The Misallocation of Finance

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Abstract

We estimate the real losses that accrue from the cross-sectional misallocation of financial liabilities by extending the framework of Hsieh and Klenow (2009) to the liabilities side of the balance sheet. Using manufacturing firm data from the United States and China, we find significant misallocation of debt and equity. Although financial liabilities appear well allocated in the United States, they are not in China. If China's debt and equity markets were as developed as those in the United States, China would realize gains of 40%-55% in terms of real firm value-added. We estimate firm-level costs of debt and equity, inclusive of allocation distortions. In China, we find markedly lower costs for larger firms and firms located in more developed cities.

JEL Codes: G32, G35

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1. Introduction

Over a decade of research in industrial organization, development, and macroeconomics has provided convincing evidence that cross-sectional misallocation of capital and labor is significant and can help explain why developing countries have lower total factor productivity (TFP).[1] Such pervasive evidence of factor misallocation begs the question of whether the financial liabilities that back the funding of capital goods and payroll are also misallocated. Are the right firms getting the right amount of external finance, and is the mix of debt and equity optimal? In this paper, we tackle these two related questions, moving from the asset side of the balance sheet to the liability side to quantify the extent of the misallocation of finance.

To this end, we turn to the empirical framework of Hsieh and Klenow (2009), so a brief outline of their approach helps clarify our own. They base their empirical work on a static model of cross-sectional factor allocation with differentiated products. In their model, a monopolistically competitive structure creates a downward-sloping demand curve, which endogenously limits firm size, even though firms have a constant returns to scale production function. In this setting, at an optimal allocation, the marginal revenue products of each factor are equal across firms in an industry. Distortions in cross-sectional allocations then break this equality and adversely affect TFP. The greater the dispersion in factor marginal revenue products within a sector, the greater the divergence of actual TFP from its potential level, and thus the greater the potential reallocation gains. Using establishment-level data on the manufacturing sectors in the United States, China, and India, Hsieh and Klenow (2009) find that China and India could realize TFP gains of 30%-50% and 40%-60%, respectively, if these countries hypothetically reallocated their factors of production to achieve the U.S. level of efficiency.

[1] Banerjee and Duflo (2005) offers an overview of the misallocation hypothesis in the development literature while Syverson (2011) and Restuccia and Rogerson (2013) survey the literature from an industrial organization and a macroeconomics perspective, respectively.

Our model is directly analogous to the setup in Hsieh and Klenow (2009), with two differences. First, while they model the factor mix that leads to potential distortions in TFP, we model the financial liabilities that back these factors and thus also potentially contribute to distortions in TFP. In particular, we specify different types of financial liabilities as the primitive inputs into the production process. Although different forms of finance are not exactly equivalent to factors of production, our modeling strategy is reasonable in the sense that firms ultimately finance their purchases of factors of production using debt and equity. The proximate factors—capital, materials, labor, and energy—can then be thought of as unmodeled intermediate inputs.

Second, Hsieh and Klenow (2009) specify capital and labor as imperfect substitutes. We extend their framework by allowing different forms of finance to be either perfect or imperfect substitutes. This extra flexibility in our model is important because it allows for a frictionless Modigliani-Miller world as a special case. This flexibility also allows us to understand whether potential reallocation gains stem from moving the type of finance or the gross flow of finance from less efficient firms to more efficient firms. This second avenue for reallocation is available even if debt and equity are perfectly substitutable.

In our framework, at an optimal allocation, the marginal contributions of debt and equity finance to nominal value-added are equal across firms in a sector, and distortions in these allocations lower productivity. Empirically, we then infer distortions by observing deviations from this first-best allocation. These deviations manifest themselves as large differences (relative to our model) in the debt-equity ratio across firms in a sector, and these large differences imply poorly developed financial markets and large potential gains from reallocation.

Using data on manufacturing firms in the United States and China, we find significant misallocation of debt and equity. Specifically, although financial liabilities appear well-allocated in the United States, they are not in China. If China's debt and equity markets were as developed as those in the United States, 40%-55% gains in real firm value would be available. Interestingly, 90% of these gains come from the misallocation of the total amount of finance, with only 10% stemming from the misallocation of the type of finance.

We produce several further interesting results. First, our framework allows us to estimate the distorted costs of debt and equity for each firm. We analyze the cross-sectional patterns in these costs in China, finding that larger firms and firms located in more developed cities face markedly lower costs. Second, we compare our financial costs with the costs of labor and capital from Hsieh and Klenow (2009). We find that while our two financial costs are highly correlated with each other, the financial and real costs are not highly correlated. This finding suggests that our results on financial misallocation do not passively reflect real misallocation. Third, our results are not concentrated in the large number of extremely small Chinese firms in our sample. In particular, we find significant potential reallocation gains even when we compare Chinese and U.S. firms of similar sizes. Although we find smaller gains, the reduction in potential gains comes from eliminating extremely large U.S. firms from our sample and not from eliminating the extremely small Chinese firms.

Because our strategy of modeling financial liabilities as factor inputs is unusual, it is worth outlining the types of market forces that might plausibly motivate this specification. We focus on financial frictions that likely spill over to a firm's real decisions, as these frictions are the ones that would lead to decreasing real marginal benefits of any particular type of finance. First, many capital structure models feature costs of financial distress that accompany too much debt, and many of these costs are real. For example, in the face of limited liability, firms with too much debt have incentives to undertake excessively risky projects (Jensen and Meckling 1976), as equity holders bear no downside risk. Also, excessive leverage can cause important employees to jump ship or deter potential workers from seeking employment (Brown and Matsa 2016). Second, too much debt can result in debt overhang, in which managers forego profitable investment projects because too few of the project proceeds would accrue to shareholders (Myers 1977). Third, informational frictions in the lending market can lead to covenants that suboptimally constrain real investment and acquisition decisions. Fourth, and in contrast, agency problems that lead managers to engage in wasteful spending can accompany too little debt (Jensen 1986). Finally, enforcement frictions lead to financial contracts in which external financing needs to be collateralized (Rampini and Viswanathan 2010). These financial frictions can therefore lead to a suboptimal mix of real factors of production.

In the end, the presence of these informational or agency frictions motivates a decreasing marginal benefit of either debt or equity, so all forms of finance are not necessarily equivalent. An optimal mix of debt and equity mitigates these frictions to the greatest extent possible. Moreover, if a firm ends up choosing an inefficient allocation of financial liabilities, these frictions would not be minimized and thus would lower total real output.

Because our strategy of modeling liabilities as factor inputs is unusual, it is also useful to outline an important advantage of our approach. It is less subject to issues of measurement and aggregation than is the modeling of capital and labor as inputs. Measuring the stocks of capital and labor is extremely difficult. Capital is an aggregate of highly heterogeneous goods with largely unobservable prices, and labor is an aggregate of highly heterogeneous individuals with unobservable skills and often unobservable wages. Such is not the case with debt and equity. Although there is heterogeneity in the financial securities classified as either debt or equity, the degree of heterogeneity is small relative to capital and labor heterogeneity. Bonds and bank loans are more similar than production robots and desk lamps. Moreover, the book values of both debt and equity are measured in local currency, so the aggregation of debt and equity securities is not hampered by the difficulty of measuring relative prices.

The literature that studies factor misallocation is extensive.[2] Within this body of work,

[2]Early work that provides the theoretical underpinnings for misallocation includes Lucas (1978), Hopenhayn (1992), Hopenhayn and Rogerson (1993), Olley and Pakes (1996) and Cooley and Quadrini (2001). More recently, several studies use firm or establishment level data and heterogeneous firm models to investigate the quantitative importance of misallocation. A partial list of more recent papers includes Jeong and Townsend (2007), Foster, Haltiwanger, and Syverson (2008), Restuccia and Rogerson (2008), Alfaro, Charlton, and Kanczuk (2009), Hsieh and Klenow (2009), Banerjee and Moll (2010), Buera, Kaboski, and Shin (2011), Song, Storesletten, and Zilibotti (2011), Petrin and Levinsohn (2012), Bartelsman, Haltiwanger,

our paper is most similar to Buera et al. (2011) and Midrigan and Xu (2014), who also consider financial frictions. However, there are substantive differences between our paper and these two. First, both papers emphasize the extensive margin of misallocation across sectors of the economy. Instead, we emphasize within-sector misallocation, as do Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Because more developed financial markets can indeed cause new firms to enter, our analysis provides a lower bound on the extent of financial misallocation that a dynamic model with entry and exit might find.

Second, both Buera et al. (2011) and Midrigan and Xu (2014) feature calibrated dynamic models, while our approach is largely empirical. For example, Buera et al. (2011) study a model in which financial frictions affect the manufacturing sector primarily on the extensive margin, as these frictions prevent talented agents from entering this sector. In Midrigan and Xu (2014), financial frictions lead to little intensive misallocation but substantial misallocation across sectors because the more productive sector requires a cost of entry that is difficult to pay in the face of financial frictions.[3]

In the finance literature, our work is related to Graham (2000), who also considers the cross-sectional allocation of debt and equity. However, there are again substantive differences between our work and his. Graham (2000) computes firm-level estimates of the point at which the marginal tax benefits of debt begin to decline. A firm that incurs interest deductions to the left of this "kink" point has an inefficiently low level of debt. Estimates of this inefficiency imply large amounts of tax benefits left on the table by underleveraged firms. One notable feature of the framework in Graham (2000) is that he takes relative prices as given and then interprets deviations from the optimal responses to these prices as suboptimal behavior. In contrast, we assume that firms behave rationally and then use our framework to back out the price distortions that lead to the financing decisions that we

and Scarpetta (2013), Chen and Song (2013), Midrigan and Xu (2014), Asker, Collard-Wexler, and Loecker (2014), Hsieh and Klenow (2014), Song and Wu (2015), Kehrig and Vincent (2016), Curtis (2016), Ai, Li, and Yang (2016), Bai, Lu, and Tian (2016), David and Venkateswaran (2016), and Wu (2017).

[3]Curtis (2016), Ai et al. (2016), and Bai et al. (2016) also use dynamic calibrated models to study how financial frictions affect misallocation.

observe in the data. This alternative perspective seems reasonable in light of the finding in Blouin, Core, and Guay (2010) that the marginal tax rate estimates of Graham (2000) imply rational behavior when the kink points are derived from more accurate estimates of future taxable income.

The rest of the paper is organized as follows. Section 2 outlines the model and shows how the model translates into an empirical framework for measuring misallocation. Section 3 describes the U.S. and Chinese data. Section 4 presents our baseline empirical results. Section 5 examines the robustness of our results to several assumptions, and Section 6 concludes. The Appendix contains all proofs.

2. Model

This section sketches our model, which closely follows Hsieh and Klenow (2009), which is in turn a closed-economy version of the model in Melitz (2003). We start with a description of the environment and technology and a statement of the optimality conditions. We then describe how to measure the benefits of reallocation. A full derivation of the model is in the Appendix.

2.1 Environment and technology

Firms in our model are financed by debt and equity. In our model, we do not distinguish between external and internal equity. Given the rarity of seasoned equity offerings (DeAngelo, DeAngelo, and Stulz 2010), and given that external equity constitutes a negligible source of funds over the last two decades in the Federal Reserve's Flow of Funds data, we view this simplification as innocuous for our purposes, which are primarily empirical.

Firms use the proceeds from these financial assets to generate the real benefit of finance, which in practice we define as real value-added. We denote the total real benefit of finance in the economy by F. We assume that the economy consists of S sectors, and in each sector s, the real benefit of finance is given by F_s. These sectoral benefits are combined using a Cobb-Douglas aggregator, as follows:

$$F = \prod_{s=1}^{S} F_s^{\theta_s}, \tag{1}$$

in which

$$\sum_{s=1}^{S} \theta_s = 1. \tag{2}$$

The Cobb-Douglas aggregator in (1) implies that increasing the size of any particular sector while holding the others constant has a decreasing marginal benefit.

