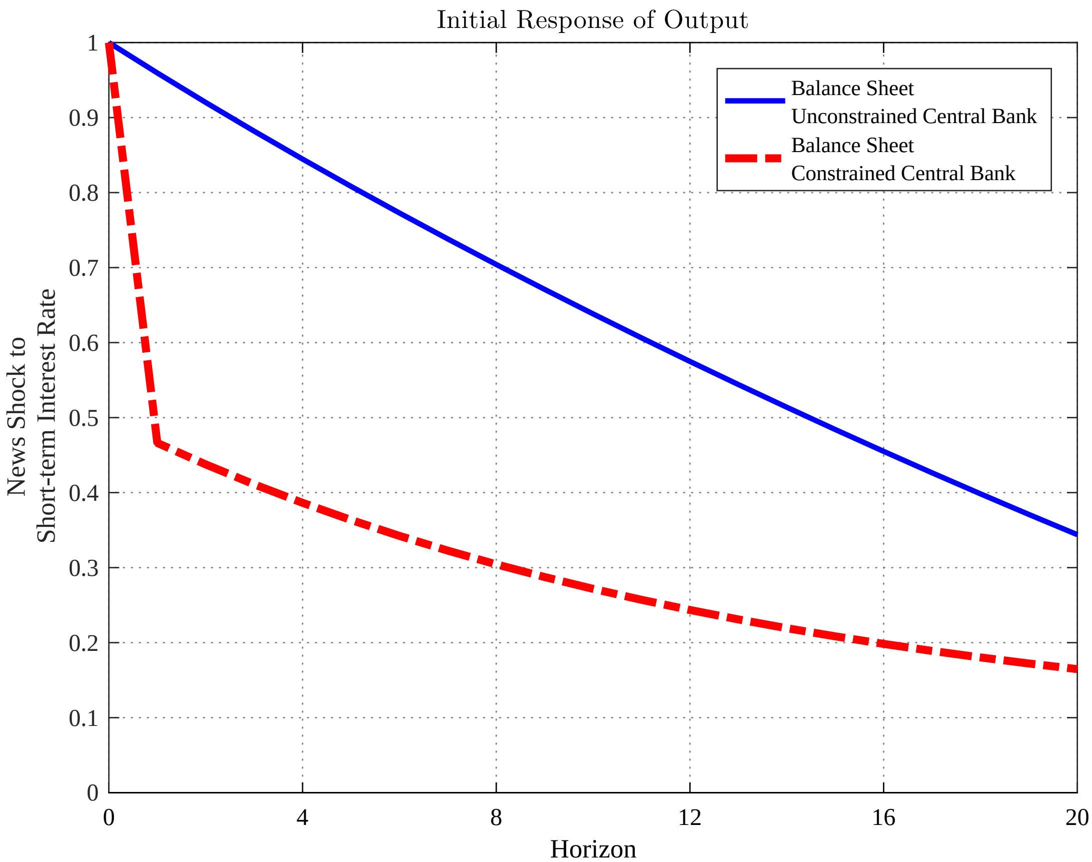

Figure 17: Comparison of initial response of output for different forward guidance horizons in the closed economy general equilibrium balance sheet unconstrained and constrained central bank models

Notes: The initial response is normalized by the output response to a current surprise shock in the two models.

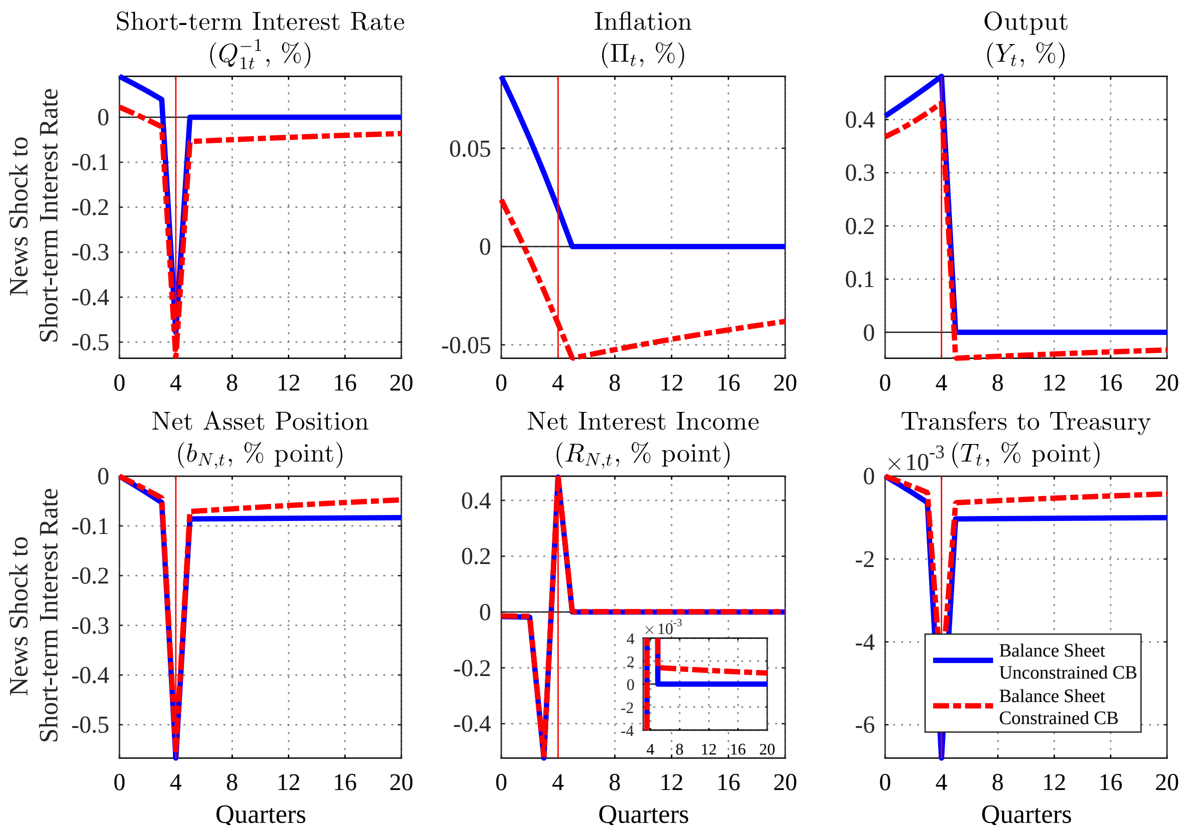

Figure 18: Non-linear impulse responses in the closed economy general equilibrium model (forward guidance at 4 quarter horizon)