We next assume that the real benefit of finance for each sector, F_s, comes from a constant elasticity of substitution (CES) aggregate of the real benefits of I differentiated firms, as follows:

$$F_s = \left(\sum_{i=1}^{I} F_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

in which F_{si} is the real benefit of firm i, and σ is the elasticity of substitution of the real benefit of finance between firms in a sector.

Finally, we assume that within an individual firm, debt and equity finance can be aggregated into the real benefit of finance using a CES function, given by:

$$F_{si} = A_{si} \left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}. \tag{4}$$

Here, D_{si} and E_{si} are the levels of debt and equity for firm i in sector s. In equation (4), $\alpha_s \in (0,1)$ denotes the weight on the importance of debt in generating this real financial benefit, γ is the elasticity of substitution between debt and equity, and A_{si} denotes total financial benefit (TFB), which is directly analogous to TFP. Note that certain variables are firm-specific, while others are sector-specific. For example, TFB, A_{si}, depends on both the sector and firm, while the weight α_s only depends on the sector. An important feature of this real benefit function is the decreasing marginal benefit to each individual financial factor input. As discussed in the introduction, a functional form with this property can stem from the frictions that underpin many trade-off models of capital structure.

It is worth noting that our use of a CES aggregator represents an important departure from Hsieh and Klenow (2009), who use a Cobb-Douglas production function. The CES functional form allows for perfect substitutability between different forms of finance and, as we show below, gives us the flexibility to distinguish between reallocation gains that come from type of finance and from the gross flow of finance. Nonetheless, equation (4) constitutes a strong functional form assumption on how debt and equity generate a real benefit to the firm. It describes the generation of benefits without explicitly showing how the proceeds raised through financing activities translate into capital and labor. In this regard, because firms ultimately finance their purchases of factors of production using debt and equity, more immediate factors such as capital and labor can be thought of as unmodeled intermediate inputs.

2.2 Optimal allocations

Next, we define the prices that enter the firm's optimization problem. First, we let r and λ be the costs associated with using debt and equity, respectively. Second, to the extent that financial market frictions distort these costs, we also need to define reduced-form cost distortions, which we refer to as "taxes." Specifically, $\tau_{D_{si}}$ is a "tax" on debt and $\tau_{E_{si}}$ is a "tax" on equity. Positive values indicate that firms face additional costs of finance. As noted in the introduction, these costs can arise from frictions such as informational asymmetry or agency problems. Negative values, on the other hand, suggest favorable financial relationships or government subsidies. As our model is ultimately an empirical measurement framework, we do not model the explicit mechanisms behind the distortions and instead assume that the they are well-encapsulated by $\tau_{D_{si}}$ and $\tau_{E_{si}}$.

Given these definitions, we now specify the nominal net benefit of finance π_{si} as:

$$\pi_{si} = P_{si}F_{si} - \left[\left(1+\tau_{D_{si}}\right) rD_{si} + \left(1+\tau_{E_{si}}\right) \lambda E_{si} \right] \tag{5}$$

Because the term in square brackets on the right-hand side of equation (5) is the cost of capital, π_{si} can be interpreted as economic value-added (EVA), which is a sensible quantity to maximize in a static model. The firm maximizes π_{si} by choosing P_{si}, D_{si}, and E_{si}, taking r, λ, $\tau_{D_{si}}$, and $\tau_{E_{si}}$ as given. P_{si} is a choice variable because of the imperfect substitutability of the real benefit, F_{si}, across firms in a sector. As in Dixit and Stiglitz (1977), in this situation, the firms are monopolistically competitive in the product market and thus have power to set prices.

To solve the optimization problem, the firm first minimizes the cost of capital, $\left(1+\tau_{D_{si}}\right) rD_{si} + \left(1+\tau_{E_{si}}\right) \lambda E_{si}$, by choosing D_{si} and E_{si} subject to setting equation (4) equal to a fixed real benefit $\bar{F}_{si}$. Intuitively, at an optimum, the marginal nominal benefits of debt and equity should be equal. As shown in the Appendix, this equality implies that the solution to this problem is given by:

$$\frac{D_{si}}{E_{si}} = \left(\frac{\alpha_s}{1-\alpha_s} \frac{\left(1+\tau_{E_{si}}\right)\lambda}{\left(1+\tau_{D_{si}}\right) r} \right)^{\gamma}. \tag{6}$$

Because α_s does not vary within firms in a sector, at an optimal allocation in which the taxes $\tau_{E_{si}}$ and $\tau_{D_{si}}$ both equal zero, the ratio of debt to equity should be equal across all firms in a sector. This stark prediction mirrors the similarly stark prediction in Hsieh and Klenow (2009) that capital-labor ratios should be equal across firms in a sector. This prediction nonetheless makes sense on an intuitive level. For example, suppose all firms in a sector have similar capital intensities and thus face similar enforcement frictions (Rampini and Viswanathan 2010). Then they should all have similar capital structures. Another example is an industry with extremely opaque technology, which makes state-incontingent debt finance prohibitively costly. All of the firms in this type of industry should be highly dependent on equity finance.

Next, the firm chooses P_{si} to maximize the nominal net benefit π_{si}. As shown in the Appendix, the solution to this second problem is given by:

$$\begin{aligned} P_{si} &= \frac{\sigma}{\sigma-1}\left[\frac{1}{A_{si}}\left(\left(1+\tau_{D_{si}}\right) r\left(\alpha_s+(1-\alpha_s) Z_{si}^{-\frac{\gamma-1}{\gamma}}\right)^{-\frac{\gamma}{\gamma-1}}\right.\right. \\ &+ \left.\left.\left(1+\tau_{E_{si}}\right) \lambda\left(\alpha_s Z_{si}^{\frac{\gamma-1}{\gamma}}+(1-\alpha_s)\right)^{-\frac{\gamma}{\gamma-1}}\right)\right], \end{aligned} \tag{7}$$

in which

$$Z_{si}=\left(\frac{\alpha_s}{1-\alpha_s} \frac{\left(1+\tau_{E_{si}}\right) \lambda}{\left(1+\tau_{D_{si}}\right) r}\right)^{\gamma}. \tag{8}$$

Note that at an optimal allocation, $Z_{si}=D_{si}/E_{si}$. Thus, from comparing equation (5) with the term in square brackets in equation (7), it is clear that this latter term is the minimized marginal cost of providing one unit of the marginal benefit of finance. Thus, equation (7) naturally shows that price is a markup over marginal cost, with the term, $\sigma/(\sigma-1)$ being the markup.

Next, we solve for the sector price P_s as a function of firm price P_{si}, by defining P_s to be the minimum price of acquiring a unit of the sector benefit. The solution is:

$$P_s=\left(\sum_{i=1}^{I} P_{si}^{-(\sigma-1)}\right)^{-\frac{1}{\sigma-1}}. \tag{9}$$

Finally, cost minimization of the Cobb-Douglas aggregator across sectors gives:

$$P = \prod_{s=1}^{S} \left(\frac{P_s}{\theta_s} \right)^{\theta_s}, \tag{10}$$

in which θ_s is the weight on industry s, and P is the minimum price of acquiring a unit of the aggregate benefit. We assume that the nominal benefit of finance satisfies value additivity at the sector level and firm level, so:

$$\sum_{s=1}^{S} P_s F_s = PF$$

and

$$\sum_{i=1}^{I} P_{si} F_{si} = P_s F_s.$$

From the derivation of P, the industry weights, θ_s, are found to be the fractions of the portfolio allocated to each industry, that is:

$$P_s F_s = \theta_s PF. \tag{11}$$

We close with one comment about preferences. Although we do not model preferences explicitly, in a more explicit general equilibrium setting, the implicit preferences that produce the above results are CES preferences over the benefit from firms in a sector and Cobb-Douglas preferences over the benefit from sectors in the economy.

2.3 Reallocation

We now demonstrate how to calculate the real gains from reallocation using this framework. One obstacle to overcome in making this calculation is that the real benefit of finance is unobservable. Although the nominal benefit, $P_{si}F_{si}$ is, in principle, observable, the real benefit is not because prices are difficult to measure with any accuracy. These measurement difficulties imply that to calculate the real benefit, we need to rely on the structure of the model. Specifically, we plug the optimal allocations of debt and equity into the firm-level real benefit function (4) to obtain the optimal, first-best, firm-level real benefit of finance. Next, we calculate an estimate of the actual firm-level real benefit of finance, which we obtain from plugging actual, observed debt and equity into (4). We then calculate economy-wide benefits by aggregating firm-level benefits into sectors and sectors into the aggregate economy. Finally, we compare optimal with actual aggregated benefits to measure aggregate gains.

Before any such aggregation can take place, inspection of (4) shows that as a first step, we need to find an expression for A_{si}. Here, we rely on the observability of the nominal benefit, $P_{si}F_{si}$, by writing A_{si} in terms of $P_{si}F_{si}$. In the Appendix, we show that we can express total financial benefit, A_{si}, as:

$$A_{si} = \eta_s \frac{(P_{si}F_{si})^{\frac{\sigma}{\sigma-1}}}{\left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s)E_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}}, \tag{12}$$

in which

$$\eta_s = \frac{1}{P_s(P_sF_s)^{\frac{1}{\sigma-1}}}. \tag{13}$$

All of the variables on the right-hand side of (12) are observable, except for η_s. However, as shown in the Appendix, the reallocation gains do not depend on η_s, as it can be divided out of the problem, so we normalize it to one. Therefore, A_{si} can be measured using available data when written in the form of (12).

The final ingredient necessary for the calculation of reallocation gains is the efficient levels of debt $\hat{D}_{si}$ and equity $\hat{E}_{si}$, in which a hat above a variable indicates the efficient level after reallocation. These two quantities can be found from the first-order conditions obtained from differentiating the expression for the sector-level aggregate benefits, (3), with respect to these two variables. These optimality conditions are given by:

$$\hat{D}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} D_s \tag{14}$$

$$\hat{E}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} E_s. \tag{15}$$

These two conditions express the intuitive result that the most productive firms in the economy, those with the highest levels of A_{si}, get the most finance at an optimal allocation.

Once we determine optimal debt and equity, we can write the optimal real benefit of finance for a firm, a sector, and the economy respectively as:

$$\hat{F}_{si} = A_{si}\left(\alpha_s \hat{D}_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s)\hat{E}_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{16}$$

$$\hat{F}_s = \left(\sum_i \hat{F}_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{17}$$

$$\hat{F} = \prod_s \hat{F}_s^{\theta_s}. \tag{18}$$

As explained above, the original, prior to reallocation, real benefit of finance can be computed by replacing $\hat{D}_{si}$ and $\hat{E}_{si}$ by the actual, observable debt D_{si} and equity E_{si} in equation (16). Therefore, we can quantify the potential reallocation gains by calculating the observed allocation as a fraction of the efficient allocation. Letting F denote the observed benefit of finance, these gains are given simply by $F/\hat{F}$.

It is worth discussing the role of the parameters σ and γ in the quantification of these gains. We first consider σ, the elasticity of substitution of the real benefit of finance between firms in a sector. The extent of misallocation and thus the magnitude of potential reallocation gains depend positively on σ. To see this point, consider a case in which firms in a sector are all the same size but their possibly inefficient allocations of debt and equity imply wide dispersion in TFB, A_{si}. In this case, if σ is high, moving to the efficient allocation would result in a great deal of dispersion in firm size, with the more productive firms receiving more finance. Conversely, a low value for σ implies that reallocating debt and equity efficiently would result in the most productive firms getting only a modest amount of finance, so reallocation gains would also be modest. Overall, reallocation gains are greater when σ is higher.