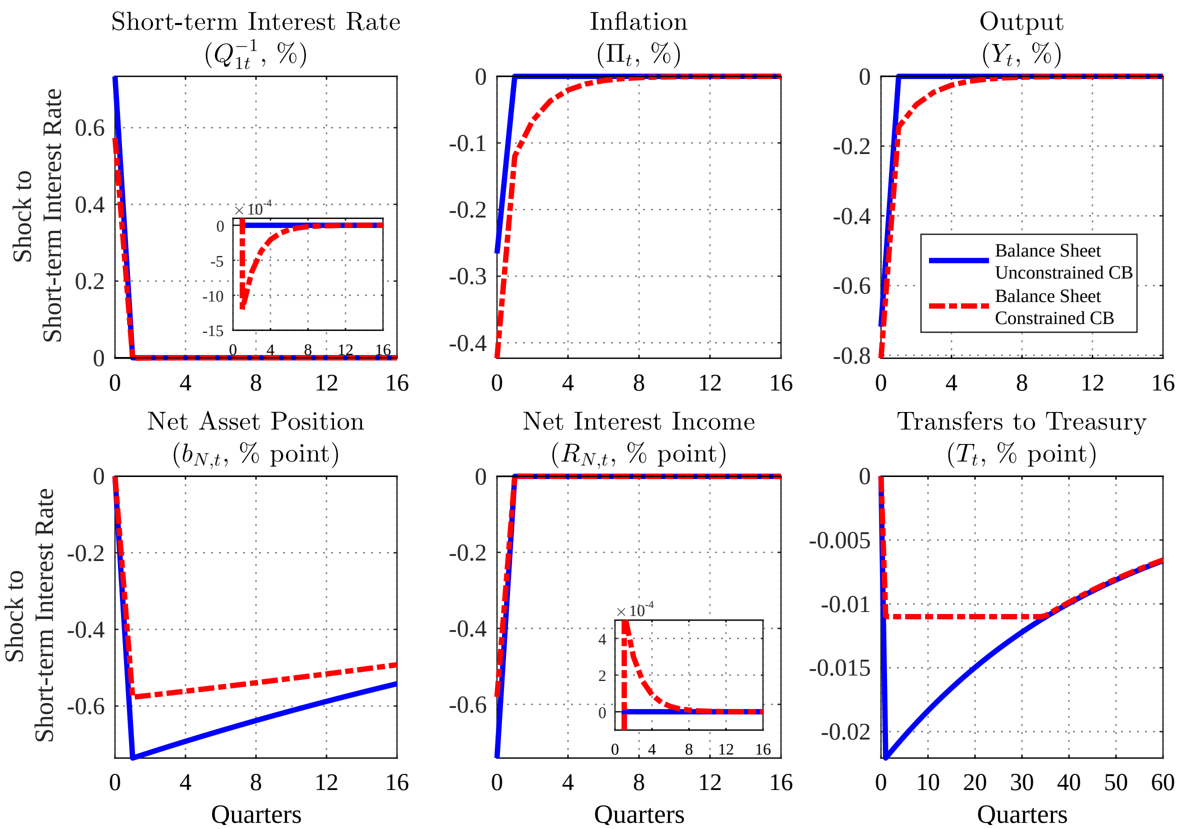

Figure 19: Non-linear impulse responses in the closed economy general equilibrium model (different transfer rule and policy regime for the balance sheet constrained central bank)

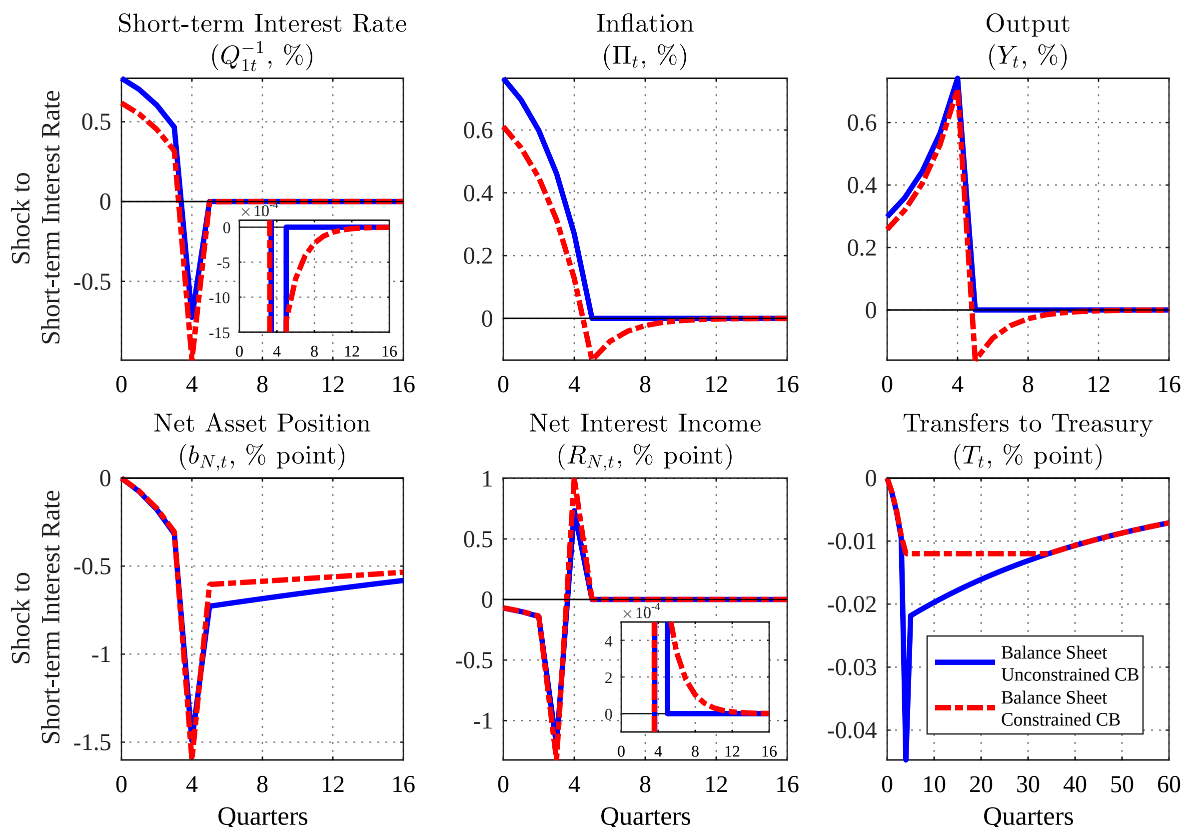

Figure 20: Non-linear impulse responses in the closed economy general equilibrium model (forward guidance at 4 quarter horizon, different transfer rule and policy regime for the balance sheet constrained central bank)

7 Appendix

I describe here the details of the model with a currency mismatched central bank balance sheet as well as additional results mentioned in the main text on the general equilibrium model.

7.1 Dynamic responses in the currency mismatch model

I present detailed results of the open economy model where the central bank balance sheet features currency mismatch. I start with describing policy rules, like in the closed economy model, and then present results on impulse responses to shocks and transition dynamics.

7.1.1 Policy rules and dynamic responses

To get further insights and to make the mechanisms described above even more concrete, like in the previous section, I now consider explicit monetary and balance sheet policy rules that can be used to model the two types of central banks. A complete equilibrium analysis of the transition dynamics associated with $b_{N,t-1} < 0$ can be undertaken in a straightforward way. Moreover, I introduce exogenous shocks to the nominal exchange rate and transfers to the Treasury in order to trace out the transmission of policy changes to model variables in a standard impulse response function analysis. This framework also allows a clear illustration of the basic logic discussed above regarding which term in (16) adjusts and which variable is set freely by the central bank.

Policy rules for the two central banks For the *open economy economy balance sheet unconstrained central bank*, as it is free to set the path of the nominal exchange rate and transfers to the Treasury can respond appropriately to the state of its balance sheet, the rules are given by

$$\hat{S}_t = \varepsilon_t, \ \hat{T}_t = \psi_T b_{N,t-1} + \xi_t \tag{35}$$

where $\psi_T > 1 - \beta$, which ensures that the central bank does not have to vary the nominal exchange rate to ensure that it satisfies (16). That is, in this case, transfers to the Treasury will co-move sufficiently positively with the net asset of the central bank. Additionally, ε_t in this simple model is simply an *iid* shock and just illustrates that the central bank is free to set the path of the nominal exchange rate, while ξ_t is an *iid* shock in the transfer rule.

For the *open economy balance sheet constrained central bank*, the path of its transfers to the Treasury is constrained (or fixed/exogenous) and as a result the net interest income will have to respond to the state of its balance sheet. This implies that it cannot freely set the path of the nominal exchange rate. The rules are then given by

$$R_{N,t} = \phi_R b_{N,t-1} + \varepsilon_t, \hat{T}_t = \xi_t \tag{36}$$

where $\phi_R < b^* \left(1 - \beta^{-1}\right)$, which ensures that (16) is satisfied in the face of exogenous transfers to the Treasury. That is, in this case, net interest income of the central bank will co-move sufficiently negatively with the net asset of the central bank.[77] Here, ξ_t is an *iid* shock in the transfer rule, which just illustrates that the path of transfers is exogenous.[78]

[77] Note again that the net interest income is given by $R_{N,t} \equiv \hat{S}_t - \beta E_t \hat{S}_{t+1}$, which is a quasi first-difference of the nominal exchange rate.

[78] These analytical boundaries for the two types of central bank regimes are again quite straightforward to derive, based on ruling out explosive dynamics of net assets of the central bank, as in the closed economy model.

Transition dynamics and responses to shocks How do variables respond to the two shocks, to the nominal exchange rate/interest income and transfers to the Treasury? How does the transition dynamics associated with $b_{N,t-1} < 0$ look like?