Turning to γ, the elasticity of substitution between debt and equity, it is intuitive to see that when γ approaches infinity, debt and equity are perfect substitutes, so the potential gains from changing the debt-equity mix are zero.

We close this section by providing the expressions for the price distortions, $\tau_{D_{si}}$ and $\tau_{E_{si}}$:

$$(1+\tau_{D_{si}})\,r = \alpha_s \frac{\sigma-1}{\sigma} \frac{P_{si}F_{si}}{\alpha_s D_{si} + (1-\alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}} D_{si}^{\frac{1}{\gamma}}} \tag{19}$$

$$(1+\tau_{E_{si}})\,\lambda = (1-\alpha_s) \frac{\sigma-1}{\sigma} \frac{P_{si}F_{si}}{\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} E_{si}^{\frac{1}{\gamma}} + (1-\alpha_s) E_{si}}. \tag{20}$$

Although the reallocation gains can be found independently of the price distortions, computing the taxes, $\tau_{D_{si}}$ and $\tau_{E_{si}}$, allows us to perform additional analysis on the model.

3. Data

We start by presenting our data from China and then repeat the analysis for the data from the United States. The data set for China comes from the National Bureau of Statistics

(NBS) of China and contains a panel of firms from 1999 to 2007. During this time period, firms with more than 5 million Chinese yuan (CNY) in sales, or approximately 600,000 U.S. dollars (USD) are required to provide detailed financial information. This information includes statistics such as employment, income statement items, balance sheet items, and, after 2004, cash flow items.

This data set contains firms only from the mining, manufacturing, and utilities sectors. We focus on the manufacturing sector because the mining sector is relatively small and operationally different from manufacturing, while the utilities sector is highly regulated in China. In addition, we also remove state-owned and collective corporations, which are also known as Township and Village Enterprises (TVEs). Each firm-year observation is classified as a private-corporation operating-year if the total state and collective paid-in capital is less than 50%.[4] We drop firms with negative and missing value-added, total liabilities, and shareholders' equity. We also drop firms with less than 5 million 1999 CNY in sales because the lack of reporting requirements for this group likely results in significant selection bias and undersampling. After applying these screens, we are left with 1,248,729 firm-year observations.

We use total liabilities as our measure of D_{si} and shareholders' equity as our measure of E_{si}. This measure of equity is the stock of book equity and thus includes both external equity finance and retained earnings. As our measure of the nominal benefit of finance, $P_{si}F_{si}$, we use value-added, which we compute as the sum of profits, indirect taxes, wages, and depreciation. These variables are all directly available in the NBS data. Note that we use total liabilities instead of debt, for two important reasons. First, debt is not an item in the NBS data, and second, using total liabilities can offer more robust estimation because there are almost no firms with zero liabilities. For a CES function without an infinite elasticity of substitution, the marginal benefit of a factor input is unbounded at zero, and

[4] This type of classification is often used because official corporate ownership registrations can lag several years behind actual ownership changes. See Guariglia, Liu, and Song (2011) for a similar approach.

this property of the CES aggregator would present omitted observation problems in the estimation. These choices for debt and equity imply that the sum of the two liabilities, debt and equity, equals total assets.

Summary statistics for the sample of Chinese firms are in Table 1. Panel A reports various statistics in the sample stratified by size, with cumulative density breakpoints of 5%, 15%, 30%, 50%, 70%, 85%, 95%, and 100%. This partition is useful because the mean of total assets roughly doubles for each size group, with the exception of the largest size group, which contains extremely large firms, which are in turn reflective of the well-known right skewness of the firm size distribution in China.

In Panel A of Table 1, we find two patterns of interest in the data. First, the ratio of liabilities to assets increases with size. Because most external finance of any type is debt for this sample of primarily private firms during this period in China, the positive dependence of leverage on size generally indicates better access to finance for the larger Chinese firms. Second, it is clear from comparing the value-added and assets columns that the smaller firms use their assets far more efficiently to produce value-added. This pattern, juxtaposed with the higher leverage across size classes, points strongly to potential misallocation of debt and equity, as more productive firms should have more access to finance.

Panel B of Table 1 presents summary statistics by year. Here, we see that the ratio of liabilities to assets is little changed over the sample period. Interestingly, average firm size shrinks somewhat from the beginning to the end of the sample period. However, these slightly smaller firms create 40% more value-added at the end of the sample period than at the beginning.

For the United States, we use Compustat data. To make the data comparable to the Chinese data, we keep only the years from 1999 through 2007 and also keep only firms in the manufacturing sector with Standard Industrial Classification (SIC) codes between 2000 and 3999. We also drop firms with missing data.

We compute value-added as in Imrohoroglu and Tuzel (2014). First, we estimate labor costs from the NBER-CES Manufacturing Industry Database by computing the mean wage per employee by three-digit SIC industry. We then multiply these wage figures by the number of employees in Compustat (EMP) to obtain a firm-level imputation of the wage bill. Value-added is then operating income before depreciation (OIBDP) plus the imputed wages. For D_{si} and E_{si}, we use total liabilities (LT) and shareholders' equity (AT - LT).

We partition the firms by size according to the same densities as for China. Although the average firm is much larger in the United States than in China, the observed patterns by firm size can still be informative. Table 2 provides summary statistics by firm size and by year. In Panel A, we present eight firm size categories, with the last three Chinese firm size categories being approximately equivalent to the first three U.S. firm size categories. As in the Chinese data, we find a positive relation between size and leverage for the U.S. firms. However, in contrast to the Chinese firms, both small and large U.S. firms have approximately the same ratio of value-added to assets. Panel B shows two more differences between the Chinese and U.S. firms. The U.S. firms grow from the beginning to the end of our sample, and they become less leveraged over time.

4. Results

Before we present our results, we explain how we set several parameters. First, we set the elasticity of substitution for the real benefit of finance between firms in an industry to $\sigma = 1.77$. This choice equates the standard deviations of the observed and efficient size distributions in the U.S. and thus provides conservative estimates of the potential reallocation gains. Second, we set the pre-distortion costs of debt and equity to $r = 0.045$ and $\lambda = 0.09$. This parameterization comes from a simple CAPM framework with a risk-free rate of 3%, a market risk premium of 6%, a debt beta of 0.25, and an equity beta of 1. Third, we set the weight on the importance of debt in a sector equal to $\alpha_s = rD_s^{1/\gamma}/(rD_s^{1/\gamma} + \lambda E_s^{1/\gamma})$, in which D_s and E_s are the observed values for total sector-level debt and equity. This expression for α_s follows from the optimality conditions (36) and (37) in the Appendix, and it represents the value of the sector-level importance of debt when there are no tax distortions. Fourth, we set the elasticity of substitution between debt and equity, $\gamma = 2$. Below we explore the robustness of our results to varying all of these parameters. Finally, we need to define sectors. Here, we use three-digit industry classifications from the Chinese NBS and three-digit Standard Industrial Classification (SIC) industries from Compustat.

We now use the framework developed in Section 2 to quantify the extent of the misallocation of finance. Table 3 contains year-by-year estimates of the potential gains from the reallocation of finance across firms in a sector.[5] Column (1) shows the observed U.S. real benefit of finance as a fraction of the optimal real benefit: $F_{\text{US}}/\hat{F}_{\text{US}}$. Column (2) shows the corresponding hypothetical percentage gain from moving from the observed to the optimal allocation of debt and equity. We find that the percent discrepancy between the optimal and observed allocation of finance for U.S. firms is 10%-13%. In other words, these firms would stand to gain 10%-13% in terms of value-added if they were to move to an optimal allocation. The potential gains appear to be less during the boom periods in 1999 and from 2002 to 2007 and greater during the recessionary period of 2000 to 2001. This result makes sense inasmuch as financial frictions are generally regarded to be more severe in recessions.

In Columns (3) and (4), we present analogous calculations for the Chinese firms. We find that the potential reallocation gains appear enormous, as value-added could be increased by over 60% if the Chinese firms were to move to an efficient allocation of debt and equity. Although these figures seem large, they are of the same order of magnitude as the estimated gains found in Hsieh and Klenow (2009) regarding capital and labor allocations. We also

[5]We calculate standard errors in two ways. First, we calculate asymptotic standard errors by stacking the influence functions for the individual components of F and $\hat{F}$ and then using the delta method to calculate the standard errors of the gains. Second, we use a simple bootstrap. The the asymptotic standard errors are an order of magnitude smaller than bootstrapped standard errors, so we report the latter to be conservative. Even in this conservative case, all of the standard errors are quite small. This result makes sense inasmuch as the figures that we present are all functions of means, which can be estimated with a great deal of precision with several thousand data points.

find, somewhat surprisingly, that the efficiency of the allocation of debt and equity falls over our sample period in China. This phenomenon can be attributed largely to the expansion of the NBS survey in the 2004 Industrial Census.[6] When we restrict our sample to firms that are in the NBS survey before and after the 2004 Industrial Census, the pattern of increasing misallocation diminishes greatly.

To put these results in perspective, we now express the estimates of misallocation in China relative to the estimates of misallocation in the United States. This comparison is motivated by the observation in Hsieh and Klenow (2009) that because a simple static model based on the framework in Melitz (2003) is likely to be misspecified, a researcher is likely to observe positive potential gains even when allocations are efficient. This observation is particularly applicable in our context of financial misallocation because U.S. financial markets are highly developed. Thus, by comparing the potential gains in China relative to the potential gains in the United States, we isolate the potential gains in China relative to an assumed efficient allocation. These results are in Columns (5) and (6) of Table 3, which present the ratio $(F_{\text{China}}/\hat{F}_{\text{China}})/(\hat{F}_{\text{US}}/F_{\text{US}})$ and the corresponding percentage gain. This normalization delivers results that are more modest. We find potential gains of approximately 40%-45% before the expansion of the NBS survey, and of approximately 50%-55% after the expansion.

Next, to understand whether the reallocation gains come from the amount of finance available to Chinese firms or to the type of finance, we compare the relative fractional benefit in Column (5) to a case in which we set $\gamma = \infty$. If $\gamma = \infty$, then the type of finance does not matter for the aggregate benefit of finance because debt and equity are perfect substitutes. This exercise produces an interesting result. As seen in Columns (7) and (8), we find that most of the potential reallocation gains come from the misallocation of scale. Before the expansion of the NBS survey in 2004, we find that only 4%-5% of the gains could

[6] Brandt, Van Biesebroeck, and Zhang (2014) discuss the impact of the 2004 Industrial Census on the NBS survey sample.

be realized by reallocating the type of finance. After the expansion, this figure rises to 6%-7%. This result is interesting because it means that the mix of debt and equity finance has little to do with the large potential TFP losses in China. What matters more is access to finance in general.

We next examine the implications of our estimates for the cross-sectional distribution of firm size. Recall that because of downward sloping demand, each firm has a well-defined optimal size, with an optimal financing mix. Therefore, deviations of the financing amount and mix from the optimal allocation affect firm size, so comparing the distributions of firm size under the actual and efficient allocations is a useful way to quantify misallocation. Figure 1 illustrates this idea with plots of the observed and efficient firm size distributions for the United States and China. We compute observed firm size as $\log(D_{si} + E_{si})$ and efficient firm size as $\log(\hat{D}_{si} + \hat{E}_{si})$. Panel A shows that the efficient U.S. firm size distribution exhibits approximately as much dispersion as the actual distribution. Of course, this result is to be expected, given our calibration of σ, with the slight discrepancy in the distributions stemming from the log transformation. In contrast, in Panel B, we see that the efficient firm size distribution for China has significantly fatter tails, with many Chinese firms being either too small or too large. These size distortions ultimately stem from misallocation of the scale and mix of finance documented in Table 3.

Although the plots in Figure 1 show the firm size distributions before and after reallocation, they do not illustrate the individual changes in firm size that accompany reallocation. Figure 2 shows these potential movements via heat maps. Panel A is a heat map of a three-dimensional histogram in which the observed U.S. firm size distribution is on the x-axis and the efficient U.S. firm size distribution is on the y-axis. The legend for the heat map z-axis is located at right of the map and represents the number of observations in each bin. Similarly, Panel B is a heat map for China. In Panel A, we find that U.S. firms are concentrated along the 45 degree line, where firm size before and after reallocation is the same. In contrast, Chinese firms are much more spread out, reflecting the substantial efficiency gains available from reallocation. Both heat maps are more concentrated towards the top right than towards the bottom left, with this contrast being somewhat more pronounced for U.S. firms. This pattern indicates that small firms are more likely to suffer from financial misallocation than large firms, and relatively more so in the United States.

Next, we calculate the distortions in the costs of debt and equity, $\tau_{D_{si}}$ and $\tau_{E_{si}}$. Table 4 summarizes the post-distortion costs of debt and equity, $(1+\tau_{D_{si}})\,r$ and $(1+\tau_{E_{si}})\,\lambda$, respectively, by year, again under the assumptions that $\gamma = 2$ and $\sigma = 1.77$. Panel A contains means and Panel B contains medians. In Panel A, we find that the costs of debt and equity decline over the sample period in the United States, while these costs rise in China over the same time period. This pattern reinforces the results in Table 3 that indicate greater misallocation after 2004, when the NBS survey samples more firms. These extra firms exhibit more misallocation and consequently greater costs of debt and equity. Finally, the figures in Panel B are uniformly much smaller than those in Panel A, especially for the Chinese firms. This result points to extreme right skewness in the distribution of the cost of finance, implying that some firms are effectively barred from financial markets.

Table 5 presents the costs of debt and equity when we stratify the sample by size instead of year. In the United States, the average cost of debt is substantially lower for large firms, while the cost of equity displays no clear pattern across firm sizes. This latter result is consistent with the well-documented lack of a size premium in equity markets in recent years. In contrast, both the costs of debt and equity are dramatically lower for large Chinese firms compared to small Chinese firms.

To finish our analysis of the costs of debt and equity, we run two descriptive OLS regressions on our sample of Chinese firms to examine how these costs vary by firm characteristics. Specifically, we regress the costs of debt and equity on location, state investment, firm size, time, and firm age. *Location* is a dummy variable that equals 1 if a firm is located in Beijing, Shanghai, Shenzhen, or Guangzhou and 0 otherwise. These four Chinese cities are also known as first tier cities and are the most developed in China.

State investment is a dummy variable that equals 1 if a firm has a non-zero percentage of paid-in-capital from state sources and 0 otherwise. *Size* is the log of total assets measured in 2005 CNY, *time* is a simple linear time trend, and *young* is a dummy variable that equals 1 if the firm is three or fewer years old and 0 otherwise.

Table 6 presents the results. We find that costs of debt and equity are significantly lower for larger firms in China, and this result confirms our cross-sectional sorts by firm size in Table 5. Firms operating in first tier Chinese cities also face lower costs. Surprisingly, firms with non-zero state investment actually face slightly higher costs on average. It is important to note that this result is conditional on firm size. If we break down the total set of firms into those with and without state paid-in-capital, we find that firms with state paid-in-capital have lower costs. However, these firms are also significantly larger, so the effect of state investment on costs reverses once we control for firm size. Foreign investment, on the other hand, is associated with higher costs of debt but lower costs of equity. We also find a positive coefficient on the time trend, which reflects the increasing costs also evident in Table 4. Finally, we find that young firms actually face slightly lower costs of debt and equity.

Next, we turn to a comparison of our financial misallocation of debt and equity with the real misallocation of labor and capital from Hsieh and Klenow (2009). Our intent is to determine whether tax distortions on debt and equity have any relation to the tax distortions on labor and capital. To this end, we first recompute the reallocation gains from the model of capital and labor in Hsieh and Klenow (2009). To make the comparison between their model and ours as close as possible, we set the price of capital, p_k, equal to the weighted average cost of debt and equity, using $r = 0.045$ as the cost of debt and $\lambda = 0.09$ as the cost of equity. We then set the wage, w, so that the capital share of income is one-third for the United States and one-half for China, in line with aggregate data. As in our own analysis, we set $\sigma = 1.77$, and as in Hsieh and Klenow (2009), the production function is Cobb-Douglas.

The results are in Table 8, which presents results by year, and where two broad patterns stand out. First, in Columns (1) and (2), we find less real factor misallocation than the financial misallocation documented in Table 3. Second, in Columns (3) through (6), we find more real factor misallocation for the Chinese firms than the financial misallocation documented in Table 3. These last figures are also slightly larger than those in Hsieh and Klenow (2009). However, these differences are due to the slightly different parameterization that we use to make their model comparable to ours.

Next, we tackle the difficult question of whether our results are passively picking up distortions in real factor markets. In other words, we are worried about a scenario in which there are no financial frictions but in which our model incorrectly attributes genuine observed shortfalls in TFP to financial, as opposed to real, misallocation. While a thorough investigation of this topic would require the estimation of a fully dynamic model of both factor accumulation and financing, we can gain some insights by examining the relations between the real costs in the model in Hsieh and Klenow (2009) and our financial costs.

In Table 9, we present each possible pairwise correlation between the log firm-level costs of debt, equity, capital, and labor, where the costs of debt and equity come from our model, and the costs of capital and labor come from the model in Hsieh and Klenow (2009), as calibrated above. All sets of costs include the tax distortions. Several interesting results stand out. First, in both the United States and China, the costs of debt and equity are more highly correlated with each other than they are with the costs of labor and capital. Because most of the financial misallocation we find stems from the misallocation of the gross flows of finance, this result suggests that financial misallocation is a separate phenomenon from real misallocation. Otherwise, one would expect much higher correlations between real and financial costs if finance were simply passive and if real misallocation were driving all economic efficiencies and thus the only force behind distortions in firm size. Second, in the United States, the correlation between the cost of debt and the price of capital is nearly the same as the correlation between the cost of debt and the wage. This result also holds for the cost of equity. However, the results for China differ sharply in this dimension. We find much higher correlations between the costs of debt and equity and the cost of capital than between either of the two financial costs and the cost of labor. This result points to financial market inefficiencies impeding capital accumulation more than the hiring of workers and thus draws a link between financial and real misallocation in China.

5. Robustness

In this section, we consider several extensions of our analysis. First, we examine the robustness of the reallocation results in Table 3 to the calibration of the parameters γ and σ, the elasticity of substitution between debt and equity, and the elasticity of substitution between firms in a sector, respectively. These results are in Table 7. First, we find that allowing γ to range between 1.5 and 10 has a negligible effect on our estimates of the percent gains in both the United States and China. By construction, γ has no effect on the misallocation of scale. However, changing γ does materially alter our estimates of the fraction of gains that comes from reallocating the type of finance, debt or equity. We allow γ to range from 1.5 to 10, and we find corresponding gains that run between 8.1% and 0.8%. Thus, our general qualitative result that the vast majority of potential gains stems from the misallocation of scale remains intact.

While varying γ has little effect on our estimated reallocation gains, varying σ does. We find that the estimated gains increase sharply when we increase σ. Intuitively, the firm size distribution becomes excessively skewed if σ is too large because, in this case, all resources flow to the most productive firms. In other words, if one were to pick the most productive Chinese firms in a sector and give them all the resources, the gains would be large because of the substantial dispersion in productivity.

However, as we argue above, the calibration of σ is conservative if σ is chosen to be on the low end. We now expand on these arguments. One way to discipline the choice of σ is to calculate the distributions of debt and equity (and thus firm size) when the allocation is efficient and compare these distributions to the realized distributions in the data. For the United States, when $\sigma = 1.77$, the standard deviation of the efficient size distribution is exactly the same as the standard deviation of the observed distribution, that is, the standard deviation of $\hat{D}_{si} + \hat{E}_{si}$ equals the standard deviation of $D_{si} + E_{si}$. This choice of σ is very conservative, because if σ is lower, the efficient size distribution would be more compressed than the observed distribution. However, when σ rises above 2, the efficient size distribution expands substantially, and the reallocation gains become quite large.

Next, a natural concern is the small overlap between the distributions of the sizes of Chinese and U.S. firms. To understand whether this property of our data drives our results, for each year, we compute the size intersection between Chinese and U.S. firms, where size is total assets in U.S. dollars adjusted by the nominal exchange rate. In computing this intersection, we drop the ten smallest U.S. firms and the ten largest Chinese firms from the sample so that extreme outliers do not affect the size intersection. We then recompute the potential reallocation gains, with the results in Table 10. We find that the relative percentage reallocation gains for the Chinese firms are approximately halved.

Interestingly and surprisingly, the factor driving most of the difference is the removal of the large U.S. firms, not the removal of the small Chinese firms. In other words, a nonoverlapping size distribution appears to mask observed inefficiencies in the U.S. firms rather than exacerbate observed inefficiencies in the Chinese firms. To understand this issue, note that we remove approximately the largest one sixth of the U.S. firms after taking the size intersection, and these firms are close to operating at an optimal allocation. Thus, the removal of these big U.S. firms increases the percentage reallocation gain in the United States from approximately 10% to over 20%. This result implies that smaller U.S. firms could make substantial gains from reallocation, but these gains appear insignificant when we consider all of the firms in Compustat, which include the top one sixth of firms that are large, productive, and already at an efficient size and financing mix. In contrast, productive

Chinese firms can be of any size, so potential reallocation gains do not come only from tiny firms. This pattern in the Chinese data is also evident in the heat map in Figure 2, in which large deviations between original and efficient sizes can be seen for both large and small firms.

Next, recall that for our benchmark results in Table 3, we include only non-state-owned firms in our sample of Chinese firms. However, in terms of pure aggregate size, state-owned firms make up a substantial fraction of the Chinese economy. Therefore, we examine the sensitivity of our results to including these firms in our sample. Table 11 reports the potential reallocation gains for state-owned Chinese firms alone and for the entire sample of Chinese firms. For reference, Column (1) of Table 11 shows the fractional reallocation benefit by year for the entire sample of U.S. firms and thus repeats Column (1) in Table 3. Columns (3) and (4) show, somewhat surprisingly, that state-owned firms stand to gain less in percentage terms than non-state-owned firms, with the fractional reallocation gains ranging from 29% to 45%, figures nearly half the corresponding values for the sample of non-state-owned Chinese firm. Two forces are at work here. On the one hand, the public status of the state-owned Chinese firms confers upon them much better access to external financial markets. On the other hand, these state-owned firms do not necessarily have a profit maximizing goal in mind when planning their activities, so they are likely to be less efficient. Our evidence suggests that the first force dominates the second. Note also that the average state-owned firm is nearly twice as large as the average non-state-owned firm. Finally, the percentage of available reallocation gains is slightly smaller for the sample of all state and non-state owned Chinese firms. However, because the measured economy is larger with the inclusion of the state-owned firms, the total gross amount of potential reallocation gains is still substantially larger.