Open economy balance sheet unconstrained central bank First, consider the *open economy balance sheet unconstrained* central bank. The model here simply consists of (15) and (35). In this case, the solution of the model is similar to conventional text-book models, at least for the nominal exchange rate shock and resulting dynamics. When there is an *iid* shock to the exchange rate rule, say a shock that appreciates the nominal exchange rate, then $\hat{S}_t$ will decrease on impact and then go back to steady-state next period. The central bank thus has complete control of the nominal exchange rate, as in conventional open economy models. Because of a temporary, one-period appreciation of the nominal exchange rate, the net interest income of the central bank decreases by the same amount on impact. This means that the net asset position of the central bank and after a period, transfers to the Treasury, also decrease. The balance sheet variables (and not exchange rate and net interest income) have endogenous dynamics as the variables evolve over time, as given clearly in (15). The result for the exchange rate shock for the *open economy balance sheet unconstrained* central bank is illustrated in panel (a) of Figure 21.[79]

Now I can analyze transition dynamics associated with $b_{N,t-1} < 0$. Here, all the adjustment comes from transfers to the Treasury, which must decline. Given that we are looking at stationary dynamics, the net asset position slowly reverts back to steady-state, as does transfers to the Treasury. The nominal exchange rate or net interest income of the central bank is not affected at all. The results for the transfer shock for the *open economy balance sheet unconstrained* central bank is illustrated in panel (b) of Figure 21.

Finally, consider an *iid* shock to the transfer rule, say a shock that increases transfers to the Treasury. Then the transfers will increase on impact. Following the present-value logic described above however, in order to satisfy (16), transfers will decrease in future. The net asset position of the central bank will decrease on impact, as transfers have increased, and then slowly transition back to steady-state. This shock does not affect nominal exchange rate or the net interest income of the central bank. Thus, other than in terms of implications for balance sheet variables, there is no effect of this shock on other variables in the model. The results for the transfer shock for the *open economy balance sheet unconstrained* central bank is illustrated in panel (c) of Figure 21.

Open economy balance sheet constrained central bank In contrast, next, consider the *open economy balance sheet constrained* central bank. The model here simply consists of (15) and (36). In this case, the solution of the model is quite different from the case above that was more in line with conventional open economy text-book models. When there is an *iid* shock to the net interest income, say a shock that decreases it, then on impact it will decrease. But, by the logic presented above, in this case, the net interest income in future must adjust as transfers to the Treasury do not. In particular, net interest income must increase in future in order to satisfy (16). Thus next period, it increases and then slowly transitions back to steady-state. What is the resulting equilibrium dynamics of the nominal exchange rate? The nominal exchange rate does not move initially, by construction, and then it depreciates, before transitioning back to steady-state with an appreciation. This is the equilibrium dynamics as the net interest income is essentially the first-difference of the nominal exchange rate. The result for the net interest income shock for the *open economy balance sheet constrained* central bank is illustrated in panel (a) of Figure 22.[80] The dynamics of the

[79]There are very few parameters in the model. I use for illustration $\beta = 0.99$, $b^* = 1$, and $\psi_T = 0.02$.
[80]There are very few parameters in the model. I use for illustration $\beta = 0.99$, $b^* = 1$, and $\phi_R = -0.02$.

nominal exchange rate in this case are thus clearly determined by central bank balance sheet considerations and are in sharp contrast to those in panel (a) of Figure 21.

Now, I can analyze transition dynamics associated with $b_{N,t-1} < 0$. Here, again, all the adjustment comes from net interest income of the central bank and none from central bank transfers to the Treasury. Thus, net interest income must increase to satisfy (16). Given that we are looking at stationary dynamics, the net asset position slowly reverts back to steady-state, as does net interest income of the central bank. The nominal exchange rate rate is now clearly affected as it has to be consistent with the dynamics of the net interest income. Specifically, the nominal exchange rate must depreciate initially, so that there is a direct reduction in the return on (domestic currency) liabilities, and then it reverts back to steady-state, appreciating along the transition (as is consistent with the net interest income, which is essentially the slope of this transition path, being positive). The results for the transition dynamics for the *open economy balance sheet constrained* central bank is illustrated in panel (b) of Figure 22 and they are in sharp contrast to those in panel (b) of Figure 21.

Finally, consider an *iid* shock to the transfer rule, say a shock that increases transfers to the Treasury. Then the transfers will increase on impact. Since this is a one-time shock, transfers go back to steady-state next period. In sharp contrast to the *open economy balance sheet unconstrained* central bank, now exchange rate dynamics get affected as the net interest income of the central bank must respond in equilibrium. In particular, here, the net interest income must increase in a present value sense. Initially however, because of the rule in (36), as initial outstanding net asset position has not changed (making this different in this respect from the transition dynamics experiment above), net interest income does not change either. In future however, net asset decreases, and consistently, net interest income will increase. The equilibrium nominal exchange rate path implies that first it depreciates, again, to ensure a direct fall in the interest paid on (domestic currency) liabilities. It also depreciates further next period, with an overshooting, before appreciating and transitioning back to steady-state.[81] Why does the nominal exchange rate depreciate further for one period? The key reason is that the net interest income in the first-period does not change. For this to be consistent with nominal exchange rate dynamics, it must be that the next period exchange rate is slightly more negative as $\beta < 1$ (note that $R_{N,t} \equiv \hat{S}_t - \beta E_t \hat{S}_{t+1}$). The results for the transfer shock for the *open economy balance sheet constrained* central bank is illustrated in panel (c) of Figure 22. These results are in sharp contrast to those in panel (c) of Figure 21, where the shock to the transfer rule for instance did not affect the nominal exchange rate or the net interest income of the central bank at all.

7.2 Robustness and Extensions in the GE model

I briefly discuss the various sensitivity analyses and extensions of the general equilibrium model. Figures 23 and 24 show results for the price stickiness comparative statics for the balance sheet unconstrained and constrained central bank models respectively. Next, I consider several values for the policy feedback parameters, both in the interest rate rule and transfer to Treasury rule. Since the balance sheet unconstrained central bank model is well understood, for concreteness, I focus on the balance sheet constrained case and also, on the standard monetary policy shock experiment. Figures 25 and 26 show the results for various values of the two policy feedback parameters for the balance sheet constrained central bank.

I next allow the two balance sheet feedback rule parameters to be different. Thus, I now use ψ_{bT} =0.009 and ψ_{lT}=0.005, whereas in the baseline they were both equal to 0.009. The equilibrium responses of variables to a positive interest rate shock for the balance sheet constrained central bank for this different model and

[81] Note that the nominal exchange rate cannot appreciate on impact. There will be no stationary dynamics in that case.

calibration are shown in Figure 27. Finally, I use a different non-linear solution method, in particular, a stochastic third-order perturbation solution method. The equilibrium responses of variables to a positive interest rate shock for the balance sheet constrained central bank for this different solution method are shown in Figure 28.

7.3 Zero lower bound

I now present results for negative demand shocks, first at positive interest rates (Figure 29, where the shock has a persistence parameter of 0.8 and is of size 1%) and then when they are big enough to make the zero lower bound bind initially (Figure 30, where the shock has a persistence parameter of 0.8 and is of size 50%) for both models of central bank .

7.4 Comparison with the sticky information model

In the sticky information model, the model on the consumer side and monetary policy is exactly the same as the balance sheet unconstrained central bank model with sticky prices. The calibration of these common parameters is therefore exactly the same. The only new component then is the (linearized) sticky information Phillips curve of Mankiw and Reis (2002) given by

$$\pi_t = \left(\frac{1-\alpha}{\alpha}\right) y_t + (1-\alpha)\sum_{k=0}^{\infty} \alpha^k E_{t-1-k}\left(\pi_t + \Delta y_t\right)$$

where I calibrate the frequency of updating information, $1-\alpha = 0.0385$, to match the slope of the Phillips curve in the sticky price model used in the rest of the paper. In Figures 31-32 I show the equilibrium response of the variables to an anticipated negative interest rate shock (of 0.5%) that will be realized 20 quarters later, where I compare the sticky information model with the balance sheet constrained and unconstrained models respectively.

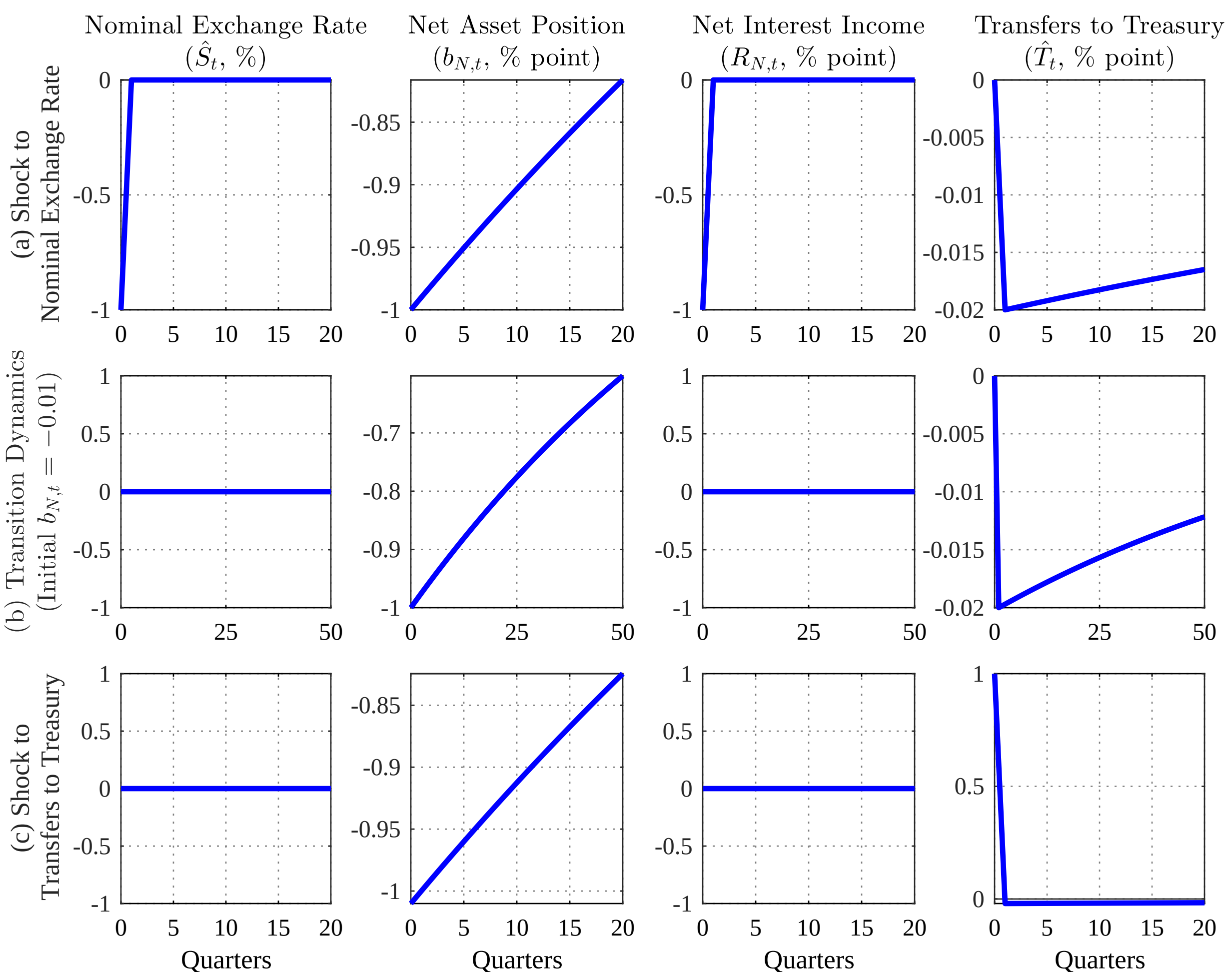

Figure 21: Impulse responses and transition dynamics in the open economy balance sheet unconstrained central bank model

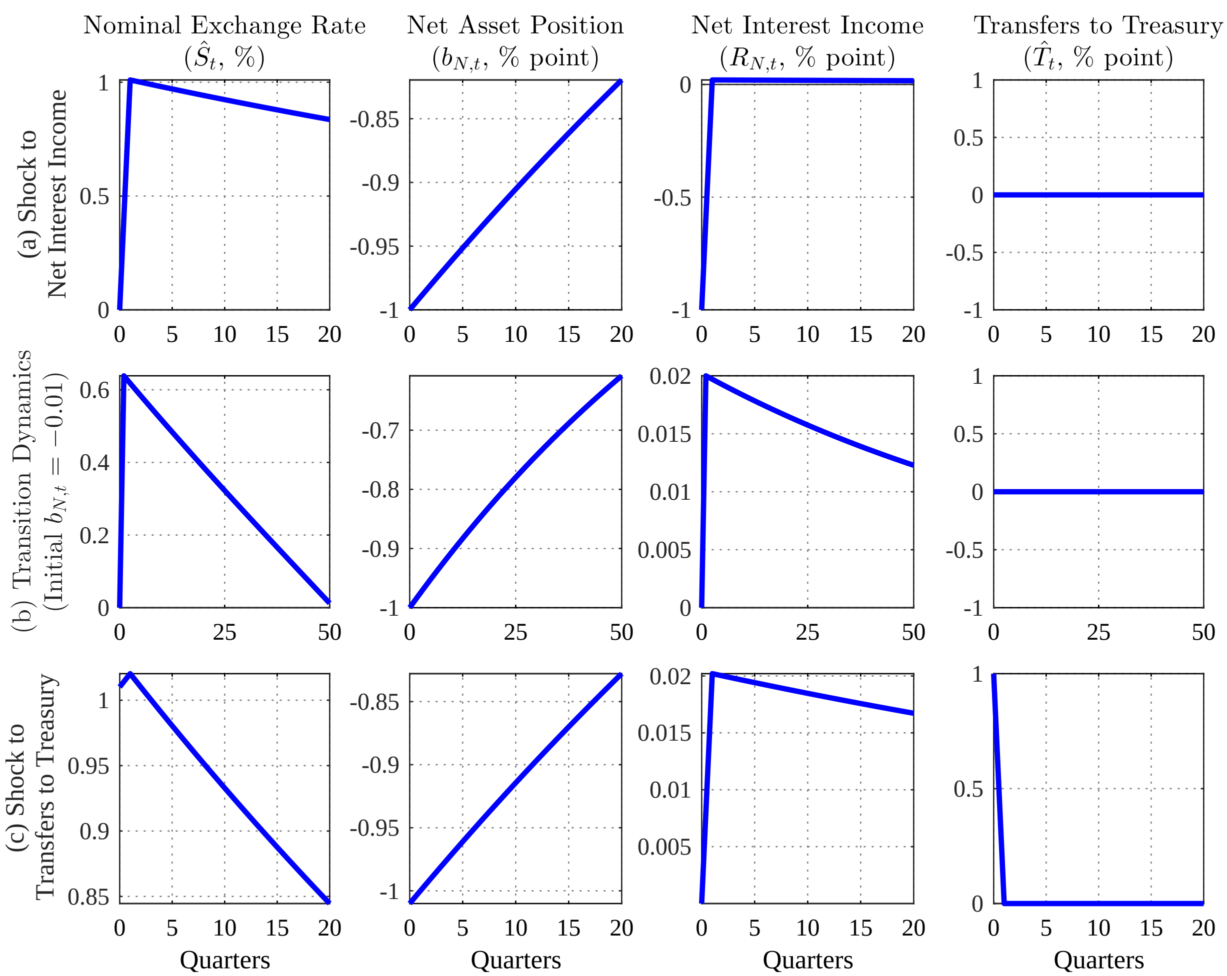

Figure 22: Impulse responses and transition dynamics in the open economy balance sheet constrained central bank model