We next consider our assumption that the costs of debt and equity, r and λ do not vary by sector. This assumption flies in the face of decades of evidence of great heterogeneity in the cost of capital. Because most of the firms in our Chinese sample are private, understanding the sensitivity of our results to a constant r and λ is difficult for this sample. However, because our U.S. firms are public, we can estimate these sectoral costs. First, we measure the sector cost of debt, r_s, as the average ratio of interest expense to total debt. For the sector equity cost, we estimate a simple market model for each three-digit industry. Specifically, we use CRSP daily stock data from 1999 to 2007 to estimate an unlevered industry-level beta. We then combine these betas with a risk-free rate of 0.03 and a market-risk premium of 0.06 to be consistent with our baseline value for λ. Table 12 shows the reallocation gains using the sector-level costs r_s and λ_s. The overall results are very similar to our benchmark for the United States. Two pieces of intuition support this result. First, sector-level costs only affect the expression for the weight on debt: $\alpha_s = r_s D_s^{1/\gamma}/(r_s D_s^{1/\gamma} + \lambda_s E_s^{1/\gamma})$. However, our estimates of the reallocation gains come from within-sector variation in debt-equity ratios and firm sizes, so a change in α_s does not have much of an impact. Second, firms only care about the after tax cost of debt $(1 + \tau_{D_{si}})\, r$ and equity $(1 + \tau_{E_{si}})\, \lambda$, so changing r or λ only changes the interpretation of the tax distortions relative to the base cost of debt and equity.

Table 13 examines whether our results depend on our measure of the nominal benefit of finance. Instead of value-added, a natural alternative measure is the sum of the market values of debt and equity. Of course, we cannot use this measure in our sample of Chinese firms, as most of these firms are not publicly traded. However, we can examine this alternative measure in our sample of U.S. firms. We find that overall reallocation gains are similar in magnitude to those in Table 3. One exception occurs during the dot-com boom, in which we find more misallocation.

6. Conclusion

This paper entertains the possibility that finance may be misallocated in the cross-section of firms. We explore this hypothesis using a tractable model of differentiated firms based on

Hsieh and Klenow (2009). Our empirical investigation is based on the intuitive result that in our framework, the optimal allocation of debt and equity equates their marginal benefits across firms within an industry. Thus, any observed dispersion in the marginal benefits of debt equity is symptomatic of misallocation.

Our evidence from U.S. data points only to modest misallocation distortions or, equivalently, modest potential reallocation gains. These firms stand to gain only 10%-13% in terms of aggregate real firm value-added if they were to move to an efficient allocation. Our results are much more dramatic for China, where firms stand to gain over 60% from moving to an efficient allocation. If China were able to achieve the more reasonable U.S. level of efficiency, gains of 40-55% would still be possible. When we break this figure down by the amount of finance versus the type of finance, we find that nearly all of this figure can be attributed to the amount of finance and little to the mix of securities used to fund a firm's operations.

Our work expands the literature on the interaction between productive and financial allocation and the puzzling persistence of productive misallocation (Hsieh and Klenow 2009). The financial misallocation we investigate in this paper may be related to productive misallocation and can help explain this puzzle. For instance, our results indicate that external financing is more important for capital accumulation than for payroll, so if financial frictions are persistent, the misallocation of productive factors should be as well. Overall, productivity losses can result both from the misallocation of debt and equity and from the misallocation of capital and labor. We leave to further work an analysis of financial and real misallocation together in a unified framework.

Appendix

Aggregate price

We begin by solving for the aggregate price P as a function of sector price P_s, where P is defined as the minimum price of acquiring a unit of the aggregate benefit. The minimization problem is mathematically stated as:

$$\min_{F_s} \left\{ \sum_s P_s F_s \right\}, \tag{21}$$

subject to:

$$\prod_s F_s^{\theta_s} = \bar{F}. \tag{22}$$

The Lagrangian is:

$$\mathcal{L} = -\sum_s P_s F_s + M \left[\prod_s F_s^{\theta_s} - \bar{F} \right], \tag{23}$$

where M is the Lagrange multiplier. The first-order condition with respect to F_s gives:

$$P_s = M \theta_s \frac{\prod_s F_s^{\theta_s}}{F_s}, \tag{24}$$

which simplifies to:

$$\frac{P_s F_s}{\theta_s} = PF \tag{25}$$

because $M = P$. After aggregation of sectors in the economy, we can write the aggregate price as a function of sector price:

$$P = \prod_s \left(\frac{P_s}{\theta_s}\right)^{\theta_s}. \tag{26}$$

Sector price

In a similar fashion, we can solve for the sector price P_s as a function of firm price P_{si}, where P_s is defined as the minimum price of acquiring a unit of the sector benefit. The minimization problem is mathematically stated as:

$$\min_{F_{si}} \left\{ \sum_i P_{si} F_{si} \right\}, \tag{27}$$

subject to:

$$\left(\sum_i F_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = \bar{F}_s. \tag{28}$$

The Lagrangian is:

$$\mathcal{L}_s = -\sum_i P_{si} F_{si} + M_s \left[\left(\sum_i F_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \bar{F}_s \right]. \tag{29}$$

where M_s is the Lagrange multiplier. The first-order condition with respect to F_{si} gives:

$$P_{si} = M_s \left(\sum_i F_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} F_{si}^{-\frac{1}{\sigma}}. \tag{30}$$

which simplifies to:

$$P_{si}^{\sigma} F_{si} = P_s^{\sigma} F_s \tag{31}$$

because $M_s = P_s$. After aggregation of firms in a sector, we can write the sector price as a function of firm price:

$$P_s = \left(\sum_i P_{si}^{-(\sigma-1)} \right)^{-\frac{1}{\sigma-1}}. \tag{32}$$

Firm's problem

A firm i in sector s chooses price P_{si}, debt D_{si}, and equity E_{si} to maximize the nominal net benefit of finance π_{si}. The debt and equity decision aims to minimize the total cost of finance for a given level of real benefit $\bar{F}_{si}$, and can be separated from the price decision. Formally, the minimization problem is:

$$\min_{D_{si}, E_{si}} \left\{ \left(1 + \tau_{D_{si}}\right) r D_{si} + \left(1 + \tau_{E_{si}}\right) \lambda E_{si} \right\}, \tag{33}$$

subject to:

$$A_{si} \left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1 - \alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} = \bar{F}_{si}. \tag{34}$$

After setting up the Lagrangian and taking the first-order conditions respect to D_{si} and E_{si}, we arrive at the following optimal debt-equity ratio:

$$\frac{D_{si}}{E_{si}} = \left(\frac{\alpha_s}{1 - \alpha_s} \frac{\left(1 + \tau_{E_{si}}\right) \lambda}{\left(1 + \tau_{D_{si}}\right) r} \right)^{\gamma}. \tag{35}$$

To simplify notation, let:

$$Z_{si} = \left(\frac{\alpha_s}{1 - \alpha_s} \frac{\left(1 + \tau_{E_{si}}\right) \lambda}{\left(1 + \tau_{D_{si}}\right) r} \right)^{\gamma}, \tag{36}$$

so that the optimal ratio can be rewritten as:

$$\frac{D_{si}}{E_{si}} = Z_{si}. \tag{37}$$

Debt and equity can thus be expressed as linear functions of the real benefit, as follows:

$$\begin{aligned} D_{si} =& \frac{\bar{F}_{si}}{A_{si}} \left(\alpha_s + (1-\alpha_s) Z_{si}^{-\frac{\gamma-1}{\gamma}} \right)^{-\frac{\gamma}{\gamma-1}} \\ E_{si} =& \frac{\bar{F}_{si}}{A_{si}} \left(\alpha_s Z_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s) \right)^{-\frac{\gamma}{\gamma-1}}. \end{aligned} \tag{38}$$

Then using the above expressions for debt and equity, the minimum cost function becomes a function of the fixed real benefit $\bar{F}_{si}$:

$$\begin{aligned} C(\bar{F}_{si}) &= (1+\tau_{D_{si}})\, r D_{si} + (1+\tau_{E_{si}})\, \lambda E_{si} \\ &= C_{si} \bar{F}_{si}, \end{aligned} \tag{39}$$

where:

$$\begin{aligned} C_{si} =& \frac{1}{A_{si}} \Bigg((1+\tau_{D_{si}})\, r \left(\alpha_s + (1-\alpha_s) Z_{si}^{-\frac{\gamma-1}{\gamma}} \right)^{-\frac{\gamma}{\gamma-1}} \\ & + (1+\tau_{E_{si}})\, \lambda \left(\alpha_s Z_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s) \right)^{-\frac{\gamma}{\gamma-1}} \Bigg). \end{aligned} \tag{40}$$

Next, we choose P_{si} to maximize the nominal net benefit of finance, that is:

$$\max_{P_{si}} \{\pi_{si}\} = \max_{P_{si}} \{P_{si} F_{si} - C_{si} F_{si}\}. \tag{41}$$

Recall from the sector price derivation that firm-level real benefit is a function of the sector price, firm price, and sector real benefit, $F_{si} = \left(\frac{P_s}{P_{si}}\right)^{\sigma} F_s$. Therefore, the firm's real benefit is just a function of price once the optimal debt-equity ratio is computed, and the firm faces a downward sloping demand curve. The maximization problem is bounded due to downward sloping demand even though the firm has constant returns to scale. From the first-order condition on price we find:

$$P_{si} = \frac{\sigma}{\sigma - 1} C_{si}. \tag{42}$$

Note that the price is a fixed markup over marginal cost and a higher elasticity of substitution between firms in a sector lowers the price the firm can charge for the real benefit it is generating.

Taxes

To solve for the tax distortions, the nominal benefit of finance should first be written as:

$$P_{si} F_{si} = P_s F_s^{\frac{1}{\sigma}} F_{si}^{\frac{\sigma-1}{\sigma}}. \tag{43}$$

The marginal nominal benefit of debt must equal the marginal nominal cost of debt for the maximizing firm, so the first-order condition with respect to D_{si} gives:

$$P_s F_s^{\frac{1}{\sigma}} \frac{\sigma - 1}{\sigma} F_{si}^{-\frac{1}{\sigma}} A_{si} \left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1 - \alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1} - 1} \alpha_s D_{si}^{-\frac{1}{\gamma}} = (1 + \tau_{D_{si}}) \, r. \tag{44}$$

which simplifies to:

$$(1 + \tau_{D_{si}}) \, r = \alpha_s \frac{\sigma - 1}{\sigma} \frac{P_{si} F_{si}}{\alpha_s D_{si} + (1 - \alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}} D_{si}^{\frac{1}{\gamma}}}. \tag{45}$$

Similarly, the first-order condition with respect to E_{si} simplifies to:

$$(1+\tau_{E_{si}})\,\lambda = (1-\alpha_s)\frac{\sigma-1}{\sigma}\frac{P_{si}F_{si}}{\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}}E_{si}^{\frac{1}{\gamma}}+(1-\alpha_s)E_{si}}. \tag{46}$$

Efficient allocation

We now turn to the derivation of the efficient allocation in a sector. Under the efficient allocation, total debt and total equity in a sector are kept the same, but debt and equity are reallocated across firms in a sector to maximize sector real benefit. The debt-to-equity ratio $Z_{si} = \frac{D_s}{E_s} = Z_s$ can be shown to be the same for all firms i in sector s when debt and equity are reallocated to achieve efficiency. The real benefit of finance can then be written as a function of $\hat{D}_{si}$:

$$\hat{F}_{si} = \left(\alpha_s + (1-\alpha_s)Z_s^{-\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} A_{si}\hat{D}_{si}, \tag{47}$$

where a hat above a variable indicates the efficient level after reallocation. The Lagrangian for maximizing sector financial benefit is:

$$\hat{\mathcal{L}}_s = \left(\sum_i \left(\left(\alpha_s + (1-\alpha_s)Z_s^{-\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} A_{si}\hat{D}_{si}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} + \hat{M}_s\left[\sum_i \hat{D}_{si} - D_s\right], \tag{48}$$

where $\hat{M}_s$ is the Lagrange multiplier. The first-order condition with respect to $\hat{D}_{si}$ and $\hat{D}_{sj}$ for firms i and j respectively rearranges to:

$$\left(\frac{\hat{D}_{si}}{\hat{D}_{sj}}\right)^{-\frac{1}{\sigma}} = \left(\frac{A_{sj}}{A_{si}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{49}$$

After aggregation, the expression above can be simplified to:

$$\hat{D}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} D_s. \tag{50}$$

The optimal equity allocation can be similarly derived as:

$$\hat{E}_{si} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} E_s. \tag{51}$$

The real benefit F_{si} is assumed to be unobservable. However, A_{si} can be expressed in terms of observable variables, such as the nominal benefit, $P_{si}F_{si}$, that is:

$$A_{si} = \eta_s \frac{(P_{si}F_{si})^{\frac{\sigma}{\sigma-1}}}{\left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s) E_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}}, \tag{52}$$

where

$$\eta_s = \frac{1}{P_s (P_s F_s)^{\frac{1}{\sigma-1}}} \tag{53}$$

because

$$F_{si} P_s (P_s F_s)^{\frac{1}{\sigma-1}} = (P_{si}F_{si})^{\frac{\sigma}{\sigma-1}}. \tag{54}$$

Reallocation gains are not affected if η_s is normalized to one for all sectors s. Because η_s does not vary across firms in a sector, it divides out of our measure of reallocation gains.