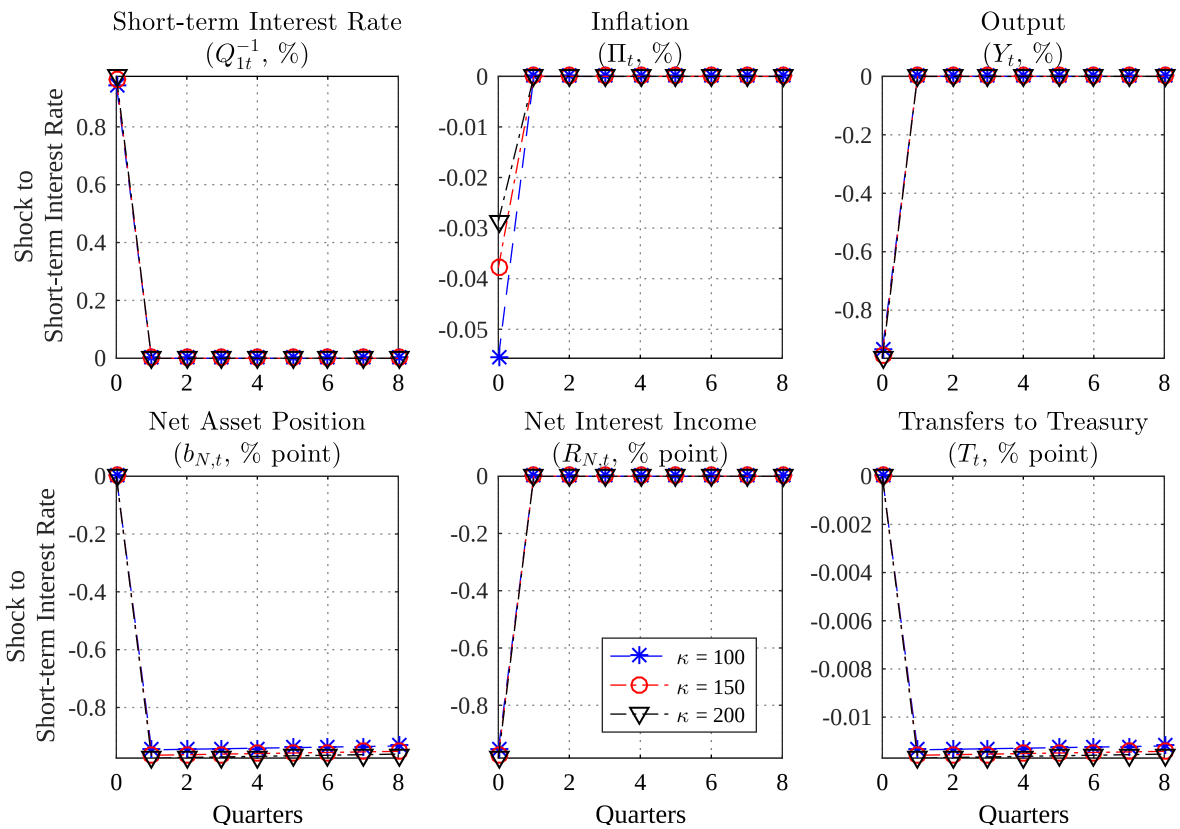

Figure 23: Non-linear impulse responses in the closed economy general equilibrium balance sheet unconstrained central bank model (different degrees of price stickiness)

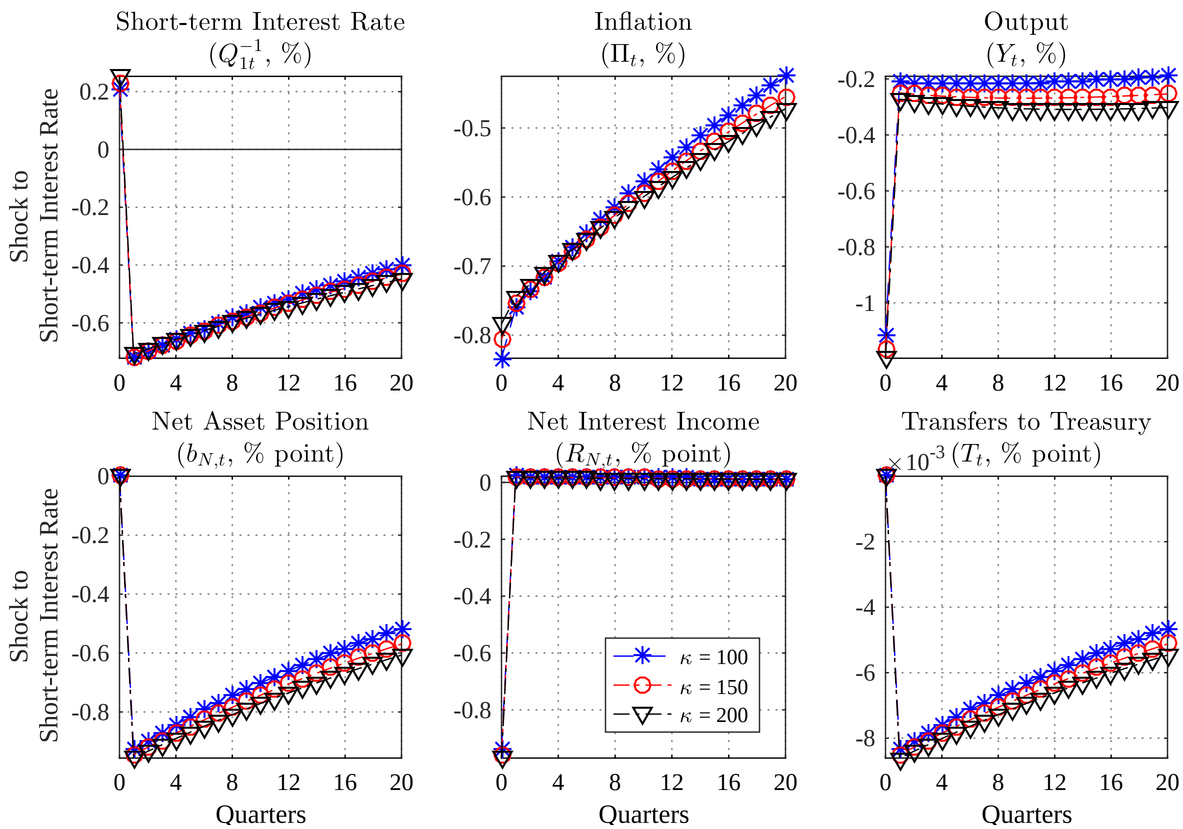

Figure 24: Non-linear impulse responses in the closed economy general equilibrium balance sheet constrained central bank model (different degrees of price stickiness)

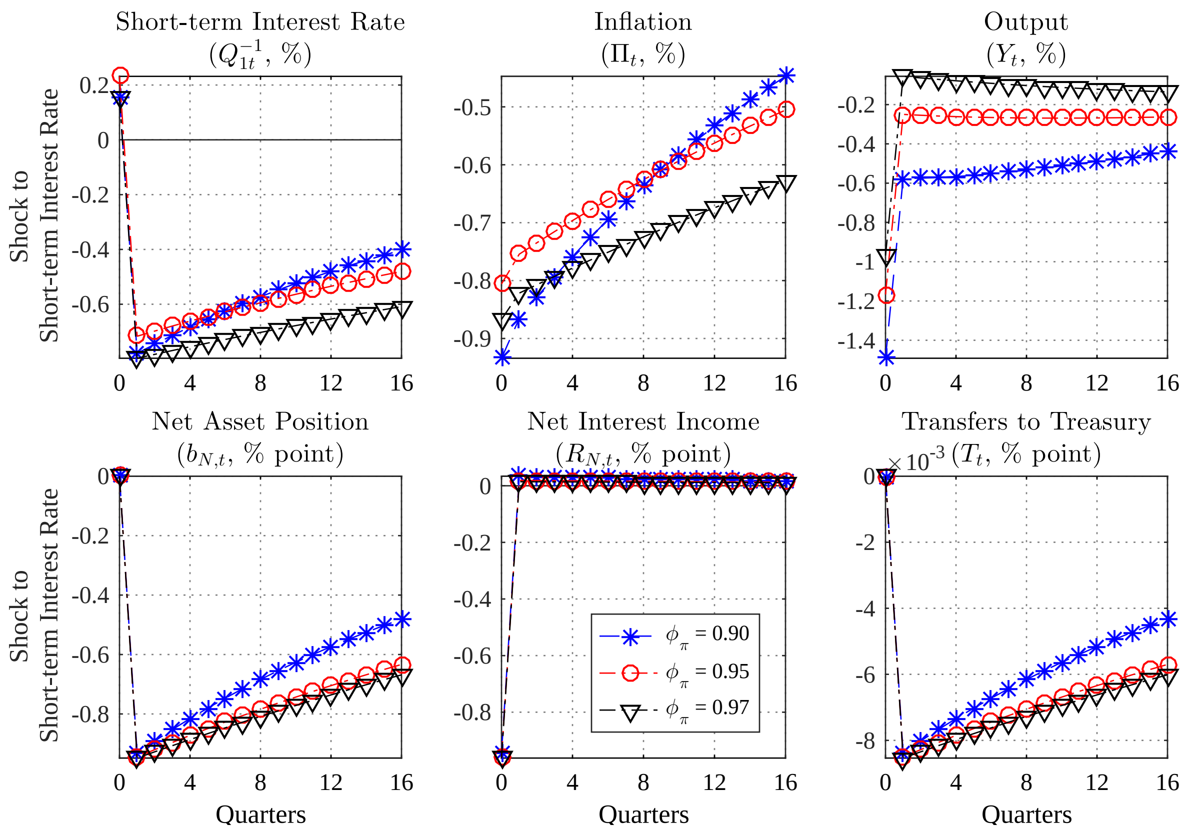

Figure 25: Non-linear impulse responses in the closed economy general equilibrium balance sheet constrained central bank model (different interest rate rule parameter)

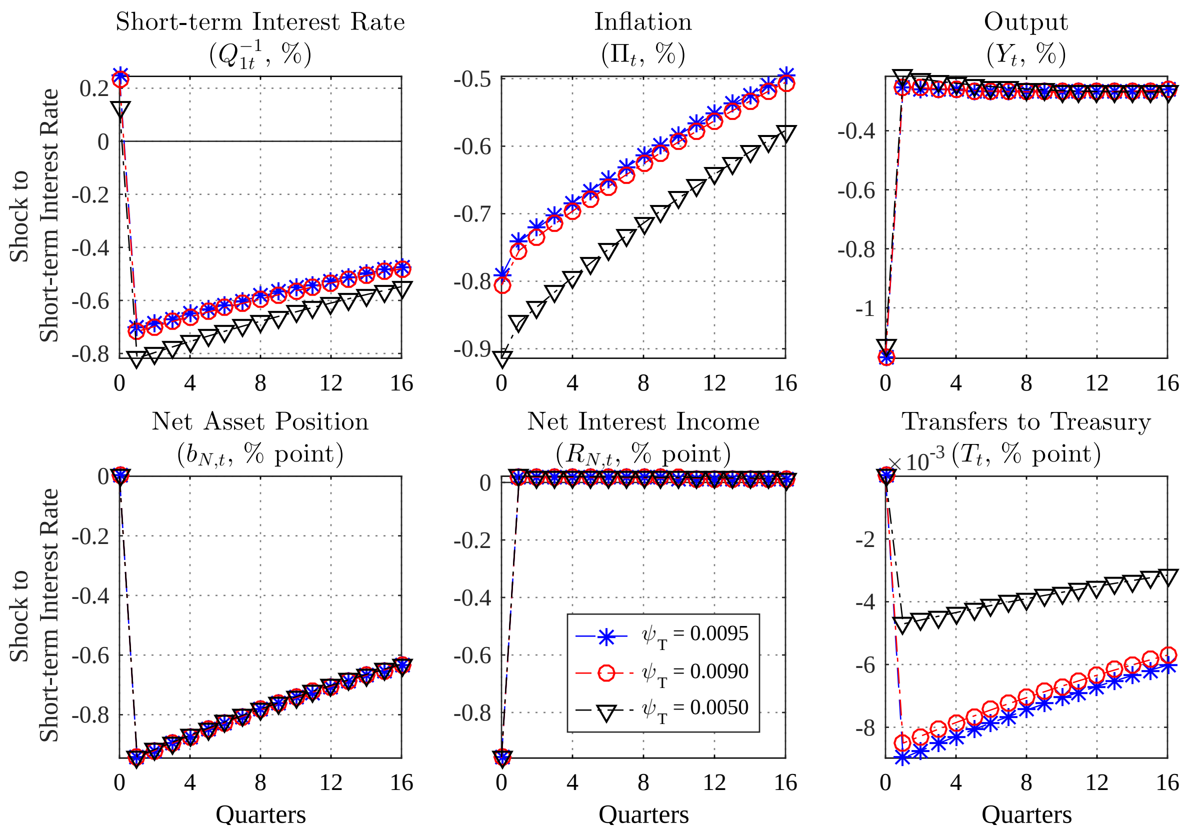

Figure 26: Non-linear impulse responses in the closed economy general equilibrium balance sheet constrained central bank model (different transfer rule parameter)

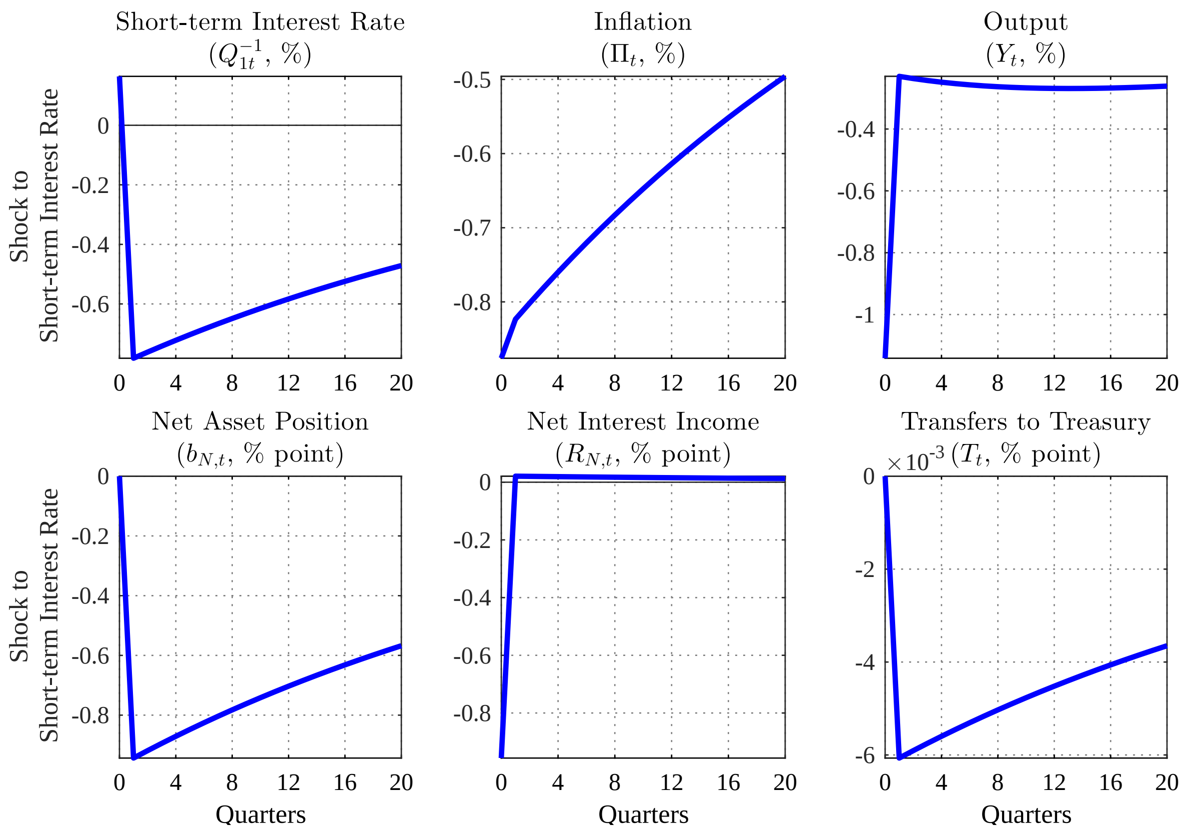

Figure 27: Non-linear impulse responses in the closed economy general equilibrium balance sheet constrained central bank model (alternative policy rule)