Aggregation

The ultimate goal is to find the ratio of the aggregate real benefit computed from data over the efficient allocation. The real benefit computed from data is given by:

$$
\begin{aligned}
F_{si} &= A_{si}\left(\alpha_s D_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s)E_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \\
F_s &= \left(\sum_i F_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
F &= \prod_s F_s^{\theta_s},
\end{aligned}
\tag{55}
$$

while the efficient allocation is given by:

$$
\begin{aligned}
\hat{F}_{si} &= A_{si}\left(\alpha_s \hat{D}_{si}^{\frac{\gamma-1}{\gamma}} + (1-\alpha_s)\hat{E}_{si}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \\
\hat{F}_s &= \left(\sum_i \hat{F}_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
\hat{F} &= \prod_s \hat{F}_s^{\theta_s}.
\end{aligned}
\tag{56}
$$

Therefore, the ratio is $F/\hat{F}$.

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Table 1: Chinese Firm Summary Statistics

Panel A

Size percentile	Observations	Assets	Liabilities	Equity	Liabilities/Assets	Value-added
0-5	62497	1.9	1.0	1.0	0.483	1.6
5-15	124849	3.8	2.1	1.8	0.536	2.0
15-30	187328	6.4	3.5	2.9	0.549	2.5
30-50	249697	11.2	6.2	5.0	0.555	3.5
50-70	249750	21.6	11.9	9.6	0.552	5.4
70-85	187304	46.3	25.5	20.8	0.551	9.7
85-95	124870	120.3	66.6	53.7	0.553	22.2
95-100	62434	859.3	493.1	365.8	0.569	138.8

Panel B

Year	Observations	Assets	Liabilities	Equity	Liabilities/Assets	Value-added
1999	51646	75.6	42.3	33.4	0.566	11.3
2000	62822	78.1	43.6	34.4	0.566	12.4
2001	78893	73.1	39.9	33.1	0.554	11.8
2002	95520	70.9	38.9	32.0	0.551	11.9
2003	119292	72.5	40.8	31.7	0.548	12.6
2004	181692	59.5	34.3	25.2	0.564	10.7
2005	190022	68.8	39.3	29.4	0.545	12.9
2006	217242	70.8	40.3	30.5	0.539	14.0
2007	251600	71.9	40.9	31.0	0.535	15.7

Calculations are based on a sample of Chinese firms from the annual survey conducted by the National Bureau of Statistics of China (NBS) from 1999 to 2007. All firms are in the manufacturing sector and all have more than 5 million Chinese yuan (CNY) in sales. There are a total of 1,248,729 firm-year observations, and all variables are reported in millions of 2005 CNY. Panel A presents summary statistics by firm size percentile. Panel B presents summary statistics by year.

Table 2: U.S. Firm Summary Statistics

Panel A

Size percentile	Observations	Assets	Liabilities	Equity	Liabilities/Assets	Value-added
0-5	713	9.9	3.3	6.6	0.330	4.7
5-15	1416	25.6	9.7	15.9	0.378	11.0
15-30	2123	63.5	22.9	40.6	0.369	25.2
30-50	2829	170.1	62.7	107.4	0.367	62.1
50-70	2833	507.8	226.8	281.0	0.438	188.0
70-85	2124	1480.5	796.7	683.8	0.540	518.0
85-95	1416	4707.0	2802.9	1904.1	0.591	1543.1
95-100	704	28128.4	16154.0	11974.5	0.596	8169.6

Panel B

Year	Observations	Assets	Liabilities	Equity	Liabilities/Assets	Value-added
1999	1936	1479.1	883.7	595.4	0.466	507.6
2000	1782	1719.4	1000.2	719.2	0.460	597.7
2001	1612	2013.2	1172.5	840.7	0.450	614.3
2002	1541	2131.0	1257.1	873.9	0.448	622.1
2003	1544	2263.7	1286.4	977.3	0.430	663.4
2004	1517	2469.3	1365.8	1103.5	0.418	750.5
2005	1470	2578.9	1411.5	1167.4	0.424	811.2
2006	1415	2860.4	1549.0	1311.4	0.426	877.7
2007	1341	3112.7	1695.0	1417.7	0.431	912.2

Calculations are based on a sample of manufacturing firms (SIC 2000 to 3999) from Compustat. The sample period is 1999 to 2007 and includes 14,158 firm-year observations. All variables are reported in millions of 2005 USD. Value-added is operating income before depreciation (OIBDP) plus imputed wages. Imputed wages are calculated by multiplying the employment of each firm with the mean wage per employee in the appropriate three-digit SIC industry. Panel A presents summary statistics broken down by firm size percentile. Panel B presents summary statistics by year.

Table 3: Reallocation Gains by Year

	United States		China		United States vs. China			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	Fractional Benefit	Percent Gain	Fractional Benefit	Percent Gain	Fractional Benefit	Percent Gain	Percent Scale	Percent Type
1999	0.903	10.7	0.617	62.1	0.683	46.4	41.2	5.2
	(0.007)	(0.9)	(0.009)	(2.2)	(0.011)	(2.3)	(2.1)	(0.3)
2000	0.886	12.8	0.605	65.3	0.683	46.4	41.9	4.5
	(0.012)	(1.4)	(0.014)	(3.6)	(0.018)	(3.8)	(3.4)	(0.6)
2001	0.892	12.1	0.619	61.5	0.694	44.0	39.4	4.6
	(0.011)	(1.4)	(0.008)	(2.0)	(0.012)	(2.5)	(2.1)	(0.5)
2002	0.885	13.0	0.607	64.6	0.687	45.6	41.0	4.7
	(0.009)	(1.1)	(0.012)	(3.1)	(0.014)	(3.0)	(2.8)	(0.5)
2003	0.886	12.9	0.628	59.1	0.709	41.0	36.4	4.6
	(0.010)	(1.3)	(0.006)	(1.4)	(0.010)	(2.0)	(1.6)	(0.5)
2004	0.900	11.1	0.596	67.9	0.662	51.2	45.2	5.9
	(0.010)	(1.2)	(0.005)	(1.3)	(0.010)	(2.3)	(1.9)	(0.5)
2005	0.900	11.1	0.585	70.9	0.650	53.8	47.4	6.3
	(0.010)	(1.2)	(0.005)	(1.5)	(0.009)	(2.1)	(1.8)	(0.5)
2006	0.894	11.9	0.591	69.3	0.661	51.3	45.0	6.2
	(0.009)	(1.1)	(0.004)	(1.2)	(0.008)	(1.9)	(1.6)	(0.5)
2007	0.892	12.1	0.574	74.2	0.643	55.4	48.8	6.6
	(0.010)	(1.2)	(0.004)	(1.1)	(0.008)	(2.0)	(1.7)	(0.7)

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat, and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007. This table presents potential reallocation gains when the substitutability between debt and equity is $\gamma = 2$. Column (1) shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal U.S. allocation: $F_{\mathrm{US}}/\hat{F}_{\mathrm{US}}$. Column (2) shows the corresponding percentage gain from moving from the observed to the optimal allocation. Columns (3)–(4) present analogous calculations for Chinese firms. Columns (5)–(6) show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio: $(F_{\mathrm{China}}/\hat{F}_{\mathrm{China}})/(\hat{F}_{\mathrm{US}}/F_{\mathrm{US}})$, and the corresponding percentage gains. Columns (7)–(8) provide a breakdown of total misallocation into the misallocation due to the scale of finance and due to the mix of debt and equity, holding scale fixed. Standard errors are in parentheses below each estimate.

Table 4: Costs of Debt and Equity by Year

		United States		China	
Panel B	Year	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$
	1999	0.144	0.257	0.102	0.204
		(0.0022)	(0.0147)	(0.0009)	(0.0018)
	2000	0.141	0.251	0.108	0.212
		(0.0025)	(0.0063)	(0.0027)	(0.0020)
	2001	0.131	0.231	0.108	0.219
		(0.0021)	(0.0061)	(0.0008)	(0.0017)
	2002	0.133	0.237	0.109	0.225
		(0.0026)	(0.0066)	(0.0007)	(0.0017)
	2003	0.133	0.231	0.122	0.223
		(0.0029)	(0.0072)	(0.0010)	(0.0011)
	2004	0.132	0.224	0.121	0.235
		(0.0026)	(0.0048)	(0.0007)	(0.0011)
	2005	0.131	0.228	0.142	0.257
		(0.0024)	(0.0047)	(0.0008)	(0.0014)
	2006	0.128	0.222	0.149	0.264
		(0.0023)	(0.0049)	(0.0010)	(0.0012)
	2007	0.124	0.218	0.171	0.291
		(0.0034)	(0.0060)	(0.0009)	(0.0010)
Panel B	Year	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$
	1999	0.125	0.212	0.061	0.130
		(0.0020)	(0.0037)	(0.0003)	(0.0006)
	2000	0.122	0.210	0.065	0.138
		(0.0016)	(0.0034)	(0.0003)	(0.0006)
	2001	0.113	0.190	0.067	0.142
		(0.0019)	(0.0038)	(0.0002)	(0.0005)
	2002	0.113	0.191	0.069	0.145
		(0.0019)	(0.0030)	(0.0002)	(0.0006)
	2003	0.113	0.182	0.072	0.147
		(0.0018)	(0.0035)	(0.0002)	(0.0004)
	2004	0.114	0.185	0.072	0.153
		(0.0022)	(0.0037)	(0.0002)	(0.0003)
	2005	0.114	0.189	0.078	0.160
		(0.0017)	(0.0032)	(0.0002)	(0.0004)
	2006	0.110	0.187	0.082	0.166
		(0.0026)	(0.0038)	(0.0002)	(0.0004)
	2007	0.104	0.177	0.088	0.178
		(0.0022)	(0.0033)	(0.0002)	(0.0004)

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat, and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007. This table displays the estimated mean (Panel A) and median (Panel B) costs of debt, $(1+\tau_{D_{si}})\,r$, and equity, $(1+\tau_{E_{si}})\,\lambda$, in the United States and China by year. The elasticity of substitution between debt and equity is set at $\gamma = 2$. Standard errors are in parentheses below each estimate.