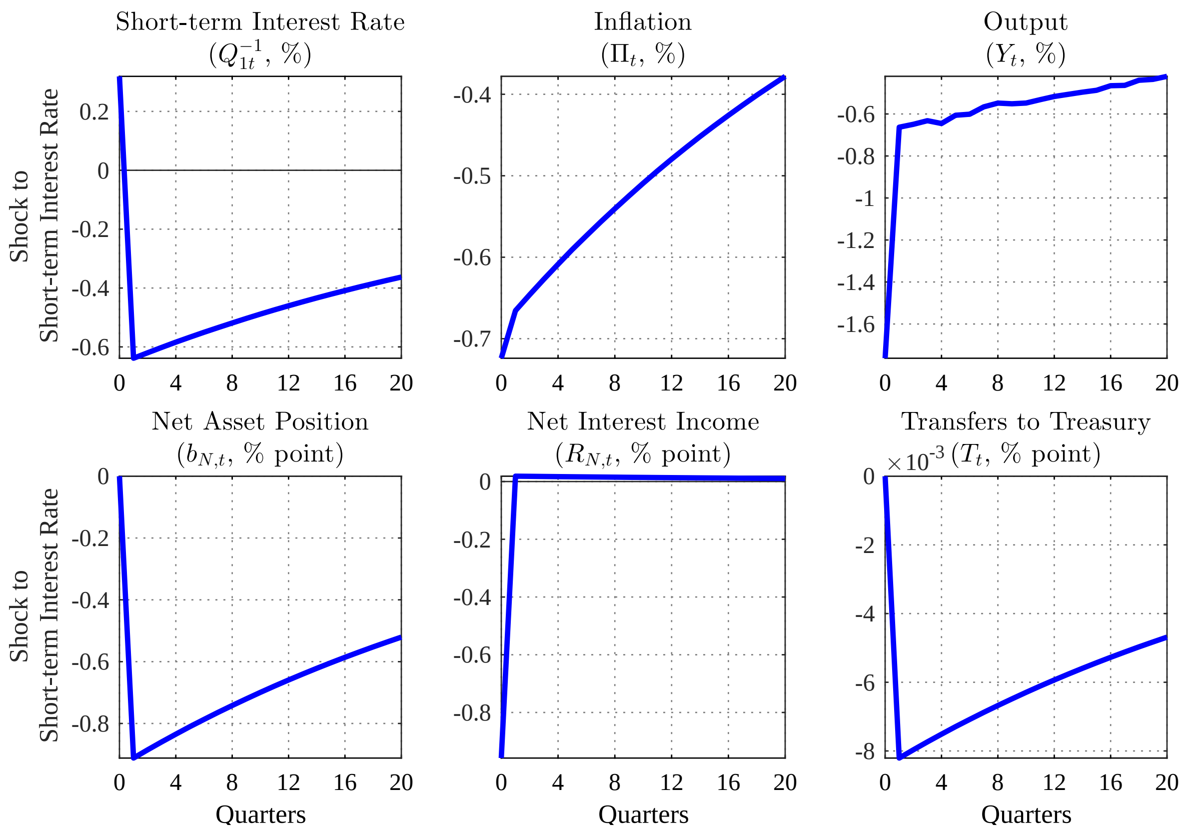

Figure 28: Non-linear impulse responses in the closed economy general equilibrium balance sheet constrained central bank model (third order stochastic solution)

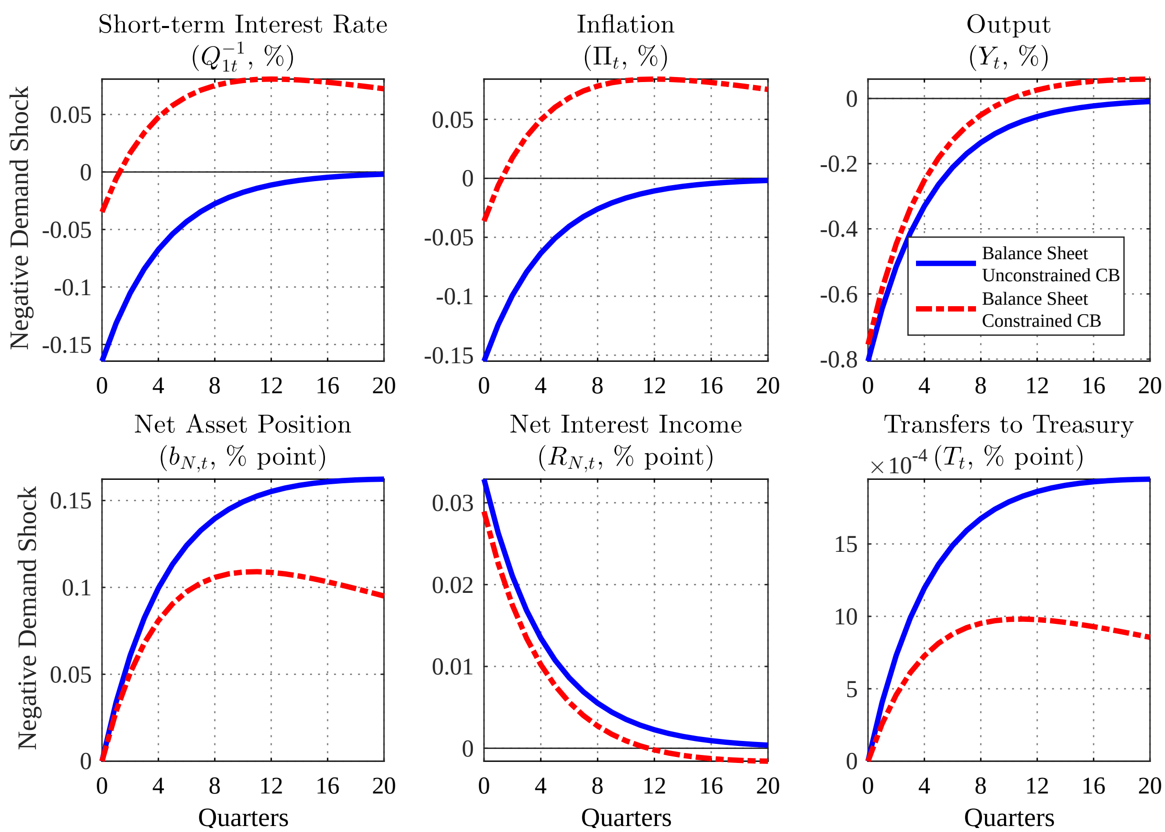

Figure 29: Impulse responses to a negative demand shock in the closed economy general equilibrium model

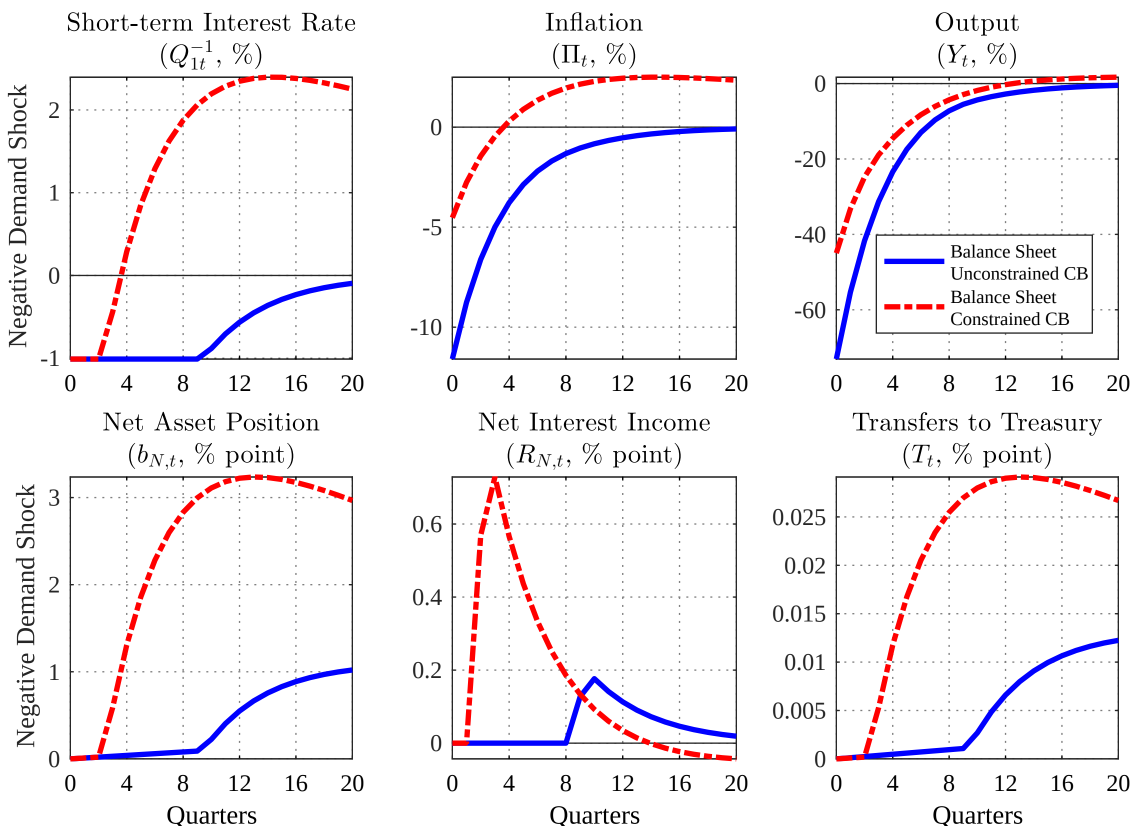

Figure 30: Impulse responses to a negative demand shock in the closed economy general equilibrium model when the zero lower bound binds

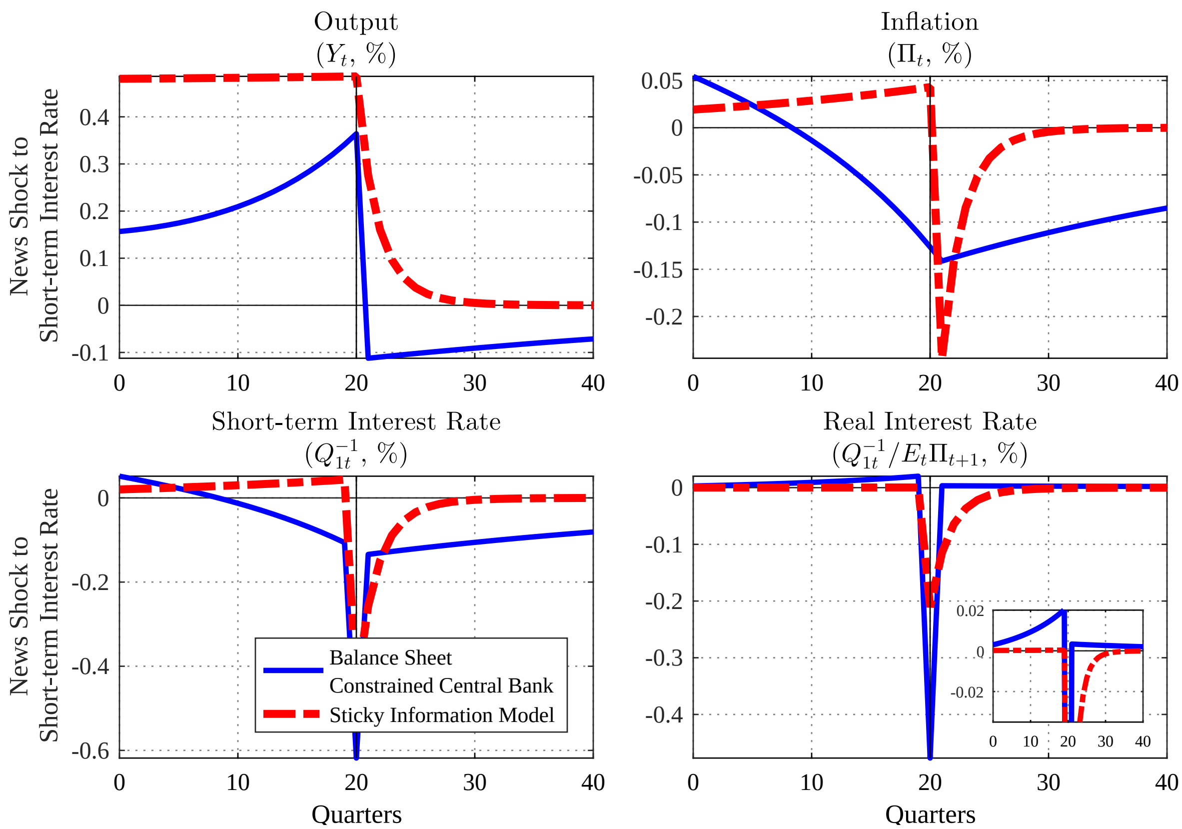

Figure 31: Impulse responses in the balance sheet constrained central bank model and the sticky information model (forward guidance)

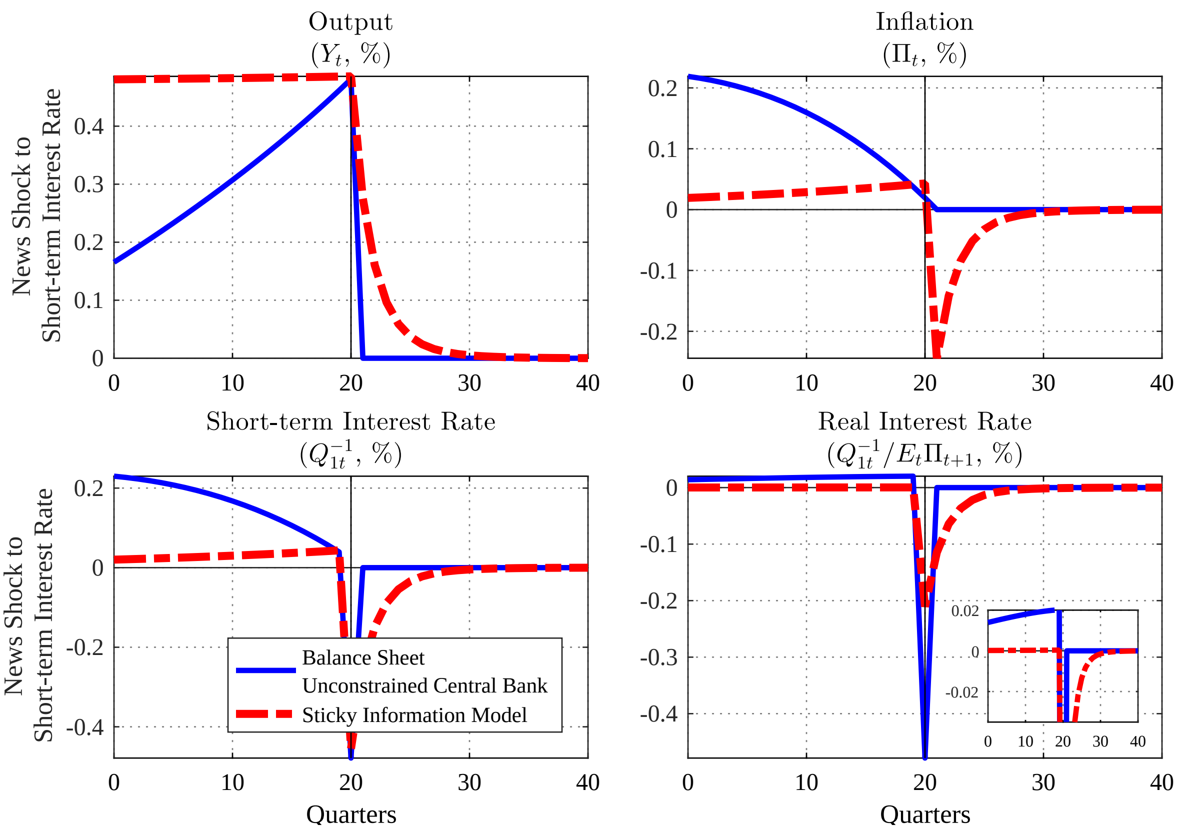

Figure 32: Impulse responses in the balance sheet unconstrained central bank model and the sticky information model (forward guidance)