Table 5: Costs of Debt and Equity by Firm Size

		United States		China	
Panel A	Percentile	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$
	0-5	0.193	0.262	0.397	0.608
		(0.0086)	(0.0137)	(0.0042)	(0.0046)
	5-15	0.165	0.258	0.219	0.395
		(0.0046)	(0.0078)	(0.0019)	(0.0023)
	15-30	0.145	0.238	0.166	0.308
		(0.0025)	(0.0054)	(0.0008)	(0.0011)
	30-50	0.143	0.225	0.128	0.247
		(0.0017)	(0.0091)	(0.0005)	(0.0009)
	50-70	0.131	0.224	0.104	0.199
		(0.0014)	(0.0034)	(0.0004)	(0.0009)
	70-85	0.113	0.245	0.086	0.163
		(0.0011)	(0.0045)	(0.0005)	(0.0008)
	85-95	0.103	0.238	0.070	0.136
		(0.0011)	(0.0046)	(0.0005)	(0.0009)
	95-100	0.096	0.201	0.062	0.118
		(0.0016)	(0.0037)	(0.0025)	(0.0007)
Panel B	Percentile	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$
	0-5	0.160	0.199	0.224	0.388
		(0.0034)	(0.0083)	(0.0009)	(0.0016)
	5-15	0.137	0.198	0.136	0.271
		(0.0034)	(0.0045)	(0.0004)	(0.0006)
	15-30	0.126	0.184	0.101	0.212
		(0.0018)	(0.0036)	(0.0002)	(0.0005)
	30-50	0.125	0.176	0.079	0.168
		(0.0022)	(0.0024)	(0.0001)	(0.0003)
	50-70	0.115	0.190	0.064	0.135
		(0.0012)	(0.0026)	(0.0001)	(0.0002)
	70-85	0.106	0.203	0.054	0.113
		(0.0014)	(0.0021)	(0.0001)	(0.0003)
	85-95	0.099	0.215	0.048	0.101
		(0.0012)	(0.0027)	(0.0001)	(0.0003)
	95-100	0.090	0.191	0.044	0.094
		(0.0021)	(0.0042)	(0.0002)	(0.0003)

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat, and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007. This table displays the estimated mean (Panel A) and median (Panel B) costs of debt $(1+\tau_{D_{si}})\,r$ and equity $(1+\tau_{E_{si}})\,\lambda$ in the United States and China by firm size. The elasticity of substitution between debt and equity is set at $\gamma = 2$. Standard errors are in parentheses below each estimate.

Table 6: The Costs of Debt and Equity and Firm Characteristics

	$(1+\tau_{D_{si}})\,r$	$(1+\tau_{E_{si}})\,\lambda$
Location	-0.014	-0.016
	(-12.6)	(-11.4)
State investment	0.015	0.014
	(5.8)	(4.4)
Foreign investment	0.029	-0.001
	(32.2)	(-0.9)
Size	-0.047	-0.072
	(-157.3)	(-196.5)
Time	0.008	0.008
	(44.8)	(38.9)
Young	-0.003	-0.010
	(-3.7)	(-10.2)

Calculations are based on a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007. This table presents two OLS regressions, in which the dependent variables are the costs of debt, $(1+\tau_{D_{si}})\,r$, and equity, $(1+\tau_{E_{si}})\,\lambda$, respectively. The regressors are location, state investment, firm size, time, and firm age. Location is a dummy variable that equals 1 if a firm is located in Beijing, Shanghai, Shenzhen, or Guangzhou and 0 otherwise. State investment is a dummy variable that equals 1 if a firm has a non-zero percentage of paid-in-capital from state sources and 0 otherwise. Foreign investment is a dummy variable that equals 1 if a firm has a non-zero percentage of paid-in-capital from foreign sources and 0 otherwise. Size is log total assets measured in 2005 CNY. Time is a linear time trend, and Young is a dummy variable that equals 1 if the firm is three or fewer years old and 0 otherwise. T-statistics are in parentheses.

Table 7: Reallocation Gains by Elasticities of Substitution

	United States		China		United States vs. China			
Parameter	(1) Fractional Benefit	(2) Percent Gain	(3) Fractional Benefit	(4) Percent Gain	(5) Fractional Benefit	(6) Percent Gain	(7) Percent Scale	(8) Percent Type
$\gamma = 1.5$	0.885	13.1	0.586	70.7	0.662	51.0	42.8	8.1
$\gamma = 2$	0.893	12.0	0.603	65.9	0.675	48.2	42.8	5.4
$\gamma = 3$	0.901	11.0	0.617	62.1	0.685	46.0	42.8	3.2
$\gamma = 5$	0.907	10.2	0.627	59.4	0.692	44.6	42.8	1.8
$\gamma = 10$	0.912	9.7	0.635	57.6	0.696	43.7	42.8	0.8
$\sigma = 1.5$	0.906	10.3	0.648	54.3	0.715	39.9	35.0	4.8
$\sigma = 1.77$	0.893	12.0	0.603	65.9	0.675	48.2	42.8	5.4
$\sigma = 2$	0.882	13.4	0.564	77.2	0.640	56.3	50.3	6.0
$\sigma = 2.5$	0.855	16.9	0.480	108.4	0.561	78.2	70.4	7.9
$\sigma = 3$	0.827	20.9	0.398	151.3	0.481	107.9	97.3	10.7

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat, and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007. This table presents potential reallocation gains averaged across all years when we allow the elasticities of substitution, γ and σ, to vary. When γ is varied, σ is set to 1.77, and when σ is varied, γ is set to 2. Column (1) shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal U.S. allocation: $F_{\text{US}}/\hat{F}_{\text{US}}$. Column (2) shows the corresponding percentage gain from moving from the observed to the optimal allocation. Columns (3)–(4) present analogous calculations for Chinese firms. Columns (5)–(6) show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio: $(F_{\text{China}}/\hat{F}_{\text{China}})/(\hat{F}_{\text{US}}/F_{\text{US}})$, and the corresponding percentage gains. Columns (7)–(8) provide a breakdown of total misallocation into the misallocation due to the scale of finance and due to the mix of debt and equity, holding scale fixed. Standard errors are in parentheses below each estimate.

Table 8: Capital and Labor Misallocation

	United States		China		United States vs. China	
	(1)	(2)	(3)	(4)	(5)	(6)
Year	Fractional Benefit	Percent Gain	Fractional Benefit	Percent Gain	Relative Fractional Benefit	Percent Gain
1999	0.936	6.9	0.554	80.4	0.592	68.8
2000	0.936	6.9	0.584	71.1	0.625	60.1
2001	0.944	6.0	0.560	78.6	0.593	68.6
2002	0.940	6.3	0.539	85.6	0.573	74.5
2003	0.939	6.5	0.548	82.4	0.584	71.3
2004	0.935	6.9	0.530	88.8	0.567	76.5
2005	0.933	7.1	0.545	83.5	0.584	71.2
2006	0.938	6.6	0.552	81.2	0.588	69.9
2007	0.932	7.3	0.540	85.2	0.579	72.6

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat, and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007. Column (1) shows observed real U.S. value-added as a fraction of optimal real U.S. value added, $Y_{\text{US}}/\hat{Y}_{\text{US}}$. Column (2) shows the corresponding percentage gain from moving from the observed to the optimal allocation. Columns (3)–(4) present analogous calculations for Chinese firms. Columns (5)–(6) columns show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio: $(Y_{\text{China}}/\hat{Y}_{\text{China}})/(\hat{Y}_{\text{US}}/Y_{\text{US}})$, and the corresponding percentage gains.

Table 9: Real and Financial Cost Correlations

Panel A

United States	$\log r_{si}$	$\log \lambda_{si}$	$\log p_{k,si}$	$\log w_{si}$
$\log r_{si}$	1.000	0.780	0.468	0.476
$\log \lambda_{si}$	0.780	1.000	0.363	0.401
$\log p_{k,si}$	0.468	0.363	1.000	0.401
$\log w_{si}$	0.476	0.401	0.401	1.000

Panel B

China	$\log r_{si}$	$\log \lambda_{si}$	$\log p_{k,si}$	$\log w_{si}$
$\log r_{si}$	1.000	0.741	0.606	0.386
$\log \lambda_{si}$	0.741	1.000	0.641	0.252
$\log p_{k,si}$	0.606	0.641	1.000	0.238
$\log w_{si}$	0.386	0.252	0.238	1.000

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat (Panel A), and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China (Panel B). The sample period is from 1999 to 2007. The base price of capital, p_k, is set to the weighted average cost of debt and equity using $r = 0.045$ as the cost of debt and $\lambda = 0.9$ as the cost of equity. The base wage, w, is set so that the capital share of income is one third for the United States and one half for China. We compute each possible pairwise correlation between the log firm-level costs of debt, equity, capital, and labor, where $r_{si} = r(1 + \tau_{D_{si}})$, $\lambda_{si} = \lambda(1 + \tau_{E_{si}})$, $p_{k,si} = p_k(1 + \tau_{K_{si}})$, and $w_{si} = w(1 + \tau_{L_{si}})$.

Table 10: Reallocation Gains for the Firm Size Intersection

	United States		China		United States vs. China	
	(1)	(2)	(3)	(4)	(5)	(6)
Year	Fractional Benefit	Percent Gain	Fractional Benefit	Percent Gain	Relative Fractional Benefit	Percent Gain
1999	0.807	24.0	0.654	52.8	0.811	23.2
2000	0.779	28.4	0.676	47.8	0.869	15.1
2001	0.787	27.1	0.672	48.9	0.853	17.2
2002	0.786	27.3	0.658	51.9	0.838	19.4
2003	0.786	27.2	0.674	48.3	0.858	16.6
2004	0.805	24.2	0.652	53.4	0.810	23.5
2005	0.811	23.3	0.665	50.4	0.820	22.0
2006	0.822	21.7	0.668	49.6	0.813	23.0
2007	0.813	23.0	0.655	52.7	0.805	24.1

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat, and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007. The size intersection between U.S. and Chinese firms is taken where size is measured by total assets. In each year, all firms above the tenth largest Chinese firm as well as all firms below the tenth smallest U.S. firm are dropped. Column (1) shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal U.S. allocation: $F_{\text{US}}/\hat{F}_{\text{US}}$. Column (2) shows the corresponding percentage gain from moving from the observed to the optimal allocation. The next two columns present analogous calculations for Chinese firms. The final two columns show the Chinese efficiency ratio as a fraction of the U.S. efficiency ratio: $(F_{\text{China}}/\hat{F}_{\text{China}})/(\hat{F}_{\text{US}}/F_{\text{US}})$, and the corresponding percentage gains.

Table 11: State-owned Firms

	United States	China State-Owned Firms			China All Firms		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Year	Fractional Benefit	Fractional Benefit	Relative Fractional Benefit	Percent Gain	Fractional Benefit	Relative Fractional Benefit	Percent Gain
1999	0.903	0.650	0.720	38.9	0.626	0.693	44.3
2000	0.886	0.646	0.729	37.2	0.612	0.691	44.8
2001	0.892	0.617	0.692	44.5	0.603	0.676	47.9
2002	0.885	0.671	0.758	31.9	0.622	0.703	42.2
2003	0.886	0.687	0.775	29.0	0.640	0.722	38.5
2004	0.900	0.681	0.757	32.2	0.613	0.680	47.0
2005	0.900	0.676	0.752	33.0	0.598	0.664	50.6
2006	0.894	0.684	0.765	30.6	0.604	0.676	47.9
2007	0.892	0.683	0.766	30.6	0.587	0.657	52.1

Calculations are based on two samples of firms. One sample constitutes U.S. firms from Compustat, and the other constitutes a sample of Chinese firms from the National Bureau of Statistics of China. The sample period is from 1999 to 2007. Column (1) shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal U.S. allocation: $F_{\text{US}}/\hat{F}_{\text{US}}$. Columns (2)–(4) report the Chinese state-owned firm efficiency ratio, $F_{\text{China}}/\hat{F}_{\text{China}}$, the Chinese state-owned firm efficiency ratio as a fraction of the U.S. efficiency ratio, $(F_{\text{China}}/\hat{F}_{\text{China}})/(\hat{F}_{\text{US}}/F_{\text{US}})$, and the corresponding percentage gains. Columns (5)–(7) repeat Columns (2)–(4), except that calculations are based on a sample of all Chinese firms, private and state-owned.

Table 12: U.S. Reallocation Gains with Sector Costs

	United States	
	(1)	(2)
Year	Fractional Benefit	Percent Gain
1999	0.898	11.3
	(0.008)	(1.0)
2000	0.875	14.3
	(0.014)	(1.8)
2001	0.880	13.6
	(0.014)	(1.7)
2002	0.869	15.0
	(0.011)	(1.4)
2003	0.874	14.4
	(0.011)	(1.4)
2004	0.889	12.5
	(0.012)	(1.4)
2005	0.889	12.5
	(0.012)	(1.5)
2006	0.881	13.5
	(0.011)	(1.3)
2007	0.881	13.4
	(0.012)	(1.5)

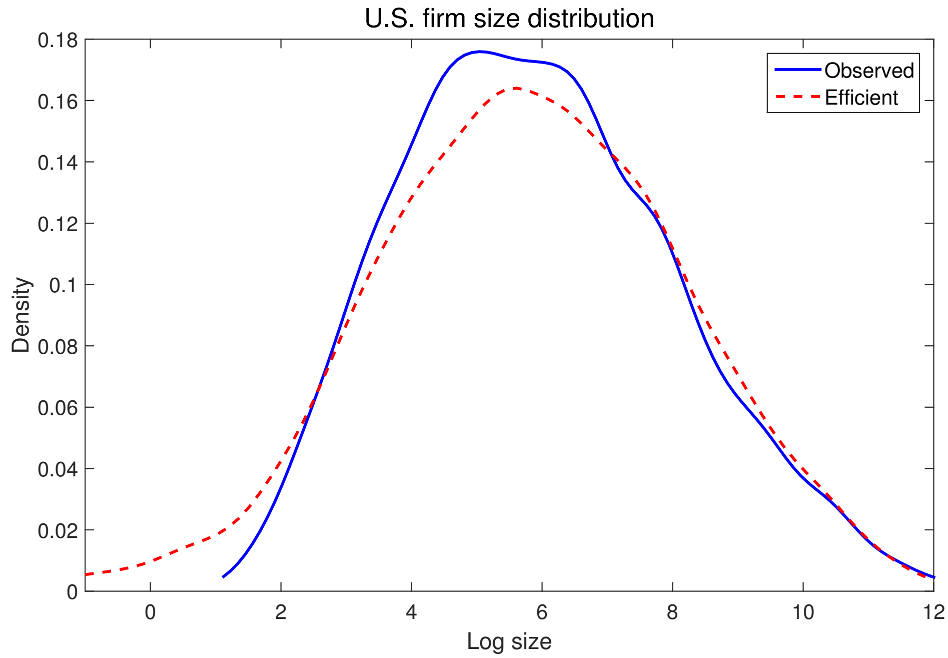
Calculations are based on U.S. firms from Compustat. The sector debt cost, r_s, is the mean sector interest rate. The sector equity cost, λ_s, is based one the sector unlevered beta, a risk-free rate of 0.03, and a market-risk premium of 0.06. We use CRSP daily stock data from 1999 to 2007 to calculate the unlevered beta. This table presents potential reallocation gains when the substitutability between debt and equity is $\gamma = 2$. Column (1) shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal U.S. allocation: $F_{\text{US}}/\hat{F}_{\text{US}}$. Column (2) shows the corresponding percentage gain from moving from the observed to the optimal allocation. Standard errors are in parentheses below each estimate.

Table 13: U.S. Reallocation Gains with Market Value Benefit

	United States	
Year	Fractional Benefit	Percent Gain
1999	0.768	30.2
	(0.023)	(3.8)
2000	0.778	28.6
	(0.022)	(3.5)
2001	0.873	14.6
	(0.010)	(1.3)
2002	0.886	12.8
	(0.011)	(1.3)
2003	0.894	11.8
	(0.010)	(1.2)
2004	0.892	12.2
	(0.012)	(1.5)
2005	0.890	12.3
	(0.012)	(1.4)
2006	0.894	11.9
	(0.010)	(1.2)
2007	0.878	13.9
	(0.010)	(1.3)

Calculations are based on U.S. firms from Compustat. The sample period is from 1999 to 2007. The nominal benefit of finance is measured as the market value of debt plus the market value of equity, instead of value-added. This table presents potential reallocation gains when the substitutability between debt and equity is $\gamma = 2$. Column (1) shows the observed U.S. allocation of the real benefit of finance as a fraction of the optimal U.S. allocation: $F_{\text{US}}/\hat{F}_{\text{US}}$. Column (2) shows the corresponding percentage gain from moving from the observed to the optimal allocation. Standard errors are in parentheses below each estimate.

Panel A



Panel B

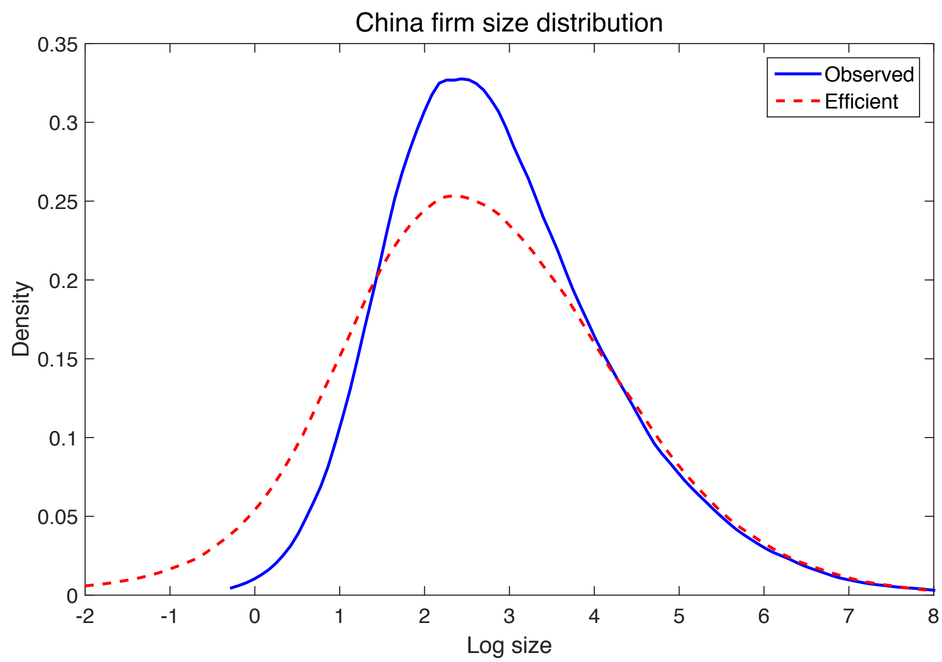
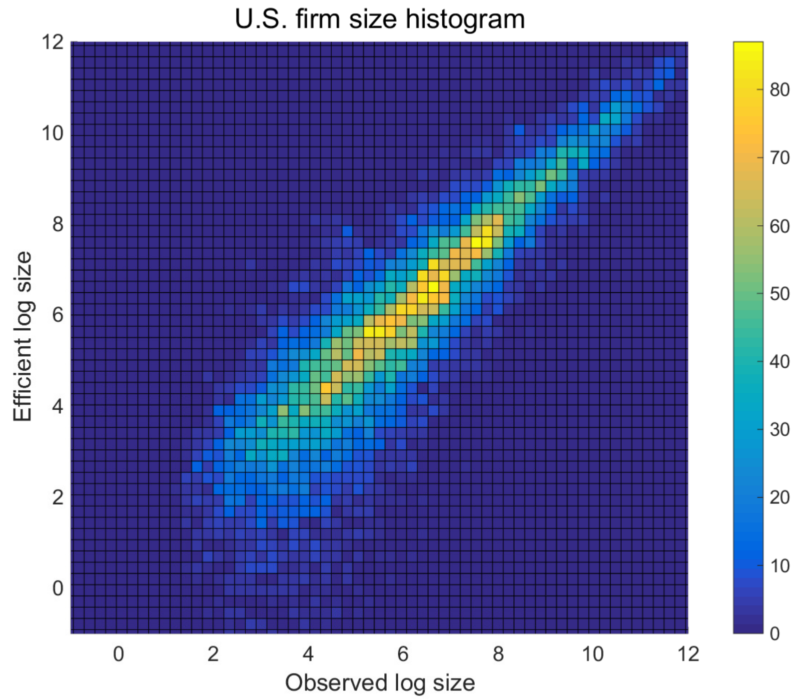


Figure 1: Panel A compares the U.S. observed and efficient firm size distributions using a kernel density estimator. Observed firm size is computed as $\log(D_{si} + E_{si})$, and efficient firm size is computed as $\log(\hat{D}_{si} + \hat{E}_{si})$, where D_{si}, E_{si}, $\hat{D}_{si}$, and $\hat{E}_{si}$ are measured in millions of 2005 USD. Panel B similarly compares the observed and efficient firm size distributions in China. Firm size is computed in the same manner, but D_{si}, E_{si}, $\hat{D}_{si}$, and $\hat{E}_{si}$ are measured in millions of 2005 CNY.

Panel A



Panel B

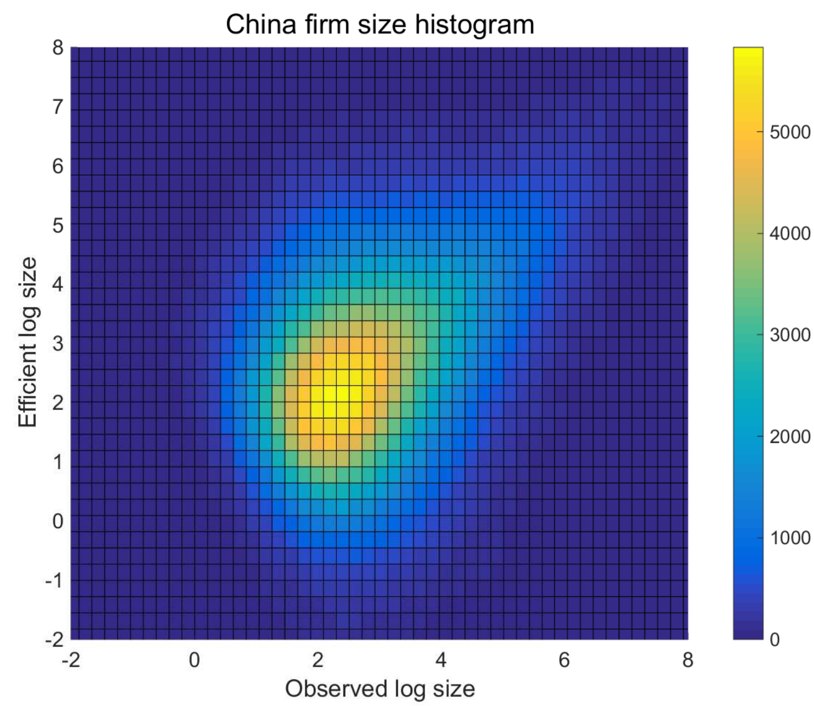


Figure 2: Panel A contains the heat map of a three-dimensional histogram where the observed U.S. firm size distribution is on the x-axis and the efficient U.S. firm size distribution is on the y-axis. The legend for the z-axis heat map is located at right of the plot and represents the number of observations in each bin. Observed firm size is computed as $\log(D_{si} + E_{si})$, and efficient firm size is computed as $\log(\hat{D}_{si} + \hat{E}_{si})$, where D_{si}, E_{si}, $\hat{D}_{si}$, and $\hat{E}_{si}$ are measured in millions of 2005 USD. Panel B similarly compares the observed and efficient firm size distributions in China. Firm size is computed in the same manner, but D_{si}, E_{si}, $\hat{D}_{si}$, and $\hat{E}_{si}$ are measured in millions of 2005 CNY.